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Differential two-body compound nuclear cross section, including the width-fluctuation corrections

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Differential two-body compound nuclear cross sections, including the width-fluctuation corrections

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We figure out the compound angular differential cross sections, following mainly Fröbrich and Lipperheide, but with the angular momentum couplings that make sense for optical model work. We include the width-fluctuation correction.

I. INTRODUCTION

Consider the reaction $x+A \to y+B$. Want to compute the differential cross section for outgoing b's (whatever they are):

$$\frac{d\sigma_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b \frac{k_b}{k_a} \left|T_{a\to b}\right|^2 \tag{1}$$

Here the reduced mass and relative momentum of the incoming system a are μ_a and $\vec{k}_a$ while the reduced mass and relative momentum of the outgoing system b are μ_b and $\vec{k}_b$. We wrote this cross section in terms of the T-matrix, $T_{a\to b} = \langle b|T(E_a)|a\rangle$, so that the in and out states are the asymptotically free two-body states:

$$|a\rangle = |I_x, M_x\rangle \, |I_A, M_A\rangle \left|\vec{k}_a\right\rangle \tag{2}$$

and

$$|b\rangle = |I_y, M_y\rangle \, |I_B, M_B\rangle \left|\vec{k}_b\right\rangle \tag{3}$$

Here, $\vec{I}_x$ is the spin operator for the projectile x, $\vec{I}_A$ is the spin operator for target A and the two particle state a is composed of the projectile x and the target A. Similarly, $\vec{I}_y$ is the spin operator for the ejectile y, $\vec{I}_B$ is the spin operator for residual nucleus B and the two particle state b is composed of the ejectile y and the residual nucleus B.

Note, the linear momentum eigenstate $\left|\vec{k}_a\right\rangle$ can be written in terms of angular momentum eigenstates as follows:

$$\left|\vec{k}_a\right\rangle = \sum_{\ell_a m_a} i^{\ell_a} Y^*_{\ell_a m_a}(\hat{k}_a) \, |E_a \ell_a m_a\rangle \tag{4}$$

where E_a is the energy of the incoming channel.

We assume that the beam and target are unpolarized and that we don't measure the outgoing spins, so really we want the following spin-averaged cross section:

$$\frac{d\overline{\sigma}_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b \frac{k_b}{k_a} \frac{1}{2I_x+1} \frac{1}{2I_A+1} \times \sum_{M_x M_A} \sum_{M_y M_B} \left|T_{a\to b}\right|^2 \tag{5}$$

In the fast region, we only concern ourselves with the energy averaged cross section as resonances are no longer resolvable. Therefore, we split the T-matrix into an energy-smooth part $\overline{T}_{a\to b}$ and a part that fluctuates with energy $T^{(fl)}_{a\to b}$:

$$T_{a\to b} = \overline{T}_{a\to b} + T^{(fl)}_{a\to b} \tag{6}$$

where

$$\overline{T}^{(fl)}_{a\to b} = 0. \tag{7}$$

So

$$\overline{|T_{a\to b}|^2} = |\overline{T_{a\to b}}|^2 + \overline{|T^{(fl)}_{a\to b}|^2} \tag{8}$$

Inserting this into eq. (5), we find that the energy average cross section is a sum of two parts:

$$\overline{\frac{d\overline{\sigma}_{a\to b}}{d\Omega_b}} = \frac{d\overline{\sigma}^{dir}_{a\to b}}{d\Omega_b} + \frac{d\overline{\sigma}^{CN}_{a\to b}}{d\Omega_b} \tag{9}$$

where the direct reaction part is associated with the energy averaged T-matrix:

$$\frac{d\overline{\sigma}^{dir}_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b \frac{k_b}{k_a} \frac{1}{2I_x+1} \frac{1}{2I_A+1} \times \sum_{M_x M_A} \sum_{M_y M_B} |\overline{T_{a\to b}}|^2 \tag{10}$$

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and the compound nuclear reaction part is associated with the energy averaged fluctuating part:

$$\frac{d\overline{\sigma}^{CN}_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b \frac{k_b}{k_a}\frac{1}{2I_x+1}\frac{1}{2I_A+1} \times \sum_{M_xM_A}\sum_{M_yM_B}\overline{|T^{(fl)}_{a\to b}|^2} \tag{11}$$

Although we formulate these the following results in terms of the T-matrix, the transition to the on-shell S-matrix is relatively straightforward. The S-matrix is

$$S_{ba} = \delta_{ba}\delta(\Omega_b-\Omega_a) - i\sqrt{\mu_\alpha\mu_\beta k_a k_b}\frac{2\pi}{\hbar^2}T_{ba} \tag{12}$$

In the remainder of this note, we will derive the expression for eq. (11) in a coupling scheme that is natural for implementation in EMPIRE.

