# ANALYZING POWER SPECTRUM CALCULATIONS MADE ON THE BOOSTER MMPS

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# Analyzing Power Spectrum Calculations Made on the Booster MMPS

## Introduction

One purpose of this technical note is to study the effects of windowing (time domain filtering), window length (sampling period), and the number of samples on the Booster MMPS power spectrum calculation. Another purpose of the paper is to explain the cause of the strong second harmonic that is seen when running the the Booster 5 hz. proton cycle and is not seen when running the Booster 7.5 hz. proton cycle.

The Booster 5 hz. power cycle used throughout this study was run in February 1993 when a group of Brookhaven Engineers[1] met with a LILCO Engineer[2] at the LILCO 8ER Brookhaven Substation which houses the Power Pulsation Monitor Relay (PPMR). The PPMR monitors the AC power spectrum on the Booster Dedicated Feed (69-861) to verify that the Booster, a pulsating load, does not create undesirable harmonics that could prove detrimental to LILCO's system or equipment. The regions monitored and their corresponding power limits are a low frequency region, .9766 - 4.6387 hz., which has a 2.5 MW Alarm Limit and a 3.0 MW Trip Limit and a high frequency region, 8.3008 - 40 hz., which has a 3.0 MW Alarm Limit and a 4.0 MW Trip Limit. Figure #1, a GE drawing obtained from LILCO's Relay Division, graphically shows the regions monitored and their corresponding power limits. Note that the drawing's upper Alarm Limit has since been modified to 3.0 MW. Also note that in the fall of 1993 software was installed on the PPMR that enables LILCO to determine the frequency bin and the corresponding power level of an Alarm or Trip if one occurs.

The main purpose of the meeting at the substation was to observe power spectrum calculations made by the PPMR. This paper will try to explain some of the observations that were made on the PPMR monitor that day. The data used in this study was taken from the Apollo computer which determines the DC outputs of the Booster MMPS. The power signal is calculated by multiplying the magnet current and voltage. See Figures #2 & #3 for the User 1 & 3 power pulses and note the power swing of each. In the absence of energy storage the power measurements that are made as a function of time on the DC side of the power supply are equivalent to the real power measurements that are made on the AC side of the power supply. A software package called Dadisp was used to analyze the data.

## Effects of Different Sampling Periods on the Power Spectrum Calculation

Since the PPMR uses a Fast Fourier Transform (FFT) to calculate the power spectrum, it requires $2^n$ sampled data points, where n is an integer. The PPMR has a data sampling rate of 1 Khz and was designed to make an FFT calculation on 4096 data points. It samples for 4.000 seconds collecting 4000 data points and appends another 96 zero data points and then calculates the FFT of the digitized power signal.[3] Note that there is no triggering on the PPMR

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[1]Andy McNerney, Jim Benson, & Joe Geller

[2]Jerry Kinalis

[3]Booster Technical Note No. 188, "AGS Booster Pulsed Power Monitoring and Interlocking", January 25, 1990.

and therefore each period of data acquisition is a random "snapshot" of the power signal. Dadisp was used to study the effects of calculating the FFT for different time length "snapshots" (window lengths) of the power signal. For this first study, no window (also known as a rectangular window) was applied to the acquired signal. Since the supercycle period for the 5 hz. power waveform used in this study is 3.8 seconds (Figure #4), any waveform analyzed with a sample time greater than this includes data points from the beginning of the next supercycle.

The sample times analyzed are 3.800 sec., 4.000 sec., 4.096 sec., and 4.000 sec. with 96 zeroes appended. Figures #4-#11 show these various length "snapshots" and their corresponding power spectrums. All the "snapshots" start at the beginning of the supercycle and thus reflect a cycle triggered at the start of the supercycle. Table 1A summarizes the results of the power spectrum calculation for these sampling times. Figures #12-#19 and Table 1B, on the other hand, reflect these different sample times but are random "snapshots" (not triggered).

**Table 1A (Triggered)**

| Sample Time | Waveform to be Fourier Analyzed | Power Spectrum of Waveform | 5 hz Fundamental Magnitude | 10 hz Second Harmonic Magnitude |
|---|---|---|---|---|
| 3.800 sec. | Figure #4 | Figure #5 | 1.3465 MW | 1.0697 MW |
| 4.000 sec. | Figure #6 | Figure #7 | 1.3270 MW | 1.0239 MW |
| 4.096 sec. | Figure #8 | Figure #9 | 1.1203 MW | 1.1372 MW |
| 4.000 sec. + 96 zeroes | Figure #10 | Figure #11 | 1.2626 MW | .97830 MW |

**Table 1B (Random)**

| Sample Time | Waveform to be Fourier Analyzed | Power Spectrum of Waveform | 5 hz Fundamental Magnitude | 10 hz Second Harmonic Magnitude |
|---|---|---|---|---|
| 3.800 sec. | Figure #12 | Figure #13 | 1.3465 MW | 1.0697 MW |
| 4.000 sec. | Figure #14 | Figure #15 | 1.5004 MW | 1.1734 MW |
| 4.096 sec. | Figure #16 | Figure #17 | 1.2093 MW | 1.2039 MW |
| 4.000 sec. + 96 zeroes | Figure #18 | Figure #19 | 1.1538 MW | 1.1076 MW |

In conclusion, after comparing the results of equivalent sample times in Table 1A and Table 1B, one can readily see that without windowing, different "snapshots" (triggered or

untriggered) tend to give similar power spectrum results. Note also that other studies (not included) were performed using the Dadisp Software Package where other random "snapshots" were analyzed which had Booster power pulses that were "cut off" at the beginning and end of the sampled time period. The resulting power levels were very similar to the power levels found in Tables 1A & 1B.

The supercycle sampled for 4.000 sec. with 96 zeroes (96 ms) appended tended to give power readings (fundamental and second harmonic) that were not noticeably larger than the readings calculated using other sample times. Since the PPMR acquires 4096 points for the FFT calculation in the same fashion, this shows that the sample time used by LILCO's PPMR is not a significant factor.

## Effects of Windowing on the Power Spectrum Calculation

Since the FFT assumes that it is processing periodic waveforms, the signals are considered to be extended beyond the window length or "snapshot" period. However, this brings out the spreading out or "leakage" effect due to the energy artificially generated by the discontinuity at the edges of the extended waveform. Windowing functions such as the Hamming, Hanninng, and Kaiser are functions that taper the endpoints of a non-periodic signal so that they match up. This causes the periodically extended signal as processed by the FFT to have a smooth transition at the edges, thereby reducing the leakage problem. A correction factor is normally applied to the calculated value of spectral power which compensates for the attenuation of the waveform. Next, this paper will try to show how different types of applied windows affect the Spectrum (FFT Magnitude) calculation.

The four types of windows that will be studied are the Rectangular window (no window), the Kaiser window, the Hanning window, and the Hamming window. For this study, a cycle that has a 4.000 sec. sample period followed by 96 zeroes (96 ms) will be used. Again, this is the same way that LILCO acquires its data. Table 2A shows the results of windowing when triggering at the start of the supercycle while Table 2B shows the results of windowing with random "snapshots."

**Table 2A (Triggered)**

| Type of Window | Waveform to be Fourier Analyzed | Power Spectrum of Waveform | 5 hz Fundamental Magnitude | 10 hz Second Harmonic Magnitude |
|---|---|---|---|---|
| Rectangular (no window) | Figure #20 | Figure #21 | 1.2626 MW | .9783 MW |
| Kaiser | Figure #22 | Figure #23 | .4032 MW | .3352 MW |
| Hanning | Figure #24 | Figure #25 | .4917 MW | .4021 MW |
| Hamming | Figure #26 | Figure #27 | .5520 MW | .4310 MW |

**Table 2B (Random)**

| Type of Window | Waveform to be Fourier Analyzed | Power Spectrum of Waveform | 5 hz Fundamental Magnitude | 10 hz Second Harmonic Magnitude |
|---|---|---|---|---|
| Rectangular (no window) | Figure #28 | Figure #29 | 1.1538 MW | 1.1076 MW |
| Kaiser | Figure #30 | Figure #31 | .2963 MW | .2480 MW |
| Hanning | Figure #32 | Figure #33 | .3520 MW | .2948 MW |
| Hamming | Figure #34 | Figure #35 | .3873 MW | .3387 MW |

From these results one can conclude that windowing has a dramatic effect on Spectrum (FFT Magnitude) calculations. Different types of windowing give different results. The Kaiser window has the most dramatic effect, followed by the Hanning window, and then the Hamming window. However, note that the FFT Magnitude decreases when any type of window is used. Comparing the results of Tables 2A and 2B shows that when no window is applied, different "snapshots" tend to give similar results (a previous conclusion in this paper). However, when a window is applied, different "snapshots" tend to give very different results. Since the PPMR is not triggered and requires a windowing function, each FFT calculation is performed on a different (untriggered) "snapshot". This probably explains the observed FFT "motion" (i.e. varying power spectrum levels shown by the fundamental and sidebands moving up and down on the PPMR monitor). "Snapshots" that have power pulses at both the beginning and end of a sample period are most affected by any type of applied window.

After discussions with a Software Engineer[4] from General Electric, the company that designed and built the PPMR, it has been determined that the PPMR uses a Hanning window in its calculation. However, to compensate for the reduction in power due to the applied Hanning window, he has also have informed us that the PPMR uses a correction scheme. If this correction scheme is overcompensating, it may be the very reason why the PPMR is calculating much higher power levels than we think actually exist. The correction algorithm that the PPMR uses is not available at this time.

