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# Imperfect World of $\beta\beta$ -decay Nuclear Data Sets

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The precision of double-beta ( $\beta\beta$ ) decay experimental half lives and their uncertainties is re-analyzed. The method of Benford's distributions has been applied to nuclear reaction, structure and decay data sets. First-digit distribution trend for  $\beta\beta$ -decay  $T_{1/2}^{2\nu}$  is consistent with large nuclear reaction and structure data sets and provides validation of experimental half lives. A complementary analysis of the decay uncertainties indicates deficiencies due to small size of statistical samples, and incomplete collection of experimental information. Further experimental and theoretical efforts would lead toward more precise values of  $\beta\beta$ -decay half lives and nuclear matrix elements.

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Double-beta decay was proposed by M. Goeppert-Mayer [1] as a nuclear disintegration with simultaneous emission of two electrons and two neutrinos

$$(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\bar{\nu}_e. \quad (1)$$

There are several double-beta decay processes:  $2\beta^-$ ,  $2\beta^+$ ,  $\epsilon\beta^+$ ,  $2\epsilon$  and possible decay modes: two-neutrino ( $2\nu$ ), neutrinoless ( $0\nu$ ) and Majoron emission ( $\chi^0$ )

$$(Z, A) \rightarrow (Z \pm 2, A) + (2e^\pm) + (2\bar{\nu}_e, 2\nu_e \text{ or } \chi^0). \quad (2)$$

The  $\beta\beta$ -process has been extensively investigated in the last 80 years [2, 3]. These efforts have led to observations of the two-neutrino decay mode and deduction of decay half lives. It is the rarest presently-observed nuclear decay. In a recent analysis of  $\beta\beta$ -decay data, A.S. Barabash [4] claimed that we could deduce precise values of experimental half lives and extract the corresponding nuclear matrix elements (NME). Unfortunately, these claims are premature and not very beneficial for the field. The main goal of this work is to investigate the present status of  $\beta\beta$ -decay research, reanalyze the data, and produce realistic assessments and recommendations that could accelerate overall progress.

The  $\beta\beta$ -decay  $T_{1/2}^{2\nu}$  are available from multiple sources [4, 5]. A brief summary of the recent National Nuclear Data Center (NNDC) evaluated or adopted values is shown in Table I and plotted in the lower part of Fig. 1. This plot contains 12 adopted half lives for nuclei of practical interest. The NNDC evaluation is completely based on standard U.S. Nuclear Data Program procedures, and its validity has been extensively scrutinized using theoretical predictions and nuclear systematics arguments [5].

To gain a complementary insight on a quality of these results, I will resort to a non-traditional approach and apply Benford's Law (BL) [6]. A data set follows BL if the leading digit  $d$  ( $d \in \{1, \dots, 9\}$ ) probability

$$P(d) = \log_{10}\left(1 + \frac{1}{d}\right). \quad (3)$$

TABLE I: Adopted  $\beta\beta$ -decay  $T_{1/2}^{2\nu}$  for  $0^+ \rightarrow 0^+$  transitions. Data are taken from the Ref. [5].

Parent nuclide	$T_{1/2}^{2\nu}$ (y)	$T_{1/2}^{2\nu+0\nu+\chi^0}$ (y)
<sup>48</sup> Ca	$(4.39 \pm 0.58) \times 10^{19}$	
<sup>76</sup> Ge	$(1.43 \pm 0.53) \times 10^{21}$	
<sup>82</sup> Se	$(9.19 \pm 0.76) \times 10^{19}$	
<sup>96</sup> Zr	$(2.16 \pm 0.26) \times 10^{19}$	
<sup>100</sup> Mo	$(6.98 \pm 0.44) \times 10^{18}$	
<sup>116</sup> Cd	$(2.89 \pm 0.25) \times 10^{19}$	
<sup>128</sup> Te		$(3.49 \pm 1.99) \times 10^{24}$
<sup>130</sup> Te	$(7.14 \pm 1.04) \times 10^{20}$	
<sup>136</sup> Xe	$(2.34 \pm 0.13) \times 10^{21}$	
<sup>130</sup> Ba		$(1.40 \pm 0.80) \times 10^{21}$
<sup>150</sup> Nd	$(8.37 \pm 0.45) \times 10^{18}$	
<sup>238</sup> U		$(2.00 \pm 0.60) \times 10^{21}$

Benford's or First-Digit law describes the frequency distribution of digits in real-life sources of data [6]. It works well for a wide variety of data sets including physical and mathematical constants.

In the current analysis, I will examine the leading digit distributions of nuclear reaction, structure and decay data sets that are shown in Fig. 1 and discuss in detail the  $\beta\beta$ -decay data sets, due to their importance in understanding fundamental symmetries and relatively small size. Visual inspection indicates that law works with a wide spectrum of nuclear physics quantities for large data sets and correctly reproduces the spectrum shape. This is its first application for evaluated thermal neutron capture cross section,  $B(E2)\uparrow$ , and  $\beta\beta$ -decay values [5, 7, 8], respectively.

