

QCD Sum Rules for Magnetically Induced Mixing between η_c and J/ψ

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We investigate the properties of charmonia in strong magnetic fields by using QCD sum rules. We show how to implement the mixing effects between η_c and J/ψ on the basis of field-theoretical approaches, and then show that the sum rules are saturated by the mixing effects with phenomenologically determined parameters. Consequently, we find that the mixing effects are the dominant contribution to the mass shifts of the static charmonia in strong magnetic fields.

Ever since the suppression of J/ψ yields due to the color screening effect was proposed as a signature of the formation of the quark-gluon plasma in ultrarelativistic heavy-ion collisions [1, 2], much attention has been paid to the properties of heavy quarkonia under extreme environments. Meanwhile, extremely strong electric and magnetic fields induced by injection of heavy-ions have been discussed recently [3–5], because they could be new ingredients affecting experimental observables at the Relativistic Heavy Ion Collider and the Large Hadron Collider. A renewed interest arises in the study of the heavy-quark (HQ) systems and their spectral densities in the strong fields [6, 7] as some of these states will likely form in an earlier time after the impact [8] where the fields still persist with large strengths.

The QCD sum rule (QCDSR) has been extensively used for investigating the spectral density of the hadrons on the basis of the fundamental quark and gluon degrees of freedom [9–11]. Remarkably, the QCD sum rules for the HQ systems predicted the small mass splitting between η_c and J/ψ of the order of 100 MeV prior to the experimental confirmation of the η_c mass [9, 12, 13]. While the properties of charmonium systems in the vacuum are well described by the Cornell potential model [14], the advantage of using the QCDSR is that effects of external environments on the correlation functions can be easily taken into account from the modifications in the operator product expansion (OPE) through moderate changes in values of quark and gluon condensates. Moreover, for HQ systems, the modification involves only the dimension-4 operators that are related to the energy momentum tensor whose matrix elements are well estimated both at finite temperature from lattice QCD [15–19] and at normal nuclear matter density from measurements in deep inelastic scatterings [20]. Recently, it has been shown that even the temperature dependence of the gauge invariant strength of the charmonium wave function at the origin

obtained from the QCDSR supports that from solving the Schrödinger equation with a finite temperature free energy potential from lattice QCD [21].

In this Letter, we apply the QCDSR to investigate the mass spectra of the lowest-lying bound states coupled to pseudoscalar (PS) and vector HQ currents in external magnetic fields (B -fields); that is, the η_c and J/ψ at rest. We put a special emphasis on how to take into account mixing effects in the spectral density, the so-called phenomenological side, and show how to distinguish nonperturbative mass modifications from hadronic mixing effects between η_c and J/ψ . Since mixing effects naturally arise in external environments, as have been known for a long time in various systems such as hydrogen atoms and positronium in external electromagnetic fields, our results can be generalized to various systems accompanied by mixing effects. We note that our treatment of the mixing effects should be applied to the very recent QCDSR analysis on B mesons in strong B -fields [22], since the B mesons are mixed with B^* mesons. Our work demonstrates how to implement mixing effects in the QCDSR method, in particular for the HQ systems where both the OPE and the phenomenological side are well under control, and thus provides a general guideline to include mixing effects in approaches based on correlation functions.

We first begin by looking at the general results of the mixing effect using effective Lagrangians. A three-point vertex which can describe a radiative decay mode, $J/\psi \rightarrow \eta_c + \gamma$, induces mass shifts caused by the mixing effects. The effective vertex can be constructed from the Lorentz invariance and the parity and charge-conjugation symmetries as

$$\mathcal{L}_{\gamma PV} = \frac{g_{PV}}{m_0} e \tilde{F}_{\mu\nu}^{\text{ext}} (\partial^\mu P) V^\nu, \quad (1)$$

where $e > 0$ is the unit electric charge, g_{PV} the dimensionless phenomenological coupling constant and $m_0 =$

$\frac{1}{2}(m_P + m_V)$ with m_P and m_V being the vacuum masses of the η_c and J/ψ , respectively. We find that effective couplings with $F_{\mu\nu}^{\text{ext}}$, such as $F_{\mu\nu}^{\text{ext}}(\partial^\mu S)V^\nu$ with a scalar field S , vanish for charmonia at rest in B -fields due to vanishing components, $\partial^i = 0$ and $F_{0i}^{\text{ext}} = -F_{i0}^{\text{ext}} = 0$, and also that the static η_c is mixed only with the longitudinal J/ψ that is polarized in parallel to the external B -fields, as $\tilde{F}_{03}^{\text{ext}}(\partial^0 P)V^3$. The coupling constant g_{PV} can be fitted to the measured radiative decay width as $g_{PV} = \sqrt{12\pi e^{-2} p_f^{-3} m_0^2 \Gamma_{\text{exp}}[J/\psi \rightarrow \gamma\eta_c]} = 2.095$ with $p_f = (m_V^2 - m_P^2)/(2m_V)$ being the magnitude of the center-of-mass momentum in the final state.