## Cause of the Strong Second Harmonic when Running the Booster 5 Hz Proton Cycle

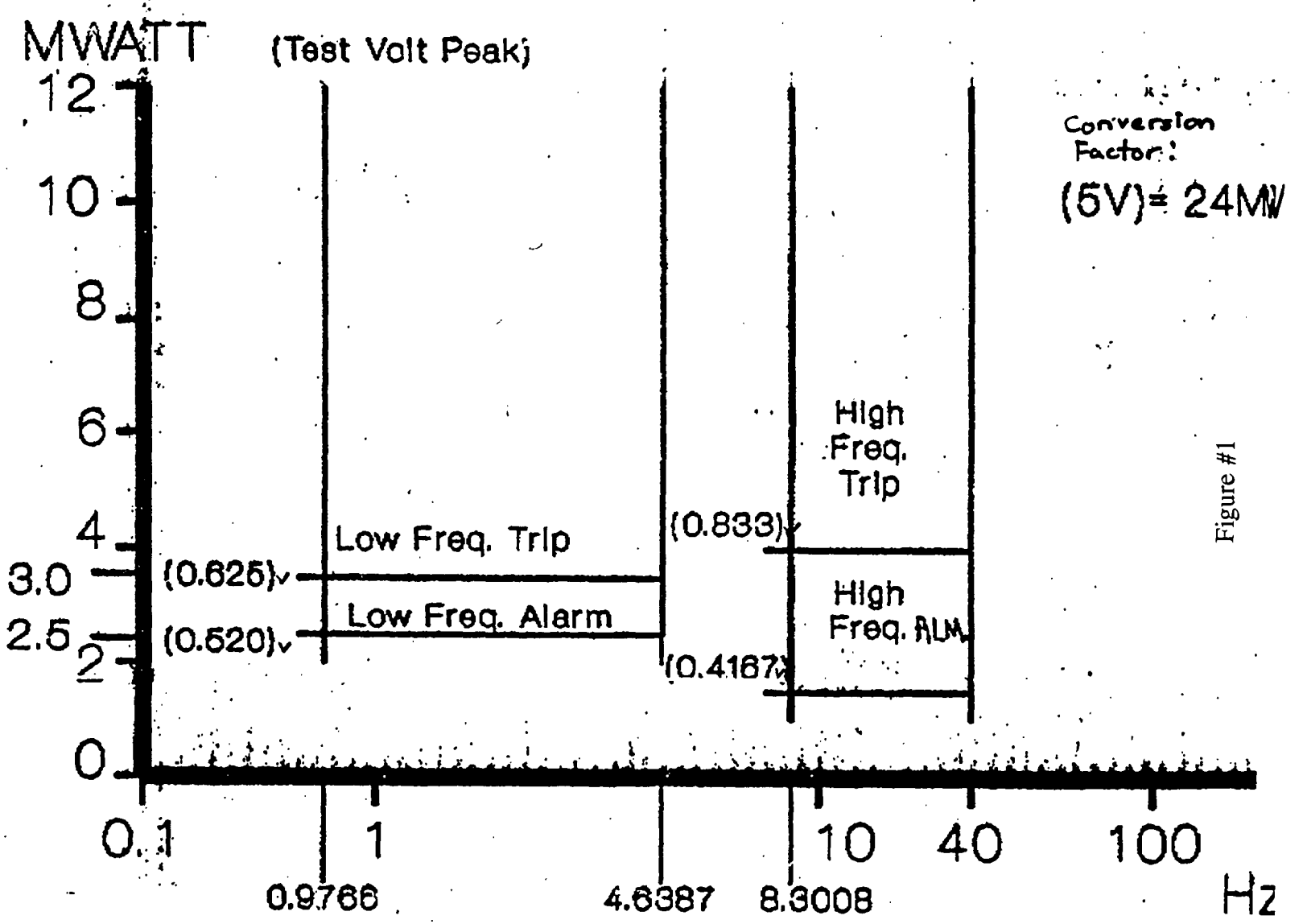
The time domain mathematical representation of the Booster power signal is a continuous running (CW) saw tooth wave multiplied, or modulated, by a rectangular wave or pulse. The modulating rectangular wave has a value of either unity or zero and a duty cycle that is in the ratio of the Booster "on" time to the AGS superperiod. The power spectrum is calculated from the saw tooth spectrum multiplied by the duty-cycle of the modulating rectangular wave. For a 7.5 hz. Booster cycle and a 3.8 second superperiod the duty cycle is 0.175 if we ignore the

[4]Colin Bowler

Booster User 3 cycle; for a 5 hz. Booster cycle, 0.263.

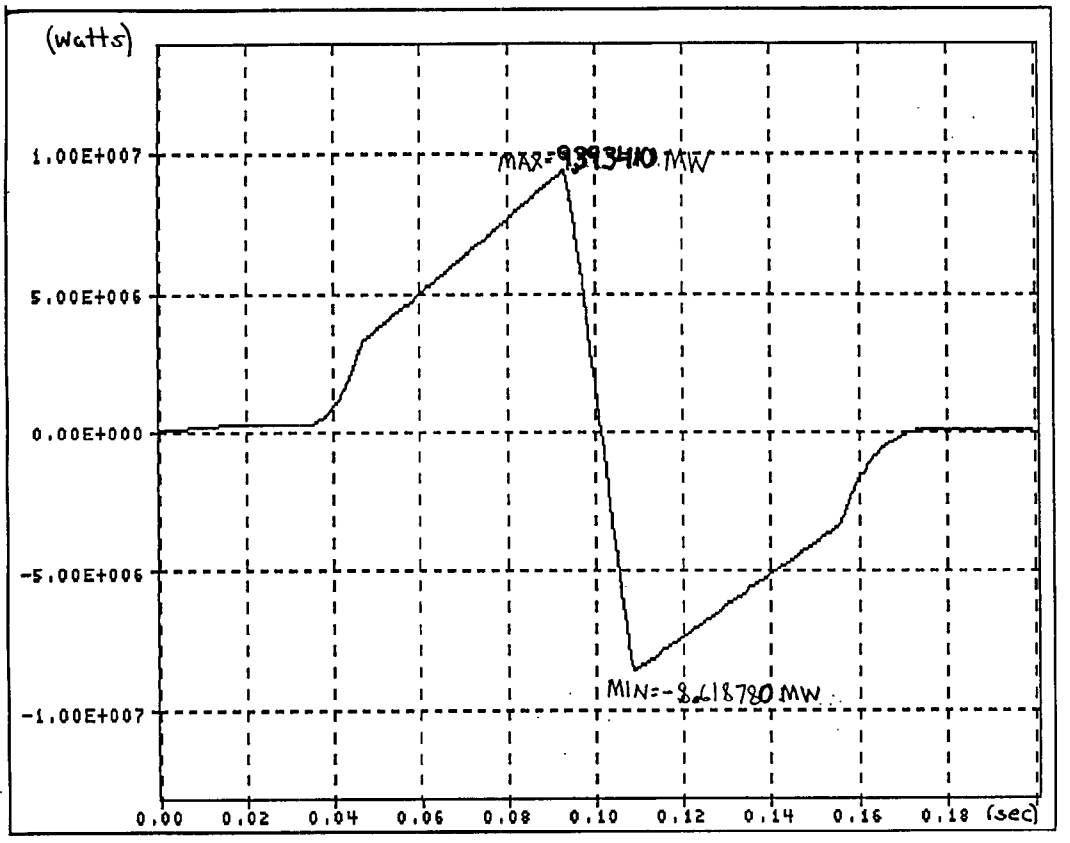
We model the Booster 7.5 hz. cycle as a saw tooth that runs continuously. The spectrum of a saw tooth wave has a harmonic distribution that varies inversely with the harmonic number. Thus the second harmonic is one-half of the fundamental. For the 5 hz cycle rate each saw tooth has a rest time. As the rest time increases, the fundamental component decreases and the higher harmonics increase in magnitude. In the limit, as the rest time approaches the Booster period, the power signal approaches a mathematical doublet. The doublet can be thought of as the derivative of the impulse function or as the second derivative of the step function. The spectrum of the step function varies inversely with frequency. The impulse has a uniform spectral distribution.[5] The doublet has a spectrum that varies directly with frequency. Thus depending on the active portion of each Booster period, the ratio of the second harmonic to the fundamental can vary between 0.5 and 2.0. For a 5 hz. cycle, where approximately 3/4 of the Booster period is active, the ratio of the second harmonic to the fundamental is 0.87. Details of the spectrum analysis of the MMPS power waveform is summarized in Appendix A.

[5]The Fourier Transform of a function that is the derivative of a waveform with a known transform is obtained by multiplying the known transform by jw; if the function is the integral of the waveform then divide the known transform by jw.




Trip and Alarm Settings

Figure #1

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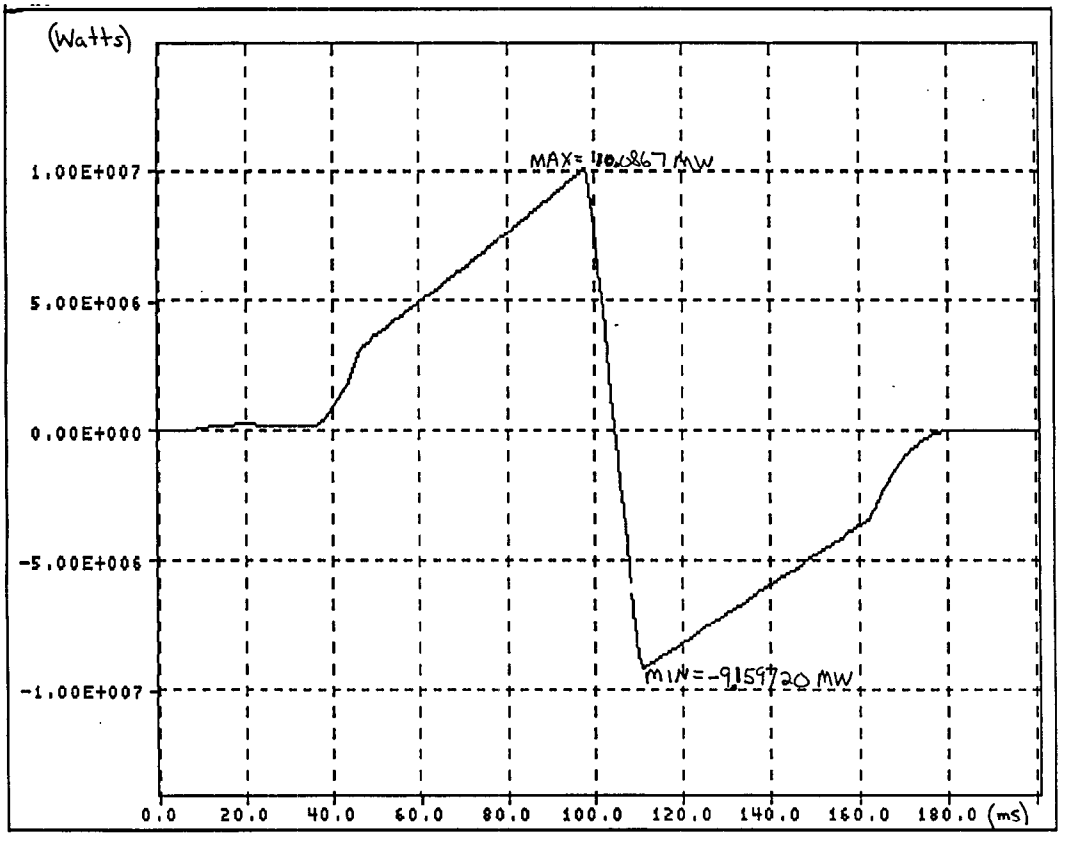


