**DEPENDENCE OF THE SPACE CHARGE LIMIT
ON THE CHOICE OF ν -VALUES**

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1. Introduction

This study investigates the effect of the choice of the ν -values, ν_x, ν_y on the space charge limit. The study finds a strong dependence of the space charge limit on the location of ν_x, ν_y relative to certain resonances, particularly the $1/4$ and $1/2$ intrinsic resonances. The study finds that the space charge limit drops sharply at ν_x, ν_y which are just above the intrinsic $1/4$ resonances, that are driven by the space charge forces. Peaks in the space charge limit are usually found at ν_x, ν_y which are just below these $1/4$ resonances. The ν_x, ν_y corresponding to these peaks may be the desirable choice of ν_x, ν_y to give the highest space charge limit. For some accelerators, the study finds that the space charge limit can be increased by a factor of 2 by proper choice of the ν -values.

Four accelerators were studied. These include the Fermilab booster, the AGS booster, the AGS and the PS booster. All four accelerators showed the above described correlation with the location of the $1/4$ intrinsic resonances. Three of the accelerators are operating accelerators. The simulation program used in this study has been used to compute the space charge limit for these accelerators,¹ and the results agree with measurements within about a factor of two.

The intrinsic $1/4$ resonances considered here are driven by the space charge forces due to the beam itself. The $1/4$ resonances are the lowest order non-linear resonance generated this way, as only even order resonances are generated. Every other $1/4$ resonance is also a $1/2$ resonance which also can be generated by the space charge forces in the beam, and this $1/2$ resonance may also be contributing to the drop in the space charge limit. Some $1/2$ resonances are also strongly driven by the magnetic fields of the accelerator magnets. This effect will also show up in the study and is a well known effect. The primary emphasis of this study is on the effect of the resonances generated by the space charge forces due to the beam itself.

2. The Space Charge Model

The following study of the dependence of the space charge limit on the choice of ν -values makes use of a model developed in some previous work¹ on the space charge limit. In this model, the intrinsic space charge limit plays an important role. The intrinsic space charge limit is the space charge limit in the absence of magnetic field errors, and is due to the forces generated by the beam itself.

In studies of three operating accelerators, which include the AGS, the PS Booster, and the Fermilab booster, it was found that the computed intrinsic space charge limit was fairly close to the experimentally observed space charge limit. This result plus studies of the effects of resonances due to magnetic field errors suggest that the intrinsic space charge limit provides an upper bound for the space charge limit which is not far from what is actually achieved by operating accelerators.

The resonances present due to magnetic field errors, if strong enough, can prevent the accelerator from achieving the intrinsic space charge limit. However, the effects of these resonances were found to be appreciable only when the beam intensity gets close to the intrinsic space charge limit. Well below the intrinsic space charge limit, there is little beam growth due to magnetic field error driven resonances, and the space charge forces tend to stabilize these resonances.

In this study, only the intrinsic space charge limit is computed. It is assumed that the space charge limit is primarily determined by the intrinsic space charge limit, and that the effects of resonances driven by magnetic field errors can be kept under control.

References

1. G. Parzen, Space Charge Effects in Proton Synchrotrons, Nucl. Inst. and Methods A281 (1989) 413-425.

Fermilab Booster

AGS

AGS BOOSTER

PS Booster

ps Booster

Effect of off diagonal V -values.

3. Off Diagonal ν -Values

Most of the studies reported in the previous sections were done with ν -values that moved along a line parallel to the $\nu_y = \nu_x$ resonance line and usually fairly close to this line. The one exception was the PS Booster where ν was varied along the line $\nu_y = \nu_x + 1.35$. For the PS Booster, the intrinsic space charge limit was also studied along the line $\nu_y = \nu_x + .1$ which is closer to the $\nu_x = \nu_y$ line. Much smaller values of $N_{b,L}$ were found along the $\nu_y = \nu_x + .1$. This is shown in Fig. 5.

Because of the results with the PS Booster, some studies were done with off-diagonal values with the other accelerators studied. It was found that the peak values of $N_{b,L}$ generally get somewhat larger for ν -values off the diagonal. Sometimes $N_{b,L}$ was considerably increased; sometimes $N_{b,L}$ changed only slightly. No consistent quantitative pattern was noticed. One important effect of space charge is to generate non-linear coupling between the horizontal and vertical oscillations. This coupling can cause an apparent increase in the beam size due to the exchange of emittance between the horizontal and vertical motions. This effect may contribute to the improvement seen at off diagonal ν -values, which are further from the $\nu_x = \nu_y$ coupling resonance.

For the AGS Booster, $N_{b,L}$ could be increased from the peak value of $N_{b,L} = 2 \times 10^{13}$ to $N_{b,L} = 2.5 \times 10^{13}$ by removing ν_x, ν_y off the diagonal to any of the following ν -values

- 1) $\nu_x = 4.82, \nu_y = 5.83$
- 2) $\nu_x = 5.64, \nu_y = 5.01$
- 3) $\nu_x = 5.92, \nu_y = 5.73$

The first choice, $\nu_x = 4.82, \nu_y = 5.83$ was found by starting at the proposed operating point $\nu_x = 4.82, \nu_y = 4.83$, and moving away from the diagonal by increasing ν_y until a peak space charge limit was reached at $\nu_y = 5.83$.

Choices 2 and 3 were found by starting at the ν -values which give the highest space charge limit in the figure on page 5, which are $\nu_x = 5.82, \nu_y = 5.83$, and then moving perpendicular to the diagonal until peaks were found on either side of the diagonal.