II. FORMULATION USING CHANNEL SPIN COUPLING

Several sources ([1–5]) couple to the channel spin $\vec{S}_a = \vec{I}_x + \vec{I}_A$:

$$\left|\underline{a}; I_xM_x; I_AM_A; \vec{k}\right\rangle = \sum_{S_aM_a}\sum_{\ell_am_a}\sum_{JM} i^{\ell_a}Y^*_{\ell_am_a}(\hat{k}_a)\,(I_xM_xI_AM_A|S_aM_a)\,(\ell_am_aS_aM_a|JM)\,|\underline{a};\ell_am_a;S_aM_a;JM\rangle \tag{13}$$

$\underline{a} = \{\alpha, E_a, I_x, I_A, T_x, Tz_x, T_A, Tz_A\}$ captures all the other quantum numbers that are unimportant for the angular momentum coupling work. Here $(\ell_am_aS_aM_a|JM)$ are the Clebsch-Gordan coefficients (a.k.a. vector coupling coefficients) [6] and are defined in terms of 3-j symbols as (Edmonds eq. (3.7.3))

$$(\ell_am_aS_aM_a|JM) = (-1)^{\ell_a-S_a+M}\sqrt{2J+1}\begin{pmatrix}\ell_a & S_a & J\\ m_a & M_a & -M\end{pmatrix}. \tag{14}$$

Note, $m_a + M_a = M$.

Following Blatt and Biedenharn [5] and Lane and Thomas [2], Fröbrich and Lipperheide [1] show (note our notation is slightly different)

$$\frac{d\overline{\sigma}_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b\frac{k_b}{k_a}\frac{1}{2I_x+1}\frac{1}{2I_A+1}\sum_{L=0}^{\infty}B_L(\underline{b},\underline{a};E_a)P_L(\mu) \tag{15}$$

and

$$B_L(\underline{b},\underline{a};E_a) = \sum_{S_a,S_b}\frac{(-)^{S_b-S_a}}{4}\sum_{J\ell_a\ell_b}\sum_{J'\ell'_a\ell'_b}\bar{Z}(\ell_aJ\ell'_aJ';S_aL)\bar{Z}(\ell_bJ\ell'_bJ';S_bL)\Re\left[T^{J*}_{\{\underline{a};\ell_aS_a\}\to\{\underline{b};\ell_bS_b\}}T^{J'}_{\{\underline{a};\ell'_aS_a\}\to\{\underline{b};\ell'_bS_b\}}\right]. \tag{16}$$

Here they used the fact that $\vec{k}_a||\hat{z}$ so $Y_{\ell_am_a}(\Omega_a\equiv 0) = \delta_{m_a0}\sqrt{(2\ell_a+1)/4\pi}$ which eliminates sums over m_a, m'_a, M_a and M'_a. We have written the T-matrix in the channel spin coupling scheme as

$$T^J_{\{\underline{a};\ell_aS_a\}\to\{\underline{b};\ell_bS_b\}} = \langle\underline{b};\ell_bm_b;S_bM_b;JM|\,T(E_a)\,|\underline{a};\ell_am_a\equiv 0;S_aM_a\equiv M;JM\rangle \tag{17}$$

It is independent of m_b, M_b and M because the Hamiltonian is rotationally invariant. The Blatt-Beidenharn coefficient $\overline{Z}$ is

$$\bar{Z}(\ell_aJ\ell'_aJ';S_aL) = \sqrt{(2\ell_a+1)(2\ell'_a+1)(2J+1)(2J'+1)}\,(\ell_a0\ell'_a0|L0)\,\mathcal{W}(\ell_aJ\ell'_aJ';S_aL) \tag{18}$$

The Racah coefficient $\mathcal{W}(\ell_aJ\ell'_aJ';S_aL)$ is defined in therms of 6-j symbols as (eq. (6.2.13) [6])

$$\mathcal{W}(\ell_aJ\ell'_aJ';S_aL) = (-1)^{-(\ell_a+\ell'_a+J+J')}\begin{Bmatrix}\ell_a & J & S_a\\ J' & \ell'_a & L\end{Bmatrix} \tag{19}$$

Note, that we write our $\overline{Z}$ in the notation of Fröhner which incorporates the time-reversal phase convention correction of Lane and Thomas (and others) [2] from Blatt and Biedenharn's original definition in [5].