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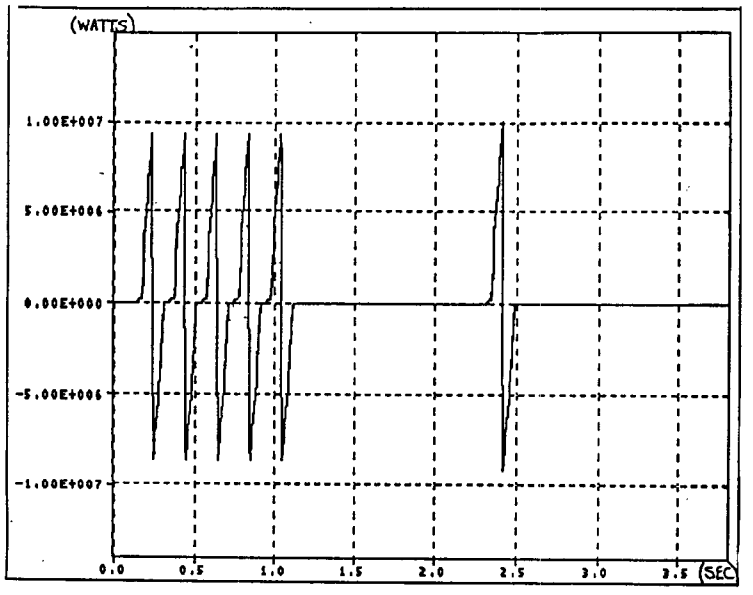


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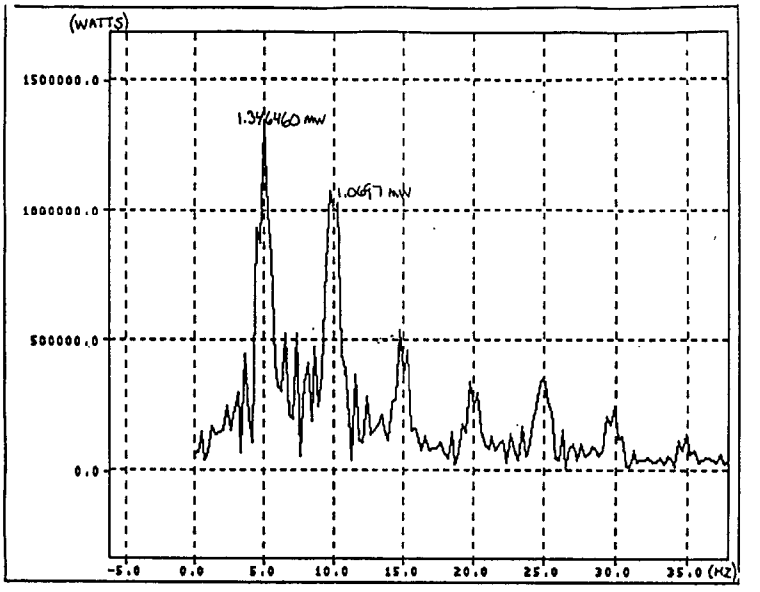


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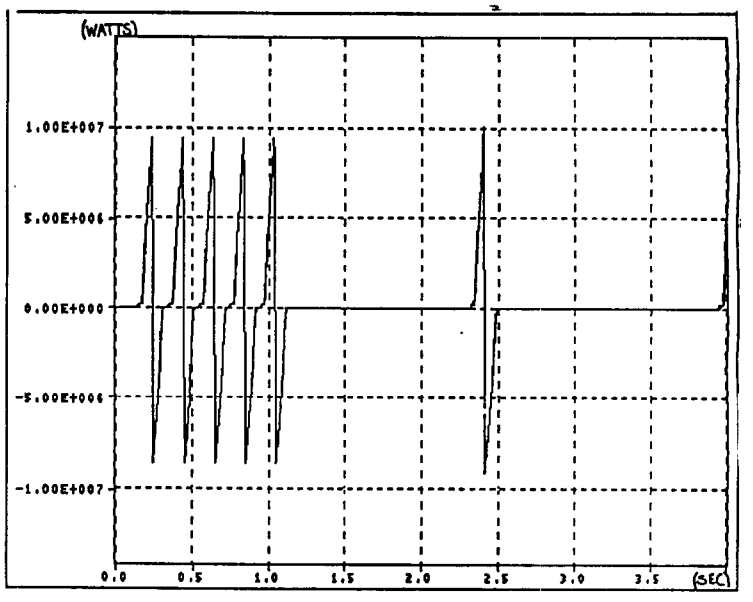


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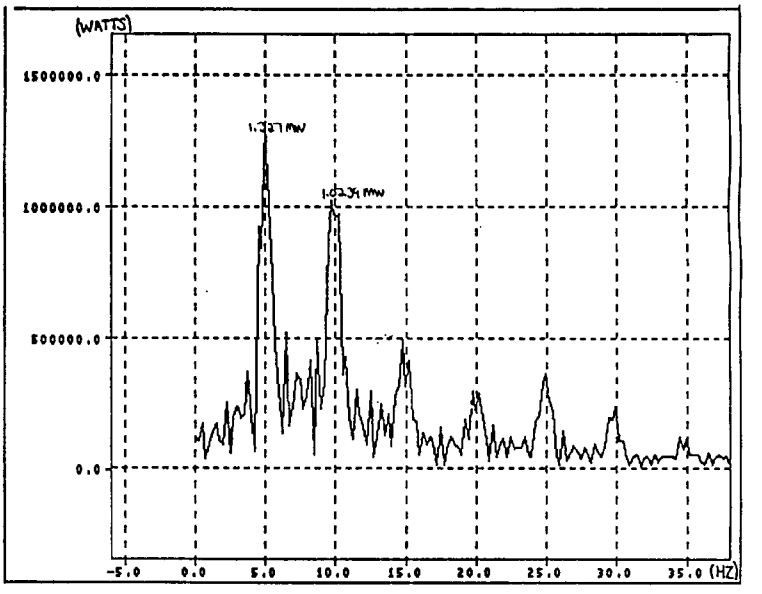


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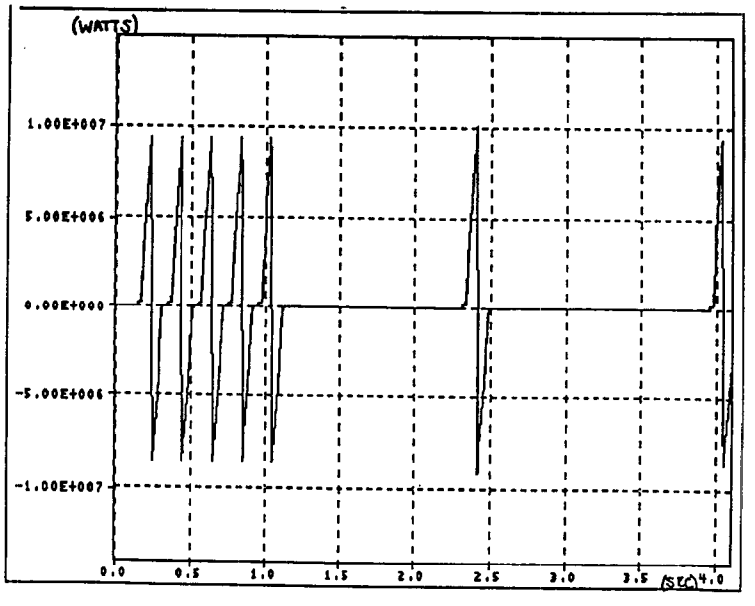


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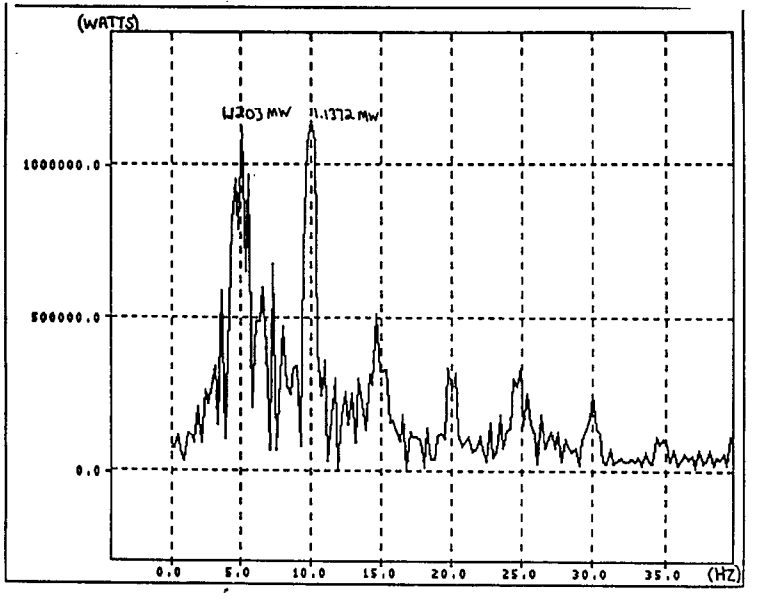