Introducing a constant B -field in Eq. (1), we solve the two-state problem for the η_c and the longitudinal J/ψ using the classical Euler-Lagrange equation of $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\gamma PV}$. We obtain the physical mass eigenstates in the presence of the mixing effects

$$m_{J/\psi, \eta_c}^2 = \frac{1}{2} \left(M_+^2 + \frac{\gamma^2}{m_0^2} \pm \sqrt{M_+^4 + \frac{2\gamma^2 M_+^2}{m_0^2} + \frac{\gamma^4}{m_0^4}} \right), \quad (2)$$

where $M_+^2 = m_P^2 + m_V^2$, $M_-^2 = m_V^2 - m_P^2$ and $\gamma = g_{PV} eB$. Expanding Eq. (2) up to the second order in γ and the leading order in $\frac{1}{2}(m_V - m_P)/m_0$, we find

$$m_{J/\psi, \eta_c}^2 = m_{V,P}^2 \pm \frac{\gamma^2}{M_-^2}, \quad (3)$$

with eigenvectors given by

$$\begin{aligned} |\eta_c\rangle_B &= \left(1 - \frac{1}{2} \frac{\gamma^2}{M_-^2}\right) |P\rangle - i \frac{\gamma}{M_-^2} |V\rangle, \\ |J/\psi\rangle_B &= -i \frac{\gamma}{M_-^2} |P\rangle + \left(1 - \frac{1}{2} \frac{\gamma^2}{M_-^2}\right) |V\rangle. \end{aligned} \quad (4)$$

These results show a decrease and an increase in the masses of the η_c and the longitudinal J/ψ , respectively. Such a level repulsion has also been found on the basis of a potential-model approach [7]. However, further mass shifts could be caused by B -fields acting on the charmed meson loops such as a $D\bar{D}$ loop and/or interactions among charmonia and two photons (B -fields) as higher-order corrections to the effective Lagrangian (1).

To examine the effects of external B -fields on the charmonia using a nonperturbative QCD formalism, we turn to the QCDSR. We consider the current correlators in external B -fields for the PS current $J^P = i\bar{c}\gamma^5 c$ and the vector current $J_\mu^V = \bar{c}\gamma_\mu c$ defined by

$$\Pi^J(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[J(x)J(0)] | 0 \rangle, \quad (5)$$

where superscripts $J = P$ and V denote the PS and the vector currents, respectively. We investigate a spin-projected scalar correlator for the longitudinal J/ψ , $\tilde{\Pi}^V = (\epsilon^\mu \Pi_{\mu\nu}^V \epsilon^\nu)/q^2$, specified by a polarization vector $\epsilon^\mu = (0, 0, 0, 1)$ in a B -field oriented in the third

spatial direction. The PS correlator is normalized as $\tilde{\Pi}^P = \Pi^P/q^2$. We will construct the sum rules for $\tilde{\Pi}^J(q^2)$.

The first step involves calculating the OPE in the presence of an external B -field. The OPE for the HQ systems is based on the expansion in the deep Euclidean region $Q^2 = -q^2 \gg 0$, where $|\langle Op \rangle| \ll 4m^2 + Q^2$, with the left-hand side being the typical scale of the vacuum and/or the external field. Thus, as long as the B -field satisfies the similar condition $|e\mathbf{B}| \ll 4m^2 + Q^2$, which is valid for a region $|e\mathbf{B}| \lesssim 10m_\pi^2$ expected up to Large Hadron Collider energies [4], we can include the effect as an additional OPE term to the conventional terms in the ordinary vacuum [10, 13]

$$\tilde{\Pi}_{\text{OPE}}(Q^2) = \tilde{\Pi}_{\text{OPE}}^{\text{vac}}(Q^2) + \tilde{\Pi}_{\text{OPE}}^{\text{ext}}(Q^2). \quad (6)$$

