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March 4-6, 2015

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in 1997 at Brookhaven National Laboratory.* RBRC is funded by "Rikagaku Kenkyosho" (RIKEN, The Institute of Physical and Chemical Research) in Japan and the United States Department of Energy's Office of Science.

A Memorandum of Understanding between RIKEN and BNL, initiated in 1997, has been renewed in 2002, 2007 and 2012.

RBRC is dedicated to the study of strong interactions, including spin physics, lattice QCD and relativistic heavy ion physics through the nurturing of new generations of young physicists. The RBRC founding Director T.D. Lee and the second Director N. P. Samios conceived and implemented this vision, which has been maintained and further developed down to the present day.

The RBRC research program has theory, lattice gauge computing and high-energy experimental nuclear physics components. Recently, an astrophysics/cosmology component has been added. The RBRC Theory, Computing and Experimental Groups presently comprise 48 researchers. Positions include full-time RBRC Fellows, half-time joint RHIC Physics Fellows and full-time postdoctoral Research Associates. The RHIC Physics Fellows hold joint appointments with RHIC and other institutions, where they have tenure track positions. To date, RBRC has over 101 graduates (Fellows and Research Associates) of whom approximately 67 have already attained tenure at major research institutions worldwide.

In 2001 a RIKEN Spin Program (RSP) was initiated at RBRC. The research staff comprises joint appointments in theory and experiment between RBRC and RIKEN, including RSP Researchers, RSP Research Associates and Young Researchers. They are mentored by senior RBRC Scientists. A number of RIKEN junior Research Associates and Visiting Scientists also contribute to the program.

In support of the lattice gauge program at RBRC and elsewhere, a series of high-performance computers has been designed and built by researchers from Columbia University, IBM, BNL, RBRC and University of Edinburgh, with the U.S. DOE Office of Science providing infrastructure support at BNL. To date, the steps in this program have been: QCDSP (0.6 TFlops, 1998-2006), which was awarded the Gordon Bell Prize for price performance in 1998; QCDOC (10 TFlops, 2005-2012); QCDCQ (600 TFlops, 2011-present). Recent $K^{\circ} \rightarrow \pi\pi$ results were awarded the Ken Wilson Prize in 2012.

A very important activity of RBRC is its active Workshop series on Strong Interaction Physics, with each workshop focused on a specific physics problem. A list of proceedings of all past Workshops can be found on the RBRC website (http://www.bnl.gov/riken/proceedings.php). The talks from many of the recent workshops can be accessed from the link at the top of the Proceedings page. To date, about 119 Workshops have taken place; the full proceedings of most of the workshops from 2005 – 2014 are available at this link.

S. H. Aronson, Director
March 2015

* Work Performed under the auspices of U.S.D.O.E. Contract No. DE-SC0012704

Introduction

The particle angular correlation measurements in small colliding systems, such as p+p, p+Pb, d+Au and ^{3}He+Au at RHIC and the LHC, have recently attracted significant interest. In particular, high-multiplicity events from such collisions exhibit a very similar pattern of long-rapidity-range azimuthal correlations (the "ridge") as in Au+Au and Pb+Pb collisions. In AA collisions it is widely accepted that the ridge reflects various harmonic flow components arising from strong hydrodynamic response to fluctuating initial conditions. Hence, it is natural to ask whether the ridge observed in small colliding systems may also be interpreted as hydrodynamic flow plus fluctuating initial conditions.

Although such a hydro-based interpretation can be made to work phenomenologically, it is unclear whether the hydrodynamic gradient expansion approach developed for A+A collisions is applicable in such small systems. Besides, there are alternative proposals in explaining the observation based on initial state effects (e.g. early gluon dynamics in a saturation picture).

More recently, multi-particle cumulants measurements (as reported at QM14) have shown strong evidence for collectivity in high multiplicity p+Pb collisions, but the exact physical meaning of these measurements needs to be clarified. This Workshop specifically aimed to have focused discussions on these issues, and hopefully progress will be made in understanding them.

Specific topics which were addressed included:

- **Meaning of experimental measurements:** What are the precise meanings of cumulants measurements? Whether/how do they unambiguously prove collectivity? What are the effects of potential event centrality selection bias, as well as the roles of jets and other short range correlations?

- **Meaning and origin of collectivity:** What are the possible explanations for observed ridge correlations? How well do they explain the features in the data? What are the possible hydrodynamic as well as non-hydrodynamic origins of collectivity?

- **Validity of hydrodynamics:** Theoretically, what is the boundary for the applicability of current viscous hydrodynamic modeling? Suppose hydrodynamics could indeed work for such small colliding systems, whether/how can we understand it in the usual hydrodynamic gradient expansion framework? What are the roles of hydrodynamic fluctuations which become significant in small systems?

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Gabriel Denicol..... *Knudsen numbers in small colliding systems*
Kevin Dusling ..... *Ridge from Glasma production in p+p and p+A collisions*
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Additional RIKEN BNL Research Center Proceeding Volumes

Presentations

Multiparticle methods for measuring anisotropic flow

From large to small systems

Nicolas BORGHINI

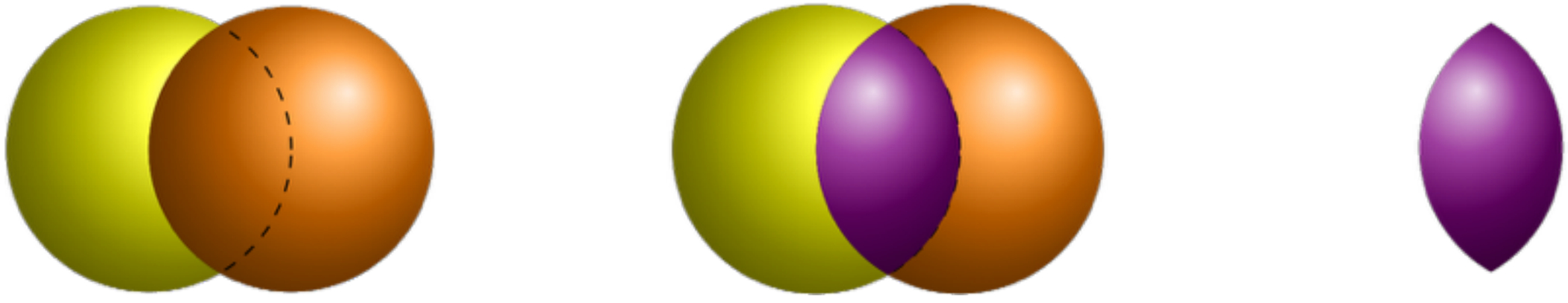
Universität Bielefeld

Collectivity in Small Colliding Systems, BNL, March 4-6, 2015

Preamble: Anisotropic (collective) flow

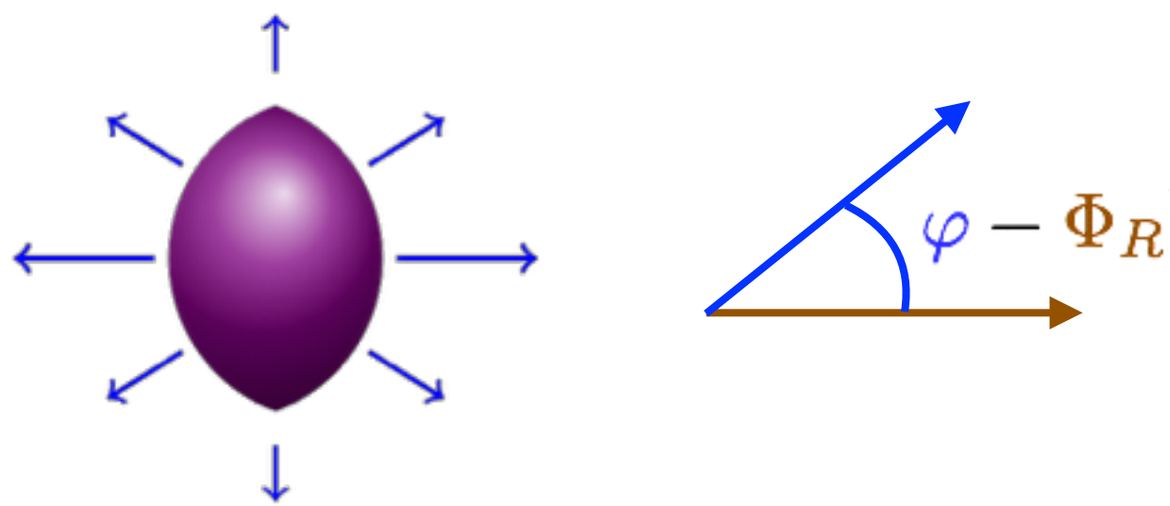
Classical picture (before ca.2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$t < 0$ $\quad$ $t = 0$ $\quad$ $t = 0^+$

$\Rightarrow$ anisotropic emission of particles



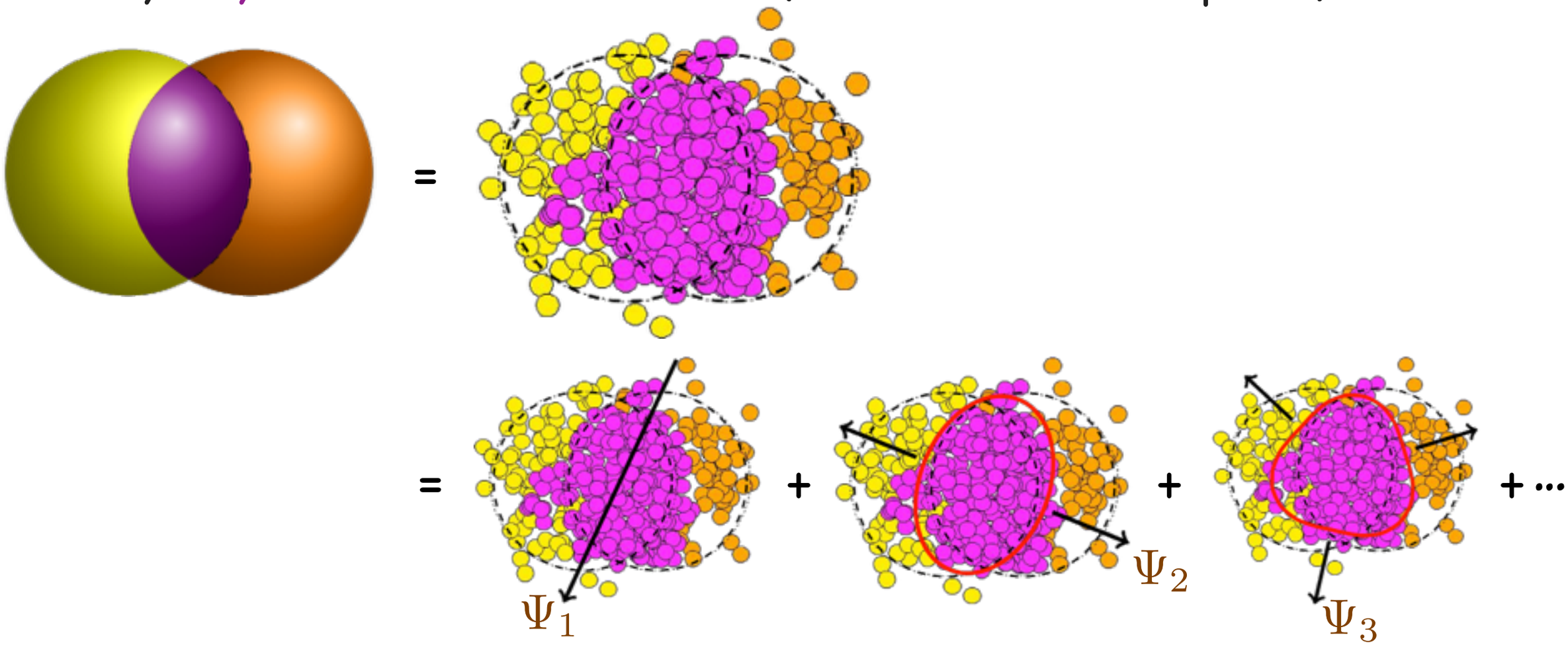
$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T \mathrm{d}p_T \mathrm{d}y}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic (collective) flow

Newer picture (since 2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



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$\Rightarrow$ anisotropic emission of particles

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T \mathrm{d}p_T \mathrm{d}y}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\varphi - \Psi_n)\right]$$

Preamble: Anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T \mathrm{d}p_T \mathrm{d}y}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\varphi - \Psi_n)\right]$$

v_n: Fourier coefficients of the single-particle distribution;

Ψ_n: n-th harmonic "event plane"

$$v_n \equiv \langle \cos n(\varphi - \Psi_n) \rangle$$

☞ even at (mathematically) fixed impact parameter, v_n & Ψ_n vary from event to event!

☛ initial conditions & system evolution

Goal: measure the v_n coefficients (in a second step: as a function of p_T, y)

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Cumulants, Lee-Yang zeroes: a reminder
  - old motivations & applications
  - newer uses
- Application to smaller(?) systems
  - overview of results
  - caveats from the theory side

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Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① Using the measured azimuths of the emitted particles, build appropriate correlators

For instance $\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle$, $\left\langle e^{in(\varphi_i+\varphi_j-\varphi_k-\varphi_l)}\right\rangle$ ...

brackets? to be discussed later

② Equate with the (theoretical) value of these correlators for events with anisotropic flow only

☛ yields flow estimate(s)

For instance $\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle = (v_n)^2$ <u>if only flow in the system</u> ☛ $v_n\{2\}$

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If only flow was present, all correlators would be equal.

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For instance $\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle$, $\left\langle e^{in(\varphi_i+\varphi_j-\varphi_k-\varphi_l)}\right\rangle$ ...

If only flow was present, all correlators would be equal.

Due to the presence of additional sources of interparticle correlations ("nonflow") in the system, some correlators are more equal than the others.

☛ Cumulants, in which the impact of the other sources is minimized

e.g. $c_n\{4\} \equiv \left\langle e^{in(\varphi_i+\varphi_j-\varphi_k-\varphi_l)}\right\rangle - \left\langle e^{in(\varphi_i-\varphi_k)}\right\rangle\left\langle e^{in(\varphi_j-\varphi_l)}\right\rangle - \left\langle e^{in(\varphi_i-\varphi_l)}\right\rangle\left\langle e^{in(\varphi_j-\varphi_k)}\right\rangle$

Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① Using the measured $\frac{\text{azimuths of the emitted particles}}{\text{multiplicity } M}$, build appropriate correlators

☛ Old motivation of cumulants: minimize the bias from nonflow.

N.B., P.M.Dinh, J.-Y.Ollitrault, PRC **63** (2001) 054904, **64** (2001) 054901

Measuring anisotropic flow with (multiparticle) "cumulants"

Principle:

① U build appro

☛ Ol

54901

Nonflow effects?

- Quantum-statistics effects
- Resonance decays
- Momentum conservation
- (Mini)jets
- Strong & Coulomb interaction
- ...

k-particle correlator generally scales as

$$\mathcal{O}\left(\frac{1}{M^{k-1}}\right)$$

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② Equate with the theoretical value of the cumulants for events with anisotropic flow only:

$c_n\{2\} = (v_n)^2$, $c_n\{4\} = -(v_n)^4$, $c_n\{6\} = 4(v_n)^6$, $c_n\{8\} = -33(v_n)^8$ ...

☛ yields flow estimate $v_n\{k\}$ if $c_n\{k\} \gg \mathcal{O}\left(\frac{1}{M^{k-1}}\right)$

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N.B., P.M.Dinh, J.-Y.Ollitrault, PRC **63** (2001) 054904, **64** (2001) 054901

(AGS), SPS, early-RHIC era!

☛ yields flow estimate $v_n\{k\}$ if $v_n \gg \mathcal{O}\left(\frac{1}{M^{1-1/k}}\right)$

... and concern

Measuring anisotropic flow with "Lee-Yang zeroes"

Principle of the method:

① Using the measured azimuths of the emitted particles, build a generating function $G_n(z)$ (which generates multiparticle averages)

② Look for the position of the first zero of $G_n(z)$ (in practice, the first minimum of its modulus) ☞ z_0

③ Under the assumption of events with anisotropic flow only, deduce from z_0 an estimate of v_n:

$$v_n\{\infty\} \equiv \frac{j_{01}}{M z_0}$$

j_{01} = 2.40483...

R.S.Bhalerao, N.B., J.-Y.Ollitrault, NPA **727** (2003) 373

Measuring anisotropic flow with "Lee-Yang zeroes"

"Lee-Yang zeroes" shared the same original motivation as cumulants: minimize the bias from nonflow.

☛ yields flow estimate $v_n\{\infty\}$ if $v_n \gg \mathcal{O}\left(\frac{1}{M}\right)$

From a theorist's point of view, LYZ-method is more aesthetic, since it directly measures "many-body collectivity" in the system under study. (The position of the first zero controls the asymptotic behavior of cumulants)

Measuring anisotropic flow with multiparticle methods

Let us discuss a few assumptions & points which I left aside till now.

The meaning of angular brackets? represent an average!

- On the experimental side, over many (all) detected particles in an event (M), then over many events ($N_{\rm ev}$)...

☛ Both M and $N_{\rm ev}$ are finite, there will be statistical fluctuations!

☞ nightmarish appendices / sections in

PRC **63** (2001) 054904, PRC **64** (2001) 054901, & NPA **727** (2003) 373

☞ were much talked about in 2002-05 ("limitation of the methods") yet have gone missing since then thanks to large M and $N_{\rm ev}$.

Measuring anisotropic flow with multiparticle methods

- Average over M detected particles in $N_{\rm ev}$ events

  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$

  - for "large" $\chi \gtrsim 1$, life is easy, the statistical fluctuations decrease like $1/\sqrt{M\,N_{\rm ev}}$ both on the $v_n\{k\}$ and on $v_n\{\infty\}$

    ☞ corresponds to the regime at LHC & high RHIC energies in collisions of large systems (measured with reasonable detectors...)!

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  - when $\chi < 1$, things become more involved...

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The meaning of angular brackets? represent an average!

- On the experimental side, over many (all) detected particles in an event, then over many events...
- On the theoretical side — as hidden in the relations between flow and the cumulants or the position of the first zero:
  - first over the particles in an event, with an azimuthal distribution modulated by v_n(!);
  - then over events, assuming an isotropic distribution of Ψ_n and a constant v_n.

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can be corrected for if it only reflects the detector properties

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is it so obvious when there is physics at play?
(☛ engineered events)

Measuring anisotropic flow with multiparticle methods

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too idealistic!

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Cumulants, Lee-Yang zeroes: a reminder
  - old motivations & applications
  - newer uses
- Application to smaller(?) systems
  - overview of results
  - caveats from the theory side

Anisotropic flow fluctuations

... in an experimental centrality bin are unavoidable; and physical!

$\Rightarrow$ Introduce a probability distribution $p(v_n)$ at fixed Ψ_n.

☛ to first approximation, mean value $\langle v_n \rangle$ and standard deviation δv_n.

- if $\delta v_n \ll \langle v_n \rangle$ then $v_n\{2\} = \langle v_n \rangle + \delta v_n$, $v_n\{4\} = v_n\{6\} = v_n\{8\} = \langle v_n \rangle - \delta v_n$...

  ☞ use cumulants to estimate $\langle v_n \rangle$ and δv_n.

- if $\delta v_n \gtrsim \langle v_n \rangle$ then the above identities no longer hold

  ☞ argue that differences between higher-order cumulant estimates $v_n\{4\}$, $v_n\{6\}$, $v_n\{8\}$... reflect non-Gaussianities of $p(v_n)$.

more in Jiangyong's talk!

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☛ minimize nonflow

☛ statistics not an issue (2002-1?)

☛ reconstruct flow fluctuations

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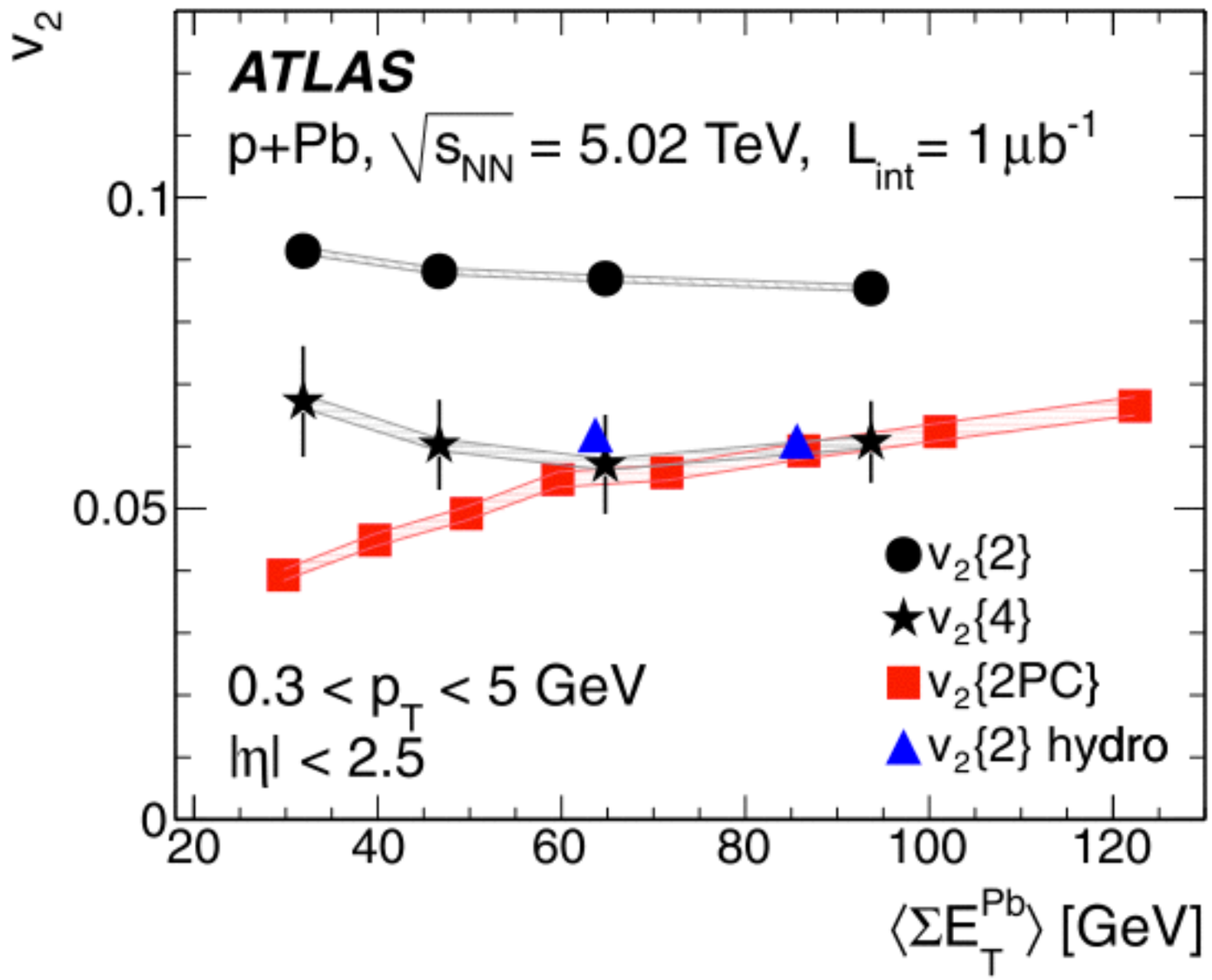
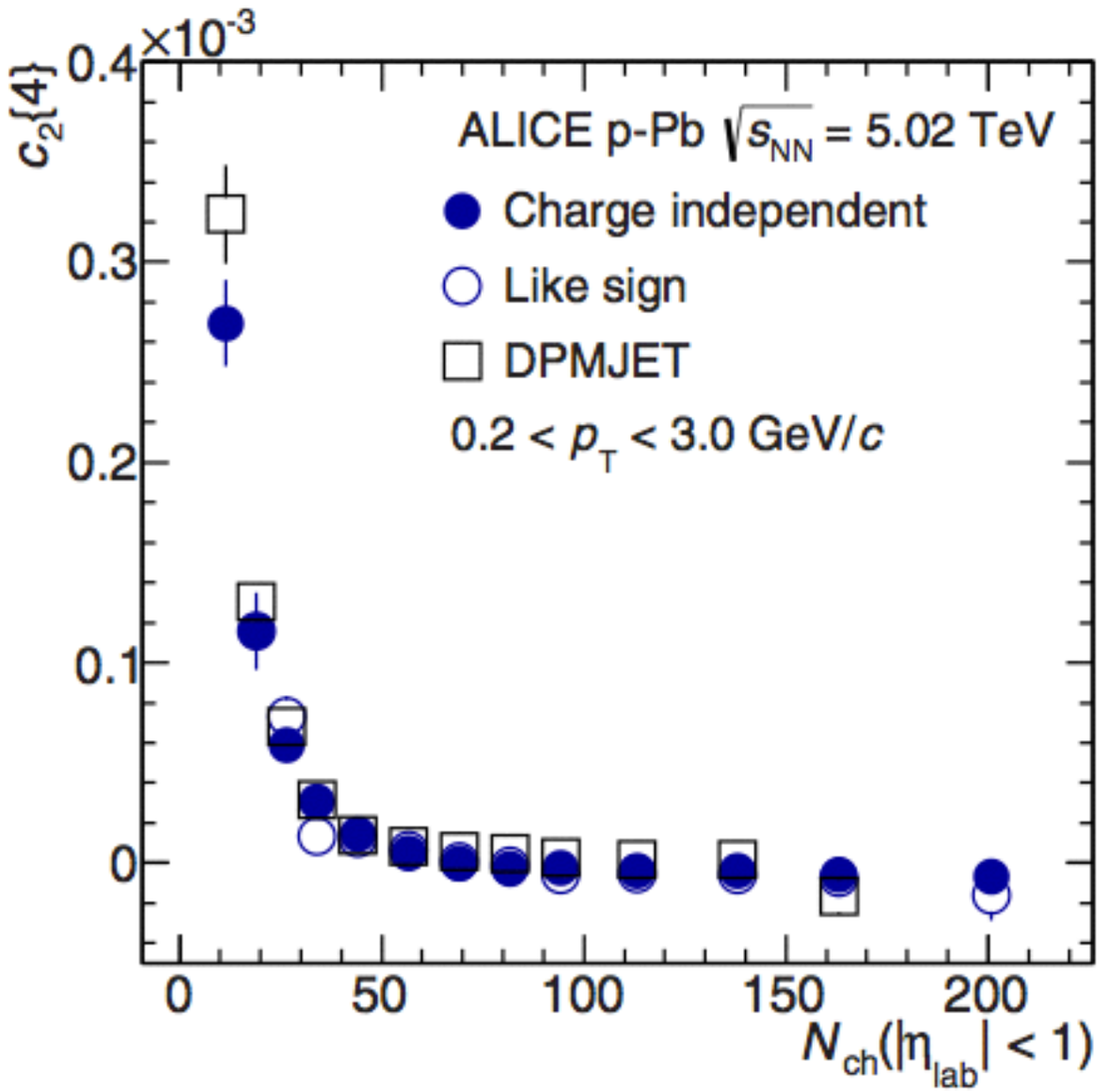
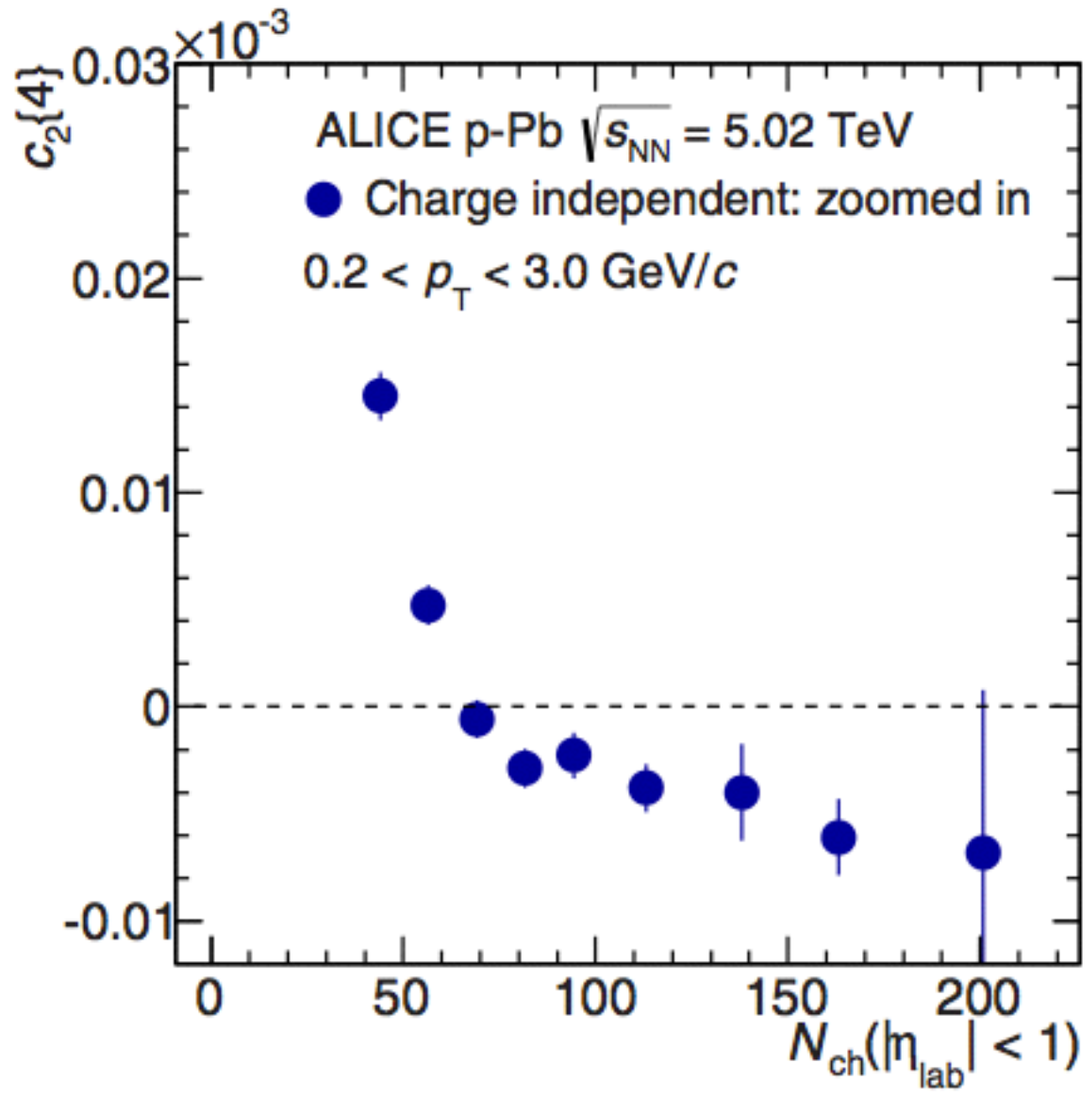
Multiparticle measurements of anisotropic flow in small systems



ATLAS Coll., Phys. Lett. B **725** (2013) 60

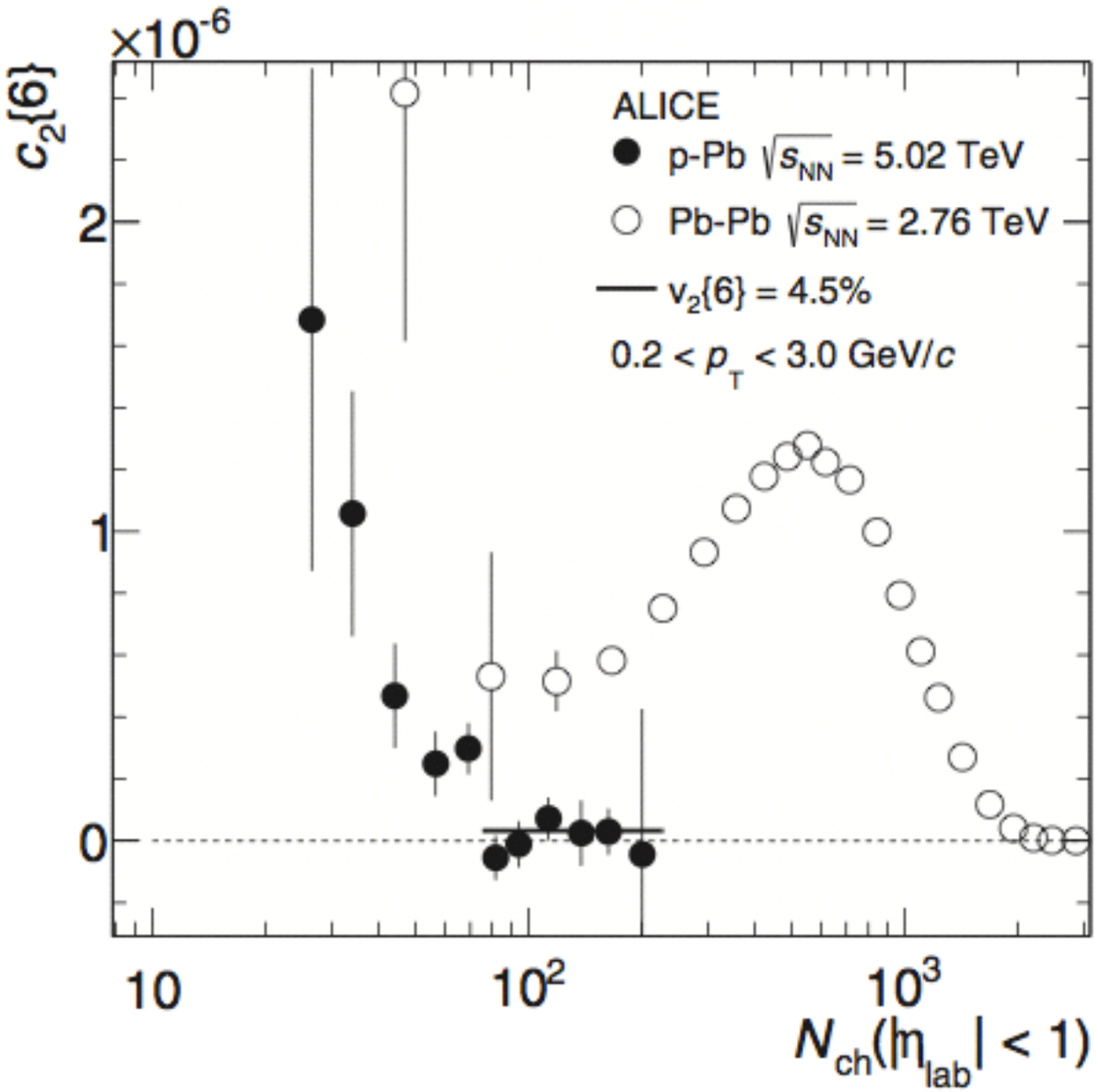
Multiparticle measurements of anisotropic flow in small systems

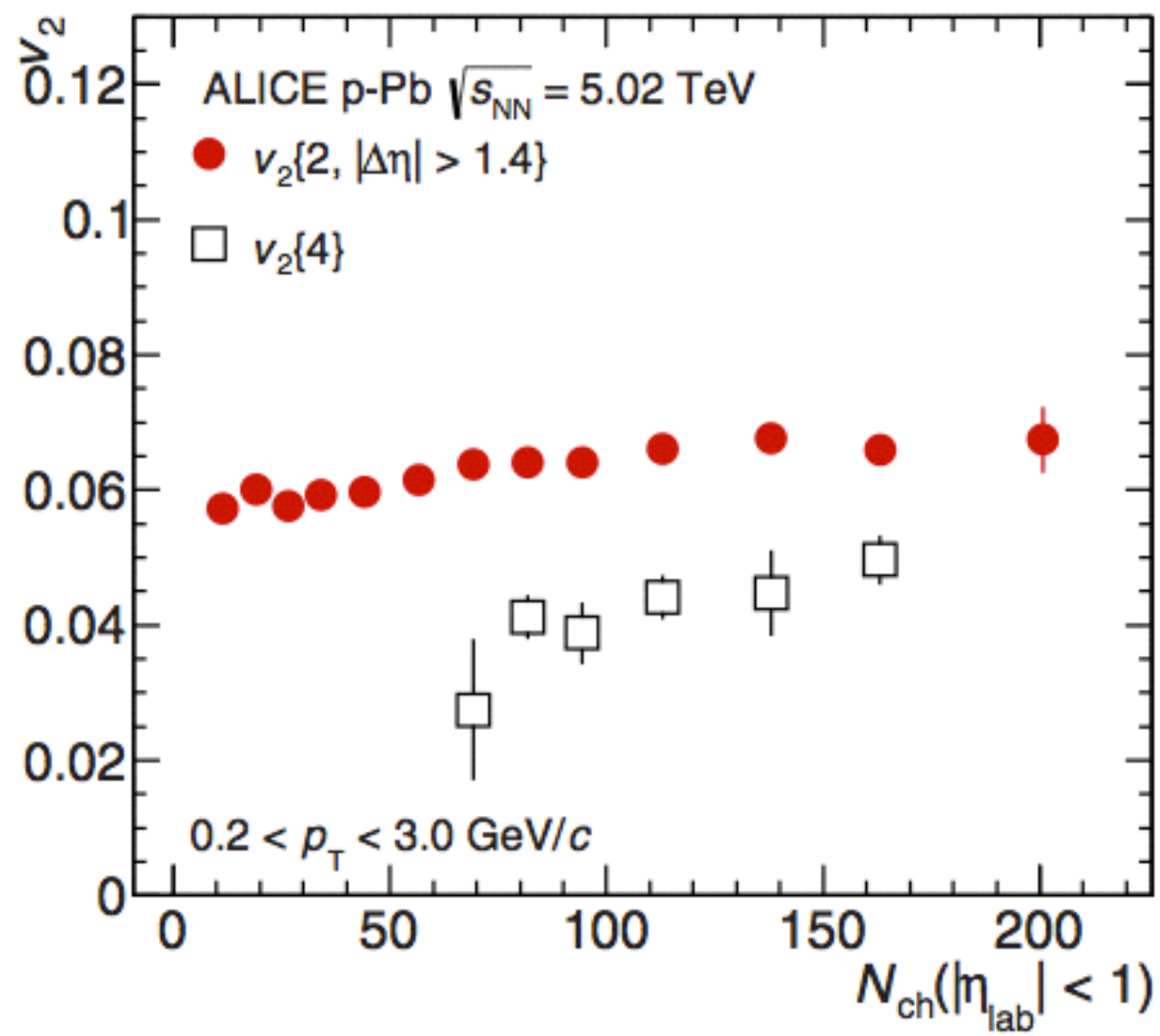




ALICE Coll., Phys. Rev. C **90** (2014) 054901

Multiparticle measurements of anisotropic flow in small systems





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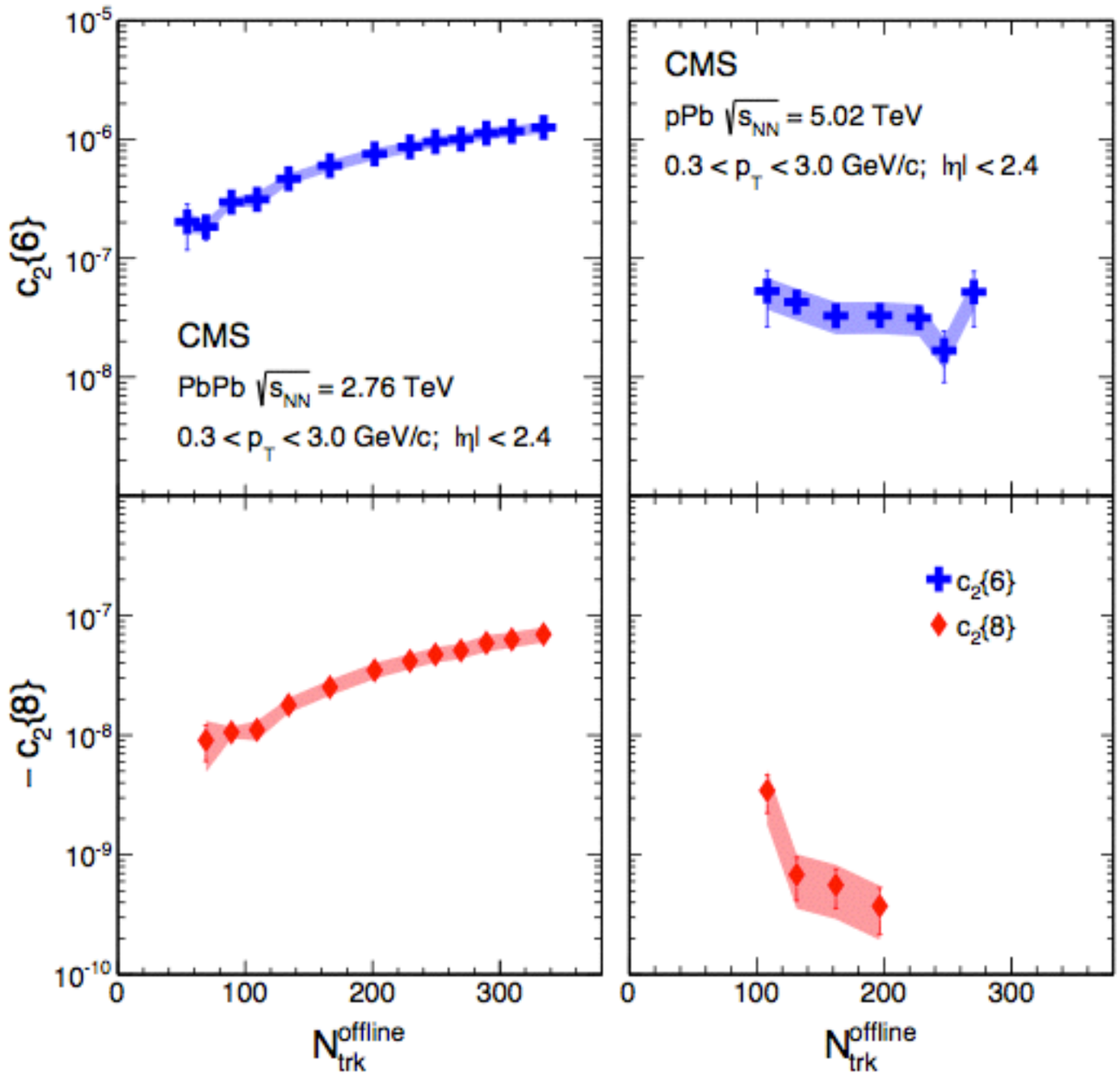
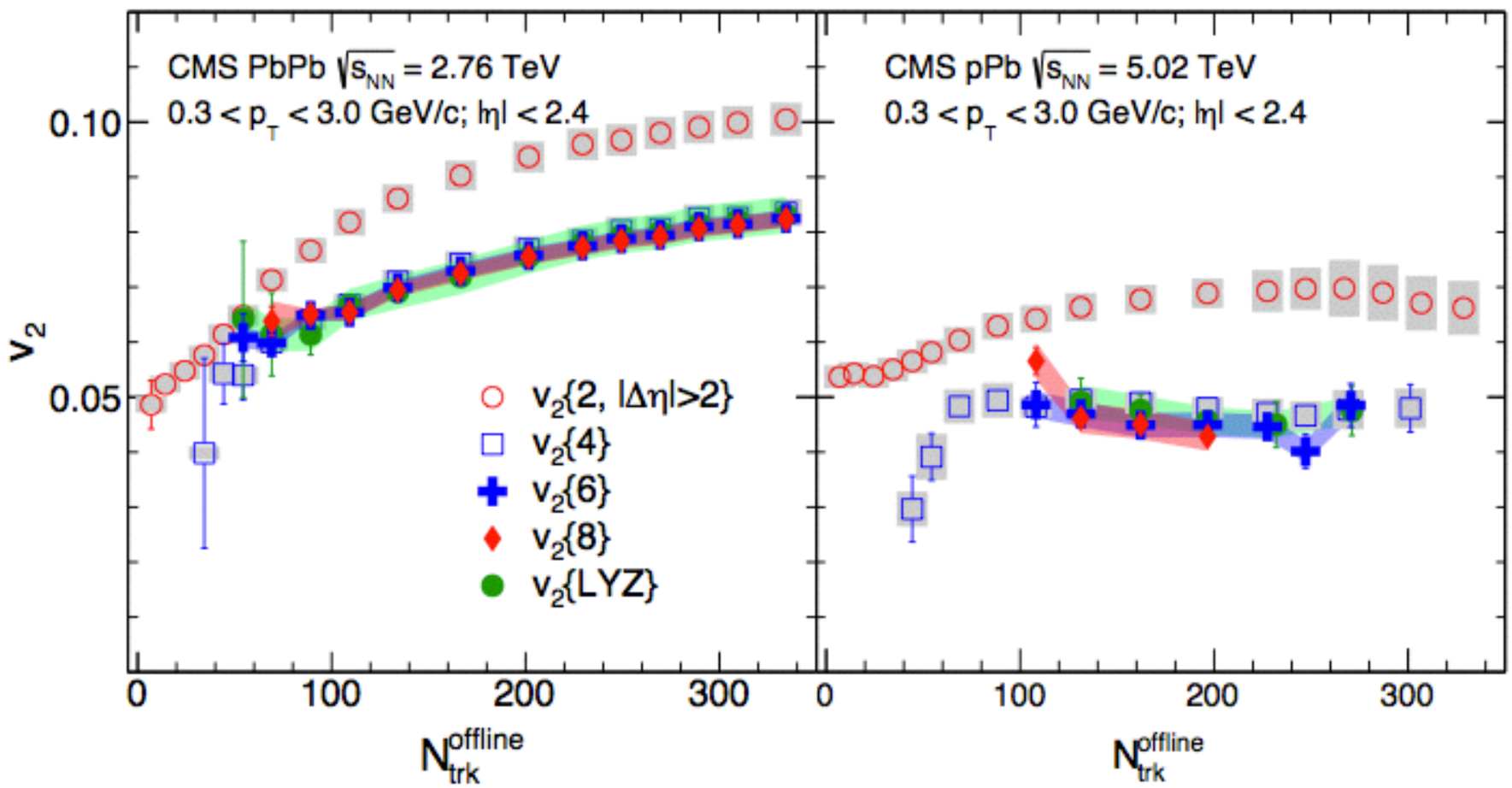


CMS Coll., arXiv:1502:05382

Multiparticle measurements of anisotropic flow in small systems



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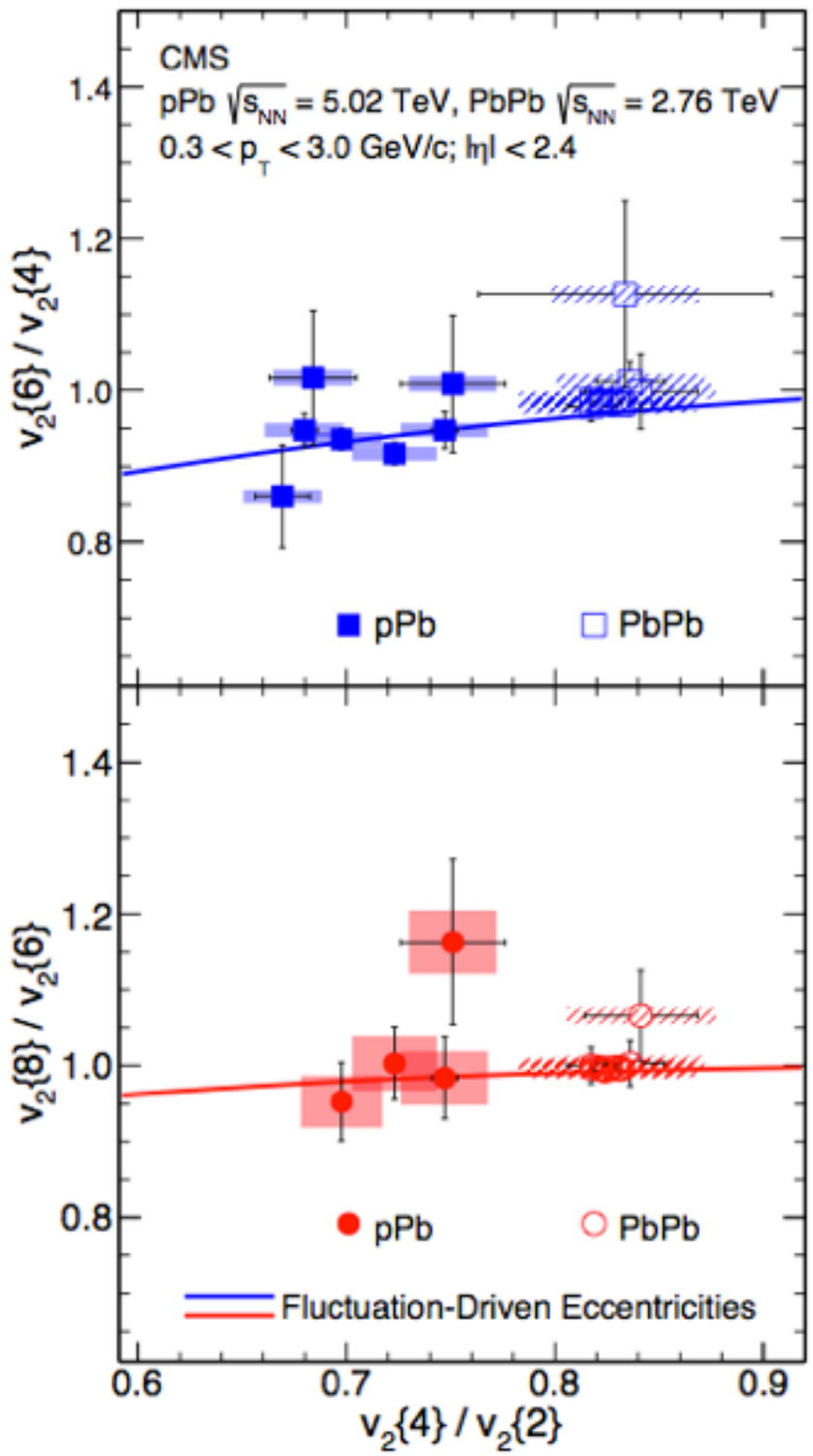

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- Average over M detected particles in $N_{\rm ev}$ events

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    ☞ corresponds to the regime at LHC & high RHIC energies in collisions of large systems (measured with reasonable detectors...)!

  - when $\chi < 1$, things become more involved...

    ☞ what does this mean for small systems?

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in N_{ev} events
  - ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$
  - when χ < 1, things become more involved...

Table 3 from NPA **727** (2003) 373 (page 411...)

Comparison between different methods: the relative statistical error on the integrated flow V_n is shown for the cumulant method [34], and various cumulant orders (denoted by $V_n\{2k\}$ with integer k), and for the present method ($V_n\{\infty\}$)

	$\chi = 0.6$	$\chi = 0.7$	$\chi = 0.8$	$\chi = 1$	$\chi = 1.5$
$\delta V_n\{2\}/V_n$	1.3%	1.0%	0.83%	0.62%	0.37%
$\delta V_n\{4\}/V_n$	4.5%	2.7%	1.8%	1.00%	0.43%
$\delta V_n\{6\}/V_n$	7.7%	3.7%	2.1%	0.99%	0.41%
$\delta V_n\{8\}/V_n$	9.9%	4.1%	2.1%	0.95%	0.41%
$\delta V_n\{\infty\}/V_n$	10.9%	3.9%	2.0%	0.94%	0.41%

As in the previous table, the number of events is $N_{\rm evts} = 20000$ events, and the resolution parameter χ takes several values.

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in N_{ev} events

  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$

  - when χ < 1, things become more involved:

    at small χ:

    - Gaussian errors on the cumulants $c_n\{2k\}$, with width $\dfrac{1}{\sqrt{M^{2k} N_{\text{ev}}}}$

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in $N_{\rm ev}$ events

  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$

  - when χ < 1, things become more involved:

    at small χ:

    - Gaussian errors on the cumulants $c_n\{2k\}$, with width $\dfrac{1}{\sqrt{M^{2k} N_{\rm ev}}}$

      ☞ measurement of v_n becomes more difficult;

      ☞ may spoil the attempts to pinpoint $p(v_n)$ (= the "physical" fluctuations of the measured signal).

Multiparticle methods for measuring anisotropic flow in small systems

- Average over M detected particles in N_{ev} events

  ☛ define the "resolution parameter" $\chi \equiv v_n \sqrt{M}$

  - when χ < 1, things become more involved:

    at small χ:

    - Gaussian errors on the cumulants $c_n\{2k\}$, with width $\dfrac{1}{\sqrt{M^{2k} N_{\text{ev}}}}$
    - error on $v_n\{\infty\}$ grows exponentially!

Multiparticle methods for measuring anisotropic flow in small systems

An over-simplified rule of thumb, which emphasizes the role of χ would be the following:

from NPA **727** (2003) 373 (LYZ-method page 385)

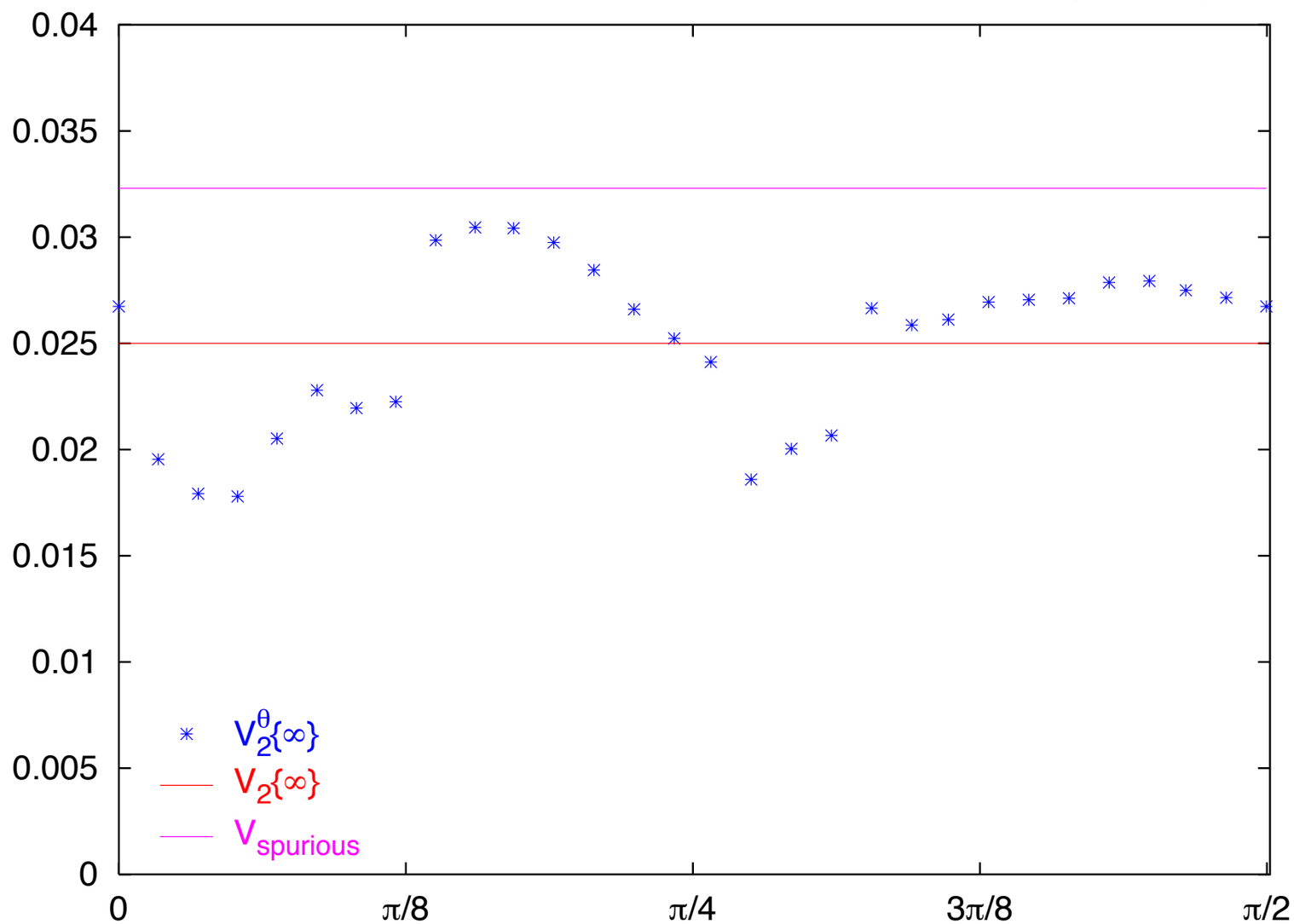
- If $\chi > 1$, the statistical error on the flow is not significantly larger than with the standard method (by a factor of at most 2, see Tables 3, 6 and 7), while systematic errors due to nonflow effects are much smaller. Thus, the present method should be used, and statistics will not be a problem;
- if $0.5 < \chi < 1$, the method can be used, but it really is most important to optimize weights, so as to increase χ, possibly by performing two analyses of the same data sets: in the first analysis, adopting (educated) guesses for the weights; and in the second pass, using as weights w_j the differential flow values obtained in the first analysis;
- if $\chi < 0.5$, statistical errors are too large, and the present method cannot be used; increasing the number of events barely helps here; in this case, one should use the cumulant methods of Refs. [34,35], which still apply if the number of events is large enough [18].

☛ statistical fluctuations (finite N_{ev}) of the generating function!

Multiparticle methods for measuring anisotropic flow in small systems

Simulations (N_{ev} = 20000 — yet slow $\sqrt{\ln N_{ev}}$-dependence —, M = 300) without flow, analyzed with Lee-Yang zeroes

from NPA **727** (2003) 373



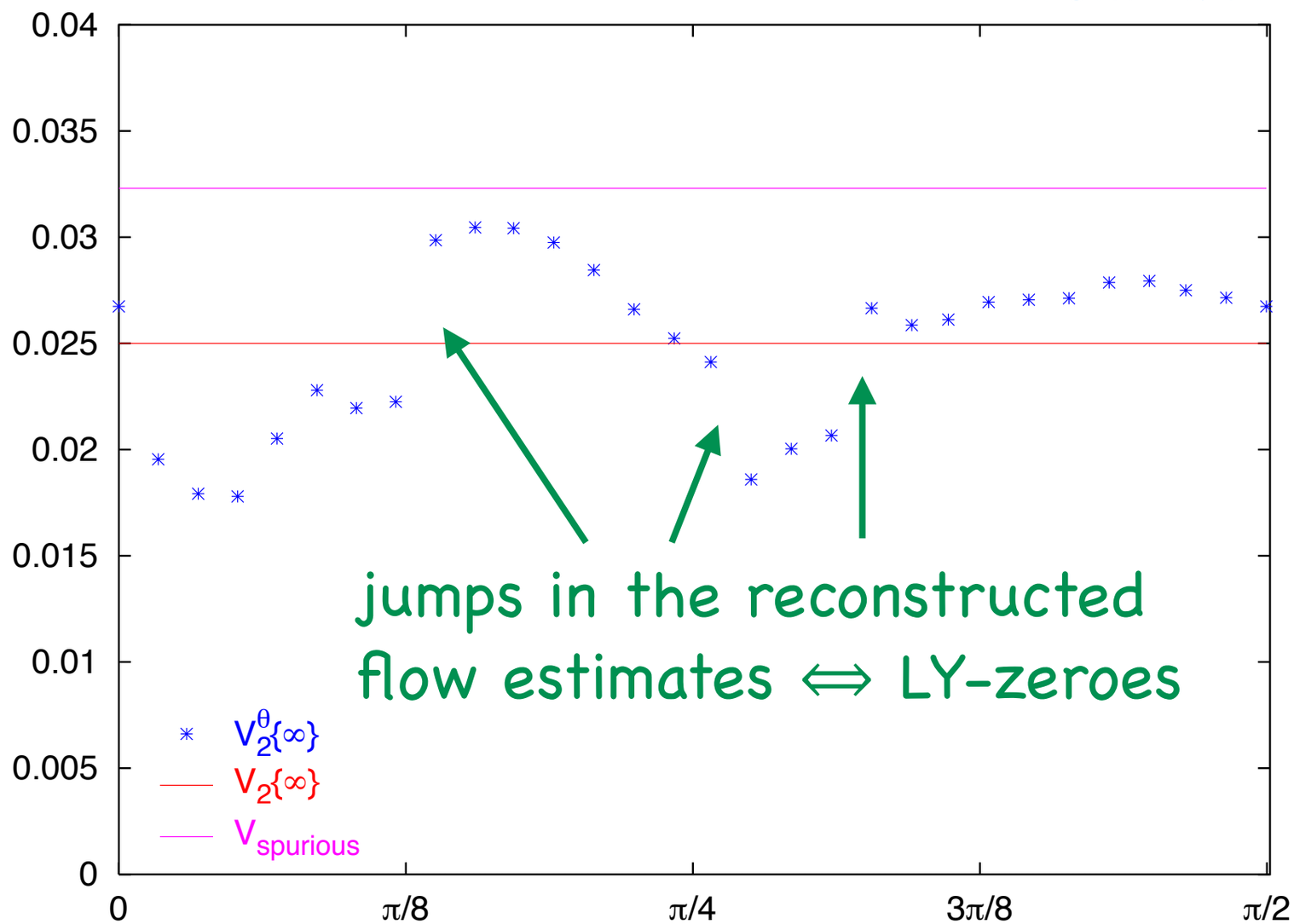
$$\chi \equiv v_n \sqrt{M}$$

$$\chi = \frac{j_{01}}{\sqrt{2 \ln N_{ev}}}$$

Multiparticle methods for measuring anisotropic flow in small systems

Simulations (N_{ev} = 20000 — yet slow $\sqrt{\ln N_{ev}}$-dependence —, M = 300) without flow, analyzed with Lee-Yang zeroes

from NPA **727** (2003) 373



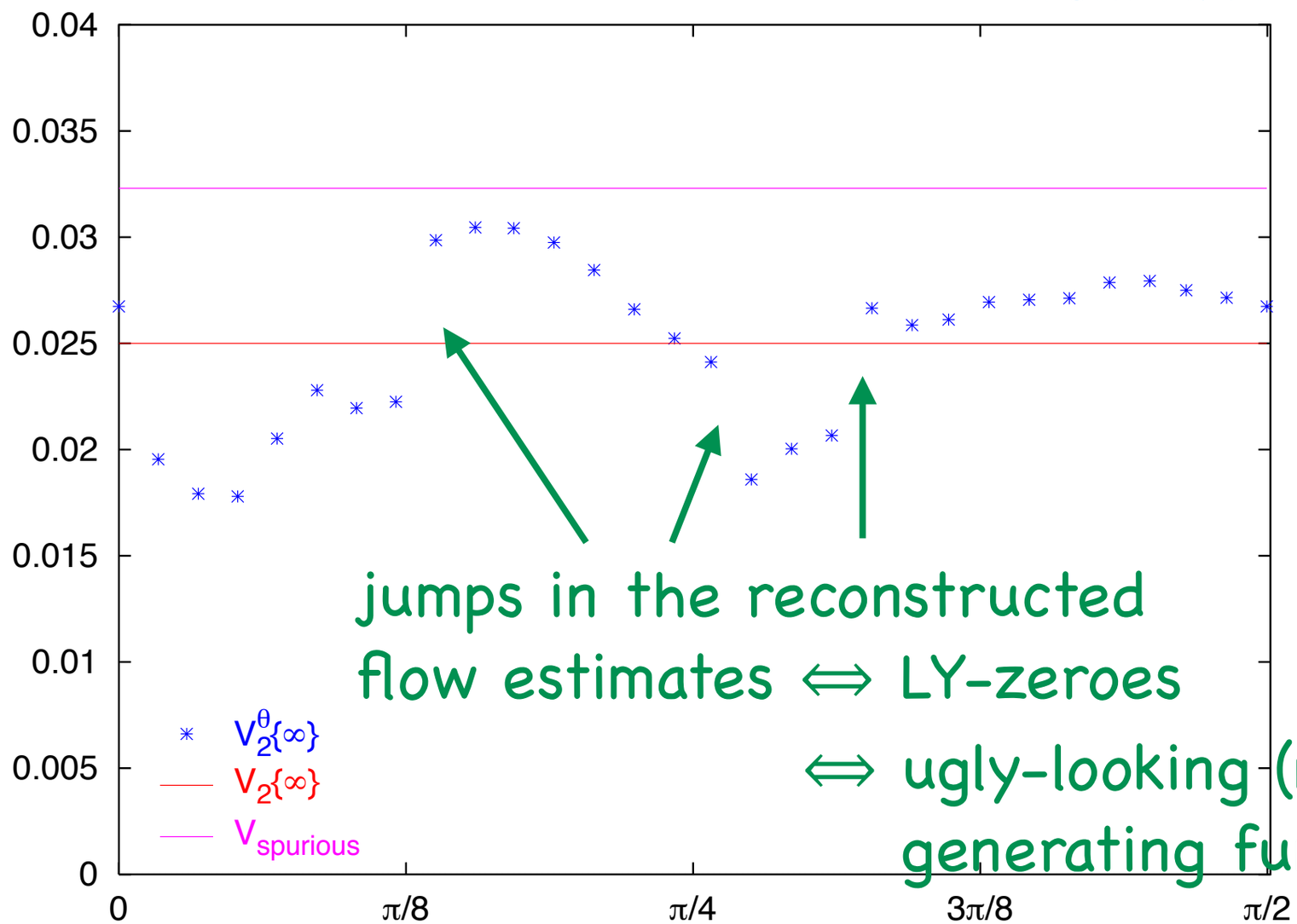
$$\chi \equiv v_n \sqrt{M}$$

$$\chi = \frac{j_{01}}{\sqrt{2 \ln N_{ev}}}$$

Multiparticle methods for measuring anisotropic flow in small systems

Simulations (N_{ev} = 20000 — yet slow $\sqrt{\ln N_{ev}}$-dependence —, M = 300) without flow, analyzed with Lee-Yang zeroes

from NPA **727** (2003) 373



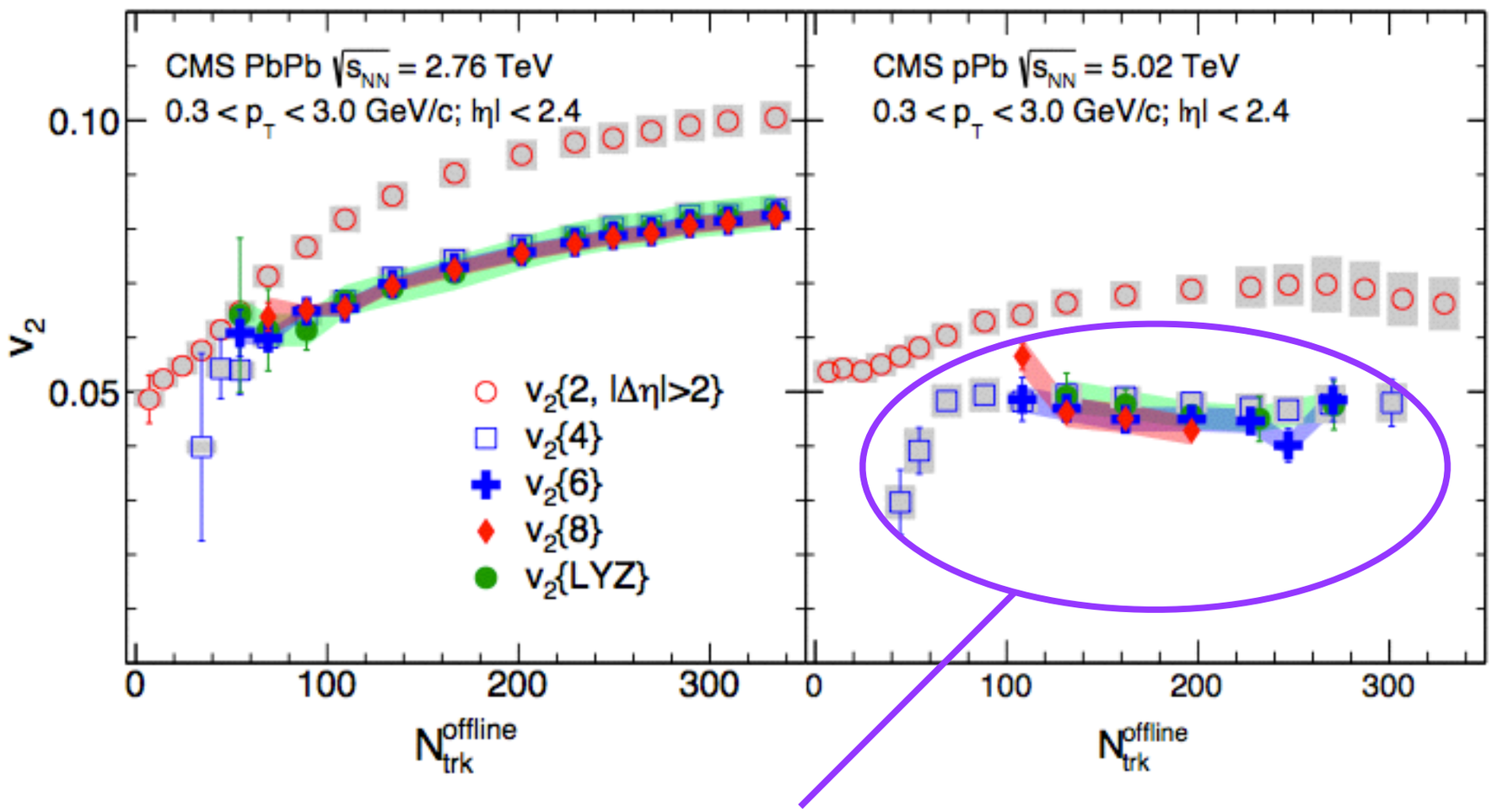
$$\chi \equiv v_n \sqrt{M}$$

$$\chi = \frac{j_{01}}{\sqrt{2 \ln N_{ev}}}$$

jumps in the reconstructed flow estimates ⟺ LY-zeroes

⟺ ugly-looking (not v_n-driven) generating functions

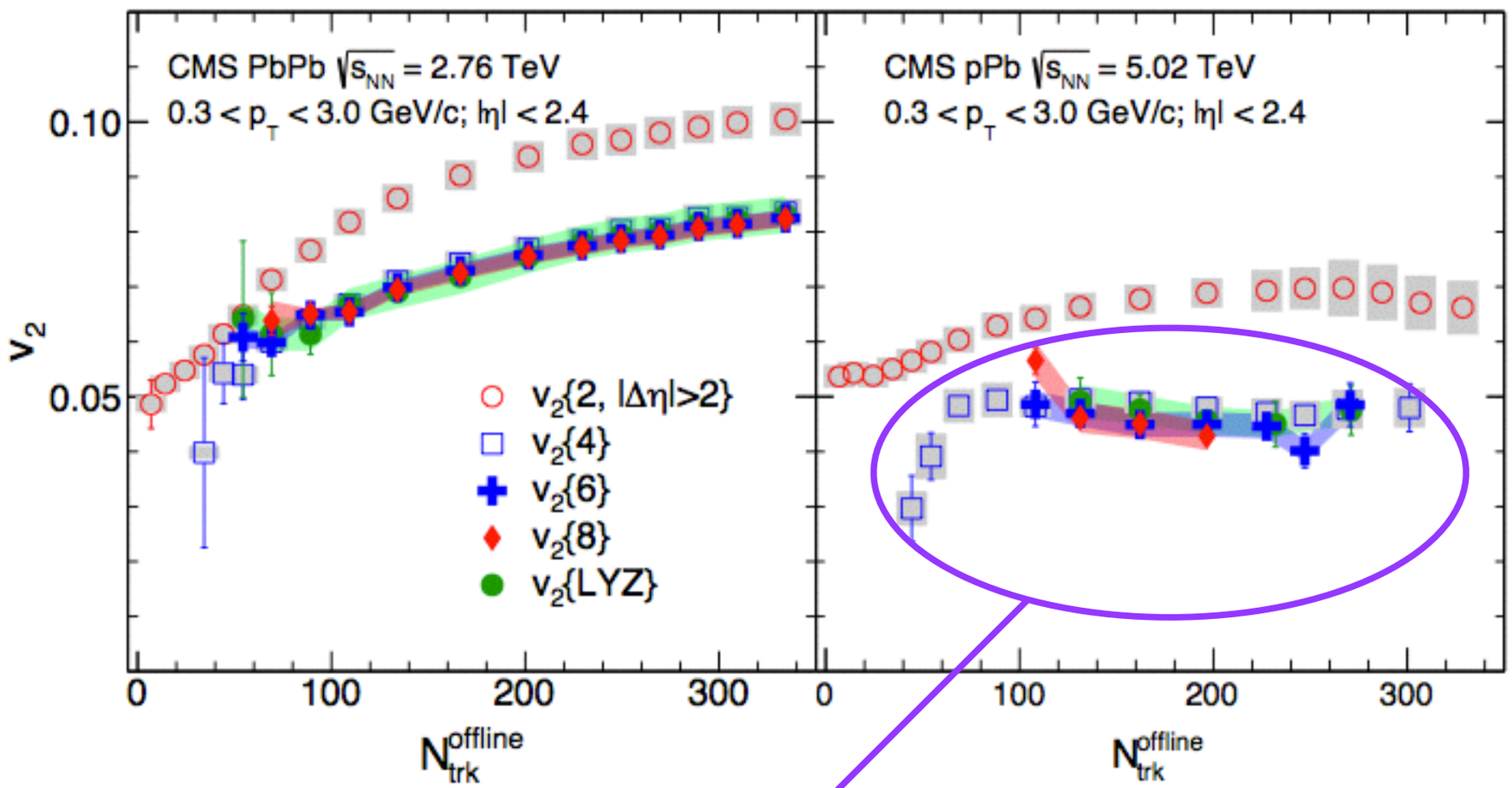
Multiparticle measurements of anisotropic flow in small systems



@CMS: Are you sure of your error bars?

remember $\chi \equiv v_n \sqrt{M}$

Multiparticle measurements of anisotropic flow in small systems



Do not ask cumulants / Lee-Yang-zeroes to provide you with any meaningful "onset of collectivity" (unfortunately!)

remember $\chi \equiv v_n \sqrt{M}$

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Cumulants, Lee-Yang zeroes: a reminder
  - old motivations & applications
  - newer uses

☛ minimize nonflow
☛ statistics not an issue (2002-?)
☛ reconstruct flow fluctuations

- Application to smaller(?) systems
  - overview of results
  - caveats from the theory side

☛ minimize nonflow
☛ statistics will strike back!

Multiparticle methods for measuring anisotropic flow

From large to small systems

- Event-plane method:
  - born 1984, refinements in 1993, 1997; many successful applications
  - criticized (nonflow) in 1999-2000, when applied to small v_n
- Cumulants (resp. Lee-Yang zeroes):
  - born 2000 (resp. 2003); many successful applications
  - now 15 (resp. 12) year old... Beware of limitations in small systems!

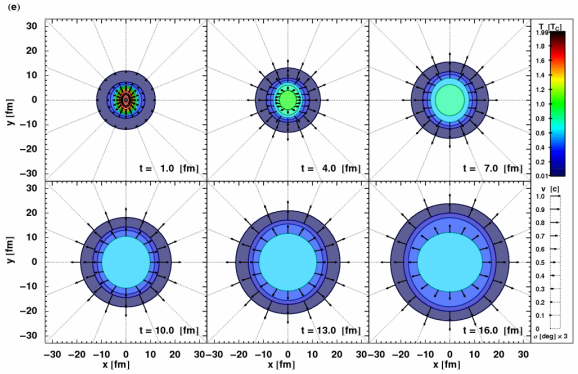
Hydrodynamic simulations in small colliding systems

Piotr Bożek

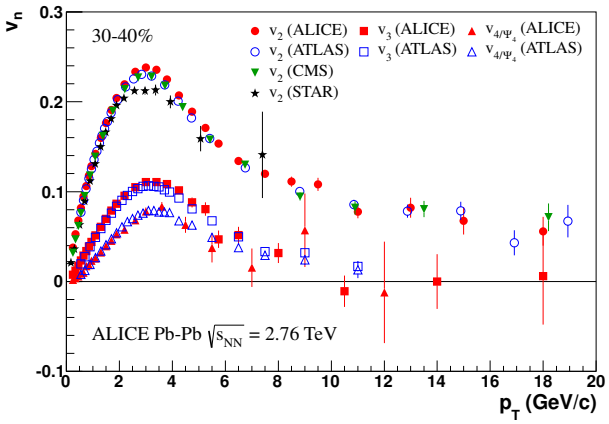
AGH University of Science and Technology, Kraków

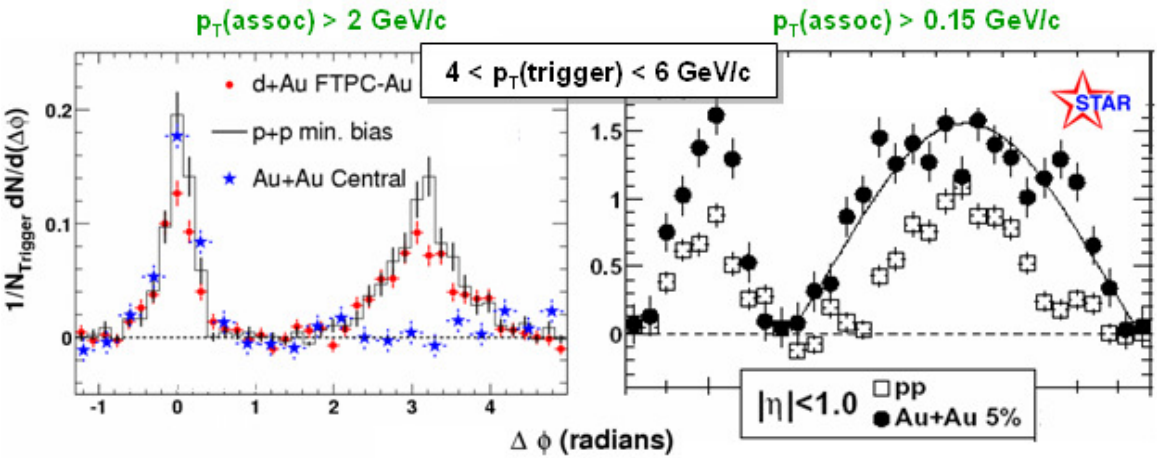
QGP formed in A-A collisions

sQGP



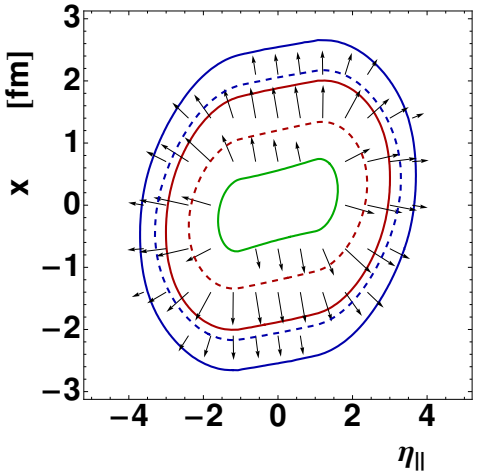
Jet quenching



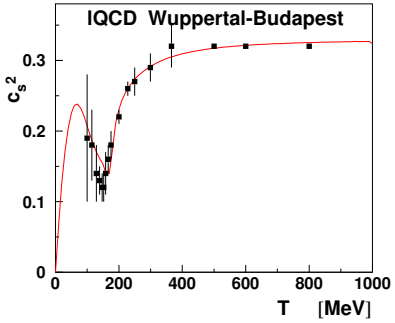


$$\frac{dN}{d\phi} \propto 1 + 2v_2 cos(2(\phi - \Psi_2)) + 2v_3 cos(3(\phi - \Psi_3)) + \dots$$

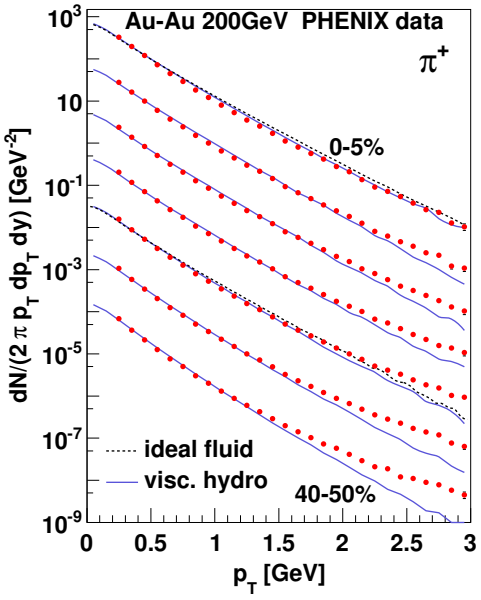
3+1D hydrodynamics

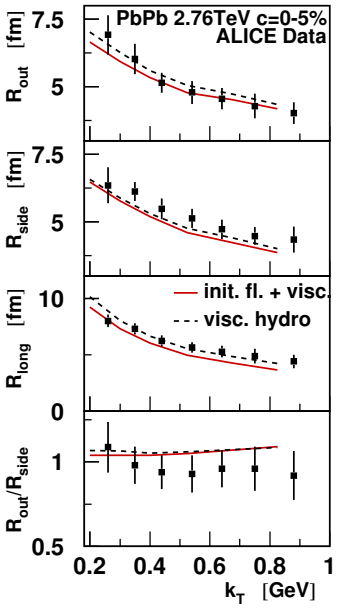


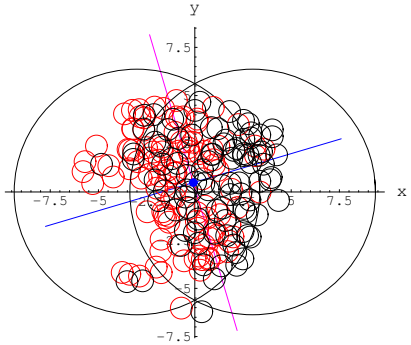
3+1D visc. hydro



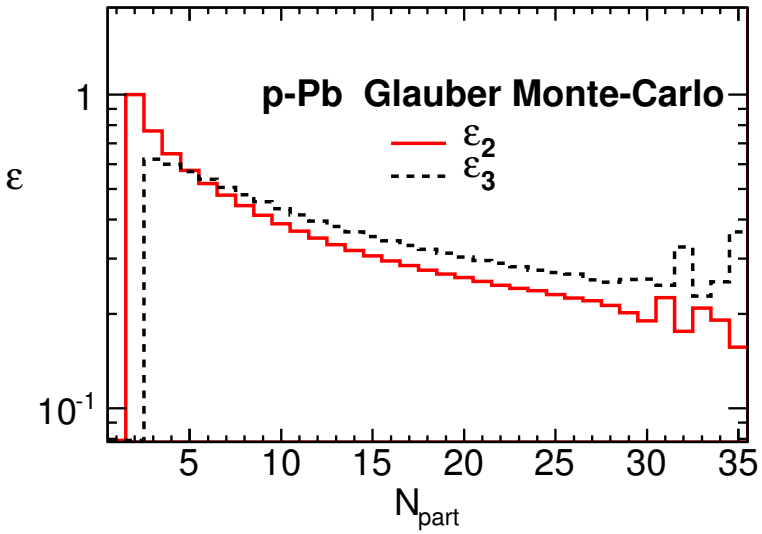
IQCD + Hadron Gas



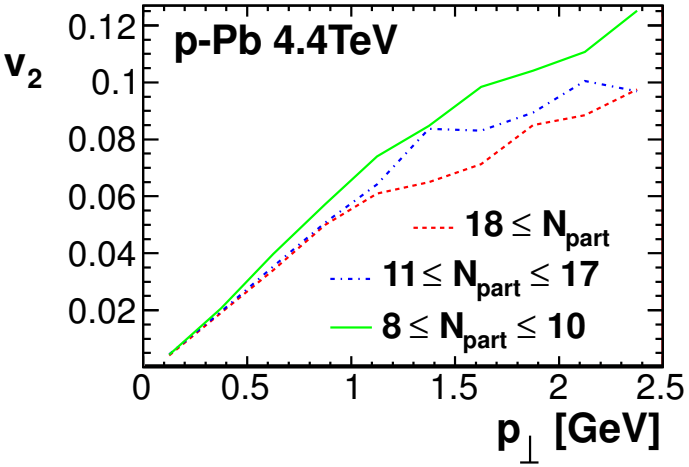


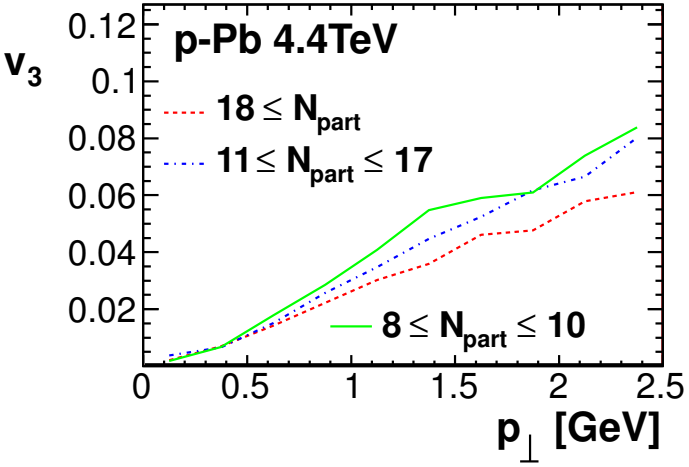


Fireball and flow in p-Pb

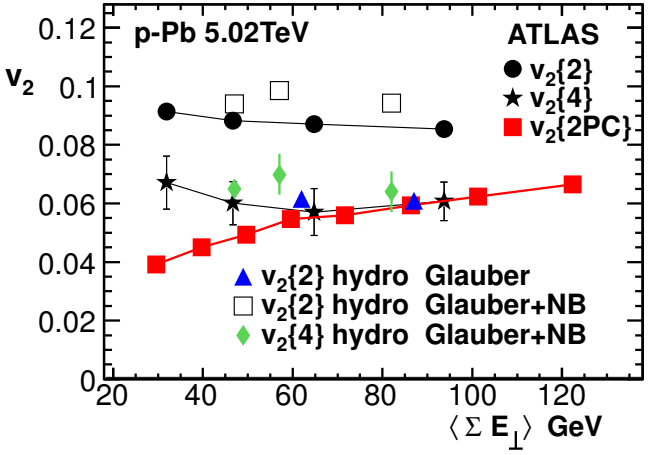


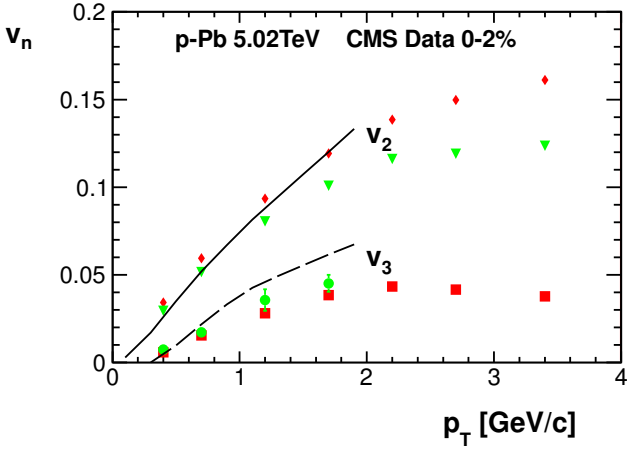
PB, arXiv:1112.0912

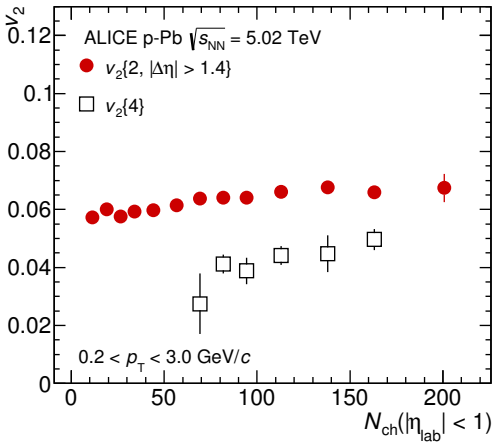


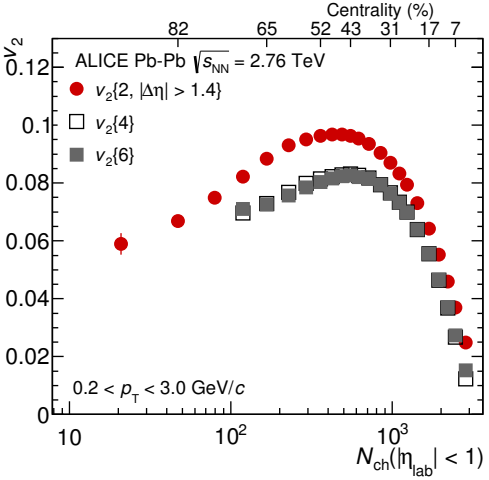


1) Elliptic and triangular flow observed in p-Pb

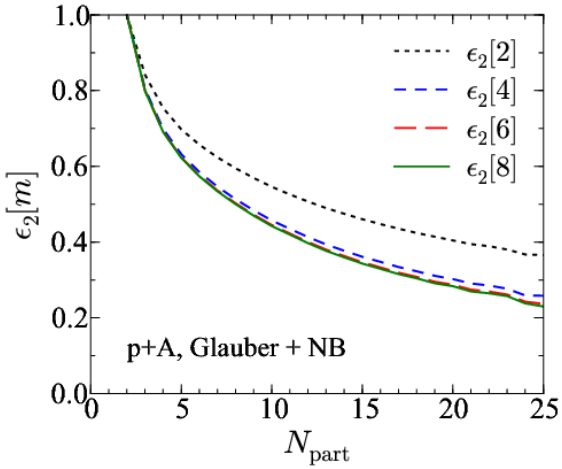


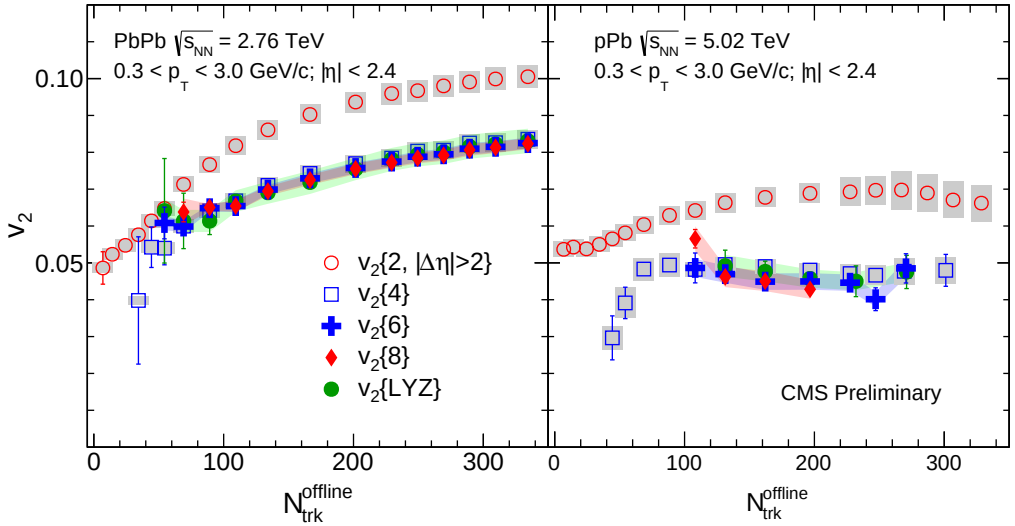






2) Flow from higher cumulants





Bzdak,PB, McLerran, 1311.7325

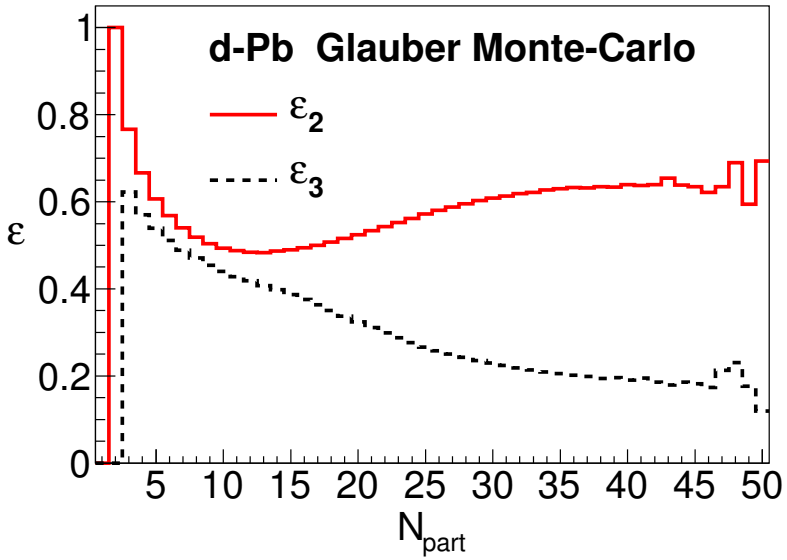
Yan, Ollitrault, 1312.6555

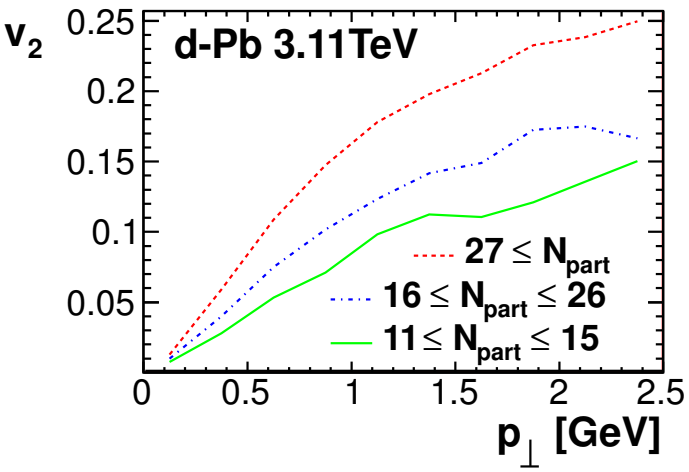
$v_2\{4\} \simeq v_2\{6\} \simeq v_2\{8\} < v_2\{2\}$

multiplicity dependence -

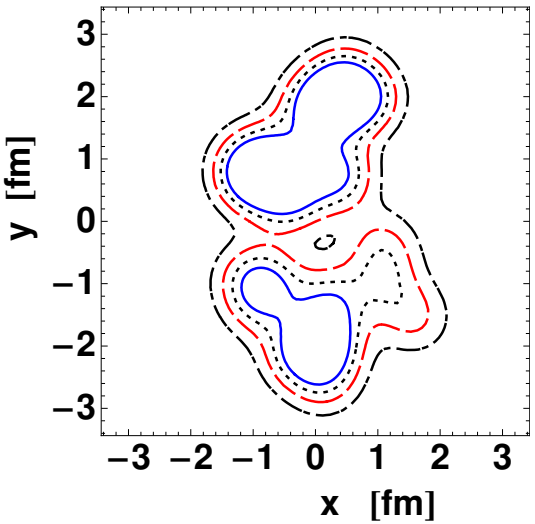
Kozlov, Luzum, Denicol, Jeon, Gale, 1405.3976

small system with large deformation d-Pb





large elliptic flow

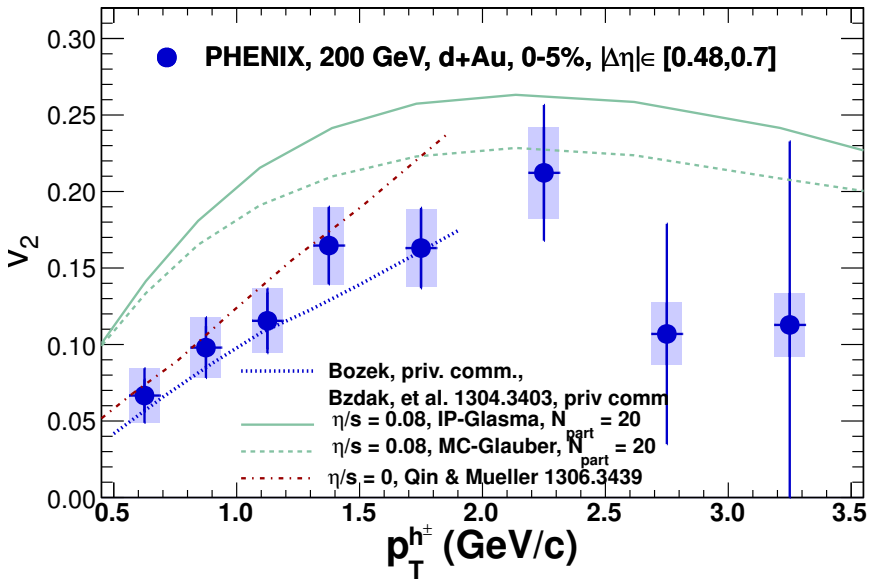


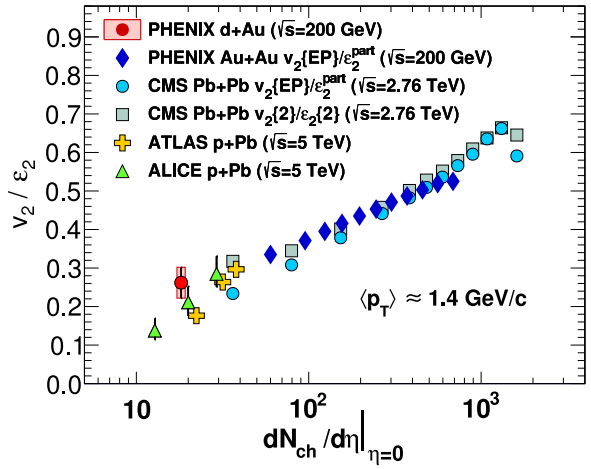
PB, arXiv:1112.0912

control of initial geometry,

central collisions - deformed fireball

3a) Elliptic flow observed in d-Au at 200GeV

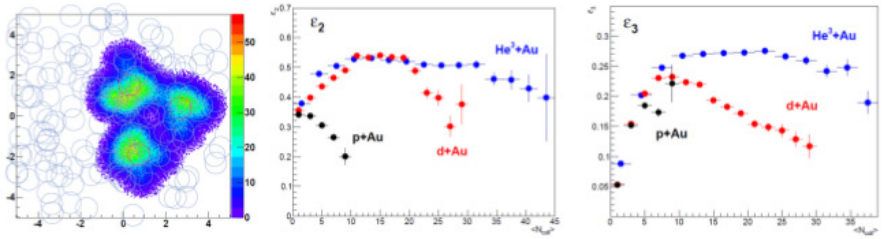




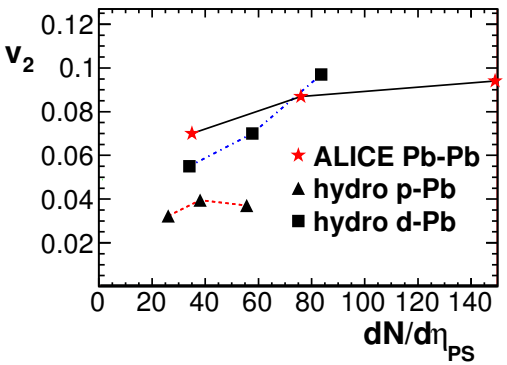
PHENIX, arXiv:1303.1794

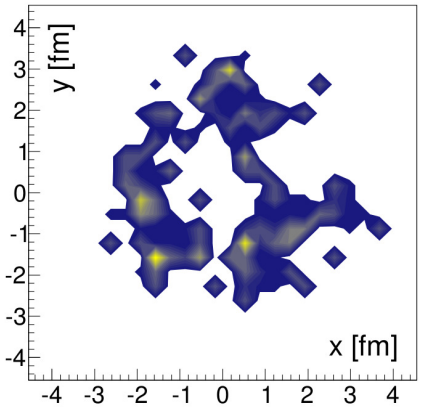
large eccentricity - large elliptic flow

small on big collisions



PHENIX proposal $\longrightarrow$ v_3, Sickles et al. arXiv:1401.2432





α clusters in ^{12}C Broniowski, Arriola arXiv:1312.0289

PB, arXiv:1112.0912

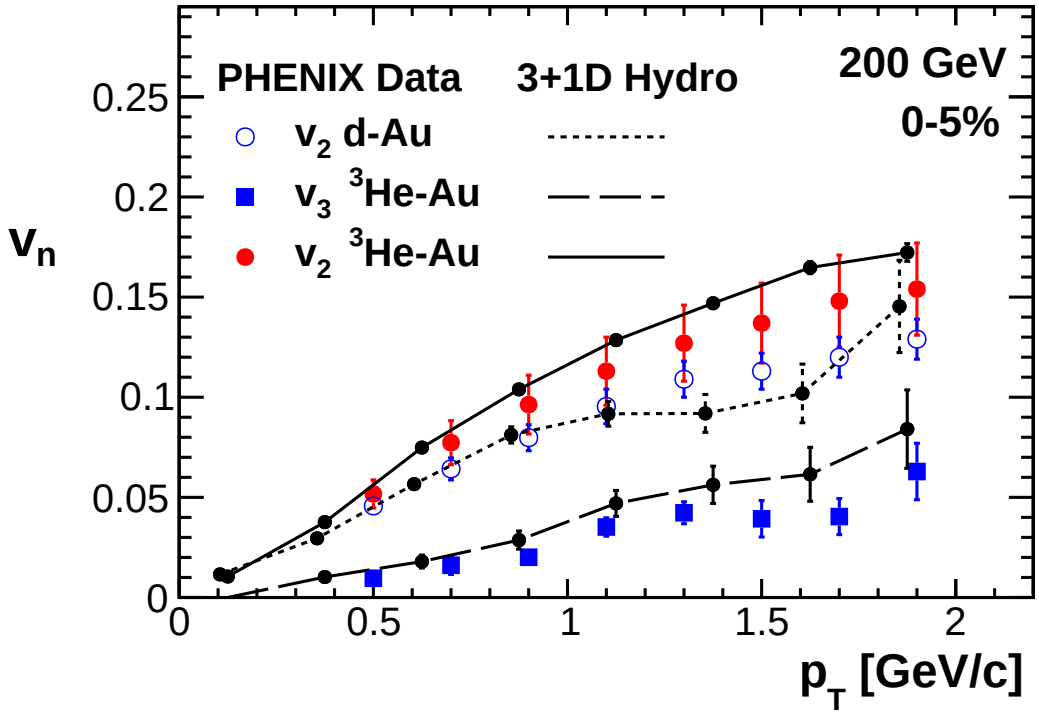
strong effect for d-A

intrinsic deformation dominates over fluctuations

effect for v_3 in ^3He-A,

Nagle et al. arXiv:1312.4565

3b) Triangular flow in ^{3}He-Au



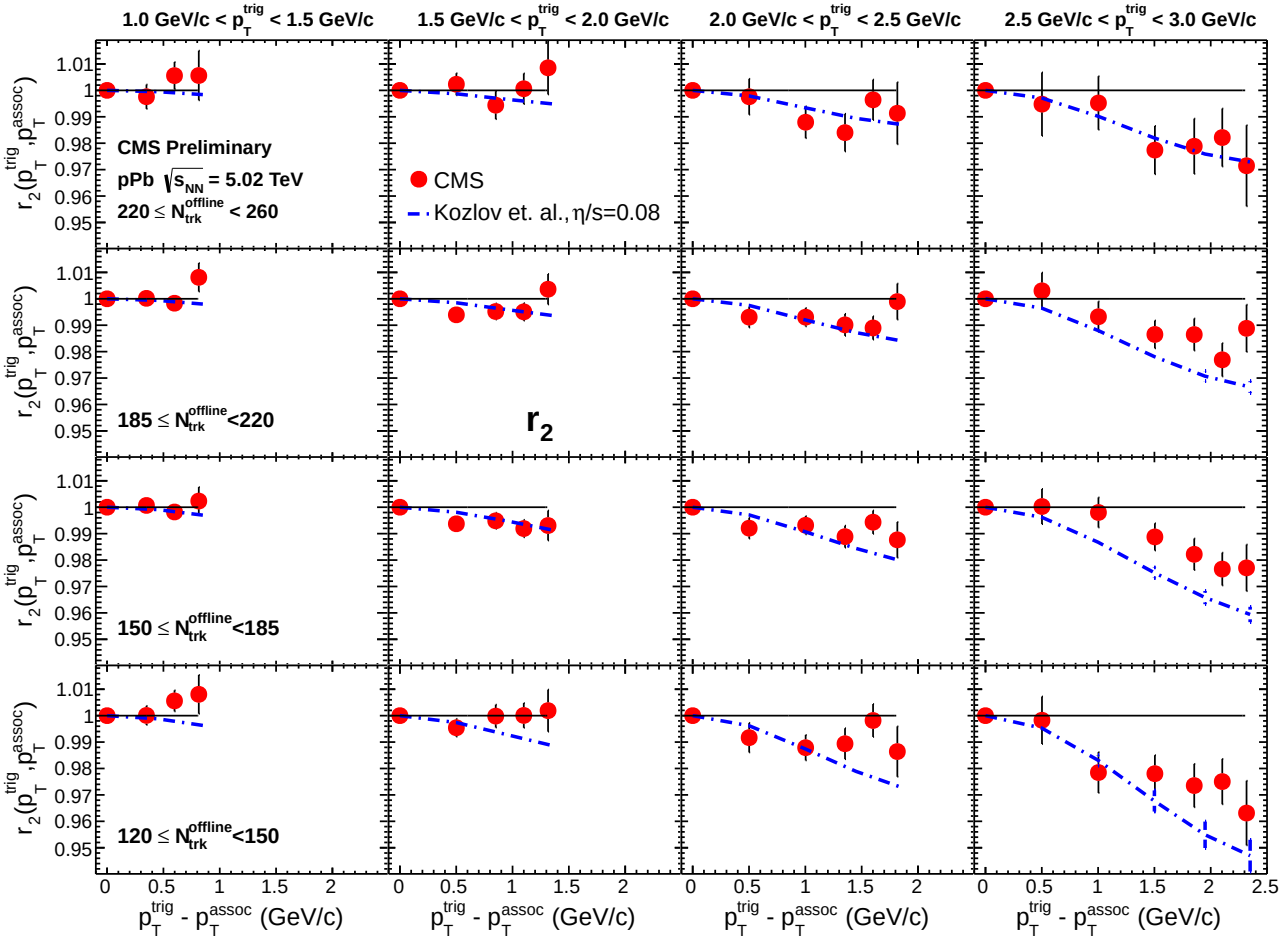
-strong v_2 in d-Au **and** ^{3}He-Au

-strong v_3 in ^{3}He-Au

- observed $v_3 \longrightarrow$ collectivity
- hierarchy of v_2 and v_3 consistent with collective response on fireball geometry
- large v_2 in He-Au

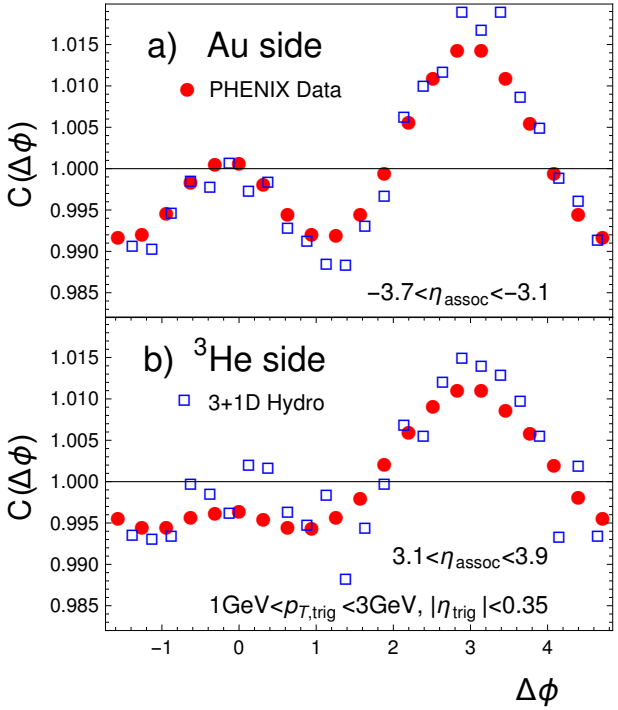
Consistent with hydrodynamics

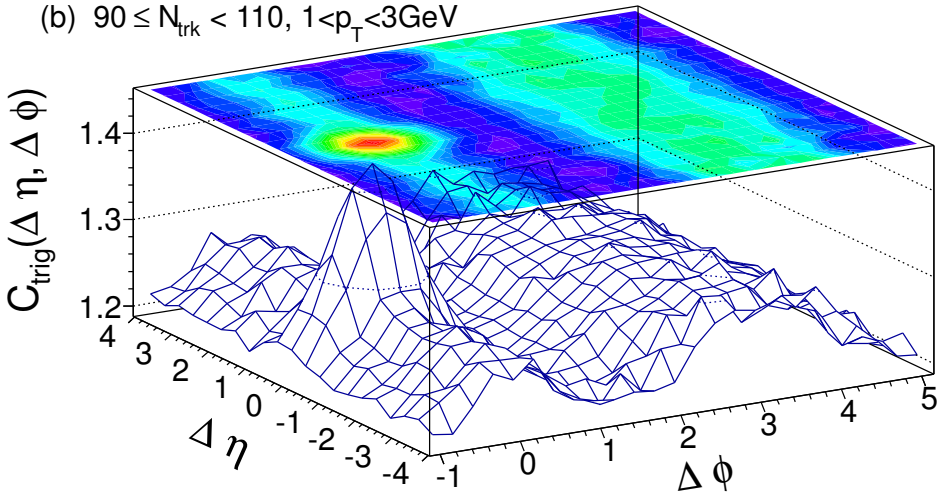
4a) Factorization at intermediate $p_\perp$



- factorization holds for v_2 and v_3 up to 3 GeV
- **small** deviations explained by hydro+Glauber Kozlov, Luzum, Denicol, Jeon, Gale, 1405.3976
- geometry driven (??) origin of correlations at small **and** intermediate $p_\perp$

4b) Correlations at large $\Delta\eta$ (Ridge)



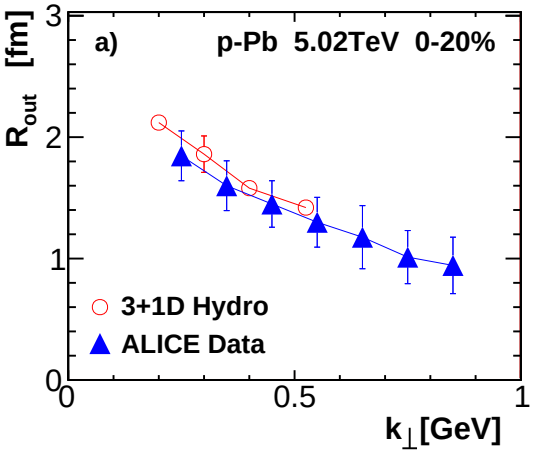


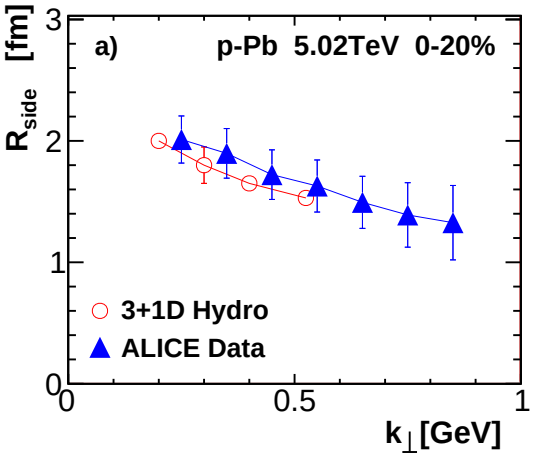
PB, Broniowski 1211.0845

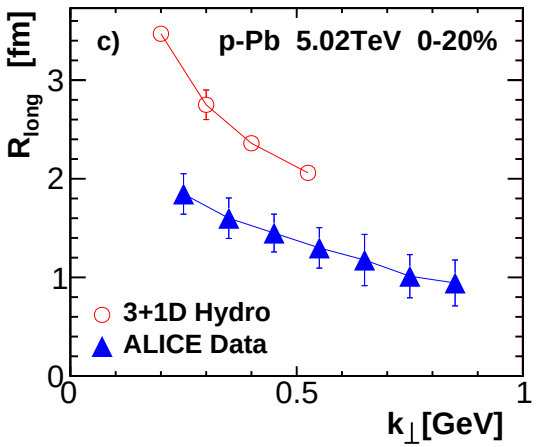
- factorization holds for large $\Delta\eta$
- geometry driven (??) origin of correlations at central, forward and backward rapidities

- similar mechanism in AMPT: Ma, Bzdak-arXiv: 1404.4129, Koop, Adare, Nagle-arXiv: 1501.06880

5) Interferometry radii

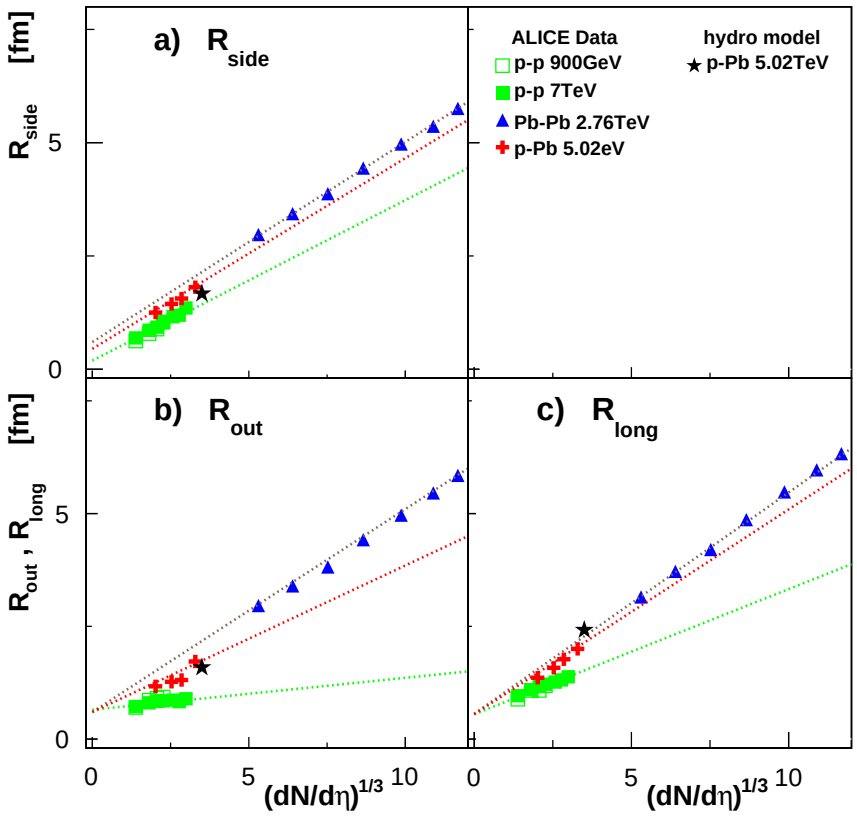






$k_\perp$ dependence of $R_{o,l,s}$
R_{side}, R_{out} consistent with hydro

Interferometry radii - pp, pA, AA

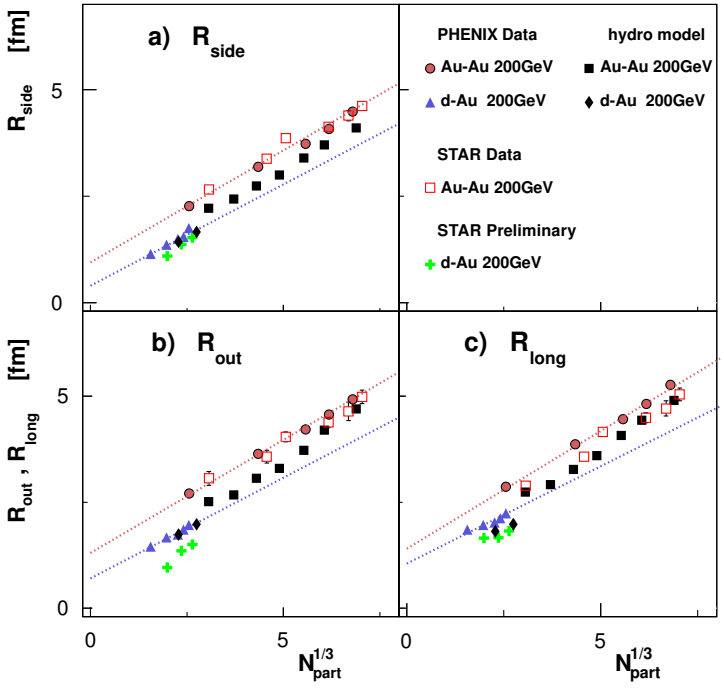


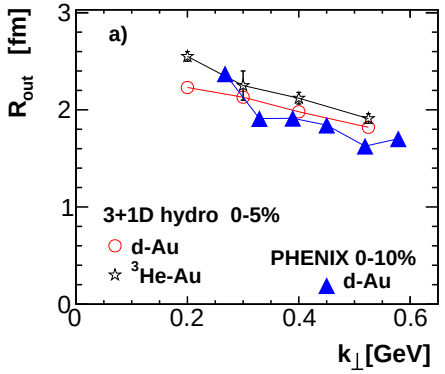
- pA system size in between pp and AA
(differences in flow strength, initial size, flow profile)

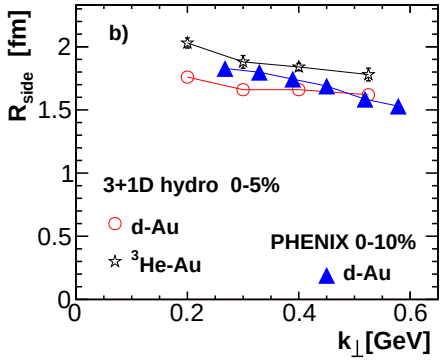
- HBT in pA consistent with hydrodynamics

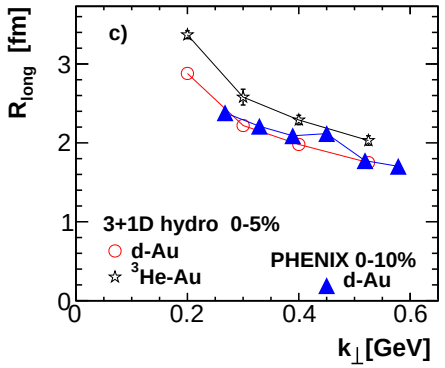
Interferometry radii (d-Au, He-Au)

d-Au (PHENIX data)









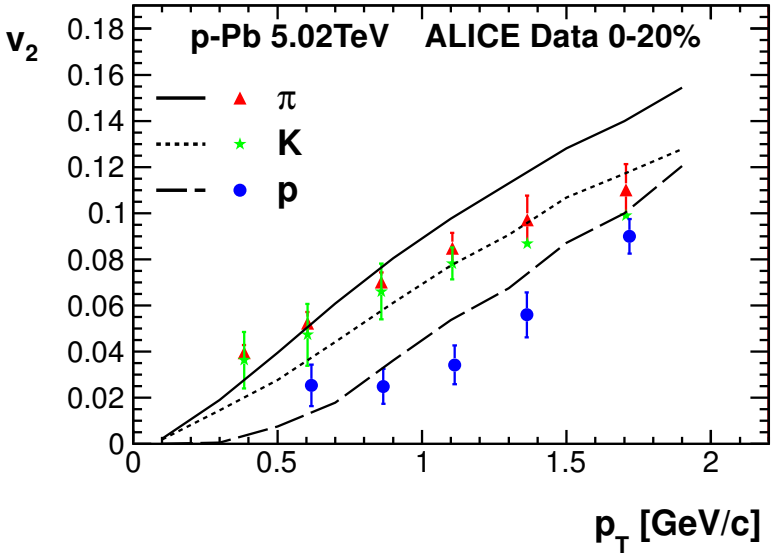
$k_\perp$ dependence of $R_{o,l,s}$

R_{side} consistent with hydro

PB, 1408.1264

Romatschke 1502.04745

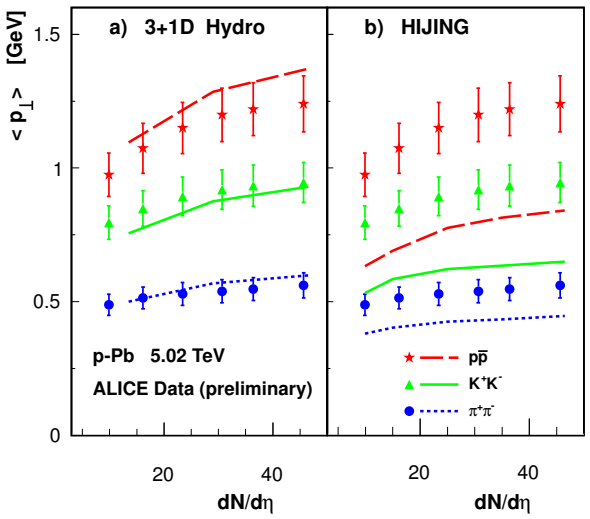
6) Mass splitting of v_2

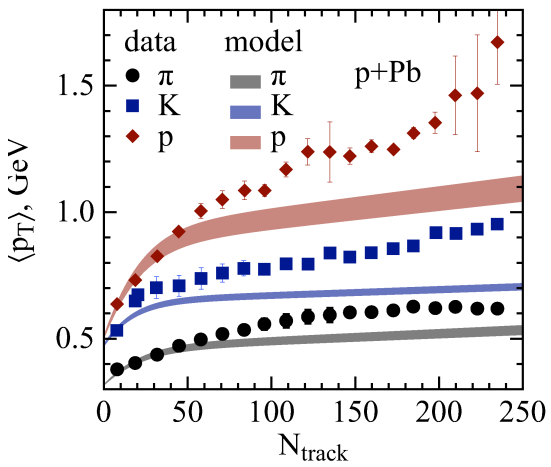


PB, Broniowski, Torrieri, 1307.5060

Werner, Bleicher, Guiot, Karpenko, Pierog, 1307.4379

7) Mass hierarchy of $< p_\perp >$





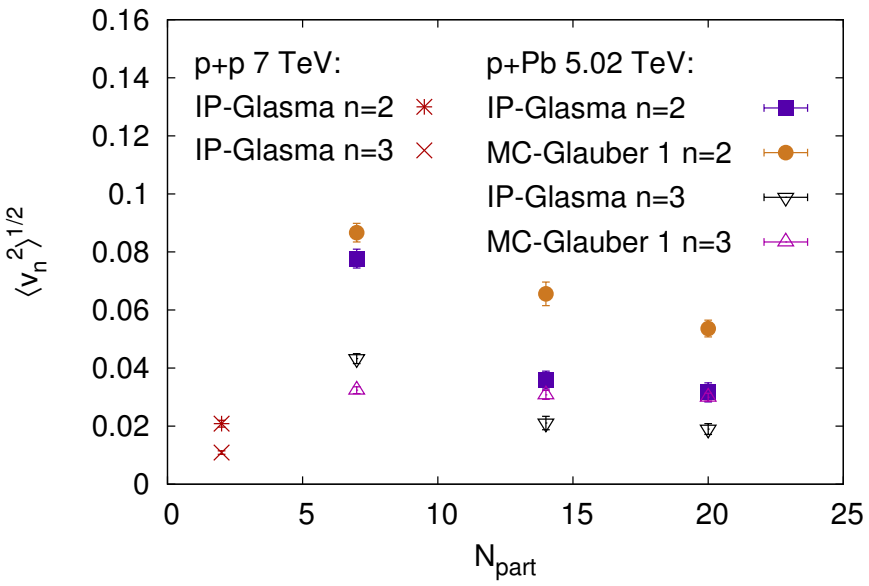
PB, W.Broniowski, G. Torrieri arXiv:1306.5442

larger $< p_\perp >$ in smaller systems

Bzdak, Skokov, arXiv:1306.5442

Hydrodynamic flow in p-p?

- Humanic-nucl-th/0612098 (pythia, cascade)
- Romatschke, Luzum-arXiv:0901.4588 (overlap)
- Prasad, Roy, Chattopadhyay, Chaudhuri -arXiv: 0910.4844 (overlap)
- Bozek-arXiv: 0911.2393 (flux-tubes)
- Yan, Dong, Zhou, Li, Ma, Sa- arXiv: 0912.3342 (transport)
- Werner, Karpenko, Pierog, Bleicher, Mikhailov-arXiv: 1010.0400 (EPOS)
- Deng, Xu, Greiner-arXiv: 1112.0470 (hot-spots, transport model)
- Shuryak, Zahed-arXiv:1301.4470 (symmetric)
- Bzdak, Schenke, Tribedy, Venugopalan-arXiv: 1304.3403 (IP-Glasma)
- Ma, Bzdak-arXiv: 1404.4129 (AMPT)

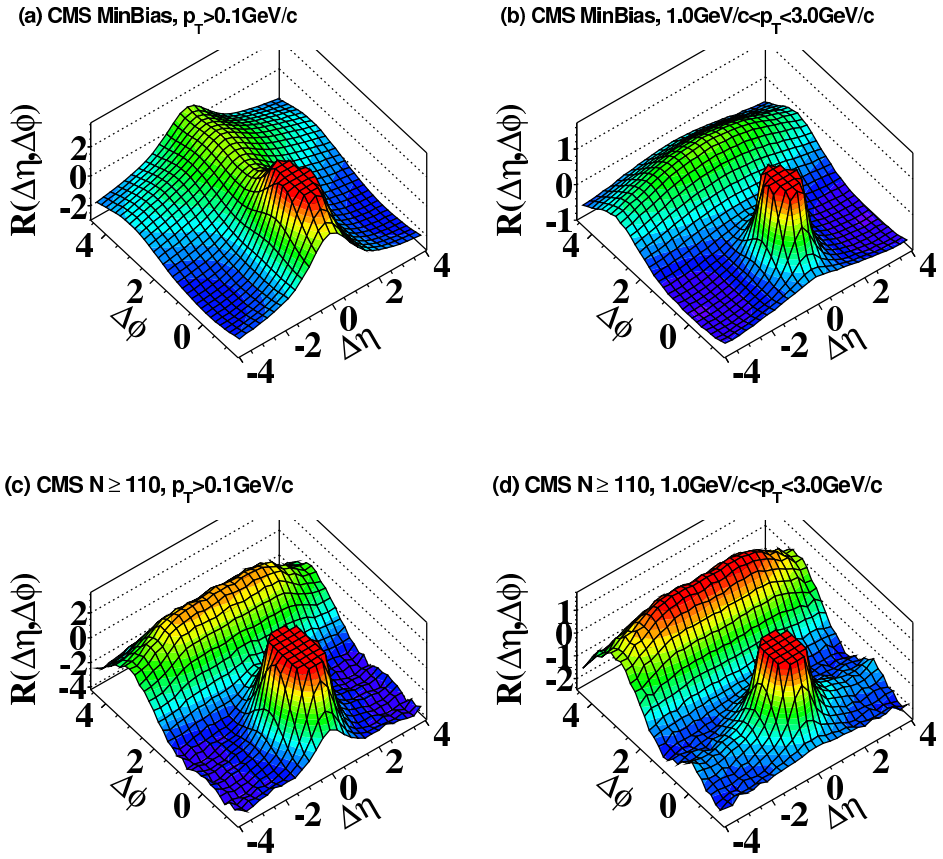


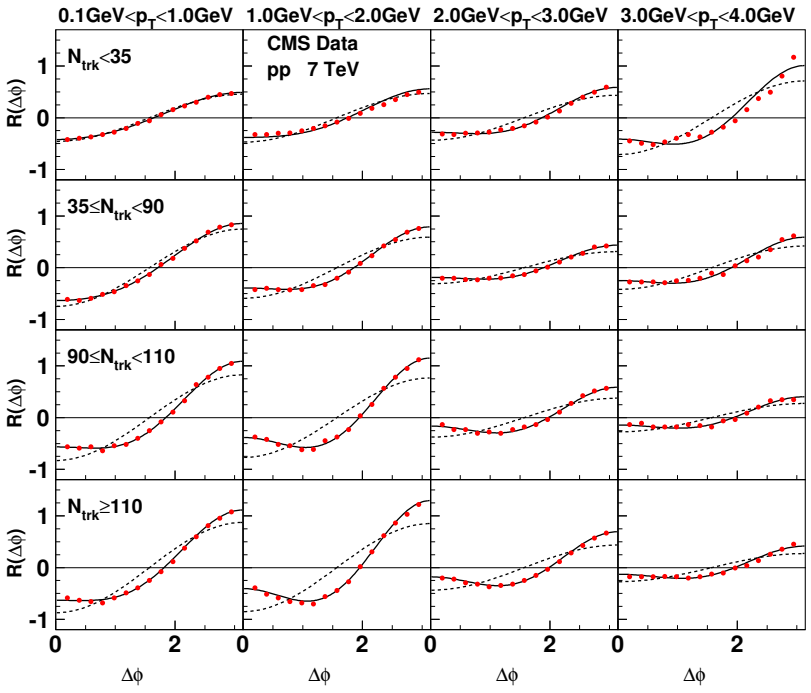
Bzdak et al. arXiv: 1304.3403

-Is hydrodynamics valid?

-What is the initial eccentricity?

Ridge in pp



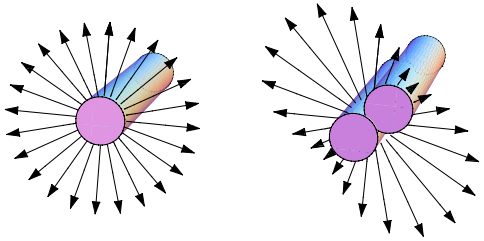


PB arXiv:1010.0405

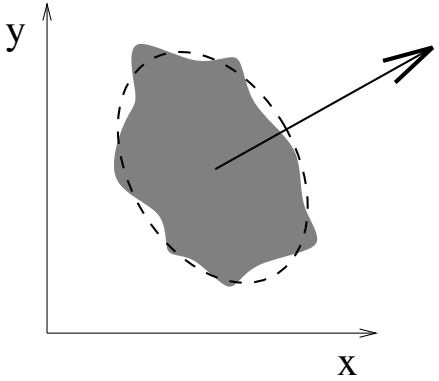
can we measure (calculate) v_2

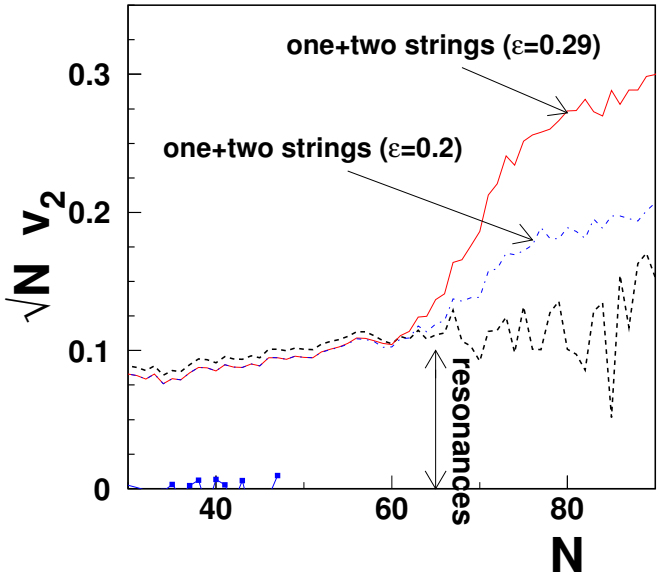
Fireball shape in pp

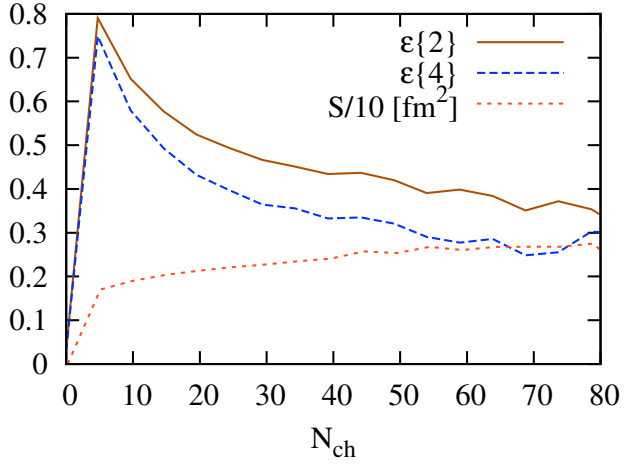
flux-tubes



granular source







Bozek, 0911.2397

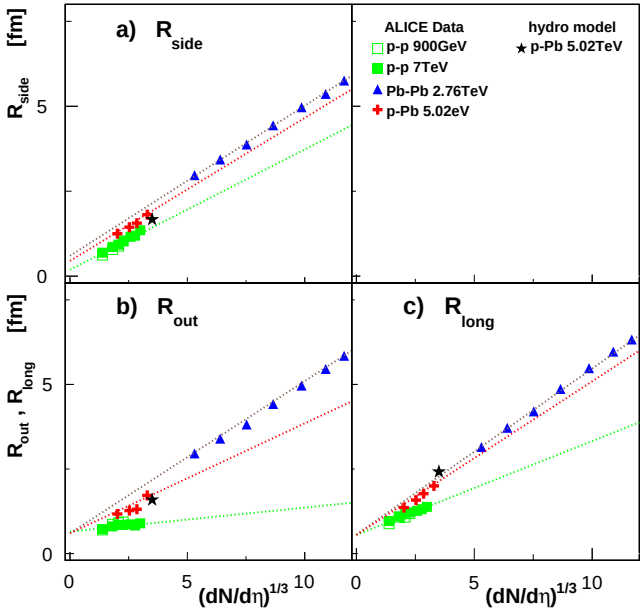
Bjorken, Brodsky, Goldhaber, 1308.1435

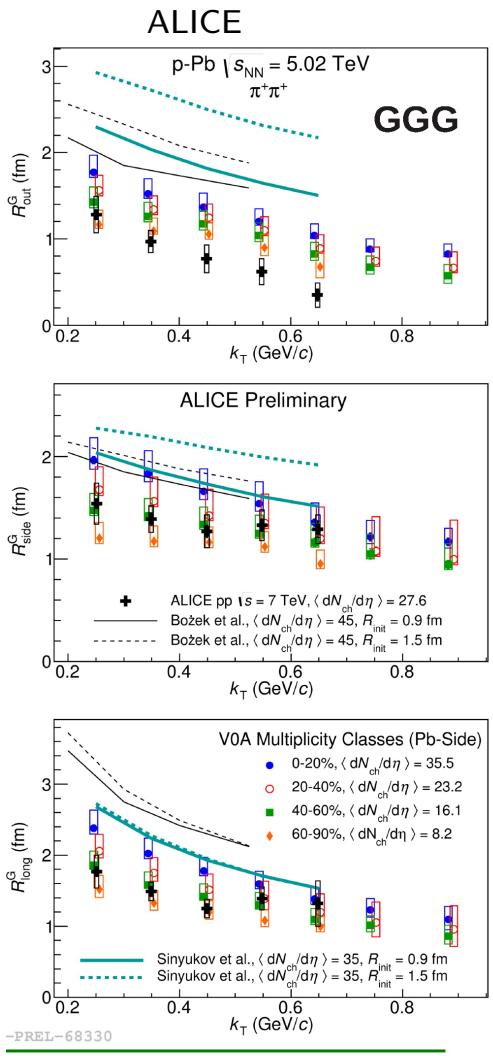
Schlichting, Schenke, 1407.8458

E.Asar et al., 1009.5643

Casalderrey-Solana, Wiedemann, 0911.4400

Stronger flow in p-p ?





stronger transverse flow in p-p !

Kisiel 1012.1517

Hirono, Shuryak 1412.0063

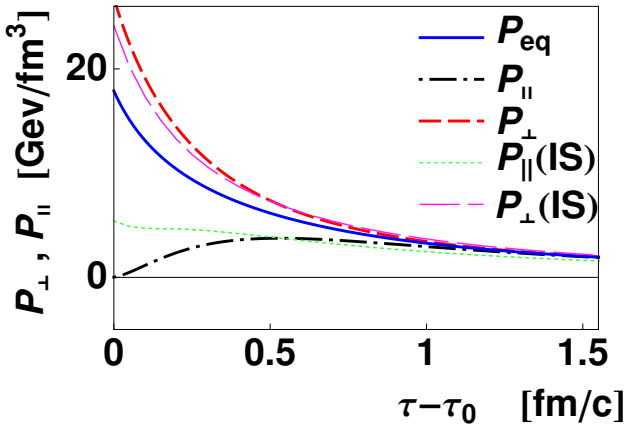
- SIGNS FOR COLLECTIVITY IN SMALL SYSTEMS

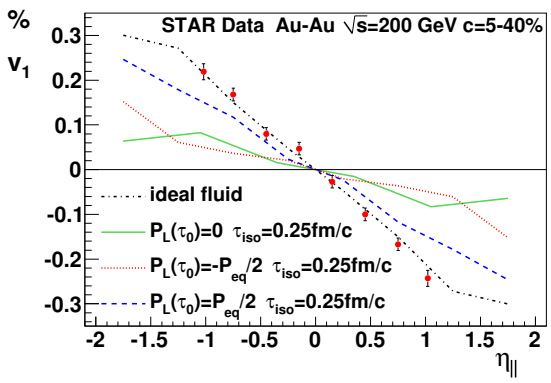
  (in order of significance, personal view)

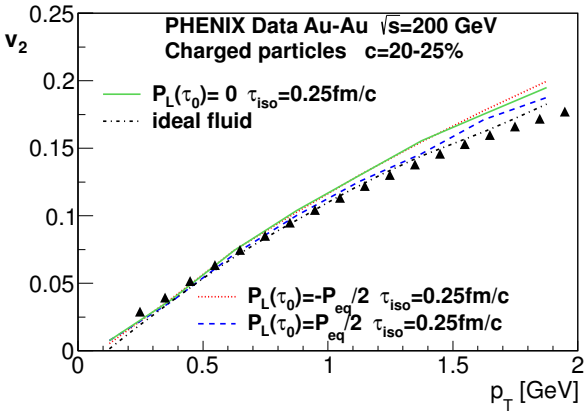
  1. **Elliptic and triangular flow**
  2. **Flow from higher cumulants**
     all particles flow
  3. **Hierarchy of v_2 and v_3 in p-A, d-A, He-A**
     collective response to geometry (final state effect)
  4. **Factorization at intermediate $p_\perp$ and large $\Delta\eta$**
     particles at intermediate $p_\perp$, large η, correlated to geometry
  5. $k_\perp$ **dependence of HBT radii**
  6. **Mass splitting of v_2**
  7. Mass hierarchy of $< p_\perp >$

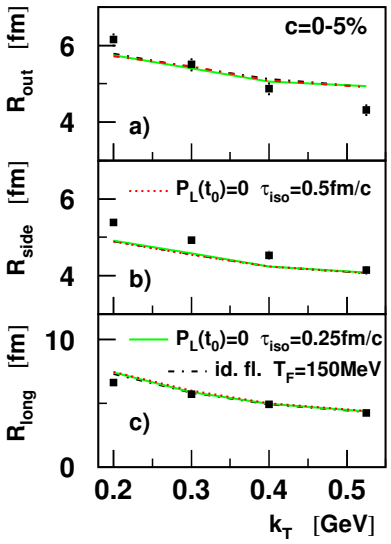
- WHAT IS IT
  - Final state interaction
  - Geometry driven :
    hydrodynamics, preequilibrium flow, surface emission, jet quenching, uncertainity principle ...
  - Probably a combination of several mechanisms

pressure anisotropy - why hydrodynamics works









PB, I. Wyskiel - arXiv:1009.0701

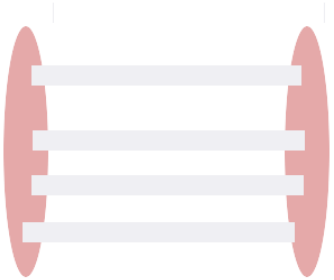
- early pressure anisotropy irrelevant!

Vredevoogd, Pratt, 0810.4325

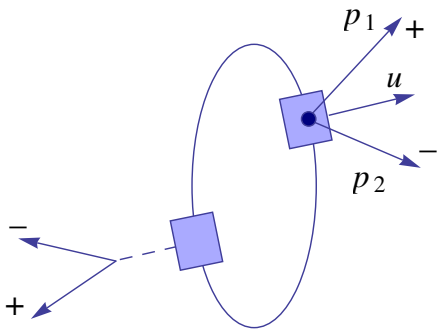
FSI scenarios

fields+thermalization

color fields

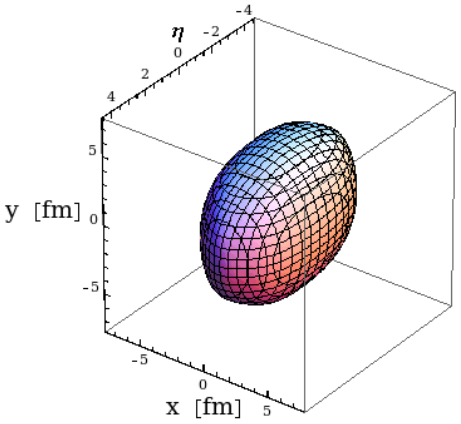


local thermalization $\rightarrow$ hadronization

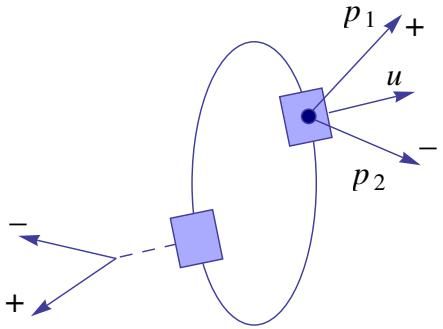


hydrodynamics

hydrodynamic expansion

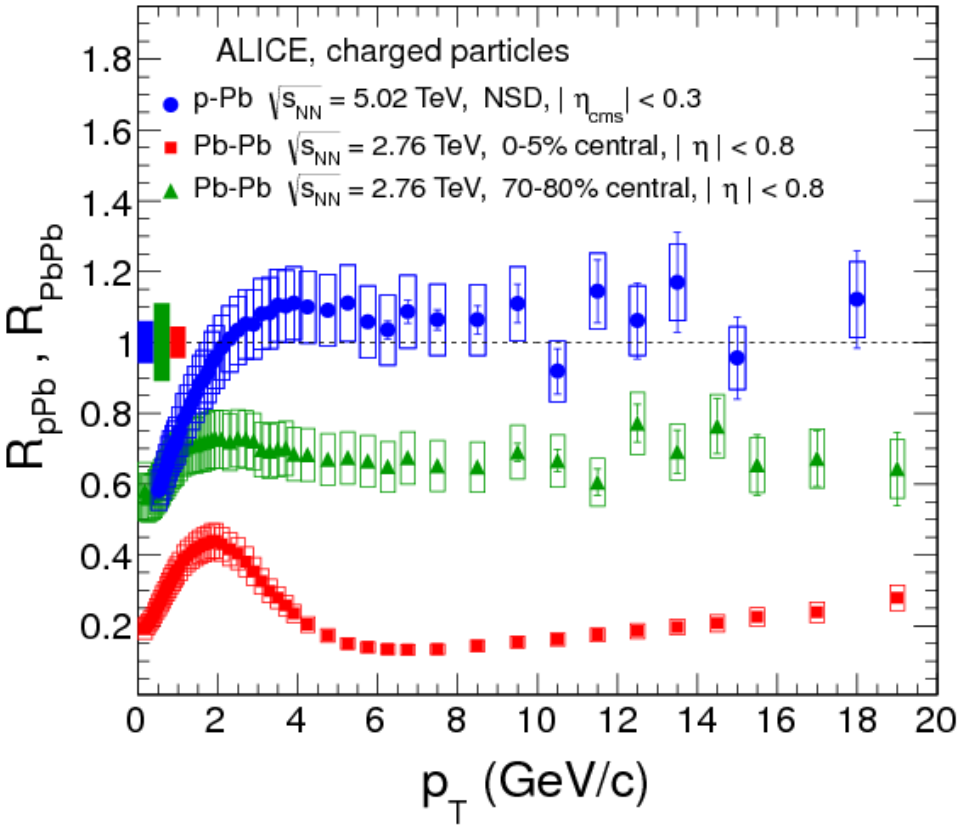


hadronization, statistical emission



Hydro, Asymmetric-Hydro, preequilibrium expansion **give similar flow**

Flow without jet quenchinq?



energy-momentum tensor

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p+\Pi & 0 & 0 \\ 0 & 0 & p+\Pi & 0 \\ 0 & 0 & 0 & p+\Pi \end{pmatrix} + \pi^{\mu\nu}$$

- shear viscosity

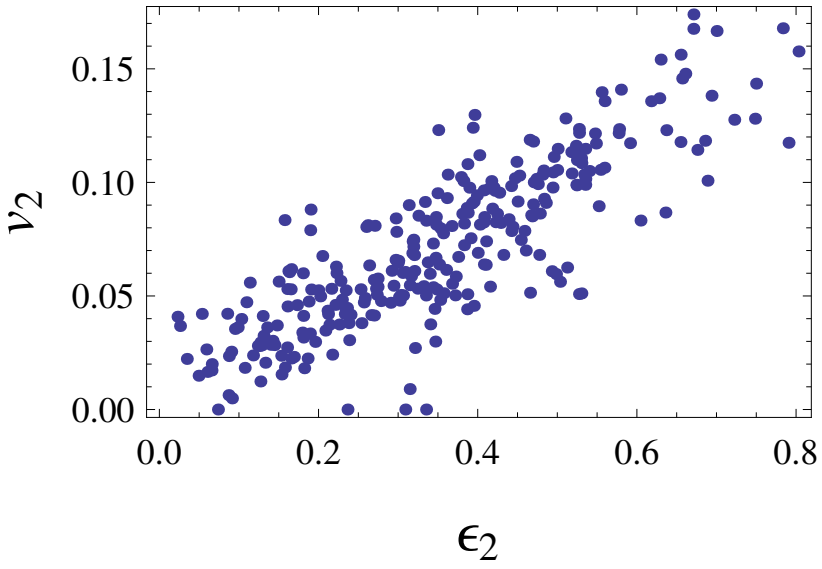
$$\Delta^{\mu\alpha}\Delta^{\nu\beta}u^{\gamma}\partial_{\gamma}\pi_{\alpha\beta} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_{\pi}} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_{\pi}}\partial_{\alpha}\left(\frac{\tau_{\pi}u^{\alpha}}{\eta T}\right)$$

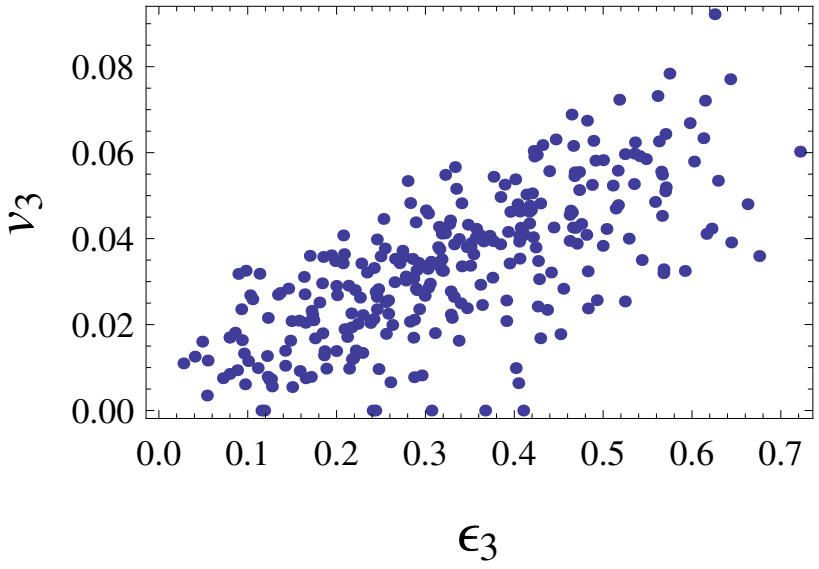
- bulk viscosity

$$u^{\gamma}\partial_{\gamma}\Pi = \frac{-\zeta\partial_{\gamma}u^{\gamma} - \Pi}{\tau_{\Pi}} - \frac{1}{2}\Pi\frac{\zeta T}{\tau_{\Pi}}\partial_{\alpha}\left(\frac{\tau_{\Pi}u^{\alpha}}{\zeta T}\right)$$

- viscosity corrections from velocity gradients
- **initial** stress tensor - pressure anisotropy
- equation of state

fireball asymmetry - flow asymmetry

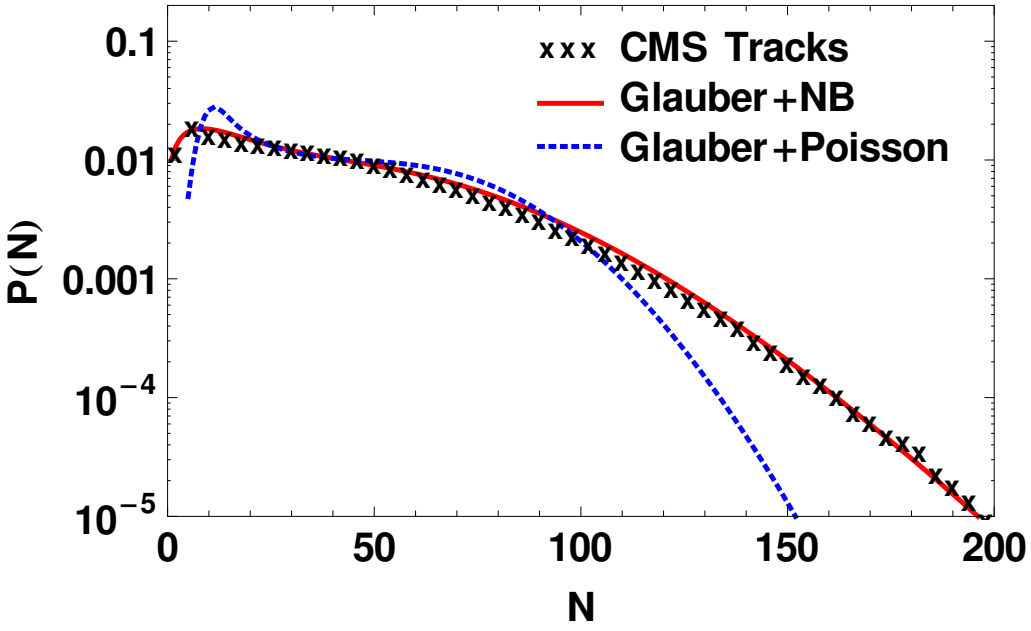




- Ev-by-Ev hydro response to geometry valid
- response strength depends on details

Glauber+NB

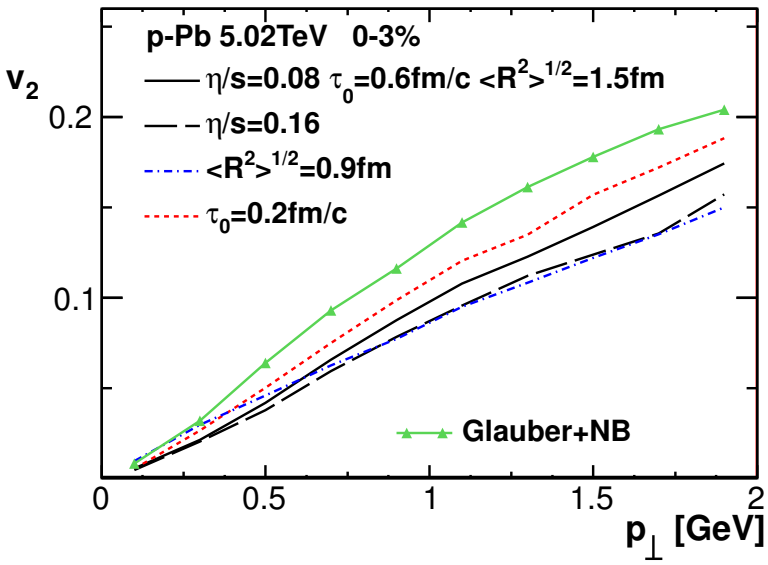
fluctuations from subnuclear dynamics

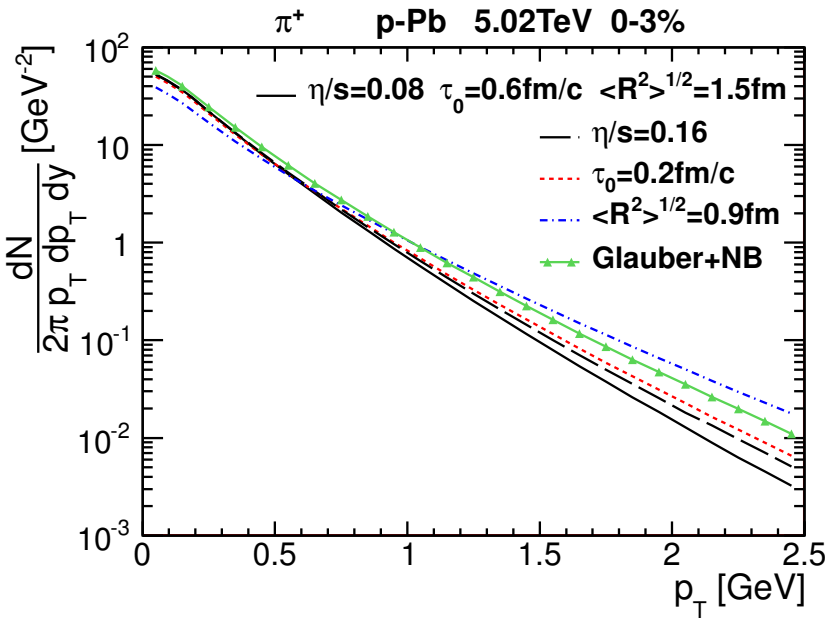


$$P(n) = \sum_i P_{part}(i) N p \lambda i, \kappa i(n)$$

Additional fluctuations of density (compared to Glauber)

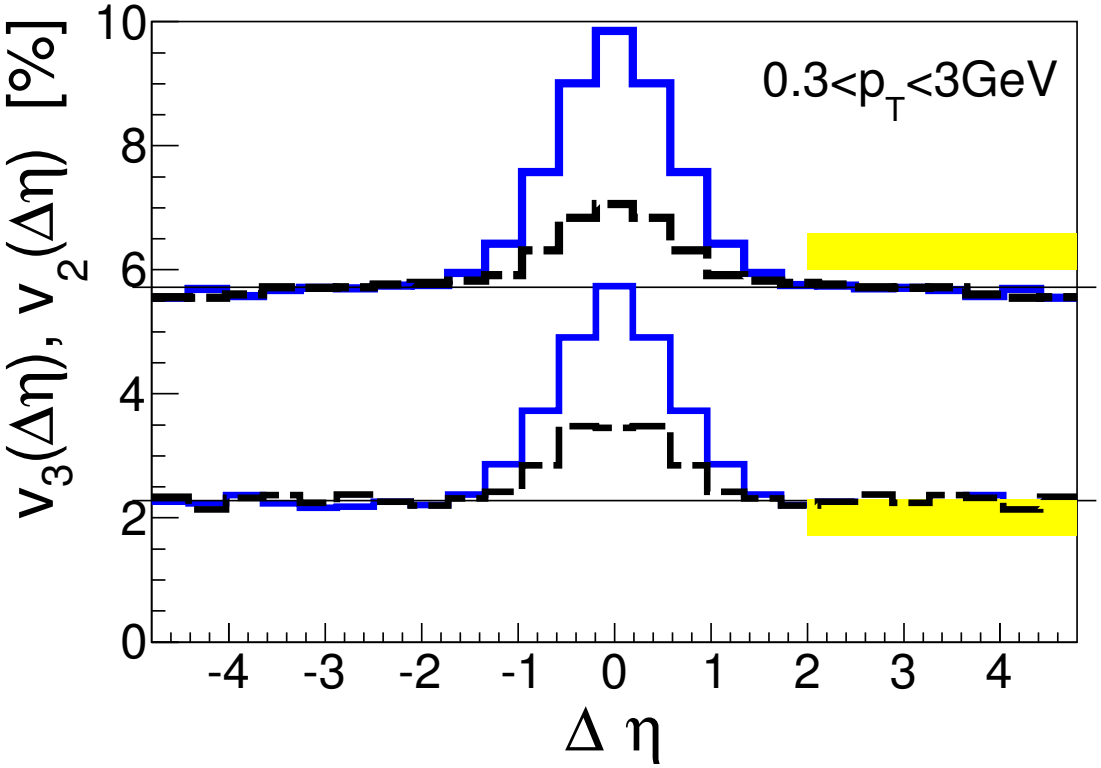
dependence on model details



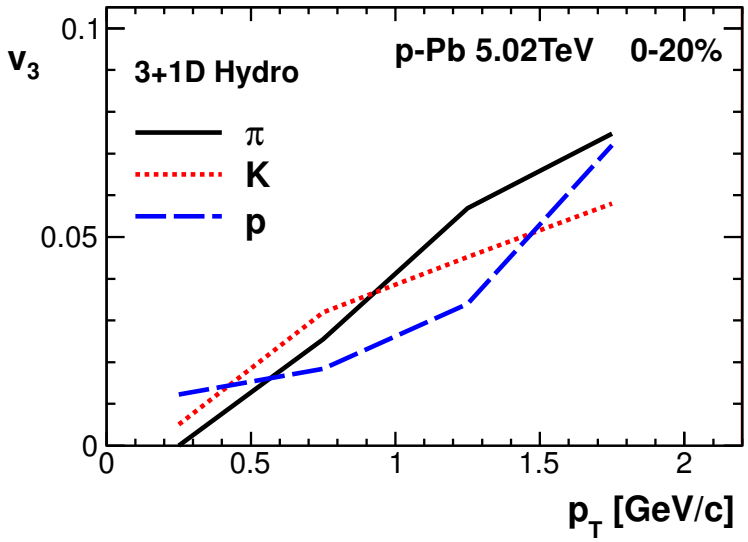


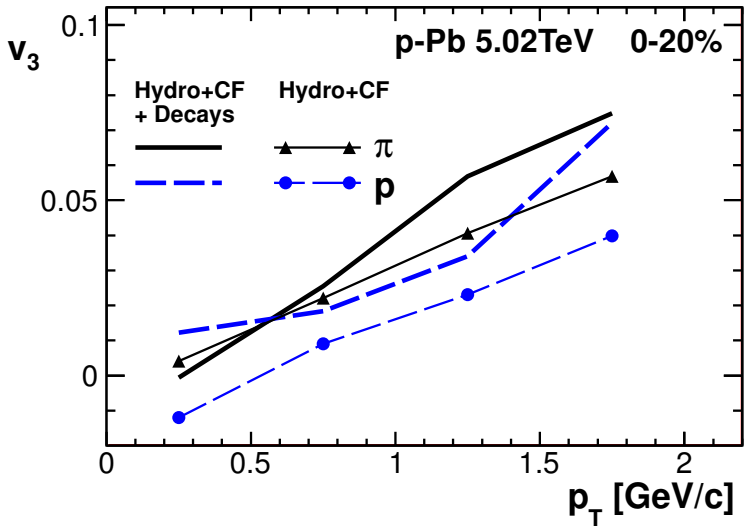
- response strength depends on details, initial eccentricity

Extracting the flow correlations



v_3 - small mass splitting

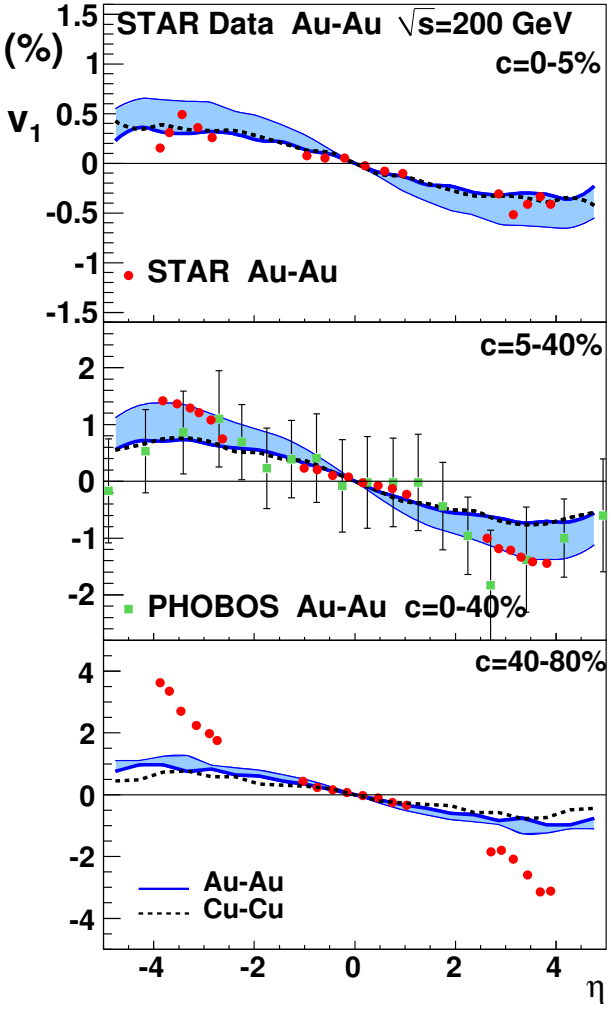


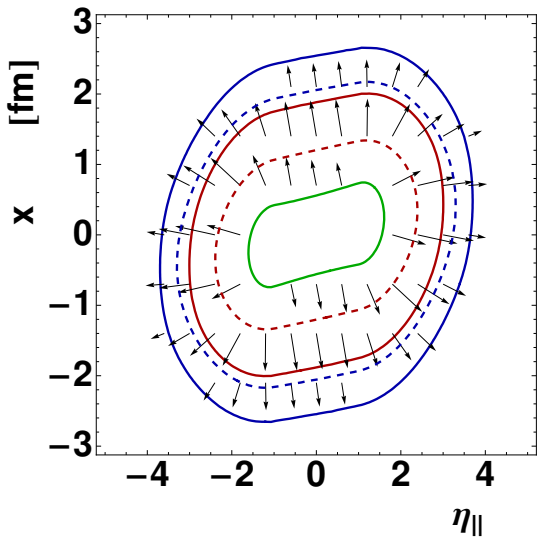


limited mass splitting

resonance decays spoil mass ordering

Directed flow- tilted source





Bozek, Wyskiel, Phys. Rev. C81, 054902 (2010)

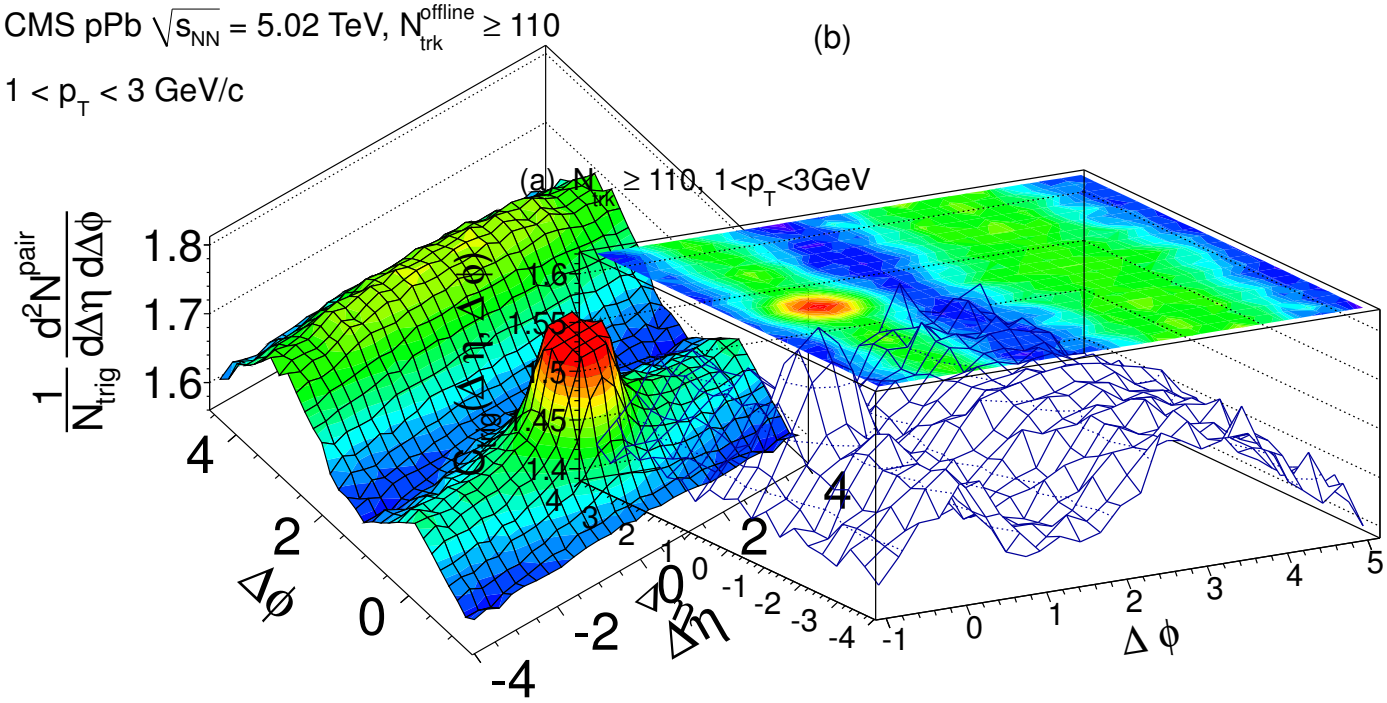
$$\partial_\tau u_x = -\frac{\partial_x p_\perp}{p+\epsilon}$$

$$\partial_\tau Y = -\frac{\partial_\eta p_\parallel}{\tau(p+\epsilon)}$$

tilted source $\rightarrow$ transverse pressure + longitudinal pressure

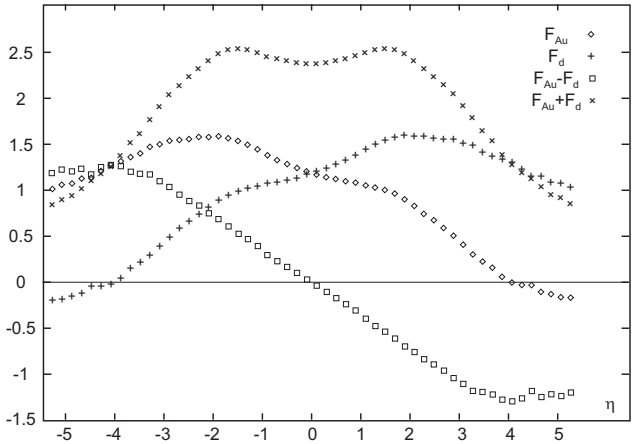
Glauber model

Ridge in p-Pb



PB, W.Broniowski, arXiv:1211.0845

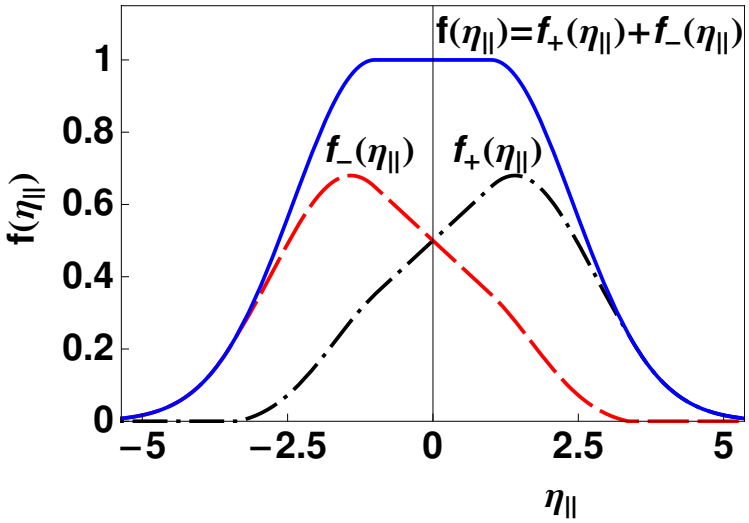
symmetric ridge also from CGC, K.Dusling, R. Venugopalan, arXiv:1210.3890, 1211.3701, 1302.7018

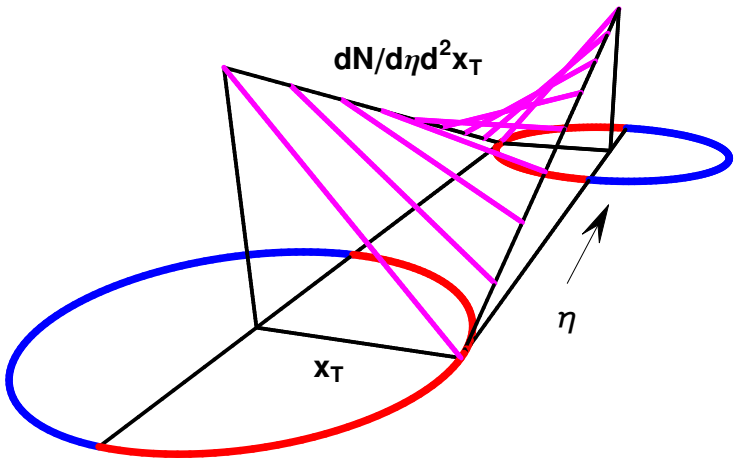


Asymmetric emission

(Białas, Czyż, Acta Phys.Polon.B36, 905 (2005)

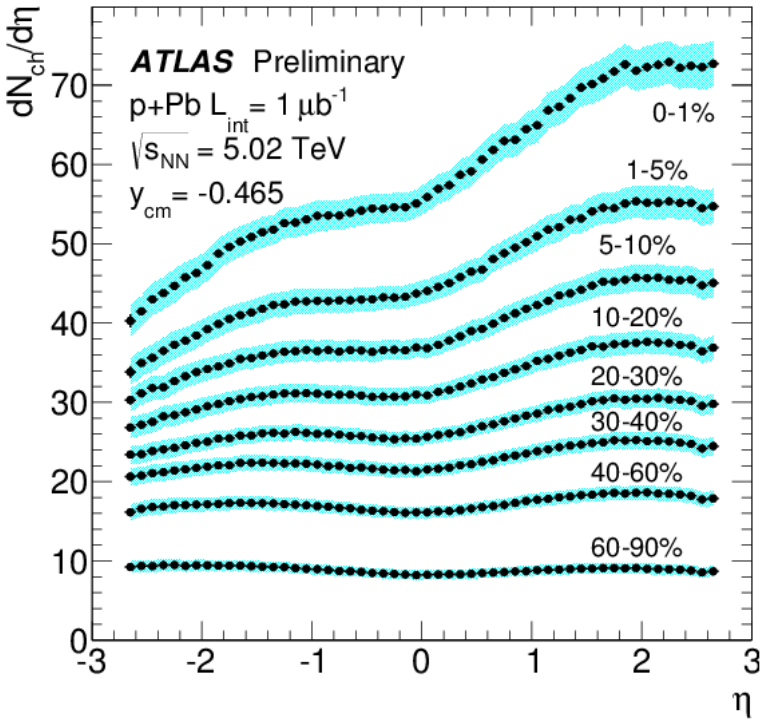
$$\rho(\eta, x, y) \propto f_+(\eta)N_+(x, y) + f_-(\eta)N_-(x, y)$$

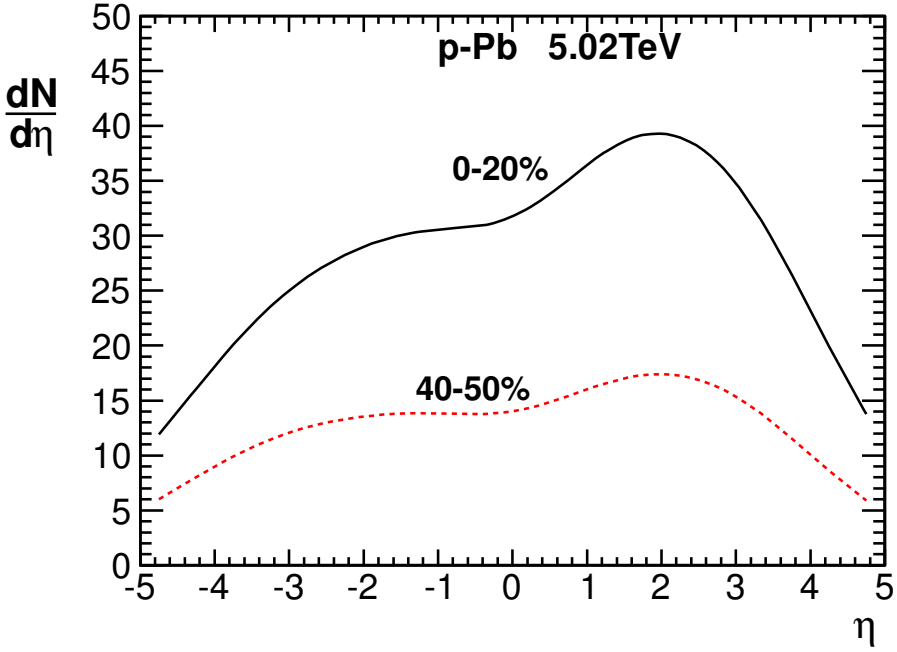




bremsstrahlung (Adil Gyulassy, Phys. Rev. C72, 034907 (2005)

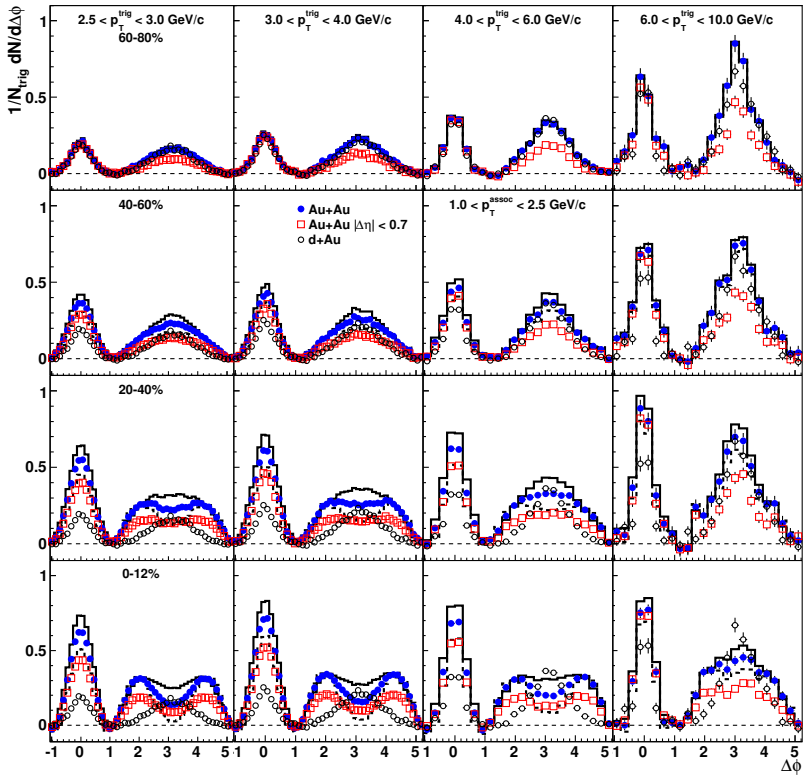
Asymmetric distributions



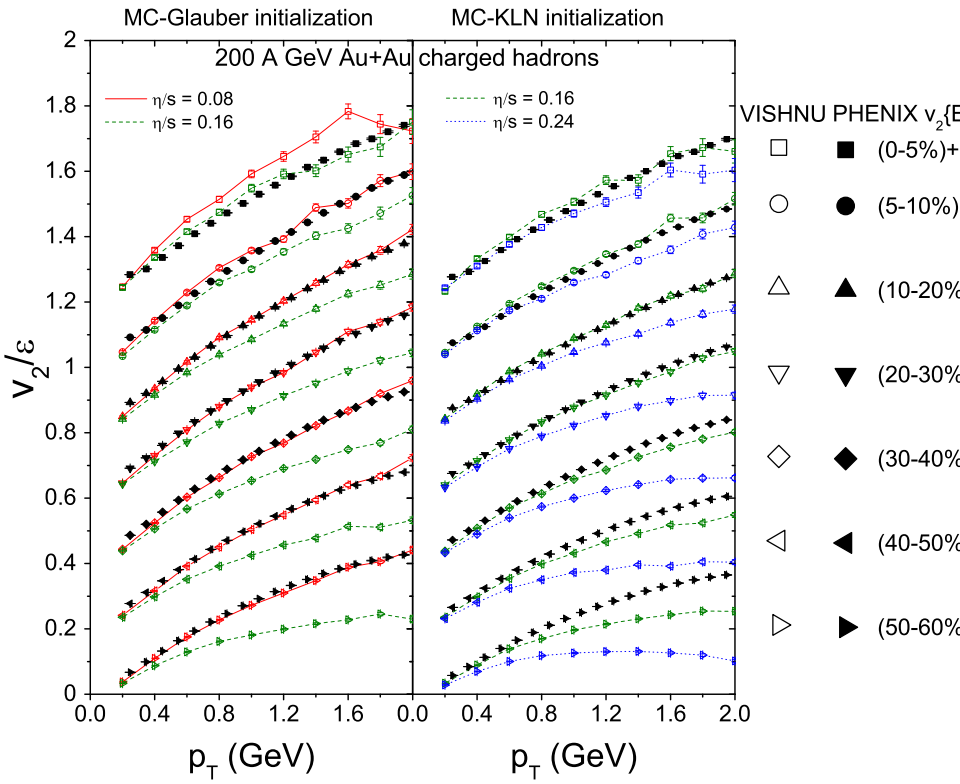


Can we reduce uncertainties?

go back to very peripheral A-A and smaller systems



STAR-arXiv:1004.2377



Song, Bass, Heinz, Hirano, Shen-arXiv:1101.4638

also jet modification, dijet asymmetry, PID flow, HBT

New observables in p+A & Ridge from cascade

Adam Bzdak

AGH University of Science and Technology, Kraków

Situation
New observables
Multi-particle cumulants
AMPT, escape mechanism
Conclusions

The situation in p+A:

$C_2(\Delta\varphi)$	hydro	glasma diagr.
$v_2(p_t)$	hydro	dipole-domain
$v_3(p_t)$	hydro	dipole-domain
$v_{2,3}$ vs N_{track}	hydro	
$\langle p_t \rangle$ vs N_{track}	hydro	geom. scaling
v_2 mass splitting	hydro	
$v_2\{2,4,6,8\}$	hydro	dipole-domain
$v_2(p_t)$ in d+Au	hydro	
$v_3(p_t)$ in He3+Au	hydro	

gluon bremsstrahlung

The situation in p+A:

$C_2(\Delta\varphi)$	hydro	glasma diagr.	
$v_2(p_t)$	hydro	dipole-domain	
$v_3(p_t)$	hydro	dipole-domain	$v_3 = 0$ in glas-dia
$v_{2,3}$ vs N_{track}	hydro		
$\langle p_t \rangle$ vs N_{track}	hydro	geom. scaling	
v_2 mass splitting	hydro		
$v_2\{2,4,6,8\}$	hydro	dipole-domain	$v_2^4\{4\} < 0$ in glas-dia
$v_2(p_t)$ in d+Au	hydro		problem for dip-dom
$v_3(p_t)$ in He3+Au	hydro		problem for dip-dom

gluon bremsstrahlung

Hydro is running strong (uncertain status in p+p!). Hydro at high p_t is very questionable.

The dipole-domain model sounds good but it seems (?) to have problems with d+Au and He3+Au (the more dipoles the smaller v_n).

Glasma diagrams might not be relevant in high-multiplicity p+A ($v_3 = 0$, $v_2^4\{4\} < 0$).

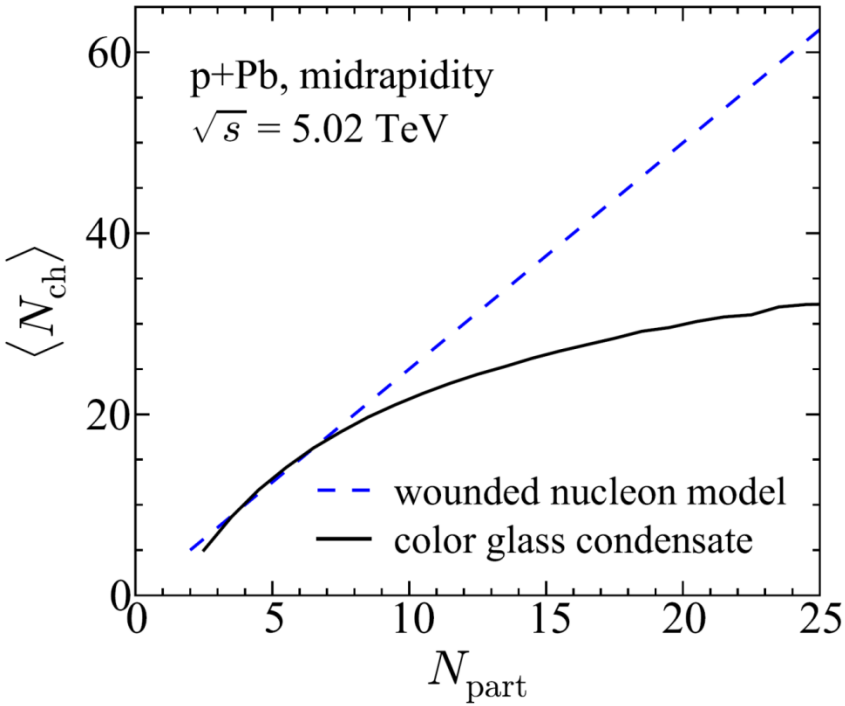
P. Bozek, PRC 85 (2012) 014911
K. Dusling, R. Venugopalan, PRD 87 (2013), 094034
A. Dumitru, A.V. Giannini, NPA 933 (2014) 212
V. Skokov, arXiv:1412.5191

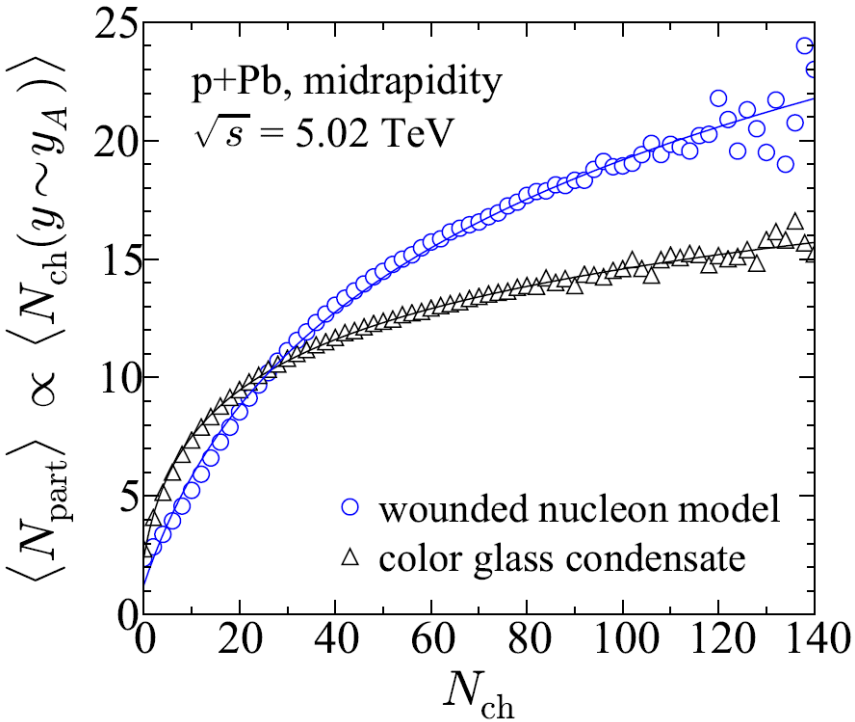
Other possible tests of initial vs. final state physics.

CGC: $\langle N_{pA} \rangle \sim \ln(N_{part})$ checked in rcBK, IP-Glasma, KLN

thanks to B.Schenke

WNM: $\langle N_{pA} \rangle \sim N_{part}$

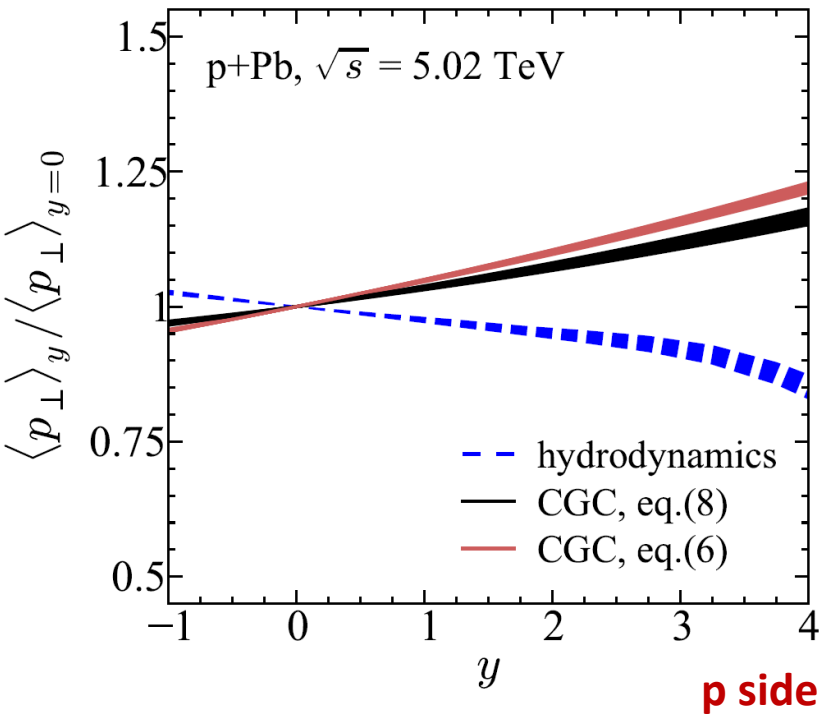




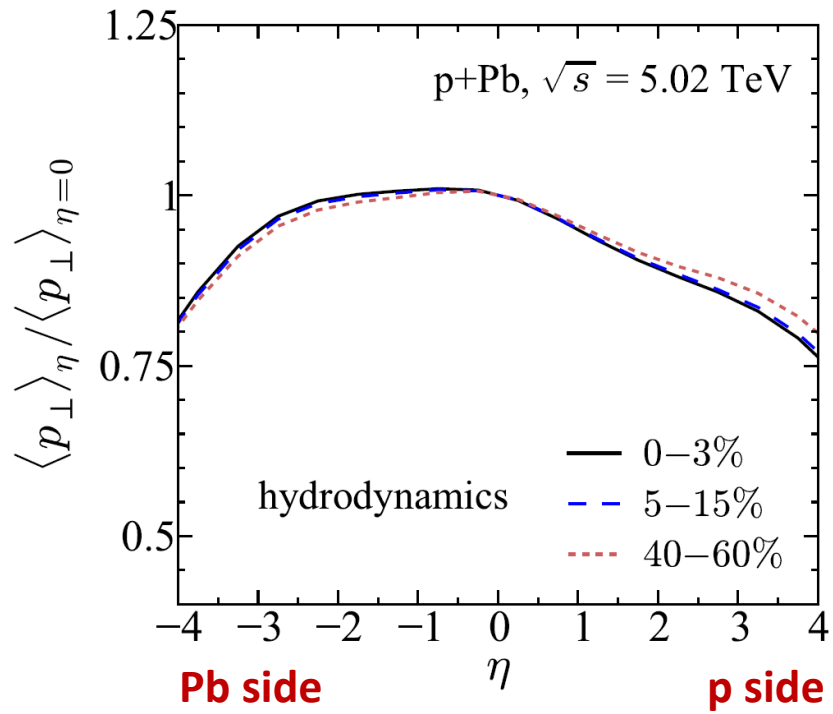
AB, V.Skokov, PRL 111 (2013) 182301

$\langle p_T \rangle$ versus η on proton side

$\langle p_t \rangle \sim Q_s^A$, Q_s^A is growing with y



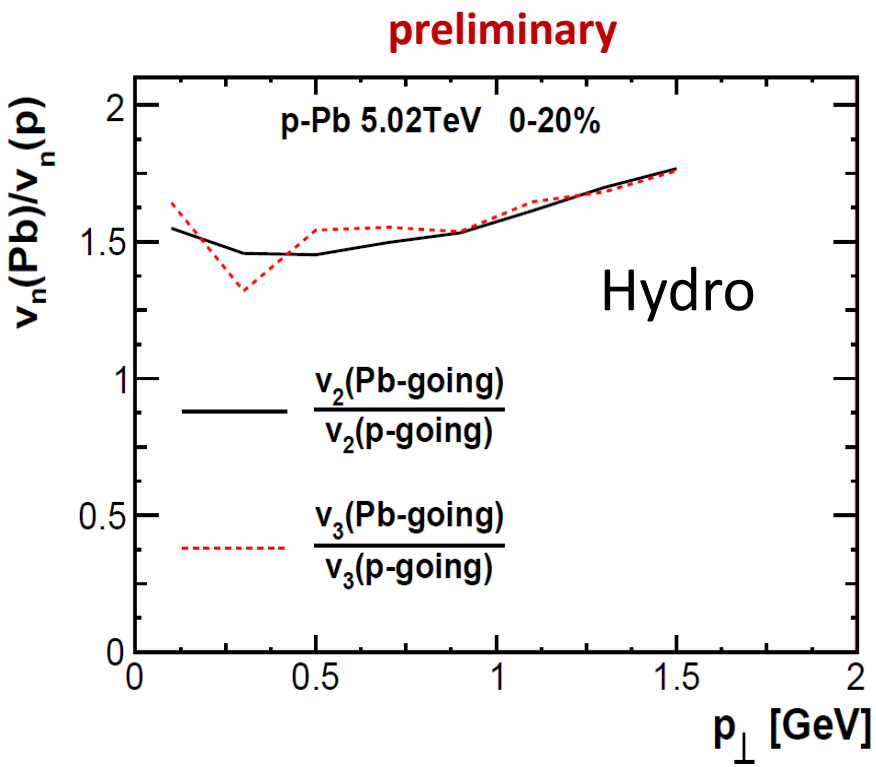
less stuff on proton side

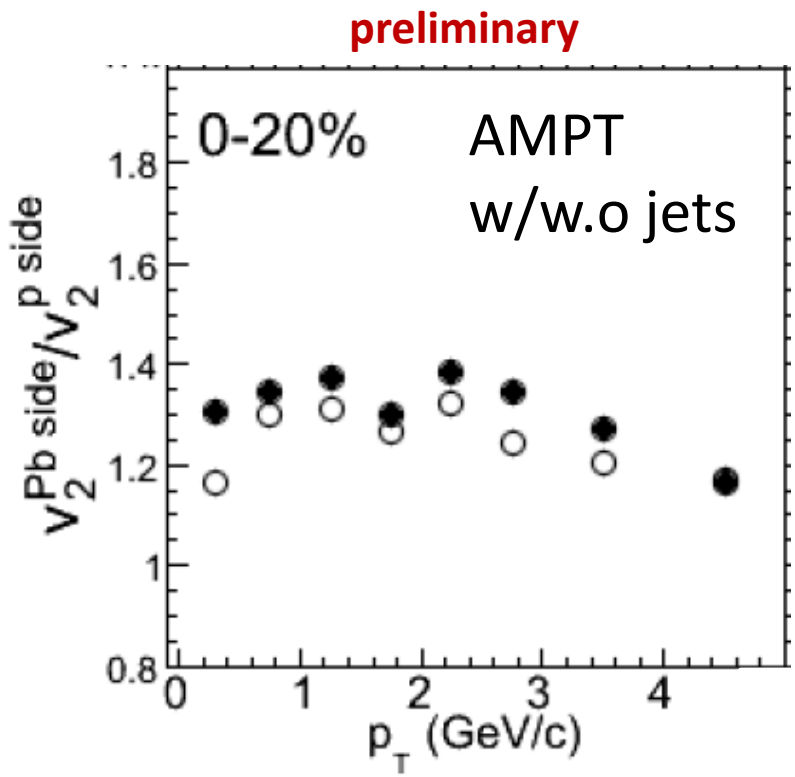


CGC predictions are not so clear, many uncertainties.
Anyway, it is worth giving a try.

P.Bozek, AB, V.Skokov, PLB 728 (2014) 662

v_n on proton ($-4 < \eta < -2.5$) and nucleus ($2.5 < \eta < 4$) sides





for 40-60% the ratio is 1

The dipole-domain model predictions are warranted.

P. Bozek, AB, G.L. Ma, in preparation

Correlations:

N.Borghini, P.M.Dinh, J.-Y.Ollitrault, PRC 63 (2001) 054906

$$(v_2\{2\})^2 = \left\langle e^{i2(\varphi_1-\varphi_2)} \right\rangle$$

$$= \left\langle v_2^2 \right\rangle + c_2$$

c_m is non-flow

$$(v_2\{4\})^4 = -\left\langle e^{i2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \right\rangle + 2\left\langle e^{i2(\varphi_1-\varphi_2)} \right\rangle^2$$

$$= -\langle v_2^4 \rangle + 2\left\langle v_2^2 \right\rangle^2 + c_4$$

etc.

if in each event $v_2 = \overline{v_2}$
and no non-flow

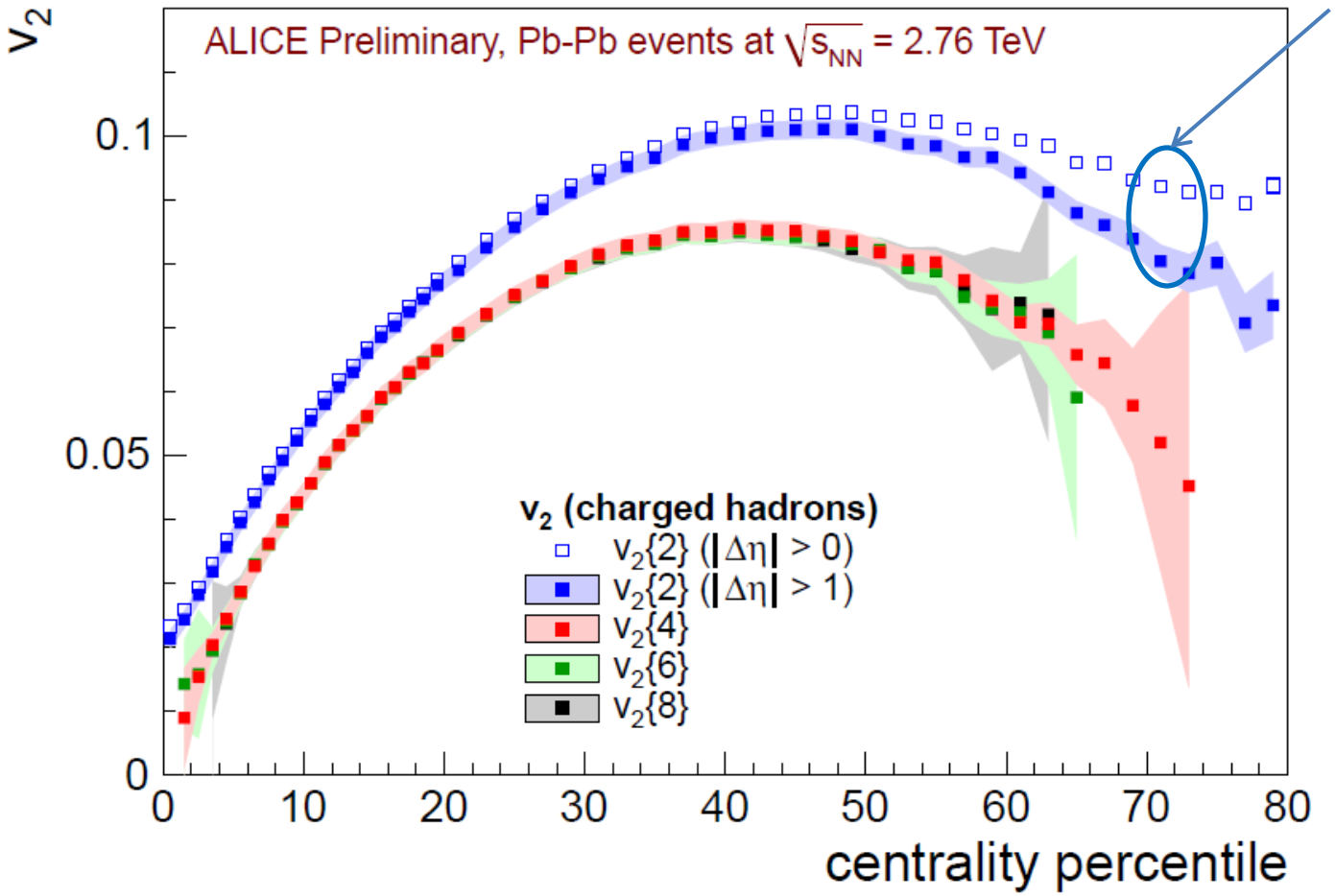
$\rightarrow$

$$v_2\{2\} = \overline{v_2}$$

$$v_2\{m\} = \overline{v_2}$$

Pb-Pb

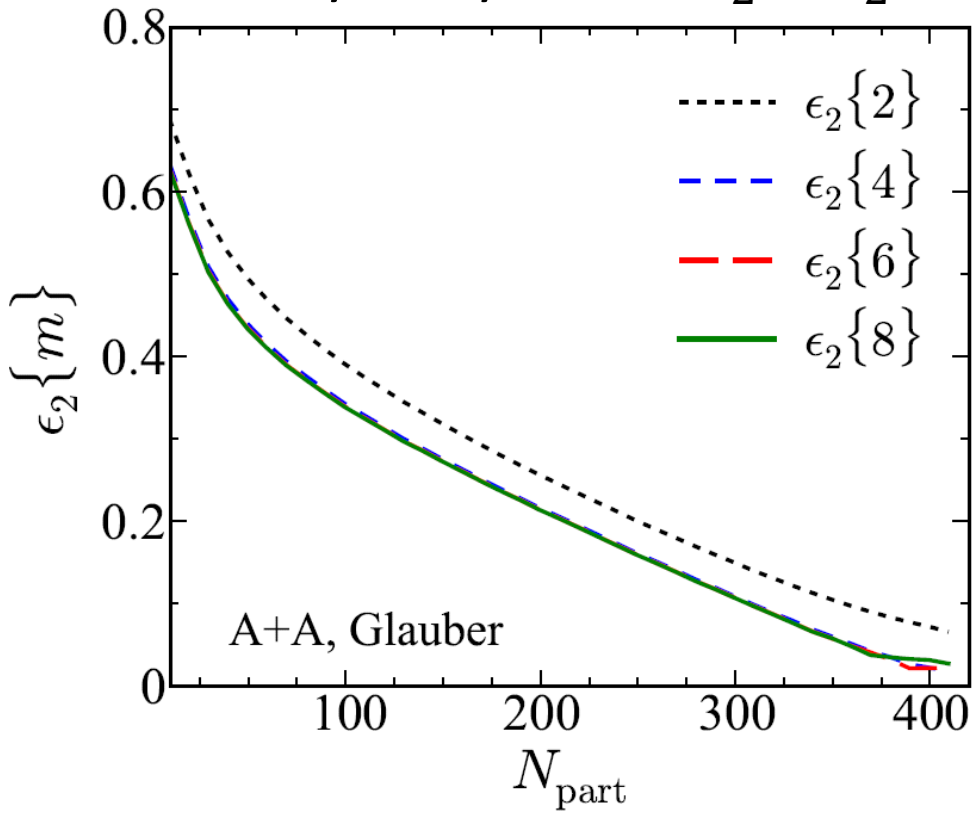
this difference is due to c_2

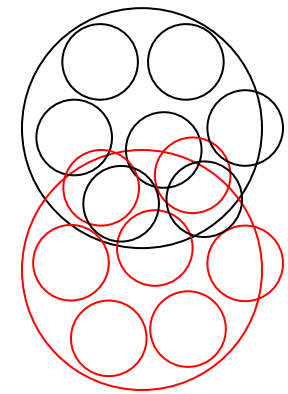


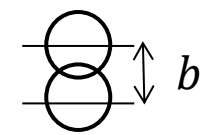
difference between $v_2\{2\}$ and $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ is due to flow fluctuations

MC calculation for ellipticities (so called Glauber MC)

in hydrodynamics $v_2 \sim \epsilon_2$







$\sigma_{in}(b) \sim e^{-b^2/\gamma}$

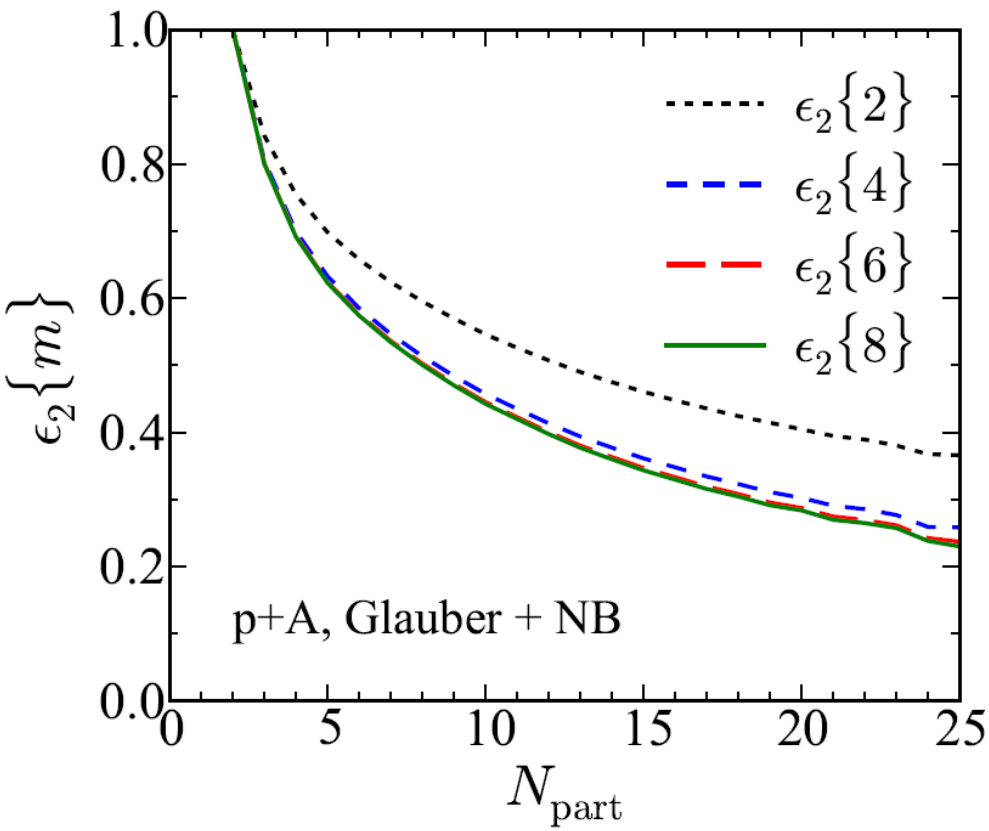
inelastic cross-section

Indication that $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ in A+A follows from initial geometry. What about p+A ?