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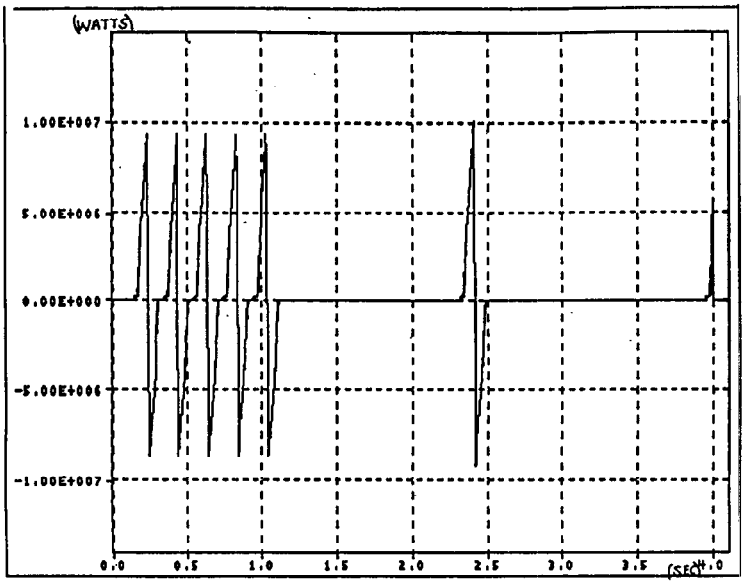


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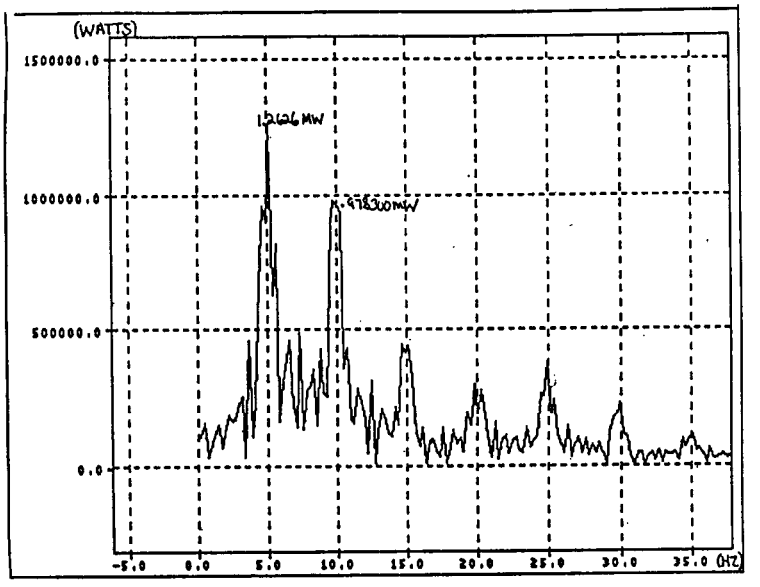


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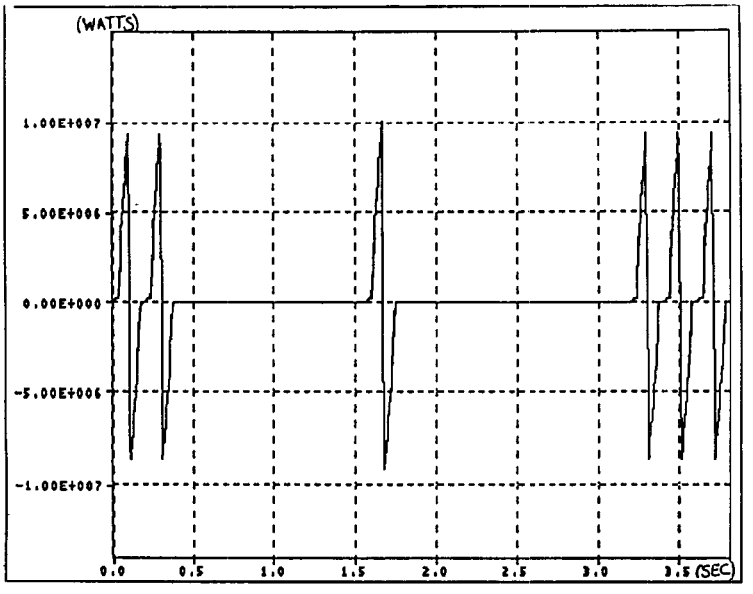


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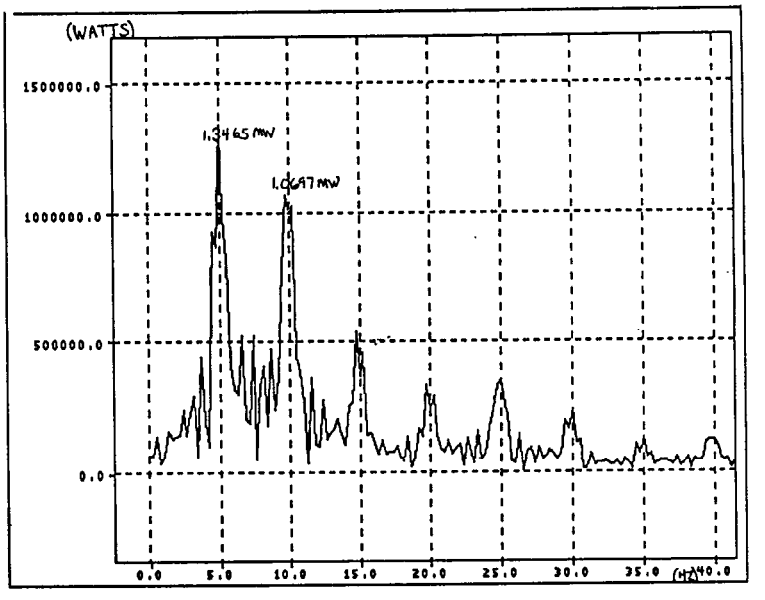


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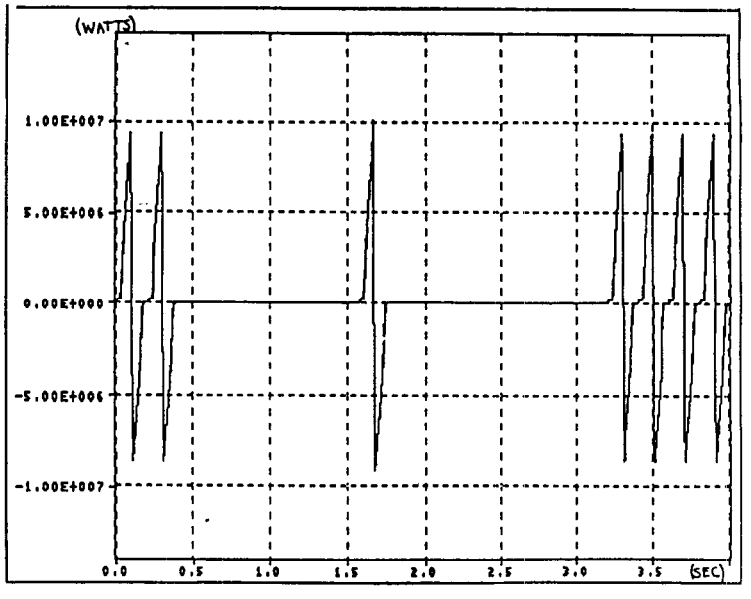


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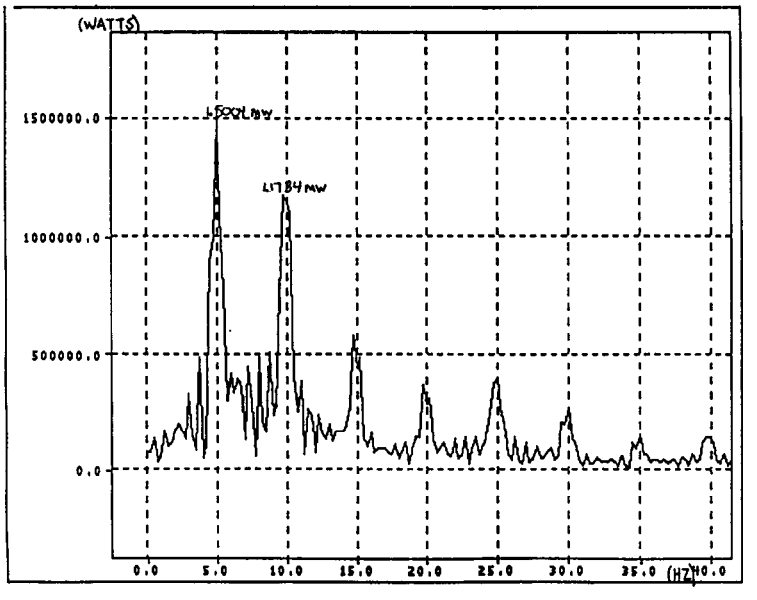


