**DETERMINING THE ORBIT OF A BEAM IN A TRANSFER LINE
FROM BEAM POSITION MEASUREMENTS**

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Introduction

Luccio^[1] has described a procedure for determining the coordinates of the beam at the entrance to the transfer line (LTB), from HEBT to the Booster. The procedure involves measuring the position of the beam at a beam position monitor (BPM) as a function of current in an upstream quad.

In this technical note, a different approach is suggested. The readings of all BPMs are recorded for the same beam pulse, and, with the known tune of the line, used to find the entrance coordinates. We show how to include steering magnets in the analysis. At least two BPMs are required; with more than two, the least squares technique is used to find the entrance coordinates. After the entrance coordinates have been found, the orbit is constructed using the transfer matrices and kicks of the steering magnets.

The advantage of this approach is that the tune of the line is not disturbed while the data is acquired, hence the beam trajectory can be monitored regularly during operation.

We have tested the method in simulation and also applied it to two sets of measurements made in the early stages of commissioning of LTB. The results are presented.

Theory

If X_i are the BPM readings, then, in the absence of any beam steering by orbit correctors or misaligned quads,

$$\text{If } X_i = a_i(1,1) \cdot XO + a_i(1,2) \cdot XPO \quad (1)$$

where XO and XPO are the coordinates (in either plane) at the entrance of the beam line, and $a_i(n,m)$ are the appropriate elements of the 2×2 transfer matrix from the entrance of the line to the i th BPM. Note that in Luccio's work, the transfer matrix is to the same BPM, but for different quad currents. These matrix elements are readily calculated from the Twiss parameters, which are available from MAD^[2] for a run with the MAD model of the beam line.

With only two BPMs, the equations can be inverted to obtain XO and XPO . With more than two, a linear least squares method is used^[3].

Treatment of Steering Magnets

When the steering magnets are energized, some, or all, of Eq. (1) may no longer be true. The observed positions are now given by

$$X_i = x_i + dx_i \quad (2)$$

where x_i would be the positions if all steering is turned off, and dx_i are the contributions of the steering magnets to X_i . The X_i , if they can be found, can then be used in Eq. (1) to solve for SO and XPO.

It can be shown that:

$$dx_i = \sum_k a_k (1,2) \cdot \theta_k \quad (3)$$

where θ_k is the kick of the k th steerer, and the summation is over steerers upstream of the BPM. The matrix elements $a_k (1,2)$ belong to the transfer matrices between these steerers and the position monitor. They are computed similarly to those in Eq. (1).

A summary of the method is:

- (i) Run MAD to obtain the Twiss parameters.
- (ii) Calculate the transfer matrices between steerers and BPMs.
- (iii) Calculate the corrections to the BPM readings.
- (iv) Solve the set of Eq. (1) for XO and XPO by the least squares method.
- (v) Construct the orbit using XO and XPO, the transfer matrices, and the kicks at the steerers.

Applicability to LTB

A schematic layout of LTB is shown in Figure 1. The line is conceptually divided into three parts, as indicated. We have one set of readings for the BPMs and profile monitor, MW035, in LTB1 and LTB2, with the steerers off, and another set for the entire line with the steerers turned on. (The tune of the line was different for both measurements.) Using the profile monitors to obtain beam position data necessarily slows down the process because of the time needed to insert and retract the devices and to analyze the profiles.

Simulation

To generate simulated beam position data, we ran MAD using an input file made up of the section of HEBT (the beam transfer line from the Linac to the AGS) up to the kicker magnet where LTB begins, and LTB itself. The beam was given small horizontal and vertical kicks in the HEBT section of the combined beamline. After the MAD run, the centroid coordinates at the entrance to LTB and at the BPMs were extracted from the MAD output. Two simulation runs were performed with these data. In one, the beam positions at the BPMs from the MAD run were used to determine the orbit, and in the other, the MAD data were randomly altered between ± 0.5 mm, to simulate measurement errors. The results given in the following table for the second case are the average of 20 runs.

Results of the Simulation

The centroid coordinates at the entrance of DH1, from the MAD run and calculated from the simulated beam positions in LTB, are:

	MAD result	Calculated	
		MAD Data	Modified MAD Data
XO =	-2.24	-2.12 $\pm$ 0.03	-2.16 $\pm$ 0.09 mm
XPO =	-1.43	-1.44 0.01	-1.44 0.03 mrad
chi ² =		0.035	
YO =	2.19	2.86 $\pm$ 0.15	1.71 $\pm$ 0.20 mm
YPO =	-0.65	-0.75 0.02	-0.61 0.03 mrad
chi ² =		0.153	

The MAD data and the calculated orbits are plotted in Figure 2a.