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III. A COUPLING SCHEME MORE CONVENIENT FOR HAUSER-FESHBACH WORK

Because the optical model does not care about the spin of the target or residual nuclei, the resulting transmission coefficients (and by extension the S- and T-matrices) do not depend on $\vec{I}_A$ or $\vec{I}_B$. Therefore, for Hauser-Feshbach and optical model work, it is more convenient to couple using $\vec{j}_a = \vec{I}_x + \vec{\ell}_a$:

$$\left|\underline{a}; I_x M_x; I_A M_A; \vec{k}\right\rangle = \sum_{j_a M_a} \sum_{\ell_a m_a} \sum_{JM} i^{\ell_a} Y^*_{\ell_a m_a}(\hat{k}_a) \left(I_x M_x \ell_a m_a | j_a M_a\right) \left(I_A M_A j_a M_a | JM\right) |\underline{a}; \ell_a m_a; j_a M_a; JM\rangle \tag{20}$$

Again, $\underline{a} = \{\alpha, E_a, I_x, I_A, T_x, Tz_x, T_A, Tz_A\}$ captures all the other quantum numbers that are unimportant for the angular momentum coupling work.

To use the results in eq. (15) and (16) for this alternate coupling scheme, we must switch bases from that in (13) to that in (20). Start by equating equations (13) and (20) and applying the spherical harmonic addition theorem:

$$\begin{aligned}&\sum_{S_a M_a} \sum_{JM} \left(I_x M_x I_A M_A | S_a M_a\right) \left(\ell_a m_a S_a M_a | JM\right) |\underline{a}; \ell_a m_a; S_a M_a; JM\rangle = \\ &\sum_{j_a M'_a} \sum_{JM} \left(I_x M_x \ell_a m_a | j_a M'_a\right) \left(I_A M_A j_a M'_a | JM\right) |\underline{a}; \ell_a m_a; j_a M'_a; JM\rangle\end{aligned} \tag{21}$$

Summing over the z-axis projection of the target and projectile spin, M_x, M_A, and applying the Clebsch-Gordan coefficient unitarity condition (Edmonds eq. (3.5.4) [6]), we find

$$\begin{aligned}&\sum_{JM} \left(\ell_a m_a S_a M_a | JM\right) |\underline{a}; \ell_a m_a; S_a M_a; JM\rangle = \\ &\sum_{M_x M_A} \sum_{j_a M'_a} \sum_{JM} \left(I_x M_x I_A M_A | S_a M_a\right) \left(I_x M_x \ell_a m_a | j_a M'_a\right) \left(I_A M_A j_a M'_a | JM\right) |\underline{a}; \ell_a m_a; j_a M'_a; JM\rangle\end{aligned} \tag{22}$$

The fact that $M_x + M_A + m_a = M$ coupled with eq. (2.5.6) of Edmonds gives us

$$\begin{aligned}&|\underline{a}; \ell_a m_a; S_a M_a; JM\rangle = \\ &\sum_{M_x M_A} \sum_{j_a M'_a} \left(\ell_a m_a S_a M_a | JM\right) \left(I_x M_x I_A M_A | S_a M_a\right) \left(I_x M_x \ell_a m_a | j_a M'_a\right) \left(I_A M_A j_a M'_a | JM\right) |\underline{a}; \ell_a m_a; j_a M'_a; JM\rangle\end{aligned} \tag{23}$$