AB, P. Bozek, L. McLerran, NPA 927 (2014) 15

p+A with negative binomial distribution

for many other implementations it looks similar



Hydro calculations:

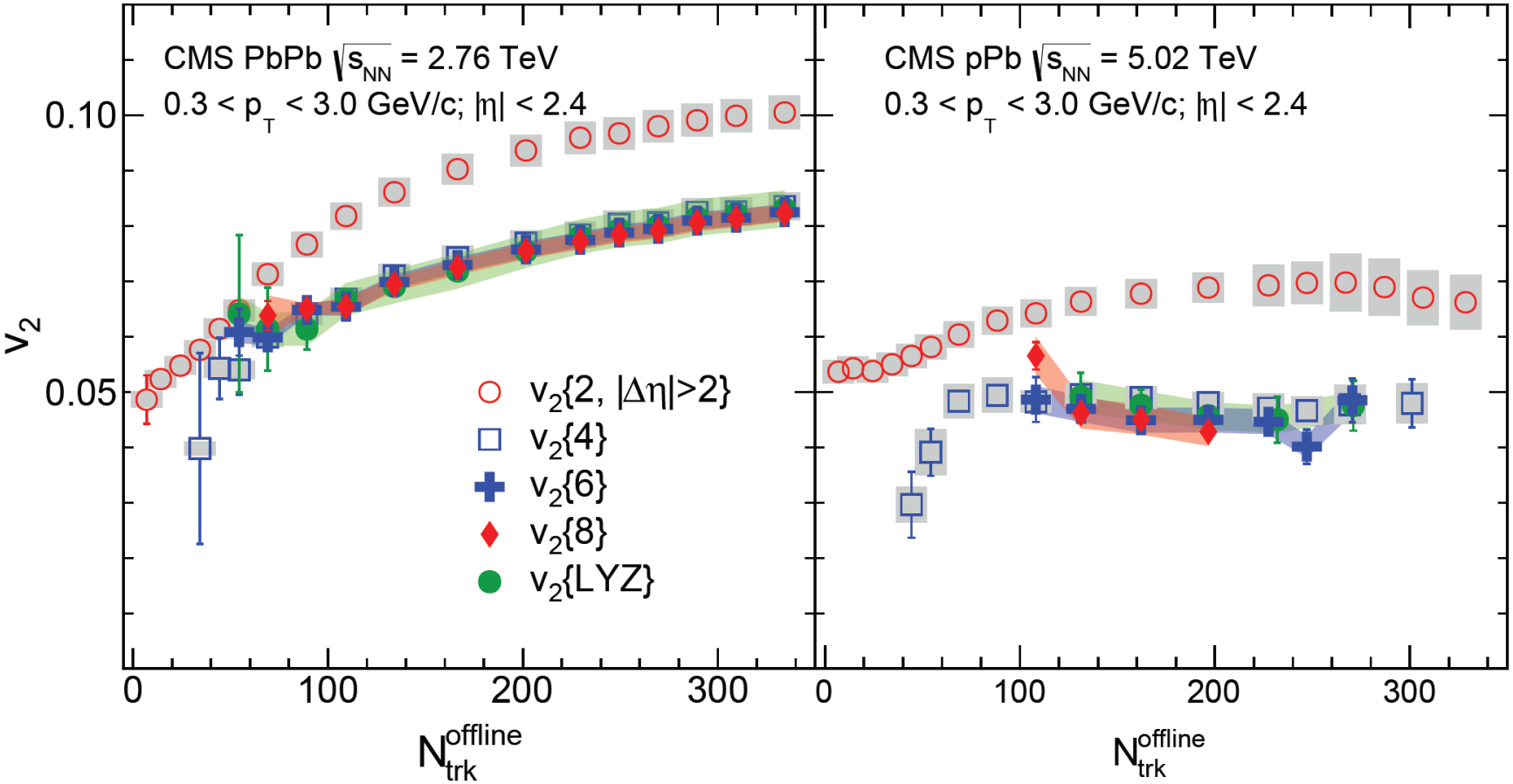
$$v_2\{2\} = 0.082 \pm 0.002$$
$$v_2\{4\} = 0.055 \pm 0.004$$
$$v_2\{6\} = 0.052 \pm 0.005$$

$$\left\langle \epsilon_2^{2n} \right\rangle = \frac{n! \lim\limits_{z\to 0} \frac{d^n}{dz^n} \left\langle I_0\left(2\sqrt{z}r^2\right)\right\rangle^N}{\lim\limits_{z\to 0} \frac{d^{2n}}{dz^{2n}} \left\langle e^{-r^2 z}\right\rangle^N}$$

AB, P. Bozek, L. McLerran, NPA 927 (2014) 15
AB, V. Skokov, arXiv:1312.7349

CMS data

CMS Coll., arXiv:1502.05382



$$v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$

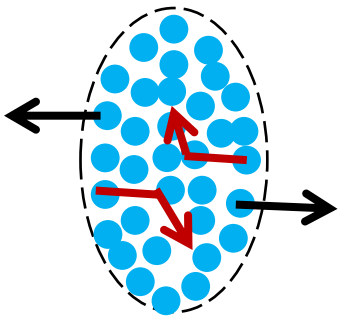
We can keep arguing why hydrodynamics cannot work in p+A even if all data show it works.

Or perhaps we should try to understand why hydrodynamics can describe all data even if *technically speaking* it might not be the right framework.

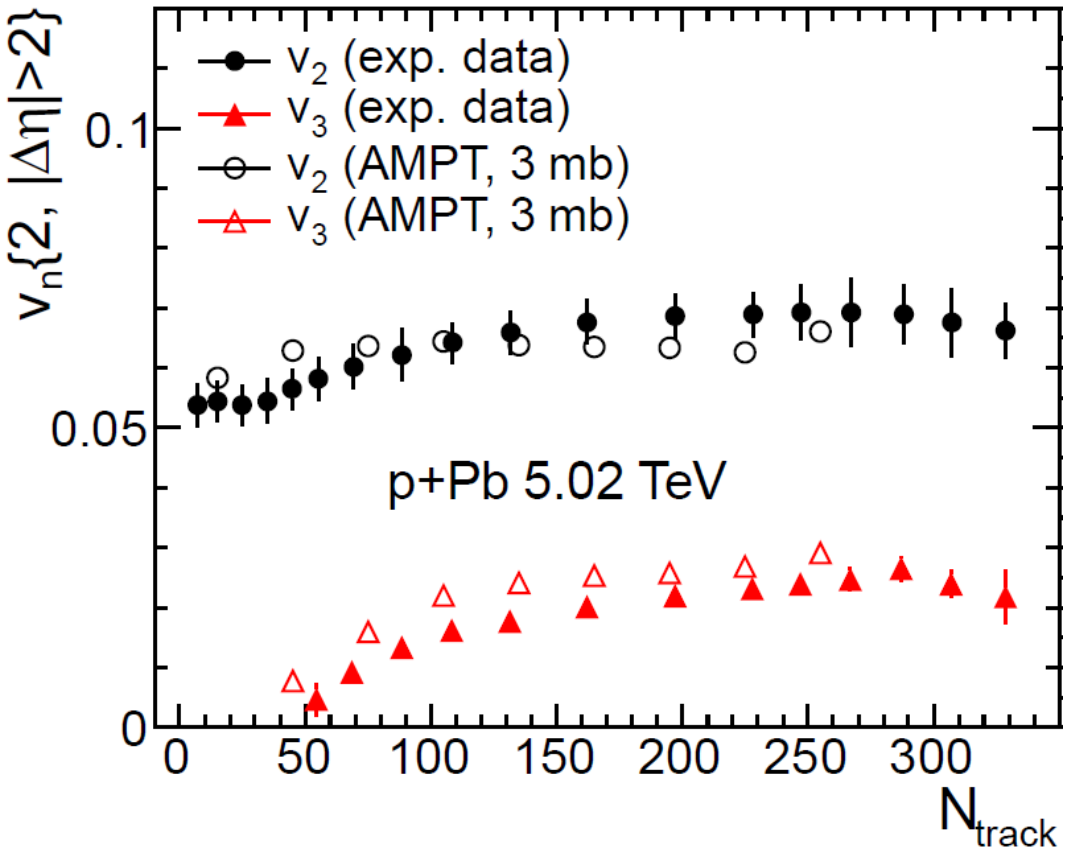
AMPT model (elastic scatterings of *partons*)

Z.W. Lin, C.M. Ko, B.A. Li, B. Zhang, S. Pal, PRC 72 (2005) 064901

$$\frac{d\sigma_{gg}}{dt} \simeq \frac{9\pi\alpha_s^2}{2(t-\mu^2)^2}$$

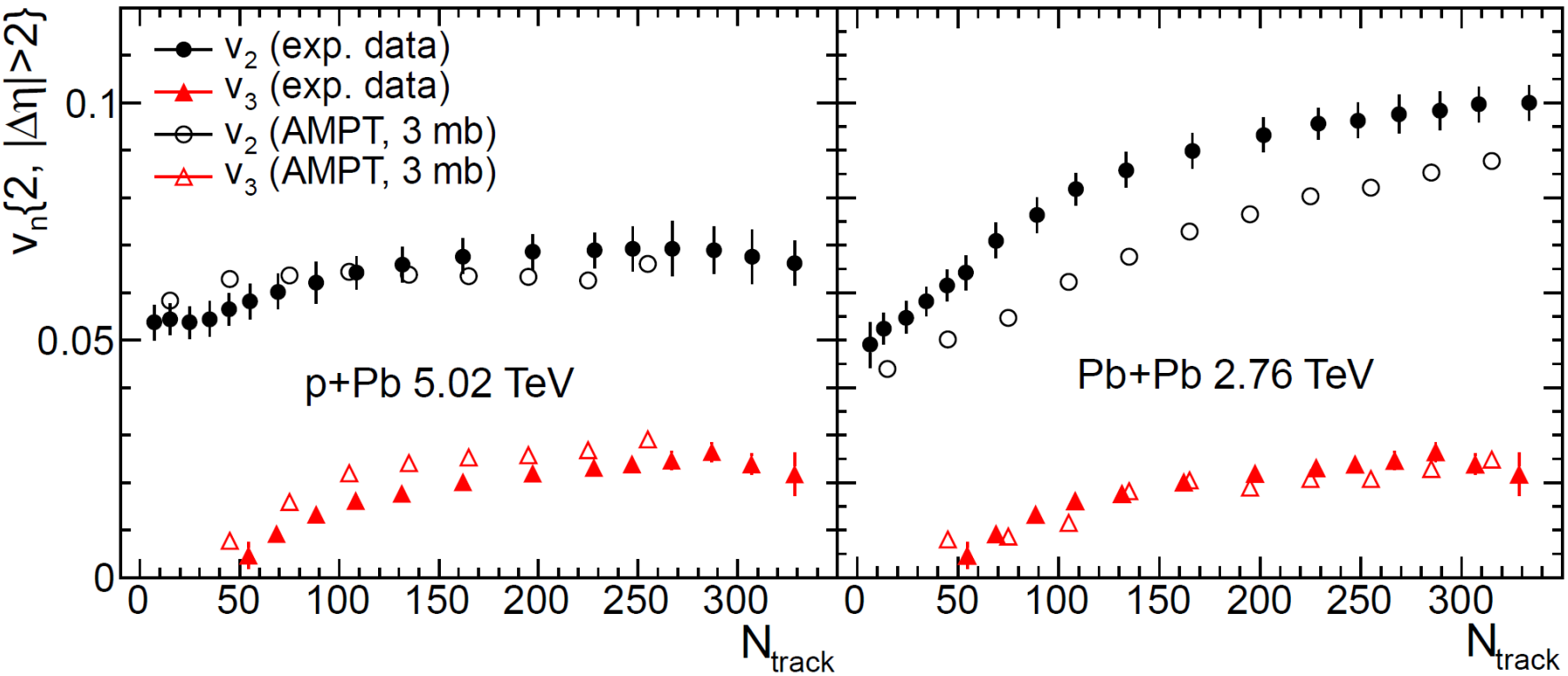


$\boldsymbol{\sigma = 1.5 - 3}$ **mb**



AB, G.L. Ma, PRL 113 (2014) 252301
G.L. Ma, AB, PLB 739 (2014) 209

p+A and peripheral Pb+Pb



v_2 (but not v_3) in Pb+Pb is underestimated

The only calculation (so far) that compares apples with apples

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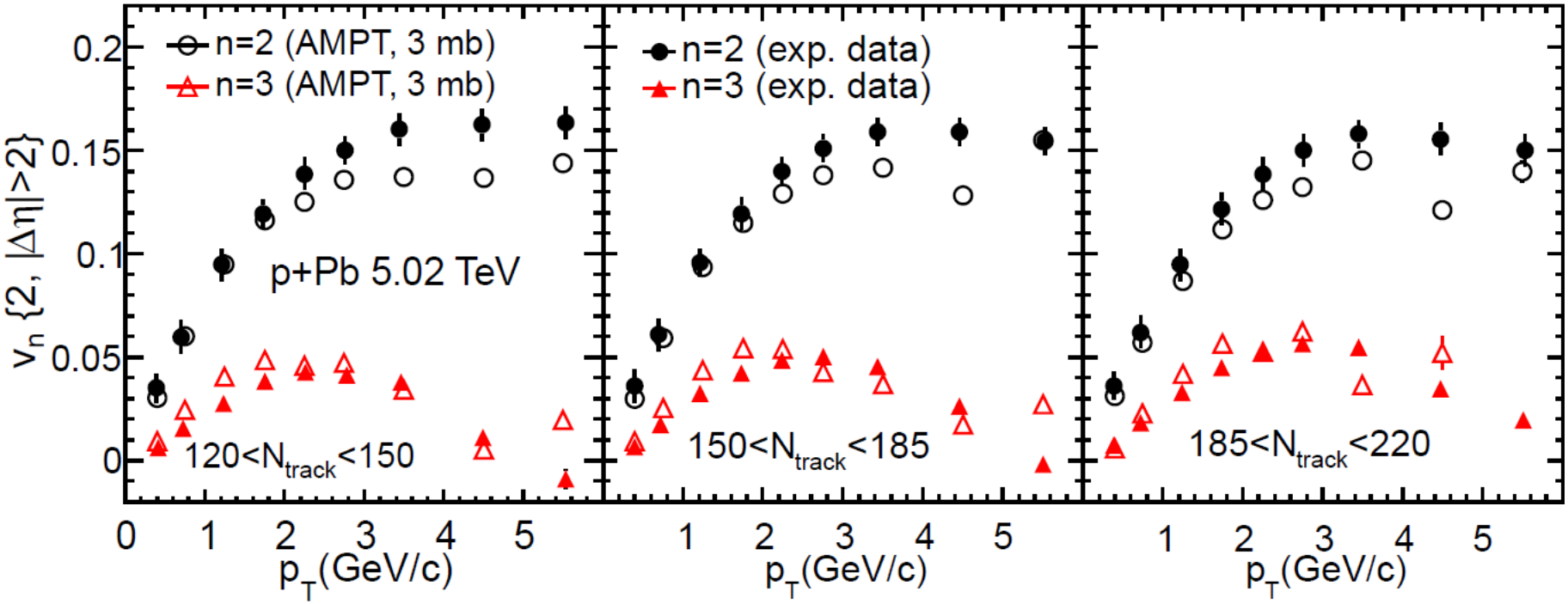
G.L. Ma, AB, PLB 739 (2014) 209

$v_2(p_t)$ and $v_3(p_t)$ in p+Pb



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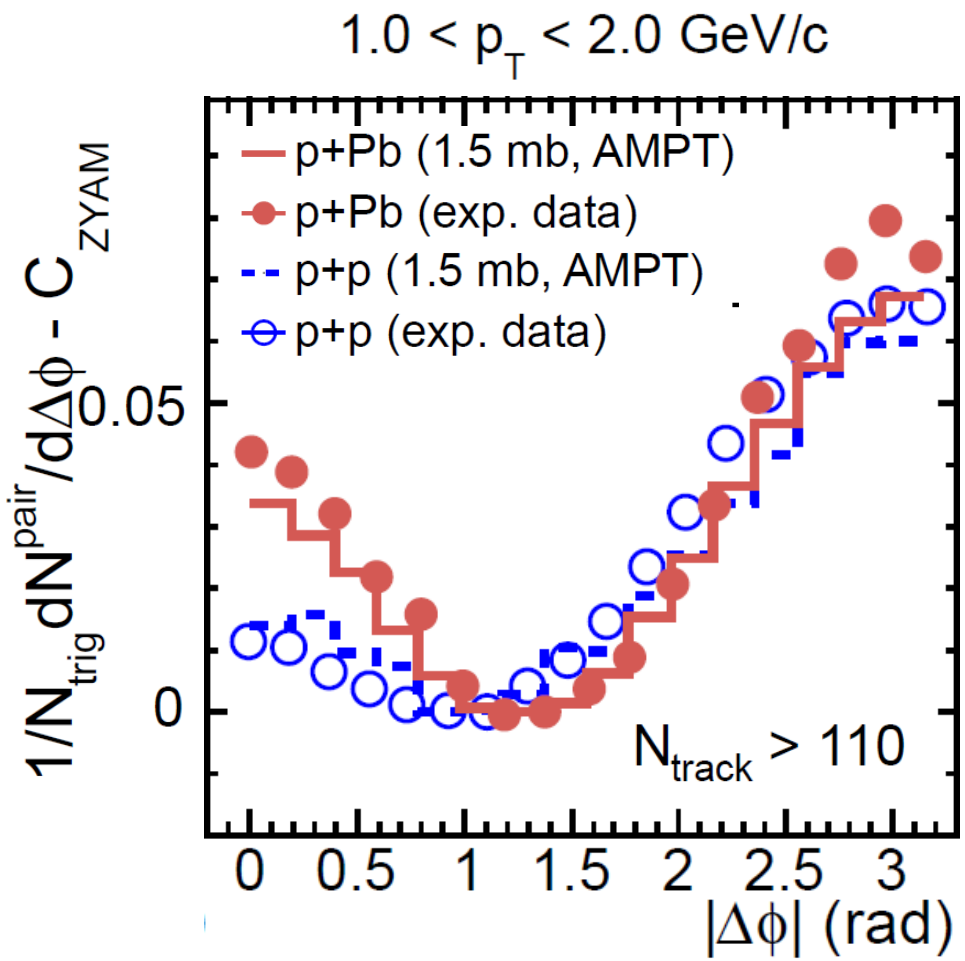
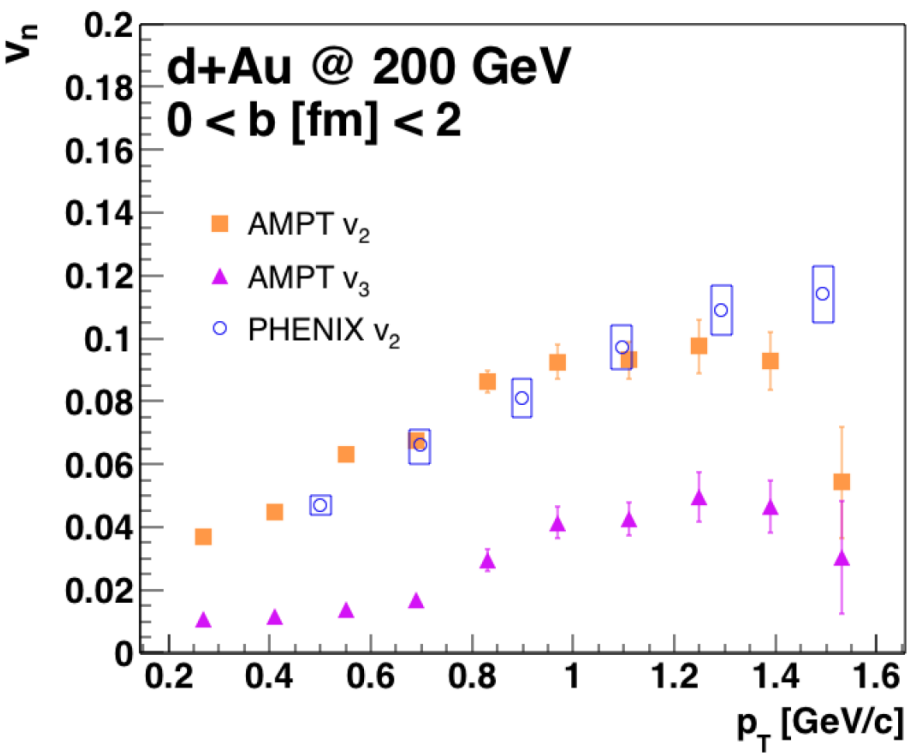
G.L. Ma, AB, PLB 739 (2014) 209

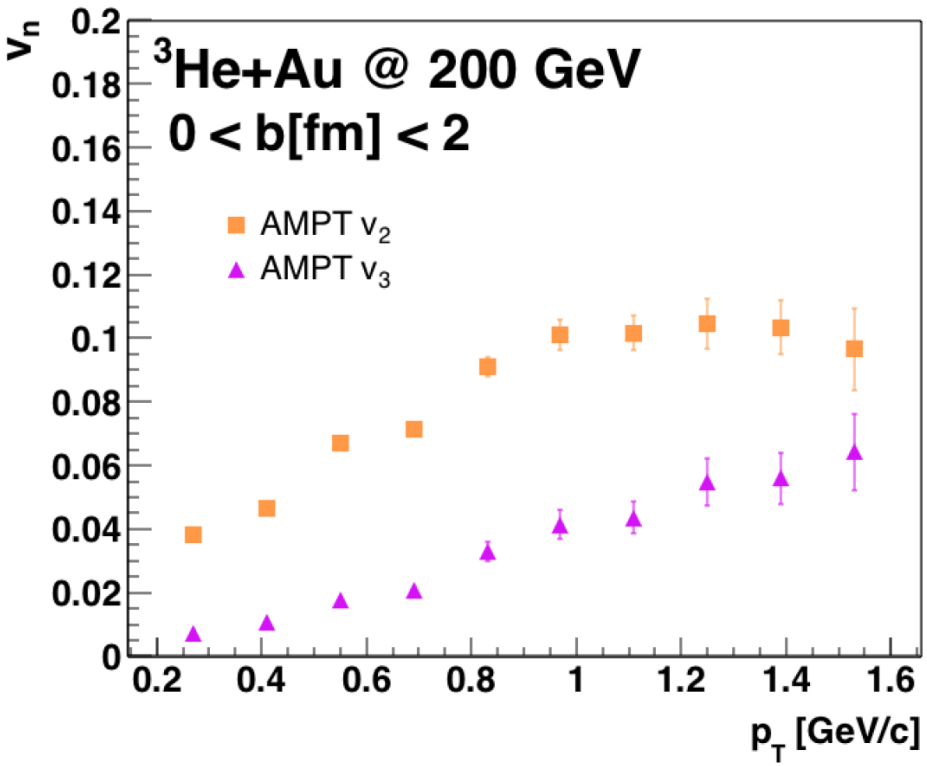
Two-particle correlation function in p+p and p+Pb



G.L. Ma, AB, PLB 739 (2014) 209

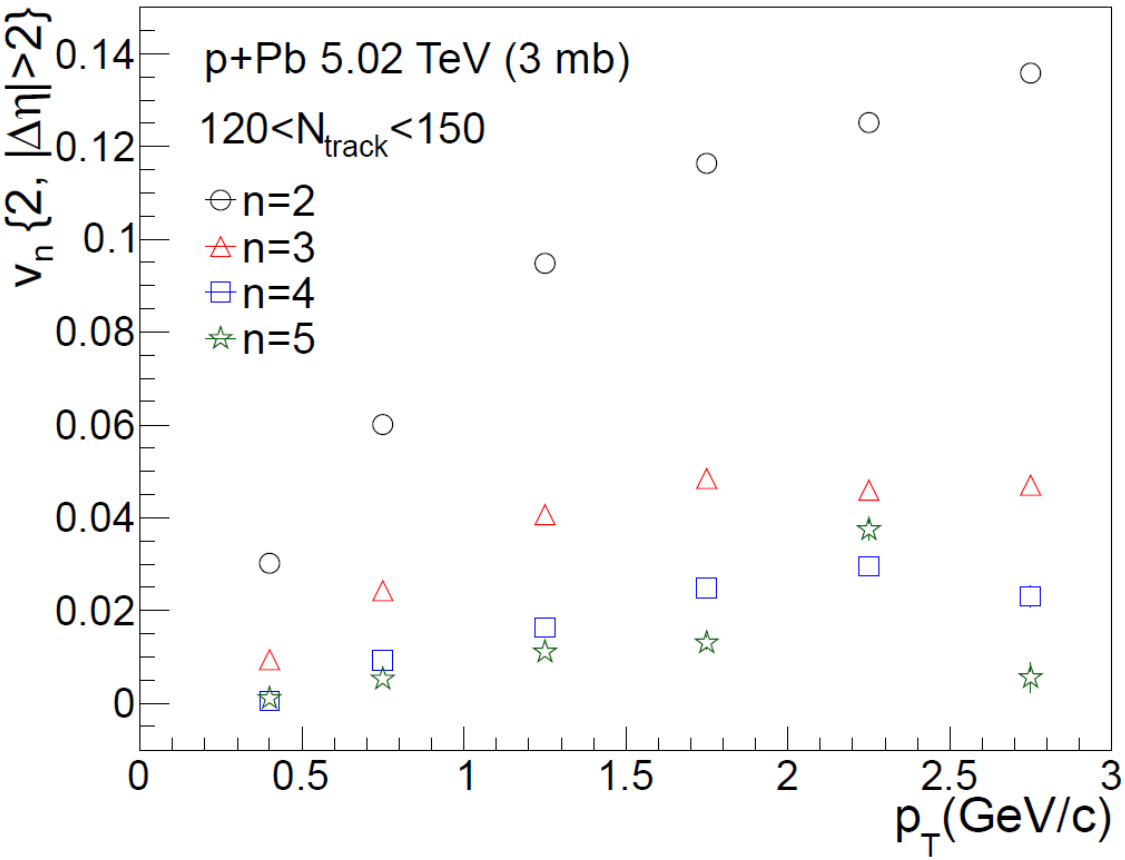
d+Au and He3+Au





J.D. Orjuela Koop, A. Adare, D. McGlinchey, J.L. Nagle, arXiv:1501.06880

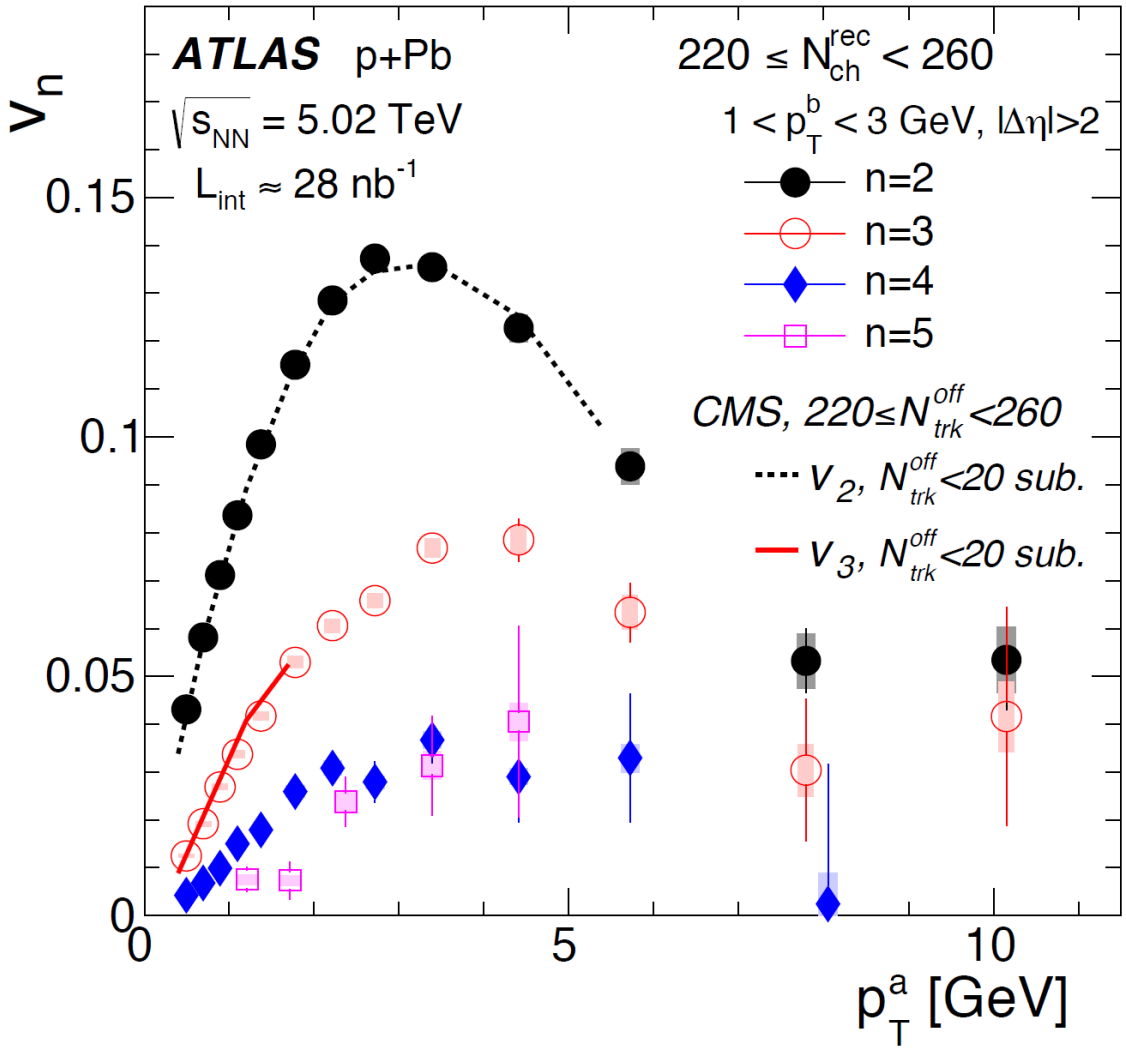
Predictions for higher $v_n(p_t)$ in p+Pb



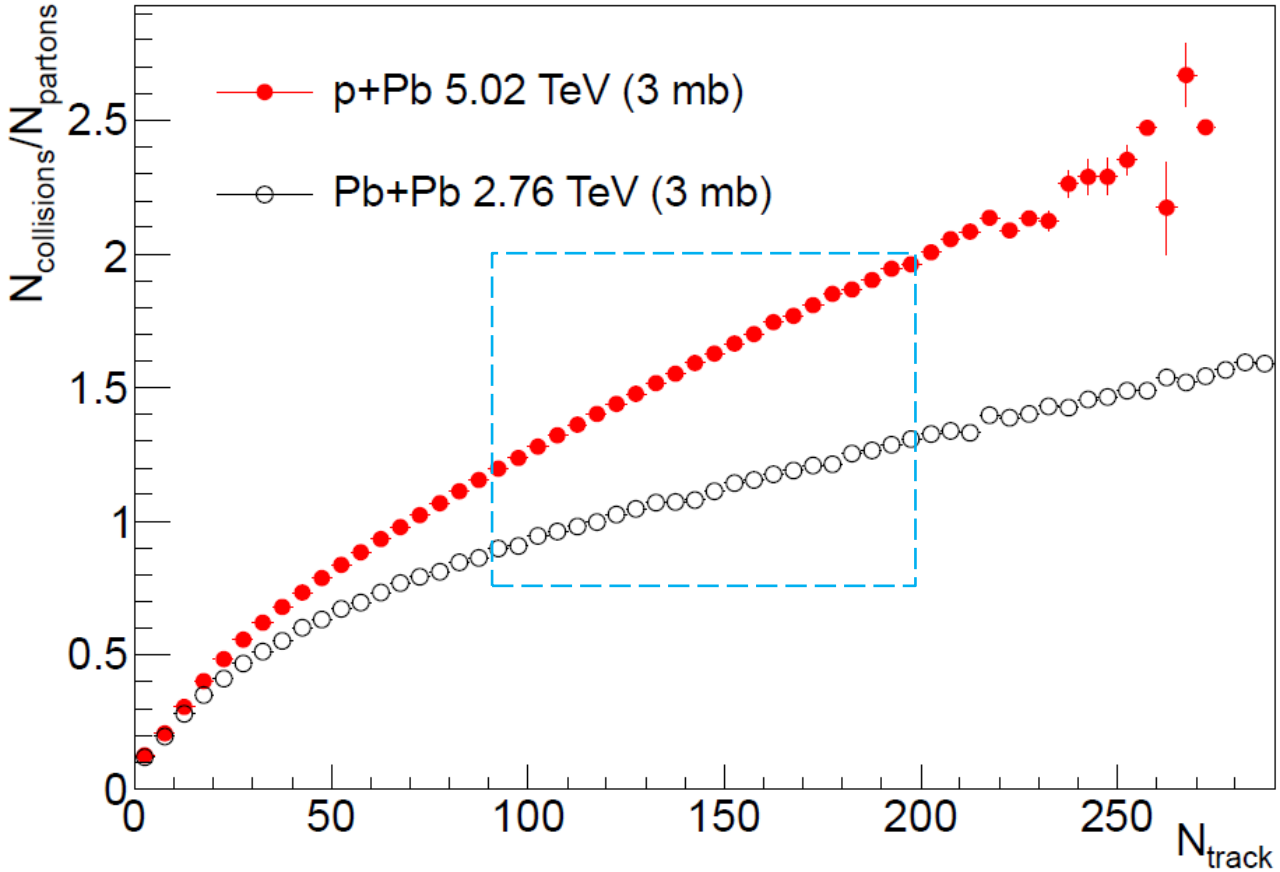
v_4 and v_5 **roughly equal and smaller by a factor of two than** v_3

AB, G.L. Ma, PRL 113 (2014) 252301
G.L. Ma, AB, PLB 739 (2014) 209

As seen by ATLAS ...



We get the right signal with the minimal number of scatterings



Roughly $1-2$ collisions per parton

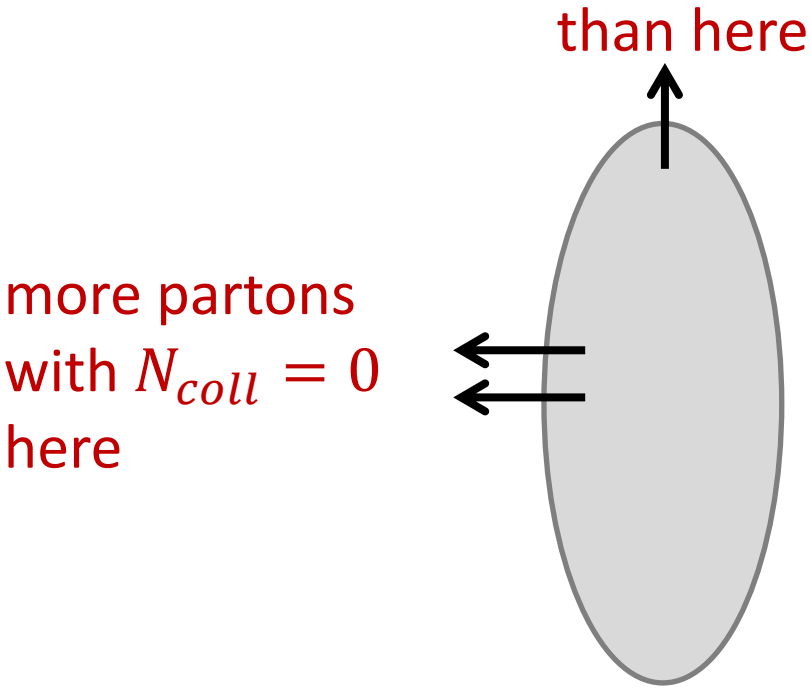
With $1-2$ collisions per parton it is natural to question hydro in p+p, p+A and peripheral A+A collisions

Hydro still capable to fit the data (it is flexible enough, η/s)

Jets are not modified in p+A, perhaps it indicates that the number of collisions is indeed small

$1-2$ collisions per parton is consistent with escape mechanizm

Hydrodynamics is difficult to distinguish from **escape mechanizm**.
v_2 is carried, e.g., by non-interacting partons.



larger probability for partons to escape along the short axis

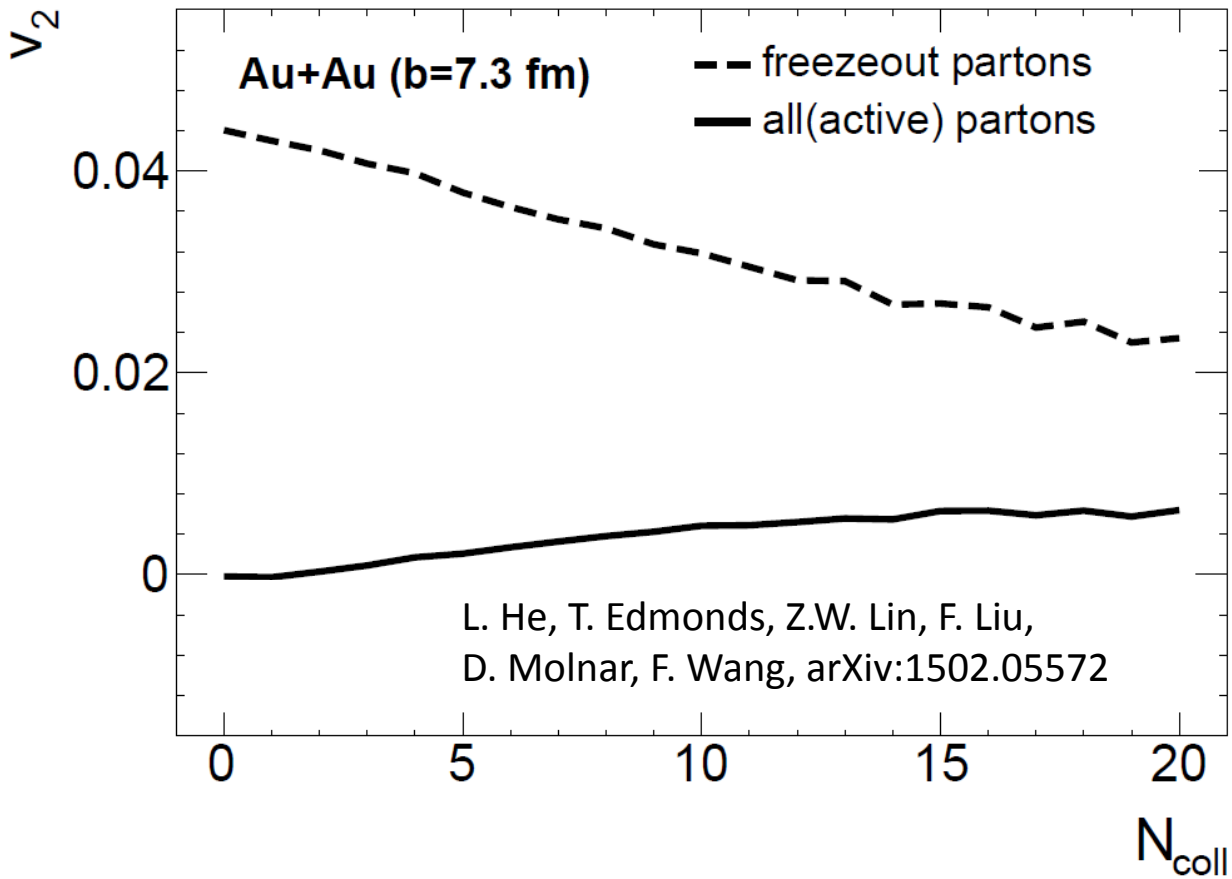
L. He, T. Edmonds, Z.W. Lin, F. Liu,
D. Molnar, F. Wang, arXiv:1502.05572

Obvious v_2, v_3 for larger p_t.

As shown recently, this is dominant source of elliptic and triangular *flow* in AMPT

Partons that interacted exactly 1, 2, …, collisions carry large v_n.

Partons that interacted $N+1$ (or $2, 3, 4 \ldots$) times carry very small v_2 just after collision number N.



Conclusions:

Hydro survives all data in p+Pb, d+Au and He3+Au

Hydro questionable for higher p_t

New predictions: $\langle p_t \rangle, v_2, v_3$ vs. η; $\langle N_{ch} \rangle$ vs. N_{part}

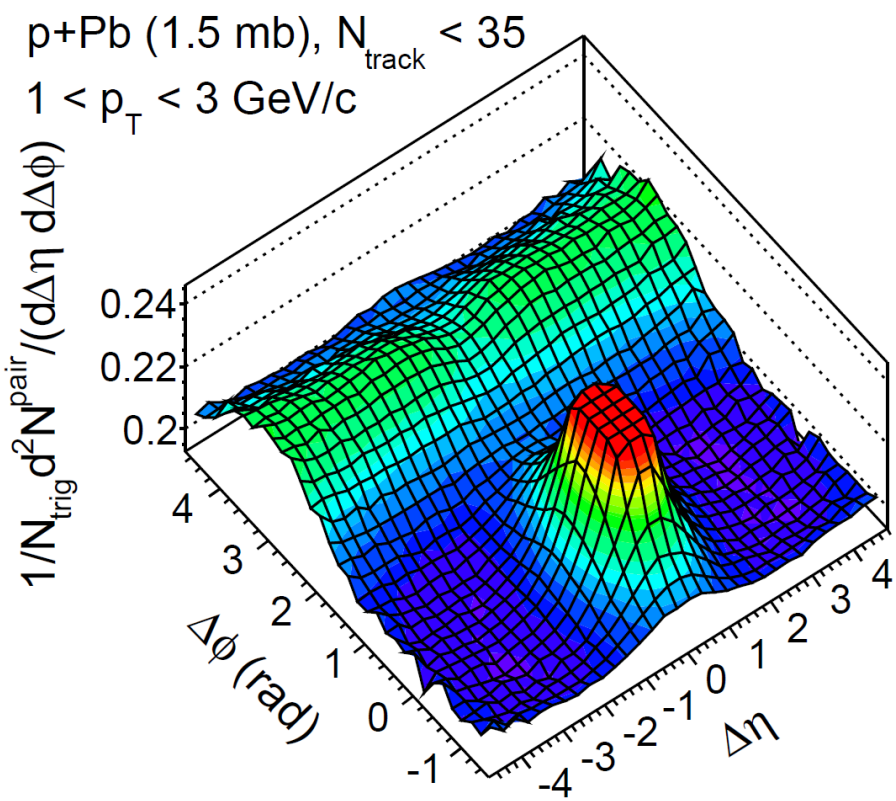
Cascade with small cross-section and minimal number of scatterings works pretty well for high and low p_t

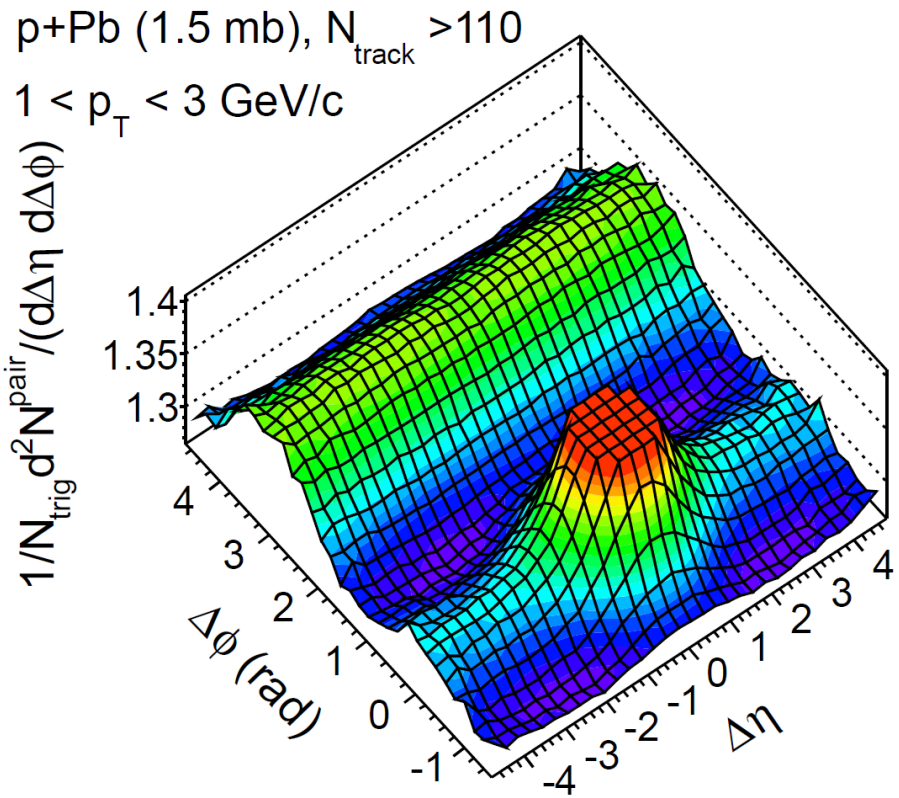
All we need is $\mathbf{1-2}$ collisions per parton

This mechanism results in hydro-like results but it is not hydro

Backup

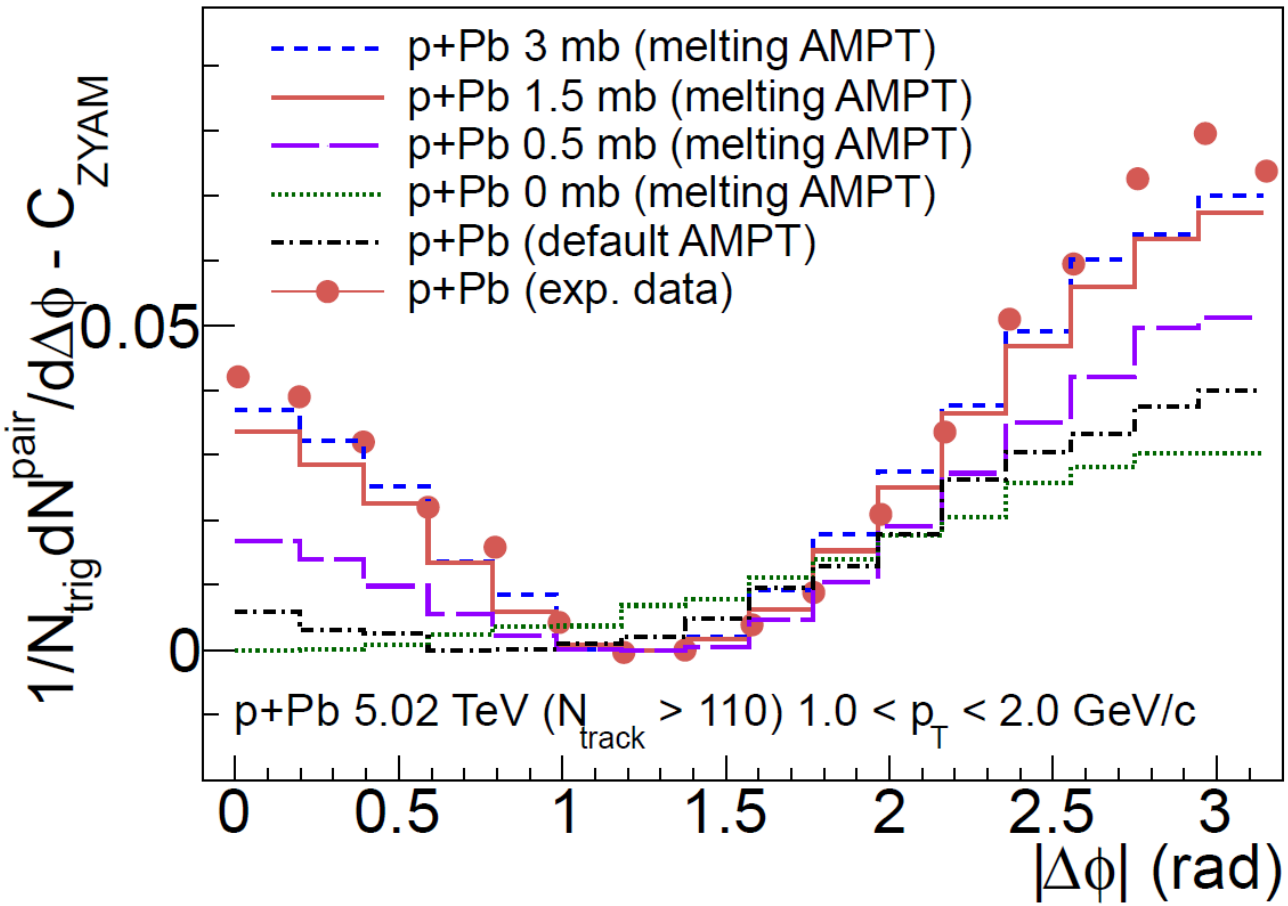
AMPT results for low- and high-multiplicity events in p+Pb





G.L. Ma, AB, PLB 739 (2014) 209

AMPT results for different cross-sections



$\sigma = 1.5 - 3$ mb works good enough

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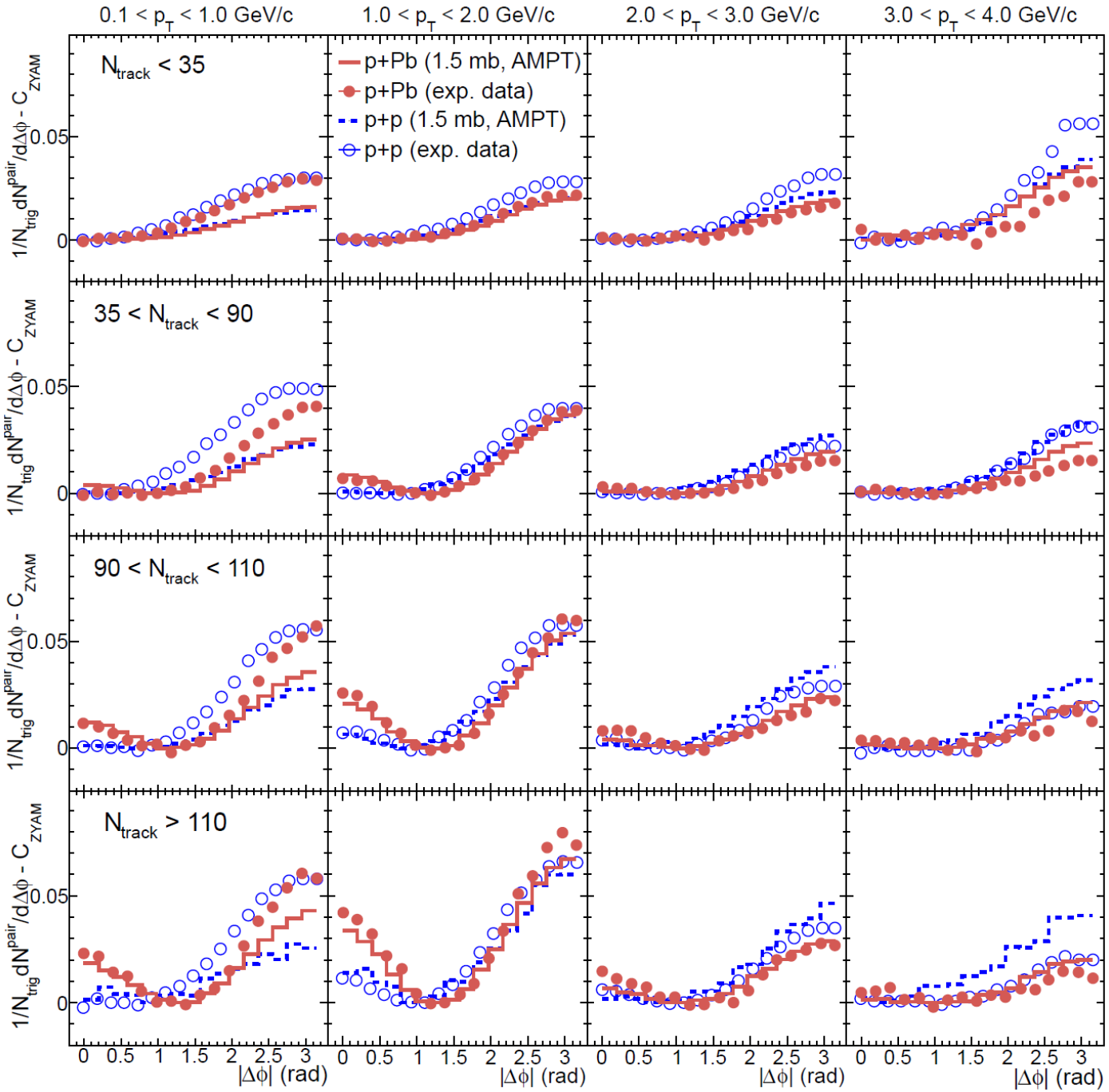
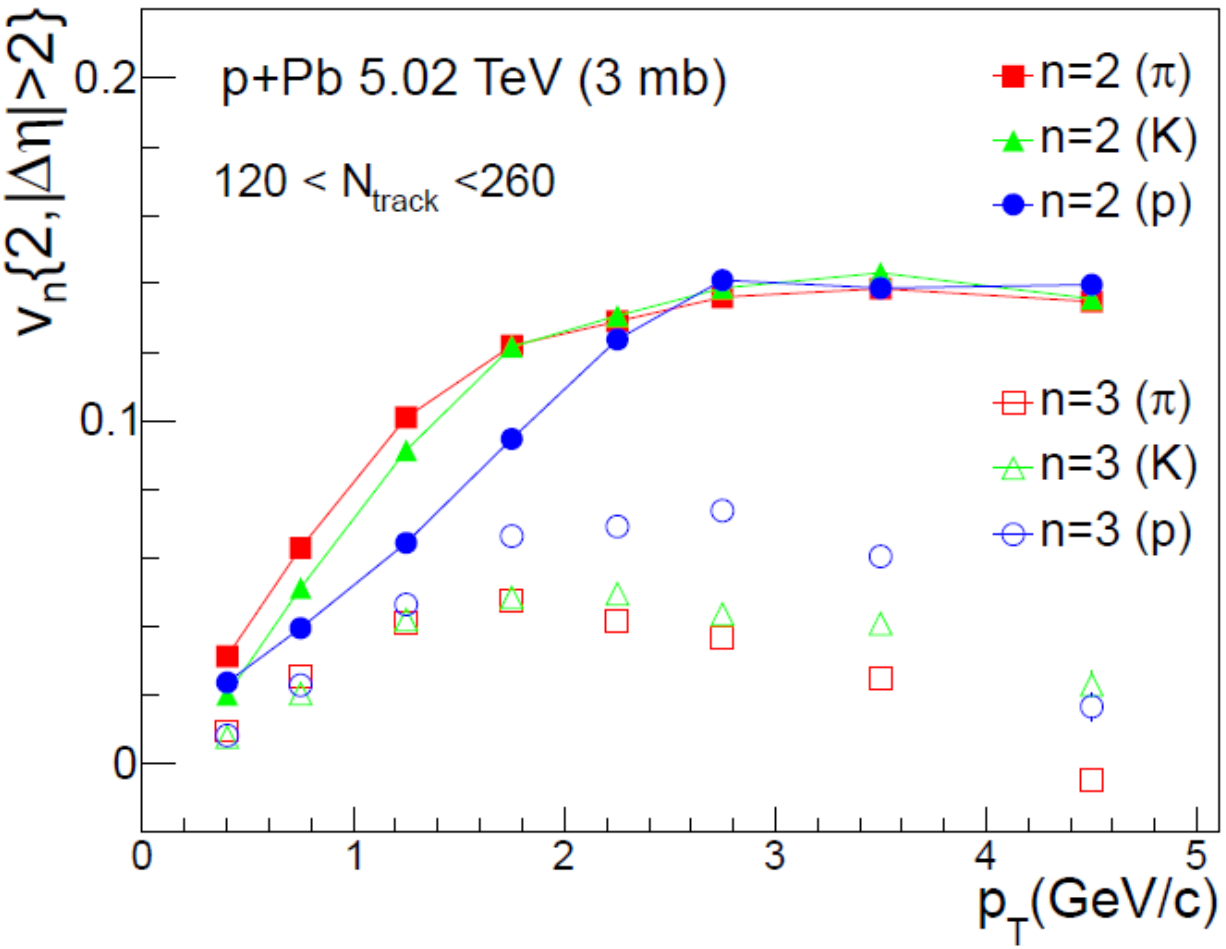


G.L. Ma, AB,
PLB 739 (2014) 209

AMPT reproduces well near-side peaks in p+p and p+Pb

AMPT and mass ordering of v_2 in p+Pb



AB, G.L. Ma, PRL 113 (2014) 252301

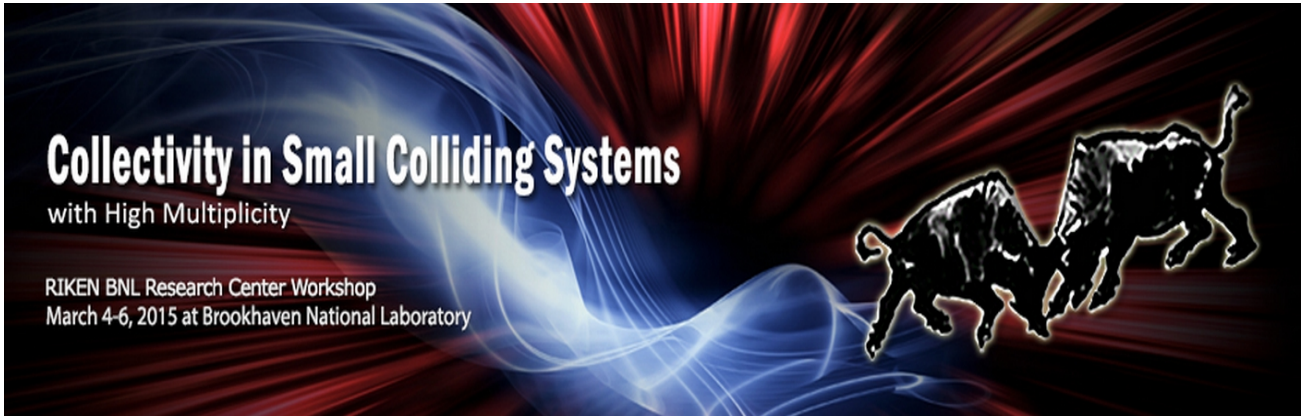
Bourses postdoctorales
Banting
Postdoctoral Fellowships

McGill

Knudsen numbers in small colliding systems

Gabriel S. Denicol

with C. Gale, S. Jeon and C. Shen



Flow in p-N collisions ?

When does hydro break down?

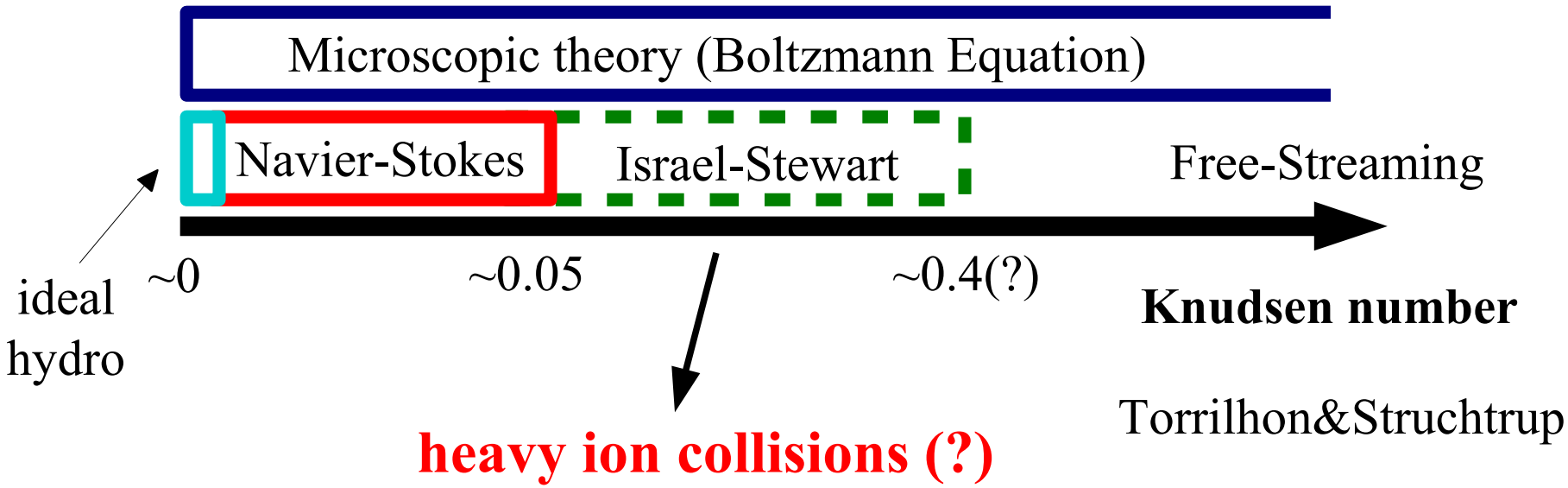
Phenomenological approach: compare model with data and see if an agreement is reached

Theoretical approach: check if hydrodynamics is applicable in the conditions created in pA

Real theoretical approach: prove it thermalizes ...

Validity of fluid dynamics

In terms of Knudsen number $\mathrm{Kn} = \dfrac{\ell_{\mathrm{micro}}}{L_{\mathrm{macro}}}$



In heavy ion collisions, hydrodynamics is *expected* to work for Kn ~ 0.5

Bouras *et al*, PRC 82, 024910 (2010).
Huovinen *et al*, PRC 79, 014906 (2009).
Denicol *et al*, PRL, arXiv:1408.5646.

What are the scales ?

$$\mathrm{Kn}_i = \tau_\pi / L^i_{\mathrm{macro}}$$

Microscopic scales

$$\tau_\pi = 5\frac{\eta}{\varepsilon + P}$$

$$\tau_\Pi = \frac{1}{15}\frac{\zeta}{\left(\frac{1}{3} - c_s^2\right)^2(\varepsilon + P)}$$

Inverse Macroscopic scales

expansion rate

$$\frac{1}{L^\theta_{\mathrm{macro}}} = \theta$$

$$\frac{1}{L^\varepsilon_{\mathrm{macro}}} = \frac{1}{\varepsilon_0}\sqrt{\nabla_\mu \varepsilon_0 \nabla^\mu \varepsilon_0}$$

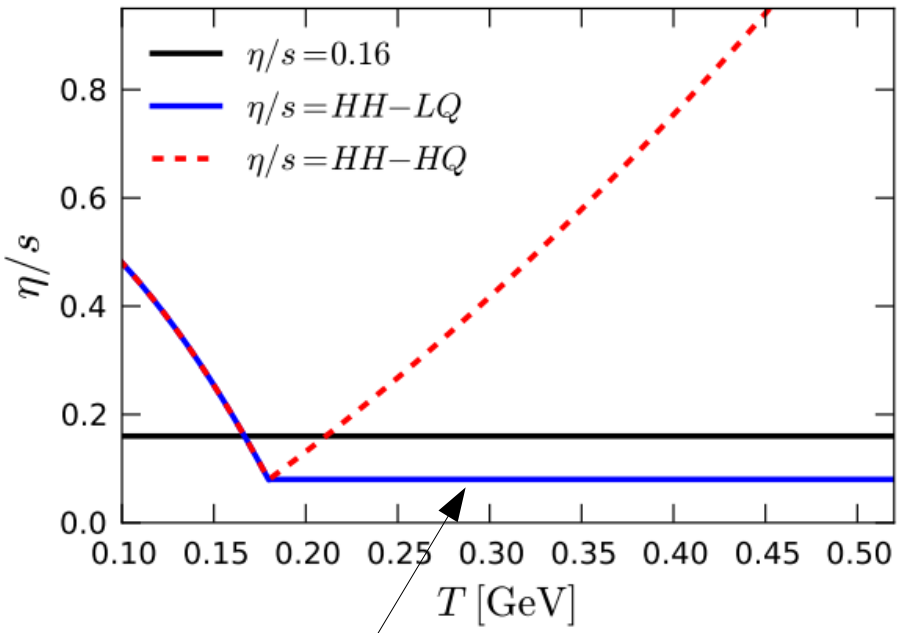
$$\frac{1}{L^\sigma_{\mathrm{macro}}} = \sqrt{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}$$

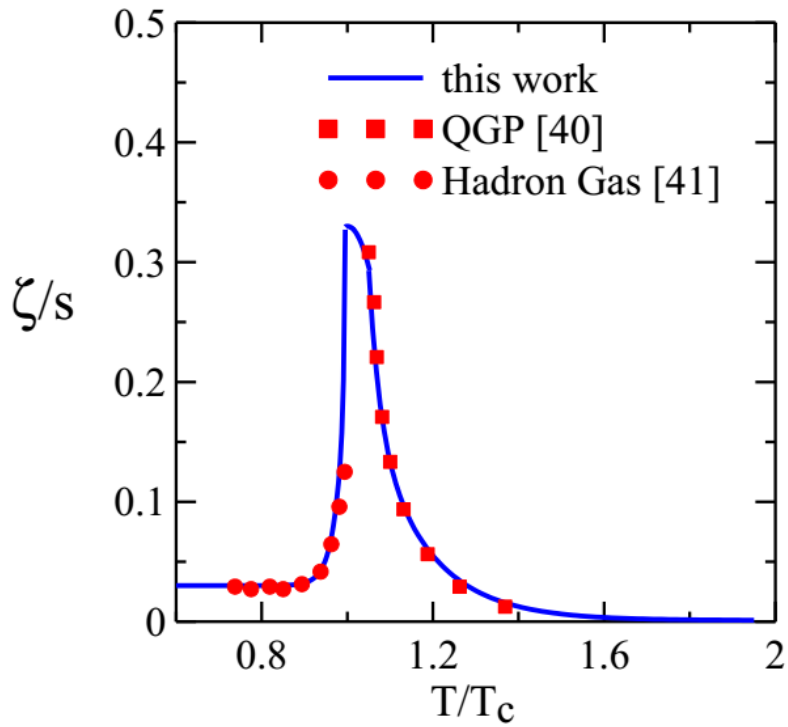
$$\frac{1}{L^\omega_{\mathrm{macro}}} = \sqrt{\omega_{\alpha\beta}\omega^{\alpha\beta}}$$

$$\frac{1}{L^{\dot{u}}_{\mathrm{macro}}} = \sqrt{\dot{u}_\alpha \dot{u}^\alpha}$$

Microscopic scale strongly depends on the viscosity ansatz

Shear and bulk viscosities



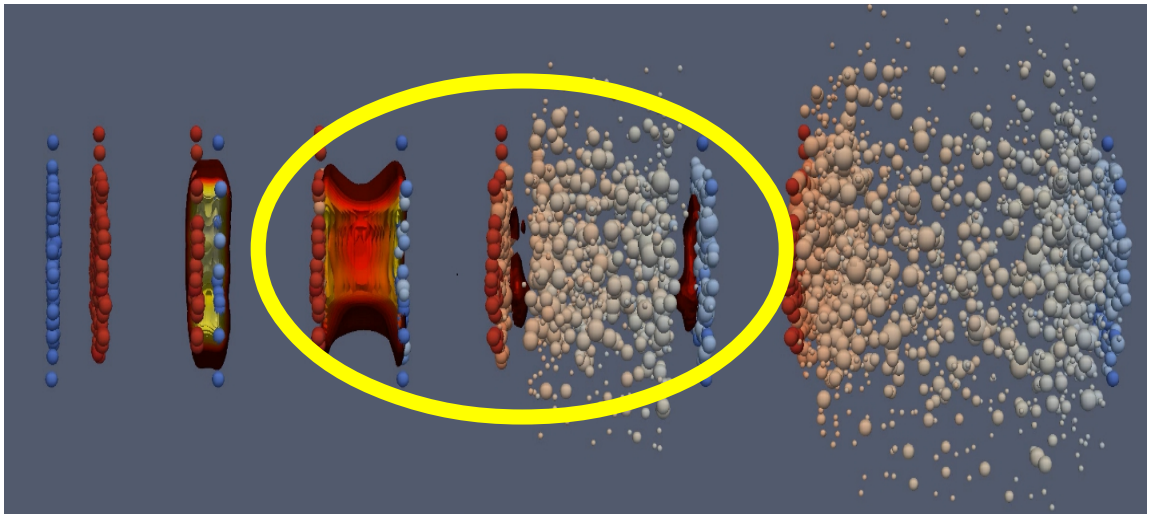


this talk

best case scenario: small viscosity in QGP phase

maximum ζ/s is 0.3

Fluid-dynamical model



Basics of fluid dynamics

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

$$N^\mu = n u^\mu + \underbrace{n^\mu}_{\text{Particle diffusion current}},$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}\left(P_0 + \underbrace{\Pi}_{\text{Bulk viscous pressure}}\right) + \underbrace{\pi^{\mu\nu}}_{\text{Shear stress tensor}}$$

Always true, regardless of the applicability fluid dynamics

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$u_\mu u^\mu = 1$$

Fluid-dynamical equations

GSD&Niemi&Molnar&Rischke, arXiv:1202.4551

Inclusion of bulk viscous pressure, shear-stress tensor, and all couplings

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \,,$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu} .$$

For collisions at lower energies, baryon number and electric charge diffusion should also enter

Results

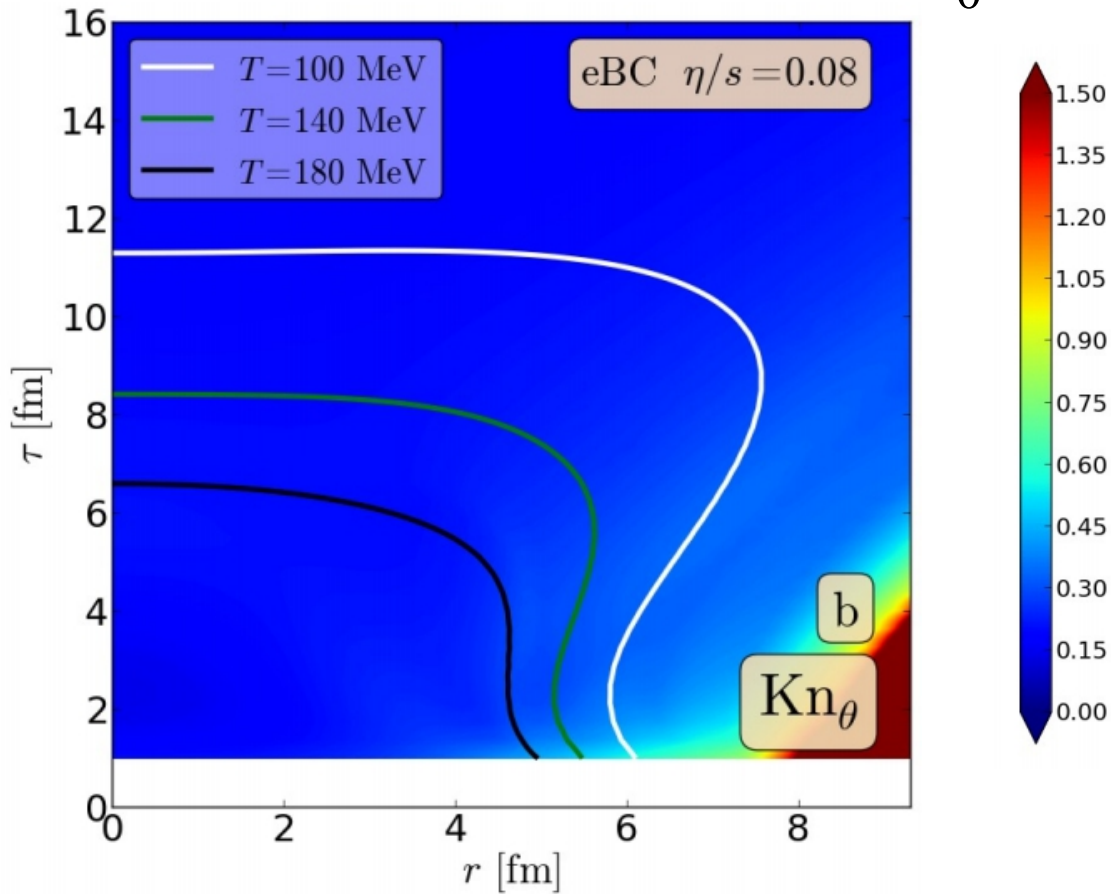
shear viscosity only

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}\left(P_0 + \cancel{\Pi}\right) + \pi^{\mu\nu}$$

Niemi&GSD, arXiv:1404.7327

Knudsen numbers – AA

Optical Glauber
20-30%, x=y, LHC
τ_0=0.6 fm, **η/s=0.08**



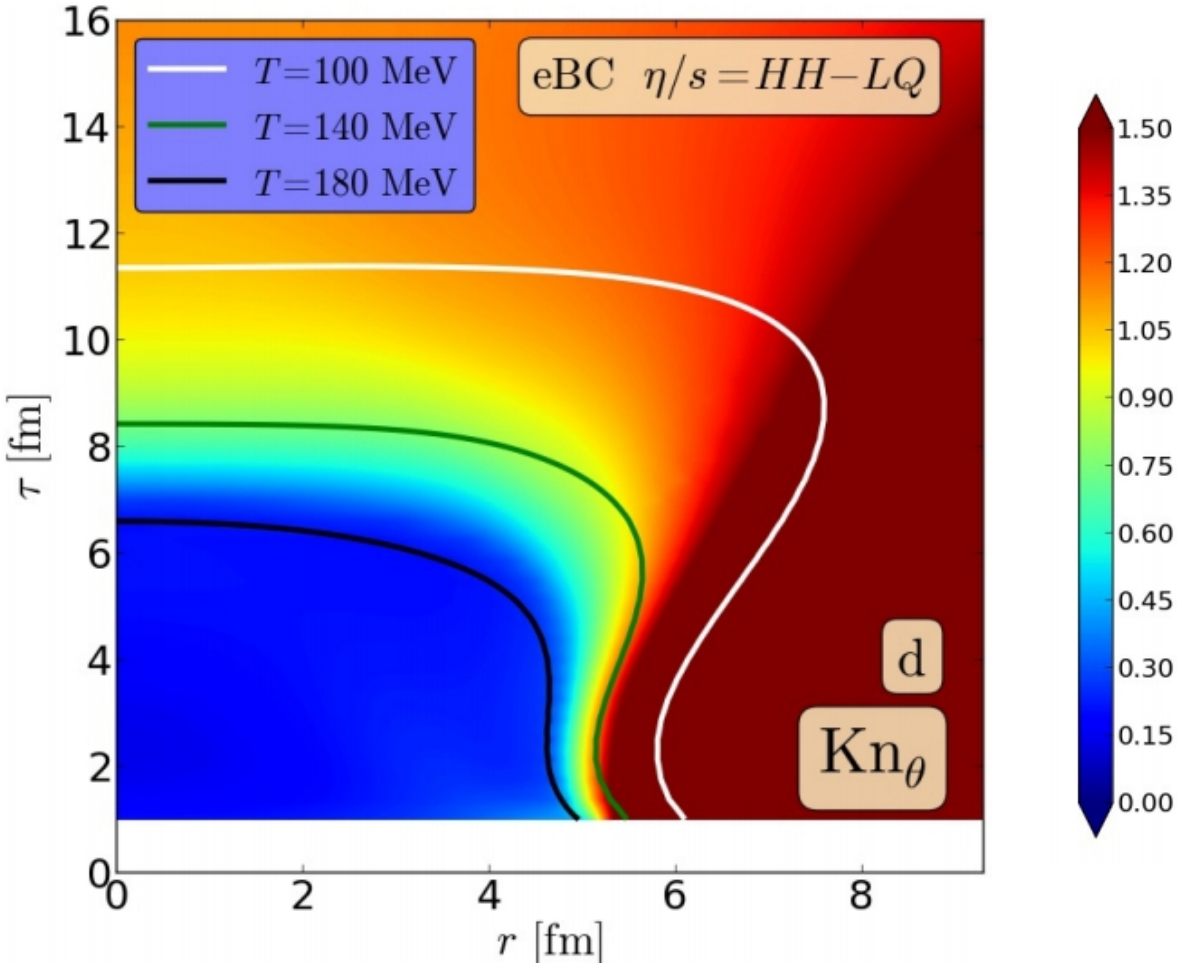
Hydrodynamics works, even where it should not.
Why is there no freezeout?

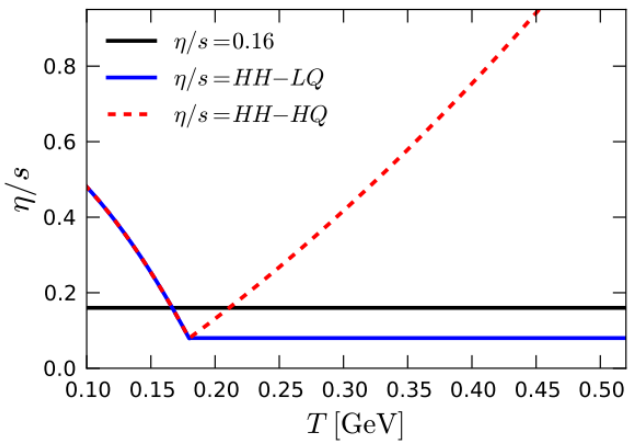
Note that viscosity is unphysical.

Niemi&GSD, arXiv:1404.7327

Knudsen numbers – AA

Optical Glauber
20-30%, LHC
τ_0=0.6 fm, $\eta/s(T)$



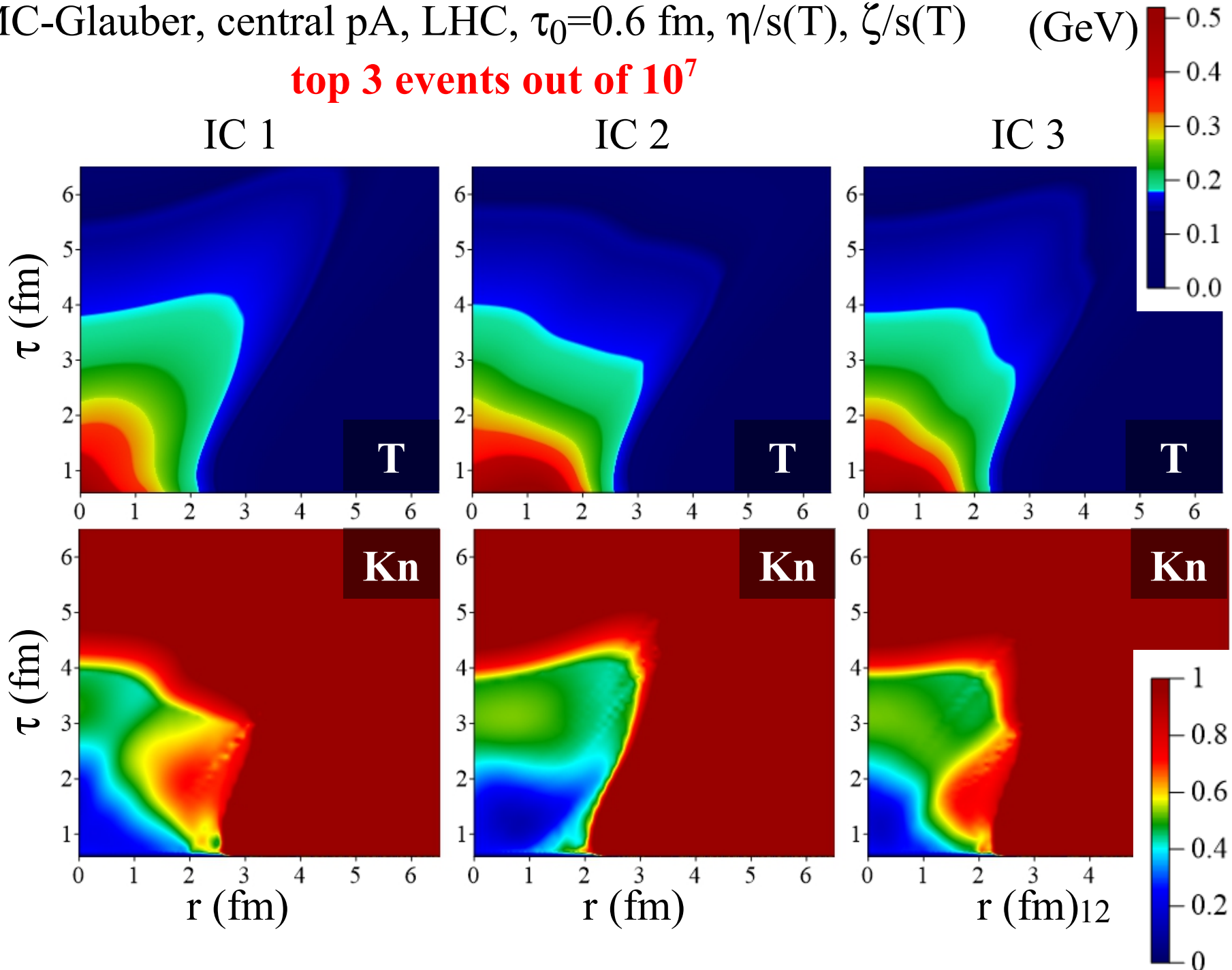


If $\eta/s(T)$ is used in hadronic phase, then freezeout occurs.

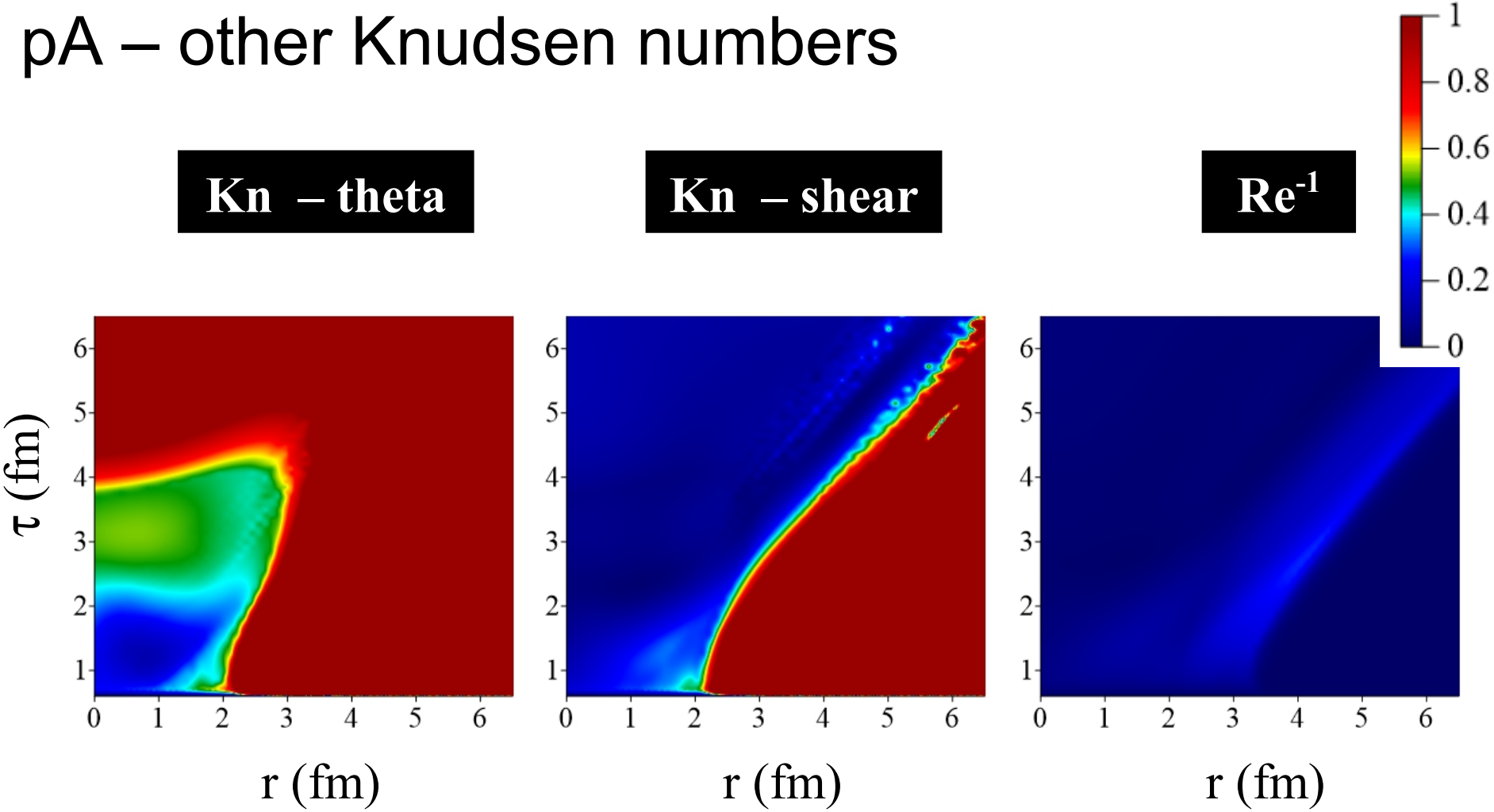
Early stages, hydro works (but we used a small η/s there)

MC-Glauber, central pA, LHC, τ_0=0.6 fm, $\eta/s(T)$, $\zeta/s(T)$

top 3 events out of 10^7



pA – other Knudsen numbers



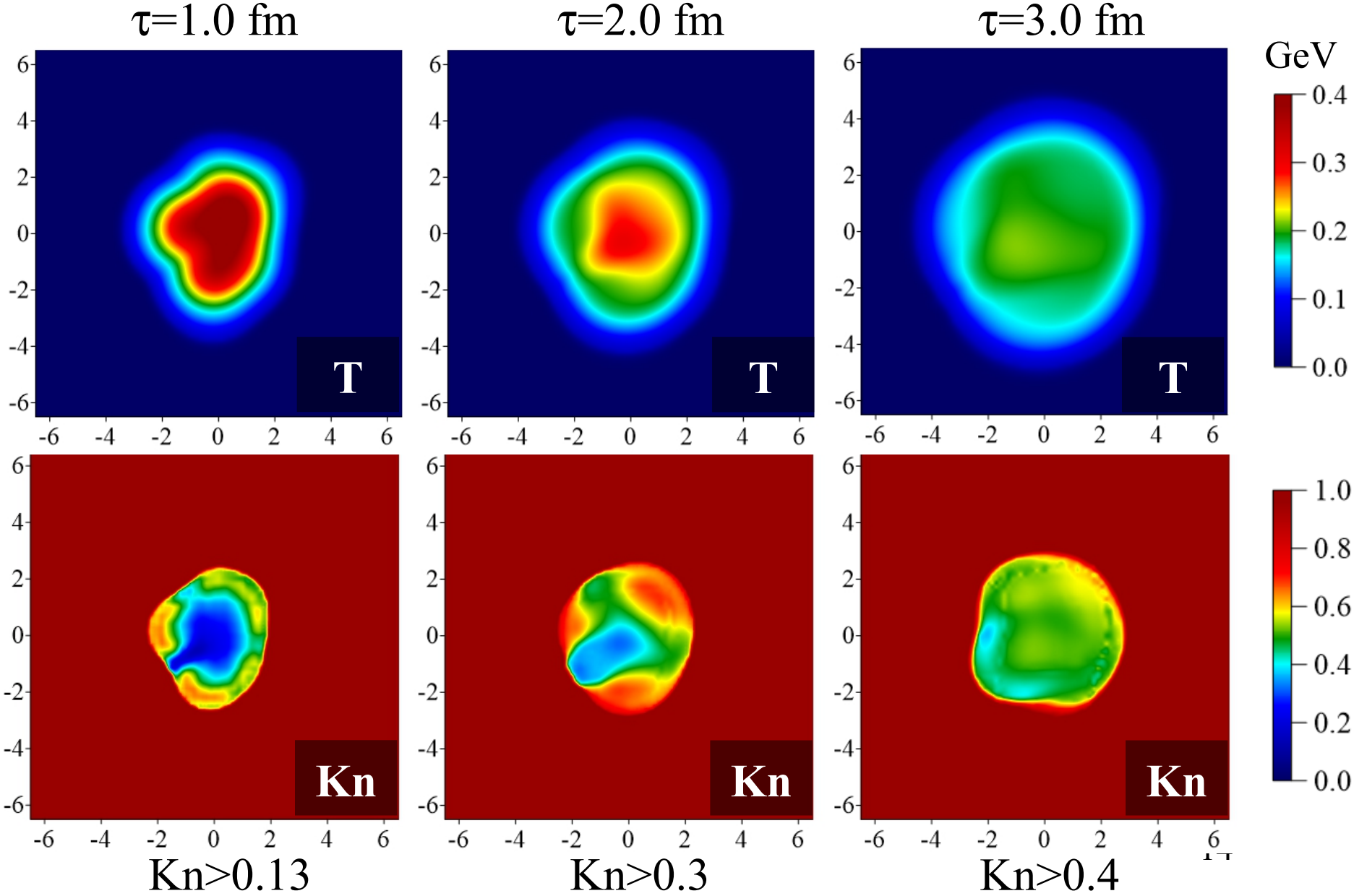
$$\tau_\pi \theta$$

$$\tau_\pi \sqrt{\sigma^{\mu\nu}\sigma_{\mu\nu}}$$

$$\frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\varepsilon_0 + P_0}$$

Profile in transverse plane

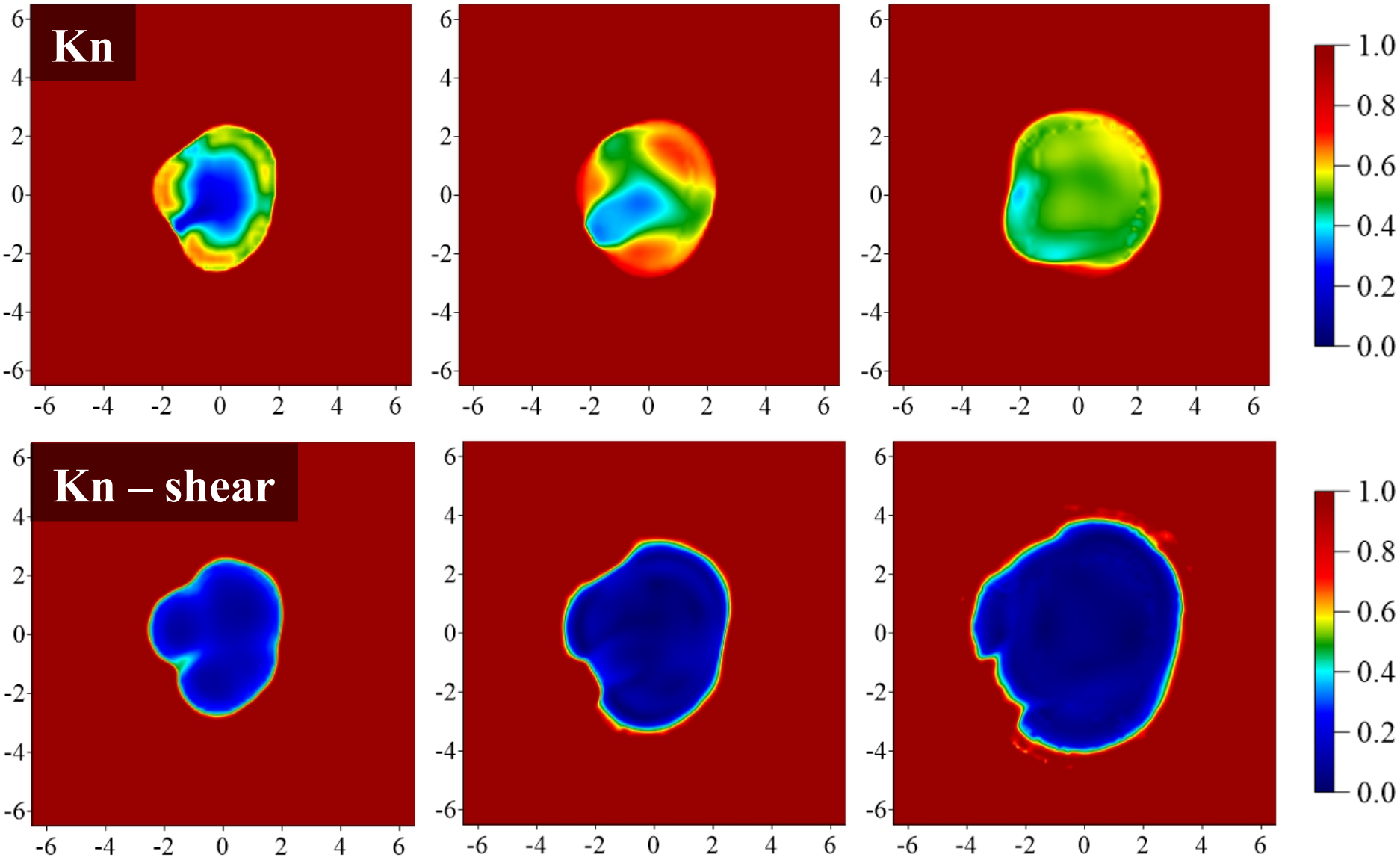
MC-Glauber, central pA, LHC
τ_0=0.6 fm, $\eta/s(T)$, $\zeta/s(T)$



Profile in transverse plane

MC-Glauber, central pA, LHC
τ_0=0.6 fm, $\eta/s(T)$, $\zeta/s(T)$

τ=1.0 fm τ=2.0 fm τ=3.0 fm



Effects of Bulk viscosity in pA collisions

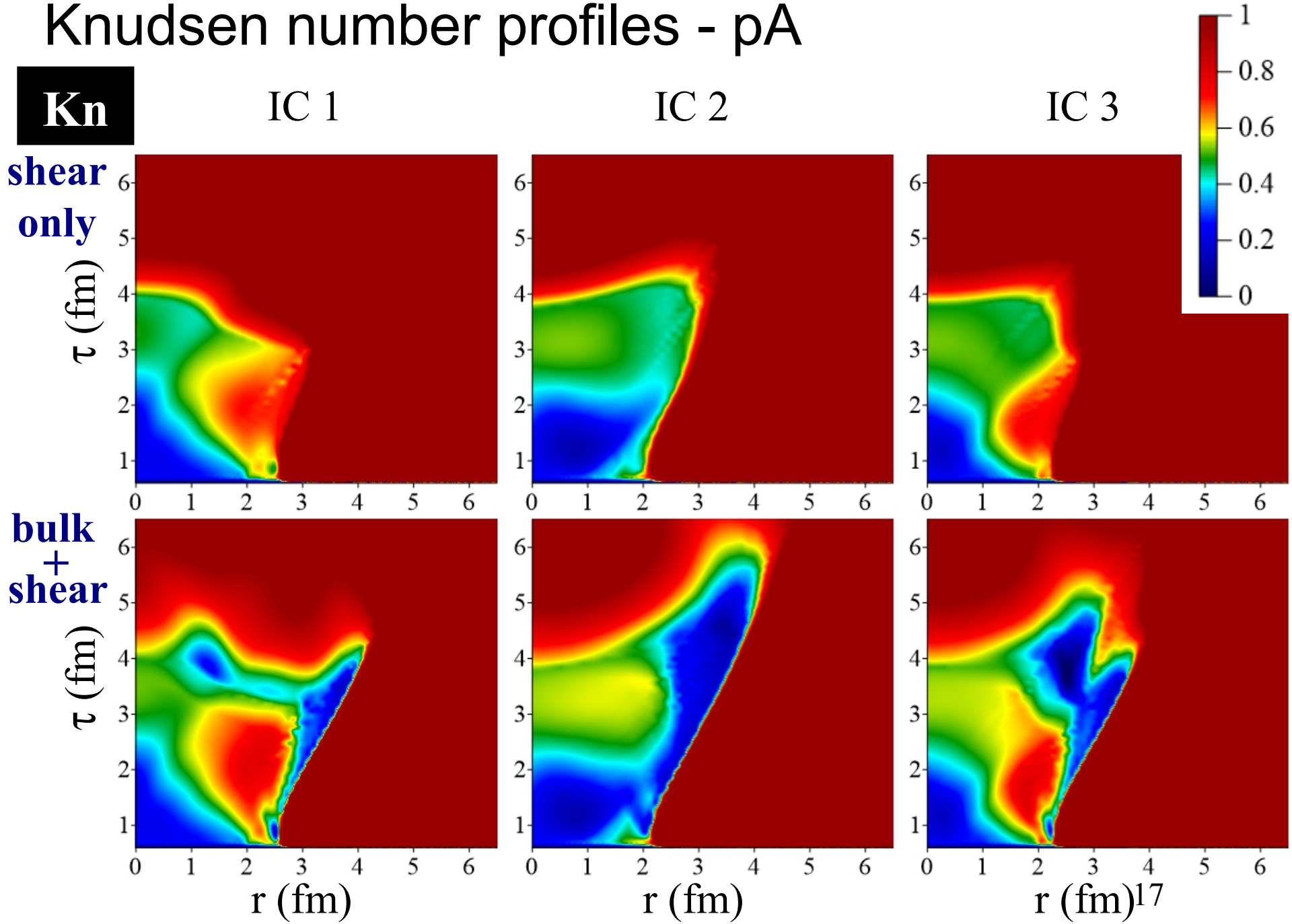
$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - \Delta^{\mu\nu} \underline{(P_0 + \Pi)} + \pi^{\mu\nu}$$

Effective pressure

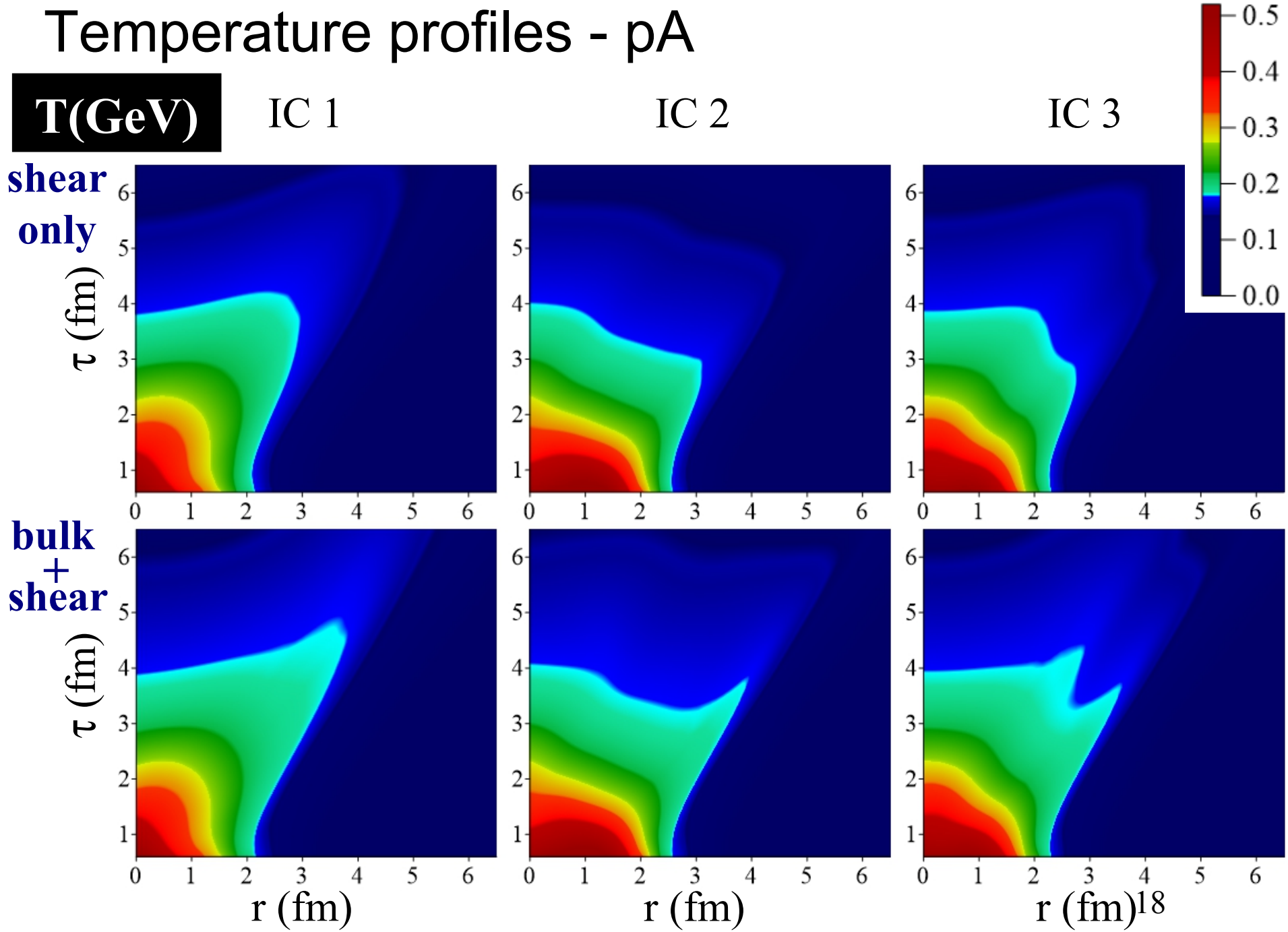
↓

Can become negative

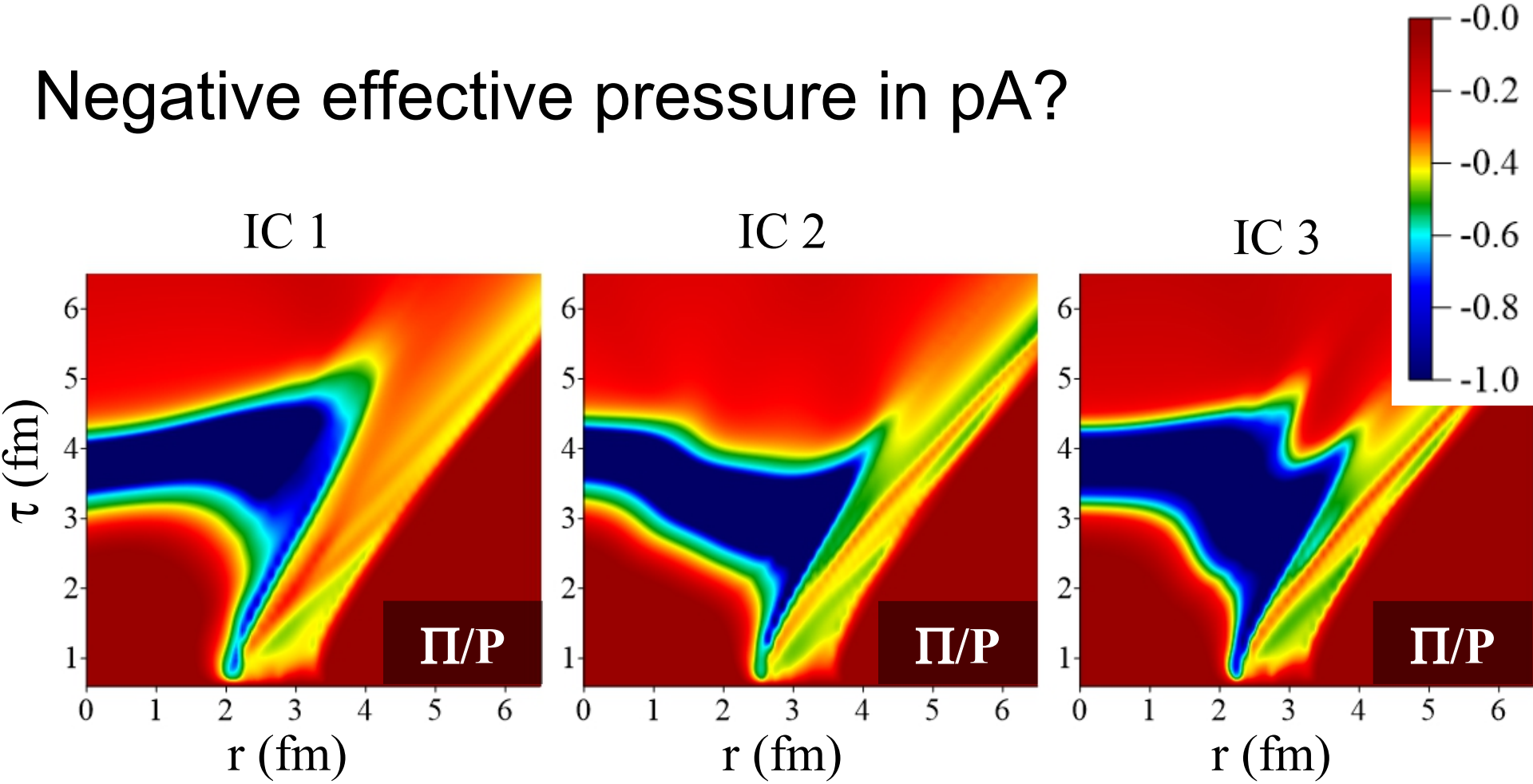
Knudsen number profiles - pA



Temperature profiles - pA

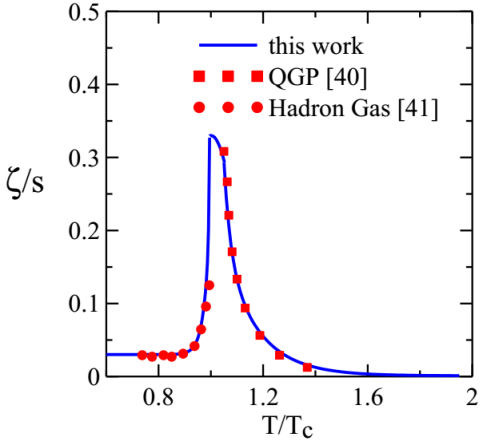


Negative effective pressure in pA?

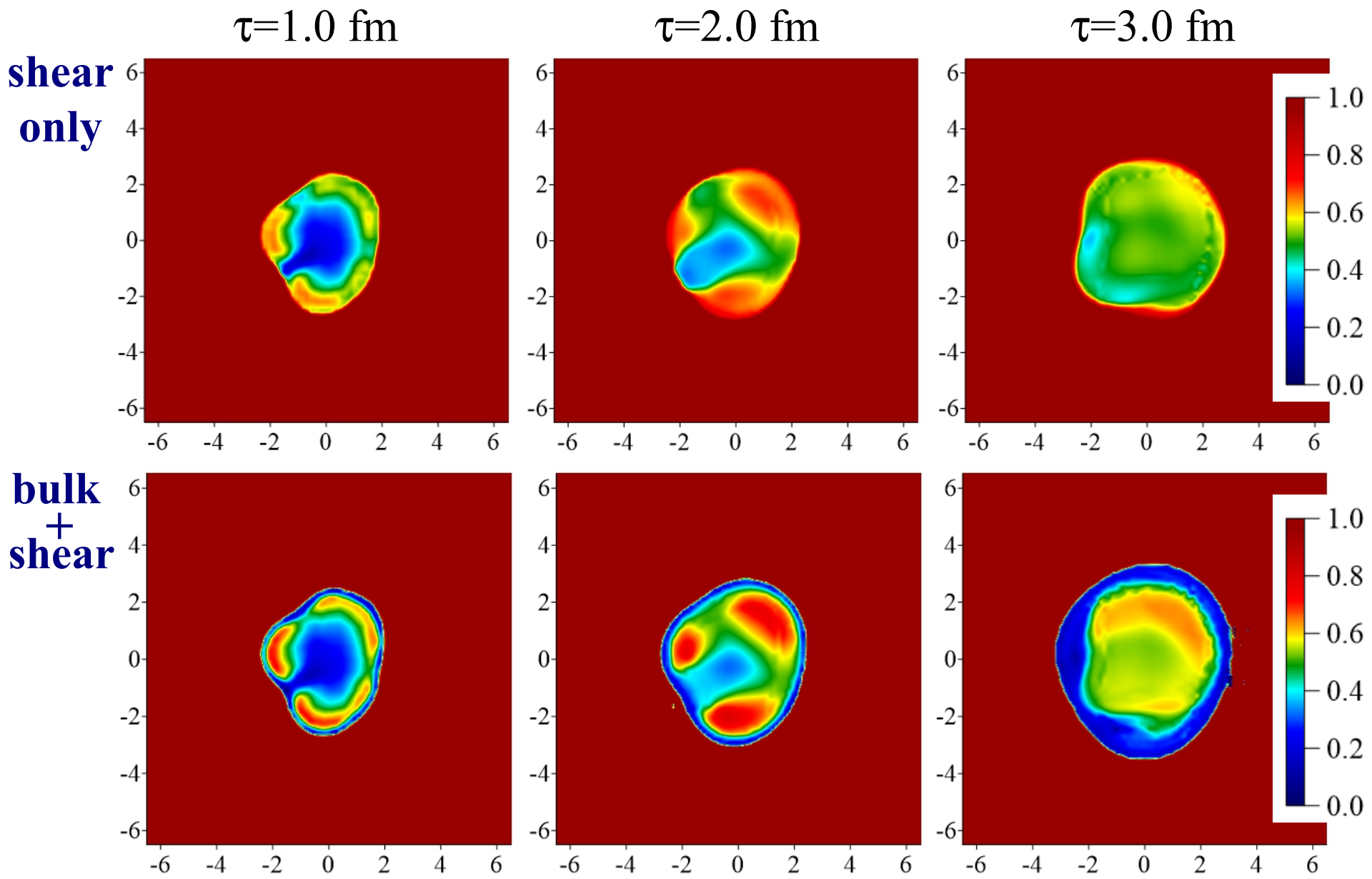


Even when $\zeta/s < 0.3$ around Tc, the effective pressure can become negative

This did not happen in AA

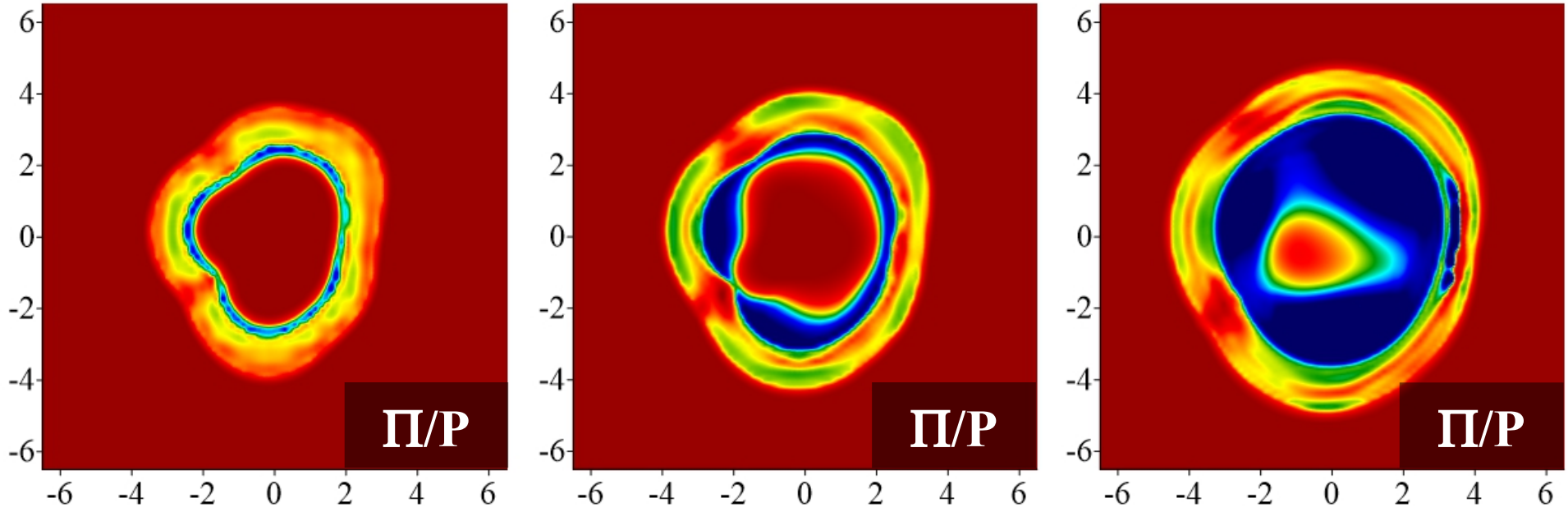


Kn profile – transverse plane



Π/P_0 – transverse plane profile

τ=1.0 fm τ=2.0 fm τ=3.0 fm



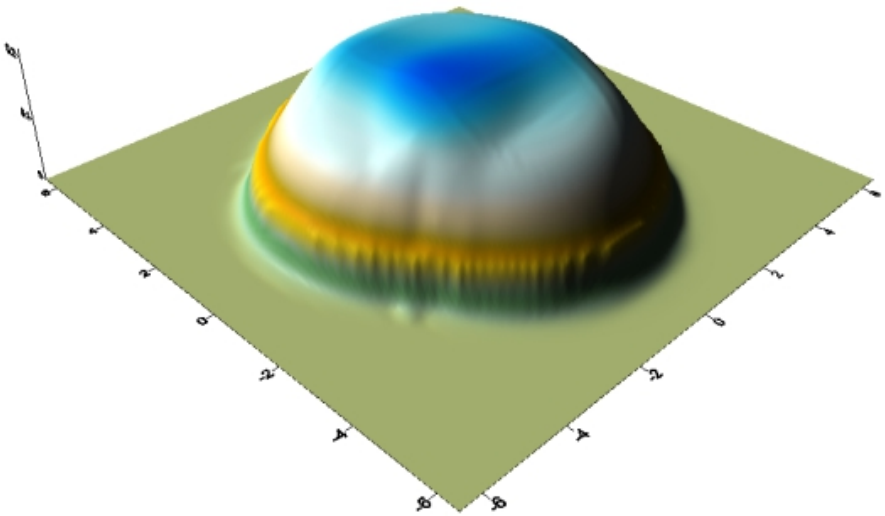
Even when $\zeta/s < 0.3$ around Tc, the effective pressure can become negative

This did not happen in AA

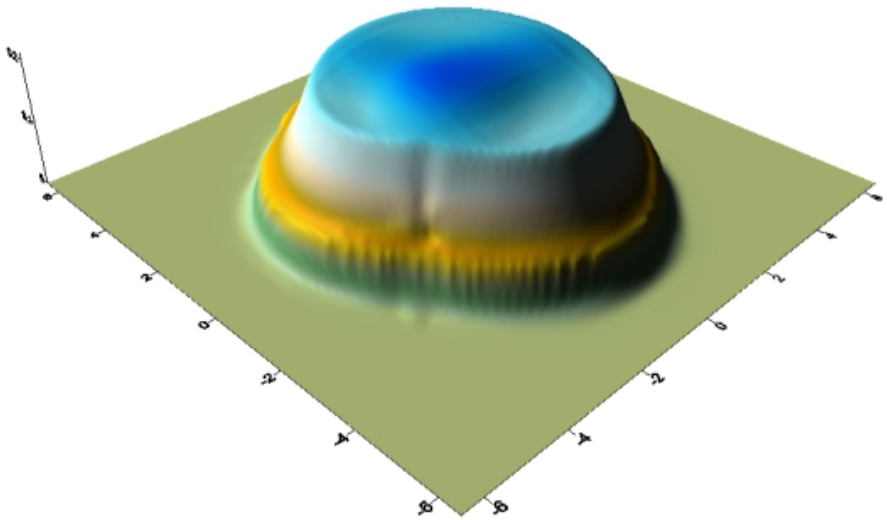
Temperature profile – transverse plane

τ=3.0 fm

no bulk



with bulk



Clearly, system is stopped by the bulk viscosity

Conclusions

We estimated the Knudsen numbers reached in a pA

- The **Knudsen numbers reached in pA collision are much larger** than in AA collisions

- The effect of **bulk viscosity is much larger in pA collisions** when compared with AA collisions

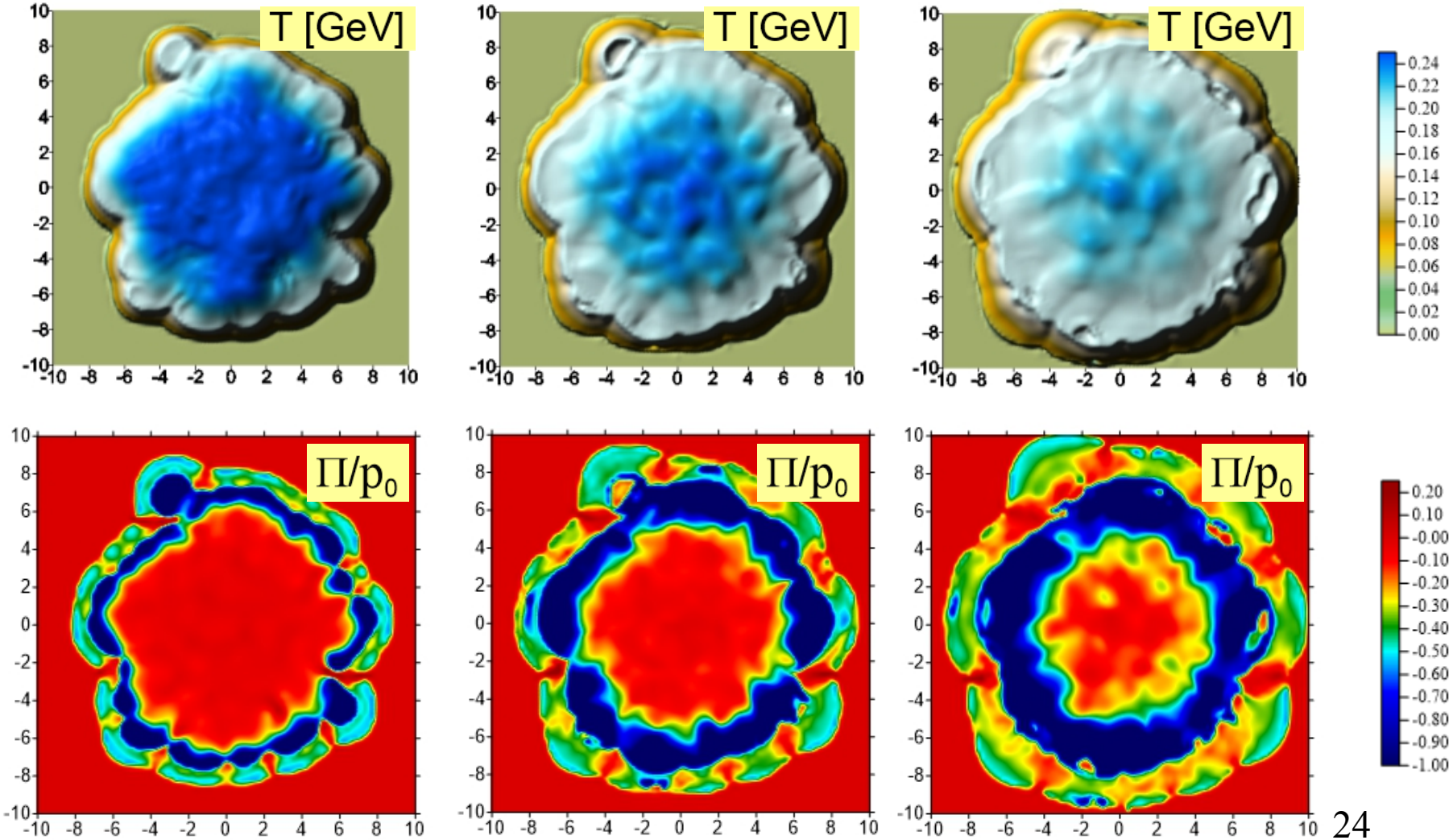
- **Bulk viscosity reduces the expansion rate** → smaller Kn
  but bulk viscosity leads to negative effective pressure ...

Even at its limit, can hydro still provide a reasonable description?

cavitation ?

MC-Glauber, central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$, $\zeta/s(T)$

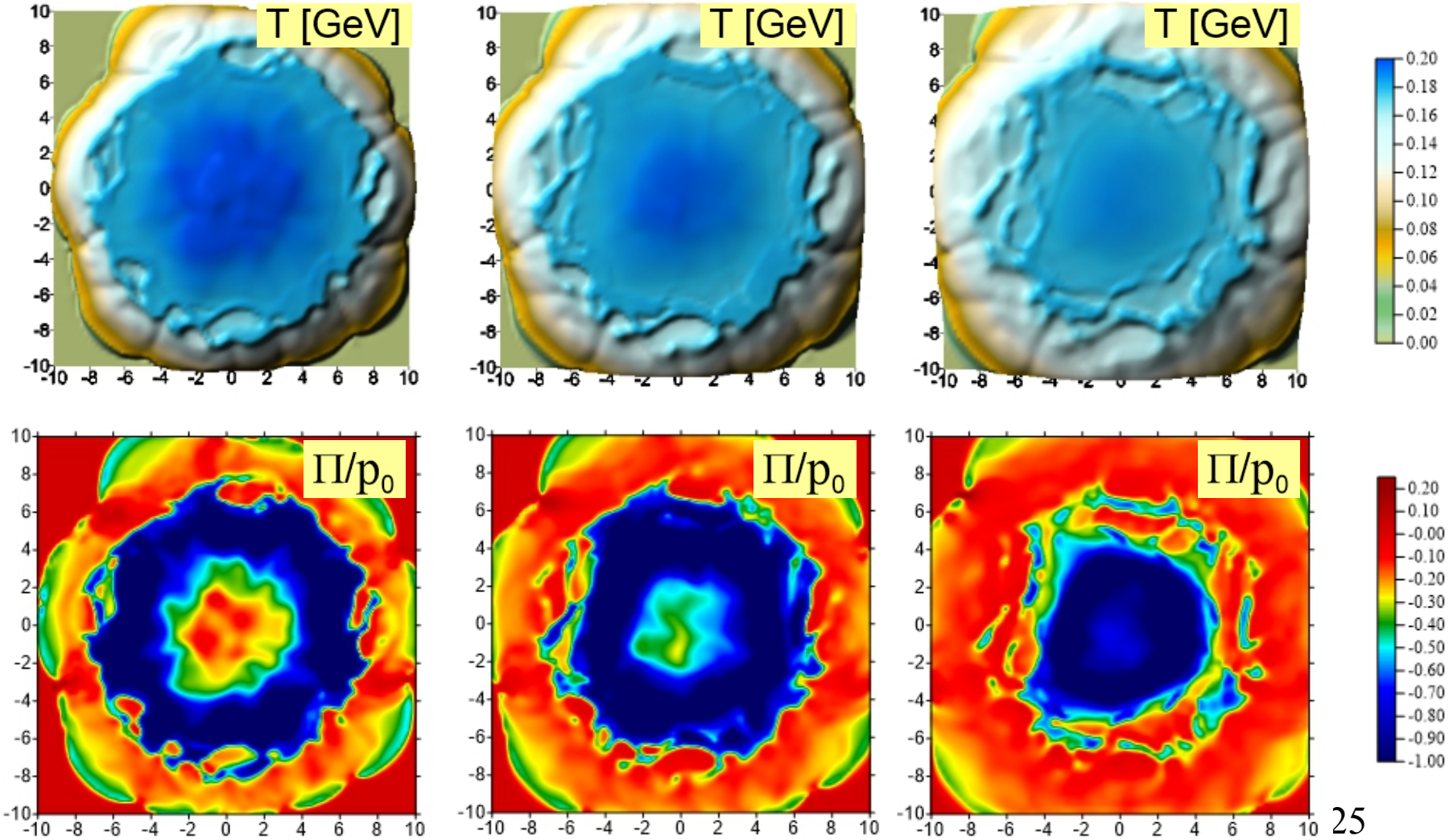
τ=1.6 fm τ=2.6 fm τ=3.6 fm



cavitation ?

MC-Glauber, central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$, $\zeta/s(T)$

τ=4.6 fm τ=5.6 fm τ=6.6 fm

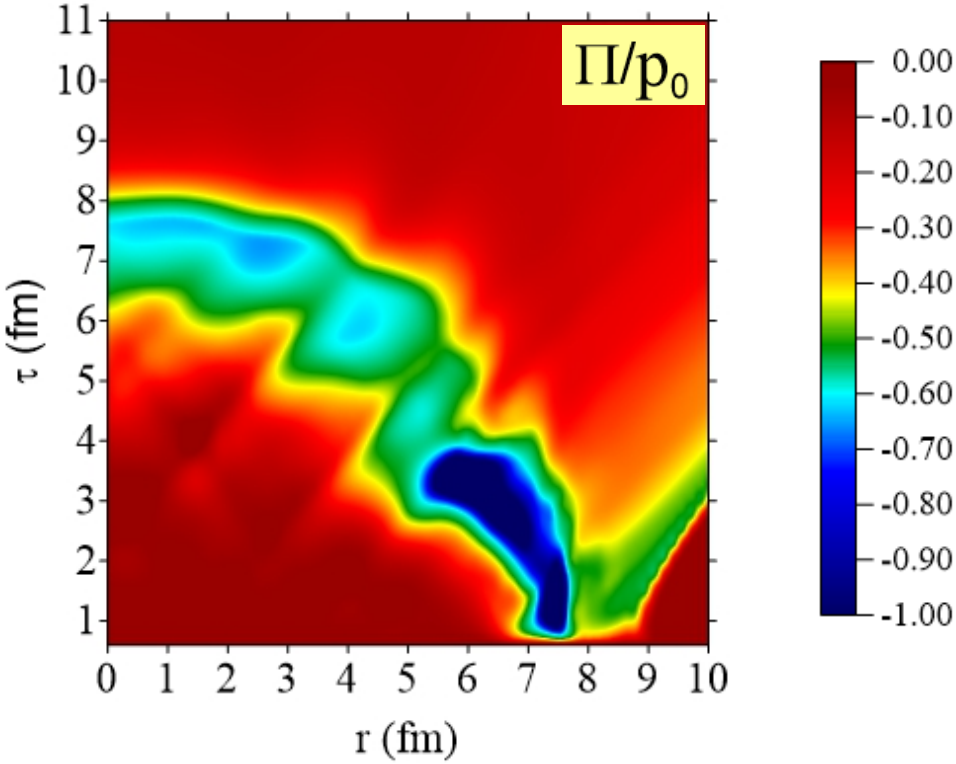


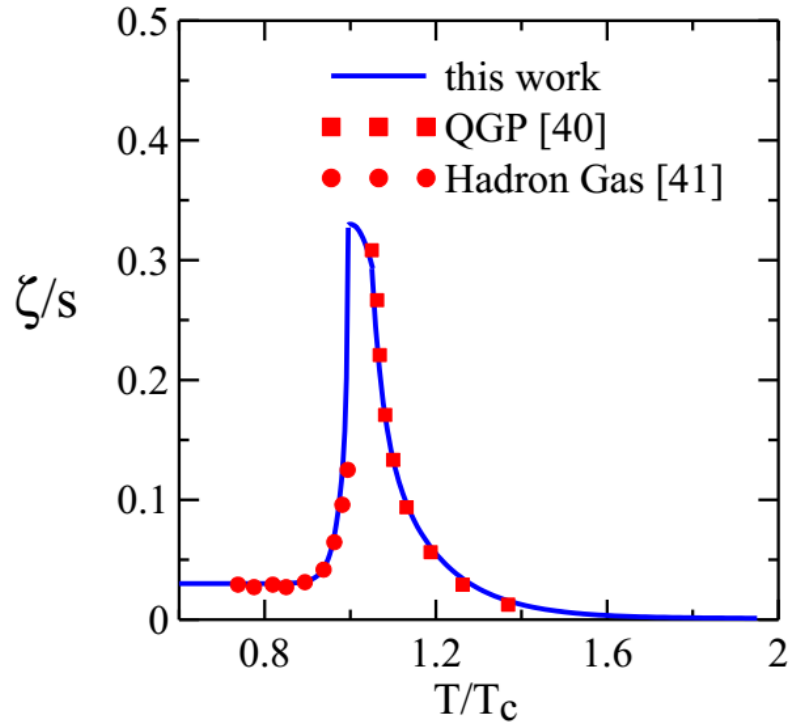
Negative pressure in AA?

central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$

MC-Glauber

Tc=180 MeV



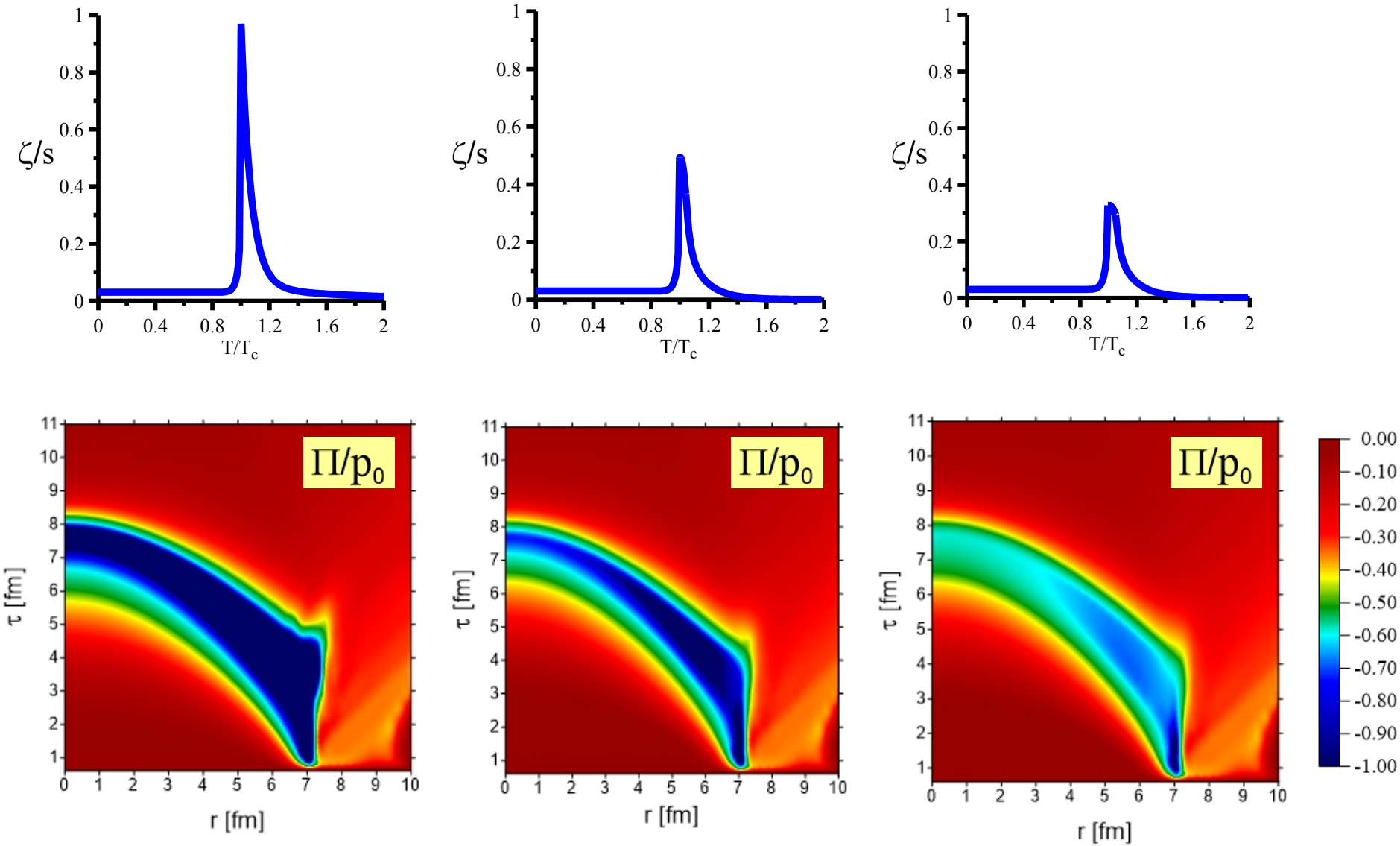


Large effect, but the effective pressure is mostly positive

Negative pressure?

central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$

Tc=180 MeV

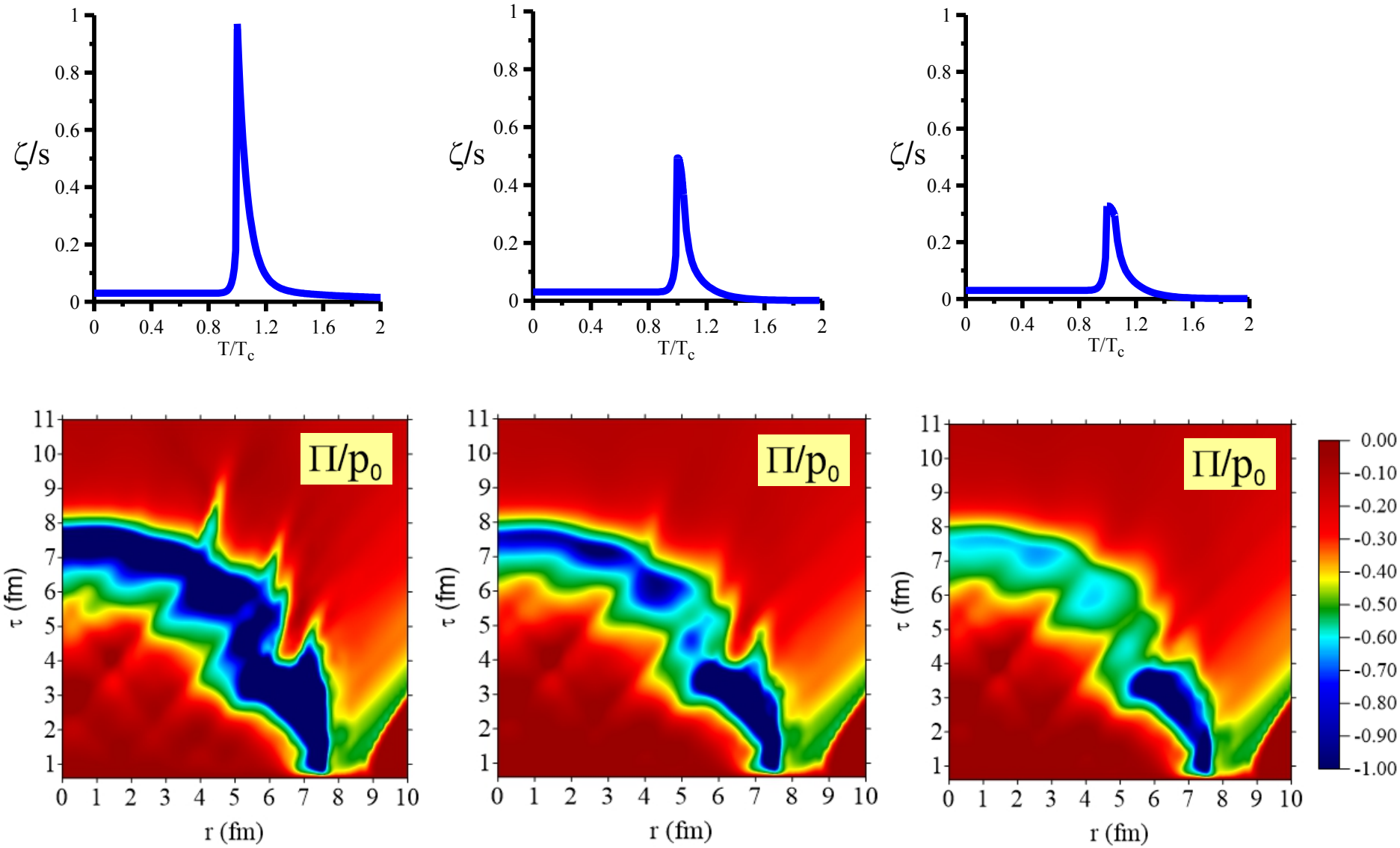


Negative pressure?

central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$

MC-Glauber

Tc=180 MeV

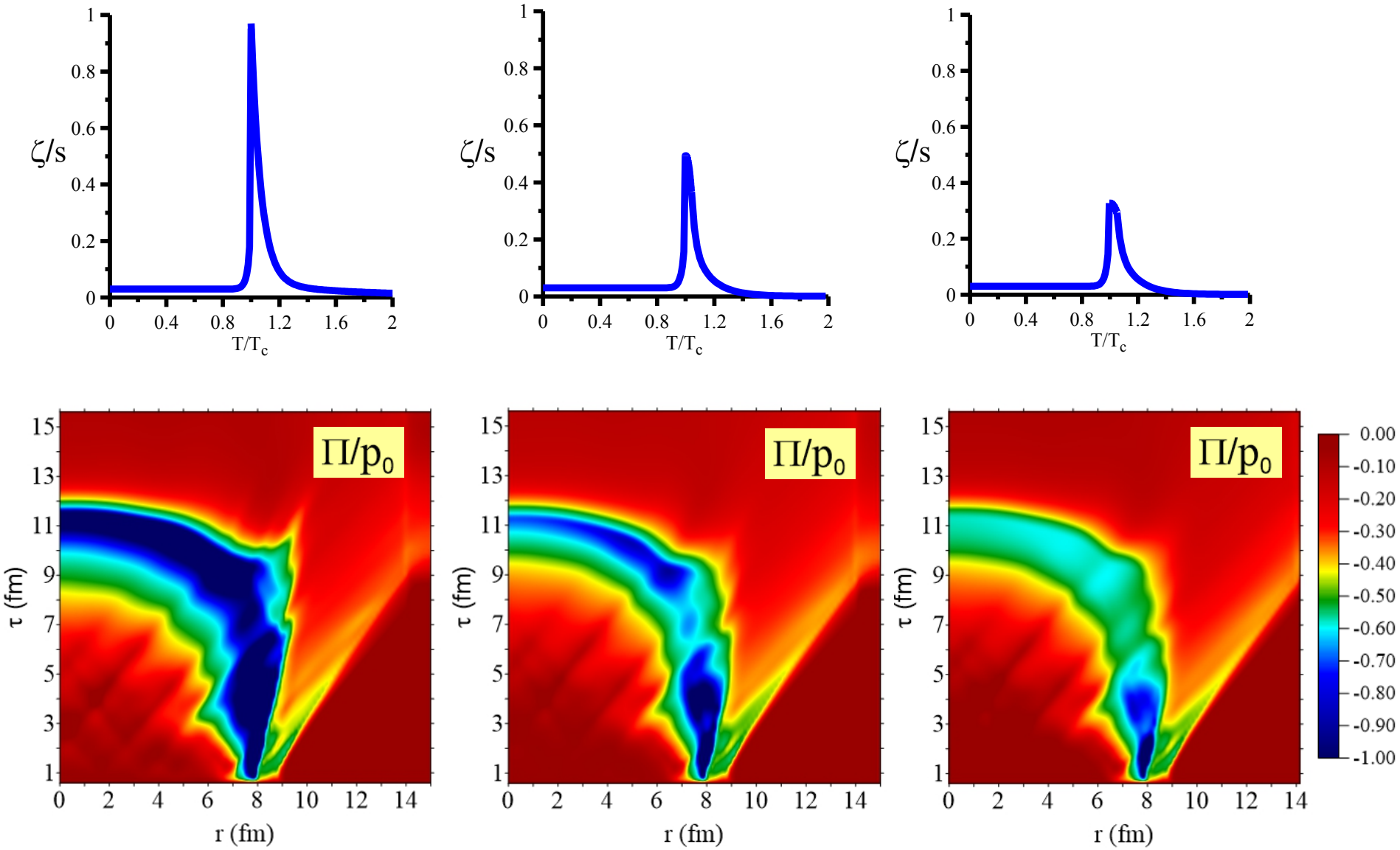


Negative pressure?

central collision, LHC
τ_0=0.6 fm, $\eta/s(T)$

MC-Glauber

Tc=150 MeV



Coefficients from kinetic theory

GSD&Jeon&Gale, arXiv:1403.0962

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \ ,$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu} .$$

Transport coefficients computed within the 14-moment approximation

$$\beta_\pi = \frac{\varepsilon_0 + P_0}{5}, \ \delta_{\pi\pi} = \frac{4}{3} \quad , \quad \tau_{\pi\pi} = \frac{10}{7} \quad , \quad \varphi_7 = \frac{9}{70 P_0} \quad .$$

$$\beta_\Pi = \frac{\zeta}{\tau_\Pi} = 14.55 \times \left(\frac{1}{3} - c_s^2\right)^2 (\varepsilon_0 + P_0) + \mathcal{O}\left(z^5\right) ,$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = 1 - c_s^2 + \mathcal{O}\left(z^2 \ln z\right) \ ,$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(z^4\right) \ ,$$

$$z \equiv m/T,$$

Fluid-dynamical equations

GSD&Niemi&Molnar&Rischke, arXiv:1202.4551

Theory constructed from a truncation scheme

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} ,$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu}.$$

First Order terms

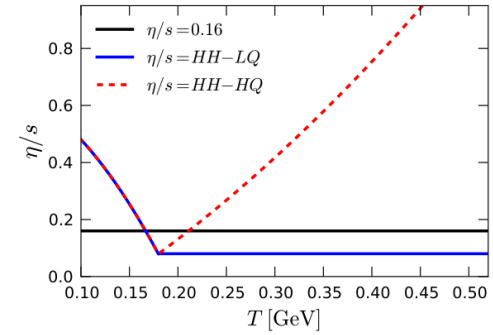
Second Order terms

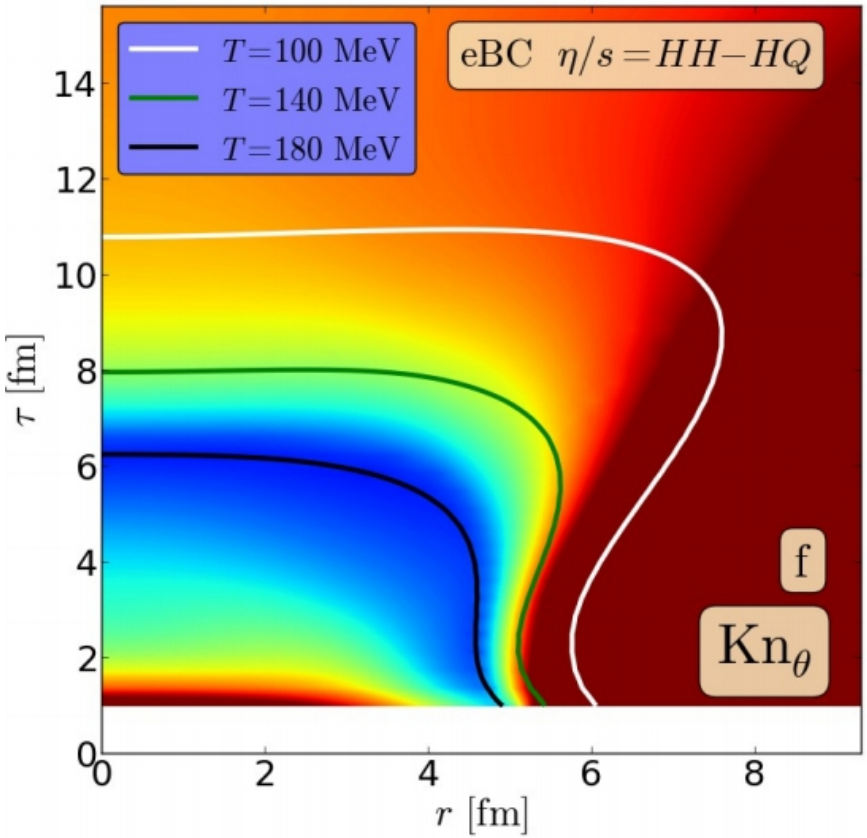
Higher Order terms?

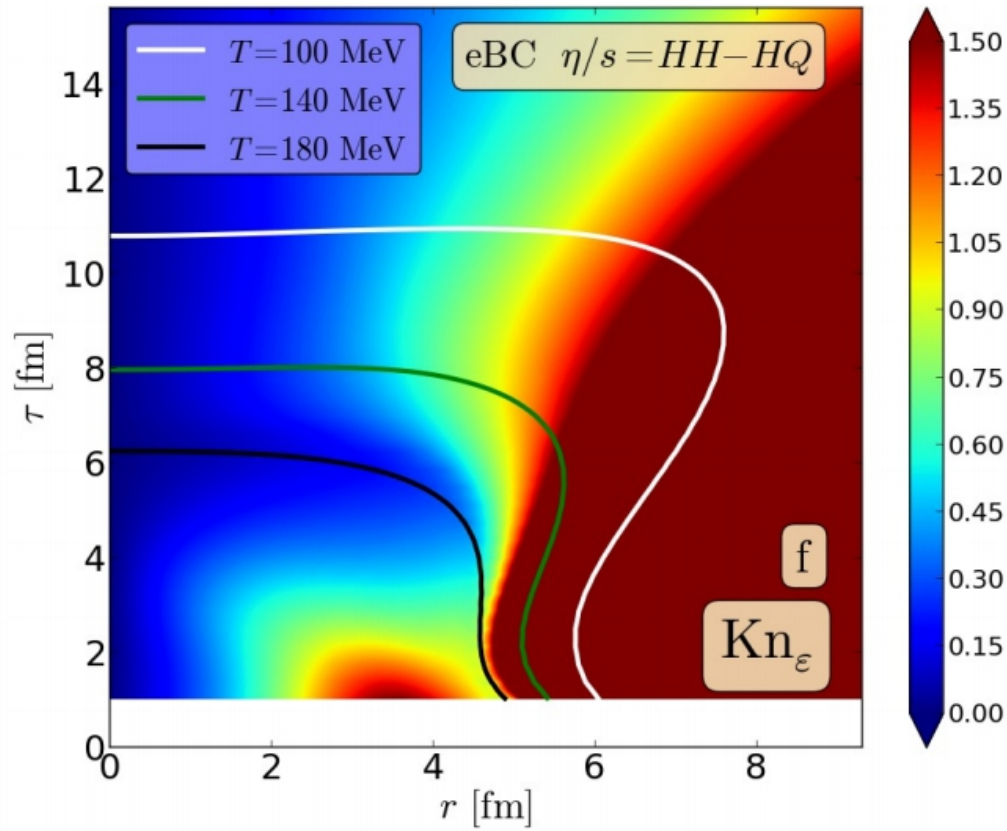
we can check if this truncation scheme works

Niemi&GSD, arXiv:1404.7327

Knudsen numbers

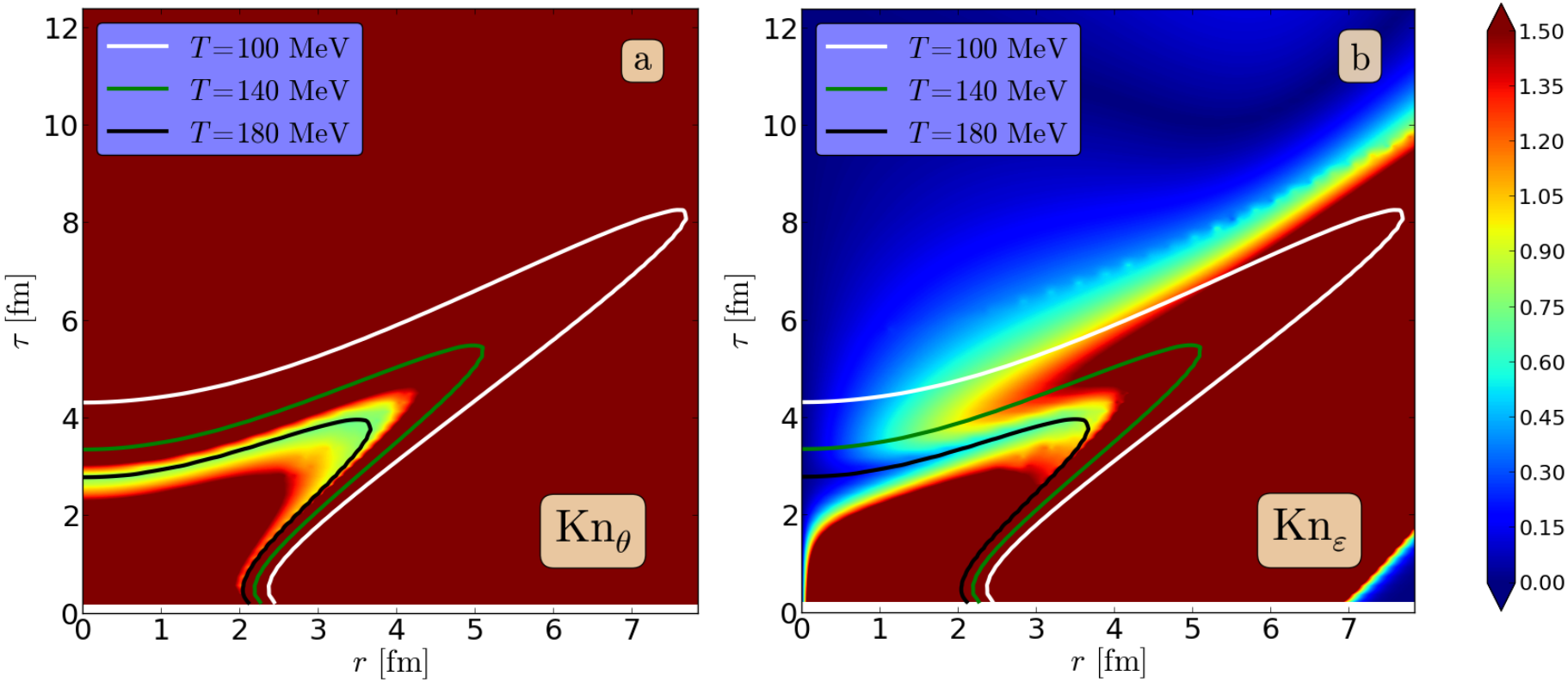






If full η/s(T) is used, domain of validity becomes limited.
Applicability of hydro at early times worsens.

Knudsen numbers – pA



For a full η/s(T), hydrodynamics is basically out of its domain of validity

Ridge from glasma production in p+p and p+A collisions

Kevin Dusling
Physical Review Letters
Brookhaven National Laboratory

Overview

1> A review of the ridge

from (connected) glasma graphs

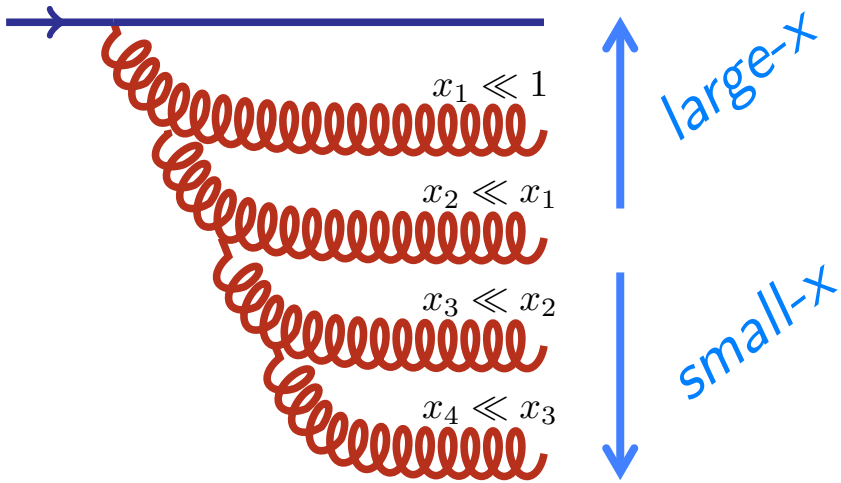
at high p_T (i.e. $> 1 \div 2$ GeV --- *hard* ridge)

2> $\cos(3\phi)$ in the hard & soft ridge

(and some difficulties with hydro)

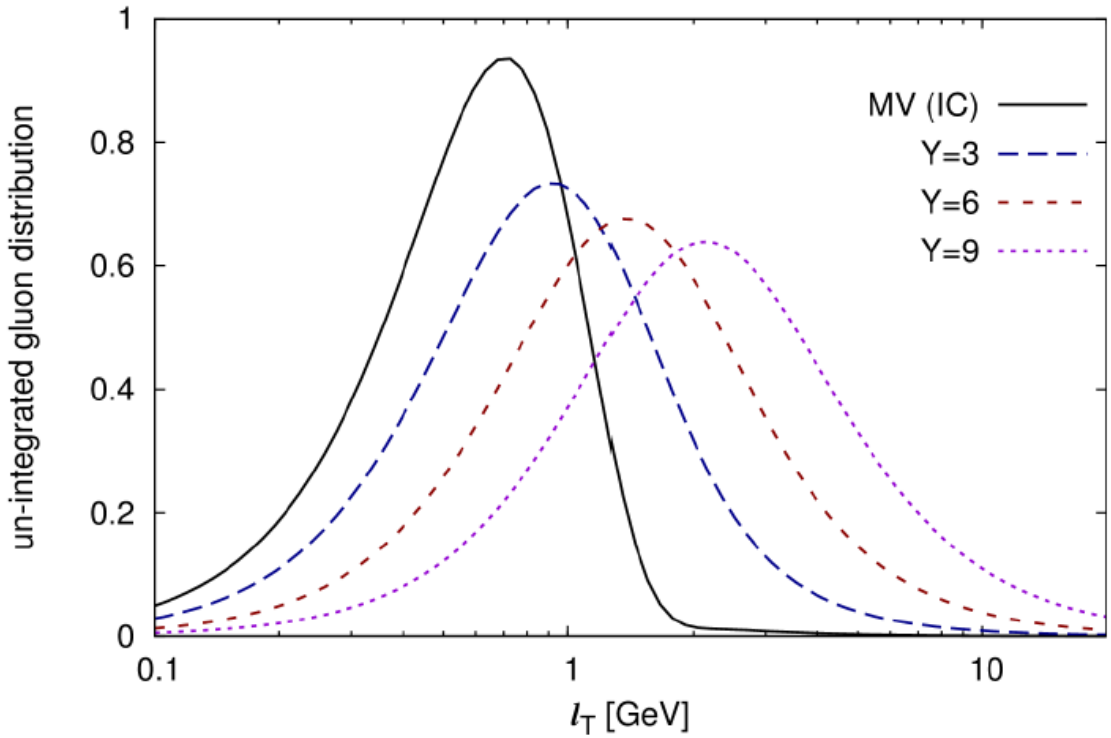
3> observation of collectivity in the soft ridge

Many-body high energy QCD: The Color Glass Condensate (CGC)



Observables must be independent from how the large-x and small-x degrees of freedom are separated:
Functional Renormalization Group equation

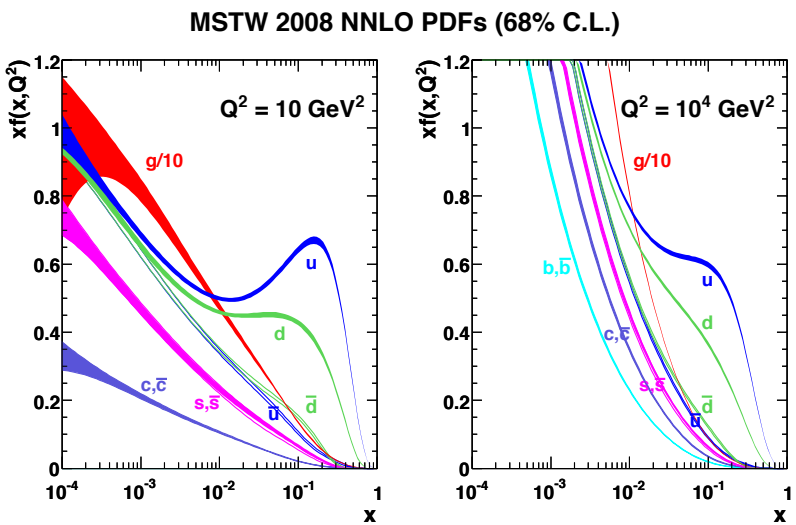
(JIMWLK - Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner).

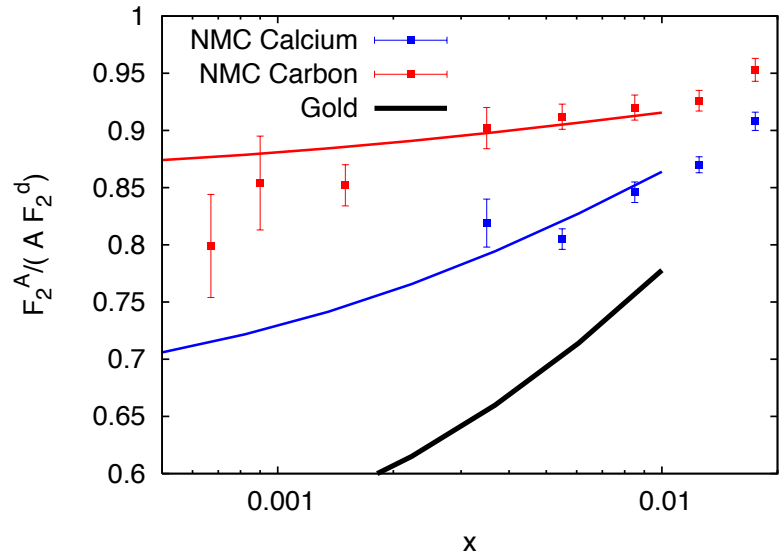


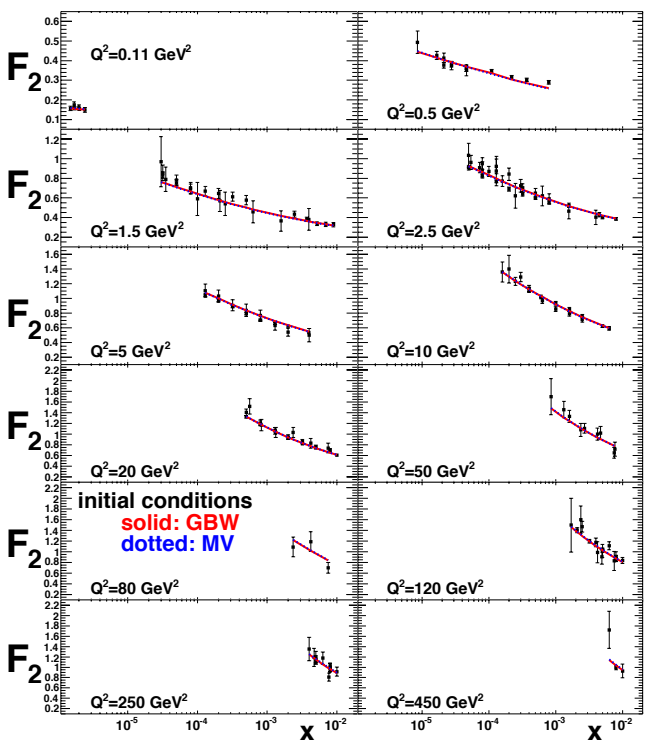
For reviews see:
McLerran, Lect. Notes Phys.583:291-334 (2002), arXiv:hep-ph/0104285

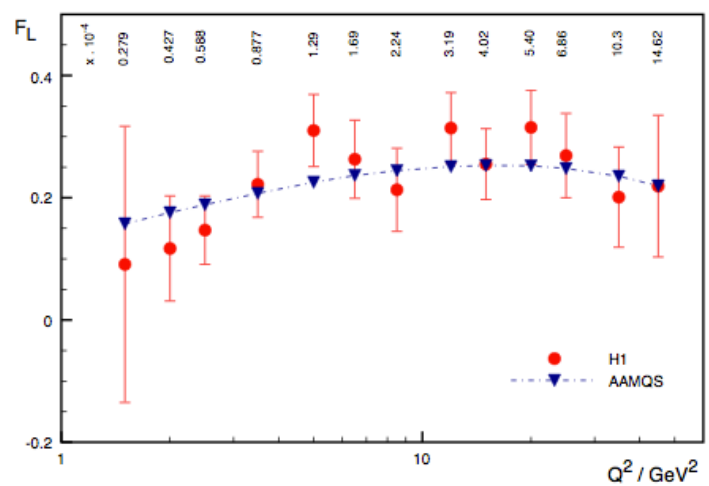
Gelis, Iancu, Jalilian-Marian, Venugopalan: Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333

There's a wealth of information on the proton wavefunction









Albacete, Milhano, Quiroga-Arias ,Rojo arXiv:1203.1043 (2012).
Quiroga-Arias, Albacete, Armesto, Milhano, Salgado J.Phys.G G38 (2011) 124124.
Albacete, Armesto, Milhano, Salgado PRD80 (2009) 034031.

Factorization in the dense/dense limit

Calculation of inclusive quantities can be written in a factorized form:

$$\left\langle \frac{d^n N_n}{d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_n} \right\rangle_{\mathrm{LLog}} = \int [D\rho_1]\,[D\rho_2]\, W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] \times \left.\frac{dN}{d^3\mathbf{p}_1}\right|_{\mathrm{LO}} \cdots \left.\frac{dN}{d^3\mathbf{p}_n}\right|_{\mathrm{LO}}$$

Gelis, Lappi, Venugopalan, PRD78 (2008) 054019, PRD78 (2008) 054020, PRD79 (2009) 094017

The complete treatment of inclusive observables at leading log of x involves

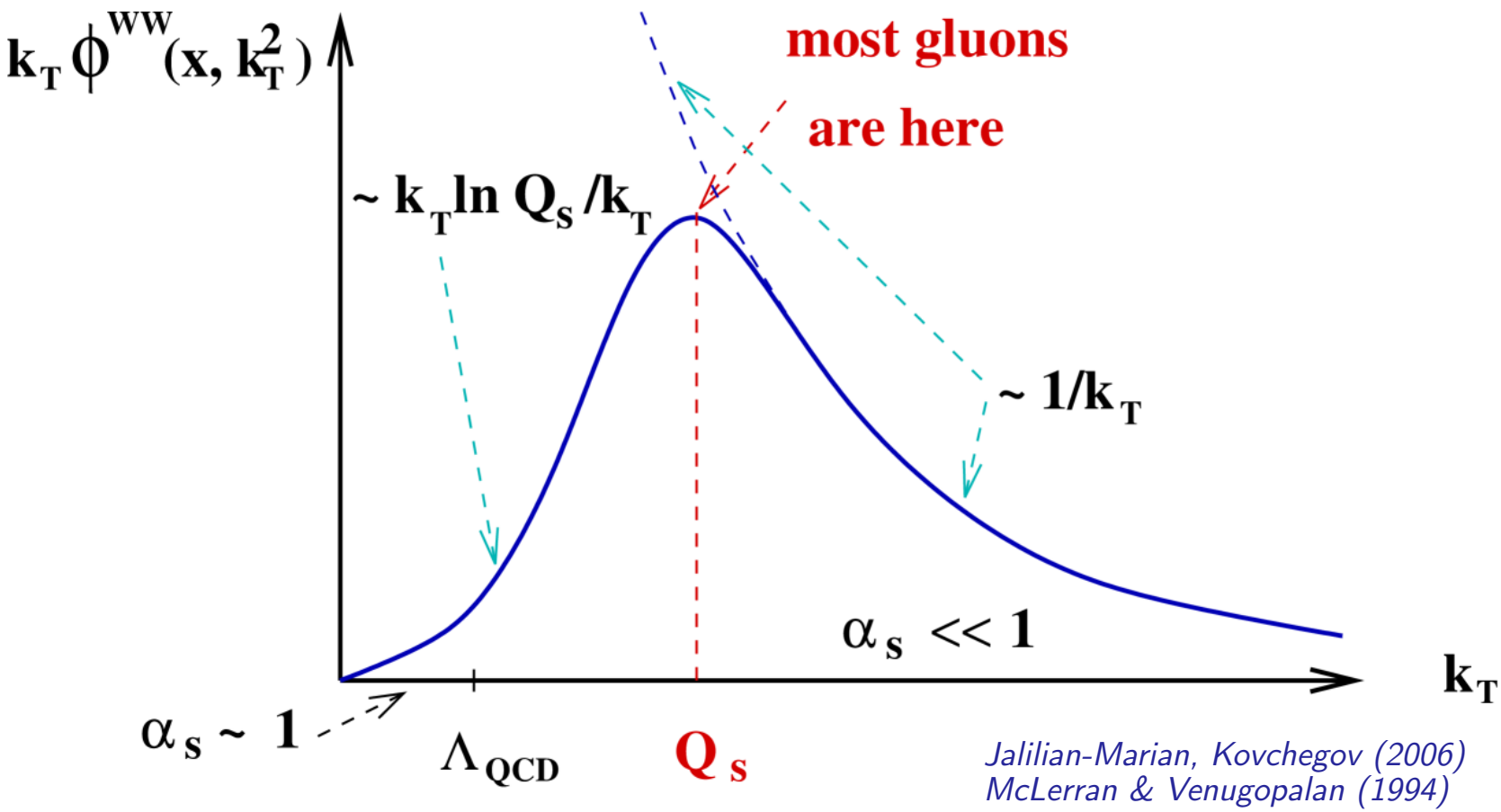
1. Solving the full JIMWLK hierarchy
2. Solving classical Yang-Mills

Our (KD, RV) ridge studies (from glasma graphs) consist of two approximations

1. Gaussian truncation: JIMWLK $\rightarrow$ rcBK
2. Perturbative limit: $p_T \gg Q_s$

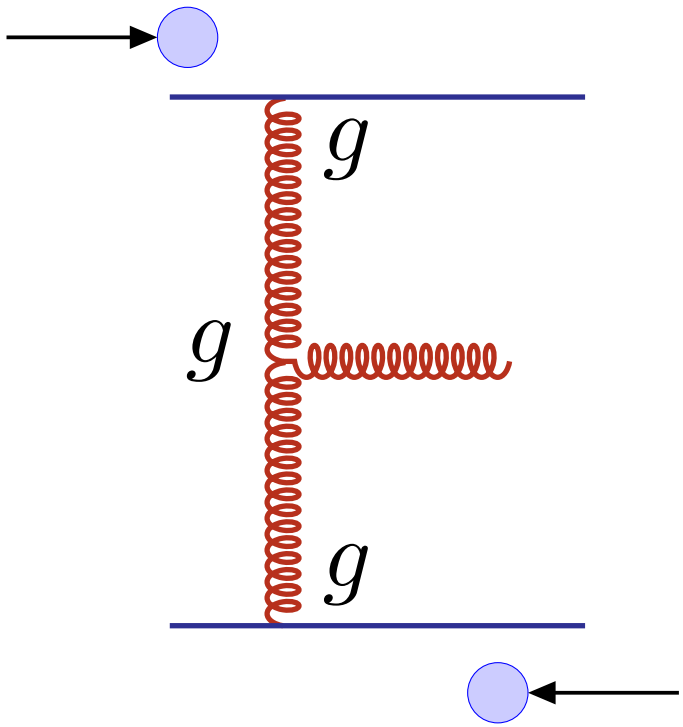
Gluon Saturation

A large nucleus, $A \gg 1$, results in a high occupation of gluons, $\rho \sim A^{1/3}$ and therefore the gluon-field can be treated classically.

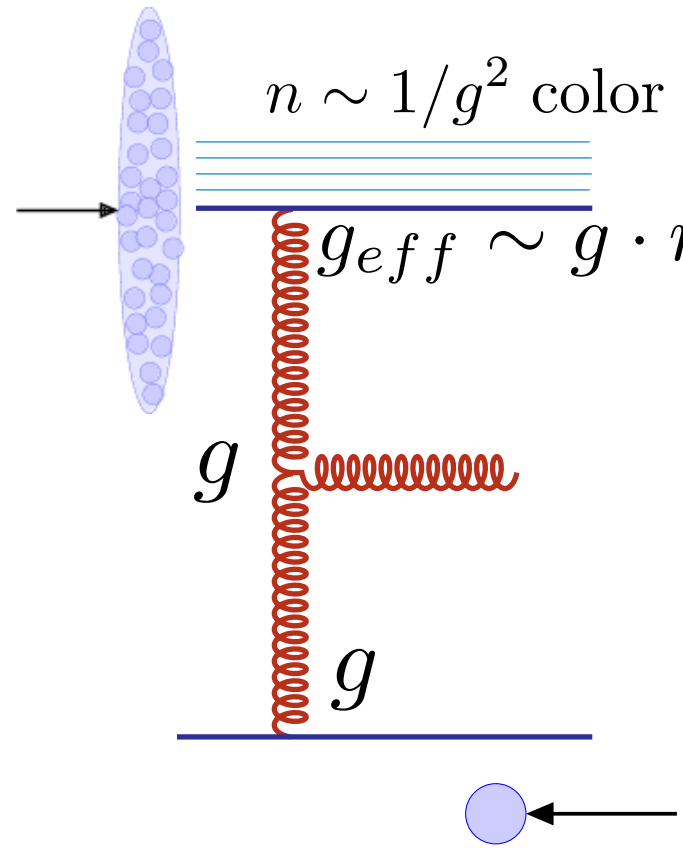


Power counting changes considerably ...

Power counting in p+A



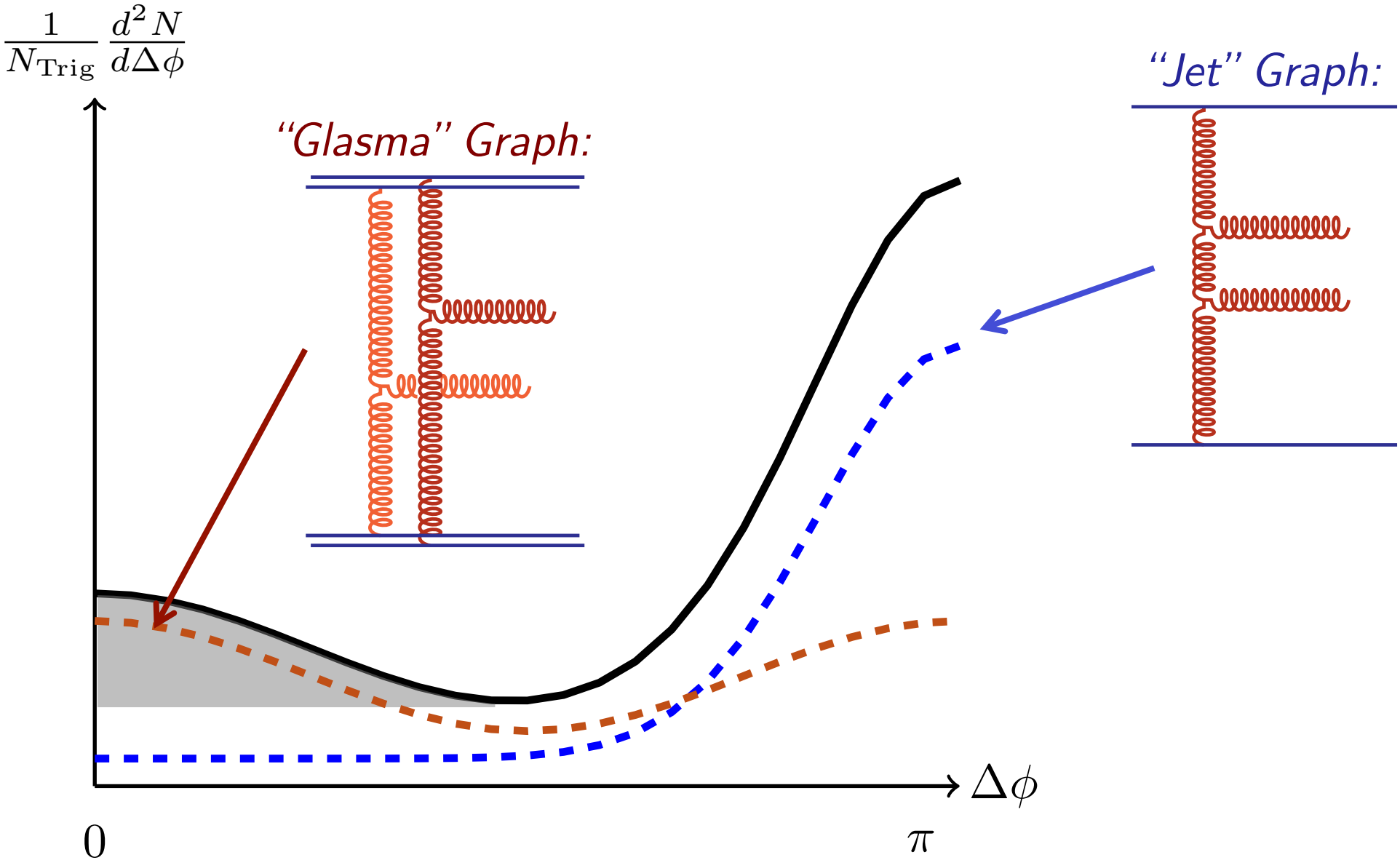
$\mathcal{O}\left(g^6\right)$



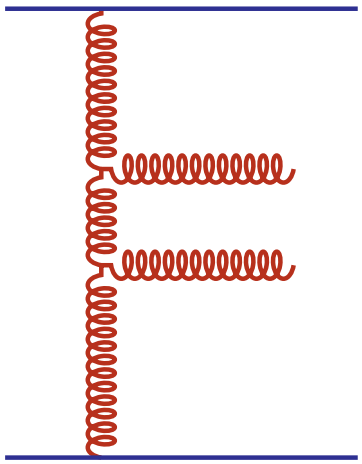
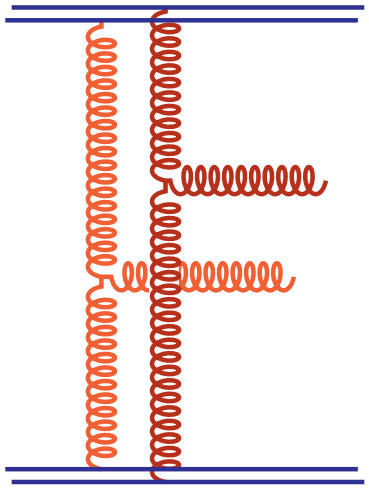
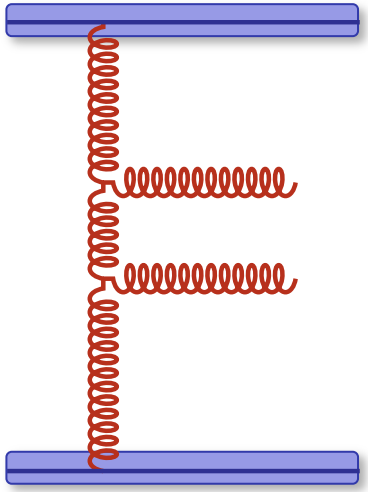
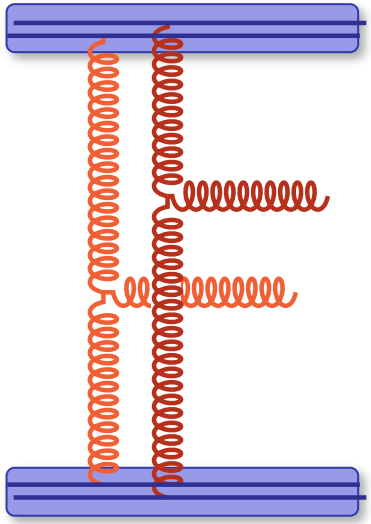
$\mathcal{O}\left(g^2\right)$

Can obtain saturation by large A, small x or high energy

Di-hadron production

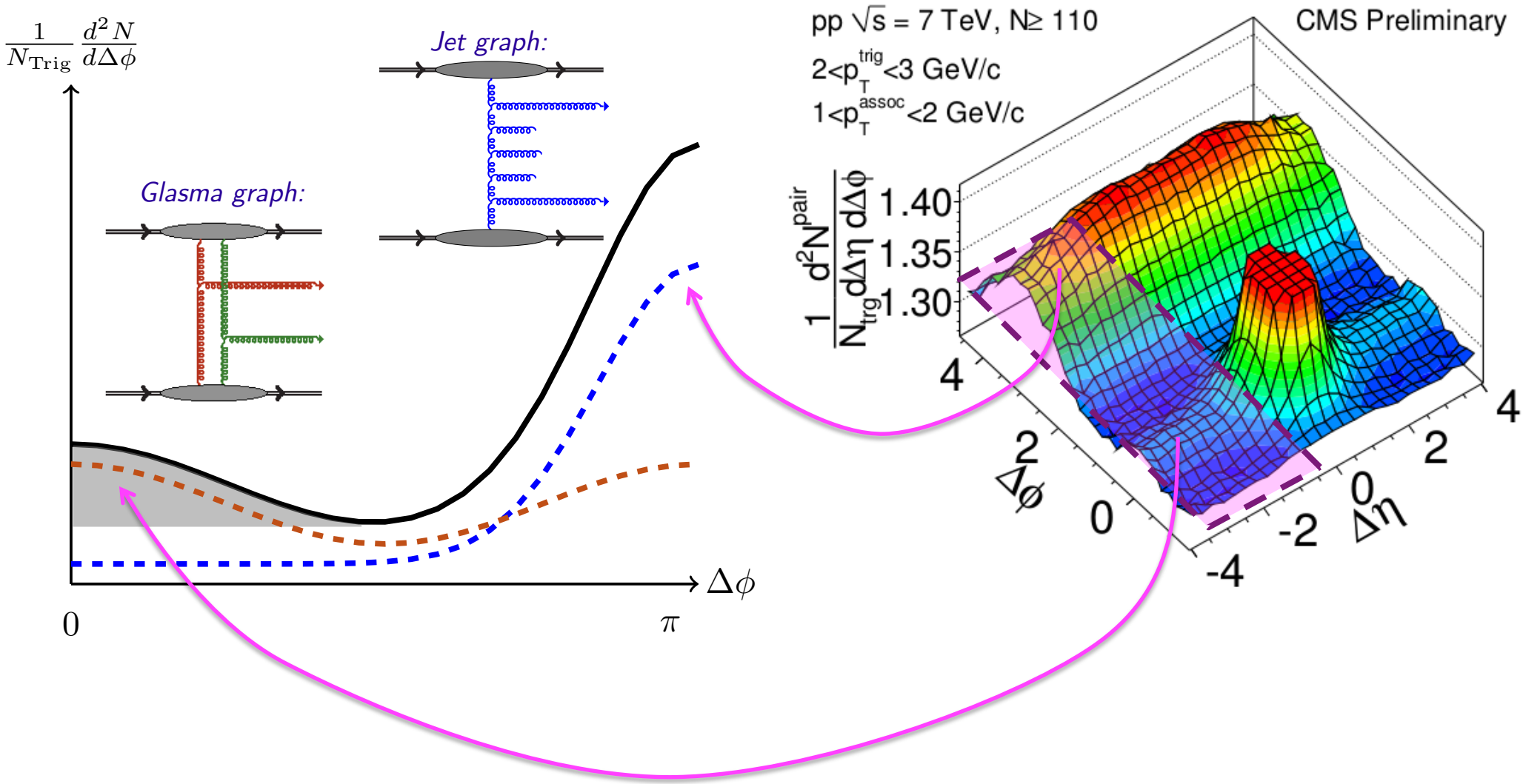


Power counting in QCD: multi-particle production

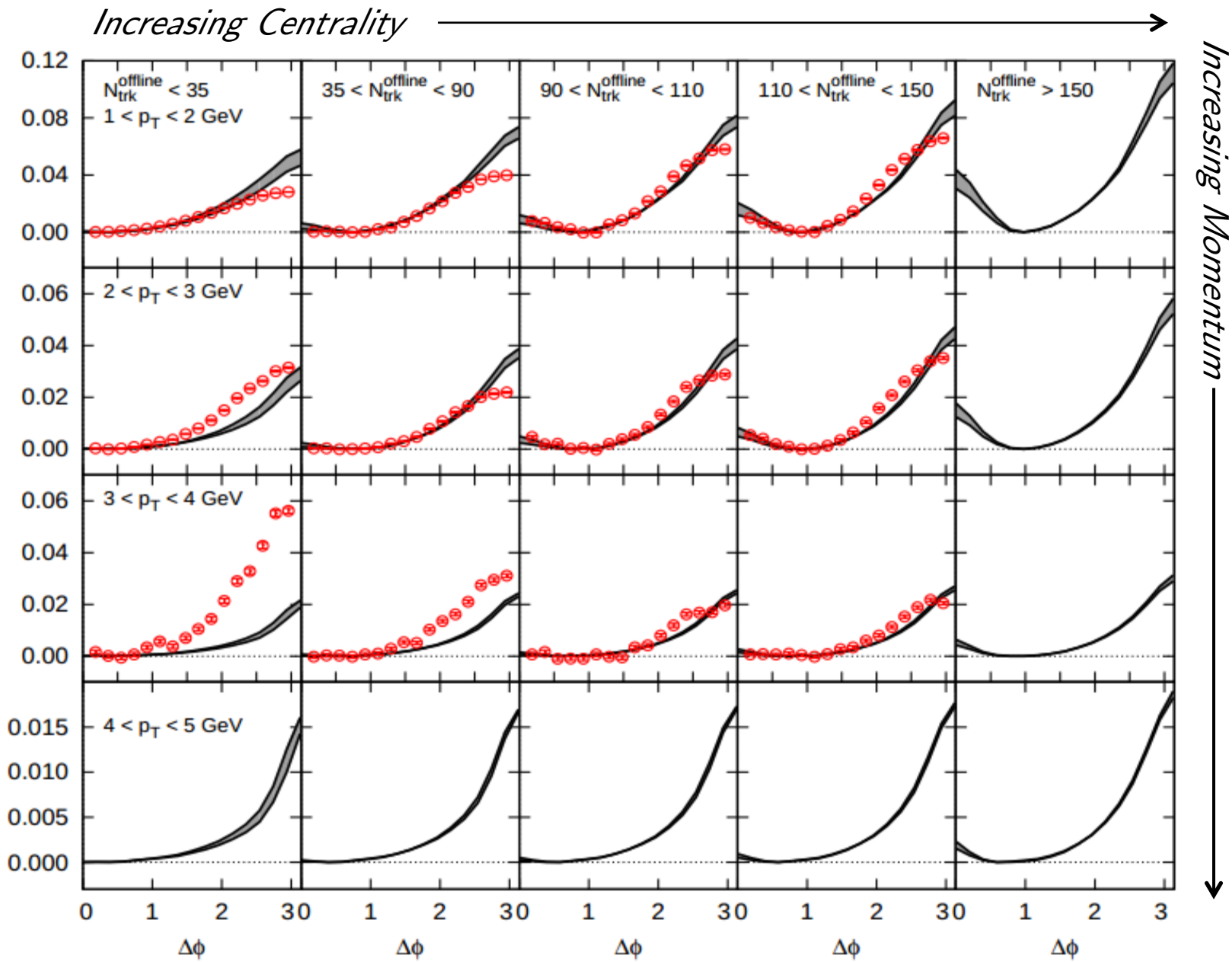
	"jet" graph	*"glasma" graph*
Low color density (min. bias events)	 $\mathcal{O}\left(g^8\right)$	 $\mathcal{O}\left(g^{12}\right)$
High color density ("central" events)	 $\mathcal{O}\left(g^0\right)$	 $\mathcal{O}\left(1/g^4\right)$

Expect large enhancement of glasma graph over jet for central events

Anatomy of a proton-proton collision

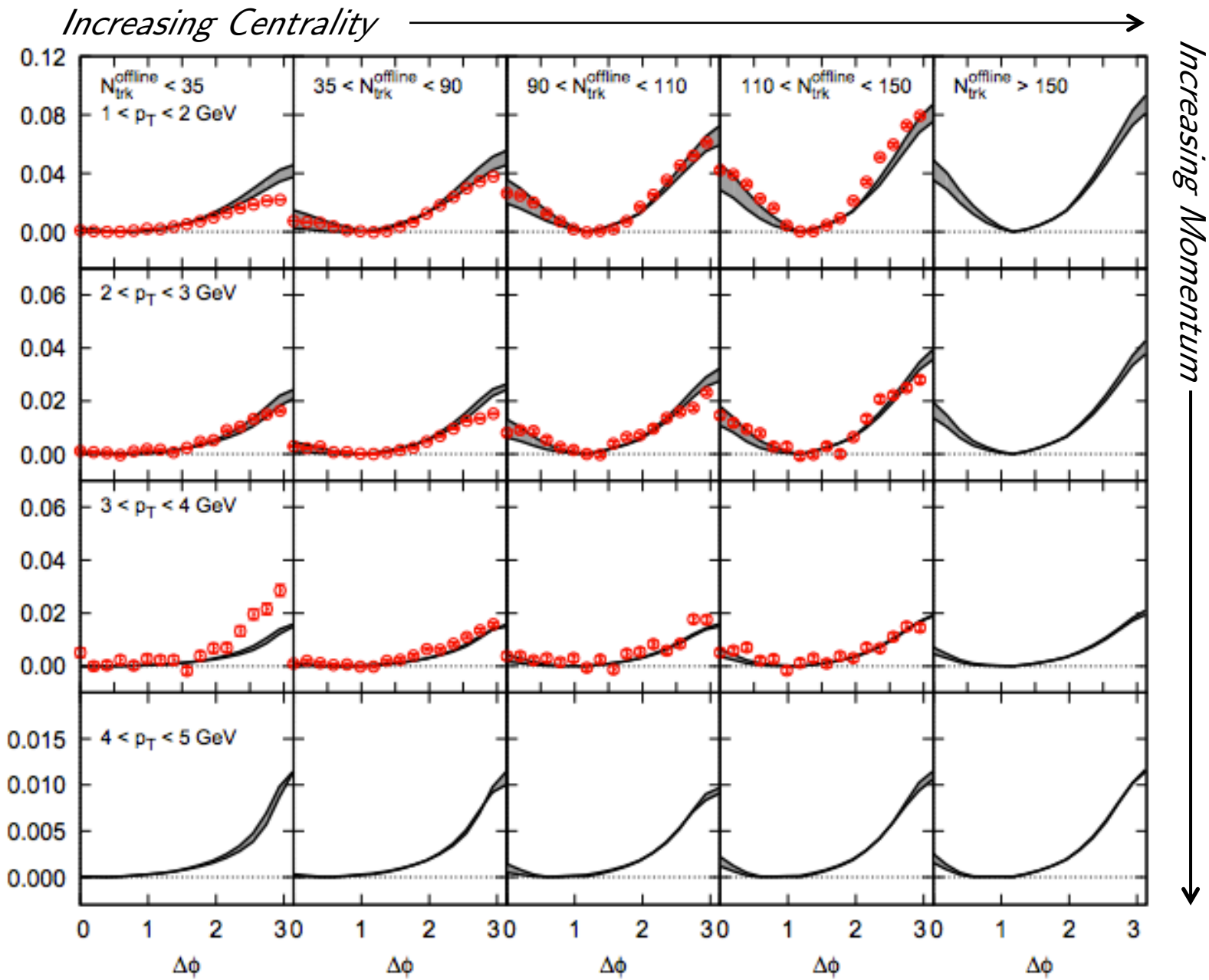


Proton-Proton systematics



Dusling, Venugopalan, Phys.Rev. D87 (2013) 094034, arXiv:1302.7018

Proton-Lead systematics

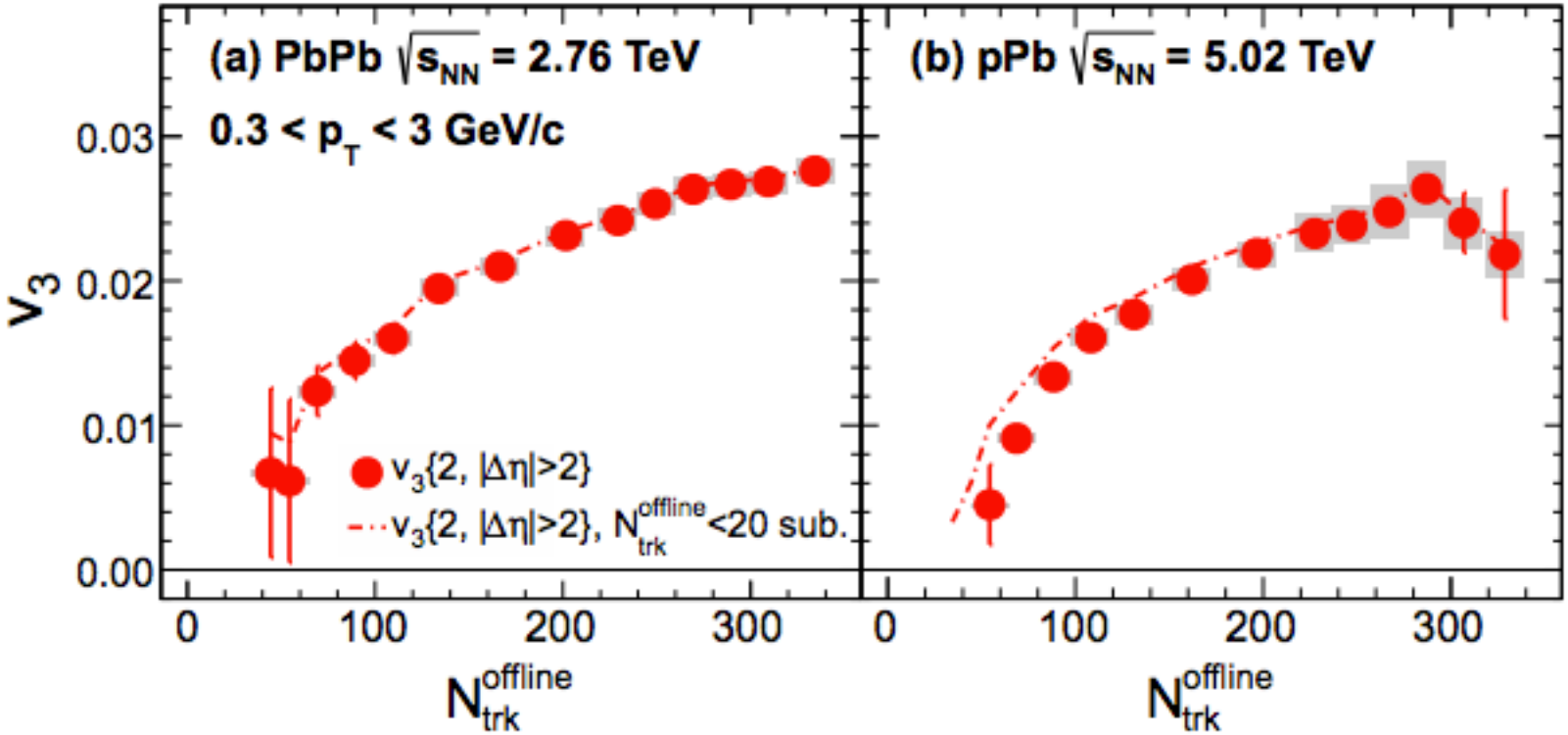


Dusling, Venugopalan, Phys.Rev. D87 (2013) 094034, arXiv:1302.7018

Part II: More Questions than Answers

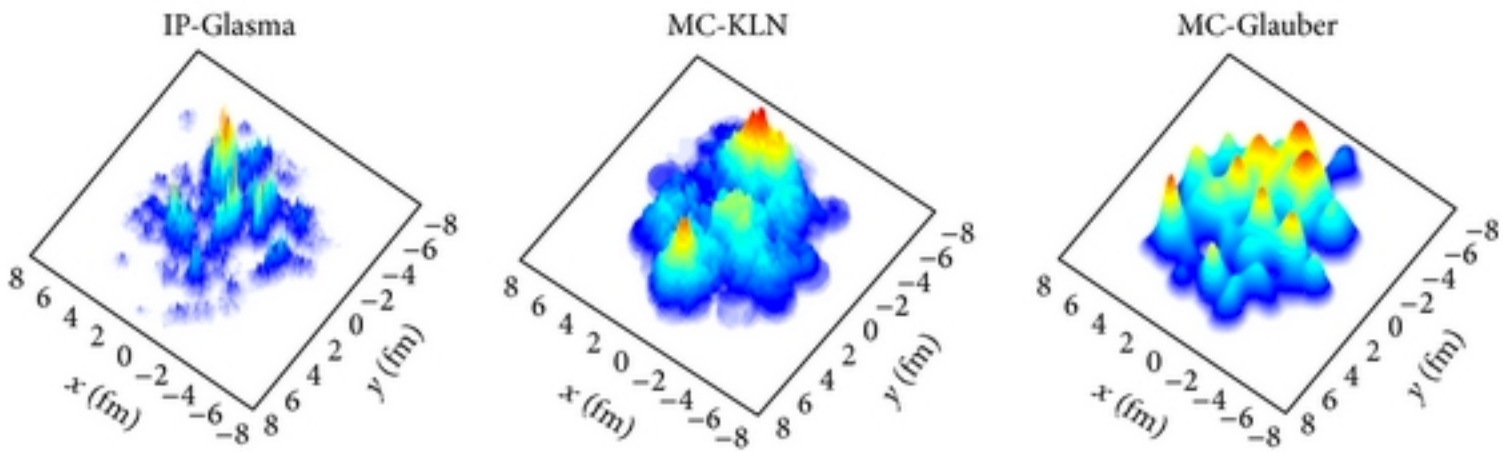
This cannot be the whole story...

Part II: The v3 puzzle

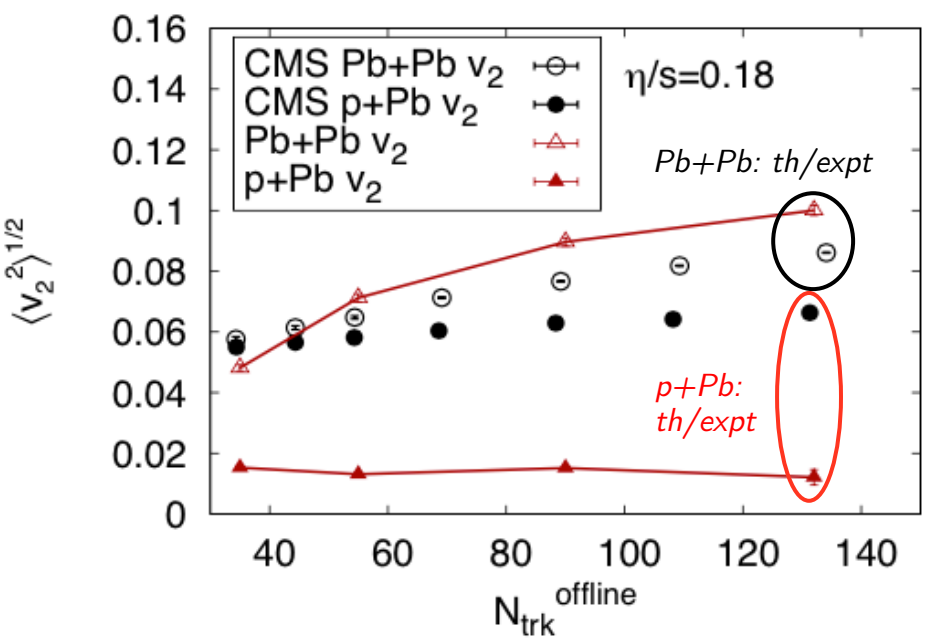


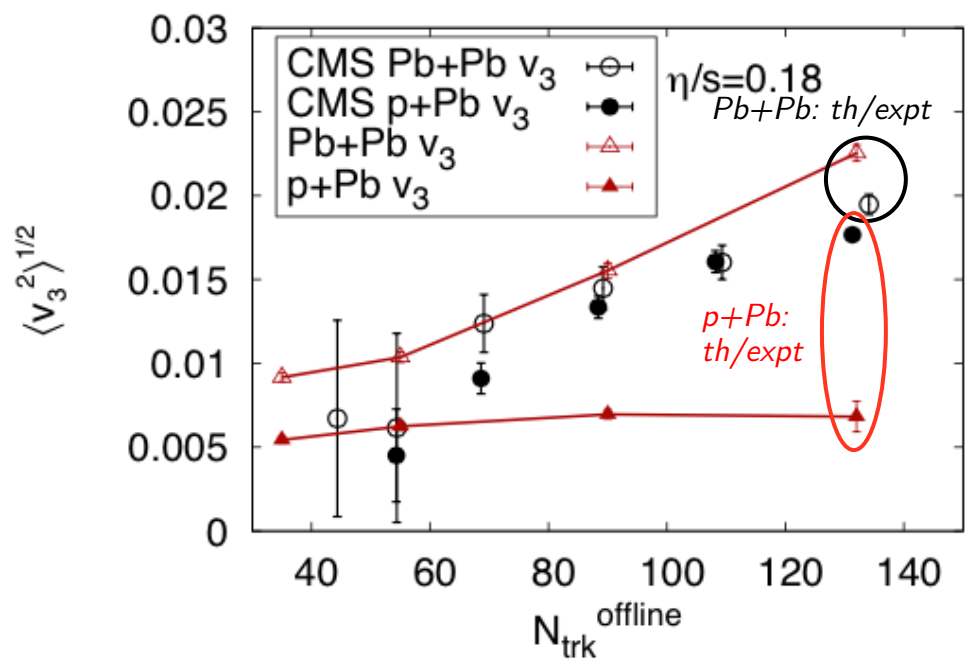
CMS Collaboration, Phys.Lett. B724 (2013) 213-240

Hydrodynamics in p+Pb ?