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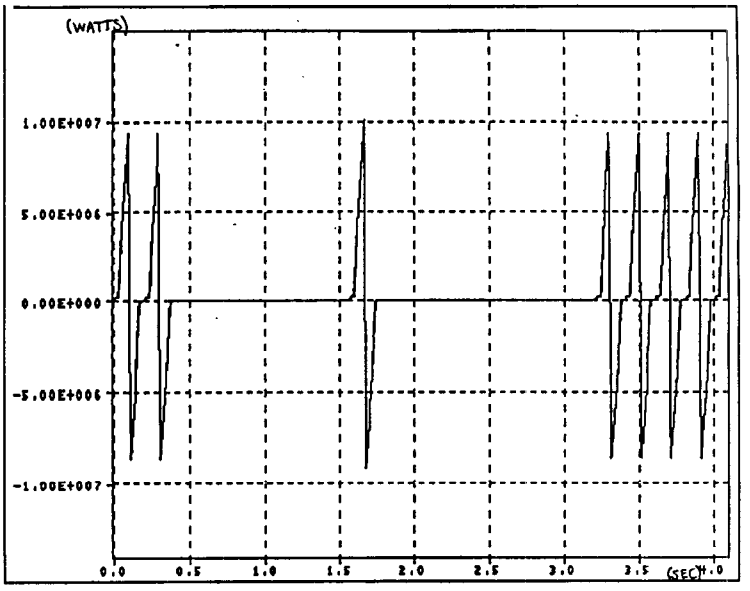


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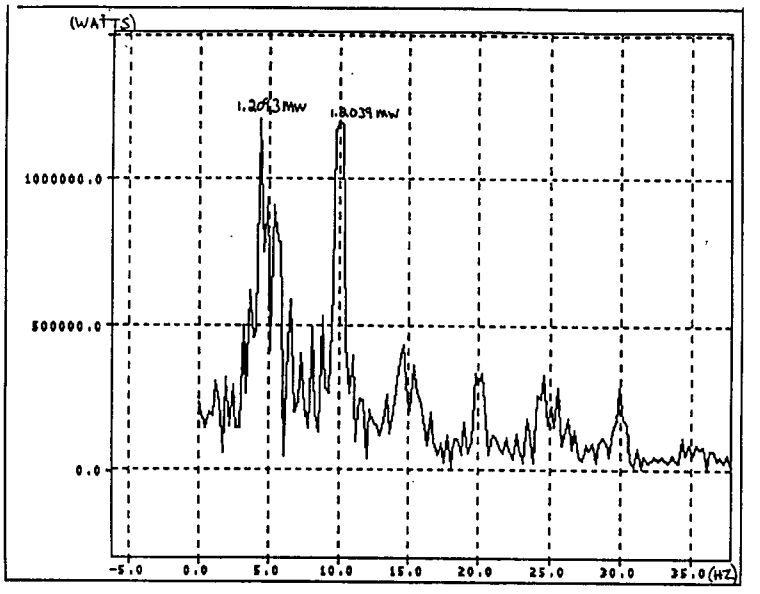


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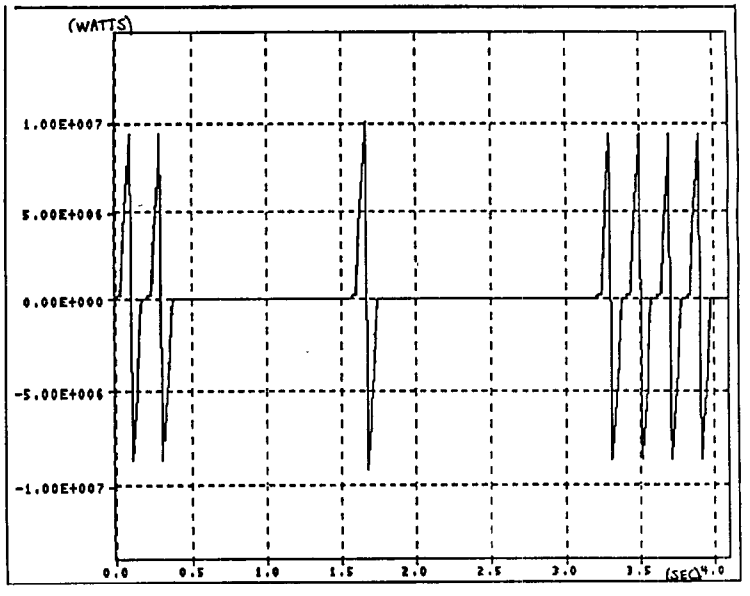


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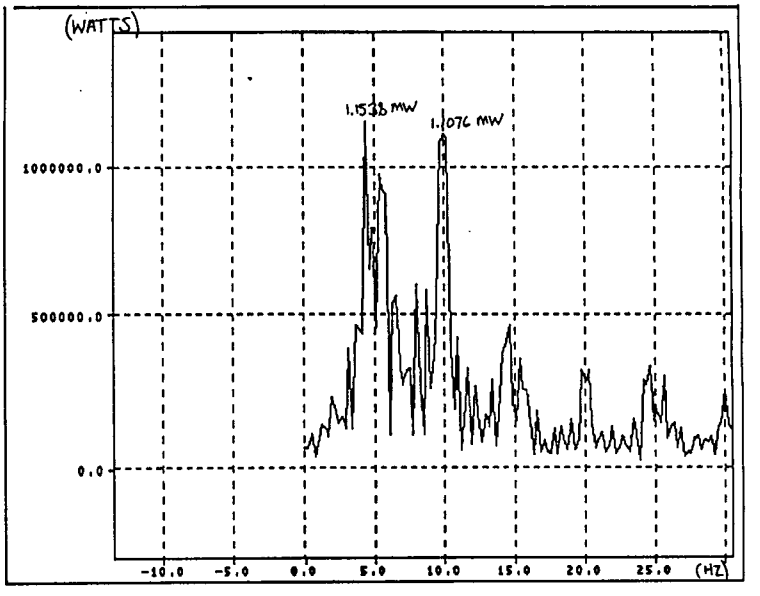


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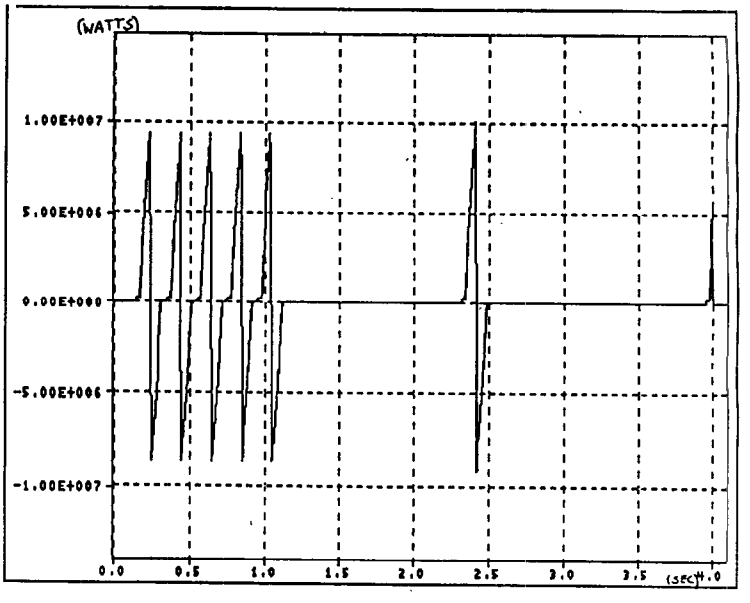


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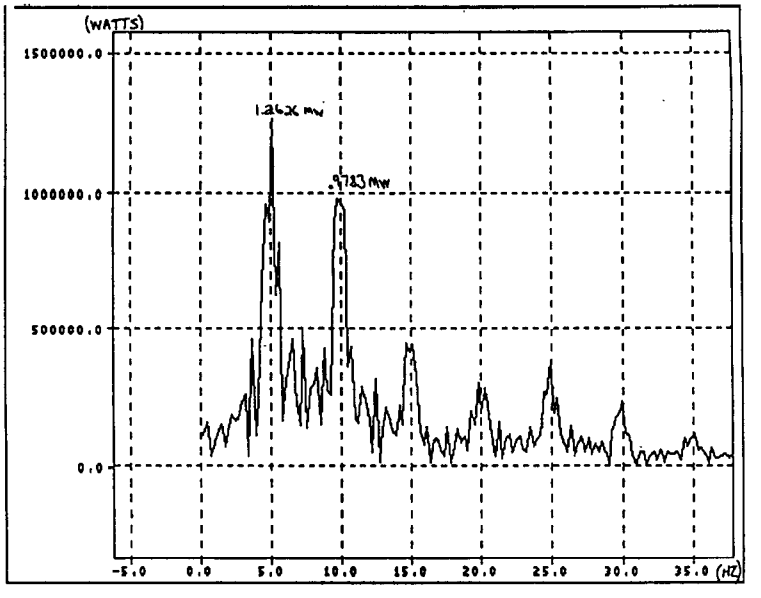


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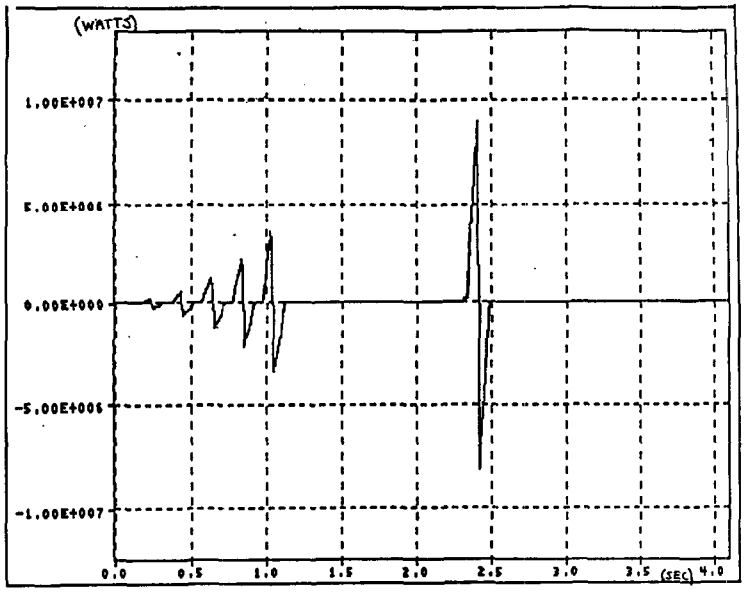