MEASURED POSITION DATA

CASE 1 (LTB commissioning run, 4/18/91. All steerers OFF.)

Monitor/Steerer	Horizontal	Vertical (mm)	Kicks (mrad)
DH015			0.000
DV017			0.000
BPM019	0.4	-3.7	
DVO26			0.000
BPM027	1.0	-2.4	
MW035	0.19	-7.4	
BPM066	-1.3	4.6	

CASE 2 (LTB commissioning run, 4/30/91. All steerers ON.)

Monitor/Steerer	Horizontal	Vertical (mm)	Kicks (mrad)
DH015			-0.006
DV017			-0.829
BPM019	-2.2	-2.7	
CH076			-0.588
BPM078	BAD.....		
DV083			1.113
DH088			-1.520
BPM090	1.9	0.0	
DV095			-0.724
BPM102	7.3	-0.8	
BPM109	6.4	0.1	
DH112			0.000

CASE 3 (LTB commissioning run, 4/29/91. All steerers ON. Data not shown.)

RESULTS FOR MEASURED POSITION DATA

The entrance coordinates at DH1 calculated from the measured beam positions are:

	CASE 1 4/18/91 (No Steering)	CASE 2 4/29/91 (Steering)	CASE 3 4/30/91 (Steering)
XO =	-0.26 $\pm$ 0.57	-2.27 $\pm$ 0.25	-2.48 $\pm$ 0.50mm
XPO =	0.15 0.07	-0.08 0.04	-0.49 0.09mrad
chi ² =	0.76	1.08	5.02
YO =	3.13 3.07	4.37 $\pm$ 1.25	3.77 $\pm$ 1.09mm
YPO =	-0.84 0.37	-0.81 0.13	-0.74 0.13mrad
chi ² =	19.71	9.93	5.86

Using these results, we have computed the trajectories of the beam centroids and plotted them in Figure 2 (b-d). The measured positions are plotted for comparison.

DISCUSSION

The results of the simulation demonstrate the validity of the method. It is not clear why the agreement in the vertical plane is not as good as in the horizontal plane.

For the case of no steering, the chi-squares and one standard deviation errors are much better in the horizontal plane than in the vertical. The plot of calculated orbits and measured positions in Figure 2b confirms this.

The two cases in which the steerers were on give consistent results for the entrance coordinates, but these do not agree well with the results with no steering. The tune of the HEBT line was the same in all three cases, hence we should expect the same entrance coordinates at DH1. On the other hand, the tune of LTB was not the same, and any quad misalignments, which our algorithm does not presently handle, will cause the extracted entrance coordinates to be different. (A quadrupole misalignment produces a "kick" proportional to the misalignment, in addition to the normal quadrupole deviation. Hence, quad misalignments, if they are known, can be treated in the same way we have treated the steering magnets.)

For all three cases of real data, the entrance coordinates are determined with better precision in the horizontal than in the vertical plane. This could be an indication that the unaccounted-for steering is more important in the vertical plane.

The results for the measured data of 4/29/91 and 4/30/91 show a clear improvement in beam steering in LTB2 and LTB3 (Figure 2c and 2d).

CONCLUSION

We have demonstrated that the BPM readings in LTB can be used to calculate the orbit of the beam in the line. The computational techniques used need to be refined to improve the accuracy of the calculations.

Similar measurements have been made in the HEBT section of the beam line⁴, to determine the beam coordinates at the entrance to the first dipole of LTB. However, the line was tuned for AGS operation. It will be interesting to repeat this measurement with the line tuned for Booster injection, and compare the results obtained for the entrance coordinates with what we obtain from the LTB measurements reported here.

References

1. A. Luccio, Booster Tech. Note 158,1190. AGS Department, Brookhaven National Laboratory, Upton, N.Y. 11973.
2. F.C. Iselin and J. Niederer, The MAD Program, Version 7.2, CERN/LEP-TH/88-38.
3. W.H. Press et al., Numerical Recipes, Cambridge University Press, 1988 Ch.14.
4. A. Kponou et al., Booster Studies Report (in preparation).

Figure 1 Schematic Layout of LTB

Figure 2

Calculated Orbits and Measured Beam Positions
 (a) Simulation with MAD Data (b) Steerers OFF - 4/18/91
 (c) Steerers ON - 4/29/91 (d) Steerers ON - 4/30/91