The coefficient decoupling that we are doing here is not new (however painful it is). In 1961, Wigner discussed this recoupling and defined the 6-j symbol to handle it. Edmonds restates Wigner's definition in his eq. (6.1.5). Using this definition, we can rewrite (23) as

$$\begin{aligned}&|\underline{a}; \ell_a m_a; S_a M_a; JM\rangle \\ &= \sum_{j_a M'_a} \left[(2S_a+1)(2j_a+1)\right]^{1/2} (-1)^{-(I_A+I_x+\ell_a+J)} \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} |\underline{a}; \ell_a m_a; j_a M'_a; JM\rangle \qquad (24) \\ &= \sum_{j_a M'_a} \left[(2S_a+1)(2j_a+1)\right]^{1/2} \mathcal{W}\left(I_A I_x J \ell_a; S_a j_a\right) |\underline{a}; \ell_a m_a; j_a M'_a; JM\rangle \qquad (25)\end{aligned}$$

Note, all we have done here is an unitary transformation of our basis states as $\left[(2S_a+1)(2j_a+1)\right]^{1/2} \mathcal{W}\left(I_A I_x J \ell_a; S_a j_a\right)$ is an orthogonal matrix (see Edmonds eq. (6.2.10)) [6].

Now that we can switch bases from a spin channel coupled basis to a more convenient one, let us rewrite eq. (17):

$$\begin{aligned}T^J_{\{\underline{a};\ell_a S_a\}\to\{\underline{b};\ell_b S_b\}} &= \langle \underline{b}; \ell_b m_b; S_b M_b; JM | \, T(E_a) \, |\underline{a}; \ell_a m_a \equiv 0; S_a M_a \equiv M; JM\rangle \\ &= \sum_{j_a M'_a} \sum_{j_b M'_b} \sqrt{(2S_a+1)(2j_a+1)(2S_b+1)(2j_b+1)} (-1)^{-(I_A+I_x+\ell_a+I_B+I_y+\ell_b+2J)} \\ &\quad \times \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} \left\{ \begin{array}{ccc} I_B & I_y & S_b \\ \ell_b & J & j_b \end{array} \right\} \langle \underline{b}; \ell_b m_b; j_b M'_b; JM | \, T(E_a) \, |\underline{a}; \ell_a m_a \equiv 0; j_a M'_a; JM\rangle\end{aligned} \tag{26}$$

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Again, the Hamiltonian is rotationally invariant so the matrix elements in eq. (27) are independent M, so we arrive at the relation between the T-matrices in the different coupling schemes:

$$T^J_{\{\underline{a};\ell_a S_a\}\to\{\underline{b};\ell_b S_b\}} = \sum_{j_a j_b} \sqrt{(2S_a+1)(2j_a+1)(2S_b+1)(2j_b+1)}(-1)^{-(I_A+I_x+\ell_a+I_B+I_y+\ell_b+2J)} \times \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} \left\{ \begin{array}{ccc} I_B & I_y & S_b \\ \ell_b & J & j_b \end{array} \right\} T^J_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}} \tag{27}$$

There is no extra factor of $(2j_b+1)$ from the sum over M_b because $M = M_b + m_b$ and m_b is fixed (but irrelevant) in the spin-coupled matrix element on the left side of the equation. Similarly, the sum over M'_b is trivial and results in no extra factors.

Now we put eq. (27) into eq. (16):

$$\begin{aligned} B_L(\underline{b},\underline{a};E_a) = &\sum_{S_a,S_b} \frac{(-)^{S_b-S_a}}{4} \sum_{J\ell_a\ell_b j_a j_b} \sum_{J'\ell'_a\ell'_b j'_a j'_b} \bar{Z}(\ell_a J \ell'_a J'; S_a L)\bar{Z}(\ell_b J \ell'_b J'; S_b L) \\ &\times (-1)^{-(\ell_a+\ell_b+\ell'_a+\ell'_b)}(2S_a+1)(2S_b+1)\sqrt{(2j_a+1)(2j'_a+1)(2j_b+1)(2j'_b+1)} \\ &\times \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} \left\{ \begin{array}{ccc} I_B & I_y & S_b \\ \ell_b & J & j_b \end{array} \right\} \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell'_a & J' & j'_a \end{array} \right\} \left\{ \begin{array}{ccc} I_B & I_y & S_b \\ \ell'_b & J' & j'_b \end{array} \right\} \\ &\times \Re\left[T^{J*}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}} T^{J'}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}} \right]. \end{aligned} \tag{28}$$