The law provides an important verification tool for nuclear data analysis and horizontal nuclear data evaluations (evaluation of a specific quantity across the nuclear chart) because the undistorted central values of nuclear physics quantities should closely follow it. Any significant deviation from the law on large statistical samples may indicate that distortion has been introduced and evaluation procedures or experimental measurements have to be revisited. At the same time, not all nuclear data sets

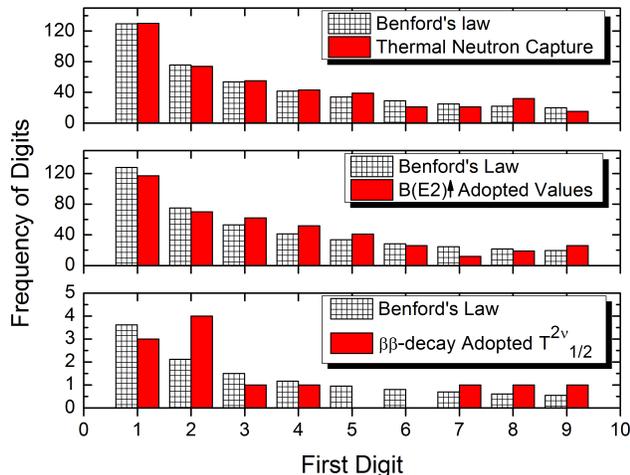


FIG. 1: Benford's distributions for adopted values of thermal neutron capture cross sections,  $B(E2)^\dagger$  and  $\beta\beta$ -decay  $T_{1/2}^{2\nu}$  data sets [5, 7, 8].

TABLE II: Upper-tail critical values of  $\chi^2$  distribution with  $\nu$  degrees of freedom [9]. This table content is courtesy of the National Institute of Standards and Technology (NIST).

$\nu$	Probability less than the critical value				
	0.90	0.95	0.975	0.99	0.999
8	13.362	15.507	17.535	20.090	26.125

would necessarily follow the law, for example constant-step tabulated cross sections or multiple measurements of a nuclear physics quantity for the same nucleus.

Figure 1 illustrates that relatively-large statistical samples for thermal neutron capture cross sections and quadrupole collectivities of even-even nuclei of 430 and 425 nuclei [7, 8], respectively, are in agreement with the BL. To extend the analysis further, I will consider a Pearson's cumulative test statistic, which asymptotically approaches a  $\chi^2$  distribution. For nuclear reaction and structure data sets, the cumulative values are 9.37 and 16.35, respectively, and the Benford's distribution number of degrees of freedom is equal to 8. These cumulative values could be compared with upper-tail critical values of  $\chi^2$  distribution [9], as shown in Table II.

The relatively high cumulative for the adopted  $B(E2)^\dagger$  values indicates the evaluation is slightly distorted. This distortion has been introduced by the quadrupole collectivity measurements in exotic even-even nuclei. These measurements are often unique and, occasionally, model dependent. Further research and additional measurements would improve the quality and reduce the corresponding cumulative values. A similar analysis for the adopted values of  $\beta\beta$ -decay  $T_{1/2}^{2\nu}$  [5] is not possible because one is supposed to have at least 5 counts in each of

the nine bins [10]. Therefore, I will apply the method of visual inspection to the lower part of Fig. 1. The distribution shape indicates that evaluation is consistent with actual data and initial deviation from the law is due to a small sample size. In the hypothetical case of a large statistical sample, one could deduce that half lives of  $^{96}\text{Zr}$ ,  $^{116}\text{Cd}$ ,  $^{136}\text{Xe}$  and  $^{238}\text{U}$  have to be reexamined.

From the dawn of the  $\beta\beta$ -decay era researchers knew the importance of complete experiments when both released energy and angular distribution of decay products have been recorded. The first direct observation of two-neutrino decay mode in  $^{82}\text{Se}$  [11] and subsequent observations in  $^{150}\text{Nd}$  and  $^{48}\text{Ca}$  [12, 13] were made using such techniques. These discoveries employed the time projection chambers (TPC) that contained small amounts of target material and, consequently, generated limited statistics. The experiments were very complex; however, observation of two-electron events has provided clear evidence of the double-beta decay process.