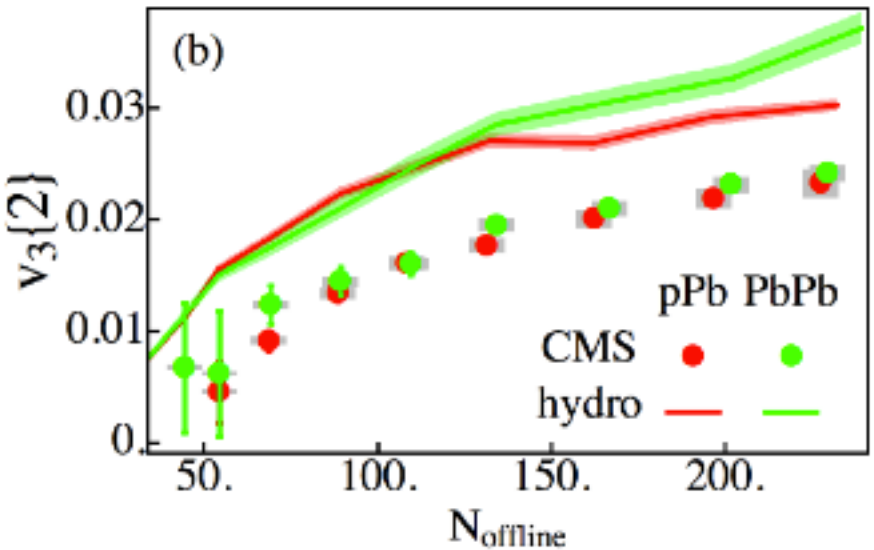
B. Schenke, P. Tribedy, R. Venugopalan
PRL 108 , 252301 (2012) , PRC 86, 034908 (2012)



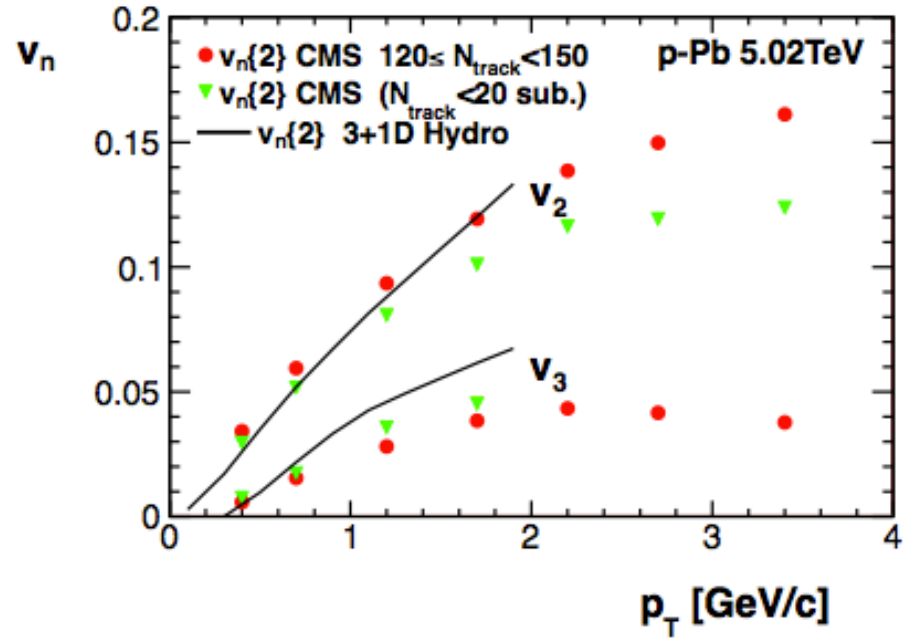


Schenke, Venugopalan PRL 113, 102301 (2014)

Hydrodynamics in p+Pb ?



Igor Kozlov, Matthew Luzum, Gabriel Denicol, Sangyong Jeon, Charles Gale arXiv:1412.3147



P. Bozek, W. Broniowski, G.Torrieri
Phys.Rev.Lett. 111 (2013) 172303

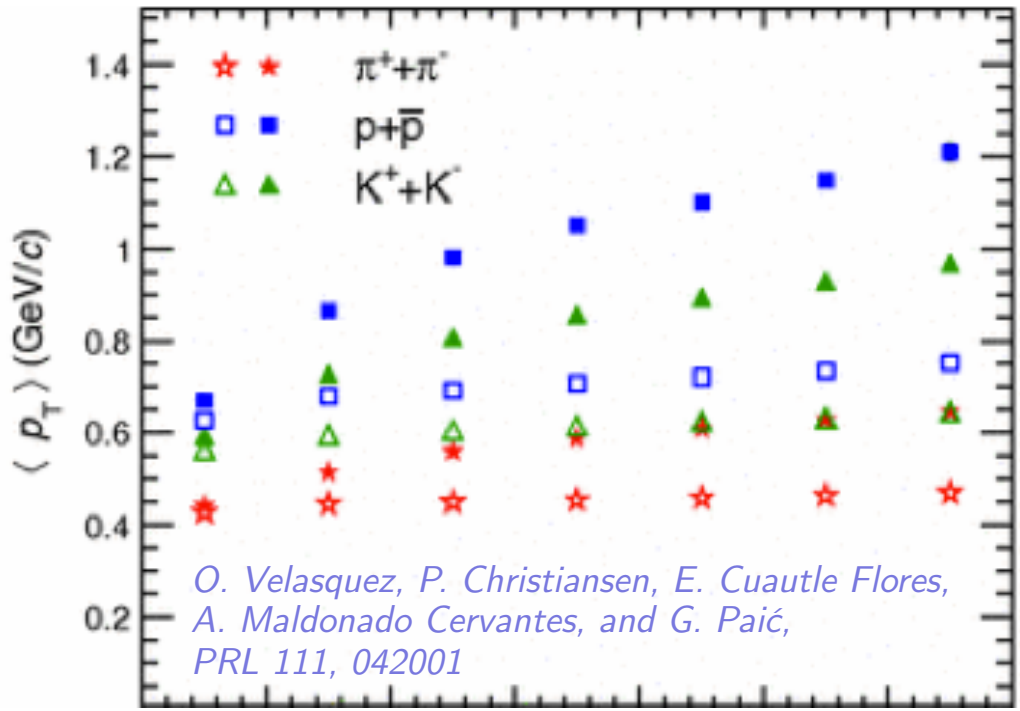
See also:
K. Werner, M. Bleicher, B. Guiot, Iu. Karpenko, T. Pierog,
Phys.Rev.Lett. 112 (2014) 23, 232301

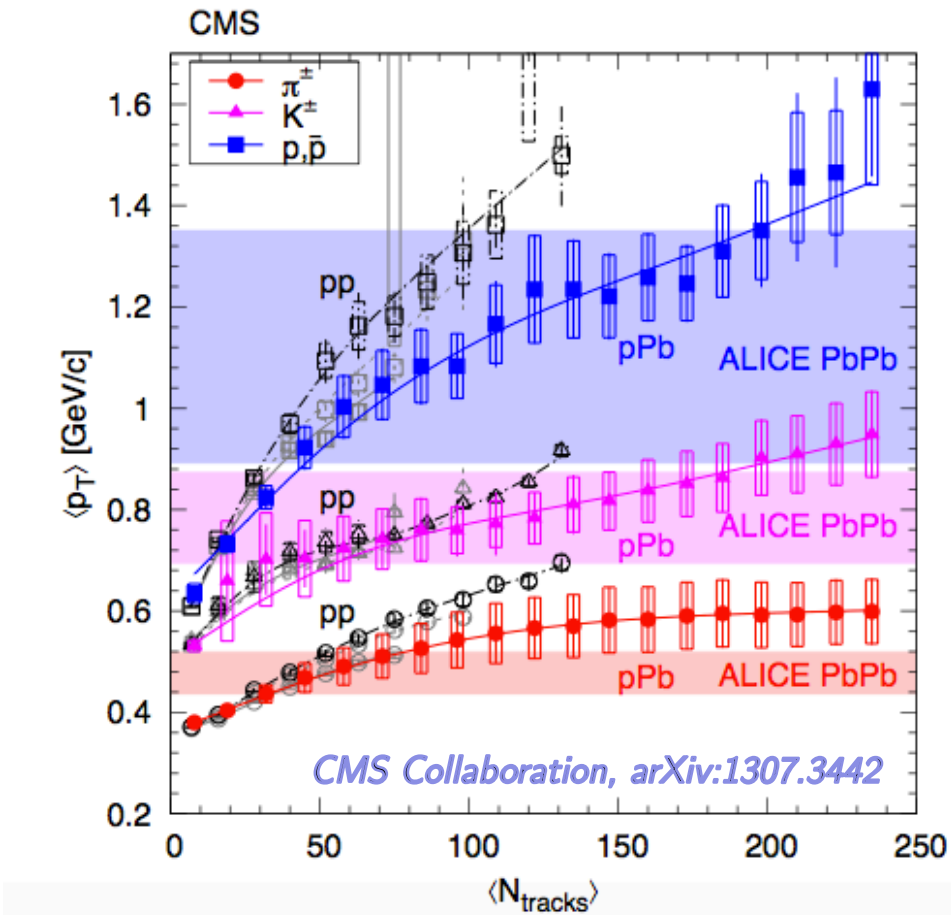
1> Which initial state is correct?

2> What about v_2 fluctuations, δv_2 ?

3> Tuning of Glauber initial conditions between p+A and A+A required to get correct two particle correlation matrix $V_{n\Delta}\left(p_T^a, p_T^b\right)$

The radial flow





1> Mass splitting observed in peripheral pp

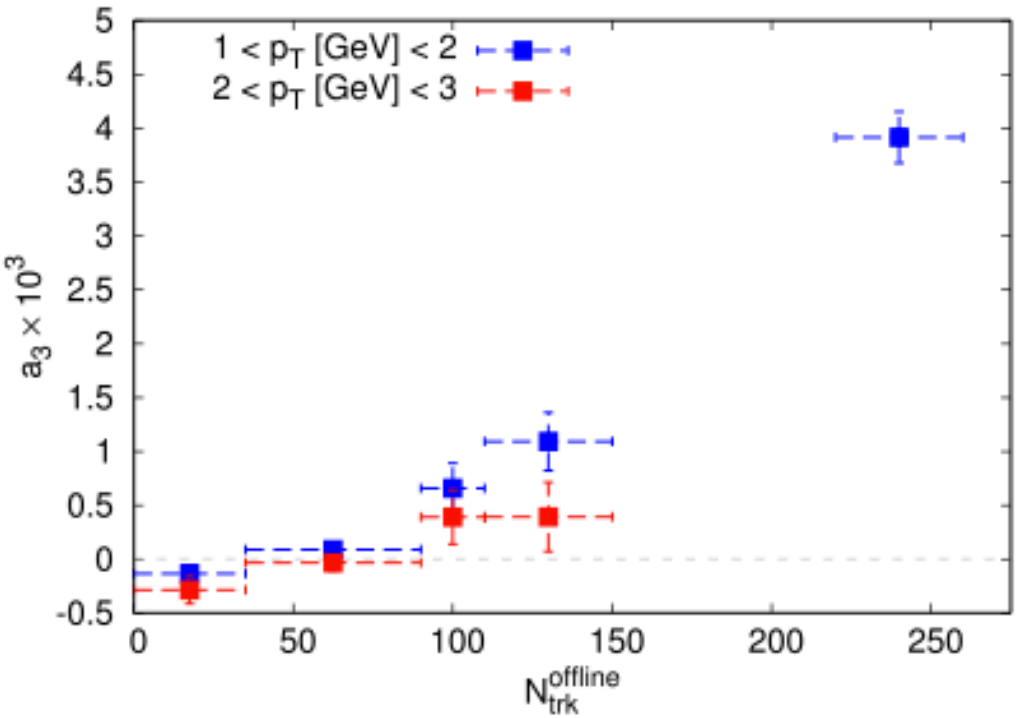
2> Rate of change of mass splitting largest for pp:
Larger flow in pp or color reconnection?

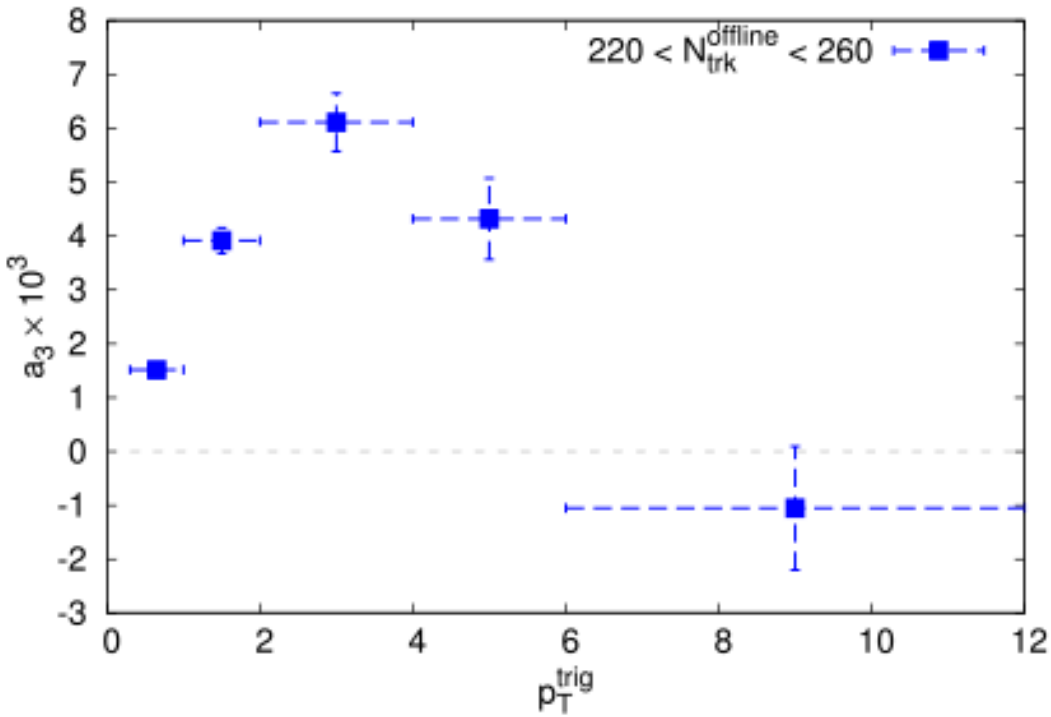
3> Largest effect is for protons:
We don't understand the proton yields in central Pb+Pb

4> v_2 mass splitting really isn't an independent piece of evidence:
The push out of protons can be driven by denominator.

But v3 persists to very high momentum...

$$\frac{1}{N_{\rm trig}}\frac{d^2N}{d\Delta\phi} = a_0 + 2\sum_{n=1} a_n \cos(n\Delta\phi)$$

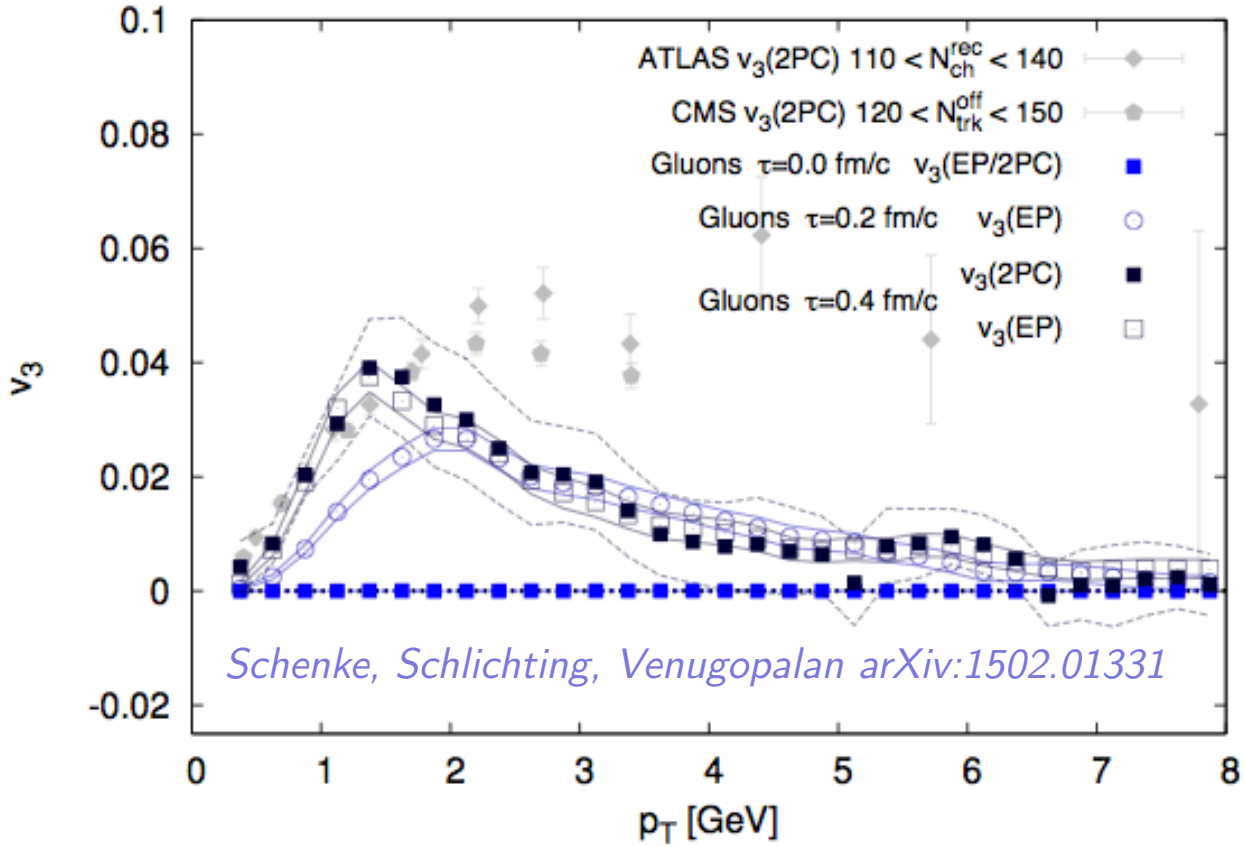




Can "flow" generate a_3 out to 5 GeV ?

Dusling, Venugopalan, Nucl.Phys. A931 (2014) 283-287

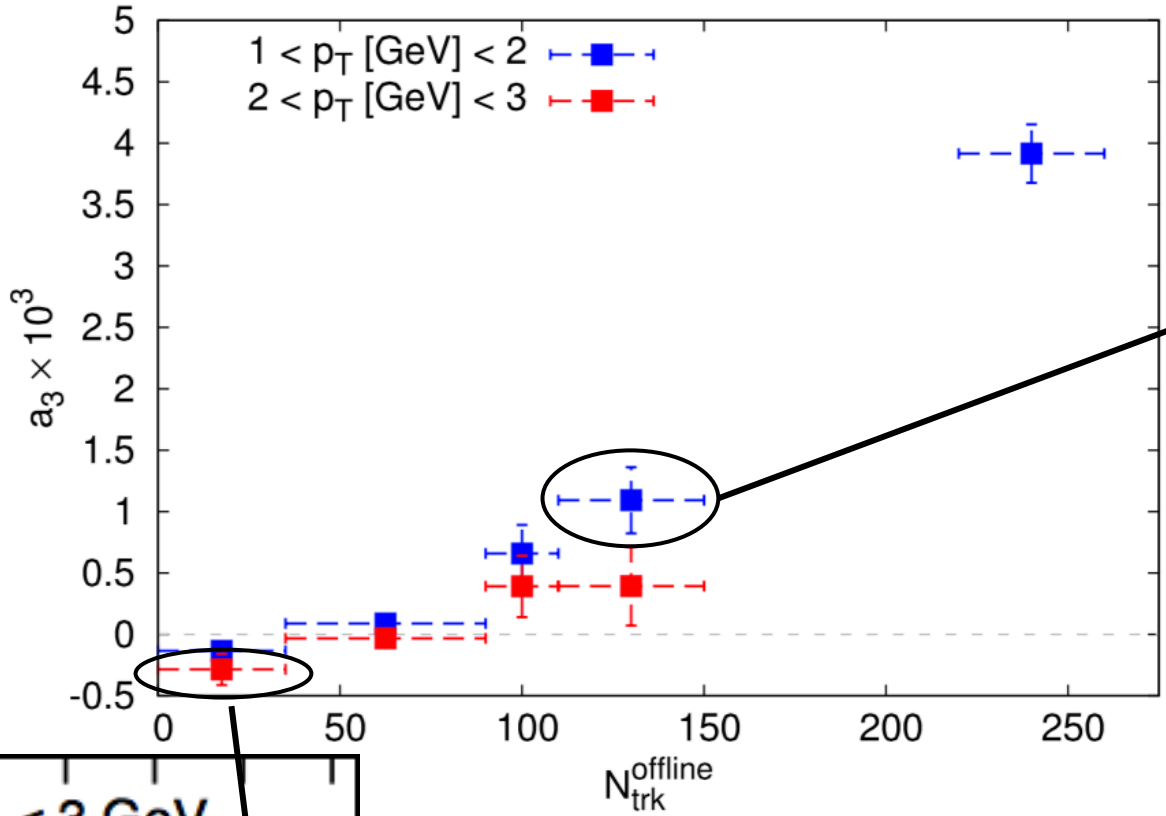
Multiple re-scatterings can generate v3 in the soft ridge ... so why not at high p_T?

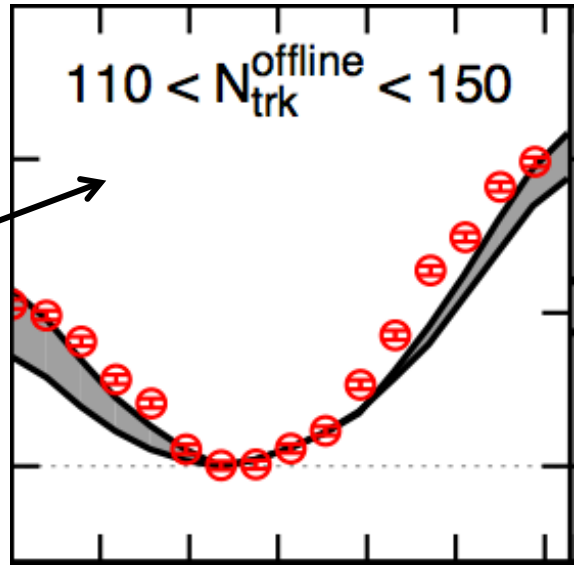


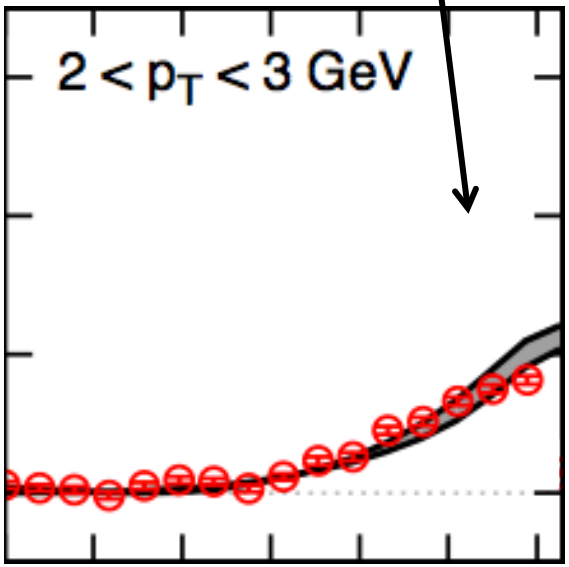
1> v_3 identically zero at τ=0 (consistent with glasma diagrams)

2> v_3 not correlated with ε_3

CYM is consistent with glasma graphs?





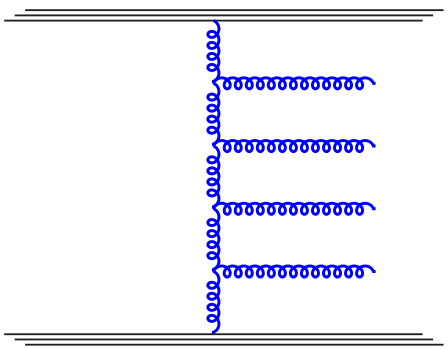


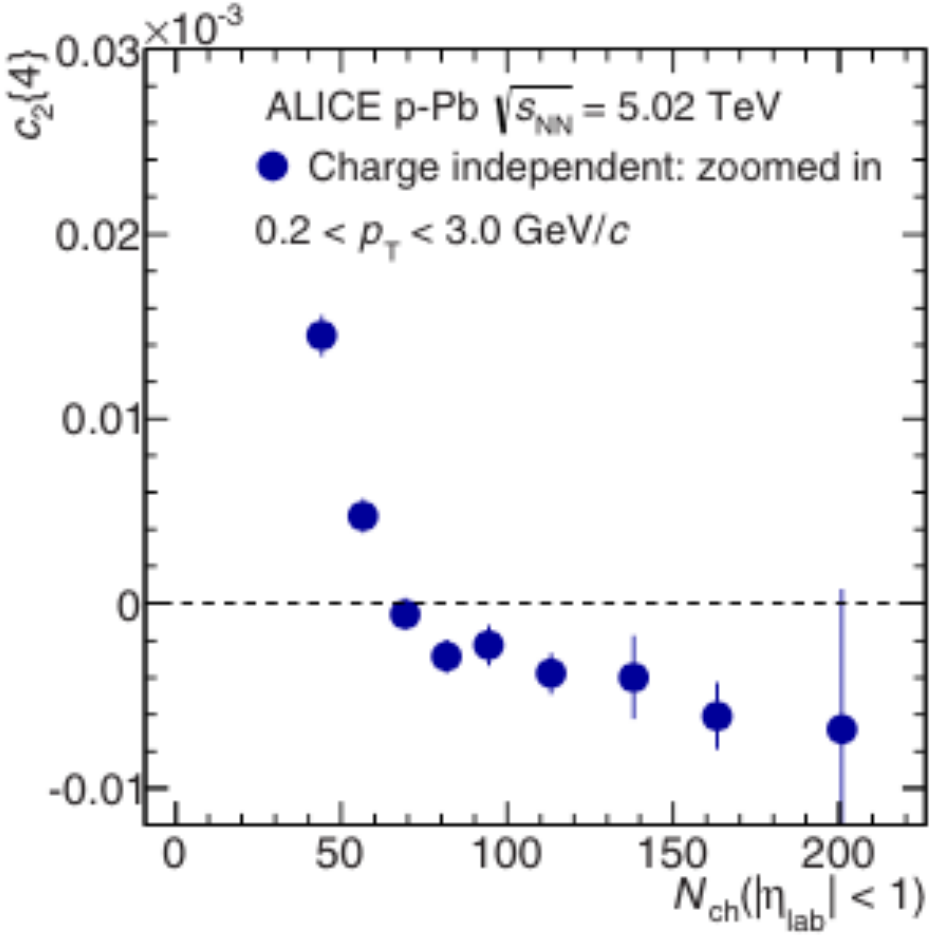
This is a small correction!

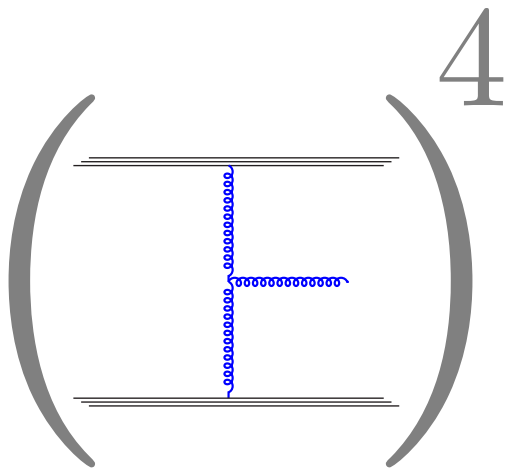
I see no reason why a single soft rescattering (off jet or glasma graph) would not reconcile this.

But the calculation needs to be done.

Four and more particle correlations





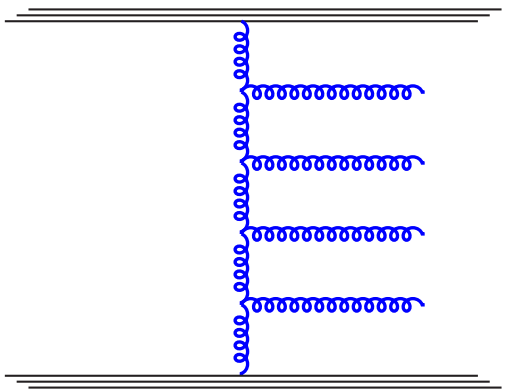


$$c_2\{4\} \sim \left\langle e^{2i(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle_c - v_2^4$$

Dumitru, McLerran, Skokov, Phys.Lett. B743 (2015) 134-137

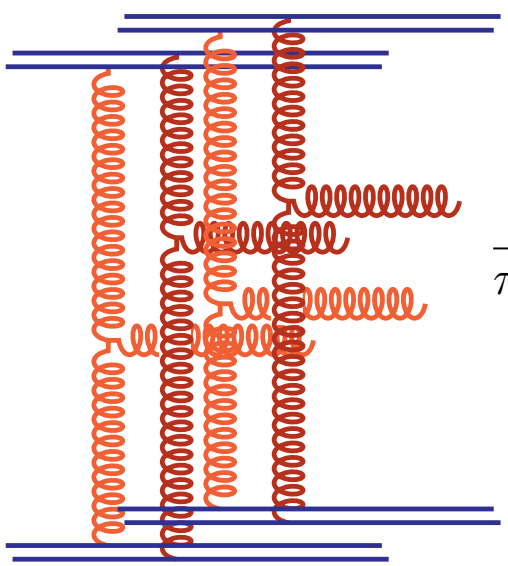
Intrinsic (connected) contributions

"Jet" Graph:



$$\frac{\alpha_s^n N_c^n}{4\pi^{2n+4}(N_c^2-1)}\left(\prod_{i=1}^{n}\frac{1}{\mathbf{k}_{i\perp}^2}\right) S_\perp \int d^2\mathbf{k}_{0\perp}\Phi(\mathbf{k}_{0\perp})\Phi\left(\mathbf{k}_{0\perp}+\mathbf{k}_{1\perp}+\cdots+\mathbf{k}_{n\perp}\right)$$

"Glasma" Graph:



$$\frac{(n-1)!\ N_c^n}{\pi^{4n}(N_c^2-1)^{2n-1}}\frac{\alpha_s^n S_\perp}{\mathbf{k}_{1\perp}^2\cdots\mathbf{k}_{n\perp}^2}\int\frac{d^2\mathbf{p}_\perp}{(2\pi)^2}\Phi^n(\mathbf{p}_\perp)\left[\Phi(\mathbf{k}_{1\perp}\pm\mathbf{p}_\perp)\cdots\Phi(\mathbf{k}_{n\perp}\pm\mathbf{p}_\perp)\right]$$

Intrinsic (connected) contributions

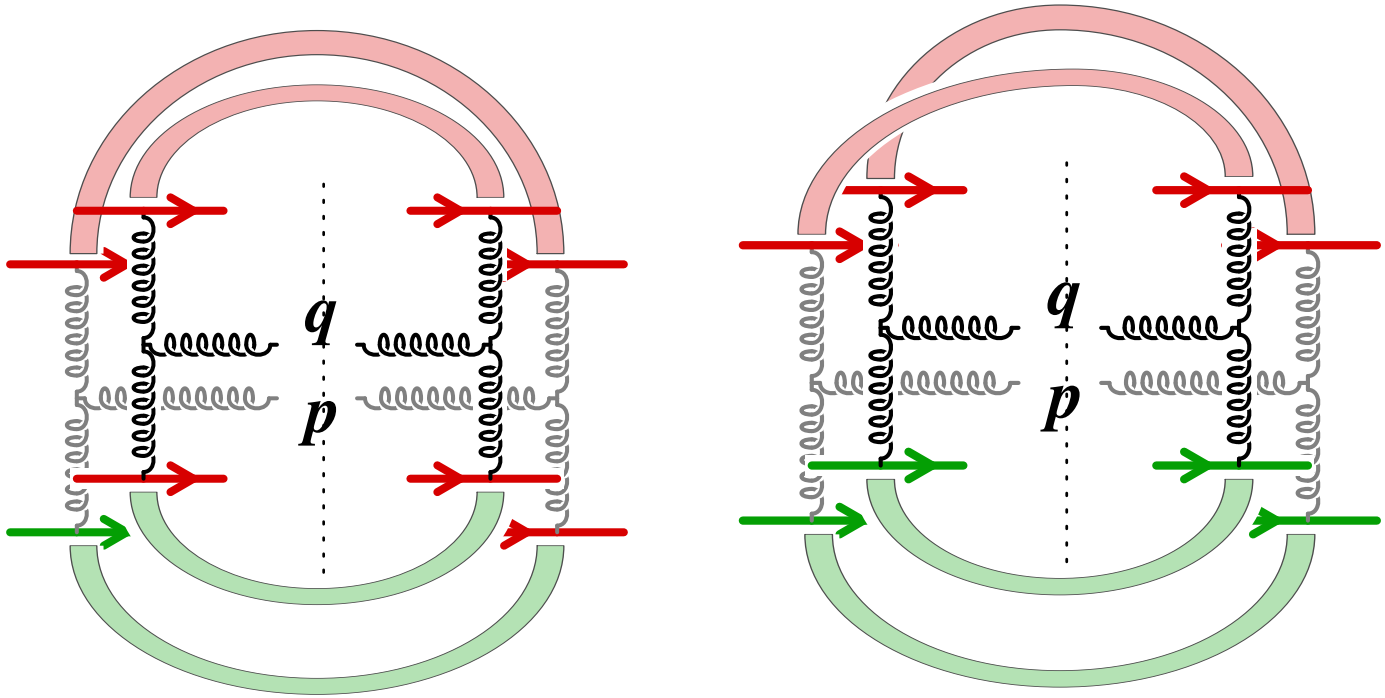
1> Both yield $c_2\{4\}>0$ at high pT where calculation reliable

2> Experimentally does $c_2\{4\}>0$ at high pT?
We expect this transition to occur but where?

3> Low multiplicity $c_2\{4,6,8\}$ contains interesting QCD physics.

4> How can we understand the negative $c_2\{4\}$ of soft particles?

Another look at glasma graphs

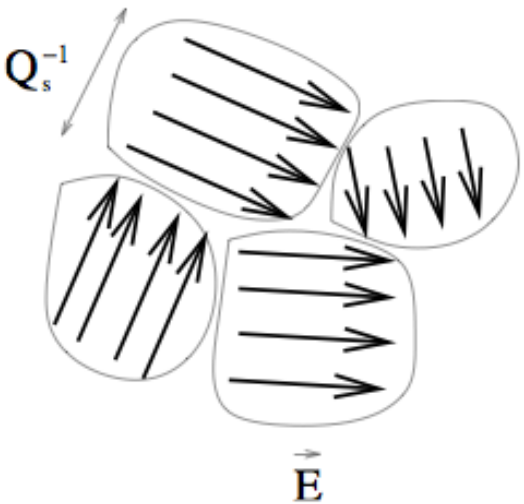


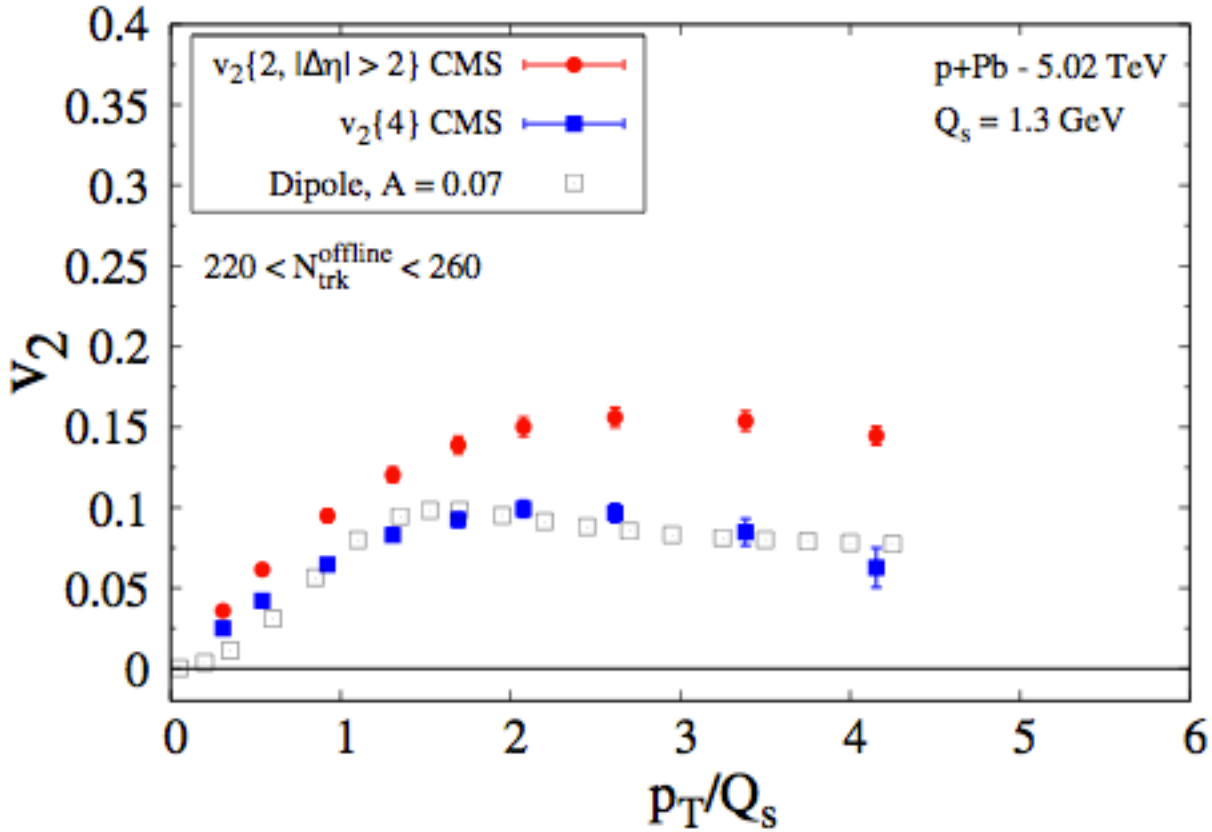
$$\langle \rho^a(\mathbf{x})\rho^b(\mathbf{y})\rangle = \delta^{ab}\delta^{(2)}(\mathbf{x}-\mathbf{y})S\left(\frac{|\mathbf{x}+\mathbf{y}|}{2}\right)$$

Dumitru,Gelis,McLerran,Venugopalan, arXiv:0804.3858

Symmetry breaking by color domains

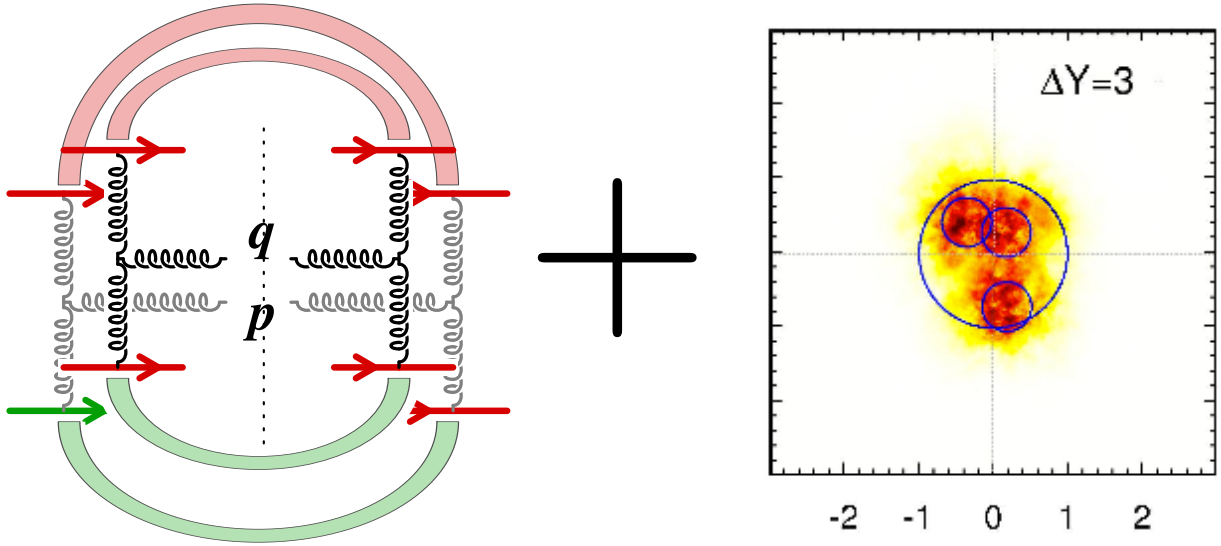
Kovner, M. Lublinsky
Phys. Rev. D84 (2011) 094011





Dumitru,Giannini, Nucl.Phys. A933 (2015) 212

Symmetry breaking by color domains



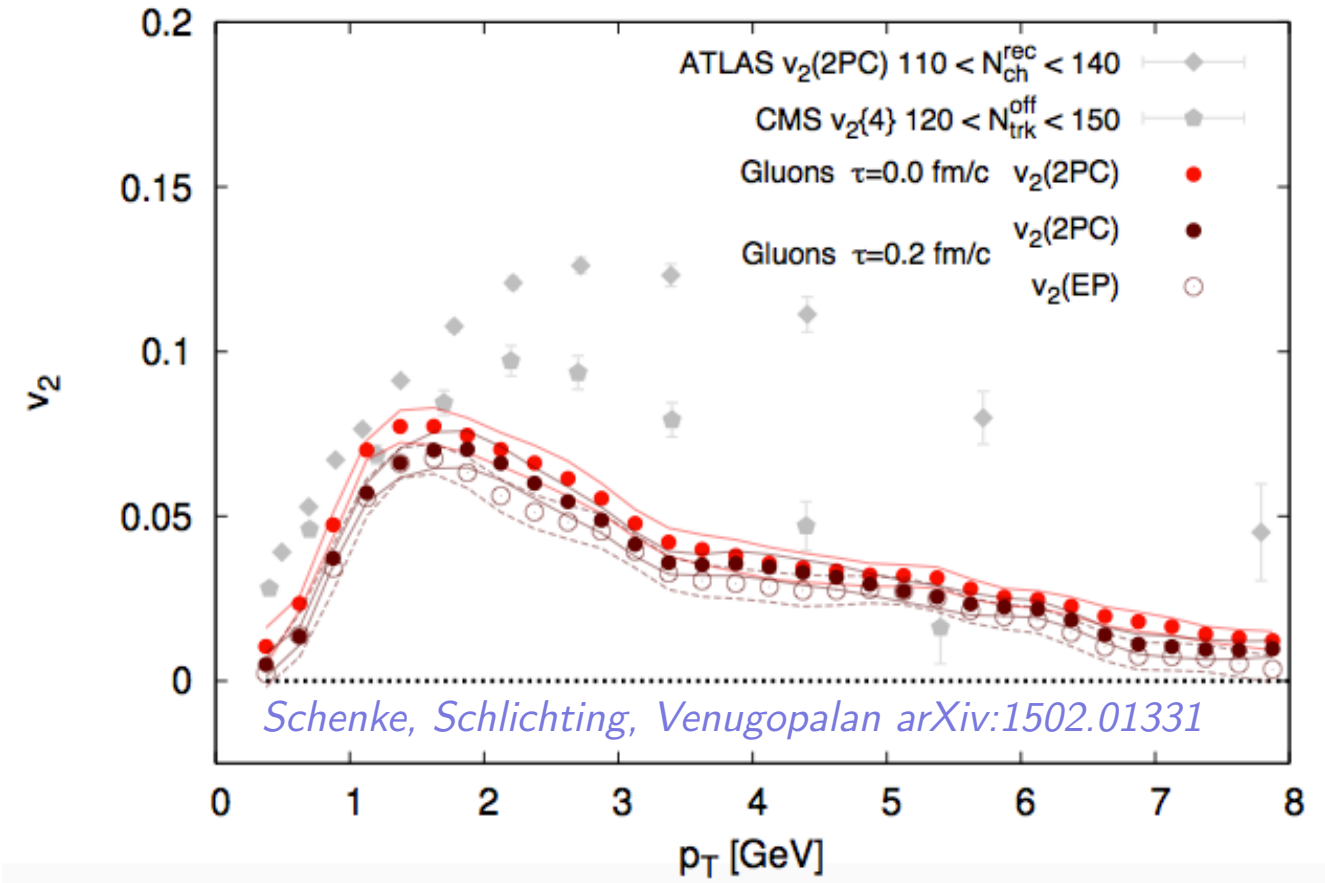
$$\langle \rho^a(\mathbf{x})\rho^b(\mathbf{y})\rangle = \delta^{ab}\delta^{(2)}(\mathbf{x}-\mathbf{y})S\left(\frac{\mathbf{x}+\mathbf{y}}{2}\right)$$

1> This symmetry breaking is enough for the disconnected glasma graphs to give v_2

[At least within the perturbative glasma calculation]

2> As the v_2 comes from the single particle spectra: $c_2\{4\}<0$

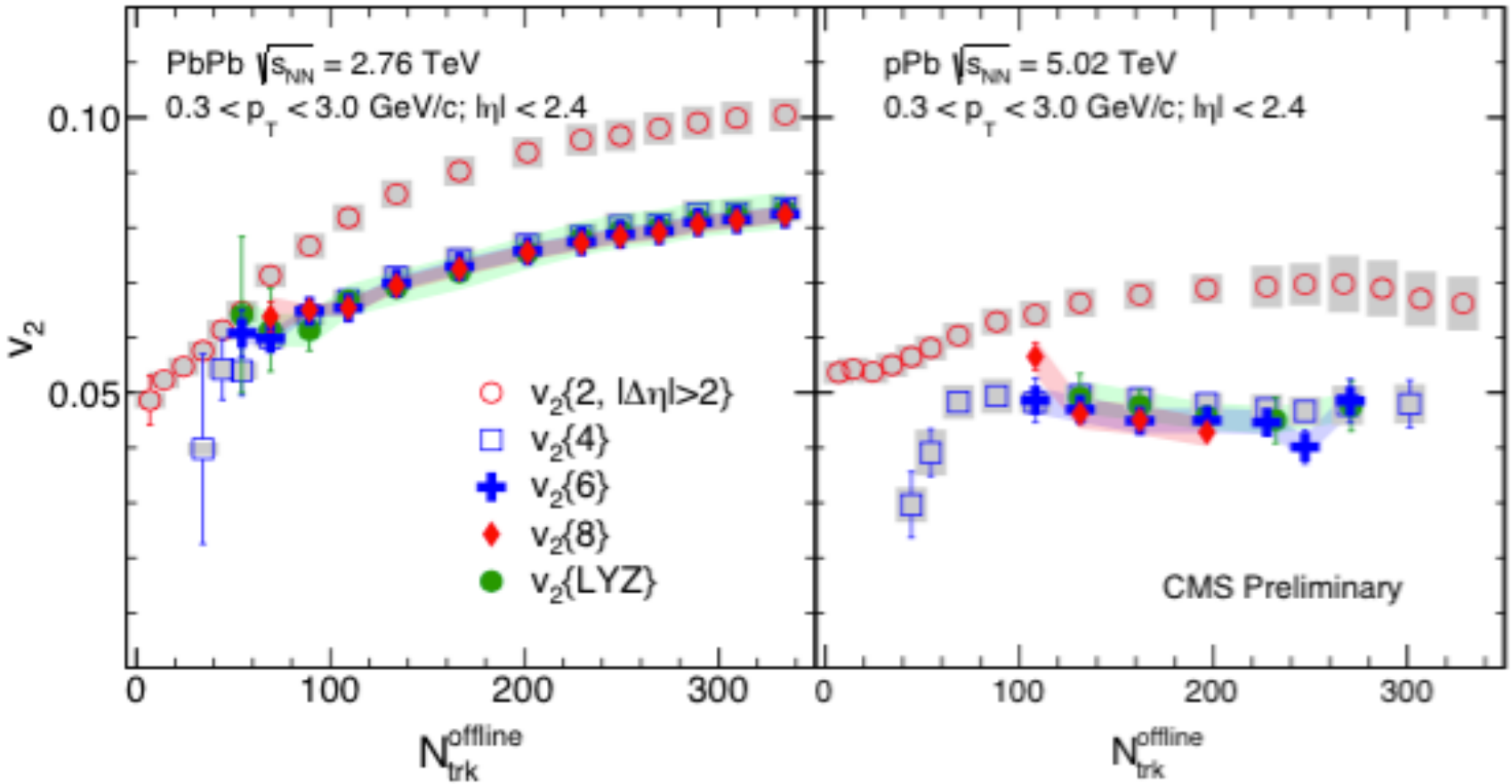
v_2 from CYM



1> Above calculation has both connected and disconnected graphs

2> Need to think about how to disentangle contributions

Further evidence for "collectivity"



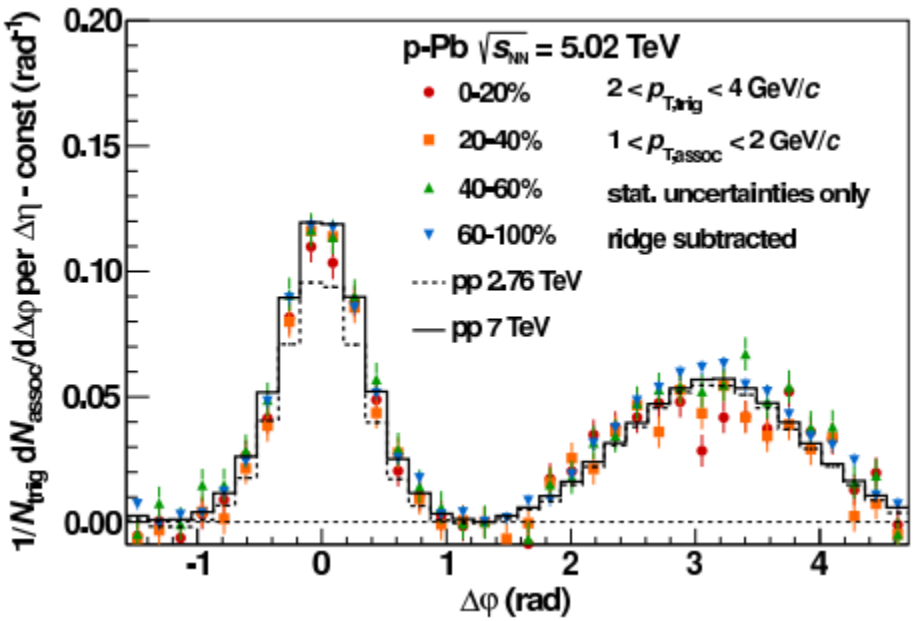
1> Connected (i.e. glasma, jet) contributions lead to a breaking of $v_2\{n\}$ scaling

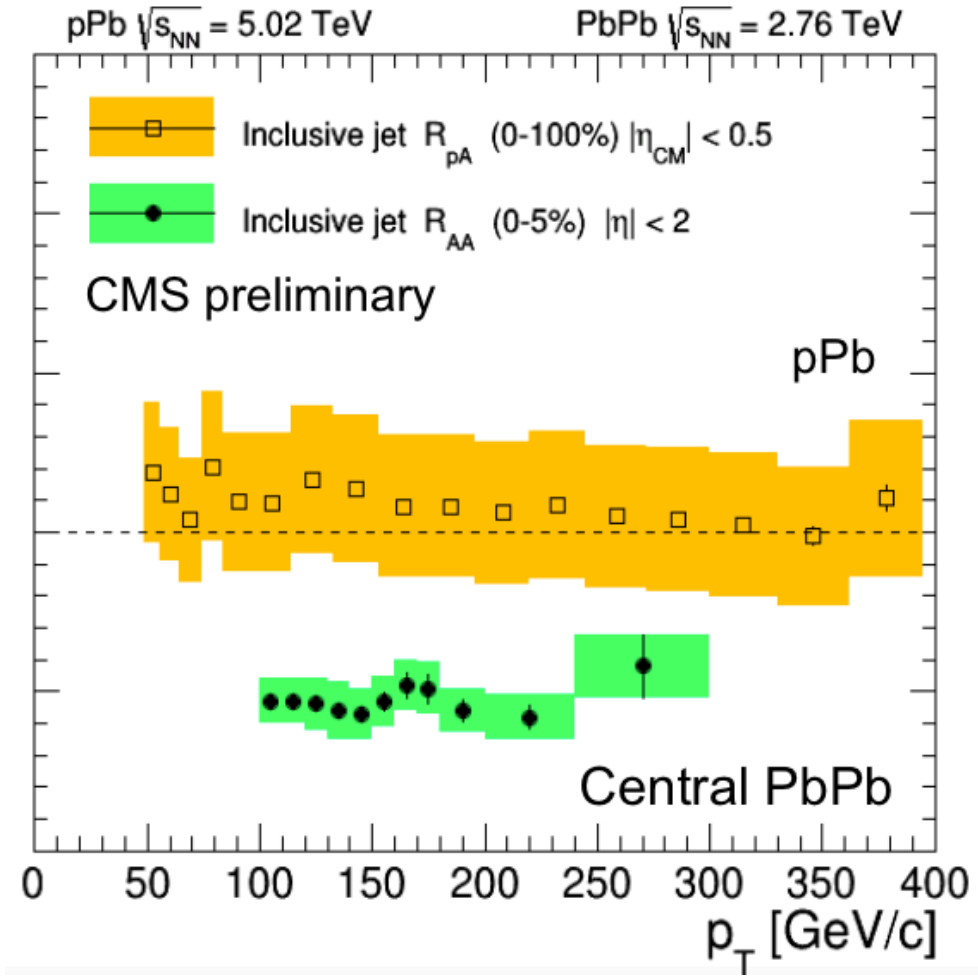
2> Does the breaking increase with higher minimum p_T?

3> Does $c_2\{4\}$ change sign at higher minimum p_T?

Mini-jets are unmodified … what about "real" jets?

ALICE Collaboration, Phys.Lett. B719 (2013) 29-41





B.G. Zakharov, Phys.Rev.Lett. 112 (2014) 3, 032301- medium effects lead to a considerable modification of the photon-tagged and inclusive jet fragmentation functions. ***For inclusive jets the magnitude of the effect is surprisingly large.***

Summary

1> Multi-particle production can be computed in a framework consist with what we know about the proton / nuclear wavefunction from DIS.

2> Glasma graphs successfully describe the *hard* ridge (i.e. $p_T. > 1 \div 2$ GeV)

3> I don't see v_3 as being a big issue with the high pT ridge

4> The soft ridge is another story: it is ~~collective~~ (i.e. $c_2\{4\}<0$)

a. Multiple scattering via CYM

b. Rotational symmetry breaking by color domains

5> Decisive measurements:

a. systematics of $c_2\{4\}$ with low pT cutoff

b. jet modification / quenching

Effects of hadronization on v_n from gluon bremsstrahlung in $p+A$

Collectivity in Small Colliding Systems with High Multiplicity

Angelo Esposito

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

March 6th, 2015 @ Brookhaven National Laboratory

Based on a work by M. Gyulassy & AE, soon to appear on the arXiv

Outline

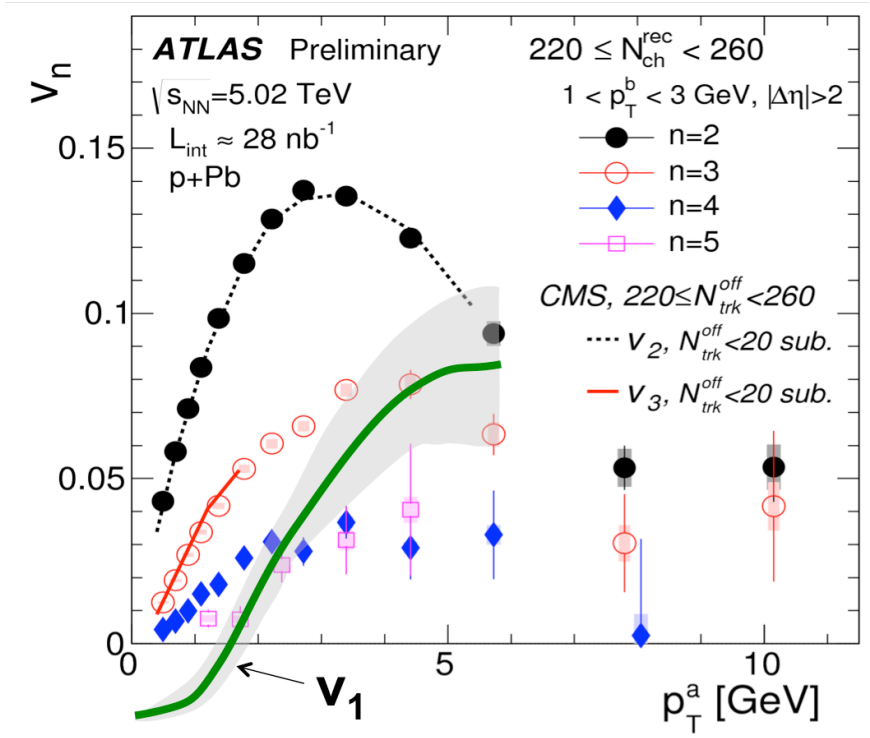
- Introduction and motivation
  - The puzzles
  - A possible QCD solution
- The Monte Carlo simulation
- Results
  - The independent fragmentation scheme
  - The Lund string model
- Conclusions and future plans

The puzzles

- Correlations among final state particles in *A+A* have been intepreted as signatures for the near "perfect fluidity" of the **Quark Gluon Plasma** (QGP)
- In the naïve picture the QGP should **arise at very high densities and/or temperatures**
- This idea has been challenged by the **recent discoveries at RICH and LHC**, e.g.:
  1. **Non-zero *p+A* $v_n(p_T)$ moments** comparable to the *A+A* ones
  2. **Near energy-independence** of the *A+A* $v_n(p_T)$ during the Beam Energy Scan (BES)
- <u>These systems were expected to be either to small or too cold to present QGP</u>!

The puzzles

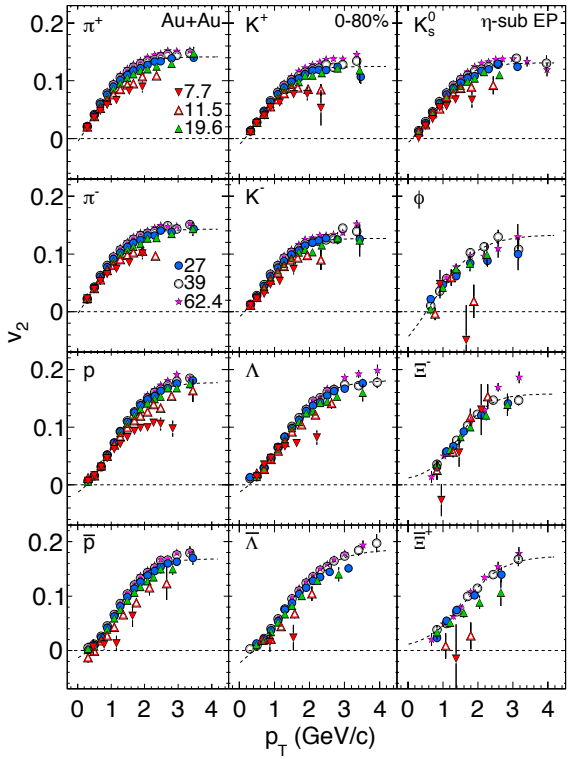
p+A Fourier harmonics



[ATLAS-CONF-2014-021 + prelim. from QM14]

- The magnitude of the *p+A* v_n is comparable to *A+A*

BES v_2 moments



[STAR Coll. — PRC**88**, 014902 (2013)]

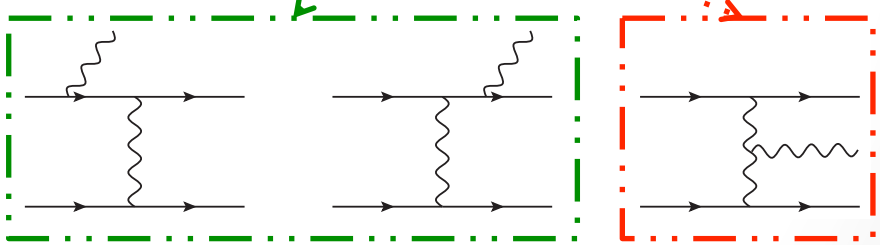
- The magnitude of v_2 does not change with energy

A possible QCD solution

- Even if hydrodinamics *can* explain the data it might not be the *only* model capable to do so
- What do all these configurations have in common? What could be the underlying physics?
- <u>In QCD when a momentum exchange happens, the distribution of the emitted gluon follows the Gunion-Bertsch (GB) distribution</u>:

$$\frac{dN_g^{(1)}}{d\eta\, d\mathbf{k}\, d\mathbf{q}} = \frac{\mu^2}{(q^2+\mu^2)^2}\,\frac{C_R\alpha}{\pi^2\mathbf{k}^2}\,\frac{\mathbf{q}^2}{(\mathbf{k}-\mathbf{q})^2}$$

Color dipole distribution

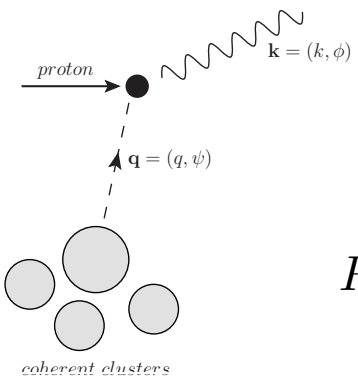


QED-like

Non-abelian QCD

A possible QCD solution

- In *p+A* the target color dipoles are longitudinally separated by $\Delta z = z_i - z_{i-1} \approx 1$ fm
- They act coherently when emitting gluons because of Lorentz contraction in CM frame and long formation time of the gluons in the lab frame
- <u>How does the emitted gluon look like after a coherent momentum exchange</u>?



$$dN_{\rm coh}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q}\, P_n^{el}(\mathbf{Q})\, dN^{GB}(\mathbf{k}, \mathbf{Q})$$

$$P_n^{el}(\mathbf{Q}) = e^{-\chi} \frac{\chi^n}{n!} \int \left\{ \prod_{j=1}^{n} \frac{d^2\mathbf{q}_j}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_j} \right\} \delta^2(\mathbf{Q} - (\mathbf{q}_1 + \cdots + \mathbf{q}_n))$$

- **Same GB distribution but with total exchanged momentum**

[Gyulassy et. al. – PRD**90**, 054025 (2014)]

A possible QCD solution

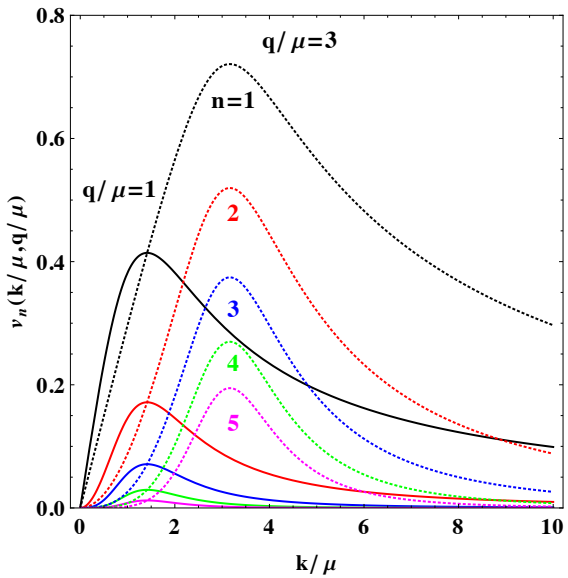
- The previous distribution has the following properties:
  1. η-independence
  2. Preferential direction (***q***)

  } "Ridge-like" structure
  3. Scaling law with *n* similar to the observed one:

$$[v_n(k,q)]^{1/n} = [v_m(k,q)]^{1/m}$$

  4. Produces non-zero gluon $v_n(p_T)$ with shape similar to the data:

[Gyulassy et. al. – PRD**90**, 054025 (2014)]



The magnitude of this moments is *huge* (by a factor ≈3) if compared to the observed ones!

Need for a smearing effect

Natural candidate: hadronization

The Monte Carlo simulation

- We performed the hadronization using JETSET 7.4
- Generated 30×10^6 events
- Each event is composed by:
  1. Two quark-antiquark pairs each with $E_{com} = 200$ GeV
  2. In between the two quarks we plug a Poissonian number of gluons ($\langle N_g \rangle = 8$) distributed according to the GB formula with $Q_{sat} = 2$ GeV and $\mu = 300$ MeV
  3. We hadronize using both the independent fragmentation scheme and the Lund string model
- ***Indep. frag.***: gluons hadronize independently from each other
- ***Lund model***: gluons "pinch" the QCD string deforming it in the transverse plane and hence generating correlations

The Monte Carlo simulation

- For initial gluons and final pions we compute the correlation function:

$$C(\Delta\phi; p_T) = \frac{S(\Delta\phi; p_T)}{B(\Delta\phi; p_T)}$$

Same event pairs

Mixed event pairs

with $|\eta_{a,b}|<2.5$, $|\Delta\eta|>2$ (long-range) and both particles in the same p_T bin

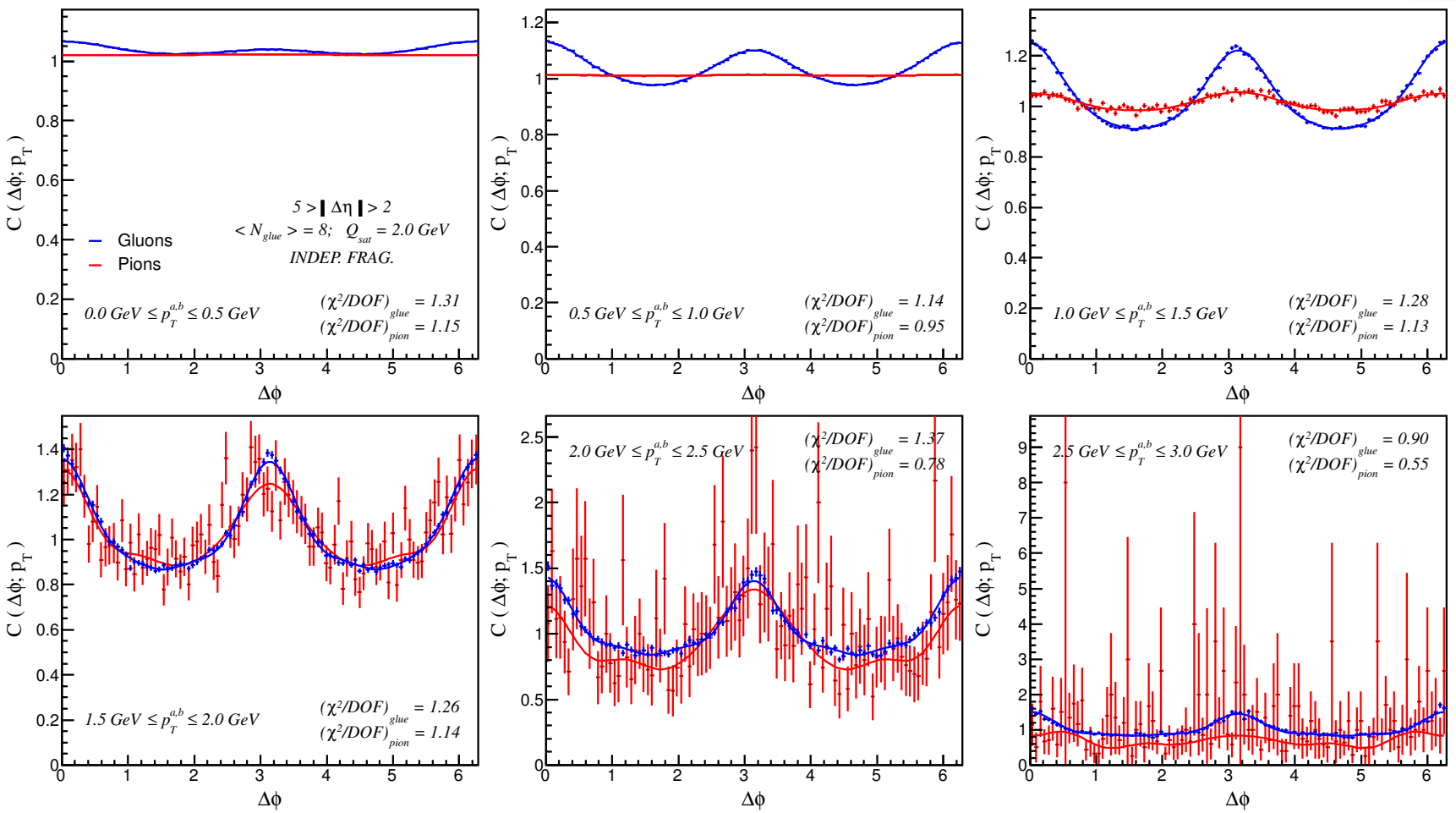
- We fit the final Δϕ distributions with a truncated Fourier expansion:

$$C_{\text{fit}}(\Delta\phi; p_T) = C_0 \left(1 + 2\sum_{n=1}^{6} v_n^2(p_T)\cos(n\Delta\phi)\right)$$

- Extract the single-particle $n \leq 6$ azimuthal moments

Results (independ. frag.)

- Gluons and pions Δϕ long-range distributions:



Results (independ. frag.)

- In the low-p_T region ($p_T < Q_{sat}$) the hadronization almost completely eliminates the initial gluonic correlation

⬇

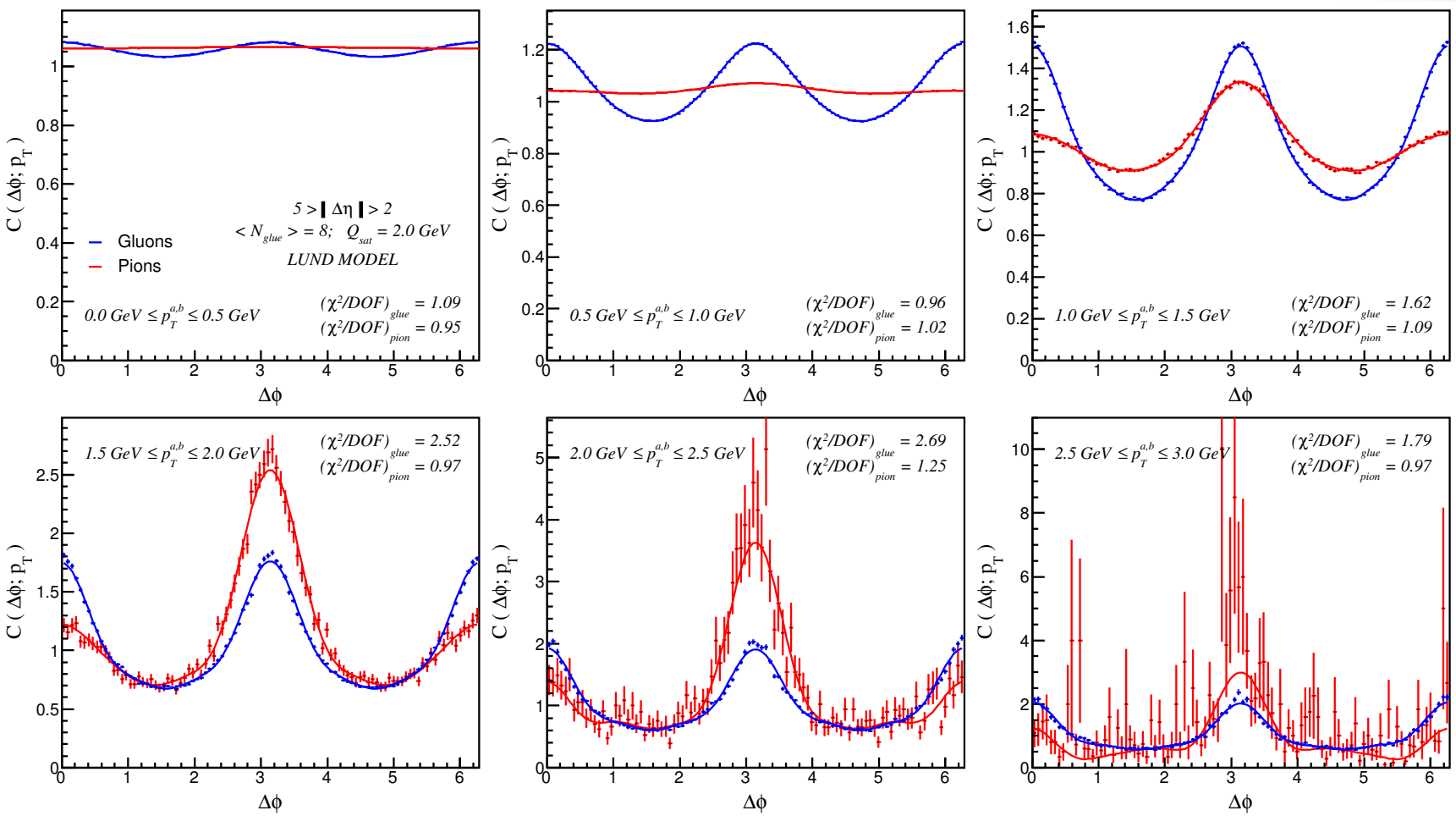
<u>This is the IR safety of the hadronization. The model is blind to the low-*k* gluons</u>

- In the higher-p_T region instead, the final pion two-particle distributions closely resemble the partonic ones

⬇

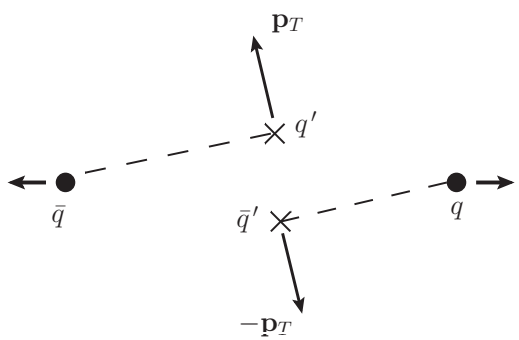
<u>Here we are simply "building" pions around the gluons. No additional source of correlation is introduced</u>

Results (Lund model)



Results (Lund model)

- Again in the low-p_T region ($p_T < Q_{sat}$) the correlation of the final hadrons is much milder than for the initial gluons
- In the **higher-p_T regime we have a striking unexpected away-side peak (several times larger than the near-side one)!**
- Where does it come from?



When the QCD string breaks, total transverse momentum is conserved

We always have a parton with $\boldsymbol{p}_T$ and one with $-\boldsymbol{p}_T$

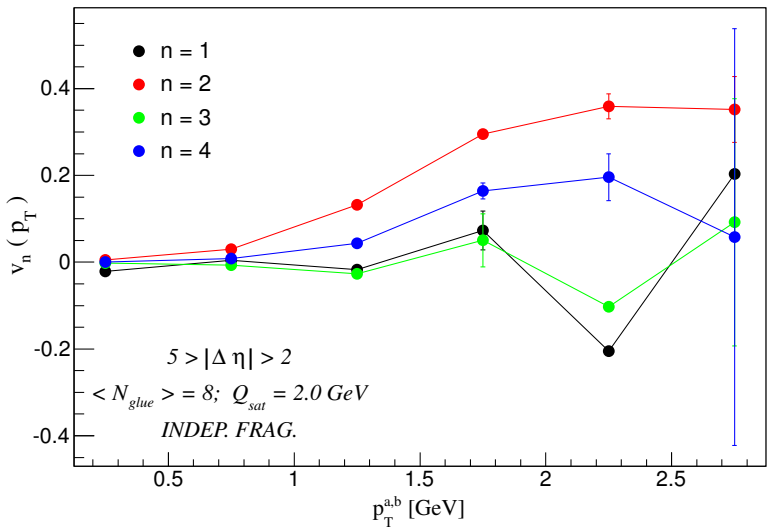
Large $\Delta\varphi \approx \pi$ peak in the two-pion distributions

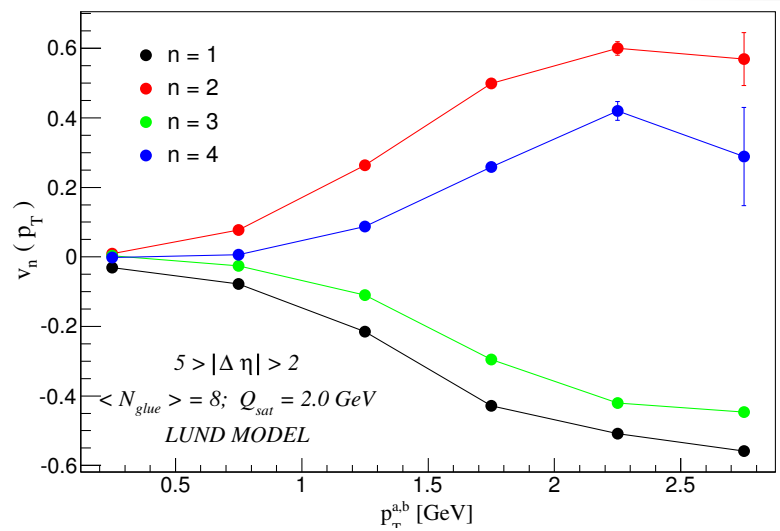
- The **string breaking** introduces deeply **non-trivial correlations** in the spectrum of the final pions → **Model dependence!**

Results (single-particle v_n)

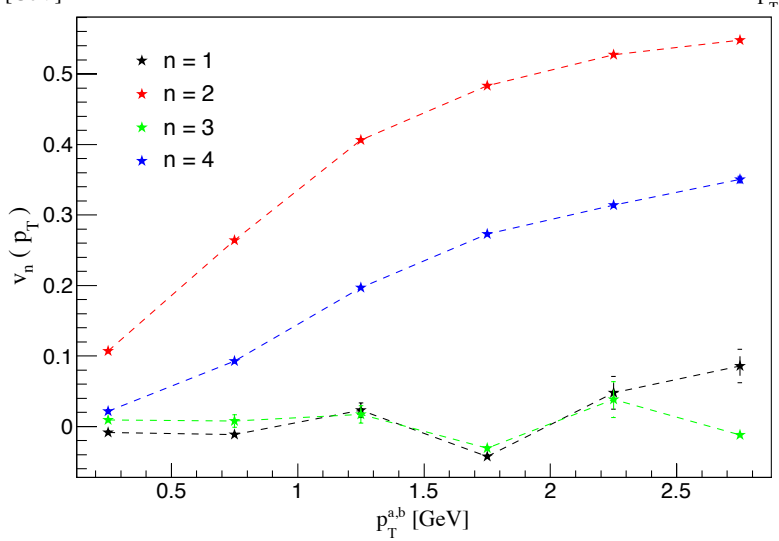
Independent Fragmentation ***Lund string model***

Final pions





Initial gluons



Conclusions (low-p_T region)

- Both indep. fragm. and Lund model show strong suppression of initial correlation for $p_T < Q_{sat}$ (**model-independent feature**)
- Consequence of the IR hadronization: large azimuthal asymmetries predicted at partonic level with $k < Q_{sat}$ are found to be greately reduced and sensitive to the hadronization scheme
- Local parton-hadron duality, independent fragm. and Lund string model differ greatly in "low-p_T" region
- Hadronization should always be taken into account by models of initial-state azimuthal anistropy
- The parton level $v_n(p_T)$ in the low-p_T region must be much larger than the experimental one since they are critically smeared out by hadronization

Conclusions (higher-p_T region)

- In the higher-p_T region:
  1. Independent fragmentation: no surprises
  2. QCD string: huge away-side correlation coming from transverse momentum conservation of the flux tube
- **IF** this feature survives to more complicated/realistic situations we have two possible interpretations:
  1. Pay particular attention to **model-dependence of final two-particle correlations**
  2. If we "buy" the Lund model as a fairly accurate description then we might have an **additional *non-flow* source of correlation**. It should be subracted.
- What about the v_n? Both models have still very large moments.
  1. Independent fragmentation: large even v_n, very small odd v_n
  2. QCD string: large even v_n, large negative odd v_n

Future plans

- **What is next?**
- Initial system is a perfectly back-to-back system (recoil)

⬇

- The final pions within the indep. fragm. model has essentially no odd v_n harmonics
- We need to implement a third quark-antiquark system. We look for a "Mercedes Benz-like" system. How do the moments look like in this configuration?
- What happens to the string breaking effect with higher order cumulants are taken into account?
- Need for a realistic description of *p+p*, *p+A* and *A+A*

⬇

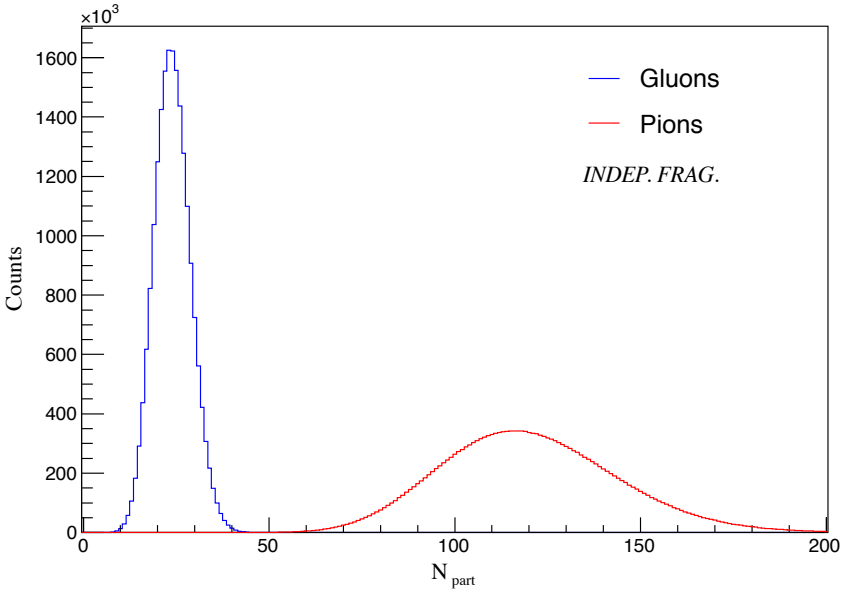
- Implement the gluon GB bremsstrahlung in HIJING

THANK YOU!

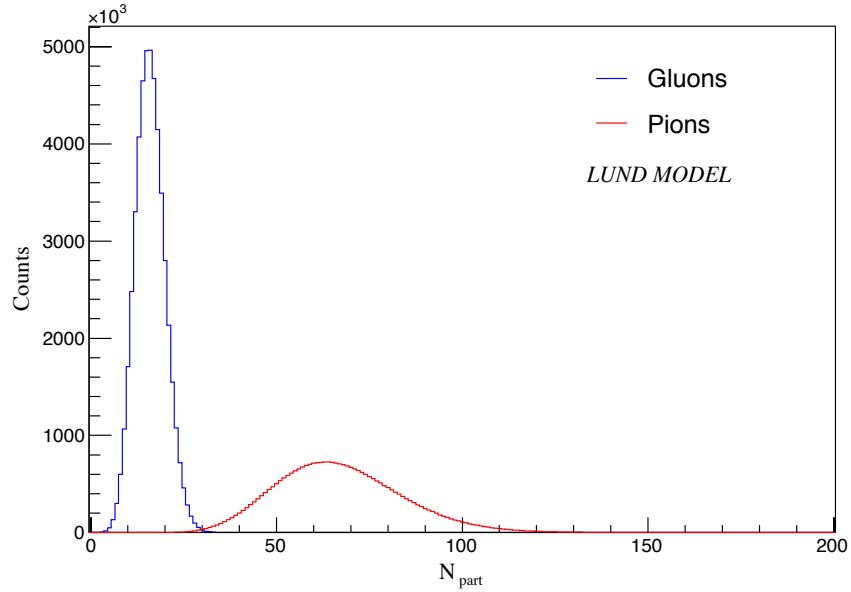
Back up

Gluons/pions multiplicity

Independent Fragmentation

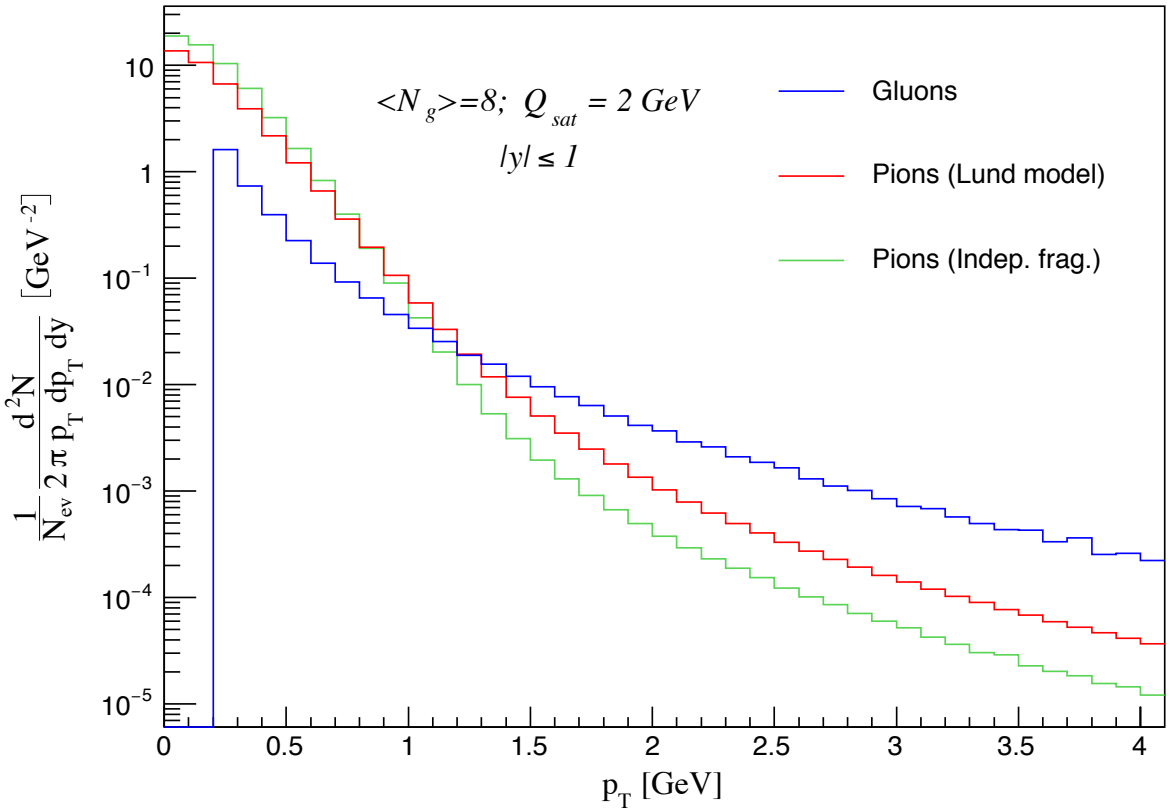


Lund model



- More particles (factor of 2) in the independent fragmentation sheme than in the Lund model

Gluons/pions p_T distributions



- The indep. fragm. subtract p_T to the gluon mostly producing soft pions
- The transverse fluctuations of the string can add p_T to the already existing gluon k, leading to harder pions

Details of the simulation

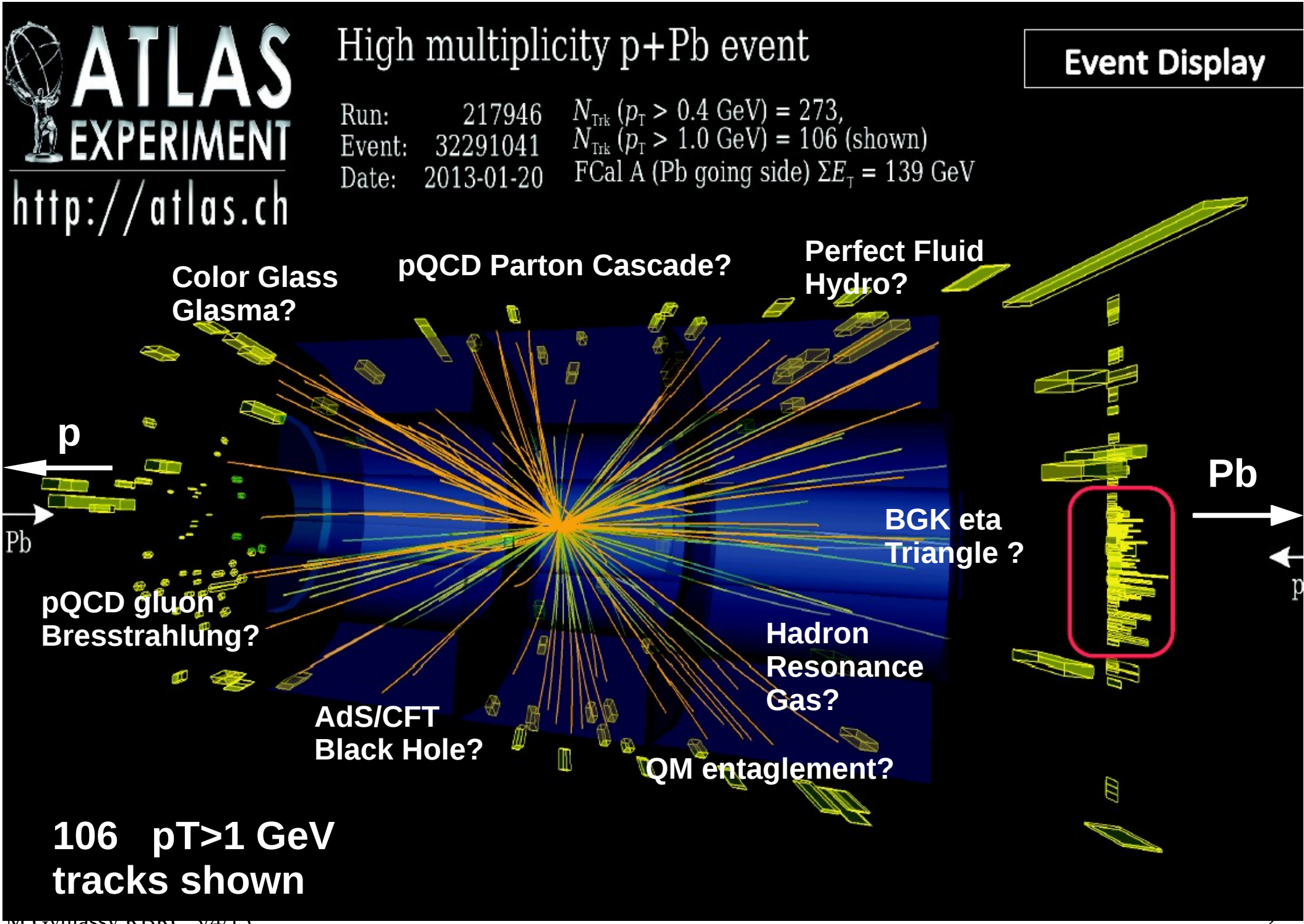
1. Use JETSET 7.4 that can implement both indep. frag. and Lund model
2. Generate 60 x 10^6 quark-gluons-antiquark systems. This means:
   - 30 x 10^6 events for a two pairs system
   - 20 x 10^6 events for a three pairs ("Merced Benz") system
3. The q-qbar of each system move along the *z*-axis with RICH center-of-mass energy (E_{com} = *200* GeV)
4. The gluons are extracted from a suitably regularized GB distribution:

$$f_{GB}^{\mathrm{reg}}(\mathbf{q},\mathbf{k}) \propto \frac{Q_{\mathrm{sat}}^2}{(q^2+Q_{\mathrm{sat}}^2)^2}\frac{q^2+\mu^2}{(k^2+\Lambda_{QCD})(k^2+q^2+\mu^2-2kq\cos(\phi-\psi))}$$

   with Q_{sat} = *2* GeV, μ = *300* MeV and Λ_{QCD} = *200* MeV
5. Prevent final π, *K* and η from decay
6. Only select final pairs pions with $|\eta_{a,b}| < 2.5$, $5 > |\Delta\eta| > 2$ and in the same p_T bin

Perspective on Large versus Small and the pA+DA+BES challenge since 2012

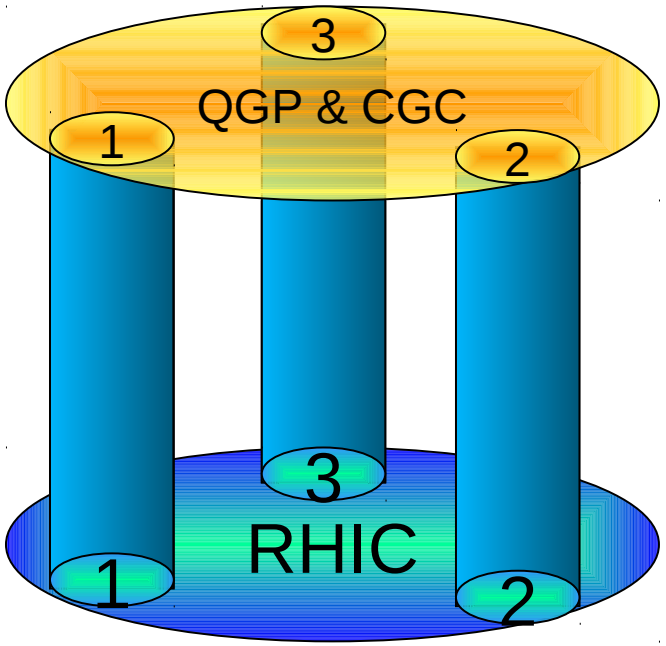
M. Gyulassy @ RBRC 3/4/15 Workshop


ATLAS EXPERIMENT
http://atlas.ch
High multiplicity p+Pb event
Run: 217946
Event: 32291041
Date: 2013-01-20
N_{Trk} (p_T > 0.4 GeV) = 273,
N_{Trk} (p_T > 1.0 GeV) = 106 (shown)
FCal A (Pb going side) ΣE_T = 139 GeV
Event Display
Color Glass Glasma?
pQCD Parton Cascade?
Perfect Fluid Hydro?
p
Pb
Pb
BGK eta Triangle ?
pQCD gluon Bresstrahlung?
Hadron Resonance Gas?
AdS/CFT Black Hole?
QM entaglement?
106 pT>1 GeV tracks shown

In 2004 Three Lines of Data Seemed to Converged to QGP and CGC

Three Stable legs

Null Control



Bulk P_{QCD}

Parton pQCD

1. Bulk P_{QCD} Collective Elliptic Flow v2 √
2. Parton pQCD Jet Quenching RAA √
3. p+p Calibration and d+A Control on CGC √ √

$QGP = P_{QCD} + pQCD + dA = v_2 + (R_{AA} + I_{AA}) + R_{dA}$

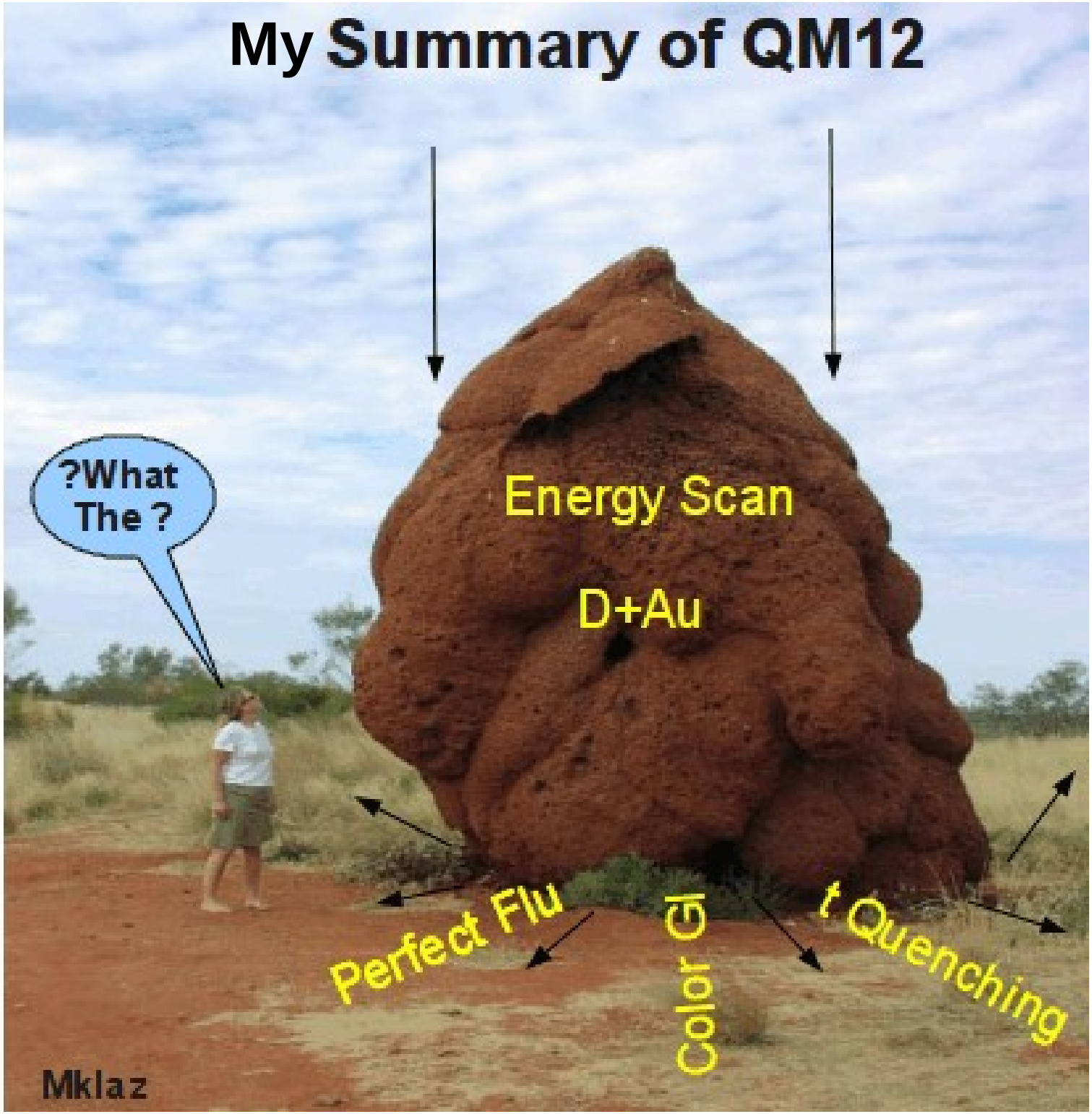
(Nucl.Phys. A750 (2005) MG,L.McLerran, E Shuryak, B Muller, XN Wang, H.Stoecker, +10^3)

Experimental Priorities

- Y=+- 3 test interplay QGP<->CGC ?
- $C_2(\underbrace{phi_1, phi_2, pt_1, pt_2, eta_1, eta_2}_{\text{6D microscope}}; \underbrace{fl_1, fl_2, Mult, A, B, Ecm}_{\text{exp. knobs}})$
- Heavy Quark tomography
- Open Charm (enhancement?); J/Psi (suppression?)
  Charm Flow?
- Direct Photons thermometer
- Tagged direct photon -quark jets!
- Turn Ecm~20-200 and A=1-100 exp. knobs

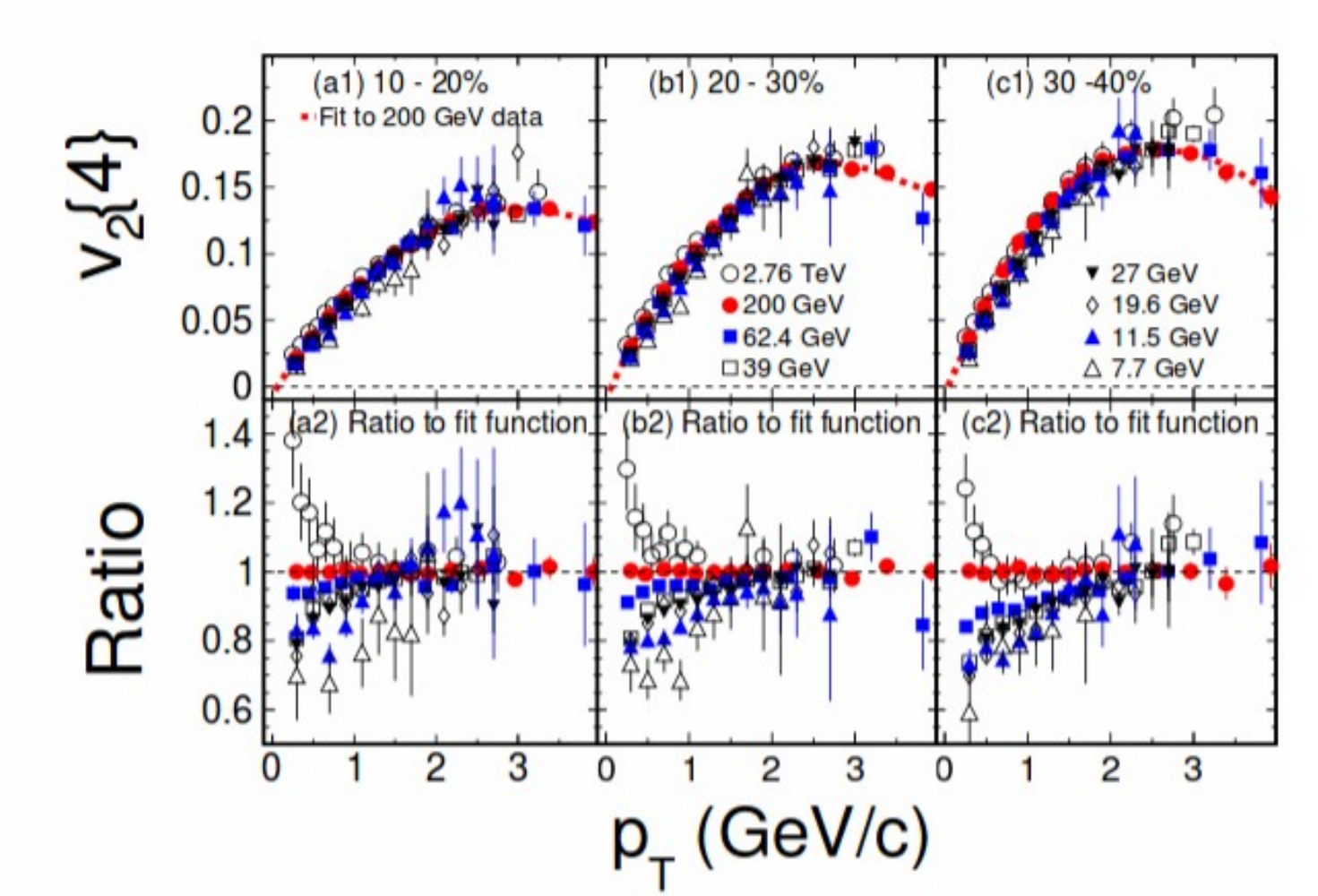
2015 RBRC workshop focuses on A=1, 2, 3 small probes of heavy nuclei

Update many remaining unsolved question since last RBRC April 2013


My Summary of QM12
?What The ?
Energy Scan
D+Au
Perfect Flu
Color Gl
t Quenching
Mklaz

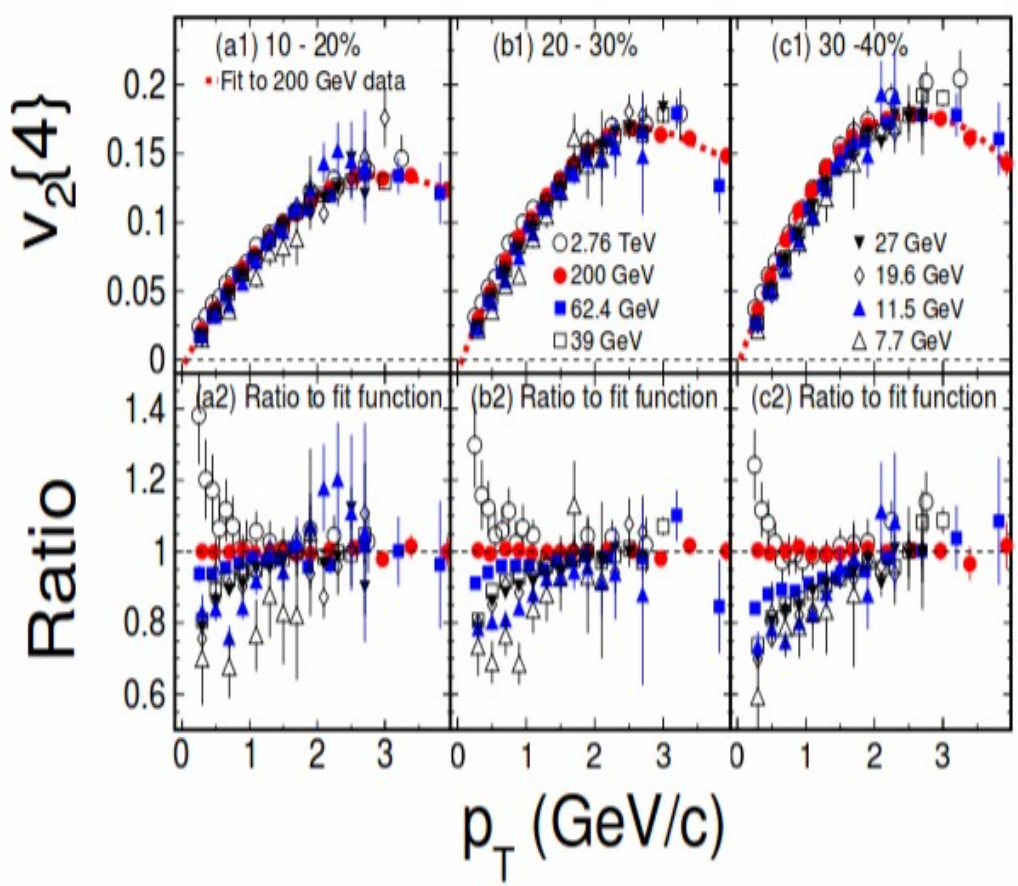
One 2012 shock for me: PHENIX and STAR Beam Energy Scan (BES) found that Azimuthal Fourier Harmonics vn are center of mass energy invariant to 20%

Is this just multi component bad luck: 1 = 1 + 1 - 1 + 1 – 1 + 1 – 1 ??
Or an AA unexpected discovery of conformal invariance to energy scale ?

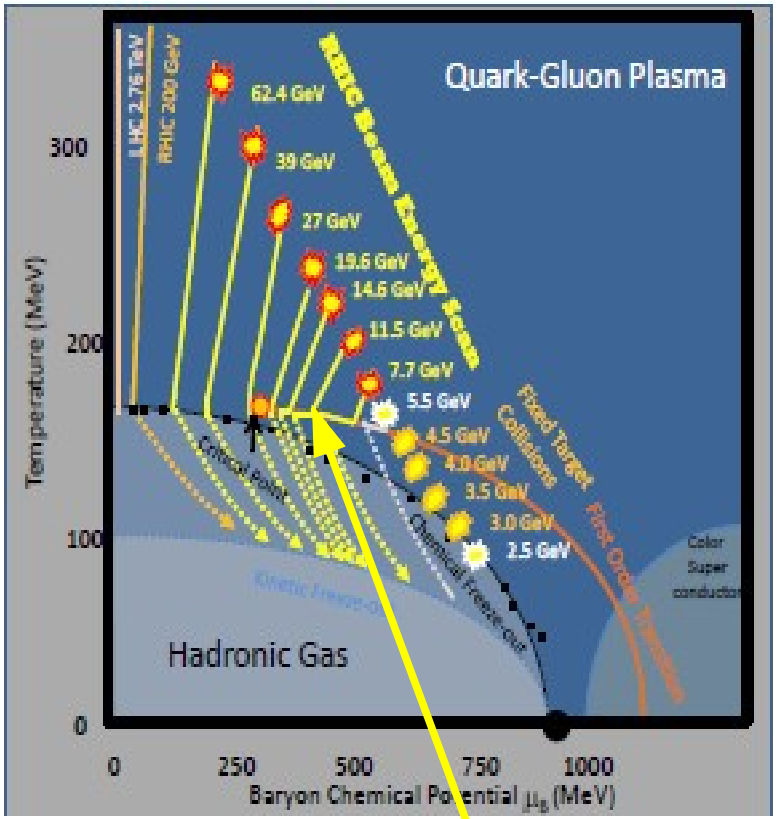


The old Bevalac, AGS, SPS, RHIC paradigm:
AA = QGP Fluid Core + HR Gas Corona
was severely challenged by beam energy independence of vn

STAR 2014 Beam energy independence of v2{4} from 8-2760 AgeV !



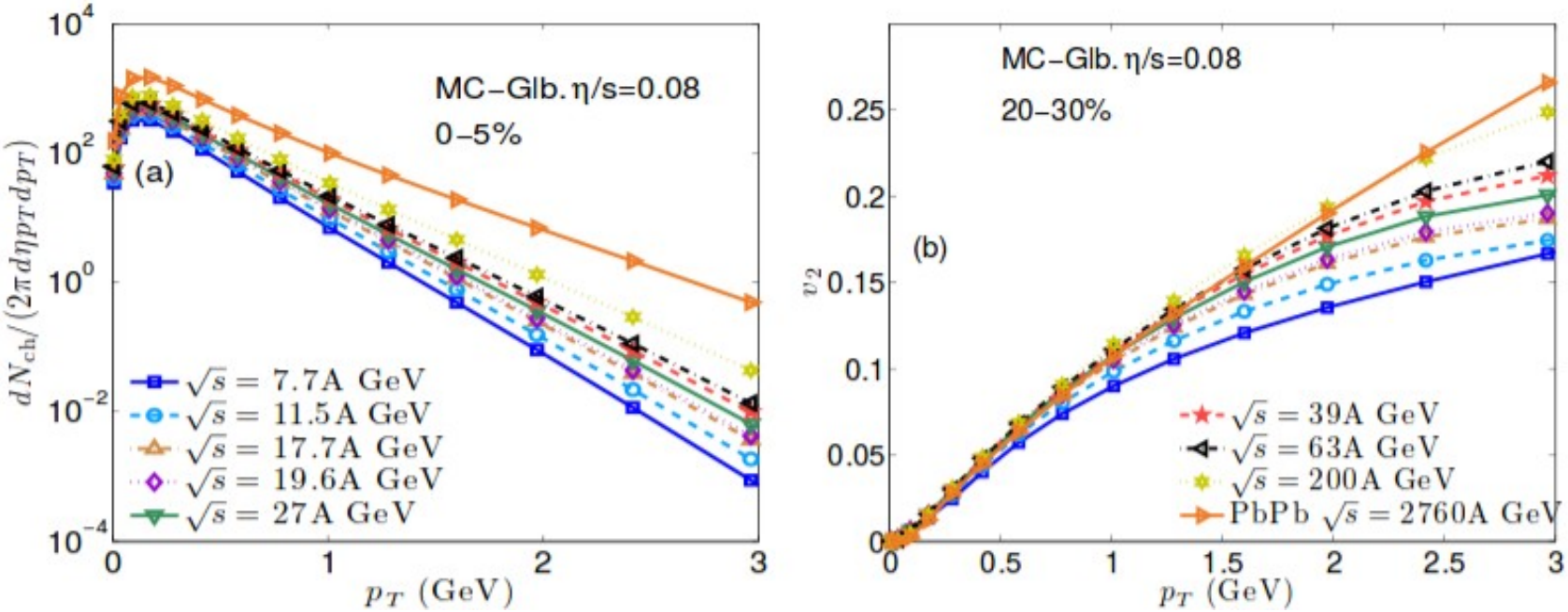
J.Steinheimer 14



But There is a tiny difference between
(v2(p) -v2(pbar))/3 ~ 0.007
Observed at ~10 AGEV
Is this the tiny signature of the critical point?

Where is all this wonderful
genanken physics hiding
In the vn data ?

Collision Energy Dependence of Viscous Hydrodynamic Flow in Relativistic Heavy-Ion Collisions
Chun Shen, Ulrich Heinz Published in Phys.Rev. C85 (2012) 054902



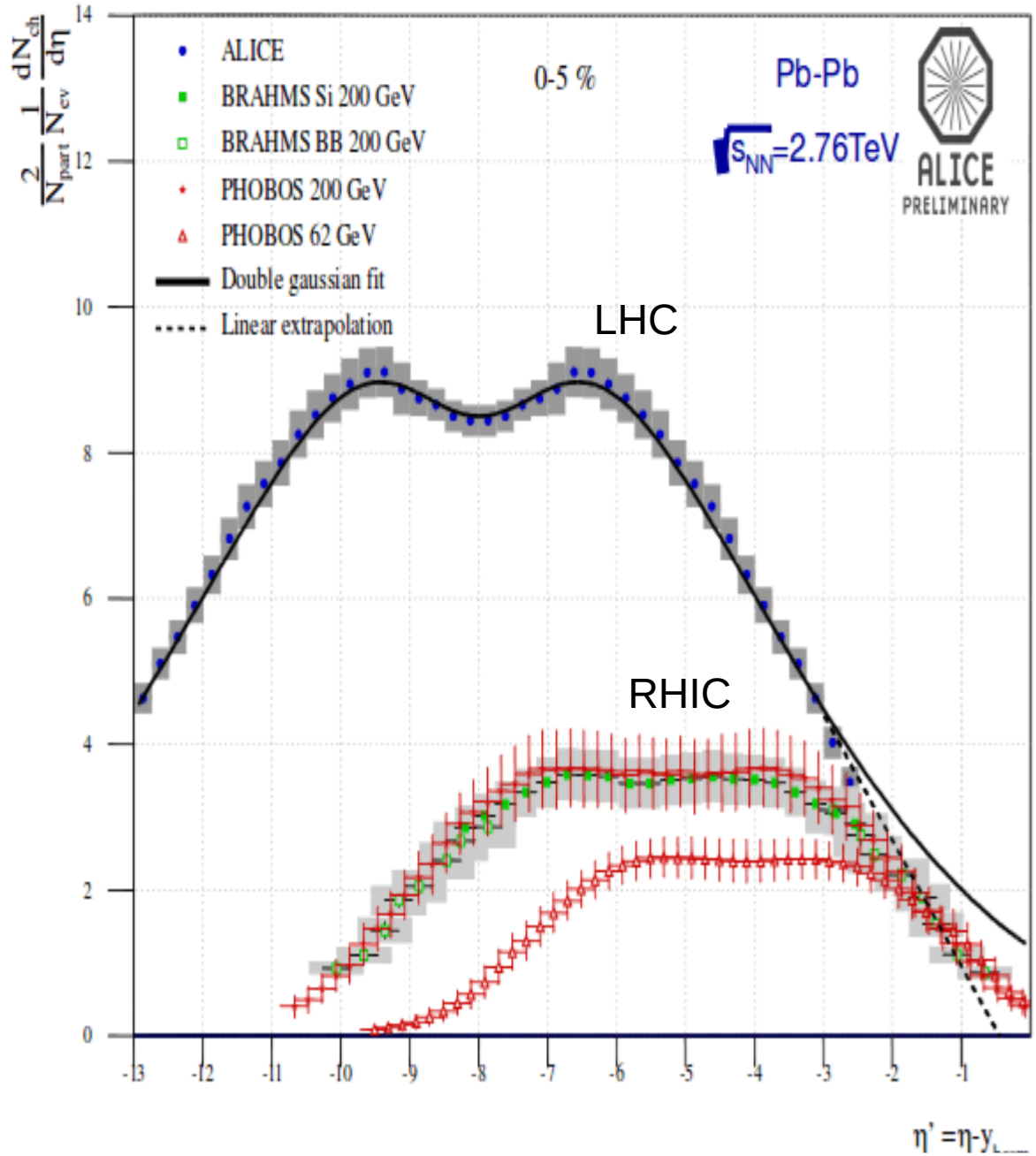
Even in pure mininal viscous hydro without a HRG corrona there should be
Be more sqrt s dependence than seen in data

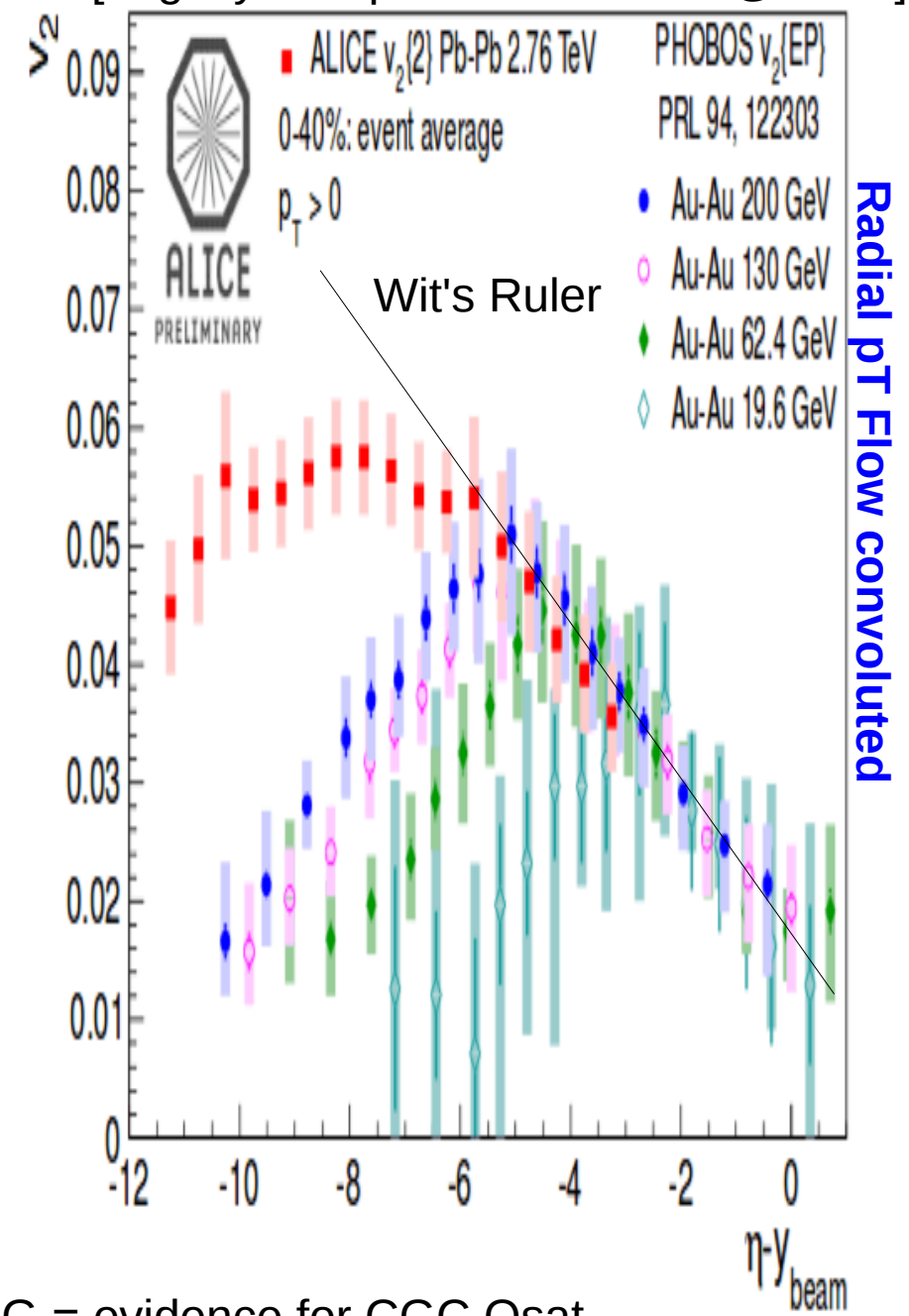
Hydro is rather sensitive to interplay of string sqrt s dependent radial pT flow
And weaker sqrt s dependence of v2(pT)

H. Peterson can fit BES by suitable adjusting initial URQMD stringy phase
Before the hydro phase. Yu. B. Ivanov, A. A. Soldatov solve BES with three
Fluid model adjusted to the data. Invariance to beam energy remains a severe
challenge for theory interpretation.

BES shock because Initial density varies greatly from LHC to RHIC to SPS

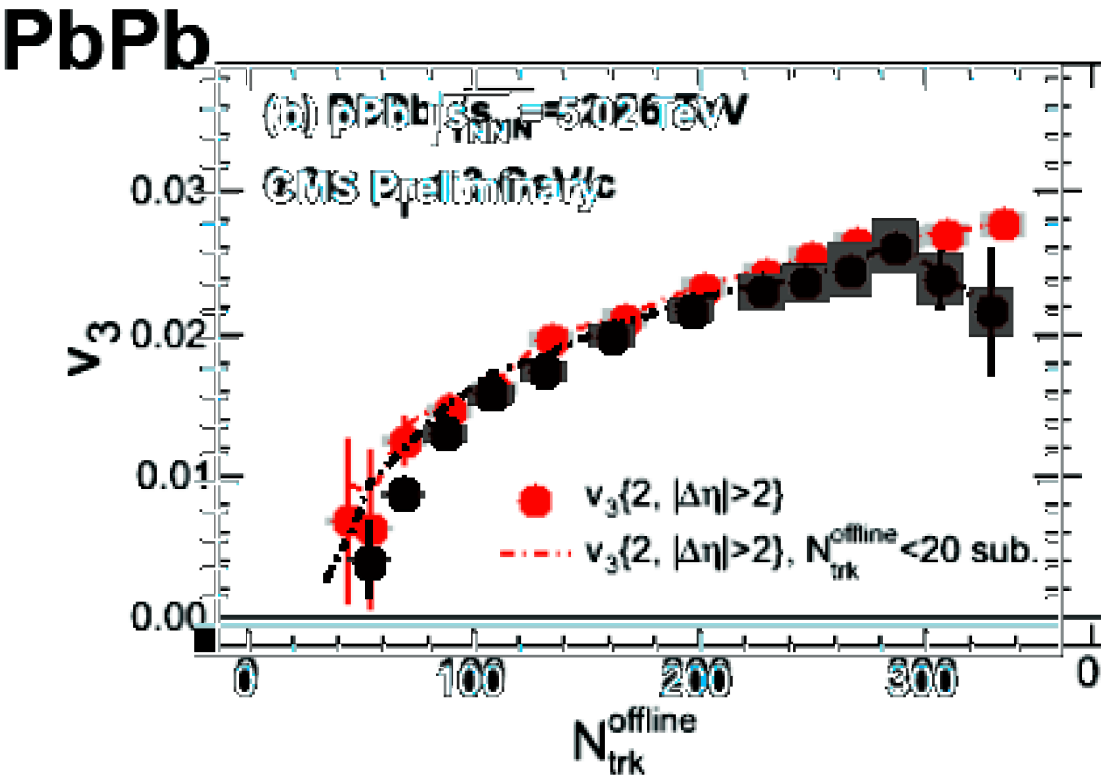
[Slightly less perfect flow than @RHIC]

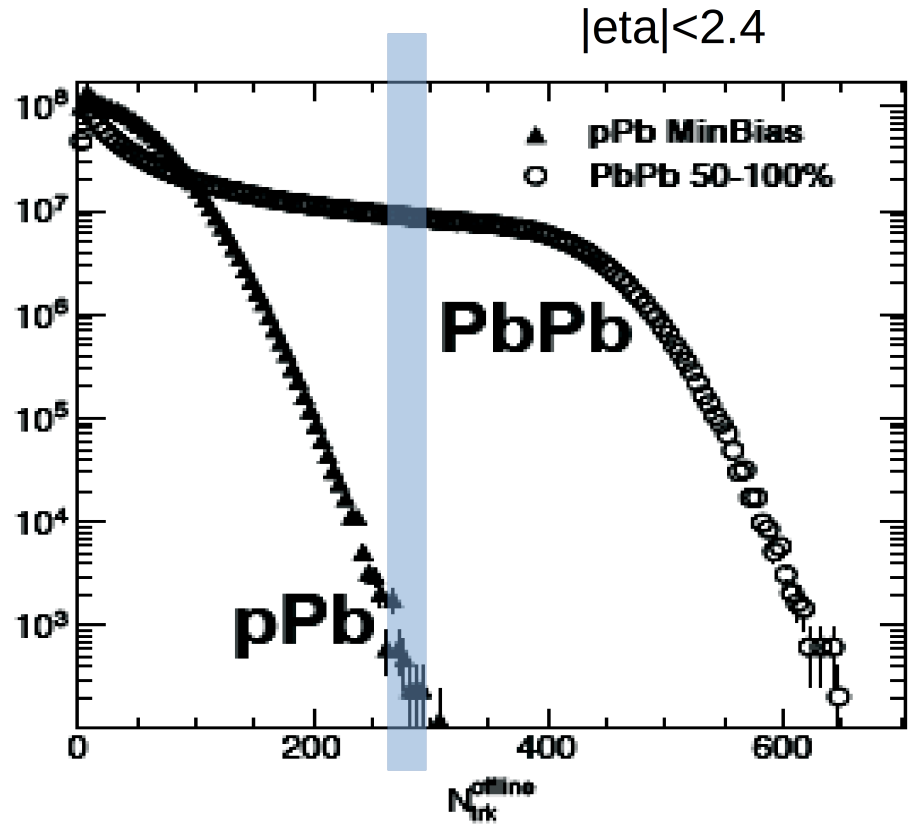




Mt. LHC is smaller than expected with HIJING = evidence for CGC Qsat
But does CGC saturation apply at low dNg/dy @ SPS??

2012 shock: v_3 in pPb and PbPb





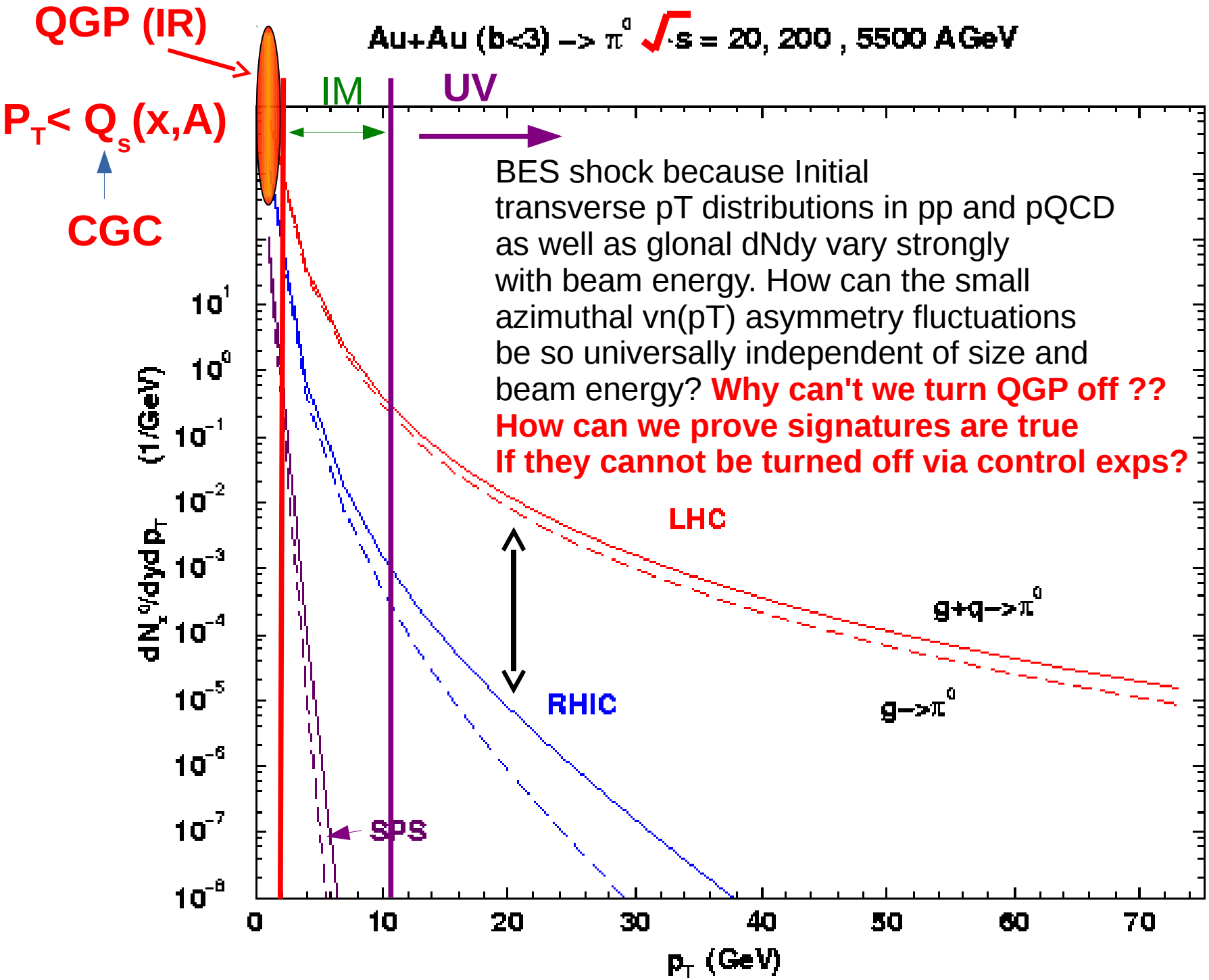
v_3 shows similar shape in pPb and PbPb; magnitude comparable

same *same*

LHC pPb data falsified the "Mouse ≠ Bull " theorem.

Is it just more bad luck that IC and FSI seem indistinguishable ??

Or a discovery of conformal invariance to spatial scale as well Beam energy scale in B+A dynamics??



X.N. Wang 1990 with my graffiti

"Wit Busza's 70th Birthday Party 2013 and His Revenge of pA"

Can the Au Pillars of QGP and CGC aka 2004 be left Standing after the pA and DA and BES data of 2013 ?

Me,Looking for the Ag Lining in the ashes of pA DA and BES

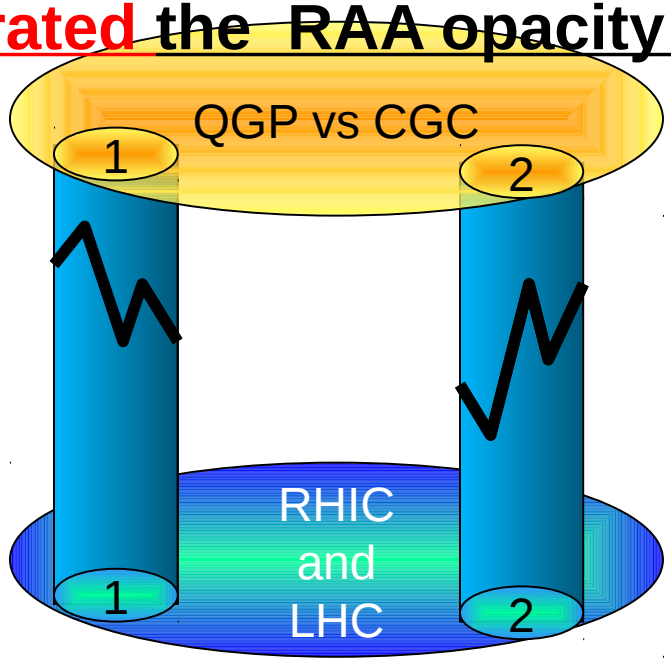
Can QGP survive the RBRC 2013 D+Au and p+Pb tsunami?

(from my RBRC workshop 4/16/2013 talk)

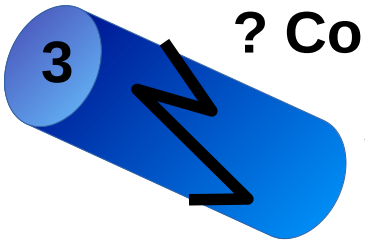
BES , D+Au and p+Pb un-calibrated the v2 Barometer of sQGP

D+Au also un-calibrated the RAA opacity meter

Why does p,DA appear To "Flow" like A+A?

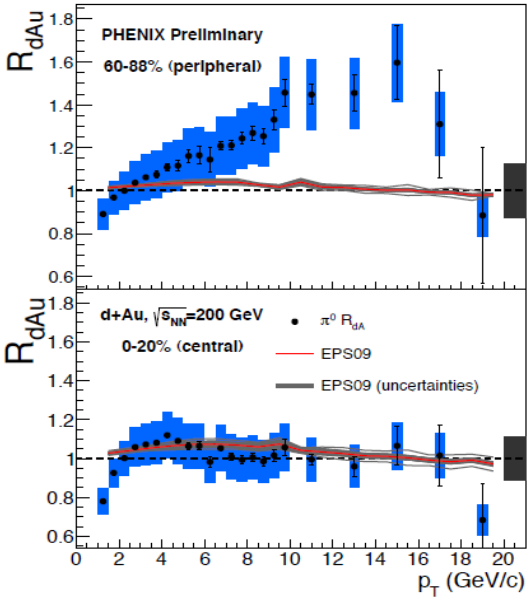


Why is R_DA modified from 1 ?

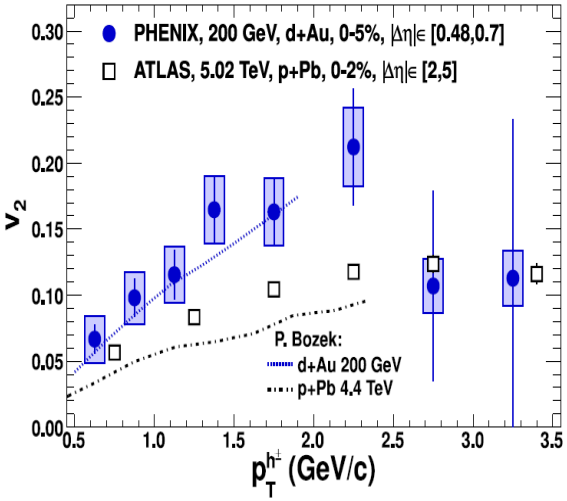


? Control Exp ?

R(DAu) ≠ 1 ??



v2(DAu) ~ v2(AuAu) ??

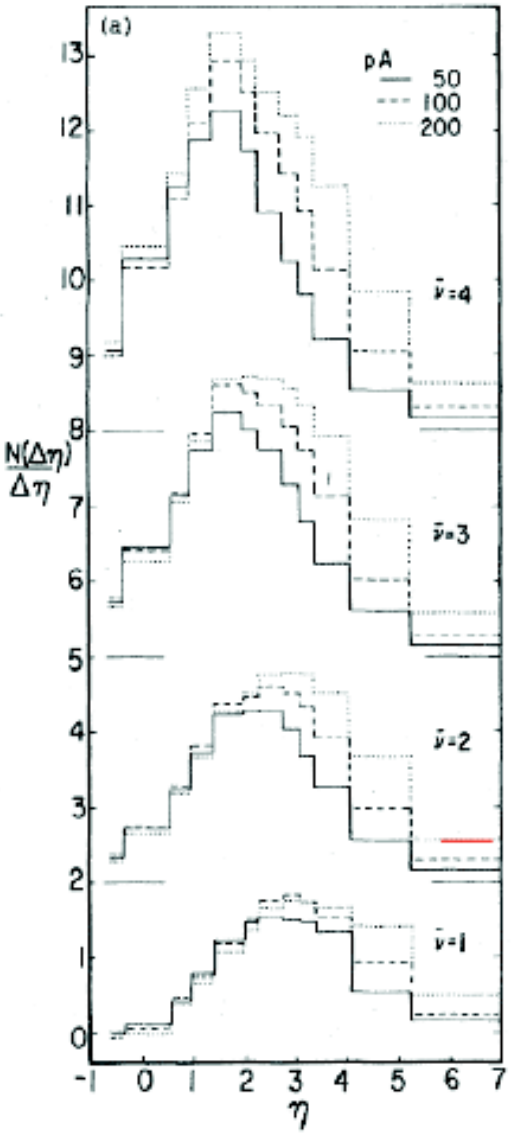


Why are Small size and Low energy observables not Null?

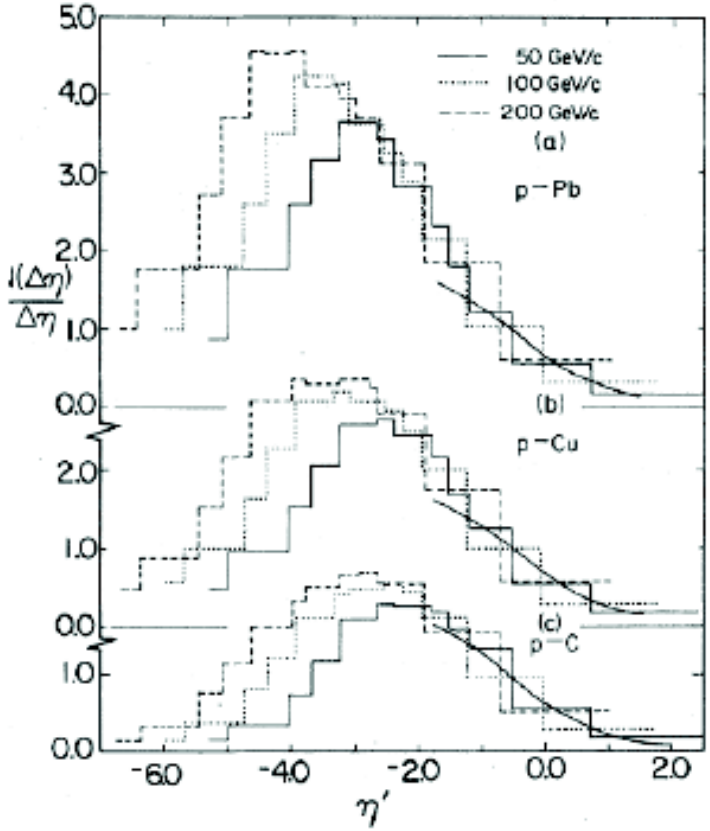
Where did the Hadron Gas Corona go?

Since 1970's Wit Busza measured p+A triangles

Extended Longitudinal Scaling in E178 Data for $\sqrt{s_{NN}}$ 10 - 20 GeV

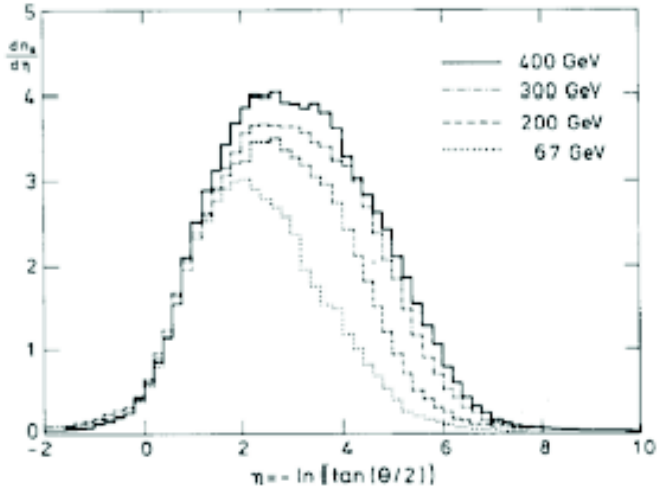


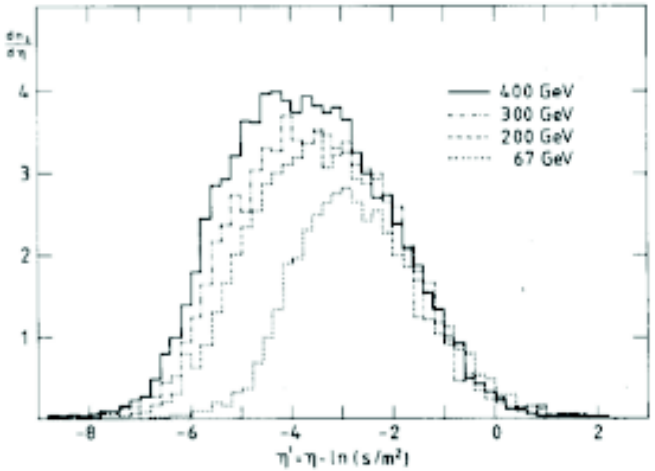
Nucleus rest frame



Projectile rest frame

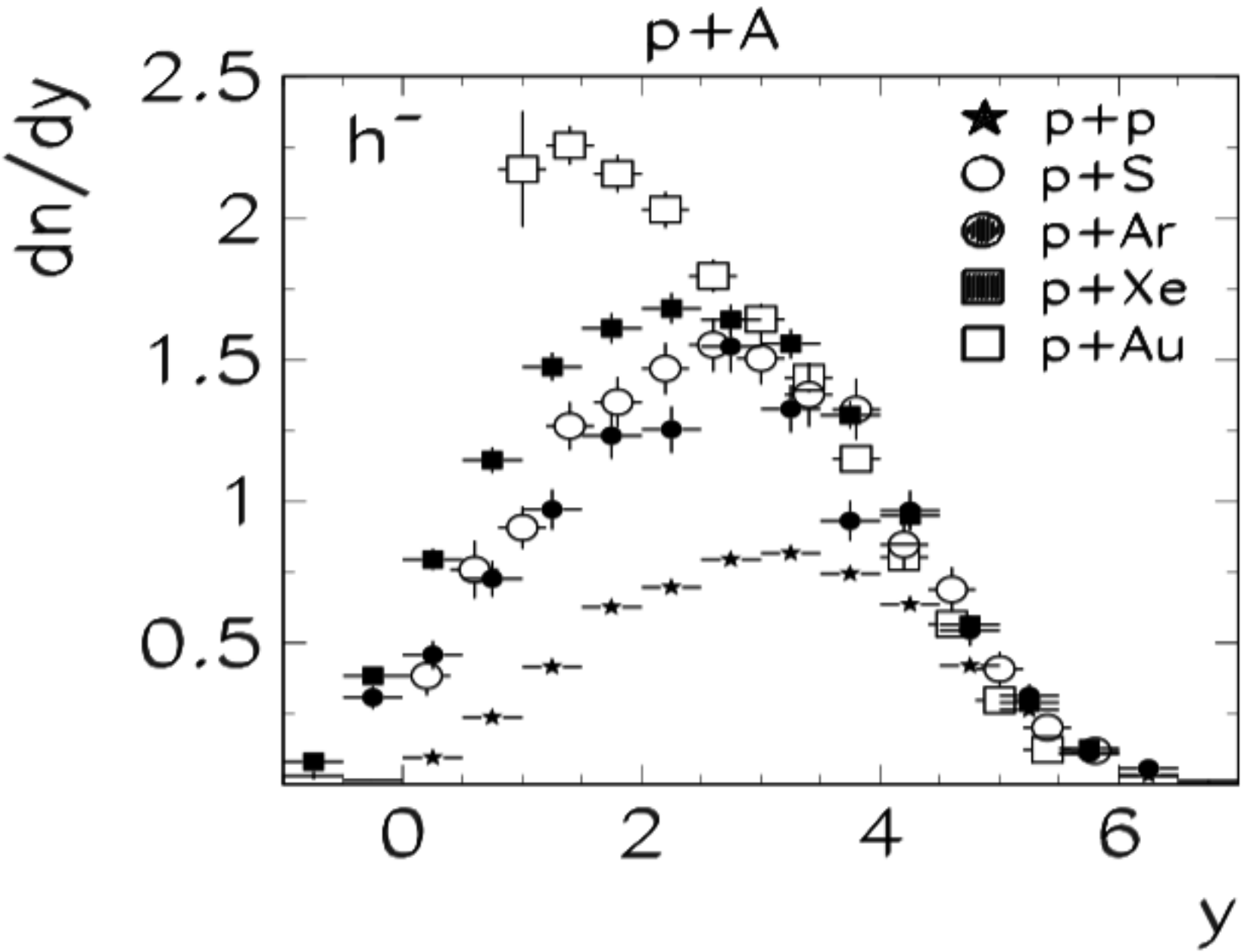
J. Elias et al., (E178) PR D22 (1980)13





pEmulsion data (Otterlund et al., compilation NP B142 (1978) 445

NA49 hep-ex/ 971101 verified Wit's FNAL triangles at SPS



Global Triangular <u>breaking</u> of boost invariance in p+A Ecm= 20 AGeV

BGK 1977 p+A "Rapidity Triangles"

- Multiple independent parton dx/x collisions liberate uniform rapidity color between p and wounded A nucleons.
- Color neutralized via pair production in flux tubes sum over "beam jets" => triangular dNd/y
- $A^{1/3}$ ~ 10 Target beam jets vs 1 Projectile beam jet
- Rapidity slope = Npart / log(s) slope RHIC ~ 2 x LHC

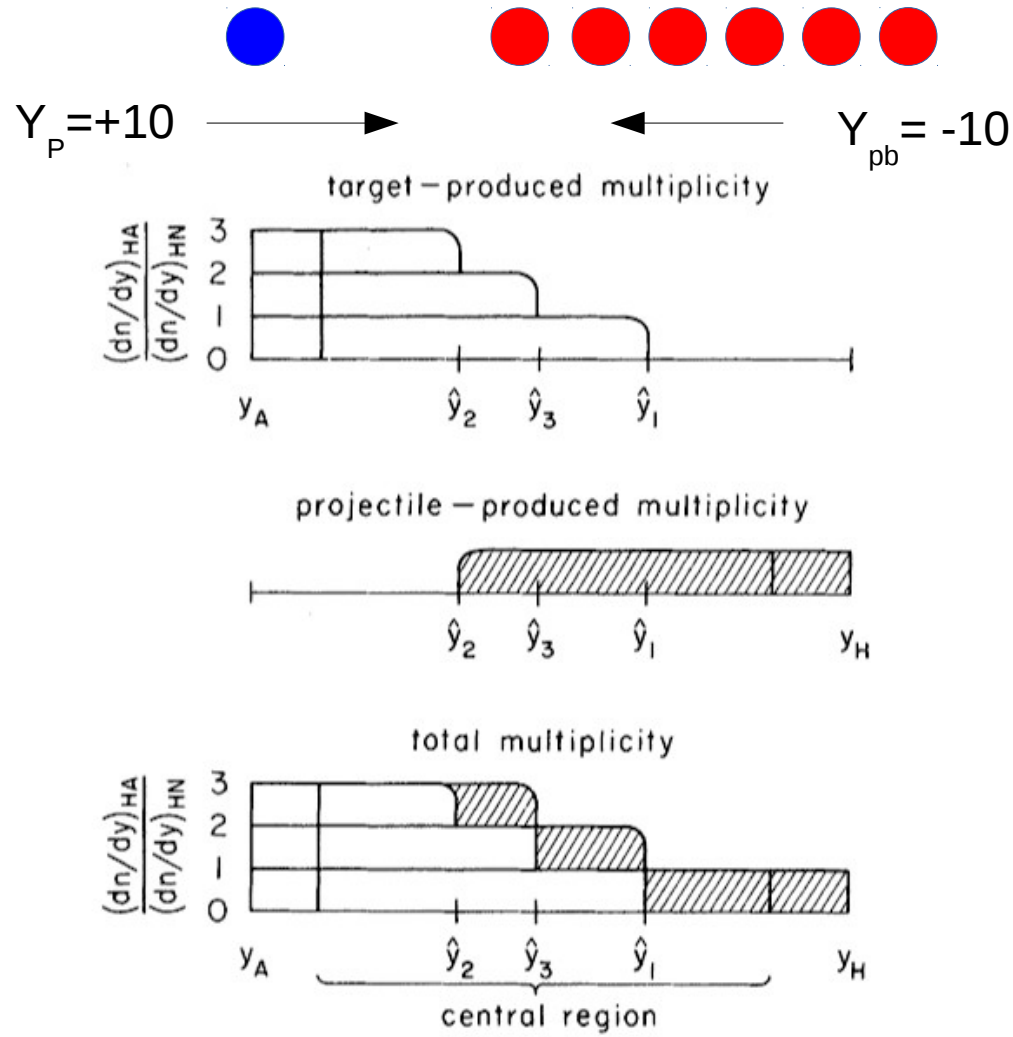
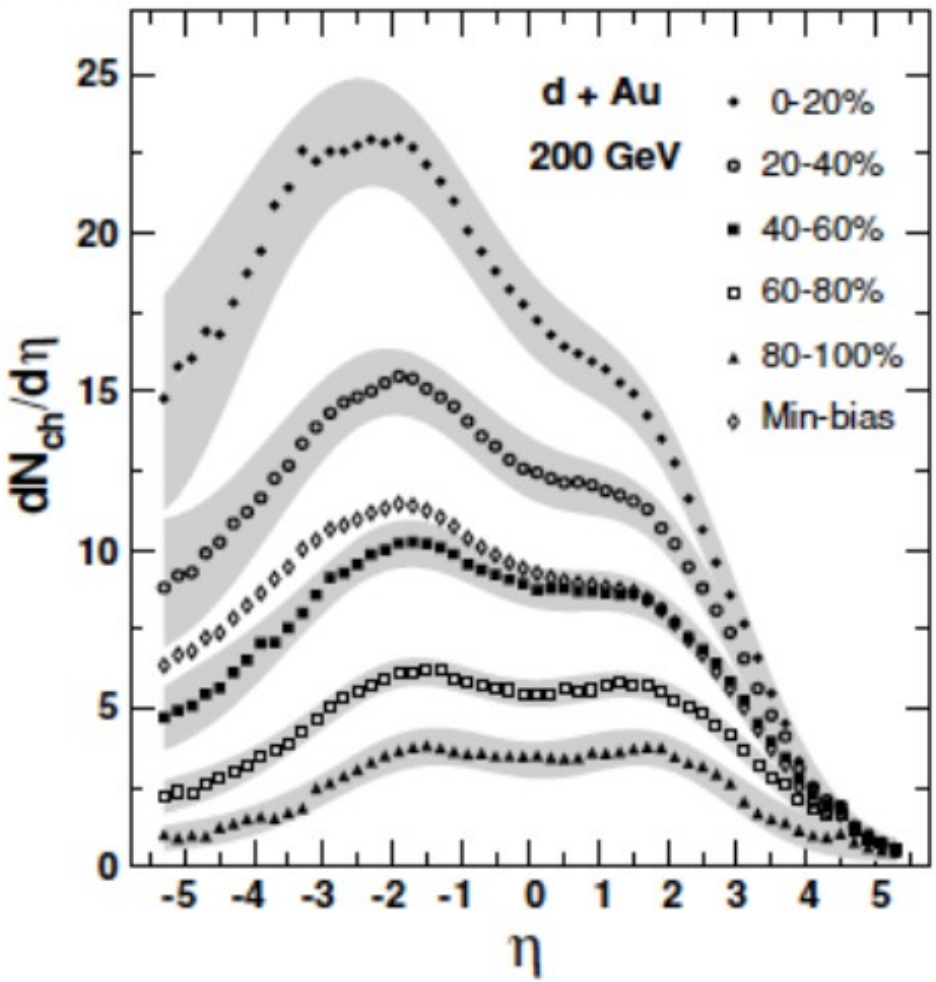


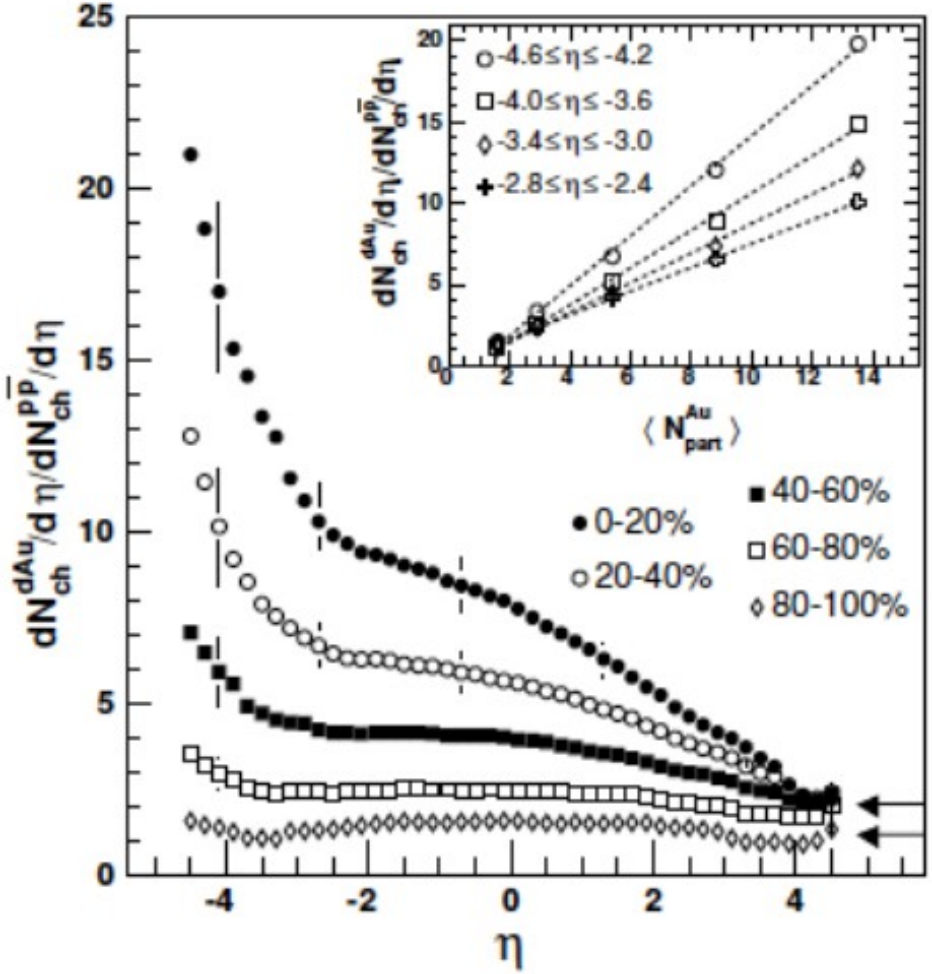
Figure from Brodsky, Gunion, Kuhn 1977

Boost invariance must be globally broken in p+A ! Generalized in 1980 Lund multi beam jet flux tube as well a 1990 CGC models

Global boost non-invariance and BGK triangles in d+A RHIC

PHOBOS

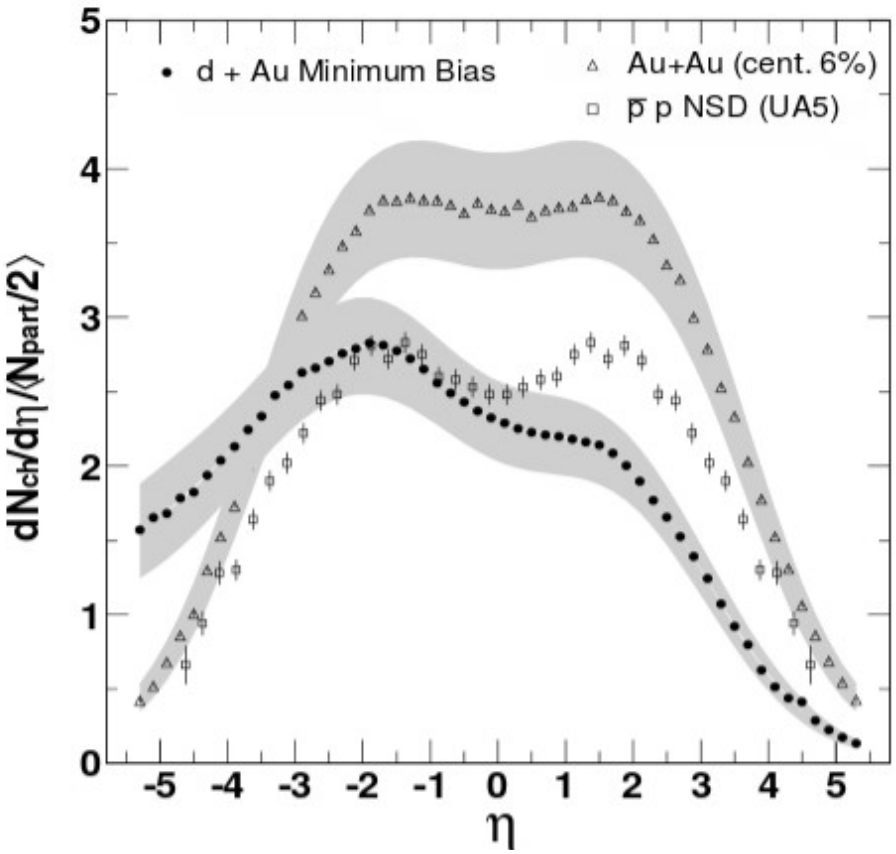




PHYSICAL REVIEW C **72**, 031901(R) (2005)

PHOBOS 0311009 pp vs Dau vs AuAu vs models

Absolute scale of dN/deta in pp and DA
Are also well understood



Monte Carlo implementation of BGK77
Via HIJING and AMPT predicted correctly
the p+p and p,D+Au dNch/dy
At SPS and RHIC. This basic feature
Understood via CGC Qsat(y,A) saturation.

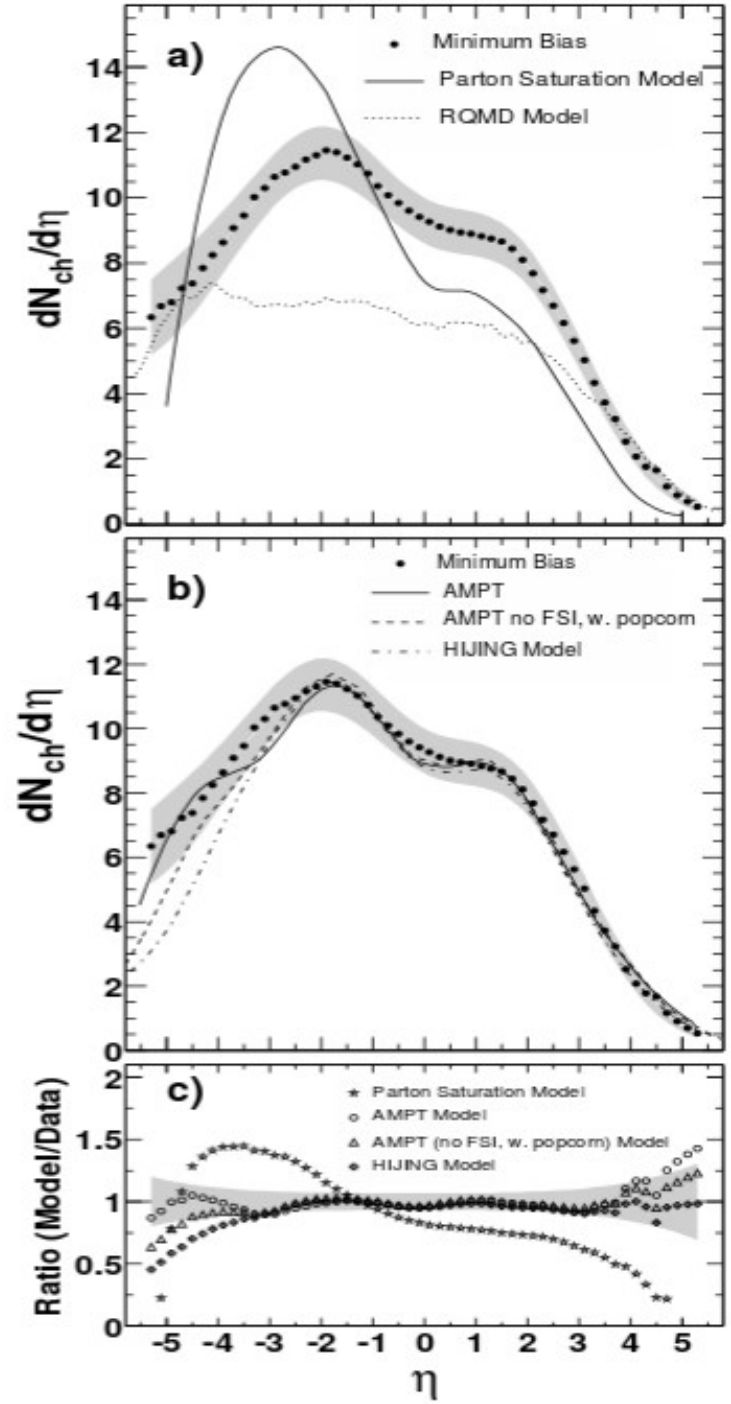
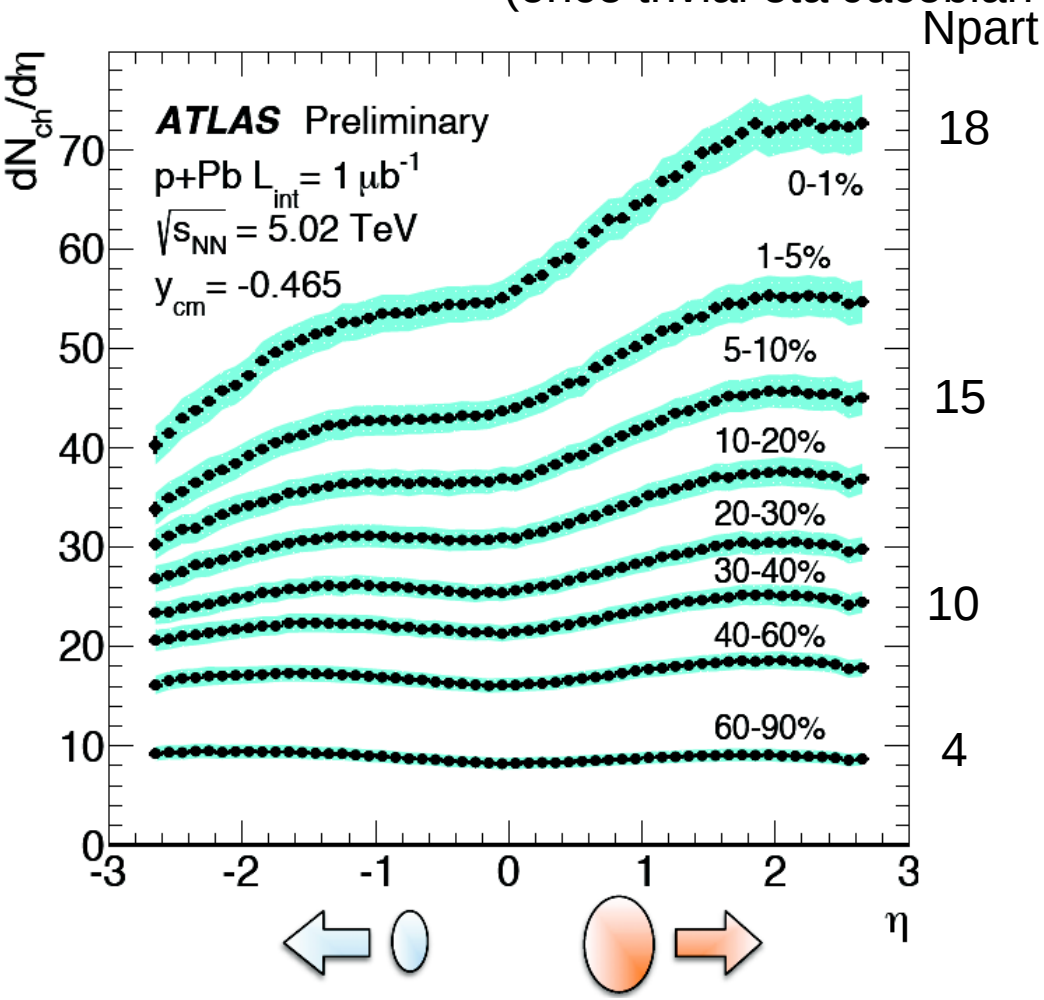
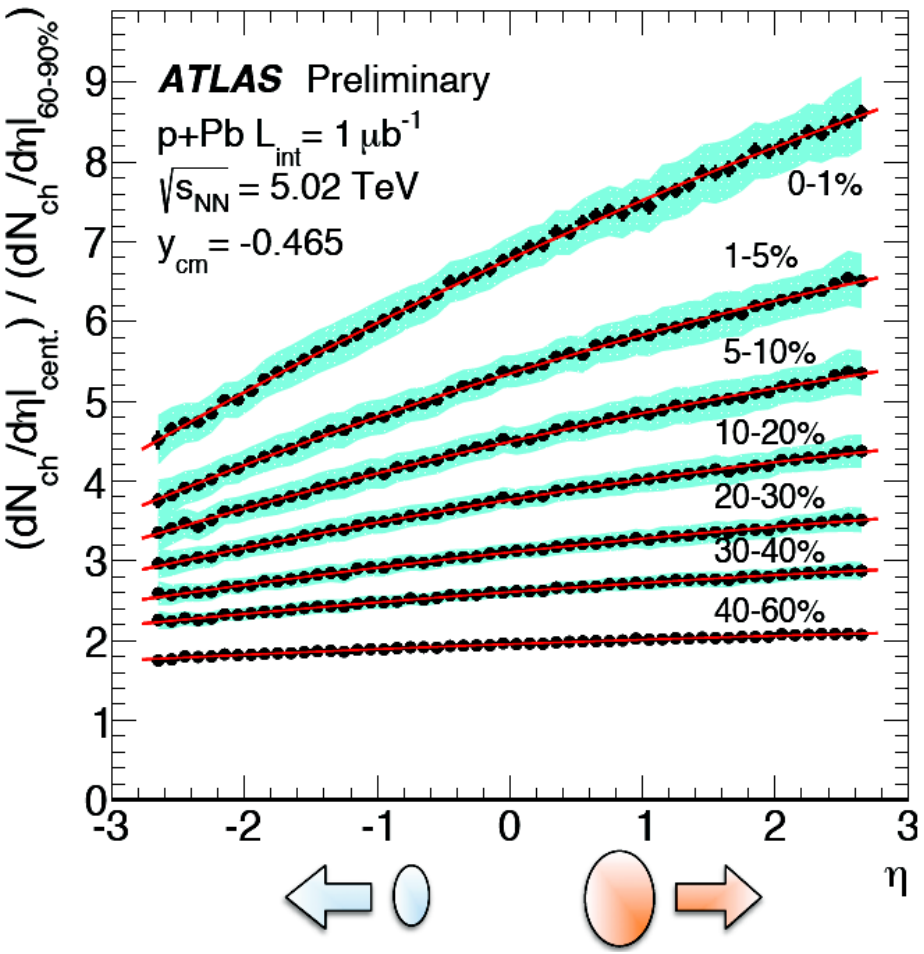


FIG. 3: Comparison of the measured minimum-bias pseudorapidity distribution for d + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV to model predictions. a) Comparison to

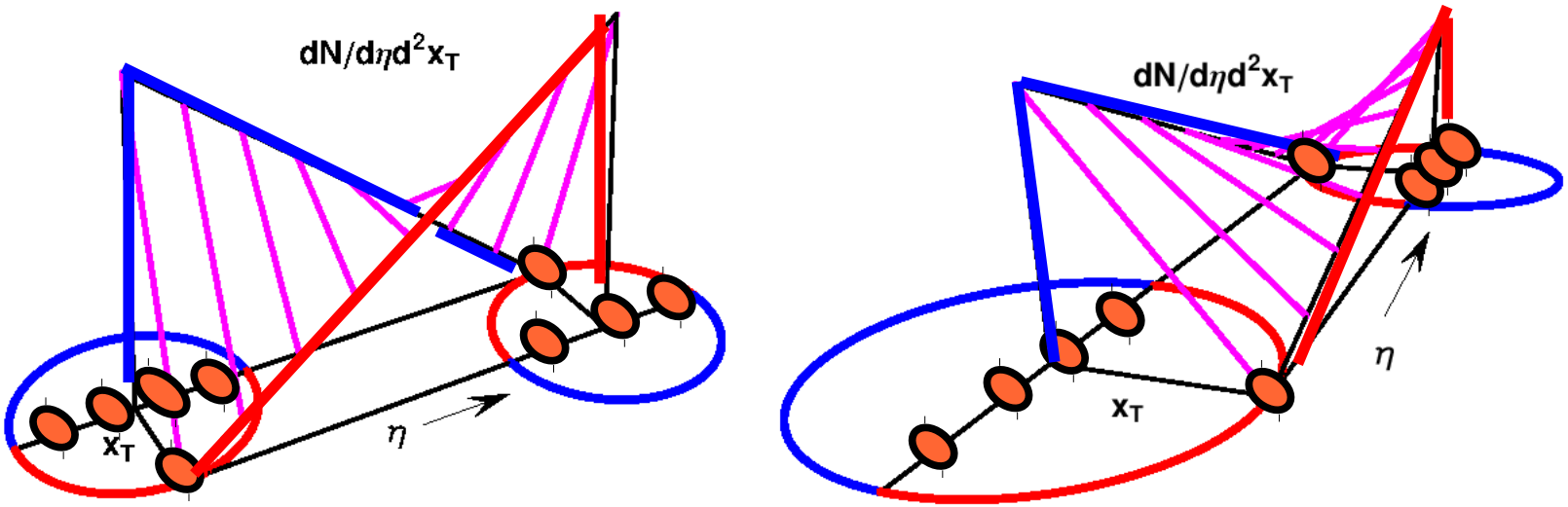
Amazingly perfect BGK77 linear rapidity triangles at LHC 5 ATeV pPb also
(once trivial eta Jacobian at eta=0 is canceled out in ratio)





3D jet tomography and the twisted color glass condensate
A. Adil, M. Gyulassy, T. Hirano , Nucl.Phys. A774 (2006) 593

A+A ~ (p+$A^{1/3}$) +($A^{1/3}$+p) + more symmetric stuff(A-$A^{1/3}$ + A-$A^{1/3}$)



AA b>0 has opposite Rapidity Triangle at p+A edges

Note the total dN/dy integrated over xT is completely flat & constant !

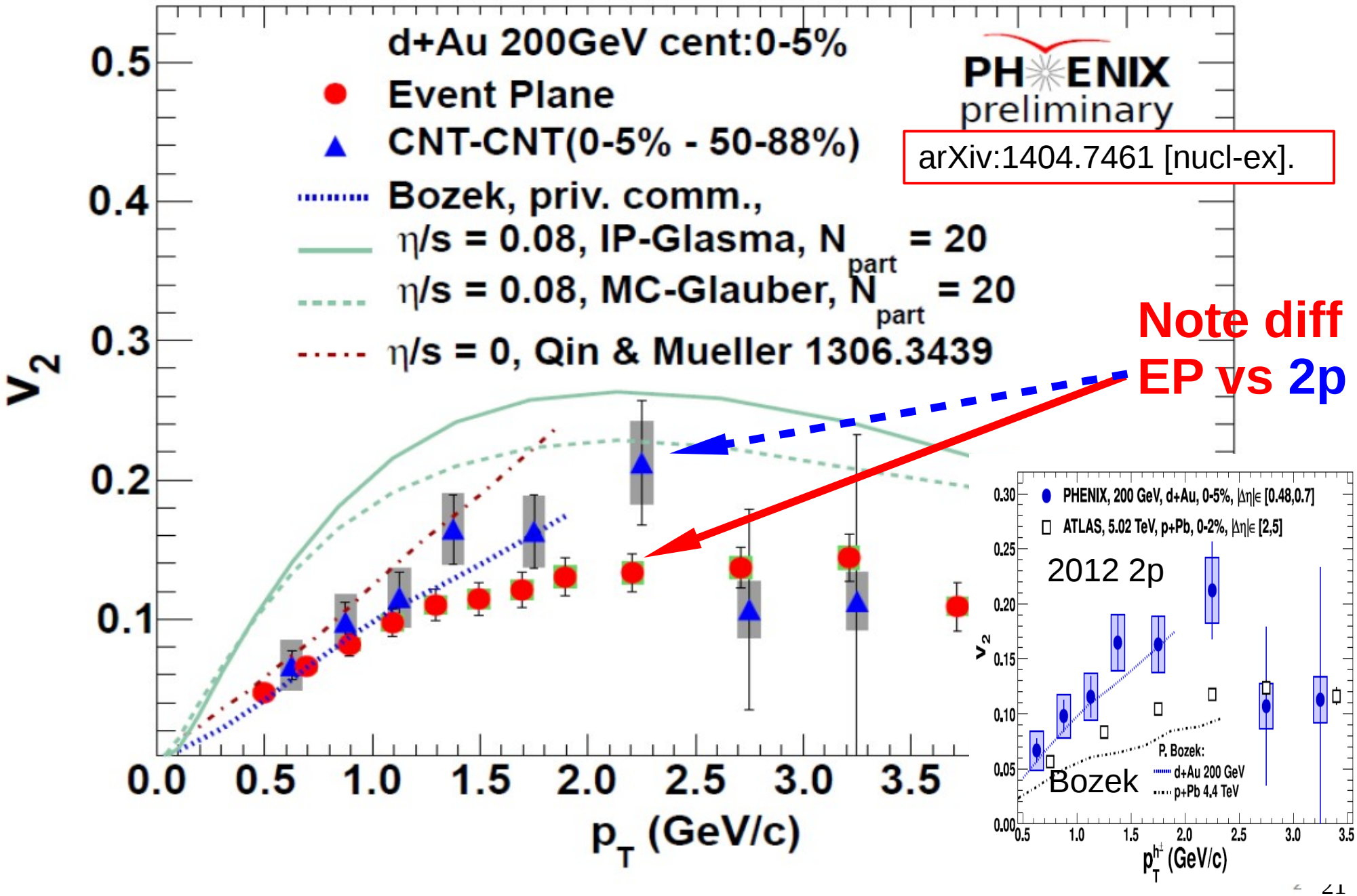
xT averaged AA *appears* boost invariant but
AA b>0 is *NOT* boost invariant *locally* except at xT=b/2

Physics of p+A, D+A, He3+A matters a lot for understanding A+A

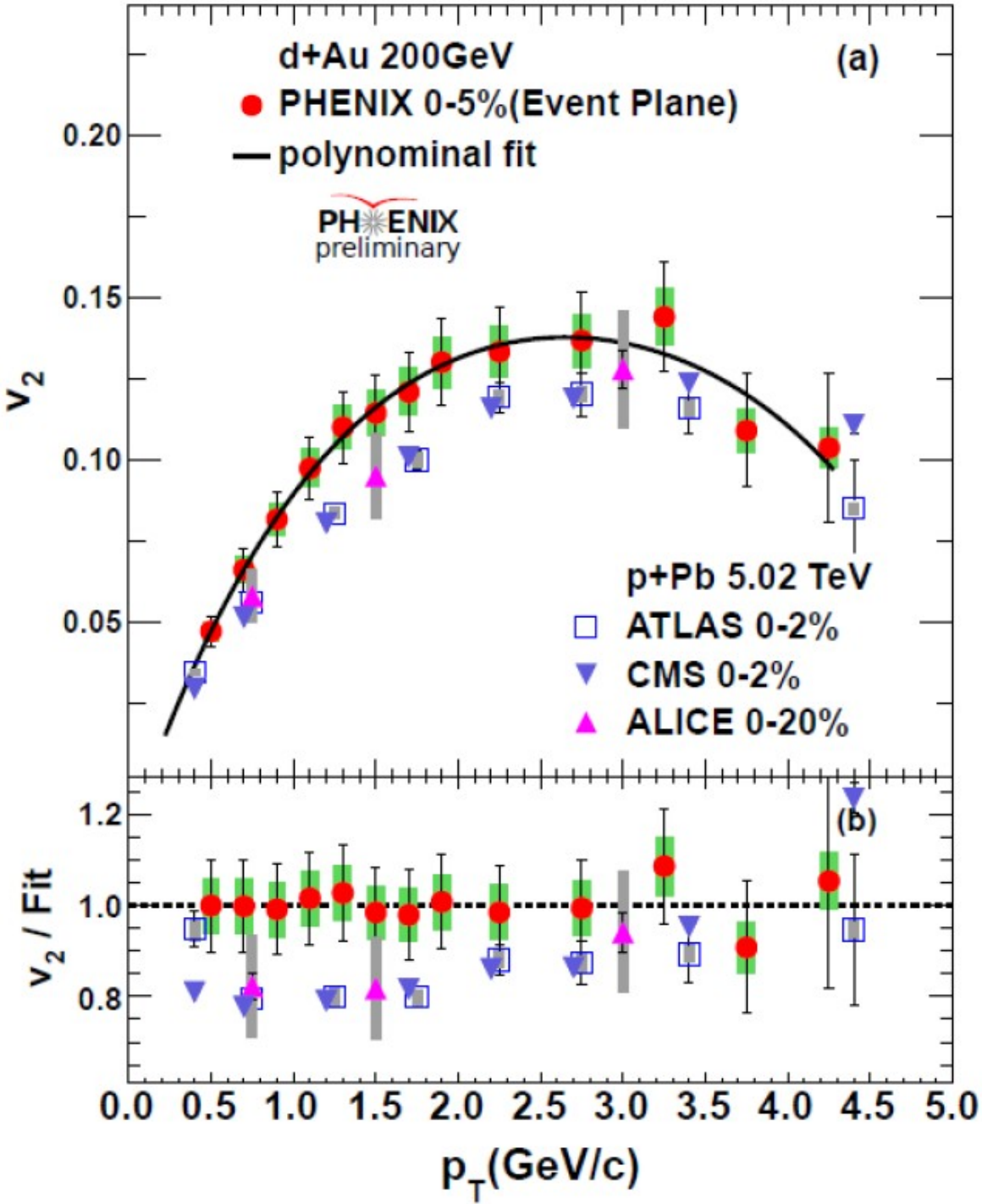
Nagle et al

A recent twist on the D+Au v2 story: Event Plane v2 significantly reduced relative to 2012 2part

Comparing with Hydro(s)

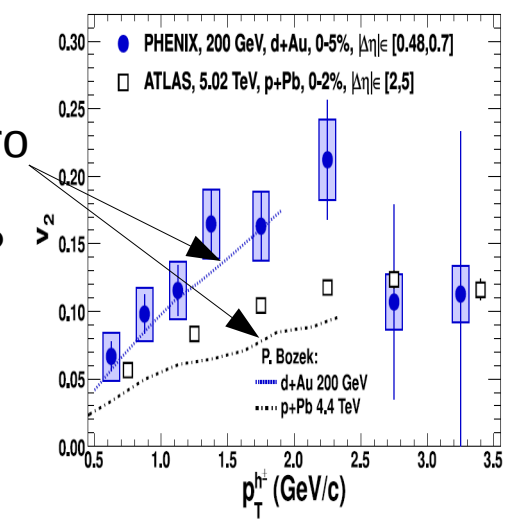


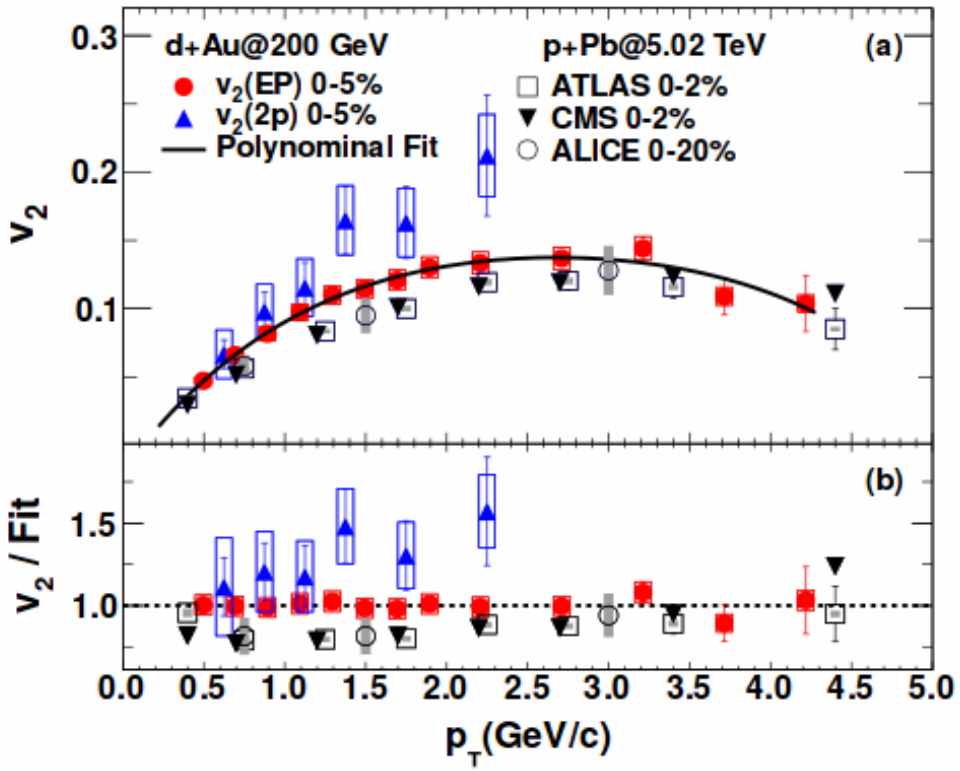
2014: A new invariance of v2: D+Au @200 = p+Pb@ 2760 ??



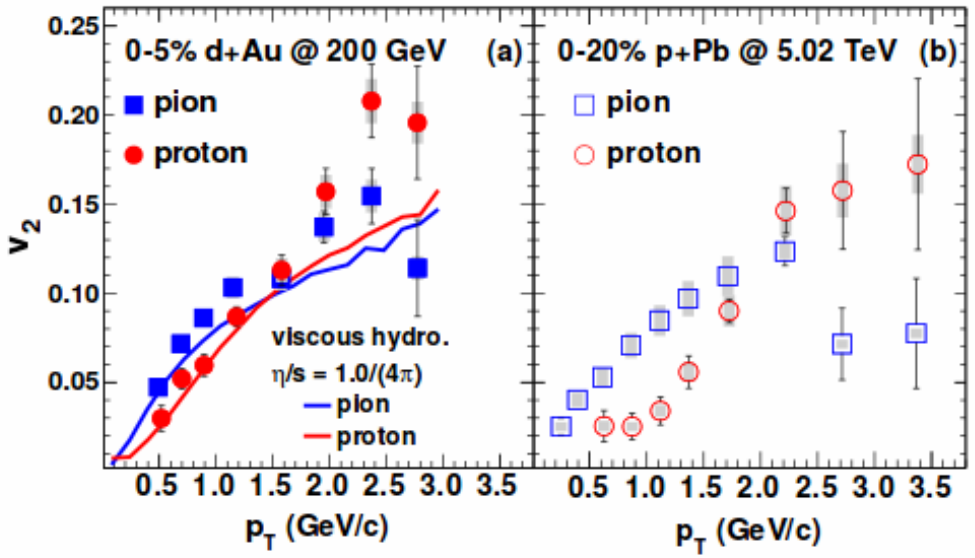
- The difference in v_2 between central d+Au and p+Pb collisions is about 20%
- The multiplicity and eccentricity are quite different between d+Au and p+Pb collisions

Is 2011
Bozek hydro
Success
Now dead?





collisions [2, 3, 5], with much improved uncertainties and extended p_T range. It is about 20% higher than that of p+Pb at $p_T = 1$ GeV/c, and the difference decreases to few percent at $p_T > 2.0$ GeV/c.



also shows calculations with Glauber initial conditions for viscous hydrodynamics starting at $\tau = 0.5$ fm/c with $\eta/s = 1.0/(4\pi)$, followed by a hadronic cascade [26–28]. The splitting at lower p_T is also seen in the calculation. Because there are no known CGC calculations available that would indicate a mass-splitting, it may be challenging – even in principle – to establish the observed mass dependence in the initial stages of the collision. The identified particle v_2 in 0%–20% p+Pb collisions are shown in Fig. 4(b) for comparison [15]. The magnitude of the mass-splitting in RHIC d+Au is smaller than that seen in LHC p+Pb, which could be an indicator of stronger radial flow in the higher energy collisions.

Note: 2014 **boost invariant** Romat hydro (not 2011 Bozek's) was used in this recent fit to new v2(EP) DAu data

Rapidity eta and radial pT dependence of absolute dN/detdpT (pi + p) must also be predicted and checked

Is this a Hydro vs CGC Skirmish?? :-)

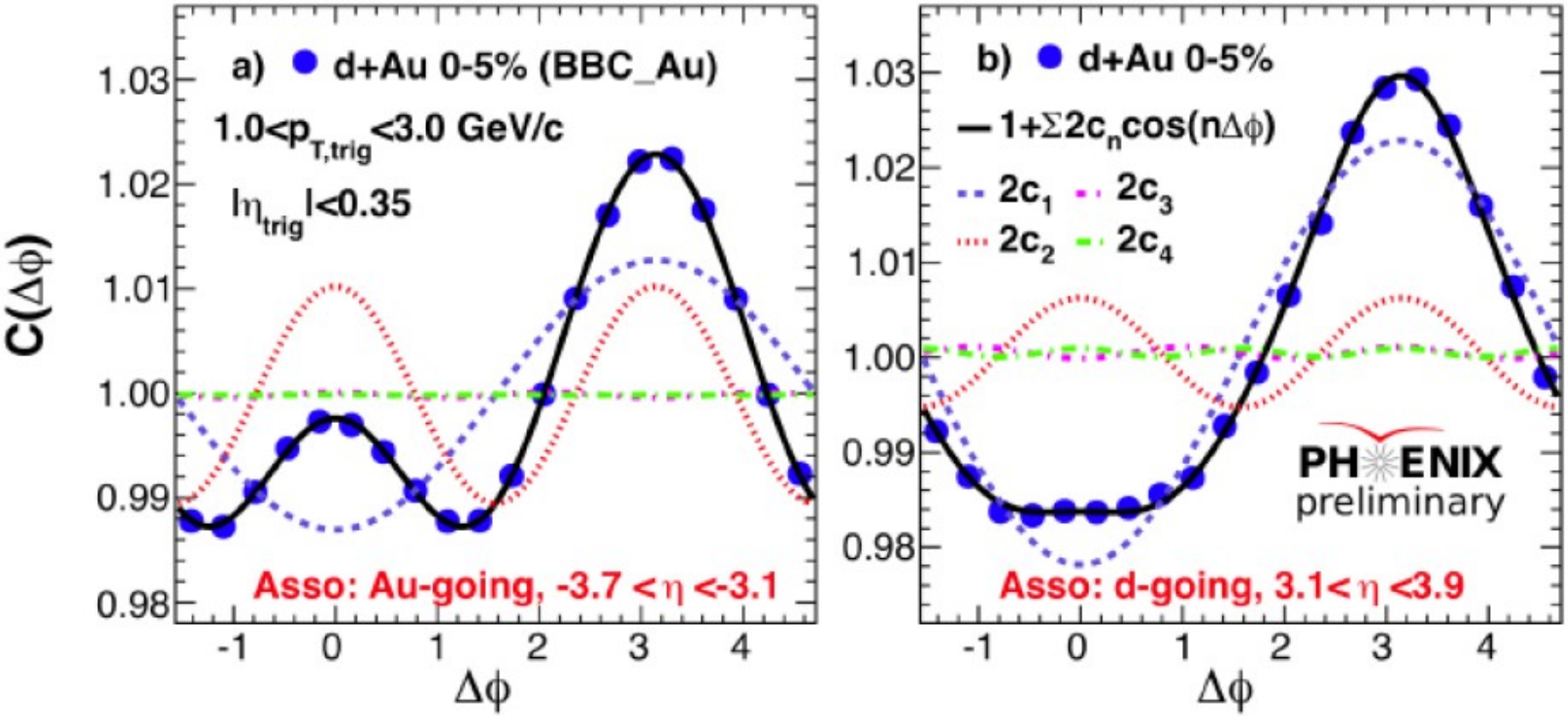
(see A. Esposito talk Friday)

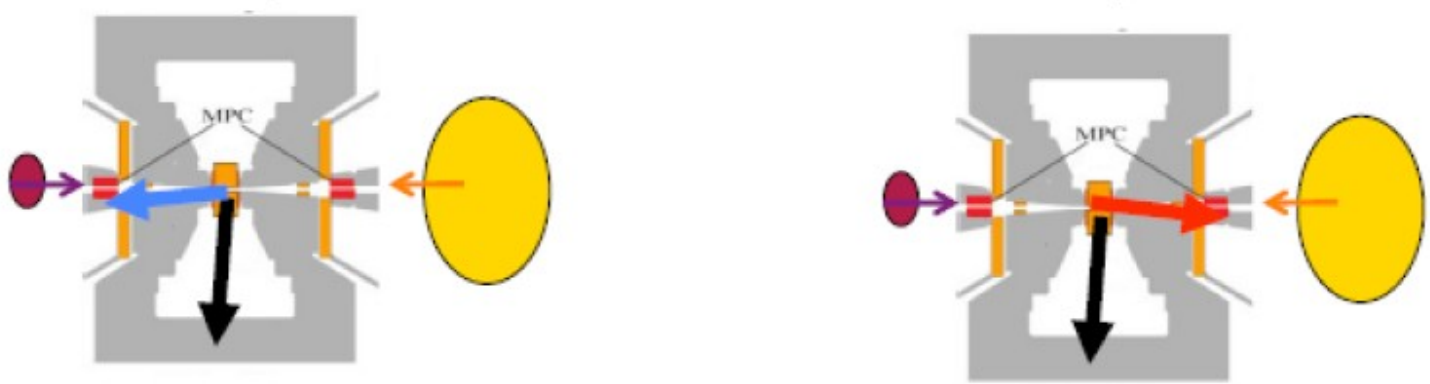
Strong rapidity asymm v2 seen Au side > D side. (differs from STAR)

"Ridge" in d+Au collisions

S. Huang, PHENIX HP13

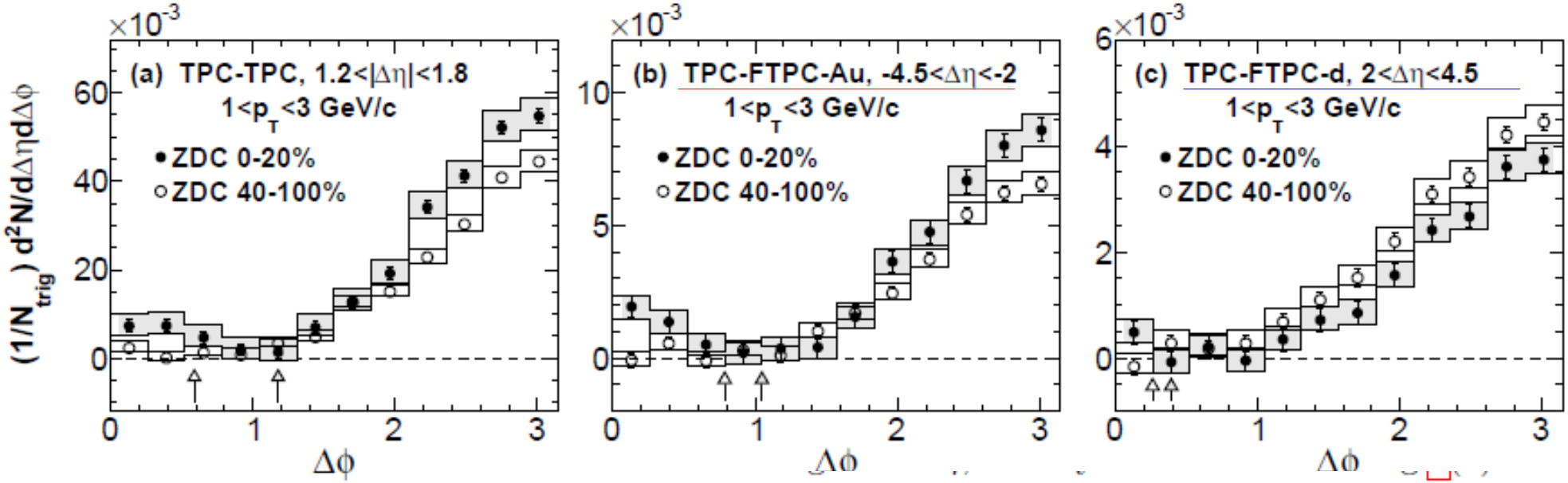
arXiv:1404.7461 [nucl-ex].





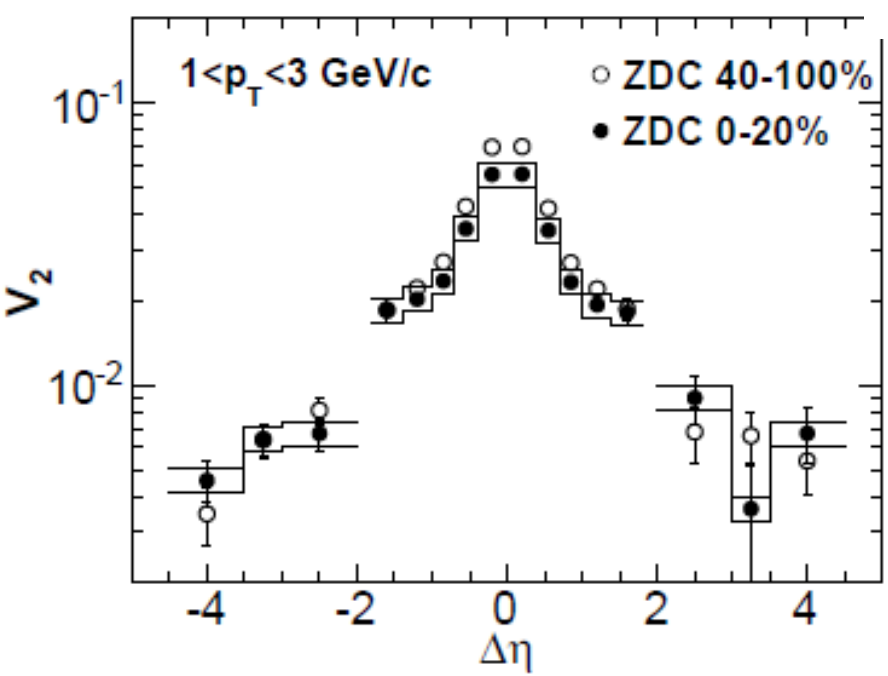
Boost noninvariant dipole v1 component at high +- rapidity masks v2(y,pT) physics

New STAR 1502.07652 D+Au large rapidity gap azimuthal asymmetries



away-side correlated yields are found to be larger in high- than low-activity d+Au collisions for TPC and FTPC-Au correlations. The opposite behavior is observed for the FTPC-d correlations, Fig. 1(c).

Strong eta dependence of v2?



STAR V2 in Au direction Deta < -2
Is similar to
V2 in D direction Deta > +2

? PHENIX v2(EP)(y,pT) seem different ?

Does strong rapidity dependence of v2 rule out hydrodynamic origin of v2 in D+Au ??
Future pA with BES should be high priority

See workshop STAR and PHENIX talks

PRL 113, 112301 (2014) PHYSICAL REVIEW LETTERS week ending 12 SEPTEMBER 2014

Exploiting Intrinsic Triangular Geometry in Relativistic ^{3}He + Au Collisions to Disentangle Medium Properties

J. L. Nagle,[1,*] A. Adare,[1] S. Beckman,[1] T. Koblesky,[1] J. Orjuela Koop,[1] D. McGlinchey,[1] P. Romatschke,[1] J. Carlson,[2] J. E. Lynn,[2] and M. McCumber[2]

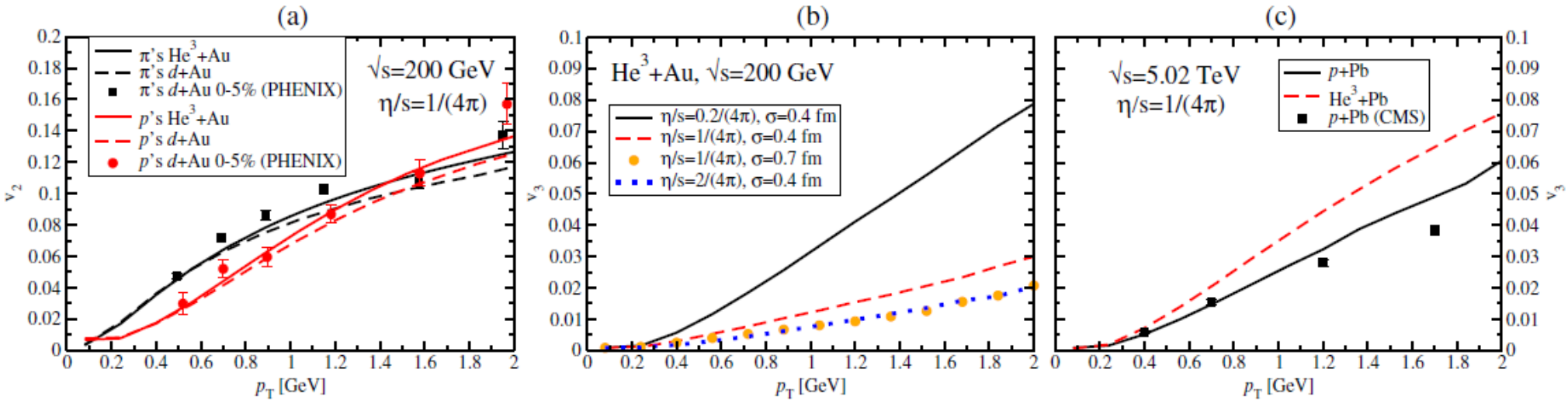


FIG. 5 (color online). (a) Pion and proton v_2 versus p_T for $d + \mathrm{Au}$ and $^3\mathrm{He} + \mathrm{Au}$ central ($b < 2$ fm) events at RHIC energies using hydrodynamics for $\eta/s = 1/(4\pi)$ in comparison to data for 0%–5% central $d + \mathrm{Au}$ data from PHENIX [24]. (b) Pion v_3 versus p_T for $^3\mathrm{He} + \mathrm{Au}$ central ($b < 2$ fm) events at RHIC energies using viscous hydrodynamics for $\eta/s = 0.2, 1, 2$ over 4π as well as the result of a different smearing parameter, σ. (c) Pion v_3 versus p_T for $p + \mathrm{Pb}$ and $^3\mathrm{He} + \mathrm{Au}$ central ($b < 2$ fm) events at LHC energies using viscous hydrodynamics for $\eta/s = 1/(4\pi)$ in comparison to data for 0.5%–2.5% central $p + \mathrm{Pb}$ data from CMS [25].