The correlator $\tilde{\Pi}_{\text{OPE}}^{\text{ext}}$ can be precisely evaluated to the second order of eB by utilizing the corresponding coefficients for the dimension-4 gluon condensates [10, 13, 20] with an appropriate correction of the color matrix factor t^a , i.e., $\text{Tr}[t^a t^a] \langle G_{\mu\nu}^a G_{\alpha\beta}^a \rangle \rightarrow \text{Tr}[\mathbb{1}_{\text{color}}] \langle F_{\mu\nu} F_{\alpha\beta} \rangle$. We show the Borel-transformed Wilson coefficients for static charmonia with $q^i = 0$ in Eqs. (15) and (16), and full accounts of the calculation in a subsequent paper [23]. In the extremely strong field limit $|e\mathbf{B}| \gg 4m^2 + Q^2$, one has to go beyond the ordinary perturbation theory and perform resummation over the all-order dimension operators as recently investigated by one of the present authors in the vector channel [24].

Another possible effect of external B -fields on the correlator is a modification of the gluon condensate $\langle G^{\alpha\mu\nu} G_{\mu\nu}^\alpha \rangle$. Recently, both a lattice QCD simulation [25] and an analytic study [26] pointed out that the gluon condensate increases with an increasing B -field at zero temperature in analogy to the growth of the quark condensate in magnetic fields known as *magnetic catalysis* [27, 28]. However, we do not take this into account in the present work, because this effect should be small without direct couplings between gluons and external B -fields, as estimated to be a change less than 10 % [25].

The current correlator (5) is connected to the physical spectral density $\rho(s) = \text{Im}\Pi(s)/\pi$ in the deep Euclidean region Q^2 through the dispersion relation

$$\tilde{\Pi}^J(Q^2) = \int ds \frac{\rho(s)}{s + Q^2} + (\text{subtraction}). \quad (7)$$

In the QCDSR, the phenomenological side for $\rho(s)$ is often modeled by the ground-state pole and the continuum. The sum rule is known to be insensitive to the structure of $\rho(s)$ in the high-energy perturbative regime after the Borel transformation of the dispersion relation. However, if there is a mixing with a state close to the ground state, it should be carefully included in the phenomenological side. Below, we show how to accomplish this in the case of the η_c ; the same calculation can be straightforwardly applied to the longitudinal J/ψ .

We start by considering the low energy states that interpolate the currents in the correlation function (5) with the PS current. Since the J/ψ mixes into the PS correlator in the second order of eB , we have

$$\tilde{\Pi}_{\text{ph}}^{\text{P}}(q^2) = \frac{|\langle 0|J^5|\eta_c\rangle|^2}{q^2 - m_{\eta_c}^2} + \frac{|\langle 0|J^5|J/\psi\rangle|^2}{q^2 - m_{J/\psi}^2}, \quad (8)$$

where the matrix element is calculated in the presence of the external B -field as follows. Let us first look at the residue of the second term. The current can either couple directly to the J/ψ or first couple to the η_c which will be subsequently converted to the J/ψ through the hadronic coupling given in Eq. (1). These can be written as

$$|\langle 0|J^5|J/\psi\rangle|^2 = f_{\text{dir}} + \frac{f_0|\langle P|J/\psi\rangle|^2}{(q^2 - m_{\text{P}}^2)^2}. \quad (9)$$

with $f_{\text{dir}} = |\langle V|J^5(q)|0\rangle|^2$ and $f_0 = |\langle P|J^5(q)|0\rangle|^2$. The effective vertex (1) leads to $|\langle P|J/\psi\rangle|^2 = \gamma^2$. Using the Bethe-Salpeter amplitudes [29] with the Coulombic wave function of the S-wave quarkonia, we compute the direct-coupling through a triangle diagram [23] as $f_{\text{dir}} = a_0^4 Q_c^2/64(eB)^2 f_0$ with the electromagnetic charge of a charm quark $Q_c = 2/3$ and the Bohr radius $a_0 = 0.811 \text{ GeV}^{-1}$, chosen to fit the root-mean-square radius of the J/ψ obtained from the Cornell potential model. After inserting Eq. (9) to Eq. (8), we find that the second term in Eq. (8) can now be decomposed as

$$\begin{aligned} & \frac{f_0\gamma^2}{(q^2 - m_{\text{P}}^2)^2(q^2 - m_{\text{V}}^2)} \\ &= \frac{f_0\gamma^2}{M_-^4} \left[\frac{1}{q^2 - m_{\text{V}}^2} - \frac{1}{q^2 - m_{\text{P}}^2} - \frac{M_-^2}{(q^2 - m_{\text{P}}^2)^2} \right], \quad (10) \end{aligned}$$

where the J/ψ mass was replaced by the vacuum mass m_{V} within the second-order correction in eB to the correlator (8). The strength of the vector single pole is found to be much larger than the direct-coupling strength, $f_{\text{dir}}/(f_0\gamma^2/M_-^4) \sim 0.0003$, so that one can safely neglect the contributions of the direct couplings, including cross diagrams in which a J/ψ converted from a η_c is directly coupled to the PS current.