We can simplify this by performing the sums over S_a and S_b. To see this, consider the parts of this expression containing only S_a:

$$\begin{aligned} &\sum_{S_a}(-1)^{-S_a}(2S_a+1)\bar{Z}(\ell_a J\ell'_a J'; S_a L) \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell'_a & J' & j'_a \end{array} \right\} \\ =&\sum_{S_a}(-1)^{-(S_a+\ell_a+\ell'_a+J+J')}(2S_a+1)\sqrt{(2\ell_a+1)(2\ell'_a+1)(2J+1)(2J'+1)}\,(\ell_a 0 \ell'_a 0|L0) \\ &\times \left\{ \begin{array}{ccc} \ell_a & J & S_a \\ J' & \ell'_a & L \end{array} \right\} \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a \end{array} \right\} \left\{ \begin{array}{ccc} I_A & I_x & S_a \\ \ell'_a & J' & j'_a \end{array} \right\} \end{aligned} \tag{29}$$

To simplify this, we would like to take advantage of Edmonds eq. (6.2.12) [6]. We can simplify the sum over S_b as we will see, but to simplify the sum over S_a, we need to change the sign in the phase factor in front. We now show that the following result is true:

$$\sum_{S_a=|I_A-I_x|}^{I_A+I_x} (-1)^{-S_a} f(S_a) = (-1)^{-2(I_A+I_x)} \sum_{S_a=|I_A-I_x|}^{I_A+I_x} (-1)^{S_a} f(S_a) \tag{30}$$

When I_A and I_x sum to an integer, then S_a is also an integer, so the expression is clearly true. So we need only consider the case when I_A and I_x sum to a half integer. Write $S_a = (I_A+I_x) - n$ where $n \in \{0,1,2,...,n_{max}\}$ and $n_{max} = (I_A+I_x) - |I_A - I_x|$. Then

$$\begin{aligned} \sum_{S_a=|I_A-I_x|}^{I_A+I_x} (-1)^{-S_a} f(S_a) &= \sum_{n=n_{max}}^{0} (-1)^{n-(I_A+I_x)} f(S_a) \\ &= \sum_{n=n_{max}}^{0} (-1)^{-n-(I_A+I_x)} f(S_a) \\ &= \sum_{S_a=|I_A-I_x|}^{I_A+I_x} (-1)^{S_a-2(I_A+I_x)} f(S_a). \end{aligned}$$

So eq. (30) is proved.

With this change, both the sums over S_a and S_b may be handled the same way. Judicious a) permutation of columns and b) flipping the upper and lower components in pairs of columns in the 6-j symbols allows us to use Edmonds

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eq. (6.2.12) [6]. Furthermore, using the fact that ℓ_a and ℓ_b are integers means we can ignore factors of $(-1)^{2\ell_a}$ and $(-1)^{2\ell_b}$ so we arrive at

$$\sum_{S_a}(-1)^{S_a}(2S_a+1)\bar{Z}(\ell_a J\ell'_a J';S_aL)\left\{\begin{array}{ccc} I_A & I_x & S_a \\ \ell_a & J & j_a\end{array}\right\}\left\{\begin{array}{ccc} I_A & I_x & S_a \\ \ell'_a & J' & j'_a\end{array}\right\}$$
$$=(-1)^{-(2J+2J'+L+I_x+I_A+j_a+j'_a)}\sqrt{(2\ell_a+1)(2\ell'_a+1)(2J+1)(2J'+1)}\,(\ell_a 0\ell'_a 0|L0)\left\{\begin{array}{ccc} j'_a & \ell'_a & I_x \\ \ell_a & j_a & L\end{array}\right\}\left\{\begin{array}{ccc} L & j'_a & j_a \\ I_A & J & J'\end{array}\right\} \tag{31}$$

$$=\sqrt{\frac{(2J+1)(2J'+1)}{(2j_a+1)(2j'_a+1)}}(-1)^{-(2J+2J'+L+I_x+I_A-\ell_a-\ell'_a)}\left\{\begin{array}{ccc} j_a & J & I_A \\ J' & j'_a & L\end{array}\right\}\overline{Z}(\ell_a j_a\ell'_a j'_a;I_xL) \tag{32}$$