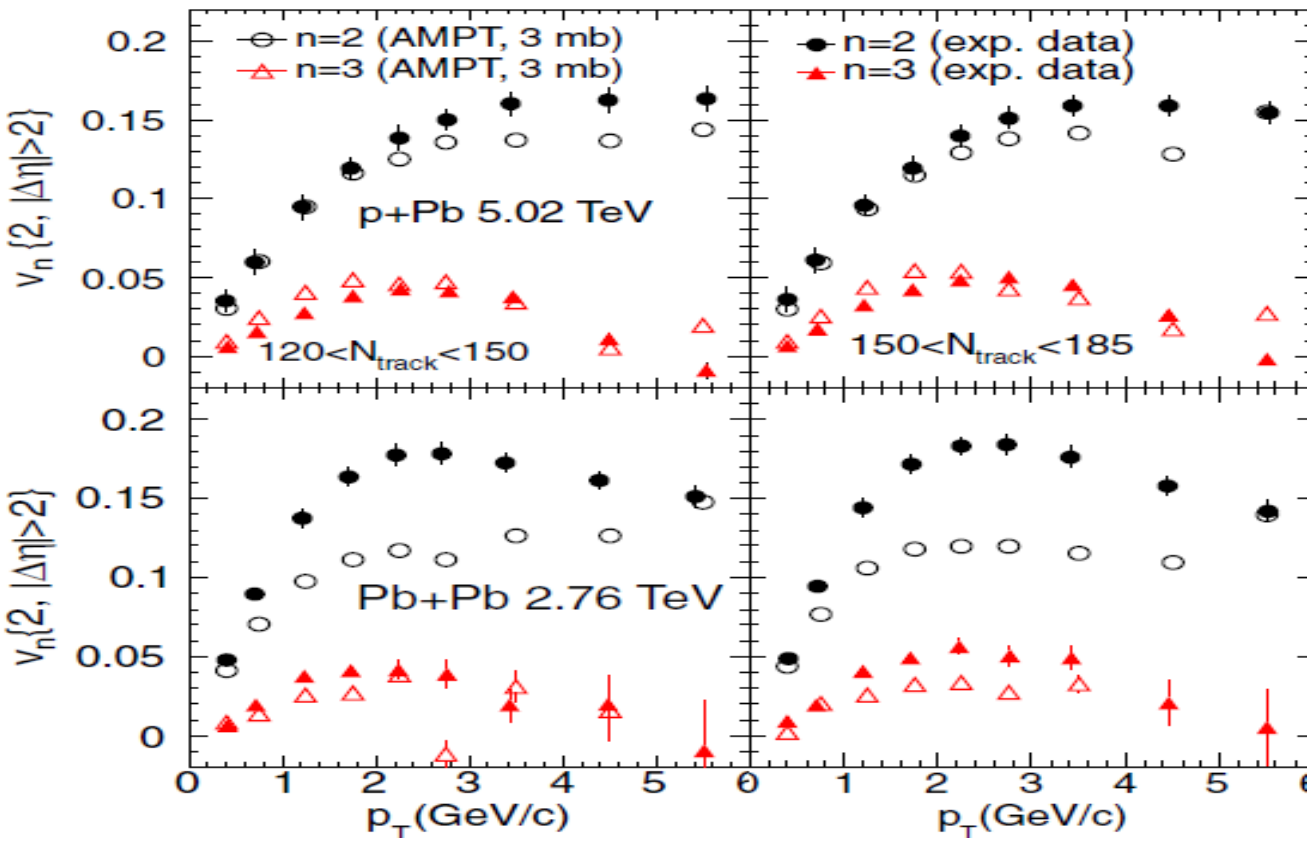
Future detailed BES with pA, DA, and He3A data at RHIC will be critically important.
Should be a very high priority for RHIC PAC

Now for a Non-hydro AMPT intermission reproducing vn in p+Pb

PRL 113, 252301 (2014) PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2014

Elliptic and Triangular Flow in *p*-Pb and Peripheral Pb-Pb Collisions from Parton Scatterings

Adam Bzdak[1,3,*] and Guo-Liang Ma[2,†]



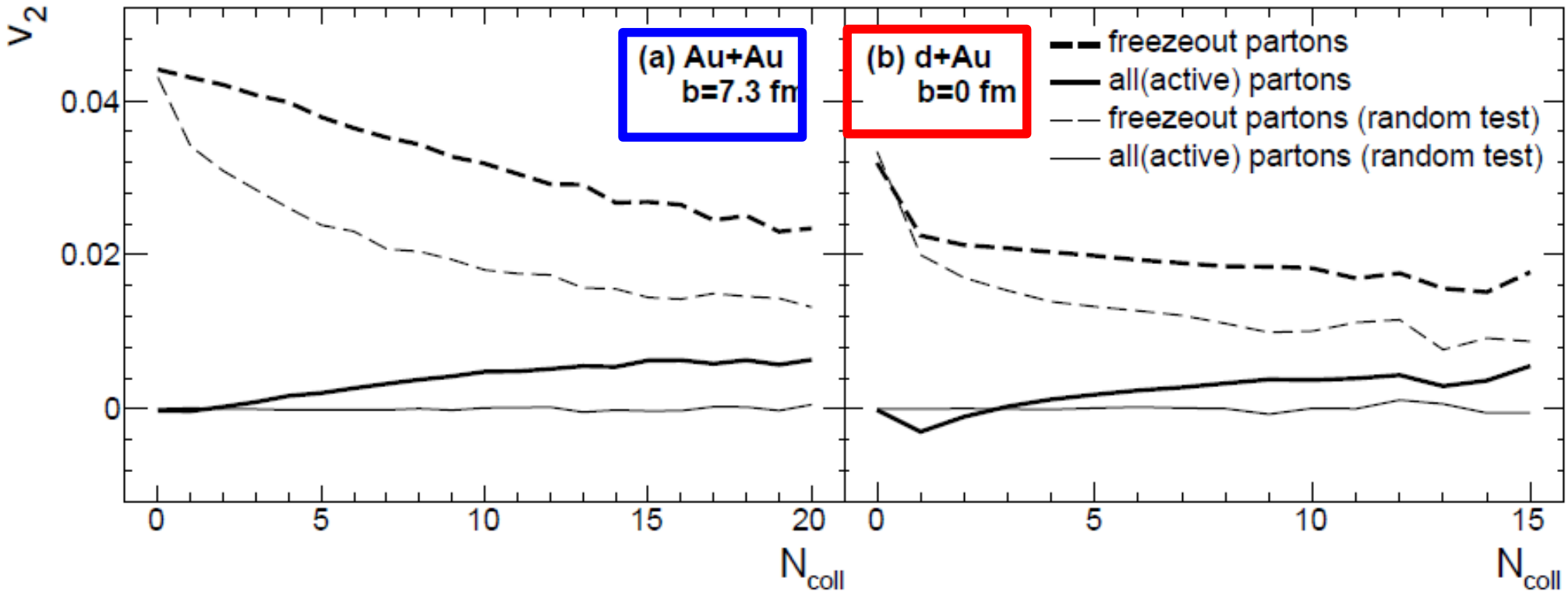
Fits pPb well ,but underestimates PbPb. PbPb is more than parton billiards

Anisotropic parton escape is the dominant source of azimuthal anisotropy from A Multi-Phase Transport

1502.05572

Liang He,[1] Terrence Edmonds,[2] Zi-Wei Lin,[3] Feng Liu,[4] Denes Molnar,[1] and Fuqiang Wang[1,4]

Analysis of AMPT evolution reveals that most of the final vn
Comes not from multiple collision “active” partons but those that freezeout
“escape” without collisions (moving away from dense regions)



AMPT somehow overcomes Shuryak's old opacity upper bound. I dont understand how.
Need a talk from D. Molnar

Discussions. The unique finding of our study is that partons escape from the overlap volume, and due to the anisotropic escape probability those escaped (freezeout) partons possess positive v_n, even without interactions. This escape mechanism contributes to the majority of the final anisotropic flow. Our study indicates that the escape mechanism is at work at both high and low $p_\perp$; there appears to be no fundamental difference in the v_n development of high- and low-$p_\perp$ partons.

ents. The azimuthal modulation of this radial flow is the anisotropic flow v_n of active partons, as shown in the solid thick curves in Figs. 2 and 4. It is the v_n of these active partons that is the most relevant for the standard hydrodynamic flow description, or the collective properties of the sQGP.

Now let's drop Final State interactions completely

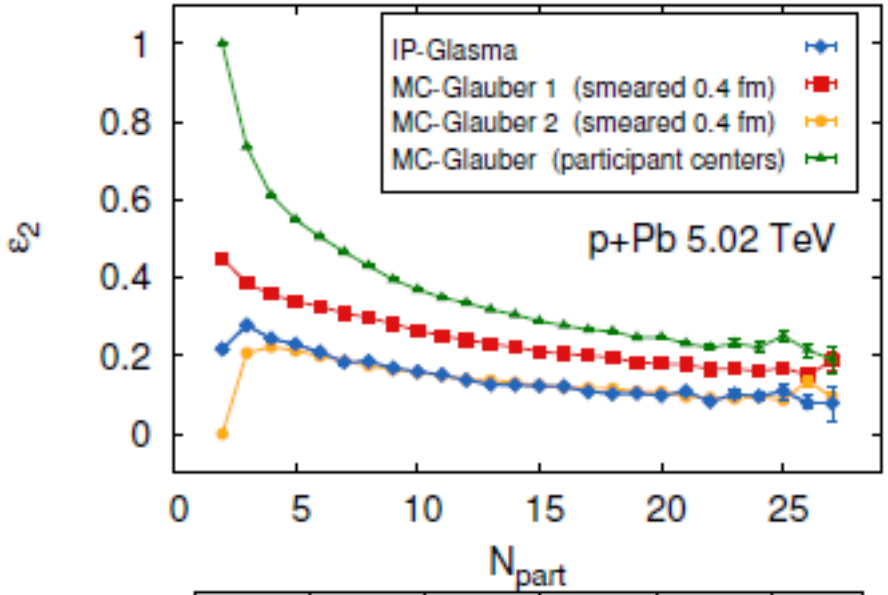
And show that p+Pb vn can also be quantitatively

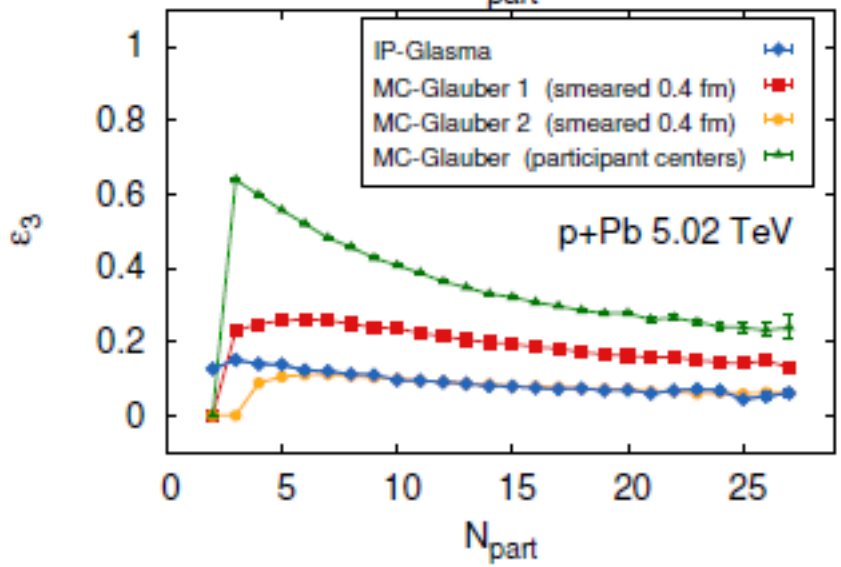
understood in terms of purely initial state classical
color field decoherence effects

The Revenge of Glasma + BFKL to quench Wit's pA Revenge

There is 100% uncertainty of hydro Initial Conditions in p+A

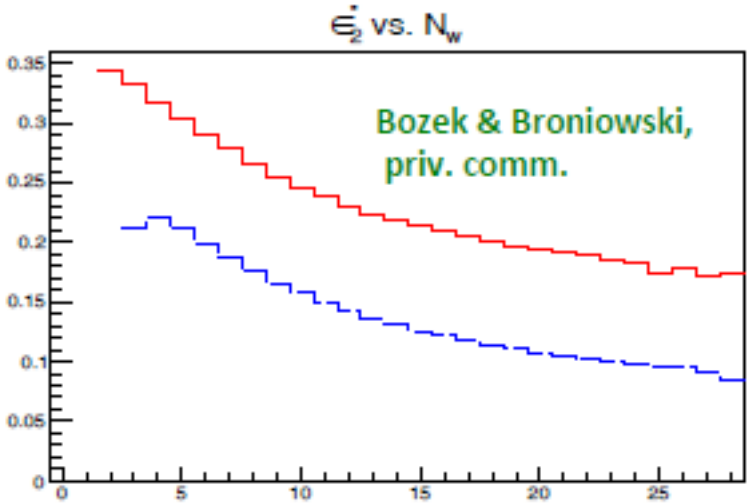
Initial conditions in p+A: IP-Glasma vs "Glauber"





Bzdak,Schenke,Tribedy,RV:1304.3403

MC-Glauber 1 MC-Glauber 2



In MC Glauber 1, big differences in hard sphere & Gaussian smearing

PHYSICAL REVIEW D **87,** 094034 (2013)

Comparison of the color glass condensate to dihadron correlations in proton-proton and proton-nucleus collisions

Kevin Dusling[1] and Raju Venugopalan[2]

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(Received 8 March 2013; published 31 May 2013)

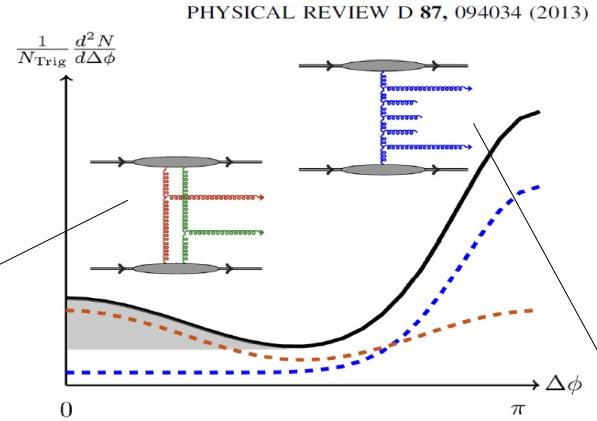
We perform a detailed comparison of long-range rapidity correlations in the color glass condensate framework to high multiplicity dihadron data in proton-proton and proton-lead collisions from the CMS, ALICE and ATLAS experiments at the LHC. The overall good agreement thus far of the nontrivial systematics of theory with data is strongly suggestive of gluon saturation and the presence of subtle quantum interference effects between rapidity separated gluons. In particular, the yield of pairs collimated in their relative azimuthal angle $\Delta\phi \sim 0$, is sensitive to the shape of unintegrated gluon distributions in the hadrons that are renormalization group evolved in rapidity from the beam rapidities to those of the measured hadrons. We present estimates for the collimated dihadron yield expected in central deuteron-gold collisions at the Relativistic Heavy Ion Collider.

The agreement of theory with data in proton-proton and proton-lead collisions over a very wide range of $p_T^{\rm trig}$, $p_T^{\rm asc}$ windows, centrality class and $\Delta\eta$ acceptance, is quite spectacular and lends strong support that (a) gluon saturation is being seen in these experiments, and (b) the data are sensitive to systematics of renormalization group evolution of UGD in the description of Glasma and "back-to-back"

Color decoherence

KDusling&RajuV, PRD 87, 094034 (2013)

Near side
"Glasma"
graphs



Away side
Mini jet "BFKL"
graphs

$$\frac{d^2N^{\rm corr}_{\rm Glasma}}{d^2\mathbf{p}_T d^2\mathbf{q}_T dy_p dy_q} = \frac{\alpha_S(\mathbf{p}_T)\alpha_S(\mathbf{q}_T)}{4\pi^{10}}\frac{N_C^2}{(N_C^2-1)^3\zeta}\frac{S_\perp}{\mathbf{p}_T^2\mathbf{q}_T^2}K_{\rm Glasma} \times\left[\int_{\mathbf{k}_T}(D_1+D_2)+\sum_{j=\pm}\left(A_1(\mathbf{p}_T,j\mathbf{q}_T)+\frac{1}{2}A_2(\mathbf{p}_T,j\mathbf{q}_T)\right)\right],$$

(1)

Input Unintegrated Gluon Structure Phi(y,kT)
BK evolved from initial Saturation Qsat(Y,A)

$$D_1 = \Phi^2_{A_1}(y_p,\mathbf{k}_T)\Phi_{A_2}(y_p,\mathbf{p}_T-\mathbf{k}_T)[\Phi_{A_2}(y_q,\mathbf{q}_T+\mathbf{k}_T) + \Phi_{A_2}(y_q,\mathbf{q}_T-\mathbf{k}_T)],$$
$$D_2 = \Phi^2_{A_2}(y_q,\mathbf{k}_T)\Phi_{A_1}(y_p,\mathbf{p}_T-\mathbf{k}_T)[\Phi_{A_1}(y_q,\mathbf{q}_T+\mathbf{k}_T) + \Phi_{A_1}(y_q,\mathbf{q}_T-\mathbf{k}_T)]. \quad (2)$$

These four terms, called the "single diffractive" and "interference" graphs in [8], constitute the leading $\mathbf{p}_T/Q_S$ behavior. Also included is the next order correction in $\mathbf{p}_T/Qs$ where we have [20] $A_1 = \delta^2(\mathbf{p}_T+\mathbf{q}_T)\times[\mathcal{I}_1^2+\mathcal{I}_2^2+2\mathcal{I}_3^2]$, such that

We now consider the double inclusive distribution from the back-to-back BFKL graphs [34] shown in Fig. 1. The double inclusive multiplicity can be expressed as [15,35]

$$\frac{d^2N^{\rm corr}_{\rm BFKL}}{d^2\mathbf{p}_T d^2\mathbf{q}_T dy_p dy_q} = \frac{32N_c\alpha_s(\mathbf{p}_T)\alpha_s(\mathbf{q}_T)}{(2\pi)^8 C_F}\frac{S_\perp}{\mathbf{p}_T^2\mathbf{q}_T^2}K_{\rm BFKL} \times\int_{\mathbf{k}_{0\perp}}\int_{\mathbf{k}_{3\perp}}\Phi_A(x_1,\mathbf{k}_{0\perp})\Phi_B(x_2,\mathbf{k}_{3\perp}) \times\mathcal{G}(\mathbf{k}_{0\perp}-\mathbf{p}_T,\mathbf{k}_{3\perp}+\mathbf{q}_T,y_p-y_q), \quad (5)$$

where $\mathcal{G}$ is the BFKL Green's function

$$\mathcal{G}(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp},\Delta y) = \frac{1}{(2\pi)^2}\frac{1}{(\mathbf{q}_{a\perp}^2\mathbf{q}_{b\perp}^2)^{1/2}}\sum_n e^{in\bar\phi} \times\int_{-\infty}^{+\infty}d\nu e^{\omega(\nu,n)\Delta y}e^{i\nu\ln(\mathbf{q}_{a\perp}^2/\mathbf{q}_{b\perp}^2)}. \quad (6)$$

Here $C_F=(N_c^2-1)/2N_c$, $\omega(\nu,n)=-2\bar\alpha_s{\rm Re}[\Psi(\frac{|n|+1}{2}+i\nu)-\Psi(1)]$ is the BFKL eigenvalue, where $\Psi(z)=d\ln\Gamma(z)/dz$ is the logarithmic derivative of the Gamma function. Further, we have $\bar\alpha_s\equiv N_c\alpha_S(\sqrt{\mathbf{q}_{a\perp}\mathbf{q}_{b\perp}})/\pi$ and $\bar\phi\equiv\arccos(\frac{\mathbf{q}_{a\perp}\cdot\mathbf{q}_{b\perp}}{|\mathbf{q}_{a\perp}||\mathbf{q}_{b\perp}|})$.

Comparison of the color glass condensate to dihadron correlations in proton-proton and proton-nucleus collisions PRD 87, 094034, Kevin Dusling and Raju Venugopalan

CMS pPb vs Glasma+BFKL

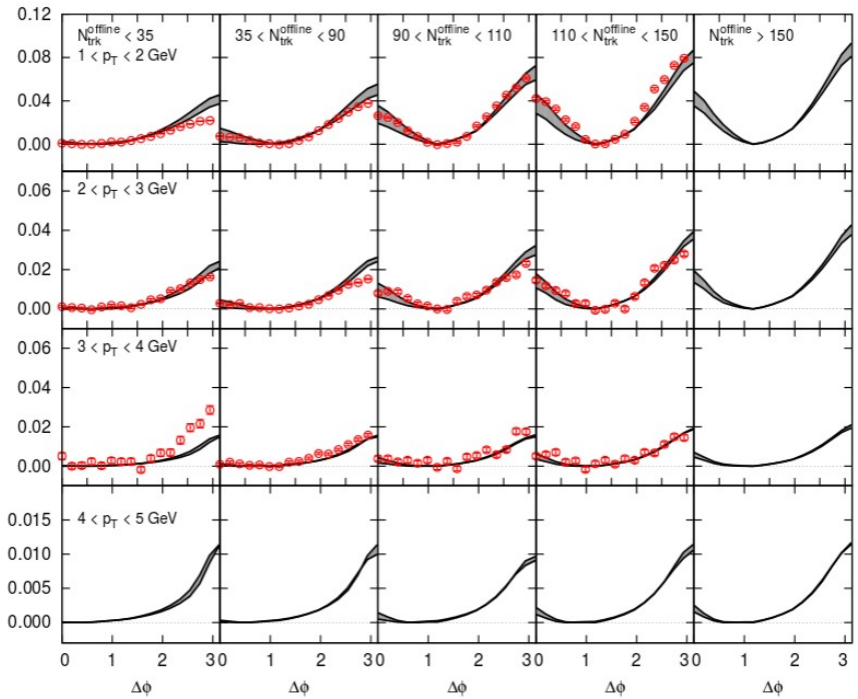


FIG. 6. Long-range $(2 \leq |\Delta\eta| \leq 4)$ per-trigger-yields $(1/N_{\rm trig} d^2N/d\Delta\phi)$ of charged hadrons as a function of $|\Delta\phi|$ in different p_T and multiplicity bins for pPb collisions at $\sqrt{s} = 5.02$ TeV with the CMS experiment. The p_T selection corresponds to both particles in the pair. The centrality dependence of the theory curves are controlled by the choice of initial saturation scale in the target

ATLAS pPb vs Glasma+BFKL

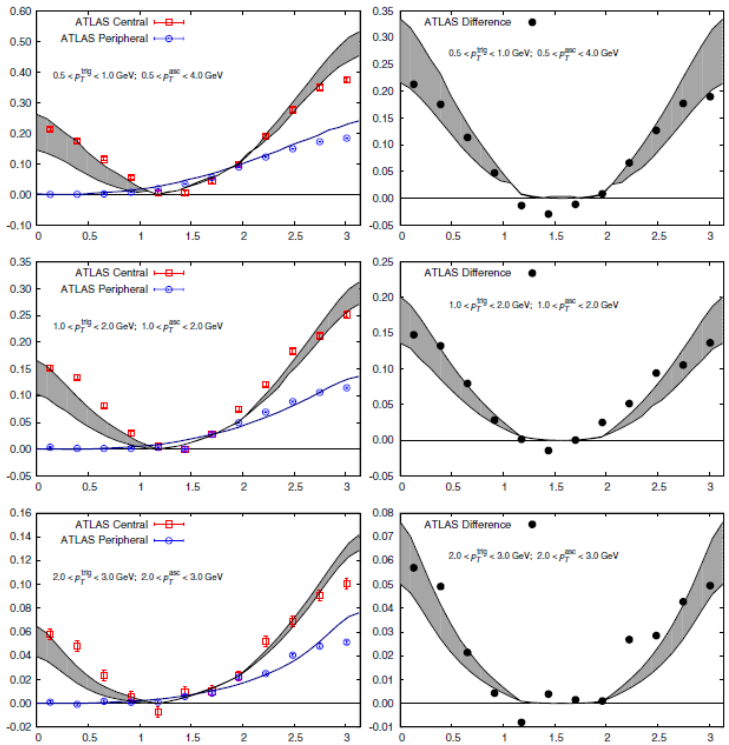


FIG. 7 (color online). Left: Data from the ATLAS Collaboration for the associated yield versus $\Delta\phi$ for the central and peripheral events identified in [18]. The blue theory curve corresponds to the comparison of Eq. (5) for the min. bias Q_0^2(proton) = 0.168 GeV^2

Quantitative description with pp constrained UGD and reasonable variations of proton and Pb
Initial Qsat(Y,A) = Npart 0.17 GeV^2 and Npart_prot ~ 3-4 and Npart_Pb~ 14-22

KDusling&RajuV, PRD 87, 094034 (2013)

Raju's Revenge:

An alternative scenario for the dynamics of long-range rapidity correlations is from flow resulting from strong final state rescattering [42,48,49]. We believe this possibility is strongly disfavored on both conceptual and phenomenological grounds—these issues will be addressed elsewhere. Our perspective based on the discussion in this paper is that the onus is now on models based on final state scenarios to present a quantitative description of data that is as transparent and competitive to the same degree as the one presented here—for instance, the matrix of data of the collimated yield versus $\Delta\phi$ for the various experiments.

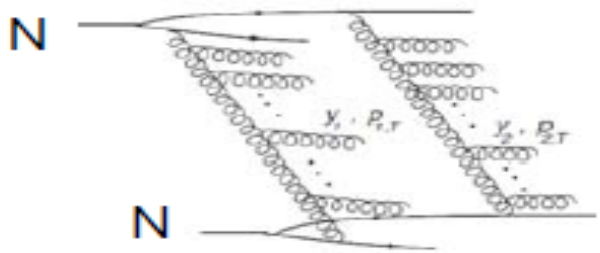
Further data from the LHC will no doubt provide clarity and challenge either or both scenarios. Regardless, unless the data presented thus far changes significantly, the results of our detailed analysis suggest that initial-state effects

[MG: must be computed from QCD and quantitatively tested on pp and pA]

(see also R.L Ray FJ00013; PRD 90, 054013 (2014))

Levin & Rezaeian BFKL* Pomeron model

E. Levin and A. H. Rezaeian, Phys. Rev. D **84**, 034031 (2011)



Two-BFKL Pomeron exchange with two-gluon emission & interference; parton showers

Single gluon density from two - parton showers

$$\frac{d\sigma}{dydp_t^2} \approx \int d^2\vec{q}_t \frac{d\sigma}{dydp_t^2}\bigg|_{Q_t=0} \left\{1 + \frac{\vec{p}_t \cdot \vec{Q}_t}{q_t'^2} + 2\frac{(\vec{p}_t \cdot \vec{Q}_t)^2}{q_t'^4} + \cdots\right\}$$ ($\vec{Q}_t$ expansion, momentum transfer)

$$(\vec{p}_t \cdot \vec{Q}_t)^2 \Rightarrow \cos 2(\phi - \hat{Q}_t) \Rightarrow v_2$$

anisotropy

Two - gluon density

$$\frac{d\sigma}{dy_1 dp_{t1}^2 dy_2 dp_{t2}^2} = \frac{1}{2}\int d^2\vec{Q}_t N_{Ph}^2(Q_t^2) \frac{d\sigma}{dy_1 dp_{t1}^2}(\vec{Q}_t) \frac{d\sigma}{dy_2 dp_{t2}^2}(-\vec{Q}_t)$$

probability for producing two parton showers

color-dipole model: Kopeliovich, Phys. Rev. D **78**, 114009 (2008);

CGC glasma and gluon interference: Dusling, Gelis, Lappi, McLerran, Venugopalan, Nucl. Phys. **A772**, 200 (2006); Acta. Phys. Pol. B **37**, 3253 (2006); Phys. Rev. D **87**, 051502(R) (2013); **87**, 094034 (2013);

BFKL-Pomeron with gluon interference:

Levin and Rezaeian, Phys. Rev. D **84**, 034031 (2011).

A Perturbative QCD Gluon Decoherence Model of vn in p+A

Multi beam jets as transverse interfering color antennas

PHYSICAL REVIEW D **90,** 054025 (2014)

Non-Abelian bremsstrahlung and azimuthal asymmetries in high energy $p + A$ reactions

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[2]*Department of Physics, Columbia University, New York, New York 10027, USA*
[3]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
(Received 22 July 2014; published 25 September 2014)

We apply the GLV reaction operator solution to the Vitev-Gunion-Bertsch (VGB) boundary conditions to compute to all orders in nuclear opacity the non-Abelian gluon bremsstrahlung of event-by-event fluctuating beam jets in nuclear collisions. We evaluate analytically azimuthal Fourier moments of single gluon, $v_n^M\{1\}$, and even numbered 2ℓ gluon distribution, $v_n^M\{2\ell\}$, inclusive distributions in high-energy $p + A$ reactions as a function of harmonic n, target recoil cluster number, M, and gluon number, 2ℓ, at the RHIC and LHC. Multiple resolved clusters of recoiling target beam jets together with the projectile beam

Goal is to understand analytically the mechanism the Glasma/BFKL initial state vn
And propose a HIJING Monte Carlo generalization

II. FIRST ORDER IN OPACITY (GB) BREMSSTRAHLUNG AND AZIMUTHAL ASYMMETRIES v_n

(Gunion Bertsch 1982)

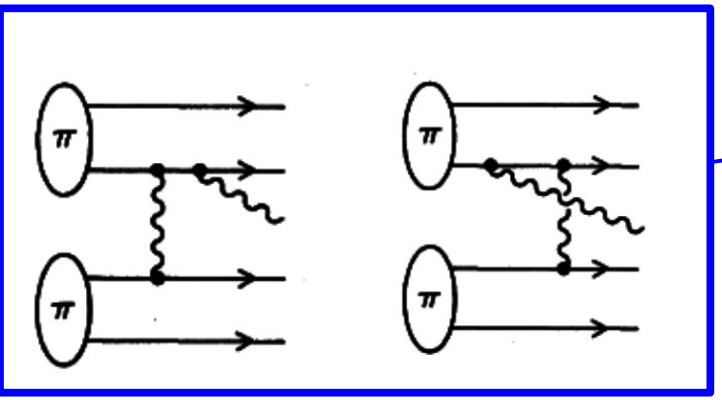
The above puzzles with BES and $D + Au$ at RHIC and with $p + Pb$ at LHC and models proposed so far motivate us to consider simpler more basic perturbative QCD sources of azimuthal asymmetries.The well known non-abelian bremstralung Gunion-Bertsch (GB) formula[29] for the soft gluon radiation single inclusive distribution is

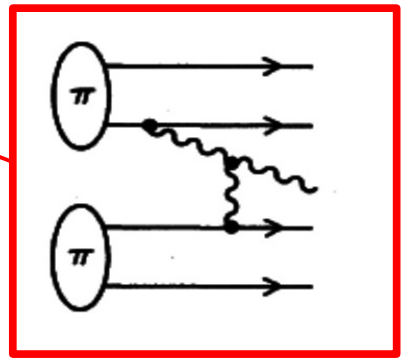
Color Dipole Form factor

$$\frac{dN_g^{(1)}}{d\eta d^2\mathbf{k} d^2\mathbf{q}} \equiv f(\eta, \mathbf{k}, \mathbf{q})$$

$$= \frac{C_R \alpha_s}{\pi^2 k^2} \frac{\mu^2 q^2}{\pi(q^2+\mu^2)^2} \frac{P_\eta}{(\mathbf{k}-\mathbf{q})^2+\mu^2}$$

$$\equiv \frac{FP}{A - \cos(\phi - \psi)},$$





Gluon Bremsstrahlung peaks in transverse direction near the total momentum transfer $\vec{Q}$ = (Q, Psi) that also defines reaction Event Plane (EP)

The Basic Non-Abelian feature: uniform *rapidity "ridge"* (unlike in QED)

And a small kT conformal peaks in beam direction 1/k^2 (as in QED)

ϕ is the azimuthal angle of $\mathbf{k}$ and ψ is the azimuthal angle of $\mathbf{q}$ and abreviations

Transverse Kinematic factors

$$A \equiv A_{kq} \equiv (k^2 + q^2 + \mu^2)/(2kq) \geq 1,$$

In pA mu^2 = qhat L = Qsat^2

$$F \equiv F_{kq} \equiv \frac{C_R \alpha_s}{\pi^2 k^2} \frac{\mu^2 q^2}{\pi (q^2 + \mu^2)^2} \frac{1}{2kq},$$

BGK Beam Jet rapidity envelope

$$P \equiv P_\eta \equiv (1 - e^{Y_T - \eta})^{n_f} (1 - e^{\eta - Y_P})^{n_f},$$

Azimuthal Harmonics of a single GB antenna

$$\begin{aligned} v_n(k,q,\psi) f_0(k,q) &= F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi - \psi)} \\ &= \cos(n\psi)\, F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi)} \quad . \end{aligned}$$

$f_0 \equiv \int d\phi f = \int d\phi \; d^7N/d\eta dk^2 d\phi dq^2 d\psi$ is the ϕ integrated single gluon inclusive

$$dN/d\eta dk^2 = F_{kq} P_\eta / (A_{kq}^2 - 1)^{1/2}$$

1/k^2 for k << q ~ mu

mu^2/k^4 for k >> q ~ mu

A single GB color antenna has analytic vn: $A_{kq} \equiv (k^2+q^2+\mu^2)/(2k\,q) \geq 1$

$$v_1^{GB}(k,q,\psi) = \cos[\psi](A_{kq} - \sqrt{A_{kq}^2 - 1})$$

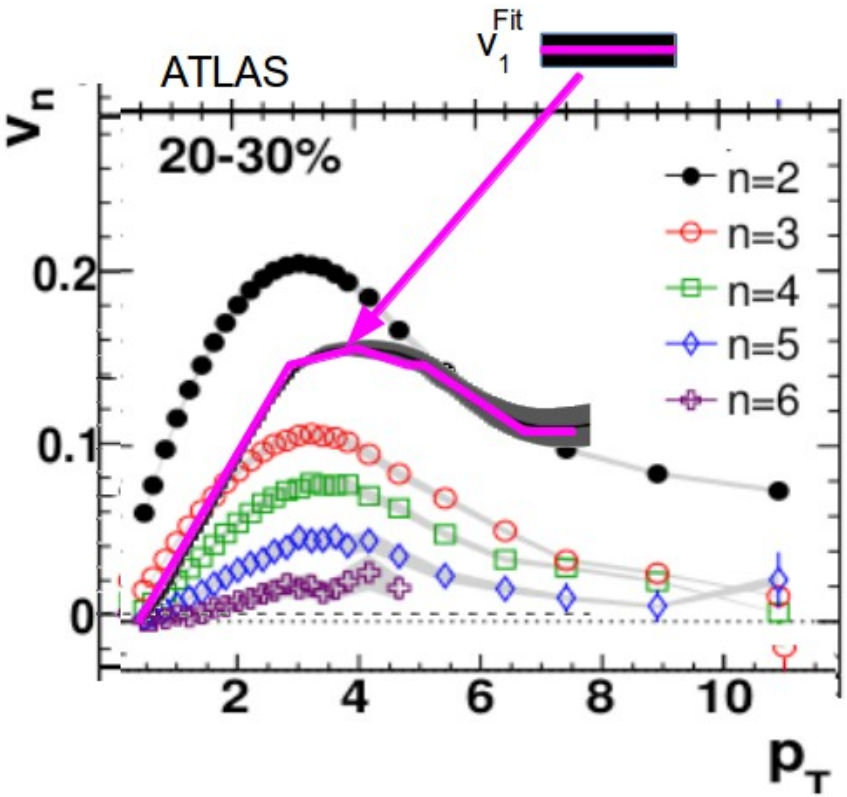
$$\lim_{\mu\to 0} v_1^{GB}(k,q,0) = (k/q)\,\theta(q-k)$$

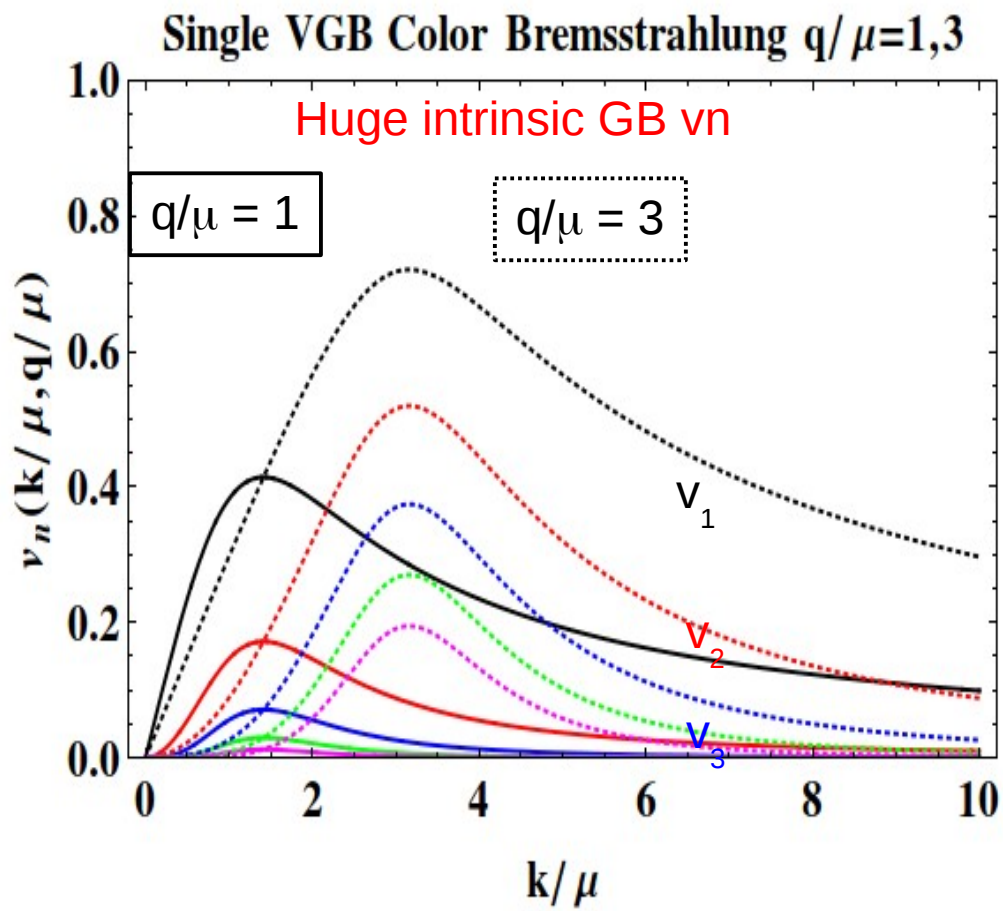
$$v_n^{GB}(k,q,\psi) = \cos[n\psi]\,(v_1^{GB}(k,q,0))^n$$

$$\lim_{\mu\to 0} v_n^{GB}(k,q,0) = (k/q)^n\,\theta(q-k)\ .$$

Perfect $v_n^{1/n} = v_1$ Scaling

Two particle vn from ATLAS





Note intrinsic huge even and odd vn peaking rear mu ~ Qsat

All order in opacity multiple scattering generalization of GB pQCD Brems

$$dN^{VGB}_{coh}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q}\, P^{el}_n(\mathbf{Q})\, dN^{GB}(\mathbf{k},\mathbf{Q})$$

$$P^{el}_n(\mathbf{Q}) \;=\; \exp[-\chi]\frac{\chi^n}{n!}\int \left\{ \prod_{j=1}^{n} \frac{d^2\mathbf{q}_j}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_j} \right\} \times\delta^2(\mathbf{Q} - (\mathbf{q}_1 + \cdots + \mathbf{q}_n))$$

Pn is total momentum transfer distribution from n coherent elastic scatterings

At n=N th order in opacity with M coherent target clusters that can be resolved by k Projectile plus Target bremsstrahlung sums to

$$dN^{M,N} \;=\; dN^{N}_{P}(\eta,\mathbf{k}_1;\mathbf{Q}_P) + dN^{M,N}_{T}(\eta,\mathbf{k}_1;\{\mathbf{Q}_a\}) \;=\; \sum_{a=0}^{M} \frac{B_{1a}}{(\mathbf{k}_1+\mathbf{Q}_a)^2+\mu_a^2}\;,$$

$$B_{ia} \equiv F_{k_i,Q_a}\, P_a(\eta_i)$$

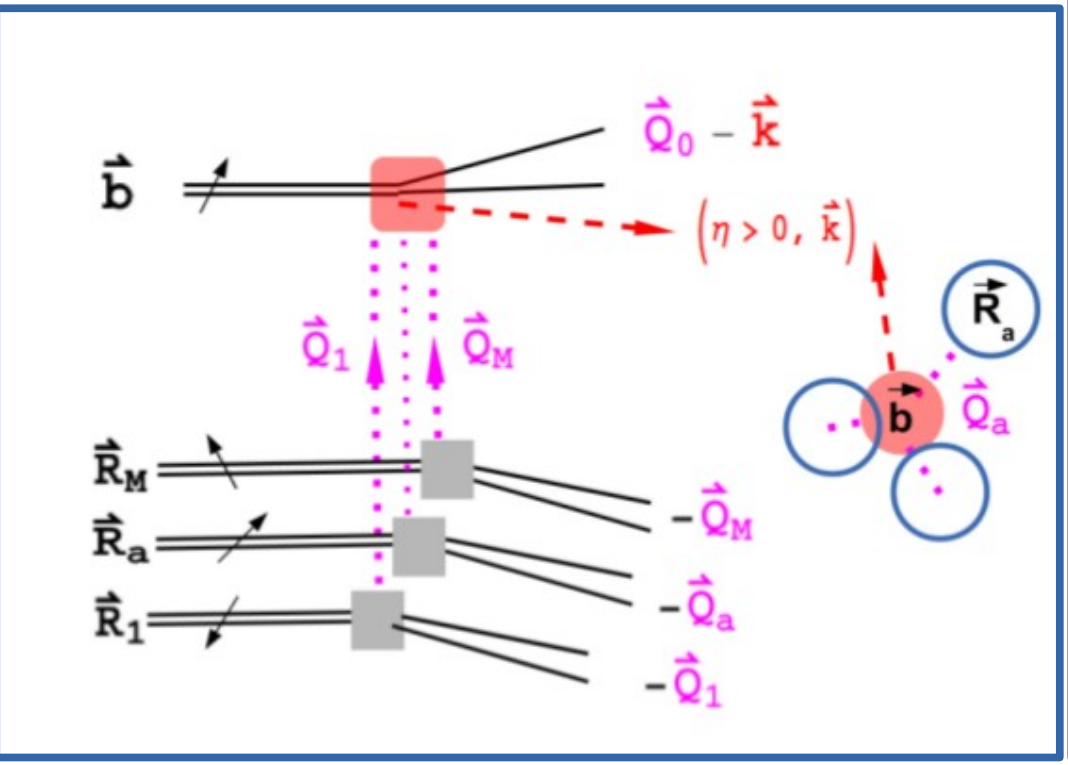
$$\mathbf{Q}_0 \equiv -\mathbf{Q}_P = -\textstyle\sum_a \mathbf{Q}_a$$

2 glue Brems in independent emission approx

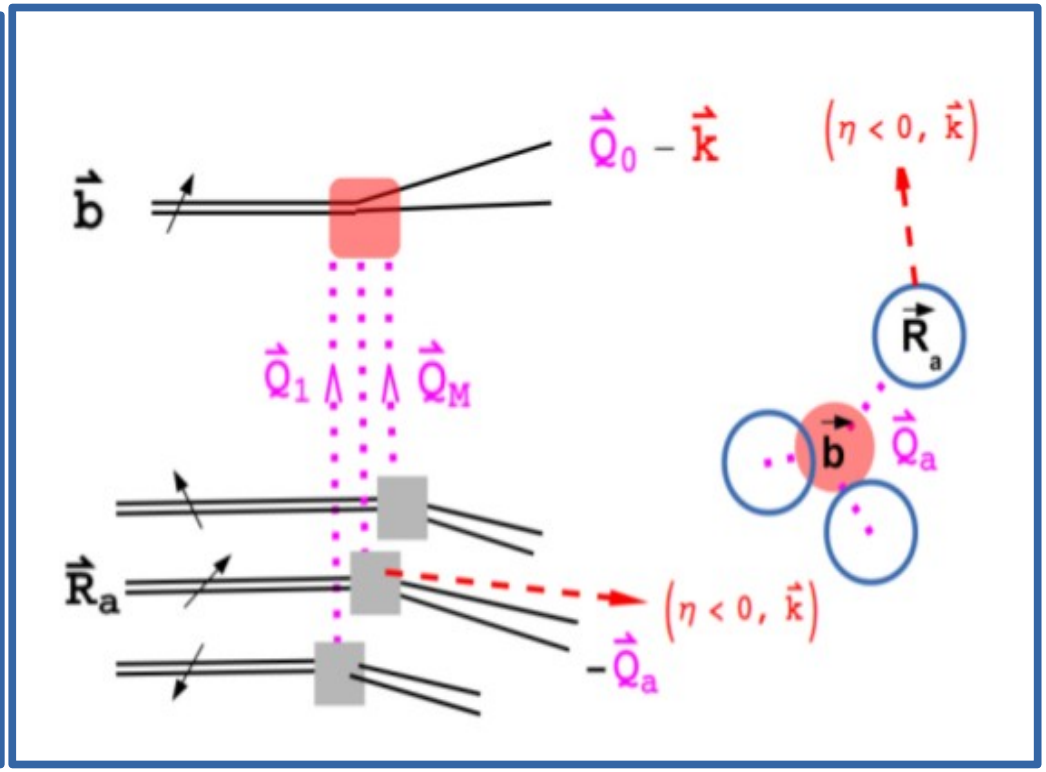
$$dN^{N,M}_2(\mathbf{k}_1,\mathbf{k}_2) \;=\; \sum_{a=0}^{M}\sum_{b=0}^{M} \frac{B_{1a}}{A_{1a}-\cos(\phi_1+\psi_a)} \frac{B_{2b}}{A_{2b}-\cos(\phi_2+\psi_b)}$$

Combined projectile and target beam jets radiate gluons collimated
In azimuthal angles dictated by cluster recoil momenta

Projectile Beam Jet Brems



Participant Recoil Target Beam Jets Brems



Target dipoles act
Coherently if transverse
separation cannot be resolved by
radiated glue

$$R_{ij} \lesssim d(k) = \frac{c}{k}$$

1< M < N coherent target clusters (color density fluctuations) radiate
Bremsstrahlung glue uniformly in rapidity but collimated near cluster recoil momenta

Assuming that M antenna clusters out of the $N = N_T^{\text{part}}(\mathbf{b})$ target participants radiate independently—i.e., assuming that each cluster in the CSA array produces approximately a semiclassical coherent state of gluon radiation with random phase with respect to other clusters (see analogous partially coherent pion interferomentry formalism in Ref. [54]), the even-numbered 2ℓ inclusive

$$dN_{2\ell}^{M}(\eta_1, \mathbf{k}_1, \ldots, \eta_{2\ell}, \mathbf{k}_{2\ell}) = \prod_{i=1}^{2\ell} \left(\sum_{a_i=0}^{M} \frac{B_{k_i a_i}}{A_{k_i a_i} - \cos(\phi_i + \psi_{a_i})} \right),$$

Consider in detail the prototype $M = 1$ VGB antenna case again, but for 2ℓ gluon cumulant nth relative harmonic

$$\begin{aligned}
f_n^{M=1}\{2\ell\} &\equiv \langle e^{+in\{\sum_{i=1}^{\ell}\phi_i\}} e^{-in\{\sum_{j=\ell+1}^{2\ell}\phi_j\}} \rangle f_0^{M=1}\{2\ell\} \\
&= \prod_{i=1}^{\ell} \left(\int \frac{d\phi_i}{2\pi} \frac{B_{k_i Q} e^{+in\phi_i}}{A_{k_i Q} - \cos(\phi_i + \psi_Q)} \right) \prod_{j=\ell+1}^{2\ell} \left(\int \frac{d\phi_j}{2\pi} \frac{B_{k_j Q} e^{-in\phi_j}}{A_{k_j Q} - \cos(\phi_j + \psi_Q)} \right) \\
&= \prod_{i=1}^{\ell} \left(e^{in\psi_Q} (z_{k_i Q})^n f_{0,k_i,Q} \right) \prod_{j=\ell+1}^{2\ell} \left(e^{-in\psi_Q} (z_{k_j Q})^n f_{0,k_j,Q} \right) \\
&= f_0^{M=1}\{2\ell\} \prod_{i=1}^{2\ell} (v_1^{\text{GB}}(k_i, Q)^n.
\end{aligned}$$

Bremms satisfies generalized power scaling of multi gluon nth harmonic cumulants !

GLVB Conclusion: pQCD decoherence (Bremmstrahlung) from multiple
Recoiling clusters of partons (beam jets) naturally lead to multiple
Long range in eta “ridges” with large vn fluctuations event by event

And naturally account for power law scaling of
multi gluon nth harmonic cummulants

The approximate factorization and power scaling of azimuthal harmonics from CSA coherent state non-Abelian bremsstrahlung is similar to “perfect fluid hydrodynamic collective flow” factorization and scaling, but in this case no assumption about local equilibration or minimal viscosity is necessary. This work provides qualitative analytic perturbative QCD Insight into the quantitative success of Glasma/BFKL initial state decoherence explanations of observed pA and DA asymmetries.

Work in progress is to implement beam jet azimuthal asymmetries into HIJING

(see contribution by Angelo Esposito Friday for some further surprises)

The end of this introductory provocation.

Because pA is not Null Control

The theory debates must now continue.

Can we work out an objective strategy to decide

Between Hydro and CGC idealizations without

invoking the dreaded 1885 Weierstrass

“ I can postdict anything ” theorem ?

RBRC Workshop on
Collectivity in Small Colliding Systems
with High Multiplicity

HYDRODYNAMIC NOISE IN SMALL SYSTEMS

TETSUFUMI HIRANO

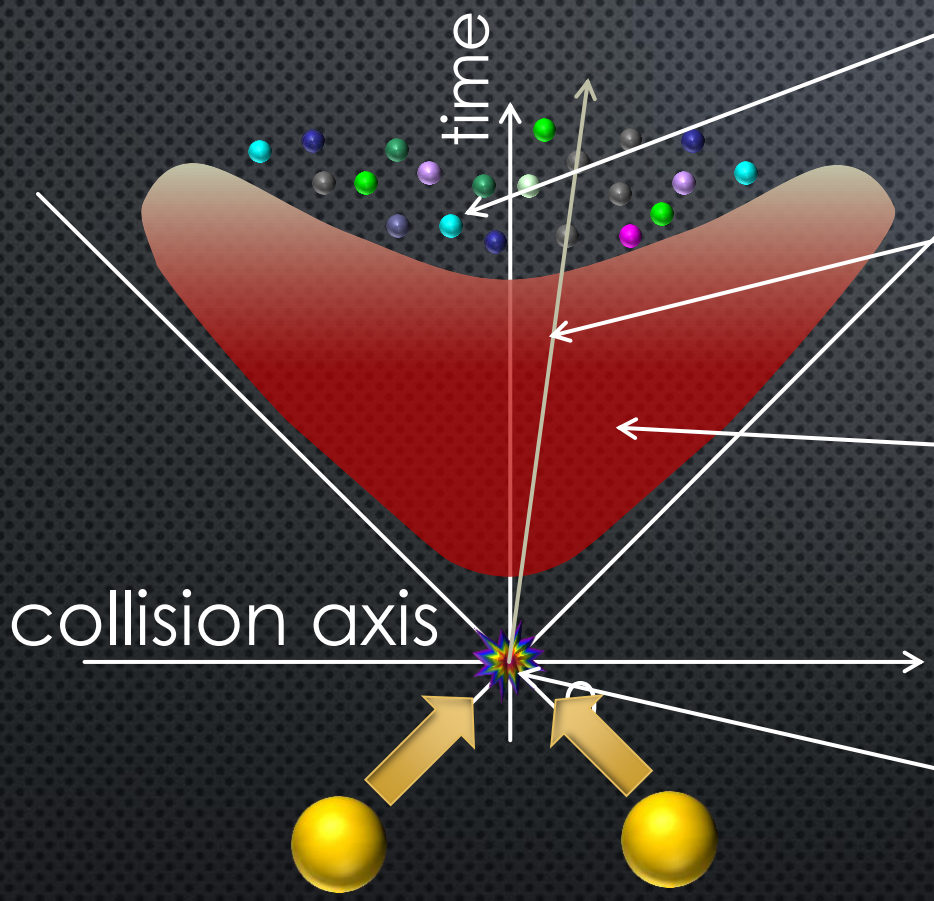
SOPHIA UNIV.

Collaborators: R.Kurita, K.Murase, K.Nagai

Outline

- Introduction
- Relativistic Fluctuating Hydrodynamics
- Fluctuating Hydrodynamics in Bjorken Expansion
- Fluctuation theorem
- Summary

INTRODUCTION



Finite number of hadrons

Propagation of jets

Hydrodynamic fluctuation

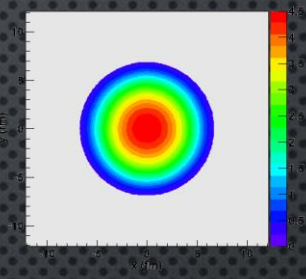
Initial state fluctuation

Fluctuations appear everywhere!
Final hadronic observables ← Whole history

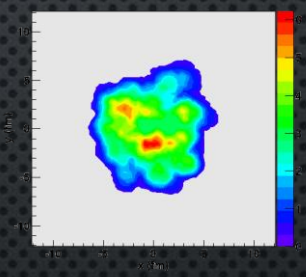
SIZE OF COARSE-GRAINED SYSTEM

- Hydro at work to describe elliptic flow (~ 2001)
- E-by-e hydro at work to describe higher harmonics (~ 2010)
- Hydro at work in p-p and/or p-A??? (2012-)

$d \lesssim 5$ fm



$d \lesssim 1$ fm



$d \lesssim 1$ fm?

Is fluctuation important in such a small system?

THERMAL FLUCTUATION

- Conventional hydrodynamics
  → Space-time evolution of (coarse-grained) thermodynamic quantities
- Microscopic information
  → Lost through coarse-graining process
- Does the lost information play an important role in dynamics in <u>small system</u> and/or on <u>an e-by-e basis</u>?
  → Thermal (Hydrodynamic) fluctuation!

Calzetta, Kapusta, Muller, Stephanov, Young, Moore, Kovtun, Romatschke, Hirano, Murase,…

RELATIVISTIC FLUCTUATING HYDRODYNAMICS

K. Murase and TH, arXiv:1304.3243[nucl-th]

LINEAR RESPONSE RELATION

$$\Pi(x) = \int d^4x' G_R(x, x') F(x')$$

	Dissipative current Π	Thermodynamic force F
Shear	Shear stress tensor	Gradient of flow
Bulk	Bulk pressure	Divergence of flow
Diffusion	Diffusion current	Gradient of μ_B

CAUSAL CONSTITUTIVE EQUATION

Retarded (Causal) response function

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

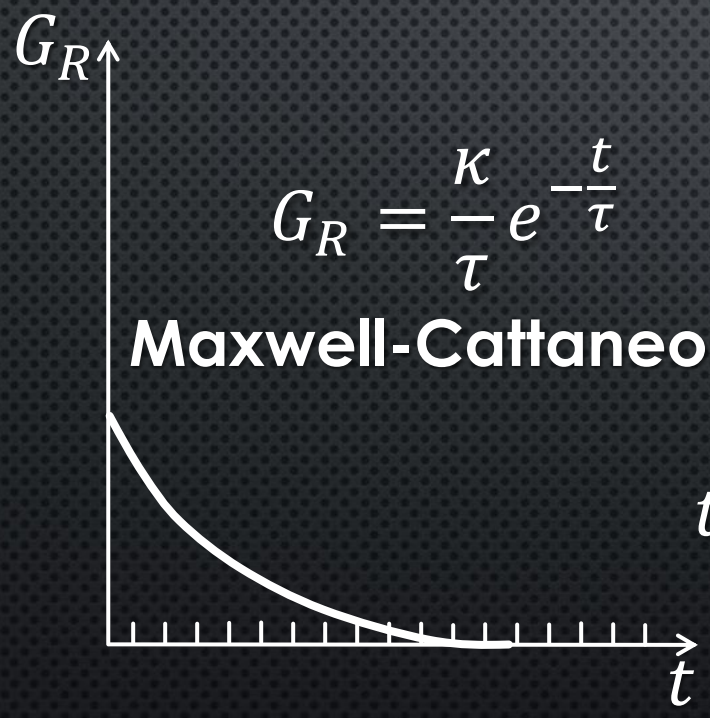
κ: **transport coefficient**

τ: **relaxation time**

$$\dot{\Pi}(t) = -\frac{\Pi(t) - \kappa F(t)}{\tau}$$

COARSE-GRAINING IN TIME

Non-Markovian



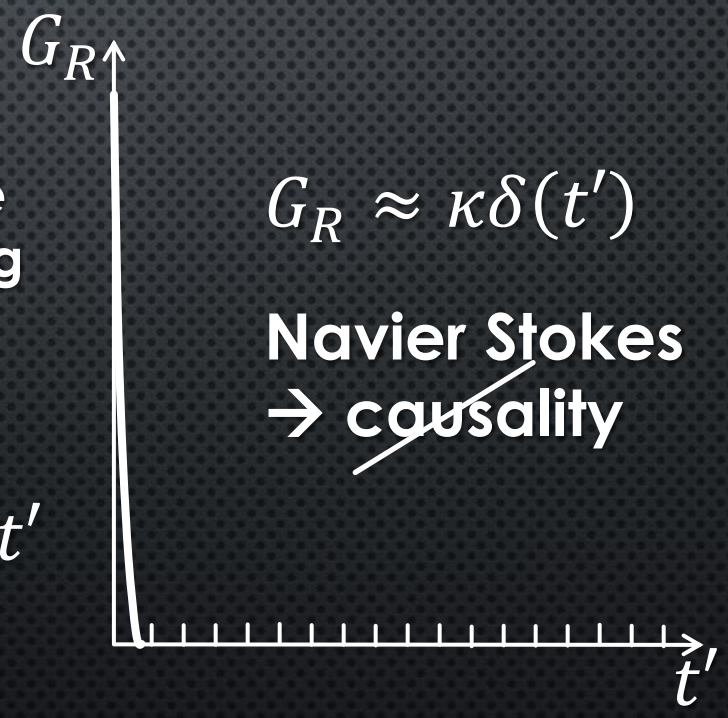
$$G_R = \frac{\kappa}{\tau} e^{-\frac{t}{\tau}}$$

Maxwell-Cattaneo

coarse graining

$$t/\lambda \to t'$$

Markovian



$$G_R \approx \kappa\delta(t')$$

Navier Stokes
→ ~~causality~~

Relaxation → Important in relativistic theory

RELATIVISTIC FLUCTUATING HYDRODYNAMICS (RFH)

- **Generalized Langevin Eq. for currents**

$$\Pi(x) = \int d^4x' G_R(x,x') F(x') + \delta\Pi(x)$$

- **Fluctuation-Dissipation Relation (F.D.R.)**

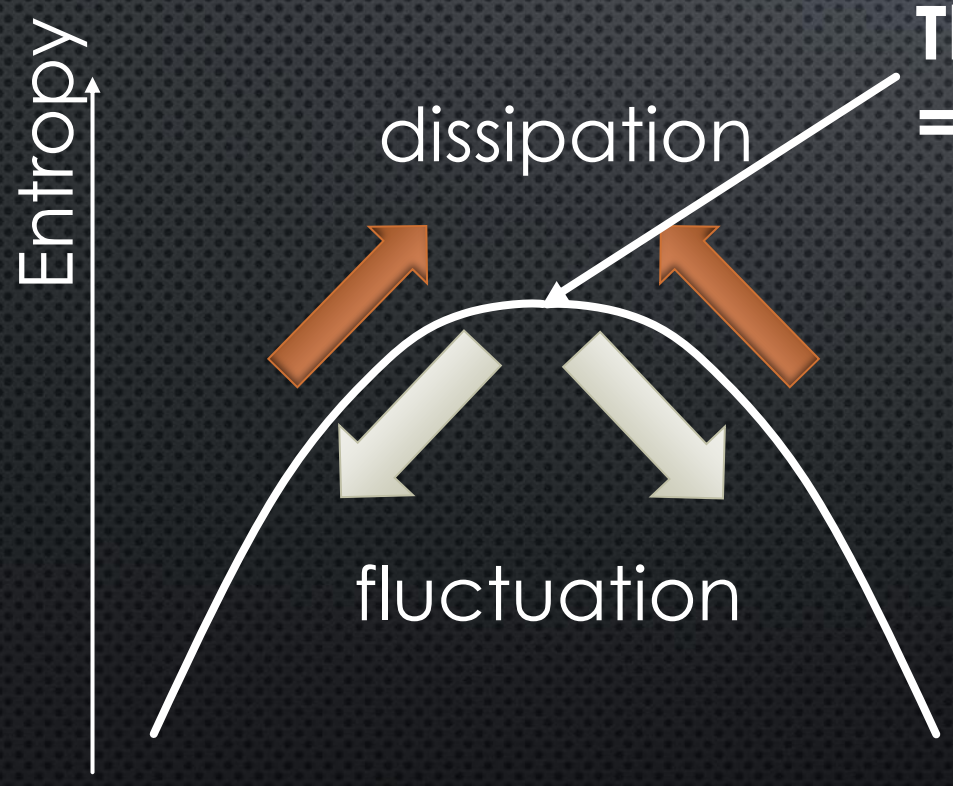
$$\langle \delta\Pi(x)\delta\Pi(x')\rangle = T G^*(x,x')$$

G^*: **Symmetrized correlation function**

$\delta\Pi$: **Hydrodynamic fluctuation**

K. Murase and TH, arXiv:1304.3243[nucl-th]

FLUCTUATION DISSIPATION RELATION



Thermal equilibrium state = Maximum entropy state

Balance between fluctuation and dissipation → Stability of the thermal equilibrium state

COLORED NOISE IN RELATIVISTIC SYSTEM

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

$$\Downarrow$$

$$\left\langle \delta\Pi^*_{\omega,\boldsymbol{k}}\delta\Pi_{\omega',\boldsymbol{k}'}\right\rangle = 2\kappa\frac{(2\pi)^4\delta(\omega-\omega')\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')}{1+\omega^2\tau^2}$$

→ Colored noise!

→ (Indirect) consequence of causality

K. Murase and TH, arXiv:1304.3243[nucl-th]

INTEGRAL FORM VS. DIFFERENTIAL FORM

Integral form

$$\Pi(x) = \int d^4x' G_R(x,x') F(x') + \delta\Pi(x)$$

Differential form

$$\tau \frac{d\Pi(x)}{dt} + \Pi(x) = F(x) + \xi(x)$$

$\xi = \mathcal{L}(d/dt)\delta\Pi$: **white noise**

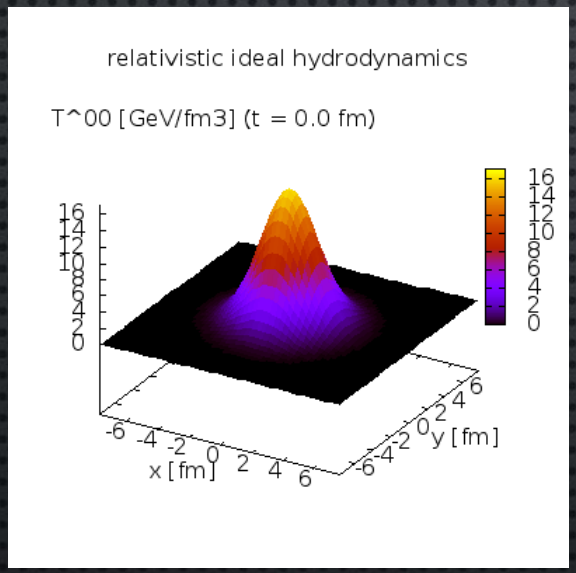
→ **No longer colored noise**

→ **Practically convenient**

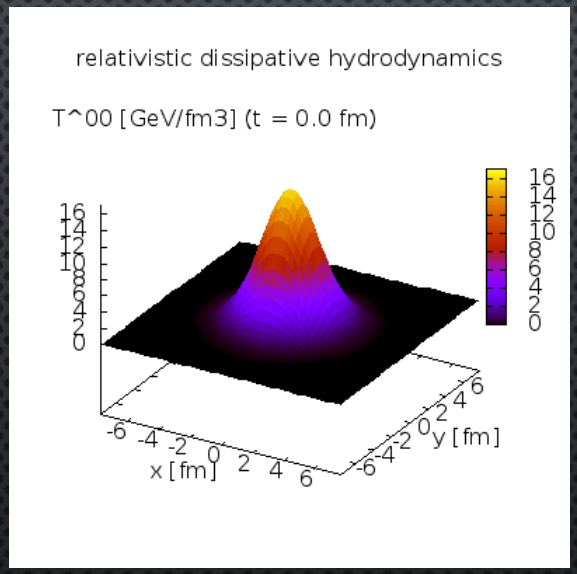
K. Murase and TH, arXiv:1304.3243[nucl-th]

QGP FLUIDS IN A BOX

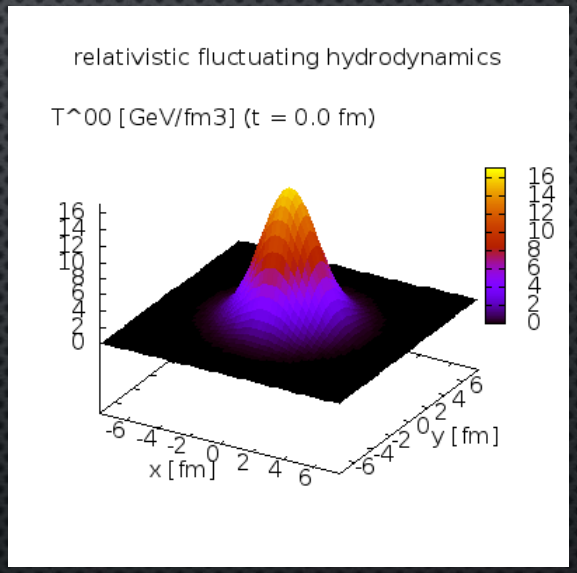
Ideal hydro (1G hydro)



Dissipative hydro (2G hydro)



Fluctuating hydro (3G hydro)

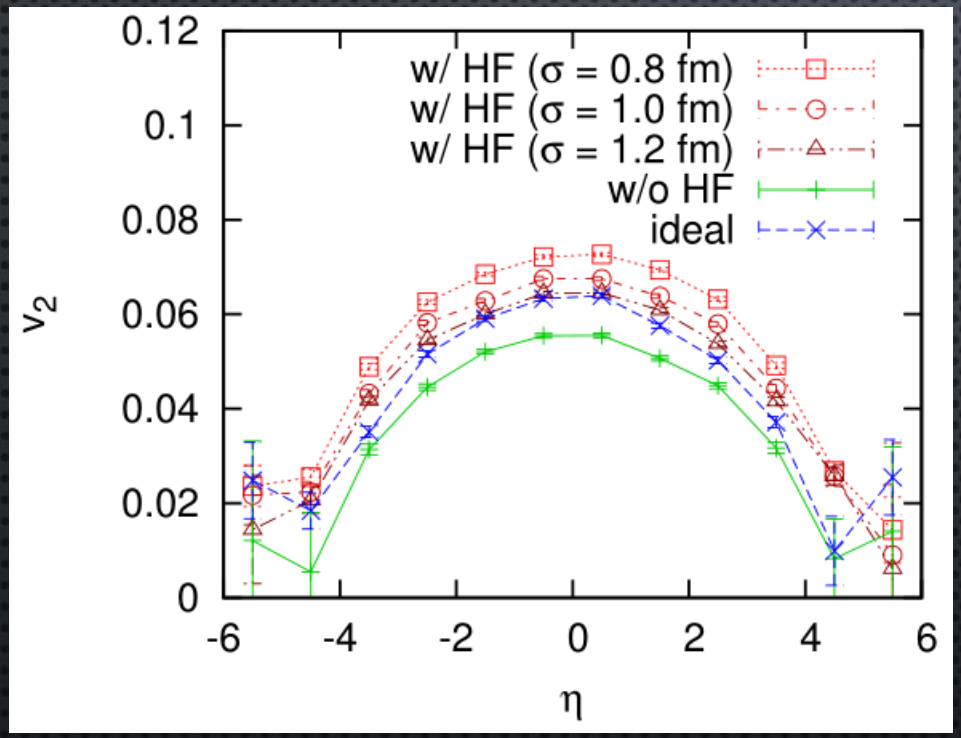


No dissipation
Wave propagation

Towards equilibrium

Fluctuating around mean value

V_2 (ETA) FROM FULL 3D FLUCTUATING HYDRO + HADRONIC AFTERBURNER



v_2 from two particle correlation/cumulant

$$(v_2\{2\})^2 = (v_2)^2 + (\delta v_2)^2$$

σ: Coarse graining size

$$v_2[\text{visc.}] < v_2[\text{ideal}] < v_2[\text{fluc.}]$$ **(!?)**

Fluctuation of v_2

→ Information about thermal fluctuation

FLUCTUATING HYDRODYNAMICS IN BJORKEN EXPANSION

R.Kurita, K.Nagai, K.Murase and TH, in preparation

BJORKEN EXPANSION WITH VISCOSITY AND FLUCTUATION

Equation of motion

$$\frac{de}{d\tau} = -\frac{e + P(e)}{\tau}\left(1 - \frac{\pi - \Pi}{sT}\right)$$

Stochastic constitutive equations

Shear: $$\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

Bulk: $$\tau_\Pi \frac{d\Pi}{d\tau} + \Pi = -\frac{\zeta}{\tau} + \xi_\Pi$$

e : **Energy density**

P : **Pressure**

s : **Entropy density**

π : **Shear stress**

Π : **Bulk pressure**

η : **Shear viscosity**

ζ : **Bulk viscosity**

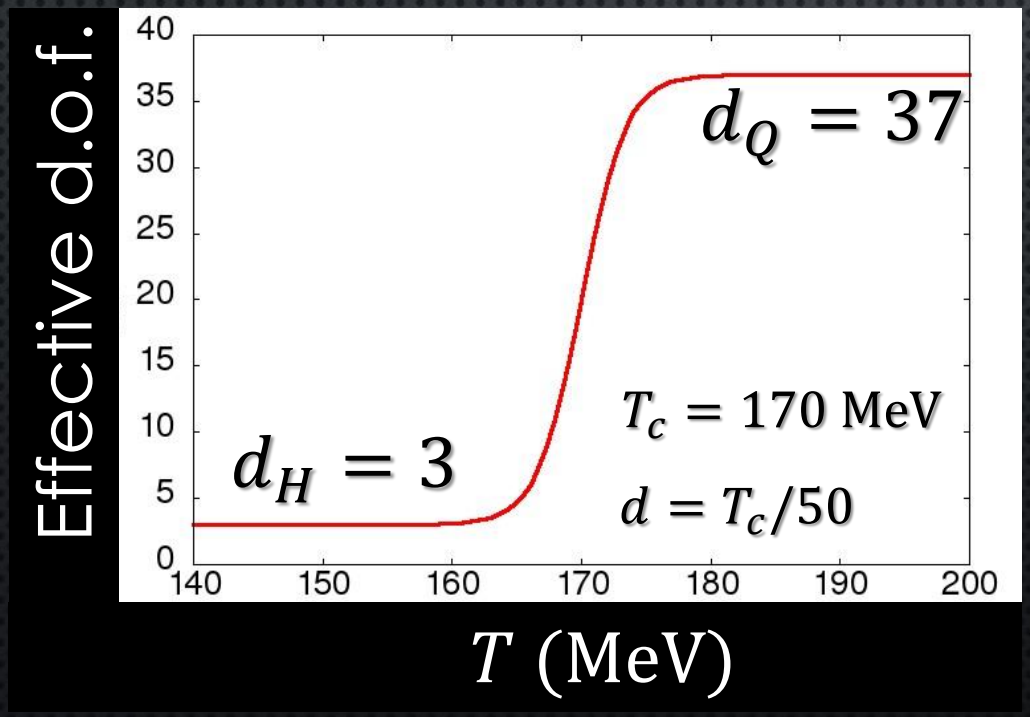
τ_π, τ_Π : **Relaxation time**

Hydrodynamic noises!

MODEL FOR EQUATION OF STATE

Effective degree of freedom

$$d_{\mathrm{eff}} = d_H \frac{1 - \tanh \frac{T - T_c}{d}}{2} + d_Q \frac{1 + \tanh \frac{T - T_c}{d}}{2}$$



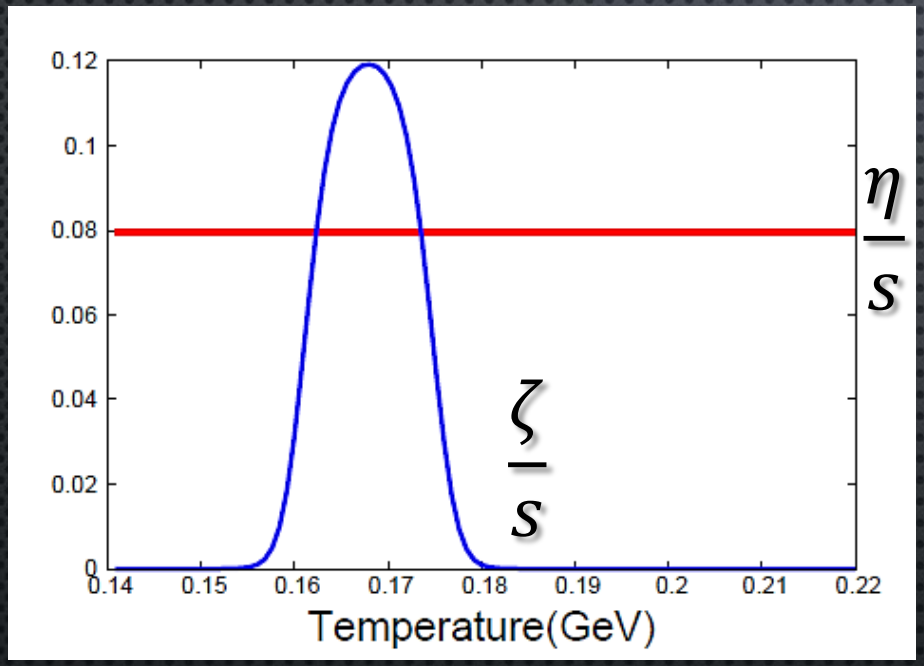
$$s(T) = d_{\mathrm{eff}} \frac{4\pi^2}{90} T^4$$

$$p(T) = \int_0^T s(T')dT'$$

$$e = Ts - p$$

Y.Asakawa, T.Hatsuda,Phys.Rev.D55, 4488 (1997)

MODELS FOR TRANSPORT COEFFICIENTS



Shear viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

P.Kovtun *et al.*, PRL94, 111601 (2005)

Bulk viscosity

$$\frac{\zeta}{s} = 15\left(\frac{1}{3} - c_s^2\right)^2 \frac{\eta}{s}$$

S.Weinberg, Astrophys.J.168, 175 (1971)

Relaxation time $\tau_\pi = \tau_\Pi = \frac{3\eta}{2p}$

Caveat: Just for demonstration!

SOME DETAILS ABOUT NOISES

F.D.R. for shear viscosity

$$\left\langle \xi_\pi{}^{\mu\nu}(x)\xi_\pi{}^{\alpha\beta}(x')\right\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x-x')$$

$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Bjorken expansion case $u^\mu = (\cosh\eta_s, 0, 0, \sinh\eta_s)$

$$\pi = \pi^{00} - \pi^{33} \Longrightarrow \xi_\pi = \xi_\pi^{00} - \xi_\pi^{33}$$

SOME DETAILS ABOUT NOISES (CONTD.)

Need F.D.R. for $\xi_\pi = \xi_\pi^{00} - \xi_\pi^{33}$

$$\langle \xi_\pi(x)\xi_\pi(x')\rangle = \langle \xi_\pi^{00}\xi_\pi^{00}\rangle - 2\langle \xi_\pi^{00}\xi_\pi^{33}\rangle + \langle \xi_\pi^{33}\xi_\pi^{33}\rangle$$

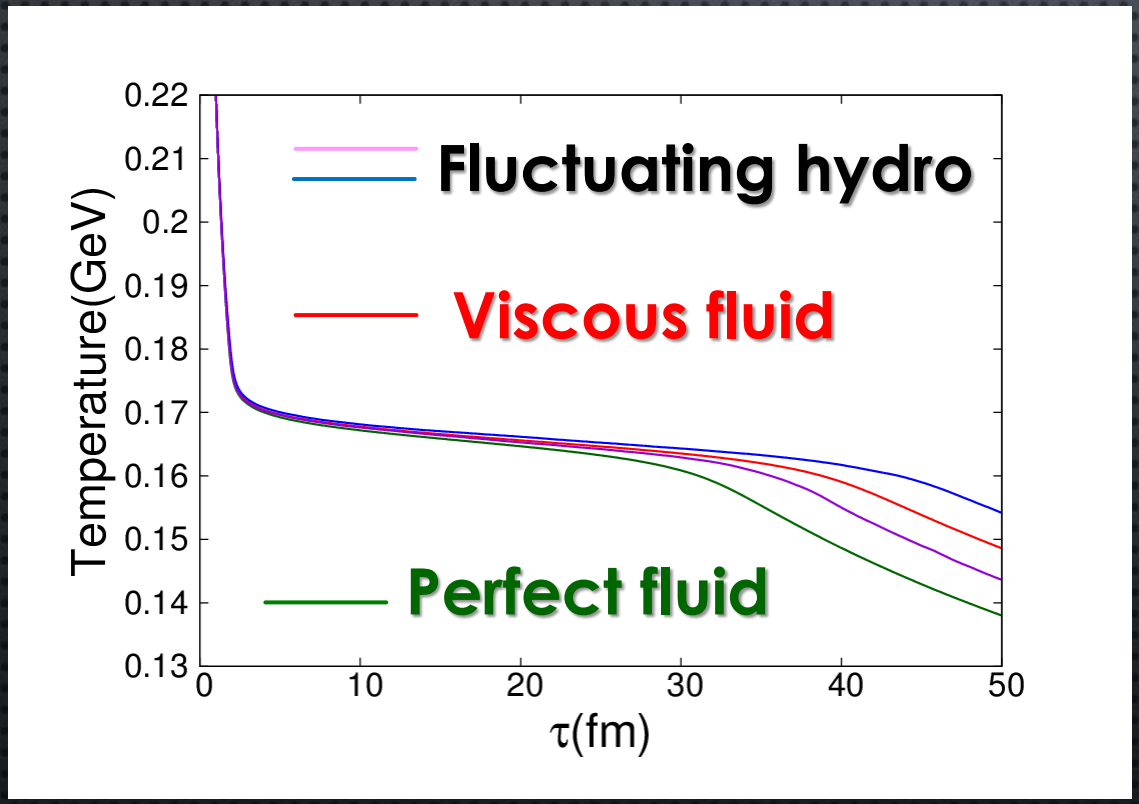
Magnitude of fluctuation in discretized space-time

$$\Longrightarrow \frac{4T\eta}{\Delta\tau\Delta V}(\Delta^{0000} - 2\Delta^{0033} + \Delta^{3333}) = \underline{\frac{8T\eta}{3\Delta\tau\Delta V}}$$

Width of Gaussian white noise → prop. to inverse volume

$$\Delta V = \tau\Delta\eta_s\Delta x\Delta y, \ \Delta\eta_s = 1, \Delta x = \Delta y = 1 \ (\mathrm{fm})$$

TIME EVOLUTION OF TEMPERATURE



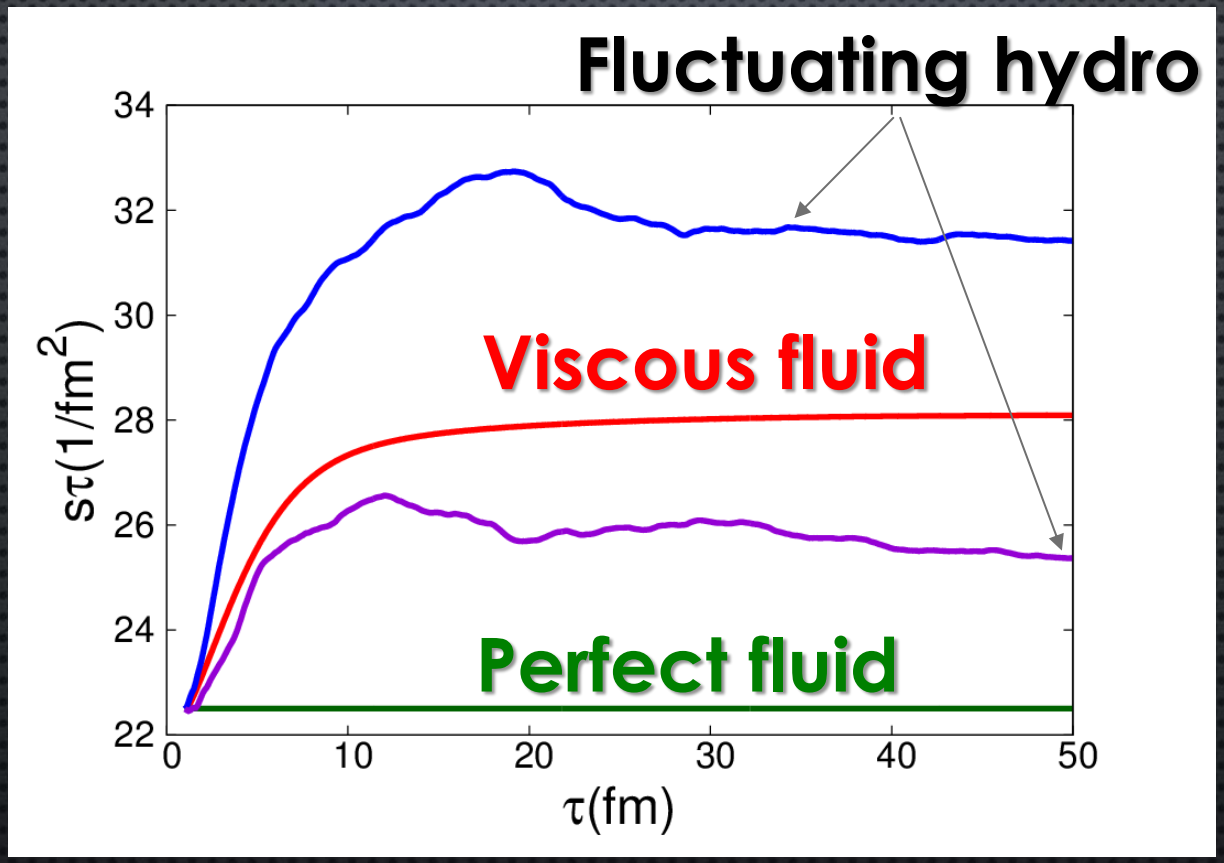
Initial conditions

$\tau_0 = 1\ \mathrm{fm}$

$T_0 = 0.22\ \mathrm{GeV}$

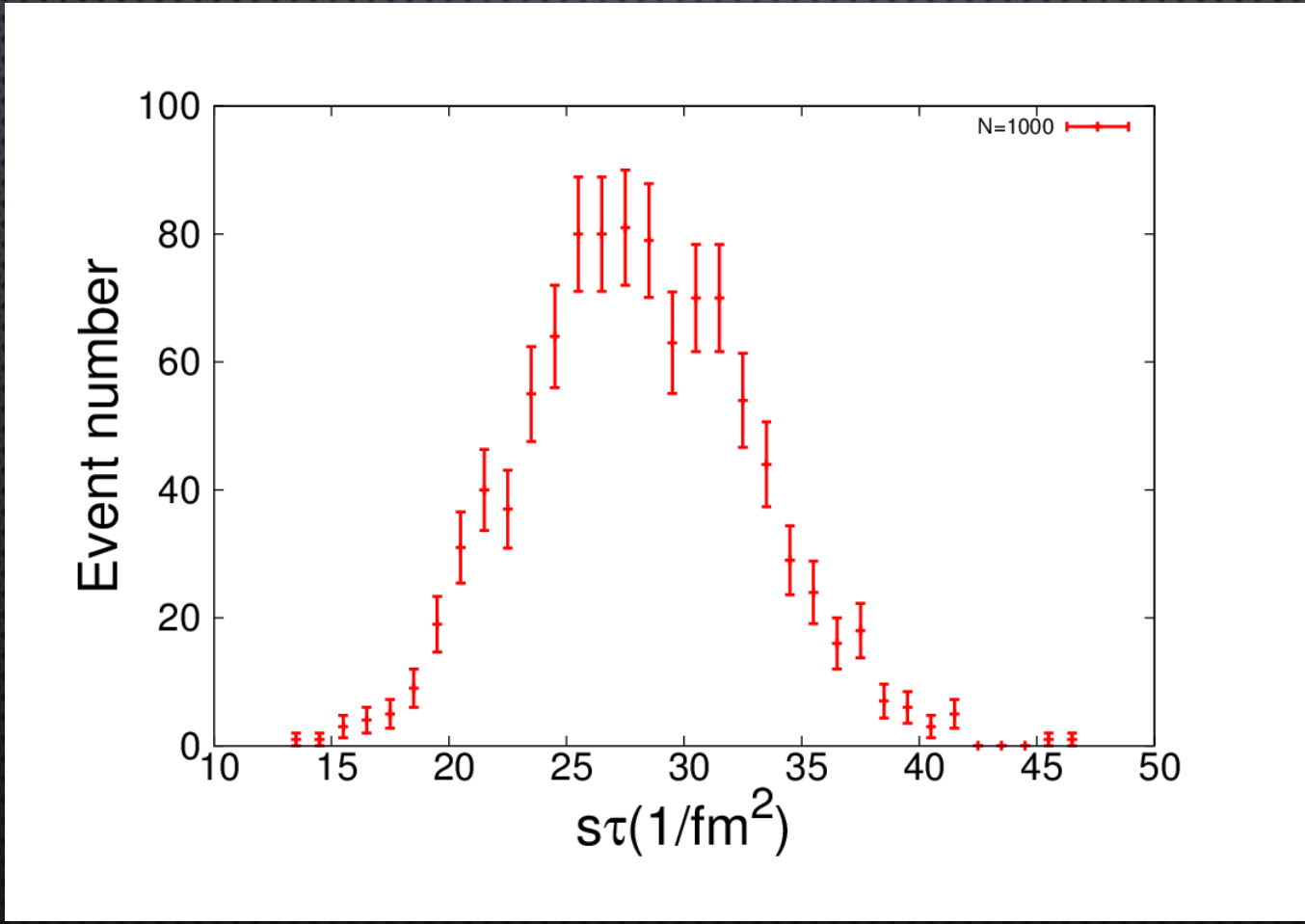
$\pi_0 = \Pi_0 = 0$

TIME EVOLUTION OF ENTROPY



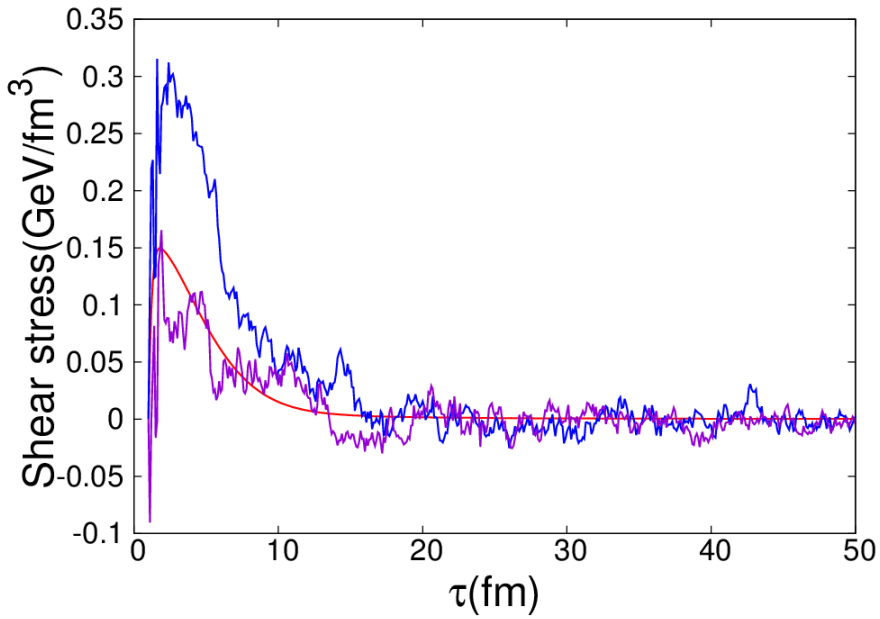
Fixed initial condition
→ Final entropy fluctuation due to hydrodynamic noises

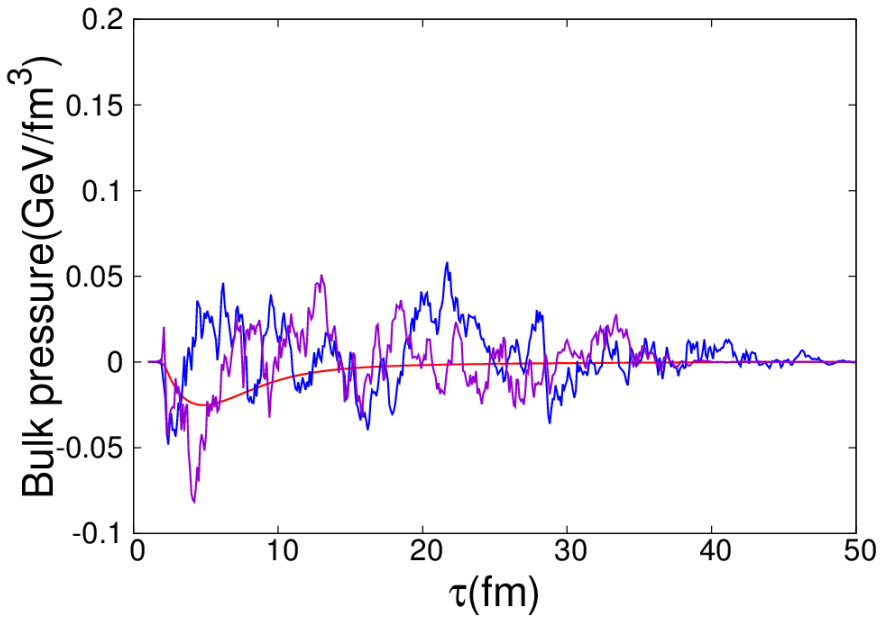
ENTROPY DISTRIBUTION



Fluctuating entropy around mean value

TIME EVOLUTION OF DISSIPATIVE CURRENT





ENTROPY PRODUCTION RATE IN BJORKEN EXPANSION

Fluctuating hydro

$$\frac{d(s\tau)}{d\tau} = \frac{\pi - \Pi}{T} \lessgtr 0$$

Entropy production
→ Not positive definite due to hydrodynamic noises
→ The 2nd law of thermodynamics *on (ensemble) average*

Israel-Stewart formalism

$$\frac{d(s\tau)}{d\tau} = \frac{\tau}{T}\left(\frac{3\pi^2}{4\eta} + \frac{\Pi^2}{\zeta}\right) \geq 0$$

Constitutive equations designed to obey the 2nd law of thermodynamics
→ Non-linear response

FLUCTUATION THEOREM

R.Kurita, K.Nagai, K.Murase and TH, in preparation

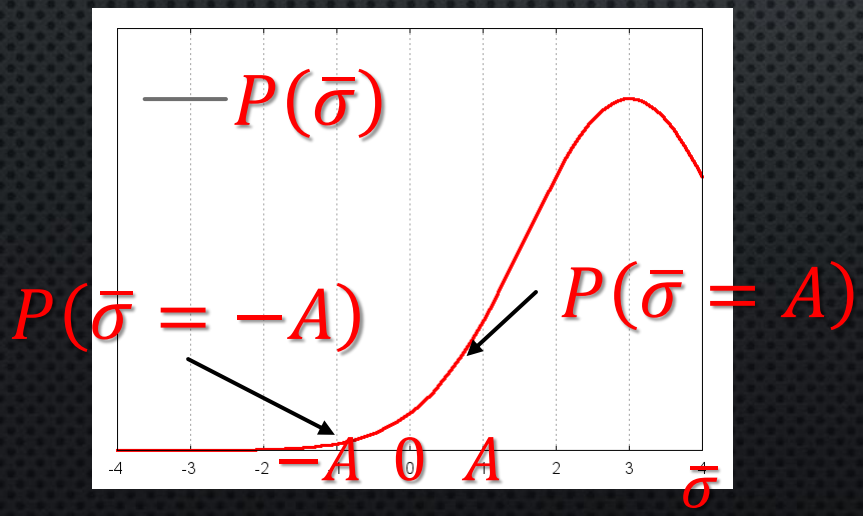
(TRANSIENT) FLUCTUATION THEOREM

$$\frac{P(\bar{\sigma} = A)}{P(\bar{\sigma} = -A)} = e^{At}$$

$$\bar{\sigma}(t) = \frac{1}{t}\int_0^t \sigma(s)ds$$

$P(\bar{\sigma})$: Probability distribution of entropy production rate

Entropy production rate averaged over $0 < s < t$



Probability of negative entropy production → Quantified through fluctuation theorem!

D.J.Evans et al., PRL71,2401(1993); G.Gallavotti and E.G.D.Cohen, PRL74, 2694(1995); D.J.Evans and D.J.Searles, PRE52, 5839(1995).

ENTROPY FLUCTUATION

Consequence of fluctuation theorem in Bjorken expansion

$$\frac{\Delta S_{\mathrm{fin}}}{S_{\mathrm{fin}}} = \frac{\sqrt{2\langle\Delta(\tau s)\rangle}}{\tau_0 s_0 + \langle\Delta(\tau s)\rangle}\frac{1}{\sqrt{\Delta\eta_s\Delta x\Delta y}}$$

$\langle\Delta(\tau s)\rangle$: **Average entropy production per volume of local thermal system**

ENTROPY FLUCTUATION (CONTD.)

The number of independent local thermal system:

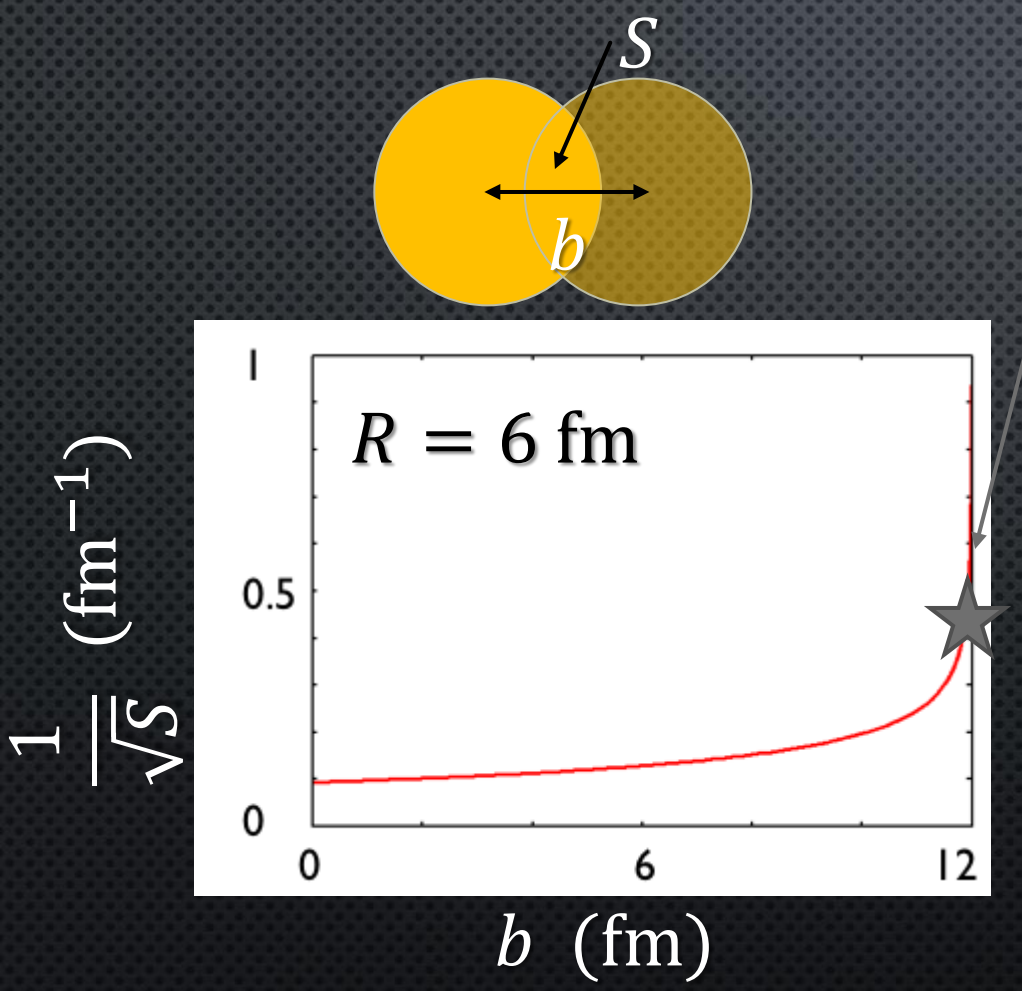
$$N = \frac{S(b)}{\Delta x \Delta y}$$

<u>Theorem</u>: Upper bound of total entropy fluctuation in Bjorken expansion

$$\frac{\Delta S_{\text{tot}}}{S_{\text{tot}}} = \frac{1}{\sqrt{N}} \frac{\Delta S_{\text{fin}}}{S_{\text{fin}}} = \frac{\sqrt{2\langle \Delta(\tau s) \rangle}}{\tau_0 s_0 + \langle \Delta(\tau s) \rangle} \frac{1}{\sqrt{\Delta \eta_s S(b)}}$$

$$\leq \frac{1}{\sqrt{2\tau_0 s_0}} \frac{1}{\sqrt{\Delta \eta_s S(b)}} = \frac{1}{\sqrt{2S_{\text{ini}}}}$$

CENTRALITY DEPENDENCE OF ENTROPY FLUCTUATION



p-p collision

$\sigma_{\text{in}} = 61$ mb at LHC

$\sigma_{\text{in}} = 42$ mb at RHIC

$$\frac{1}{\sqrt{\sigma_{\text{in}}}} \sim 0.4 - 0.5$$

Enhancement of fluctuation effects in small system

SUMMARY AND OUTLOOK

- Formulation of relativistic fluctuating hydrodynamics for consistency to describe viscous fluid evolution on an event-by-event basis
- Entropy fluctuation from hydro noise
- Fluctuation theorem in the context of heavy ion collisions
- Themal fluctuation in small system

GREEN-KUBO FORMULA

$$\eta = \lim_{\omega \to 0} \lim_{q \to 0} \frac{1}{2\omega} \int dt dx \, e^{i(\omega t - qx)}$$

$$\times \left\langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \right\rangle$$

Slow dynamics → How slow?

- **Macroscopic time scale ~** $1/\omega \leftarrow t_{\text{"macro"}}$
- **Microscopic time scale** τ

cf.) Long tail problem (liquid in 2D, glassy system, super-cooling, etc.)

RELAXATION AND CAUSALITY

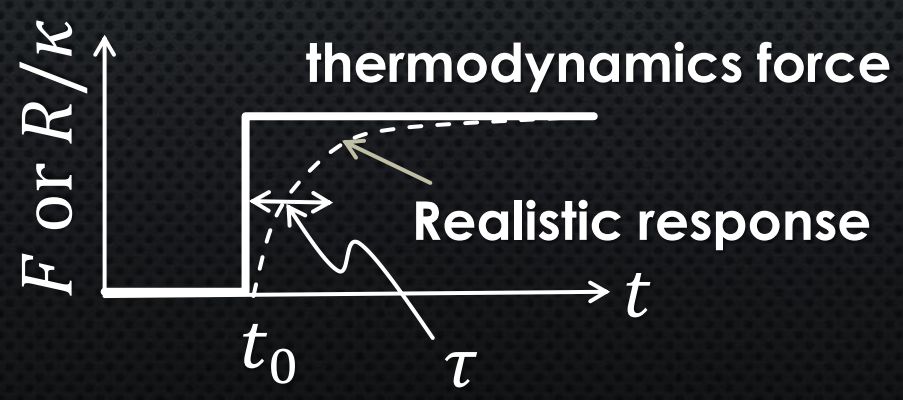
Constitutive equations at Navier-Stokes level

$$\pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$
$$\Pi = -\varsigma\partial_\mu u^\mu,$$
$$\ldots$$

Instantaneous response violates causality

→ **Critical issue in relativistic theory**

→ **Relaxation plays an essential role**



CAUSAL HYDRODYNAMICS

Linear response to thermodynamic force

$$\Pi(t) = \int dt' G_R(t,t') F(t')$$

Retarded Green function (as an <u>example</u>)

$$G_R(t,t') = \frac{\kappa}{\tau} \exp\left(-\frac{t-t'}{\tau}\right) \theta(t-t')$$

Differential form

$$\dot{\Pi}(t) = -\frac{\Pi(t) - \kappa F(t)}{\tau}, \qquad v_{\text{signal}} = \sqrt{\frac{\kappa}{\tau}} < c$$

➡ **Maxwell-Cattaneo Eq. (simplified Israel-Stewart Eq.)**

DERIVATION OF THEOREM

Suppose entropy production rate obeys Gaussian

$$P(\bar{\sigma}) = \frac{1}{\sqrt{2\pi a^2}} \exp\left[-\frac{(\bar{\sigma} - \langle\bar{\sigma}\rangle)^2}{2a^2}\right]$$

From fluctuation theorem,

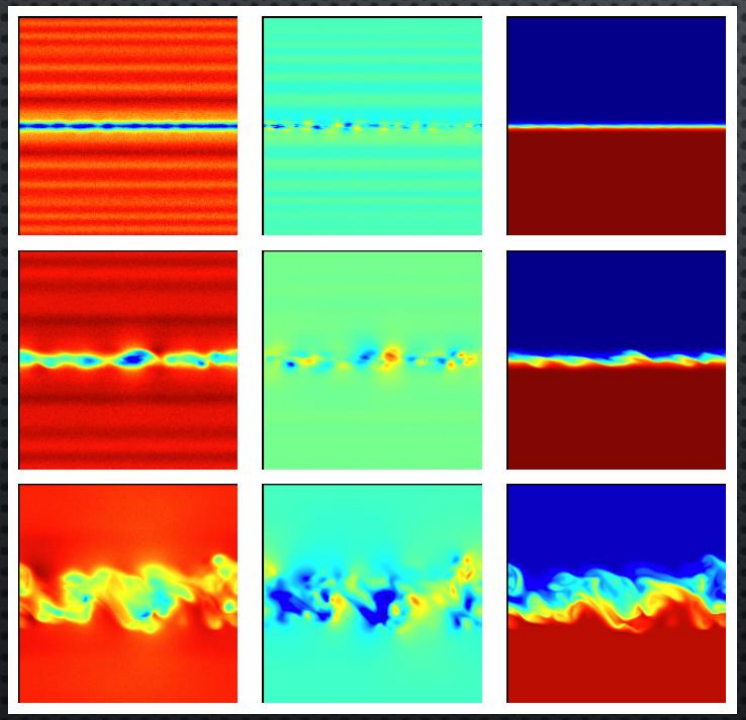
$$\frac{2\langle\bar{\sigma}\rangle}{a^2} = t \implies ta = \sqrt{2\langle\bar{\sigma}\rangle t}$$

$$P(S_{\mathrm{fin}}) \propto \exp\left[-\frac{(S_{\mathrm{fin}} - \langle S_{\mathrm{fin}}\rangle)^2}{2\left(\sqrt{2\langle\bar{\sigma}\rangle t}\right)^2}\right]$$

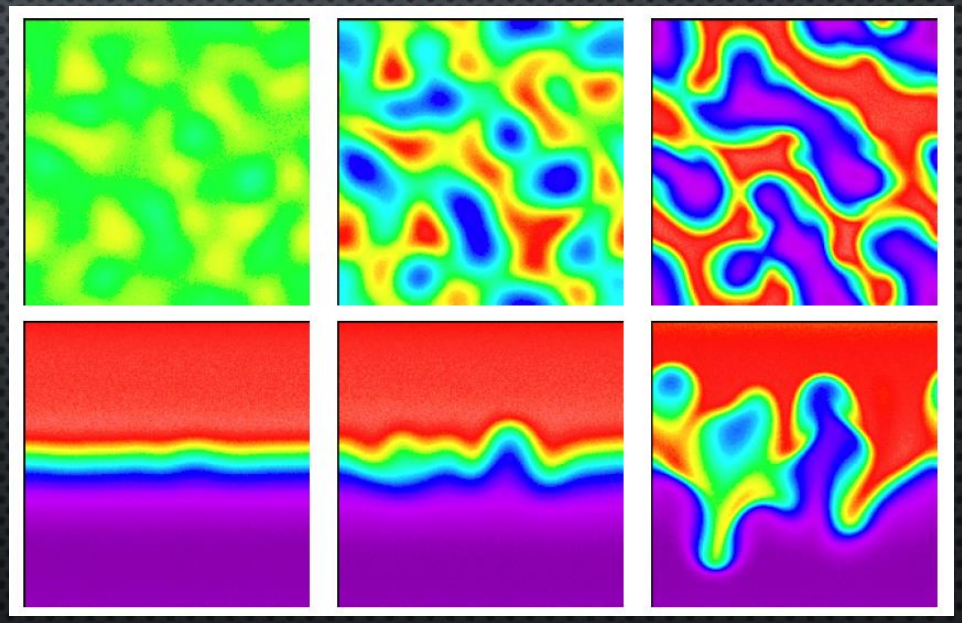
Figures from J.B.Bell *et al.*, ESAIM 44-5 (2010)1085

NEED FLUCTUATION?

Ex.) Seeds for instabilities



Kelvin-Helmhortz instability



Rayleigh-Taylor instability

Non-linearity, instability, dynamic critical phenomena,…

DISCRETIZATION

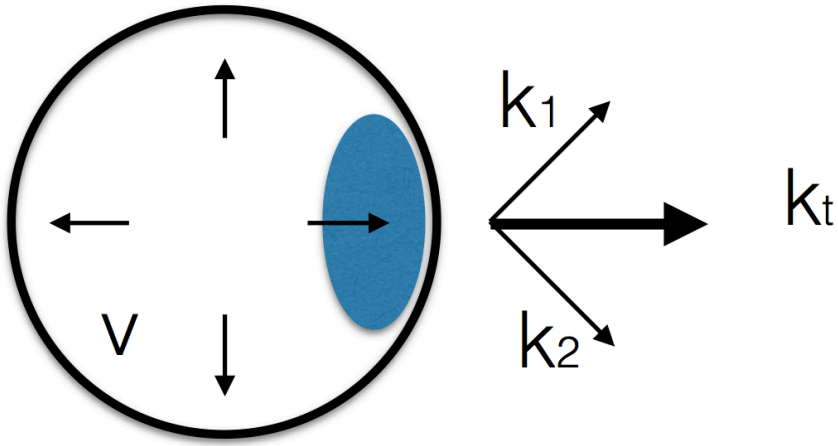
$$\tau_\pi \frac{\Delta\pi}{\Delta\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

$$\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi$$

$$\frac{\Delta\pi}{\Delta\tau} = -\frac{\pi}{\tau_\pi} + \frac{4\eta}{3\tau_\pi\tau} + \frac{\xi_\pi}{\tau_\pi}$$

Discretized

$$\pi(\tau + \Delta\tau) = \pi(\tau) + \left(-\frac{\pi}{\tau_\pi} + \frac{4\eta}{3\tau_\pi\tau} + \frac{\xi_\pi}{\tau_\pi}\right)\Delta\tau$$



Femtoscopic signature of strong radial flow in high-multiplicity pp collisions [arXiv:1412.0063]

Yuji Hirono [Stony Brook Univ.]

In collaboration with E. Shuryak [SBU]

"Collectivity in Small Colliding Systems with High Multiplicity"

Radial flow in **pp collisions** ?

HBT radii

Outline

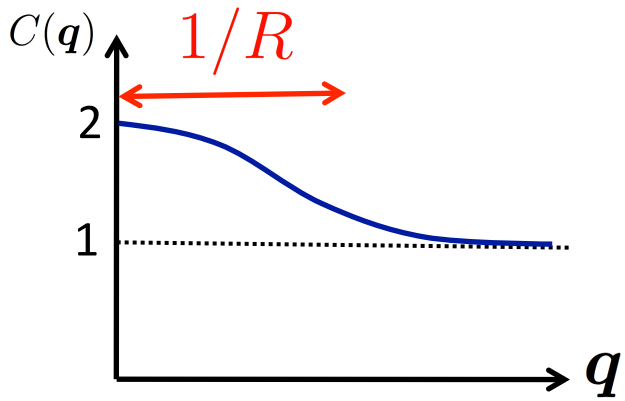
- Introduction
  - HBT interferometry
  - Data from ALICE
- Method
  - Gubser flow + lattice EOS
- Results

Hanbury-Brown Twiss interferometry

$$C\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right) = \frac{P(\boldsymbol{p}_1, \boldsymbol{p}_2)}{P(\boldsymbol{p}_1)P(\boldsymbol{p}_2)}$$

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = C(\boldsymbol{k}, \boldsymbol{q})$$

$$\boldsymbol{k} \equiv \boldsymbol{p}_1 + \boldsymbol{p}_2 \quad \boldsymbol{q} \equiv \boldsymbol{p}_1 - \boldsymbol{p}_2$$

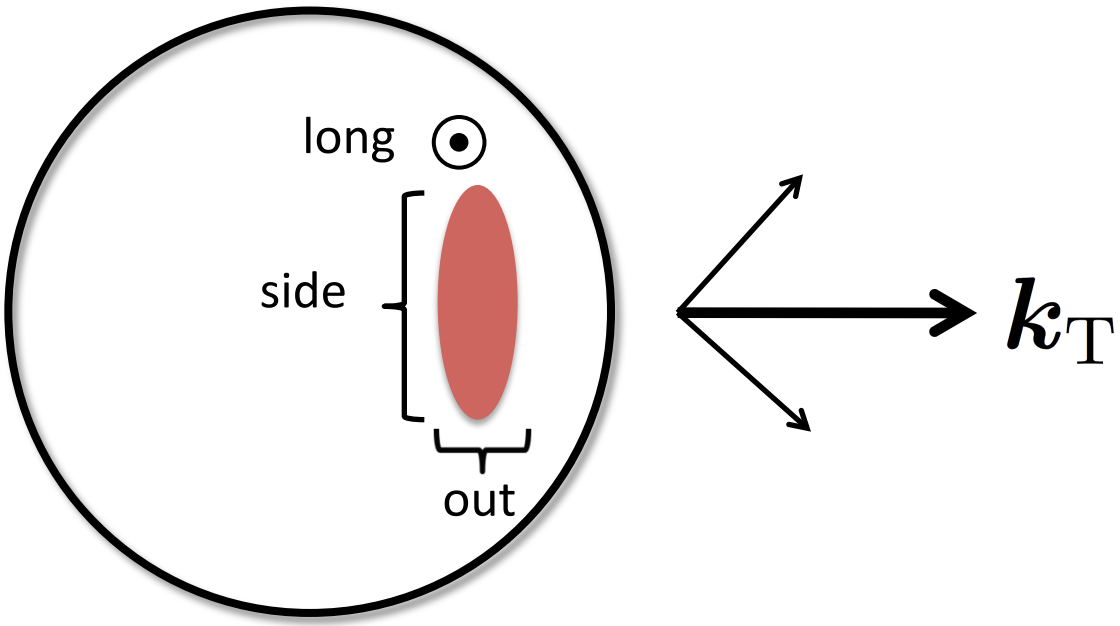


out-side-long parametrization

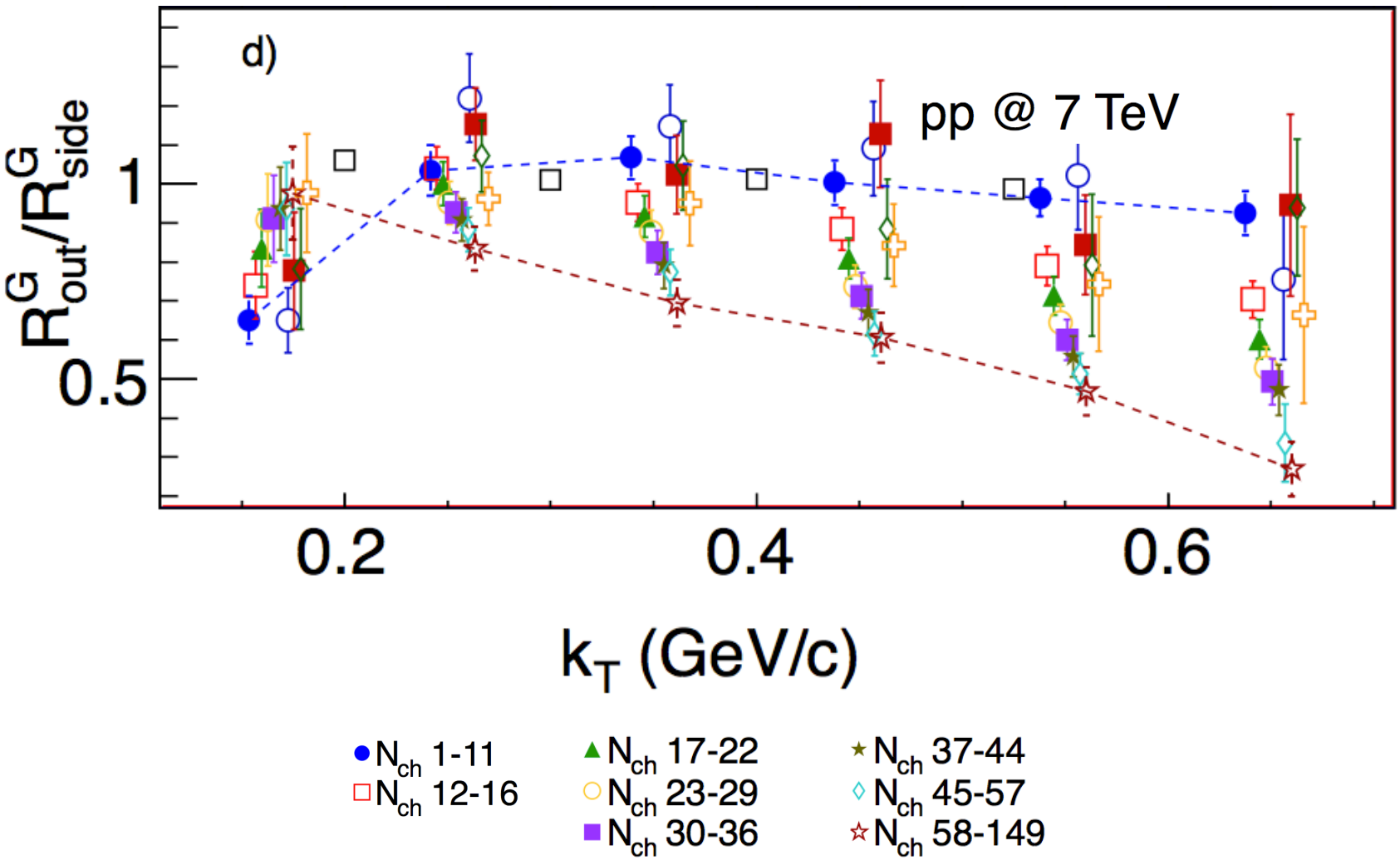
$$C(k_{\mathrm{T}}, \boldsymbol{q}) = 1 + \lambda \exp\left[-R_o^2 q_o^2 - R_s^2 q_s^2 - R_\ell^2 q_\ell^2\right]$$

$$R_{o,s,\ell} = R_{o,s,\ell}(k_{\mathrm{T}})$$

[Pratt (1986)]
[Bertsch, Gong, Tohyama (1988)]



out/side ratio @pp from ALICE (2011)



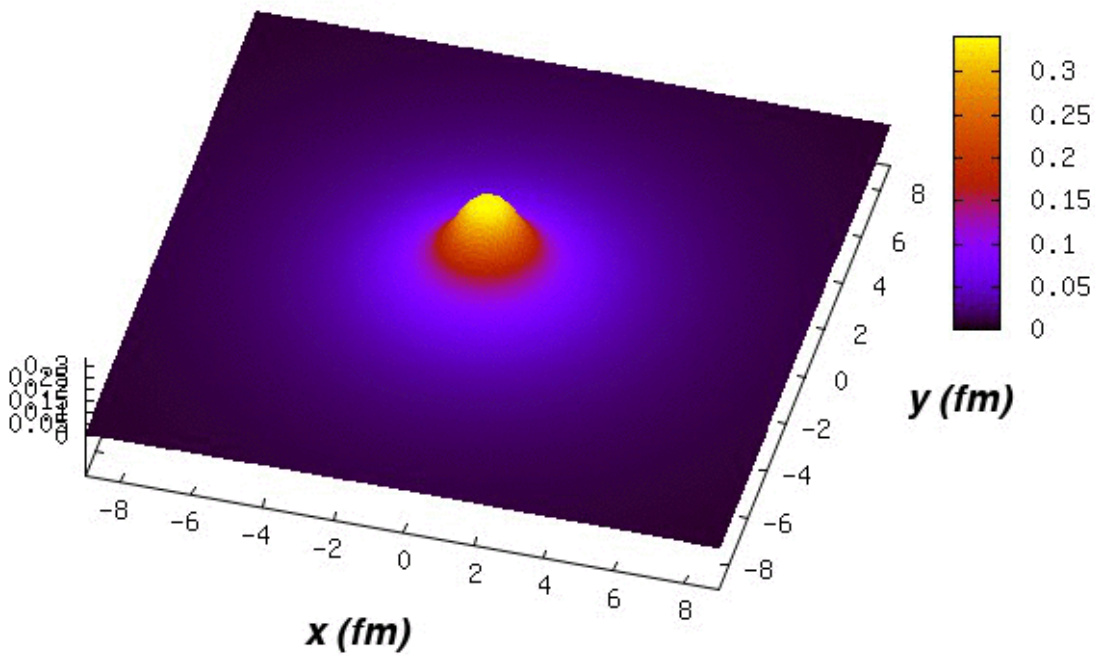
Radial flow in pp?

- We use hydro
- We have to parametrize the strength of radial flow
  - Axisymmetric & boost invariant

Radial flow in pp?

- We use hydro
- We have to parametrize the strength of radial flow
  - Axisymmetric & boost invariant

Gubser flow



Gubser flow [Gubser (2010)]

Parameters:

$$q \ [\mathrm{fm}^{-1}] \qquad \epsilon_0$$

fixed by multiplicity

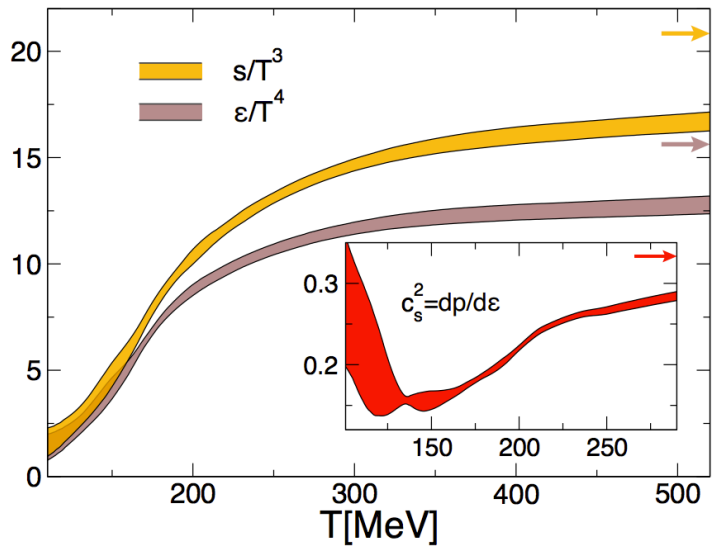
$$\epsilon(\tau, r) = \frac{\epsilon_0 (2q)^{8/3}}{\tau^{4/3}[1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2]^{4/3}}$$

$$v_\perp(\tau, r) = \frac{2q^2 \tau r}{1 + q^2\tau^2 + q^2 r^2}$$

Equation of state from lattice

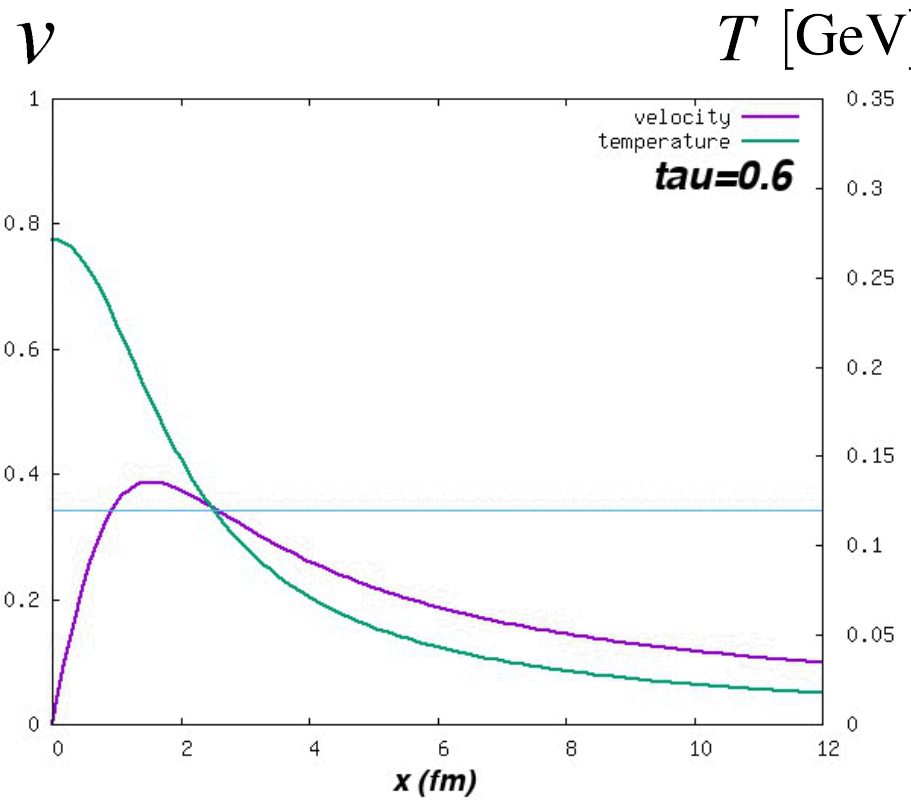
Gubser solutions are for conformal EOS $e = 3p$

Evolved with lattice EOS after 0.6 fm



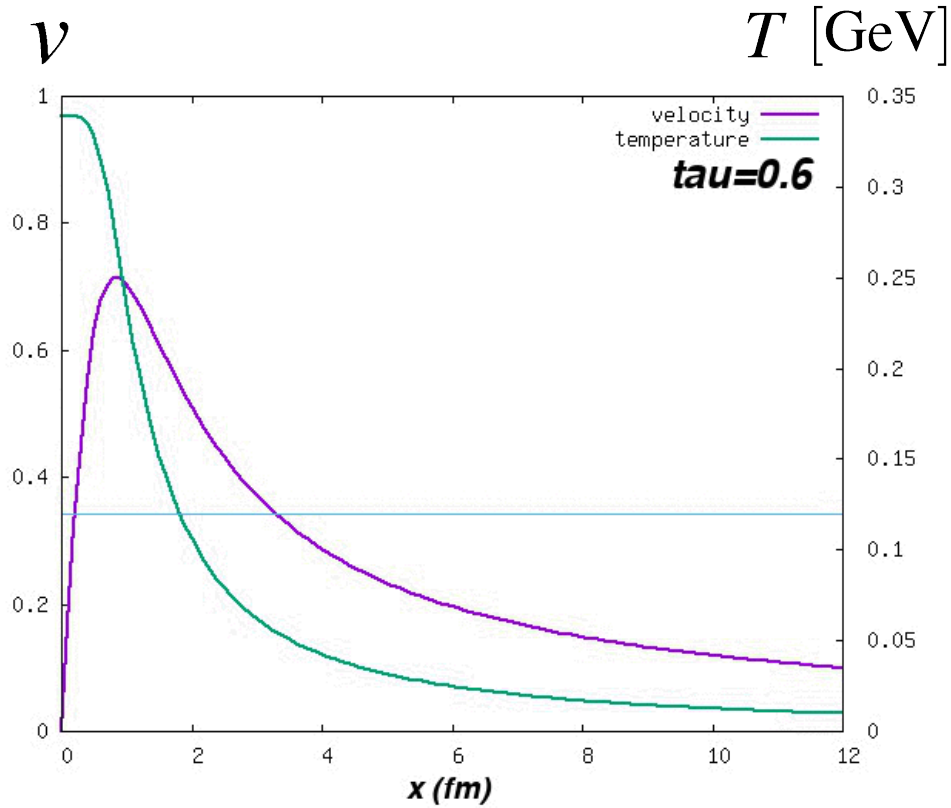
[Wuppertal-Budapest coll.(2014)]

q indicates the strength of radial flow



q=0.7/fm

Small radial flow



q=1.7/fm

Large radial flow

Two-particle Correlation function

Freezeout surface $\longrightarrow$ MC sampling of particles

Two-particle correlation

$$C(k_{\mathrm{T}}, \boldsymbol{q}) = \frac{\sum_{<i,j>\in[k_{\mathrm{T}}]} \left[1 + \cos\left(q_\mu \Delta x^\mu\right)\right]}{\sum_{<i,j>\in[k_{\mathrm{T}}]} 1}$$

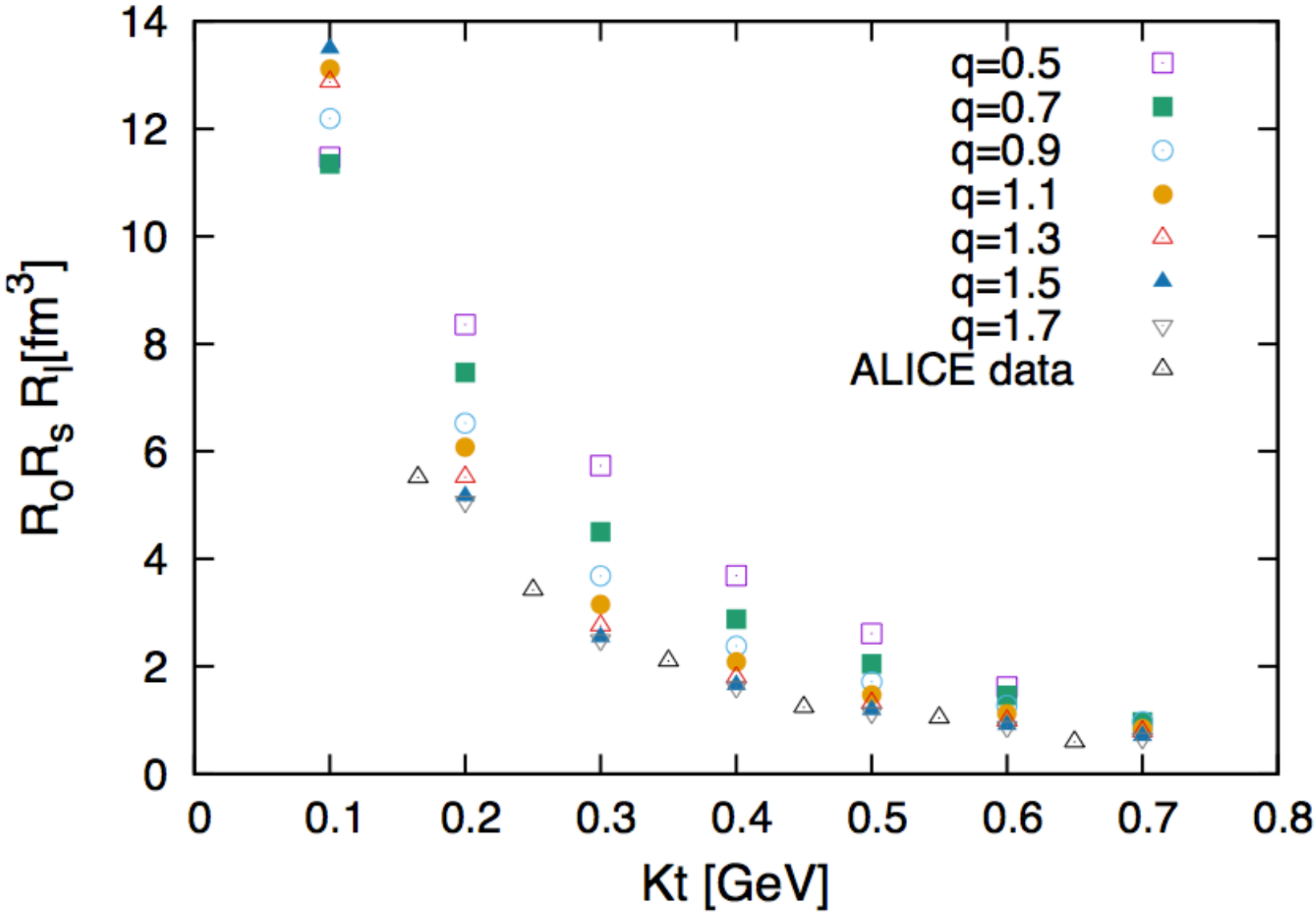
$$\Delta x^\mu \equiv x_1^\mu - x_2^\mu$$

- evaluated in the longitudinally comoving system

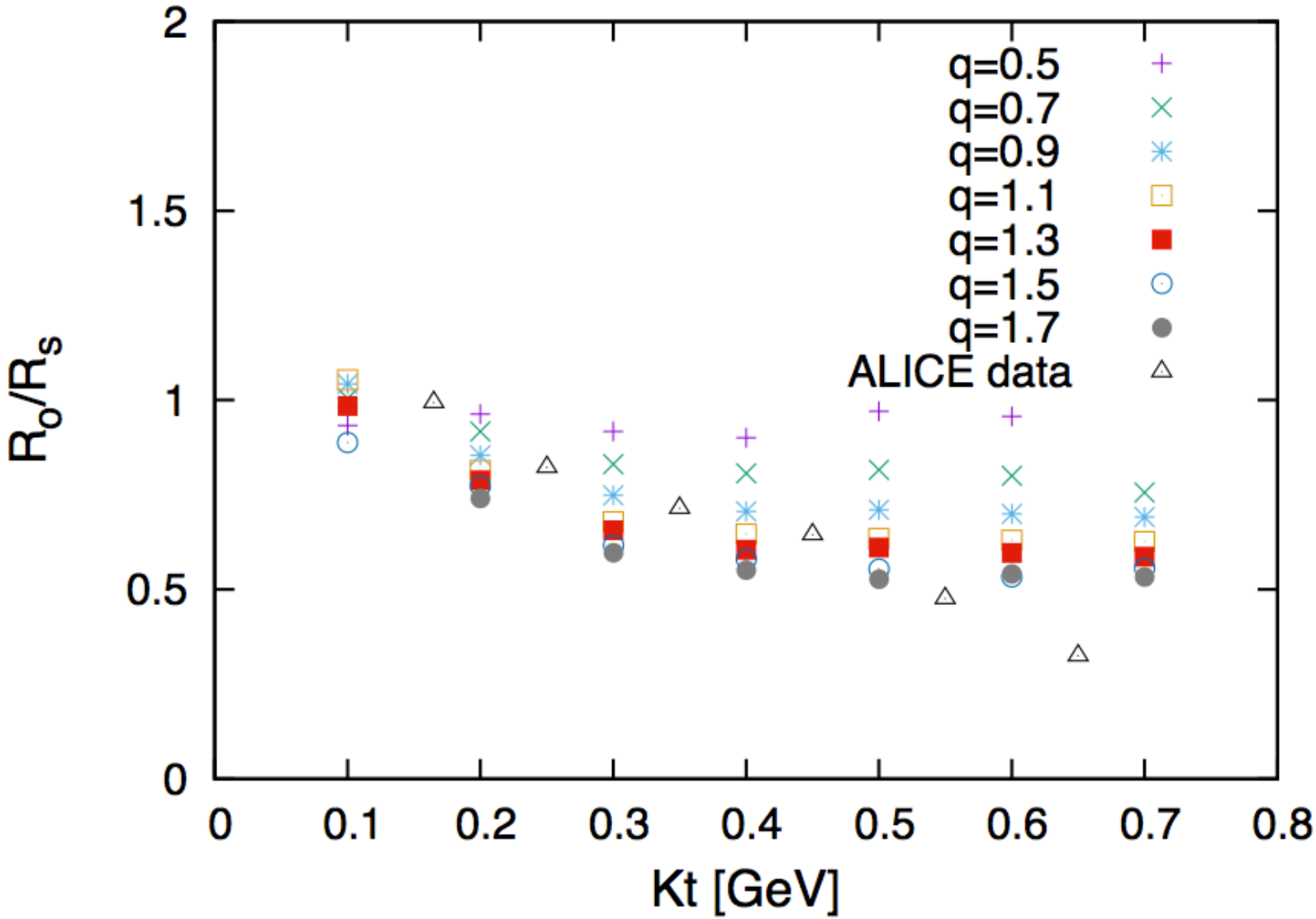
Radii are determined by 3D fitting

$$C(k_{\mathrm{T}}, \boldsymbol{q}) = 1 + \lambda \exp\left[-R_o^2 q_o^2 - R_s^2 q_s^2 - R_\ell^2 q_\ell^2\right]$$

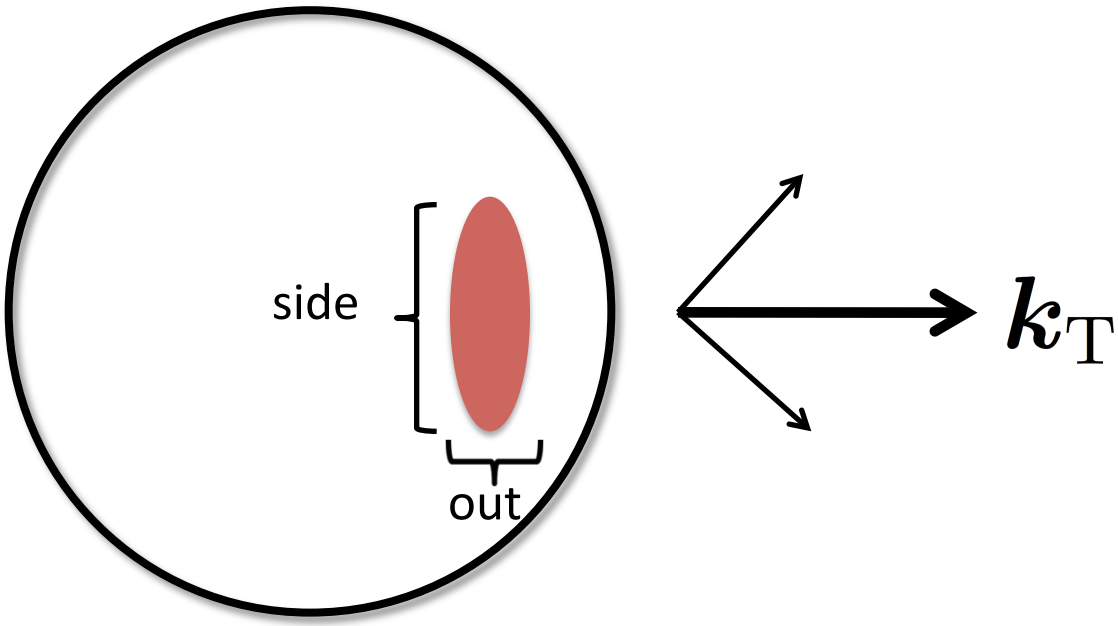
HBT volume


14
12
10
8
6
4
2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$R_o R_s R_l [fm^3]$
Kt [GeV]
q=0.5
q=0.7
q=0.9
q=1.1
q=1.3
q=1.5
q=1.7
ALICE data

out/side ratio



Why out/side ratio is small for large radial flow?

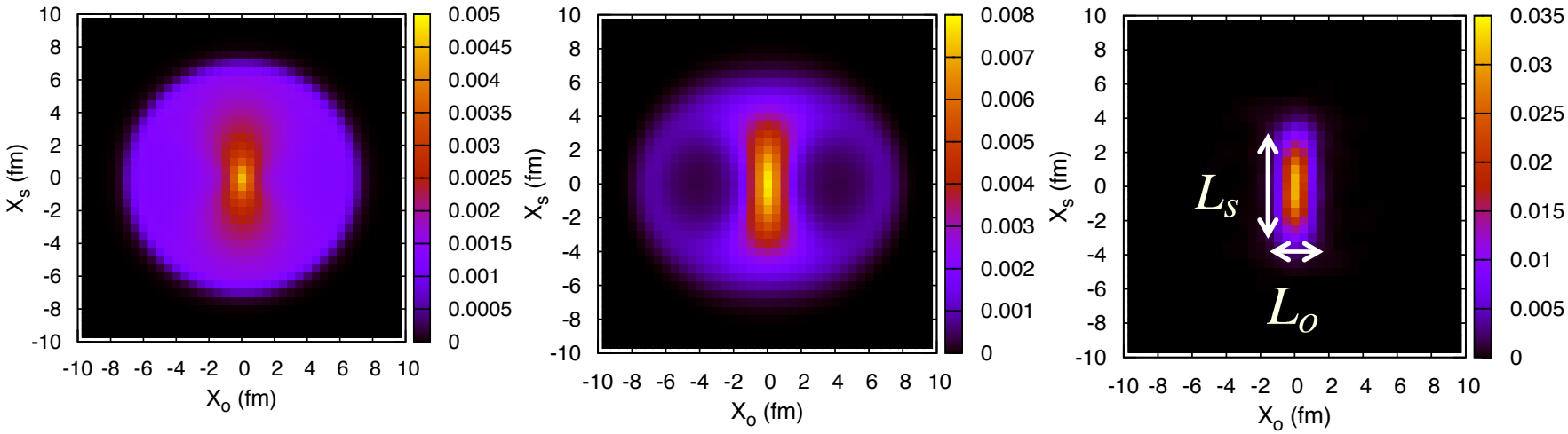


$P(\Delta x^\mu, k_{\mathrm{T}})$: dist. of pair displacements

$$\Delta x^\mu = (X_t, X_o, X_s, X_\ell)$$

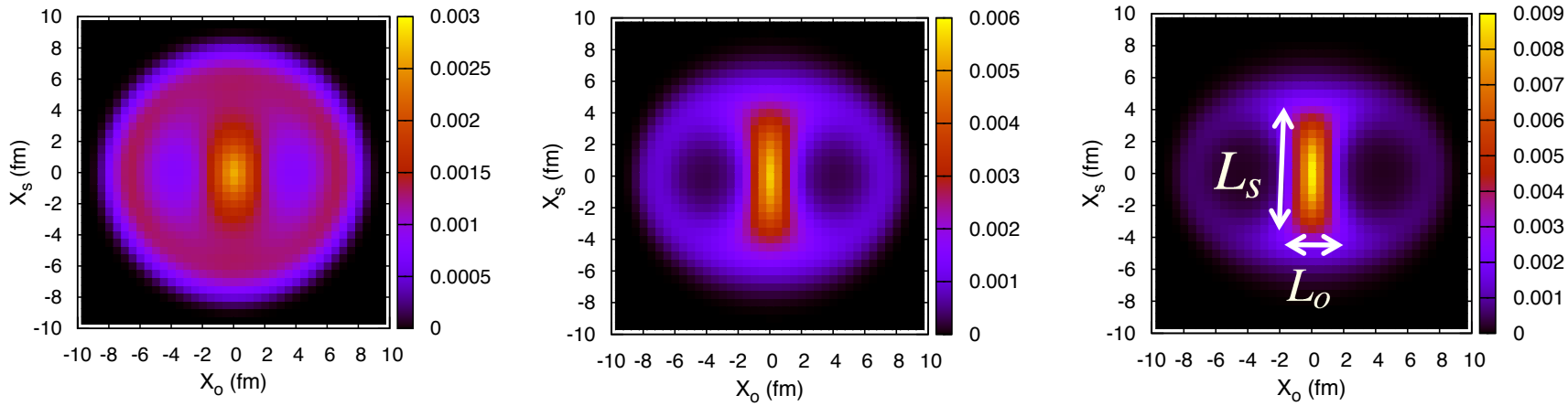
$$C(k_{\mathrm{T}}, \boldsymbol{q}) - 1 = \int dX_t dX_o dX_s dX_\ell \; P(\Delta x^\mu) \cos\left(q_\mu \Delta x^\mu\right)$$

out-side $P(X_o, X_s)$



(a) $0.05\ \mathrm{GeV} < k_\mathrm{T} < 0.15\ \mathrm{GeV}$ (b) $0.35\ \mathrm{GeV} < k_\mathrm{T} < 0.45\ \mathrm{GeV}$ (c) $0.65\ \mathrm{GeV} < k_\mathrm{T} < 0.75\ \mathrm{GeV}$

q=0.5/fm (small flow)



q=1.5/fm (large flow)

Gaussian parametrization of displacements

$$P(\Delta x^\mu) = \frac{1}{16\pi^2 V} \exp\left[-\frac{X_t^2}{4L_t^2} - \frac{X_o^2}{4L_o^2} - \frac{X_s^2}{4L_s^2} - \frac{X_\ell^2}{4L_\ell^2}\right]$$

$$C(K_{\mathrm{T}}, \boldsymbol{q}) - 1 \;\propto \exp\left[-(L_t^2\beta_{\mathrm{T}}^2 + L_o^2)q_o^2 - L_s^2 q_s^2 - (L_\ell^2\beta_{\mathrm{L}}^2 + L_\ell^2)q_\ell^2\right]$$

$$\boldsymbol{\beta} = \boldsymbol{k}/k_0$$

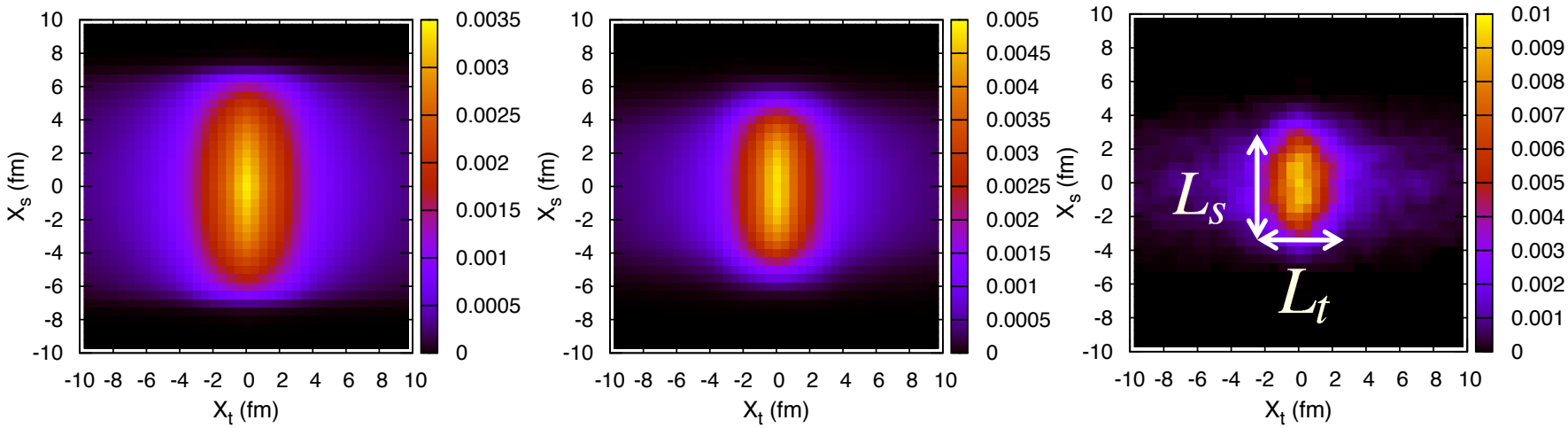
$$R_o^2 = L_o^2 + L_t^2\beta_{\mathrm{T}}^2$$

$$R_\ell^2 = L_\ell^2 + L_t^2\beta_{\mathrm{L}}^2$$

$$R_s^2 = L_s^2$$

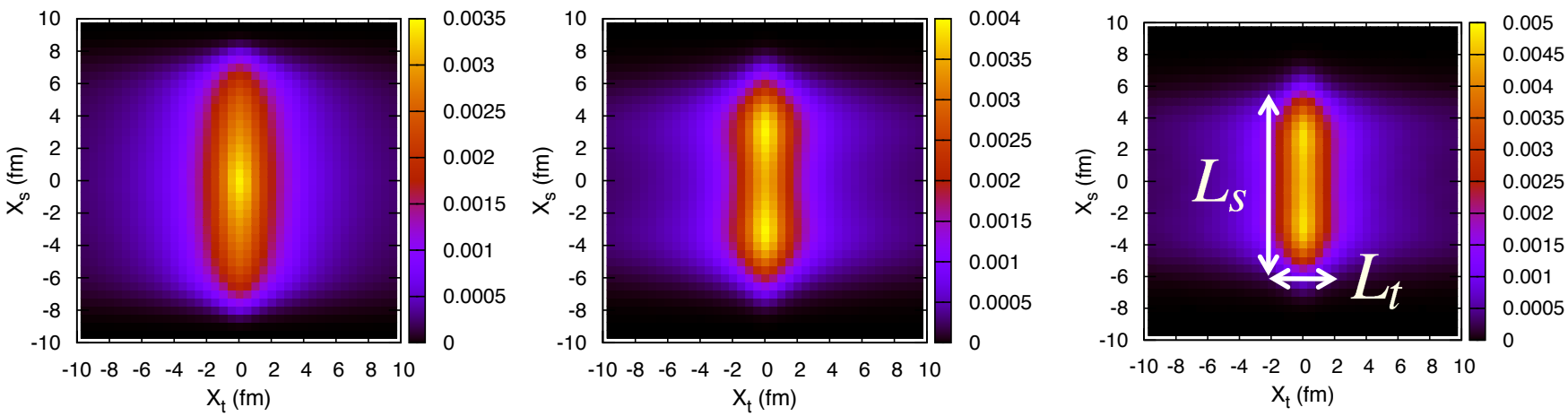
$$\left(\frac{R_o}{R_s}\right)^2 = \left(\frac{L_o}{L_s}\right)^2 + \left(\frac{L_t}{L_s}\right)^2 \beta_{\mathrm{T}}^2$$

time-side $P(X_t, X_s)$



(a) $0.05\ \mathrm{GeV} < k_\mathrm{T} < 0.15\ \mathrm{GeV}$ (b) $0.35\ \mathrm{GeV} < k_\mathrm{T} < 0.45\ \mathrm{GeV}$ (c) $0.65\ \mathrm{GeV} < k_\mathrm{T} < 0.75\ \mathrm{GeV}$

q=0.5/fm (small flow)



q=1.5/fm (large flow)

Why out/side ratio is suppressed at hight kt for large radial flow?

$$\left(\frac{R_o}{R_s}\right)^2 = \left(\frac{L_o}{L_s}\right)^2 + \left(\frac{L_t}{L_s}\right)^2 \beta_{\mathrm{T}}^2$$

$$\left(\frac{L_o}{L_s}\right)_{\text{small q}} \simeq \left(\frac{L_o}{L_s}\right)_{\text{large q}} \qquad \left(\frac{L_t}{L_s}\right)_{\text{small q}} > \left(\frac{L_t}{L_s}\right)_{\text{large q}}$$

Low kt: $\left(\frac{R_o}{R_s}\right)^2 \simeq \left(\frac{L_o}{L_s}\right)^2$

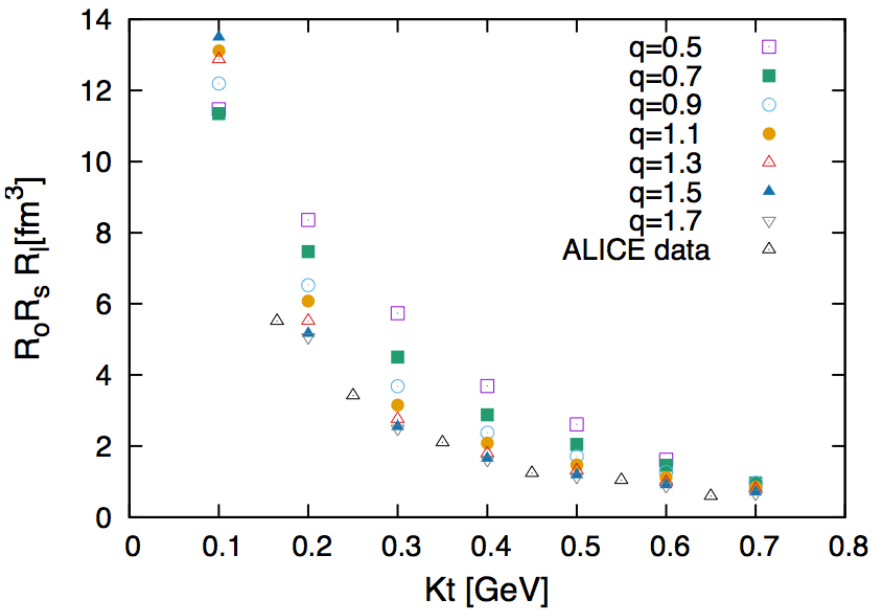
$$\boxed{\left(\frac{R_o}{R_s}\right)_{\text{small q}} \simeq \left(\frac{R_o}{R_s}\right)_{\text{large q}}}$$

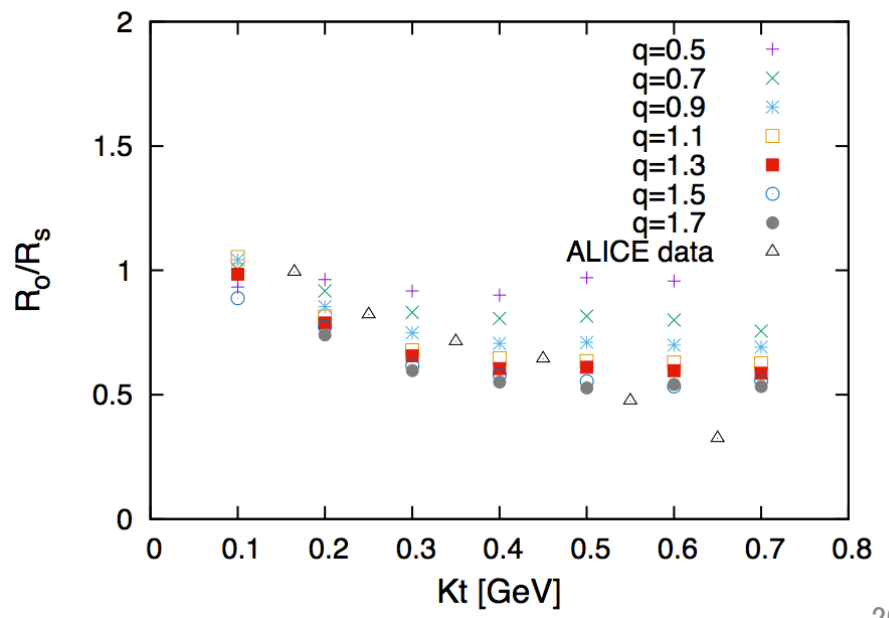
High kt: $\left(\frac{R_o}{R_s}\right)^2 \simeq \left(\frac{L_o}{L_s}\right)^2 + \left(\frac{L_t}{L_s}\right)^2$

$$\boxed{\left(\frac{R_o}{R_s}\right)_{\text{small q}} > \left(\frac{R_o}{R_s}\right)_{\text{large q}}}$$

Summary

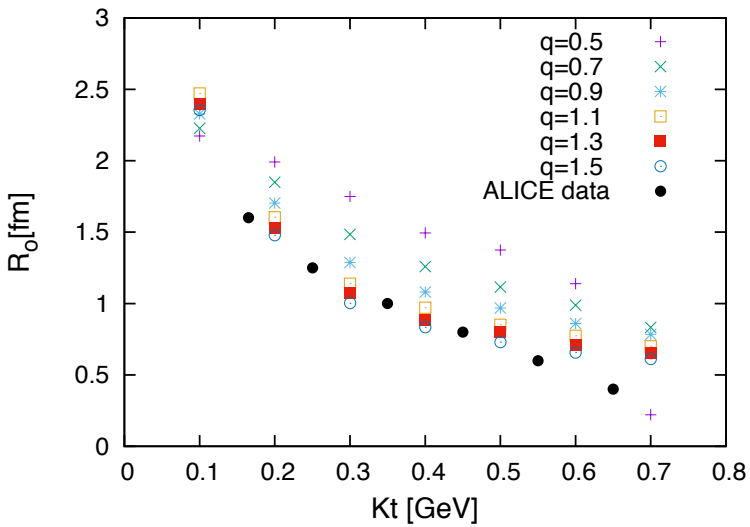
- Radial flow in high-multiplicity pp collisions?
- Calculated HBT radii @pp
  - Gubser + lattice EOS
- Pretty strong radial flow [q ~ 1.7 /fm]

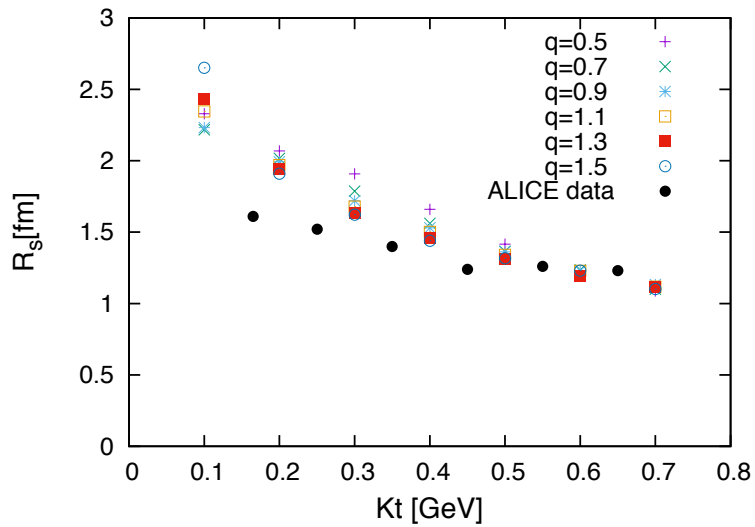


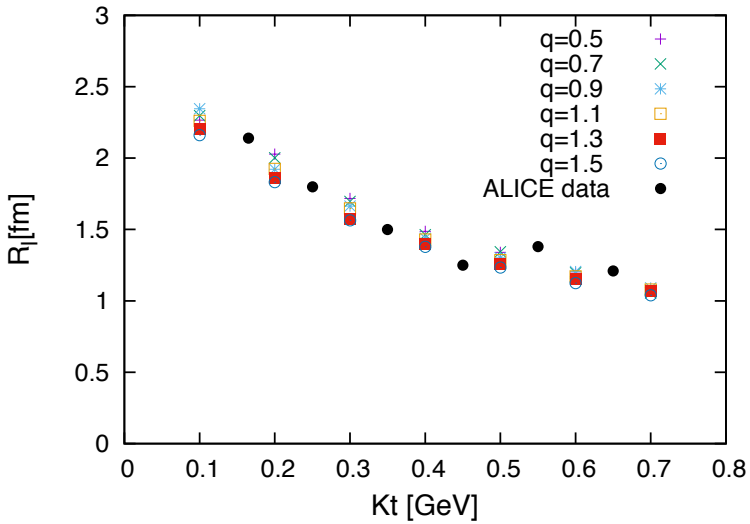


Backup slides

HBT radii


q=0.5
q=0.7
q=0.9
q=1.1
q=1.3
q=1.5
ALICE data
R_o[fm]
Kt [GeV]


q=0.5
q=0.7
q=0.9
q=1.1
q=1.3
q=1.5
ALICE data
R_s[fm]
Kt [GeV]


q=0.5
q=0.7
q=0.9
q=1.1
q=1.3
q=1.5
ALICE data
R_l[fm]
Kt [GeV]

What is the meaning of cumulants?

--what can and can't be learned from it

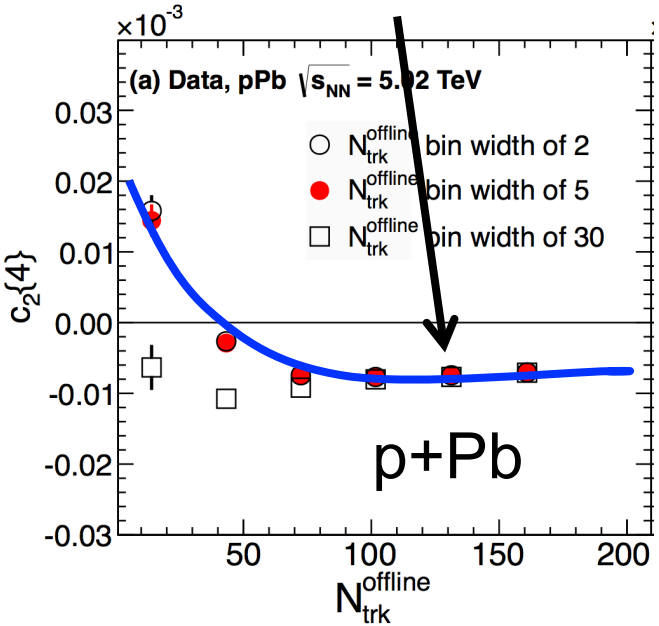
Jiangyong Jia collaboration with Sooraj Radhakrishnan

- How are cumulants affected by flow & nonflow?
- How to suppress & subtract nonflow?
- How are cumulants affected by $p(v_n)$?

The precise meaning of cumulant data?

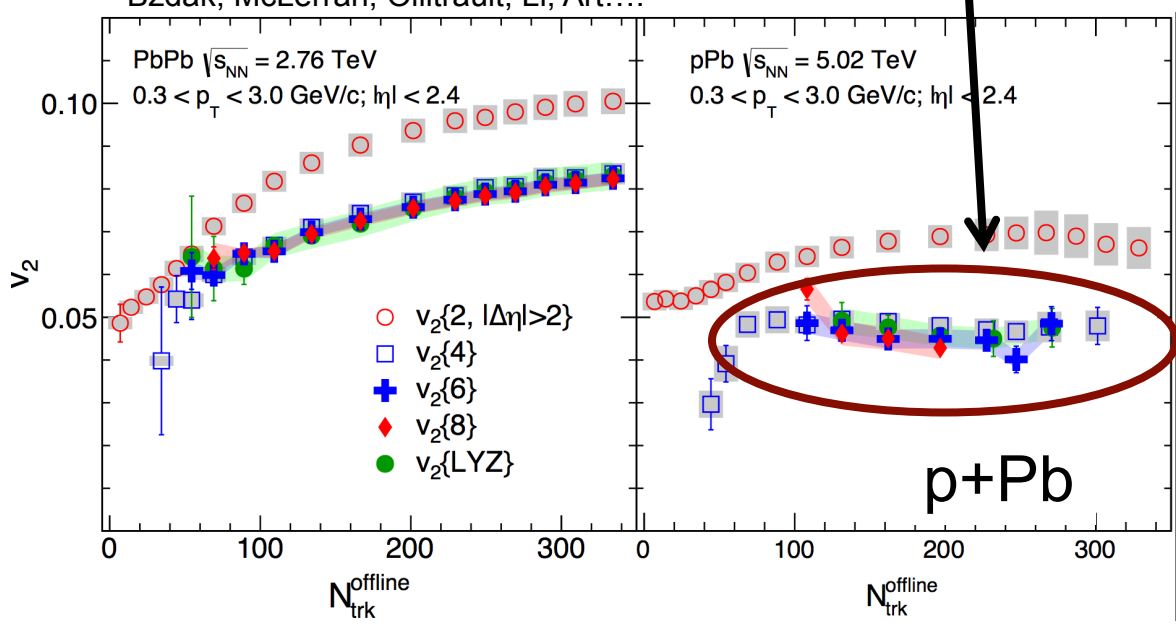
$$v_n\{2\}^2 = c_n\{2\} = \langle v_n^2 \rangle$$
$$-v_n\{4\}^4 = c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$$
$$4v_n\{6\}^6 = c_n\{6\} = \langle v_n^6 \rangle - 9\langle v_n^4 \rangle\langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3$$
$$\ldots$$

negative $c_2\{4\}$ or real $v_2\{4\}$
Onset of collectivity?



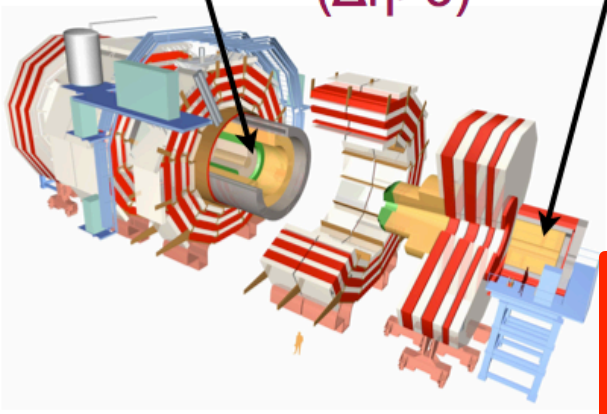
What "$v_2\{2k\} \approx$ same" means?
$p(v_n)$~Gaussian?, ~power?

Bzdak, McLerran, Ollitrault, Li, Art….



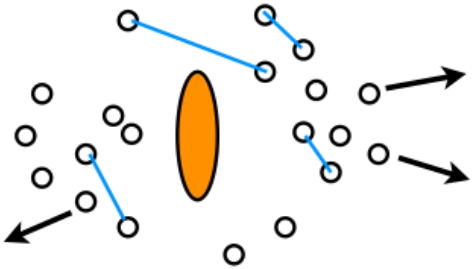
Event Plane

Correlate particles in tracker with those in HF. ($\Delta\eta>3$)



Two-particle Cumulant

Consider all two-particle correlations.

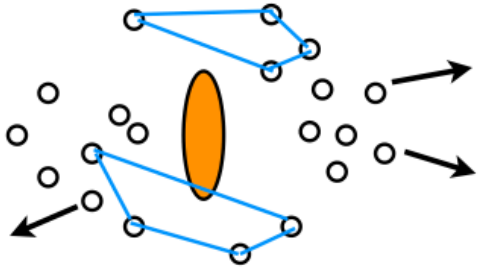


$p(v_n)$ & nonflow

Methods have different sensitivity to event-by-event fluctuations and non-flow effects.

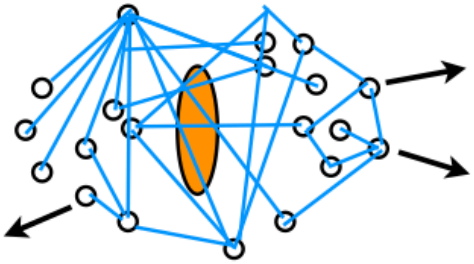
Four-particle Cumulant

Consider all four-particle correlations.



Lee-Yang Zeros

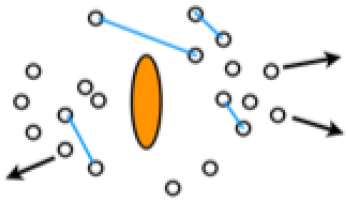
Consider all particle correlations -(Not all shown!).



The precise meaning of cumulant?

Two-particle Cumulant

Consider all two-particle correlations.

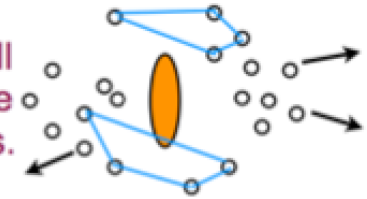


$$c_n\{2\} \equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\rm ev}\right\rangle \approx \langle v_n^2\rangle$$

$\phi_1 \neq \phi_2$

Four-particle Cumulant

Consider all four-particle correlations.



$$c_n\{4\} \equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_1-\phi_4)}\right\rangle_{\rm ev}\right\rangle - 2\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\rm ev}\right\rangle^2 \approx \langle v_n^4\rangle - 2\langle v_n^2\rangle^2$$

$\phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_4$

- Average combinations in one event, then average over events
- Q-cumulant method allow determination in one pass, but with careful removal of duplicate terms, i.e. Nicolas, Jean-Yves, Ante, Ramond,Sergi,…

$\phi_1=\phi_2\neq\phi_3\neq\phi_4$ $\phi_1=\phi_2=\phi_3\neq\phi_4$ $\phi_1=\phi_2=\phi_3=\phi_4$ + permutations

Direct cumulant or Q-cumulant

1

$$\left\langle e^{in(\phi_1+\phi_2-\phi_1-\phi_4)}\right\rangle_{\rm ev} = \frac{1}{N(N-1)(N-2)(N-3)}\sum_{\{i\neq j\neq k\neq l\}=1}^{N} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

$$= \frac{1}{N^4}\sum_{\{i,j,k,l\}=1}^{N} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)} - \text{duplicates.}$$

$$= \frac{\sum_{i=1}^{N} e^{in\phi_i}}{N}\frac{\sum_{j=1}^{N} e^{in\phi_j}}{N}\frac{\sum_{k=1}^{N} e^{-in\phi_k}}{N}\frac{\sum_{l=1}^{N} e^{-in\phi_l}}{N} - \text{duplicates.}$$

$$= \left(\boldsymbol{q}_n \boldsymbol{q}_n *\right)^2 - \text{duplicates.}$$

$$\boldsymbol{q}_n = \frac{\sum_{i=1}^{N} e^{in\phi_i}}{N} \equiv \left\langle e^{in\phi}\right\rangle_{\rm ev}$$

single particle quantity!

Non-flow?

$$q_n = |\boldsymbol{q}_n|$$

2

$$c_n\{4\} \equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_1-\phi_4)}\right\rangle_{\rm ev}\right\rangle - 2\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\rm ev}\right\rangle^2 \approx \langle q_n^4\rangle - 2\langle q_n^2\rangle^2$$

Is non-flow removed by the duplicate terms i.e. is non-flow suppressed within each event? or is it suppressed by averaging over the events?

Non-flow in this talk = “Non-flow” + statistical fluc. from finite N

Removing non-flow in each event?

- Cumulant expression

$$\langle\langle 2\rangle_{\rm ev}\rangle \equiv \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\rm ev}$$
$$\langle\langle 4\rangle_{\rm ev}\rangle \equiv \left\langle e^{in(\phi_1+\phi_2-\phi_1-\phi_4)}\right\rangle_{\rm ev}$$
$$\ldots$$

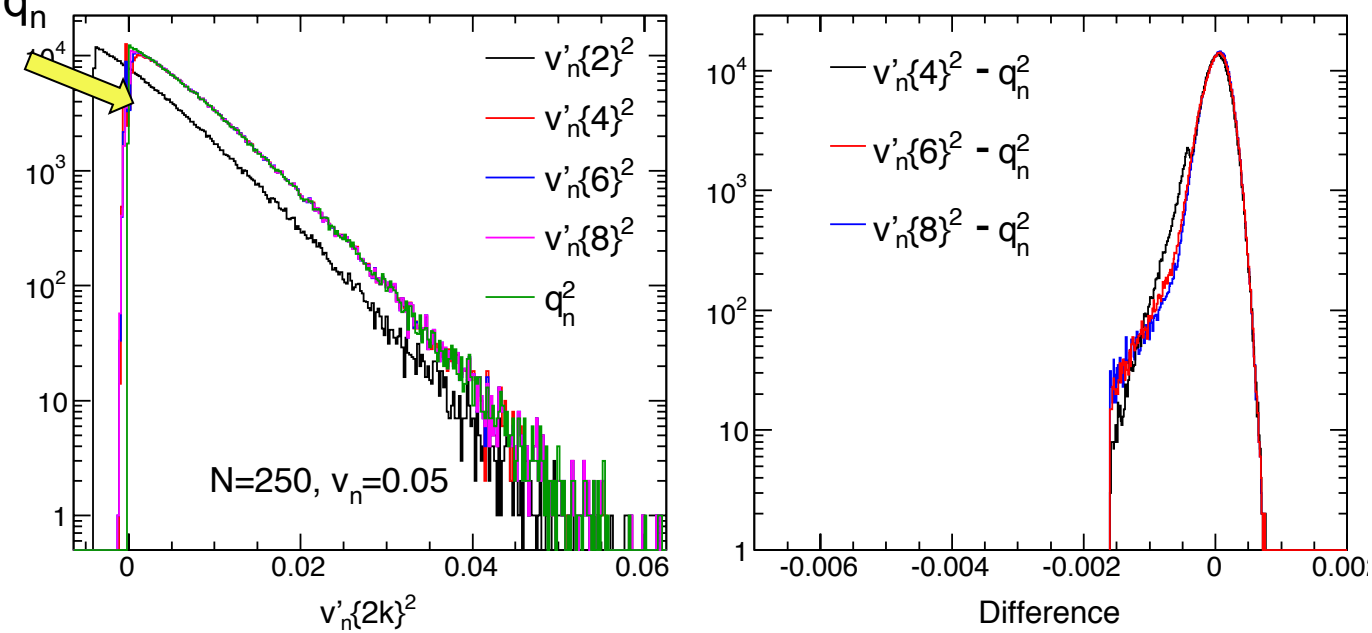
$$c_n\{2\} = \langle\langle 2\rangle_{\rm ev}\rangle \equiv v_n\{2\}^2 \approx \left\langle q_n^2\right\rangle$$
$$c_n\{4\} = \langle\langle 4\rangle_{\rm ev}\rangle - 2\langle\langle 2\rangle_{\rm ev}\rangle^2 \equiv -v_n\{4\}^4 \approx \left\langle q_n^4\right\rangle - 2\left\langle q_n^2\right\rangle^2$$
$$c_n\{6\} = \langle\langle 6\rangle_{\rm ev}\rangle - 9\langle\langle 4\rangle_{\rm ev}\rangle\langle\langle 2\rangle_{\rm ev}\rangle + 12\langle\langle 2\rangle_{\rm ev}\rangle^3 \equiv 4v_n\{6\}^6 \approx \left\langle q_n^6\right\rangle - 9\left\langle q_n^4\right\rangle\left\langle q_n^4\right\rangle + 12\left\langle q_n^2\right\rangle^3$$

- To see if one can cancel nonflow in each event, define cumulant EbyE.

$$v_n'\{2\}^2 \equiv c_n'\{2\} \equiv \langle\langle 2\rangle_{\rm ev}\rangle$$
$$-v_n'\{4\}^4 \equiv c_n'\{4\} \equiv \left\langle\langle 4\rangle_{\rm ev} - 2\langle 2\rangle_{\rm ev}^2\right\rangle$$
$$4v_n'\{6\}^6 \equiv c_n'\{6\} \equiv \left\langle\langle 6\rangle_{\rm ev} - 9\langle 4\rangle_{\rm ev}\langle 2\rangle_{\rm ev} + 12\langle 2\rangle_{\rm ev}^3\right\rangle$$
$$\ldots$$

Dominated by fluc. in q_n

non-flow not removed event by event

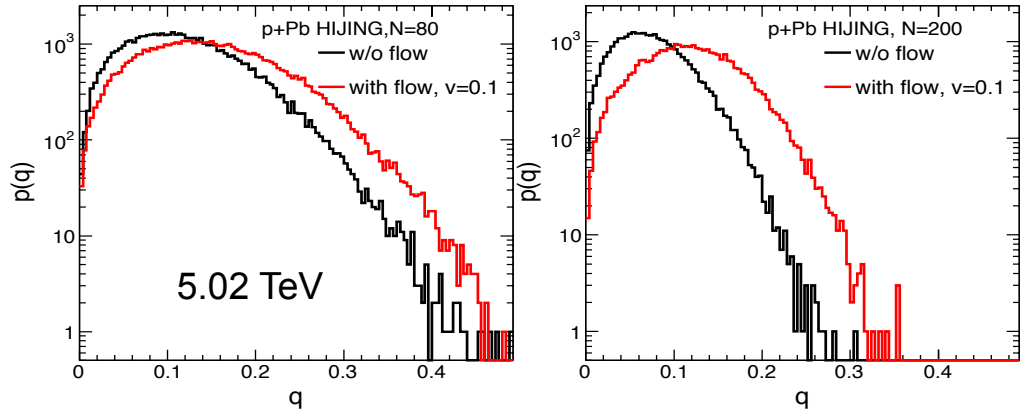


Can we estimate $v_n\{2k\}$ directly from $p(q_n)$?

- Simulation pPb hijing w/o and with v =0.1 flow.
  - Calculate EbyE flow vector q_n and group them according to multiplicity N.
  - Calculate the cumulants of flow vectors, ignore duplicate terms

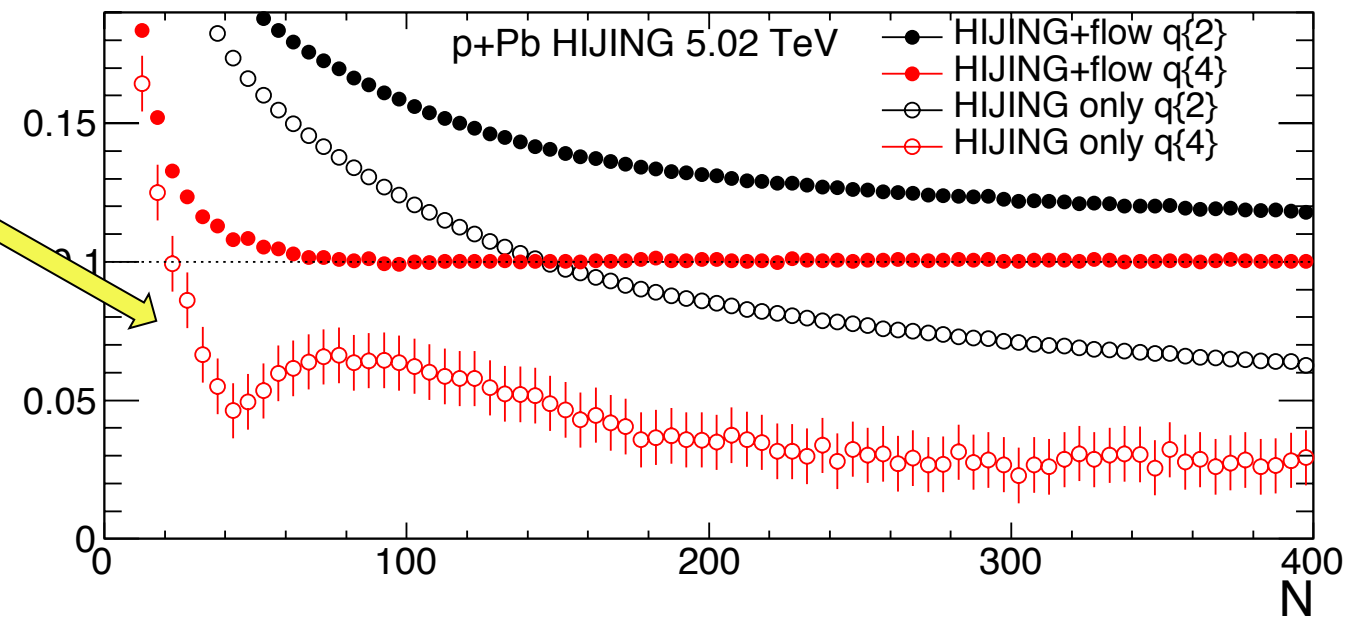
Non-flow strongly suppressed all the way to N~80 in 4p correlation

Nevertheless non-zero



$q_n\{4\}$ from pure HIJING,

$c_n\{4\}=-q_n\{4\}^4$ value is much smaller



$v_n\{2k\}$ can be calculated directly from $p(q_n)$ with good precision!

Direct cumulant or Q-cumulant

1

$$\left\langle e^{in(\phi_1+\phi_2-\phi_1-\phi_4)}\right\rangle_{\mathrm{ev}} = \frac{1}{N(N-1)(N-2)(N-3)} \sum_{\{i\neq j\neq k\neq l\}=1}^{N} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

$$= \frac{1}{N^4} \sum_{\{i,j,k,l\}=1}^{N} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)} - \text{duplicates.}$$

$$= \frac{\sum_{i=1}^{N} e^{in\phi_i}}{N} \frac{\sum_{j=1}^{N} e^{in\phi_j}}{N} \frac{\sum_{k=1}^{N} e^{-in\phi_k}}{N} \frac{\sum_{l=1}^{N} e^{-in\phi_l}}{N} - \text{duplicates.}$$

$$= \left(\boldsymbol{q}_n \boldsymbol{q}_n *\right)^2 - \text{duplicates.}$$

Single particle quantity

These terms are not important!

- Non-flow can't be suppressed within each event!
- Non-flow is suppressed by averaging over the events!

How non-flow contribute to cumulant?

- For each event $\boldsymbol{q} = \boldsymbol{v} + \boldsymbol{s}$ $\quad p(\boldsymbol{q}) = p(\boldsymbol{v}) \otimes p(\boldsymbol{s})$

Drop subscript n and use $c_x\{2k\}$ to represent $c_n\{2k\}$ calculated from p(x) distribution

- If flow **v** and nonflow **s** are uncorr, then their cumulants are additive

Generating func.:

Use slightly different notation

$$\ln\left\langle e^{z(\boldsymbol{v}+\boldsymbol{v}^*)}\right\rangle = \ln\left(\sum_{k=1}^{\infty}\frac{z^{2k}}{k!^2}\left\langle(\boldsymbol{v}\boldsymbol{v}^*)^k\right\rangle\right) = \sum_{k=1}^{\infty}\frac{z^{2k}}{k!^2}c_v\{2k\}$$

Therefore:

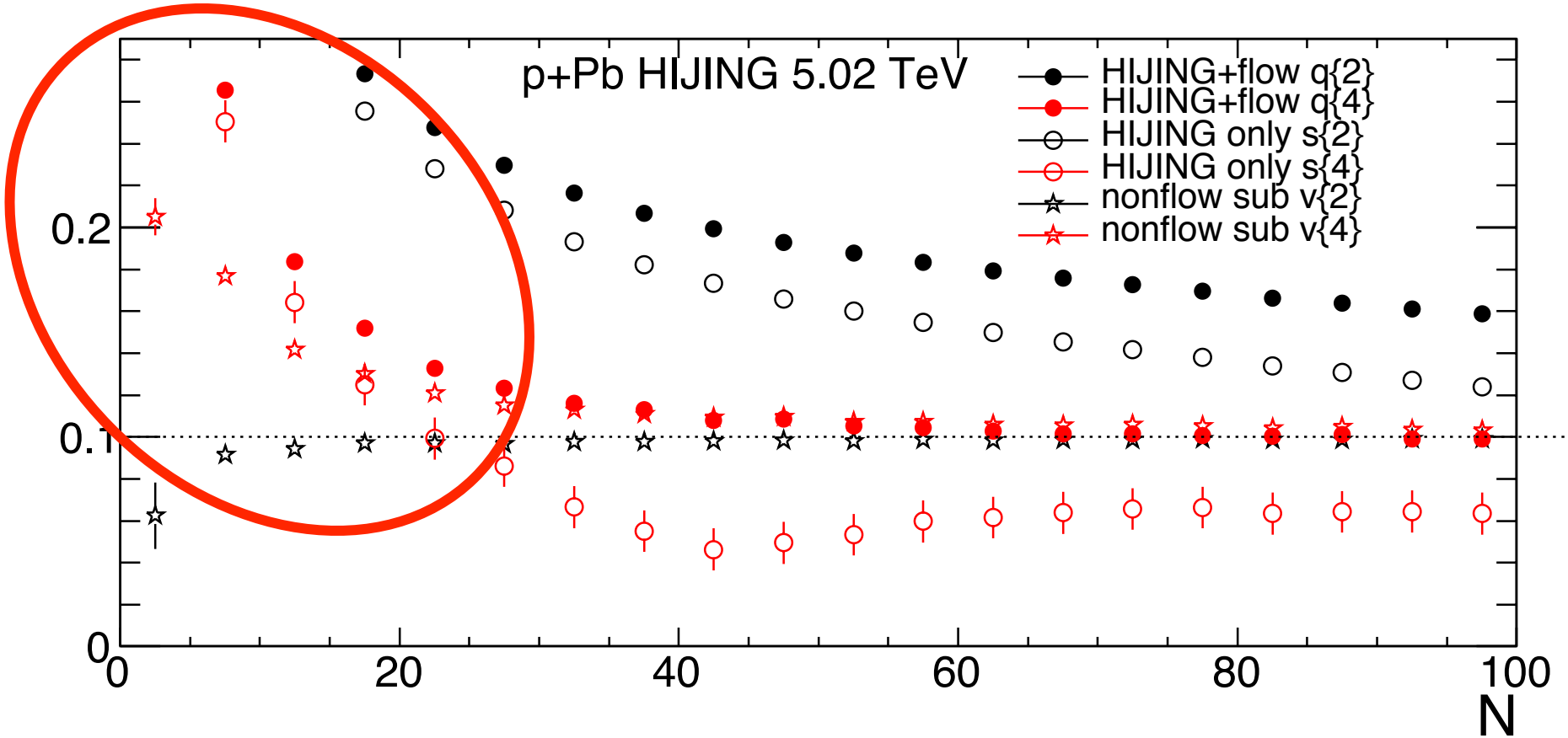
$$\ln\left\langle e^{z(\boldsymbol{q}+\boldsymbol{q}^*)}\right\rangle = \ln\left\langle e^{z(\boldsymbol{v}+\boldsymbol{v}^*)}\right\rangle + \ln\left\langle e^{z(\boldsymbol{s}+\boldsymbol{s}^*)}\right\rangle = \sum_{k=1}^{\infty}\frac{z^{2k}}{k!^2}\left(c_v\{2k\} + c_s\{2k\}\right)$$

$$\boxed{c_q\{2k\} = c_v\{2k\} + c_s\{2k\}}$$

$$c_v\{2k\} = c_q\{2k\} - c_s\{2k\} \quad \rightarrow v\{2k\}$$

Subtract non-flow contribution

$$c_v\{2k\} = c_q\{2k\} - c_s\{2k\} \qquad \rightarrow v\{2k\}$$



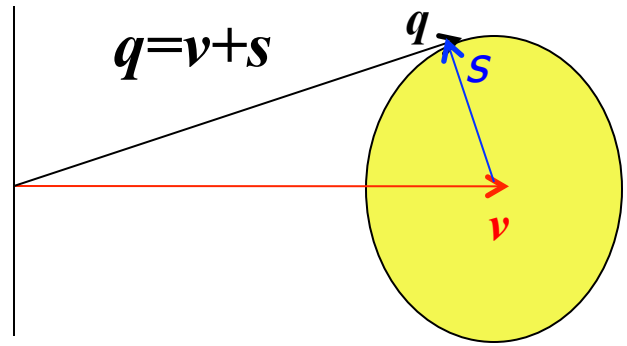
- Near perfect improvements on q{2} in general.
- Good improvement on q{2k} in small N, but not sufficient, why?

Trivial correlation between non-flow and flow

- Consider random fluc. around **v**, flow vector chosen along x-axis

$$\boldsymbol{v}=v_x+iv_y \quad \boldsymbol{s}=s_x+is_y$$

$$v_x = \langle \cos\phi \rangle = v \quad v_y = \langle \sin\phi \rangle = 0$$



- Presence of flow v suppresses statistical fluctuation in flow direction!

$$\langle s_x^2 \rangle = \mathrm{Var}^2(q_x) = \frac{\langle \cos^2\phi \rangle - \langle \cos\phi \rangle^2}{N} = \frac{1-2v^2}{2N}, \quad \langle s_y^2 \rangle = \mathrm{Var}^2(q_y) = \frac{\langle \sin^2\phi \rangle - \langle \sin\phi \rangle^2}{N} = \frac{1}{2N}$$

$$\langle s_x^2 - s_y^2 \rangle = \frac{-v^2}{N}$$

- Terms like $\langle s_x^3 \rangle, \langle s_x s_y^2 \rangle, \langle s_x^2 s_y^2 \rangle$ ... all non-zero, but 1/N,1/N^2…suppressed
  - Evaluated by relating moments for the sample with moment for the population

Resulting corrections

- For 2nd order: $\langle q^2 \rangle = \langle v^2 \rangle + \langle s^2 \rangle + 2\left(\langle v_x s_x \rangle + \langle v_y s_y \rangle\right) = \langle v^2 \rangle + \langle s^2 \rangle$

$$c_q\{2\} = c_v\{2\} + c_s\{2\}$$

NO correction for 2PC!

- 4th order:

$$c_q\{4\} = \langle q^4 \rangle - 2\langle q^2 \rangle^2 = c_v\{4\} + c_s\{4\} + 4\langle s^2 s_x v \rangle + 2\langle (s_x^2 - s_y^2) v^2 \rangle$$

$$\approx c_v\{4\} + c_s\{4\} - 4v^2\left(\frac{1}{N^2} + \frac{v^2}{2N}\right)$$

- 6th order:

$$\begin{aligned} c_q\{6\} &= c_v\{6\} + c_s\{6\} + 12\langle s^2 s_x v^3 \rangle + 6\langle (s_x^2 - s_y^2) v^4 \rangle + 6\langle (s_x^4 - s_y^4) v^2 \rangle \\ &\quad + 8\langle s_x^3 v^3 \rangle + 6\langle s_x s^4 v \rangle - 18(2\langle s^2 s_x v \rangle + \langle (s_x^2 - s_y^2) v^2 \rangle)(\langle v^2 \rangle + \langle s^2 \rangle) \\ &\approx c_v\{6\} + c_s\{6\} + 4v^4\left(\frac{3v^2}{N} + \frac{9}{2N^2}\right) \end{aligned}$$

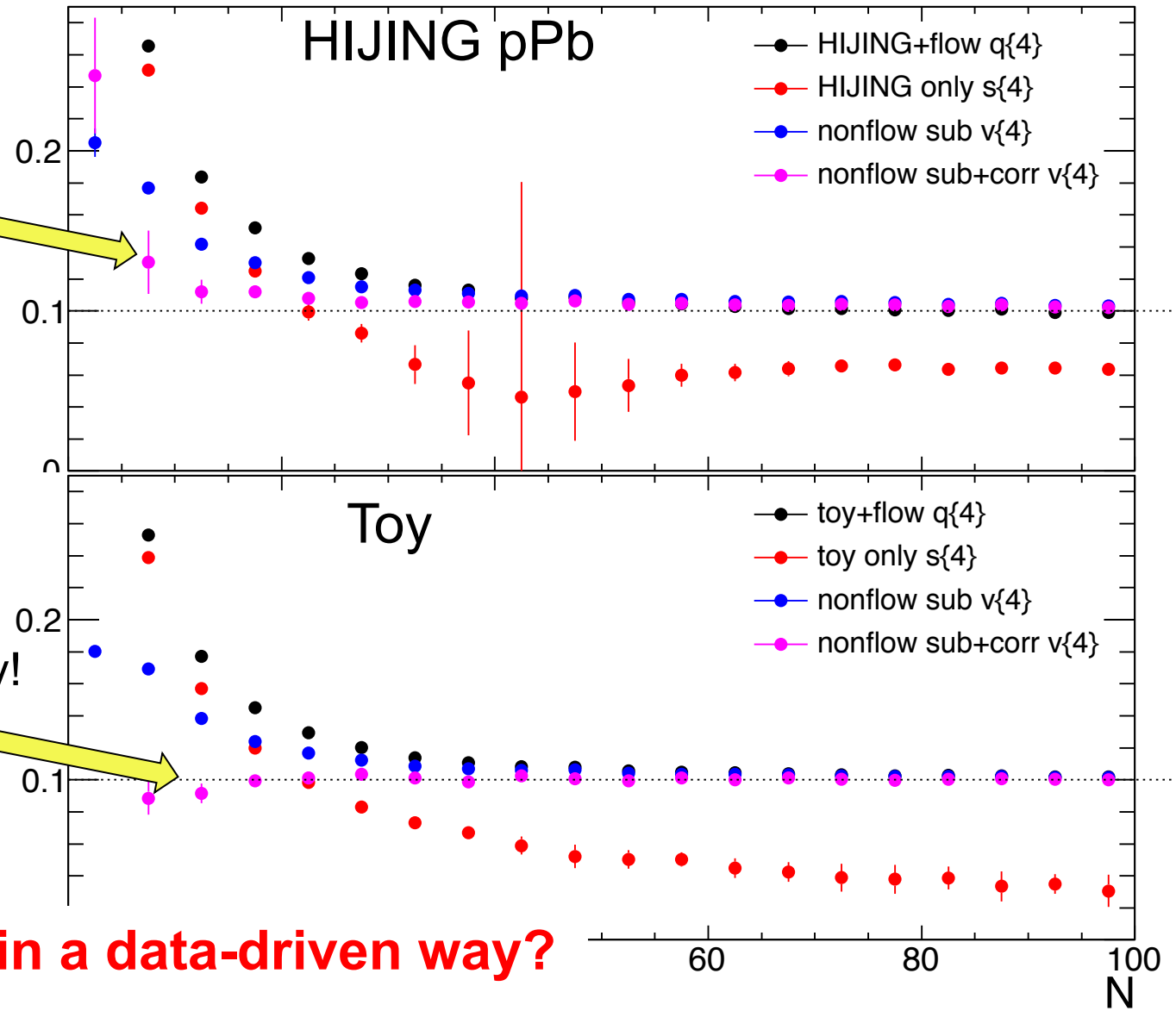
Applying corrections in simulation for $c_n\{4\}$

- Improves the behavior at small N

Shown by magenta

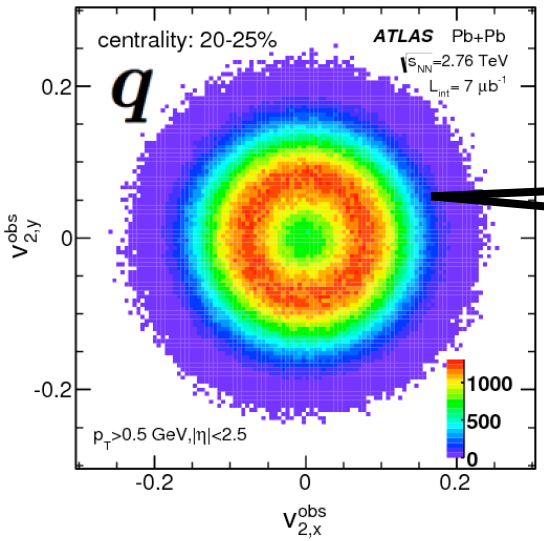
4% residual derivations → about 20% for c{4}

Perfect recovery in Toy!

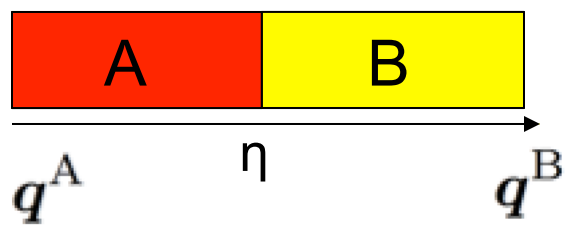


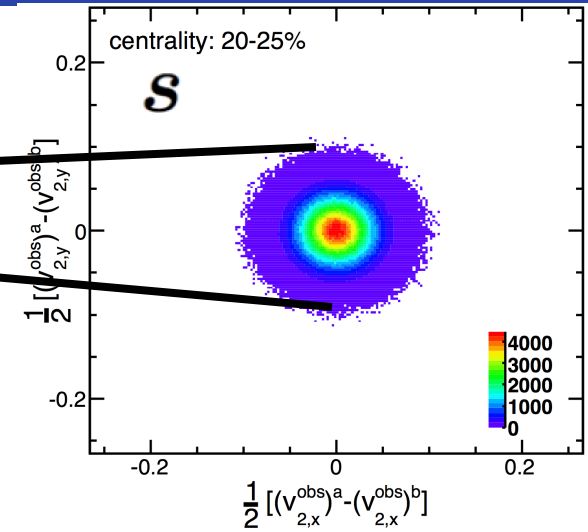
Can we correct in a data-driven way?

Estimating non-flow distribution in data



$$v = q - s$$





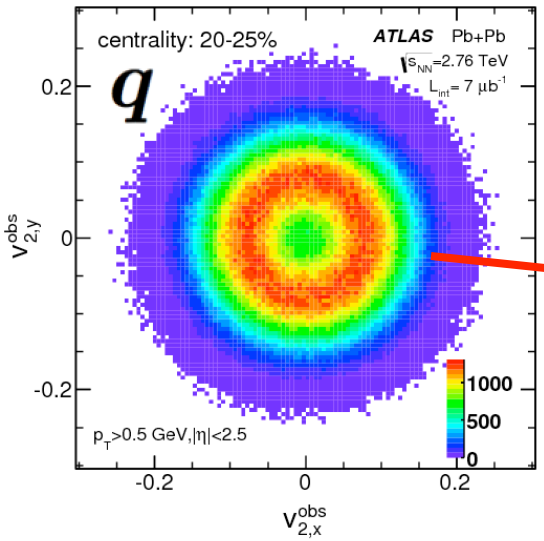
$$q = \left[q^{\mathrm{A}} + q^{\mathrm{B}}\right]/2$$
$$= \text{nonflow} + v$$

$$s \simeq \left[q^{\mathrm{A}} - q^{\mathrm{B}}\right]/2$$
$$= \text{nonflow}$$

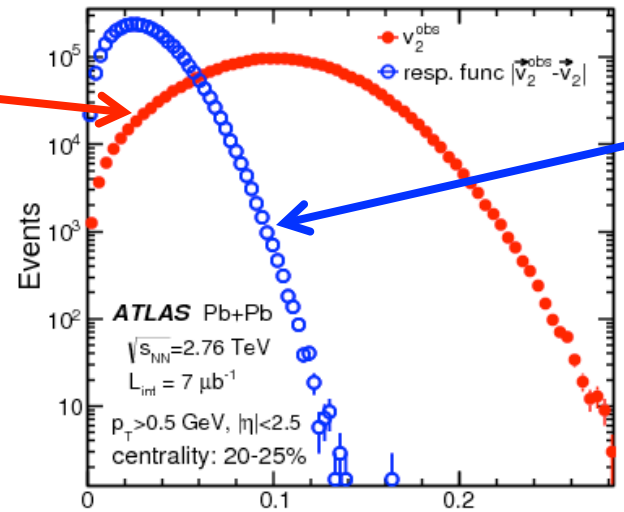
JJ, S. Mohapatra 1304.1417

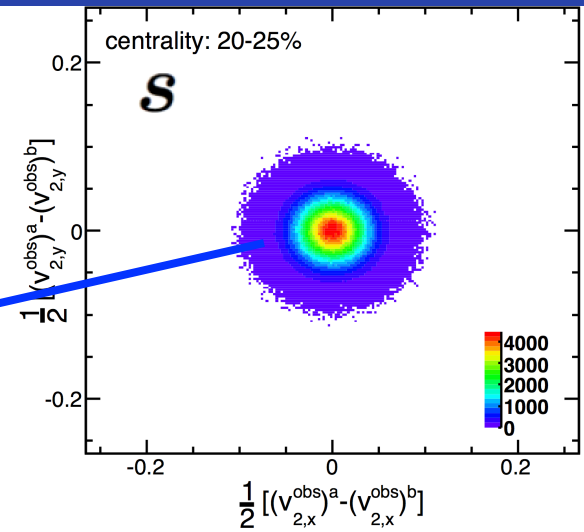
- The nonflow fluctuations estimated from difference of two sub-events.
  - It works as long as long-range-nonflow is small (further suppressed by Δη gap)
  - Gap not important for s{4} and higher-order, but could be for s{2}.

Subtracting non-flow distribution



$$v = q - s$$





$$q = \left[q^{\mathrm{A}} + q^{\mathrm{B}}\right]/2$$
$$= \text{nonflow} + v$$

$$s \simeq \left[q^{\mathrm{A}} - q^{\mathrm{B}}\right]/2$$
$$= \text{nonflow}$$

ATLAS obtain p(v) via unfolding p(s) from p(q): 1305.2942

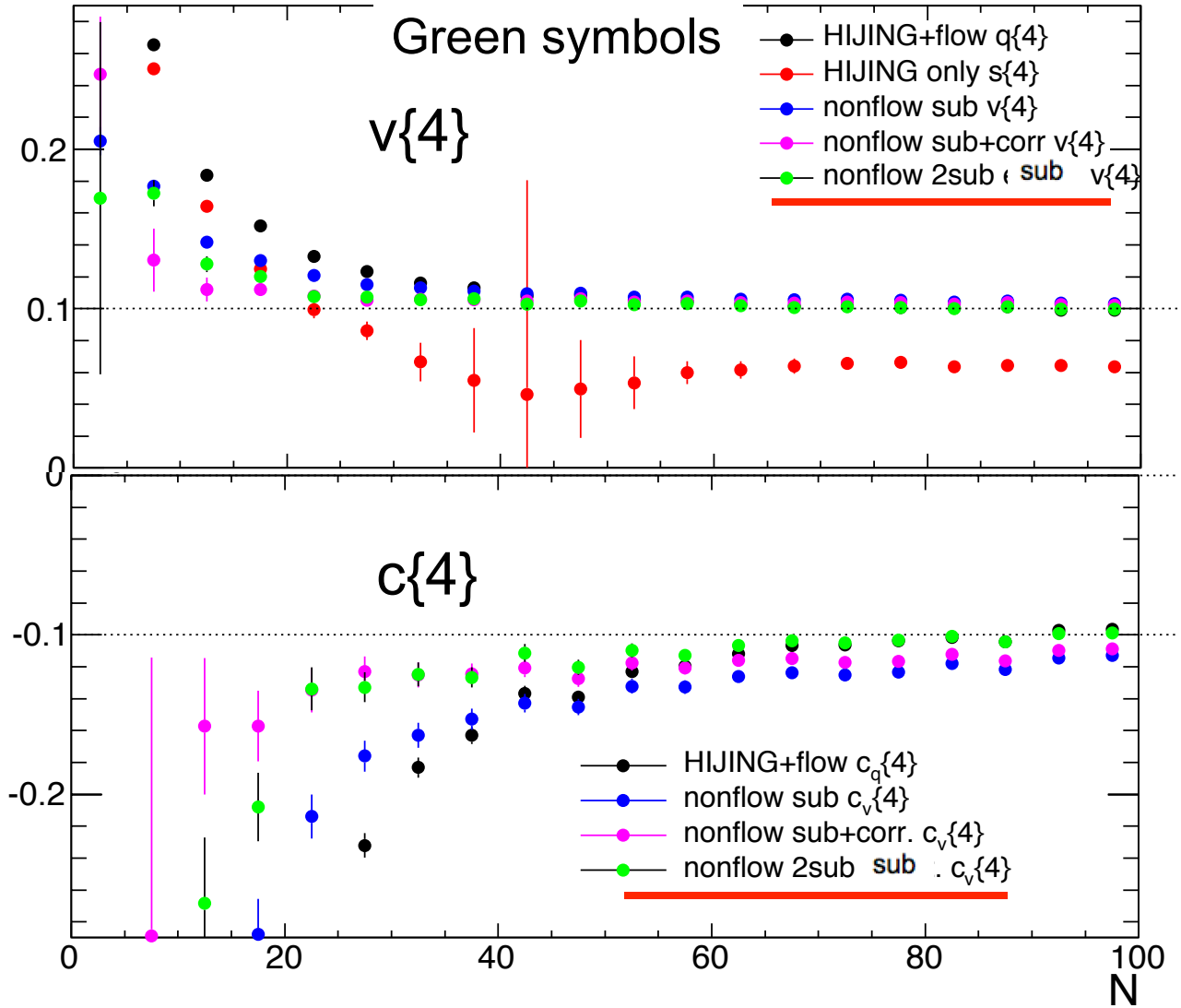
But for cumulants, just calculate:

$$c_n\{2k\} = c_n\{2k\}_q - c_n\{2k\}_s$$

Works within ~% for higher-order cumulants

Non-flow suppression via 2 subevent method

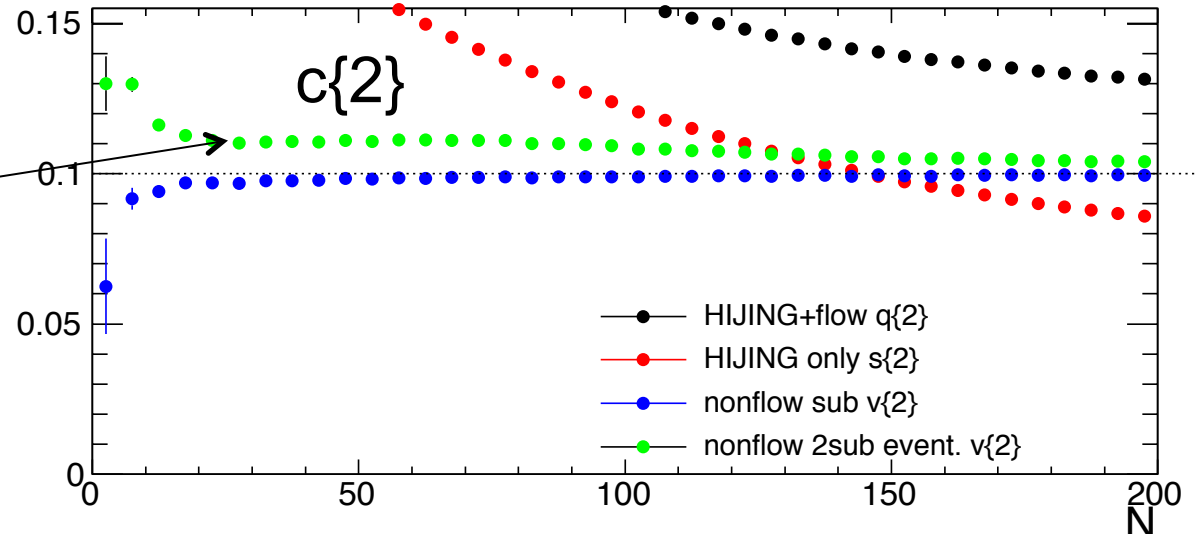
The method works well for 4 and higher-order cumulant, sometimes even better than the correction method.