One should note that the phenomenological side discussed above can be obtained by first converting the current to the pseudoscalar meson with the strength f_0 and then using the second-order perturbation theory shown in Eqs. (3) and (4), in which the correlator is given by

$$\Pi_{2\text{nd}}(q^2) = f_0 \left[\frac{|(P|\eta_c)_B|^2}{q^2 - m_{\eta_c}^2} + \frac{|(P|J/\psi)_B|^2}{q^2 - m_{J/\psi}^2} \right]. \quad (11)$$

All three of the terms in Eq. (10) are reproduced by expanding the rhs in Eq. (11) up to the second order in eB . Interpretation of the terms in Eq. (10) are as follows. The first term corresponds to production of an on-shell J/ψ from the PS current via an off-shell η_c . The second term

with a negative sign is needed to preserve the normalization, because the coupling of η_c to the current must be reduced to balance the occurrence of the coupling to J/ψ . This is confirmed in Eq. (11), where these two terms come from overlaps between the properly normalized unperturbed and perturbed states obtained as $|(P|\eta_c)_B|^2 \sim 1 - (\gamma/M_-^2)^2$ and $|(P|J/\psi)_B|^2 \sim (\gamma/M_-^2)^2$. The third term has a double pole on the η_c mass with a factor M_-^2 , corresponding to a virtual transition to J/ψ state between on-shell η_c states, which is nothing but the origin of the mass shift due to the mixing effect. In Eq. (11), this term comes from an expansion with respect to the mass correction shown in Eq. (3). Clearly, if one includes this mixing term in the phenomenological spectral function, its effect is subtracted out from the total mass shift obtained from the QCDSR, and thus can be separated from the residual effects of B -fields, *not described in the hadronic level*.

Now we evaluate the mass spectra of η_c and J/ψ using the standard Borel transformation method. With a transformation parameter M^2 called the Borel mass, the dispersion relation (7) is transformed to

$$\mathcal{M}^J(M^2) = \int_0^\infty ds e^{-s/M^2} \text{Im}\tilde{\Pi}^J(s), \quad (12)$$

so that the sum rule can be expressed as ($\nu = 4m_c^2/M^2$)

$$\mathcal{M}_{\text{OPE}}(\nu) = \mathcal{M}_{\text{ph}}^{\text{pole}}(\nu) + \mathcal{M}^{\text{cont}}(\nu) + \mathcal{M}_{\text{ph}}^{\text{ext}}(\nu). \quad (13)$$

A transform of the OPE side (6) is then obtained as

$$\begin{aligned} \mathcal{M}_{\text{OPE}}(\nu) = & \pi e^{-\nu} A(\nu)[1 + \alpha_s(\nu)a(\nu) \\ & + b(\nu)(\phi_b + \phi_b^{\text{ext}}) + c^{\text{ext}}(\nu)\phi_c^{\text{ext}}]. \quad (14) \end{aligned}$$

While explicit forms of the coefficients $A(\nu)$, $a(\nu)$ and $b(\nu)$ are given in Refs. [17, 30], the Lorentz-breaking part $c^{\text{ext}}(\nu)$ is obtained for static charmonia ($q^i = 0$) to be

$$c^{\text{P,ext}}(\nu) = \frac{4}{3}b(\nu) - \frac{16}{3}\nu \frac{G(-\frac{1}{2}, \frac{1}{2}, \nu)}{G(\frac{1}{2}, \frac{3}{2}, \nu)}, \quad (15)$$

$$\begin{aligned} c^{\text{V,ext}}(\nu) = & \frac{2\nu}{3G(\frac{1}{2}, \frac{5}{2}, \nu)} \left[6G\left(\frac{1}{2}, \frac{5}{2}, \nu\right) \right. \\ & \left. + 6G\left(-\frac{1}{2}, \frac{5}{2}, \nu\right) - G\left(-\frac{3}{2}, \frac{5}{2}, \nu\right) \right], \quad (16) \end{aligned}$$

with $G(a, b, \nu)$ being the Whittaker function. Operator expectation values ϕ_b^{ext} and ϕ_c^{ext} account for magnitudes of the external B -fields, and are defined by $\phi_b^{\text{ext}} = \frac{4}{3} \frac{Q_c^2}{16m_c^2} (eB)^2$ and $\phi_c^{\text{ext}} = -\frac{Q_c^2}{16m_c^2} (eB)^2$.