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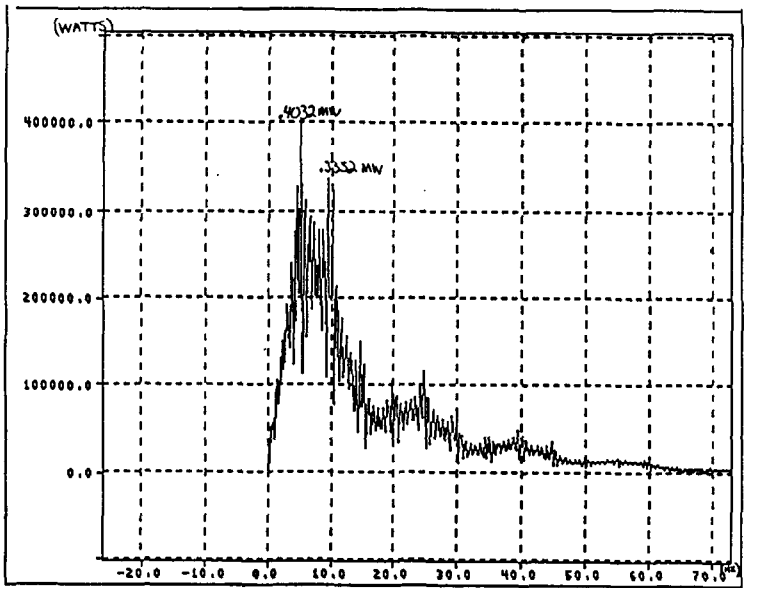


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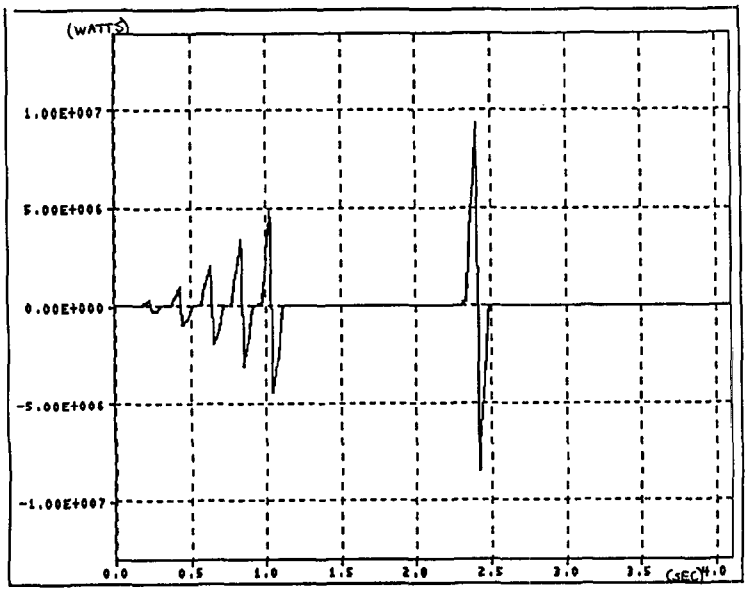


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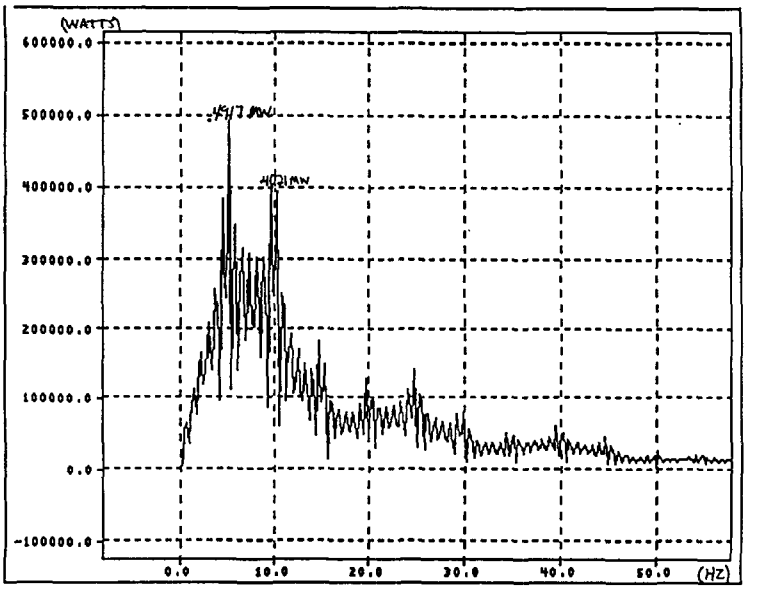


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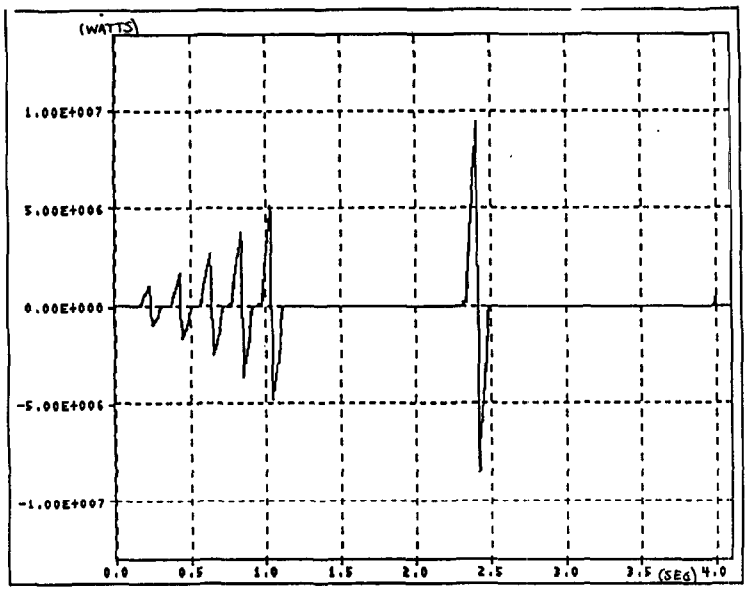


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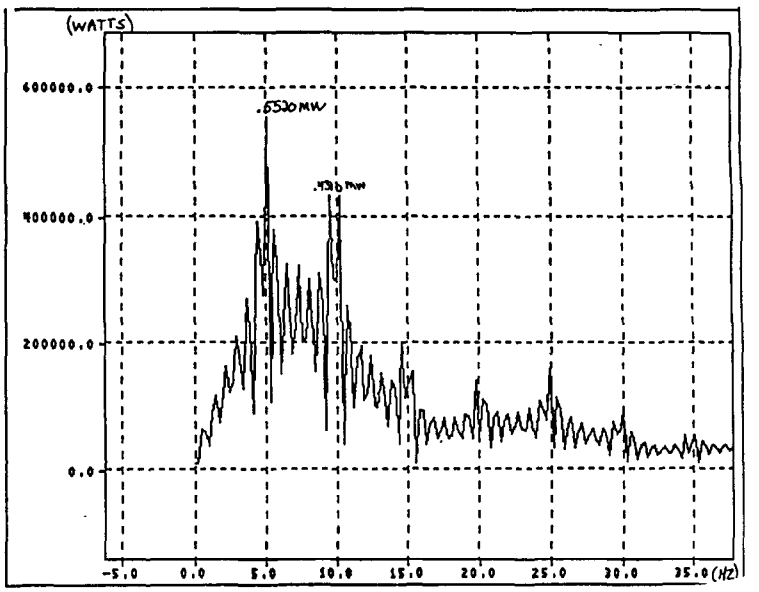


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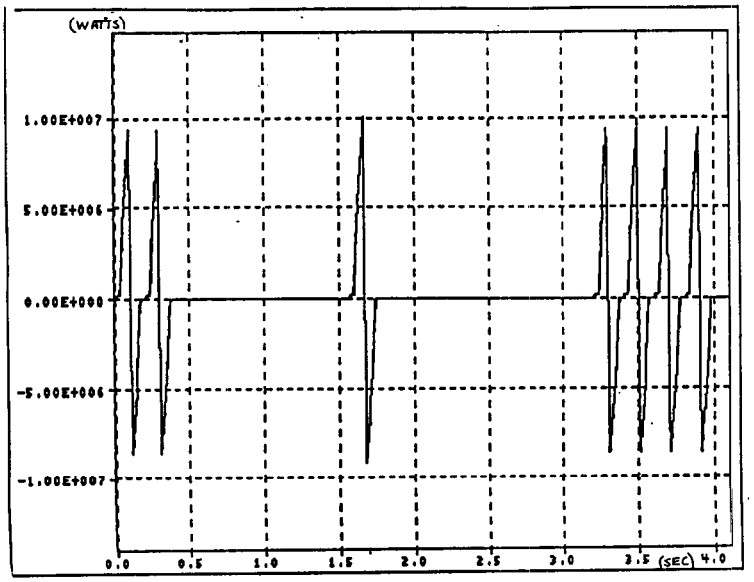


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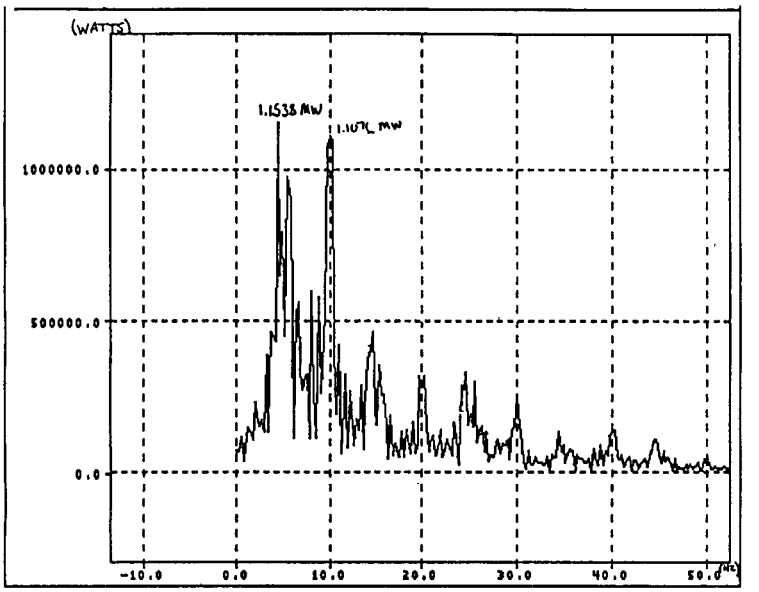