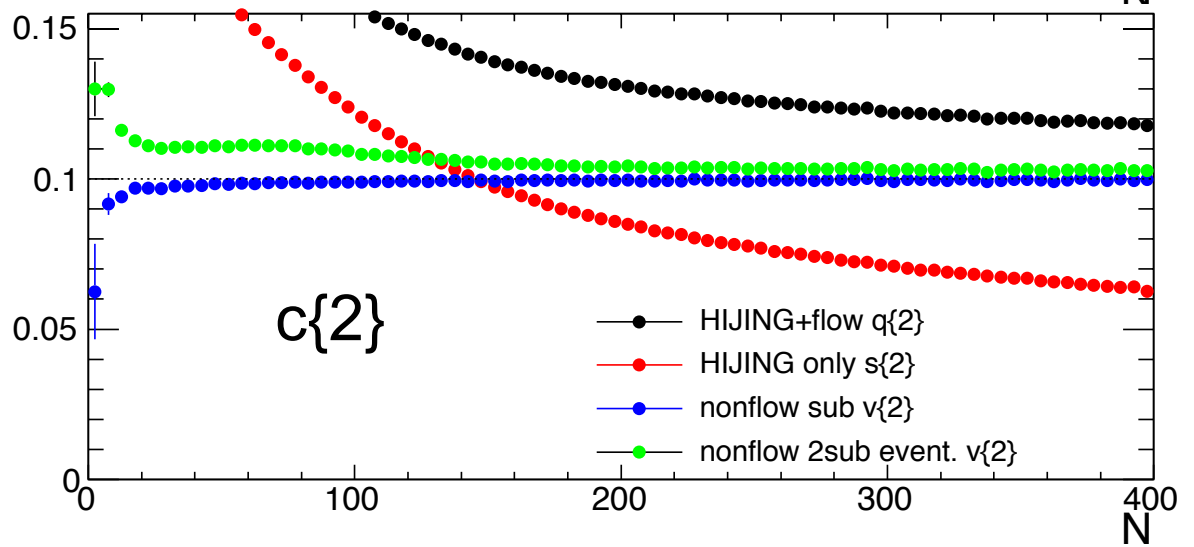


Non-flow suppression via 2 subevent method

- Performance is not as good for v{2}: due to dominance of non-flow, imprecision from 2sub event method can not be neglected

The residual effects is about 10% of v{2} for N<100, and decrease as 1/√N at large N





Correlation is only needed for non-flow, but not for flow!

How are cumulants affected by $p(v_n)$?

The precise meaning of cumulant data?

$$v_n\{2\}^2 = c_n\{2\} = \langle v_n^2 \rangle$$
$$-v_n\{4\}^4 = c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$$
$$4v_n\{6\}^6 = c_n\{6\} = \langle v_n^6 \rangle - 9\langle v_n^4 \rangle\langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3$$
$$\ldots$$

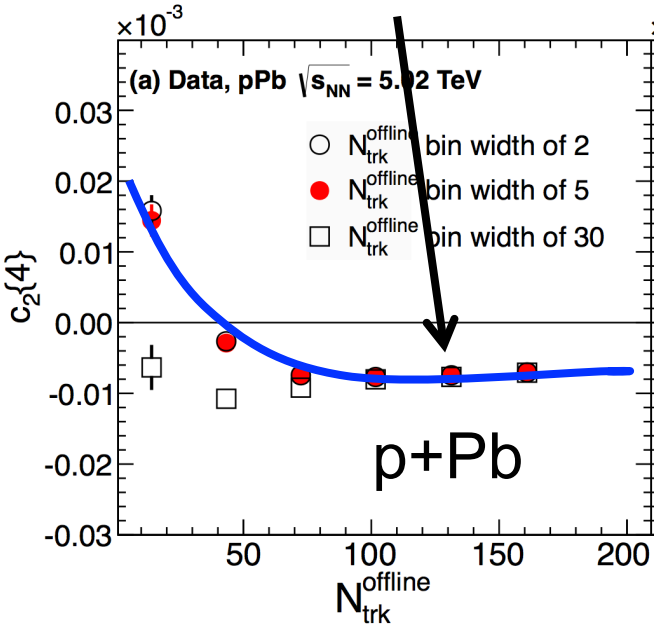
v{2k} depend on p(v) shape!

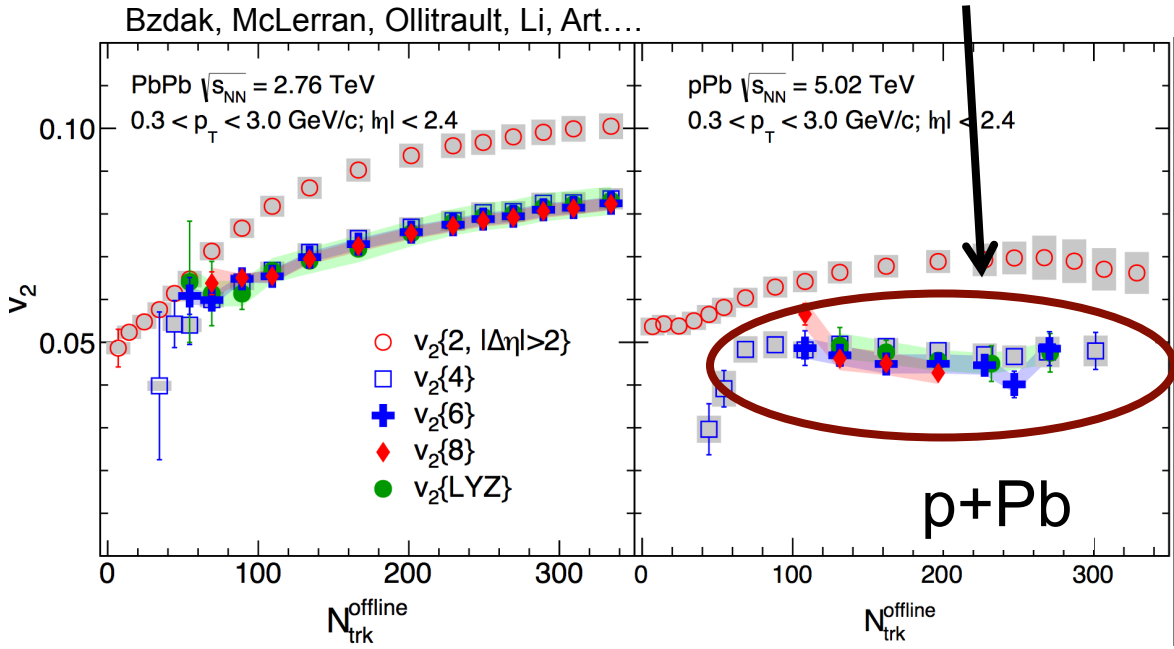
$$\langle v_n^4 \rangle \lesssim 2\langle v_n^2 \rangle^2 \ ?$$
$$\langle v_n^6 \rangle + 12\langle v_n^2 \rangle^3 \lesssim 9\langle v_n^4 \rangle\langle v_n^2 \rangle ?$$

negative $c_2\{4\}$ or real $v_2\{4\}$
Onset of collectivity?

What "$v_2\{2k\} \approx$ same" means?
$p(v_n)$~Gaussian?, ~power?

Bzdak, McLerran, Ollitrault, Li, Art….





For narrow distributions

JJ, S. Radhakrishnan 1412.4759

- Simple central-moment expansion

$$\theta_j = \int \left(\frac{v_n - \langle v_n \rangle}{\langle v_n \rangle} \right)^j p(v_n) dv_n$$

$$\langle v_n^{2k} \rangle = \langle v_n \rangle^{2k} \int \left(1 + \frac{v_n - \langle v_n \rangle}{\langle v_n \rangle} \right)^{2k} p(v_n) dv_n = \langle v_n \rangle^{2k} \left(1 + \sum_{j=2}^{2k} C_{2k}^j \theta_j \right)$$

- Keep up to θ_4:

$$\begin{aligned}
v_n\{2\}/\langle v_n \rangle &\approx \boxed{1 + \frac{1}{2}\theta_2} - \frac{3}{8}\theta_2^2 \\
v_n\{4\}/\langle v_n \rangle &\approx \boxed{1 - \frac{1}{2}\theta_2} - \theta_3 - \frac{1}{4}\theta_4 + \frac{1}{8}\theta_2^2 \\
v_n\{6\}/\langle v_n \rangle &\approx \boxed{1 - \frac{1}{2}\theta_2} - \frac{2}{3}\theta_3 + \frac{1}{4}\theta_4 - \frac{11}{8}\theta_2^2 \\
v_n\{8\}/\langle v_n \rangle &\approx \boxed{1 - \frac{1}{2}\theta_2} - \frac{7}{11}\theta_3 + \frac{31}{132}\theta_4 - \frac{117}{88}\theta_2^2
\end{aligned}$$

- $v\{4\} \approx v\{6\} \approx \ldots$

Cumulants mainly sensitive to mean and width of p(v).

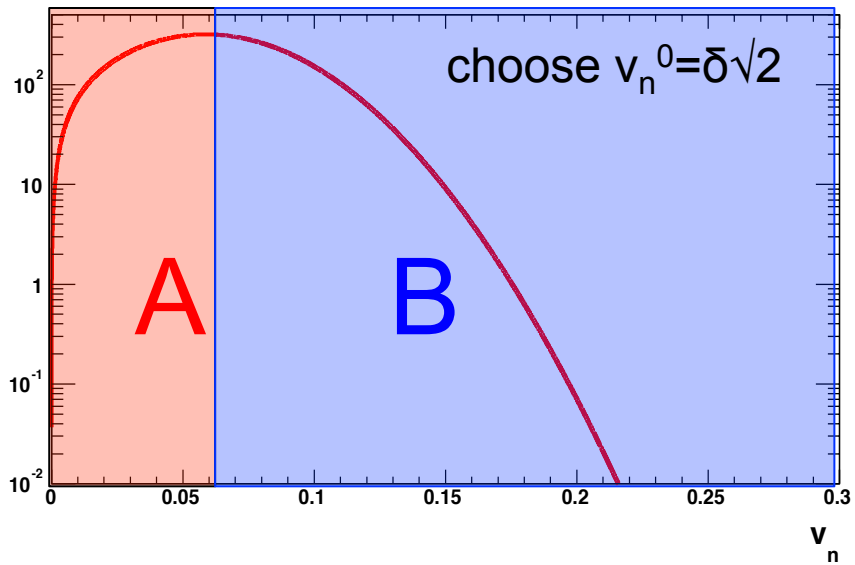
Other details lead to only tiny differences between v{4},v{6},…

Are cumulants sensitive to non-Gaussianity?

$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2+(v_n^0)^2}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^0}{\delta_n^2}\right)$$

$$v_n\{4\} = v_n\{6\} = v_n\{8\} = .. = v_n^0$$

$$v_n\{2\}^2 = \left(v_n^0\right)^2 + 2\delta^2$$



- Divide B-G distri. to 2 equal parts, and calculate cumulants separately.

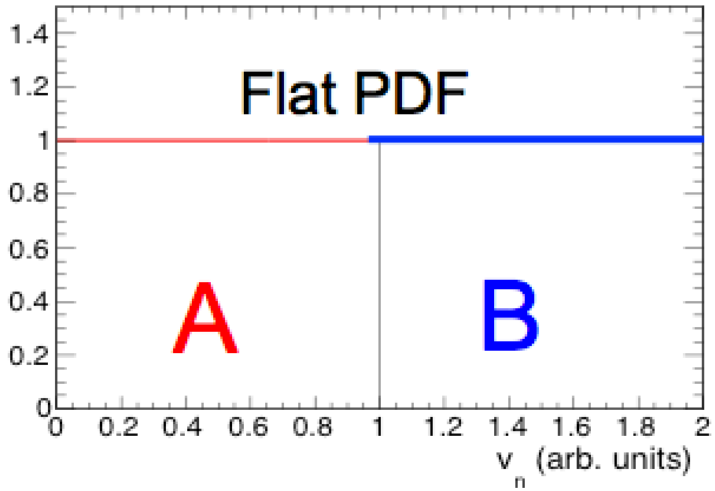
	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$
all	1.414	1	1	1
A	0.851	0.759	0.746	0.744
B	1.809	1.690	1.701	1.701

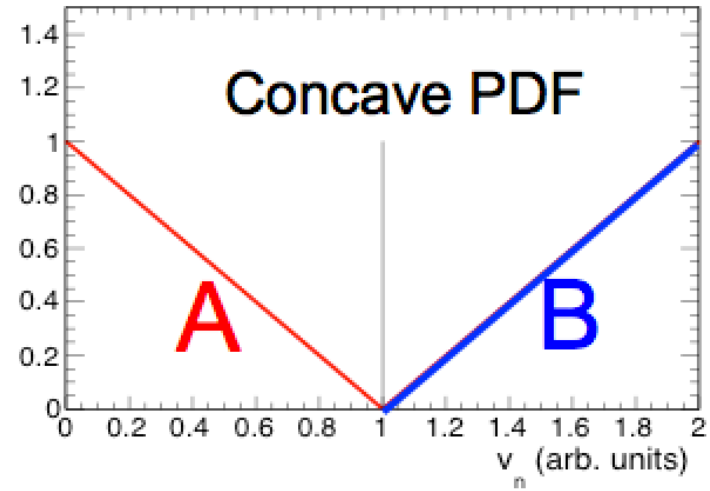
In units of v_n^0

- Differences between v{4}, v{6},… are very small

Higher-order cumulants not very sensitive to p(v) beyond mean and width?

For very broad distribution (example)





Negative v{2k} ➔ $c_n\{2k\}$ wrong sign

	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$
A	0.5774	0.3861	-0.3834	-0.4457
B	1.5275	1.4715	1.47016	1.47015

	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$
A	0.4083	-0.3247	-0.3602	-0.3060
B	1.6833	1.6524	1.6513	1.6512

$v_n\{2k\}$ similar when central-moments<<moments

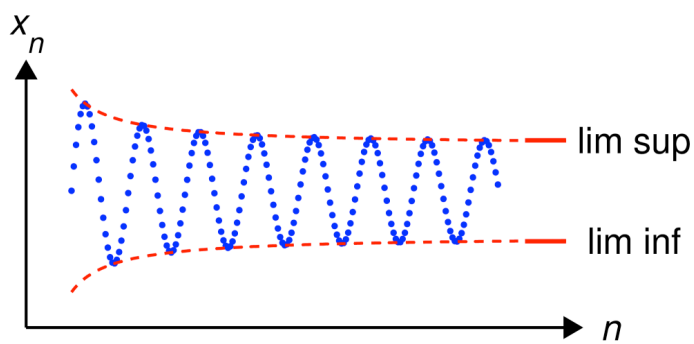
- In principle can't use the $c_2\{4\}<0$ as sign of onset of collectivity
  - Unless we have some a priori knowledge of its shape, i.e. close to BG

How cumulants depends on p(v)?

$I_0(2z) = J_0(2iz)$

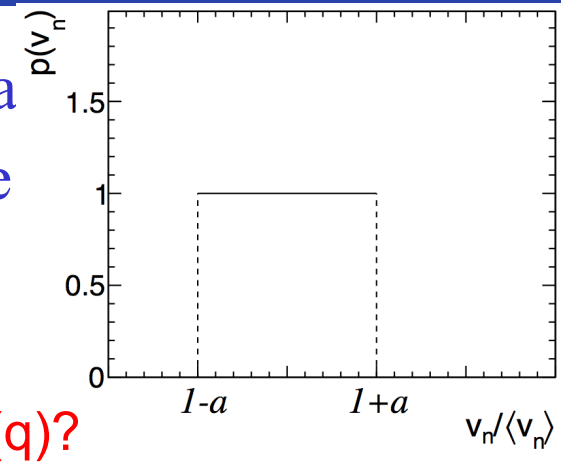
- Cumulant defined via: $\ln\langle I_0(2vz)\rangle = \ln\left\langle e^{z(\boldsymbol{v}+\bar{\boldsymbol{v}})}\right\rangle = \sum_{k=1}^{\infty}\frac{z^{2k}}{k!^2}c\{2k\}$
- Redefine to absorb sign: $-\ln\langle J_0(2vz)\rangle = \ln\left\langle e^{iz(\boldsymbol{v}+\bar{\boldsymbol{v}})}\right\rangle = \sum_{k=1}^{\infty}\frac{z^{2k}a_k}{k!^2}\chi\{2k\}$
  - $a_k = 1, 1, 4, 33, 456 \ldots$ defined by $-\ln J_0(2z) = \sum_{k=1}^{\infty}\frac{z^{2k}a_k}{k!^2}$
- This way the flow values can be calculated as $v\{2k\} \equiv \sqrt[2k]{\chi\{2k\}}$
- Convergence of $\lim_{k\to\infty} v\{2k\}$ requires $\langle J_0(2vz)\rangle = \int J_0(2vz)p(v)dv$ has 0 in complex plane, i.e. LYZ method.
  - **Necessary but not sufficient** condition when p(v) is non-Gaussian.

$f(z) = \sum_{k=0}^{\infty} a_n z^k$ ➔ $c = \limsup_{k\to\infty}\sqrt[k]{|a_k|} = R_{conv}$, but $c = \lim_{k\to\infty}\sqrt[k]{|a_k|}$ may not exist

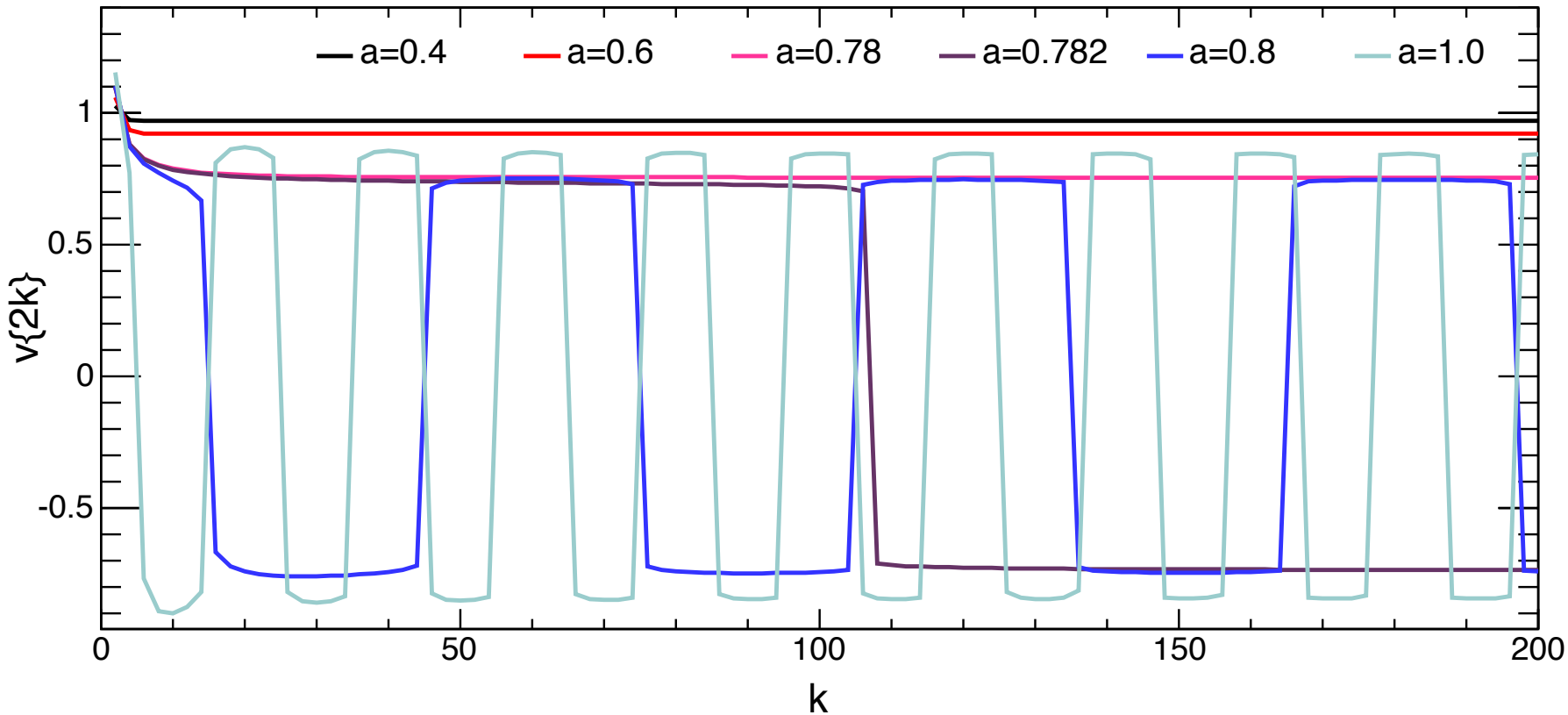


Behavior of v{2k} from narrow to broad

- Consider v{2k} for a flat p(v), width dialed by a
- As p(v) become broader, v{2k} converges more slowly, and eventually flip sign.



Need all cumulants to understand p(v) if it is far from Gaussian. So why not just measure/calculate p(v) or p(q)?



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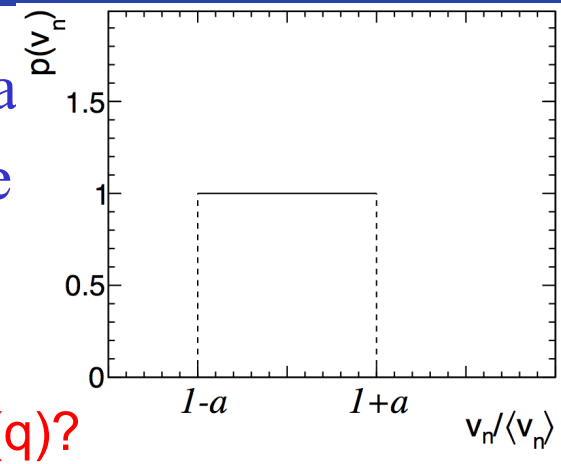

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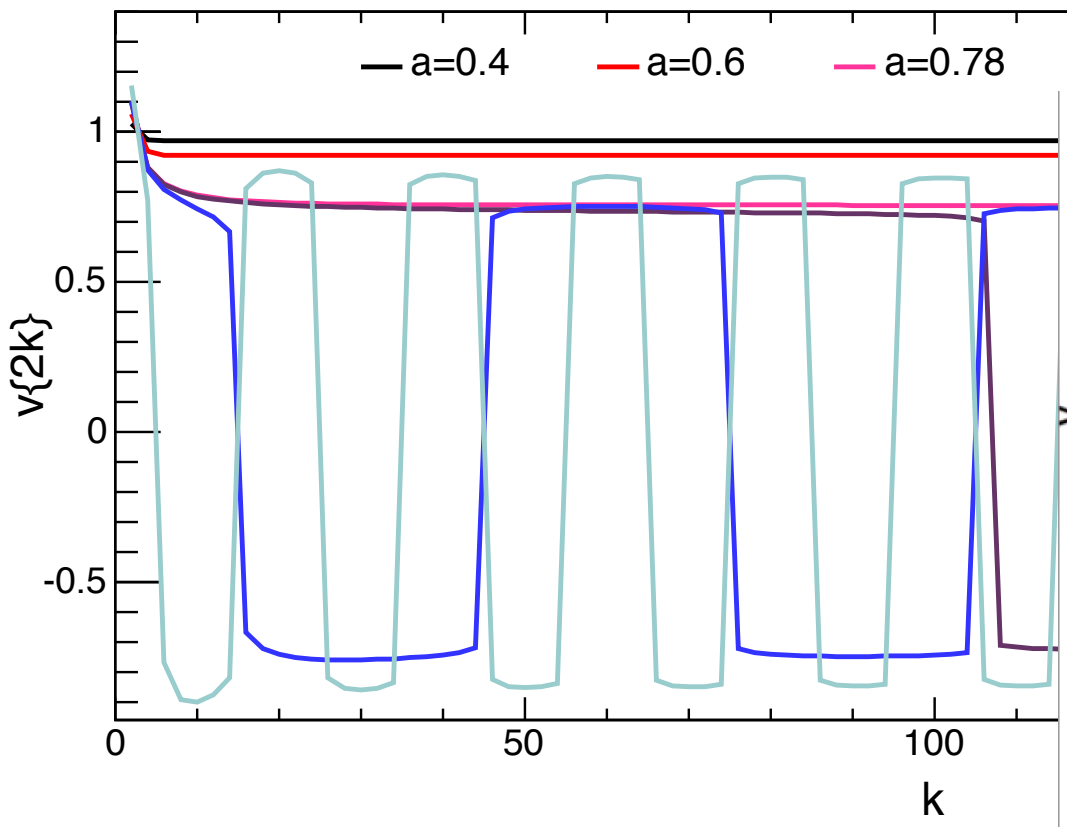
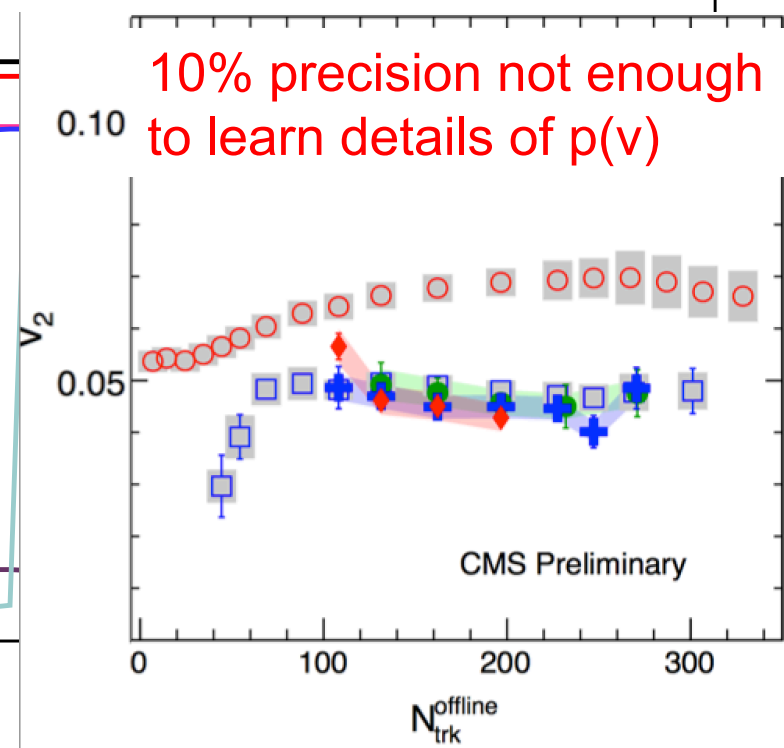


10% precision not enough to learn details of p(v)



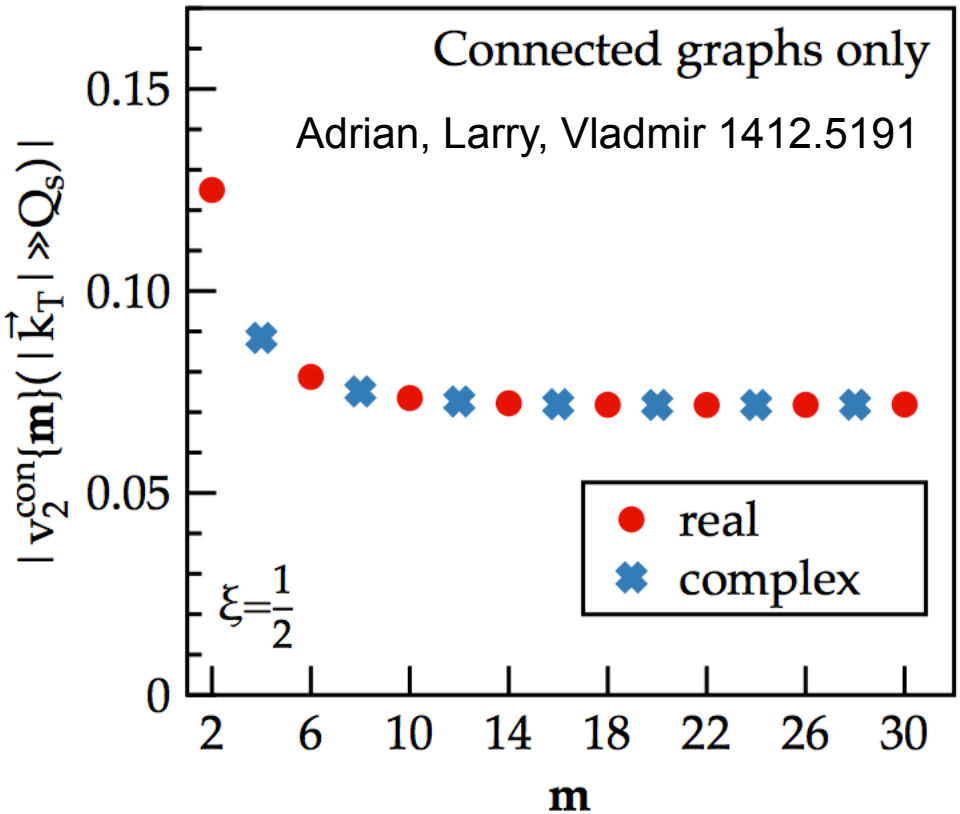
p(q) form initial state color fluc?

- May imply the related p(v) is strongly non-Gaussian
- To understand the correlations from coherent color field, one could just calculate p(q), since $c_q\{2k\} \cong c_v\{2k\}$!

$$c_2\{2\} = \frac{1}{N_D}\left(\mathcal{A}^2 + \frac{1}{4(N_c^2-1)}\right)$$

$$c_2\{4\} = -\frac{1}{N_D^3}\left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right)$$

A is q without connected terms



Summary

- Studied how cumulants depends on flow and nonflow
- Non-flow suppressed by averaging over the events, not in each event
- Hence $p(q_n)$ contains ≈ full information of flow & nonflow
  - q_n is a single particle quantity: $q_n = \left|\left\langle e^{in\phi}\right\rangle_{\mathrm{ev}}\right|$
- Since $\boldsymbol{v}_n = \boldsymbol{q}_n - \boldsymbol{s}_n$ for each event, non-flow contribution to cumulants can be subtracted order-by-order $c_n\{2k\} = c_n\{2k\}_q - c_n\{2k\}_s$
  - Data-driven two-subevent method allow subtraction nonflow and coupling between flow and non-flow due to trivial statistics
  - Correlations are need only for nonflow removal, not for flow.
- $v_n\{2k\}$ is influenced or even dominated by non-Gauss. of $p(v_n)$
  - Especially important in small system where flow & nonflow (initial color field) can be large.
  - In this case, probably $p(q_n)$ is a simpler quantity to measure/calculate.
  - $v_2\{2k\}$ data need to have much smaller uncertainty to have meaningful constraints on $p(v_2)$.

Size, shape, and dissipative length in small colliding systems

Yin Jiang

In collaboration with Jinfeng Liao

Indiana University

Outline

- Motivation
- Wave function
- Density distribution
- Eccentricity
- Dissipative length
- Conclusions

Motivations

- pA and pp collisions' high multiplicity probe rare fluctuations.
- Proton's spatial eccentricity may be reflected in the final-state flow.
- The validity of the fluid description for small systems.

➢ Proton size and shape fluctuation (density, radius and eccentricity distribution). Motivated by 1301.5911

➢ Dissipative length for a small system

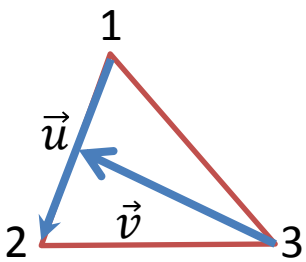
Wave function

- Cornell potential $V(r) = -\frac{\sigma}{r} + \kappa\, r$
- Two body wave function behavior
- $r \to 0, \phi \to e^{-a\, r}$
- $r \to +\infty\ , \phi \to e^{-b\, r^{\frac{3}{2}}}/(c\, r)^{1/4}$
- Interpolation
- $\phi = \exp[-a\, r - \left((b\, r)^{\frac{3}{2}} - a\, r\right)\frac{(c\, r)^2}{(c\, r)^2+1} - \ \frac{1}{2}\ln(\sqrt{c\, r} + 1)]$

Wave function

- 3-body ground state, variational method
- $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3; a, b, c)$

$$= \phi(|\vec{r}_2 - \vec{r}_1|)\phi(|\vec{r}_3 - \vec{r}_2|)\phi(|\vec{r}_1 - \vec{r}_3|)$$

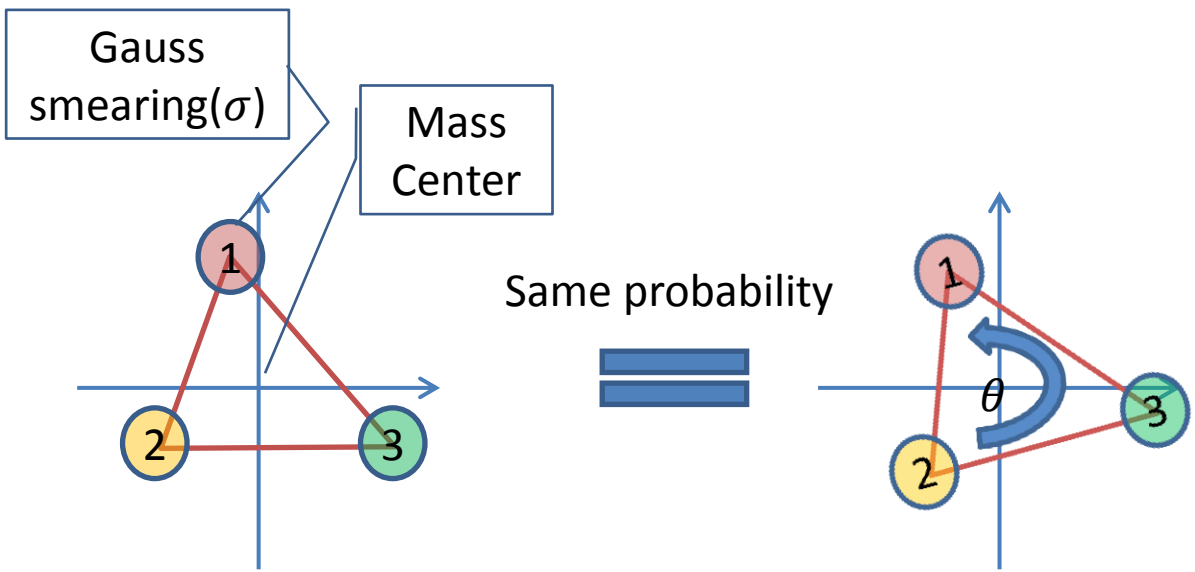


$$= \phi(|\vec{u}|)\phi\left(\left|\frac{1}{2}\vec{u} + \vec{v}\right|\right)\phi\left(\left|\frac{1}{2}\vec{u} - \vec{v}\right|\right)$$

- Given $u = |\vec{r}_2 - \vec{r}_1|$, $v = \left|\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right|$ and angle $\vec{u} \cdot \vec{v} = u\, v \cos(\varphi)$, the shape of the triangle and the wave function is determined.

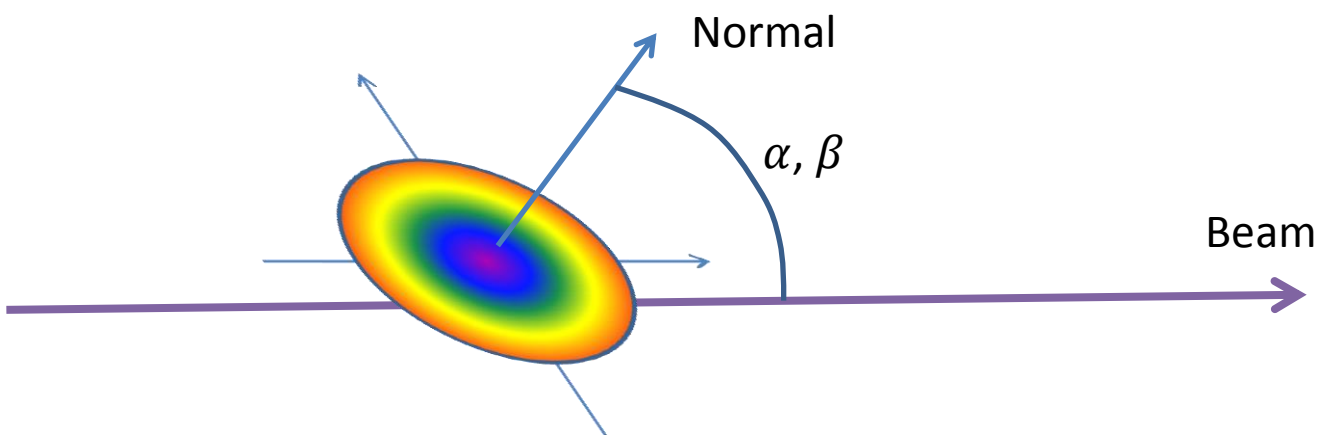
Transverse density

- Given u, v and angle φ, the shape of the triangle and the value wave function are determined.



Transverse density

- The orientation of the triangle plane normal $(\sin\alpha\cos\beta\,, \sin\alpha\sin\beta\,, \cos\alpha)$ is random.

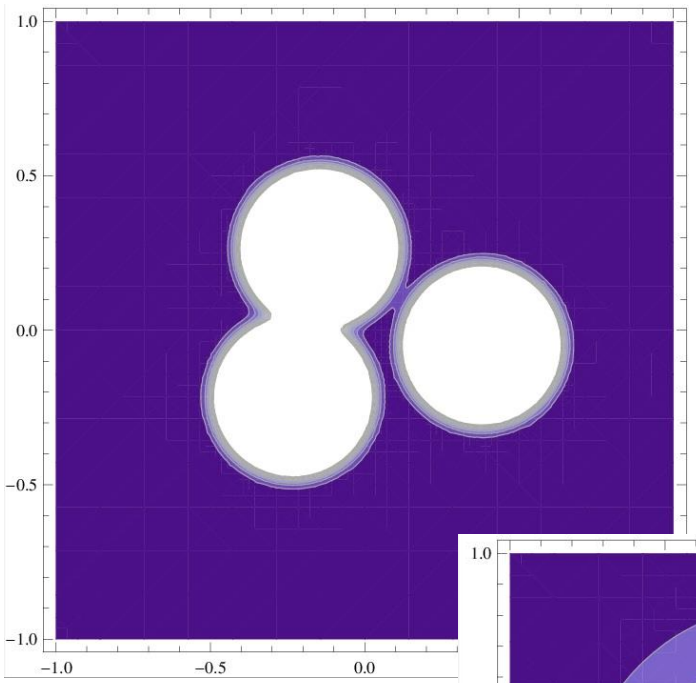


- We get the final distribution

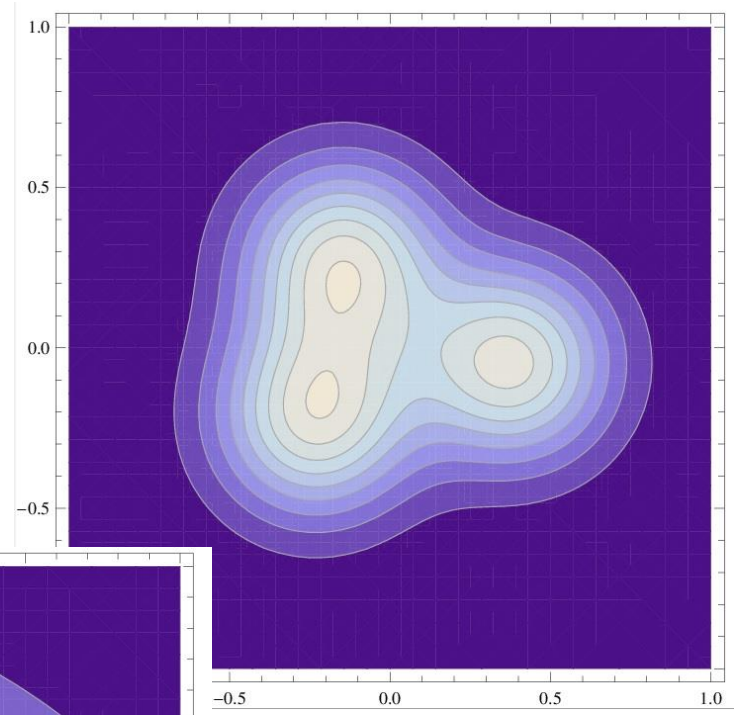
$$f(x, y, z; u, v, \cos\varphi\,, \theta, \cos\alpha\,, \beta) \xrightarrow{\text{Boost}}$$

$$\mathrm{g} = \int dz\, f(x, y, z, u, v, \cos\varphi\,, \theta, \cos\alpha\,, \beta)$$

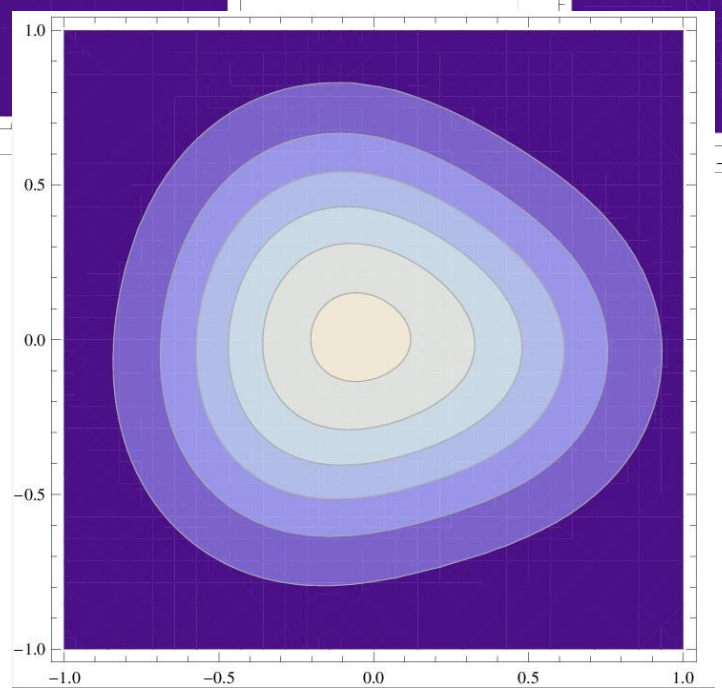
Some events



σ=0.1fm

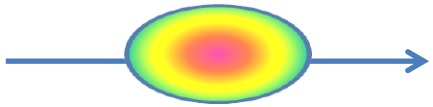


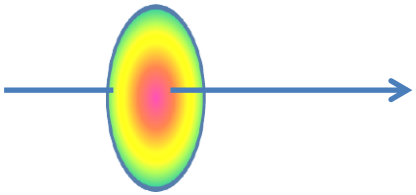
σ=0.3fm

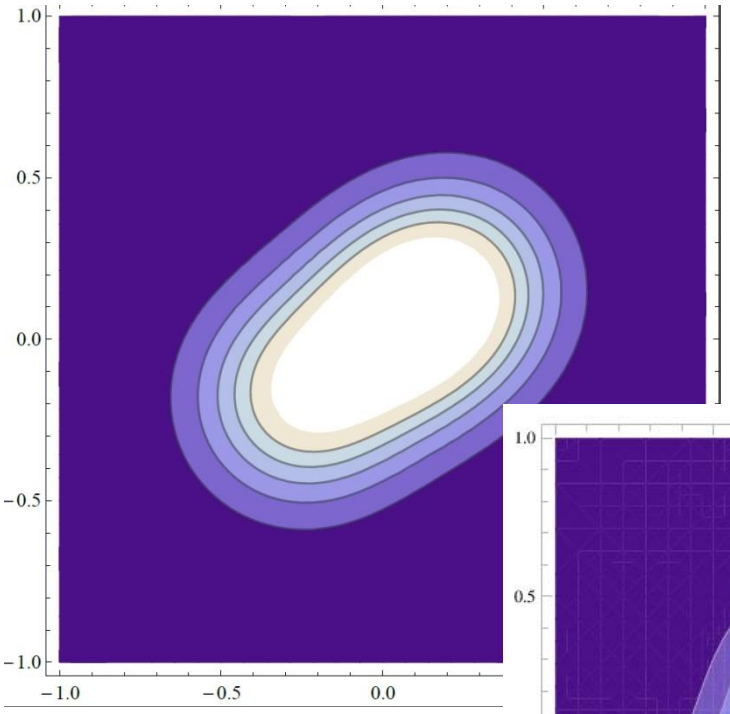


σ=0.5fm

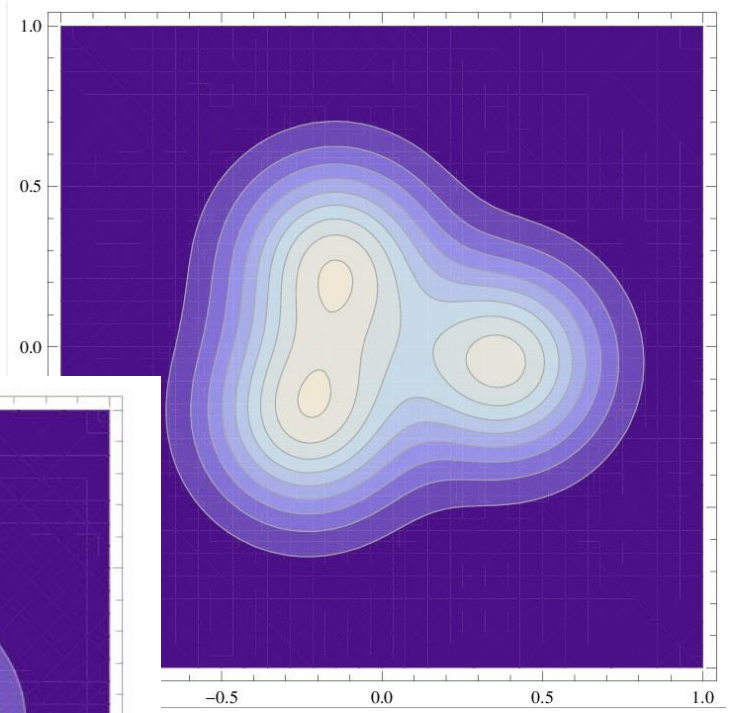
Some events



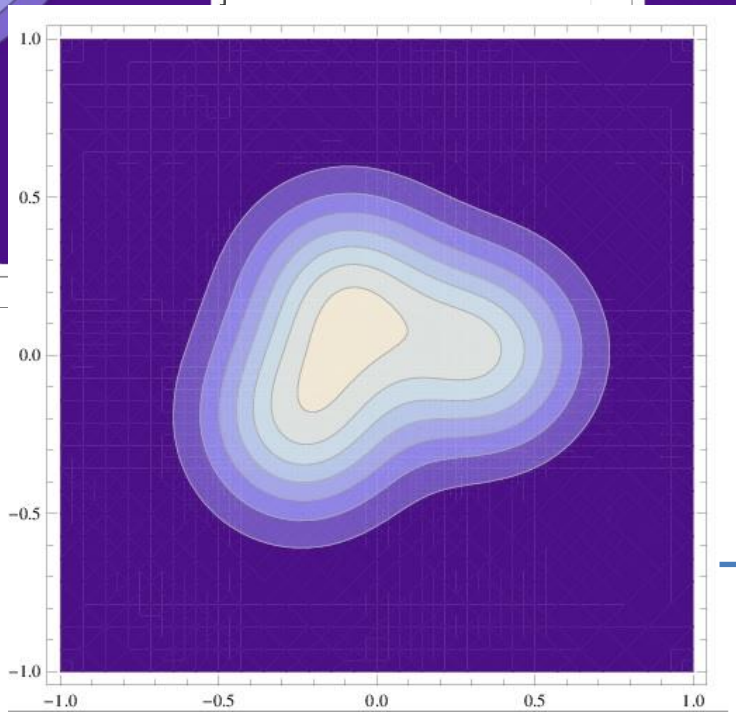




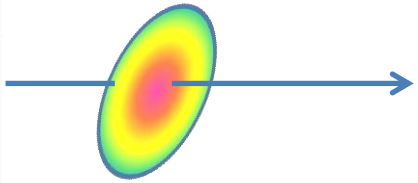
$\cos\alpha$=0



$\cos\alpha$=1



$\cos\alpha$=0.5



Nucleon thickness function

- The averaged 2D density distribution of the proton as

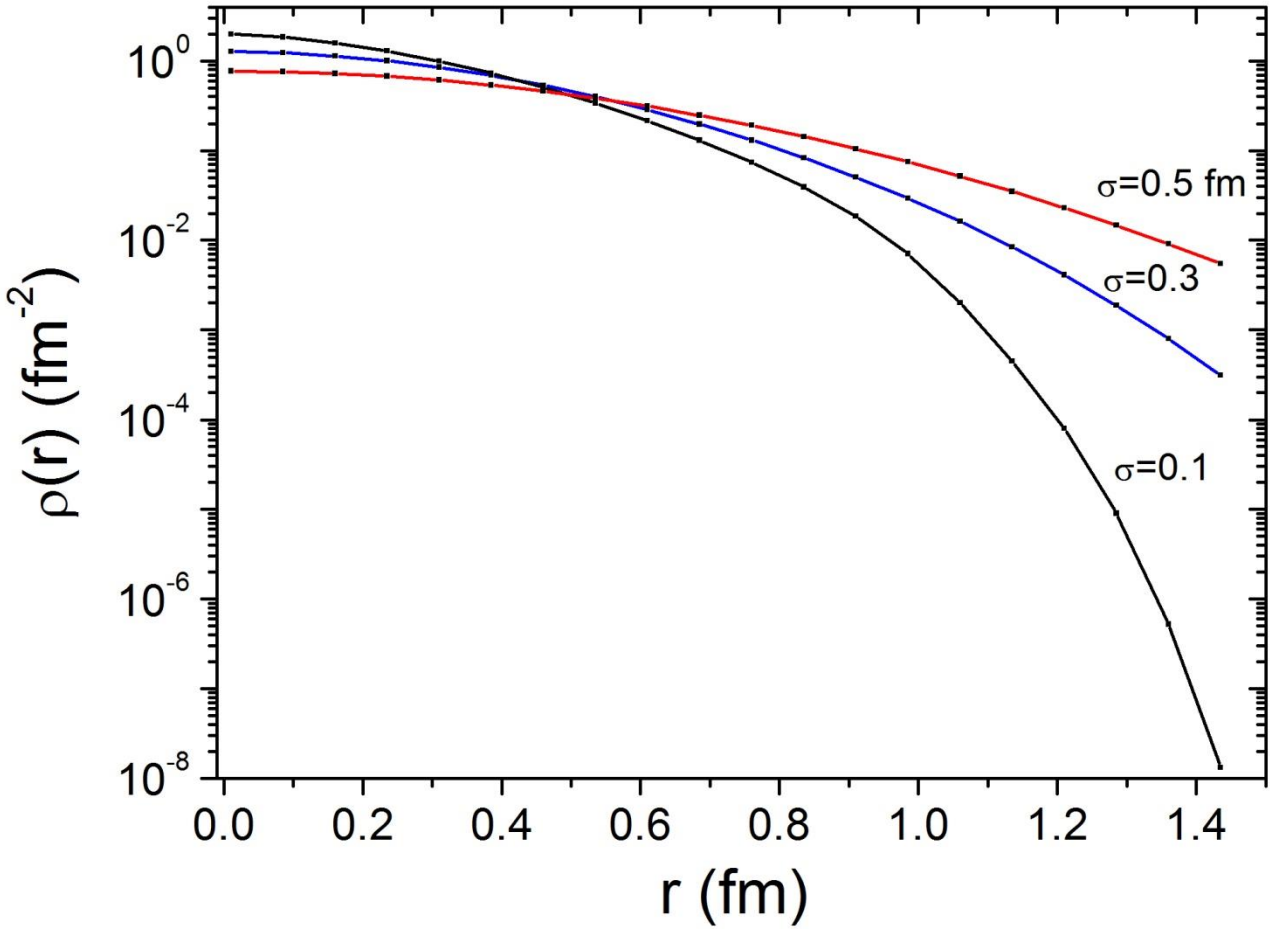
$$\rho\left(r=\sqrt{x^2+y^2}\right)$$

$$=\int du\, dv\, d\cos\varphi\, u^2 v^2\, \frac{1}{8\pi^2}\int d\cos\alpha\, d\beta\, d\theta\, g\, \Psi^2(u,v,\varphi)$$

- $\{u, v, \cos\varphi\}$ obey distribution $u^2 v^2\, \Psi^2(u,v,\varphi)$
- $\{\alpha, \cos\beta\, , \theta\}$ obey uniform distribution.

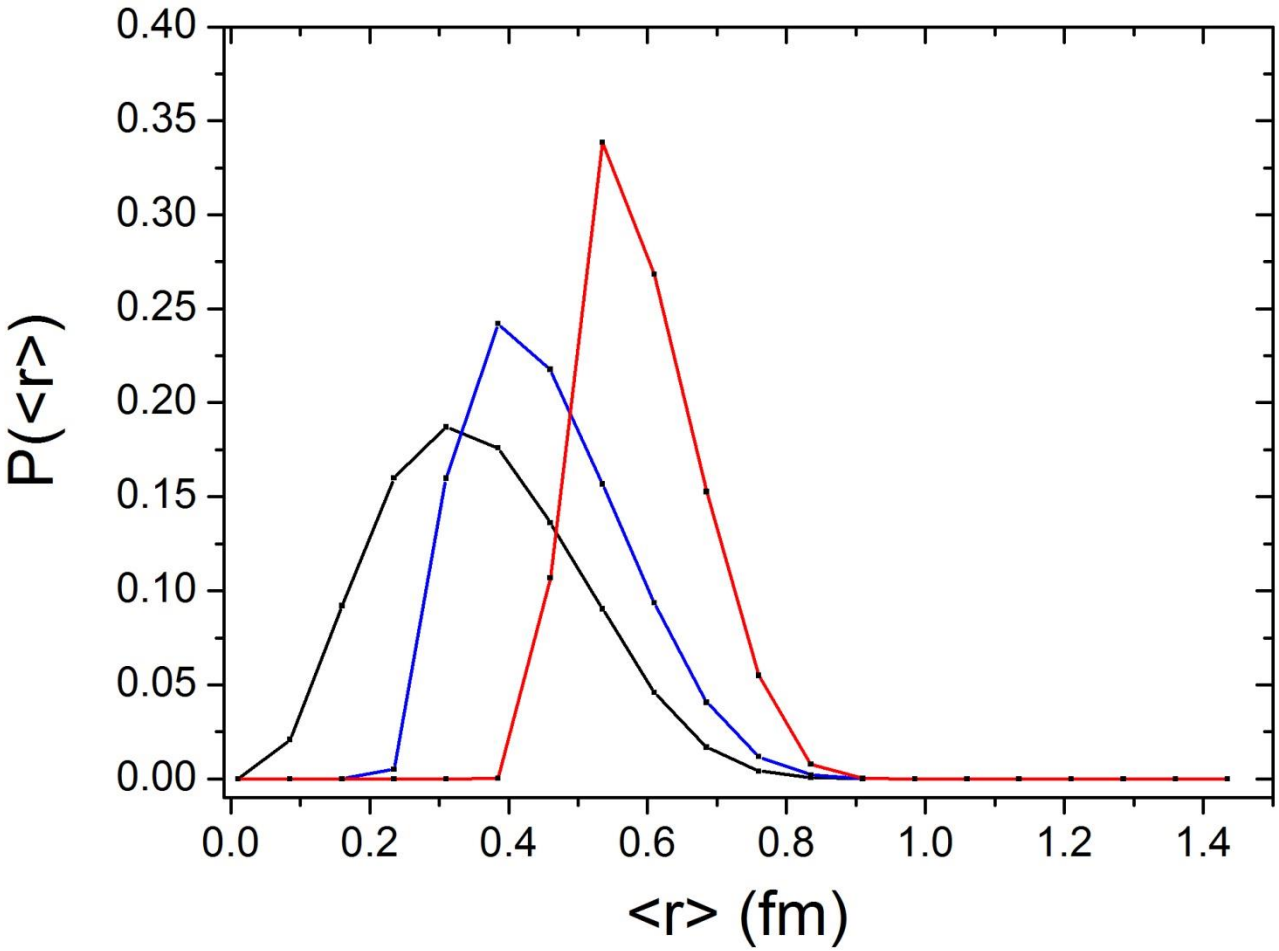
Nucleon thickness function

- Gauss smearing width σ dependence.



Average radius distribution

- Gauss smearing width σ dependence.



Eccentricity

- For a given event $\{u, v, \cos\varphi\,, \alpha, \cos\beta\,, \theta\}$, transverse distribution $\rho(r, \xi)$ is anisotropic.
- Eccentricity

$$\epsilon_n = \langle \cos(n\,\xi - n\,\Phi_n) \rangle$$

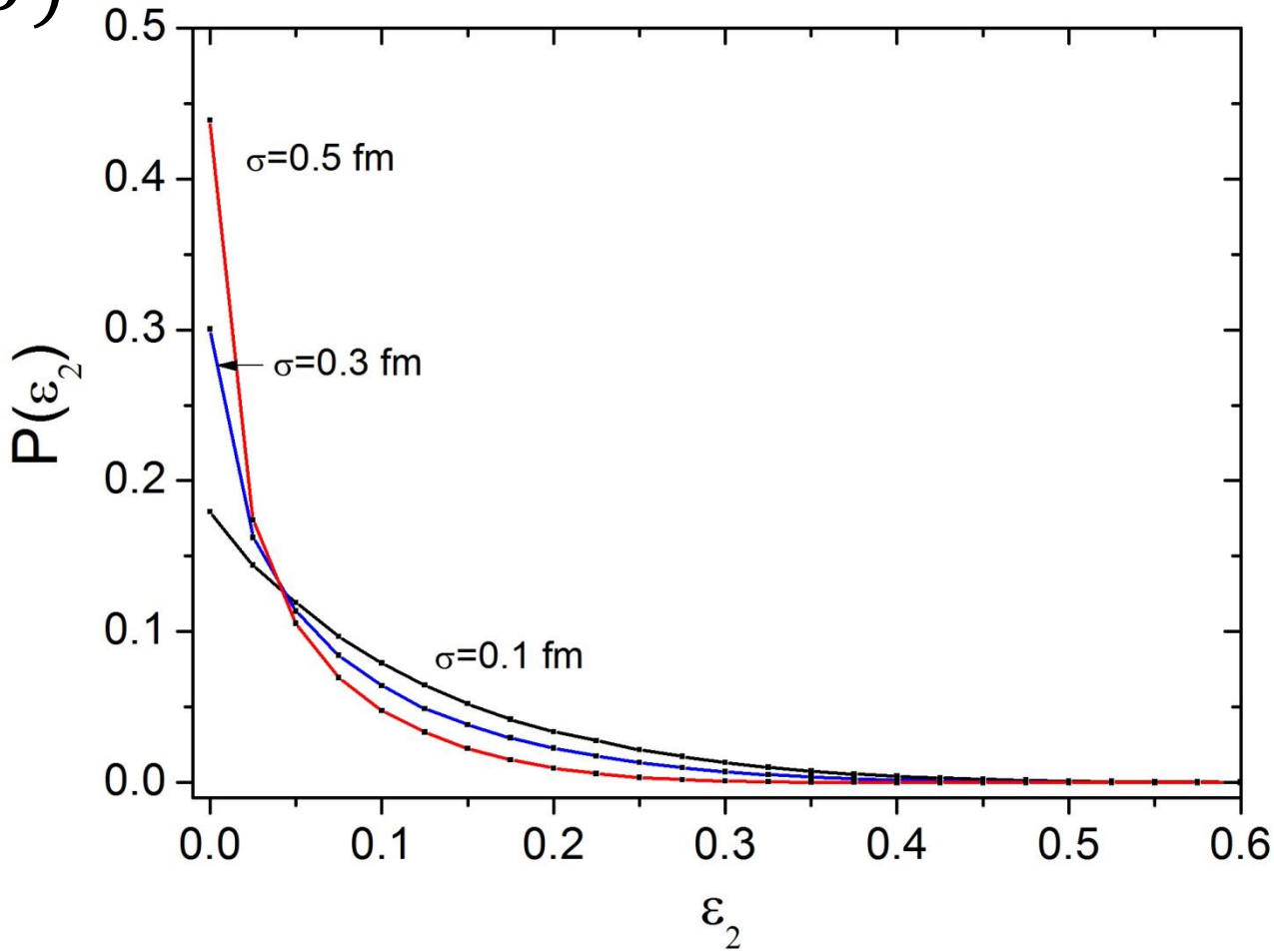
- Event plane angle

$$n\,\Phi_n = \arctan\langle \sin(n\,\xi) \rangle / \langle \cos(n\,\xi) \rangle$$

- Determining the $\epsilon_n(u, v, \cos\varphi\,, \alpha, \cos\beta\,, \theta)$ by using the distribution of $\{u, v, \cos\varphi\,, \alpha, \cos\beta\,, \theta\}$.

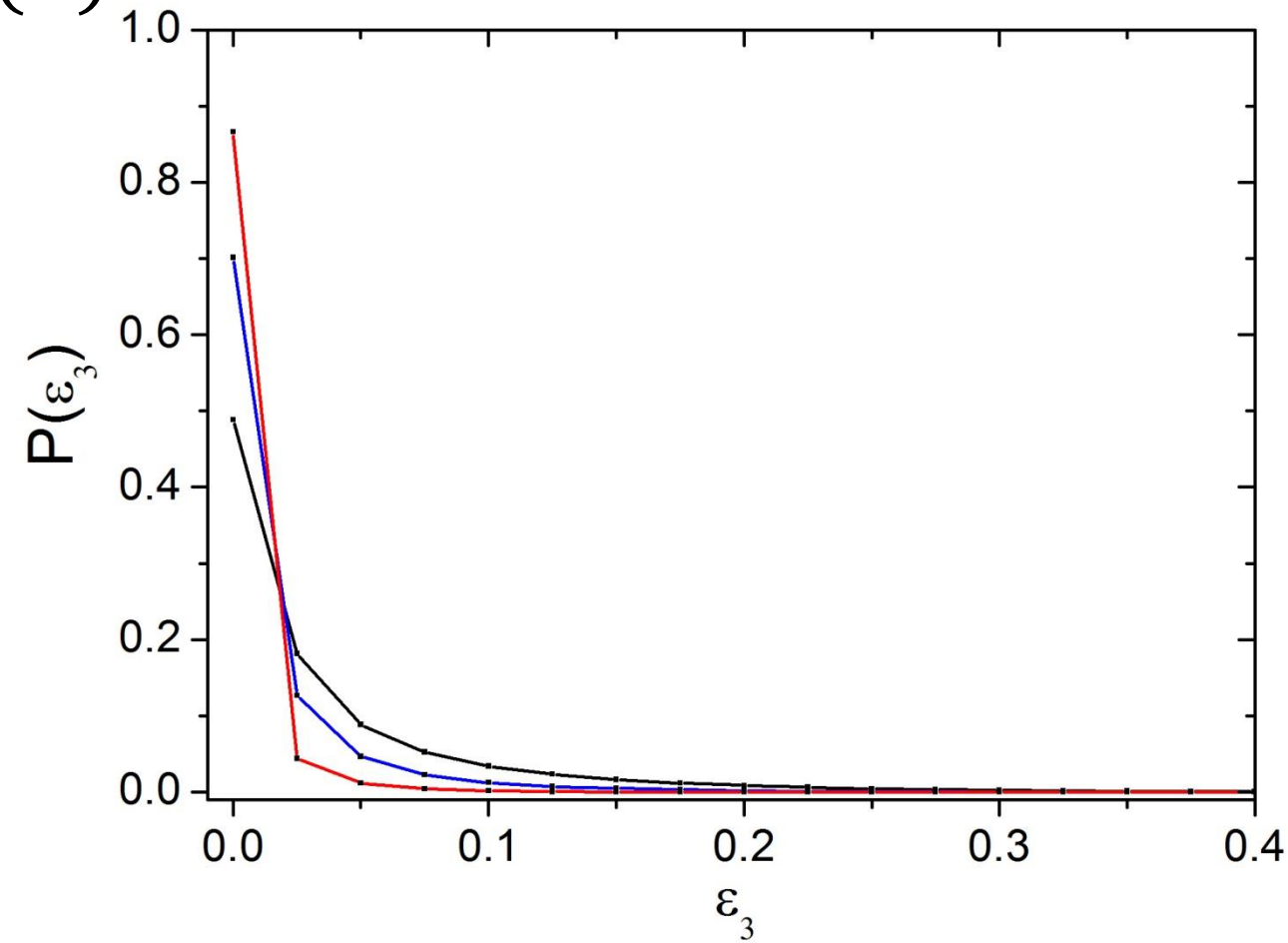
Eccentricity

- Gauss smearing width σ dependence.
- $\epsilon_2(\sigma)$



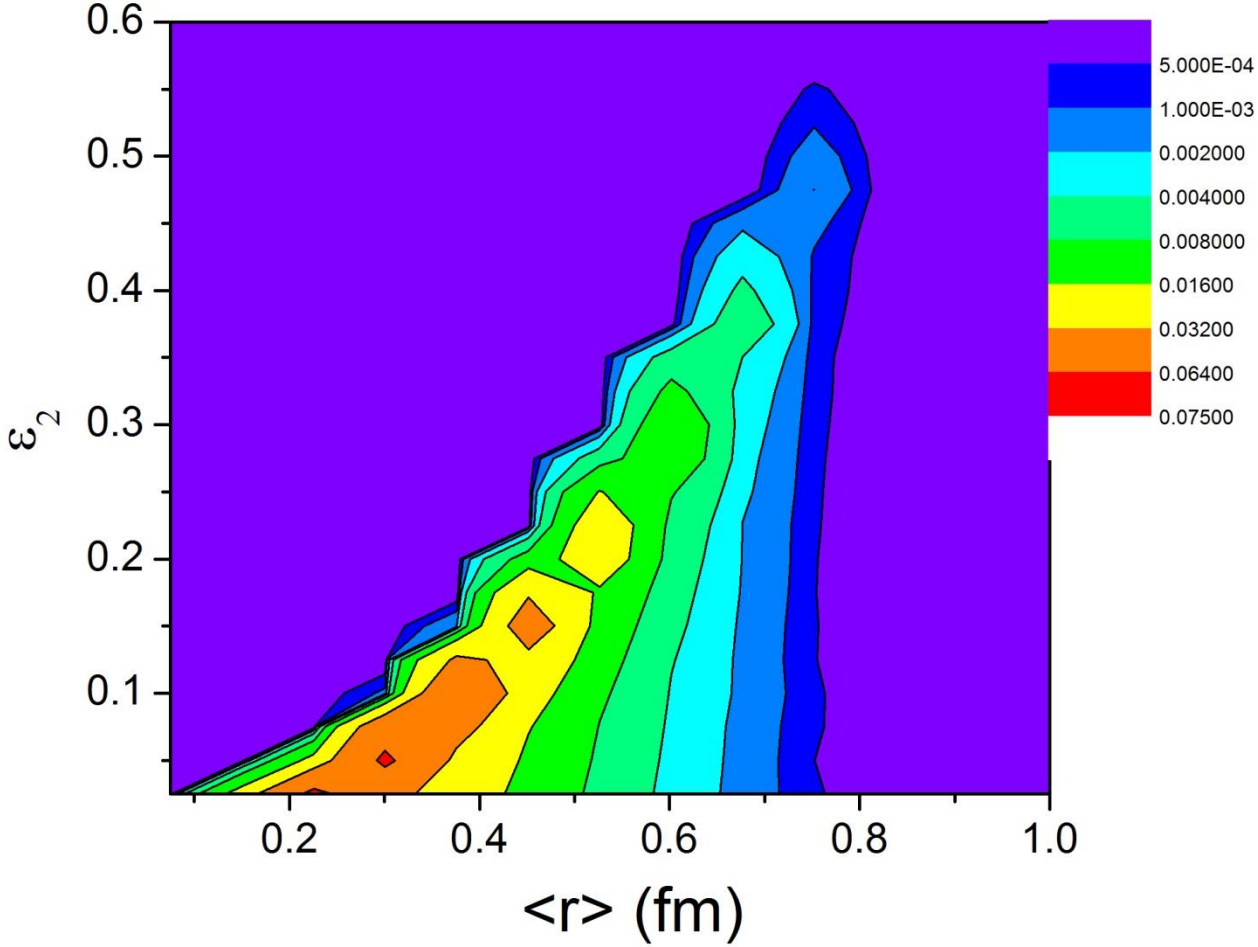
Eccentricity

- Gauss smearing width σ dependence.
- $\epsilon_3(\sigma)$



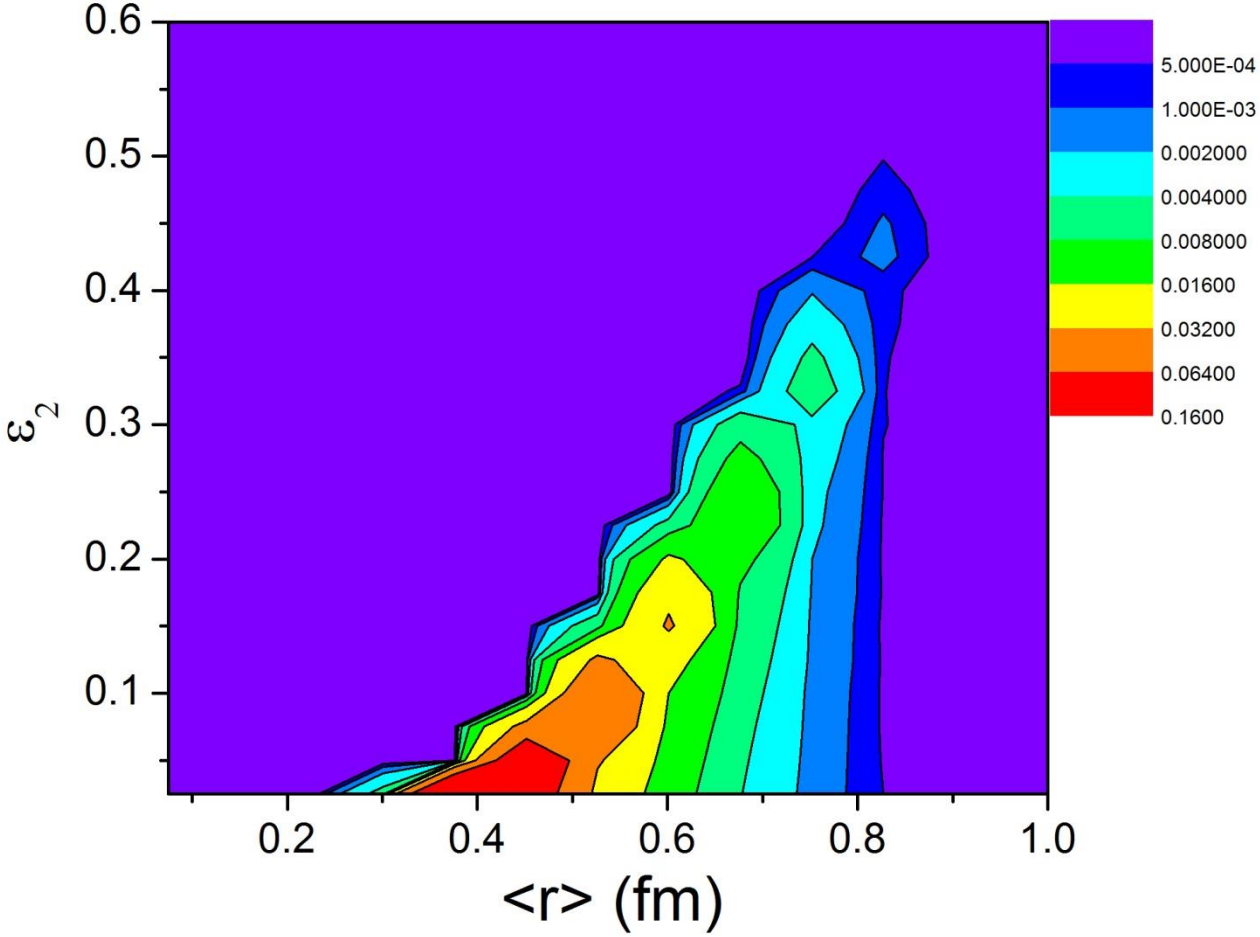
Correlations

- σ=0.1fm



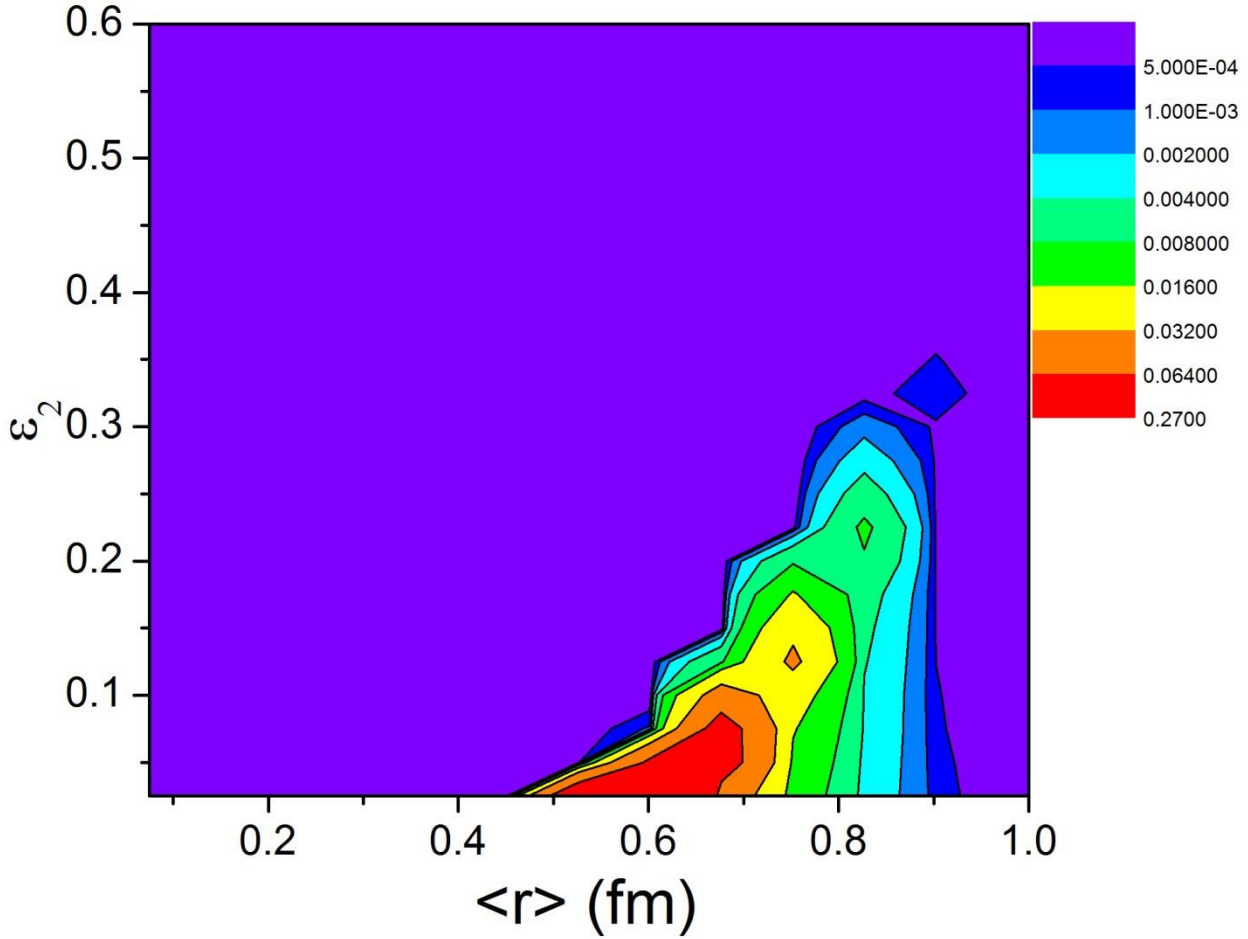
Correlations

- σ=0.3fm



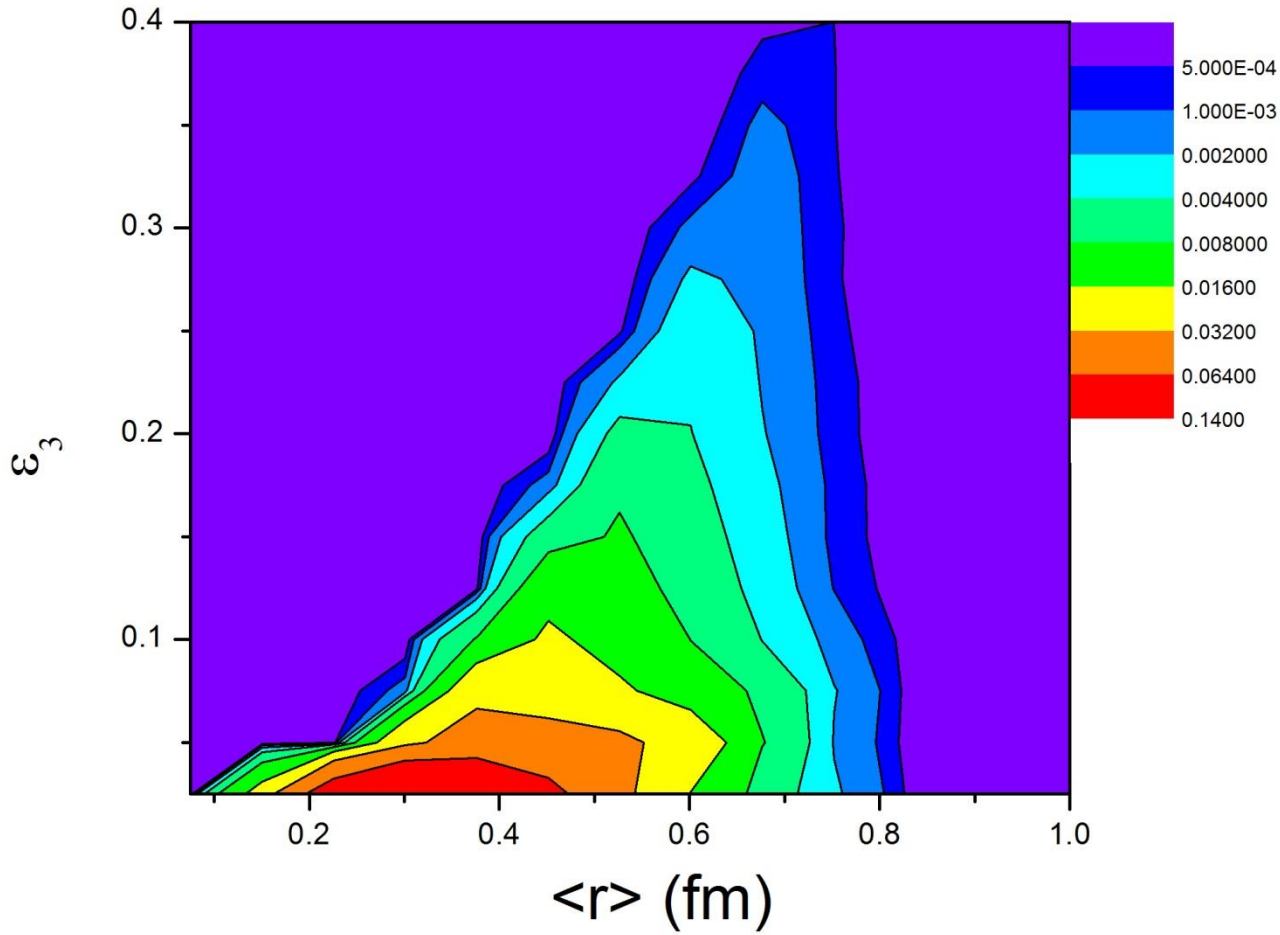
Correlations

- σ=0.5fm



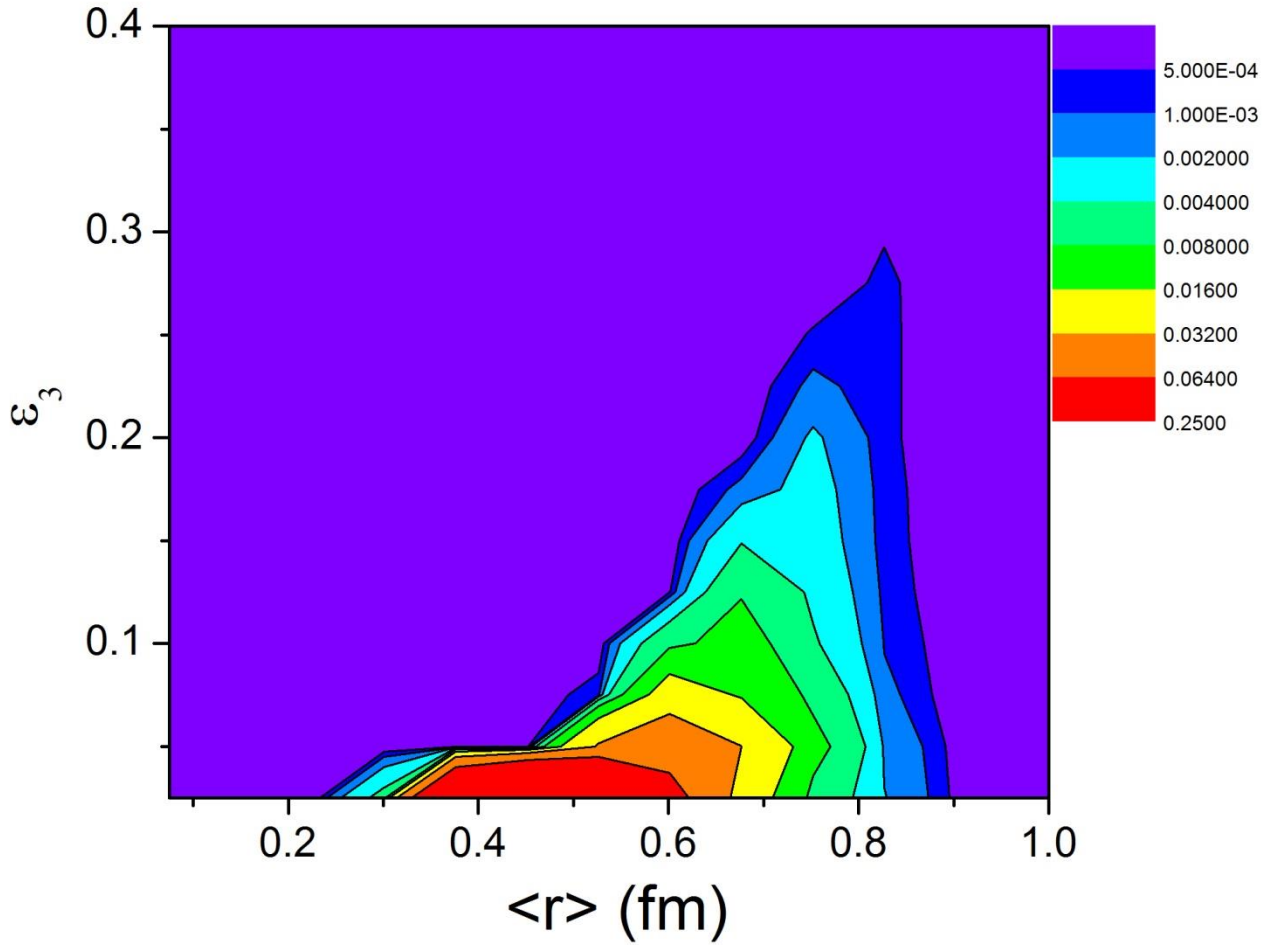
Correlations

- σ=0.1fm



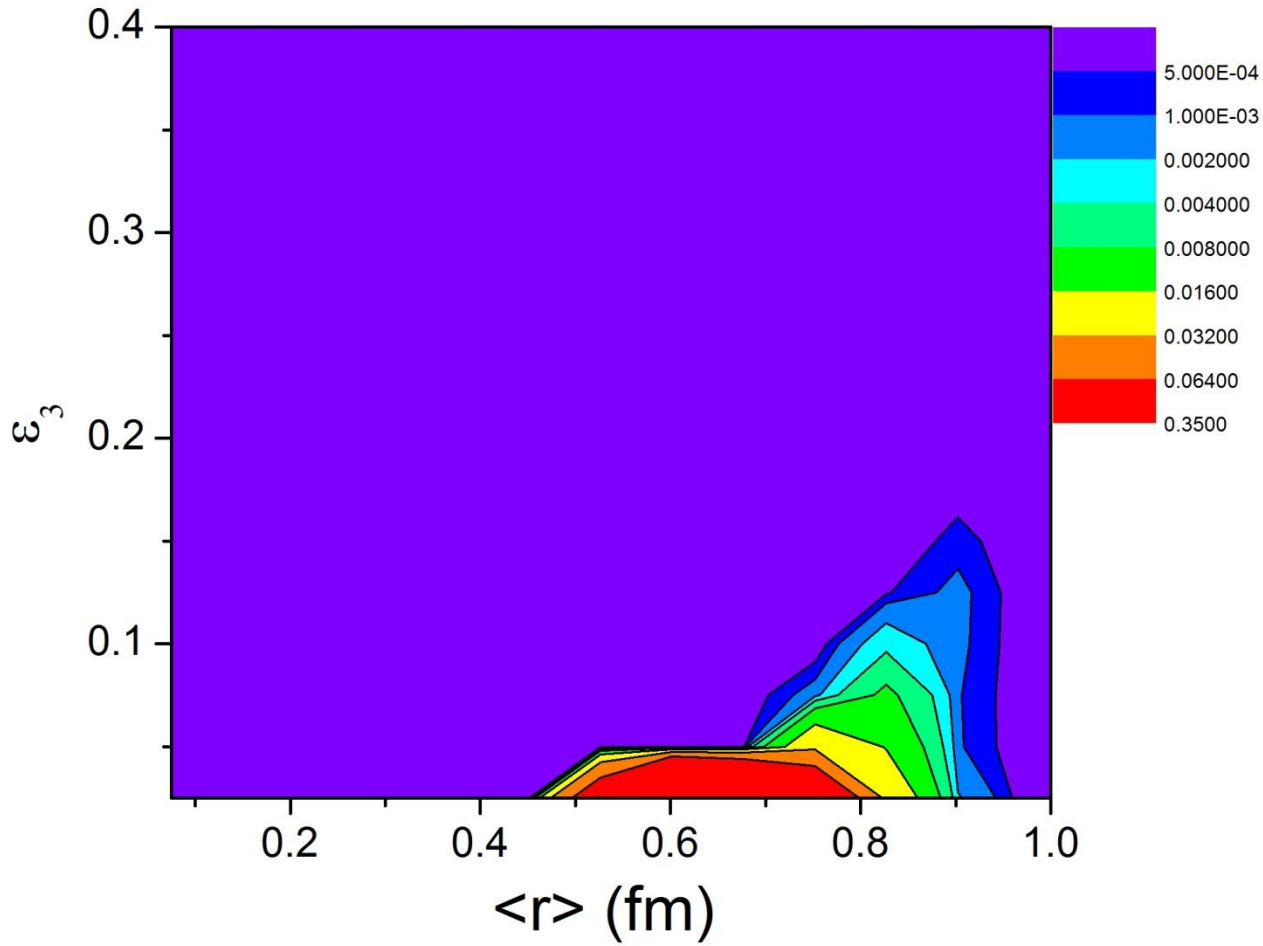
Correlations

- σ=0.3fm



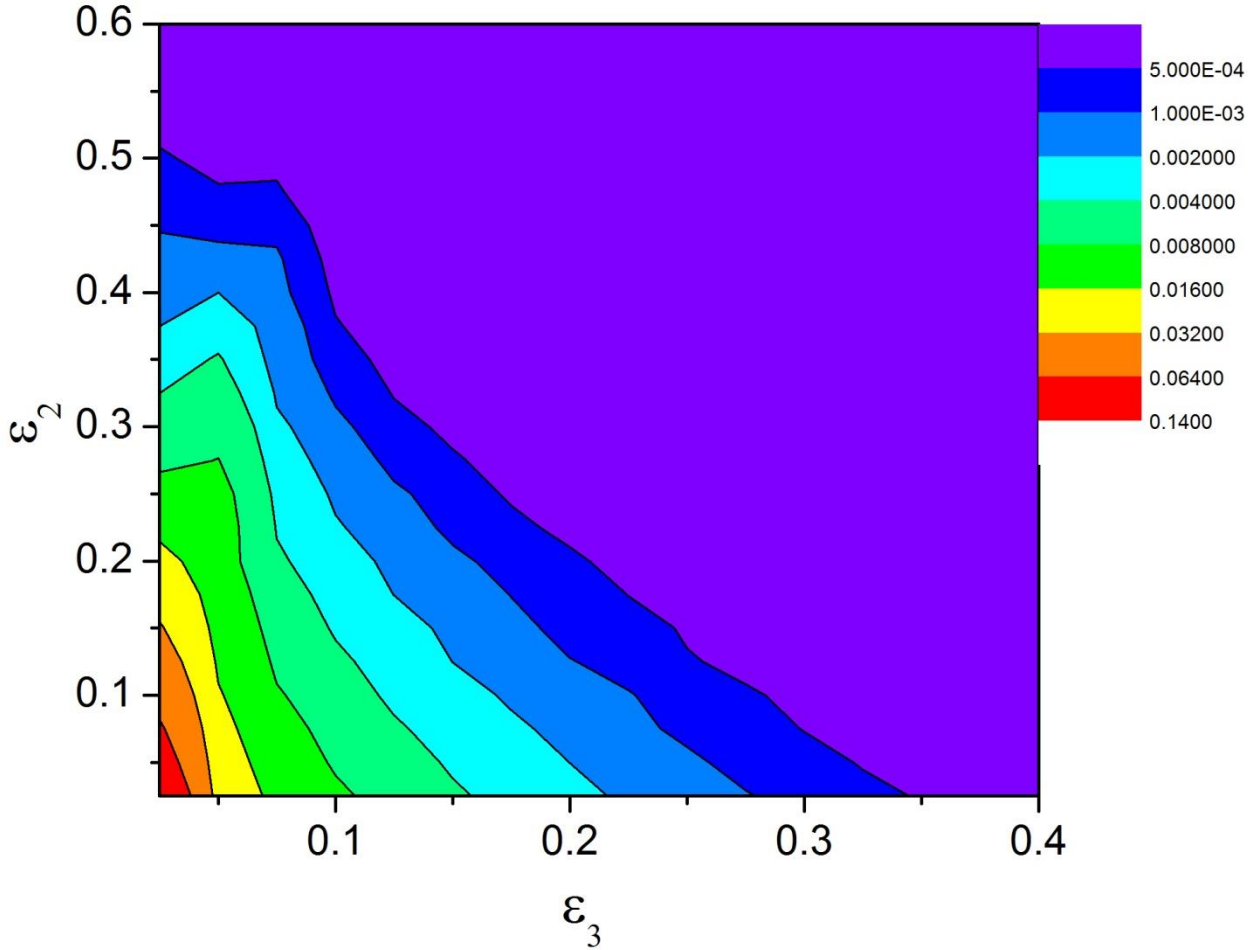
Correlations

- σ=0.5fm



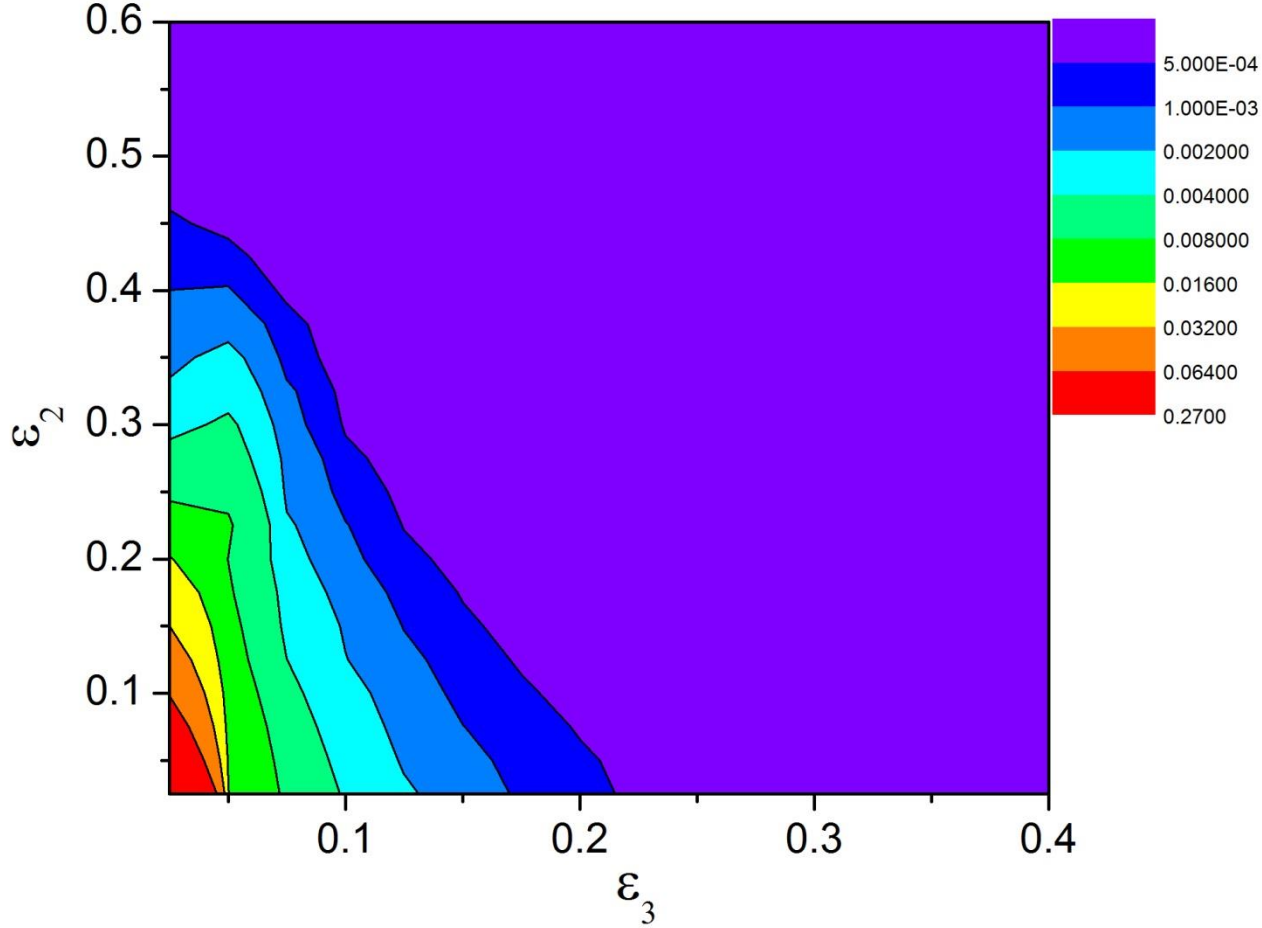
Correlations

- σ=0.1fm



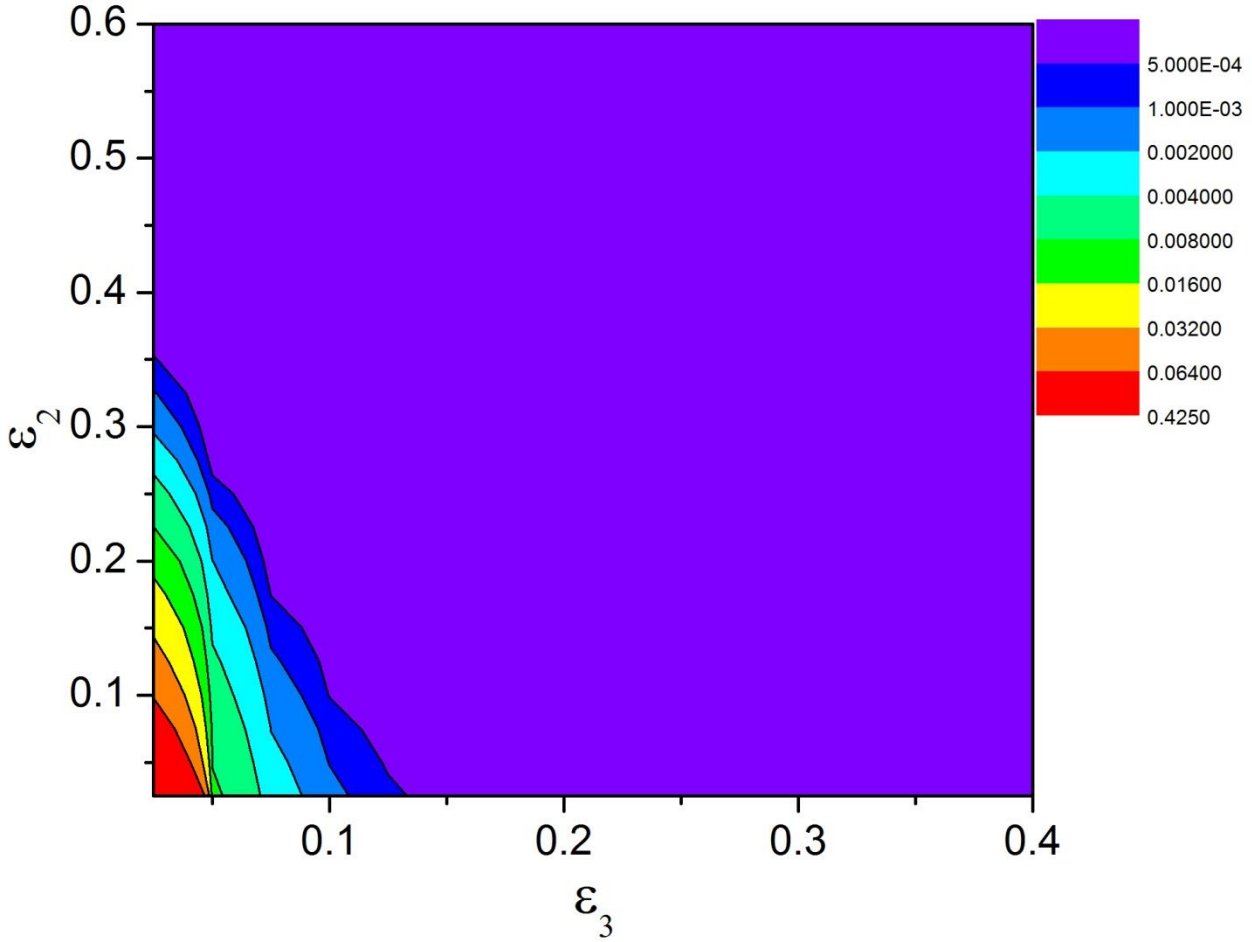
Correlations

- σ=0.3fm



Correlations

- σ=0.5fm



Dissipative length

- Hydrodynamic valid with small Knudsen number

$$\mathrm{Kn} = l/L$$

- Dissipative length in relativistic fluid

Phys. Rev. C **81**, 014902

$$l = \left(\frac{\eta}{s}\right)\frac{1}{c_s T} \Longrightarrow \mathrm{Kn} = \left(\frac{\eta}{s}\right)\frac{1}{c_s T L}$$

- Keeping $S = sL^3 \approx const$

- High temperature $s/T^3 \approx const$, $c_s \approx const$.

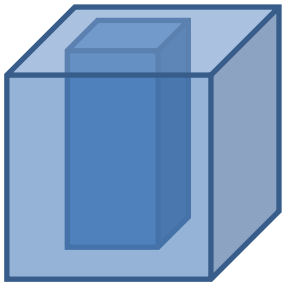
Dissipative length

- Squeezing the system $L' = L/\lambda, \lambda > 1$

$$l = \left(\frac{\eta}{s}\right)\frac{1}{c_s T}$$

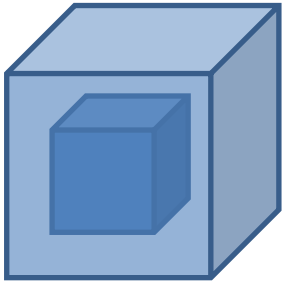
- 2D case $s' = s\,\lambda^2, T' = T\,\lambda^{2/3}, l' = \left(\frac{\eta}{s}\right)_{T'}\frac{\lambda^{-2/3}}{c_s T}$

then $Kn' = \lambda^{1/3}\left(\frac{\eta}{s}\right)_{T'} / \left(\frac{\eta}{s}\right)_{T} Kn$



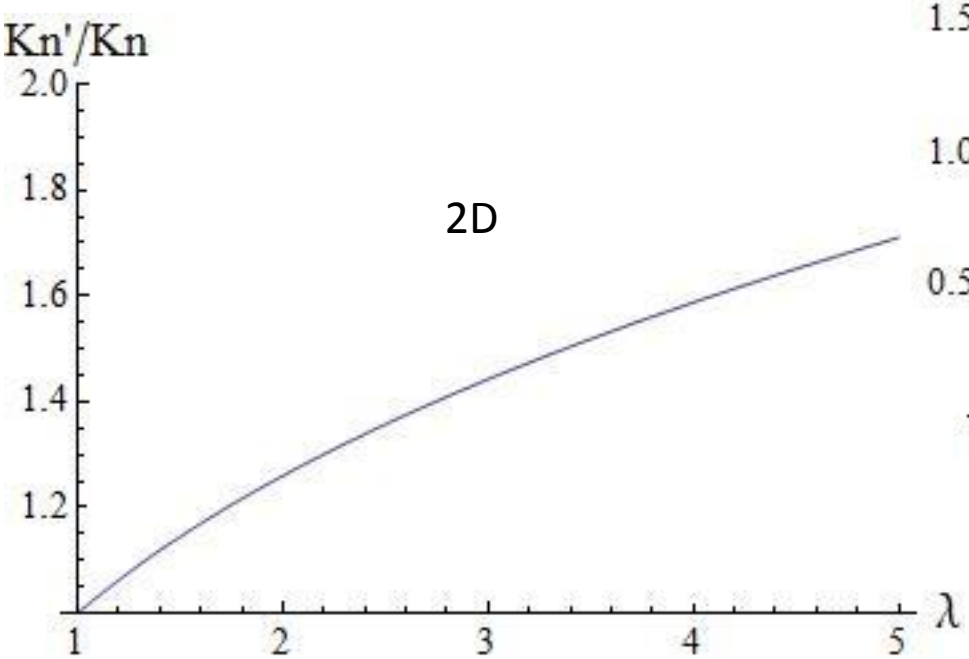
- 3D case $s' = s\,\lambda^3, T' = T\,\lambda, l' = \left(\frac{\eta}{s}\right)_{T'}\frac{\lambda^{-1}}{c_s T}$

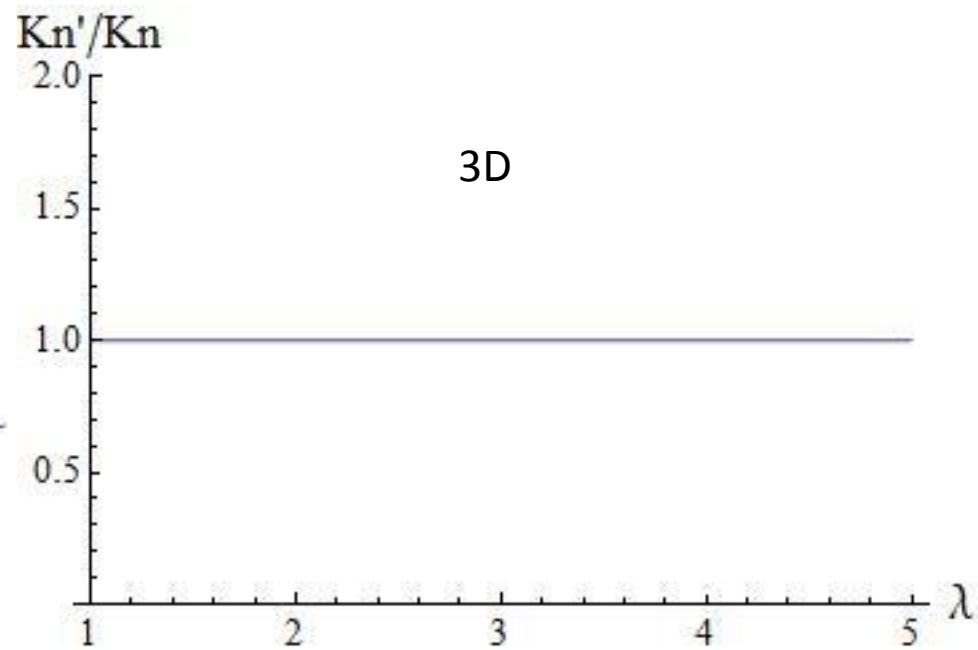
then $Kn' = \left(\frac{\eta}{s}\right)_{T'} / \left(\frac{\eta}{s}\right)_{T} Kn$



Conformal case

- In conformal case η/s is a constant.

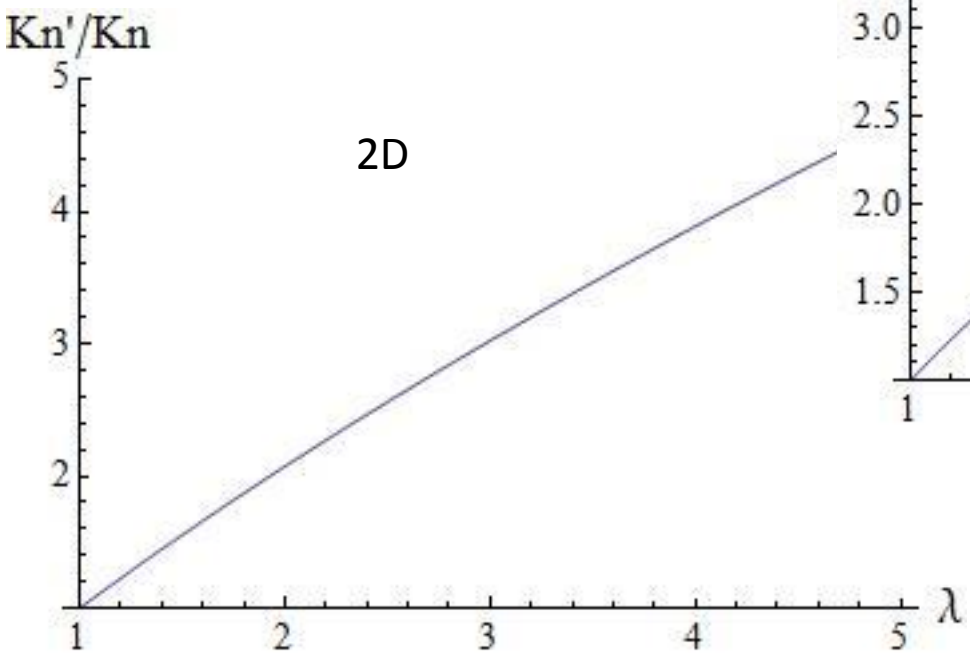


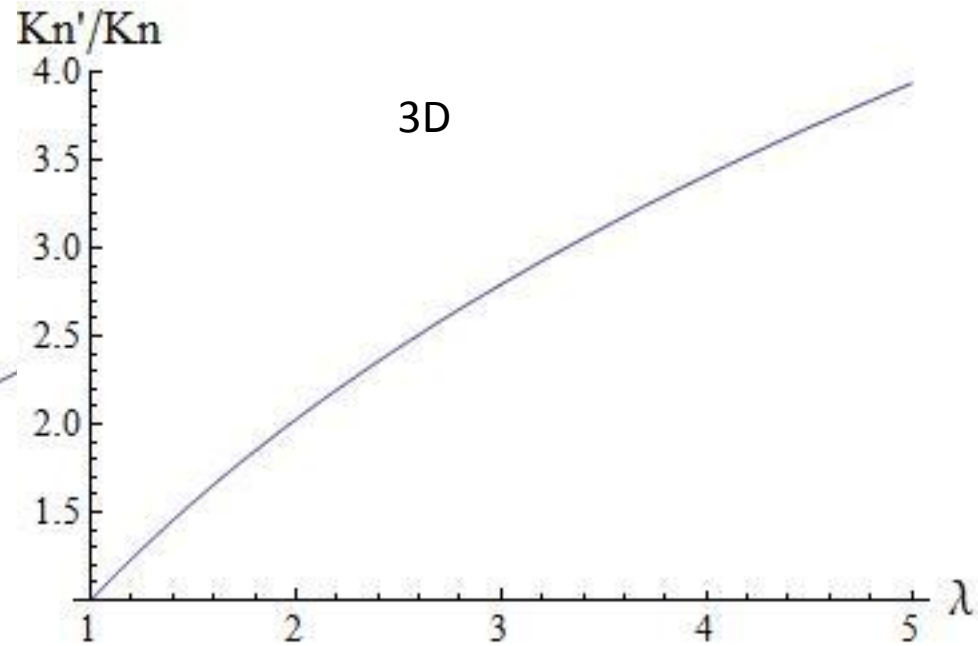


Temperature dependence

- We set $T = 2T_c$.

η/s Nucl.Phys. A769 (2006)





Conclusions

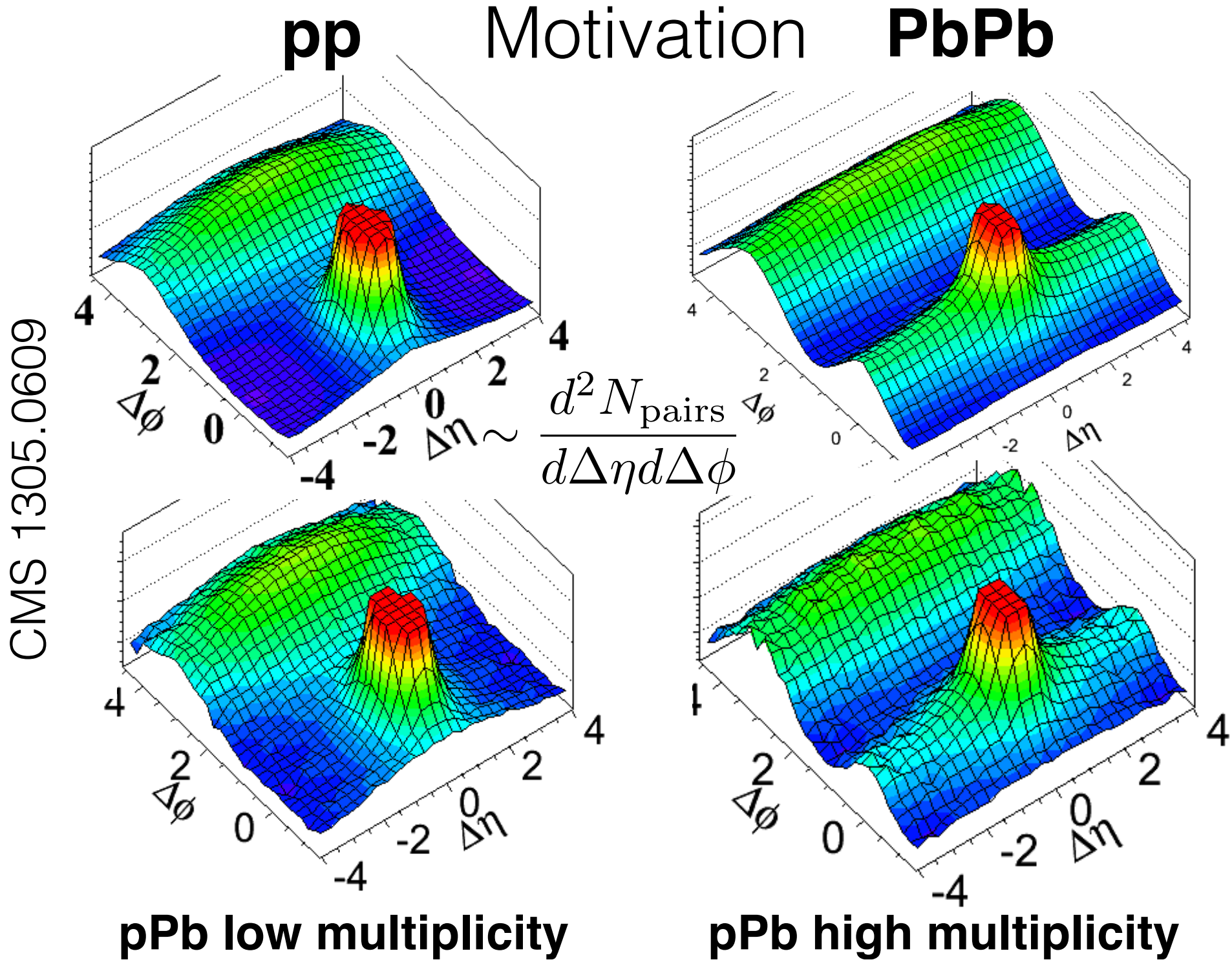
- We calculated proton's ground state wave function with variational method.
- The large radius events may have a observable probability. Eccentricities are not so small.
- The Knudsen number is mainly controlled by the temperature dependence of η/s. Fluid could be applicable for small systems.

Thank you!

Collective behavior in small colliding systems

I. Kozlov, M. Luzum, G. Denicol, S. Jeon, C. Gale
(1405.3976,1412.3147)

Motivation


pp
PbPb
CMS 1305.0609
$\sim \frac{d^2 N_{\text{pairs}}}{d\Delta\eta d\Delta\phi}$
$\Delta\phi$
$\Delta\eta$
pPb low multiplicity
pPb high multiplicity

Are particle azimuthal anisotropies due to hydro?

- Modify Glauber model + use MUSIC by Schenke, Jeon & Gale, PRL 106 (2011)
- Check whether experimental data can be described with hydrodynamics
- Introduce a more stringent test on hydrodynamics (observable Υ_n), which gives another handle to explore HIC

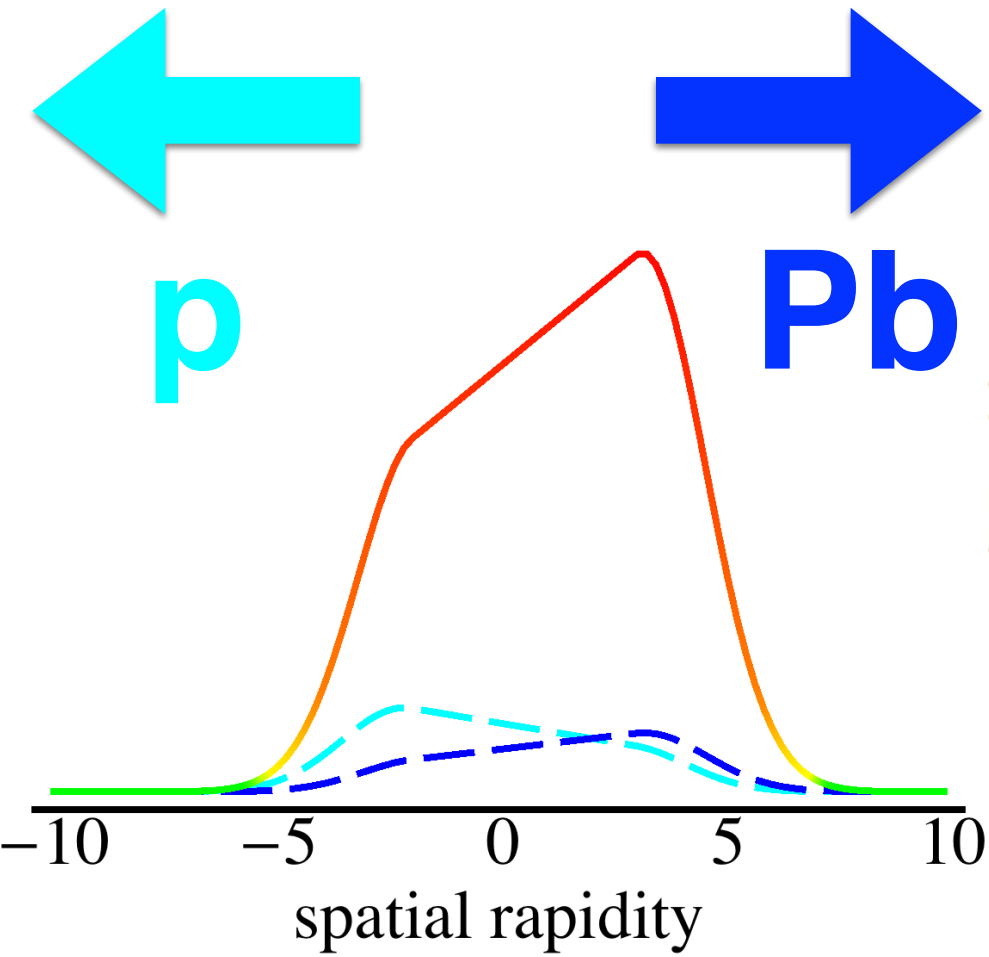
CMS 1305.0609

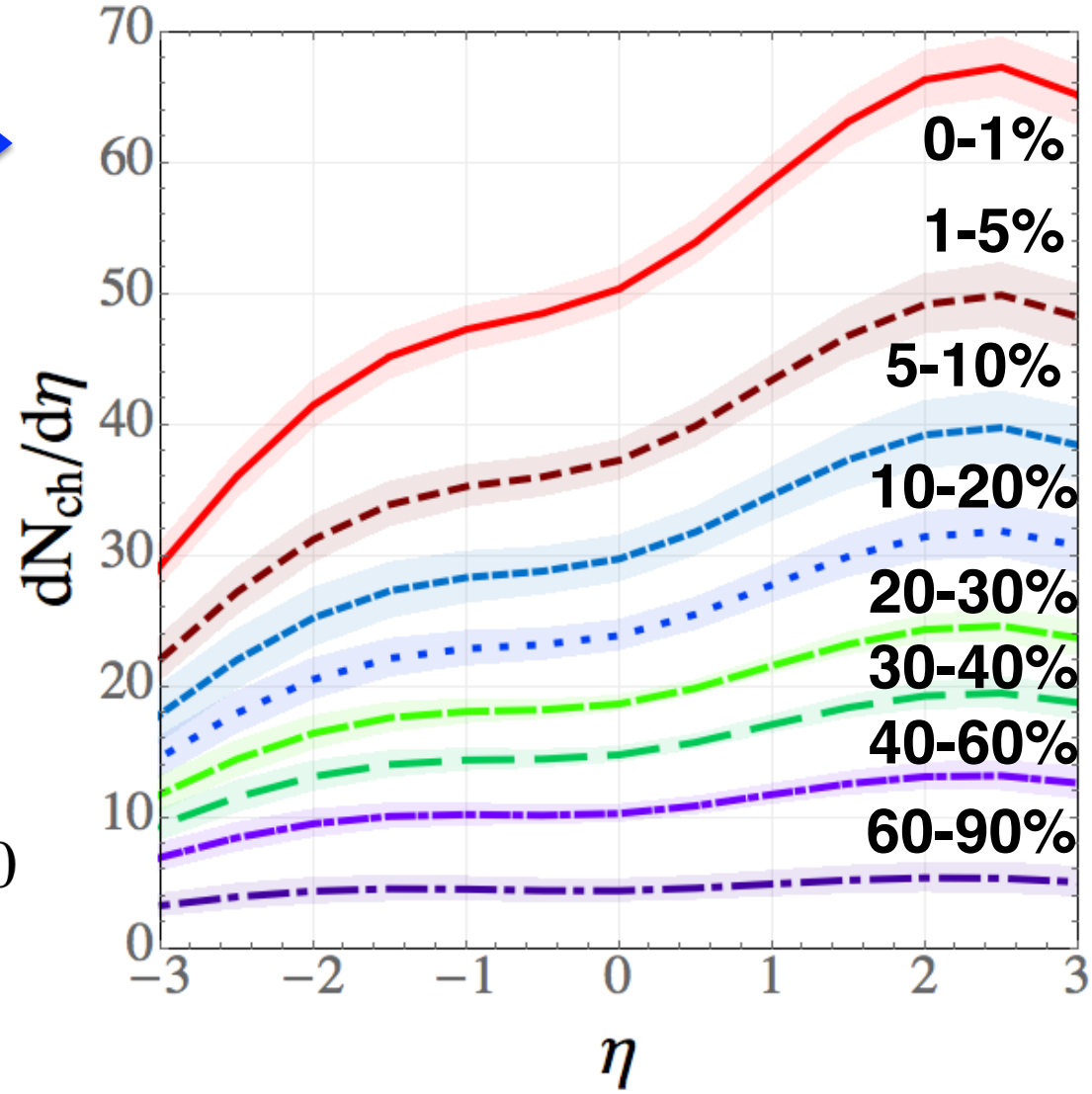
pPb low multiplicity

pPb high multiplicity

Glauber + rapidity distribution

initial entropy profile → final multiplicity profile

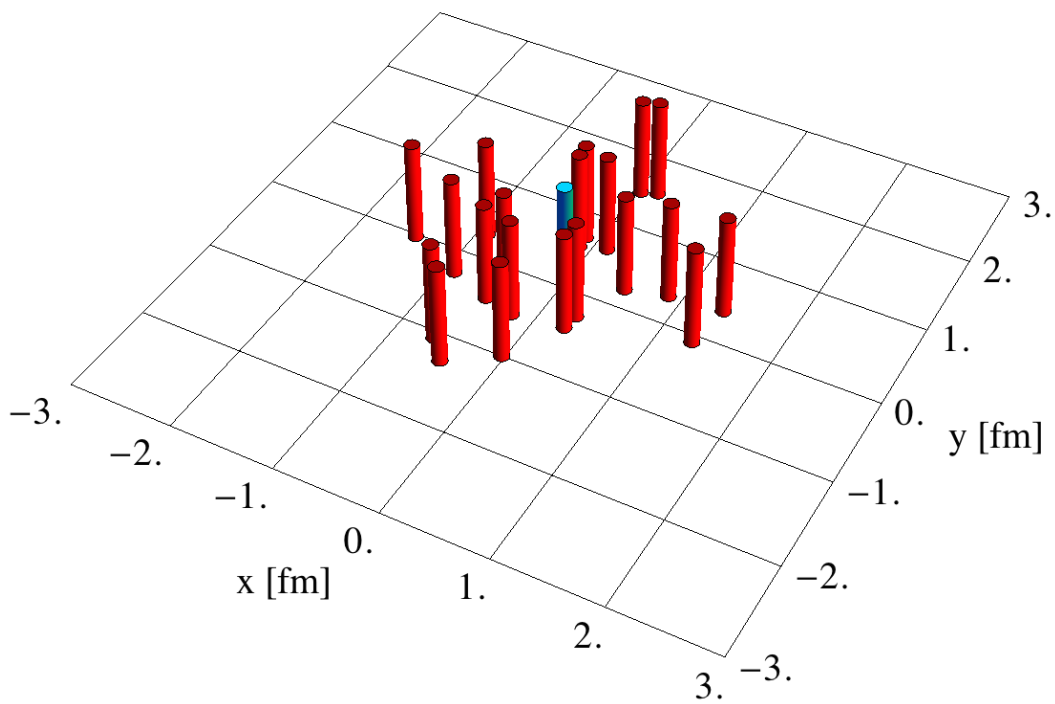




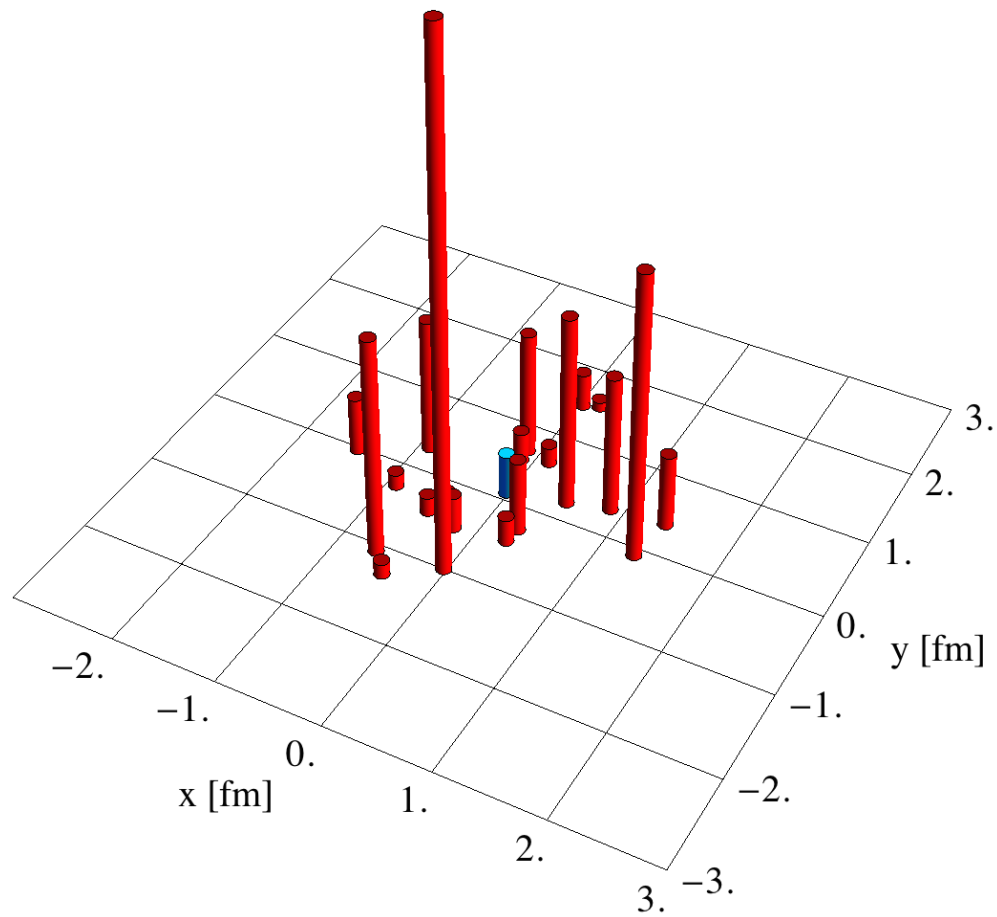
Bozek, Wyskiel 1002.4999

McGill, LBNL 1405.3976

Glauber

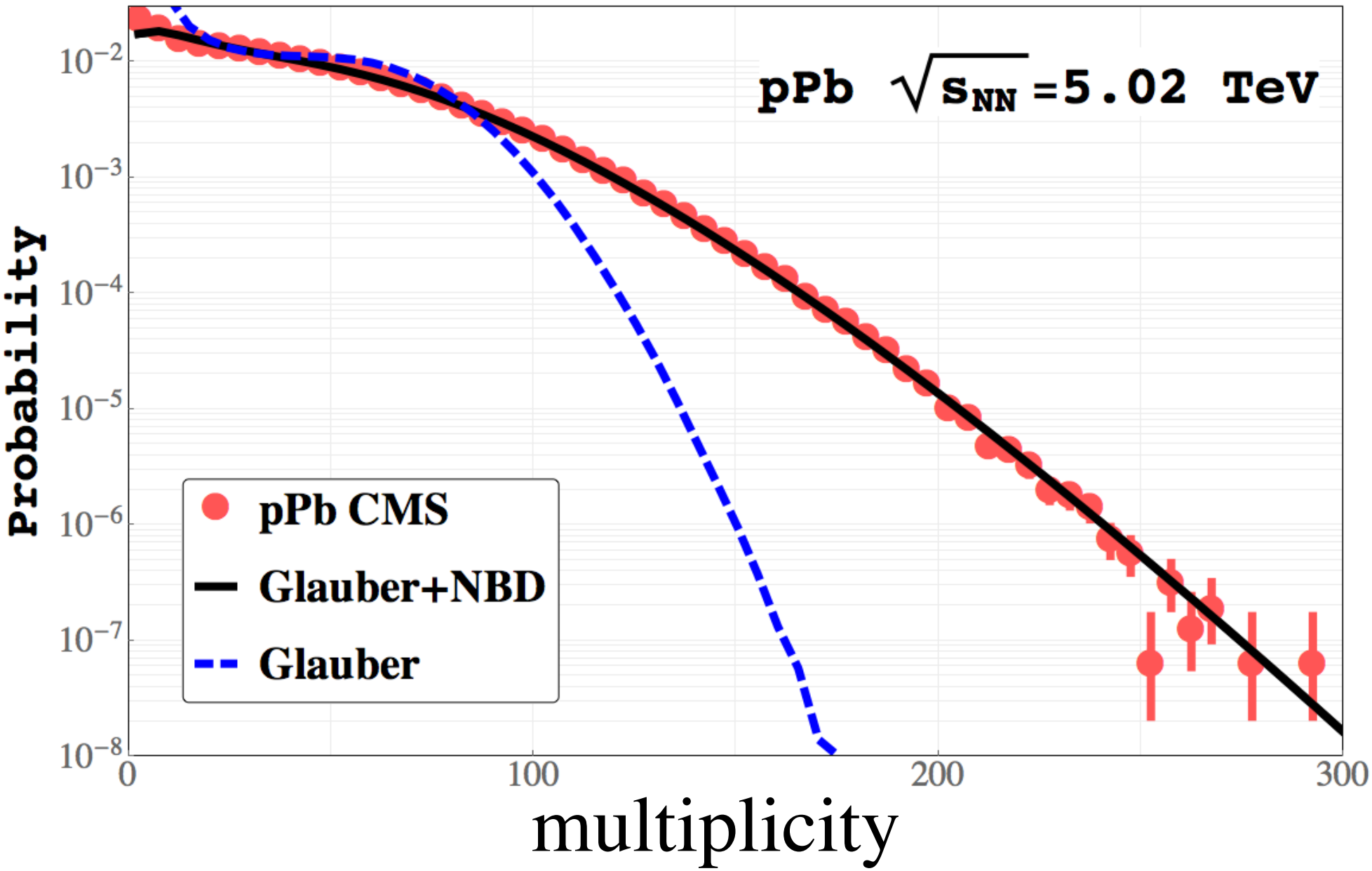


Glauber+NBD



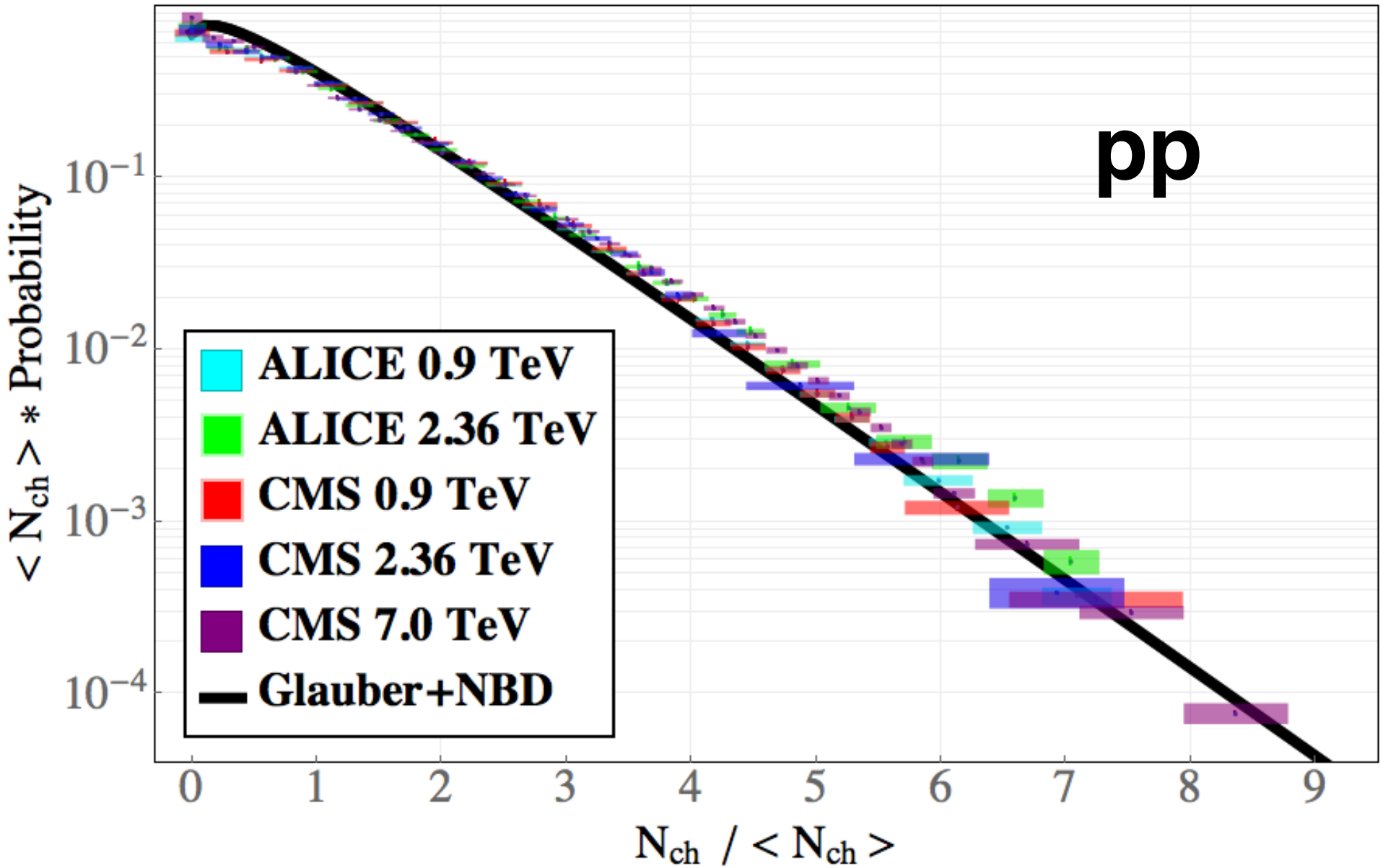
McGill, LBNL 1405.3976

Glauber+NBD



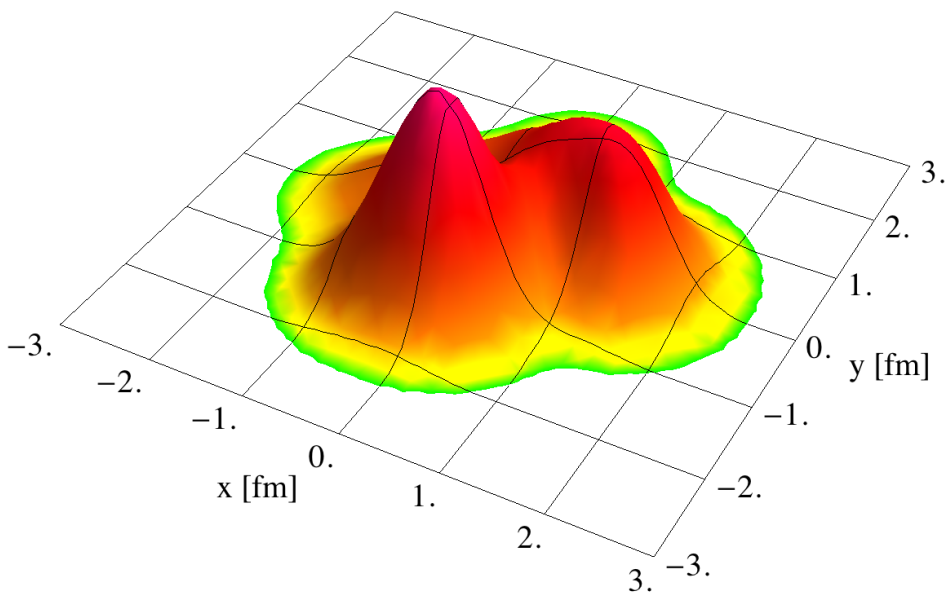
CMS twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIN13002

pp multiplicity with **Glauber+**NBD

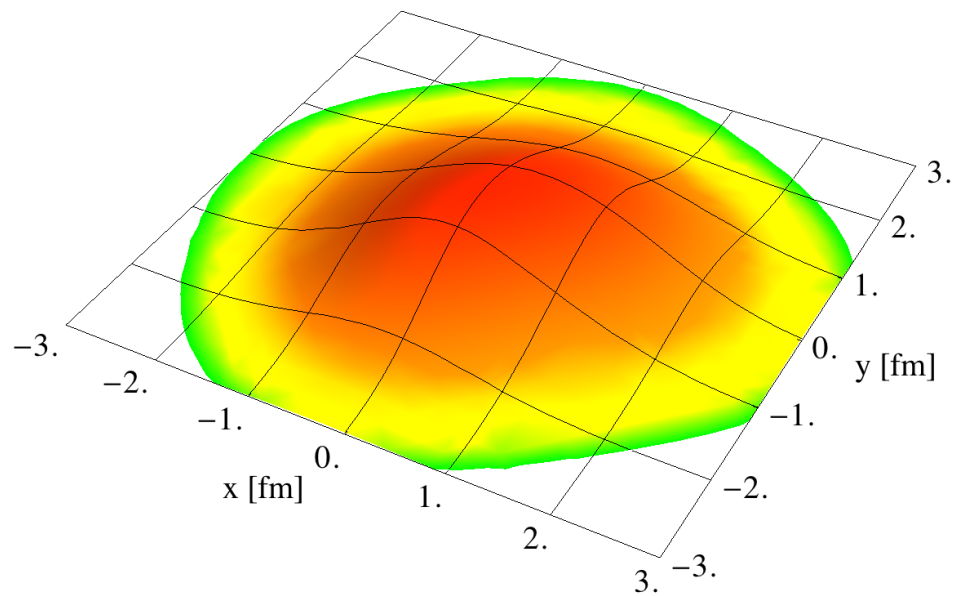


Transverse granularity

$\sigma = 0.40$ fm



$\sigma = 0.80$ fm



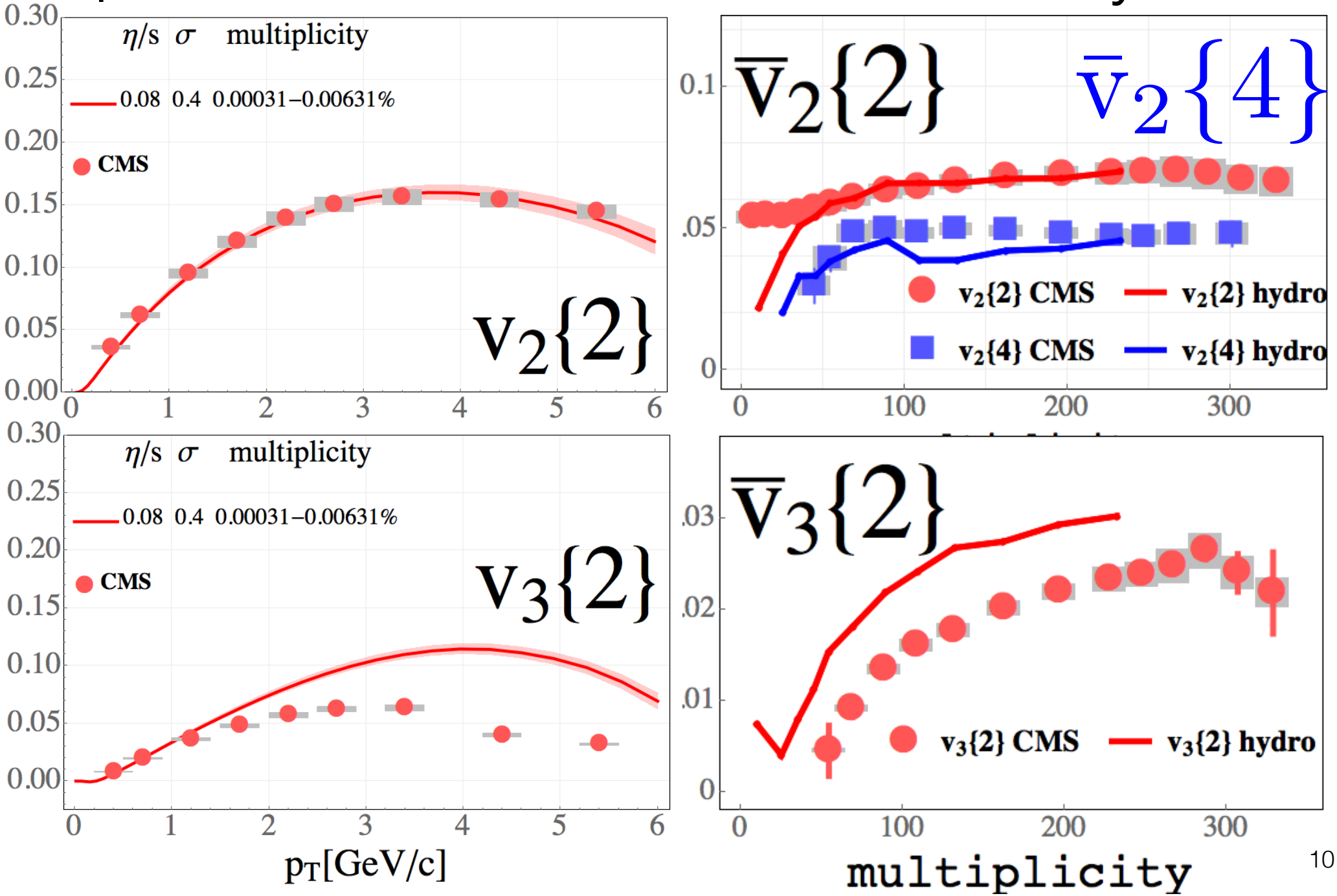
In small systems σ affects both fluctuations and system size

Comparing hydro calculations to existing pA data

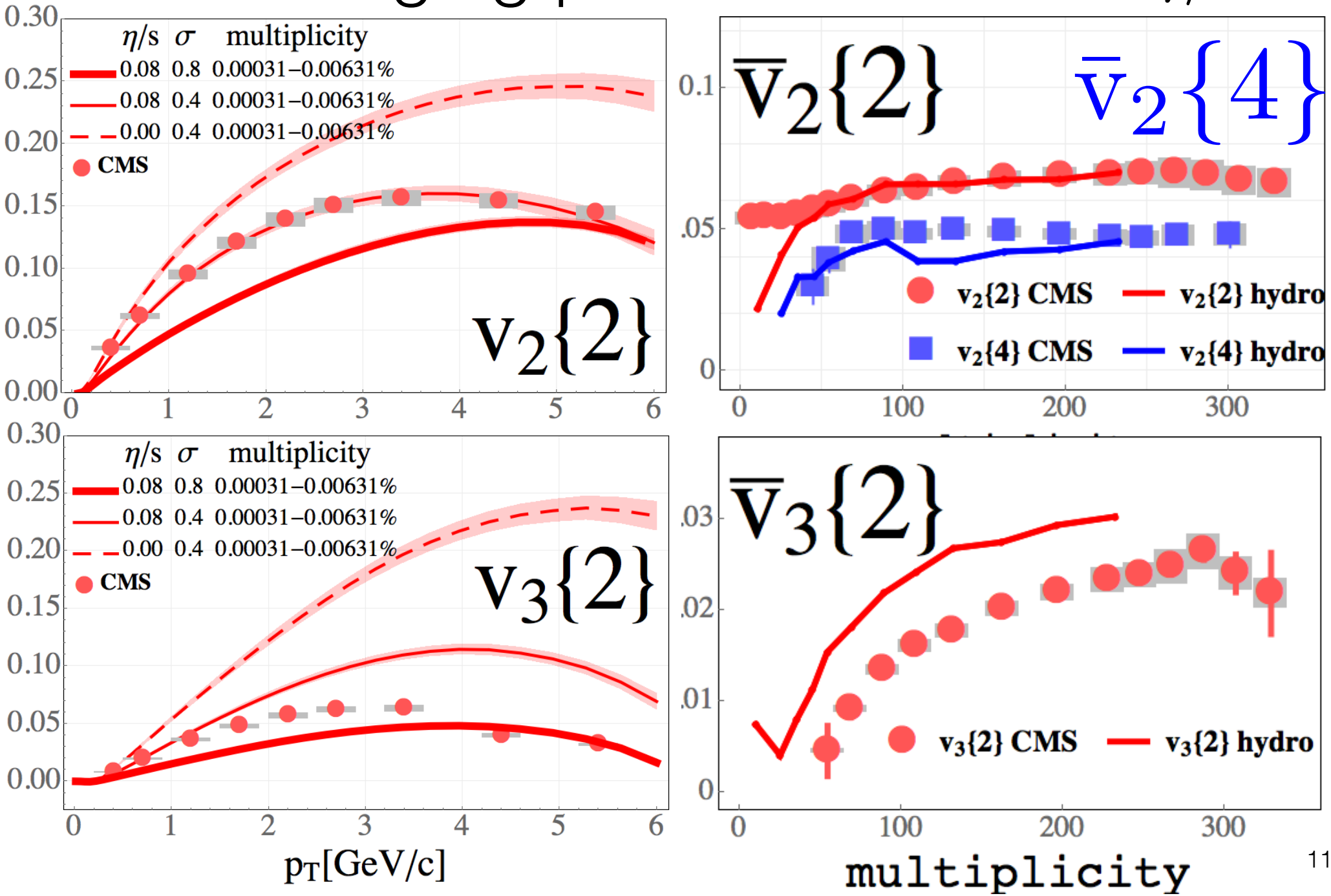
Start with: $\sigma = 0.4\mathrm{fm}$ $\eta/s = 0.08$

CMS 1211.0989 McGill, LBNL 1405.3976

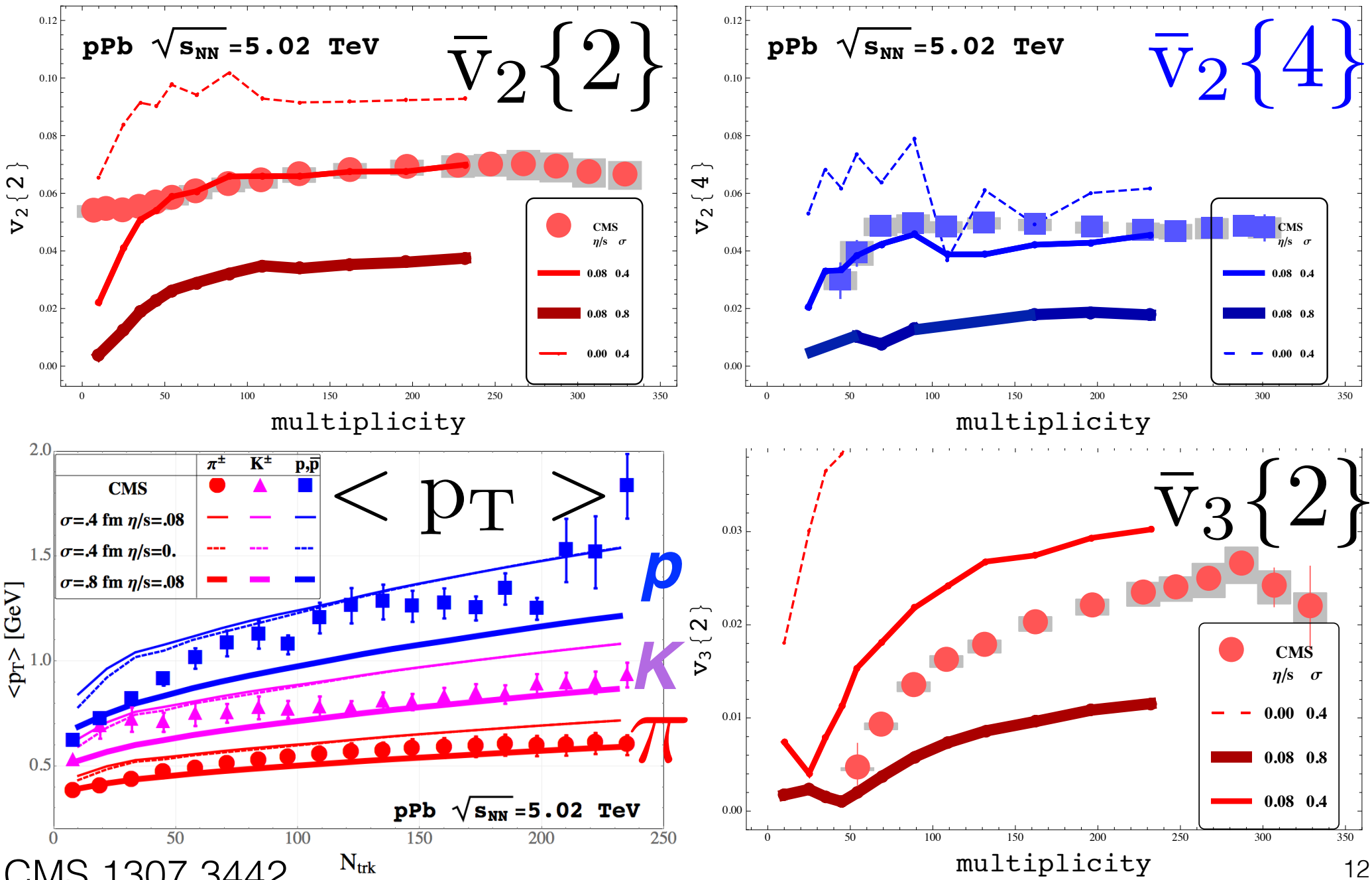
pPb flow observables: CMS & hydro



Changing parameters σ and η/s



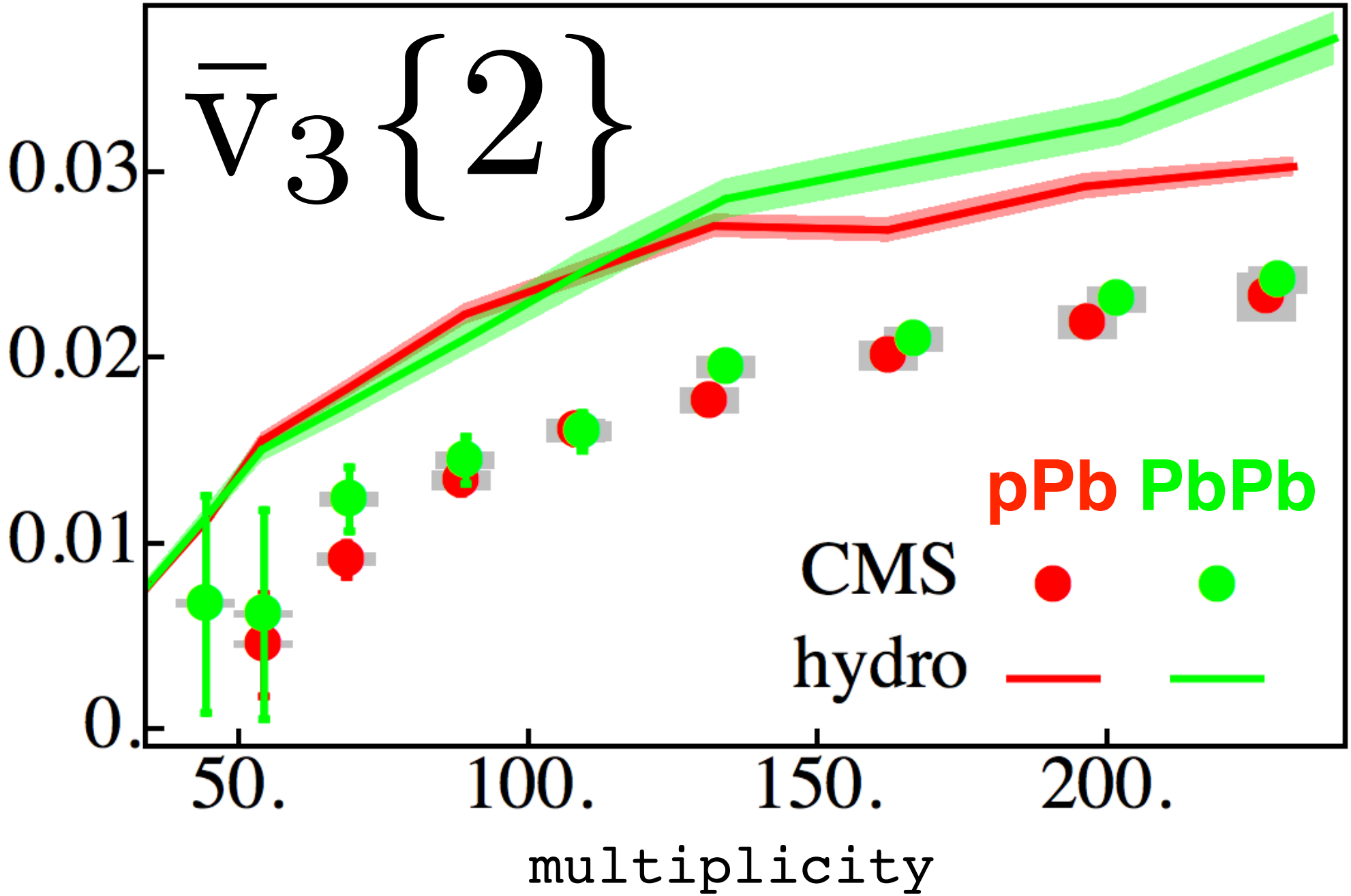
Flow observables dependence on σ and η/s



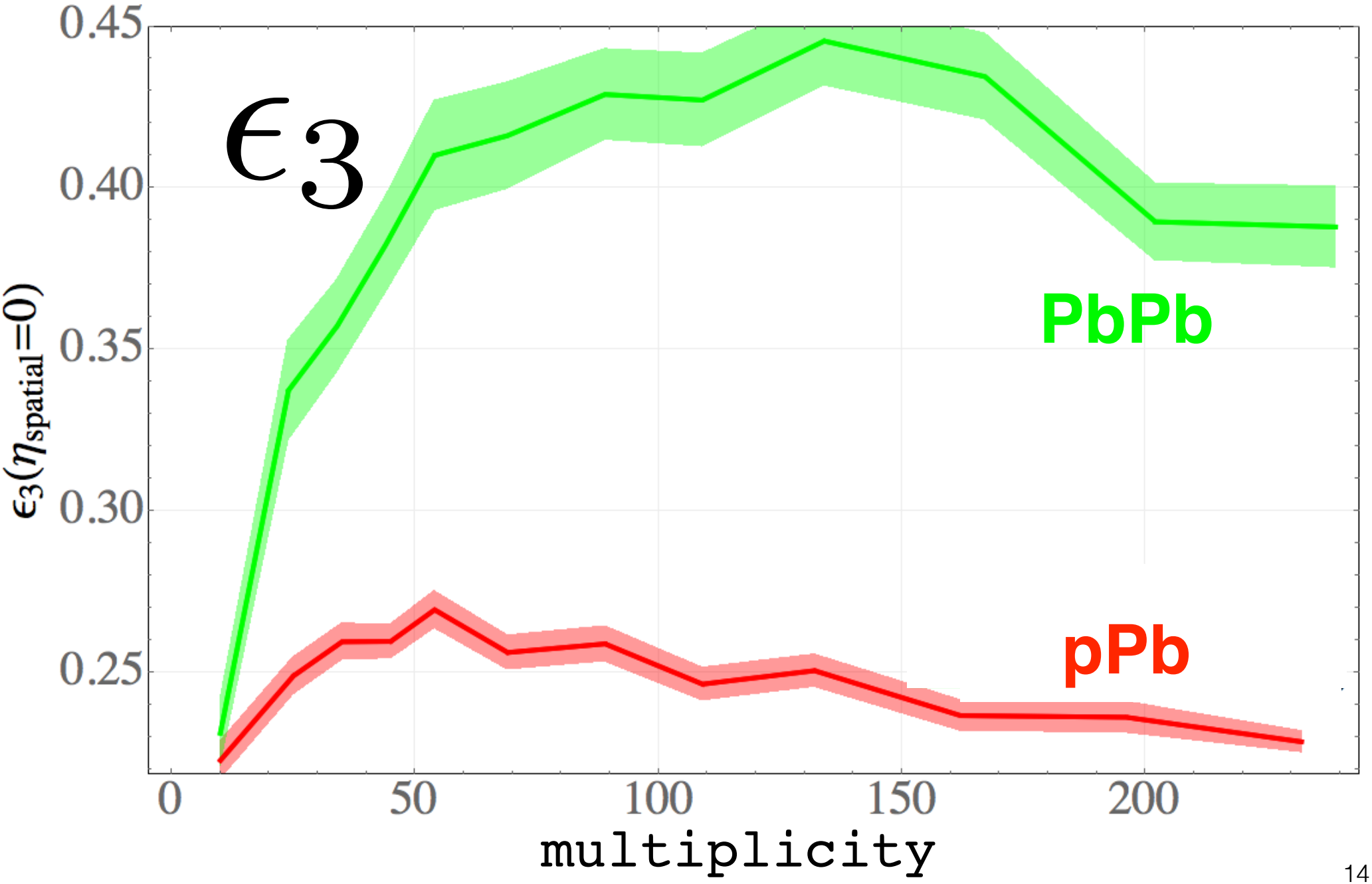
CMS 1307.3442

CMS 1211.0989 McGill, LBNL 1405.3976

CMS finding in hydro



pPb and PbPb eccentricities are **different**



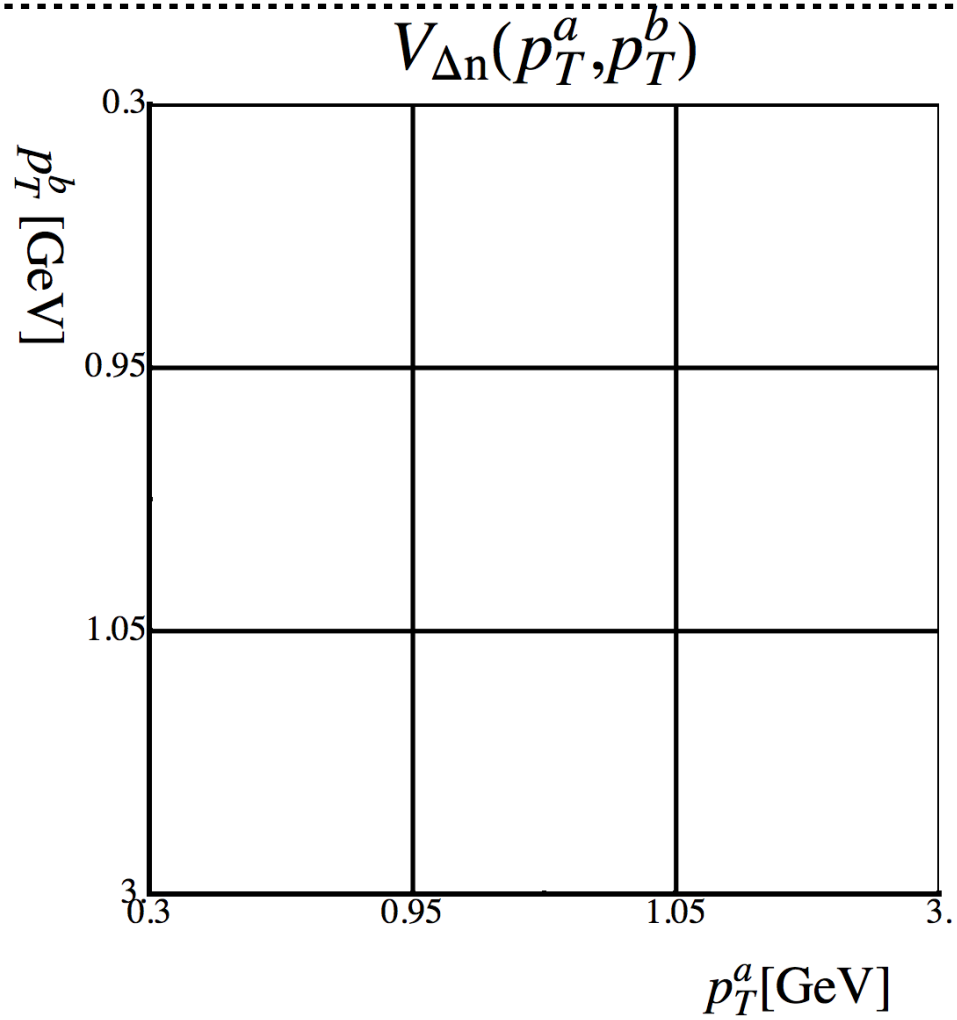
All two-particle correlation observables

theory:

$$\frac{2\pi}{N}\frac{dN}{d\phi^a} = 1 + 2\sum_{n=1}^{\infty} v_n(p_T^a)\cos n(\phi^a - \Psi_n)$$

experiment:

$$V_{n\Delta}(p_T^a, p_T^b) \equiv \langle\overline{\cos n(\phi^a - \phi^b)}\rangle$$



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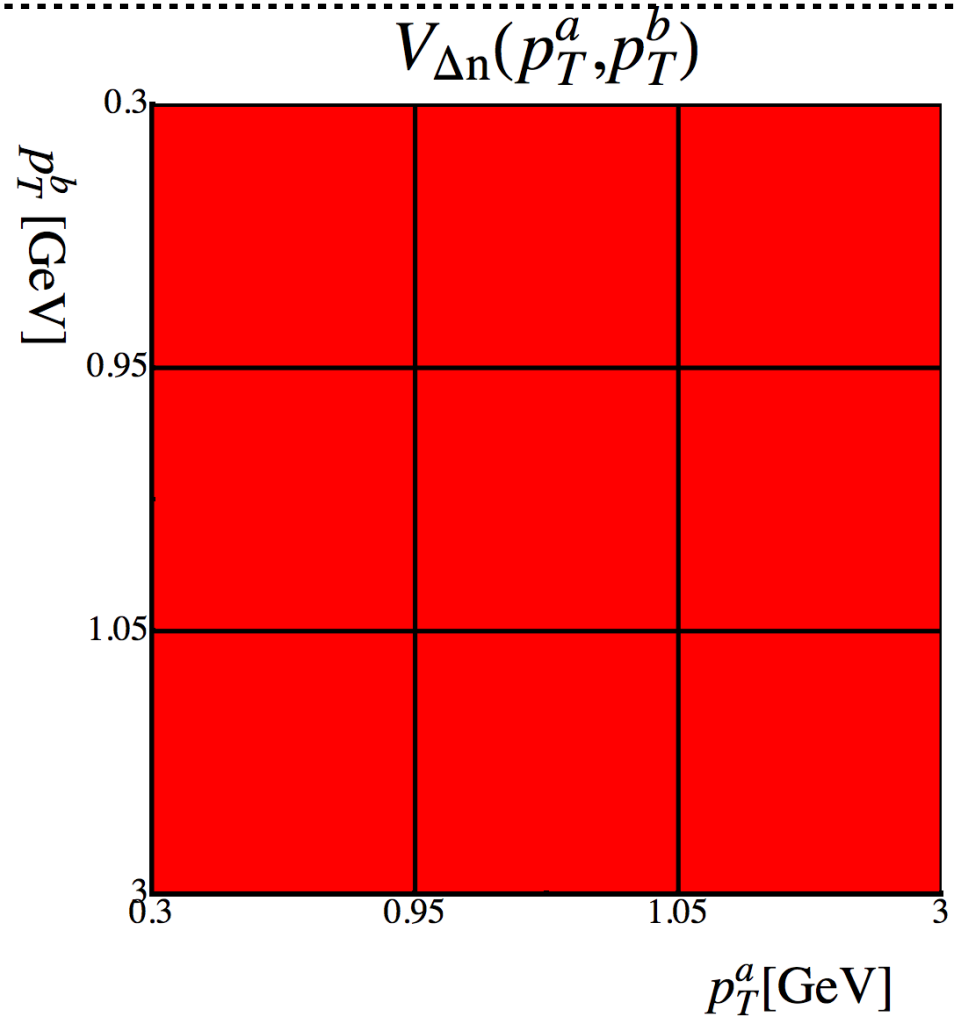

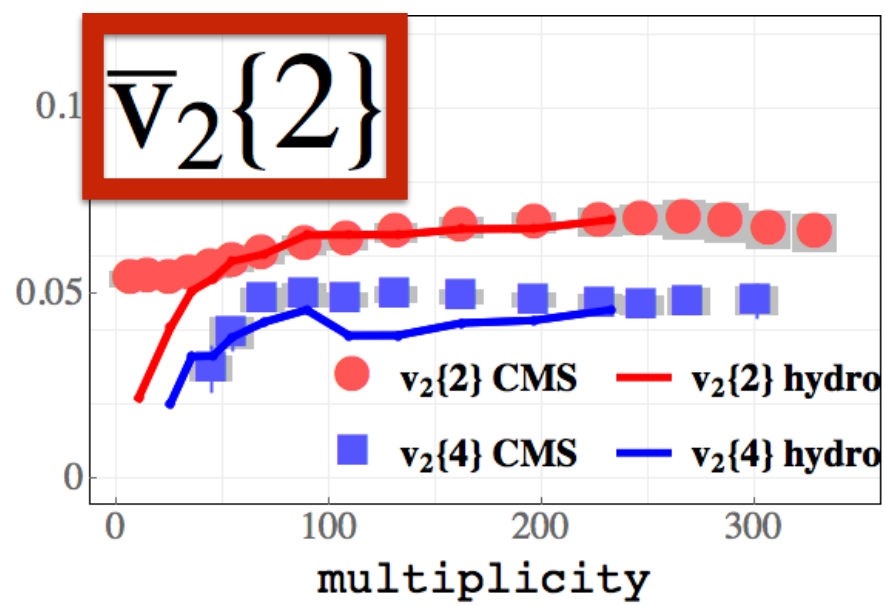

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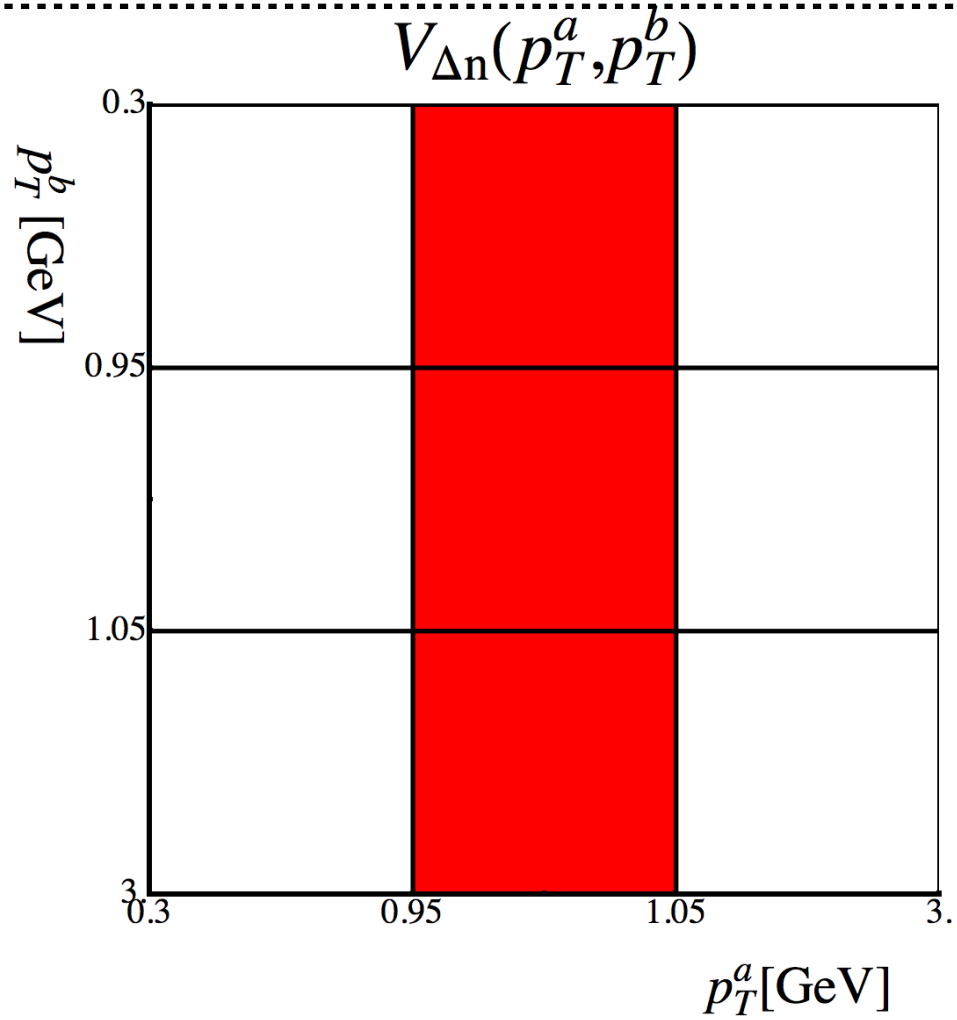
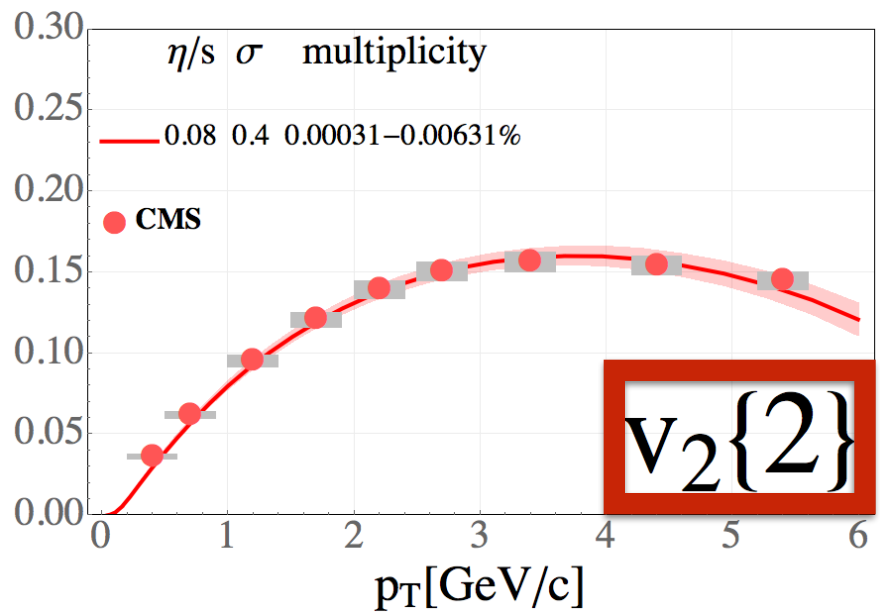




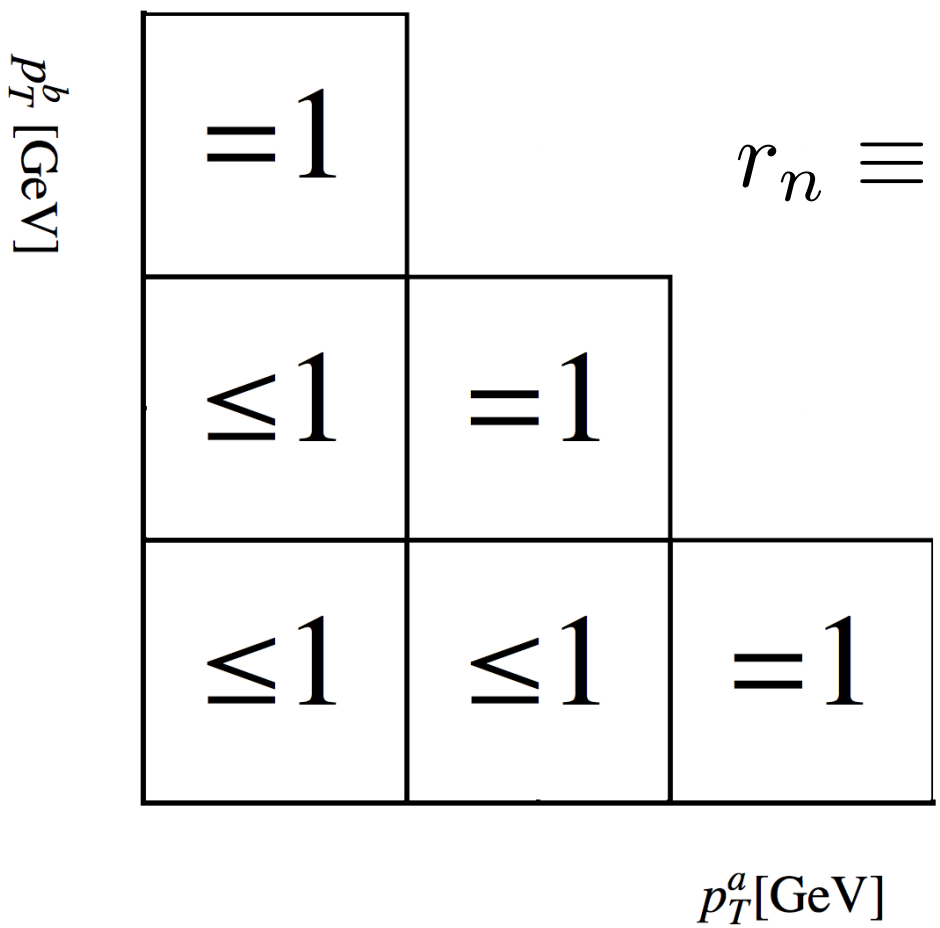
Two-particle correlation in hydro

theory:

$$\frac{2\pi}{N}\frac{dN}{d\phi^a} = 1 + 2\sum_{n=1}^{\infty} v_n(p_T^a)\cos n(\phi^a - \Psi_n)$$

experiment:

$$V_{n\Delta}(p_T^a, p_T^b) \equiv \langle \overline{\cos n(\phi^a - \phi^b)} \rangle$$

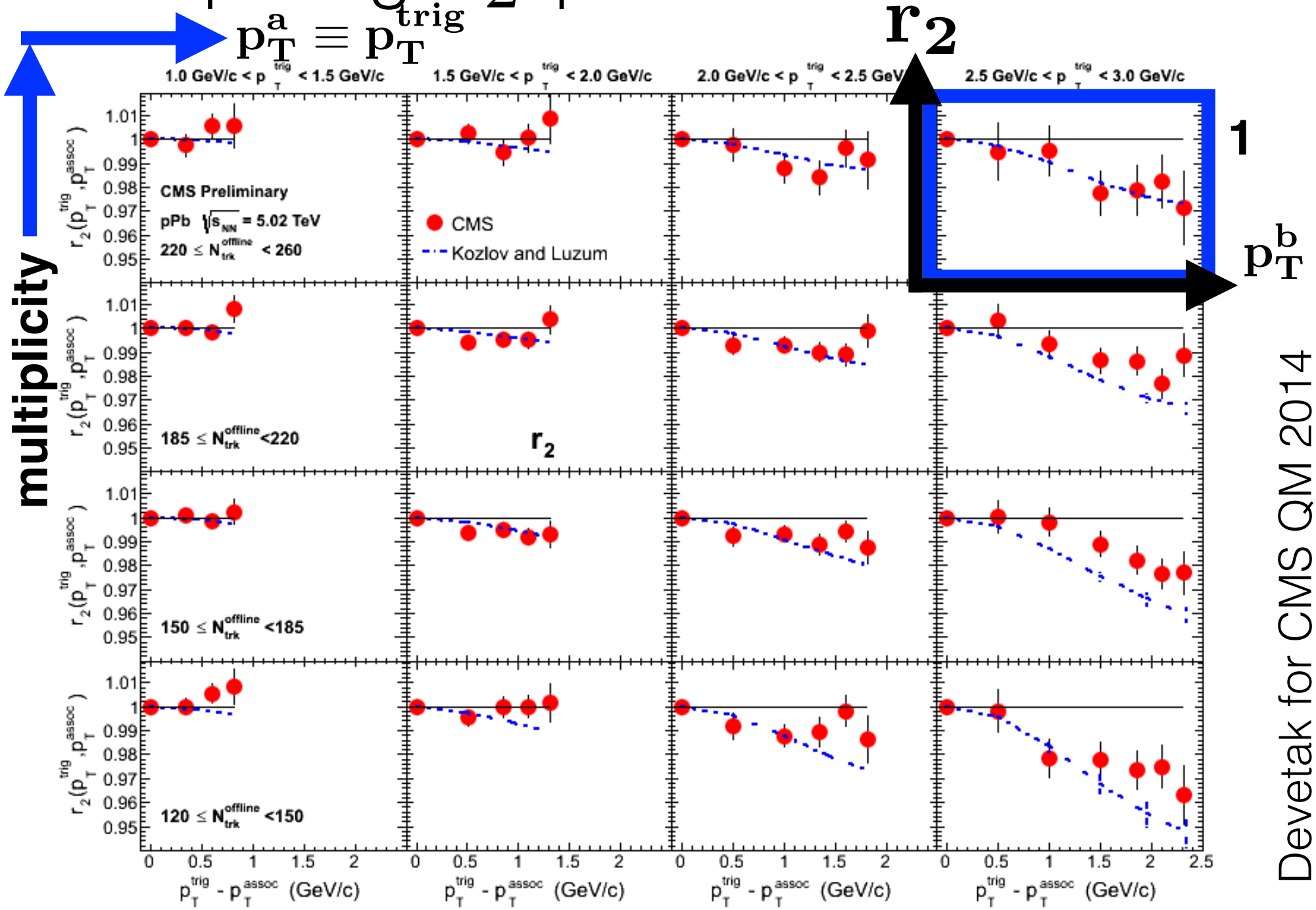


$$r_n \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a)V_{n\Delta}(p_T^b, p_T^b)}}$$

$\longleftarrow$ $\mathrm{Abs}[r_n]$

Gardim, Grassi, Luzum, Ollitrault 1211.0989

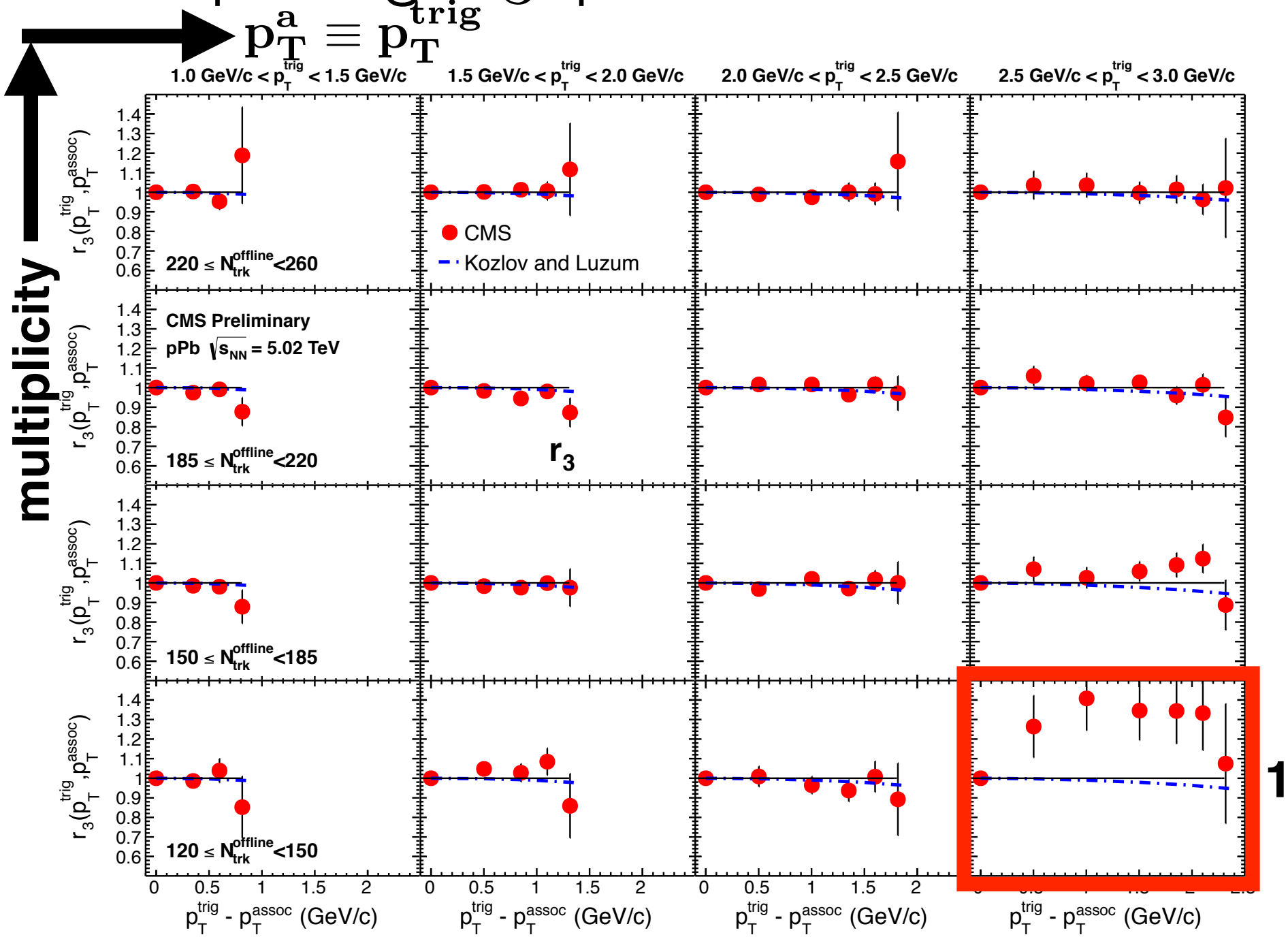
Comparing r_2 predictions to CMS data



Devetak for CMS QM 2014

twiki.cern.ch/twiki/pub/CMSPublic/PhysicsResultsHIN14012

Comparing r_3 predictions to CMS data



Devetak for CMS QM 2014

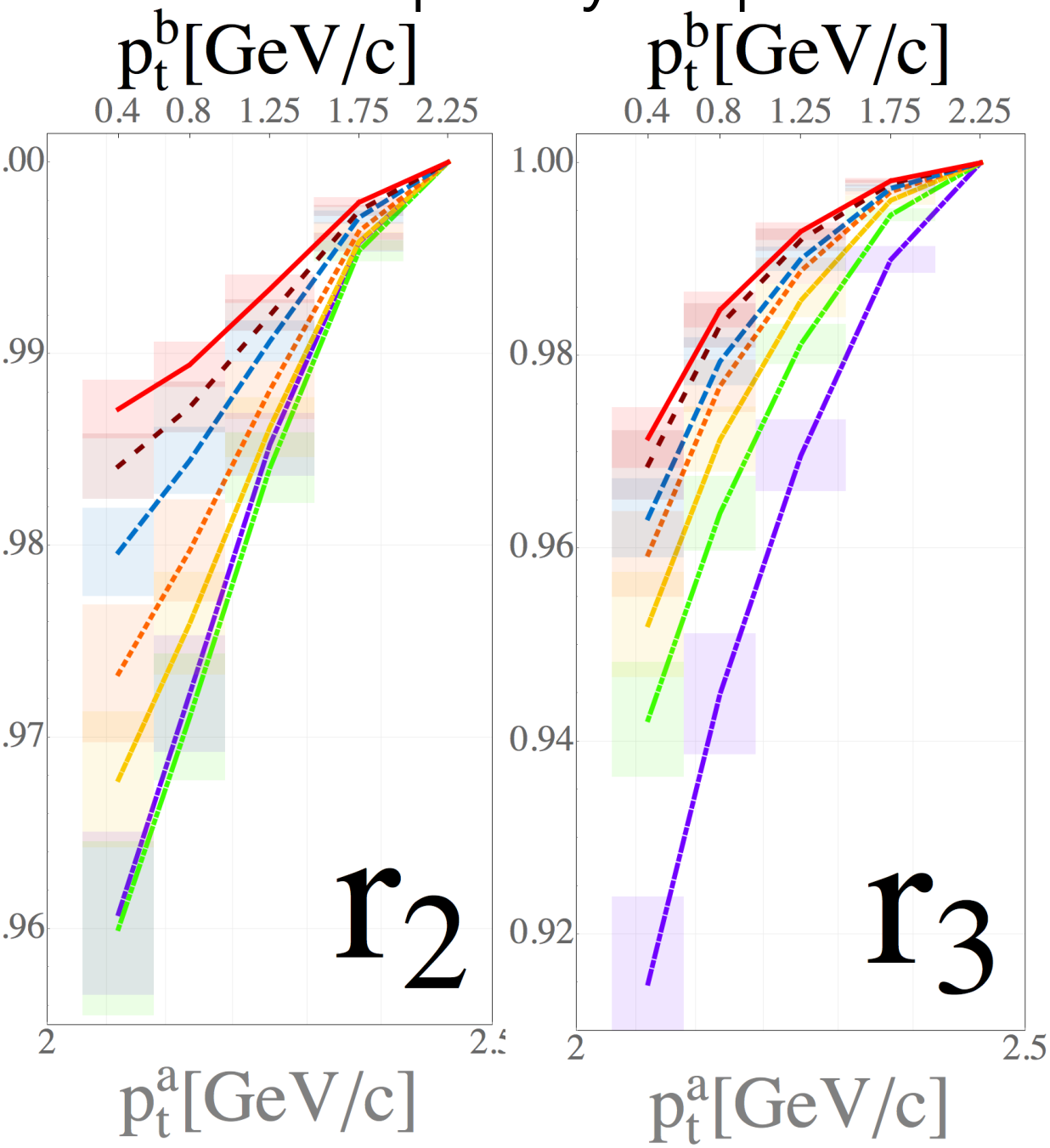
twiki.cern.ch/twiki/pub/CMSPublic/PhysicsResultsHIN14012

Conclusion

- Hydrodynamics can reasonably describe:
  - **pPb** system at high multiplicity
    $v_2\{2\}, v_3\{2\}, v_2\{4\}$ and r_n
  - **pPb** and **PbPb** effects with one set of parameters
    e.g. different ϵ_3 lead to remarkably similar $v_3\{2\}$