As for the phenomenological side on the rhs of Eq. (13), $\mathcal{M}_{\text{ph}}^{\text{pole}}$ and $\mathcal{M}^{\text{cont}}$ have the same form as in the conventional QCDSR analyses. While $\mathcal{M}_{\text{ph}}^{\text{pole}}$ corresponds to the transform of the first term in Eq. (8) given by $\mathcal{M}_{\text{ph}}^{\text{pole}} = f_0 e^{-m_{\eta_c}^2/M^2}$, $\mathcal{M}^{\text{cont}}$ stands for a perturbative

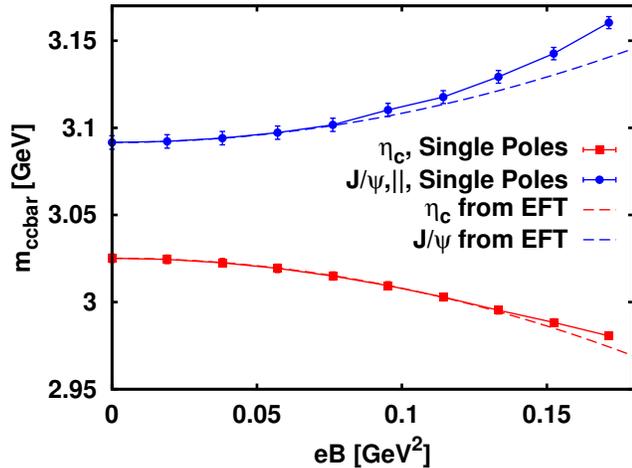


FIG. 1. Mass of the charmonium states from the QCD sum rules (closed symbols with solid lines) and the effective Lagrangian (3) (dashed lines) as functions of eB .

continuum contribution $\theta(s - s_0)\text{Im}\tilde{\Pi}(s)$ up to $\mathcal{O}(\alpha_s)$ with s_0 being the effective threshold parameter. The B -dependent part $\mathcal{M}_{\text{ph}}^{\text{ext}}$ considered above is, by inserting the correlator (8) into Eq. (12), obtained as

$$\mathcal{M}_{\text{ph}}^{\text{ext}, \eta_c}(M^2) = f_0(eB)^2 \left[Q_c^2 \frac{a_0^4}{64} e^{-\frac{m_{\tilde{\chi}}^2}{M^2}} + \frac{g_{\text{FV}}^2}{M_-^4} \left(e^{-\frac{m_{\tilde{\chi}}^2}{M^2}} - e^{-\frac{m_{\tilde{\psi}}^2}{M^2}} + \frac{M^2}{M^2} e^{-\frac{m_{\tilde{\psi}}^2}{M^2}} \right) \right]. \quad (17)$$

The corresponding formula for J/ψ can be obtained by interchanging $m_{\tilde{\psi}}$ and $m_{\tilde{\chi}}$. Following from a sign flip in M_-^2 , we find that the double-pole contribution in the vector channel has the opposite sign to that of the last term in Eq. (17). Inserting these results into the Borel-transformed dispersion relation (13), the mass of the lowest-lying pole can be evaluated from an equation,

$$m_{\eta_c}^2(M^2) = -\frac{\partial}{\partial(1/M^2)} \ln[\mathcal{M}_{\text{OPE}} - \mathcal{M}_{\text{ph}}^{\text{cont}} - \mathcal{M}_{\text{ph}}^{\text{ext}}]. \quad (18)$$

We examine a stability of m^2 with respect to the M^2 -dependence, called the Borel curve, within a range of M^2 which satisfies two competing conditions, that is, less than 30% contribution from the dimension-4 operators to the OPE and more than 70% lowest-pole dominance in the dispersion integral (12), specifying the Borel window. The effective threshold parameter s_0 is so tuned to make the Borel curve the least sensitive to M^2 . Finally, we average the value of the mass over the Borel window and calculate the variance to estimate a systematic error. See Ref. [31] for the details of the systematic framework.