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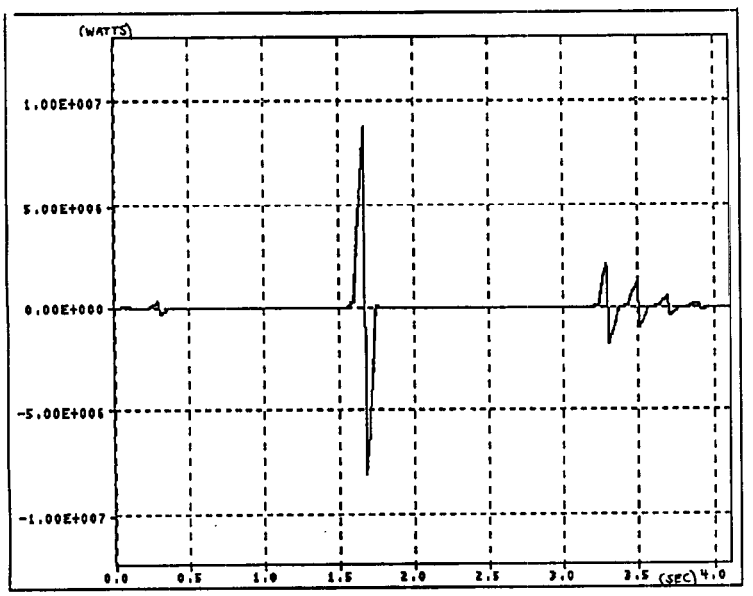


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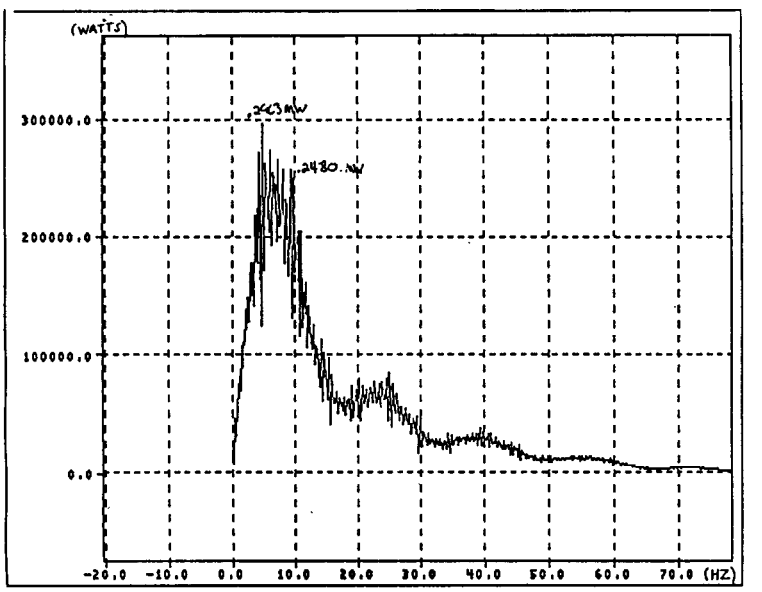


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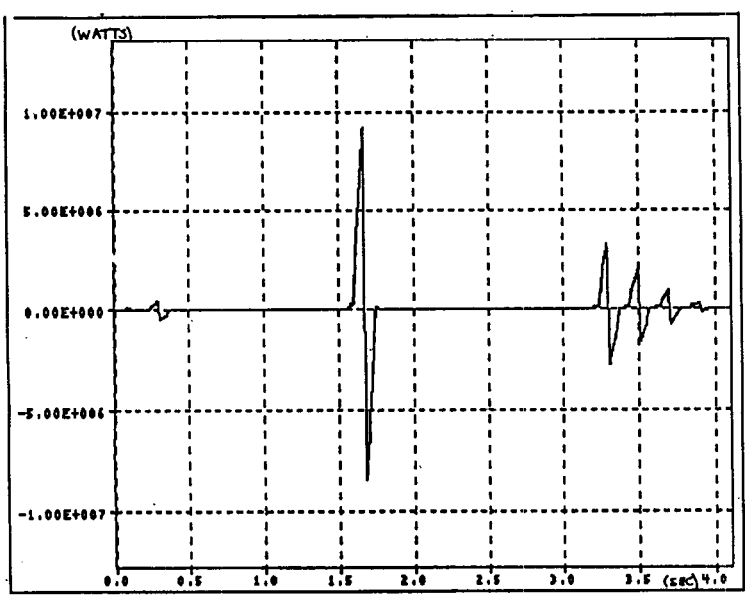


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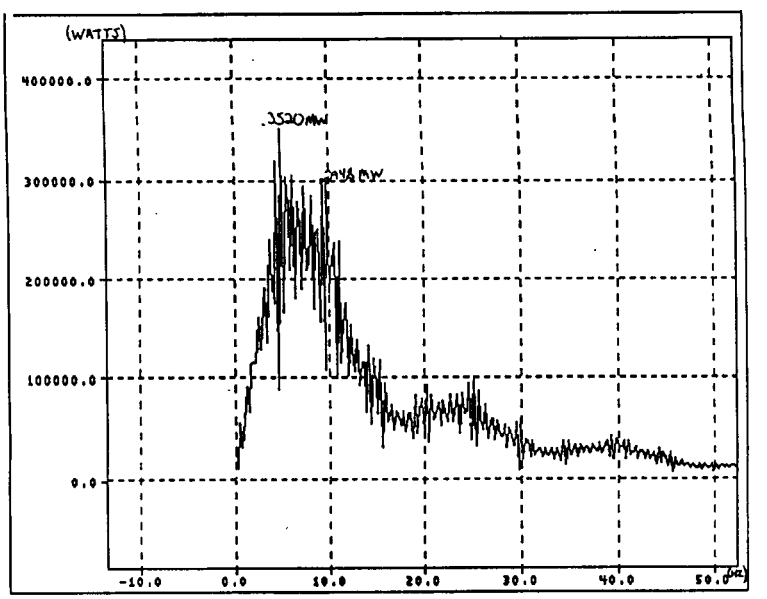


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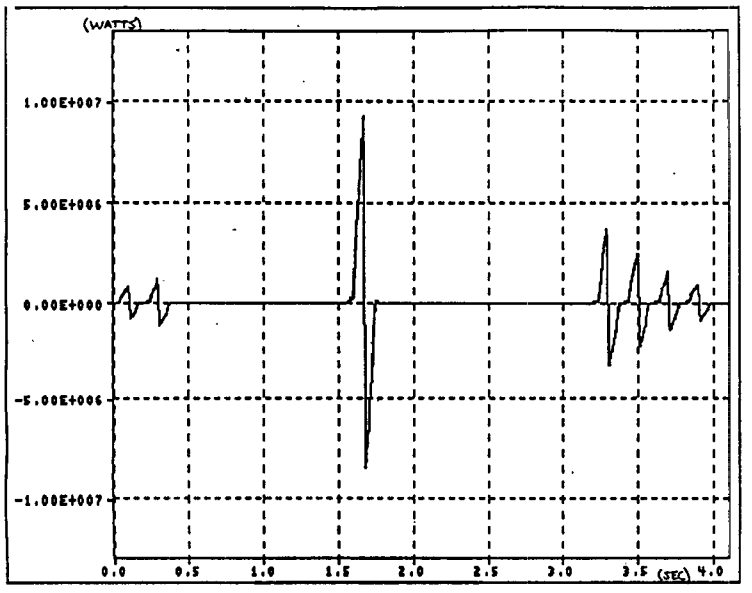


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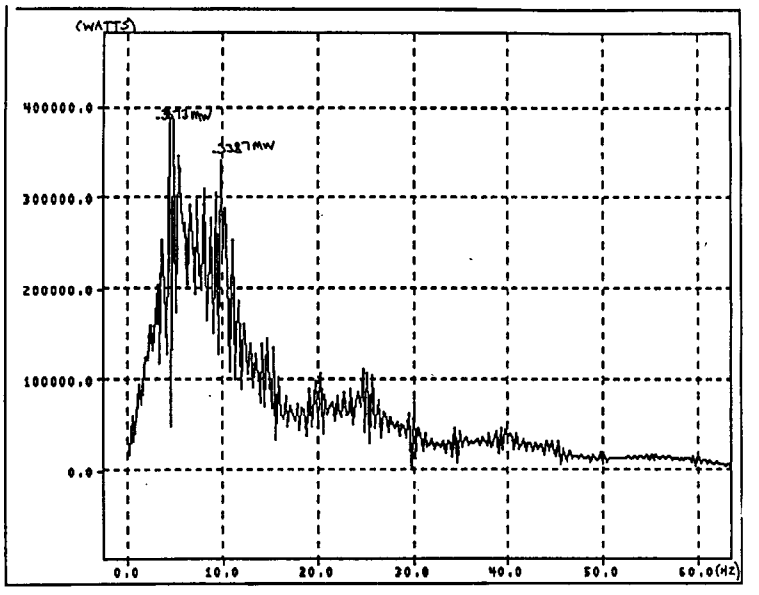


Figure #35

## Appendix A

## Waveform Analysis

The power waveform for the MMPS can be seen in Figure A-1 where the parameter k represents the fraction of time that the magnet is energized for each Booster cycle. The Fourier Series expansion of the waveform in the region $-\pi < x < \pi$ is given by

$$f(x) = -\left(\frac{2E}{\pi}\right) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \left[1 - \frac{\sin nk\pi}{nk\pi}\right] \sin nx$$

The magnitude of the fundamental component $a_1$ and the second harmonic component $a_2$ is given by

$$a_1 = \left(\frac{2E}{\pi}\right) \left[1 - \frac{\sin k\pi}{k\pi}\right]$$

$$a_2 = \left(\frac{E}{\pi}\right) \left[1 - \frac{\sin 2k\pi}{2k\pi}\right]$$

Thus the ratio of $a_2/a_1$ is given by

$$\frac{a_2}{a_1} = \left(\frac{1}{2}\right) \left[\frac{\left(1 - \frac{\sin 2k\pi}{2k\pi}\right)}{\left(1 - \frac{\sin k\pi}{k\pi}\right)}\right]$$

Plots of $a_1$, $a_2$, and the ratio of $a_2/a_1$ as a function of parameter k can be seen in Figures A-2, A-3, and A-4 respectively.