Inserting eq. (32) into eq. (28) (including the additional $(-1)^{-2(I_A+I_x)}$ phase factor), we arrive at our main result

$$B_L(\underline{b},\underline{a};E_a)=\frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b}\sum_{J'\ell'_a\ell'_b j'_a j'_b}\bar{Z}(\ell_a j_a\ell'_a j'_a;I_xL)\bar{Z}(\ell_b j_b\ell'_b j'_b;I_yL)\Re\left[T^{J*}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}T^{J'}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}\right]$$
$$\times(-1)^{-I_A-I_x+I_B+I_y}(2J+1)(2J'+1)\left\{\begin{array}{ccc} j_a & J & I_A \\ J' & j'_a & L\end{array}\right\}\left\{\begin{array}{ccc} j_b & J & I_B \\ J' & j'_b & L\end{array}\right\} \tag{33}$$

The lower line in the previous equation is really two orthogonal matrices (see Edmonds eq. (6.2.10) [6]) and are a pair of unitary transforms that recouple the angular momenta in the first line up to the total angular momenta J and J'.

IV. ANGULAR DISTRIBUTIONS FOR COMPOUND NUCLEAR REACTIONS IN ℓj COUPLING SCHEME

It is straightforward to repeat the algebraic steps from eqs. (6–8) on the quantity $\Re\left(T^{J*}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}T^{J'}_{\{\underline{a};\ell'_a j_a\}\to\{\underline{b};\ell'_b j_b\}}\right)$, arriving at

$$\Re\left(\overline{T^{J*}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}T^{J'}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}}\right)$$
$$=\Re\left(\overline{T^{J*}}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\,\overline{T^{J'}}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}+\overline{T^{J*(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}T^{J'(fl)}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}}\right) \tag{34}$$

The first term on the right hand side of the previous equation is the direct part in our alternative coupling scheme. The righthand term is the compound nuclear part. We make the usual compound nuclear reaction approximation and assume that the components with different angular quantum numbers are uncorrelated giving

$$\overline{T^{J*(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}T^{J'(fl)}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}}=\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}\delta_{JJ'}\delta_{\ell_a\ell'_a}\delta_{\ell_b\ell'_b} \tag{35}$$

So the B_L coefficient for compound reactions is

$$B_L(\underline{b},\underline{a};E_a)=\frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b}\bar{Z}(\ell_a j_a\ell_a j_a;I_xL)\bar{Z}(\ell_b j_b\ell_b j_b;I_yL)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}$$
$$\times(-1)^{-I_A-I_x+I_B+I_y}(2J+1)^2\left\{\begin{array}{ccc} j_a & J & I_A \\ J & j_a & L\end{array}\right\}\left\{\begin{array}{ccc} j_b & J & I_B \\ J & j_b & L\end{array}\right\} \tag{36}$$

$$=\frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b}\bar{Z}(\ell_a j_a\ell_a j_a;I_xL)\bar{Z}(\ell_b j_b\ell_b j_b;I_yL)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}$$
$$\times(-1)^{-I_A-I_x+I_B+I_y+2(j_a+j_b)}(2J+1)^2\mathcal{W}\left(j_aJj_aJ;I_AL\right)\mathcal{W}\left(j_bJj_bJ;I_BL\right) \tag{37}$$

In the Moldauer approach to the width fluctuation correction [7], the average square T-matrix in terms of transmission coefficients $T^J_{\overline{c}\ell_c j_c}$ is

$$\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}=(const)\frac{T^J_{\overline{a}\ell_a j_a}T^J_{\overline{b}\ell_b j_b}}{\sum_c T^J_{\overline{c}\ell_c j_c}}W^J_{\overline{a}\overline{b}}$$