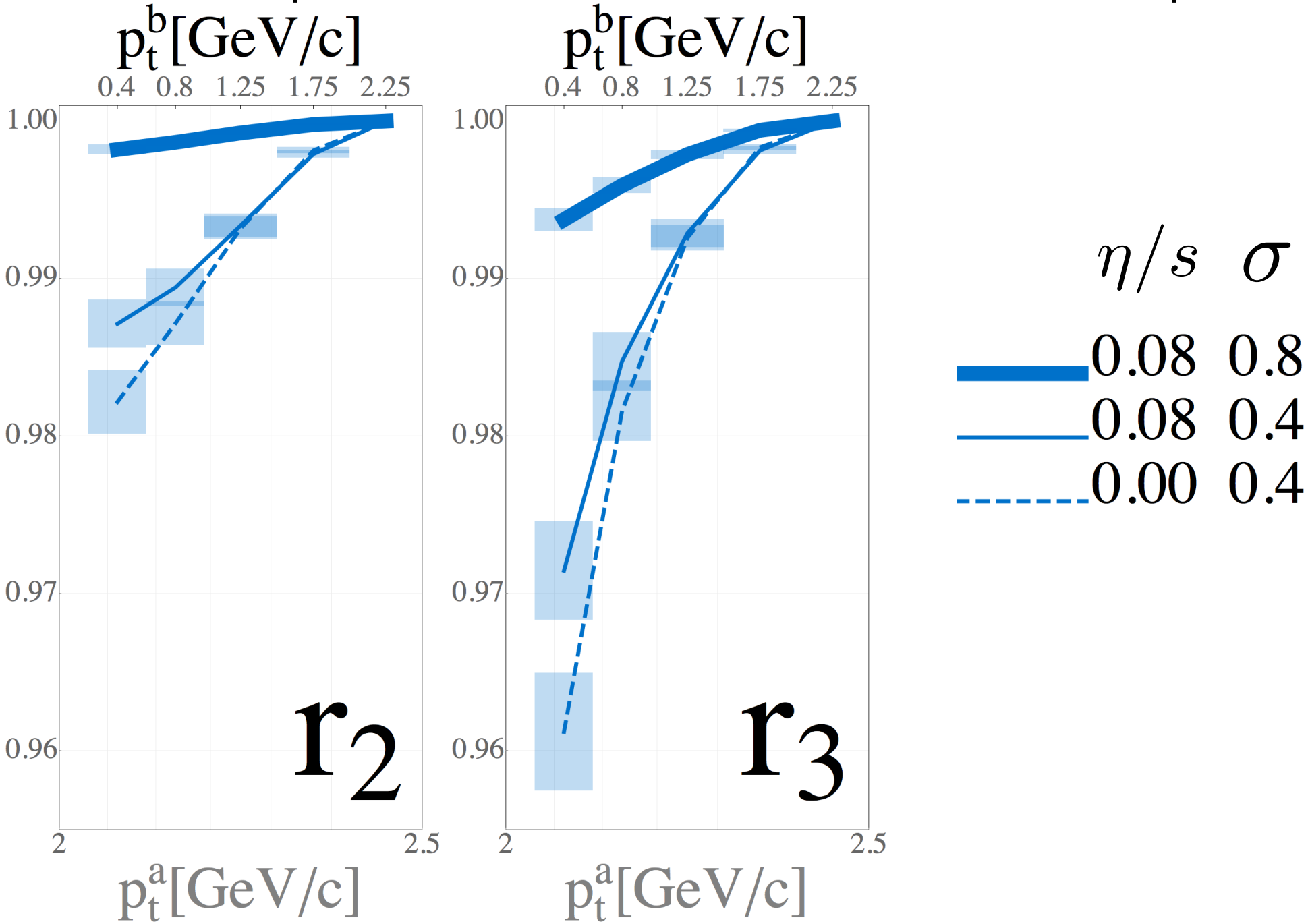
Another handle to study HIC

multiplicity dependence prediction in pPb

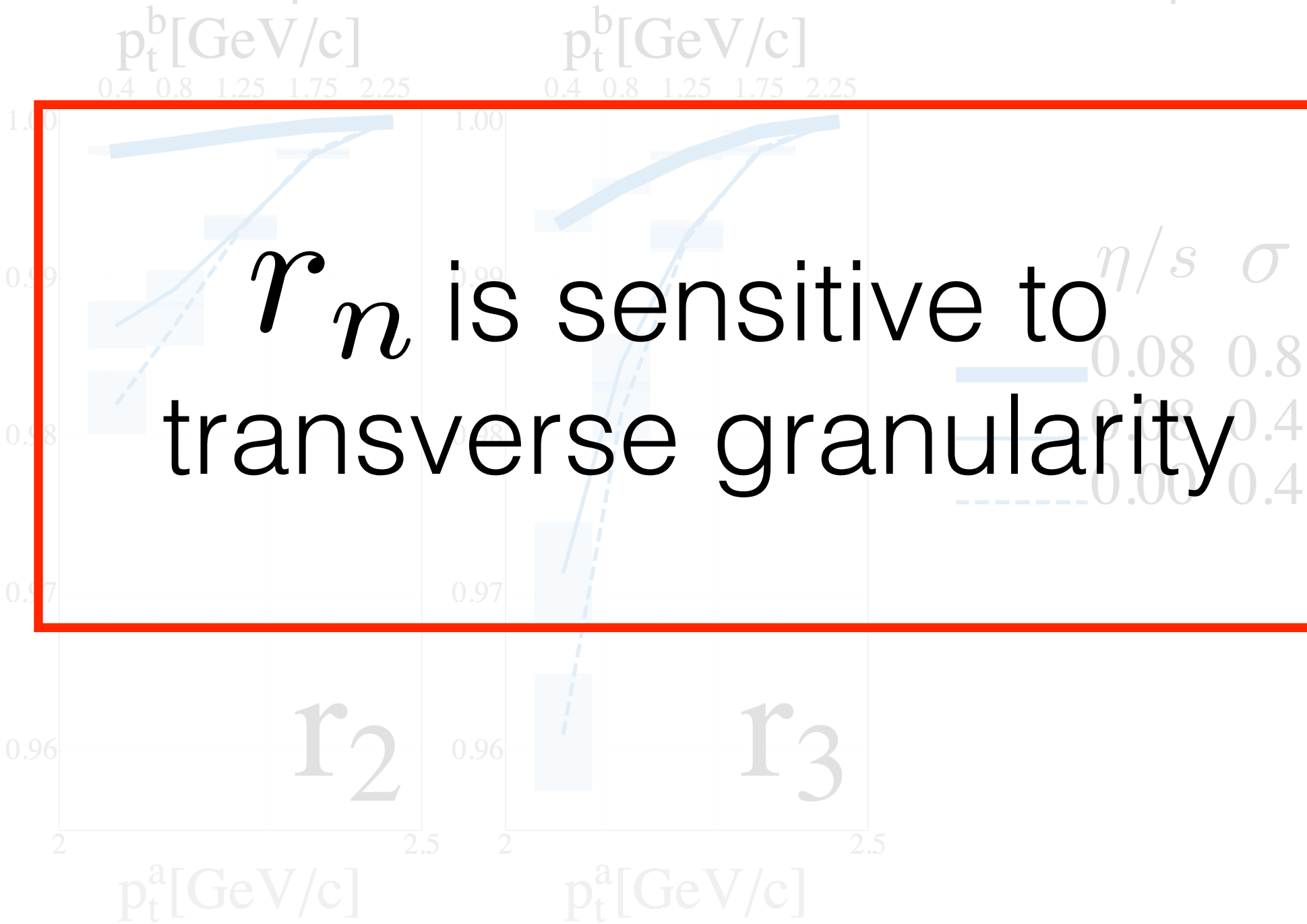


0.00031–0.00631%
0.00631–0.05631%
0.05631–0.45631%
0.45631–2.%
2.–5.%
5.–12.%
12.–24.%

r_n dependence on σ and η/s in pPb

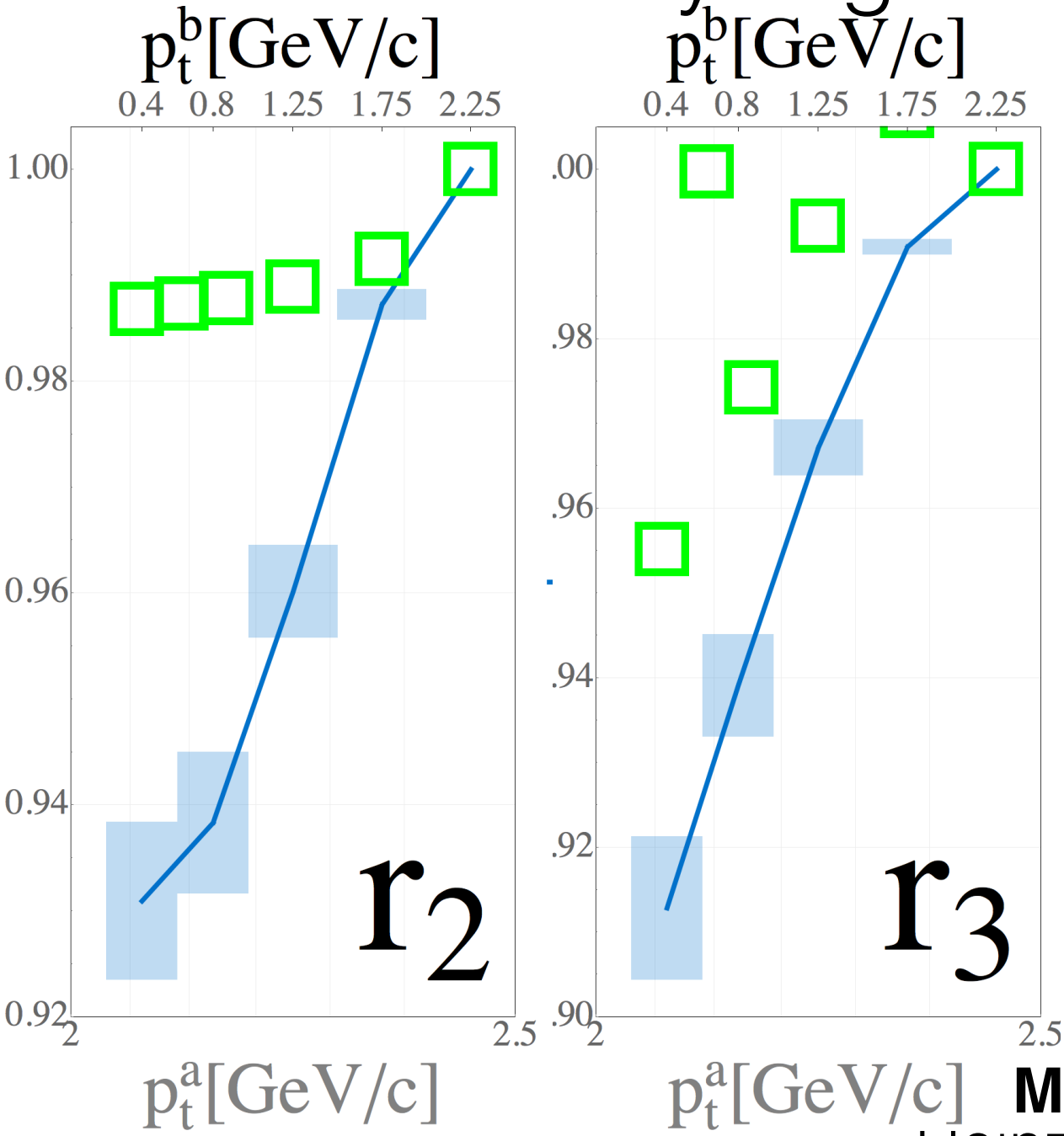


r_n dependence on σ and η/s in pPb



r_n is sensitive to transverse granularity

Reanalyzing PbPb data



ALICE 1109.2501

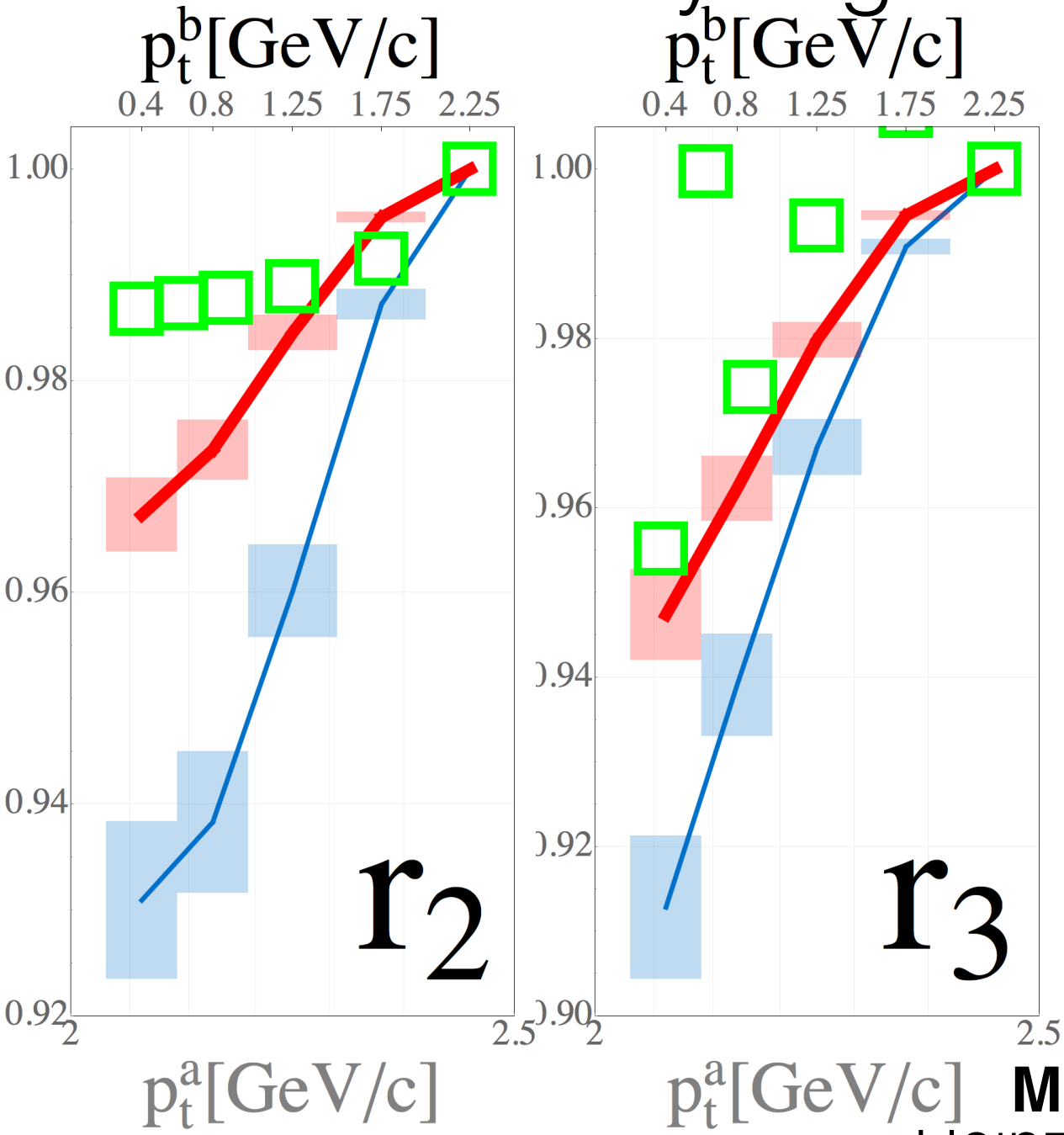
η/s σ **fluct**

0.08 0.4 NBD

McGill LBNL 1405.3976

Heinz, Qiu, Shen 1302.3535

Reanalyzing PbPb data



ALICE 1109.2501

η/s σ **fluct**

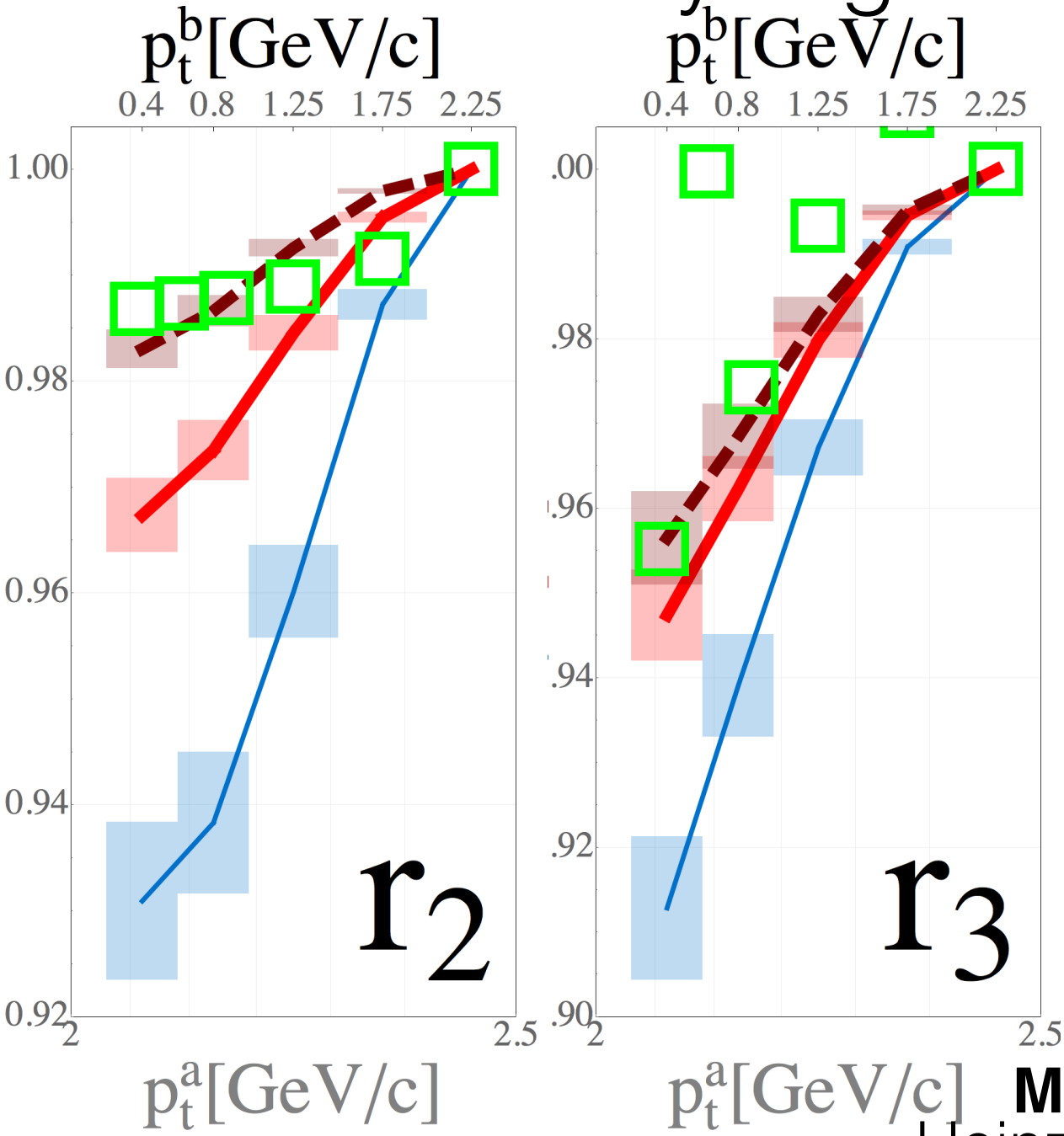
0.08 0.8 NBD

0.08 0.4 NBD

McGill LBNL 1405.3976

Heinz, Qiu, Shen 1302.3535

Reanalyzing PbPb data



ALICE 1109.2501

η/s	σ	**fluct**
0.08	0.8	
0.08	0.8	NBD
0.08	0.4	NBD

McGill LBNL 1405.3976

Heinz, Qiu, Shen 1302.3535

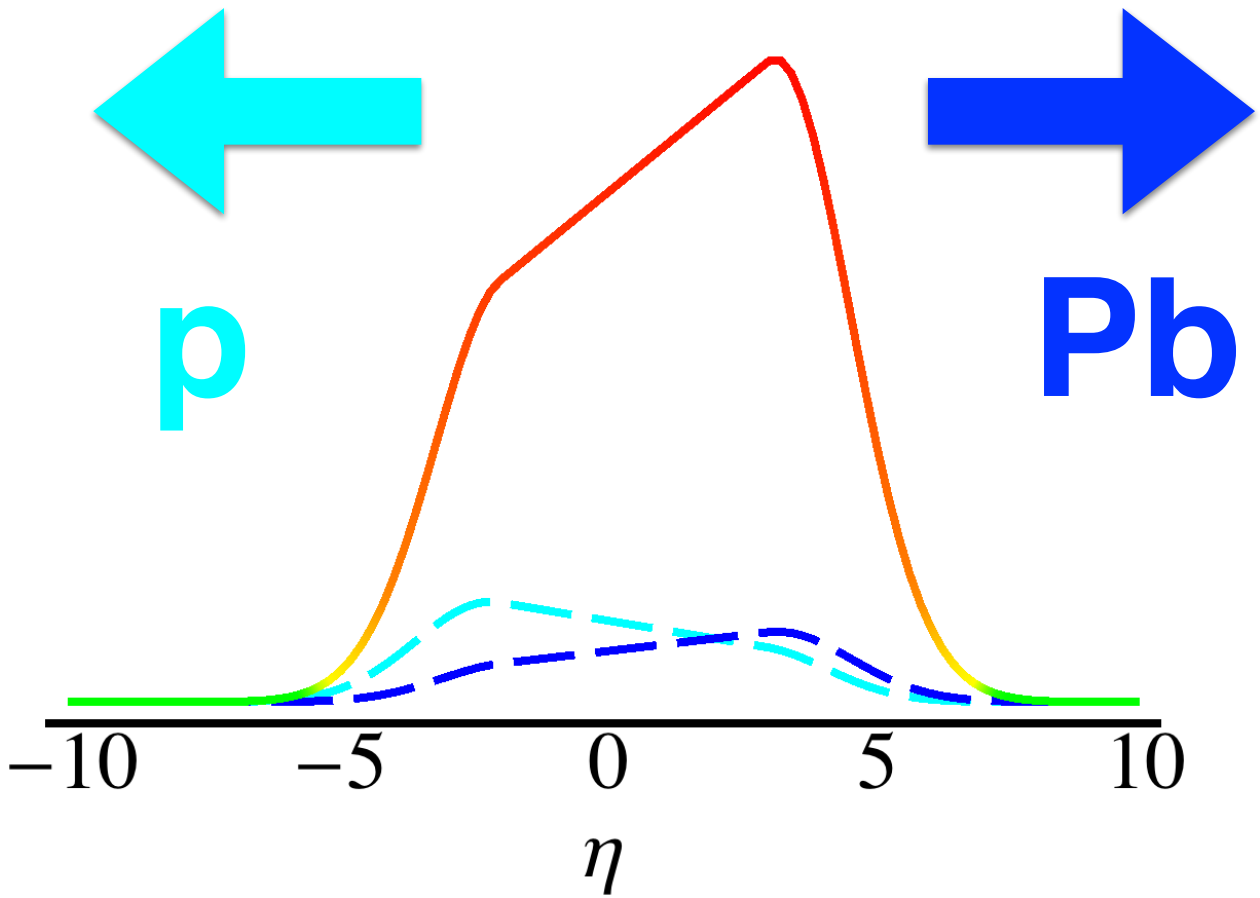
Conclusion

- Hydrodynamics can reasonably describe:
  - **pPb** system at high multiplicity
    $v_2\{2\}, v_3\{2\}, v_2\{4\}$ and r_n
  - **pPb** and **PbPb** effects with one set of parameters
    e.g. different ϵ_3 lead to remarkably similar $v_3\{2\}$

- r_n predictions provide another handle to explore HIC
  - it tells us where hydro breaks down
  - a way to probe initial conditions (granularity)
  - a way to study differences between pA and AA

Backup

Initial conditions: longitudinal profile

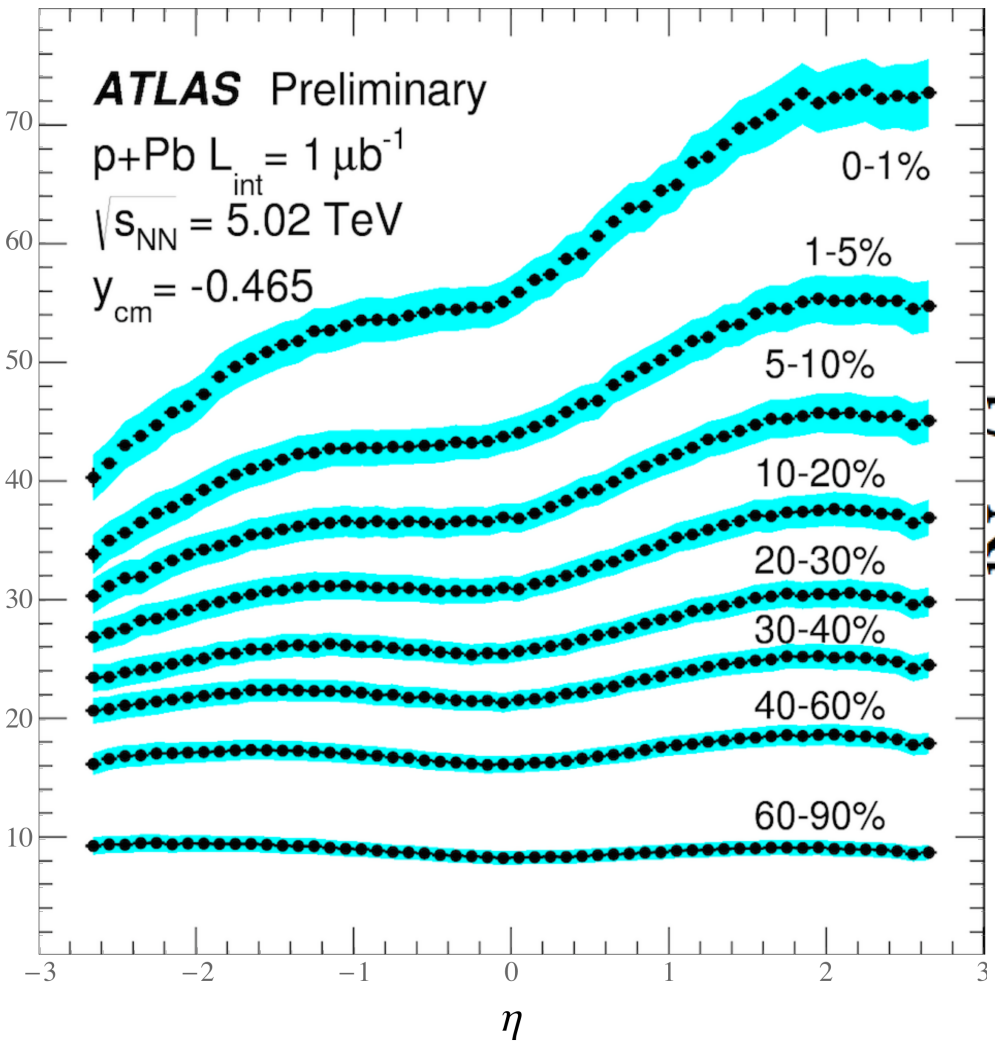


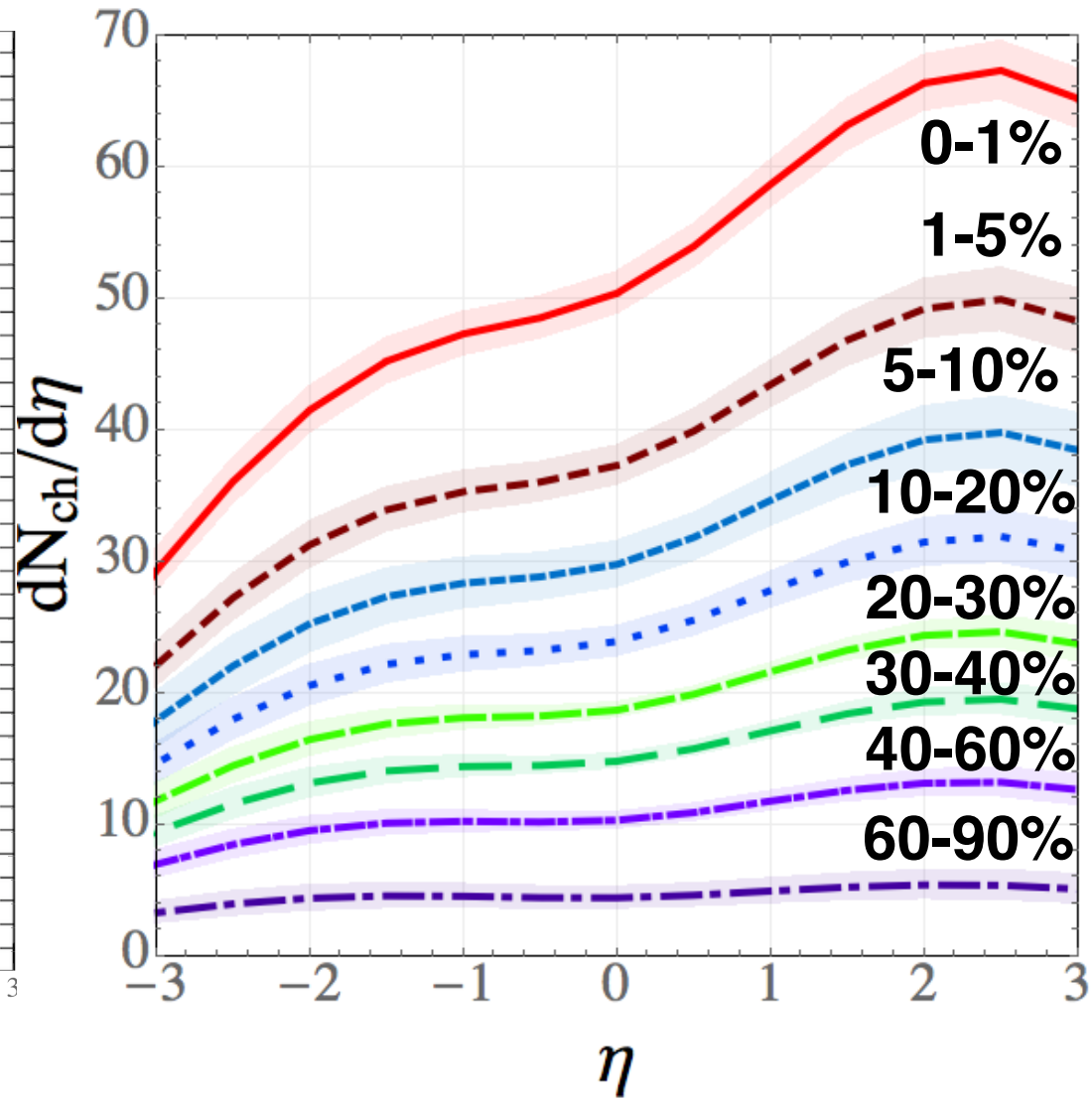
$$\left(1 \pm \frac{\eta}{y_{\mathrm{beam}}}\right) \exp\left(-\frac{(|\eta| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta| - \eta_0)\right)$$

Bozek, Wyskiel 1002.4999

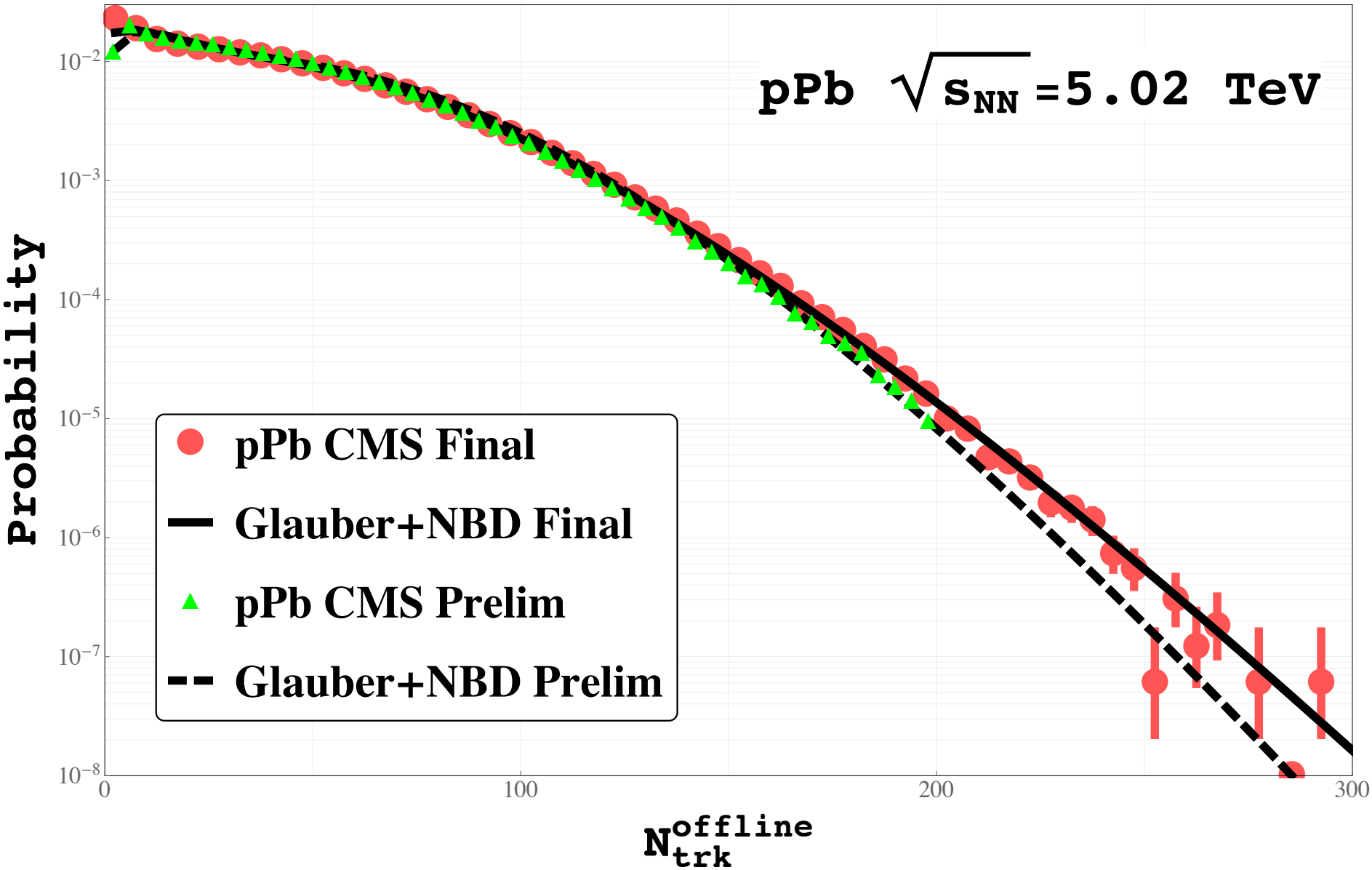
Pseudorapidity distribution

ATLAS arXiv/1403.5738



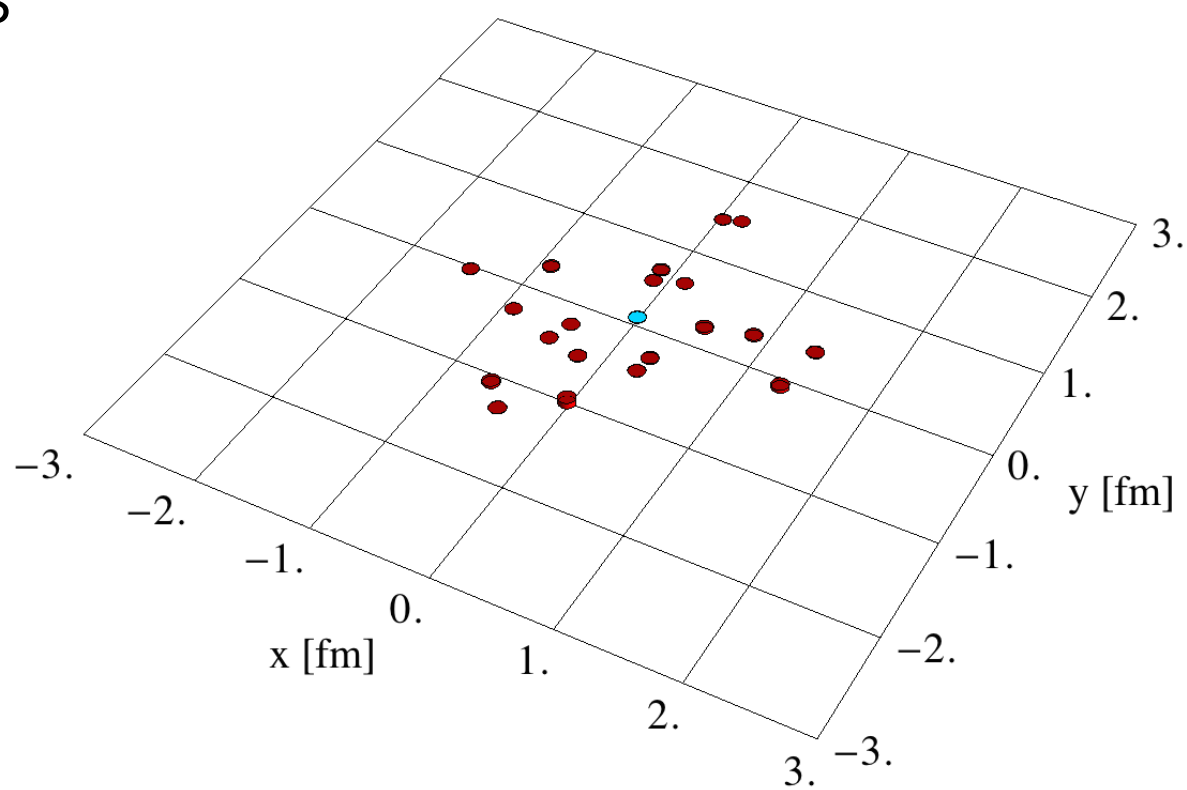


Initial conditions: Glauber+NBD



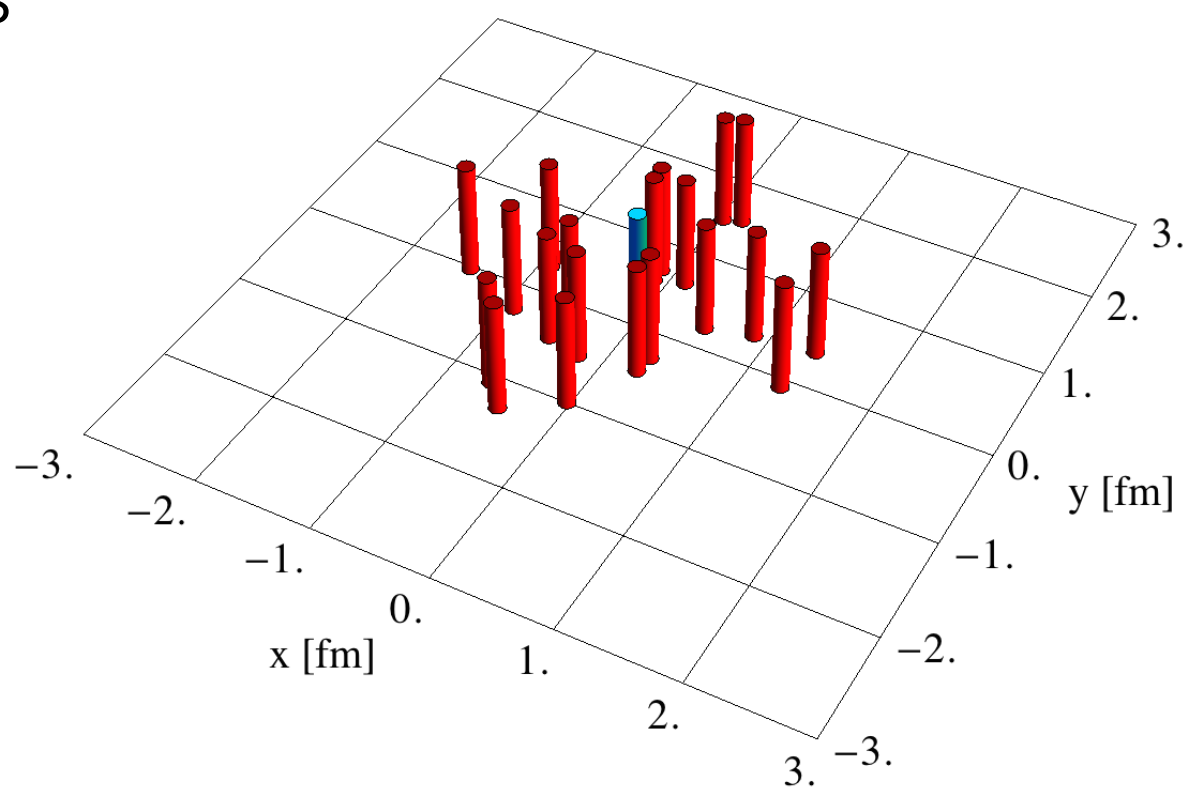
Initial conditions: Glauber

- sample participants



Initial conditions: Glauber

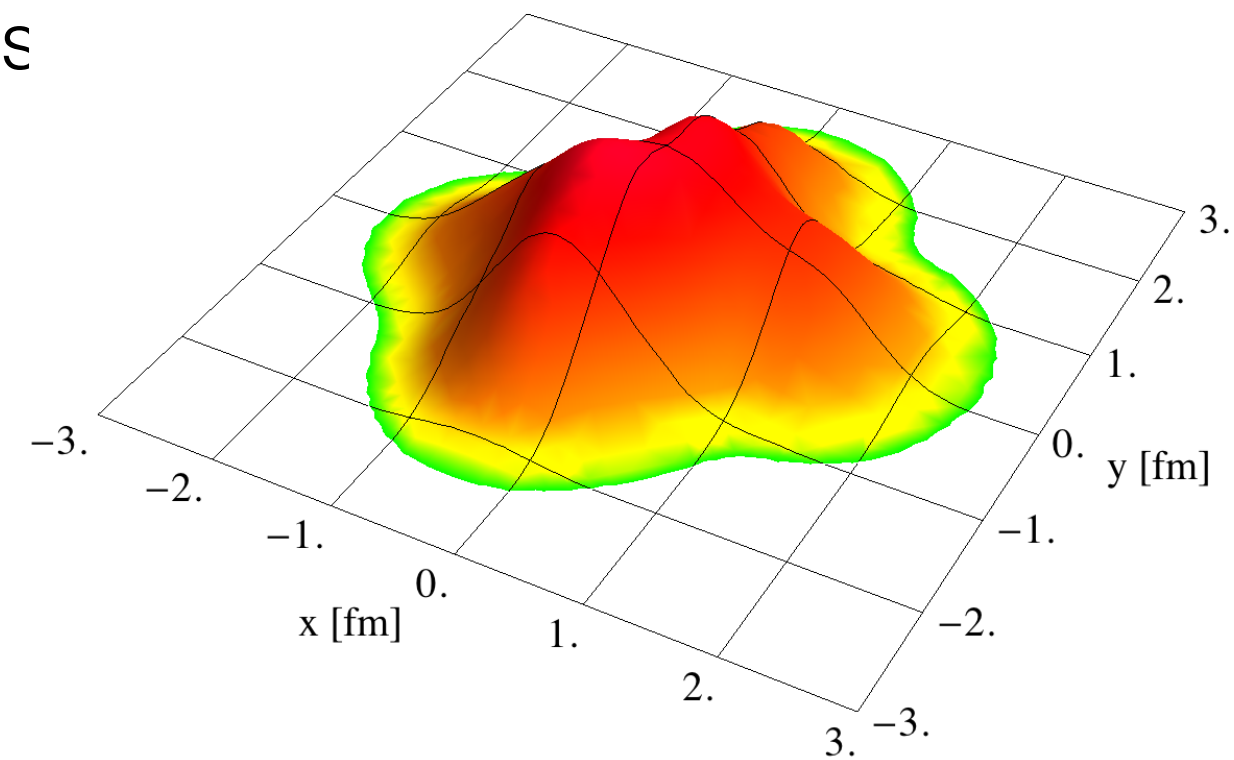
- sample participants
- add sources



Initial conditions: Glauber

$$\sigma = 0.40 \text{ fm}$$

- sample participants
- add sources
- increase σ



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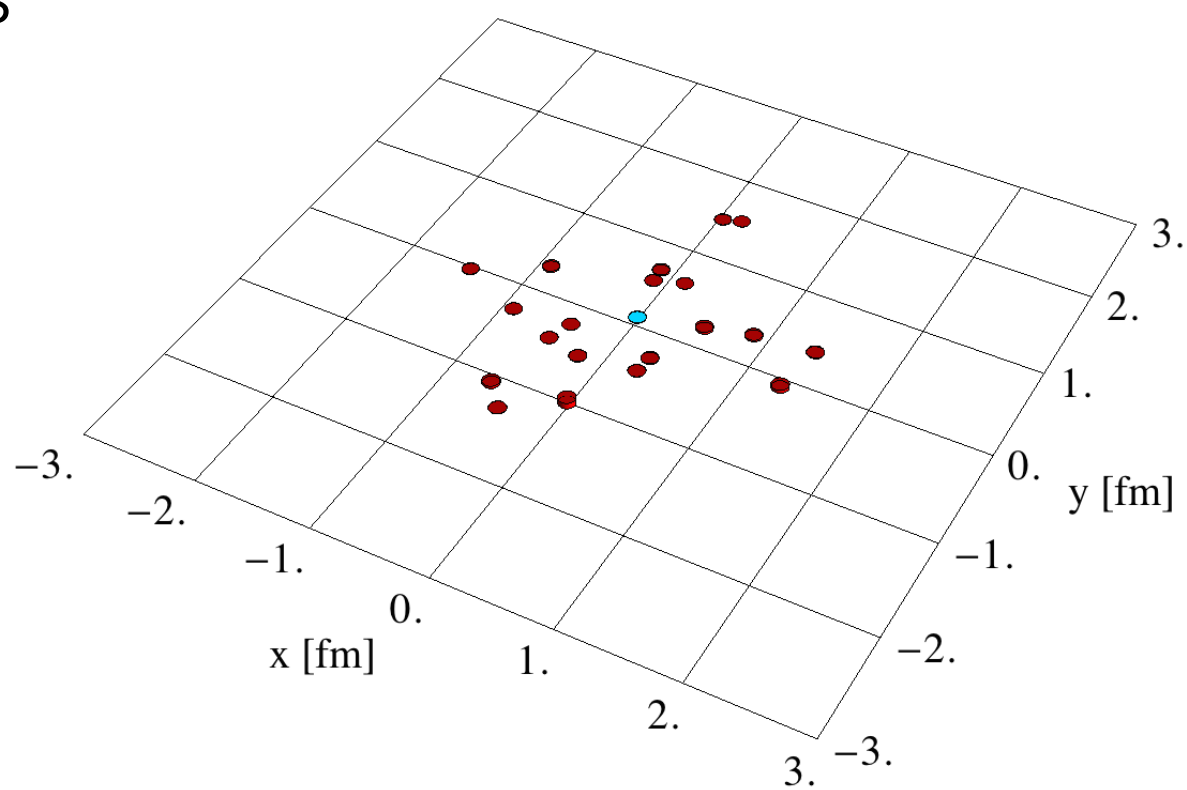

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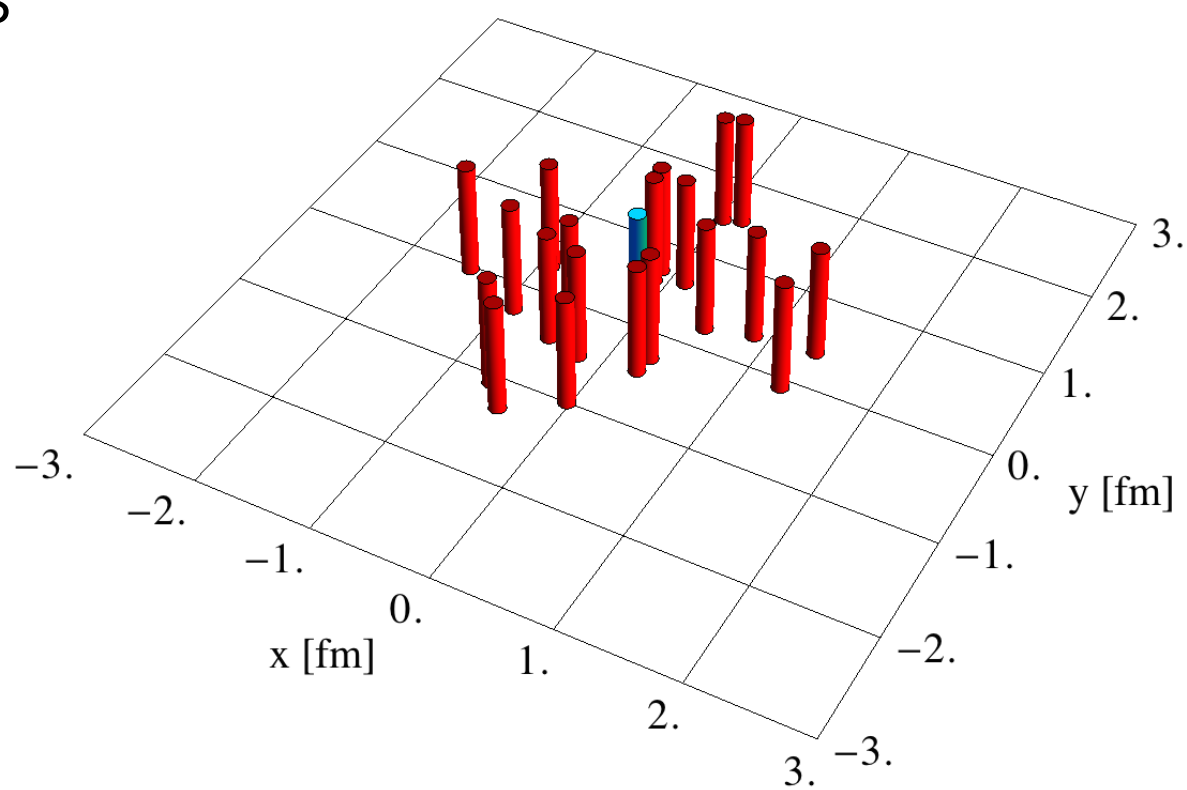
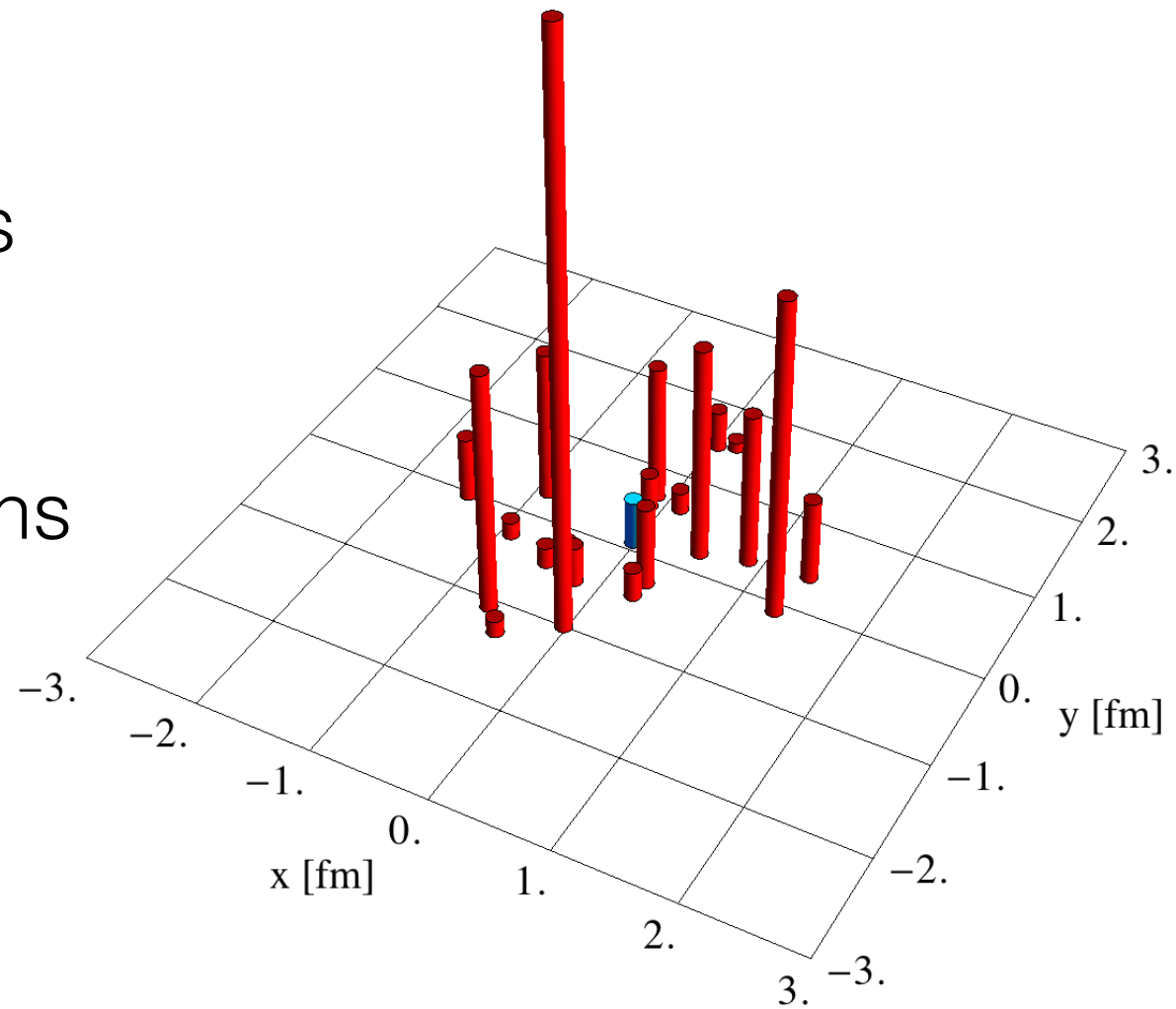


Initial conditions: Glauber+NBD

- sample participants
- add sources
- add NBD fluctuations



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- sample participants
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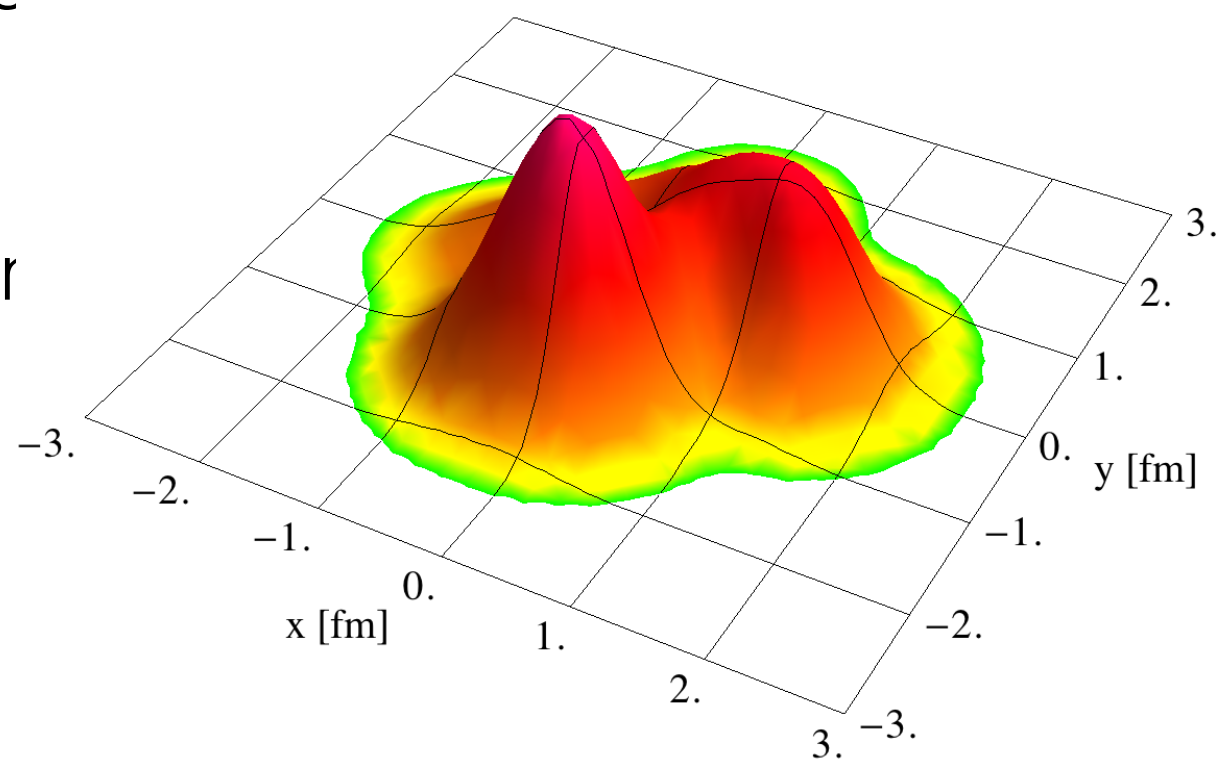
- add NBD fluctuation
- increase σ



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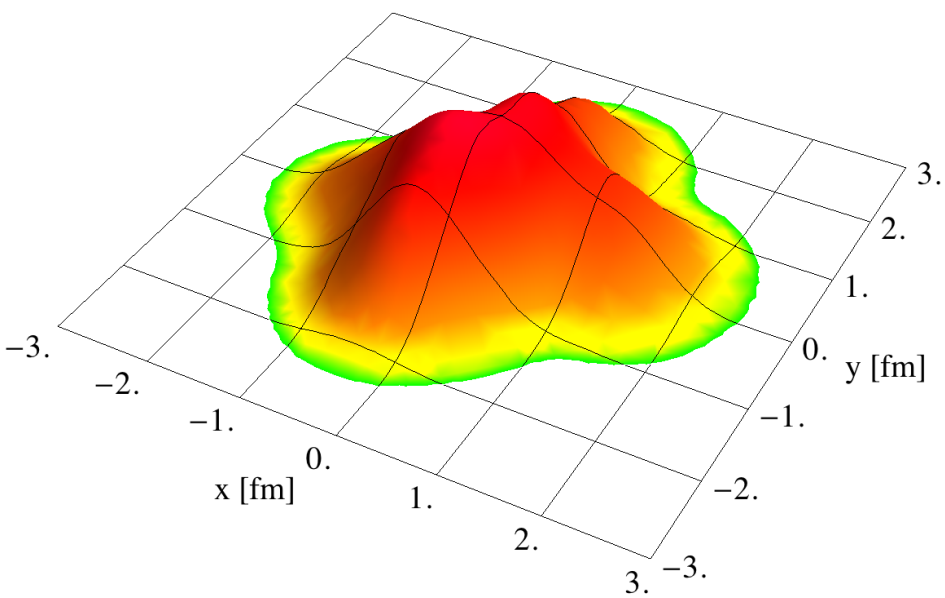
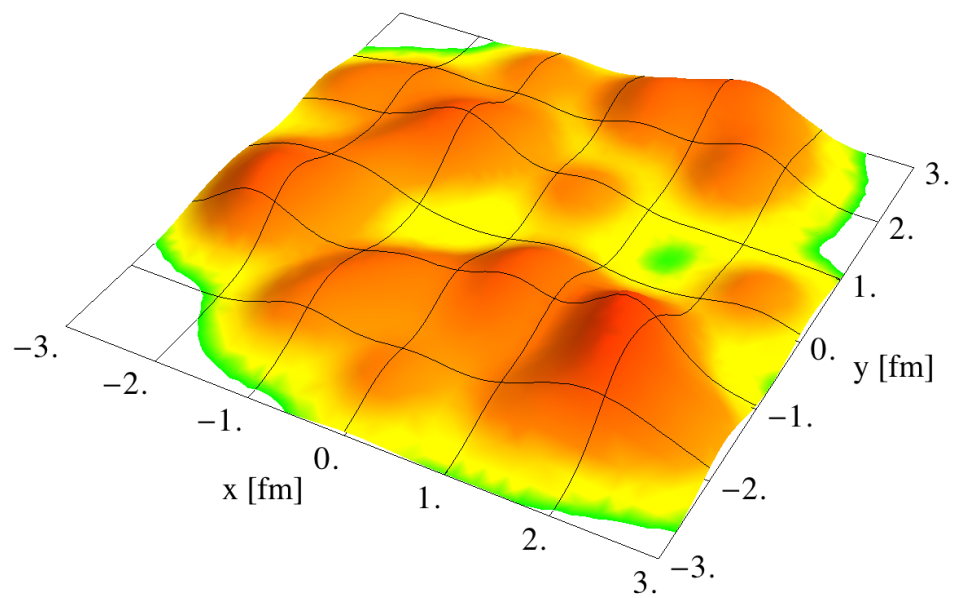


pPb

$\sigma = 0.40$ fm



PbPb

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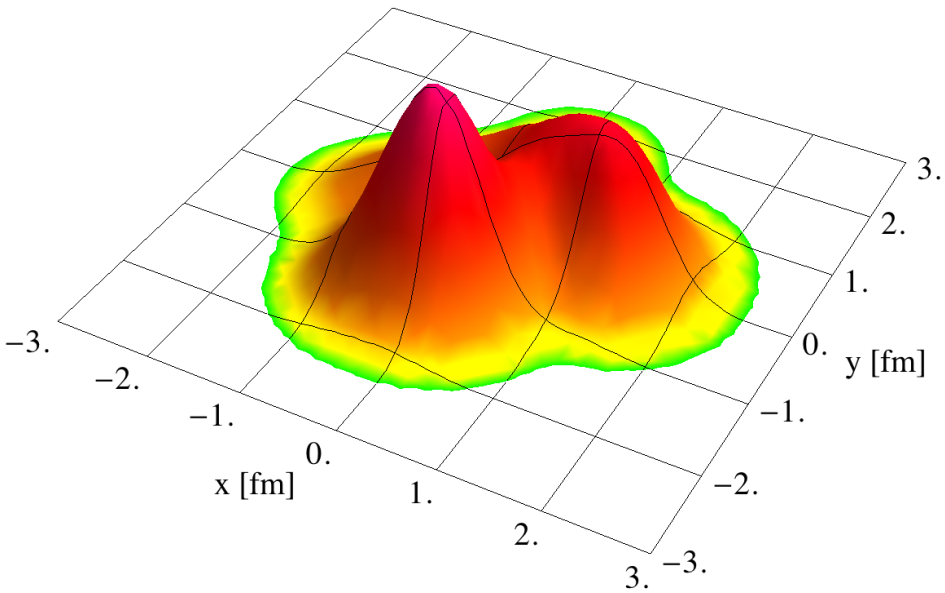

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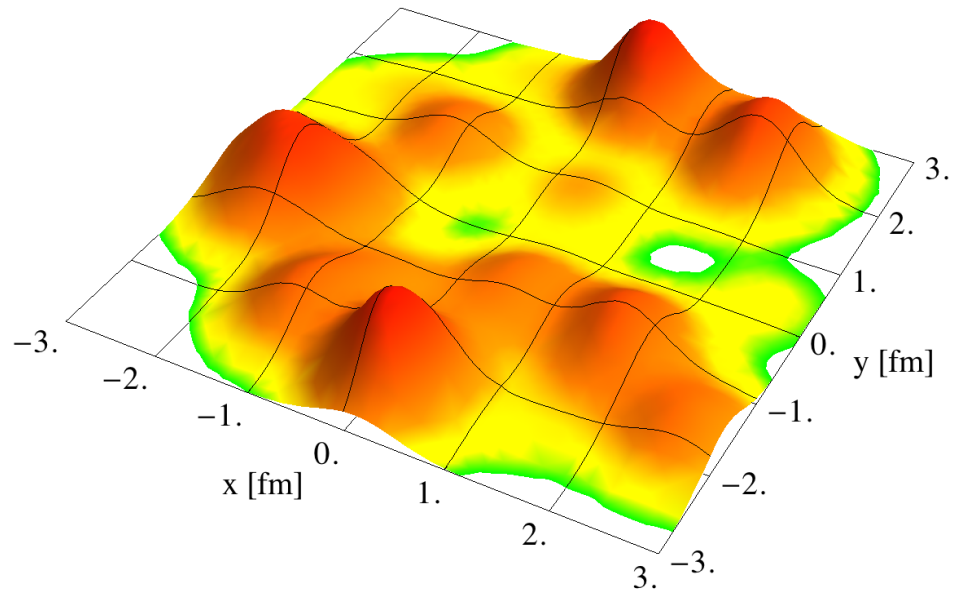
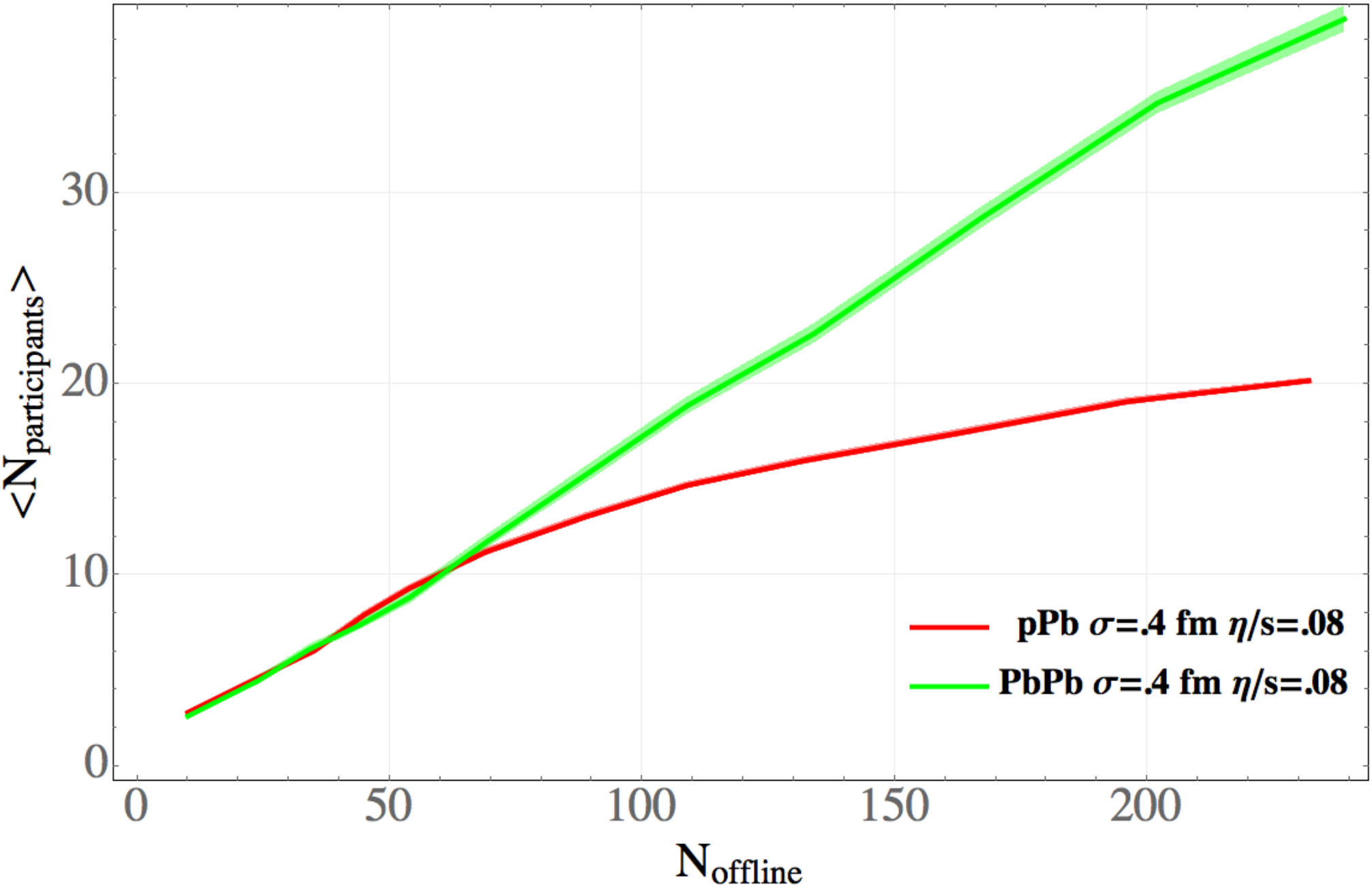
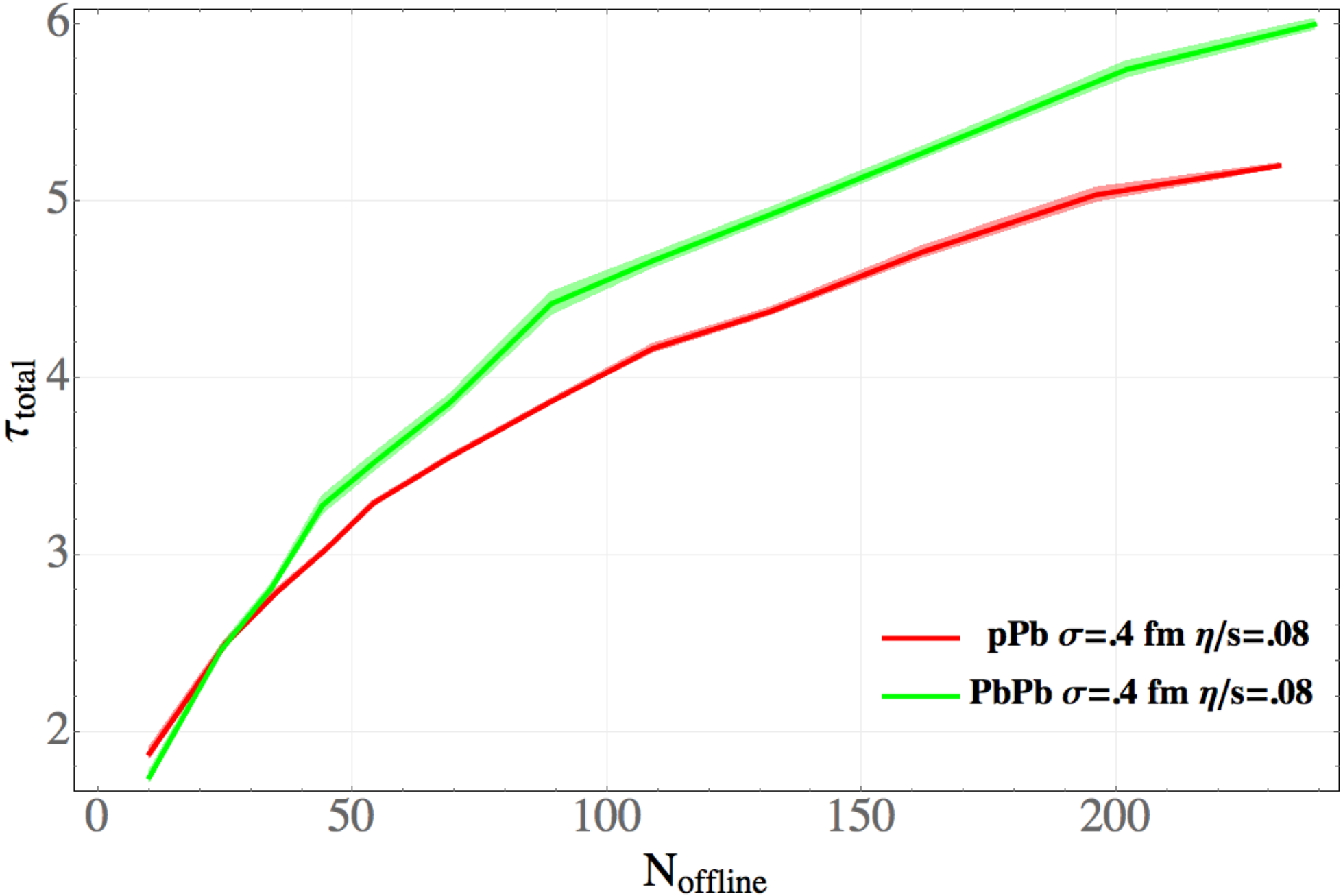


PbPb

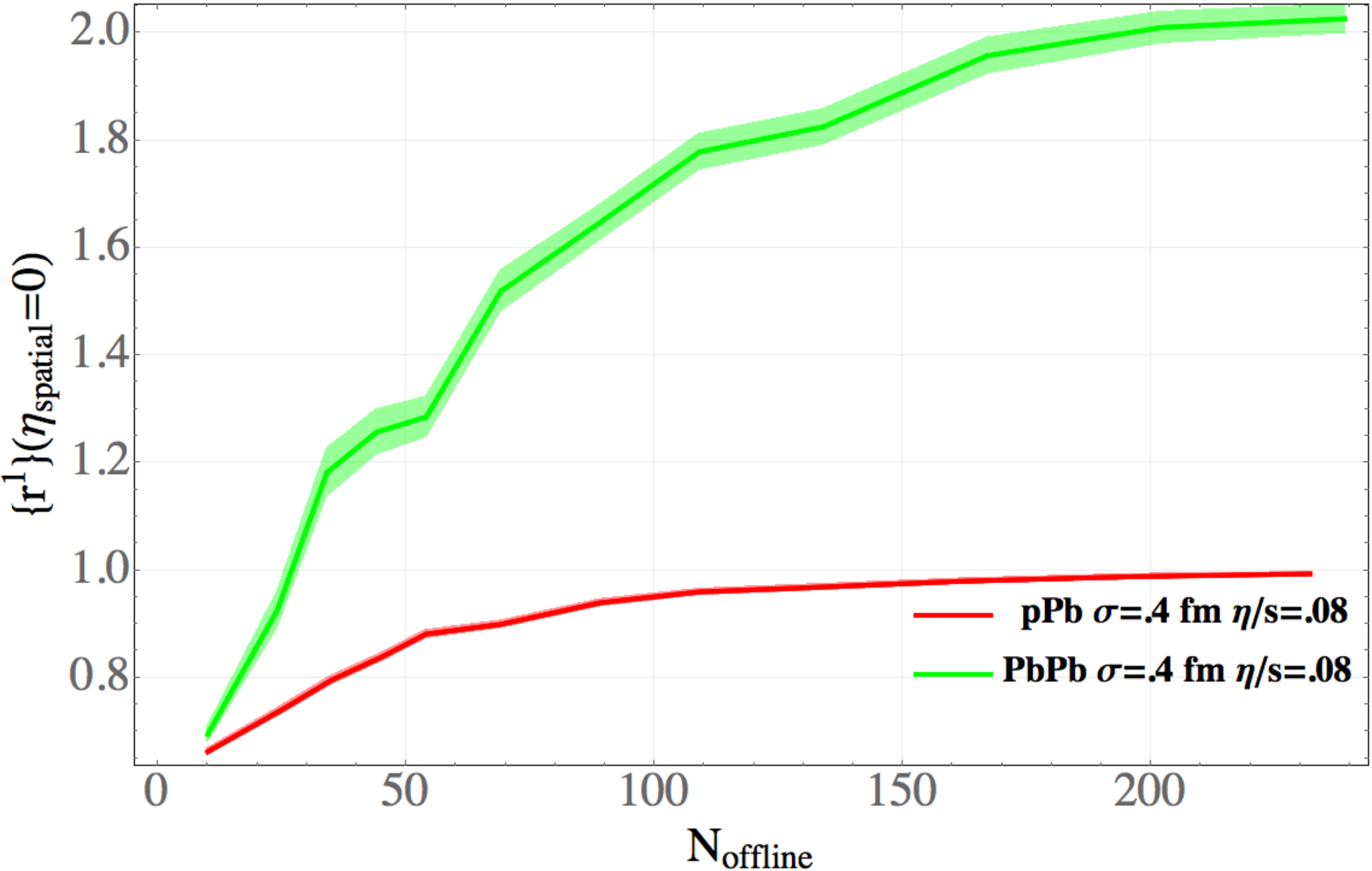
Number of participants in pPb and PbPb systems



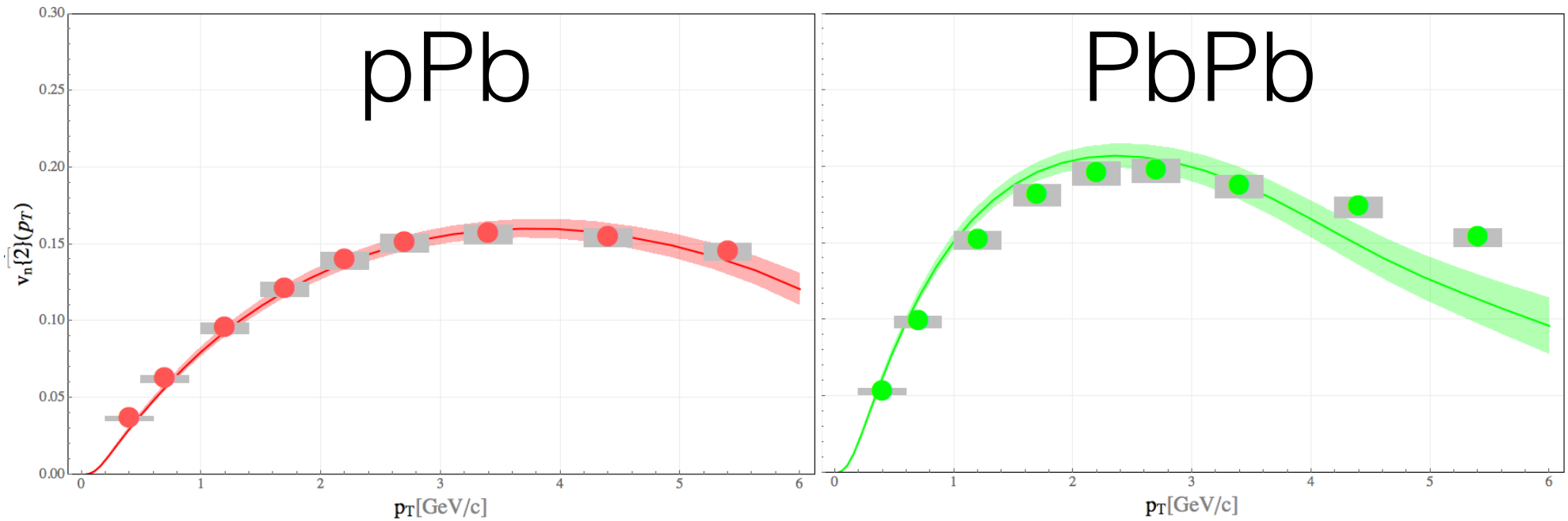
Evolution time in pPb and PbPb systems



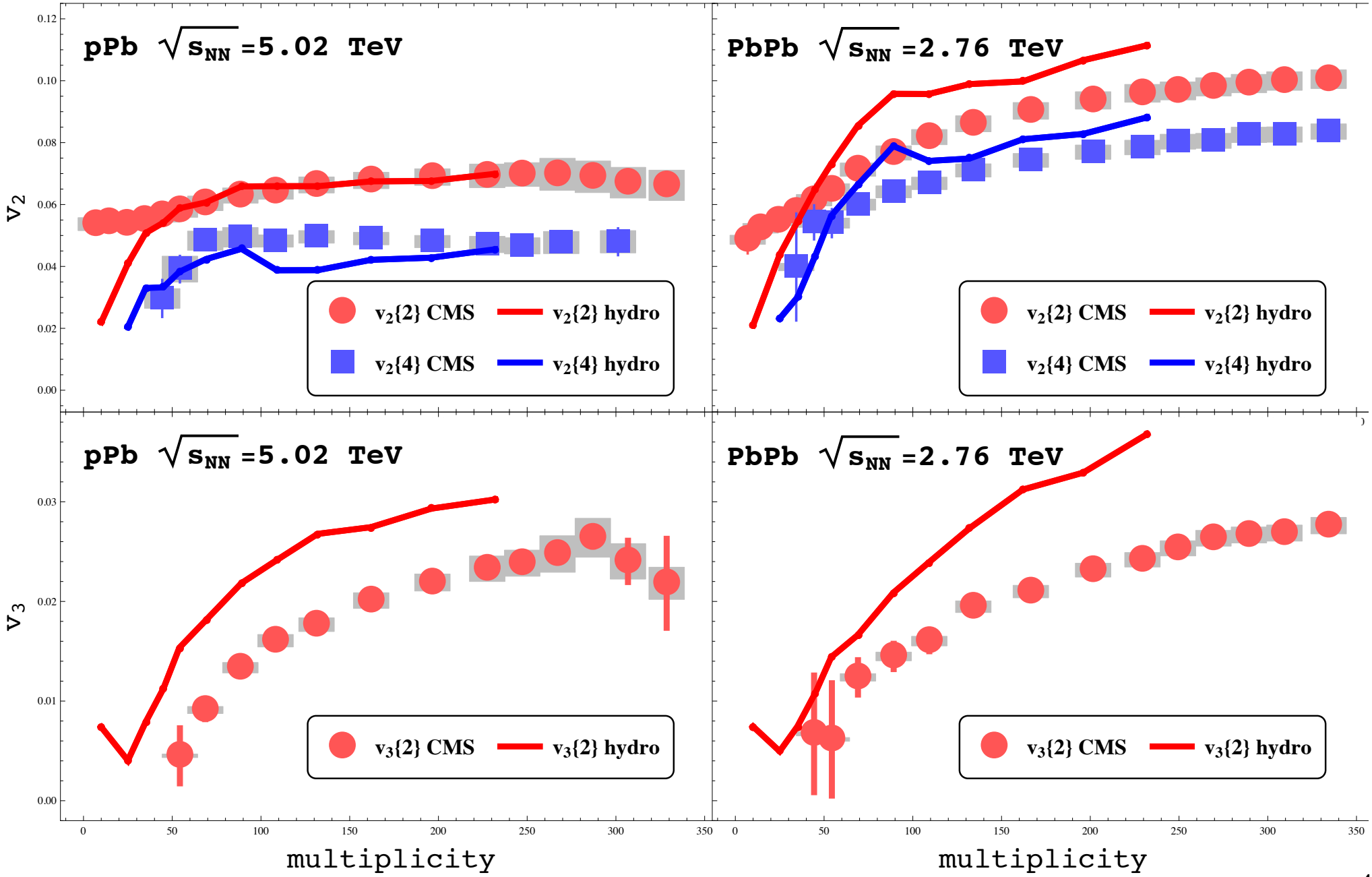
pPb and PbPb systems size



Flow observables for pPb and PbPb



Flow observables for pPb and PbPb



experiment:

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$$r_n \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}}$$

$$\frac{dN_{pairs}}{d^3p^a d^3p^b} \overset{\text{(flow)}}{=} \frac{dN}{d^3p^a} \times \frac{dN}{d^3p^b}$$

$$\langle \overline{e^{in(\phi^a - \phi^b)}} \rangle = \langle \overline{e^{in\phi^a}} \cdot \overline{e^{-in\phi^b}} \rangle$$

$$v_n^a e^{in\Psi_n^a} \equiv \overline{e^{in\phi^a}}$$

$$V_{n\Delta}^{ab}(p_T^a, p_T^b) = \langle v_n^a v_n^b e^{in(\Psi_n^a - \Psi_n^b)} \rangle$$

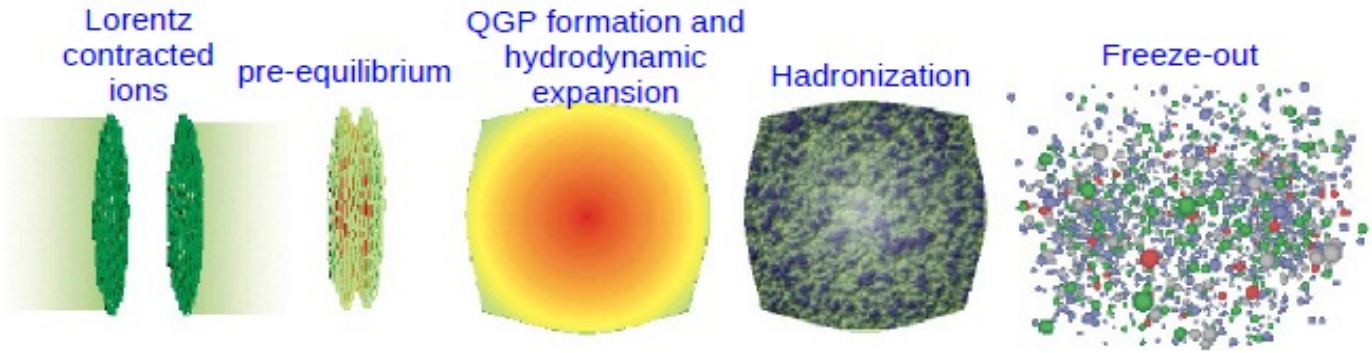
$$V_{n\Delta}(p_T^a, p_T^a) \geq 0$$

$$V_{n\Delta}(p_T^a, p_T^b)^2 \leq V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)$$

Insights from Acoustic Scaling

Roy A. Lacey
Stony Brook University

Backdrop



Essential Questions

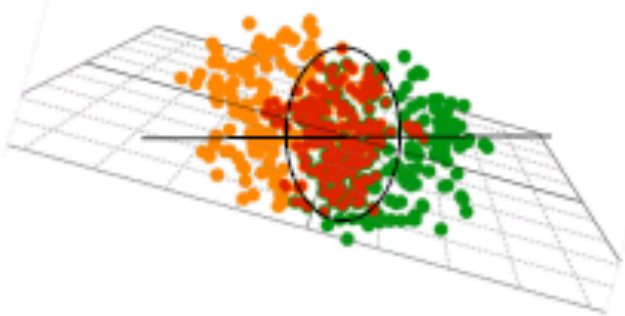
- *Do we have potent experimental constraints for initial-state fluctuations and what do they tell us?*
- *Is there a fundamental change in the reaction dynamics with reduced system size?*
  - *What are the Implications for η/s extraction?*

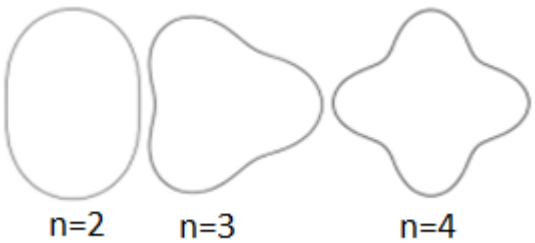
Acoustic scaling provide invaluable insights on these questions!!

Matter Flow

Initial Geometry characterized by many shape harmonics (ε_n) → drive v_n

Flow is acoustic





$$\frac{dN}{d\phi} \propto \left(1 + 2\sum_{n=1} v_n \cos\left[n\left(\phi - \Psi_n\right)\right]\right)$$

Acoustic viscous modulation of v_n

$$\delta T_{\mu\nu}(t,k) = \exp\left(-\frac{2}{3}\frac{\eta}{s}k^2\frac{t}{T}\right)\delta T_{\mu\nu}(0)$$

Staig & Shuryak **arXiv:1008.3139**

$$k = n / \bar{R} \qquad t \propto \bar{R}$$

$$\delta T_{\mu\nu}(n,t) = \exp\left(-\beta n^2\right)\delta T_{\mu\nu}(0), \quad \beta = \frac{2}{3}\frac{\eta}{s}\frac{1}{\bar{R}^2}\frac{t}{T}$$

Scaling expectations:

n^2 dependence

$$\left(\frac{v_n(p_T)}{\varepsilon_n}\right)^{1/n} \propto \exp\left(-\beta' n\right)$$

v_n is related to v_2

$$\frac{\left(v_n(p_T)\right)^{1/n}}{\left(v_2(p_T)\right)^{1/2}} \sim \frac{\left(\varepsilon_n\right)^{1/n}}{\left(\varepsilon_2\right)^{1/2}} \cdot \exp\left(-\beta'(n-2)\right)$$

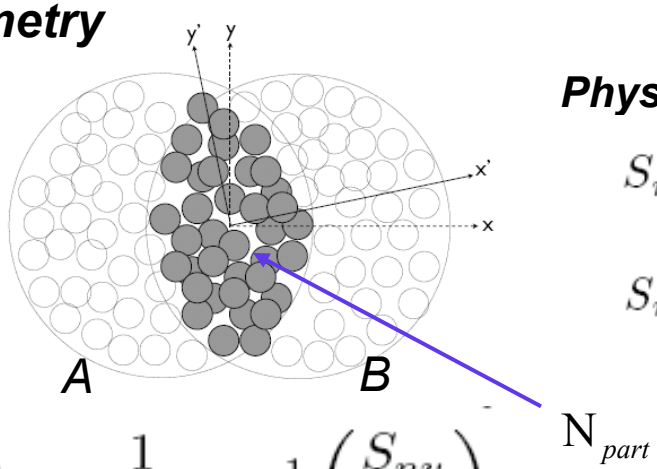
System size dependence

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Each of these scaling expectations has been exquisitely validated

Geometric quantities for scaling

Geometry



Phys. Rev. C 81, 061901(R) (2010)

$$S_{nx} \equiv S_n \cos(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp)\omega(\mathbf{r}_\perp)\cos(n\phi)$$

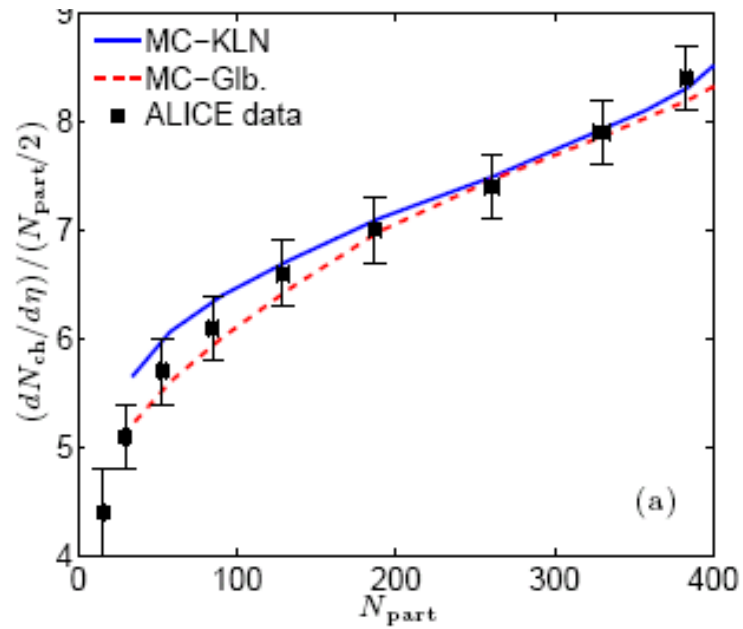
$$S_{ny} \equiv S_n \sin(n\Psi_n^*) = \int d\mathbf{r}_\perp \rho_s(\mathbf{r}_\perp)\omega(\mathbf{r}_\perp)\sin(n\phi),$$

$$\Psi_n^* = \frac{1}{n}\tan^{-1}\left(\frac{S_{ny}}{S_{nx}}\right)$$

$$\varepsilon_n = \left\langle \cos n\left(\phi - \psi_n^*\right)\right\rangle$$

$$\frac{1}{\bar{R}} = \sqrt{\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}\right)}$$

arXiv:1203.3605



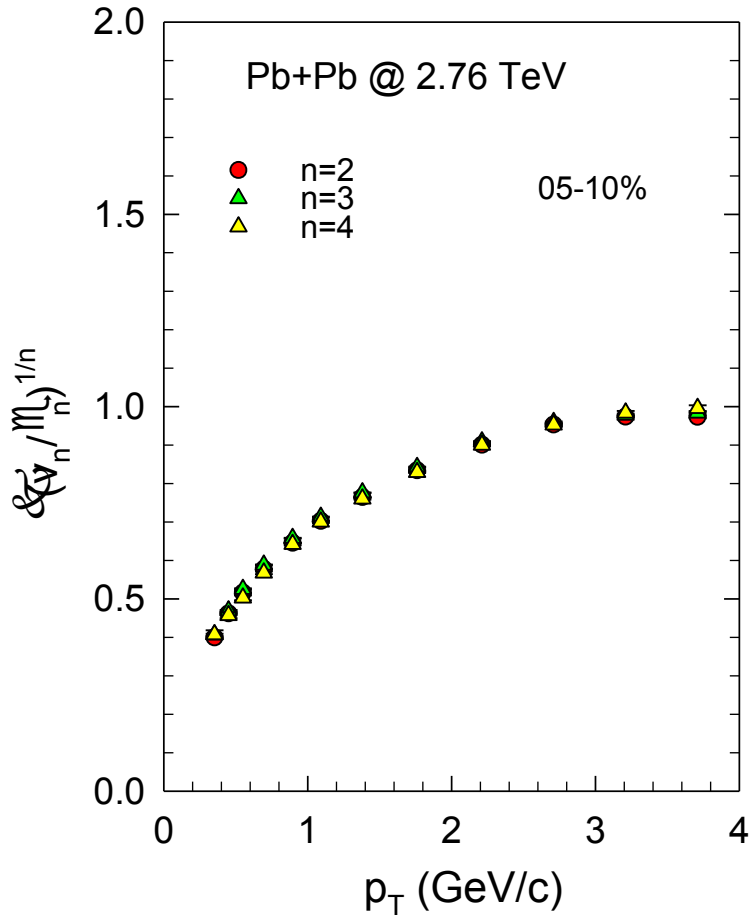
σ_x & σ_y → RMS widths of density distribution

- ***Geometric fluctuations included***
- ***Geometric quantities constrained by multiplicity density.***

Scaling properties of flow

***Acoustic Scaling* - n²**

$$\left(\frac{v_n(p_T)}{\varepsilon_n}\right)^{1/n} \propto \exp(-\beta' n)$$

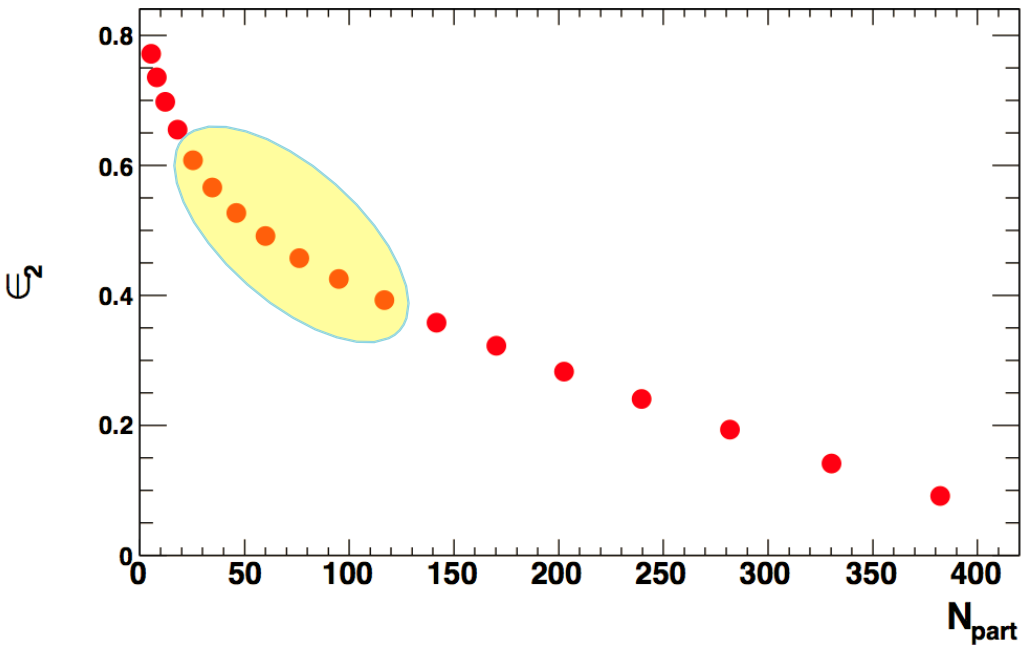


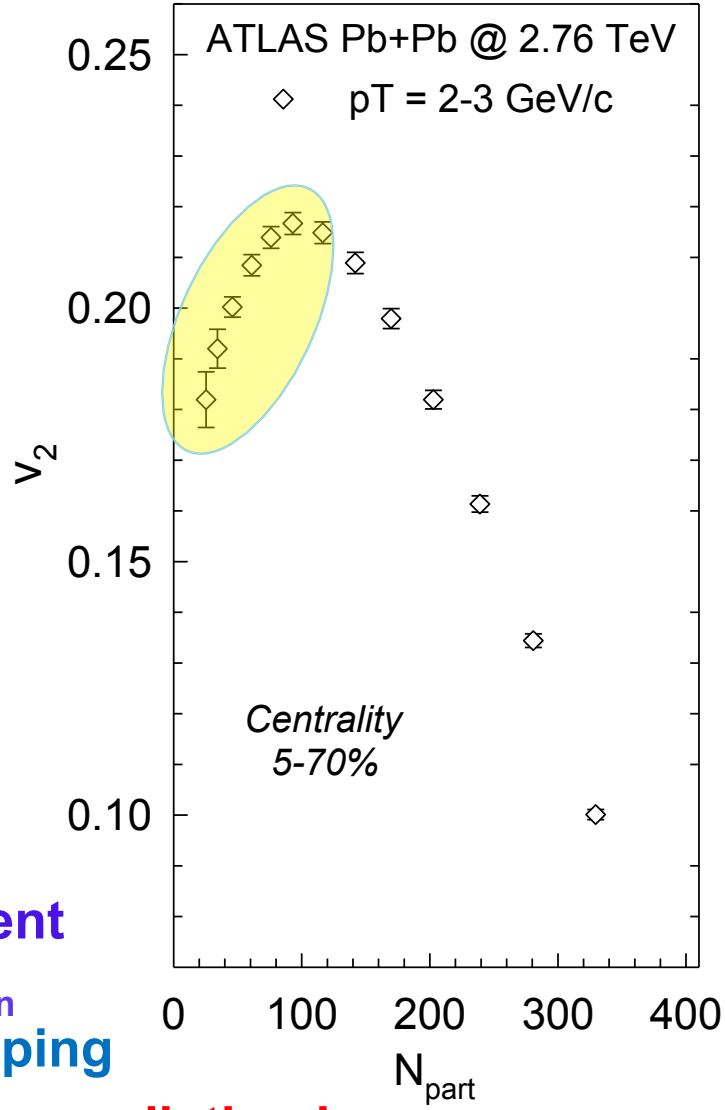
- ✓ **Characteristic n^2 viscous damping validated**
- ✓ **Similar patterns for other centrality selections**
- ✓ ***Important constraint for η/s***

Scaling properties of flow

Acoustic Scaling – 1/R

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$





- **Eccentricity change alone is not sufficient To account for the N_{part} dependence of v_n**

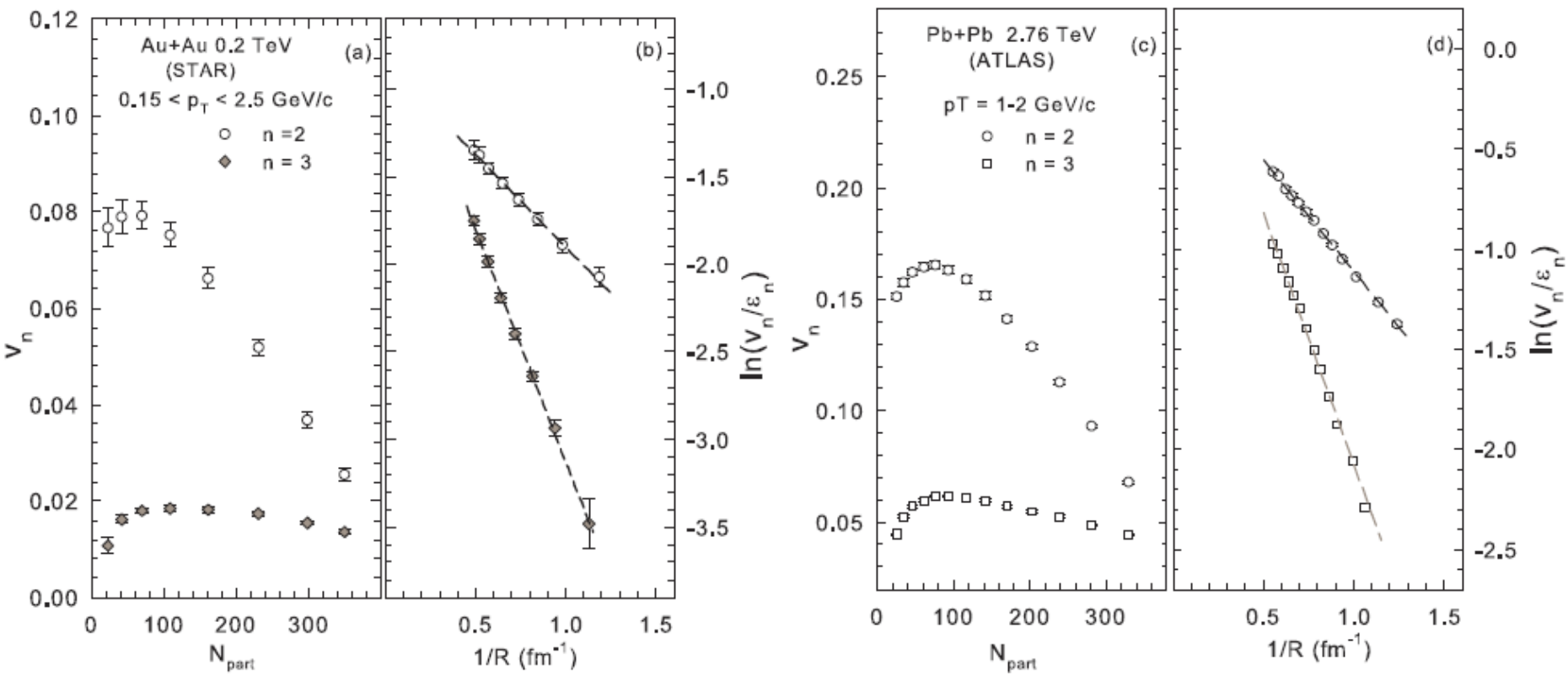
Transverse size (R) influences viscous damping

- ✓ **Characteristic $1/R$ scaling prediction is non-trivial**

Scaling properties of flow

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Acoustic Scaling – $1/R$



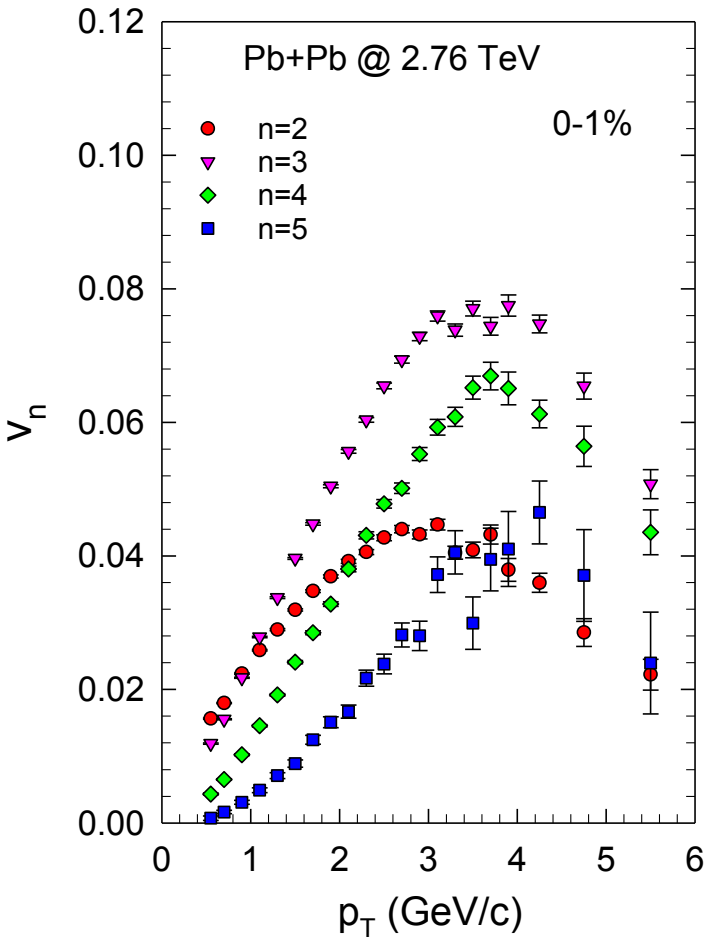
- ✓ **Characteristic $1/R$ viscous damping validated with n^2 dependence at RHIC & the LHC**
- ✓ ***Important constraint for η/s***

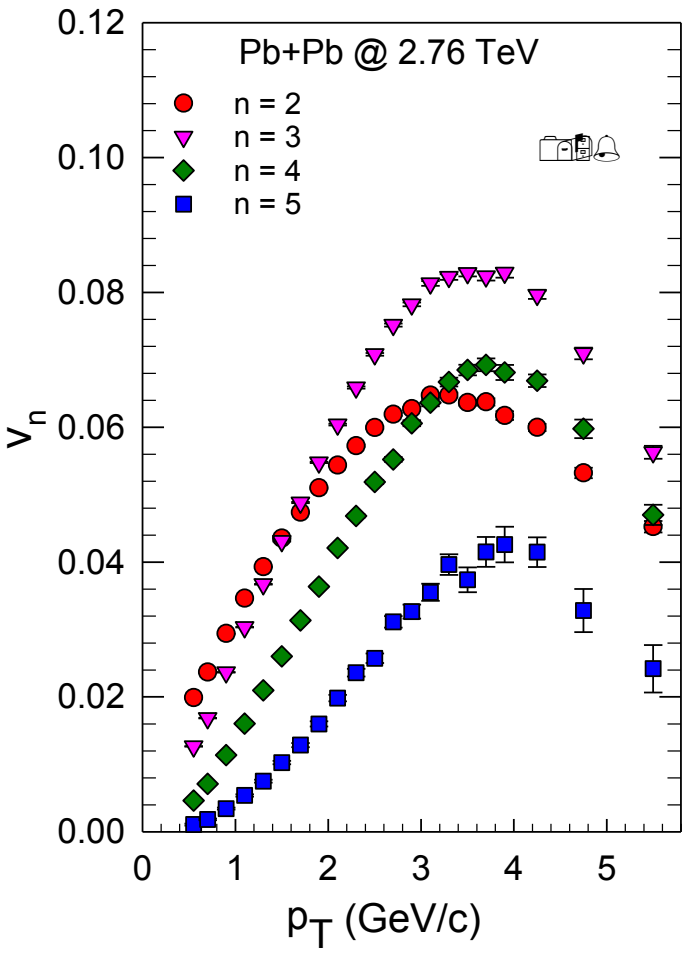
Experimental Constraints for initial-state fluctuations

Acoustic scaling in

- ✓ ***Ultra-central collisions***
- ✓ ***Shape-engineered events***

Ultra-central collisions



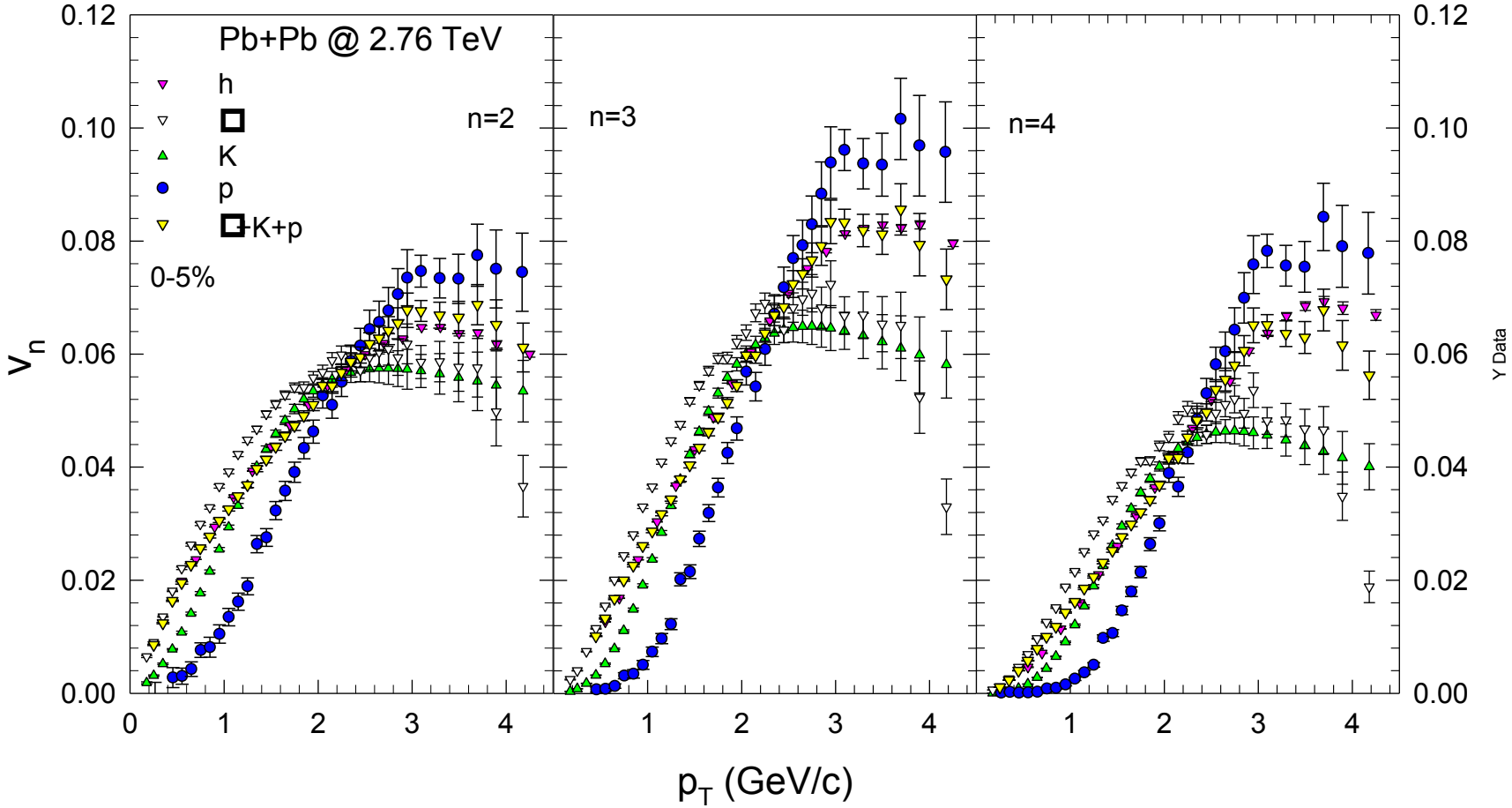


- ✓ ***Specific magitudes & ordering of vn provides a characteristic constraint***
  - ✓ ***Construct v_n from PID v_n via acoustic scaling ansatz***

Ultra-central collisions

$$\frac{\left(v_n(p_T)\right)^{1/n}}{\left(v_2(p_T)\right)^{1/2}} \sim \frac{\left(\varepsilon_n\right)^{1/n}}{\left(\varepsilon_2\right)^{1/2}} \cdot \exp\left(-\beta'(n-2)\right)$$

+ p/π & K/π



- ✓ ***Acoustic validation of vn magnitudes & trends!***
- ✓ ***Similar results for 0-1% centrality selection***

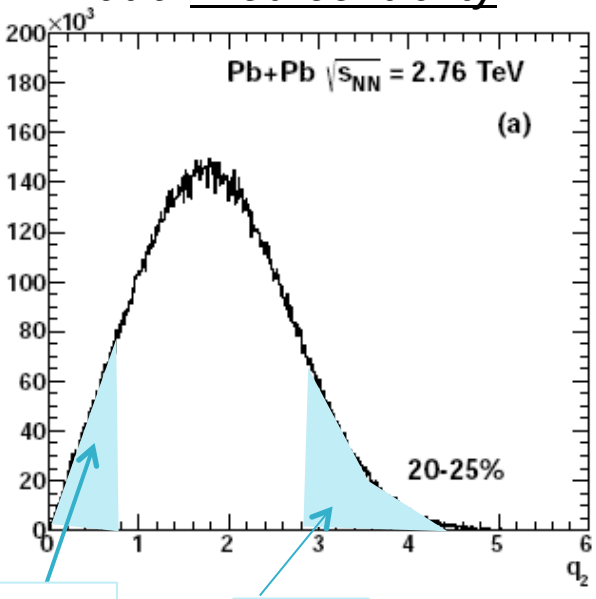
Shape-engineered Events

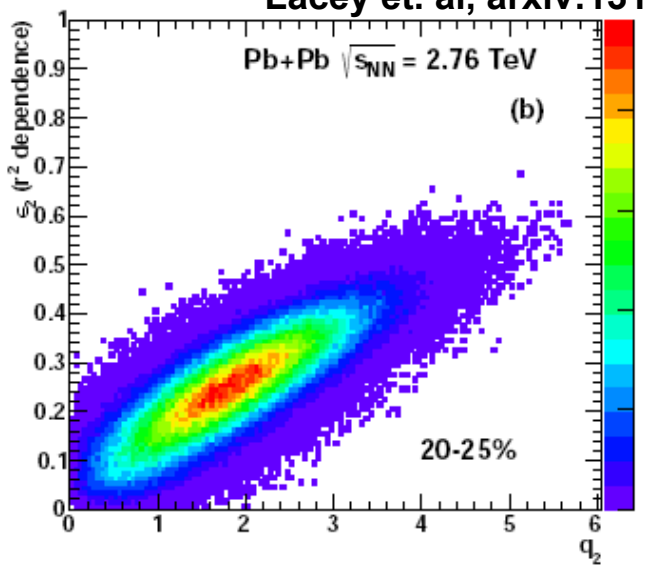
Shape fluctuations lead to a distribution of the Q vector at a fixed centrality

$$Q_{n,x} = \sum_i^M \cos(n\phi_i);\ Q_{n,y} = \sum_i^M \sin(n$$

$$q_n = Q_n/\sqrt{M},$$

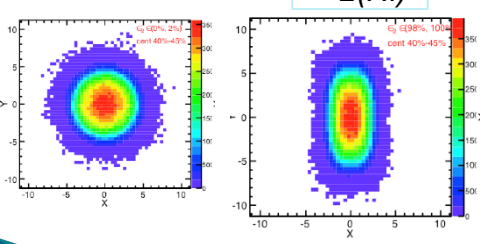
Lacey et. al, arxiv:1311.1728





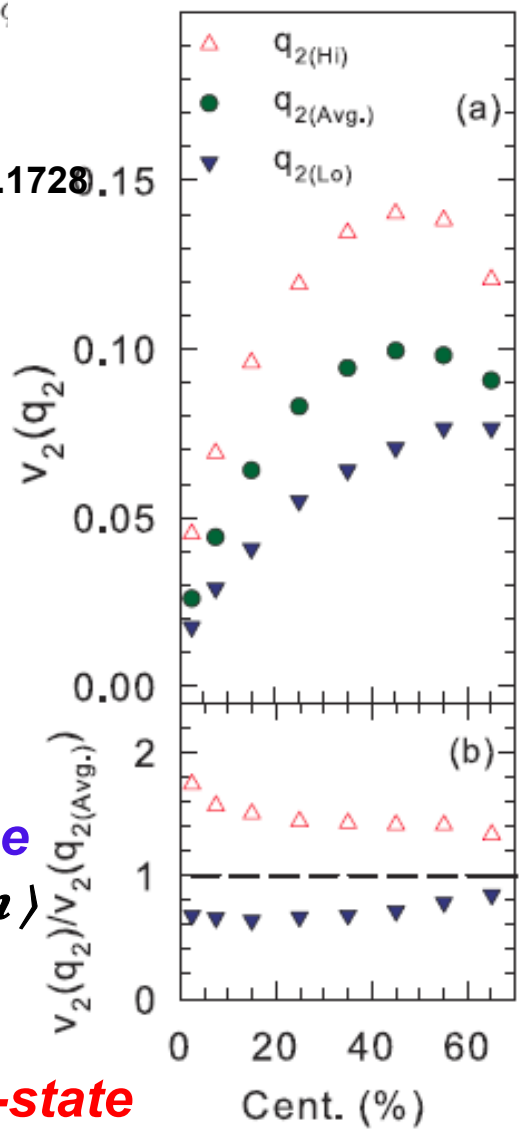
$q_{2(Lo)}$

$q_{2(Hi)}$



- ***Cuts on q_n should change the magnitudes ⟨∈↓n⟩, ⟨v↓n⟩, ⟨R↓n⟩ at a given centrality due to fluctuations***
  - ***Viable models for initial-state fluctuations should still scale***

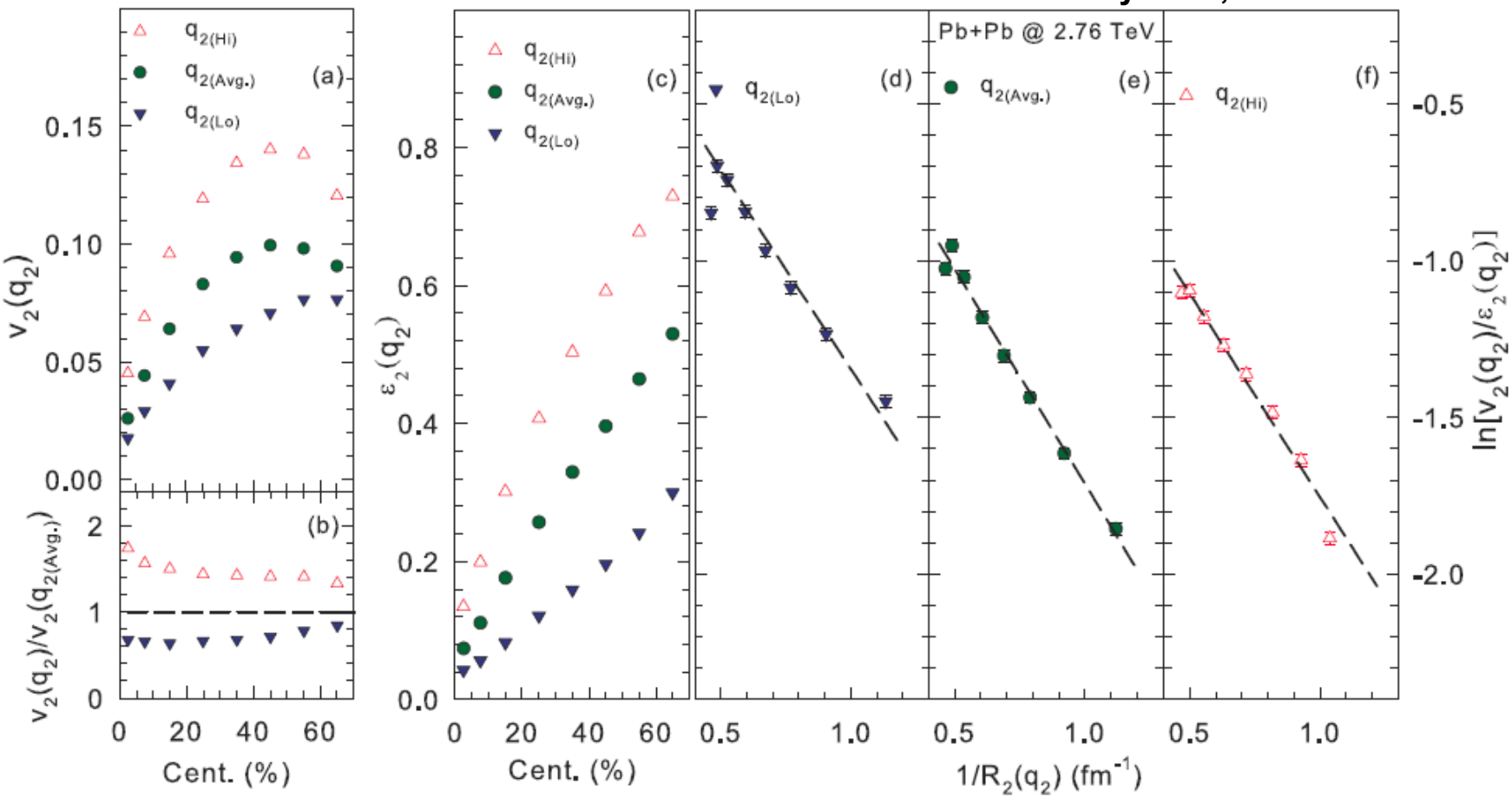
ALICE data



Scaling properties of flow

Acoustic Scaling of shape-engineered events

Lacey et. al, arxiv:1311.1728



- ✓ **Characteristic $1/R$ viscous damping validated for different event shapes at the same centrality**
- ✓ ***A further constraint for initial fluctuations model and η/s***

- *Is there a fundamental change in the reaction dynamics with reduced system size?*

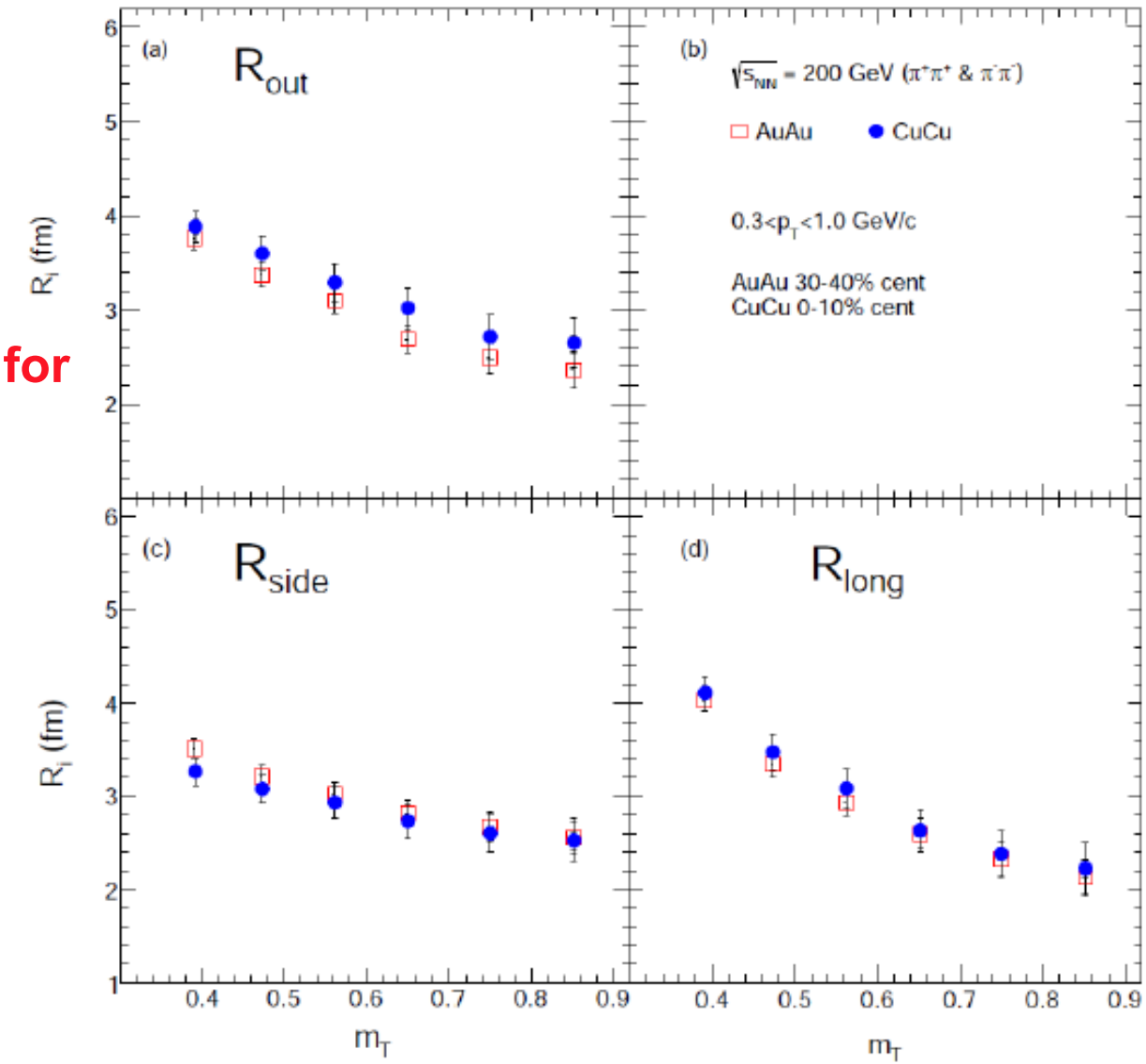
Compare patterns sensitive to the reaction dynamics

- ✓ ***HBT radii***
- ✓ ***Acoustic scaling of v_n***

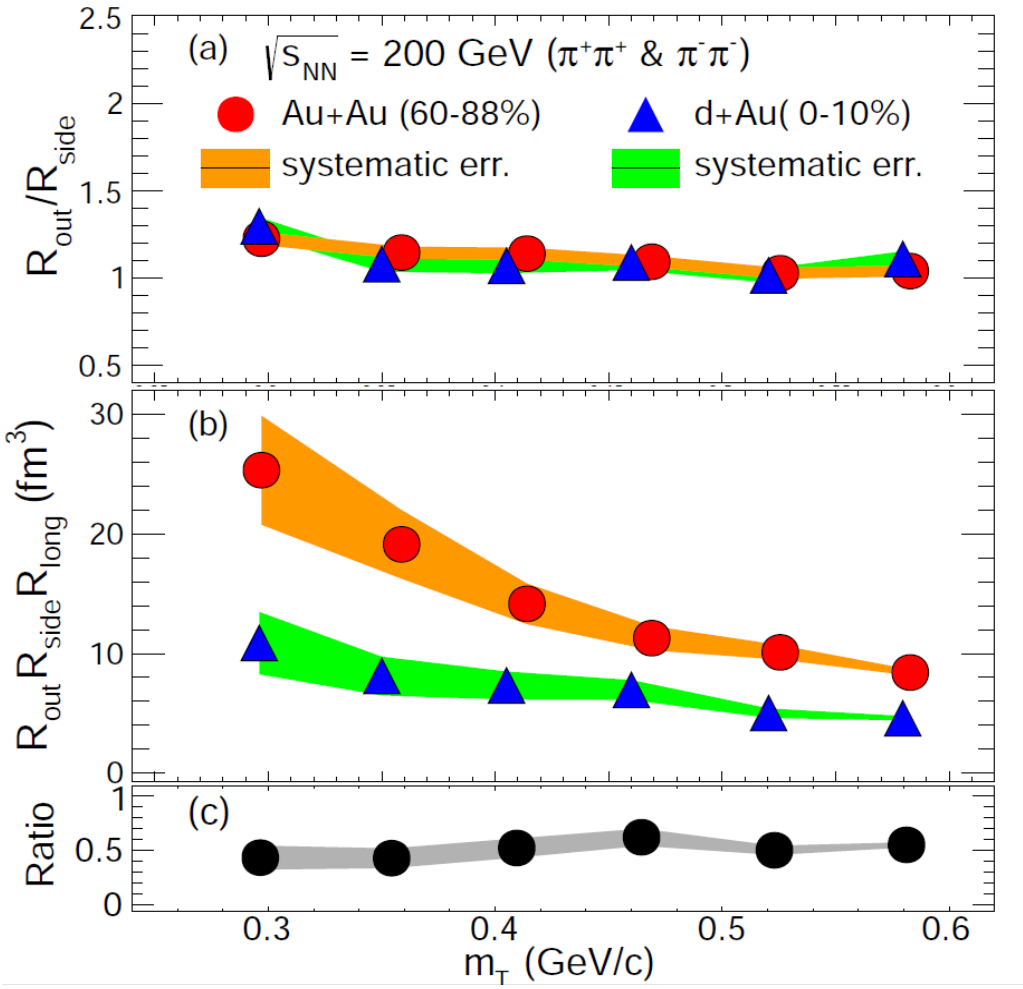
Comparison of Au+Au and Cu+Cu collisions – m_T dependence

Similar m_T dependence for Both systems

Similar influence of Space-momentum Correlations!



Comparison of Au+Au and d+Au collisions – m_T dependence



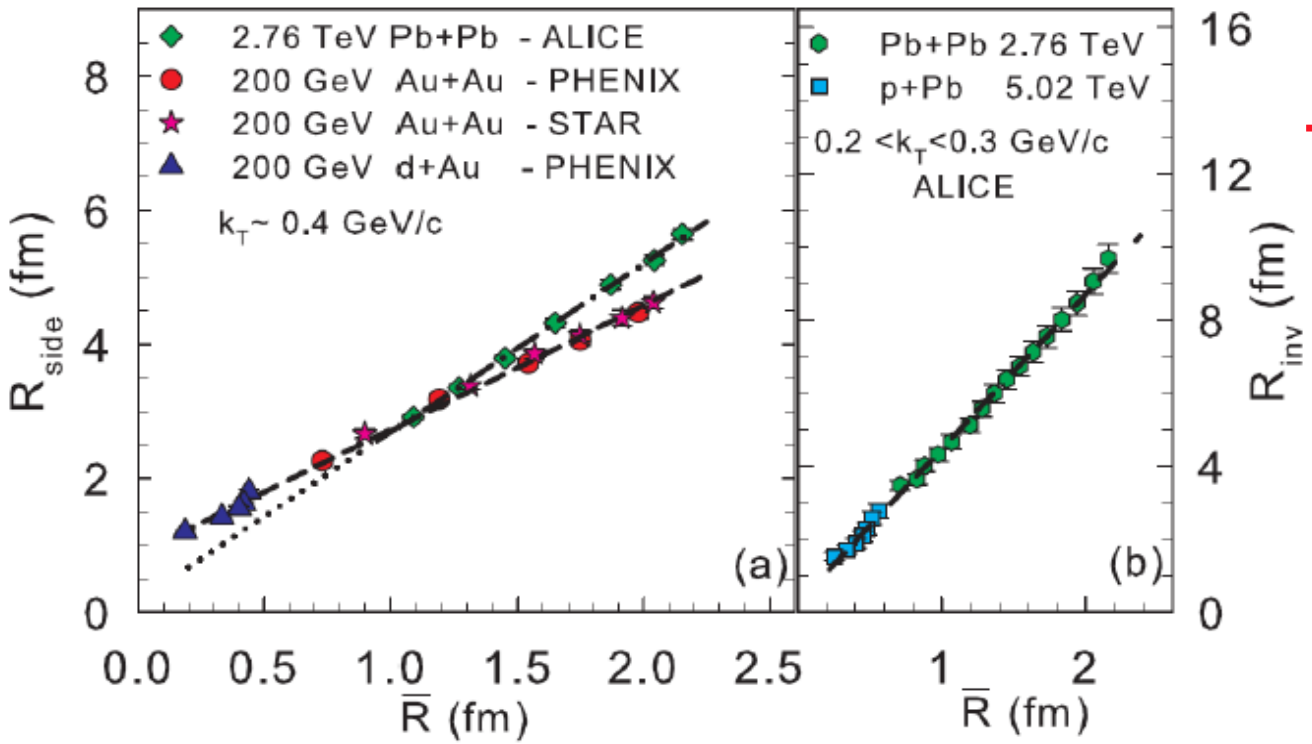
$$R_{out}^2 = \frac{R_{geom}^2}{1+\frac{m_T}{T}\upsilon^2} + \frac{1}{2}\left(\frac{T}{m_T}\right)^2 \beta_T^2 \tau_0^2 \qquad R_{side}^2 = \frac{R_{geom}^2}{1+\frac{m_T}{T}\upsilon^2}$$

$$\frac{R_{out}}{R_{side}} \propto \Delta\tau$$

Similar m_T dependence for Both systems

d+Au indicates a smaller freeze-out volume!

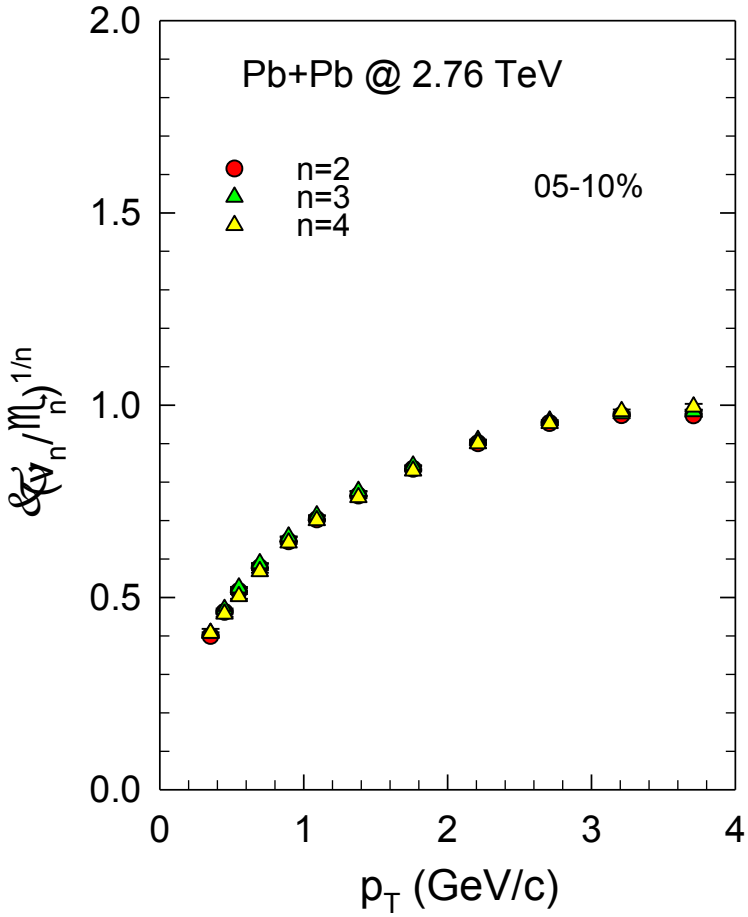
Scaling of the transverse radii

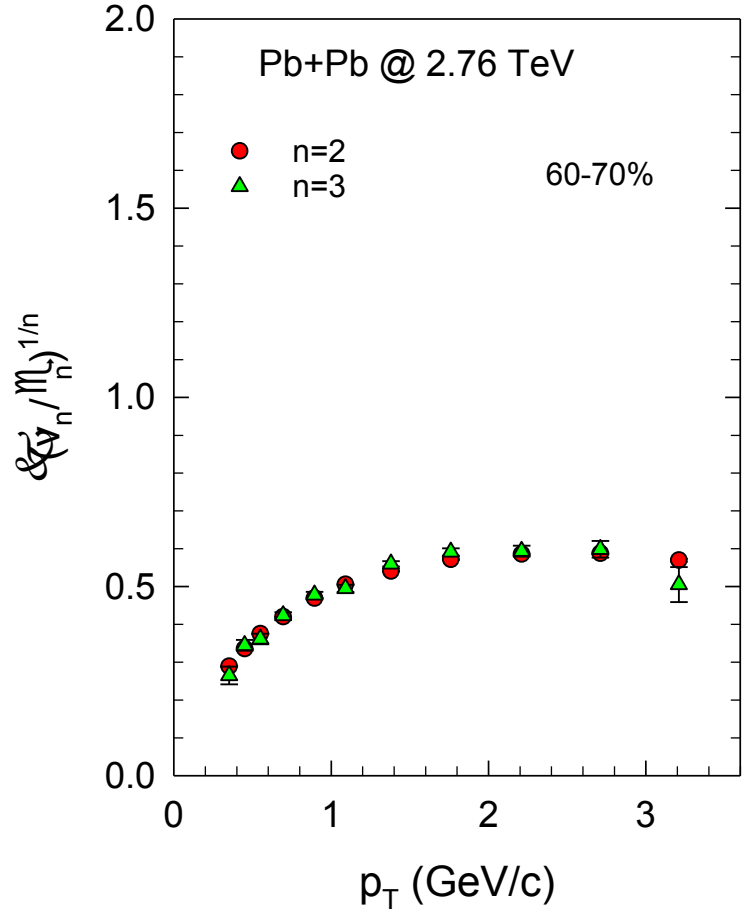


d+Au, Cu+Cu and Au +Au radii scale for the same beam energy

- Expansion dynamics of the d+Au system appears to be strongly influenced by final state effects
- Larger expansion rate at the LHC

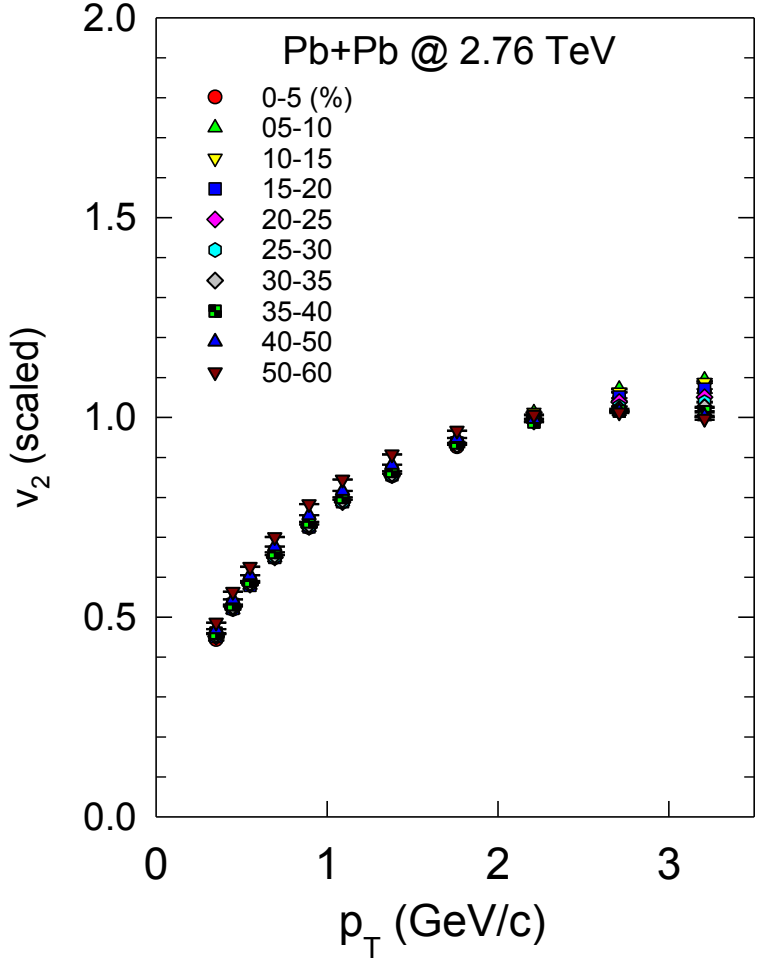
System-size dependence





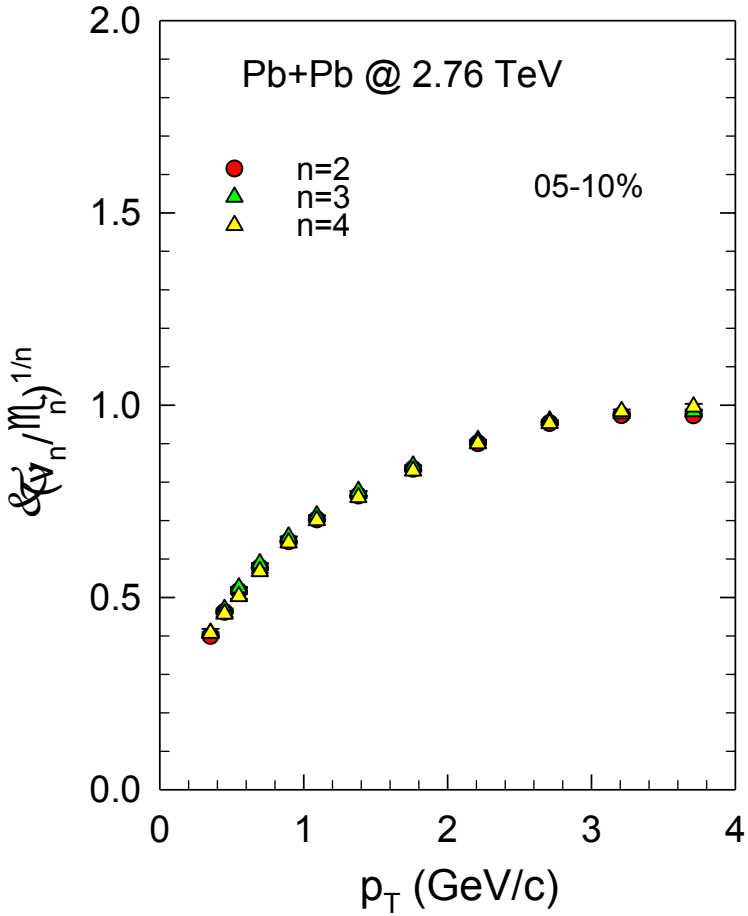
- ✓ **Expected scaling patterns validated**
- ✓ ***Important constraint for η/s***

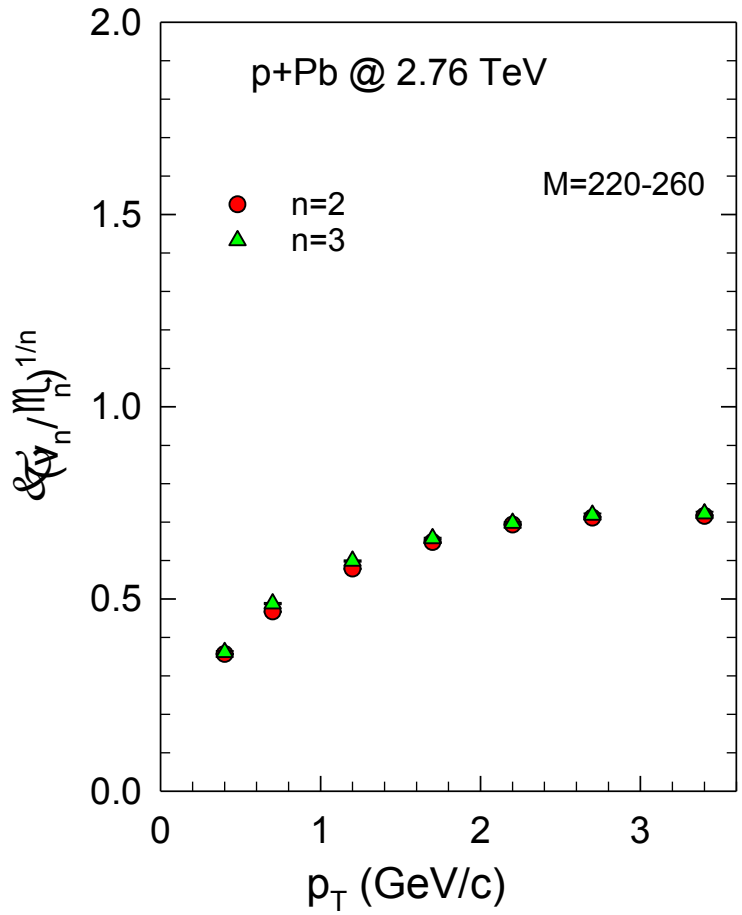
Scaling properties of flow



✓ ***Combined scaling understood***

System-size dependence





- ✓ **Similar acoustic patterns for p+Pb and Pb+Pb**
- ✓ ***Similar results for other small systems***

Answer:

- *No strong indication for a fundamental change in the reaction dynamics with reduced system size?*

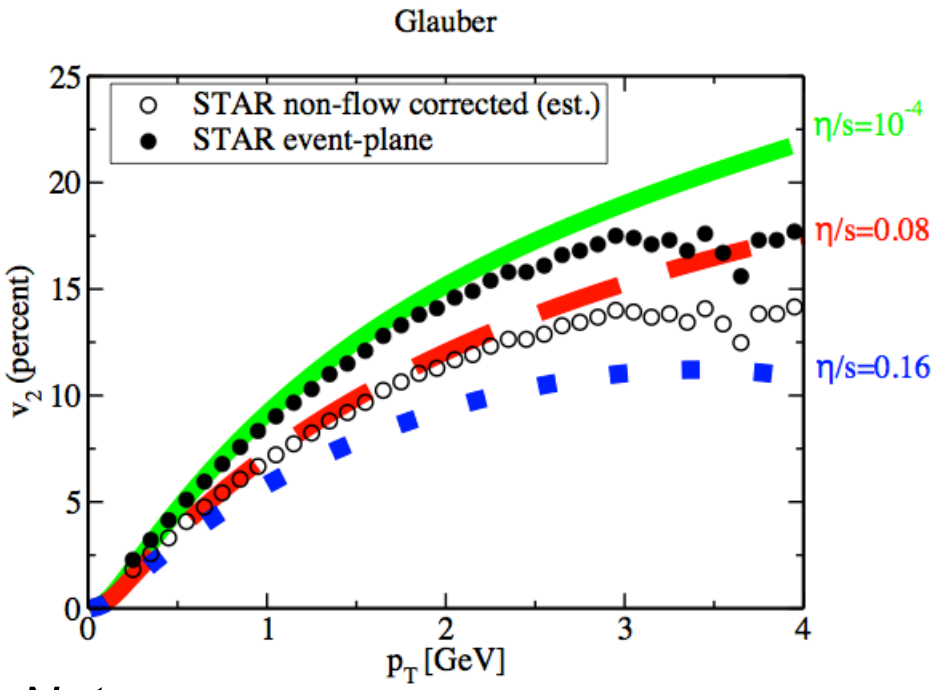
Other system combinations – including deformed nuclei - fully predictable → testable

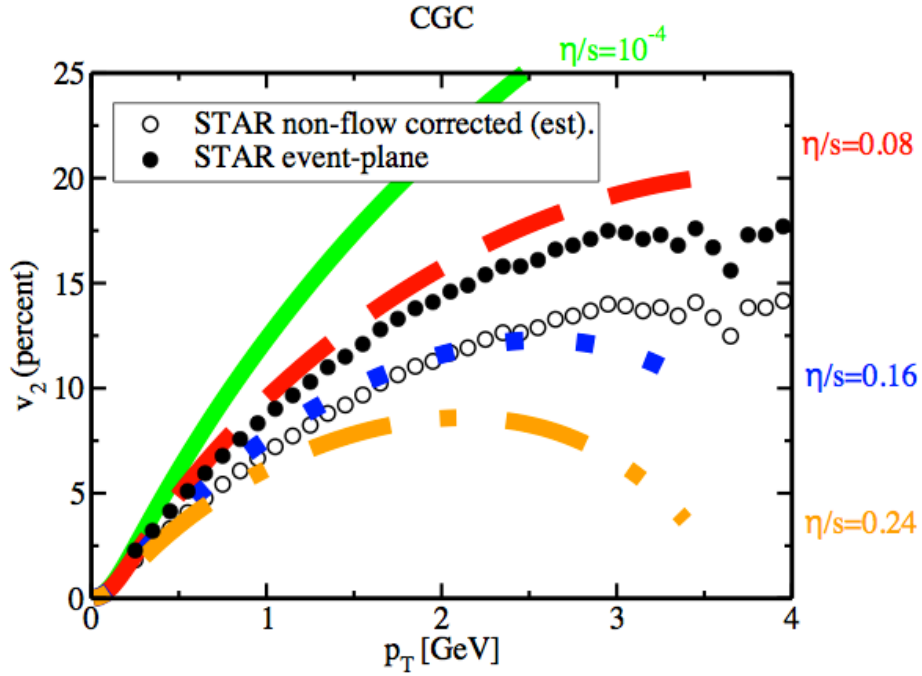
- *Implications for η/s extraction?*

Better constraints via acoustic scaling

Status Quo

Luzum et al. arXiv 0804.4015





<u>Note</u>

The Initial-state eccentricity difference between MC-KLN and MC-Glauber is ~ 20% due to fluctuation differences in the models!

ε_n – η/s interplay?

η/s is a property of the medium and should not depend on initial geometry!
This is <u>NOT</u> an uncertainty; It is a failure of the method of extraction

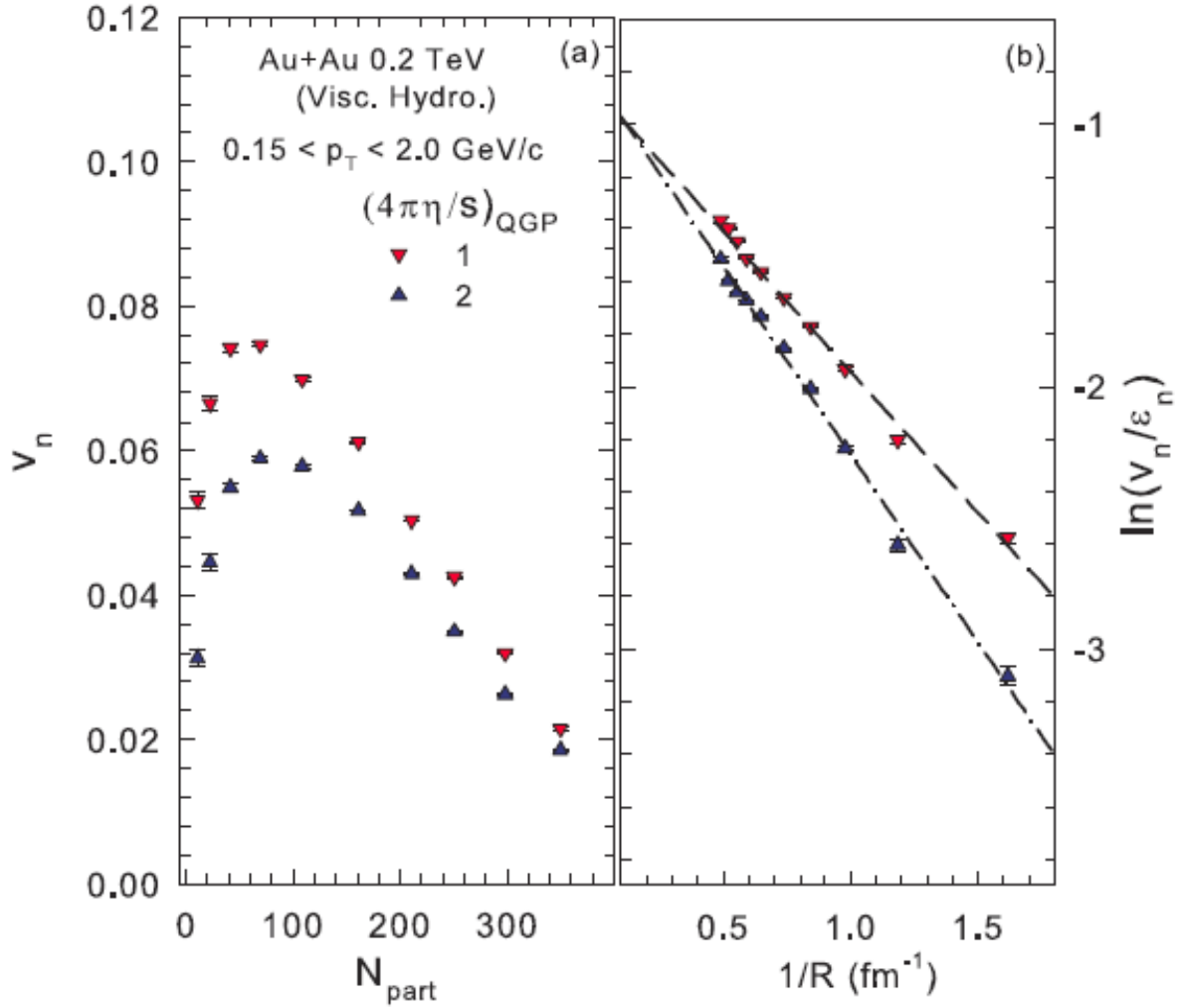
Improved methodology required

Scaling properties of flow

- Viscous Hydrodynamics

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

✓ **Characteristic a*coustic scaling validated for viscous hydrodynamics***



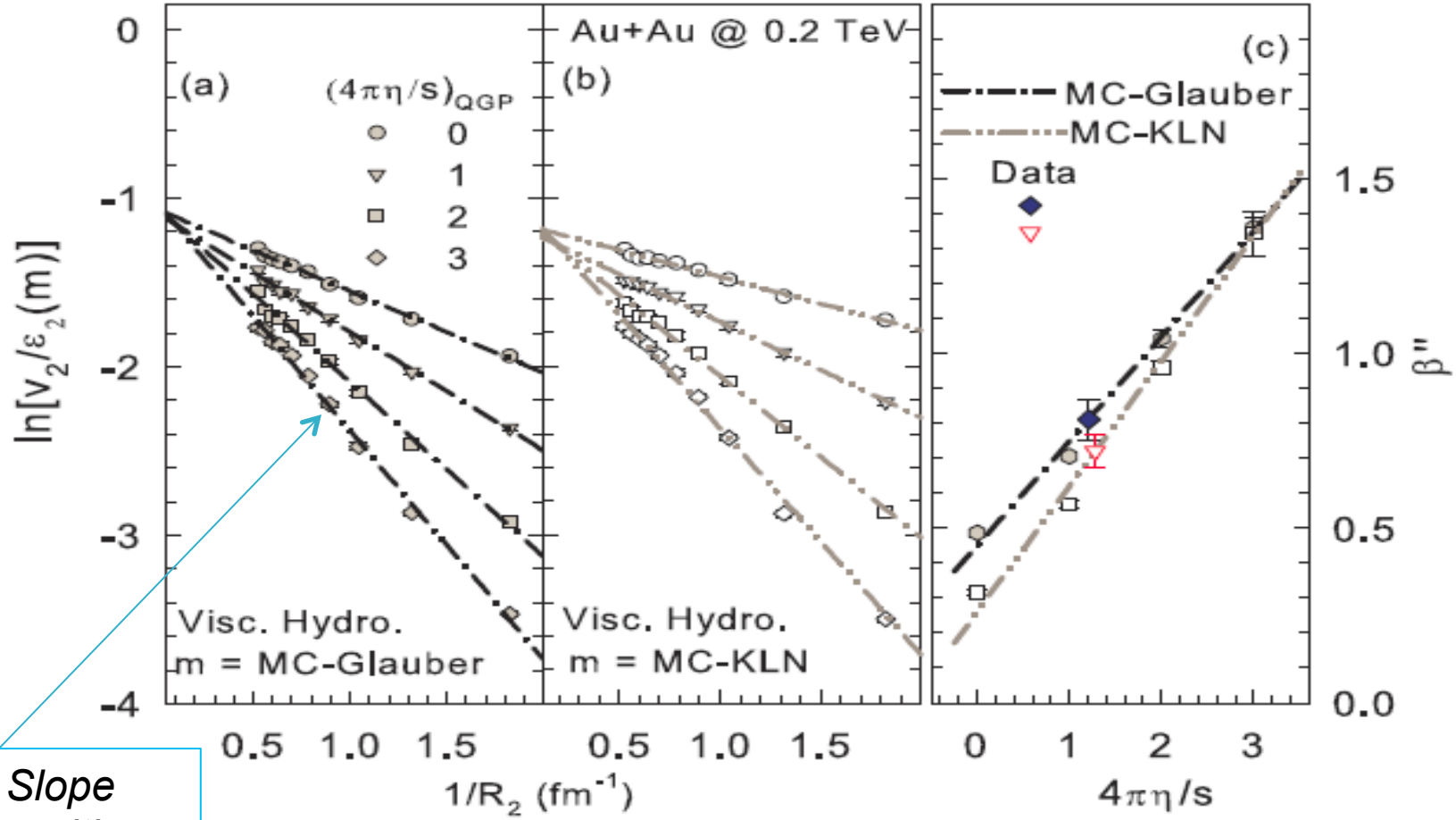
✓ ***β↑ ↑ shows clear sensitivity to η/s***

✓ ***Viscous hydrodynamics can be used for calibration***

Extraction of η/s

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Lacey et. al, arxiv:1311.1728



Slope sensitive to $4\pi\eta/s$

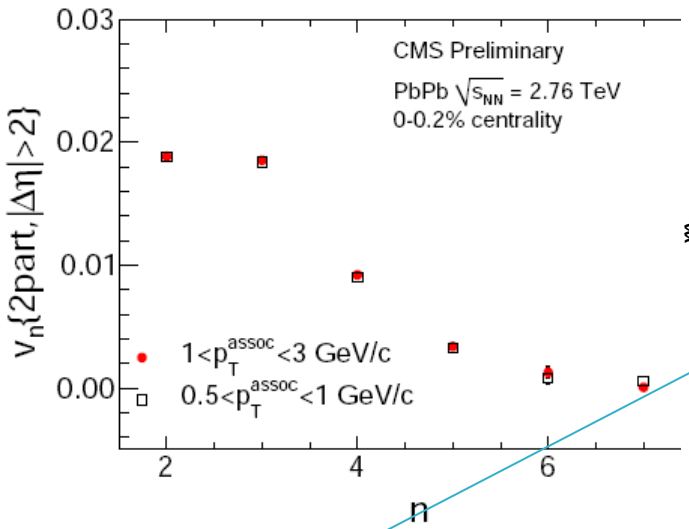
Characteristic $1/R$ viscous damping validated in viscous hydrodynamics; calibration → $4\pi\eta/s \sim 1.3 \pm 0.2$

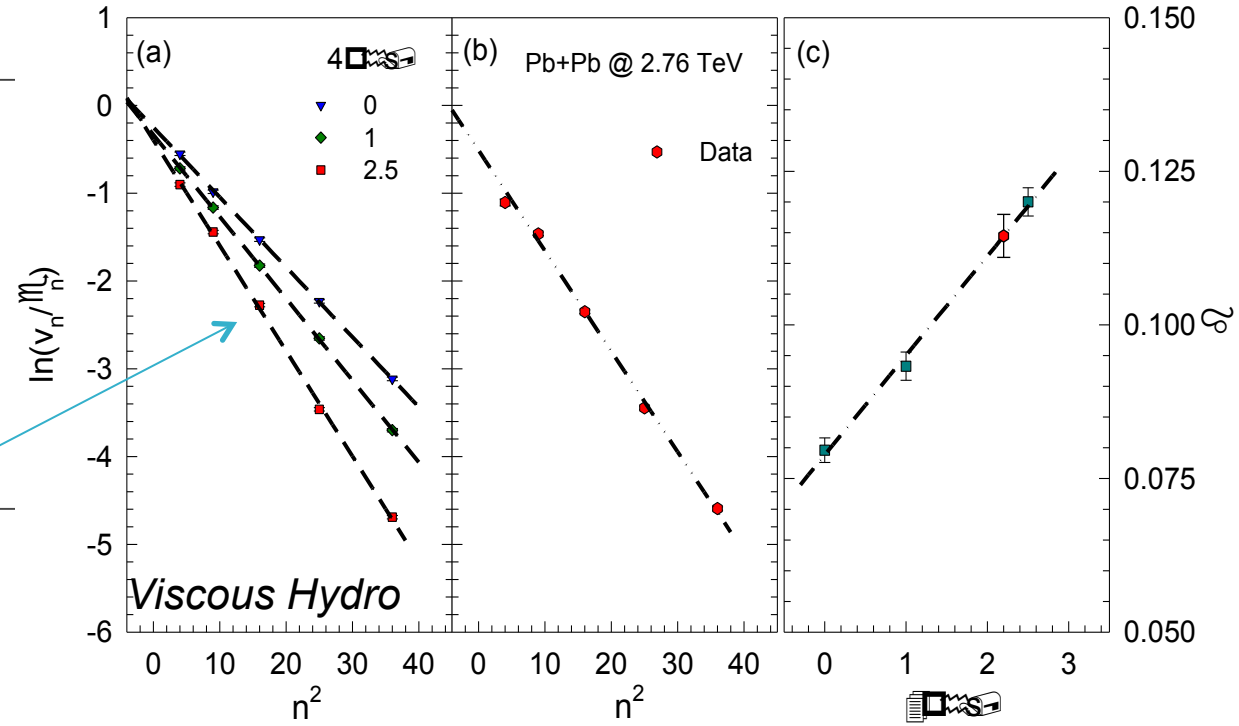
Extracted η/s value insensitive to initial conditions

Extraction of η/s

$$\frac{v_n(p_T)}{\varepsilon_n} \propto \cdot \exp\left(-\beta' n^2\right)$$

arXiv:1301.0165 & CMS PAS HIN-12-011





Slope sensitive to η/s

Characteristic $n2$ viscous damping validated in viscous hydrodynamics; calibration → 4πη/s ~ 2.2±0.2

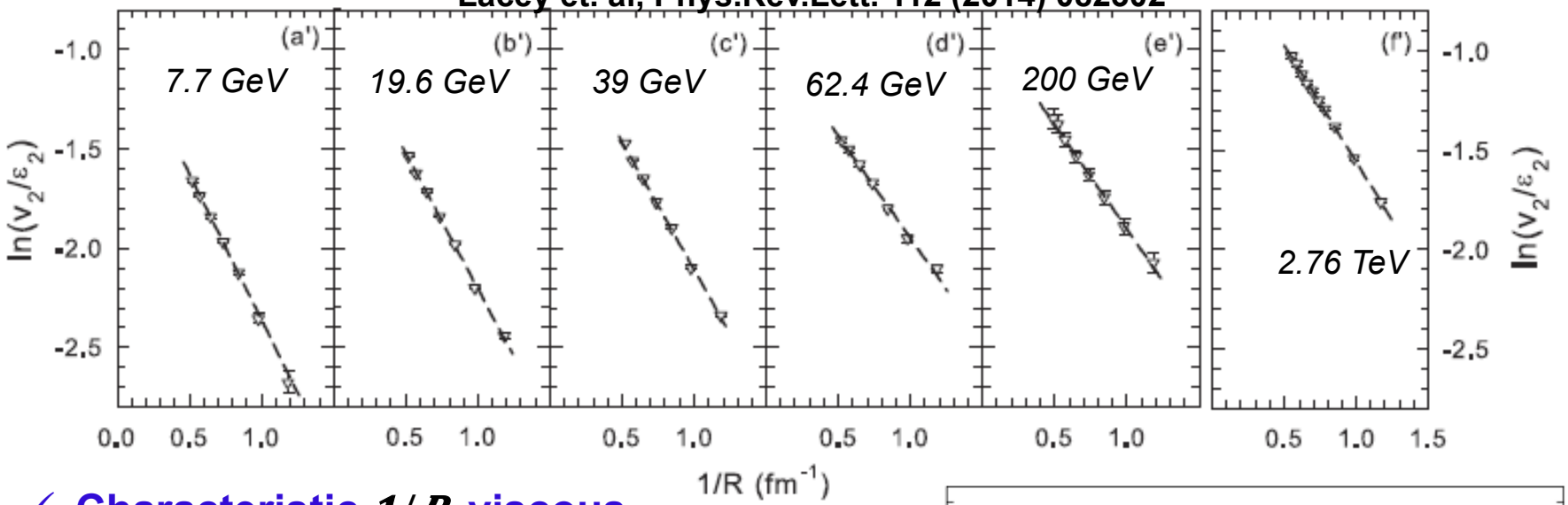
Extracted η/s value (LHC) insensitive to initial conditions

Scaling properties of flow

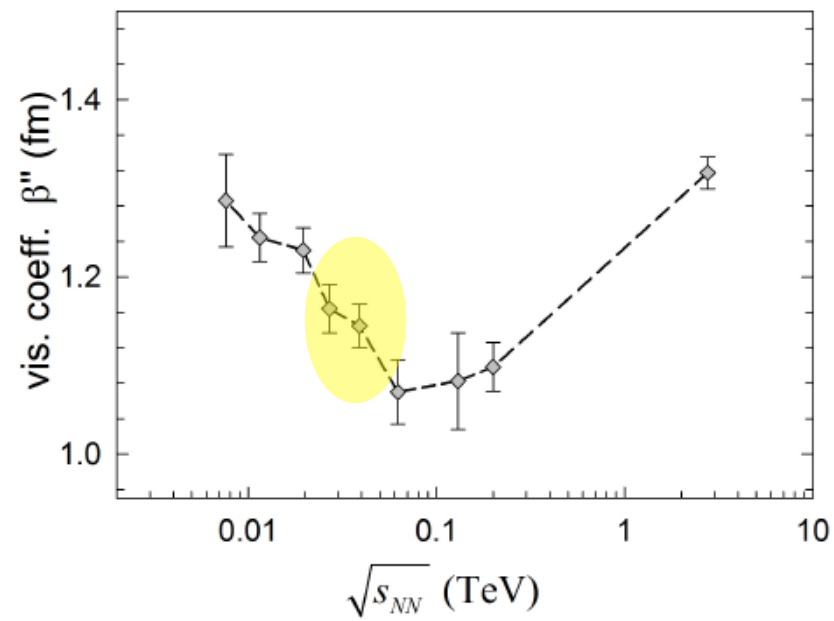
Acoustic Scaling – $1/R$ *Scaling for the Beam Energy Scan*

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Lacey et. al, Phys.Rev.Lett. 112 (2014) 082302



- ✓ Characteristic $1/R$ viscous damping validated across beam energies
- ✓ Experimental indications for *η/s variation in the* $(T,\mu\downarrow B)$-plane
- ✓ CEP?



Epilogue

- ***Acoustic dynamics validated in "large" and "small" systems***
  - ✓ ***Strong evidence for the important role of final-state interactions.***
  - ✓ ***Better estimates for η/s;***
    - ✓ ***η/s estimate insensitive to initial geometry***
    - ✓ ***$(\eta/s)_{LHC} > (\eta/s)_{RHIC}$***
    - ✓ ***Constraints for η/s (μ_B, T)***

Experimental summary and future measurements in pp and pA collisions

– *some personal perspectives*

Wei Li
Rice University

What is collectivity?

a group of entities that share a common property

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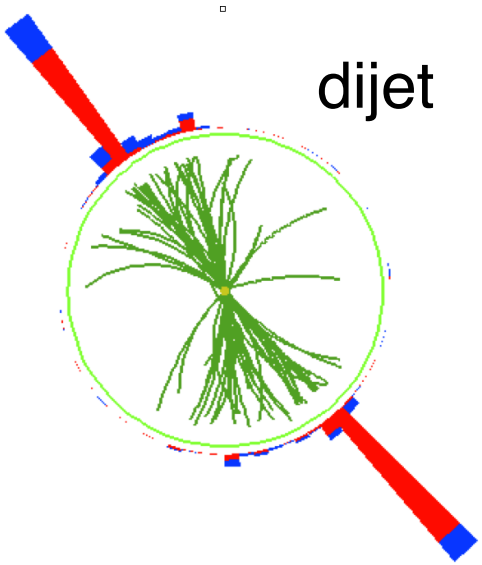
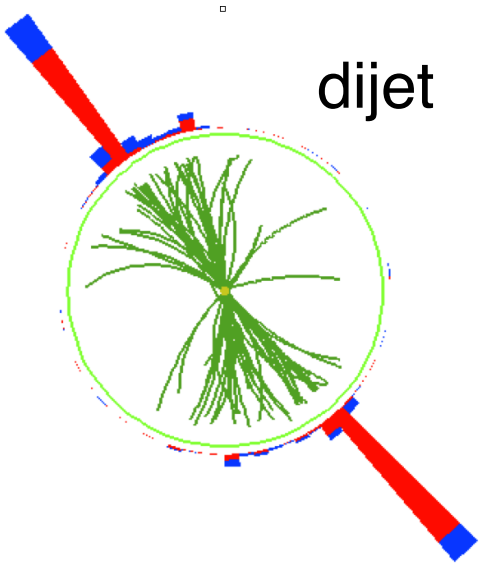
a group of entities that share a common property

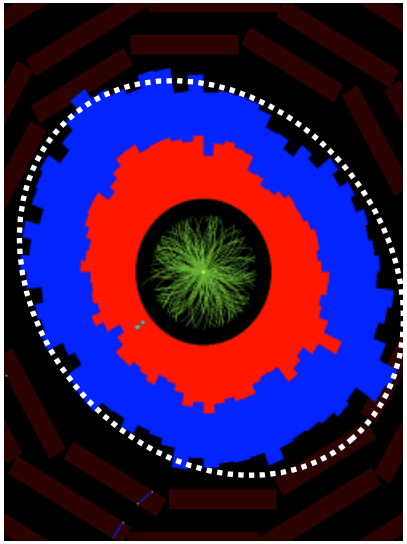


What is collectivity?

a group of entities that share a common property



dijet



PbPb event display

What is collectivity?

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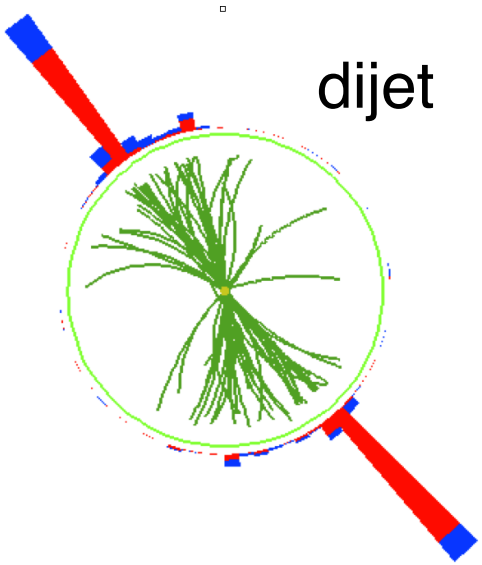

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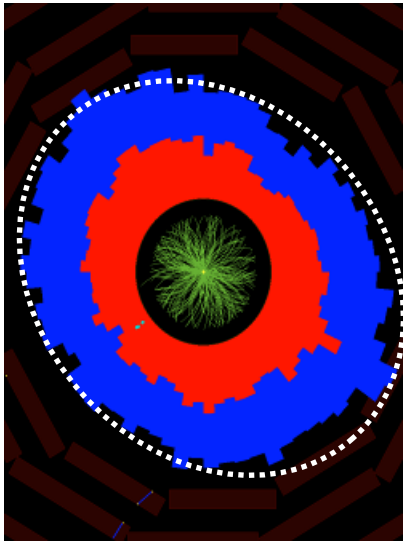


PbPb event display

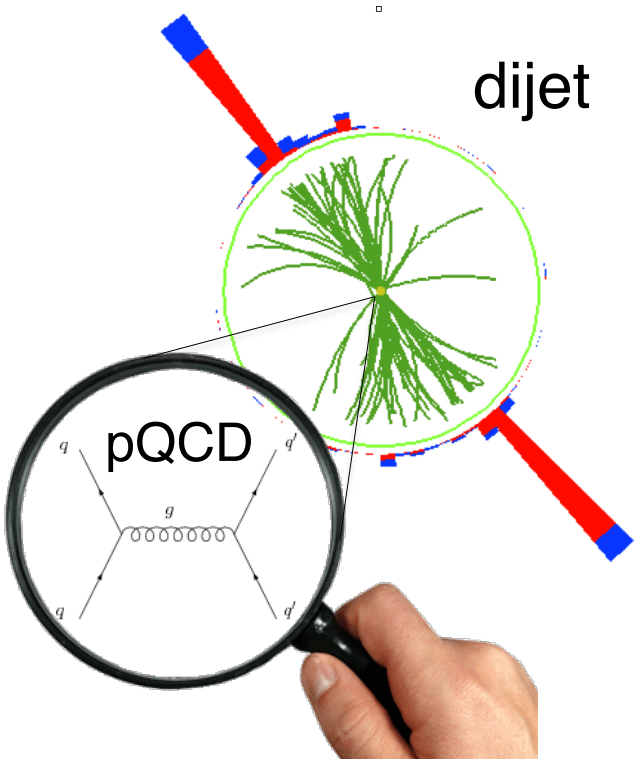
The more interesting questions:

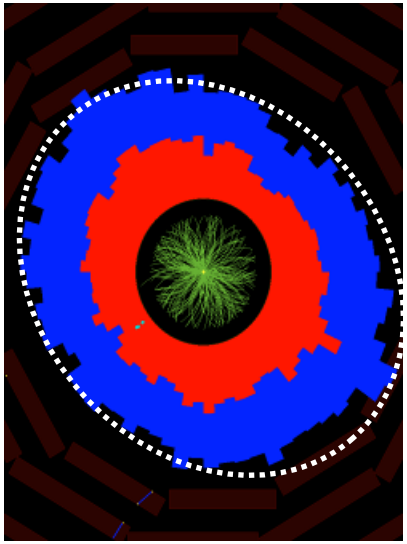
What is behind such collectivity?

What is the emergent phenomenon?

What is collectivity?

a group of entities that share a common property





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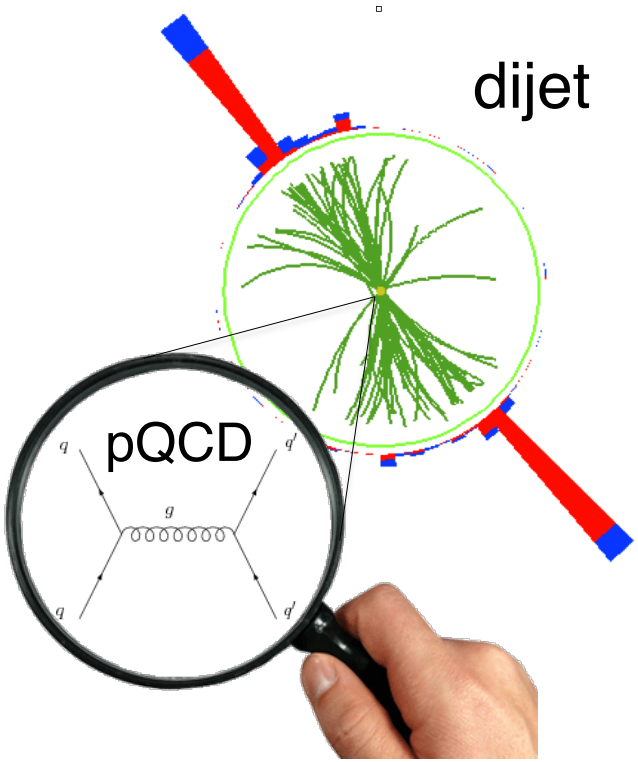

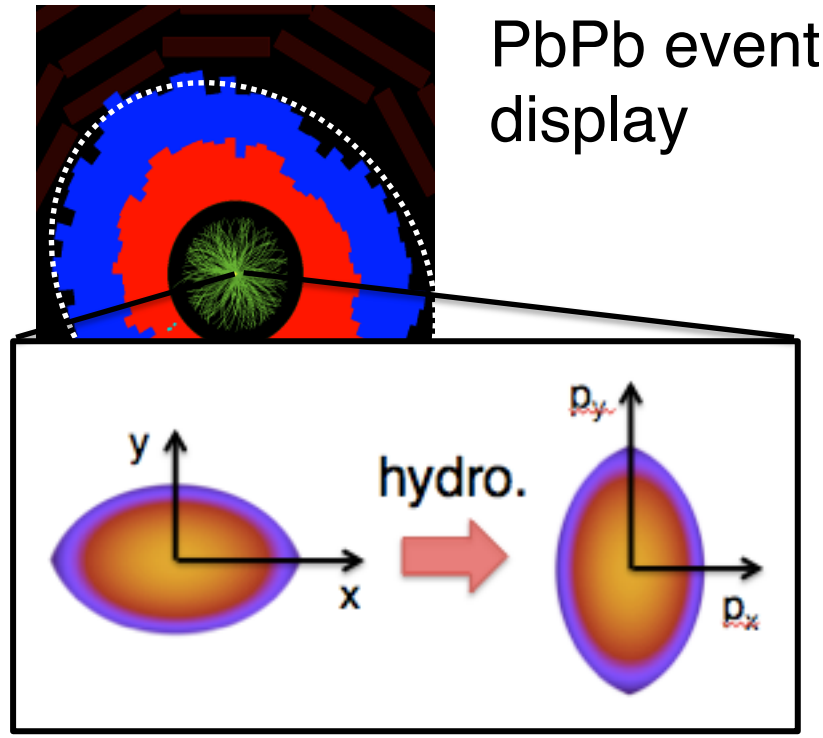

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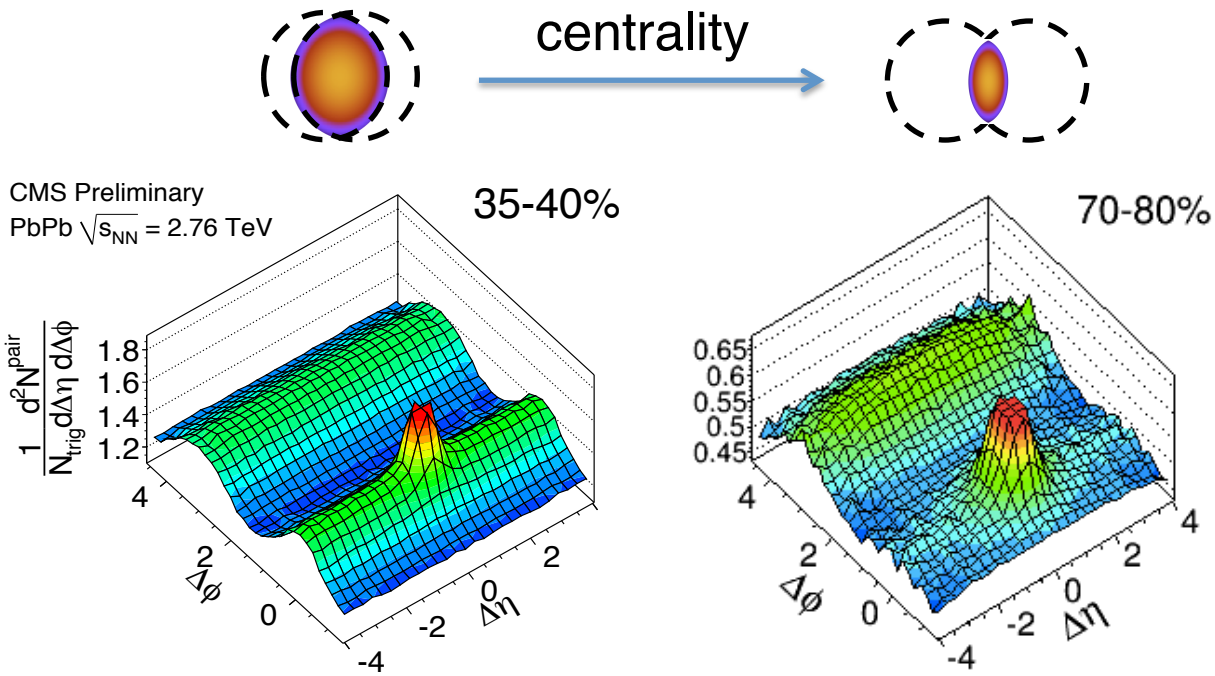
What is behind such collectivity?

What is the emergent phenomenon?

How small a QCD fluid can be?

Collectivity diminishing as ***L*** decreases:



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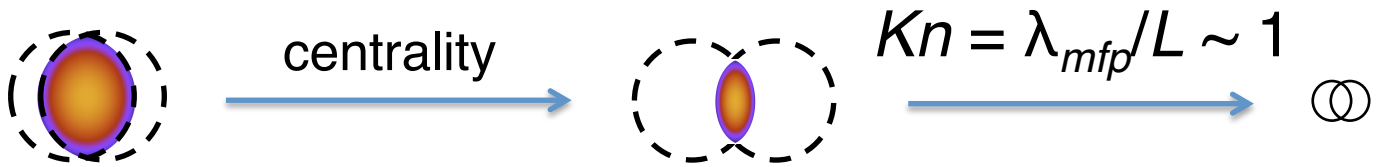
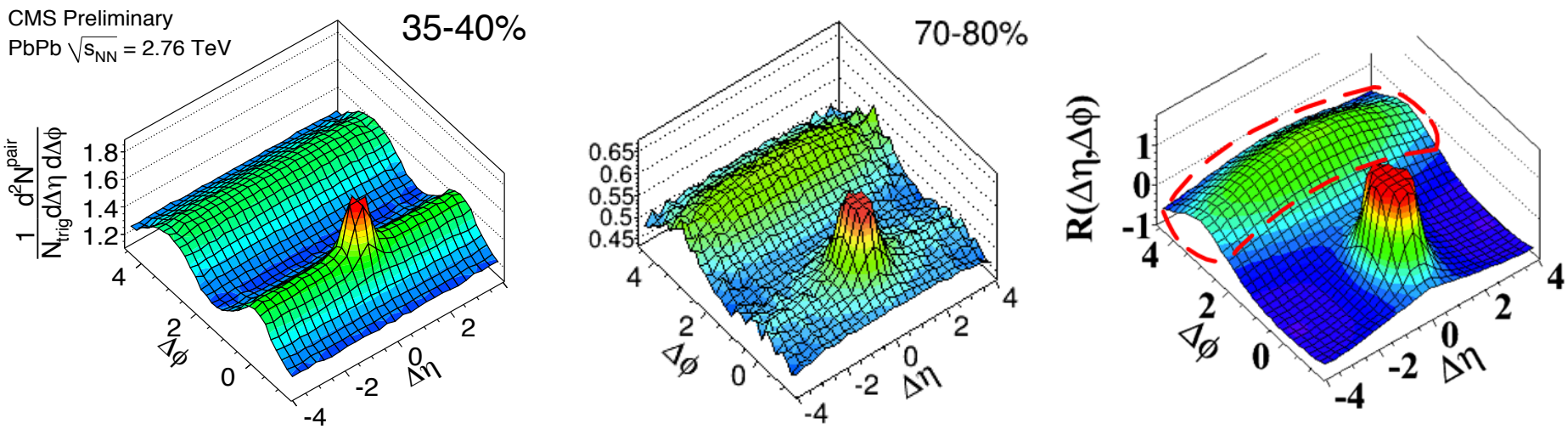
Collectivity diminishing as ***L*** decreases:





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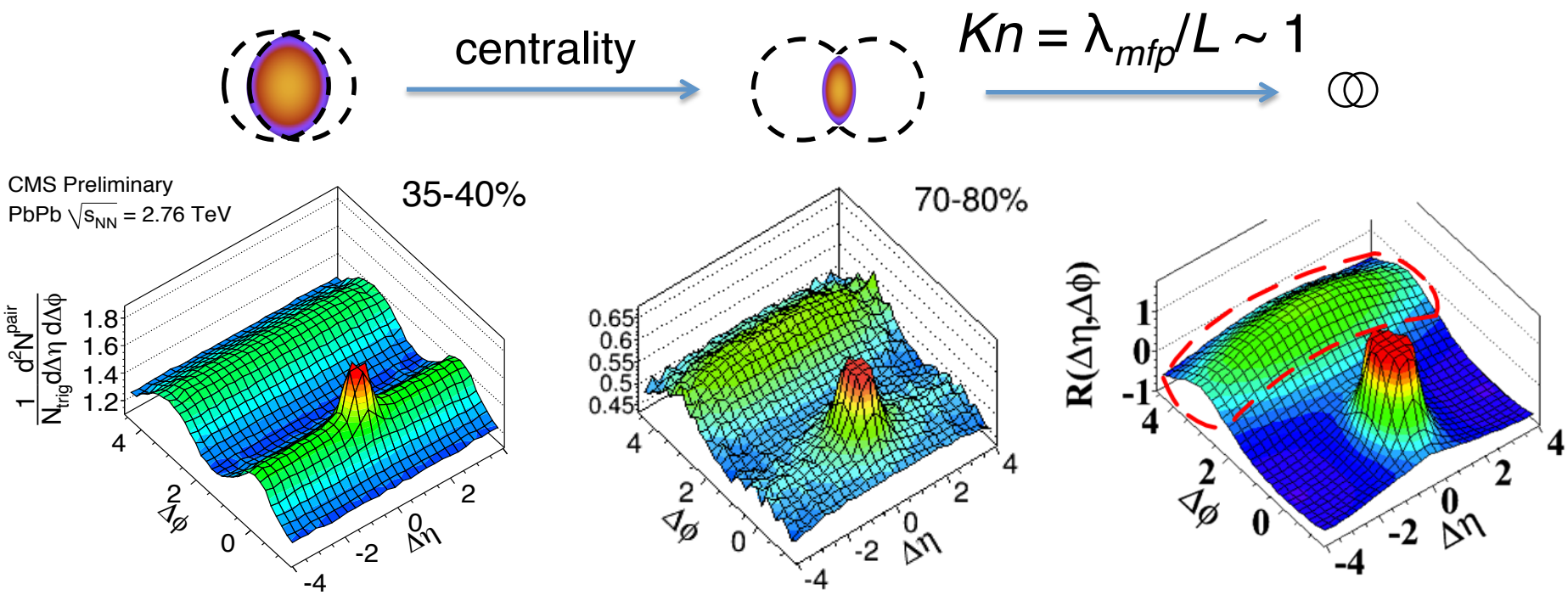

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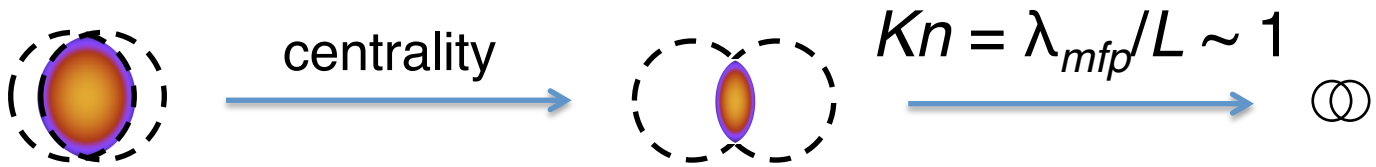

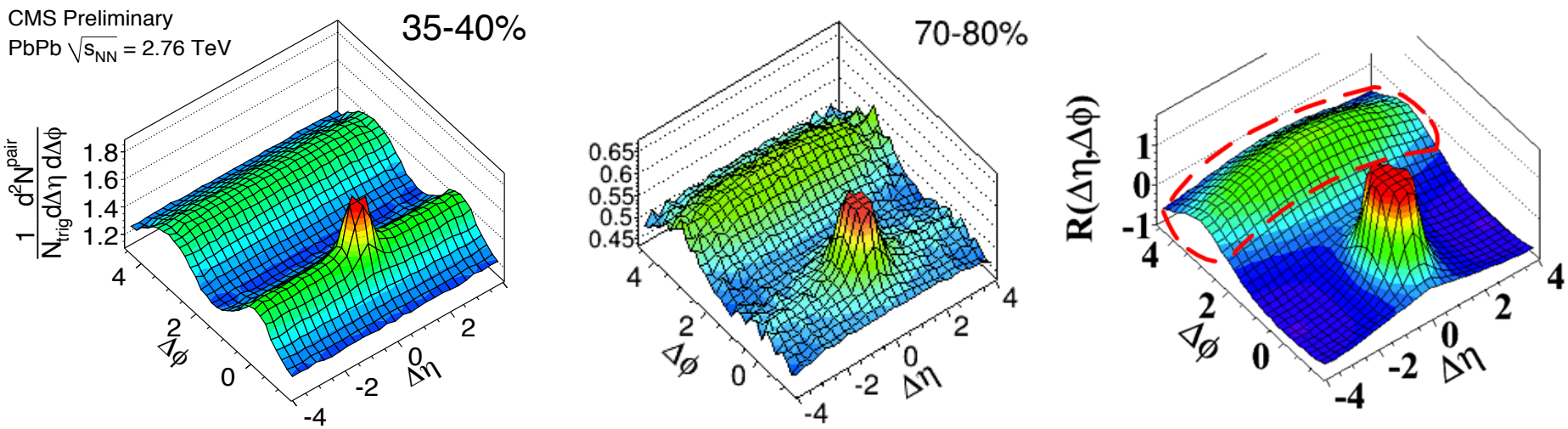

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But what if making it denser (reducing $\boldsymbol{\lambda_{mfp}}$)?

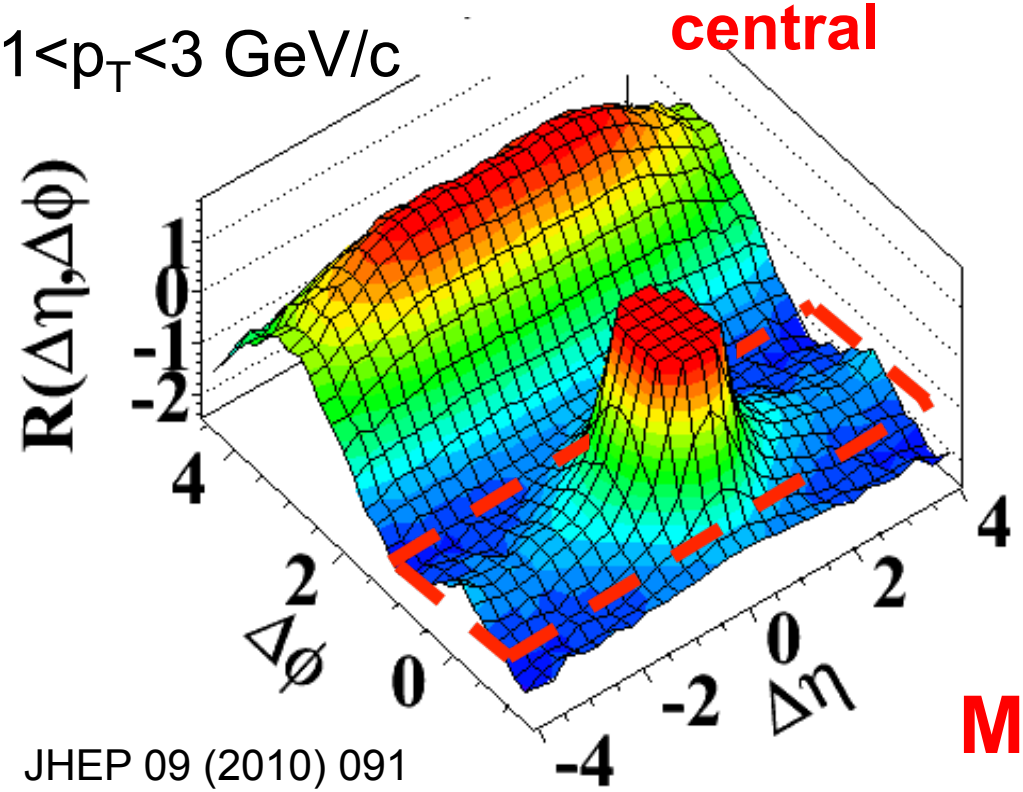
➔ a smaller but hotter QGP fluid?!

First surprise in small system

Breaking news in 2010:

pp 7 TeV, **N_{trk}>=110** **0-0.0007% central**

$1<p_T<3$ GeV/c



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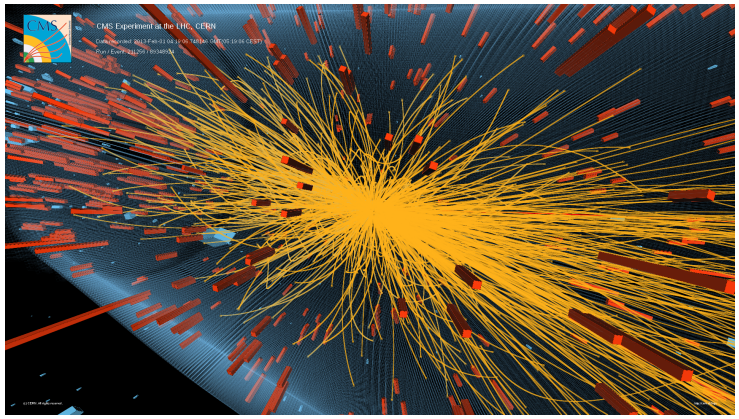
High-multiplicity pp (N^{ch}_{trk}>200)



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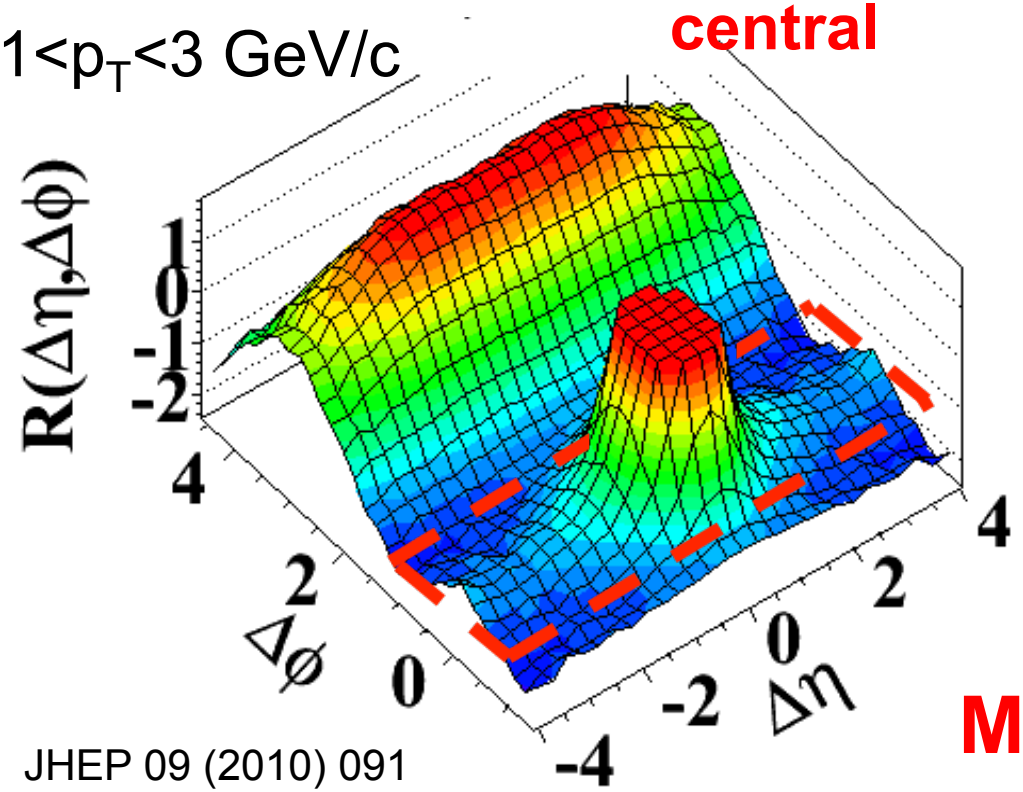

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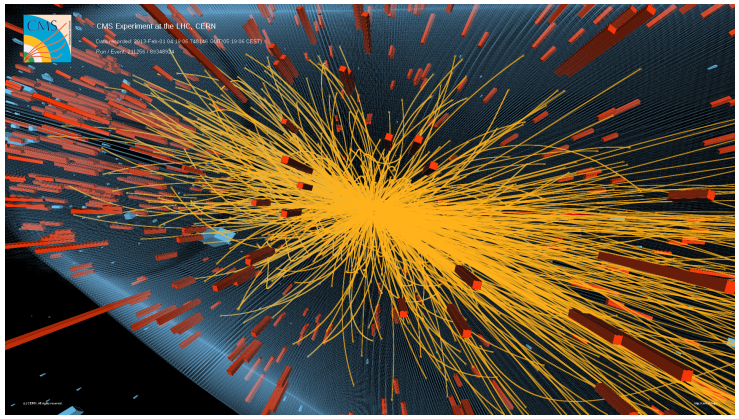

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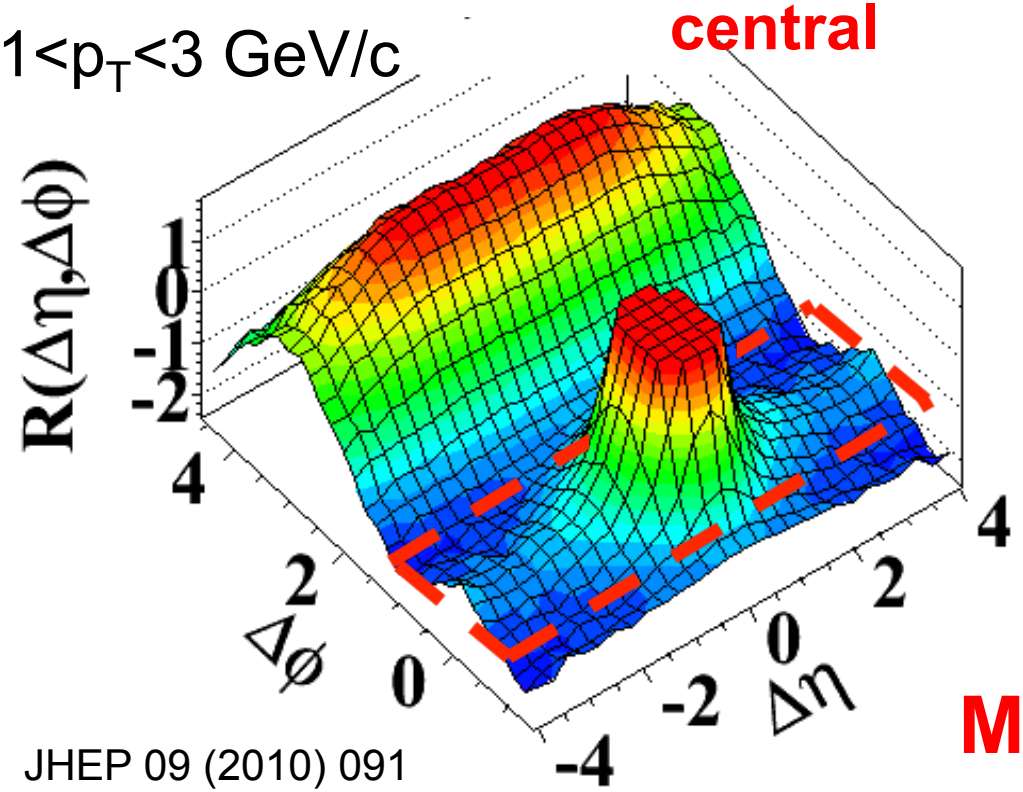

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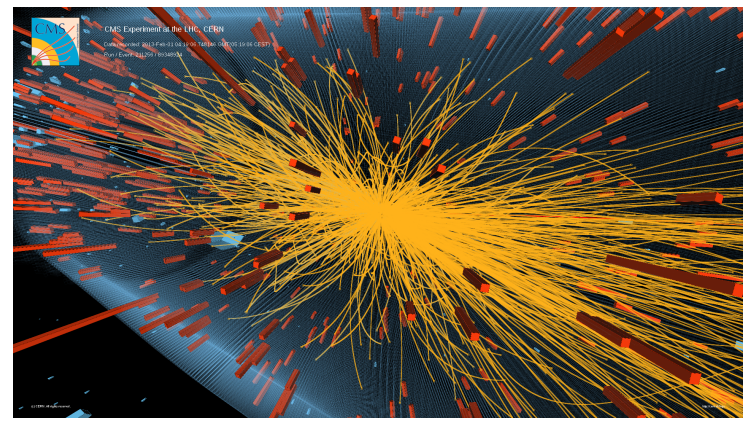

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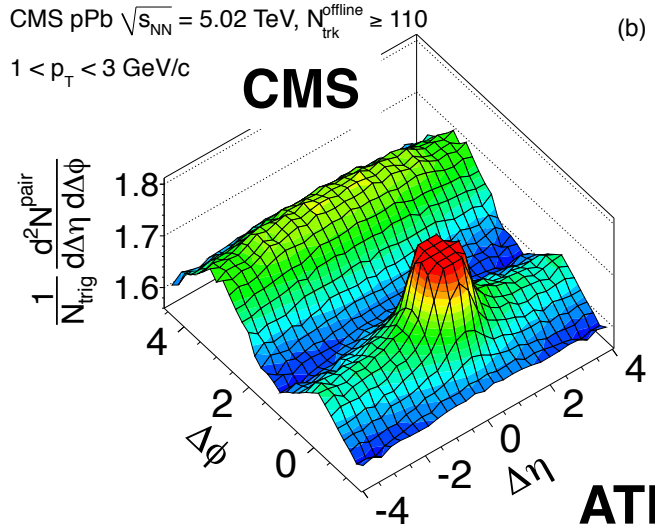
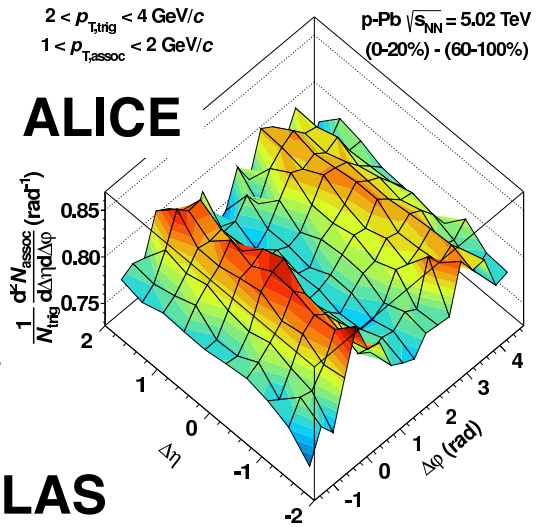
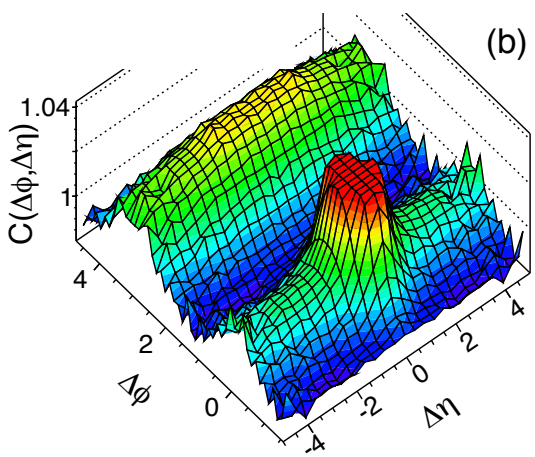
Other contenders, e.g., CGC glasma

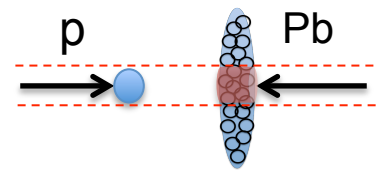
The “ridge” tsunami in pPb at the LHC



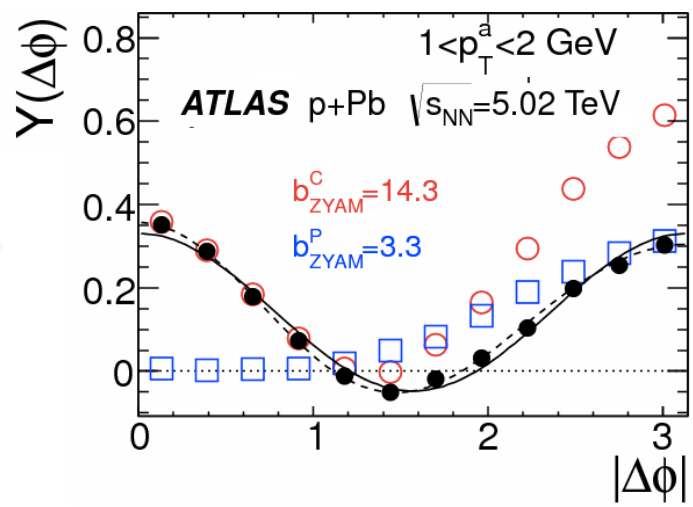




~ a few hours of pPb beam on Sep. 21, 2012



followed by a longer run in 2013



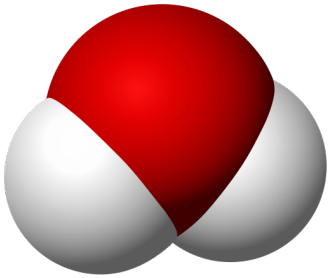
The “ridge” seen in all systems at the LHC!

➔ Everything “flows”?

How small a QCD fluid can be?

In QED matter, there is a smallest scale

cannot be smaller than a



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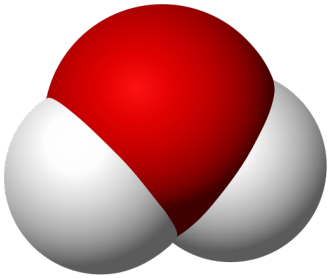

Not obvious in QCD as partons are point-like

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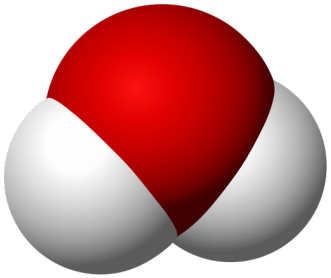

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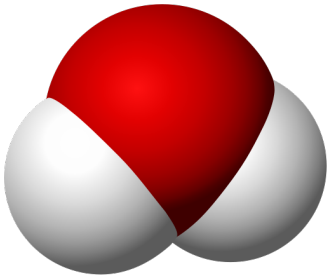

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Then what is it? Does it have an intrinsic size from the fundamental principle?

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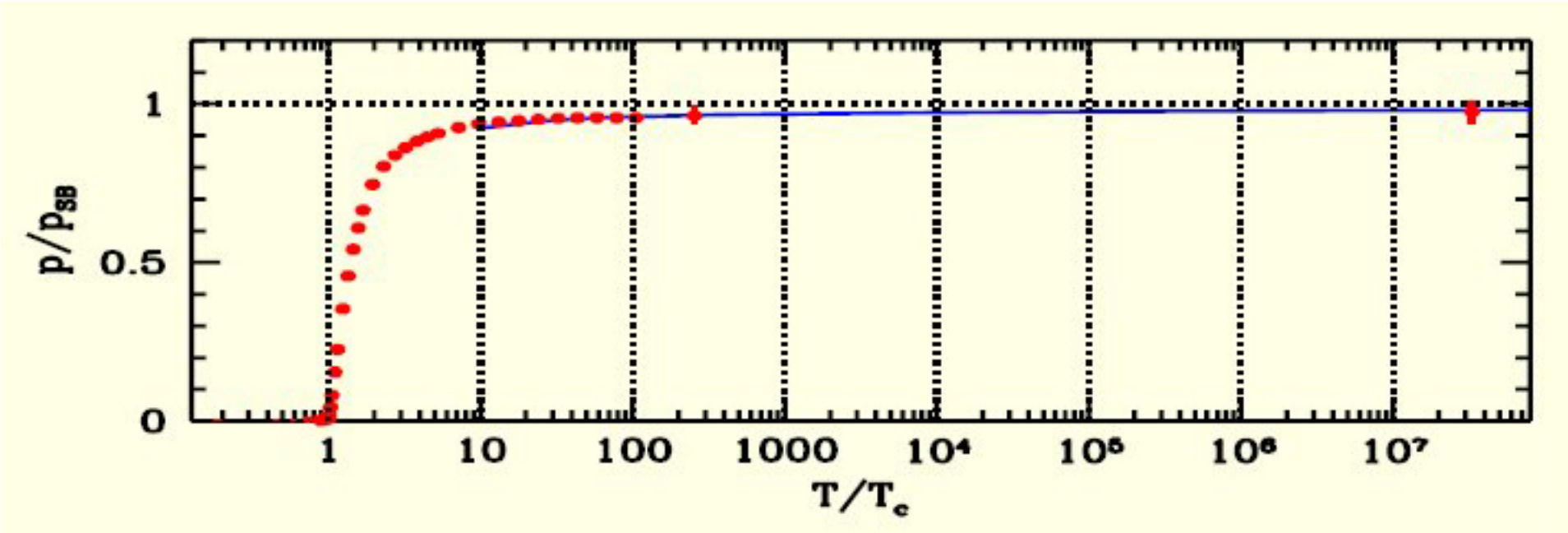

Endrodi et al., 0710.4197

$$\varepsilon \sim \frac{N_{trk}}{V}$$

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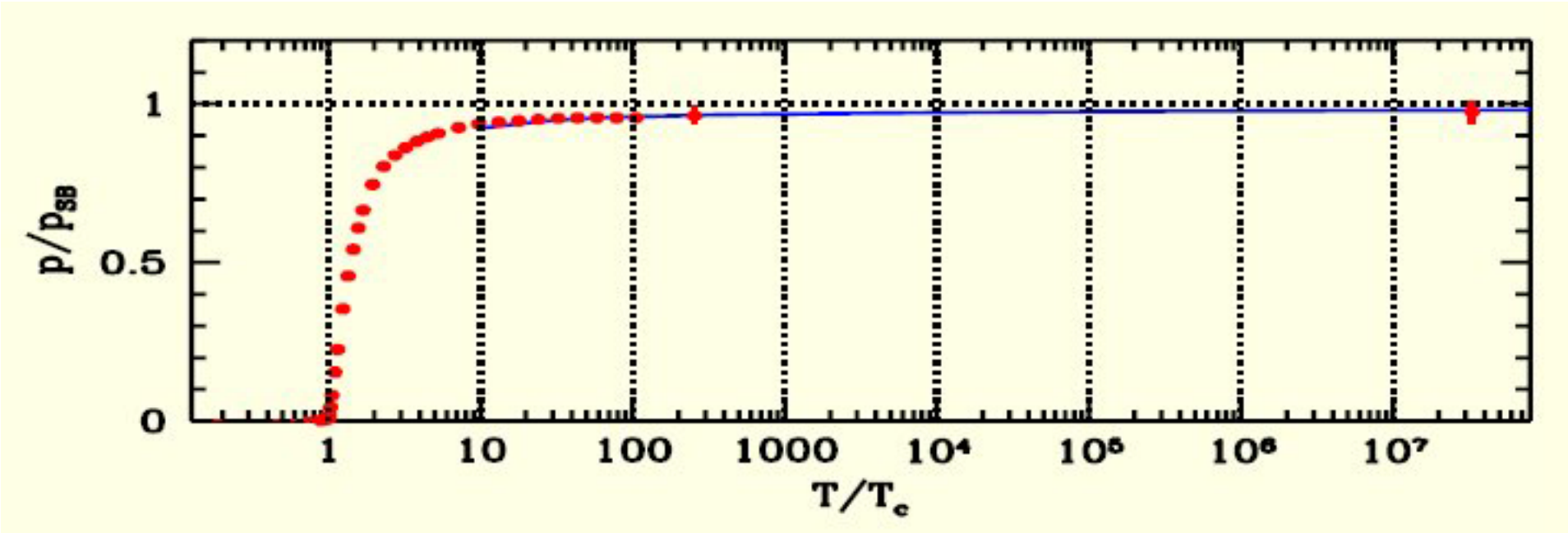

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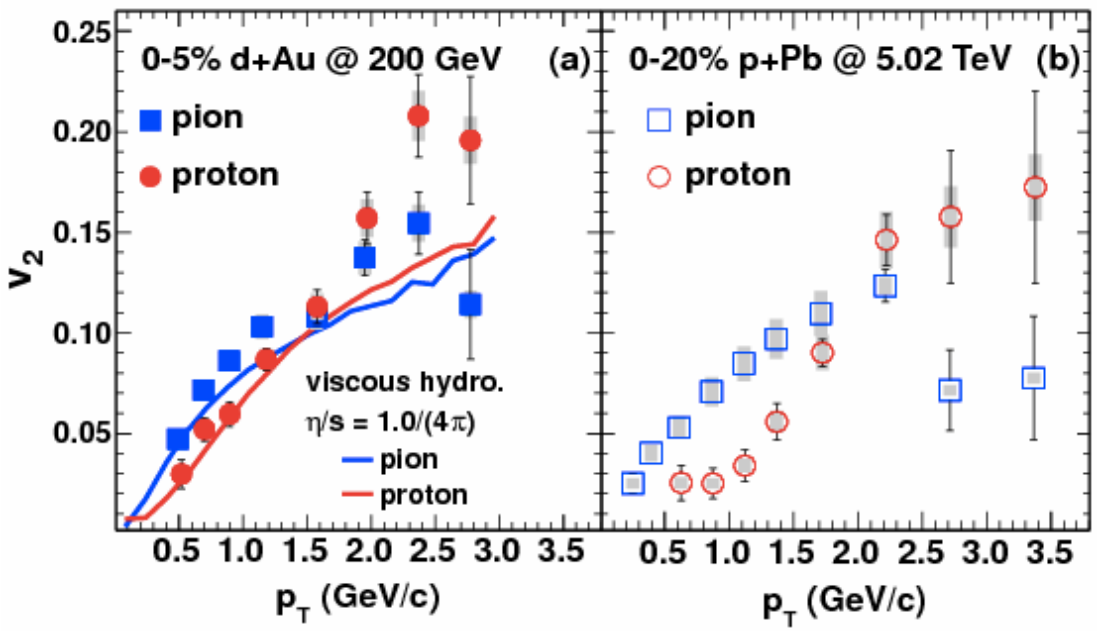
pp and pA should be well in the "fluid" regime

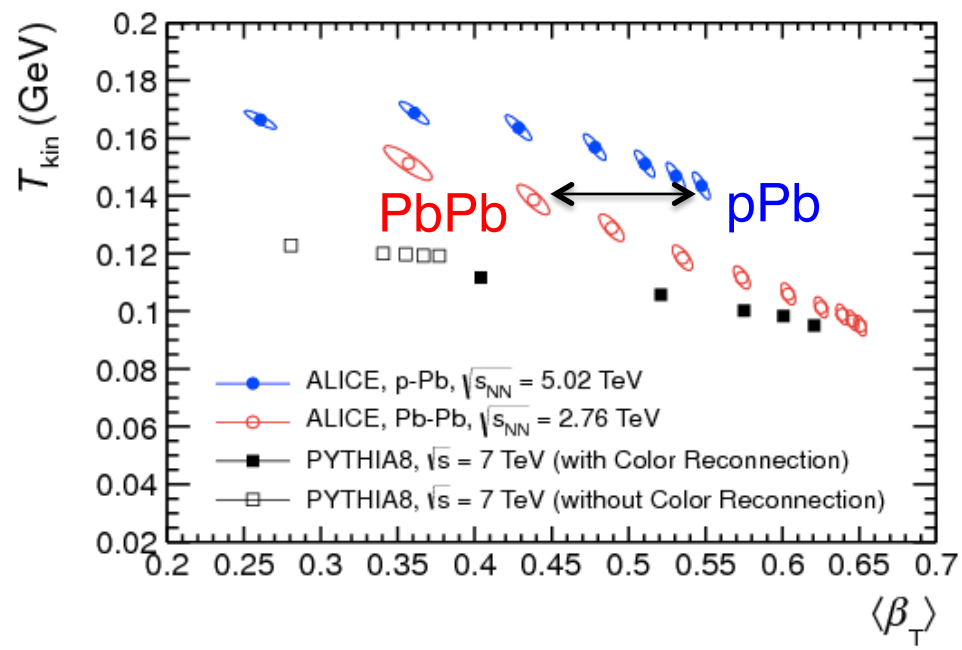
Mass ordering of PID spectra and v_n

PHENIX ALICE



Hydro wins again!