With $\alpha_s(8m_c^2) = 0.24$, $m_c(p^2 = -2m_c^2) = 1.26$ GeV and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.35 \text{ GeV})^4$, the vacuum mass of J/ψ and η_c are found to be 3.092 GeV and 3.025 GeV, respectively. To compare results from the QCDSR with those from the effective Lagrangian (1), we insert these vacuum masses into $m_{\tilde{\psi}, \tilde{\chi}}$ in Eq. (3). The effective coupling

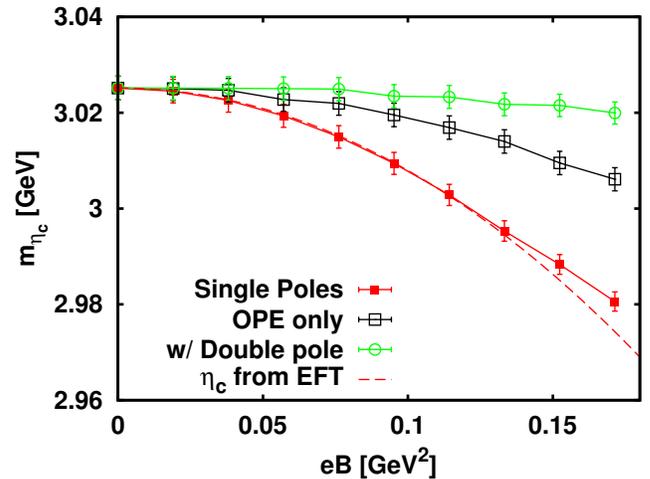


FIG. 2. Mass of η_c from the QCD sum rule with different implementations of the phenomenological side.

g_{FV} obtained above is used in both approaches. Figure 1 displays the results from the QCDSR with the phenomenological side shown on the rhs in Eq. (13), but without including the double-pole term responsible for the mixing effect in $\mathcal{M}_{\text{ph}}^{\text{ext}}$ [see Eq. (17)]. Remarkably, one finds perfect agreement between the two approaches in $eB < 0.1 \text{ GeV}^2$, followed by a slight deviation as eB is further increased. The agreement indicates that the B -dependent terms in Eq. (17) are essential ingredients to obtain physically meaningful results in QCDSR, where the level repulsion is understood as a consequence of the different signs of the single-pole terms in Eq. (17), $e^{-m_{\tilde{\chi}}^2/M^2} < e^{-m_{\tilde{\psi}}^2/M^2}$, owing to the vacuum mass difference.

In order to understand the role of each term in $\mathcal{M}_{\text{ph}}^{\text{ext}}$, we perform the QCDSR analyses in two cases employing the phenomenological sides without $\mathcal{M}_{\text{ph}}^{\text{ext}}$ and with all the terms of $\mathcal{M}_{\text{ph}}^{\text{ext}}$ including the double-pole term. In Fig. 2, we show results for the η_c in these two cases with open symbols. Without any B -induced poles, one obtains the open squares (“OPE only”), which show somewhat heavier mass than the final results obtained by including the single poles (filled squares in Figs. 1 and 2). Since the conventional one-peak spectral ansatz cannot account for the occurrence of the J/ψ pole induced by B -fields, the resultant η_c mass is an average of η_c and J/ψ , giving the artificially heavier η_c mass. On the other hand, if one includes all the terms of $\mathcal{M}_{\text{ph}}^{\text{ext}}$, the η_c mass becomes almost constant, despite the fact that the magnetic field contribution is included in the OPE. This means that the double-pole term on the phenomenological side in Eq. (17) almost exclusively accounts for the B -dependence on the OPE side. The residual mass shift, albeit tiny for the η_c , is an effect that cannot be explained by the mixing effect.

In conclusion, we have discussed effects of strong mag-

netic fields on the mass spectra of η_c and J/ψ with an elaborate treatment of the mixing effects on the phenomenological side in the QCDSR. We found that the mass shifts are dominated by the level repulsion coming from the mixing effect in precise agreement with those from the effective Lagrangian approach. While the residual mass shift is found to be small for the charmonia, our analysis indicates that, to obtain the correct results, one has to take into account effects of the magnetic fields on the phenomenological side as well as the OPE side. Therefore, a similar approach should be adopted when investigating light mesons by QCDSR or even any other systems involving the spectral density by means of the correlation functions in constant magnetic fields.

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