Further developments have progressed using advanced commercially-available detectors [14, 15] and large quantities of enriched isotopes. Unfortunately, the usage of commercial detectors often leads to incomplete experiments when only energy release information has been collected. To illustrate this concept, we should consider  $\beta\beta$ -decay search in  $^{76}\text{Ge}$ . The lower part of Fig. 2 depicts a chronological record of the measurements [14]. The different half life values with large uncertainties for two-neutrino mode of decay are often explained by the relatively high background in the earlier experiments. These experiments were based on an essentially-single source of enriched  $^{76}\text{Ge}$  isotope and slightly different detector fabrication and shielding technologies. All of these measurements relied heavily on an excellent energy resolution of Ge detectors and suffered from the lack of electron tracking information. Consequently, a final  $\beta\beta$ -decay spectrum could contain the contribution of single-electron events. High energy resolution is absolutely essential for observations of neutrinoless of  $\beta\beta$ -decay, when decay will produce a sharp peak corresponding to a Q-value between parent and daughter nuclei. Unfortunately, the quantum world is very diverse, and background processes may affect the results. It has been demonstrated recently that  $\gamma$ -ray transitions in Pb and  $^{76}\text{Ge}$  could produce a 2039 keV signal and obscure the decay signature [18, 19].

Electron tracking information is crucial in  $\beta\beta$ -decay research. An observation of two-electron events in addition to energy release information would help to suppress the radioactive background and decisively prove  $\beta\beta$ -decay process in a particular nucleus. Perhaps, in addition to HPGe technologies, it could be interesting to remeasure  $^{76}\text{Ge}$   $T_{1/2}^{2\nu}$  in a NEMO experiment that is based on a more extensive set of observables [20]. Similar scenarios have been developing with  $\beta\beta$ -decay search in  $^{136}\text{Xe}$  [21–23].

In light of this disclosure, it becomes clear that excel-

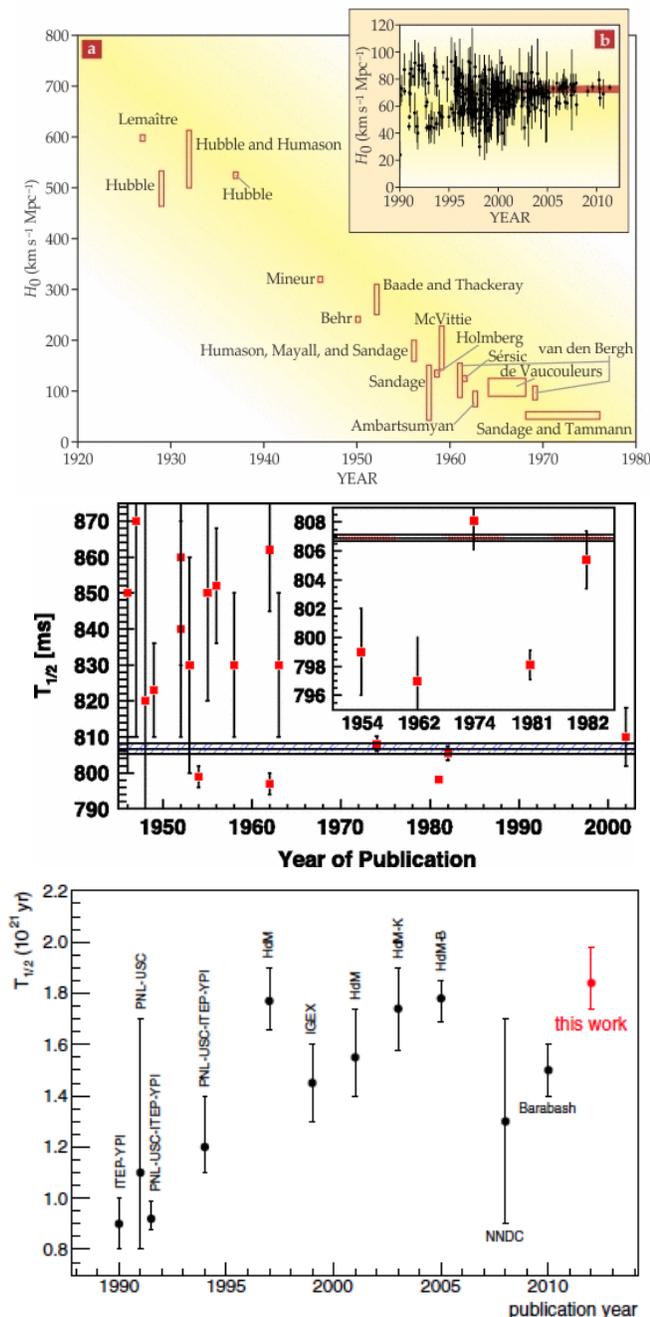


FIG. 2: Chronological record of three different groups of measurements: Hubble constant [16],  $T_{1/2}$  of  ${}^6\text{He}(\beta)$  [17] and  ${}^{76}\text{Ge}^{2\nu}(\beta\beta)$  [14]. These graphs are courtesy of the cited publications.

lent energy resolution of HPGe detectors alone may not be sufficient for an observation of neutrinoless mode in  ${}^{76}\text{Ge}$ . Therefore, the present day state-of-the-art GERDA and Majorana experiments [14, 15] may not be able to provide an ultimate answer due to the incomplete nature of the data recording.