Larger $\langle\beta_T\rangle$ for smaller systems



Color reconnection + initial-state anisotropy could lead to the same effect?

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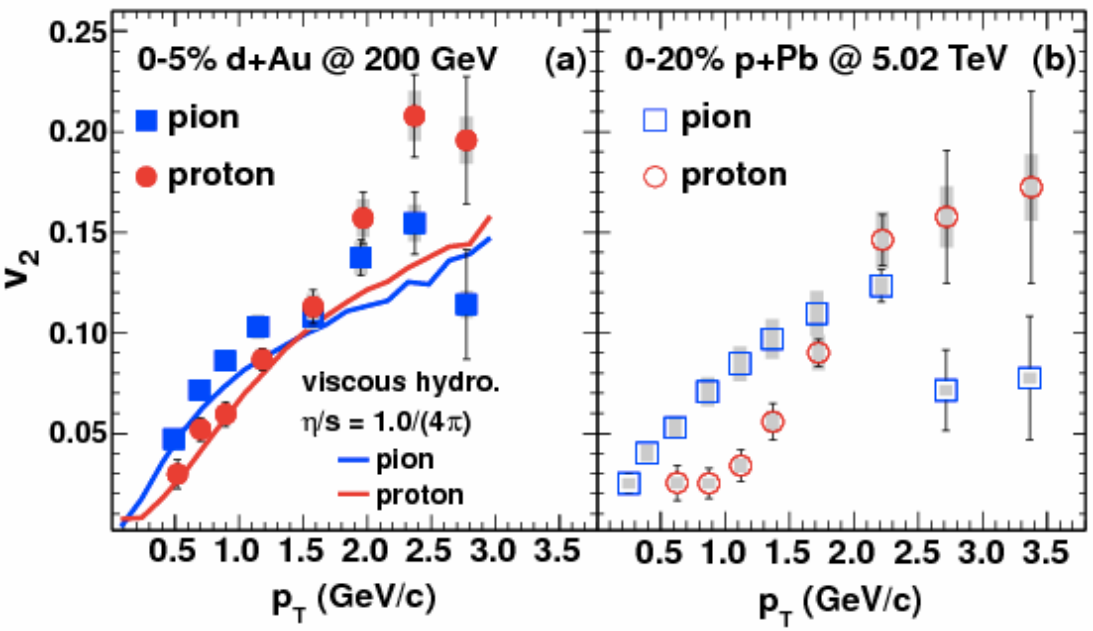

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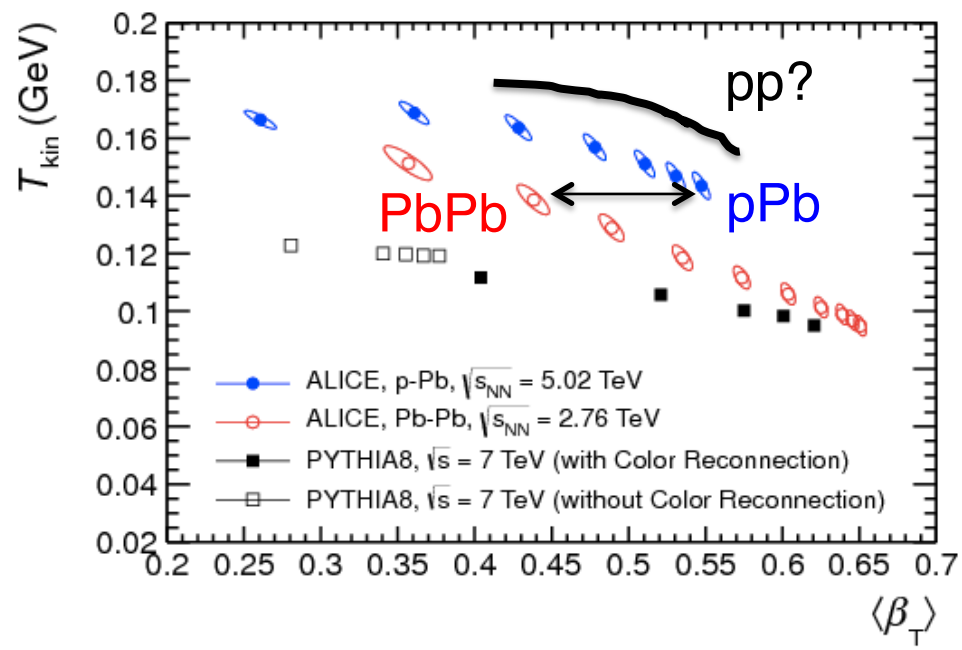

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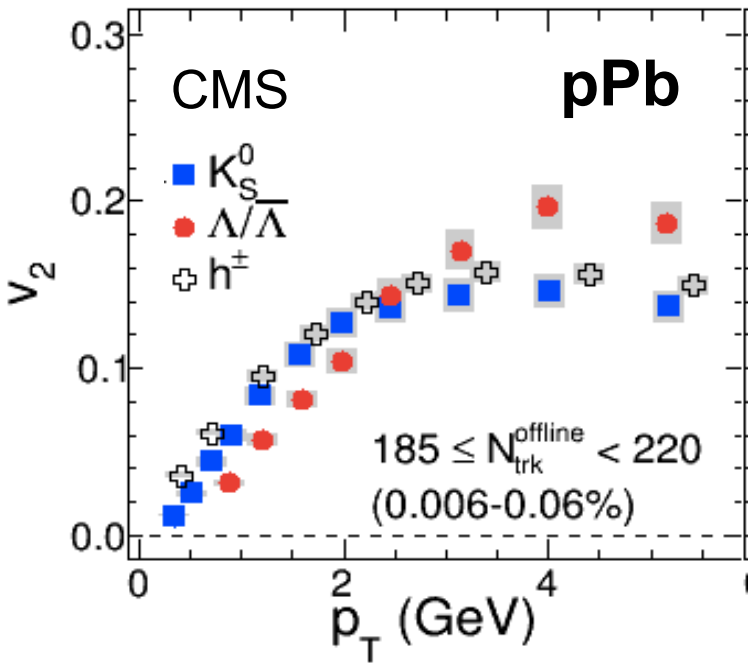
NCQ scaling in pPb system!



Baryon/meson crossing

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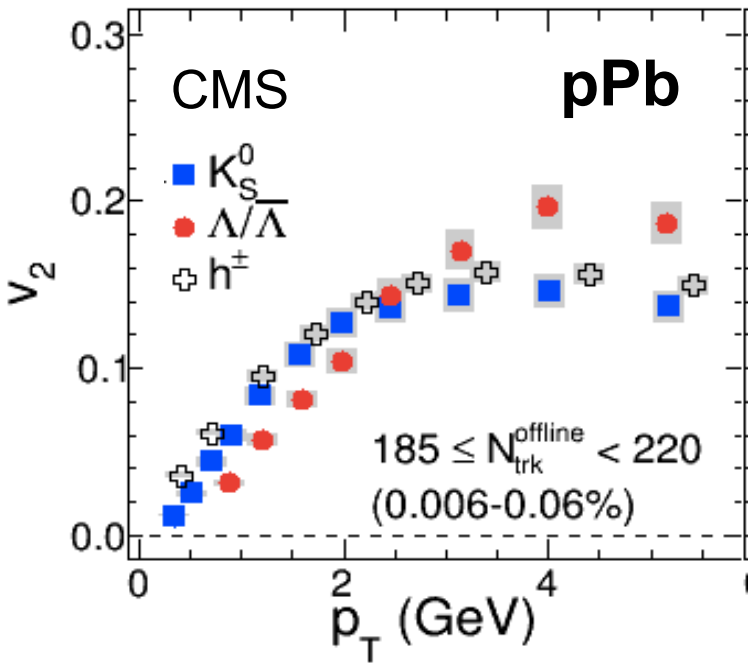

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scaled by n_q

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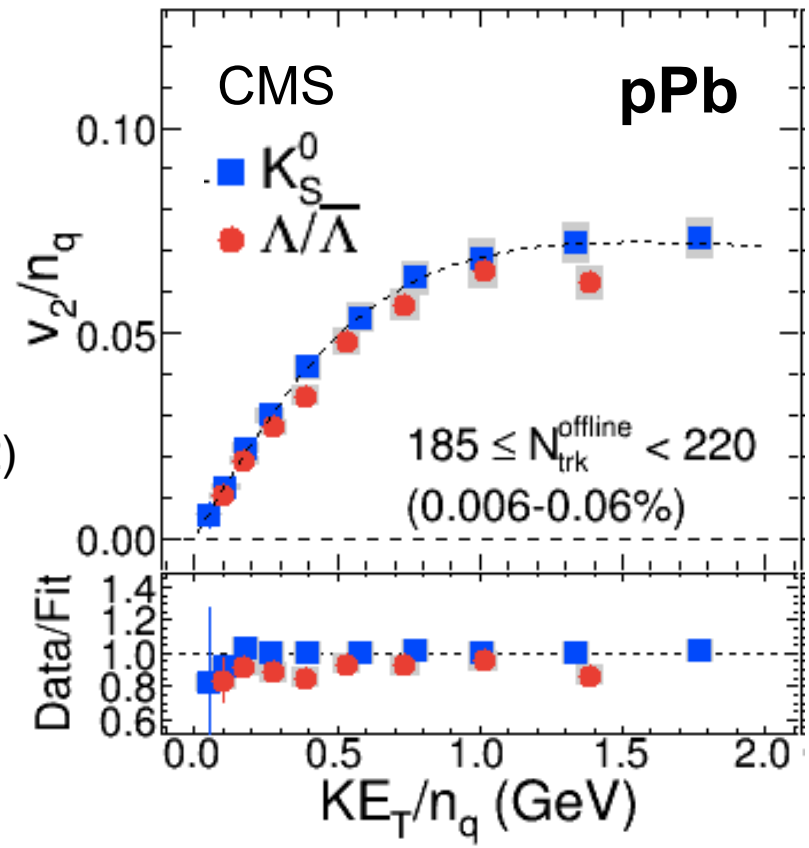

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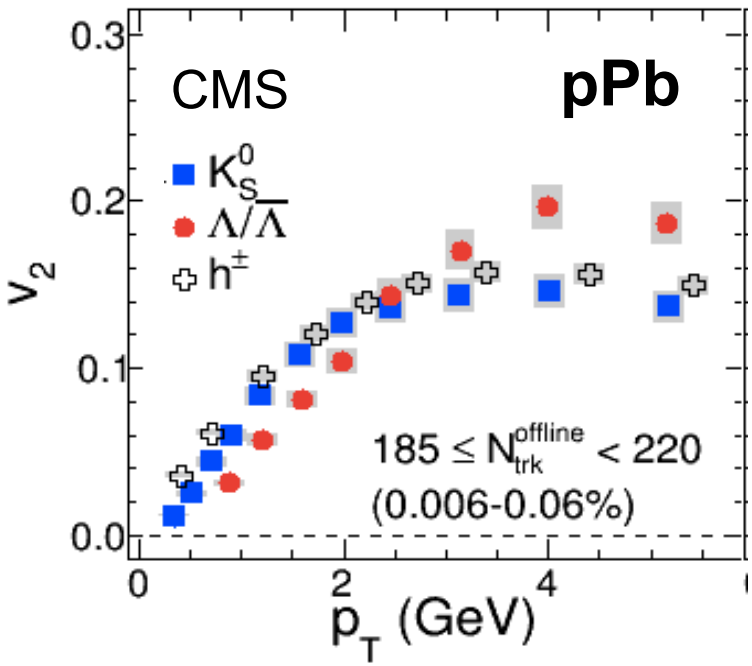

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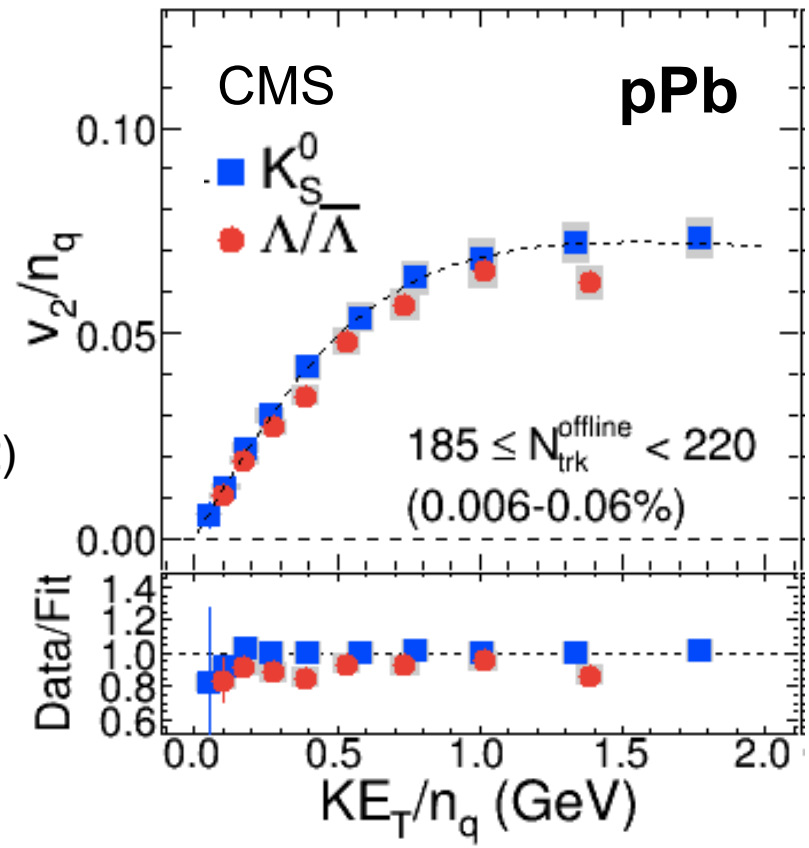


Flow developed at partonic level!?

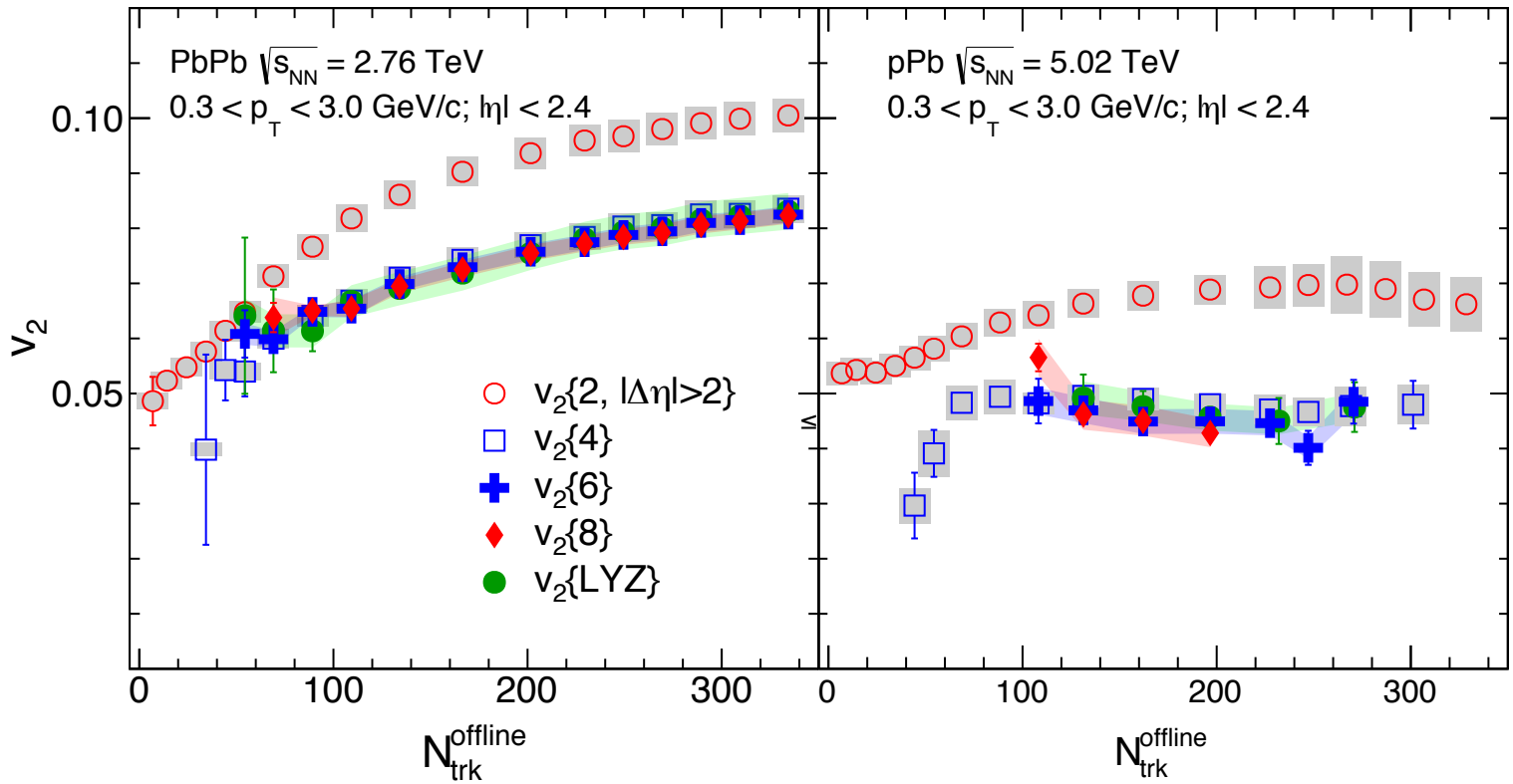
Expected or surprising in pPb?

Amazing scaling in AA discovered 10 yrs ago in quest of explanations, esp. in light of pPb data

Direct evidence of collectivity in pPb

$v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \approx v_2\{\infty\}$

arXiv:1502.05382



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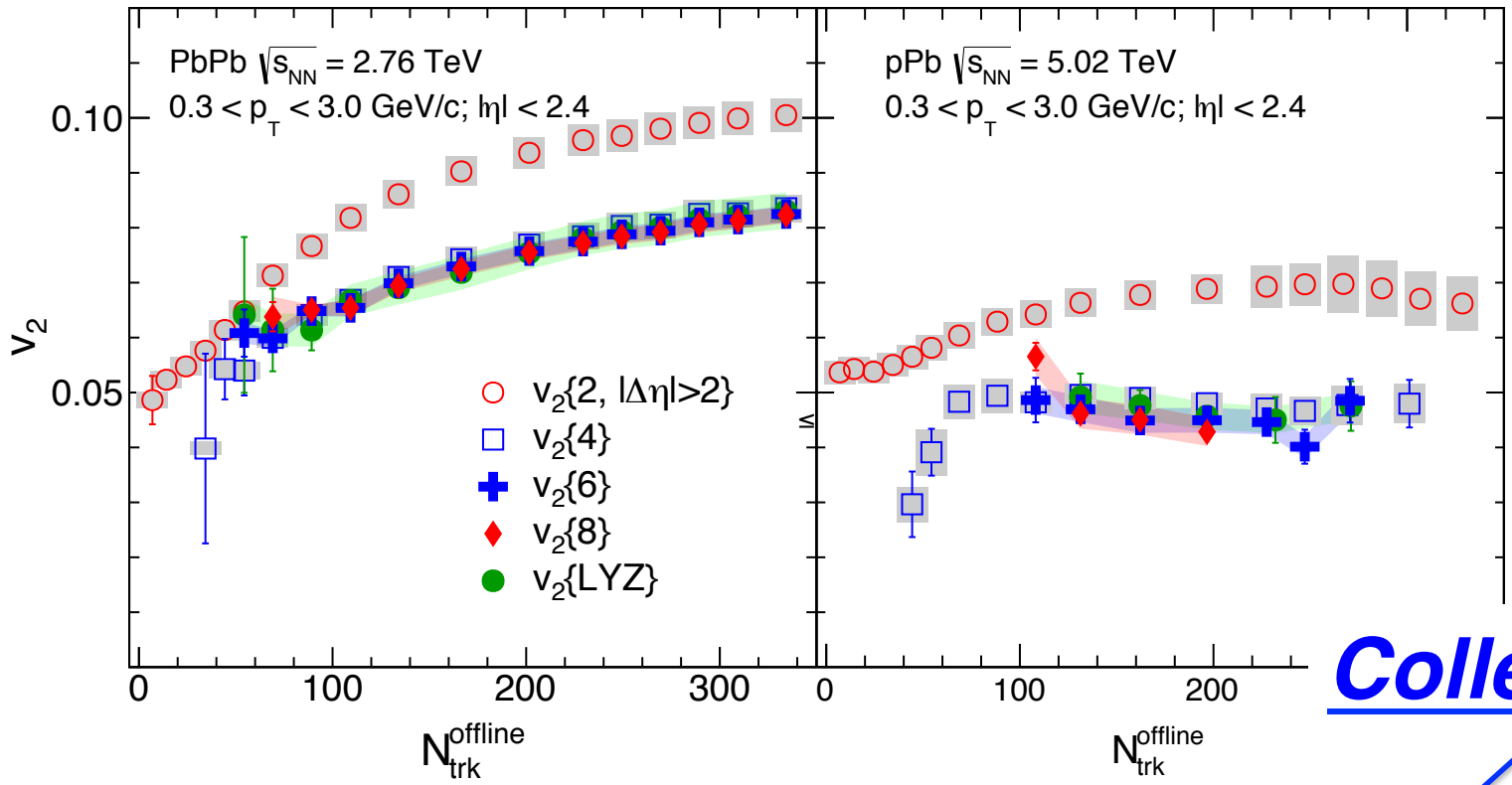

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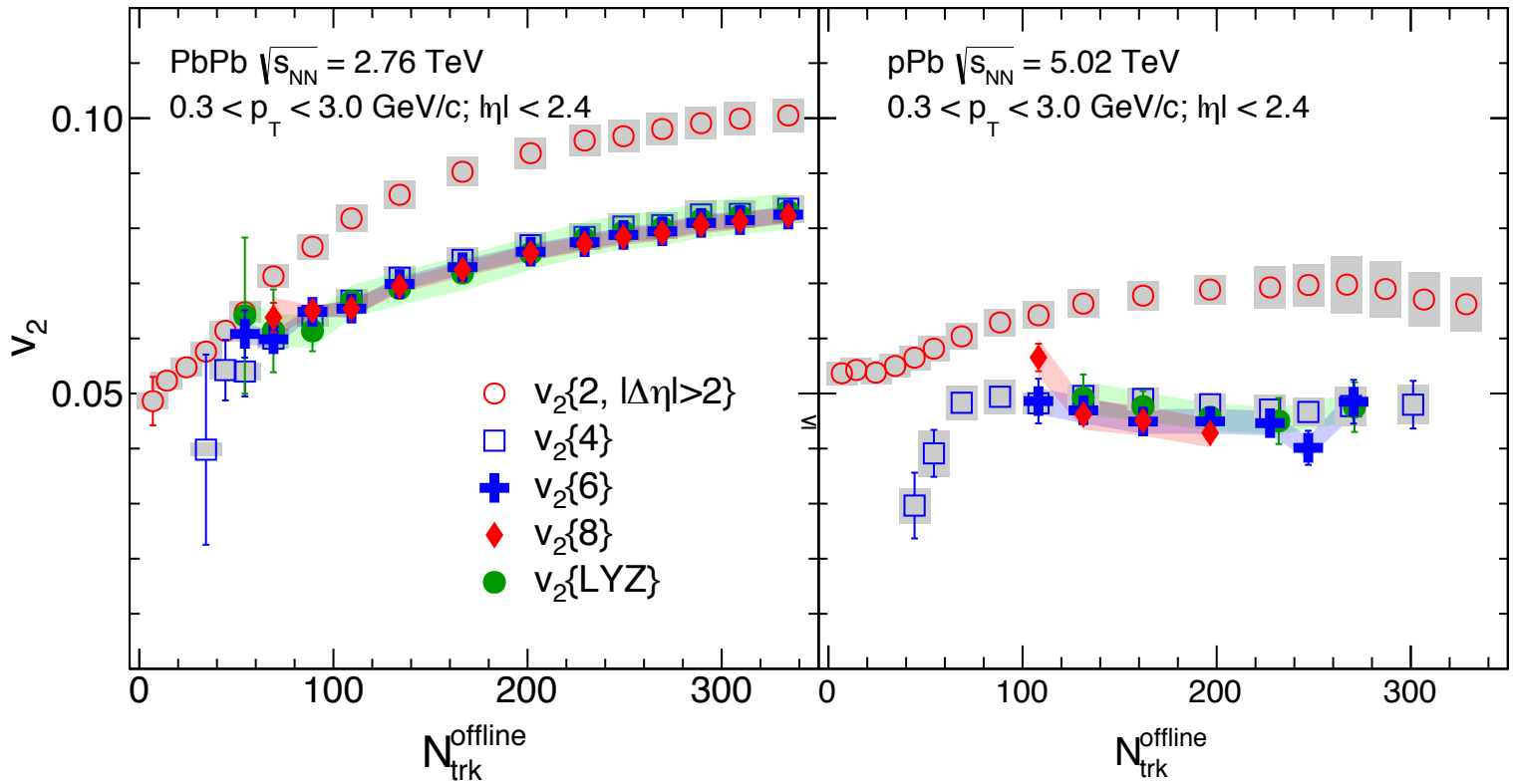

A question:100 particles with a v_2. Now adding one more particle randomly in φ, will v_2(LYZ) be

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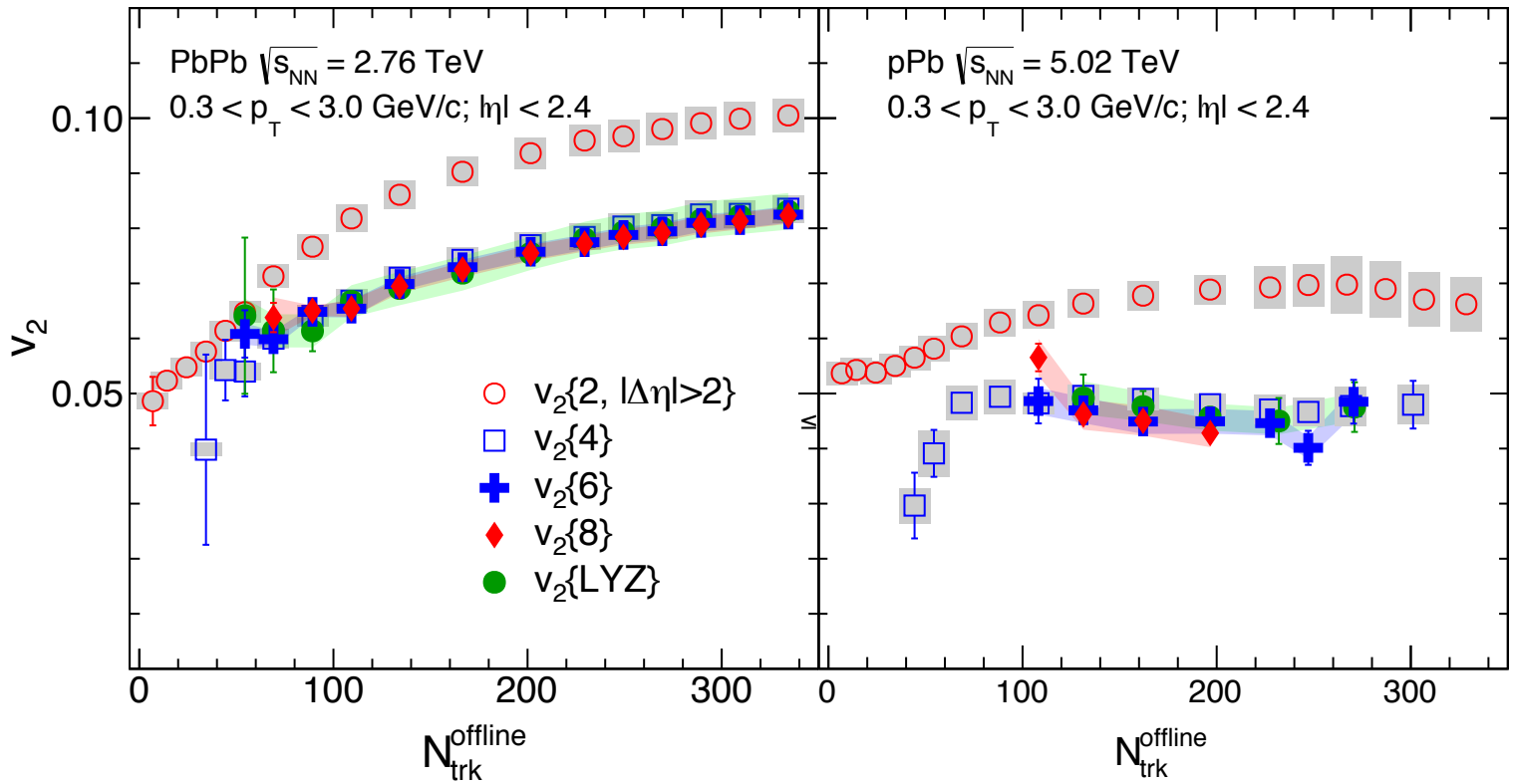

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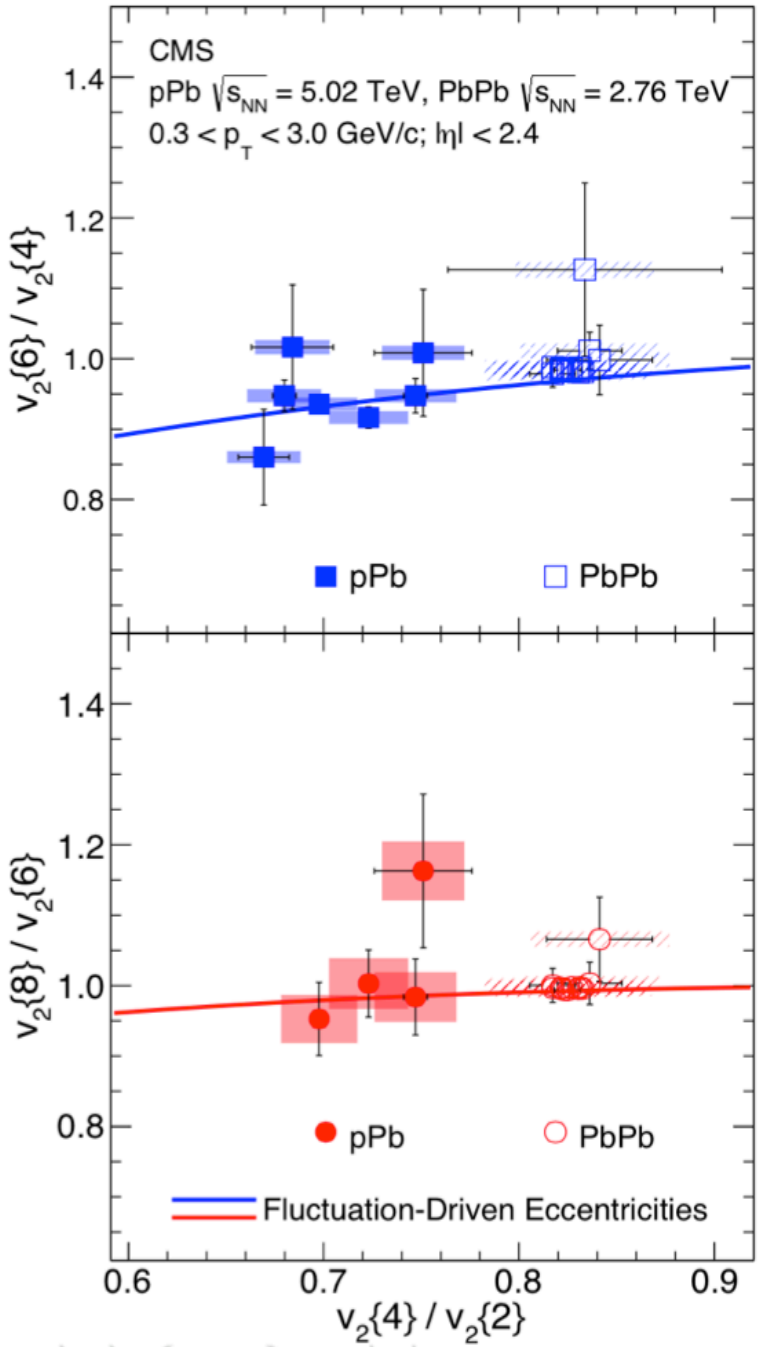
✔

Hydro predicted more than just collectivity



a quantitative relation of $v_2\{2\}$, $v_2\{4\}$, $v_2\{6\}$, $v_2\{8\}$ …

(assuming $v_2\{m\} \propto \varepsilon_2\{m\}$)

Yan, Ollitrault, Bzdak, Mclerran

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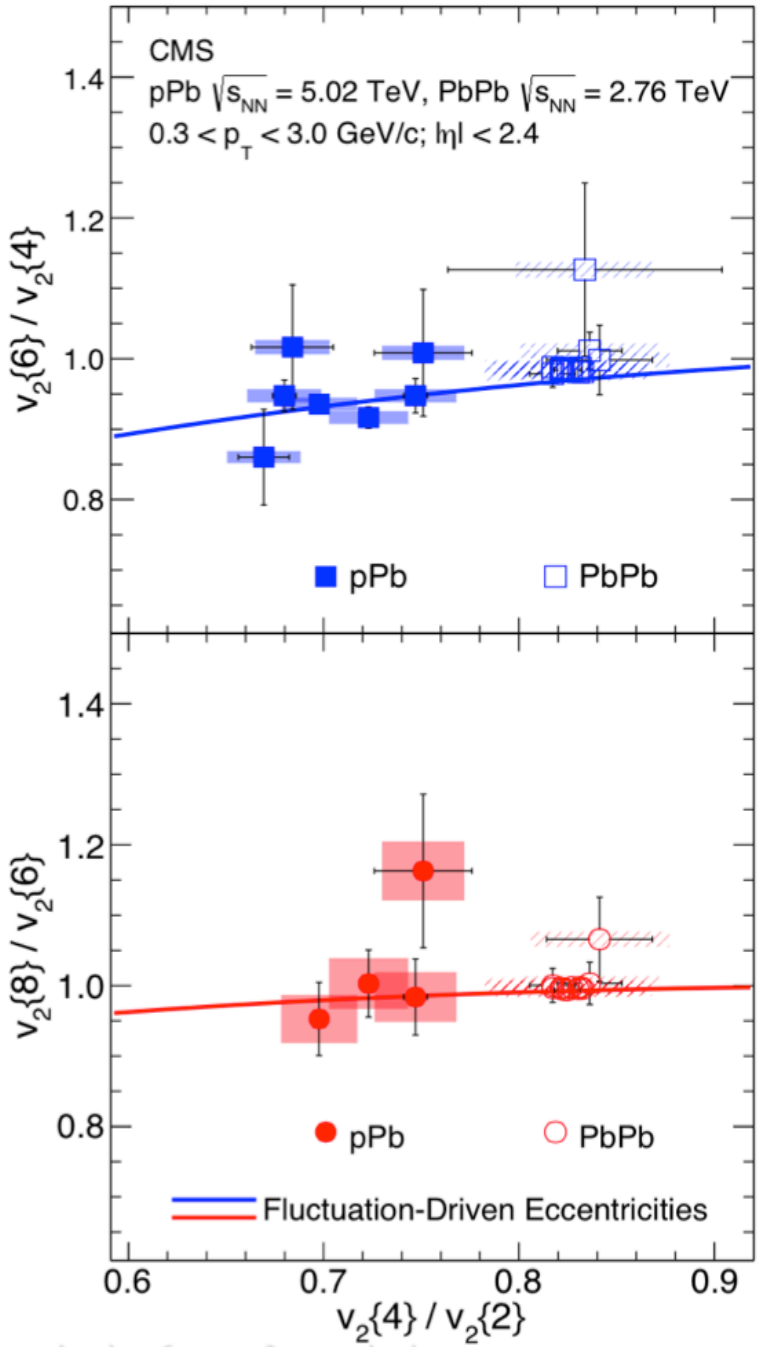

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With more precise data, hydro. model is falsifiable!

Summary of experimental facts

High-multiplicity pPb exhibits "hydro-like" behavior as PbPb in many ways

- Similar shape of $v_n(p_T)$
- Mass ordering of v_2 and NCQ scaling
- $T_{slope} \propto$ mass; stronger radial flow in small system
- Flow factorization breaking
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Results in dAu/He^3A also suggestive of hydro.

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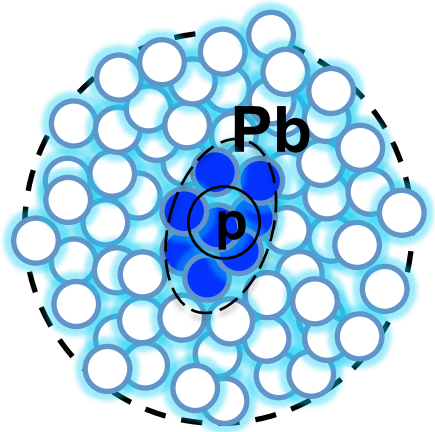
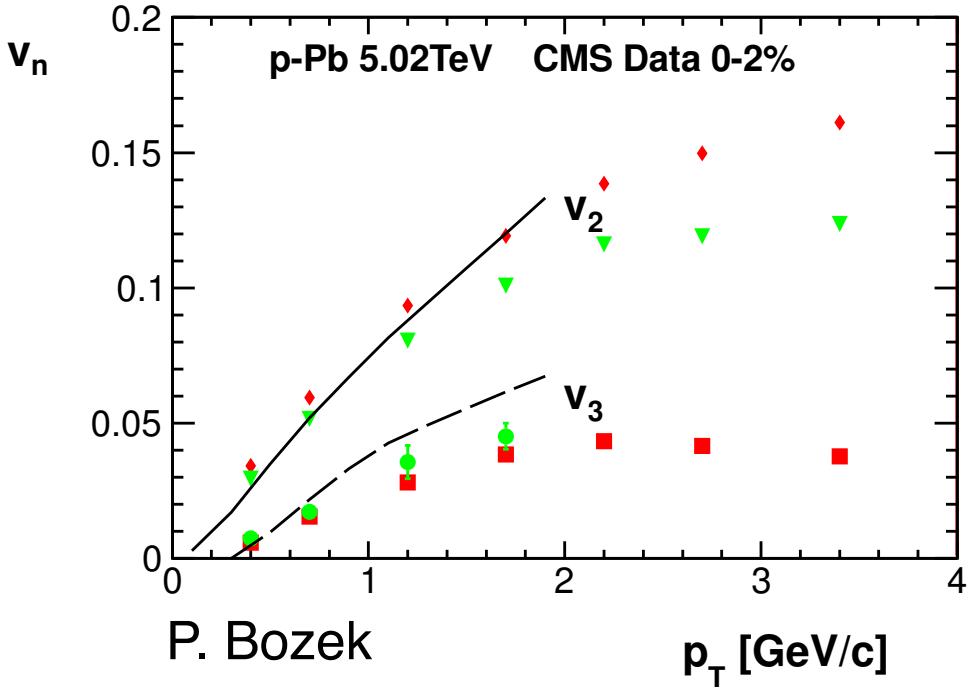
Any observable qualitatively contradicts hydro?

Hydro. has its problem that initial-state “geometry” in pA is very uncertain

Glauber-like



Works “surprisingly” well!



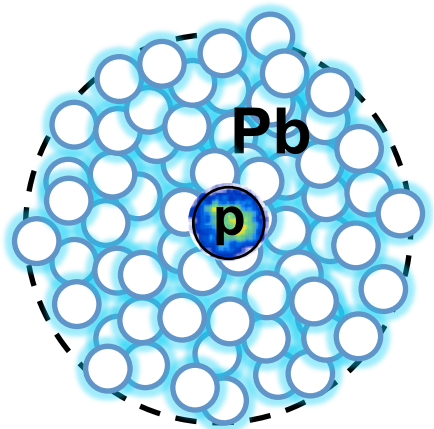
P. Bozek

What does it imply?

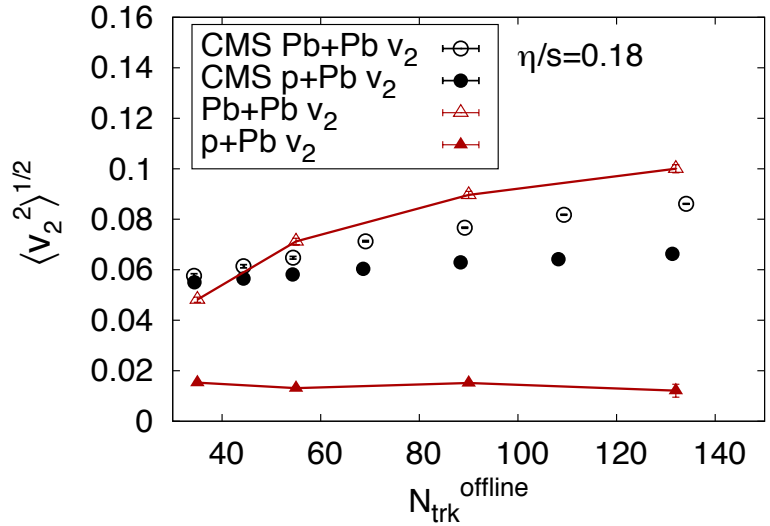
Coincidence? (Maybe just picked the “right” η/s)

Hydro. has its problem that initial-state "geometry" in pA is very uncertain

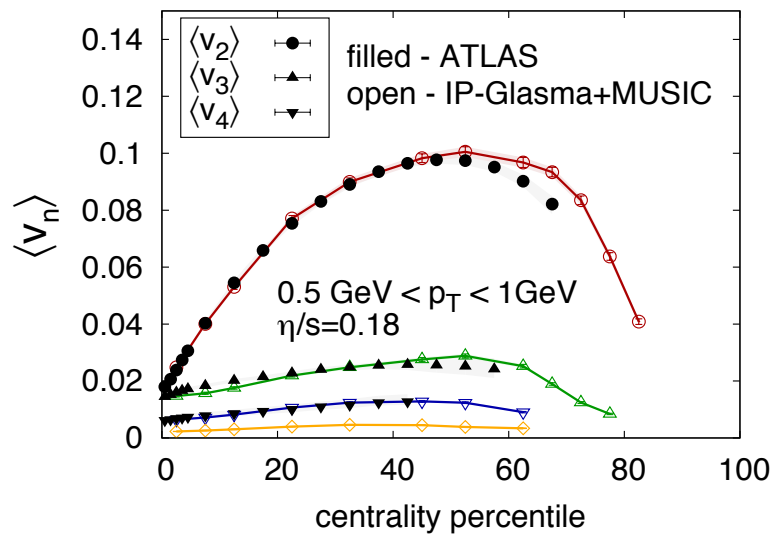
IP-glasma pPb



Not working well in pPb



But Perfect in AA!



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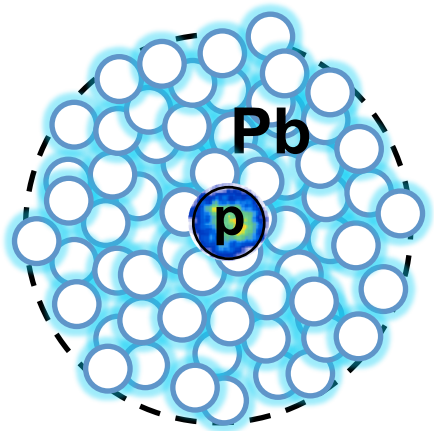
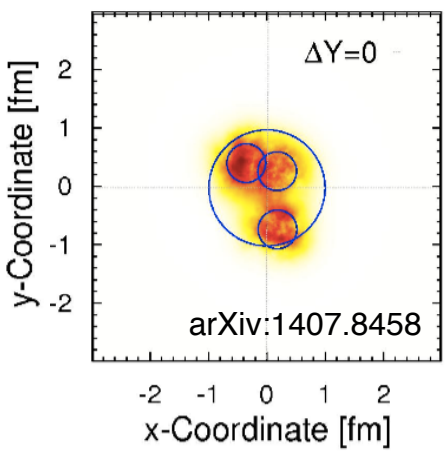
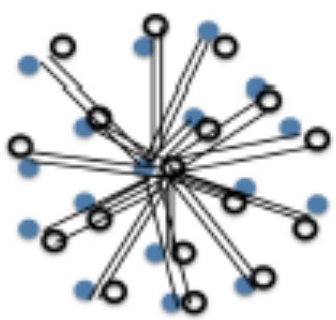


eccentric proton?



or

pomerons



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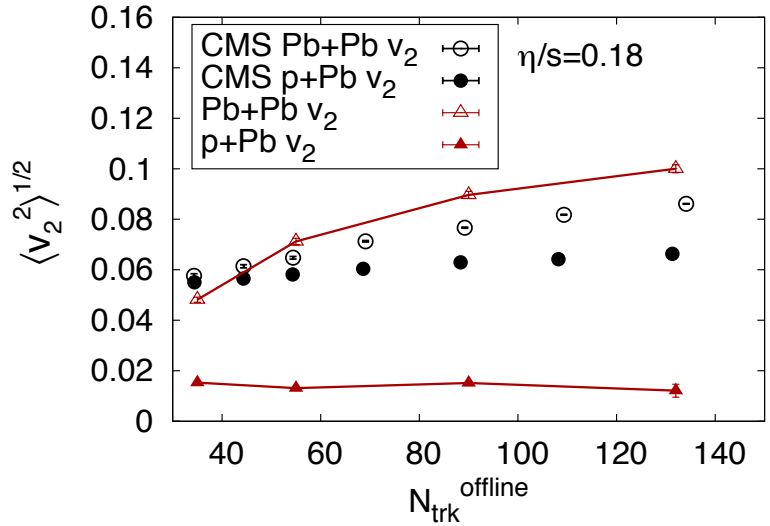
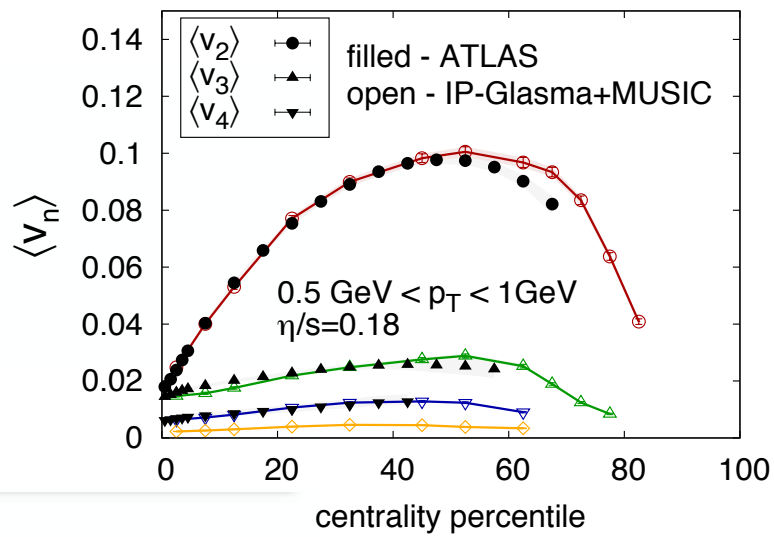
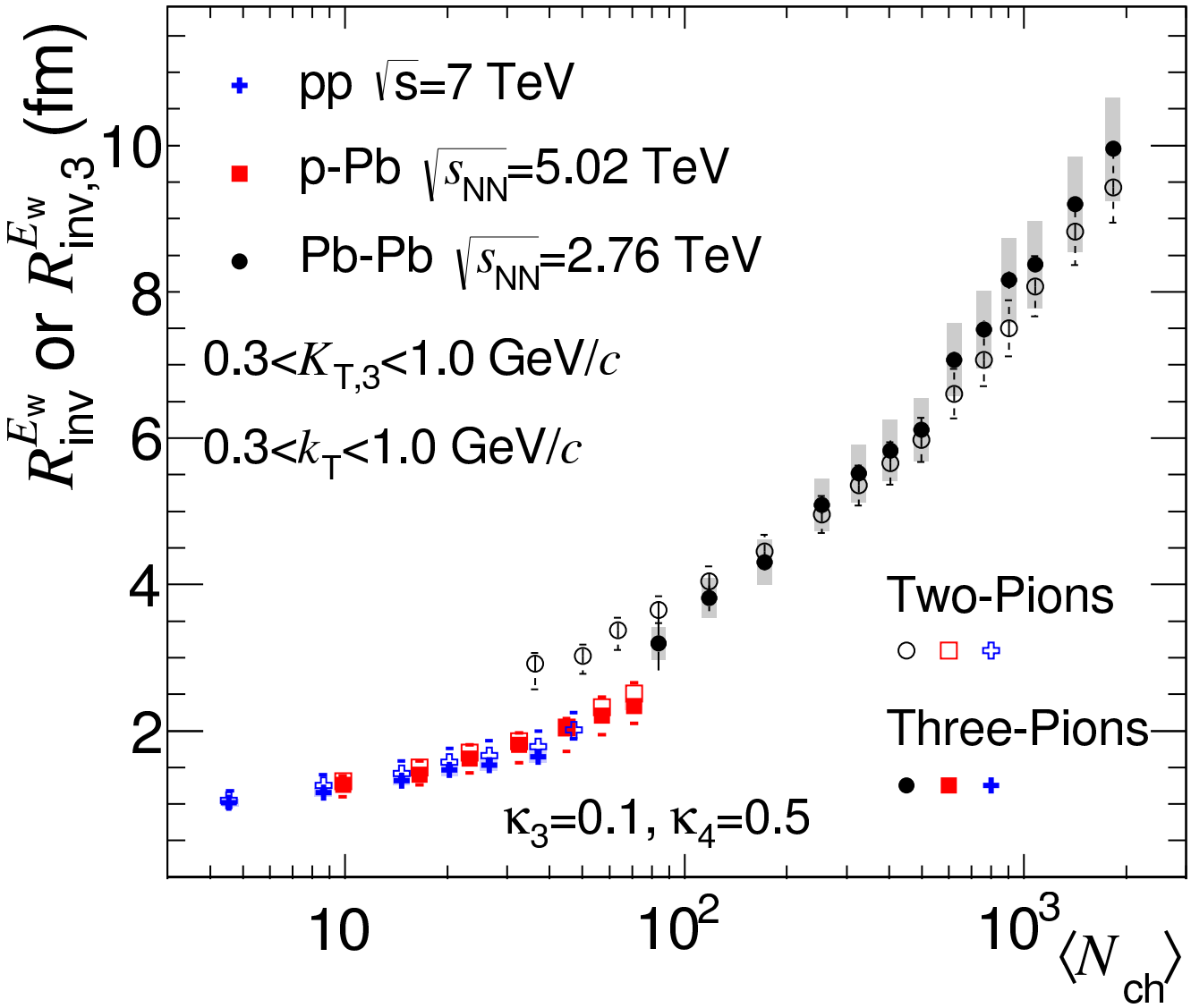


But Perfect in AA!



pp/pA sensitive subnucleonic fluctuations

HBT provide us important system size information



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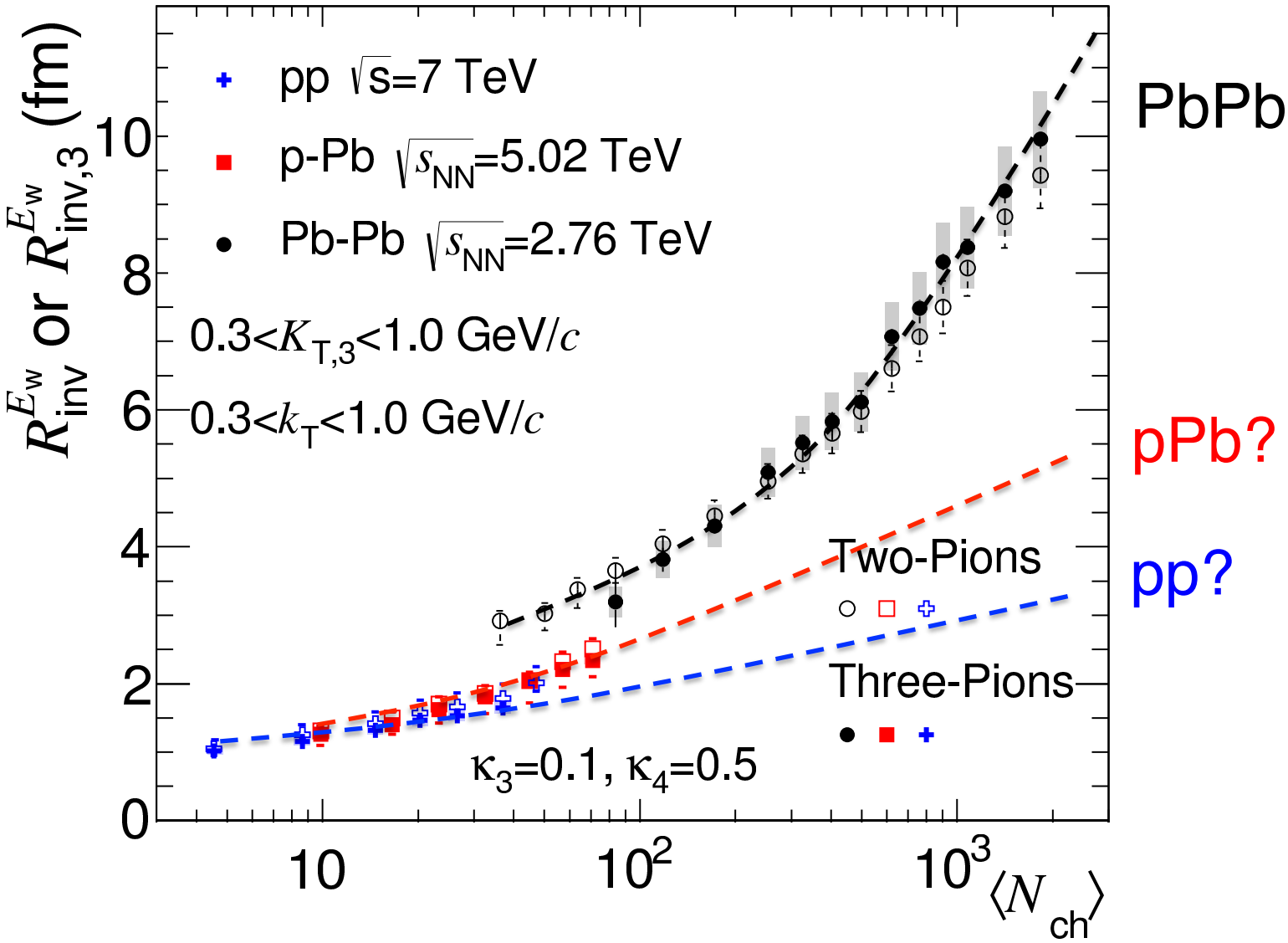

Relation among pp, pA and AA at fixed N_{ch} unclear

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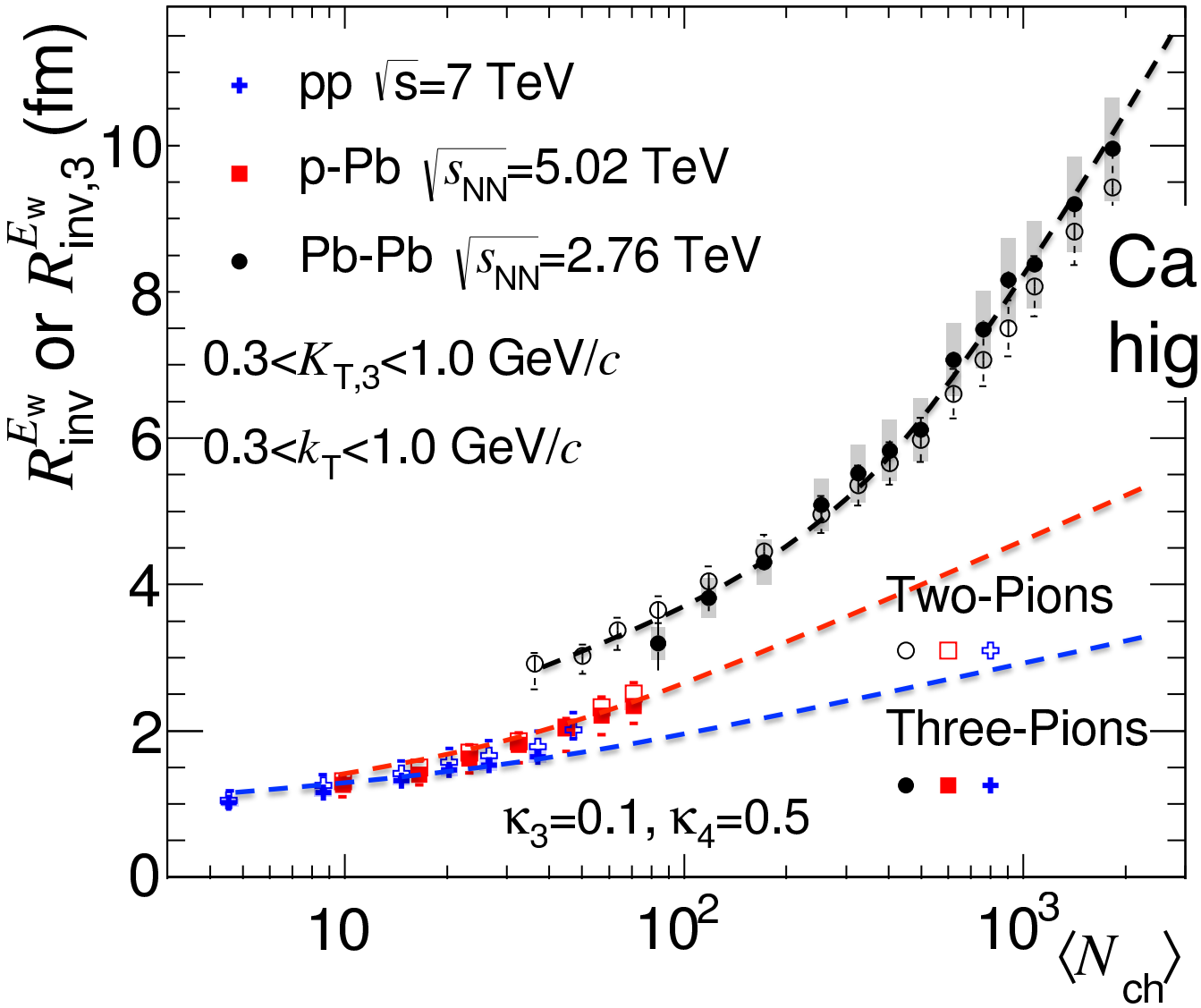


PbPb

Can we push to higher multiplicity?

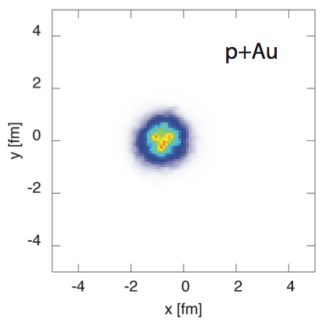
pPb?

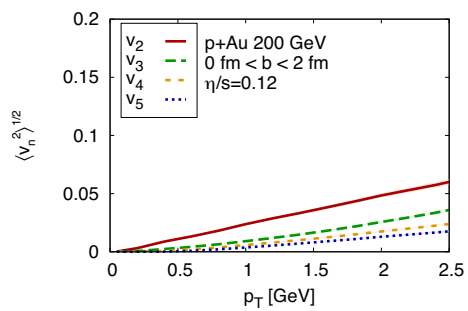
pp?

Relation among pp, pA and AA at fixed N_{ch} unclear

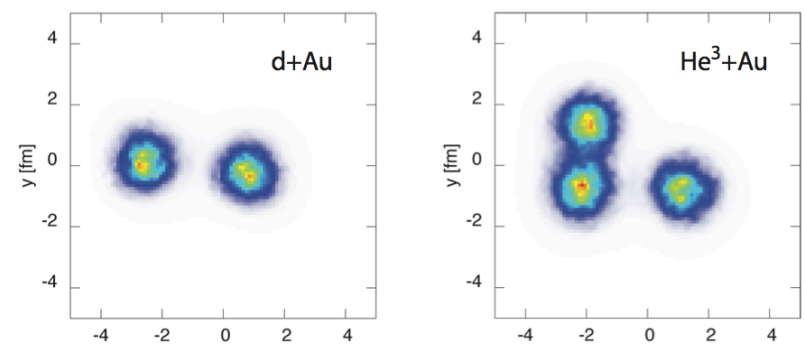
System Size Scan (SSS) at RHIC

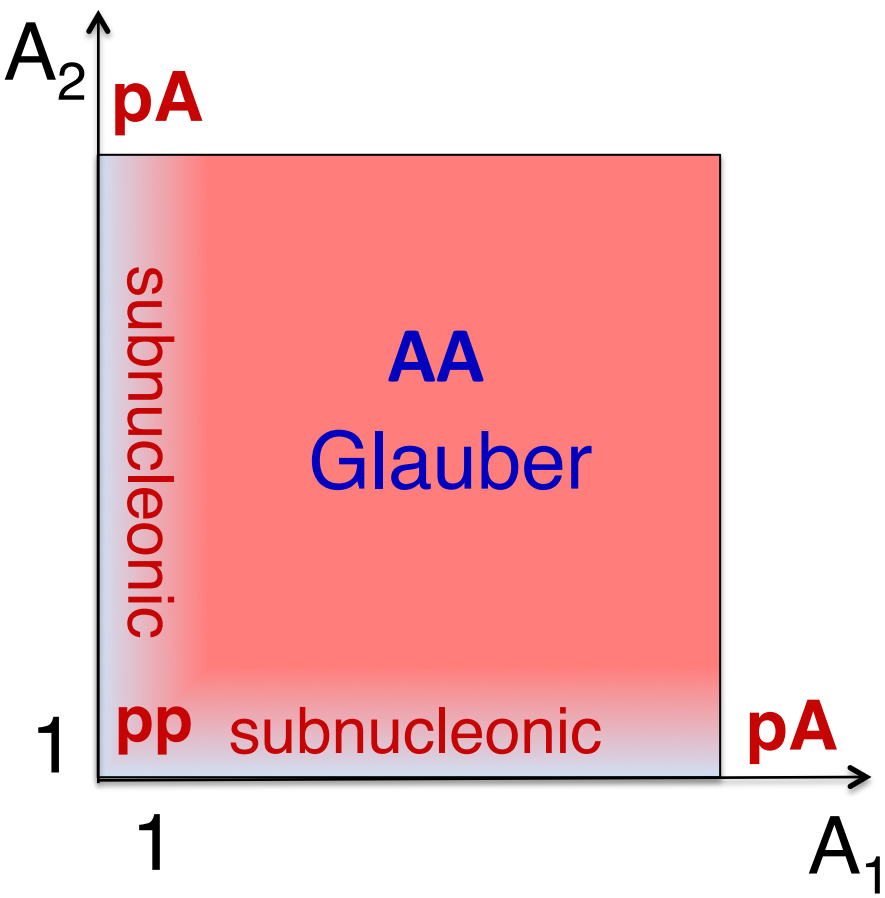
Diverge for $A_1(A_2) = 1$

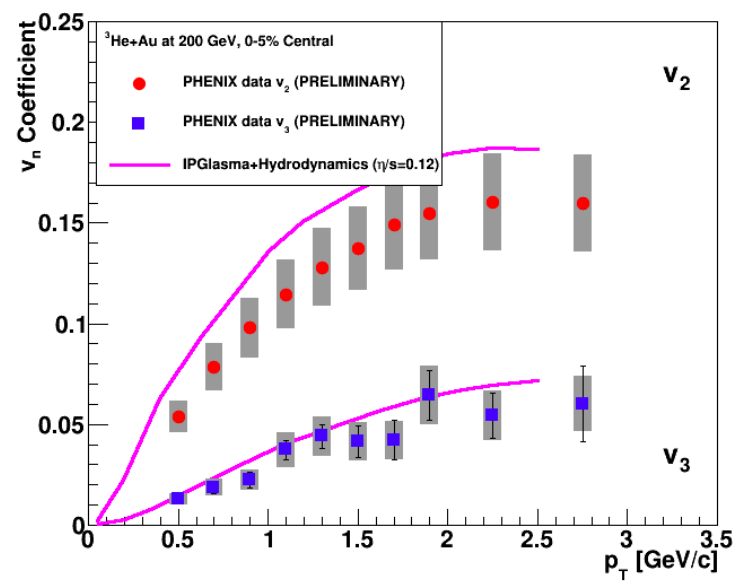




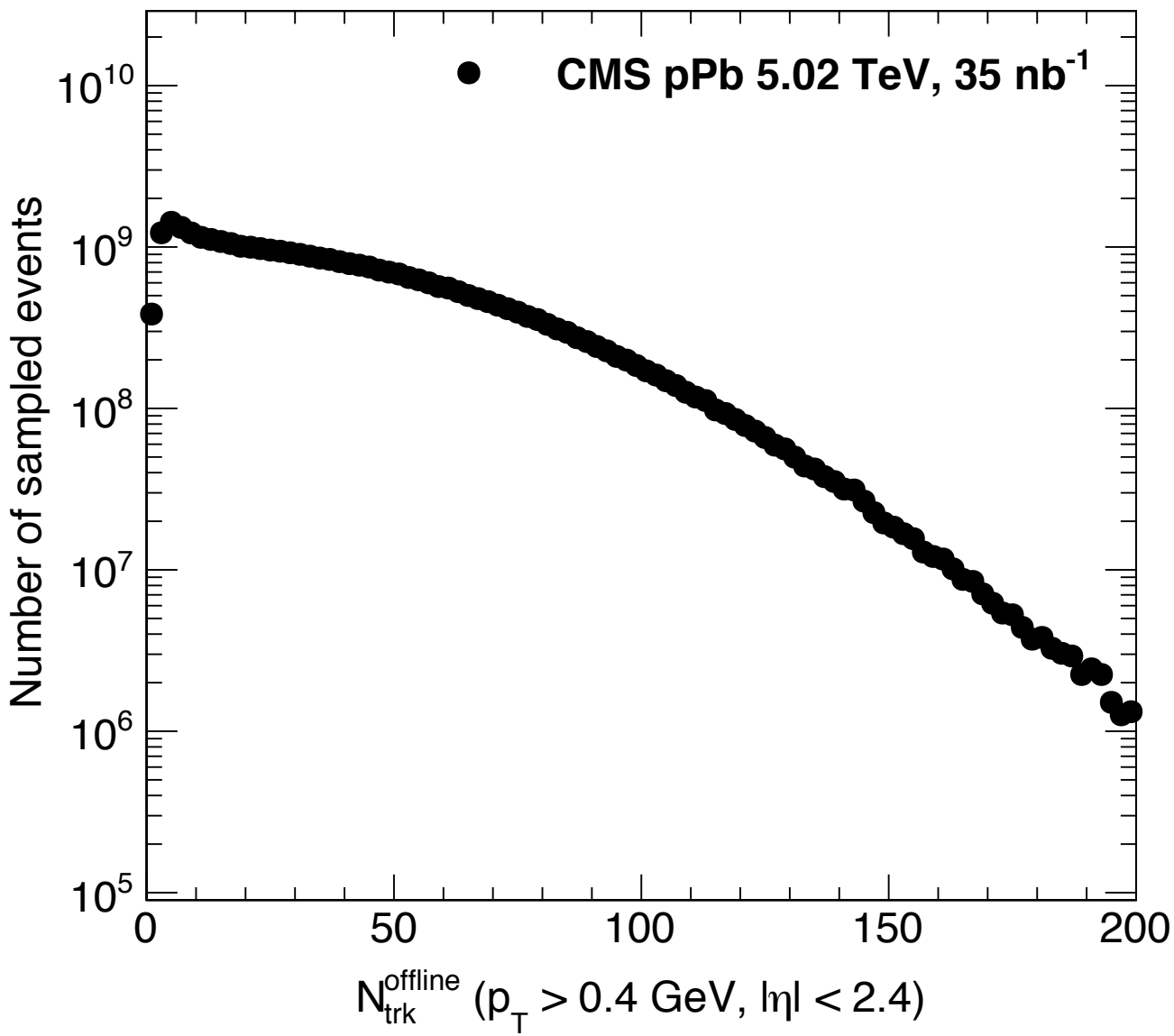
For $A_1(A_2) \geq 2$, all hydro models tend to converge







pA (and pp?) program at RHIC



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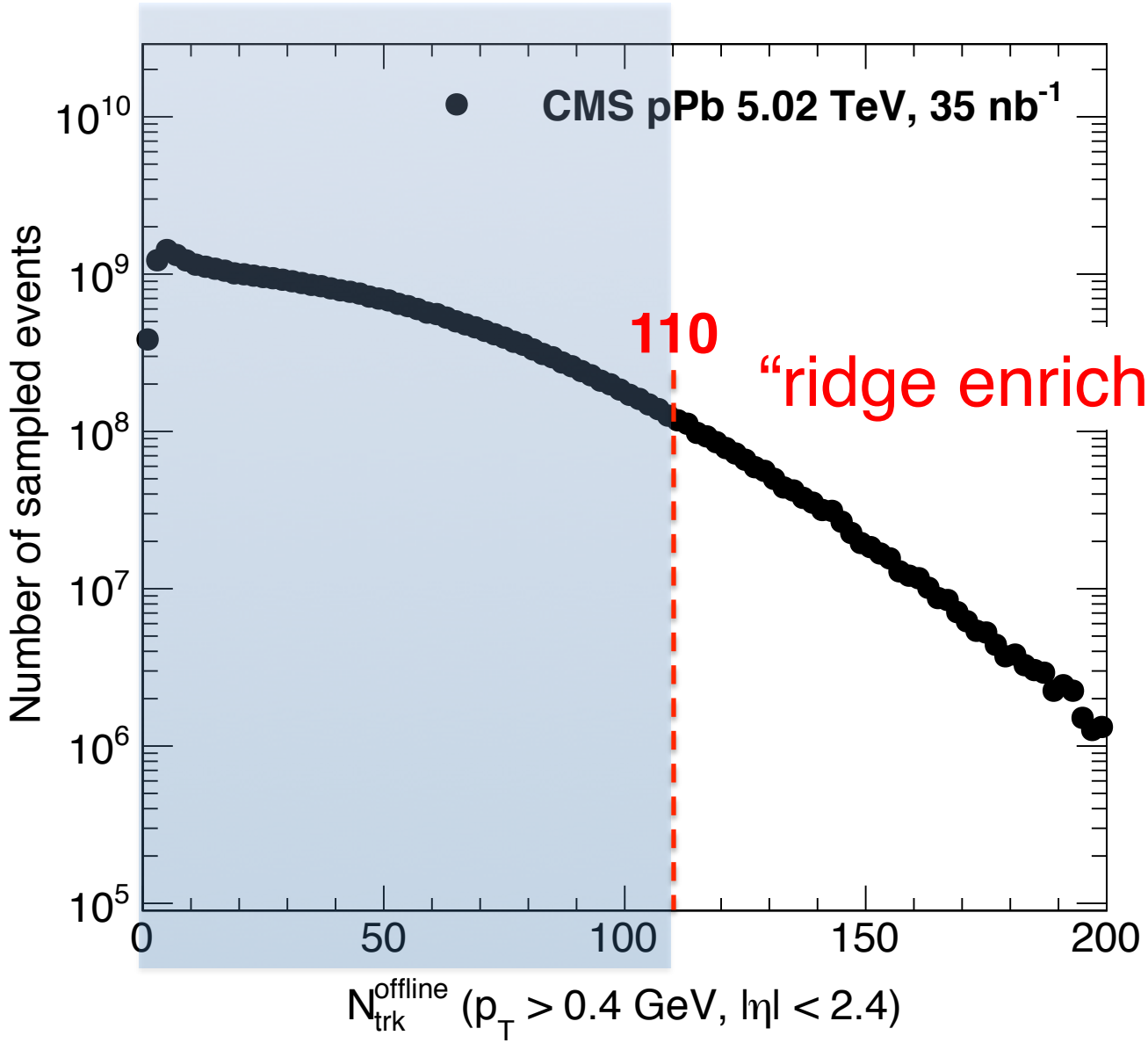

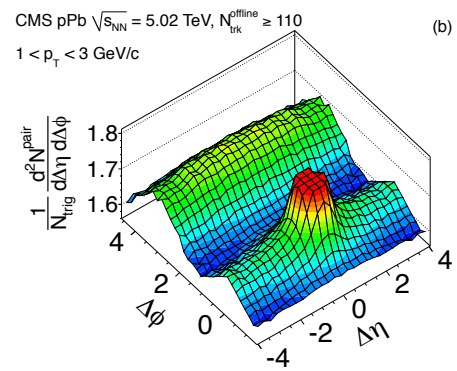

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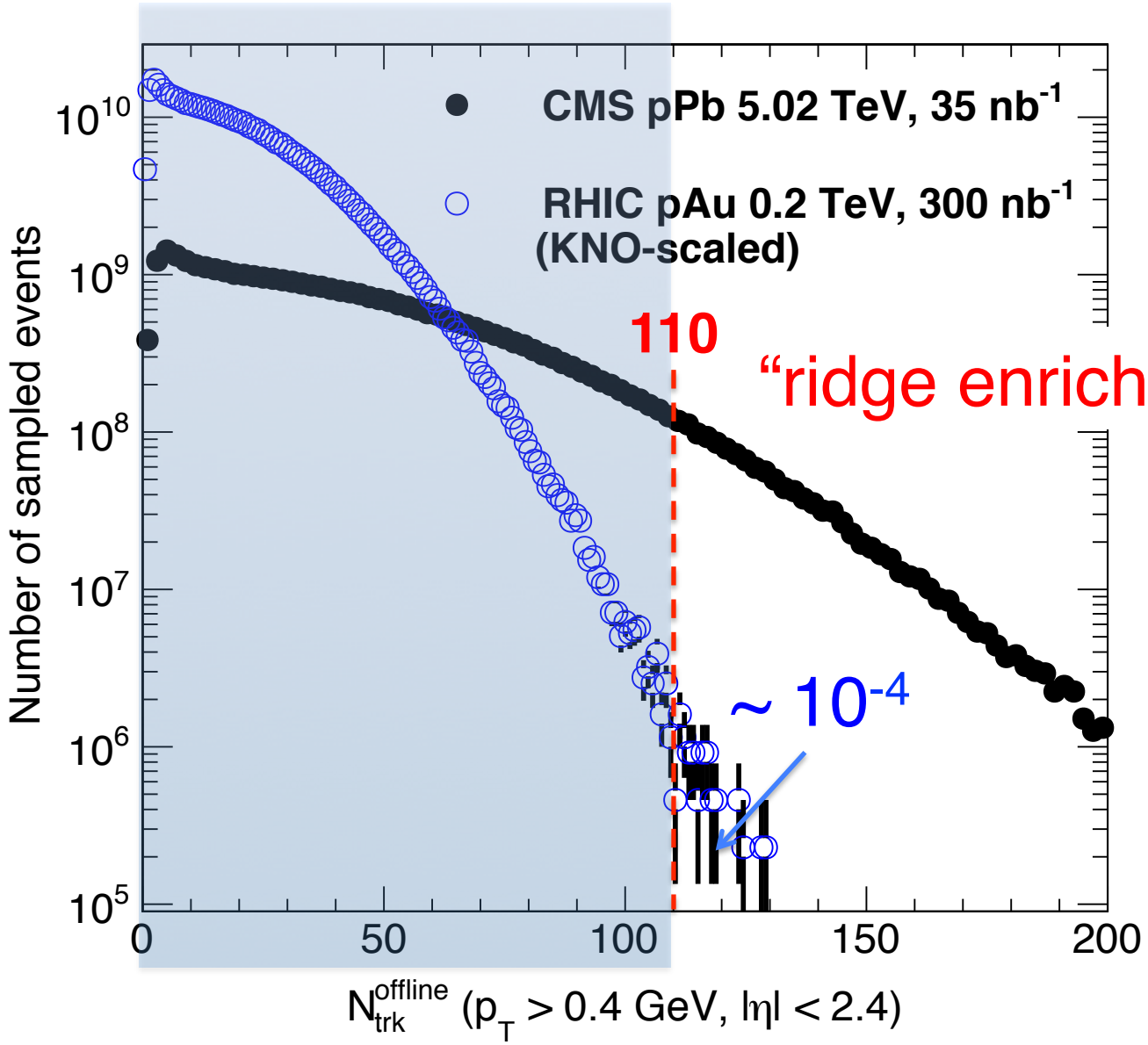

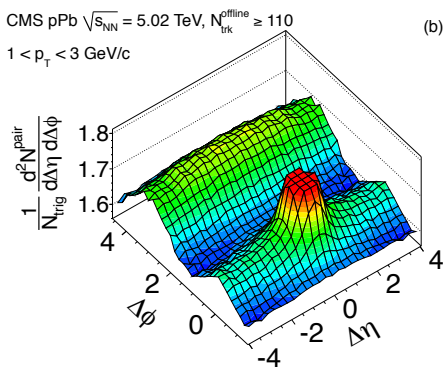

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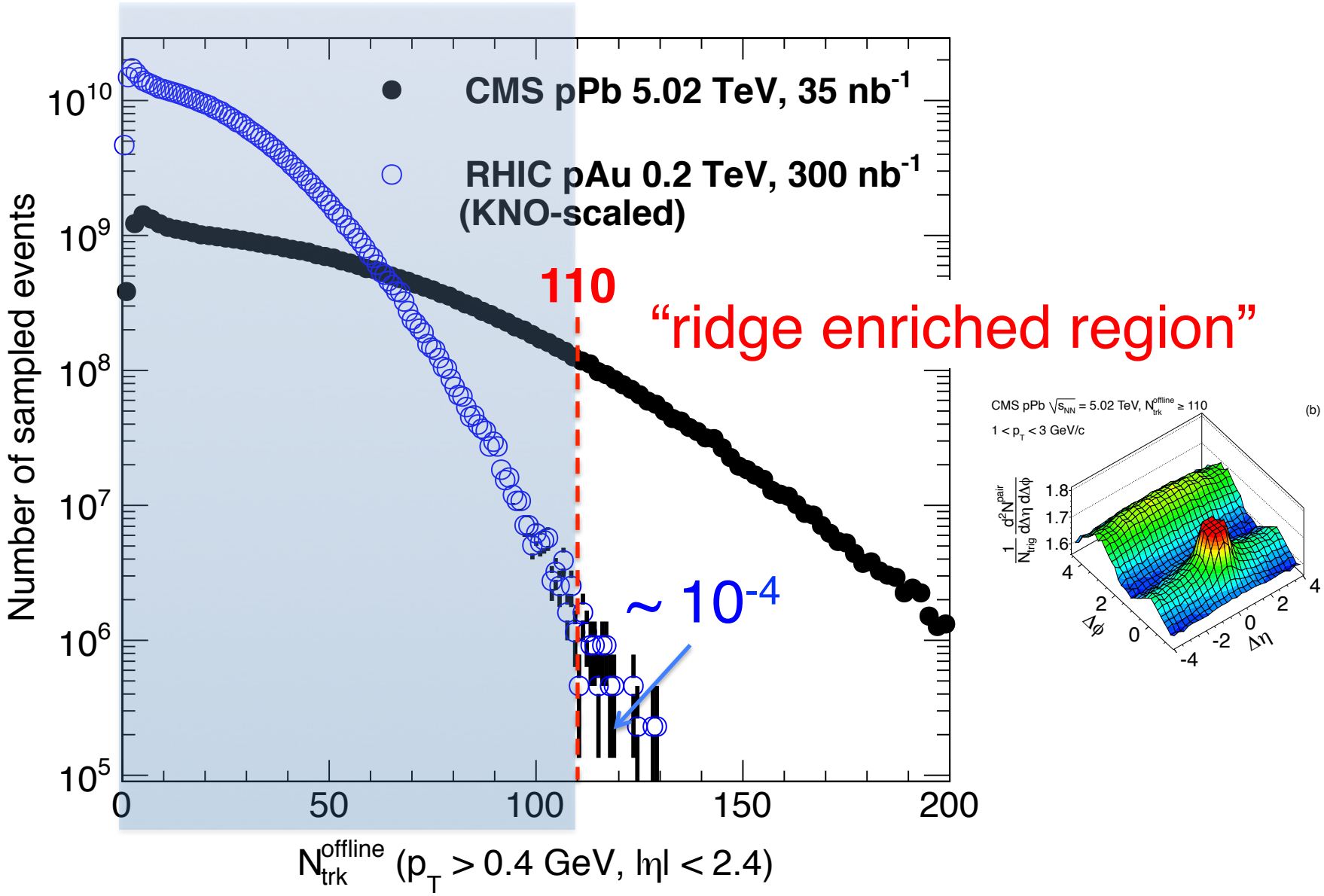


Look forward to pAu (and pAl) results from RHIC!

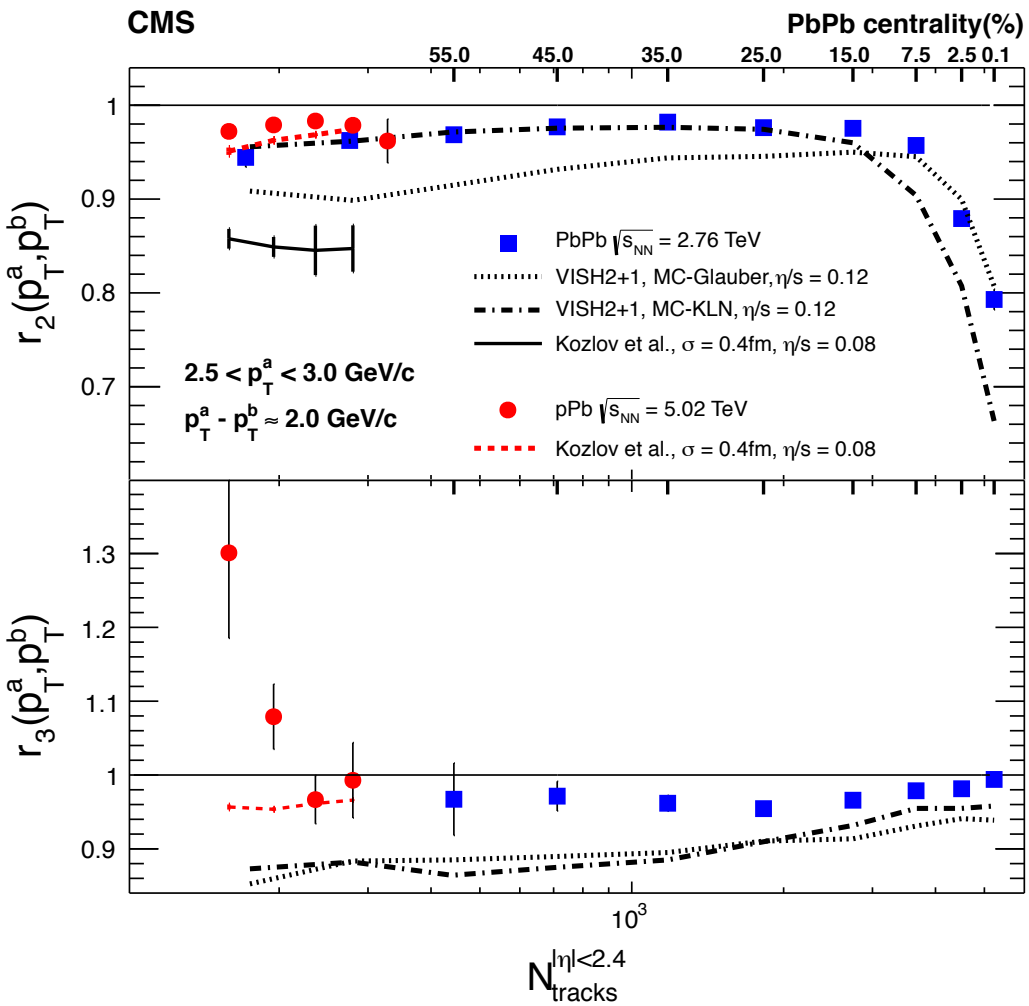
How to further scrutinize hydro.?

Decouple **initial fluctuations** and **final dynamics**

Initial-state lumpiness – flow factorization

$$V_{n\Delta}(p_T^a,\eta^a;p_T^b,\eta^b) \neq v_n(p_T^a,\eta^a)\times v_n(p_T^b,\eta^b)$$

(subleading flow)



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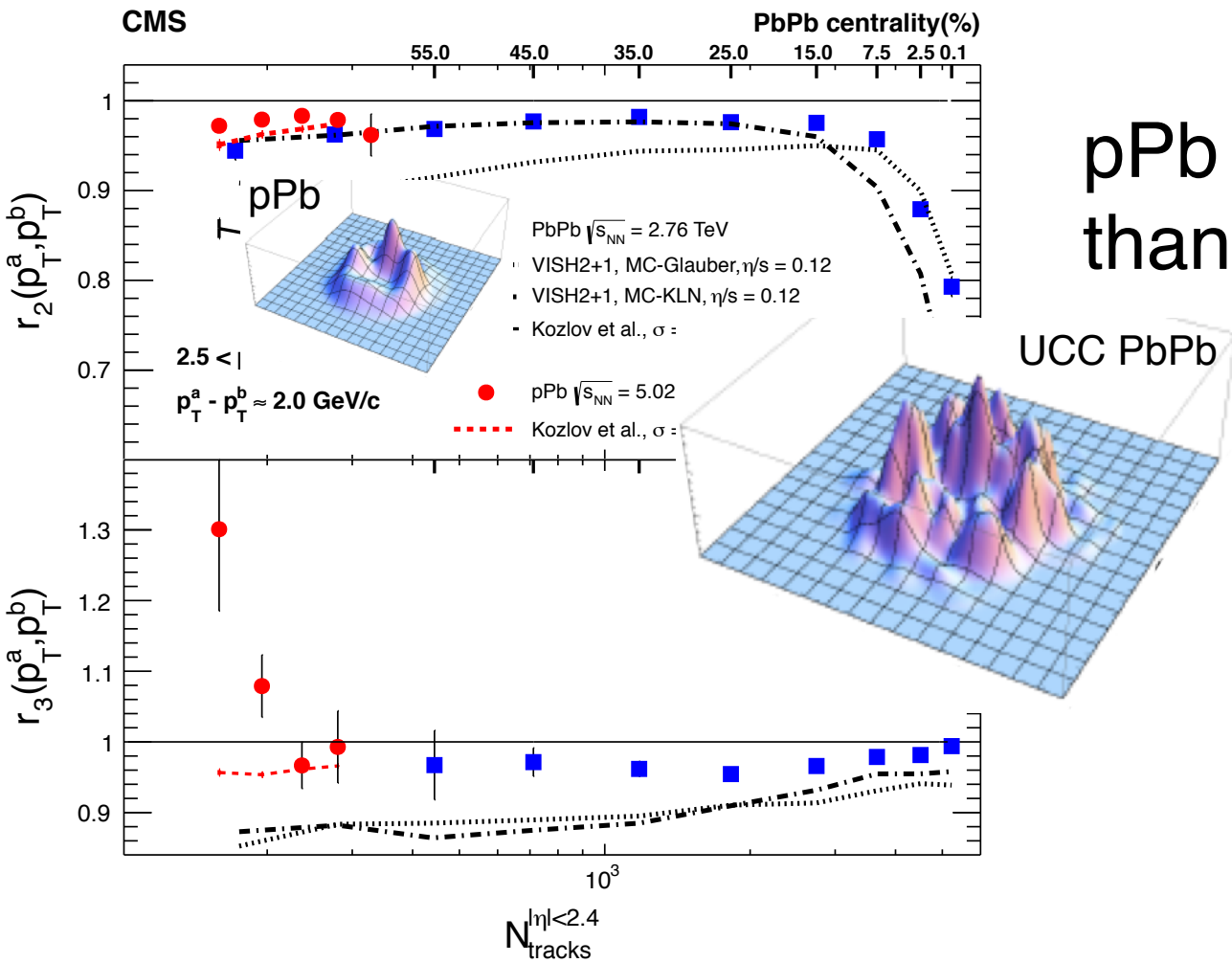
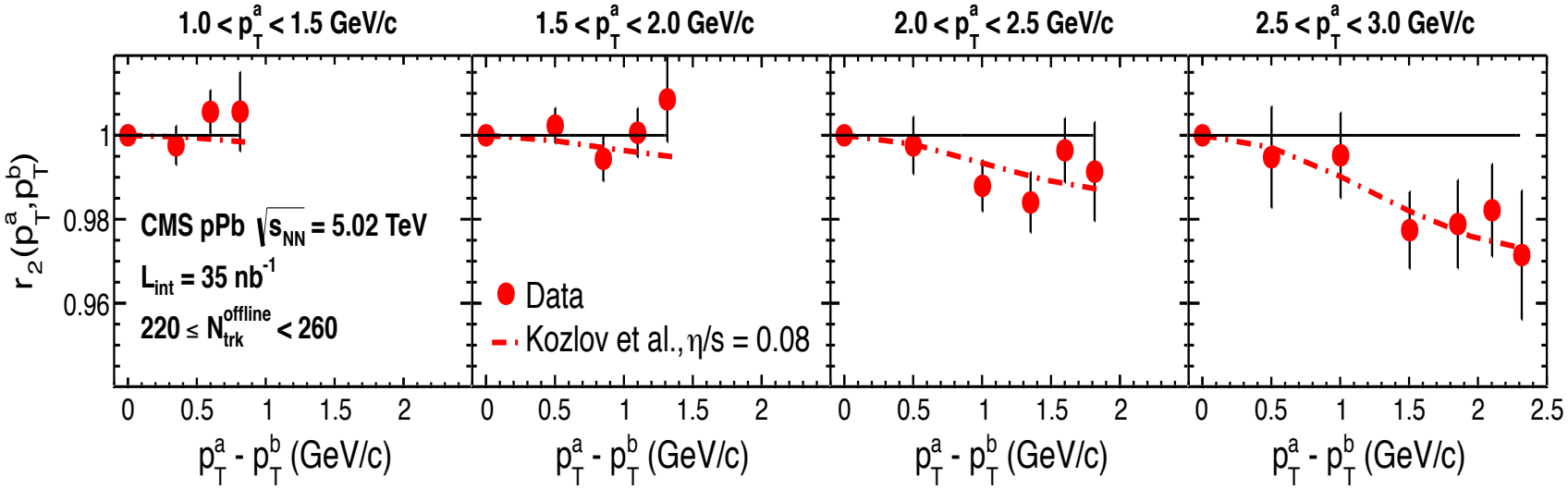


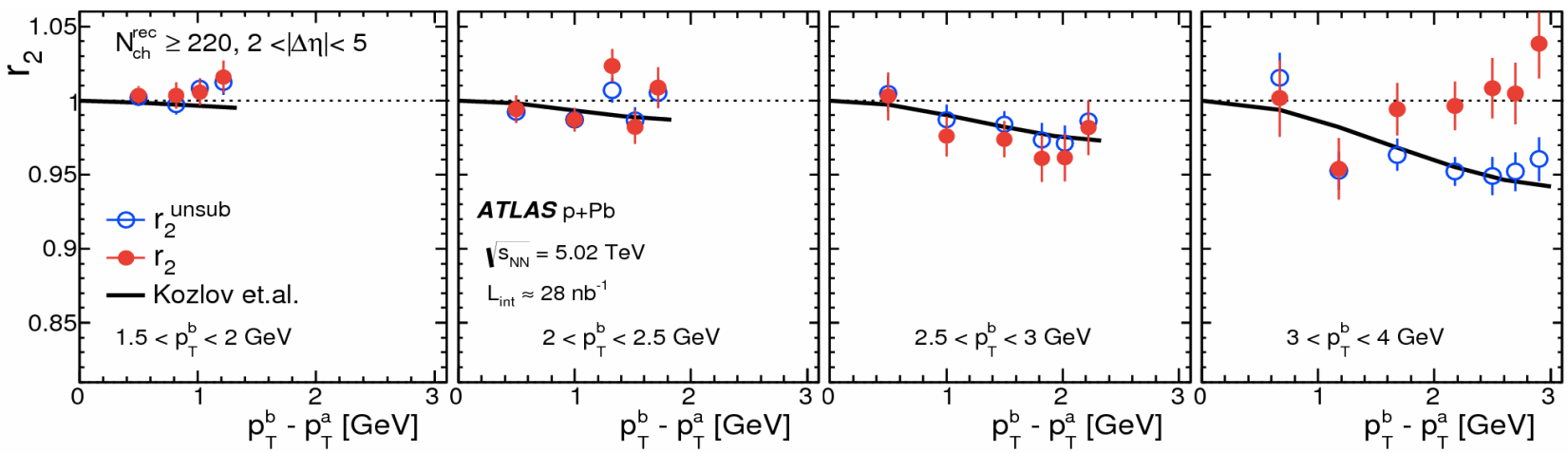
pPb is smoother than UCC PbPb

How to further scrutinize hydro.?

CMS

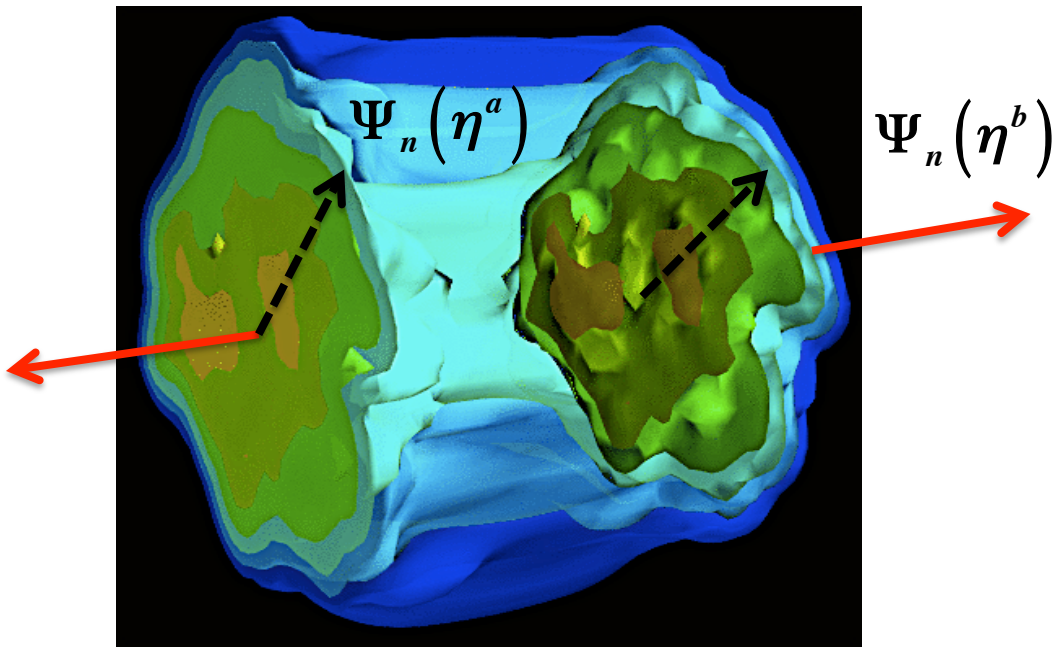


ATLAS



How to further scrutinize hydro.?

3D expansion

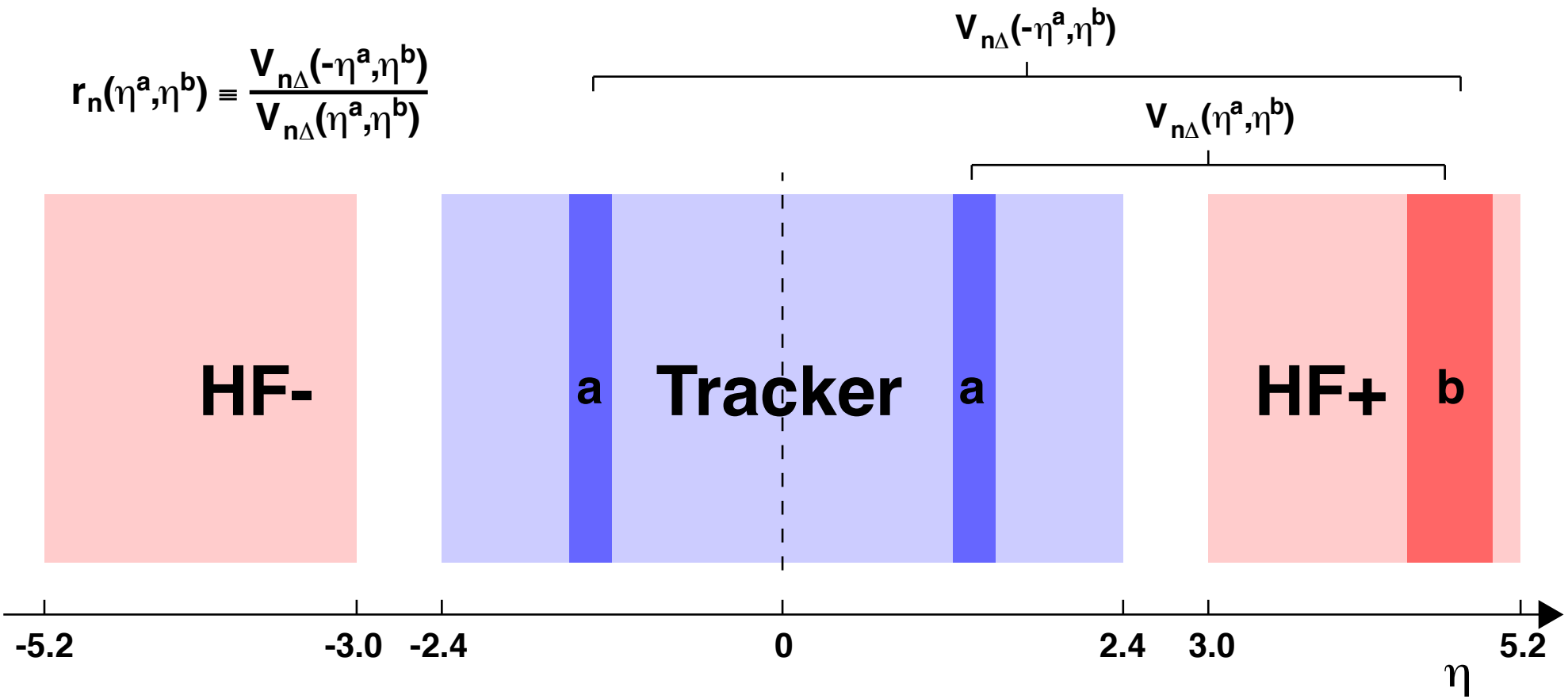


$$f(p_T,\phi,\eta) \sim 1 + 2\sum_{n=1}^{\infty} v_n(p_T,\eta)\cos\left[n\left(\phi - \Psi_n(p_T,\eta)\right)\right]$$

Gateway to a full 3D description of initial-state fluctuations and dynamics of system evolution

How to further scrutinize hydro.?

$$r_n(\eta^a,\eta^b) \equiv \frac{V_{n\Delta}(-\eta^a,\eta^b)}{V_{n\Delta}(\eta^a,\eta^b)}$$

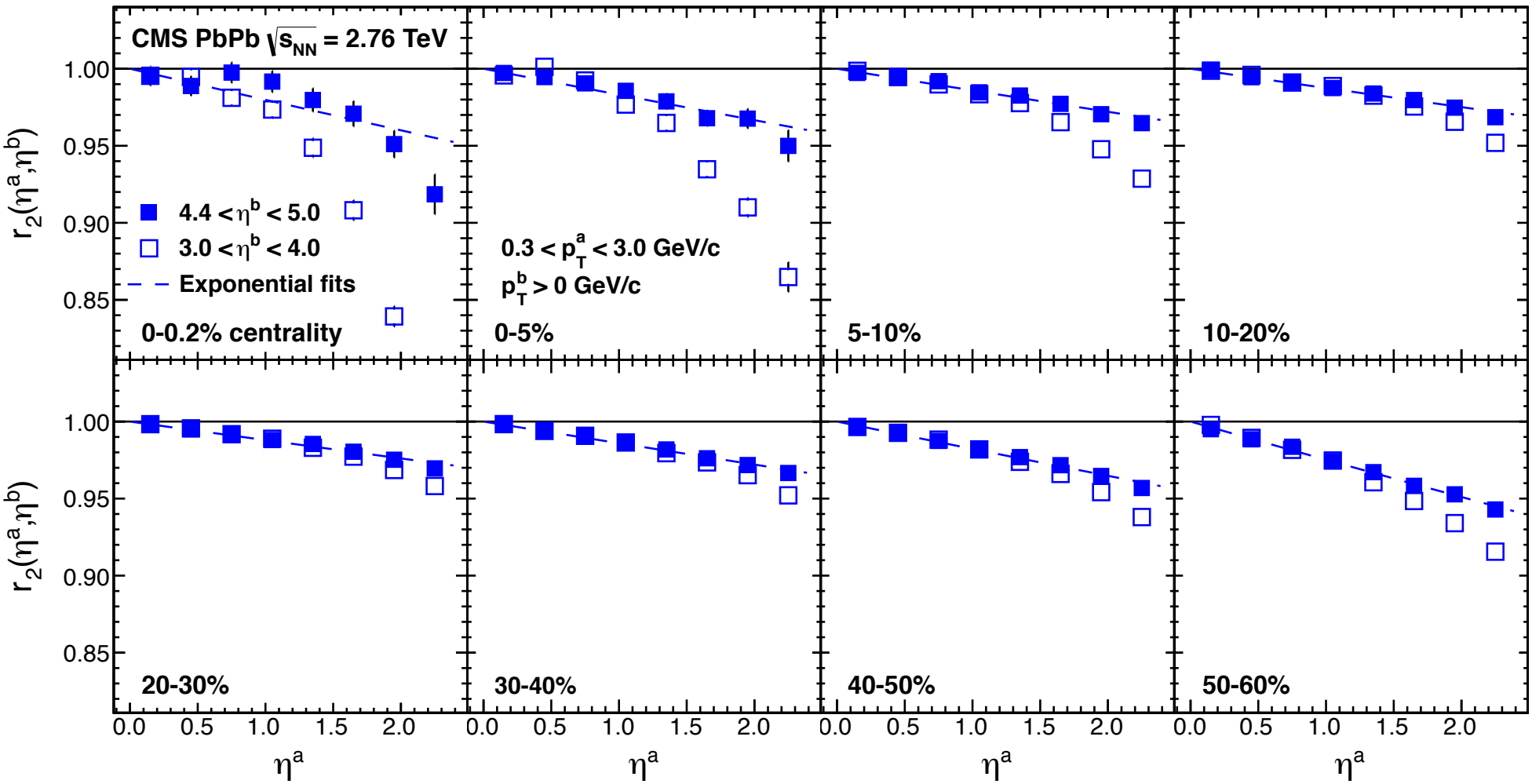


All pairs have η gap > 2 units

arXiv:1503.01692

$$r_n(\eta^a,\eta^b) \approx \left\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \right\rangle$$

How to further scrutinize hydro.?



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η gap: $2\eta_a$

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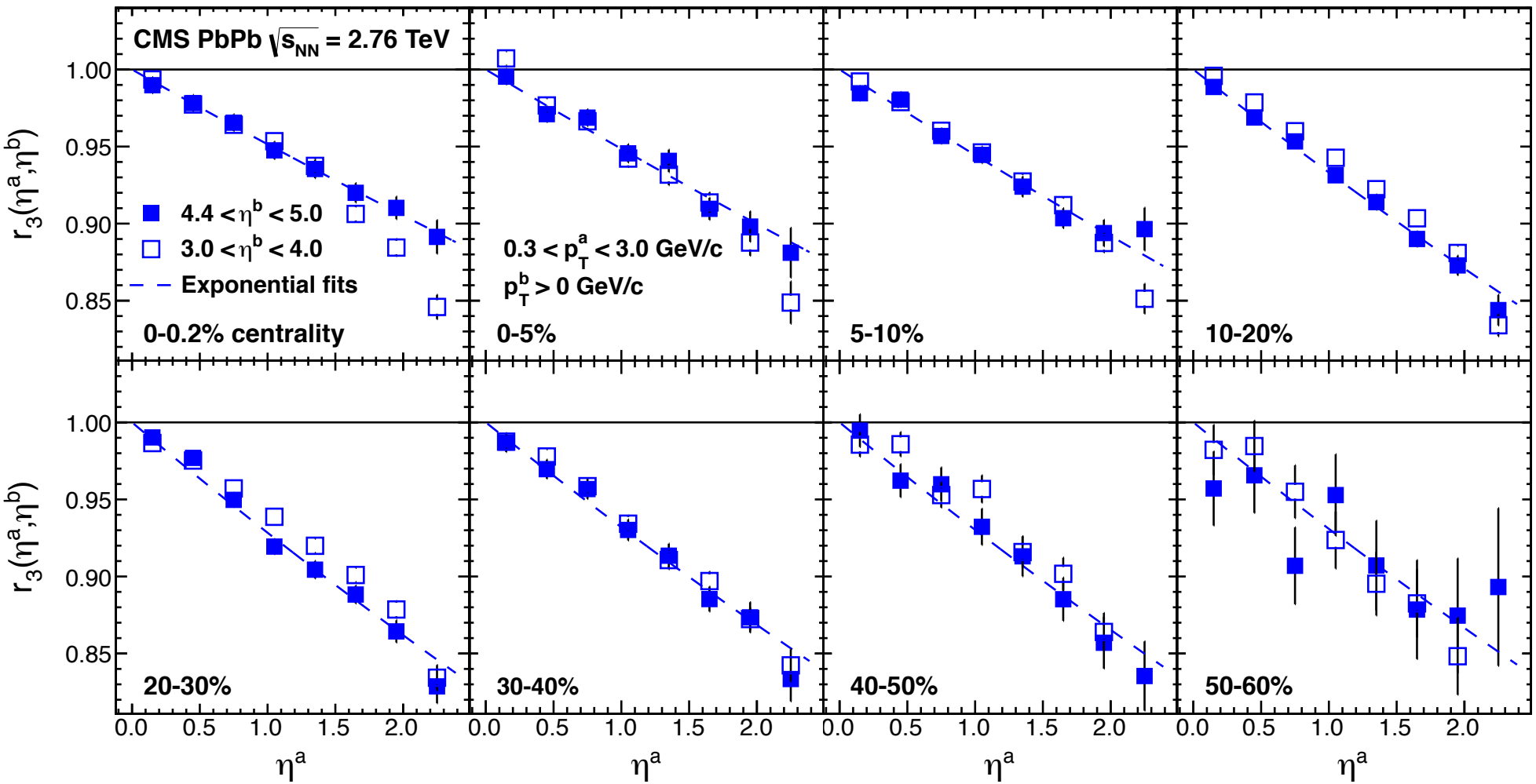

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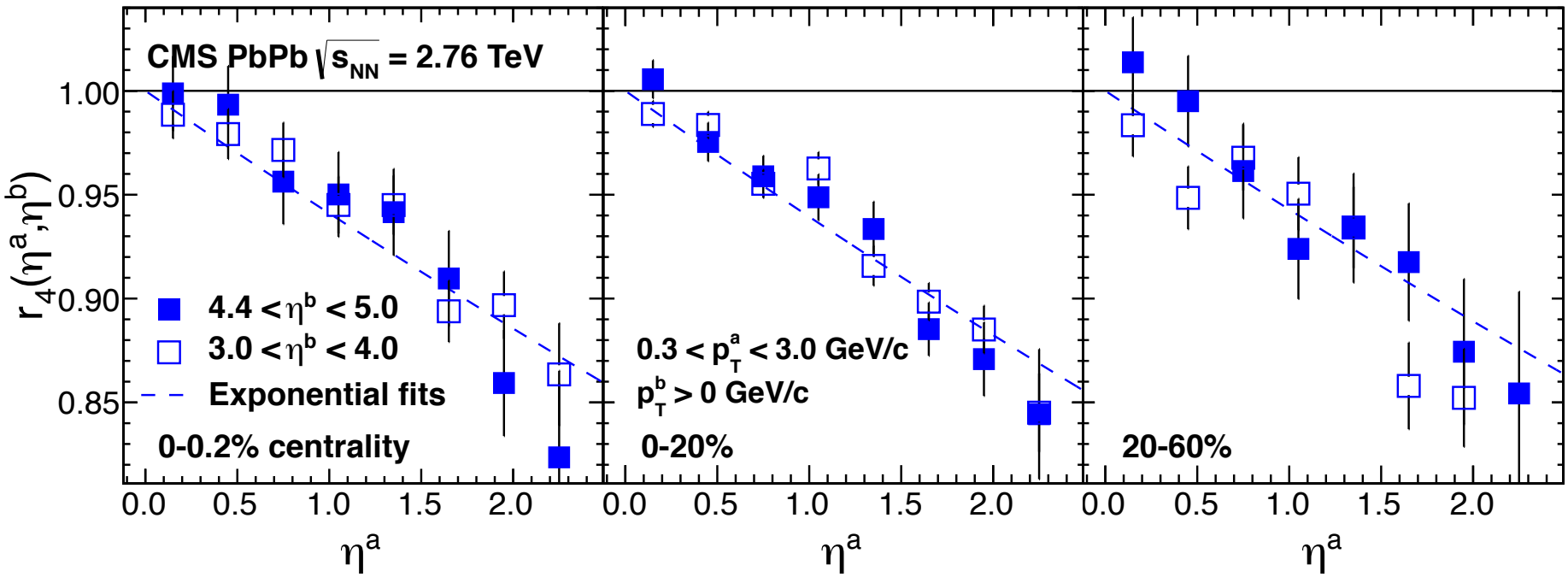

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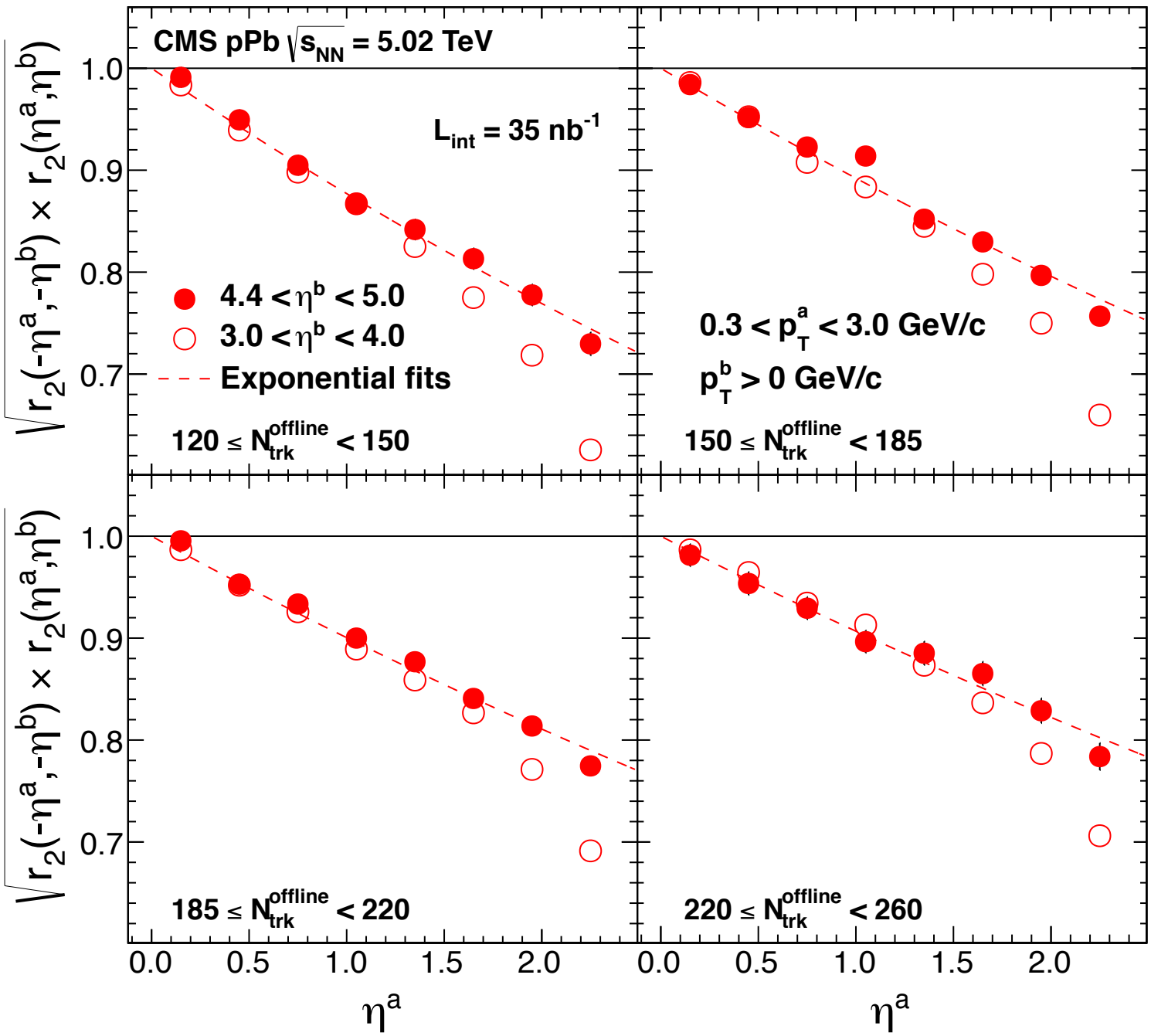
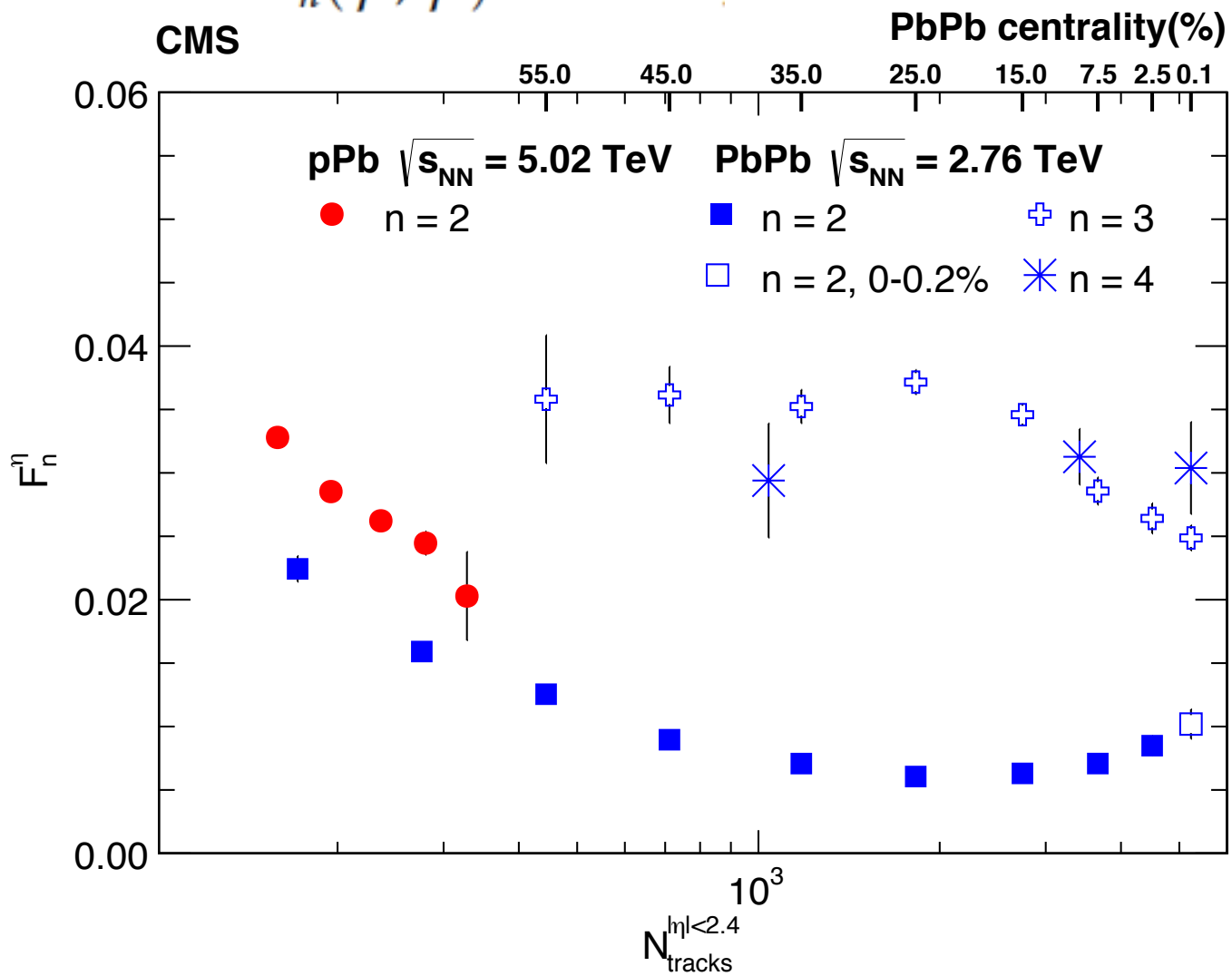


How to further scrutinize hydro.?

$$r_n(\eta^a, \eta^b) \approx e^{-2F_n^\eta \eta^a}$$



Many implications to 3D hydro, η dependence of initial state, how to reinterpret previous $v_n(\eta)$ measurement etc.

Some puzzles raised by Raju

Puzzles for the collectivity picture in p+A

- **Why is there no sign of "mini-jet" quenching in p+A ?**
- **Why is v_2 large up to p_T of 9 GeV in p+A ?**
- **Why is v_2 much smaller in p+p than p+A for the same N_{ch} and HBT radius ?**
- **Why is mass ordering in $\langle p_T \rangle$ seen even at $N_{ch} = 6$?**

Raju at RHIC/AGS users' meeting 2014

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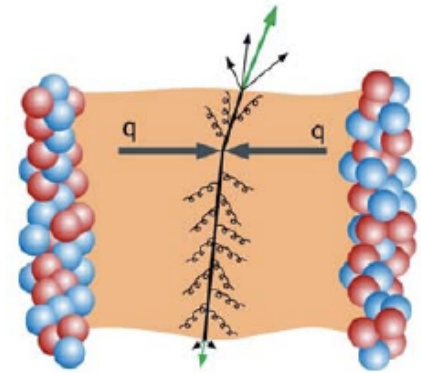
Raju at RHIC/AGS users' meeting 2014

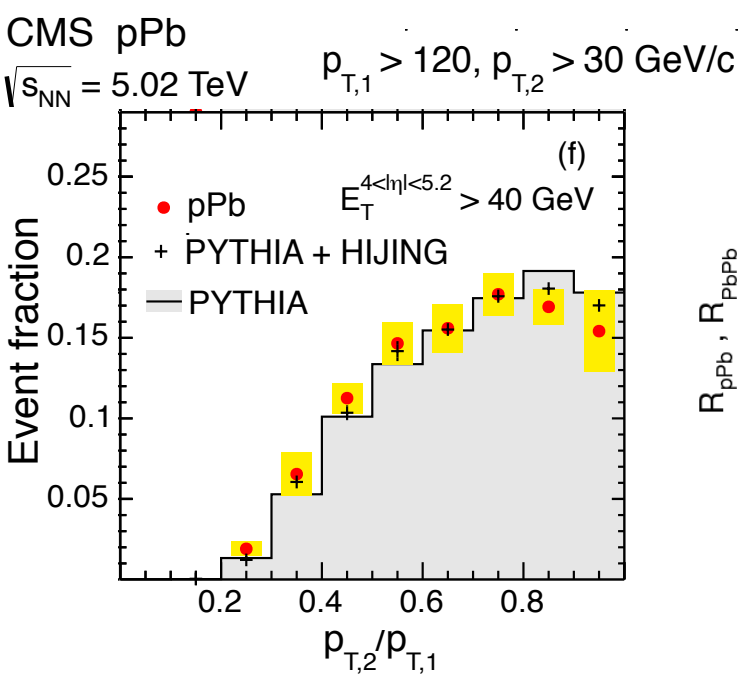
Jet quenching in pPb and pp (?)

If ridge is flow ➔ strongly interacting

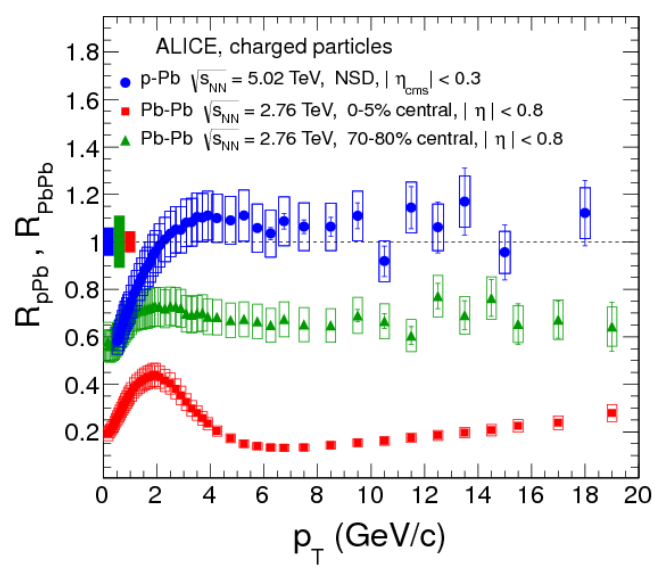
➔ **presence of jet quenching?**



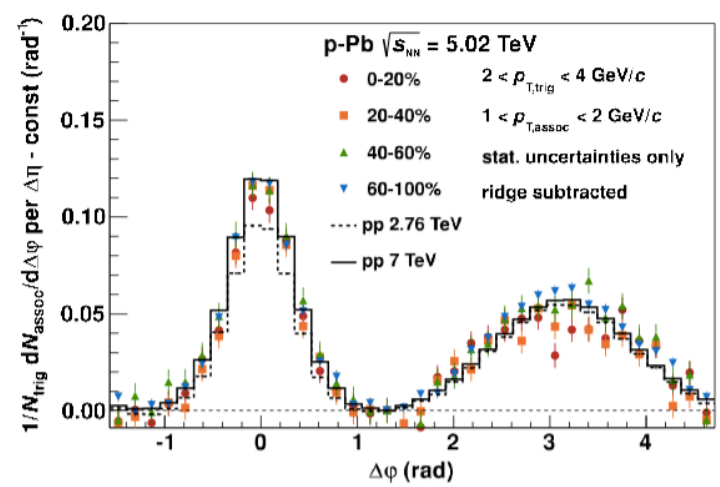
No dijet asymmetry



No leading hadron suppression



Dihadron unmodified



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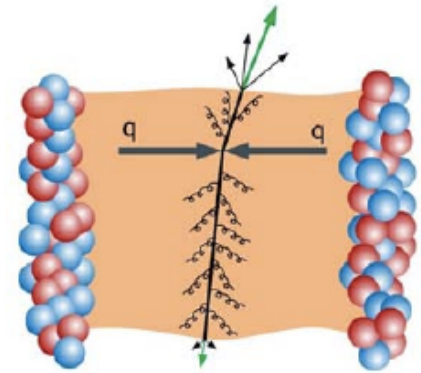

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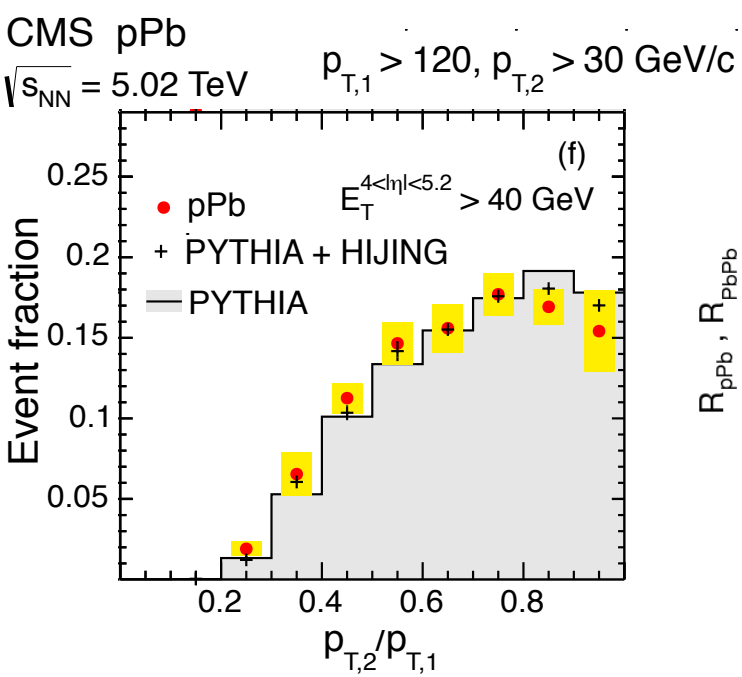
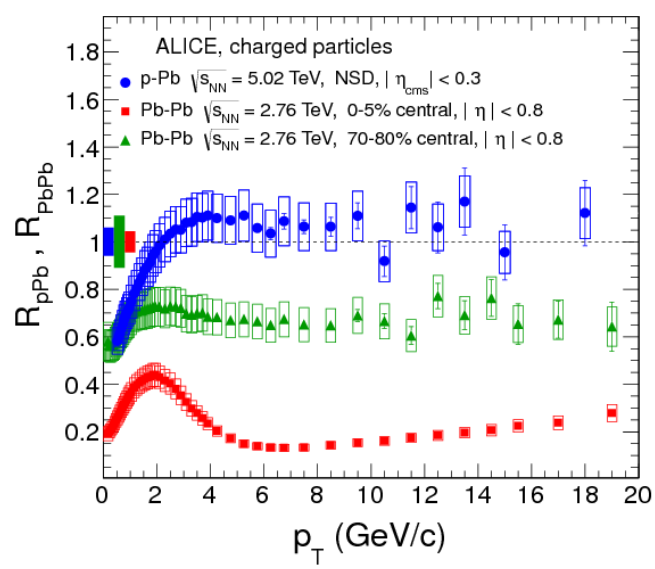


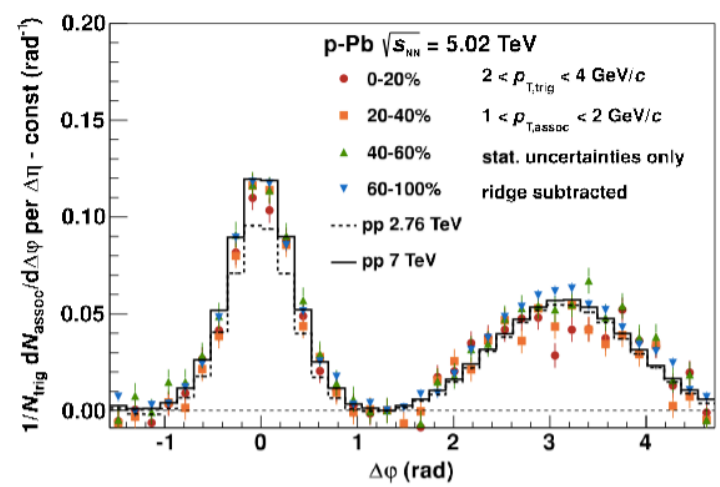
But for very high-p_T jets

No leading hadron suppression



But in MB events

Dihadron unmodified



How is "v_2" subtracted

What is the expectation of parton energy loss as system size decreases?

$$\Delta E \sim \alpha_s(T)\hat{q}(T)L^2$$

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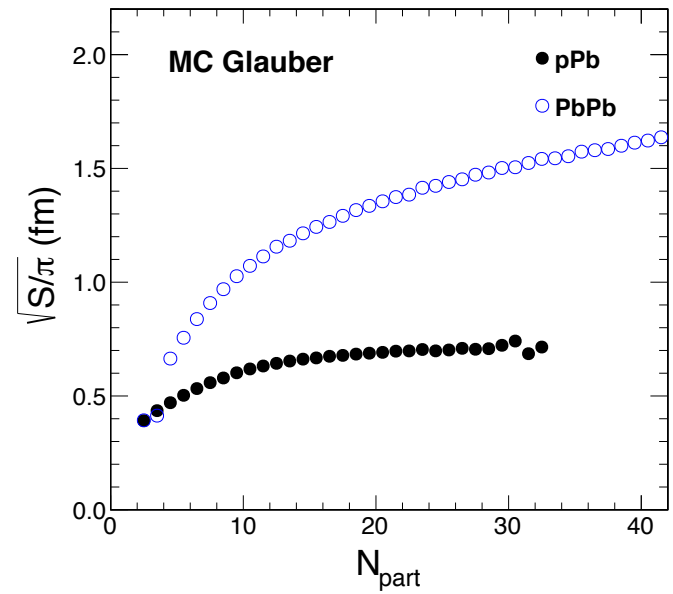
In small system

L ↓ but T ↑ so $\hat{q} \sim T^3$ ↑

Who wins?



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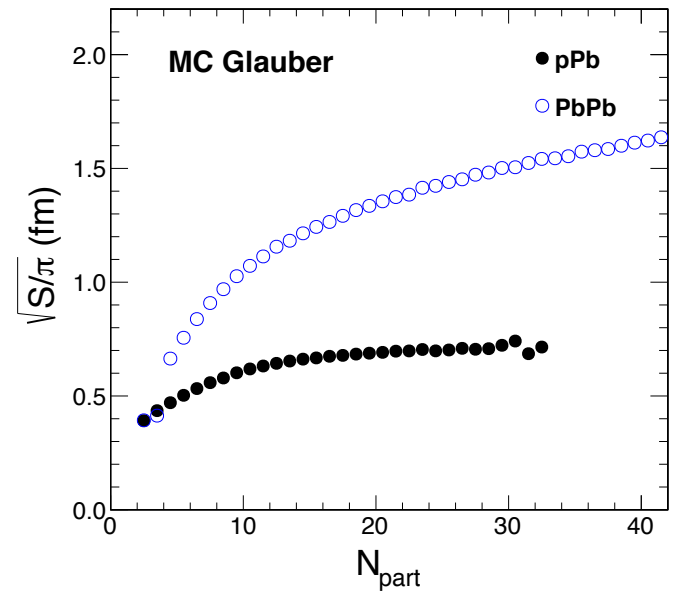

$$\begin{cases} s \sim T^3 \\ \hat{q} \sim T^3 \\ s \sim \dfrac{N_{trk}}{\pi R^2} \end{cases}$$

fix N_{trk} →

$$\boxed{\hat{q} \sim \frac{1}{R^2} \text{ or } \frac{1}{<L>^2}}$$

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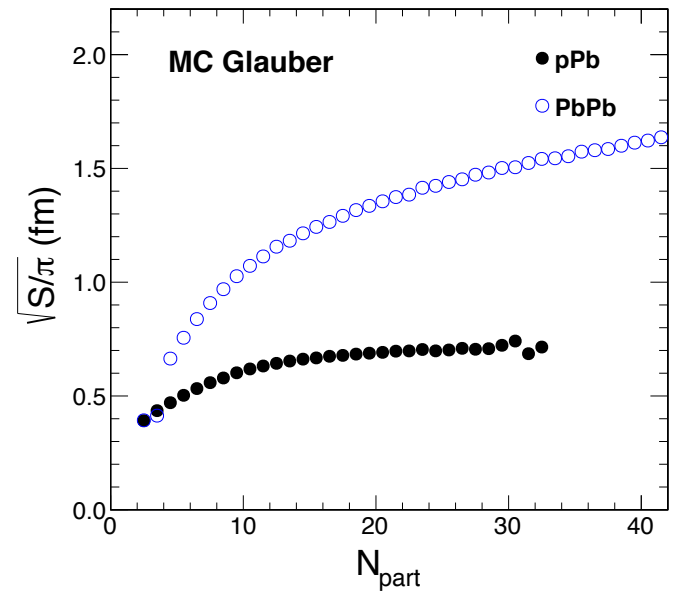

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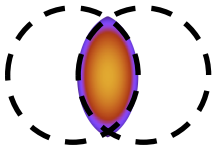
ΔE nearly indep. of systems size, if fixing N_{trk}?!

Why we are not seeing parton energy loss in pPb?



50-70%

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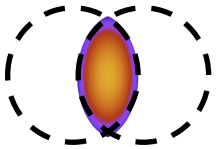

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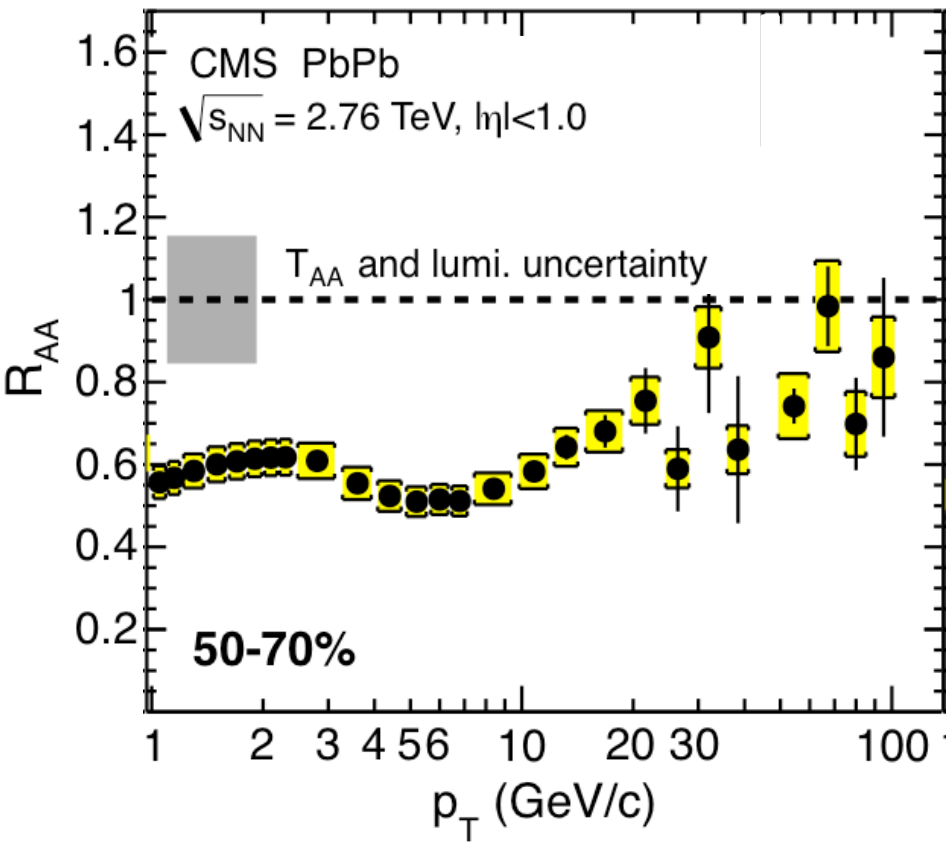

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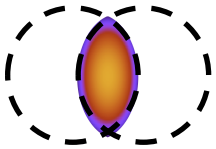

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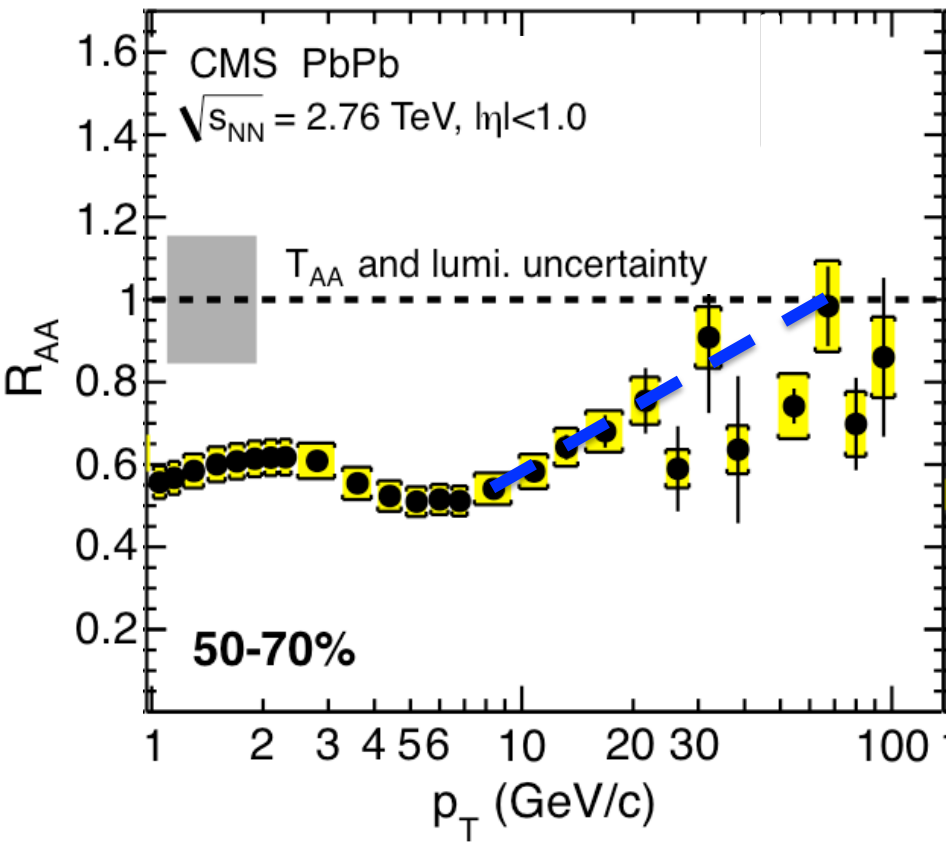
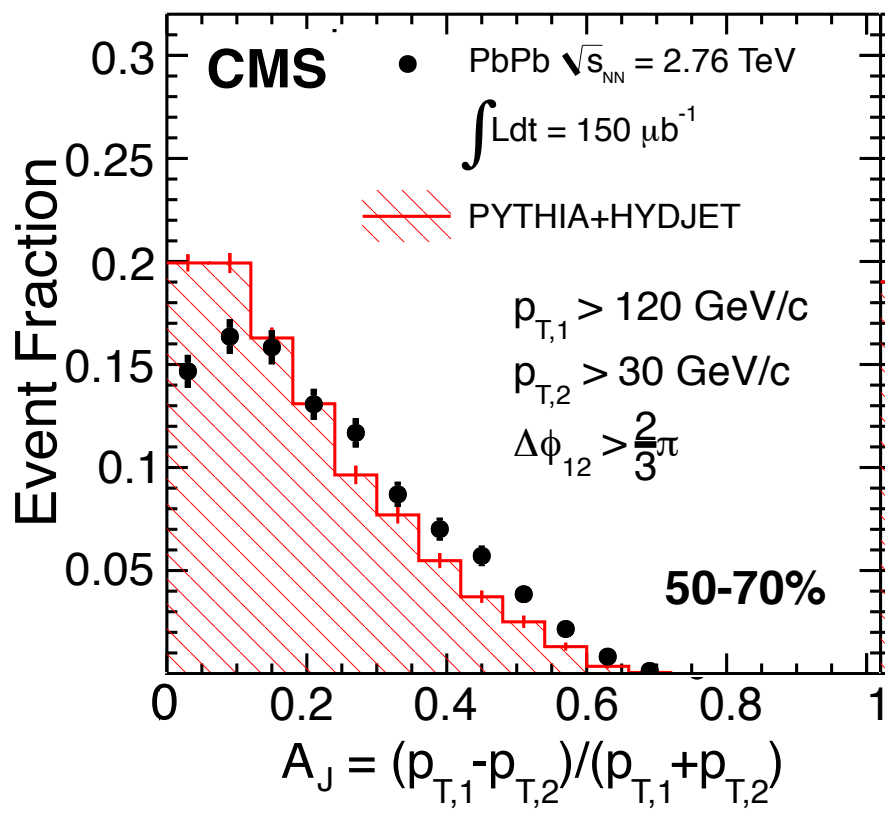
Suppression going away at high p_T





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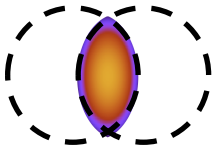

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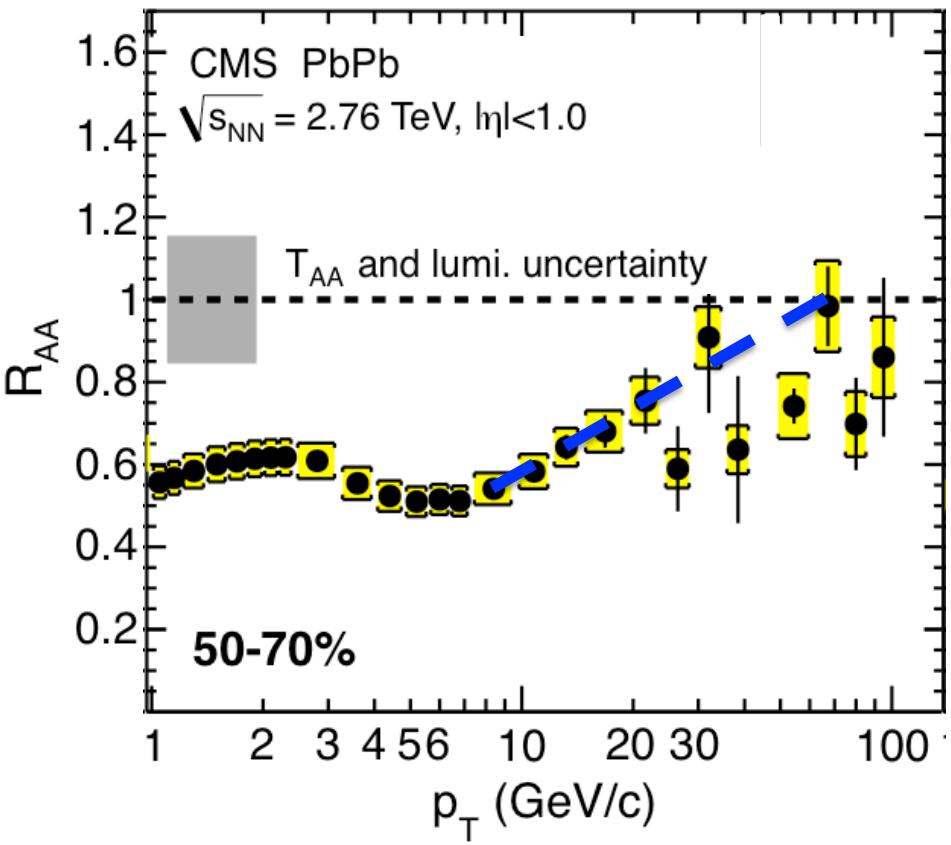

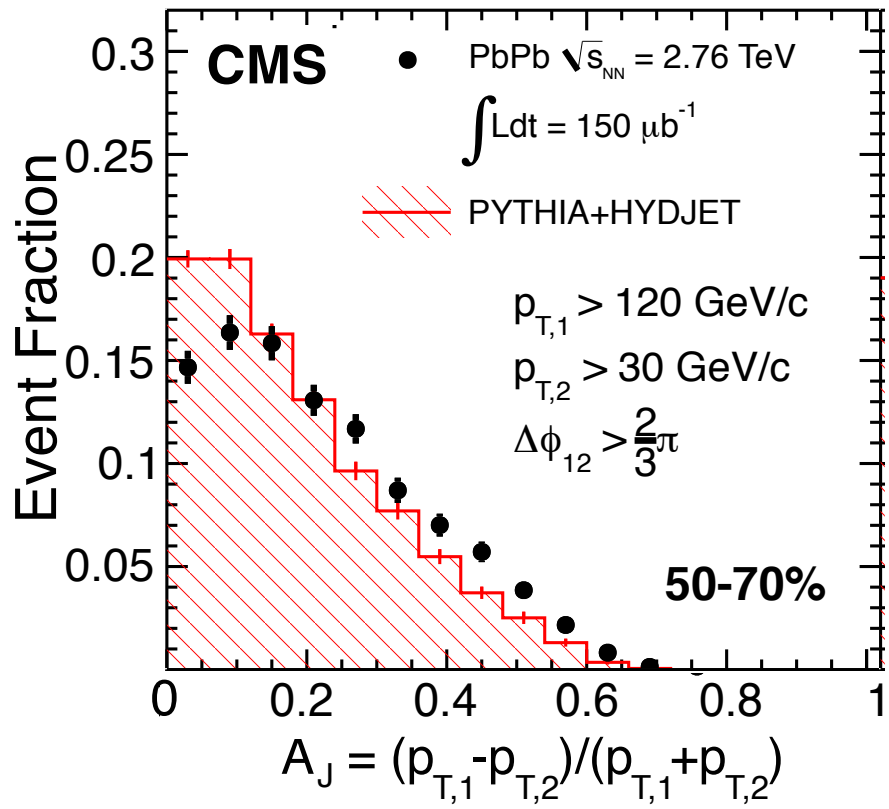

Still sizable suppression at p_T ~ 8-15 GeV/c

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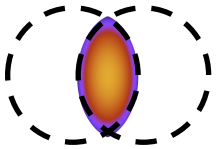
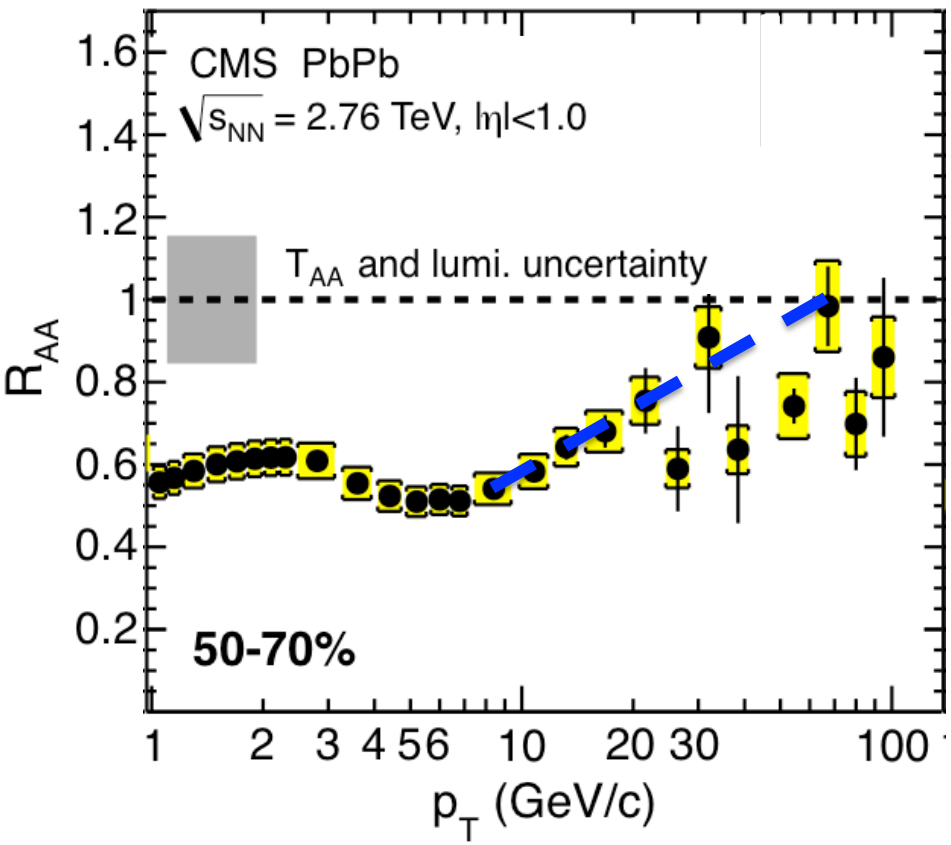
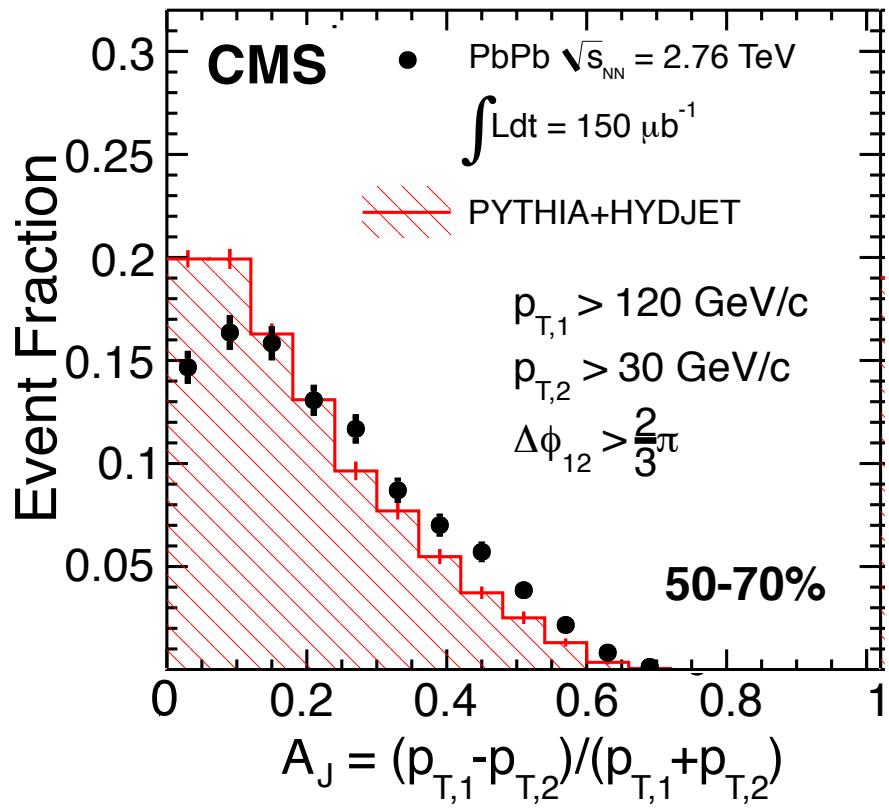


50-70%

Suppression going away at high p_T

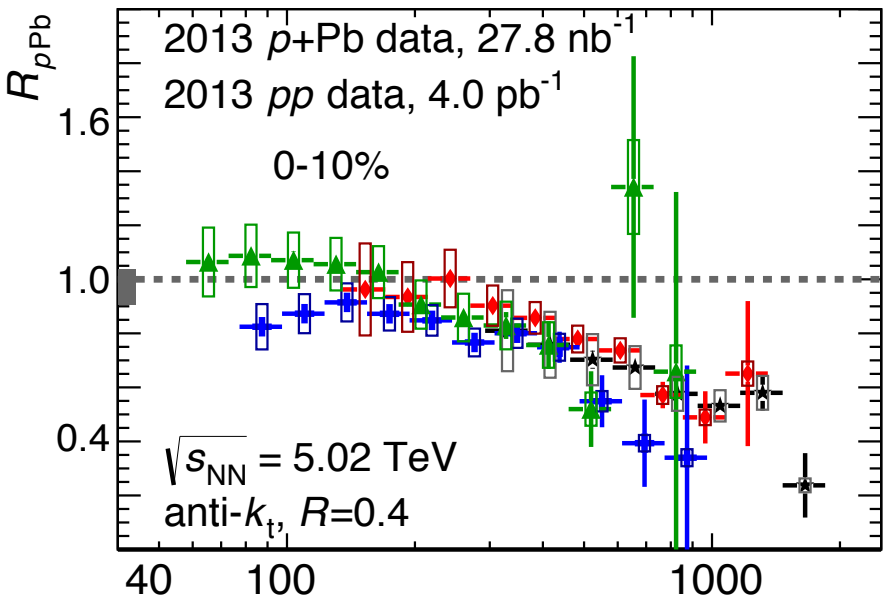


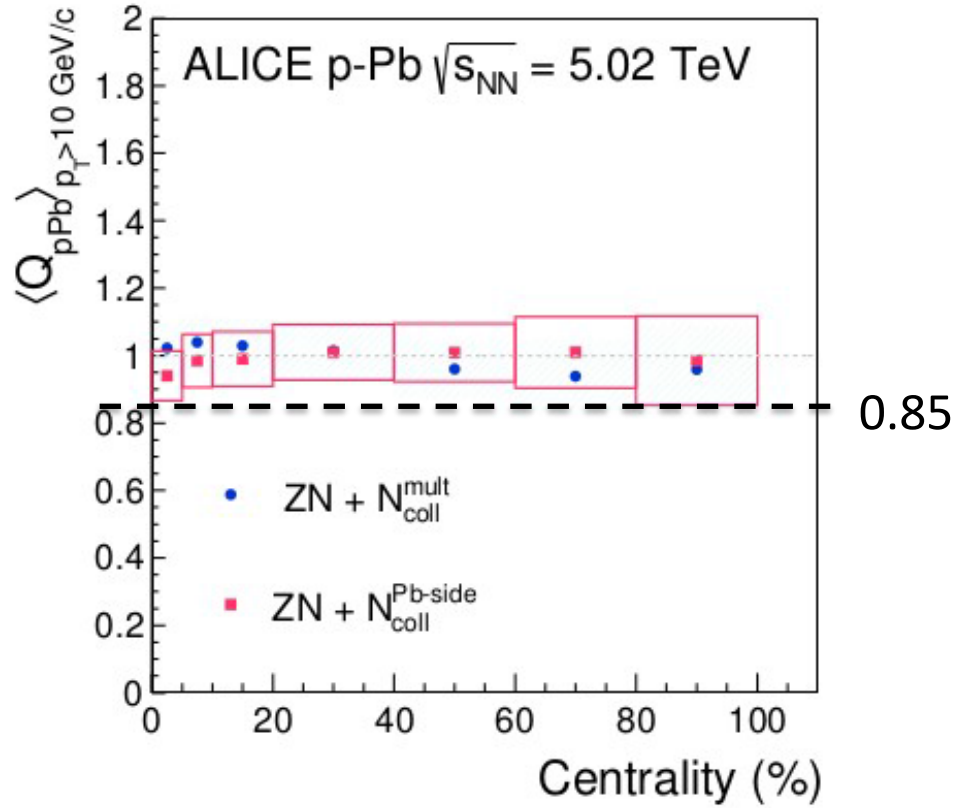


Still sizable suppression at p_T ~ 8-15 GeV/c

Best shot: high-multiplicity pPb + intermediate p_T

What is the in-vacuum reference?





Jets in pPb modified but not like quenching

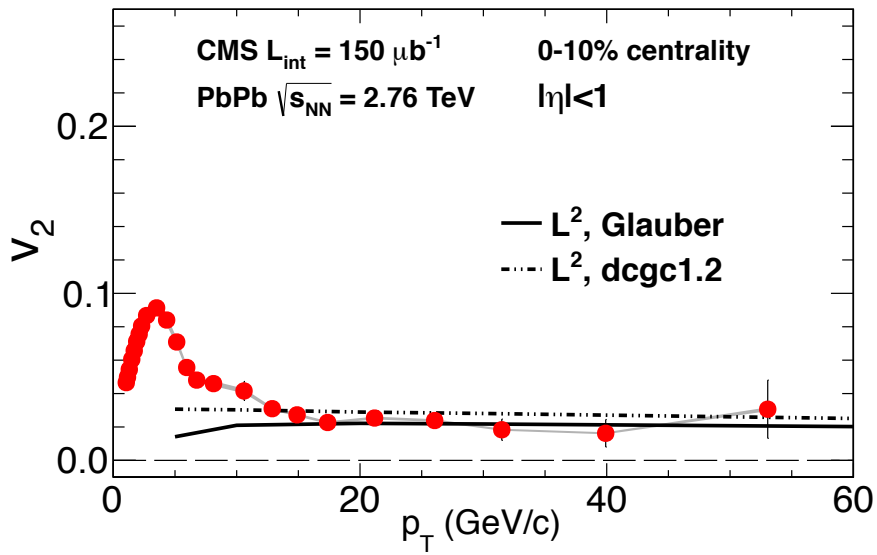
(color fluctuations? energy conservation?)

How solid the assumption is?

Major progress on pPb centrality but still pretty tough

Jet/hadron anisotropy at high p_T

$$v_2 \sim \frac{Y^{in} - Y^{out}}{Y^{in} + Y^{out}}$$ self-referenced!



Jet/hadron anisotropy at high p_T

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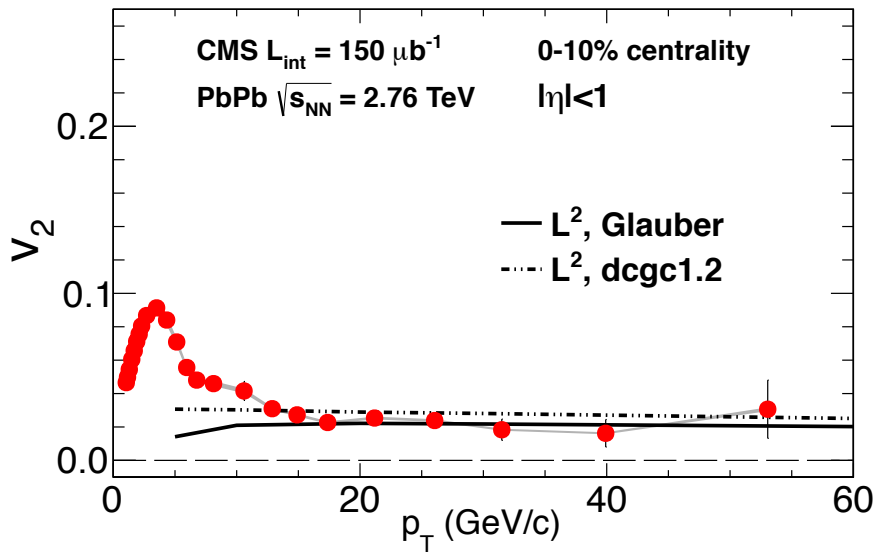
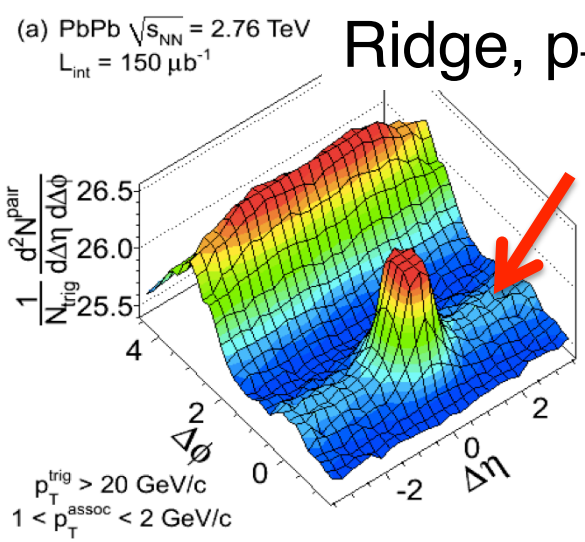
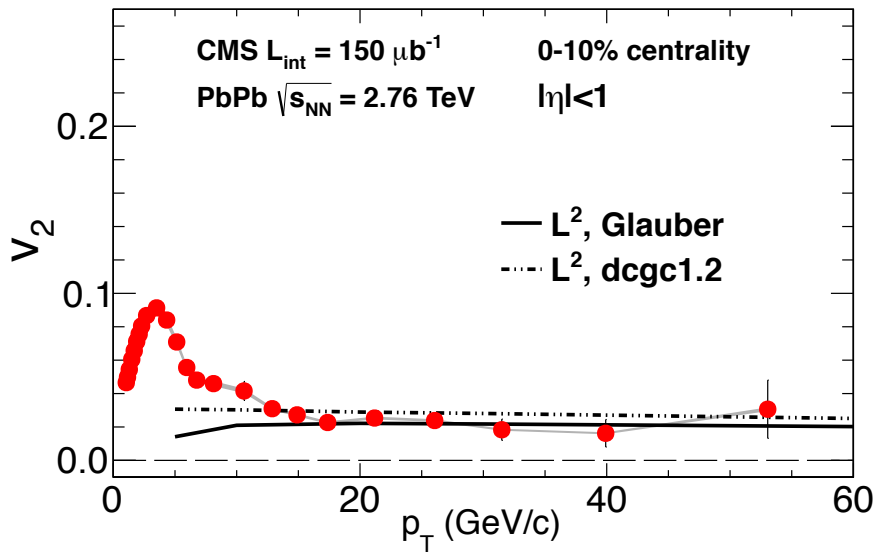




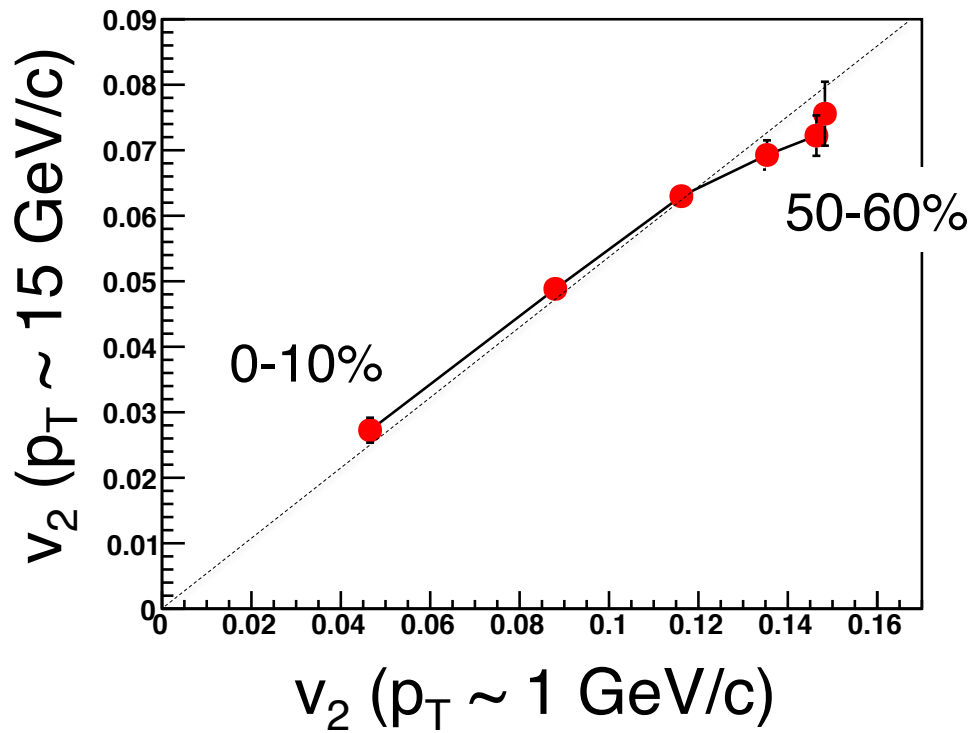
Ridge, $p_T > 20$ GeV

Jet/hadron anisotropy at high p_T

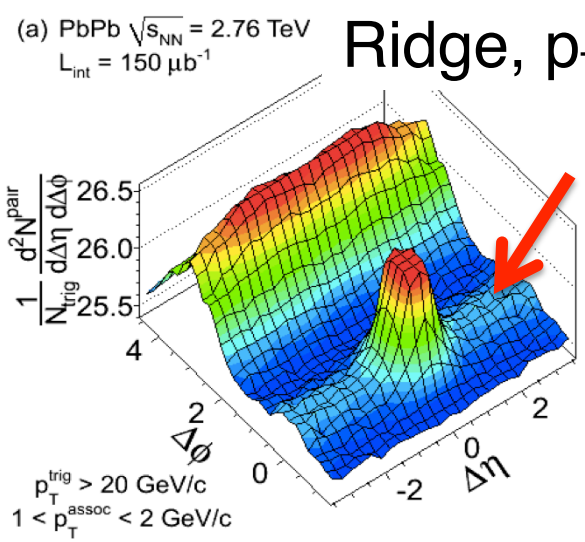
$$v_2 \sim \frac{Y^{in} - Y^{out}}{Y^{in} + Y^{out}}$$ self-referenced!



"soft" vs "hard" v_2



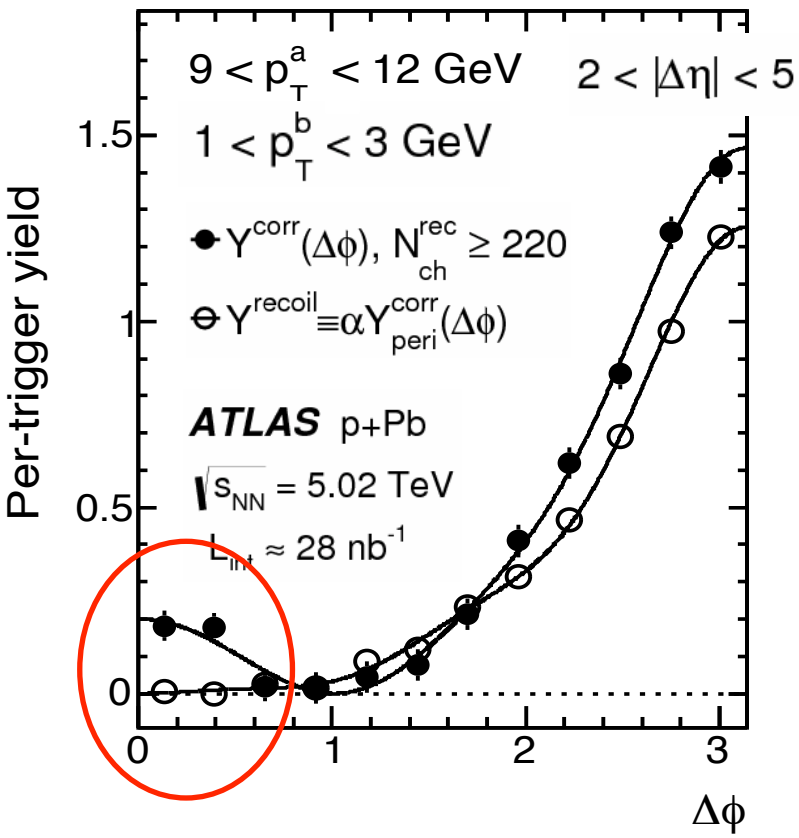
Ridge, $p_T > 20$ GeV

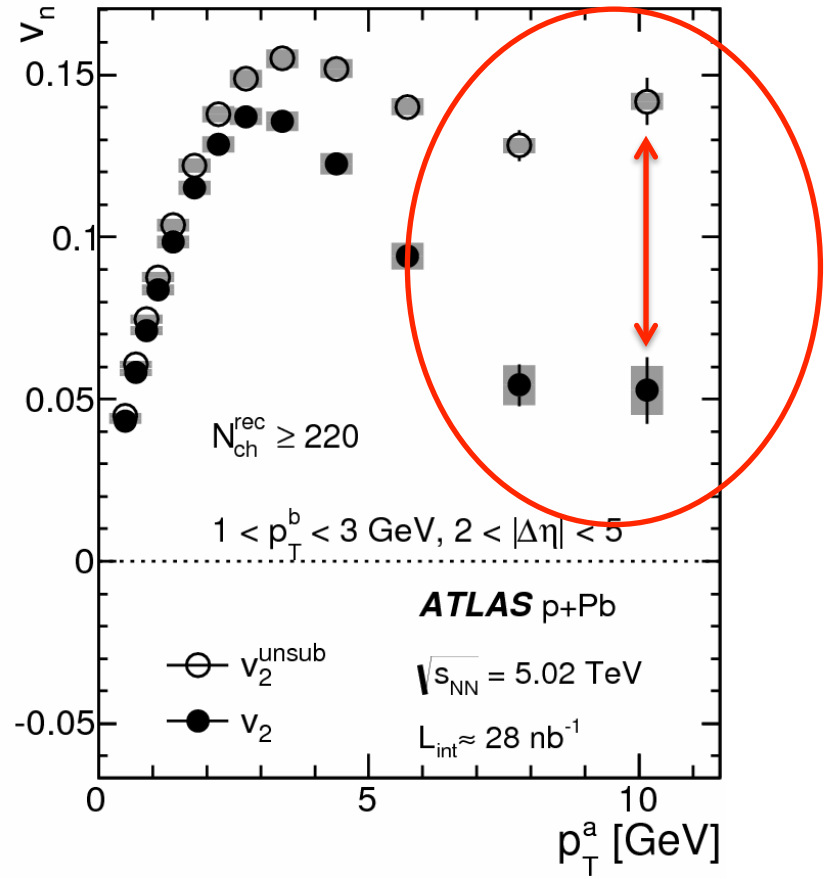


In PbPb, high-p_T v_2 related to the geometry

Predictions in pPb needed!

Ridge in pPb persists up to at least several GeV/c



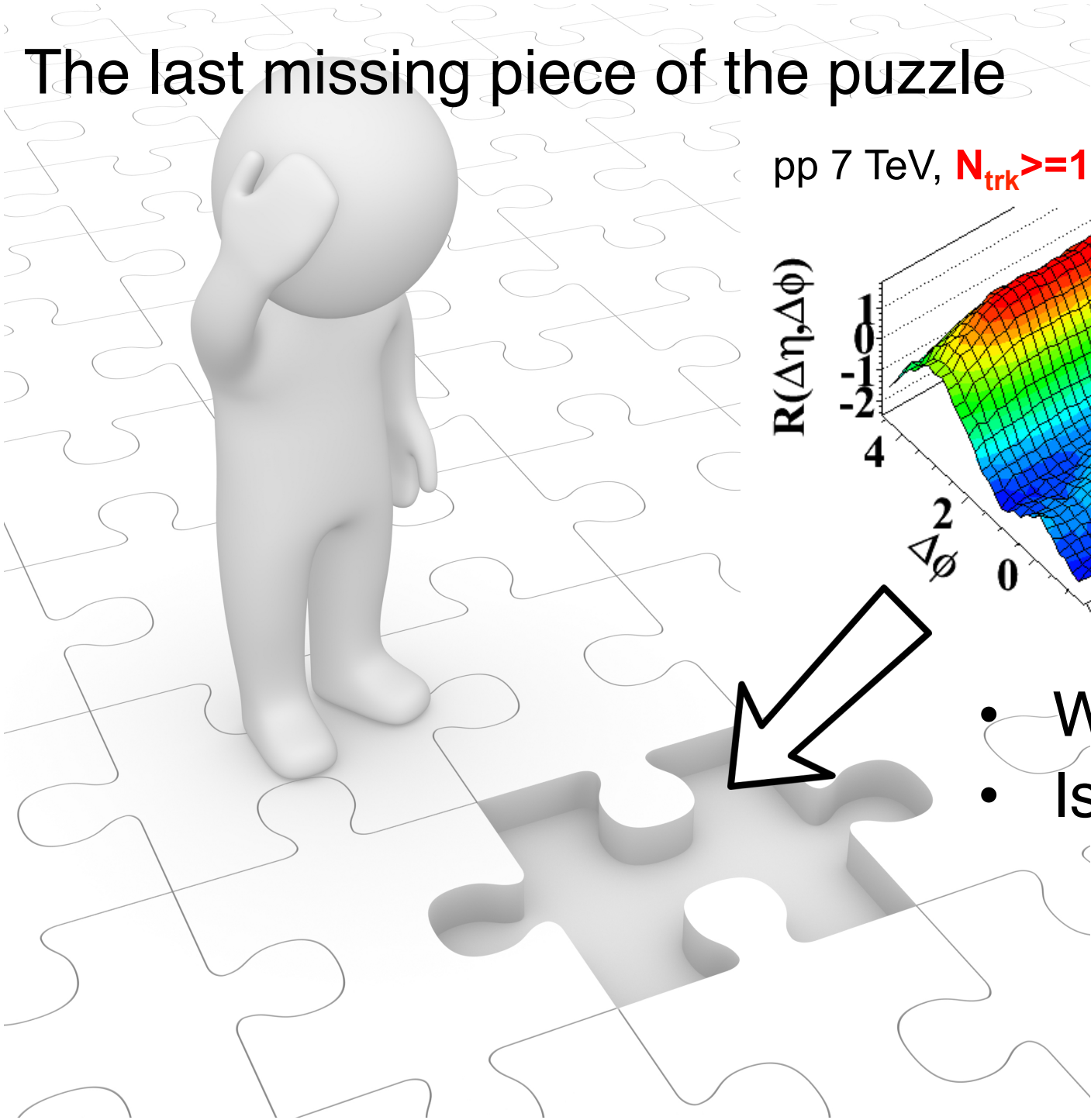


Sizeable $v_2 \sim 5\%$ (after a large subtraction)

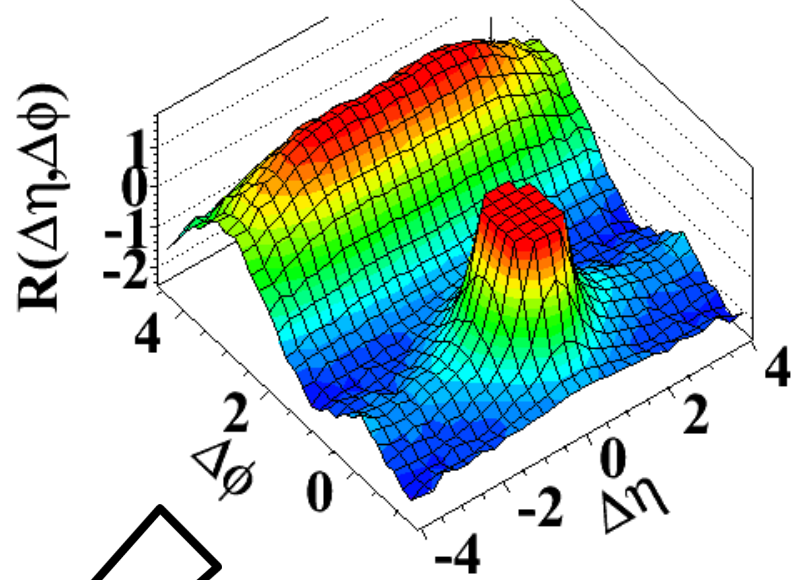
Multiparticle correlations to test collectivity of high p_T particles

Ridge in pp

The last missing piece of the puzzle



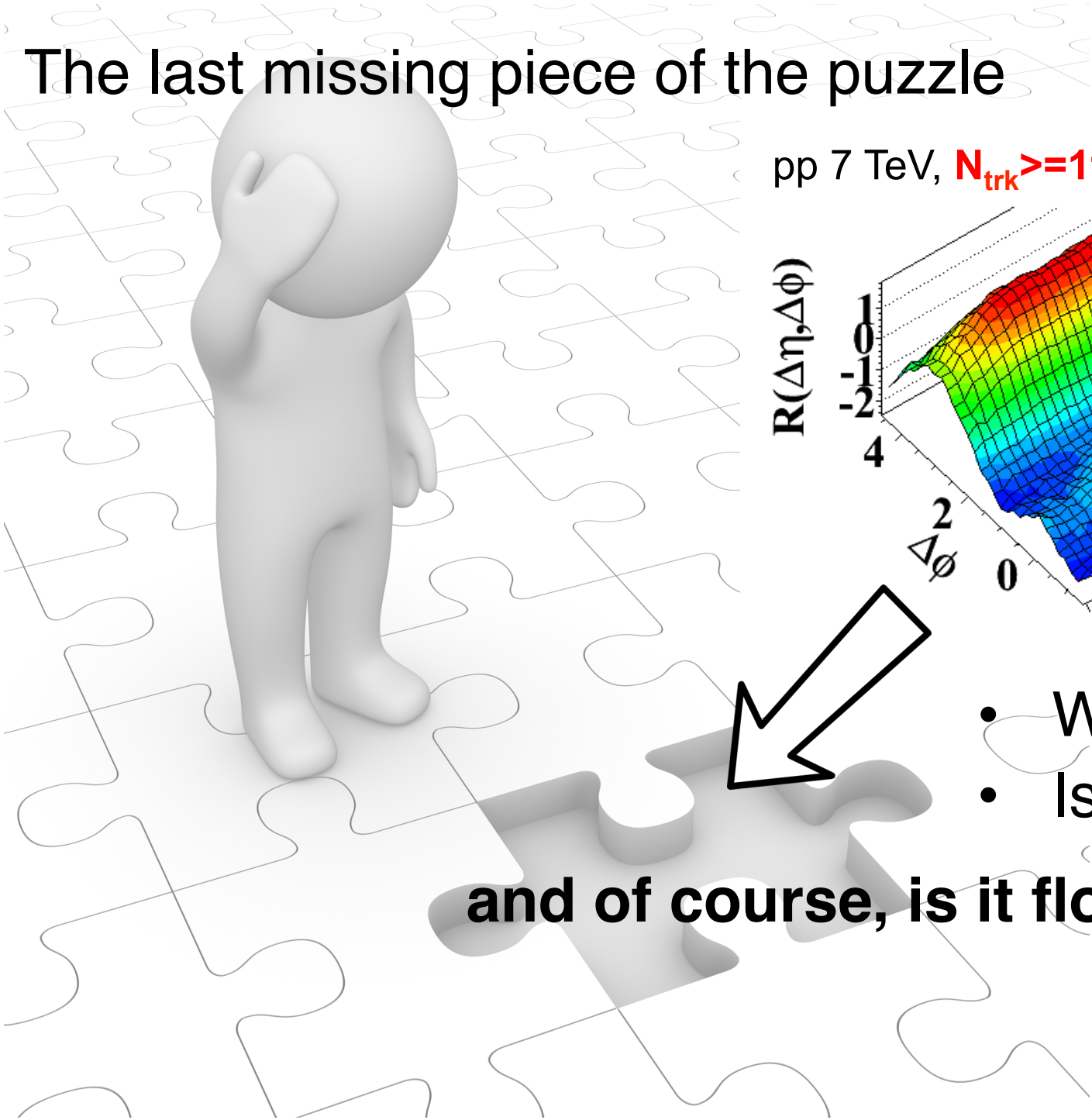
pp 7 TeV, N_{trk}>=110 1<p_T<3 GeV/c



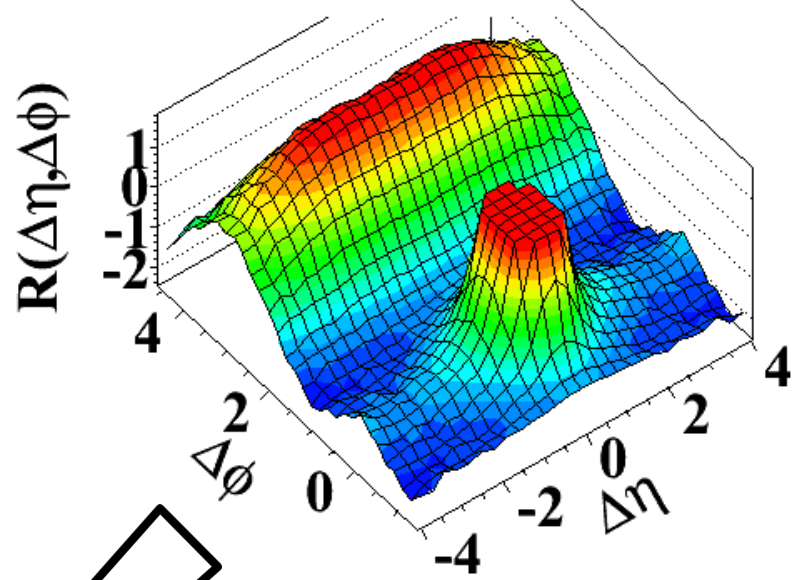
- What is the v_n?
- Is it collective?

Ridge in pp

The last missing piece of the puzzle



pp 7 TeV, N_{trk}>=110 $1<p_T<3$ GeV/c

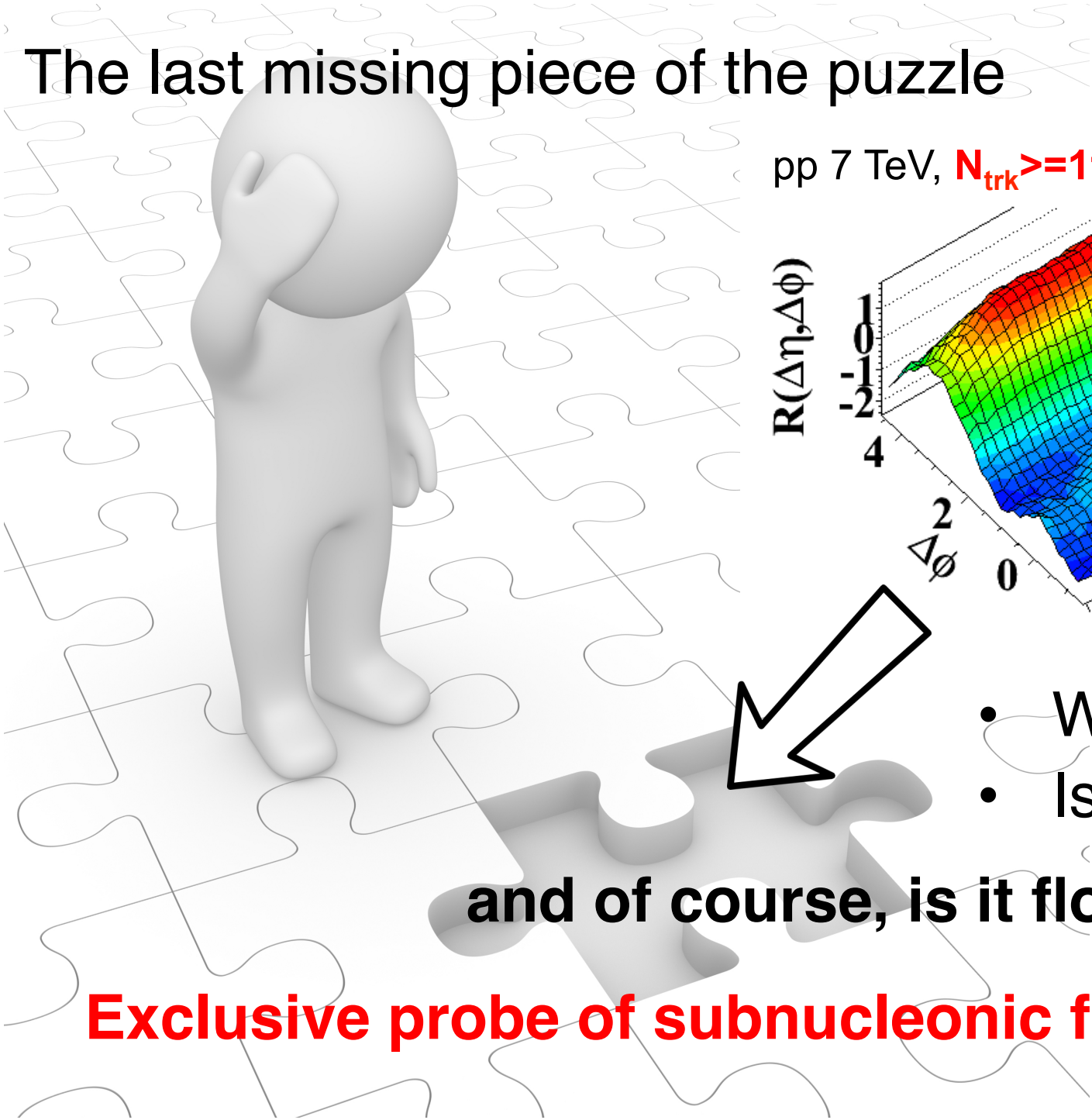


- What is the v_n?
- Is it collective?

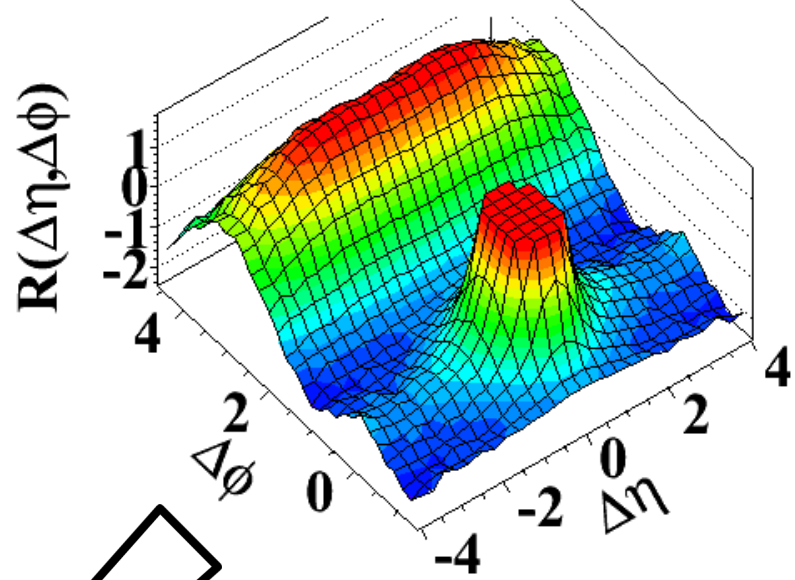
and of course, is it flow, CGC, or …?

Ridge in pp

The last missing piece of the puzzle



pp 7 TeV, N_{trk}>=110 $1<p_T<3$ GeV/c



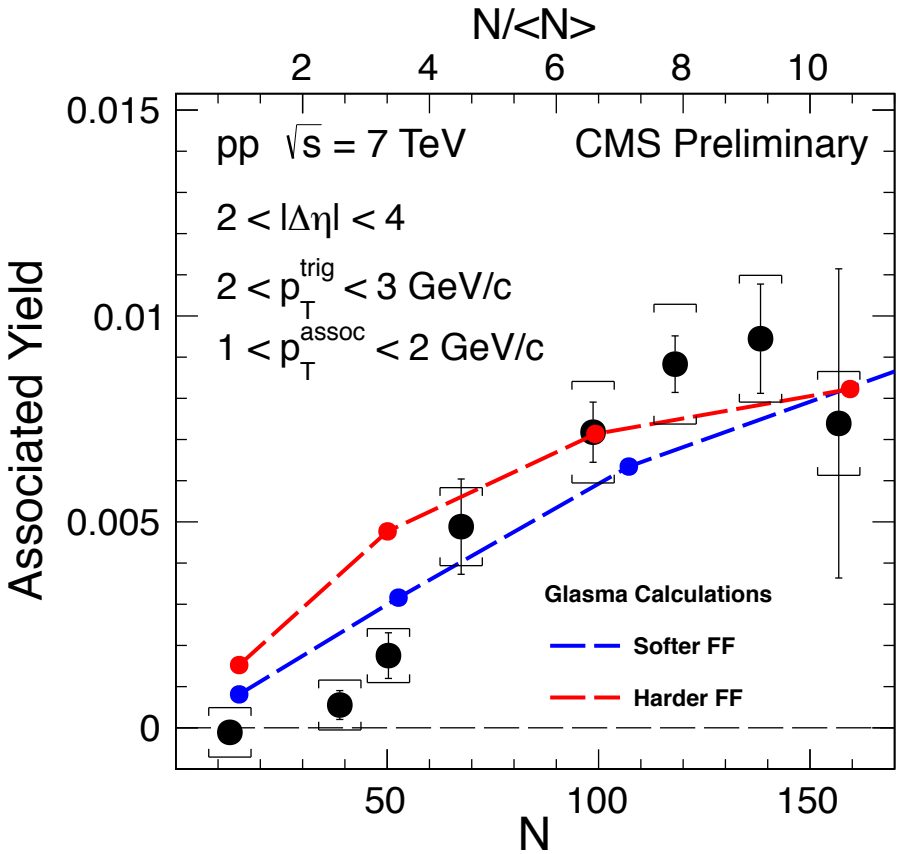
- What is the v_n?
- Is it collective?

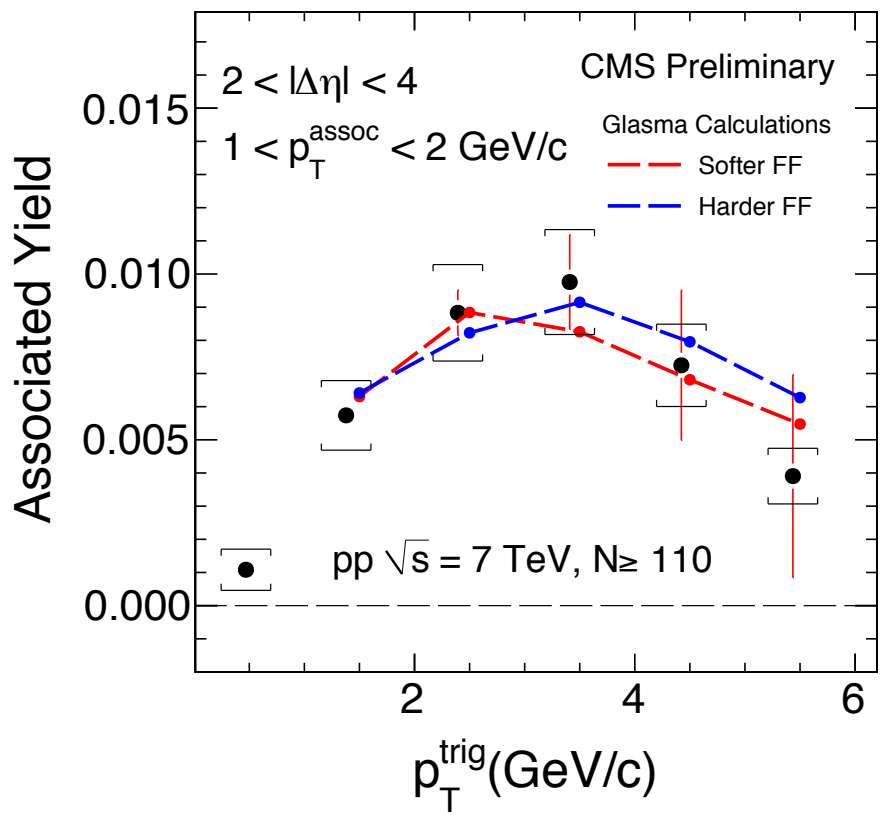
and of course, is it flow, CGC, or …?

Exclusive probe of subnucleonic fluctuations!

Ridge in pp

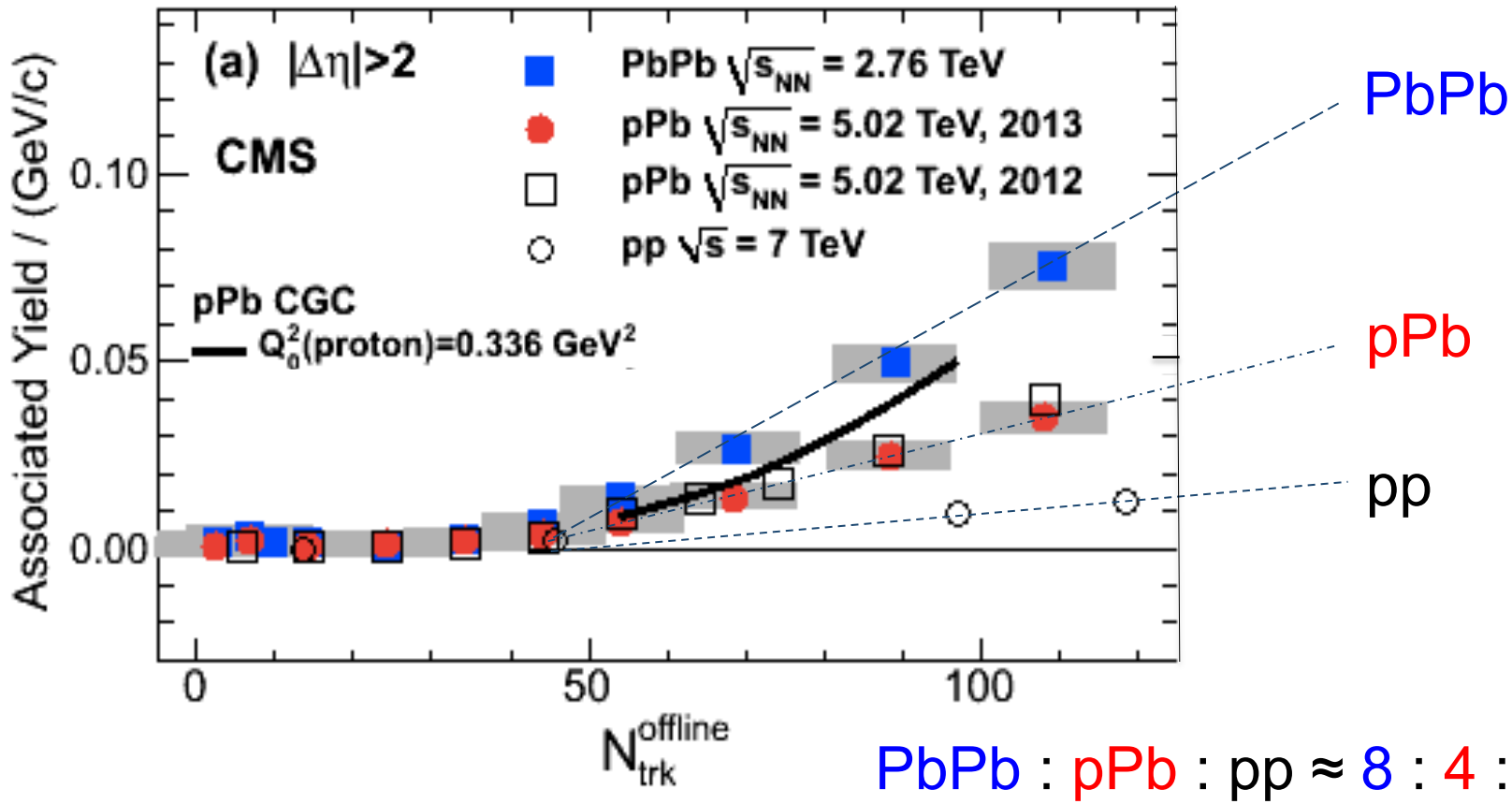
CMS PAS HIN-11-006
arXiv: 1206.0148





Intriguing N_{trk} and p_T dependence of ridge yield!

Ridge in **pp**

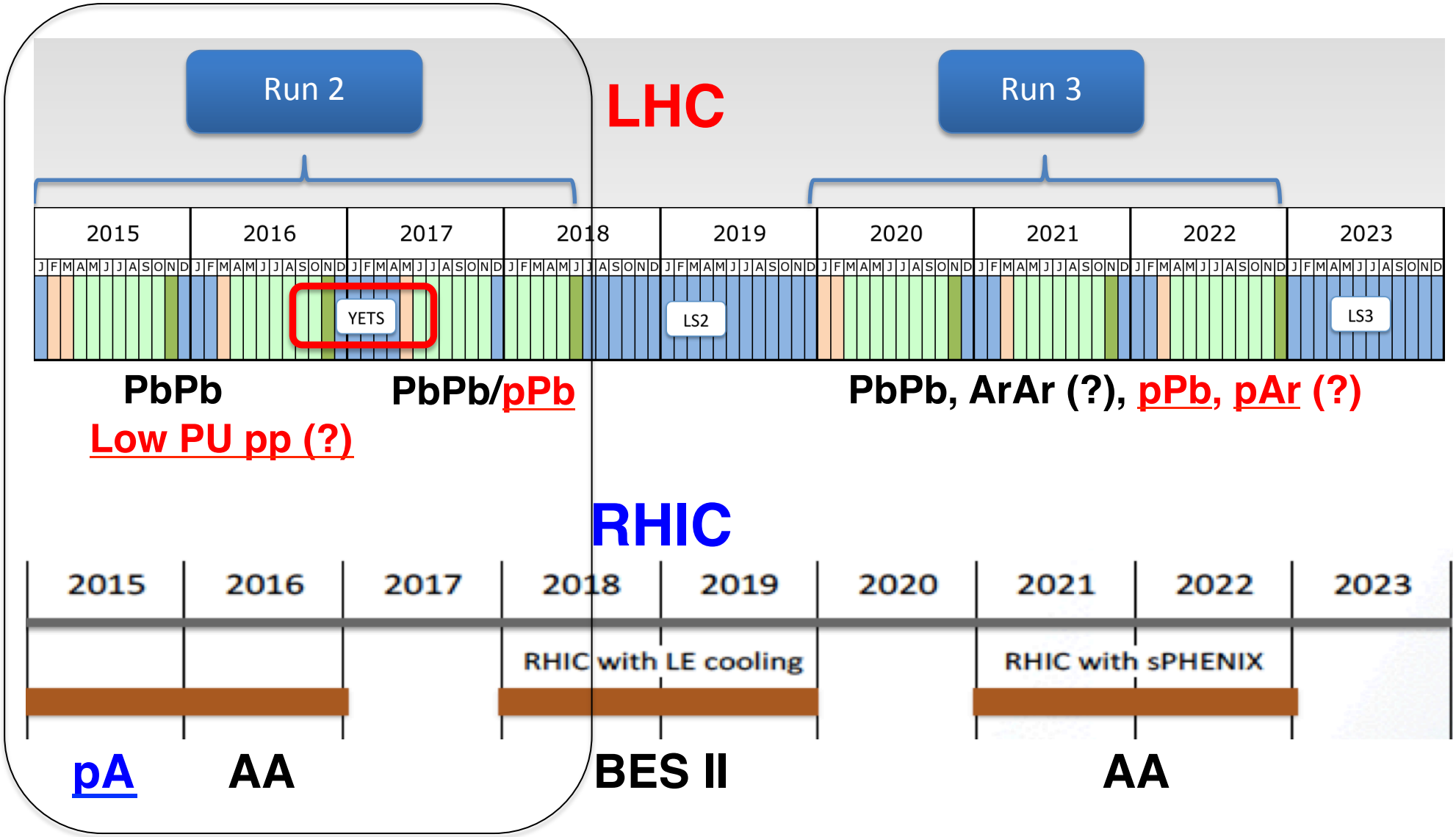


PbPb : pPb : pp ≈ 8 : 4 : 1

Need more low PU pp (**~ 50 pb^{-1}**)

- *Higher N_{trk} (≥150) ➔ away-side jets less dominant*
- *Multiparticle correlations*

RHIC and the LHC schedules

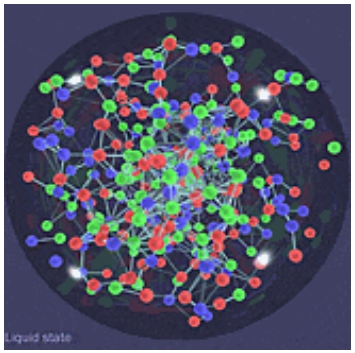


pA program next 2-3 yrs will provide important guide

The big questions from small systems

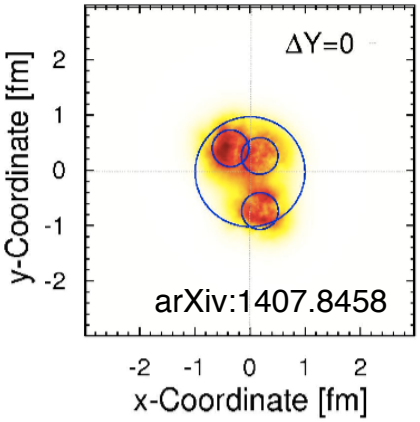
"Perfect fluid" paradigm

- Consolidate it (or destroy?)