For the Booster 5 hz. proton cycle the parameter k is in the region of 0.7 to 0.8; for the 7.5 hz. cycle k is in the region of 0.9.

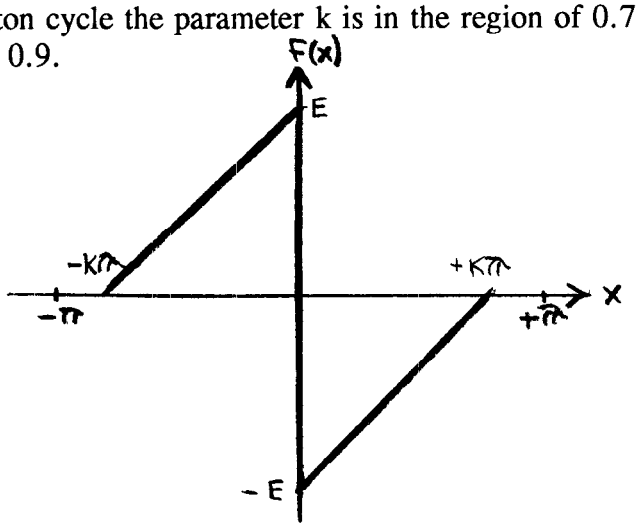


Figure A-1

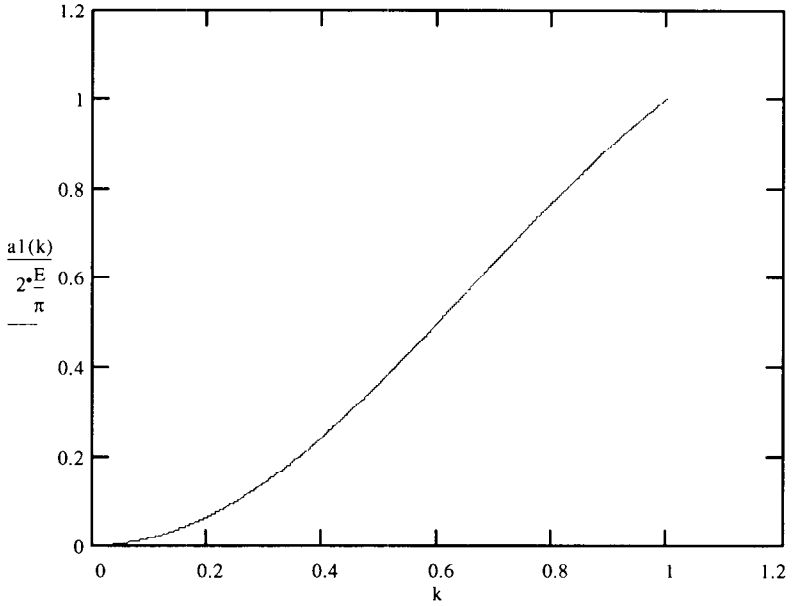

Figure A-2

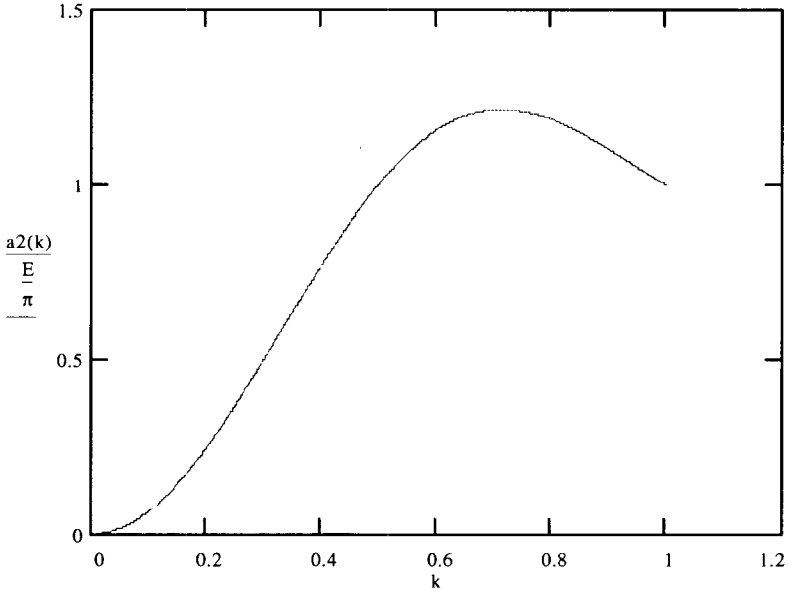

Figure A-3

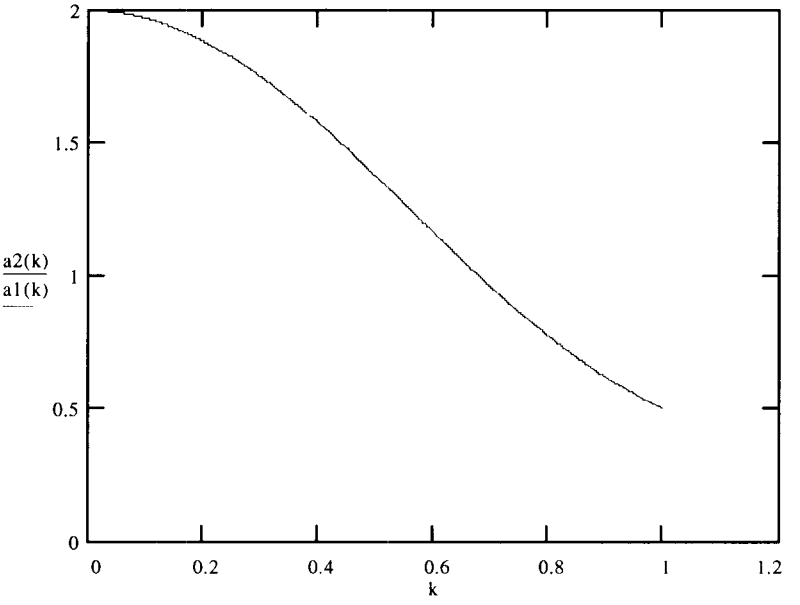

Figure A-4

## Appendix B

## Allowable Power Swing

### Proton Cycles

The maximum allowable power swing has two limiting factors, induced line voltage flicker and the harmonic or frequency components of the power swing. The induced line voltage flicker has been previously calculated to limit the power swing to a maximum value of 32 MW peak to peak.[6] The flicker induced on the power grid is an instantaneous phenomenon. The harmonic components of power swing involve an integration of the power waveform. Thus, the spectrum is a function of the supercycle structure. For the 5 hz. and the 7.5 hz. proton cycle there are no limits on the magnitude of the fundamental component $a_1$. Thus, the allowable power swing is limited by the second harmonic $a_2$ for both the 5 hz. and the 7.5 hz. proton cycles. The allowable amplitude of the second harmonic is set by the Alarm limit of 3 MW. Based on this limit, with the maximum value of $a_2$ ($1.217\ E/\pi$), and the Booster duty cycle (0.263), the maximum power swing is calculated from

$$1.217\frac{E}{\pi}(0.263) = 3$$

The peak swing (E) is calculated as 29.4 MW or 58.8 MW peak to peak. With a User 3 cycle contributing an additional power pulse comparable to the Booster cycle, the peak swing is approximately 5/6 of the calculated value or 49 MW peak to peak. Thus, for all proton cycles the power swing is limited by the induced flicker.

### Heavy Ion Cycles

Due to the large value of energy dissipated within the magnet, the Booster heavy ion power waveform resembles a triangular wave where as the proton cycle resembles a sawtooth wave. Triangular waves are weaker in harmonic content than are saw tooth waves. Only odd ordered harmonics are generated by this waveform and the harmonic amplitudes vary inversely as the square of the harmonic number. Thus the amplitude of the third harmonic is 1/9 of the fundamental. The spectrum of a triangular power waveform is given by

$$f(x) = \left[\frac{8}{\pi^2}\right]E \sum_{n=1}^{\infty} \left[\frac{1}{(2n-1)^2}\right]\cos\ (2n-1)x$$

If the Booster period is greater than 1.024 seconds, then the power swing is limited by the amplitude of the third harmonic. The value of power swing to reach the Alarm limit of 2.5 MW is calculated from

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[6]Booster Technical Note No. 215, "Measurement of Power Line Flicker Induced by the AGS Booster", December 14, 1992.

$$\left(\frac{1}{9}\right)\left[\frac{8}{\pi^2}\right]E = 2.5$$

The value of E is 27.5 MW peak or 55 MW peak to peak. Again the induced flicker will limit the maximum allowable power swing to 32 MW peak to peak.

If the Booster period is less than 1.024 seconds, then the power swing is limited by the amplitude of the fundamental. The value of power swing to reach the Alarm Limit of 2.5 MW is 3.08 MW peak or 6.16 MW peak to peak and is the limit on the allowable power swing.