Visual inspection of the lower part of Fig. 2 shows that the “precise”  $T_{1/2}^{2\nu}$  value of Barabash [4] strongly contradicts the latest result of the GERDA experiment (depicted as “this work” in the Figure) [14]. This example shows that it is too early, at this point, to state a high precision of adopted half lives and NME, and large error bars of the previous NNDC evaluation [24] are more appropriate. All NNDC values were produced from the experimental half lives when new measurements would trigger the data reevaluation every 5-7 years.

A complementary analysis of the upper and lower parts of Fig. 2 shows that the presently-discussed situation with  $\beta\beta$ -decay measurements is not unique in physics. It is rather a common occurrence when initial, pioneering measurements are not very accurate and often discrepant. It definitely has happened to Hubble constant and  ${}^6\text{He}$   $\beta$ -decay experiments [16, 17], as shown in the upper and middle part of Fig. 2. This figure content is based on original graphs borrowed from the Refs. [14, 16, 17]. Other recent cases of inconsistent data include the discrepant decay scheme and half-life of  ${}^{139}\text{Ba}$  [25], and cross section values. In the recent review of neutron cross section deficiencies, M.B. Chadwick [26] compiles an impressive list that includes  ${}^{235}\text{U}$  and  ${}^{197}\text{Au}$  fast neutron capture cross sections. The last cross section value was used in calibration of the stellar nucleosynthesis KaDO-NiS database [27] and has a broad impact across neutron physics. We are surrounded by a large number of imperfect nuclear data sets and constantly work on their improvement.

The experimental  $\beta\beta(2\nu)$ -decay half-lives include contributions from the nuclear structure effects and decay kinematics.  $T_{1/2}^{2\nu}$  values are often described as follows

$$\frac{1}{T_{1/2}^{2\nu}(0^+ \rightarrow 0^+)} = G^{2\nu}(E, Z) |M_{GT}^{2\nu} - \frac{g_V^2}{g_A^2} M_F^{2\nu}|^2, \quad (4)$$

where the function  $G^{2\nu}(E, Z)$  results from lepton phase space integration and contains all the relevant constants [2]. Equation (4) highlights a direct dependency of experimental NME on the calculated values of phase space factors.

Both Barabash’s and NNDC’s evaluated NME were based on the best available phase space factor calculations in 2010 and 2013, respectively [2, 29, 30]. Table III shows the evolving values of  ${}^{76}\text{Ge}$  phase space factors, and the table data raise a question about calculation limitations and possible model dependency of the PSFs. The observed discrepancies between recent calculations of Kotila & Iachello [30] and Stoica & Mirea [31] create reason for concern. The exact values of phase space factors are needed in order to deduce the precise values of experimental NME for comparison with recent theoretical calculations of Senkov and Horoi [32]. It will be highly beneficial for the field if a third group of theorists will clarify the situation. The formula (4) gets even

TABLE III:  $^{76}\text{Ge}$   $\beta\beta$ -decay phase space factor (PSF) values in  $10^{-21} \text{ yr}^{-1}$  for  $0^+ \rightarrow 0^+$  transition.

Authors	Year	PSF
Boem & Vogel [2]	1987	130.54
Doi <i>et al.</i> [28]	1993	53.8
Suhonen & Civitarese [29]	1998	52.6
Kotila & Iachello [30]	2012	48.17
Stoica & Mirea [31]	2013	43.9

more complex for the neutrinoless mode where half life also depends on a neutrino mass, and neither neutrino mass or NME can be easily disentangled.

Finally, a novel method of Benford's distribution analysis has been extended to nuclear reaction, structure and decay data sets. BL is a simple and very powerful tool for discovering and revisiting the "outlier" data points. A re-analysis of the two-neutrino mode of  $\beta\beta$ -decay data sets indicates that central values of the adopted half lives are reasonably good; however, their uncertainties are high due to relatively small statistics and incomplete collection of decay information. We live in an era of  $\beta\beta$ -decay observations with large uncertainties. It may take years before future complete or "almost" complete experiments like low-pressure TPC [11–13] or simultaneous measurement of positrons and  $\gamma$ -rays [33] will improve the data. The precision of experimental NME strongly depends on the quality of phase space factors, and additional theoretical calculations are necessary to clarify the discrepant results.

Double-beta has been an exciting nuclear physics phenomenon that has slowly revealed its experimental signatures in the last 30 years. It may take many painstaking experimental and theoretical efforts before the process will be well understood and measured. It is not an unusual situation in a history of science; in fact, it is a very common case. Therefore, one can assume that it may take another 30 years before all decay modes, experimental half lives and NME values will be finalized.

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