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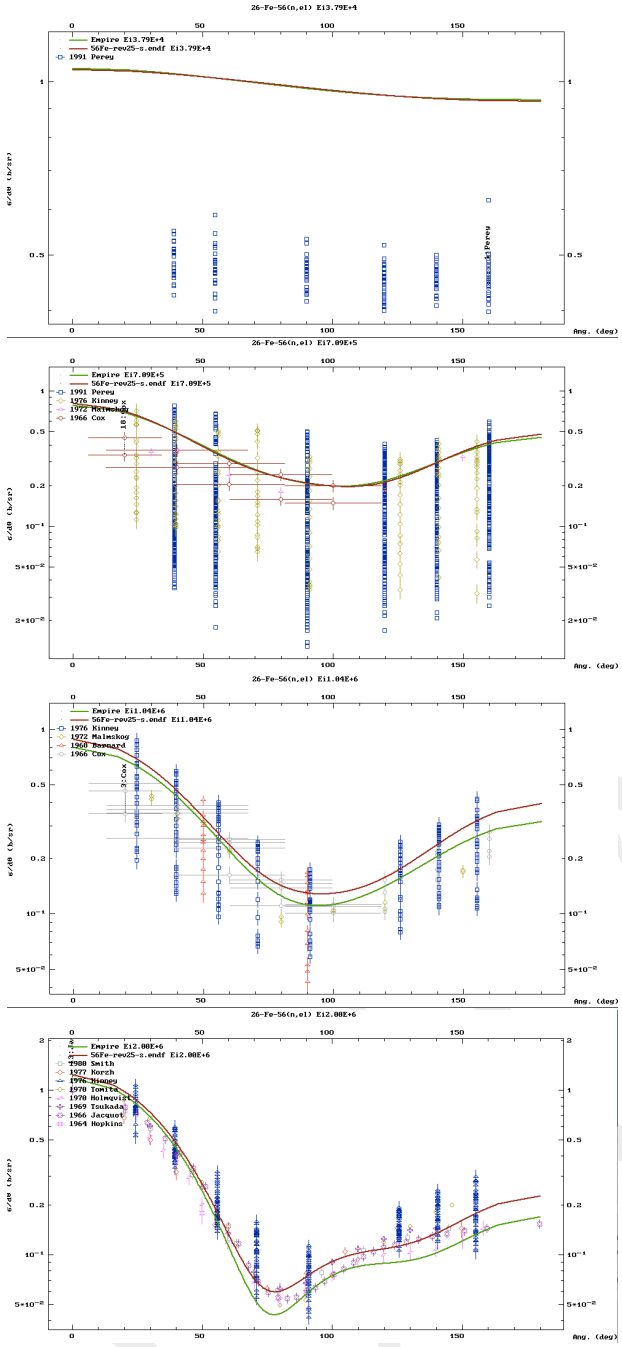

FIG. 1. Sample plots of of the outgoing neutron angular distribution from the ^{56}Fe(n,el) reaction. Each panel is a different beam energy and the beam energy increases from top to bottom. The two theoretical curves are that calculated directly in EMPIRE using the formulation in this note (green) and that calculated by ECIS (red).

(38)

where the matrix $W^J_{\overline{a}\overline{b}}$ encodes the width fluctuation correction ([7, 9]). Here the unimaginatively named constant $(const)$ is

$$(const) = \left((\mu_a\mu_b)(k_ak_b)(2\pi/\hbar)^4\right)^{-1} \tag{39}$$

In the HRTW approach, the average square T-matrix is

$$\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_aj_a\}\to\{\underline{b};\ell_bj_b\}}\right|^2} = (const)\frac{V^J_{\overline{a}\ell_aj_a}V^J_{\overline{b}\ell_bj_b}}{\sum_c V^J_{\overline{c}\ell_cj_c}}\left[1+\delta_{\overline{a}\ell_aj_a,\overline{b}\ell_bj_b}(W_{\overline{a}\ell_aj_a}-1)\right] \tag{40}$$

where $V_{\overline{a}\ell_aj_a}$ and $W_{\overline{a}\ell_aj_a}$ must be computed from $T^J_{\overline{a}\ell_aj_a}$ using the procedures outline in [8–10]. The comparison to the full triple integral result, such as carried out in Ref. [9] is not done here since the triple integral approach to the width fluctuation correction is not used in practical evaluations.

In Fig. 1, we compare results for the compound angular distribution computed with ECIS and EMPIRE, with width fluctuations turned off. Many of these energies are in the ^{56}Fe resonance region so the comparison to experiment is difficult. Nevertheless, agreement between our approach and the results from ECIS are quite good, with only minor differences likely attributable to the difference in level density prescriptions in the two codes.

As an additional check, I will try to (re)compute the compound nuclear cross section by taking the $L=0$ moment of B_L, using the Moldauer approach to the width fluctuation correction. Note,

$$\left\{\begin{array}{ccc} j & J & I \\ J & j & 0 \end{array}\right\} = (-1)^{j+J+I}\left[(2j+1)(2J+1)\right]^{-1/2} \tag{41}$$

so

$$\begin{aligned}\bar{Z}(\ell j\ell j;I0) =&(2\ell+1)(2j+1)\\ &\times(-1)^{2\ell+2j}(-1)^{\ell+j+I}\\ &\times\left[(2\ell+1)(2j+1)\right]^{-1/2}\\ &\times(-1)^{\ell}(2\ell+1)^{-1/2}\\ =&\sqrt{(2j+1)}(-1)^{I-j}\end{aligned} \tag{42}$$

and

$$\begin{aligned}&B_{L=0}(\underline{b},\underline{a};E_a)\\ =&\frac{1}{4}\sum_{J\ell_a\ell_bj_aj_b}(2J+1)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_aj_a\}\to\{\underline{b};\ell_bj_b\}}\right|^2} \qquad (43)\\ &\times(-1)^{2(J+I_B+I_y)}\\ =&\frac{1}{4}\sum_{J\ell_a\ell_bj_aj_b}(2J+1)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_aj_a\}\to\{\underline{b};\ell_bj_b\}}\right|^2} \qquad (44)\end{aligned}$$

Now

$$
\begin{aligned}
\overline{\sigma}_{a\to b} =& \int d\Omega_b \frac{d\overline{\sigma}_{a\to b}}{d\Omega_b} \\
=& 4\pi \frac{(const)^{-1}}{k_a^2} \frac{1}{2I_x+1} \frac{1}{2I_A+1} \\
& \times B_{L=0}(\underline{b},\underline{a};E_a) \\
=& \pi \frac{(const)^{-1}}{k_a^2} \frac{1}{2I_x+1} \frac{1}{2I_A+1} \\
& \times \sum_{J\ell_a\ell_b j_a j_b} (2J+1) \overline{\left| T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}} \right|^2} \\
=& \frac{\pi}{k_a^2} \frac{1}{2I_x+1} \frac{1}{2I_A+1} \\
& \times \sum_{J\ell_a\ell_b j_a j_b} (2J+1) \frac{T^J_{\overline{a}\ell_a j_a} T^J_{\overline{b}\ell_b j_b}}{\sum_c T^J_{\overline{c}\ell_c j_c}} W^J_{\overline{a}\overline{b}} \\
\equiv& \sum_J \sigma^{abs,J}_{\overline{a}} \frac{\sum_{\ell_b j_b} T^J_{\overline{b}\ell_b j_b}}{\sum_c T^J_{\overline{c}\ell_c j_c}} W^J_{\overline{a}\overline{b}}
\end{aligned}
\tag{45}
$$

Here we have identified the absorption cross section $\sigma^{abs,J}_{\overline{a}}$ as

$$
\sigma^{abs,J}_{\overline{a}} = \frac{\pi}{k_a^2} \frac{(2J+1)}{(2I_x+1)(2I_A+1)} T^J_{\overline{a}\ell_a j_a} \tag{46}
$$

V. COMMENTS ON CHARGED PARTICLE IN– AND OUT– CHANNELS

In the presence of incoming or outgoing charged particle states, the above work is changed in a few ways. First, all of the above must strictly speaking be redone using the S-matrix rather than the T-matrix because the asymptotic states in the presence of Coulomb interactions must be Coulomb wave functions. This does not affect the details of the angular momentum couplings, so the compound nuclear cross sections for charged particles as given above are correct once formulated in terms of transmission coefficients (provided that the Coulomb force is used in the calculation of the transmission coefficients). Second, for incoming charged particles, one must account for the pure Coulomb scattering amplitude. Schematically, we have

$$
\mathcal{A}_{tot} = \mathcal{A}_{coul} + \mathcal{A}_{nucl} \tag{47}
$$

so

$$
\frac{d\sigma}{d\Omega} \propto |\mathcal{A}_{tot}|^2 = |\mathcal{A}_{coul}|^2 + 2\Re\left(\mathcal{A}_{coul}\mathcal{A}_{nucl}\right) + |\mathcal{A}_{nucl}|^2 \tag{48}
$$

The individual terms in this expression have different spin and parity combinations and the first (Coulomb) term is purely analytic. The details are worked out in Lane and Thomas [2].

VI. CONCLUSION

We got it working for the most part.

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