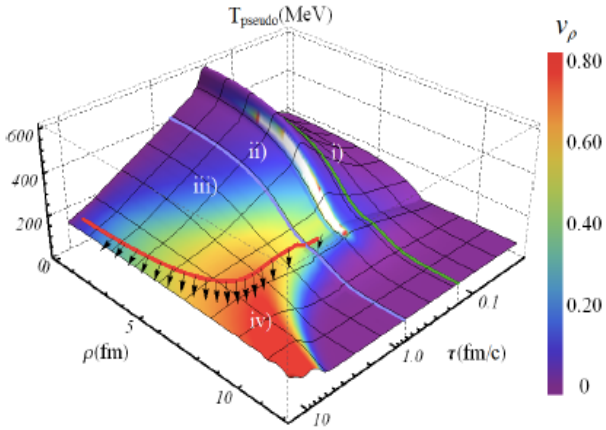
Subnucleonic fluctuations

– Test fundamental QCD



Pre-equilibruim flow

– Dynamics of thermalization



Event selection and centrality bias in pA collisions (*)

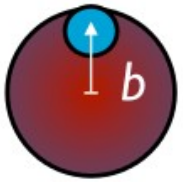
Constantin Loizides
(LBNL)
04 March 2015

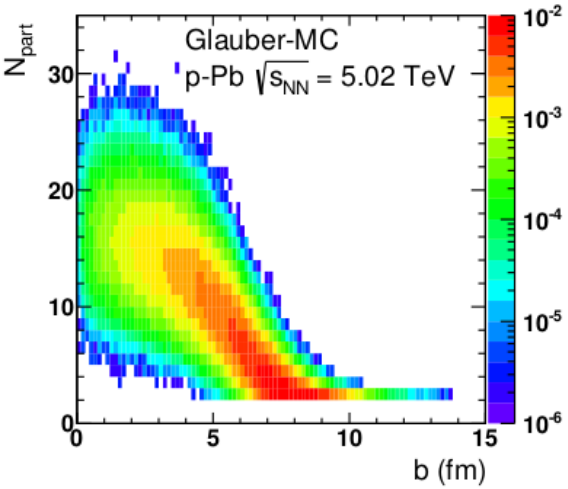
(*) based on talk by A.Morsch at IS2014
and ALICE paper arXiv:1412.6828
on pPb centrality

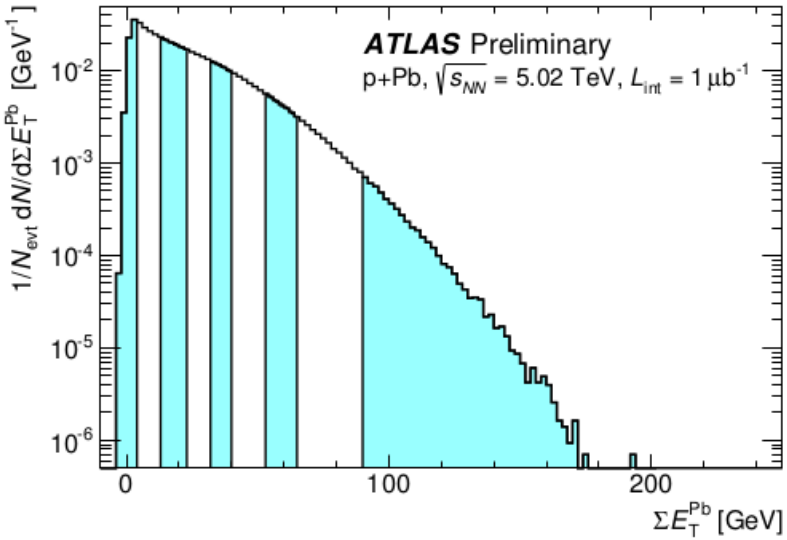
RBRC workshop on "Collectivity in small colliding systems"

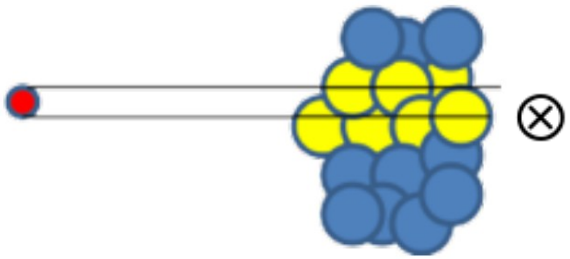
2 Basic procedure

- Impact parameter not observable
  - And for small systems only weakly correlated to number of participants (N_{part})
- Classify events in terms of event activity (or centrality estimator E)
  - E should vary monotonously with number of participants
  - Multiplicity, energy, slow neutron energy
  - Order as percentile of cross section
- Establish relation to Glauber model parameters (N_{part}, N_{coll}) via particle production model

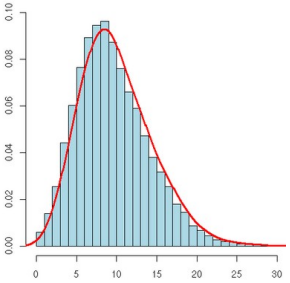






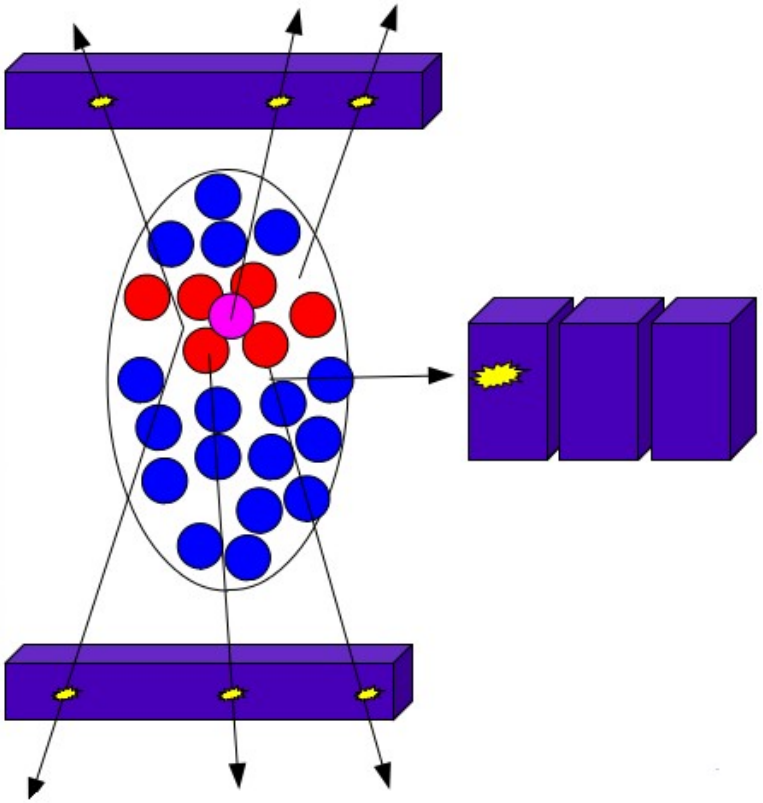


Glauber MC: $\pi(N_{col})$

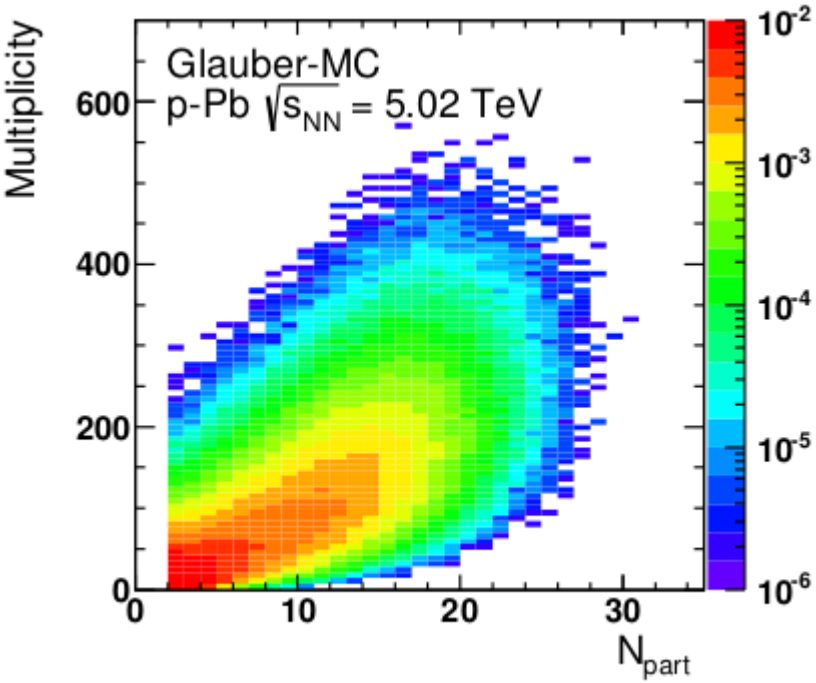


Model: $P(E \mid N_{col})$

3 Essential requirements



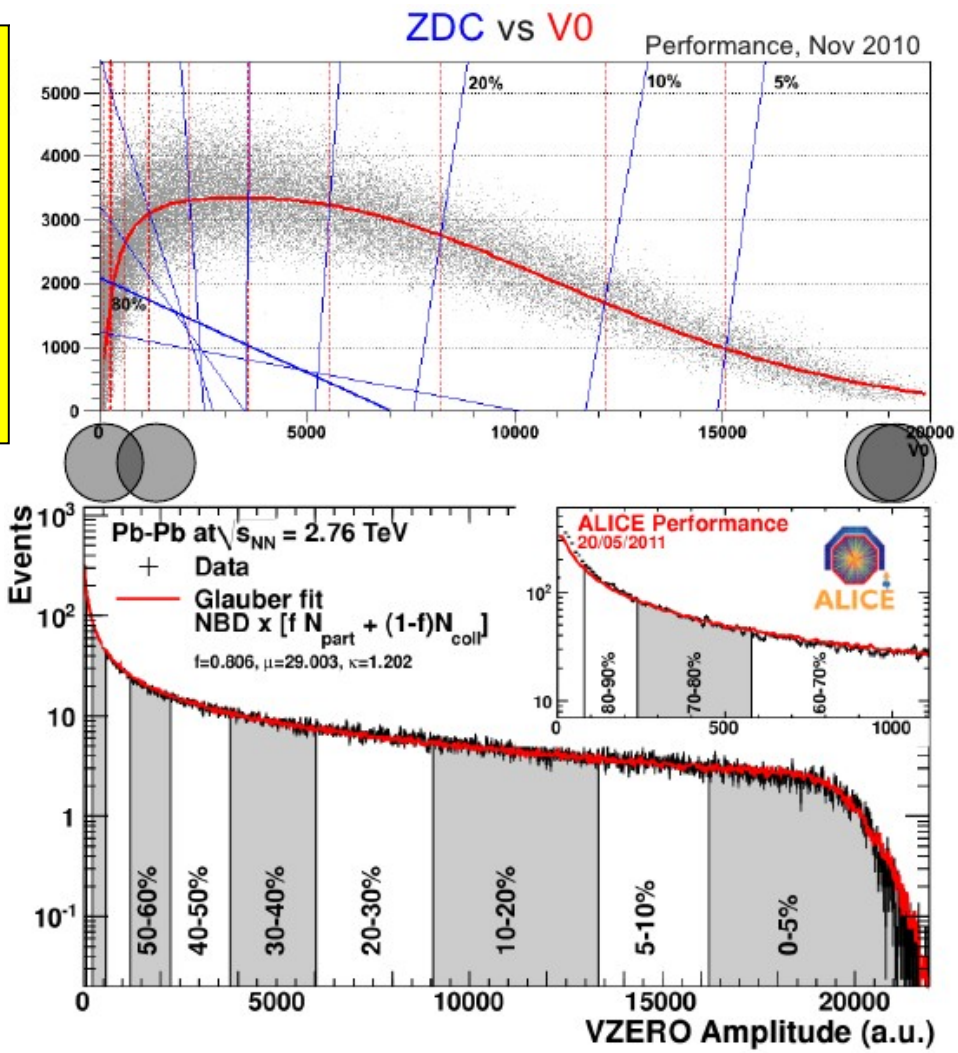
- Demonstrate correlation of measurement to collision geometry
  - Via correlation of observables that are causally disconnected after collision
- Demonstrate completeness
  - Are there other relevant geometry parameters that are biased by the selection wrt minimum bias?
  - What are their possible influence on centrality dependent measurements?
    - Importance for p(d)A: small dynamic range leads to large fluctuations



4 Example large system: ALICE Pb+Pb

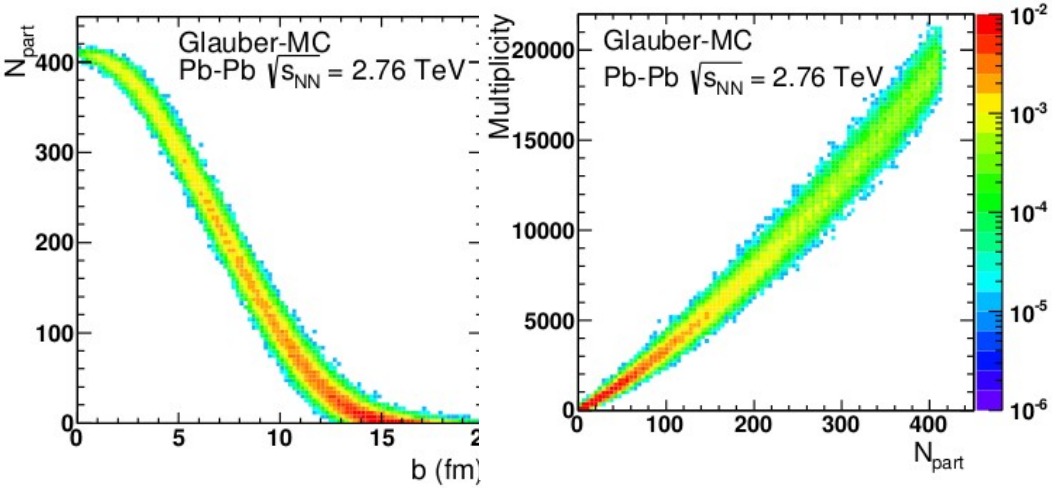
Miller et al., Ann. Rev. Nucl. Part. Sci 57 (2007) 205
ALICE, Phys. Rev. C 88 (2013) 044909

Forward neutrons



Charged hadrons η~3

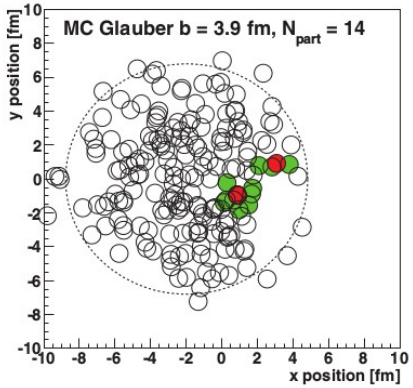
- Correlate particle yields from disconnected parts of phase space
  - Correlation arises from common dependence on collision impact parameter

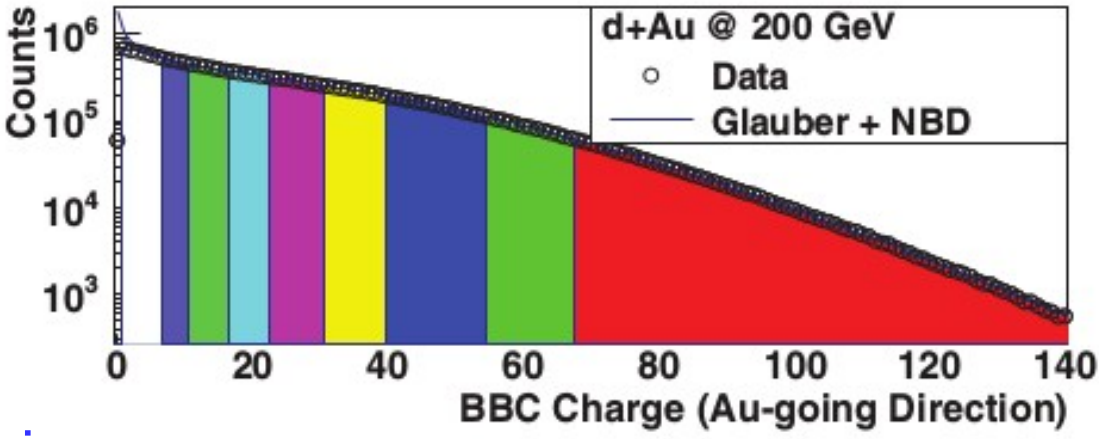


5 Example small system: PHENIX d+Au

PHENIX, PRC 90 (2014) 034902

- Probability for Ncoll binary collisions $\pi(N_{coll})$ from Glauber





- Charge distribution for one collision (Negative Binomial)

$$P_{\mathrm{NBD}}(n;\mu,k) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{(\frac{\mu}{k})^n}{(\frac{\mu}{k+1})^{n+k}}$$

- For N_{coll} collisions, assume <BBC>~<Ncoll>

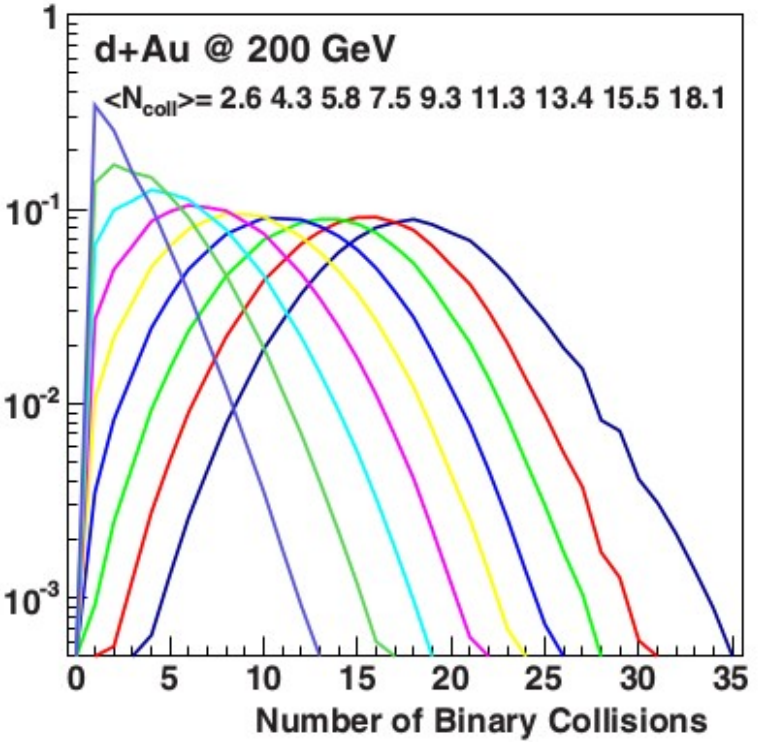
$$P(BBC|N_{\mathrm{coll}}) = P_{\mathrm{NBD}}(BBC; N_{\mathrm{coll}} \times \mu, N_{\mathrm{coll}} \times k)$$

- Fit to measured distribution

$$P_{\mathrm{BBC}}(BBC) = \Sigma_1^{N_{\mathrm{coll}}^{\max}} \pi(N_{\mathrm{coll}}) P(BBC|N_{\mathrm{coll}})$$

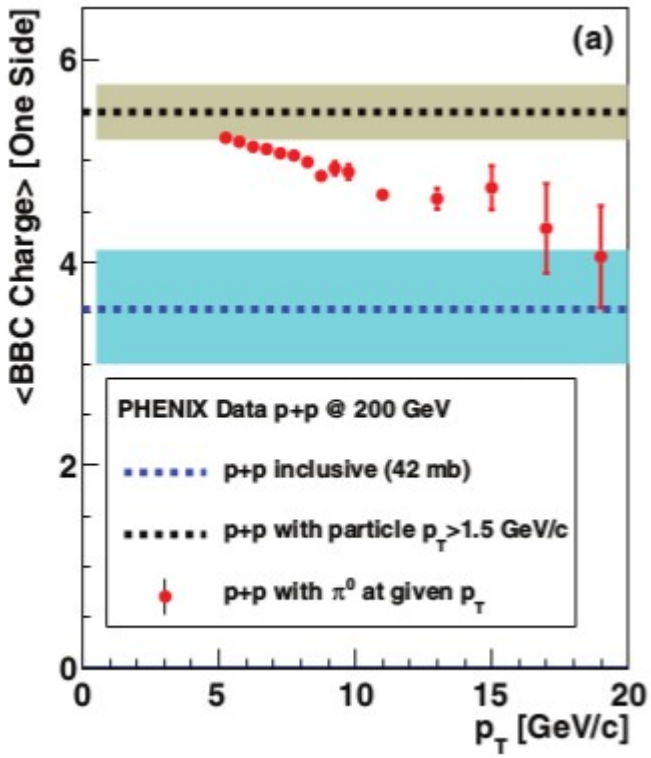
- For fixed k and μ

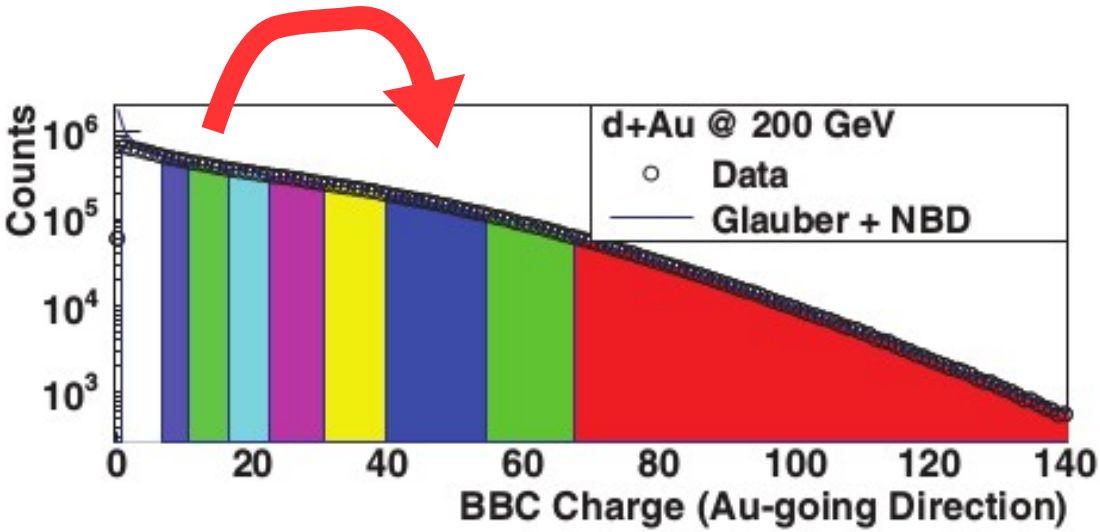
$$P(N_{\mathrm{coll}}|BBC) = P(BBC|N_{\mathrm{coll}})\pi(N_{\mathrm{coll}})/P(BBC)$$



6 High-p_T bias factor correction

PHENIX, PRC 90 (2014) 034902

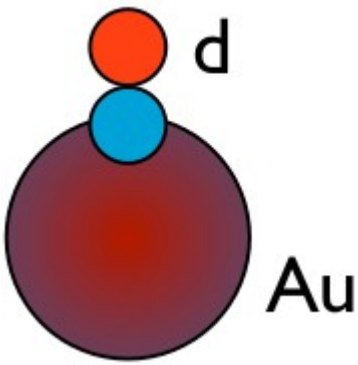


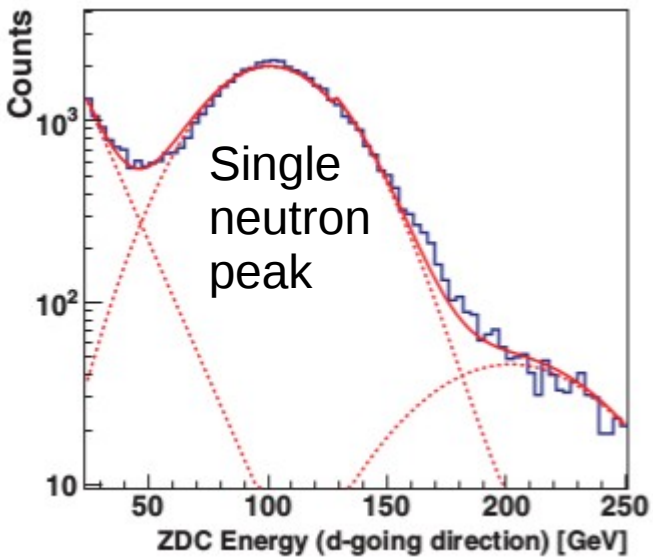


- Presence of high p_T particle at central rapidity increases BBC charge
- Quantify bias using pp data coupled with the Glauber model
  - And check with HIJING

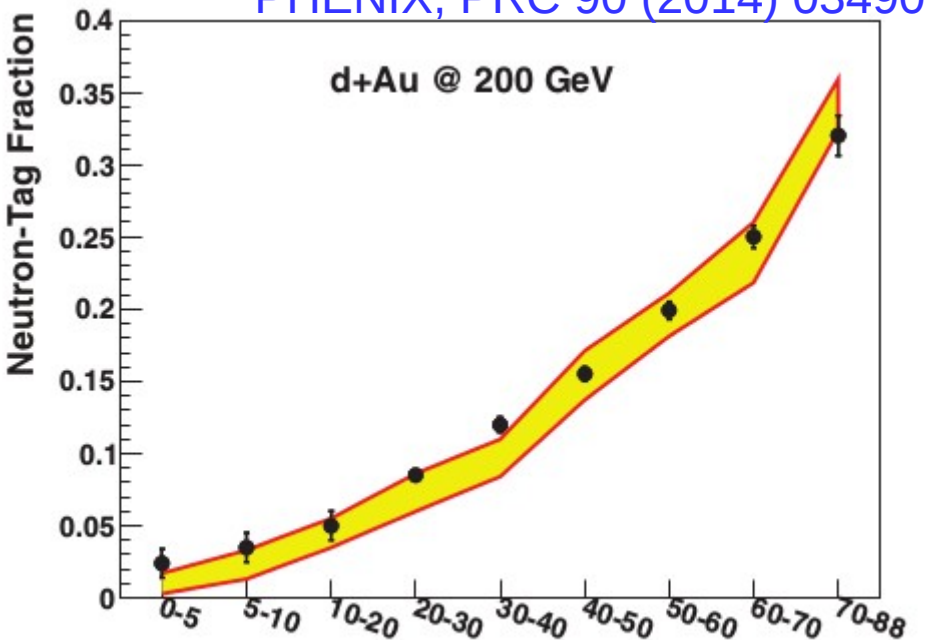
Centrality (%)	Glauber + NBD	HIJING $1 \leqslant p_T < 5$
0–20	0.94 ± 0.01	0.951 ± 0.001
20–40	1.00 ± 0.01	0.996 ± 0.001
40–60	1.03 ± 0.02	1.010 ± 0.001
60–88	1.03 ± 0.06	1.030 ± 0.001

7 Correlation with d-dissociation

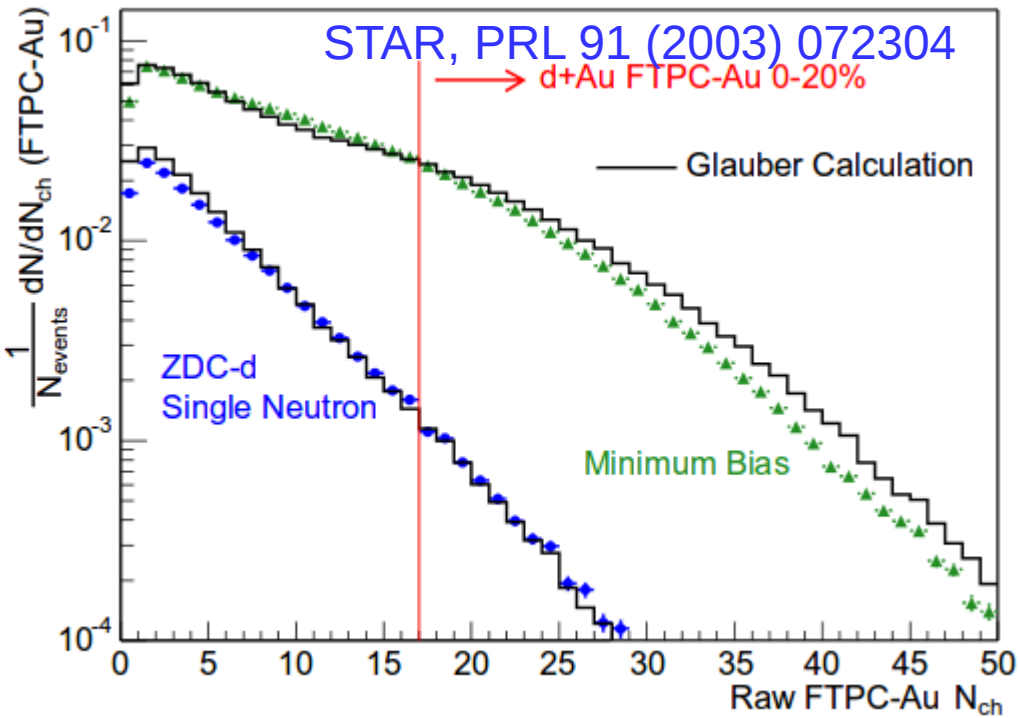




PHENIX, PRC 90 (2014) 034902



STAR, PRL 91 (2003) 072304



- Essential cross check
- Establish unambiguous relation of centrality estimator to collision geometry
- Note: d-going ZDC energy not a centrality estimator

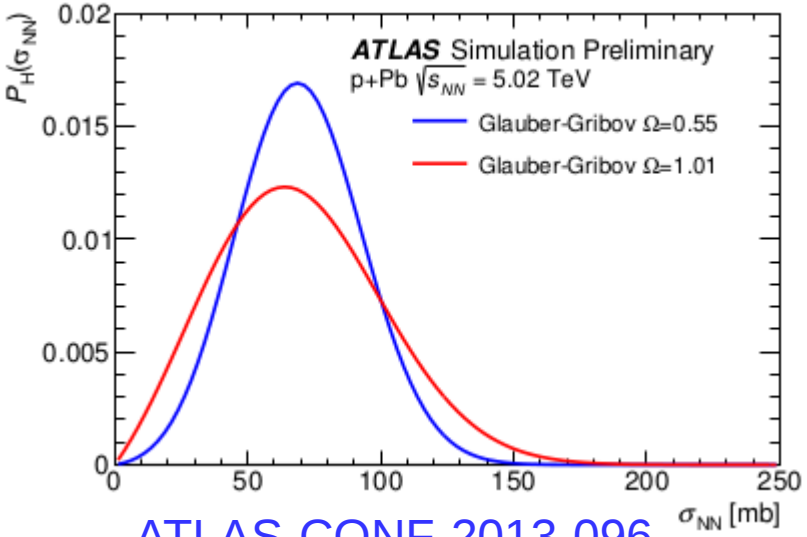
8 Remarks

- Need Glauber fit with specific particle production model because of defining centrality and determining N_{coll} from the same estimator
- Biases can be consequence of
  - Correlations of collision parameters other than N_{part}
  - Correlations induced after collision (eg. jet fragmentation in the example of PHENIX)
- Bias corrections are not necessarily corrections of N_{coll}
  - Physics origin has to be understood

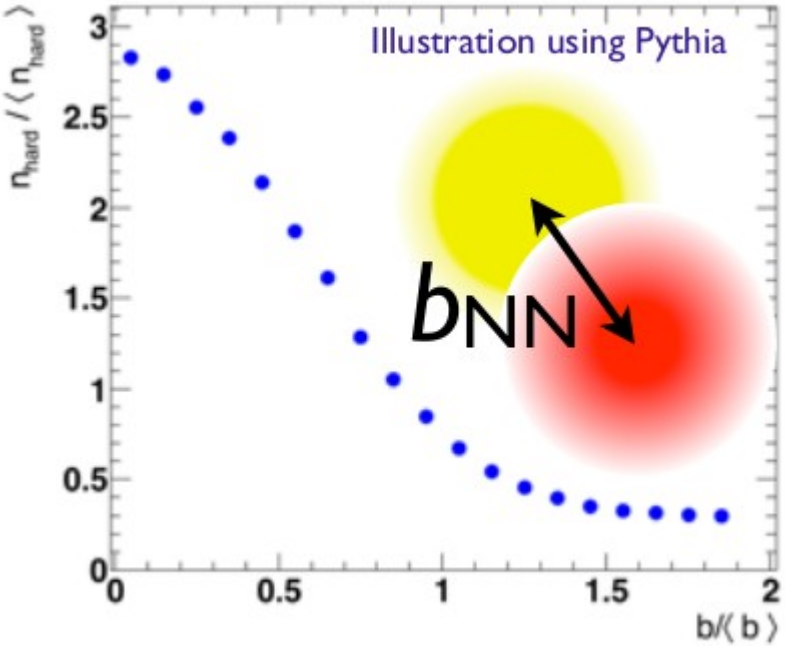
Look at non-trival extensions of the Glauber model

9 Glauber extensions

- Glauber-Gribov color fluctuations
  - Size of proton varies e-by-e
  - Configuration frozen for a single p-A collision
  - Parameter Ω equals width of Gaussian fluctuations
- HIJING Glauber
  - Mean number of hard scatterings (n_{hard}) depends on NN overlap
  - No fluctuations of spatial distribution
    - Only Poisson fluctuations of n_{hard}
- Flickering of the interaction strength
  - Generalized gluon distribution and fluctuations



ATLAS-CONF-2013-096

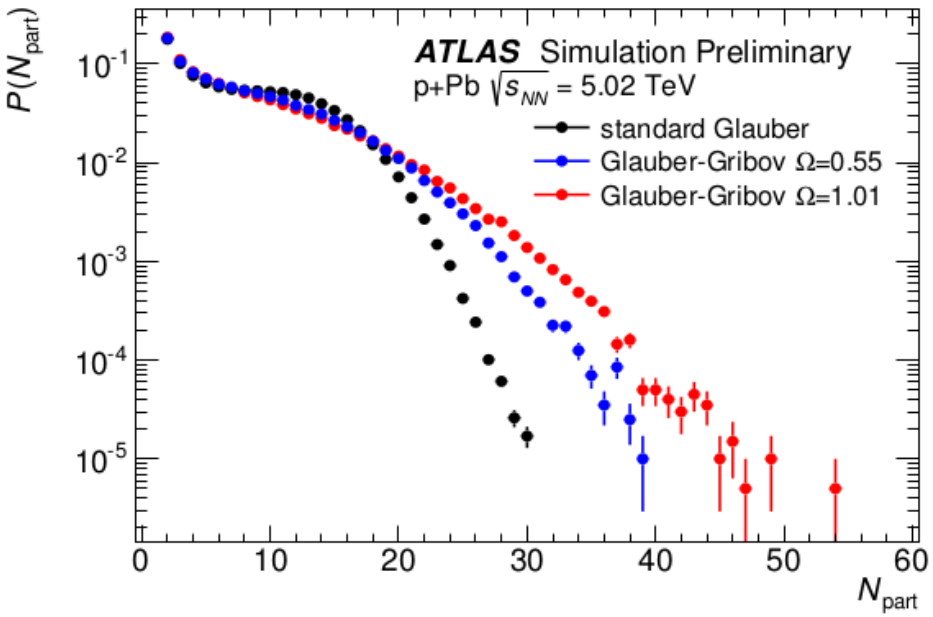


Alvioli et al., PRC 90 (2014) 034914

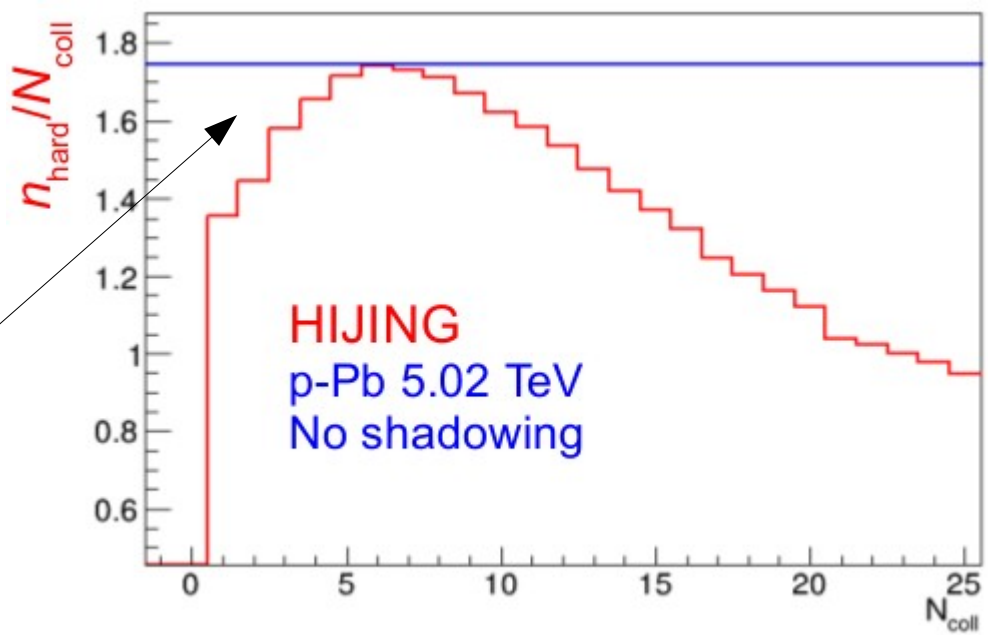
10 Glauber extensions

- Glauber-Gribov color fluctuations
  - Changes $\pi(N_{coll})$

ATLAS-CONF-2013-096



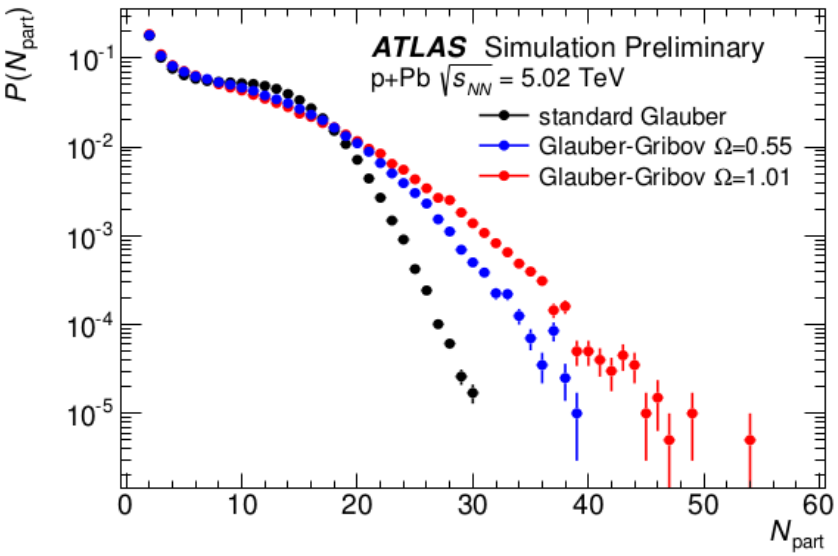
- HIJING Glauber
  - Does not change $\pi(N_{coll})$
  - Provides a correlation between hard and soft particle production
  - Long range correlation via b_{NN}
  - Note: Large n_{hard} values suppressed by energy conservation

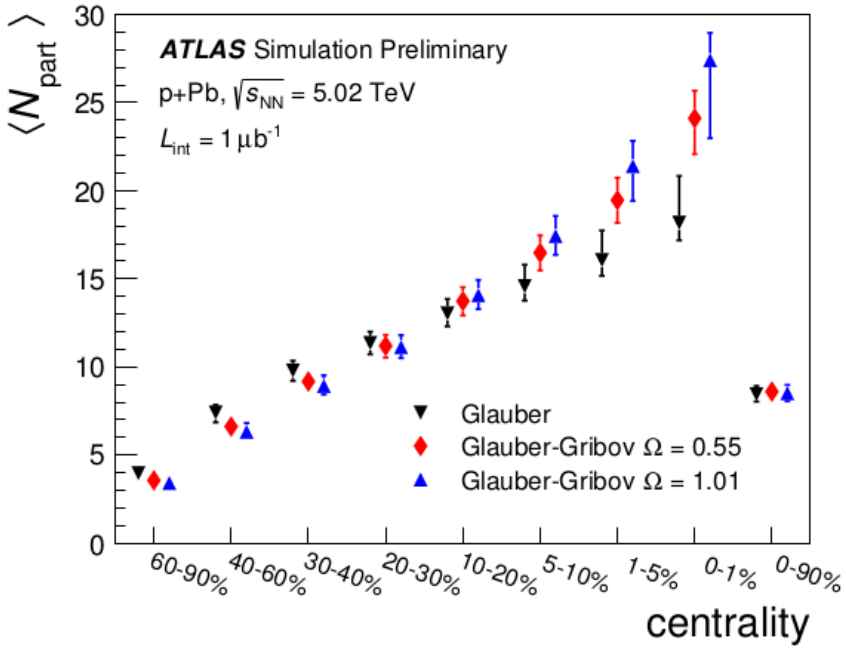


Geometric bias

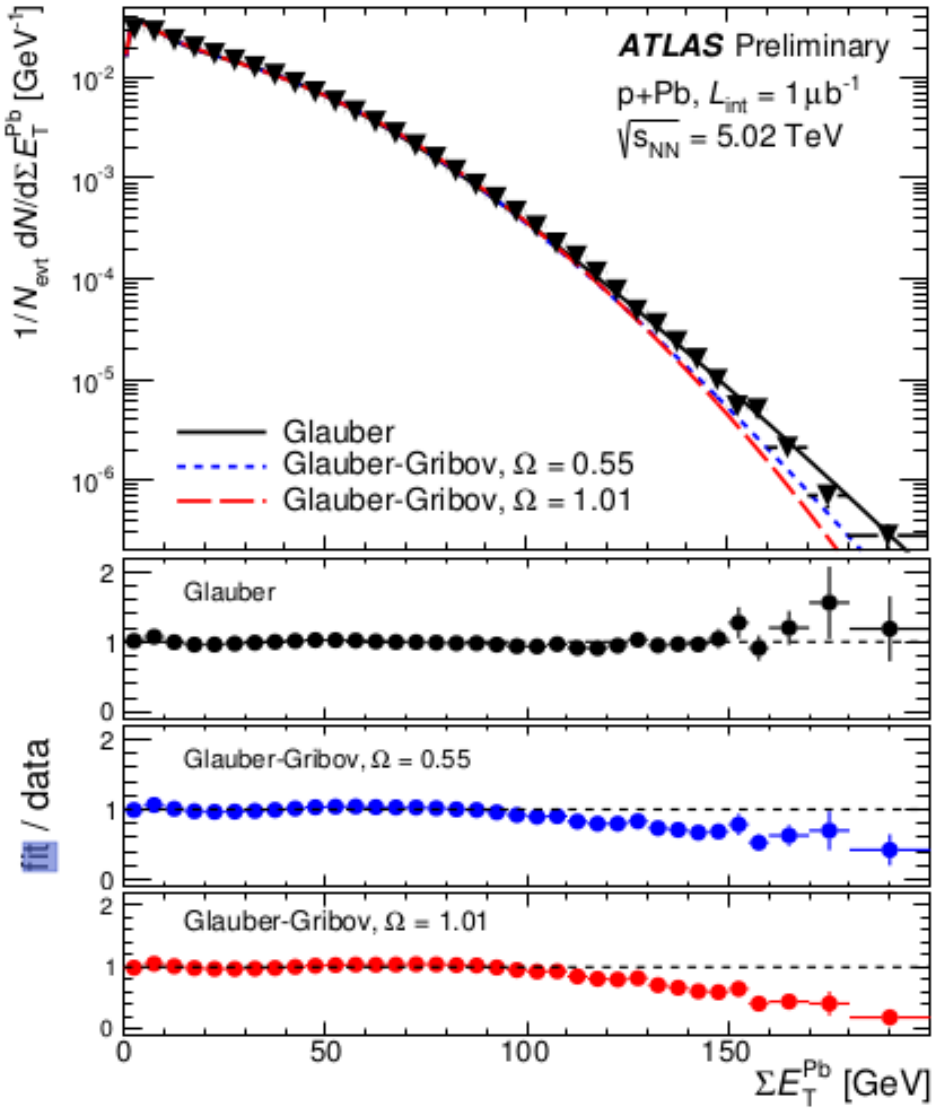
Jia, PLB 681 (2009) 320

11 Glauber-Gribov



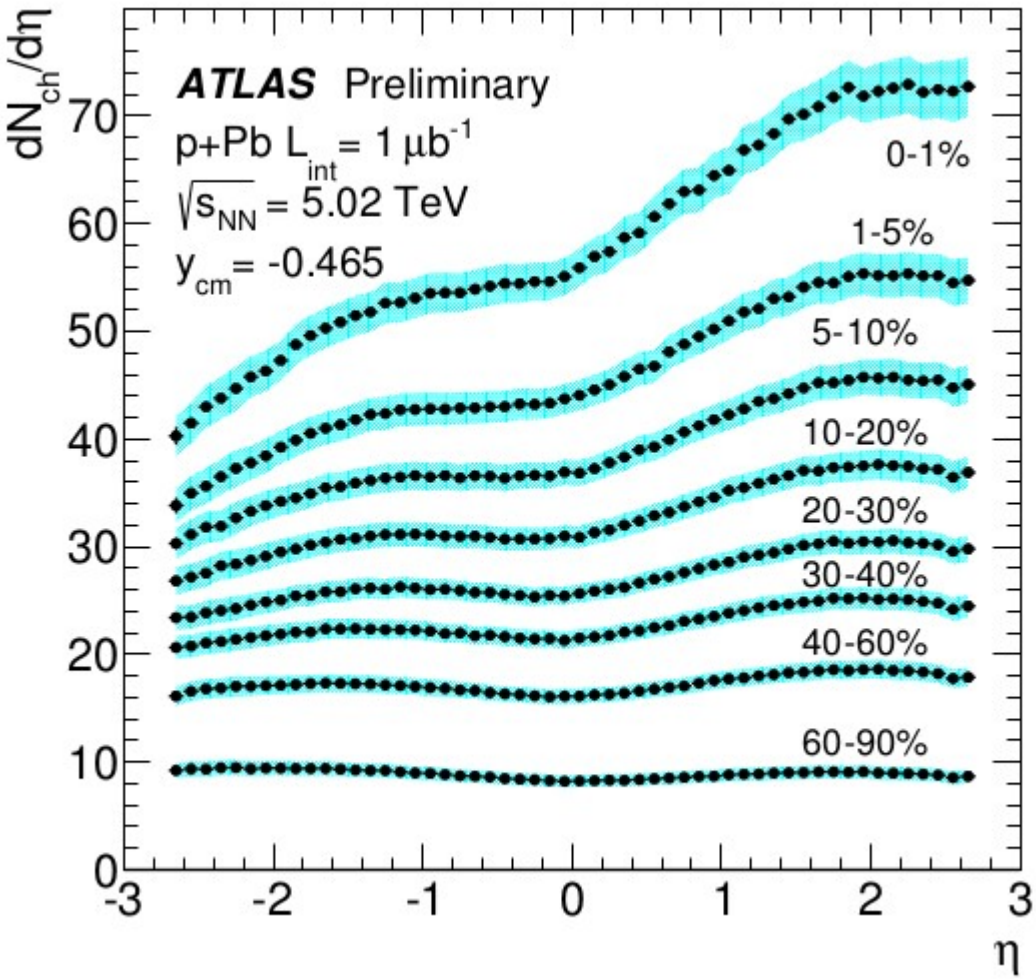


ATLAS-CONF-2013-096

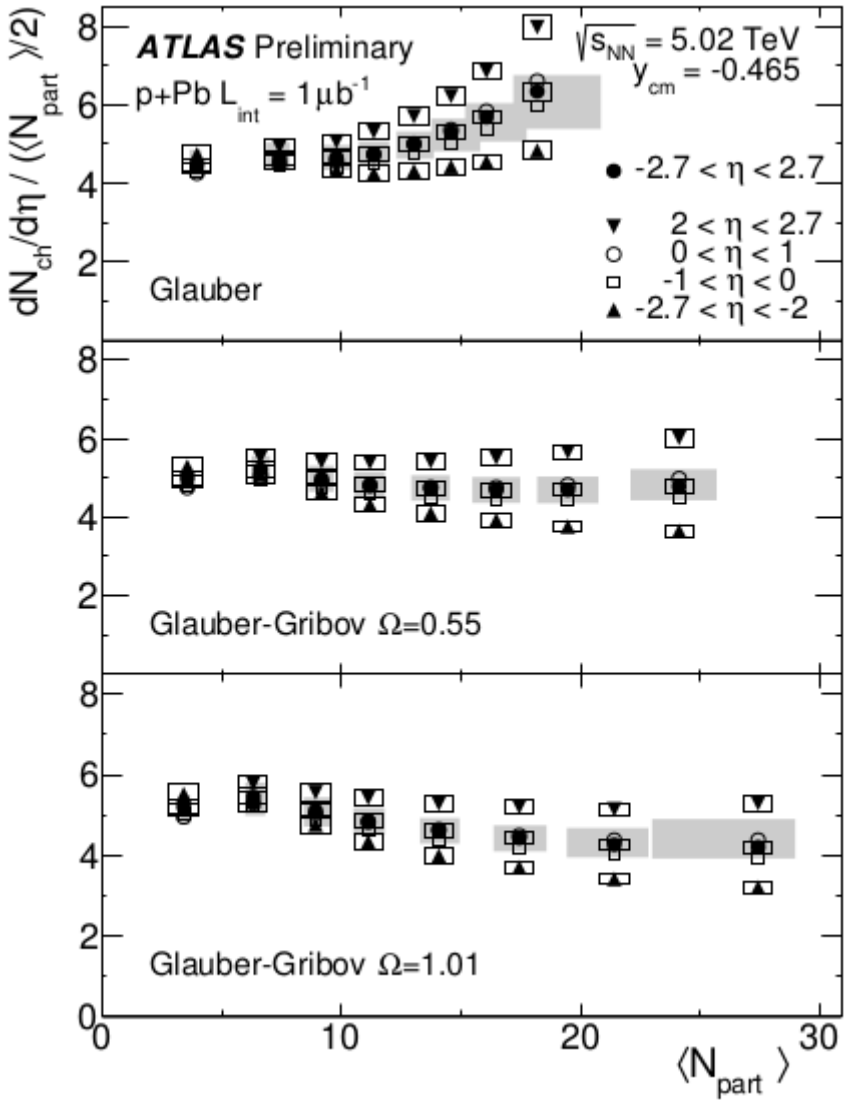


- Glauber-Gribov fits slightly worse
- However, extracted parameters closer to WN expectation

12 Centrality dependent dN/dη

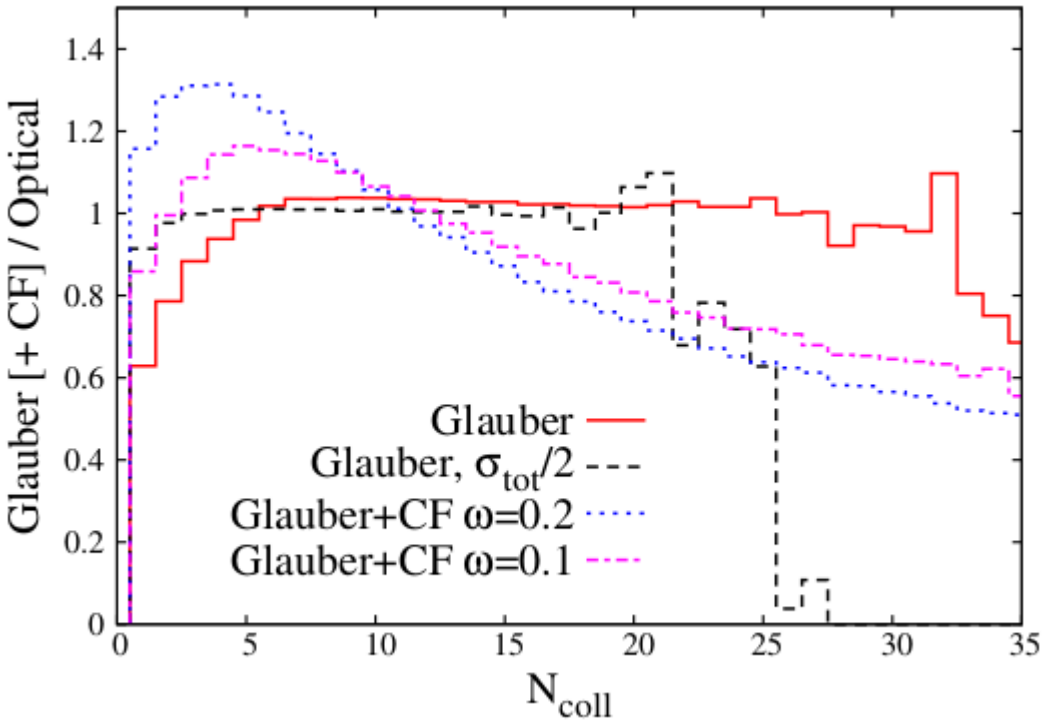


ATLAS-CONF-2013-096

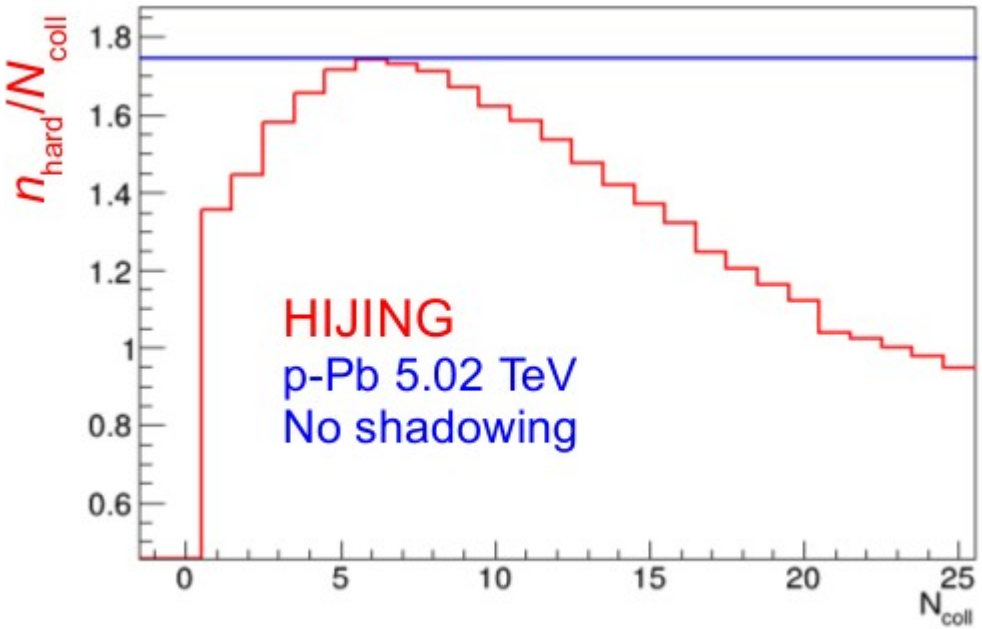


- N_{part} scaling depends on Ω
- Presence of bias open question

13 Centrality dependent measurements



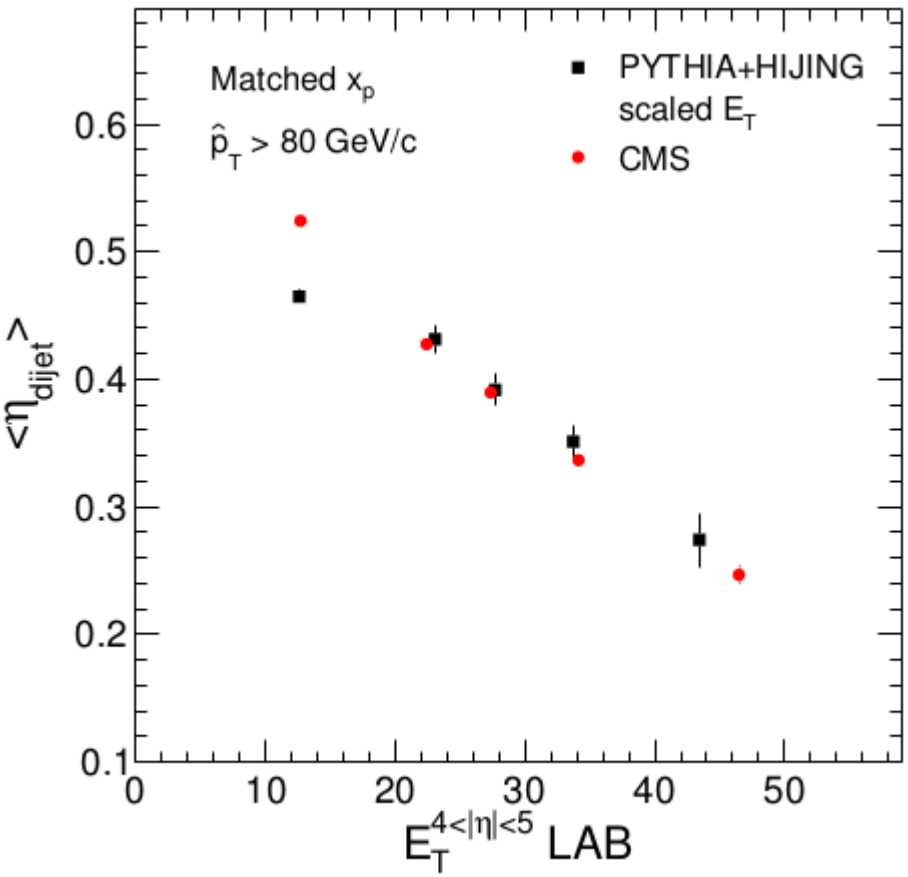
Alvioli et al., PRC 90 (2014) 034914

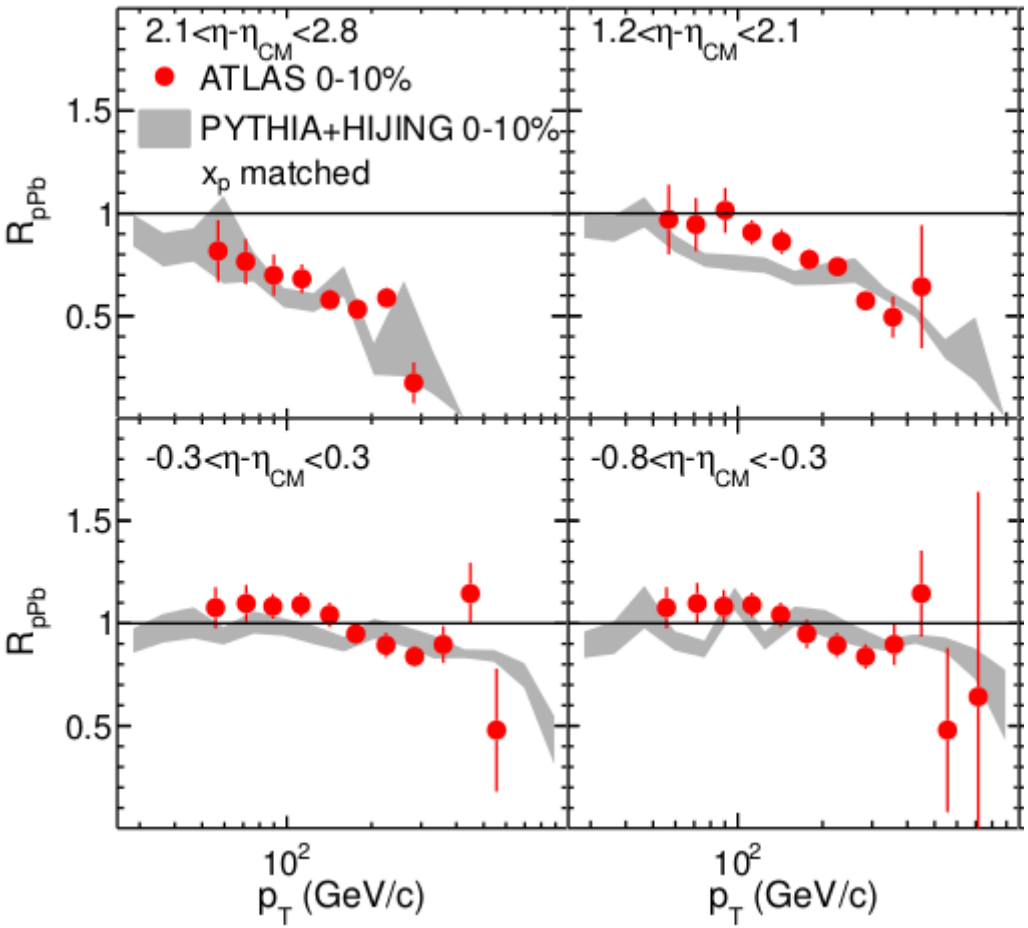


- Rich phenomenology if one trusts the measurement of N_{coll}
- However, systematics of centrality determination itself has to be discussed first in the context of particle production models

14 Kinematic bias on centrality from jets

Armesto et al., arXiv:1502.02986





Taking into account energy-momentum conservation in the proton in a toy simulation of pp (hard) PYTHIA plus pPb (UE) HIJING events describes main features of data

15 Multiple parton interactions (MPI)

Skands, arXiv:1207.2389

- Naive factorization

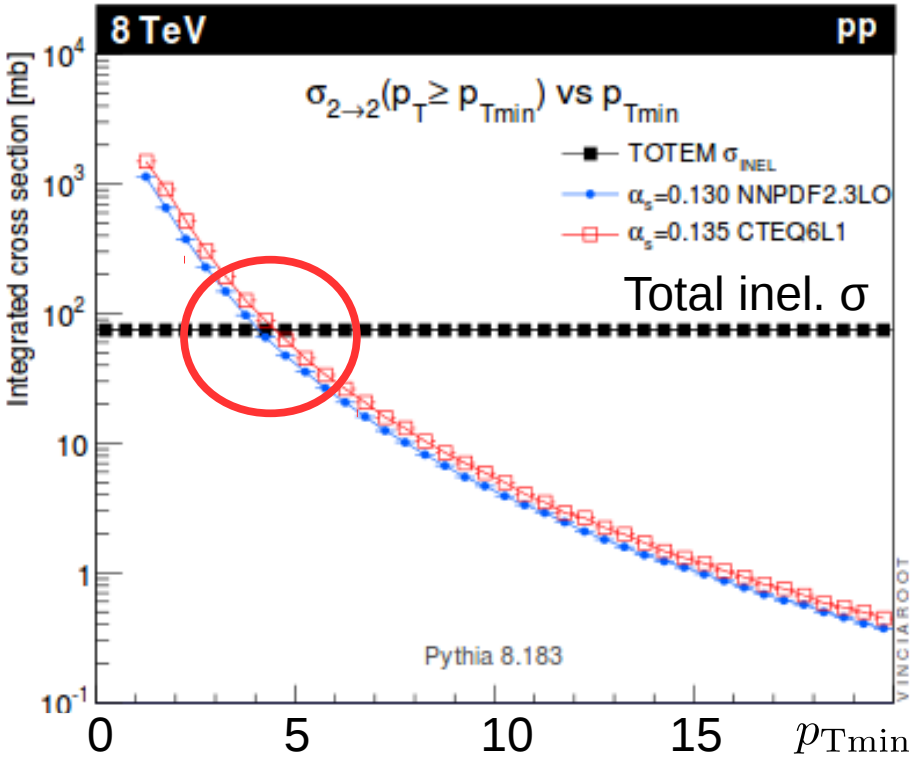
$$\langle n_{2\to 2}\rangle = \frac{\sigma_{2\to 2}}{\sigma_{\rm tot}}$$ >1 at pert. scale

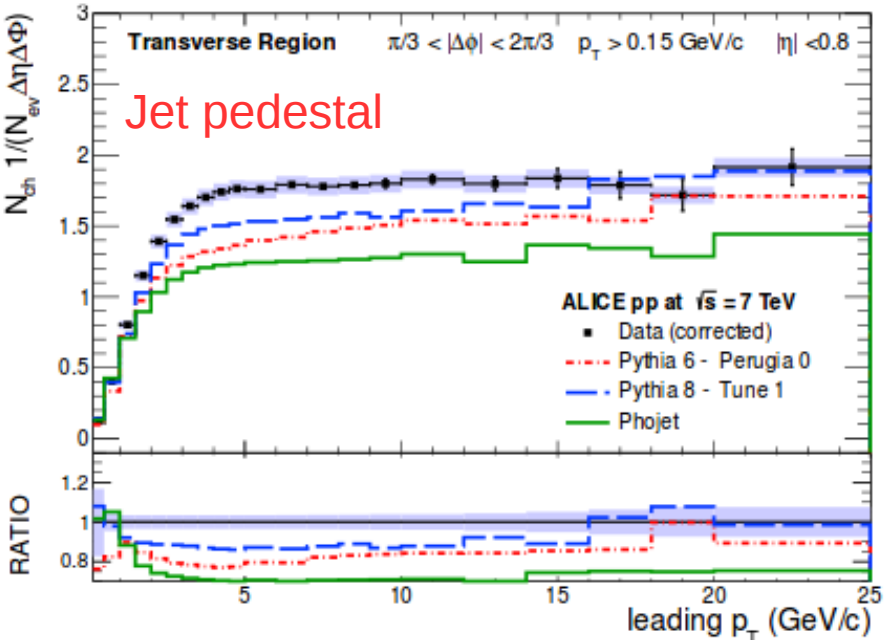
$$P_n = \frac{\langle n_{2\to 2}\rangle^n}{n!}\, exp\left(-\langle n_{2\to 2}\rangle\right)$$

- In reality
  - Color screening to regularize hard cross section at low p_T
  - Cut-off at high n because of energy conservation
  - Coherence between scatters
  - Impact parameter dependence

$$n_{\rm hard}(b) = \sigma_{\rm hard} T_{\rm p}(b)$$

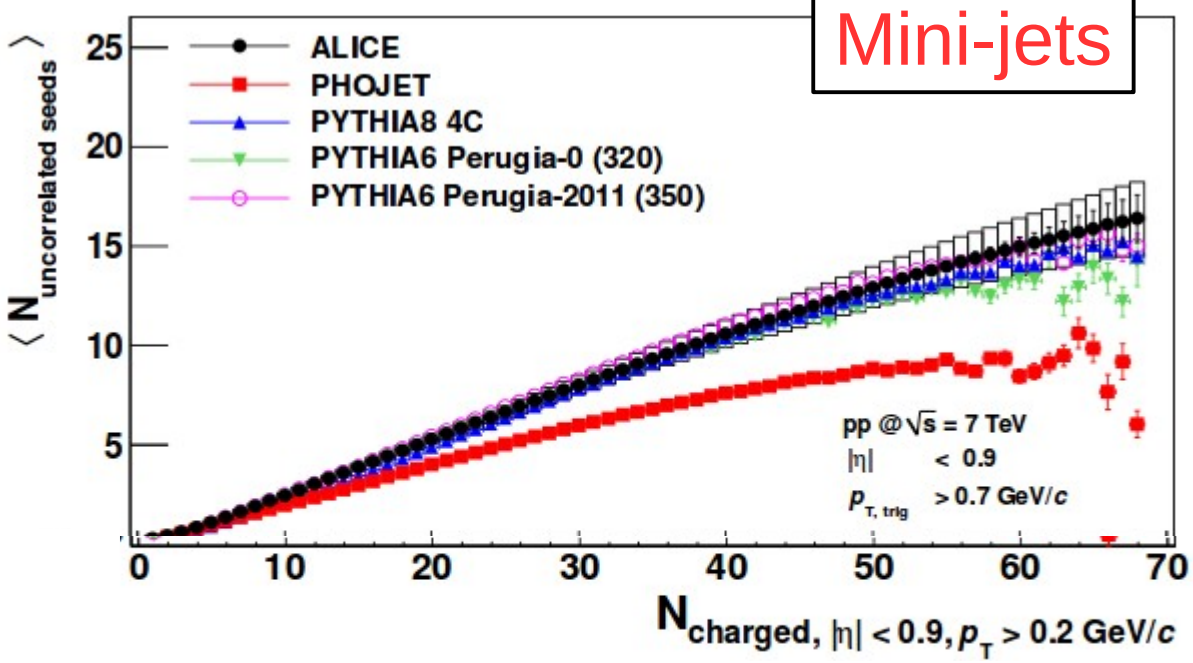
    - Leads to a correlation between hard and soft as in AA



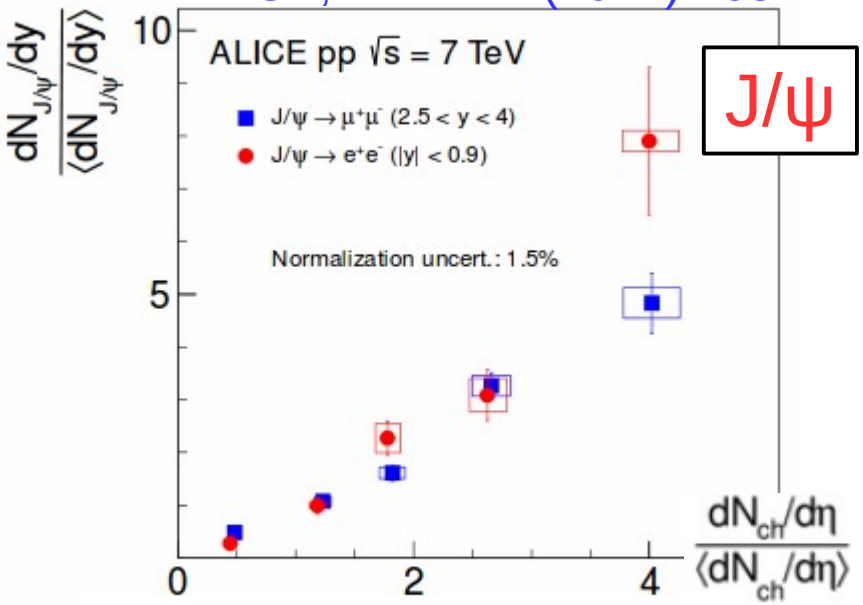


16 Scaling of hard probes with multiplicity

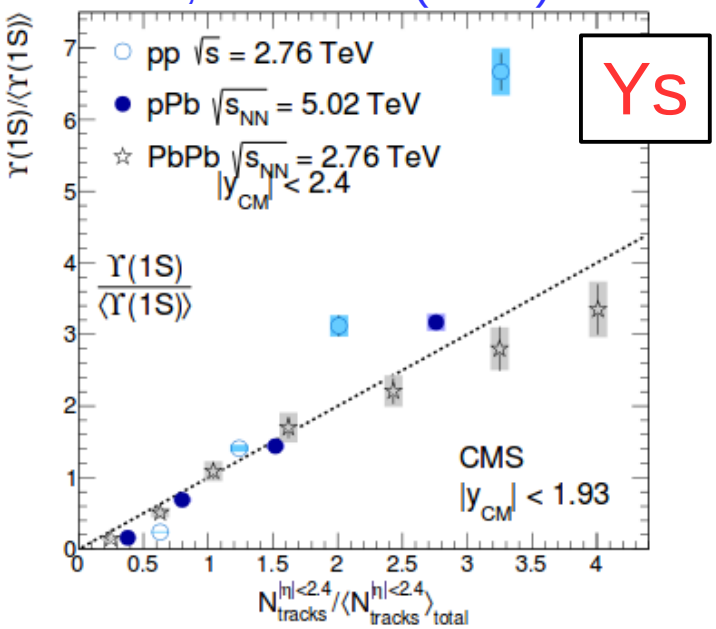
ALICE. JHEP 09 (2013) 049

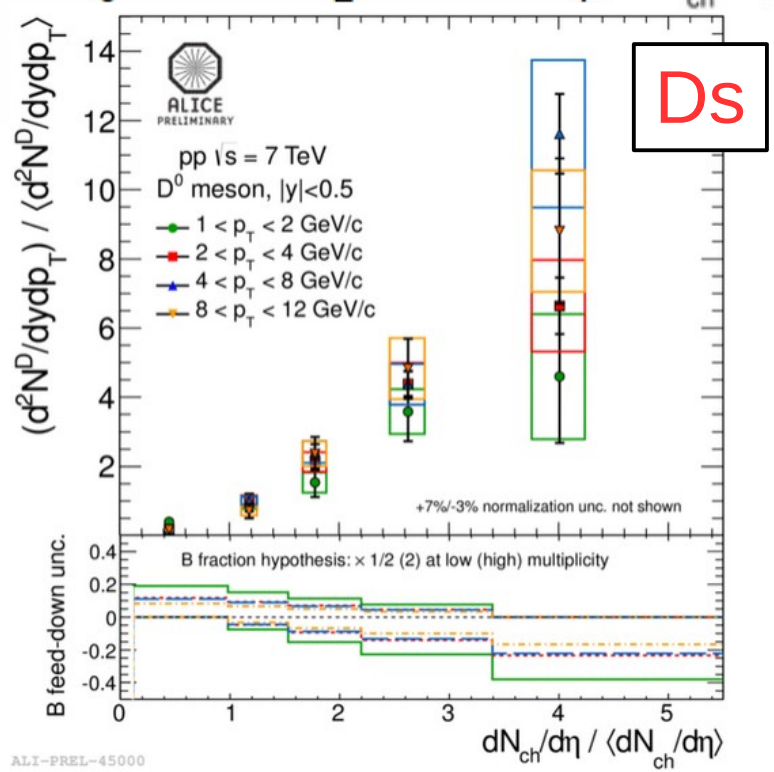


ALICE, PLB 712 (2012) 165



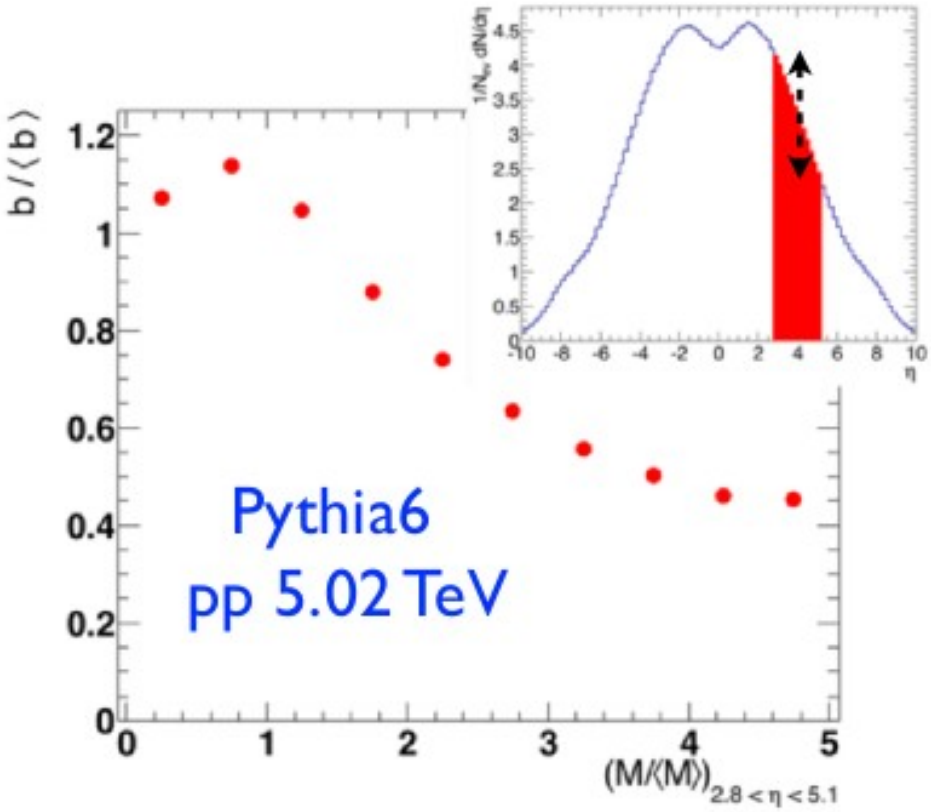
CMS, JHEP 04 (2014) 103

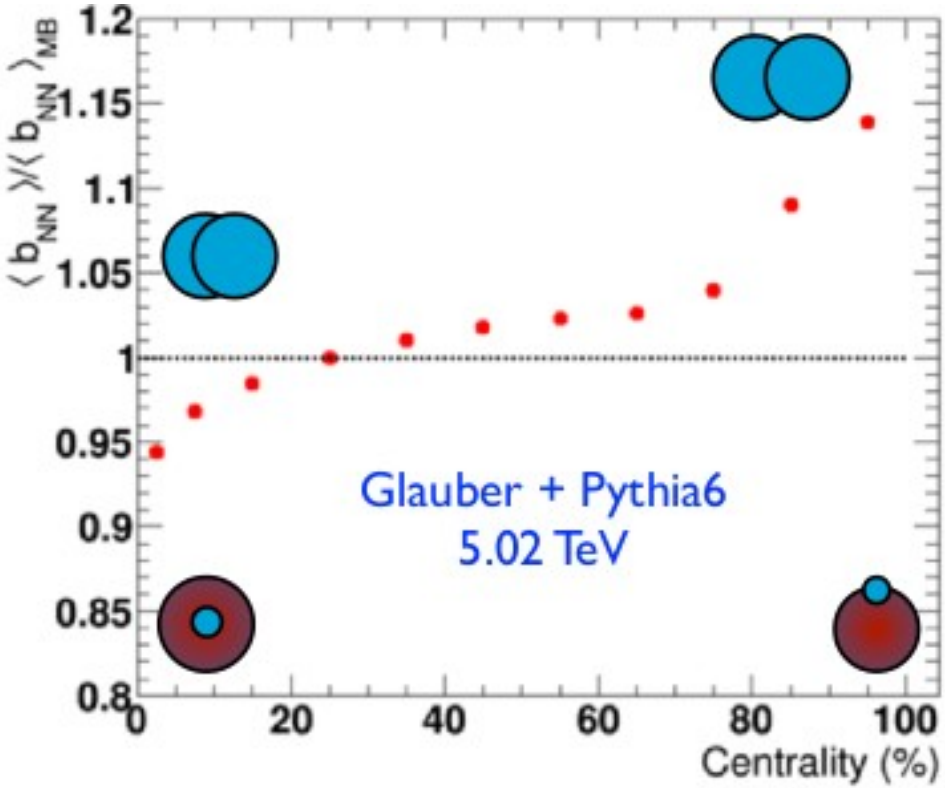




17 Nucleon-nucleon impact parameter studies

Morsch, IS2014



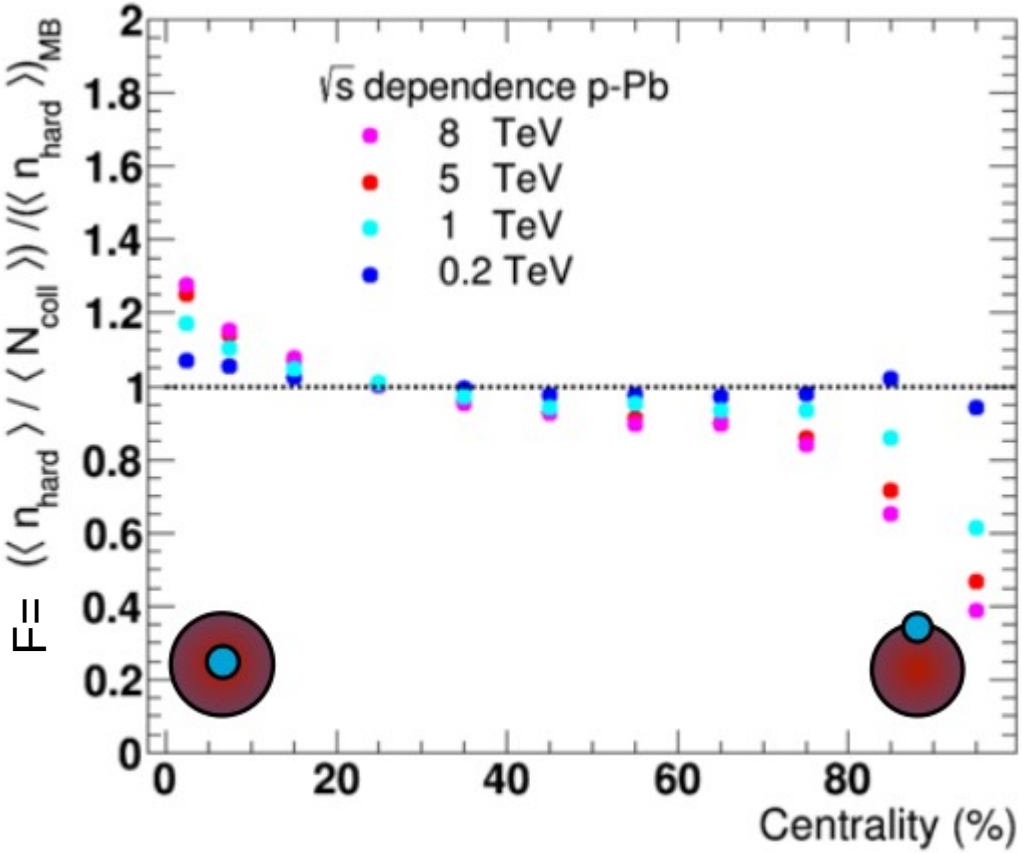


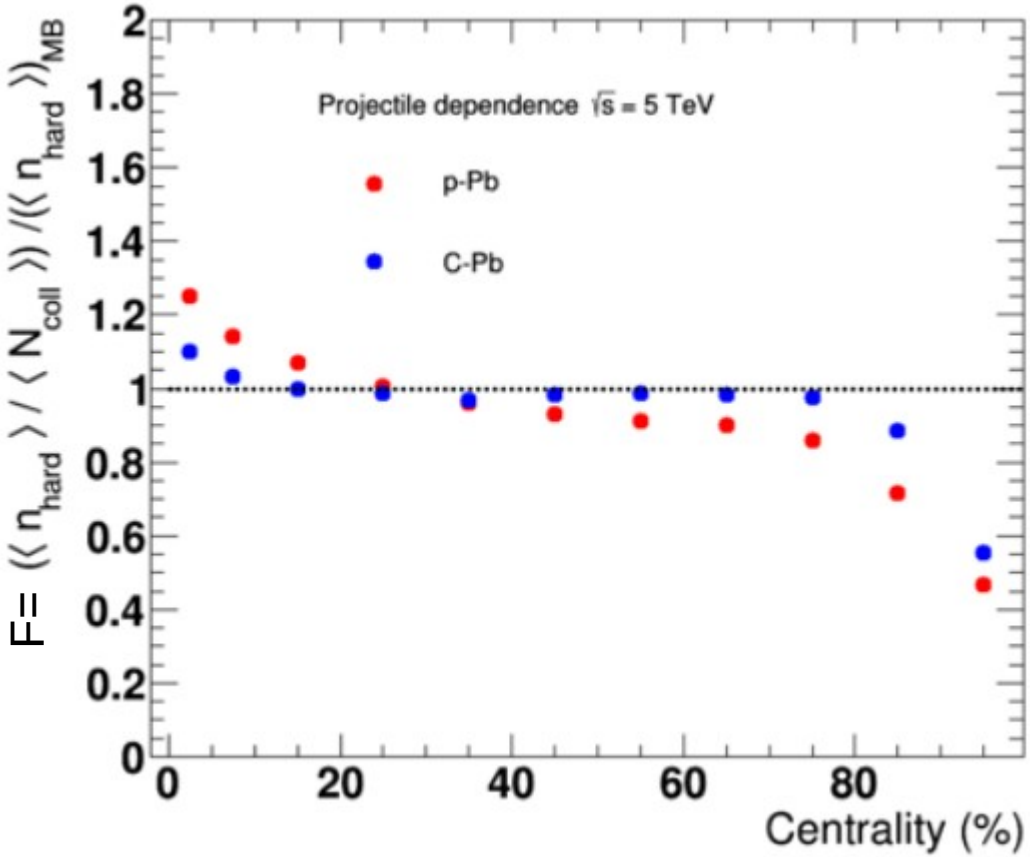
(obtained from slicing superposition of N_{coll} pp collisions in $2.8<\eta<5.1$)

Leads to long range (η) correlations.
How much of this effect survives in pPb?

18 Energy and species dependence

Morsch, IS2014



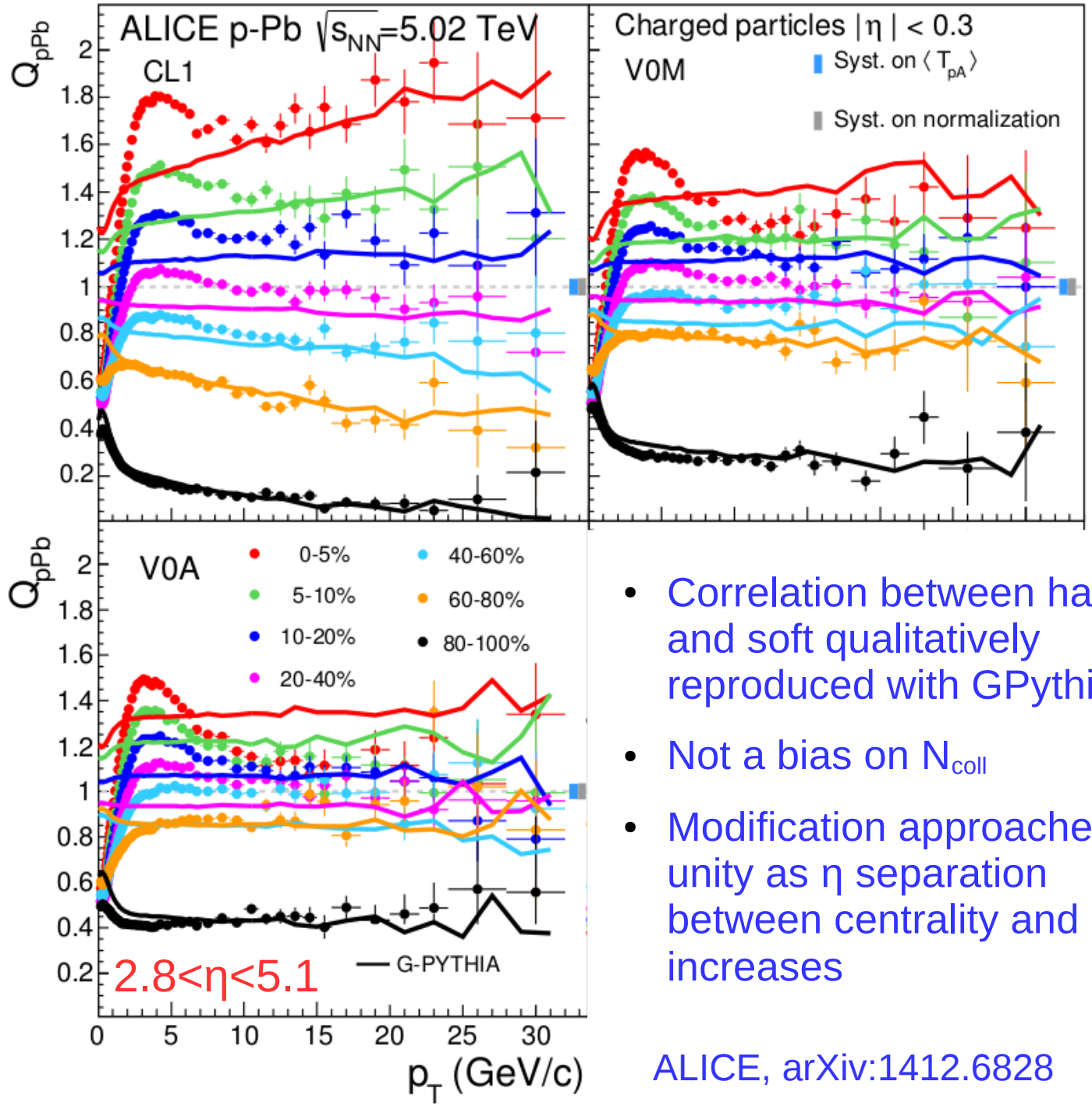


Bias on n_{hard} O(30%) at the LHC, and only O(5%) at RHIC

And decreases with projectile size

Deviation from binary scaling: $$\left.\frac{dN}{p_{\mathrm{T}}}\right|_{\mathrm{pA}} = N_{\mathrm{coll}}\, F \left.\frac{dN}{p_{\mathrm{T}}}\right|_{\mathrm{pp}}$$

|η|<1.4 -3.7<η<-1.7&&2.8<η<5.1

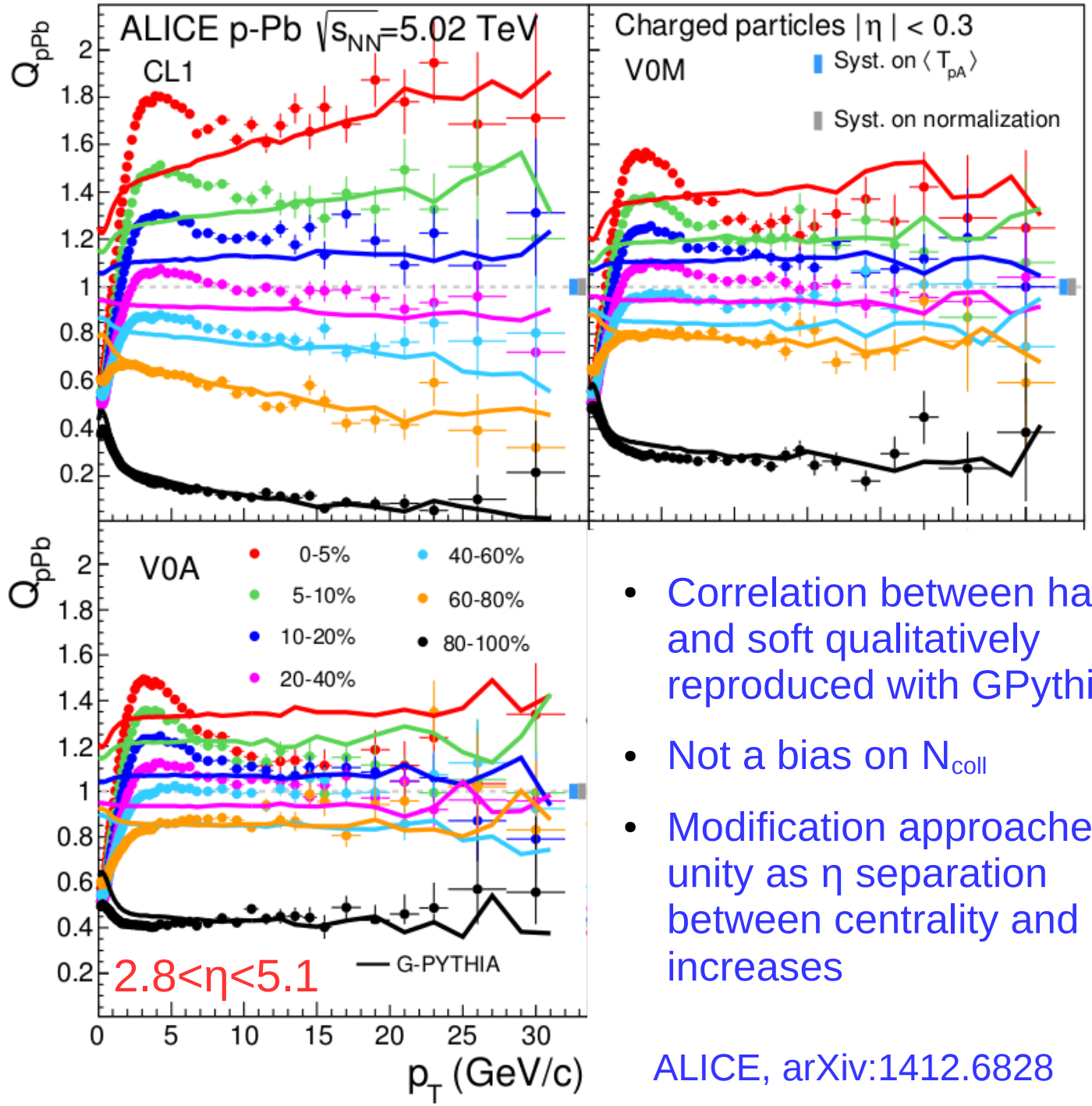


- Correlation between hard and soft qualitatively reproduced with GPythia
- Not a bias on N_{coll}
- Modification approaches unity as η separation between centrality and p_T increases

ALICE, arXiv:1412.6828

|η|<1.4

-3.7<η<-1.7&&2.8<η<5.1

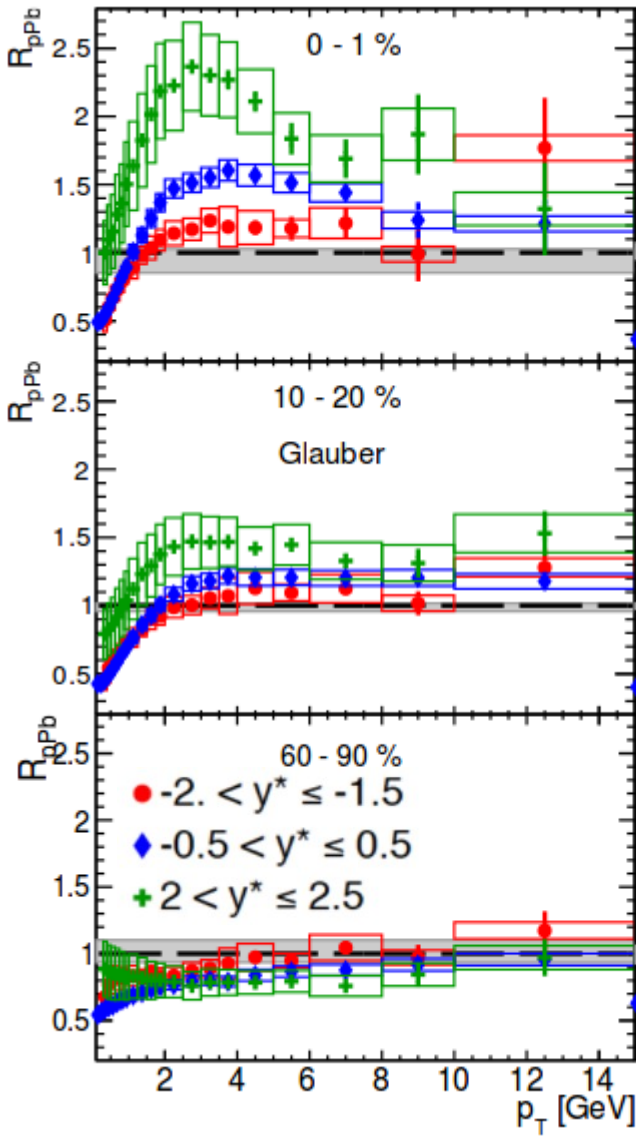


2.8<η<5.1

- Correlation between harc and soft qualitatively reproduced with GPythia
- Not a bias on N_{coll}
- Modification approaches unity as η separation between centrality and p increases

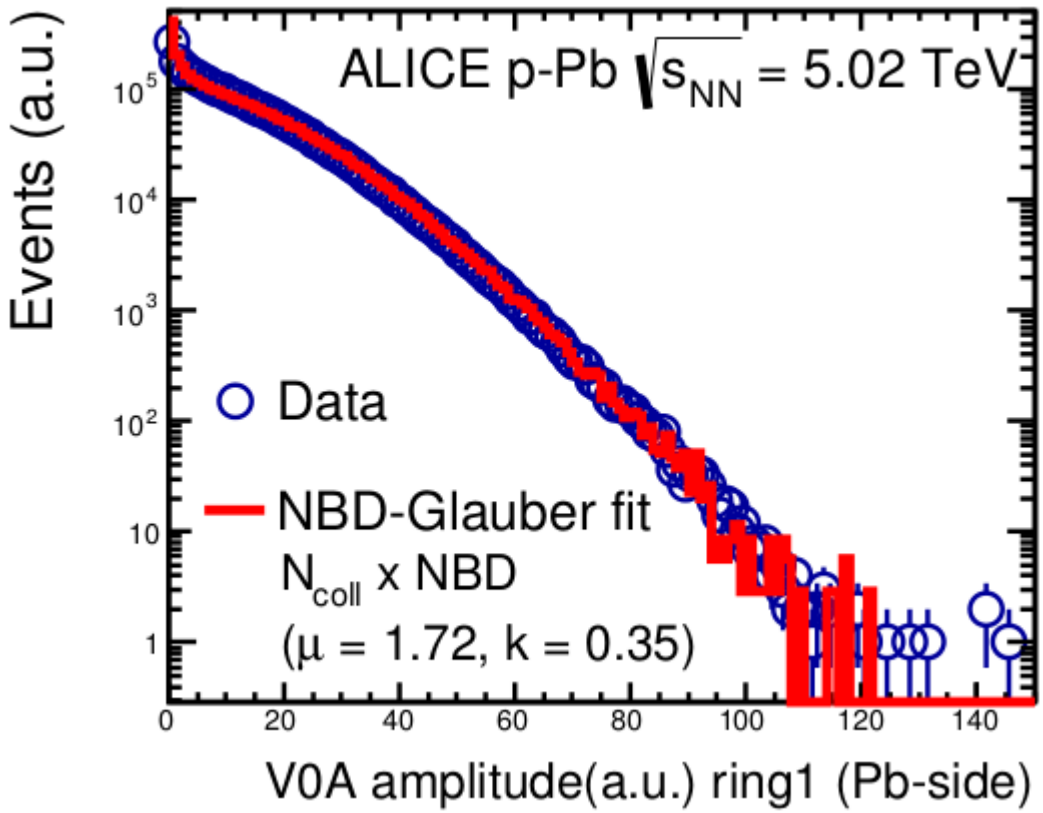
ALICE, arXiv:1412.6828

ATLAS-CONF-2013-107

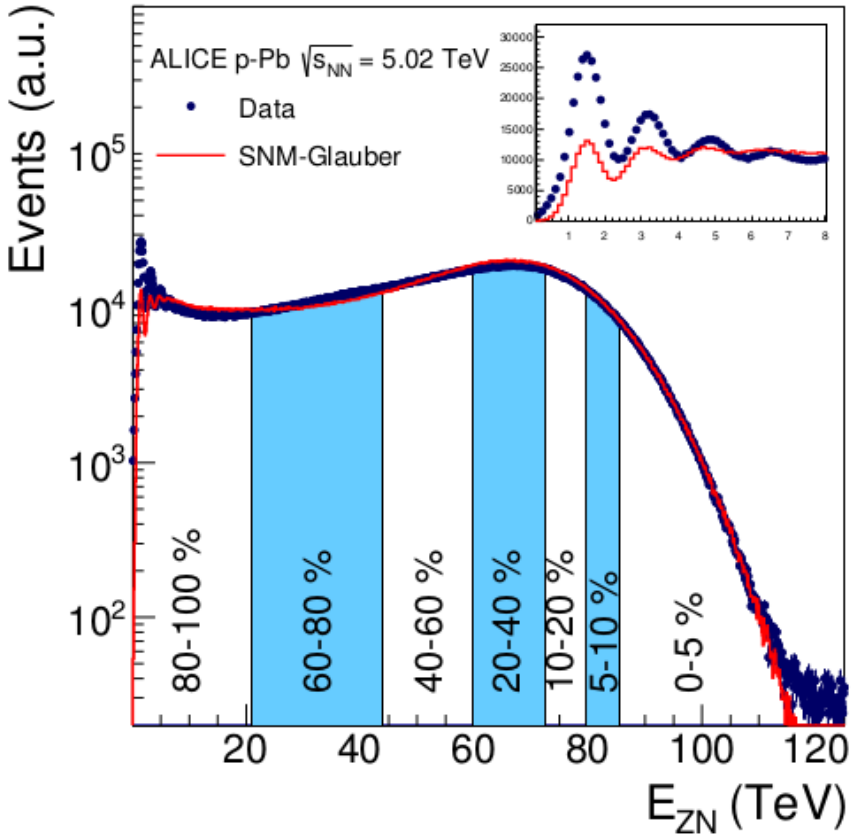


21 Forward neutron energy vs multiplicity

NBD method

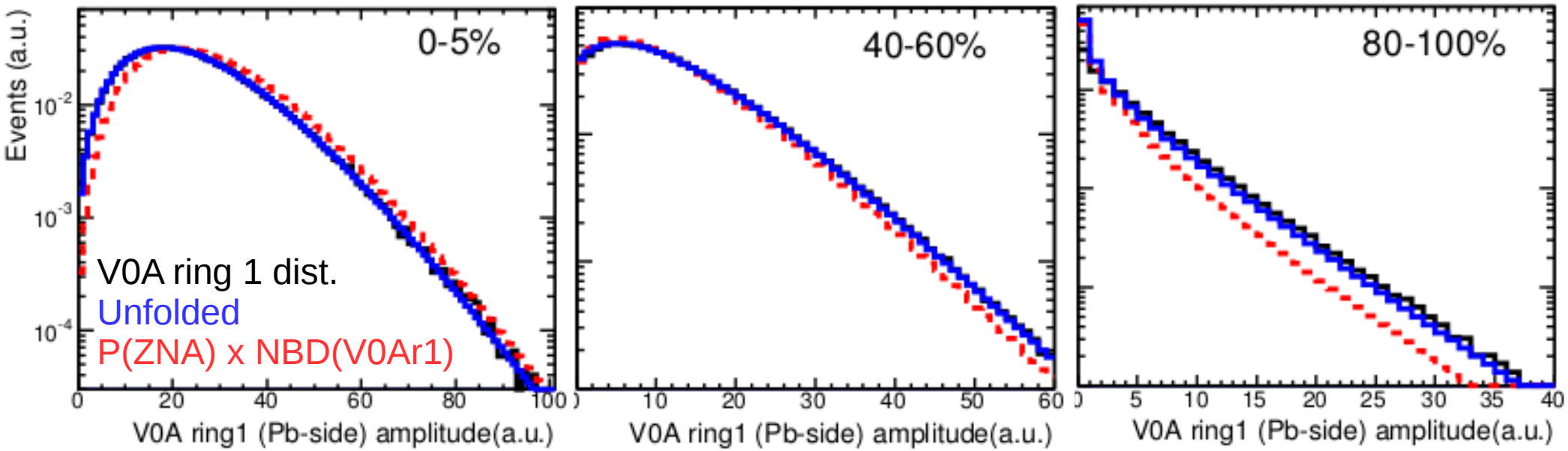


SNM method

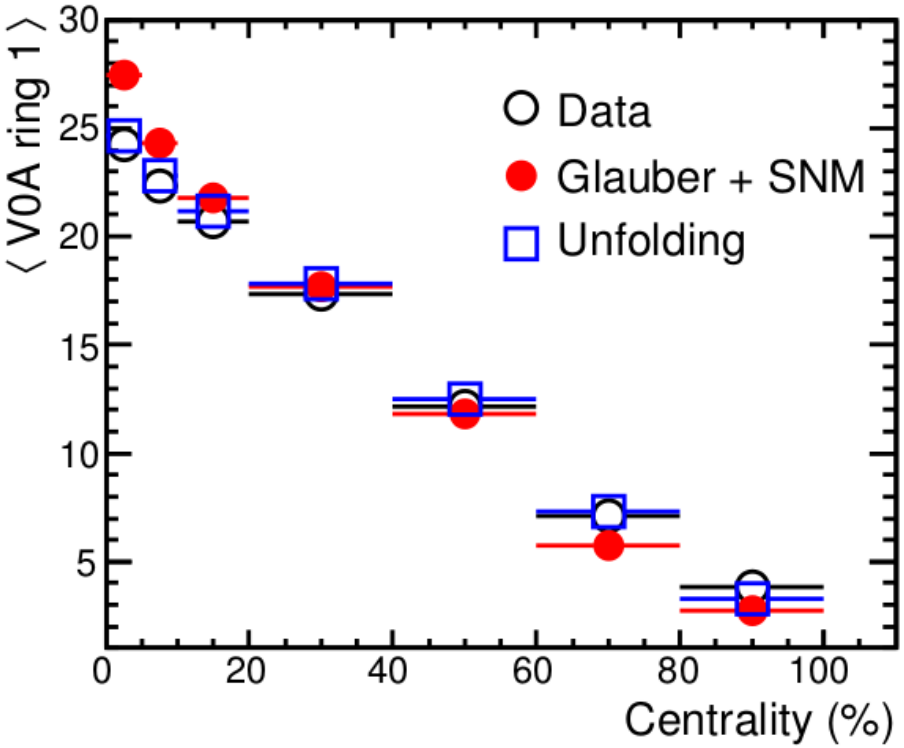


Correlation between forward neutron energy and multiplicity?

22 Correlation between ZNA and V0A



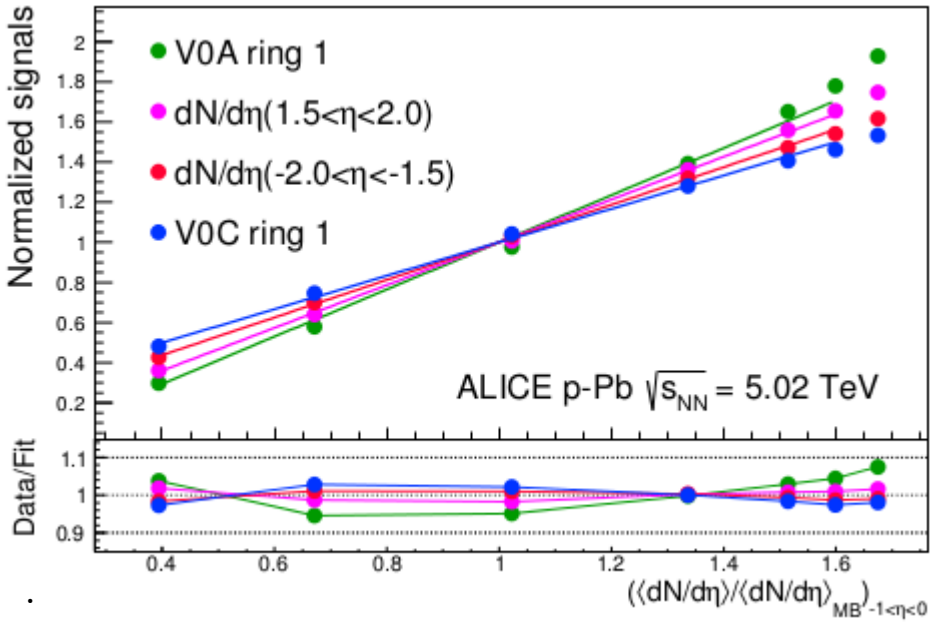
- For each ZNA centrality class
  - Plot the V0A ring1 distribution
  - Find input $\pi(N_{coll})$ distribution via Unfolding
    - Does not work for a biased estimator
  - Convolute the distribution of the SNM model with the NBD from the V0A glauber fit



ALICE, arXiv:1412.6828

23 Scaling of particle production

- $\langle S\rangle_i / \langle S\rangle_{MB}$ vs $\langle dN/d\eta\rangle_i/\langle dN/d\eta\rangle_{MB}(-1<\eta<0)$ sliced in ZN activity

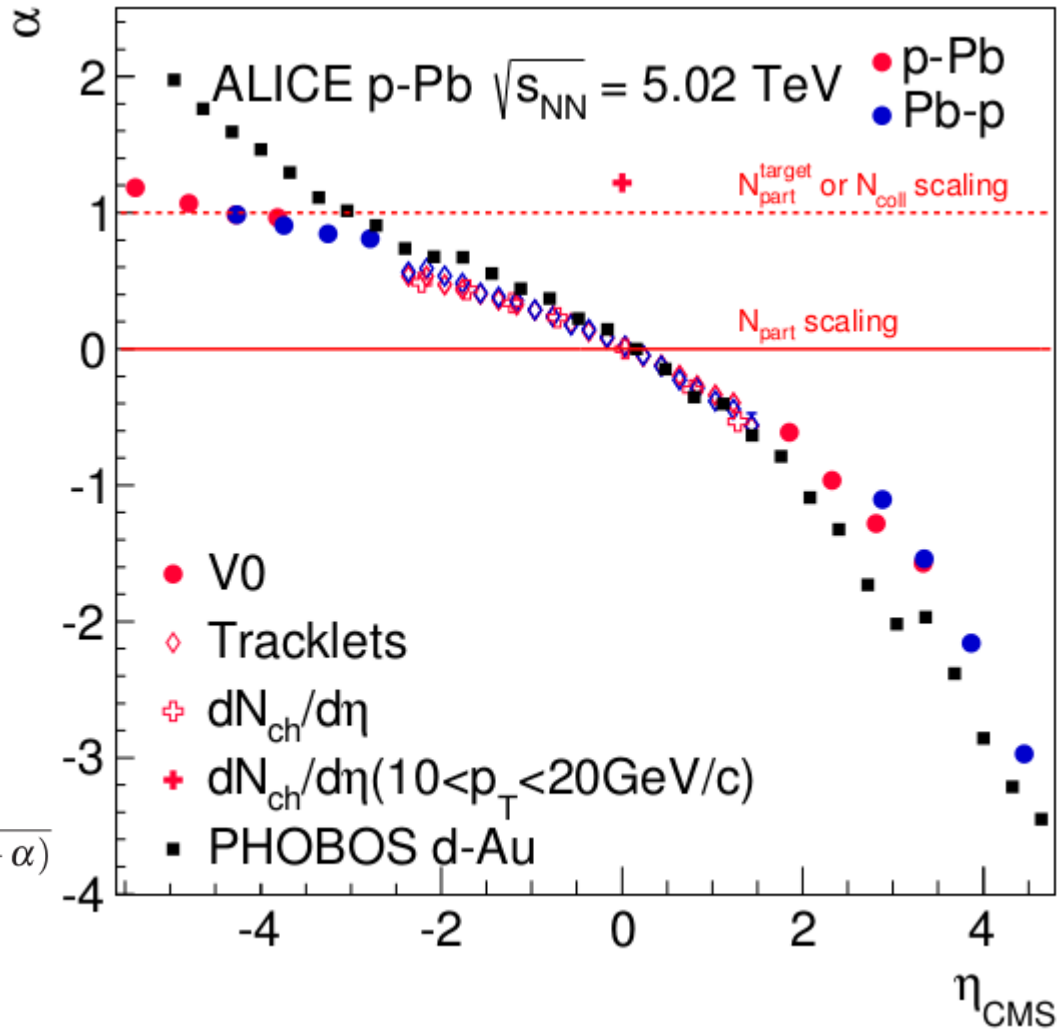


- Assume dN/dη scales with N_{part}

$$\frac{\langle S\rangle_i}{\langle S\rangle_{MB}} = \frac{\langle N_{part}\rangle_{MB}}{(\langle N_{part}\rangle_{MB}-\alpha)}\cdot\left(\frac{\langle dN/d\eta\rangle_i}{\langle dN/d\eta\rangle_{MB}}\right)_{-1<\eta<0} - \frac{\alpha}{(\langle N_{part}\rangle_{MB}-\alpha)}$$

$\alpha = 0$ – perfect N_{part} scaling

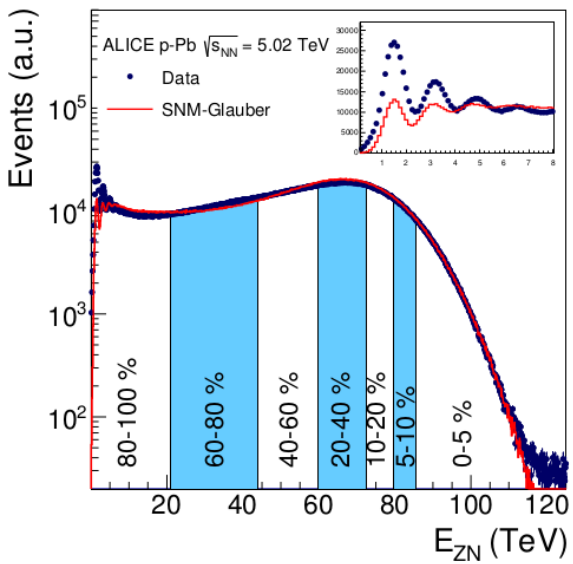
$\alpha = 1$ – perfect N_{coll} (or N_{part}^{target}) scaling



Correlation between causally disconnected observables (slow neutrons vs multiplicity)
→ **connection to geometry**

ALICE, arXiv:1412.6828

24 Centrality from Hybrid method

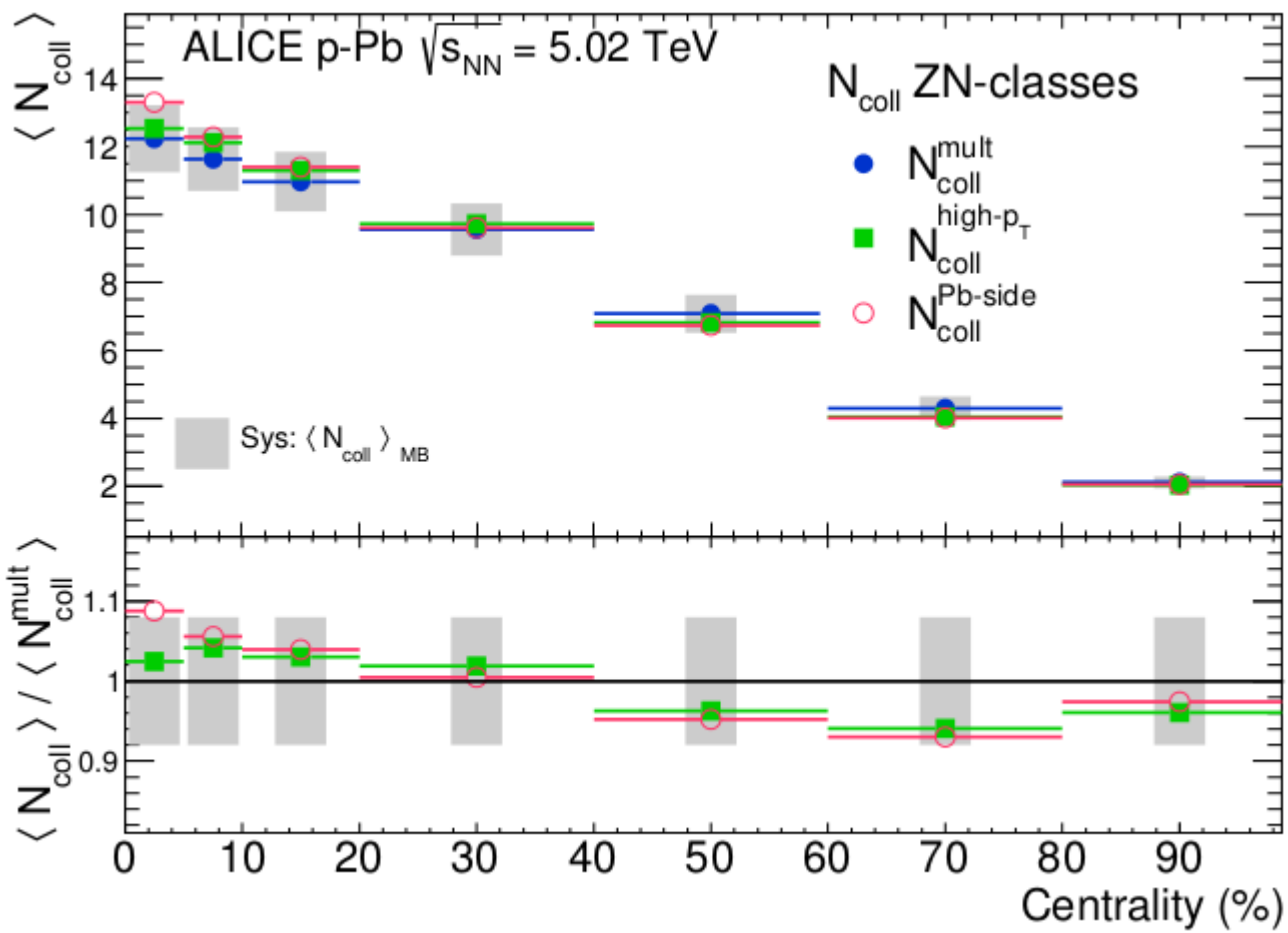


$$\langle N_{\rm coll}\rangle_i^{\rm mult} = \langle N_{\rm part}\rangle_{\rm MB} \left.\frac{\langle dN/d\eta\rangle_i}{\langle dN/d\eta\rangle_{\rm MB}}\right|_{-1<\eta<0} - 1$$

$$\langle N_{\rm coll}\rangle_i^{\rm high\ p_T} = \langle N_{\rm coll}\rangle_{\rm MB} \frac{\langle Y_{10<p_{\rm T}<20}\rangle_i}{\langle Y_{10<p_{\rm T}<20}\rangle_{\rm MB}}$$

$$\langle N_{\rm coll}\rangle_i^{\rm Pb\ side} = \langle N_{\rm coll}\rangle_{\rm MB} \frac{\langle S_{\rm V0Ar1}\rangle_i}{\langle S_{\rm V0Ar1}\rangle_{\rm MB}}$$

1) Assume ZN is bias free + define centrality classes
2) Construct similar model as for the Glauber fits

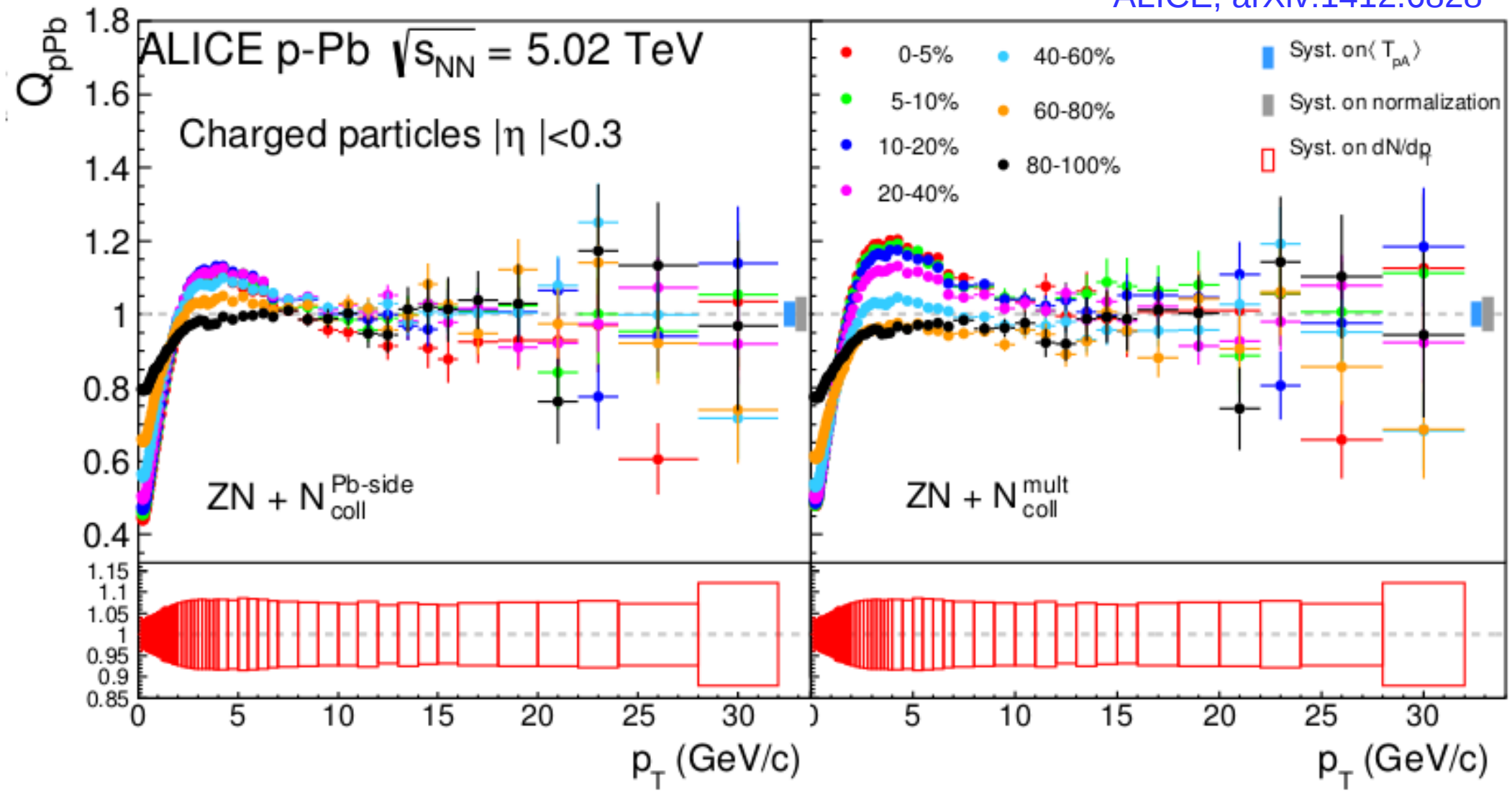


Resulting values within at most 10%

ALICE, arXiv:1412.6828

25 QpPb factors with hybrid method

ALICE, arXiv:1412.6828

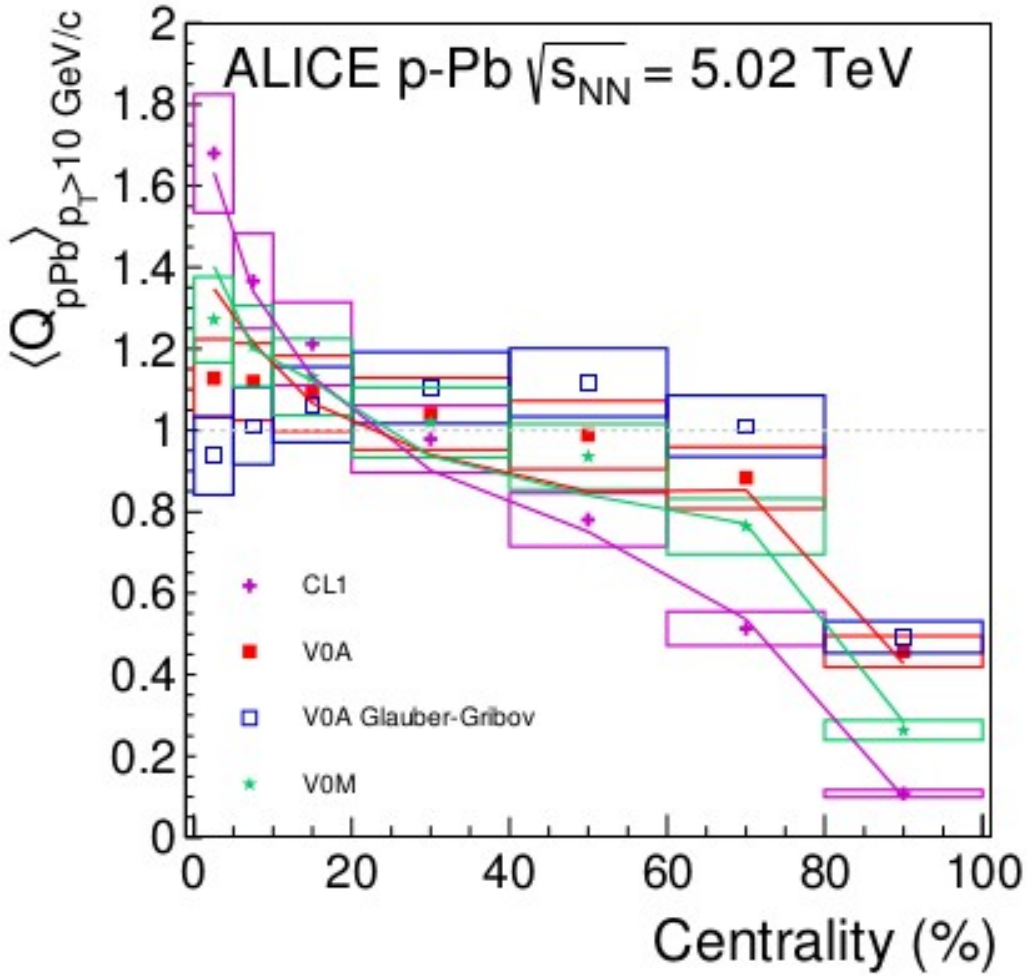


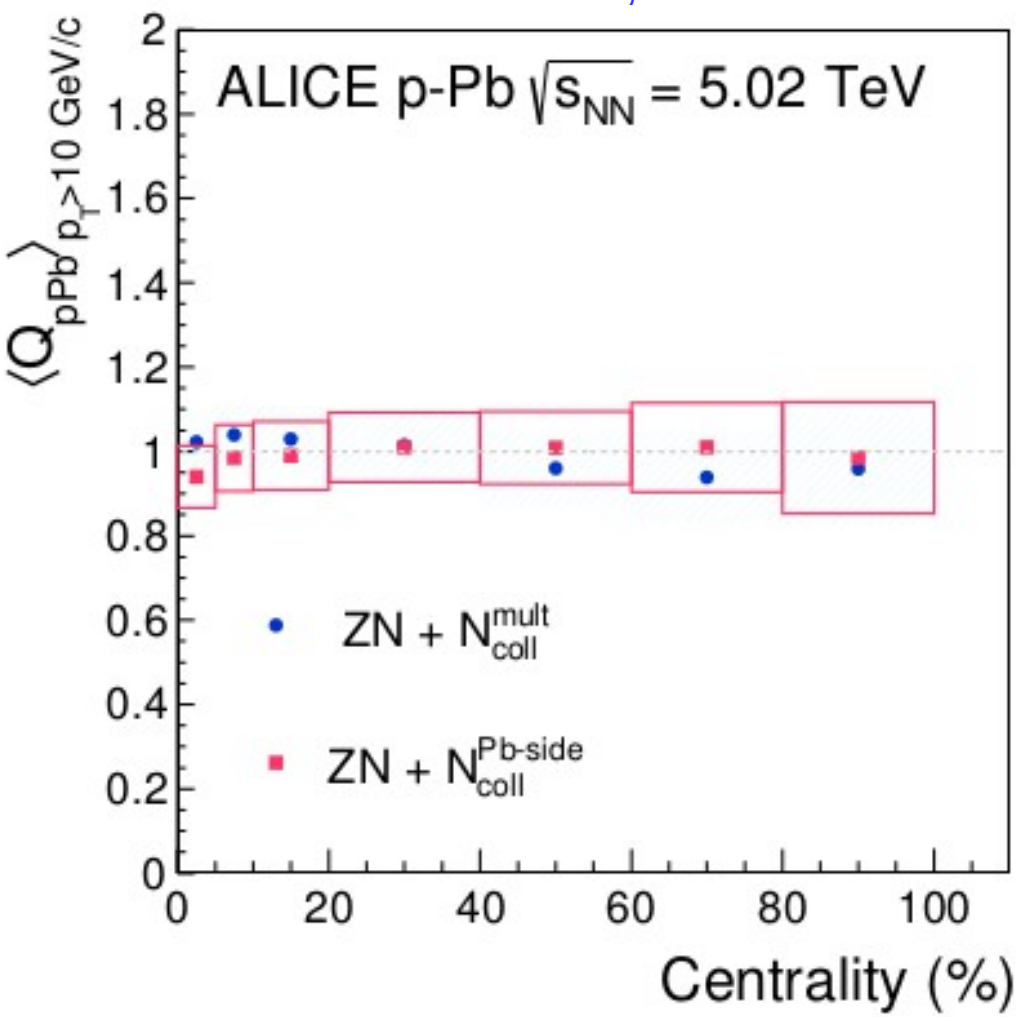
Hybrid method

- Charged particle Q_{pPb} consistent with unity at high p_T
- Cronin peak develops with multiplicity

26 Average QpPb

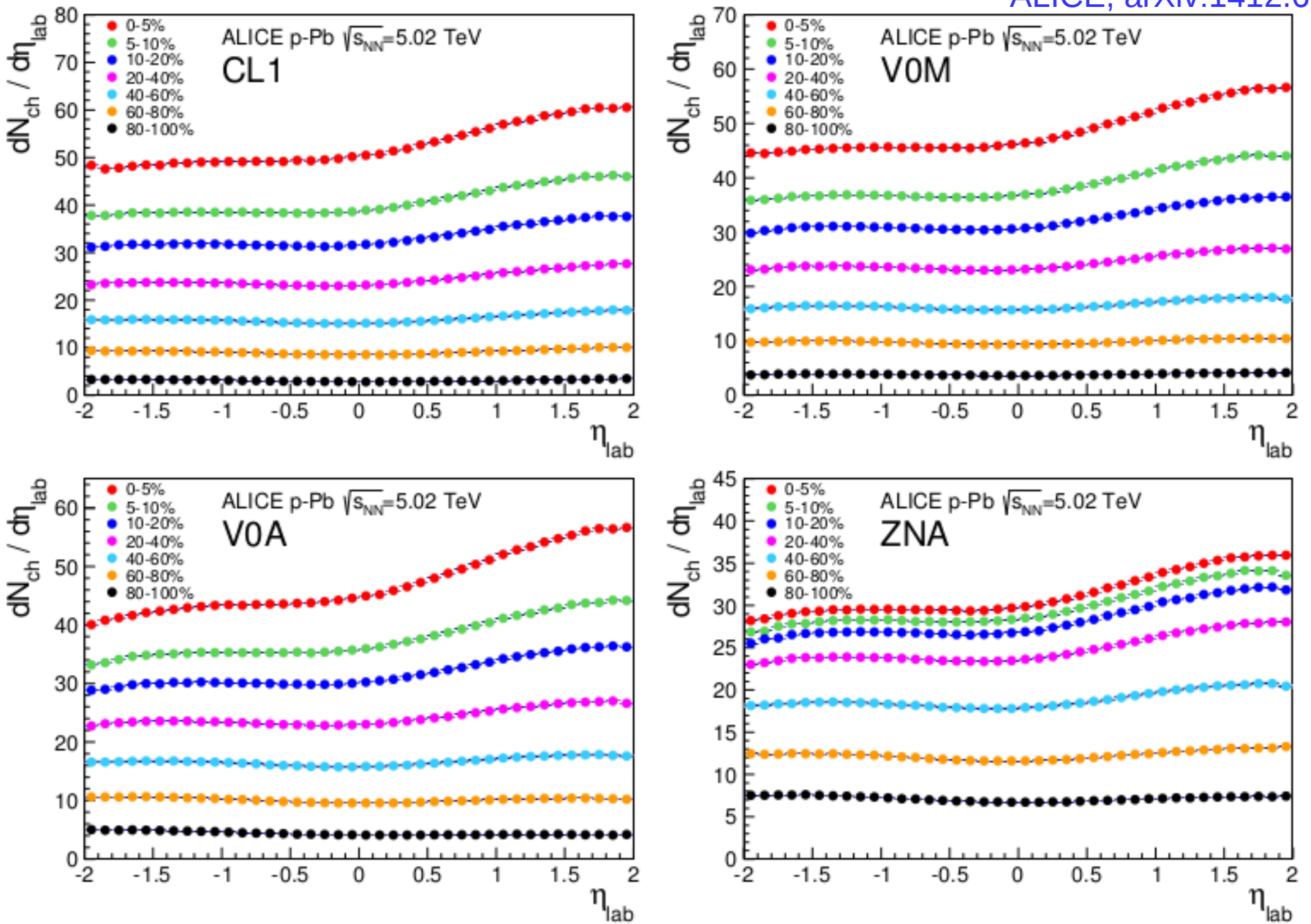
ALICE, arXiv:1412.6828



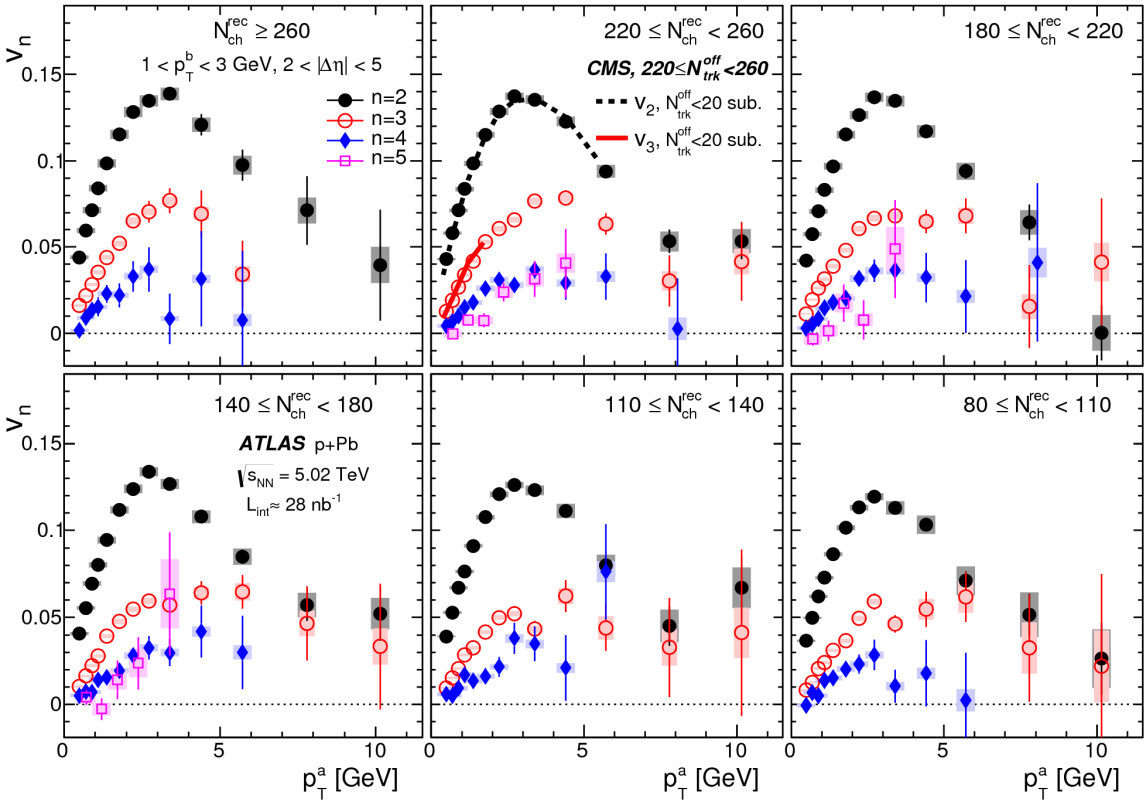


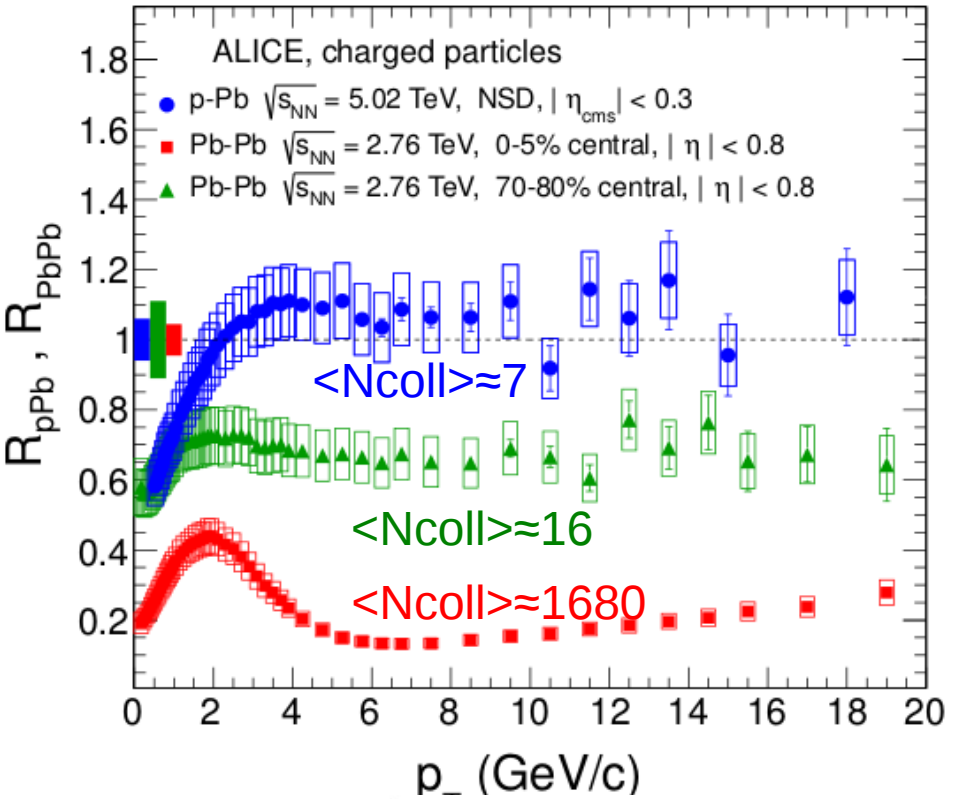
27 dN/dη measurements

ALICE, arXiv:1412.6828

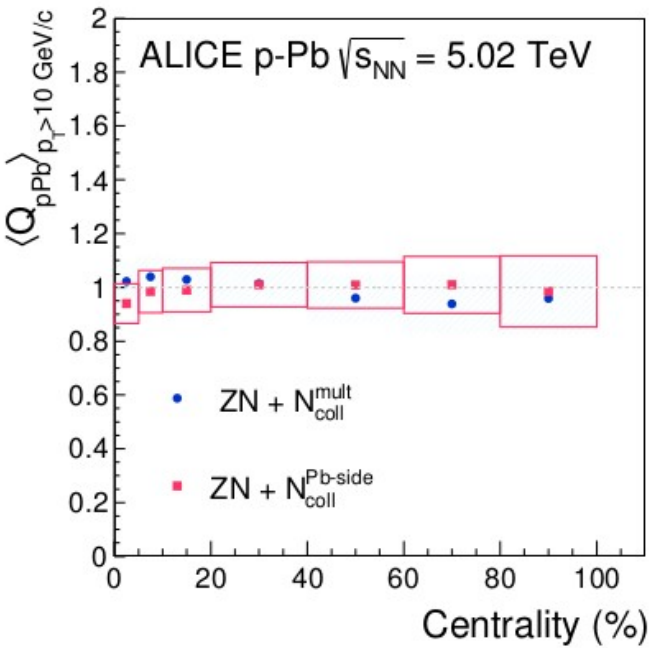


28 Wrt discussion of this morning





0-5% ZNA selection corresponds to roughly 2xminbias multiplicity, say roughly 100 tracks for ATLAS or CMS
QpPb close to 1 (but of course with uncertainty), but v2 non-zero!
Result in PbPb at Ncoll~16?



29 Conclusions

- Question of “bias vs no-bias” in general has no definite answer
- Systematics of centrality measurement and interpretation of data must be done in the same framework
- Using the hybrid approach avoids the bias (but at expense of limited dynamical range)

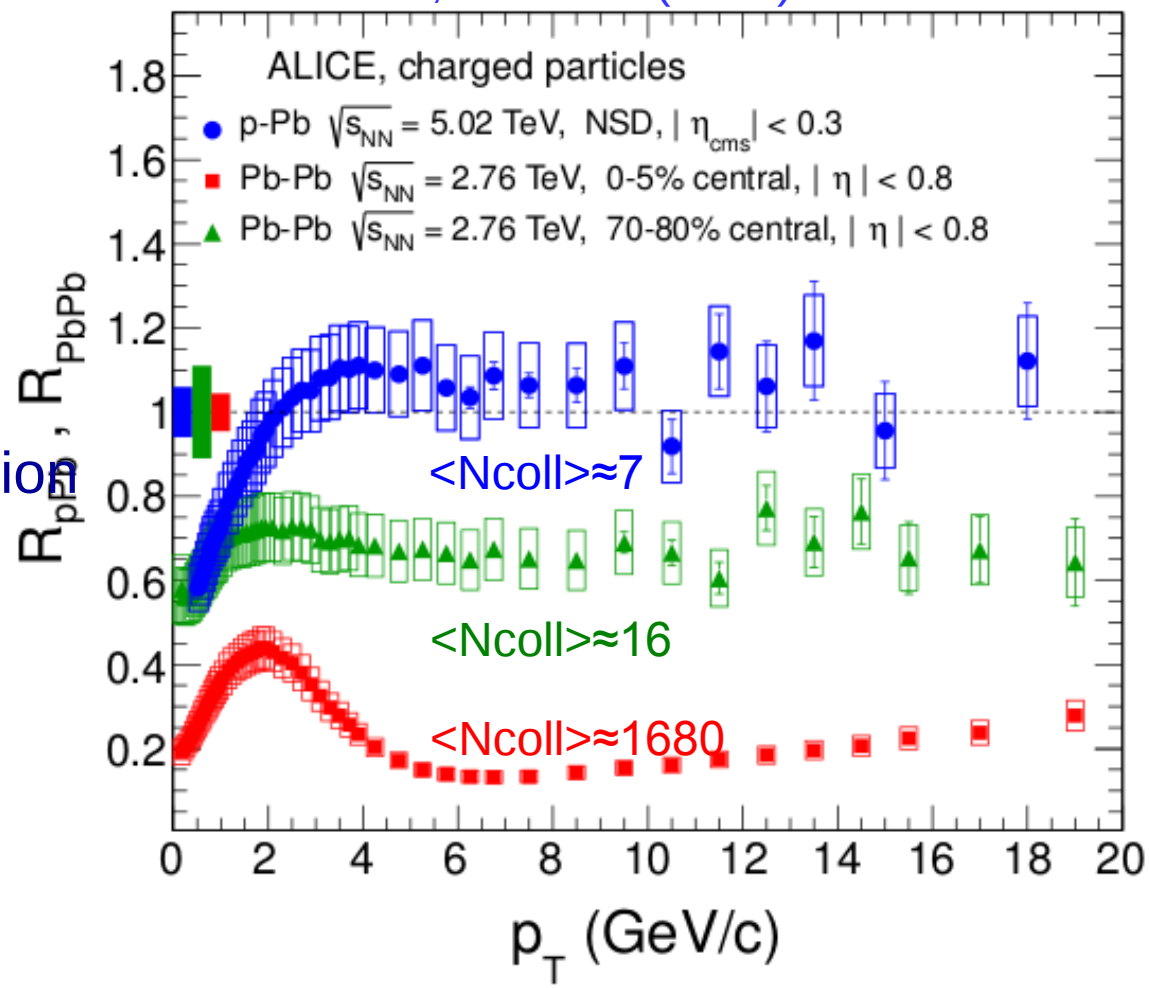
30 Extra

31 Centrality dependent nuclear modification

$$R_{AB} = \frac{\mathrm{d}N_{AB}/\mathrm{d}p_{\mathrm{T}}}{\langle N_{\mathrm{coll}}\rangle \mathrm{d}N_{\mathrm{pp}}/\mathrm{d}p_{\mathrm{T}}}$$

- $\langle N_{coll}\rangle = A\,\sigma_{pp}/\sigma_{pA} \approx 7$ with
  - σ_{pp} = 70 mb from interpolation of existing data
  - σ_{pA} = 2090 ± 120 mb from LHCb-CONF-2012-034 (or use Glauber)
- Note: $\langle N_{coll}\rangle \approx 15$ is reached in "0-5% central" pPb collisions

ALICE, PRL 110 (2013) 082302

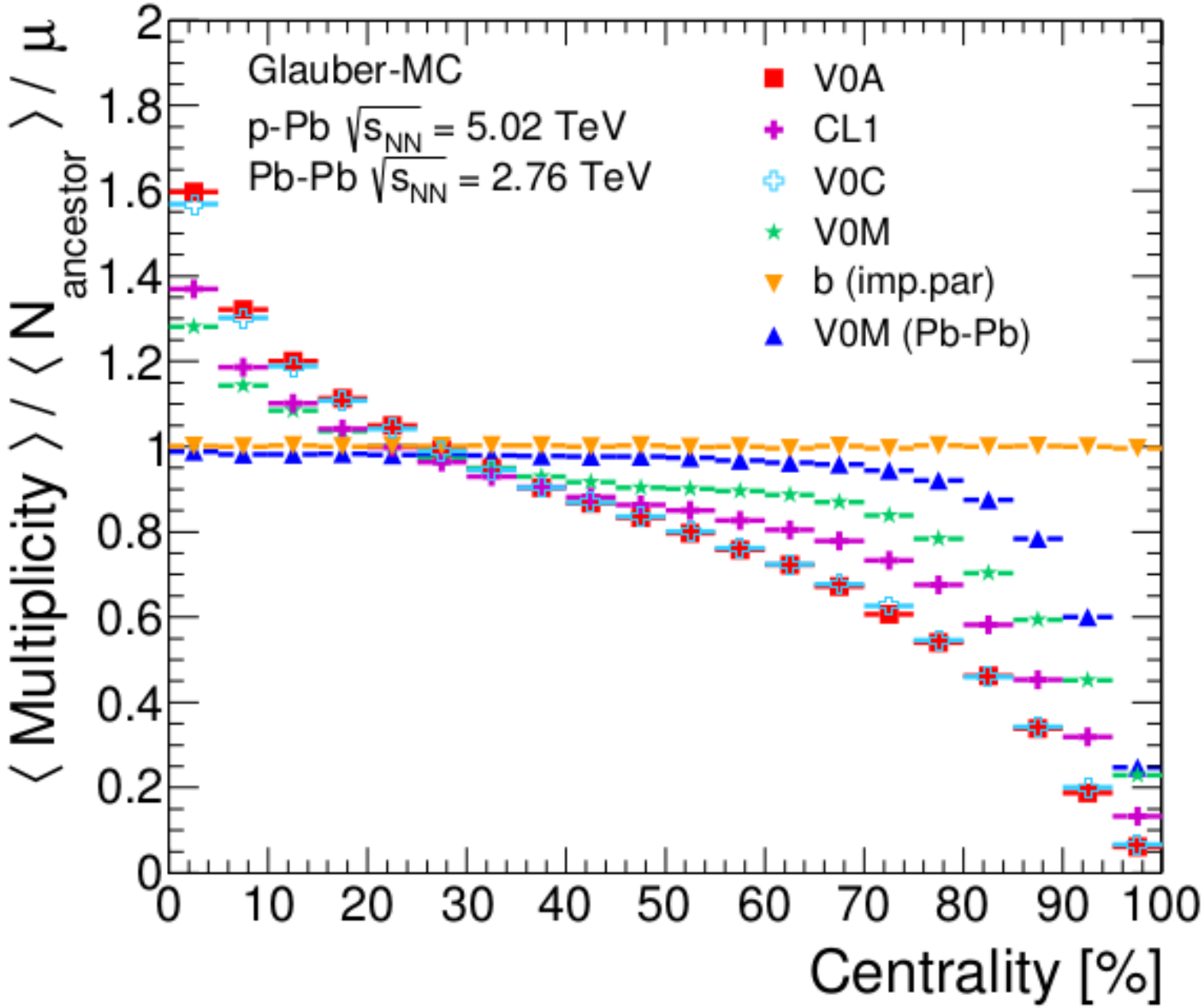


How to perform a centrality dependent measurement?

$$R_{\mathrm{pA}}^{\mathrm{cent}}(p_{\mathrm{T}}) = \frac{\mathrm{d}N^{\mathrm{pA}}/\mathrm{d}p_{\mathrm{T}}}{\langle T_{\mathrm{pA}}^{\mathrm{cent}}\rangle \mathrm{d}\sigma^{\mathrm{pp}}/\mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}N^{\mathrm{pA}}/\mathrm{d}p_{\mathrm{T}}}{\langle N_{\mathrm{coll}}^{\mathrm{cent}}\rangle \mathrm{d}N^{\mathrm{pp}}/\mathrm{d}p_{\mathrm{T}}}$$

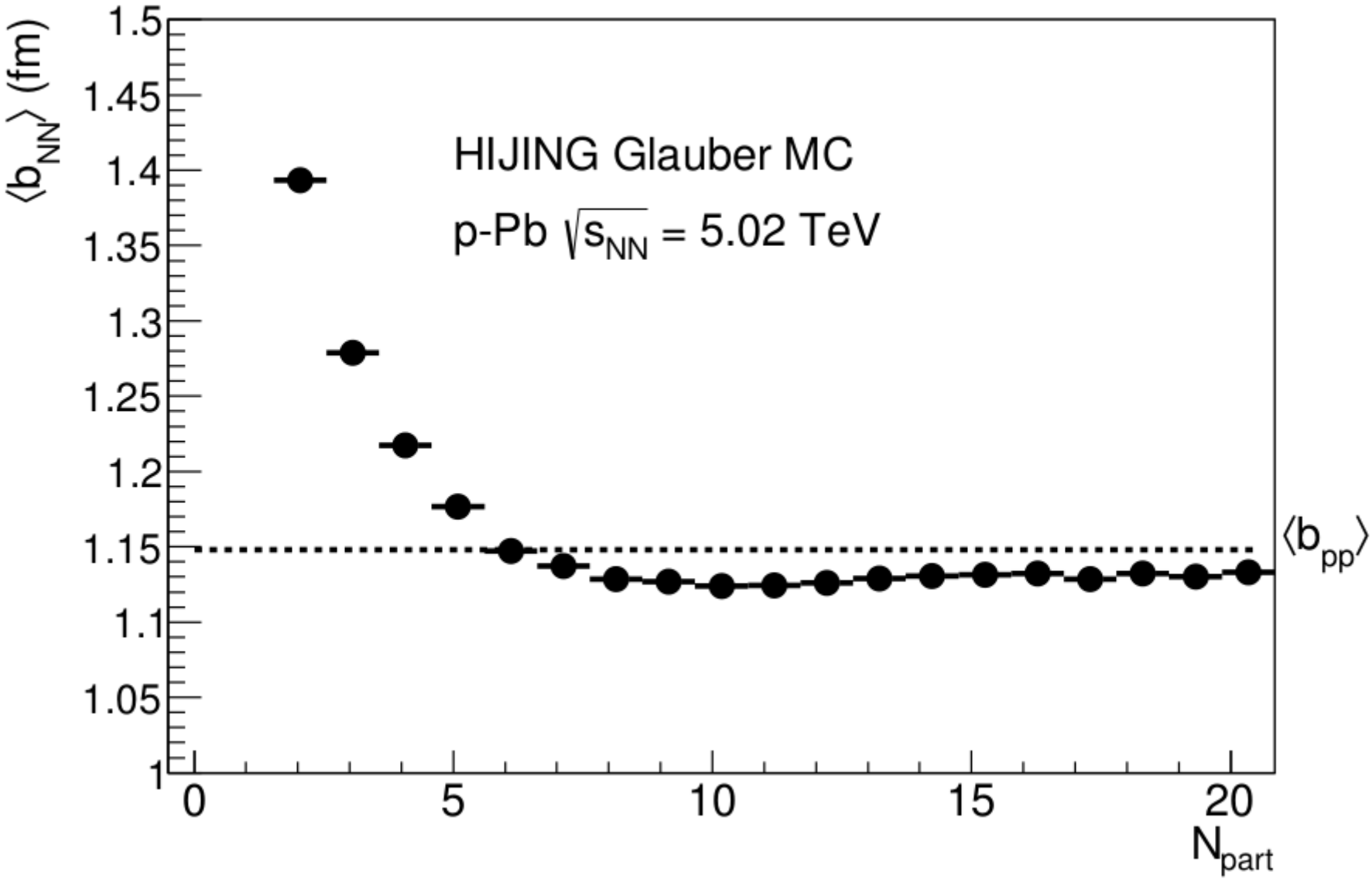
32 Multiplicity bias

ALICE, arXiv:1412.6828



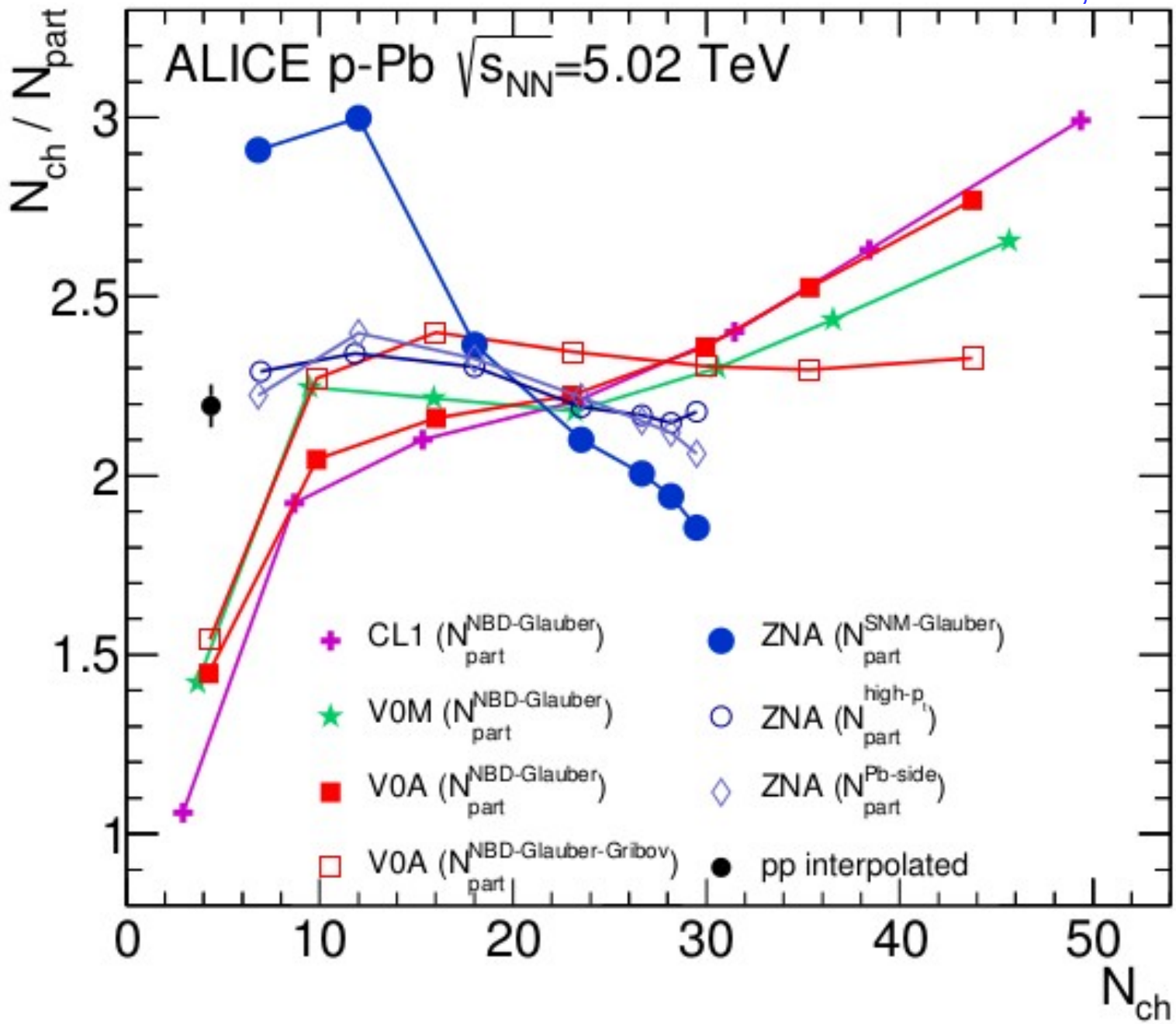
33 Geometry bias

ALICE, arXiv:1412.6828



34 Multiplicity scaled by different Npart

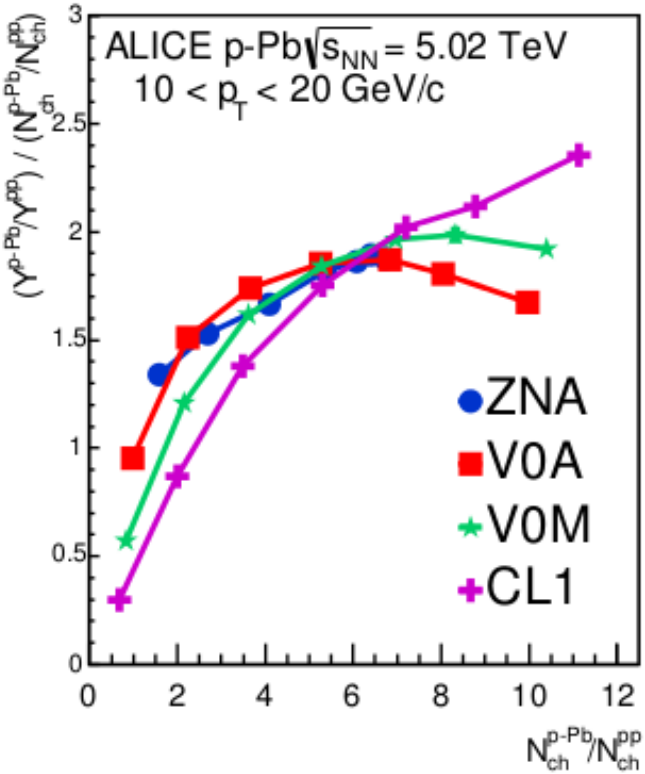
ALICE, arXiv:1412.6828



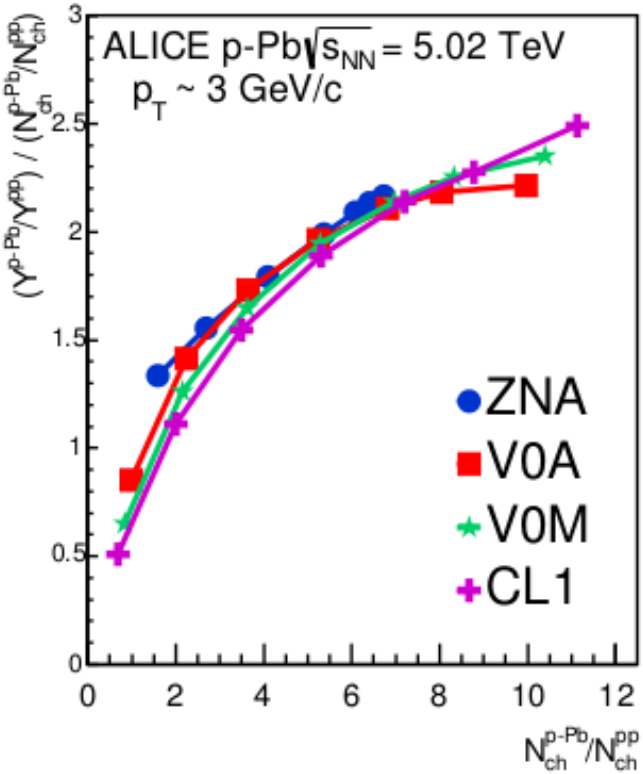
35 Cronin and high-p_T region vs Nch

ALICE, arXiv:1412.6828

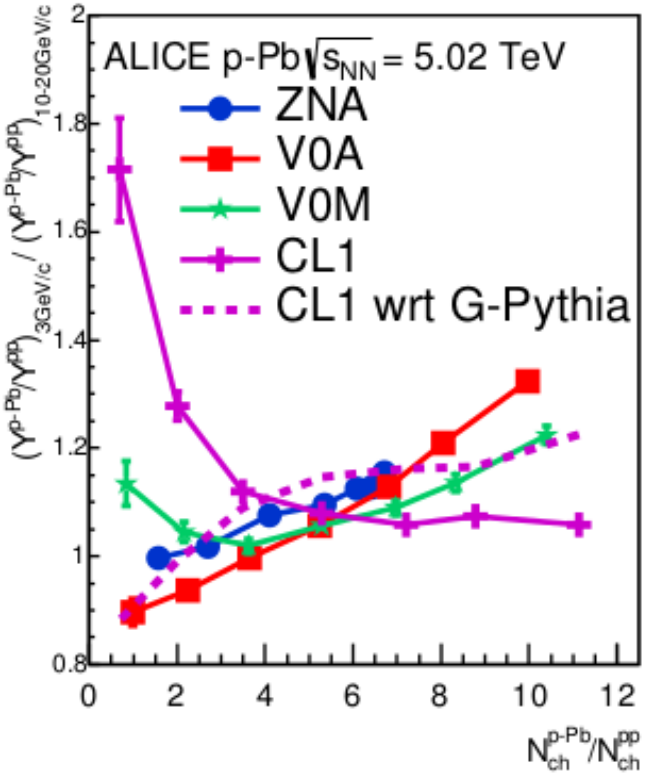
High p_T region



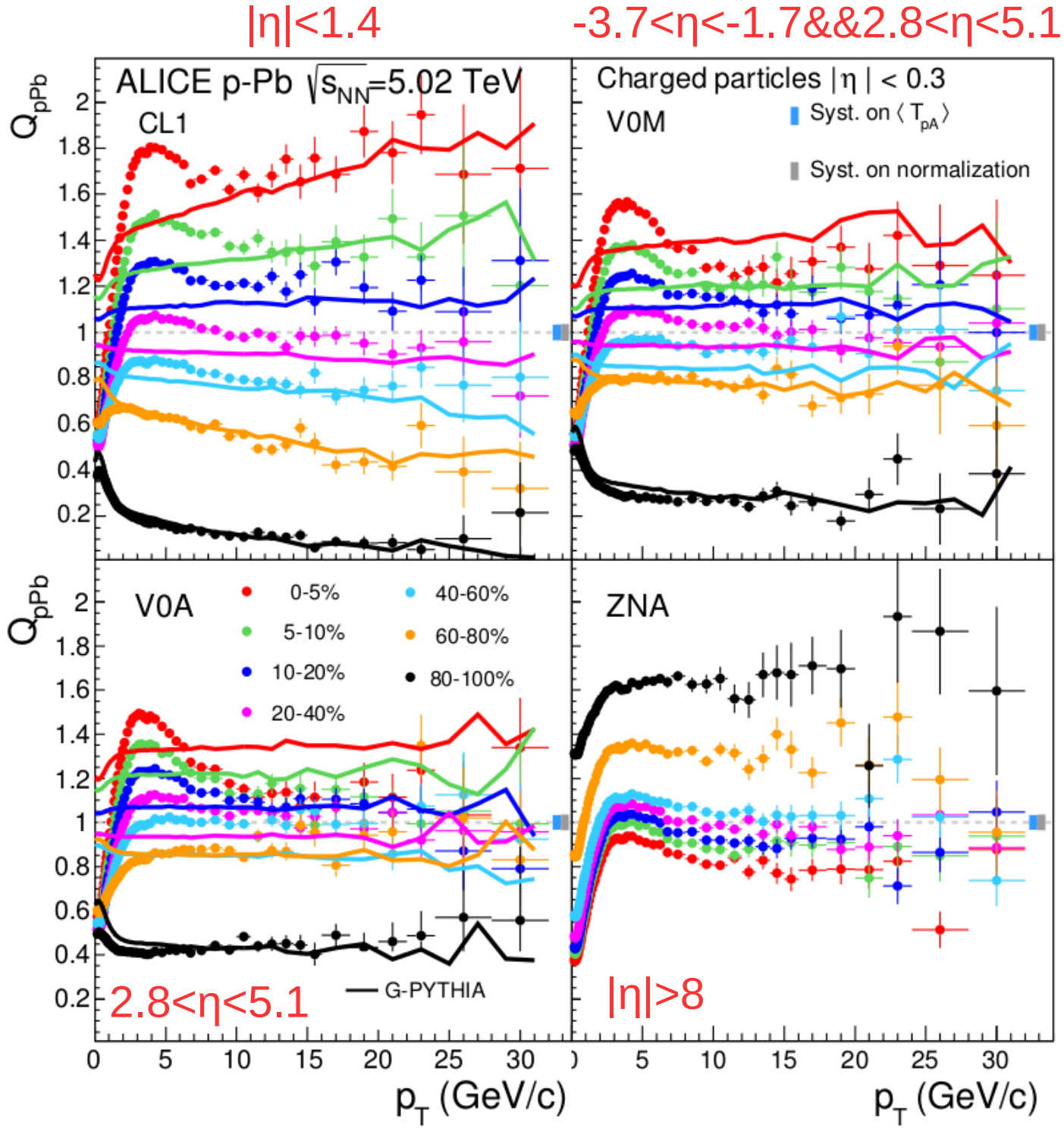
Peak region



Ratio peak / high p_T

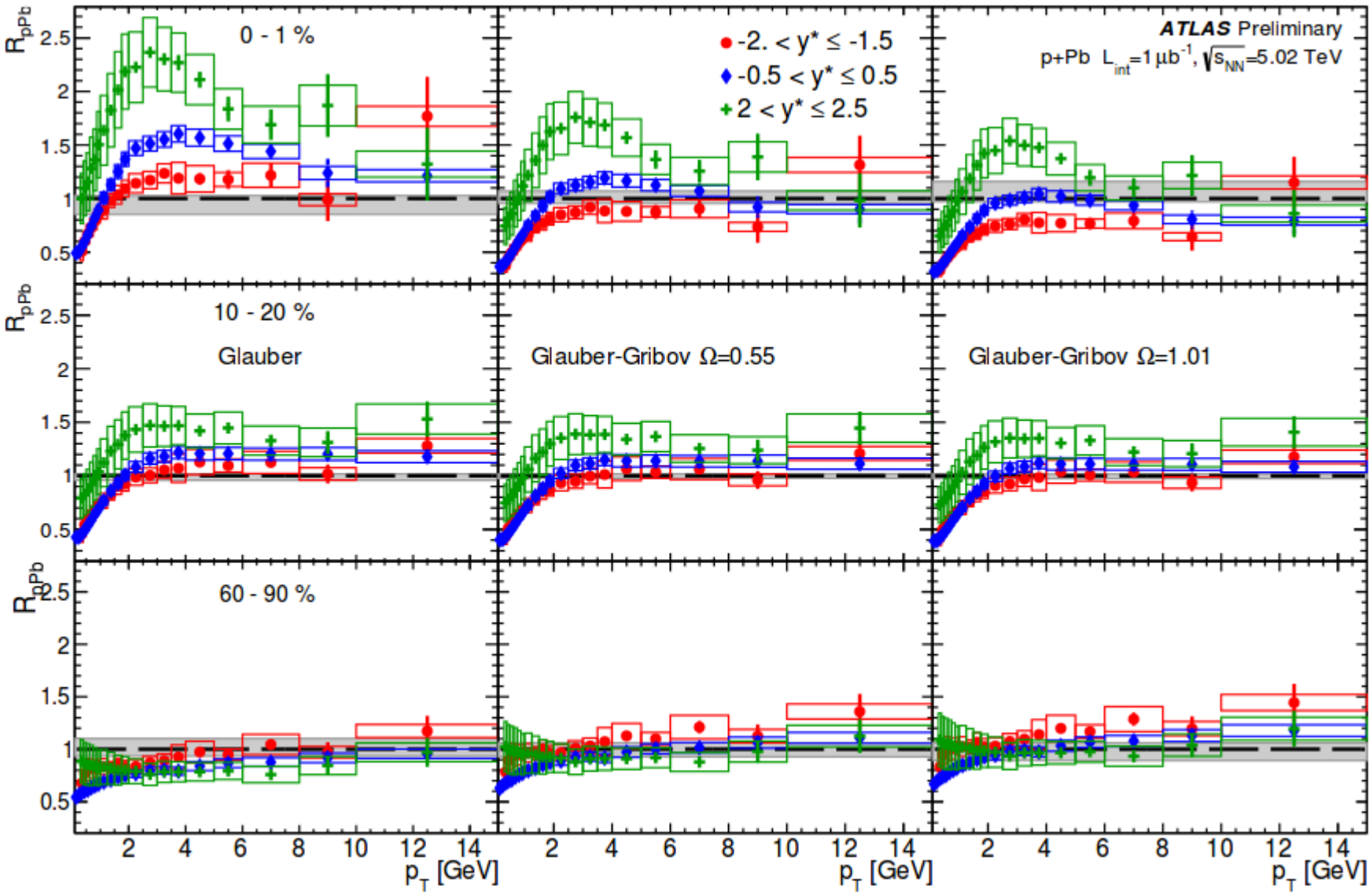


ALICE, arXiv:1412.6828



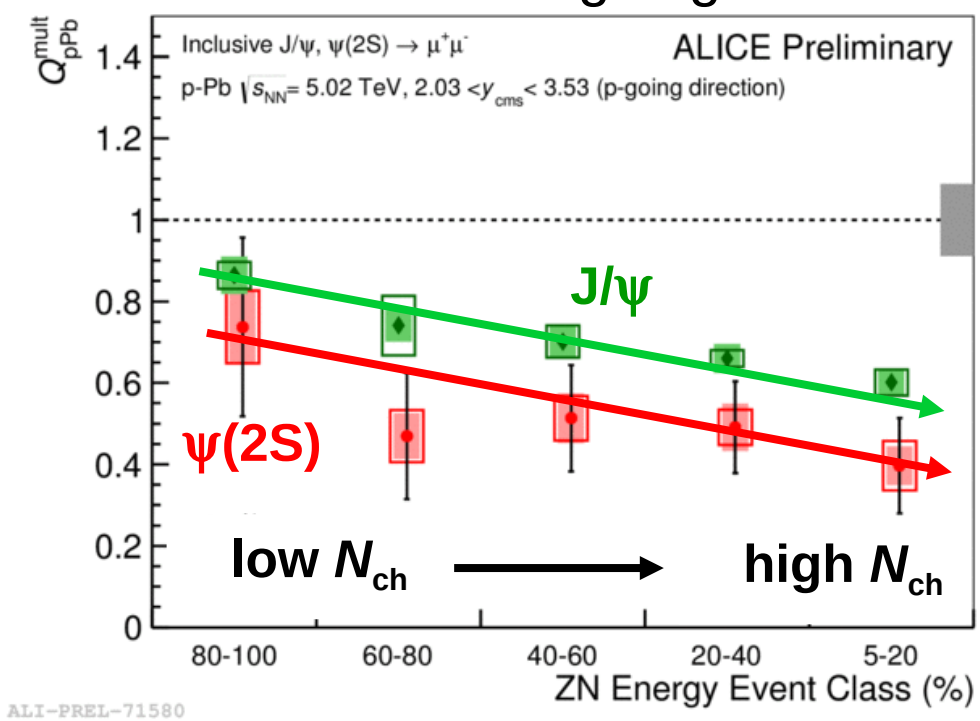
37 RpPb measurement

ATLAS-CONF-2013-107

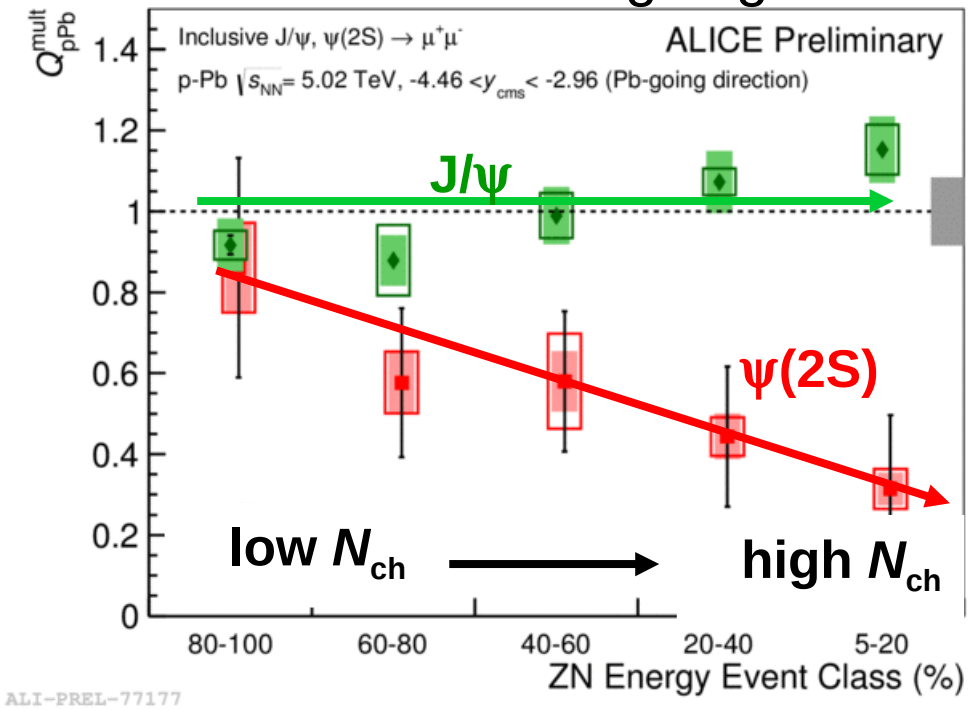


38 J/Ψ and Ψ(2S) suppression

Forward going



Backward going



- J/ψ → μμ: Multiplicity dependent suppression in p-going direction, and no suppression in Pb-going direction
  - Consistent with shadowing
- ψ(2S) → μμ: Multiplicity dependent suppression in both directions
  - Needs additional effect (Final state?)

Possibility of Superfluidity in p-A and p-p collisions at the LHC

Abhijit Majumder,
Derek Everett, Chris Zin, and Sean Gavin
Wayne State University

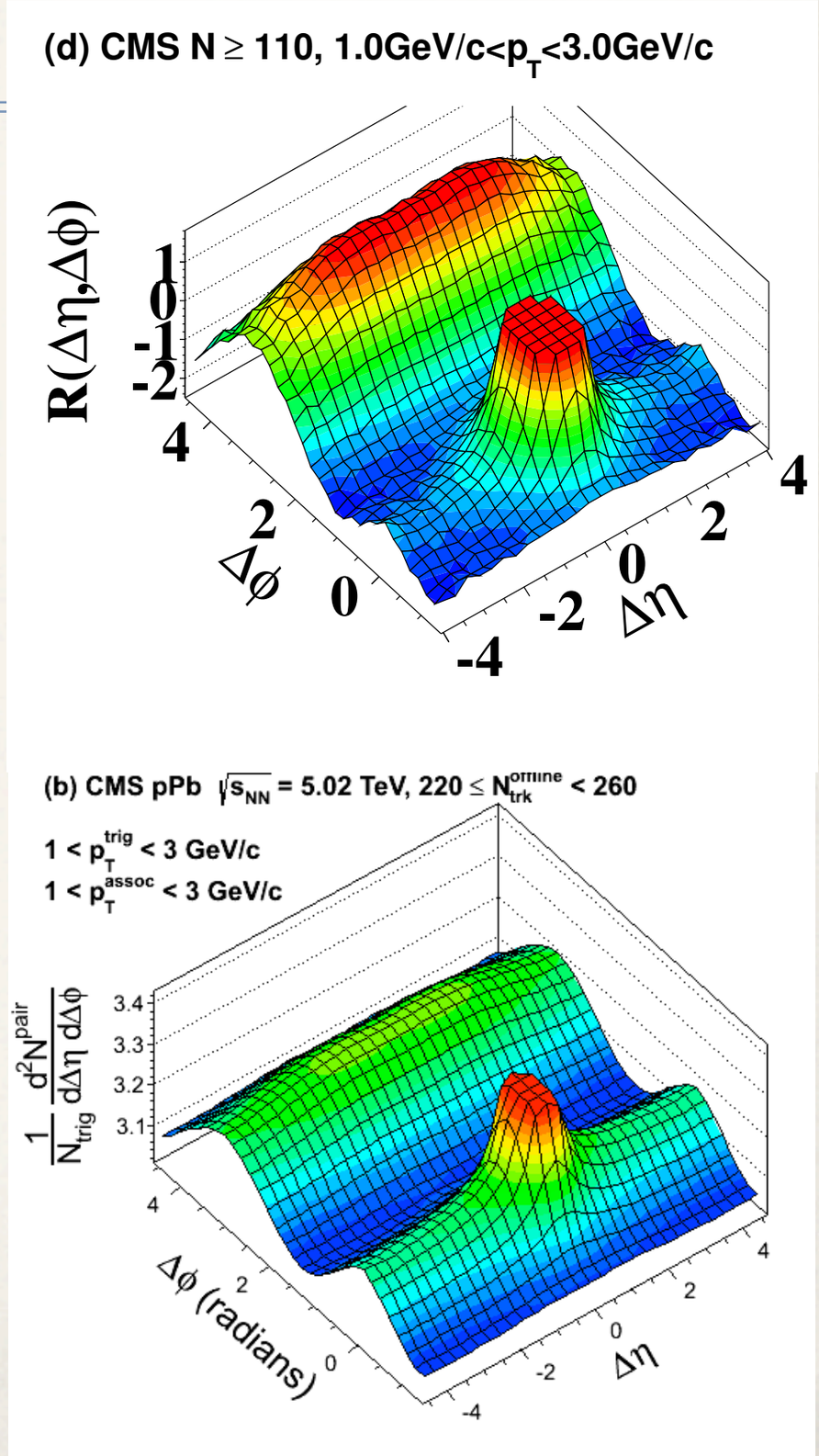
Riken BNL workshop on collectivity in small systems, Mar 4-6, 2015

Outline

- Motivation for p-p, p-A, (and also A-A)
- Hard and soft multiplicity fluctuations in p-A
- Behavior of initial state in $\lambda\phi^4$ theory
- Simulations....approximations
- Expected behavior in SU(2) gauge theory
- Conclusions and discussion

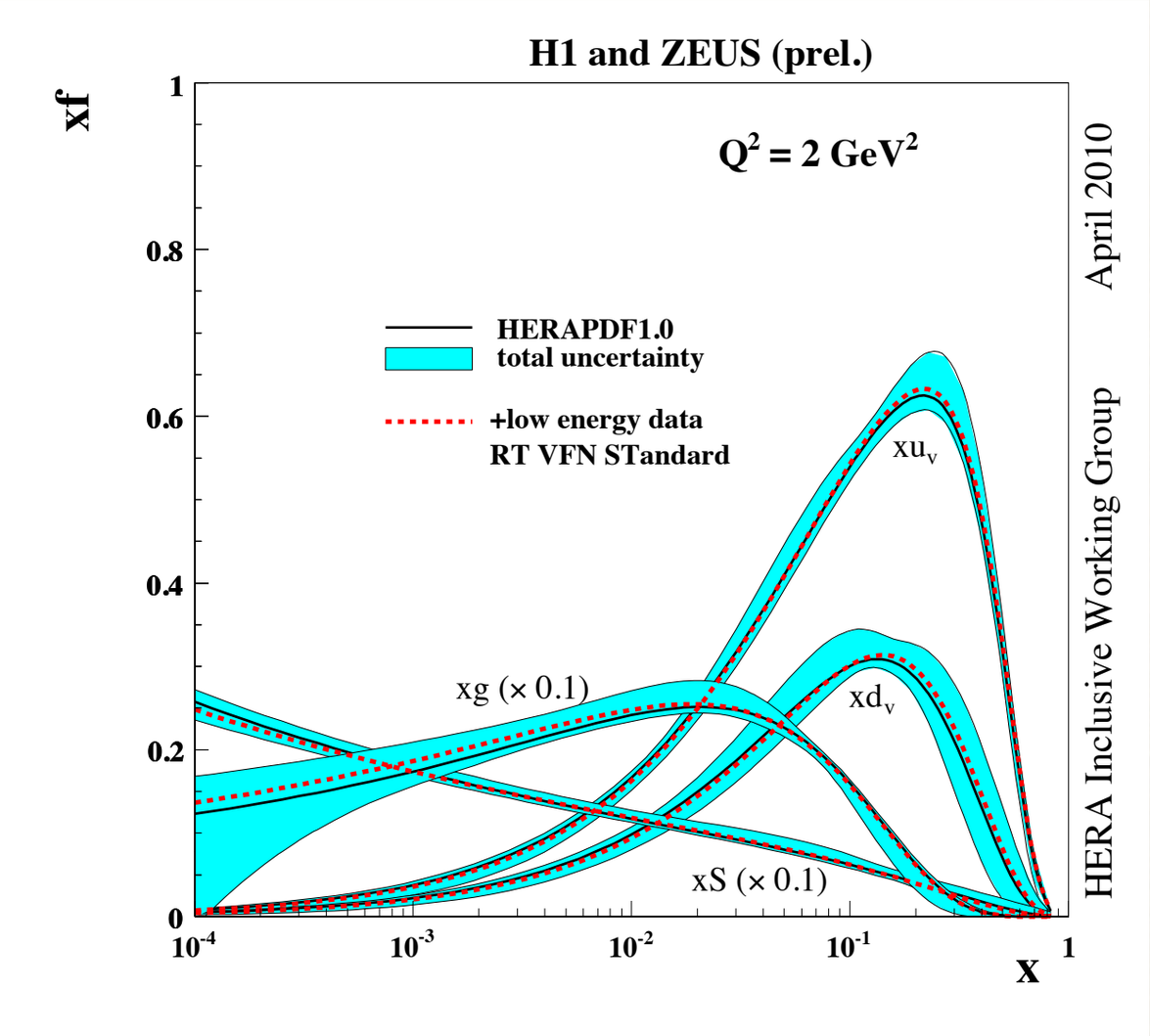
Motivation in p-p, p-A, and in A-A

- Possible collective behavior in high multiplicity p-p and p-A collisions
- Requirement of rapid thermalization in A-A ($\tau_0 \sim 0.5$ fm/c)
- Is it possible to have something like pressure driven flow but with a delayed thermalization?



Gluon density in initial state

- Mean no. of gluons very high in the initial state
- CGC / Glasma initial state
- But there could also be **large** fluctuations around this mean



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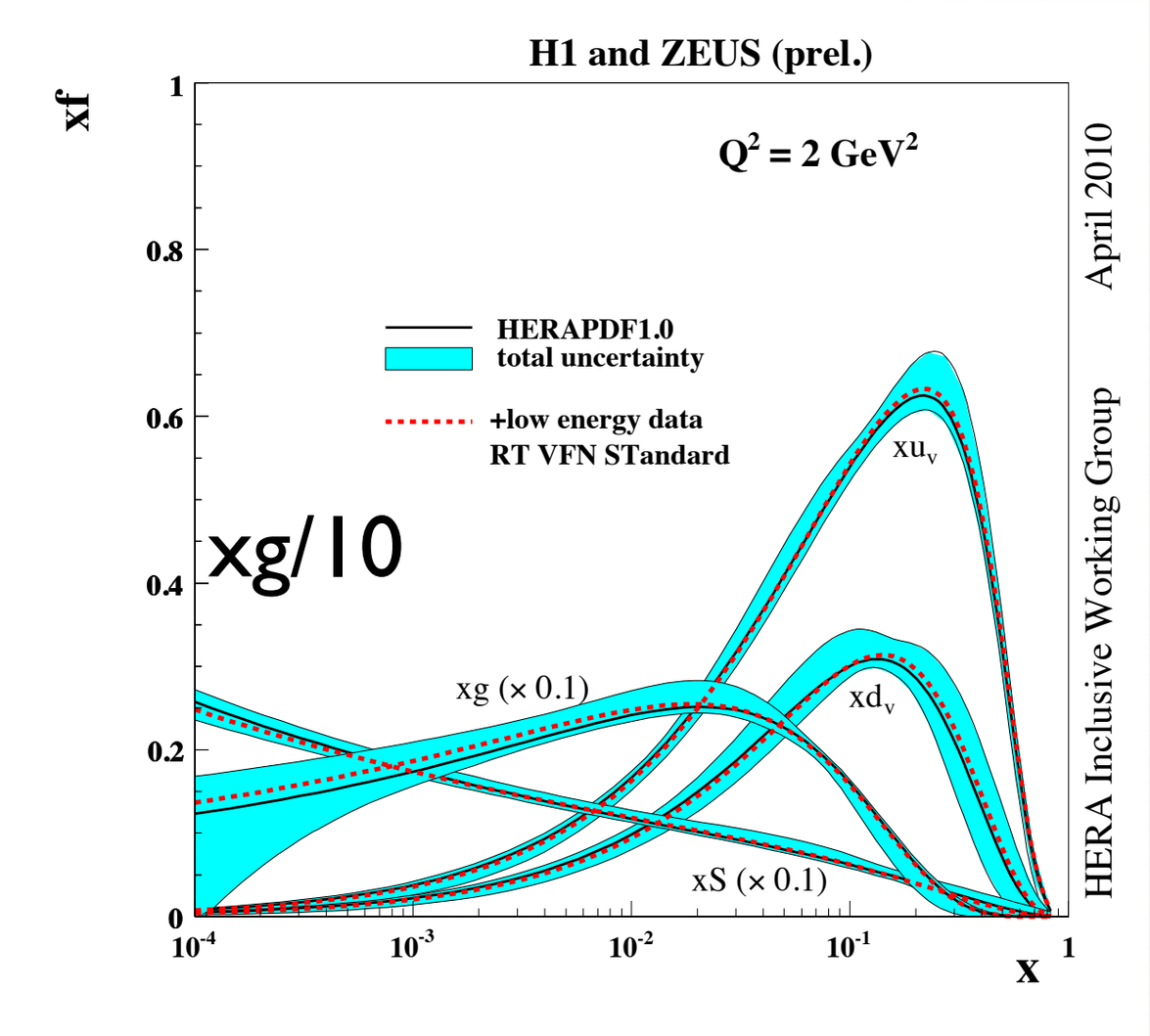
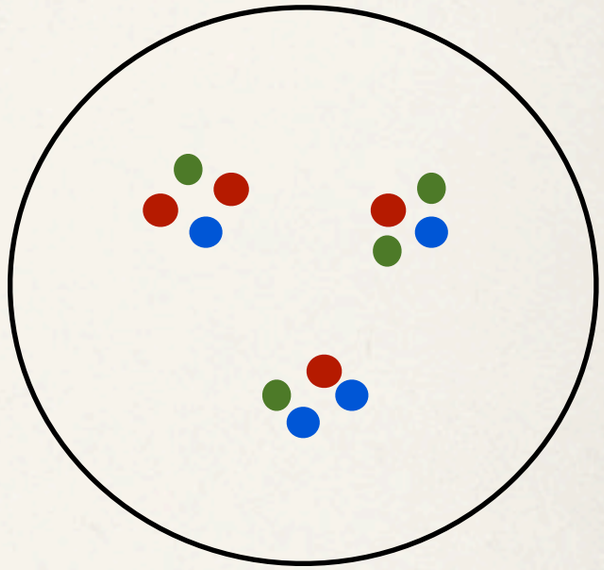
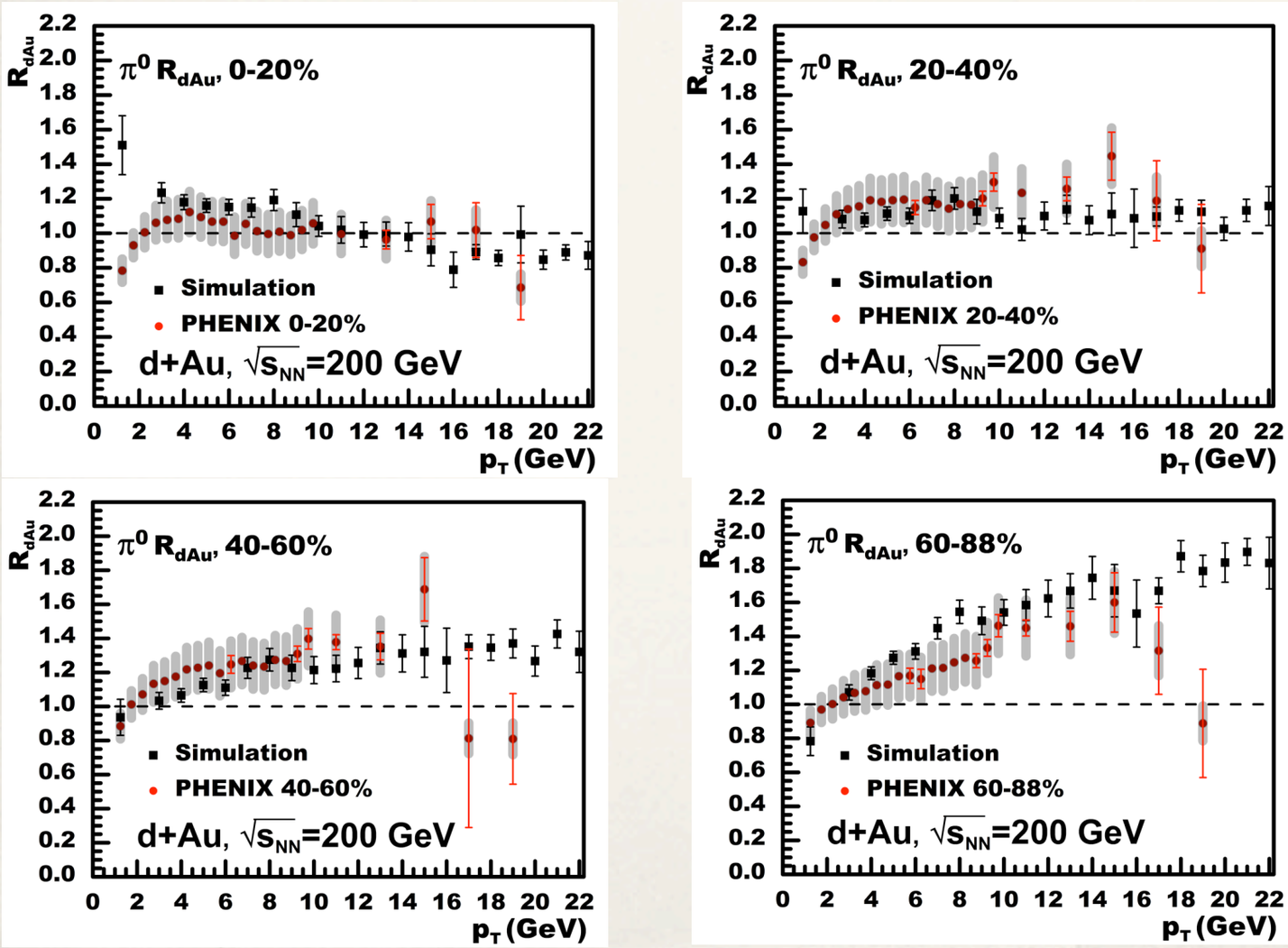
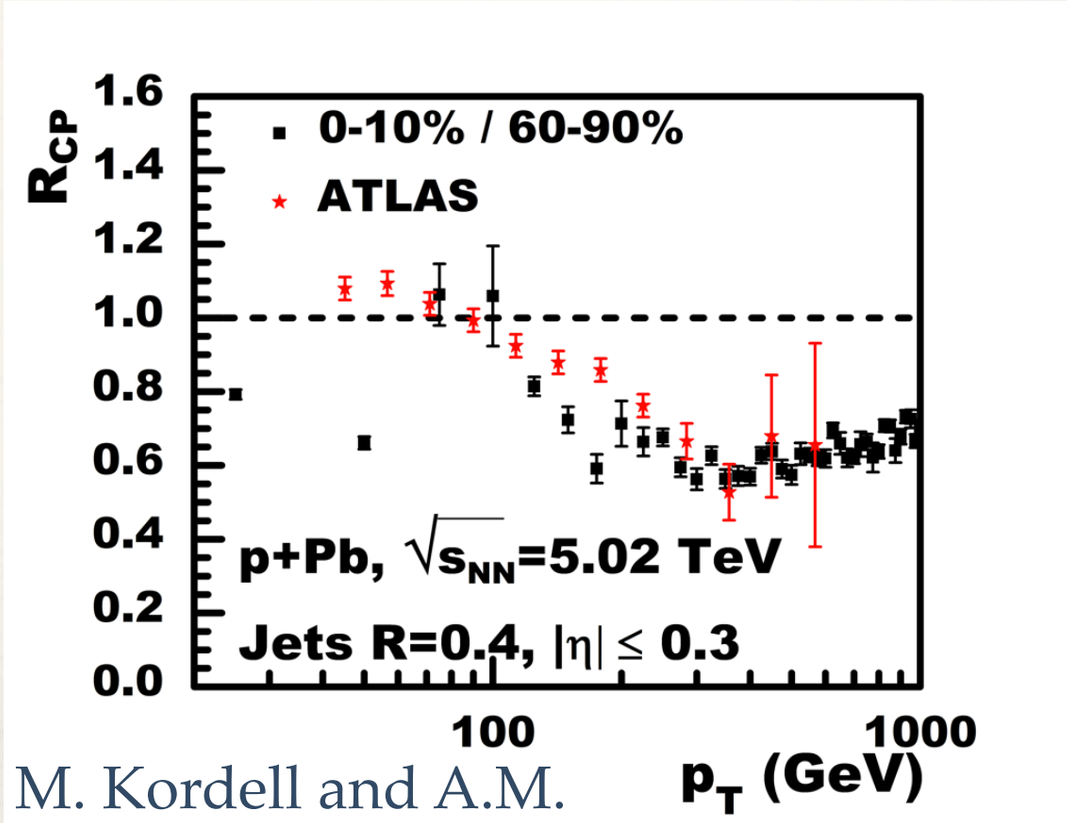


Trigger on a hard jet: high energy/low density partons

- Good evidence from ATLAS and PHENIX that a proton can fluctuate to a small object with few very high energy partons.



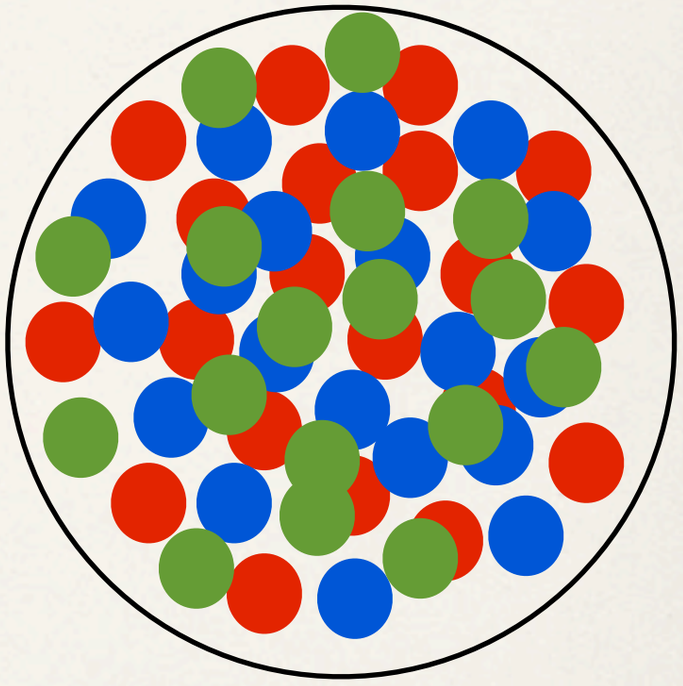




M. Kordell and A.M.

Trigger on a high multiplicity event: low energy/high density partons

- The opposite can also be true!
- There could be large fluctuations into a denser phase, which are captured by high-multiplicity events
- CGC calculations give you the mean event: There could be fluctuations to a denser state.



Comparison to previous work

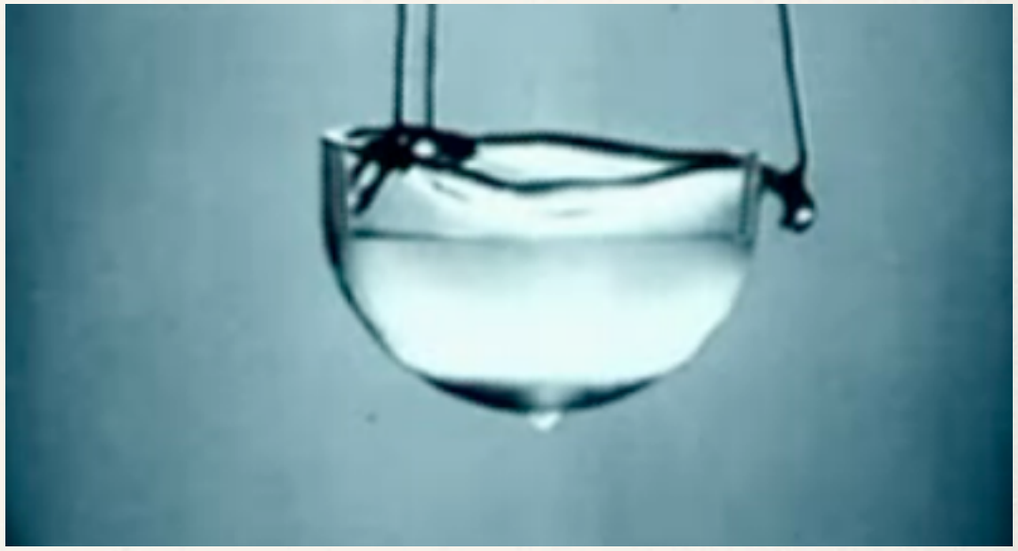
- Dusling, Epelbaum, Gelis, Venugopalan
  (focussed on post condensate phase, instability induced thermalization of fluctuations)
- Liao, Blaizot, Mclerran, Gelis
  Kinetic thermalization, followed by BEC (delayed condensation)
- Coleman-Smith and Mueller
  Proton fluctuates to a large size, followed by hydro-dynamics

An initial state with a very high density of gluons (bosons)

- Initial state formed by fusion of space-like (virtual) gluons into a light-like (real) gluon.
- Similar to the GLASMA (note: mean state!)
  [This work is not predicated on the presence of the CGC]
- Several low momentum states up to $Q \sim Q_s$ are now over occupied
- Gluons begin to appear after $\tau \sim 1/Q$
- Macroscopic occupation of some momentum states
- One condition for a super-fluid fulfilled

2 conditions for a super-fluid

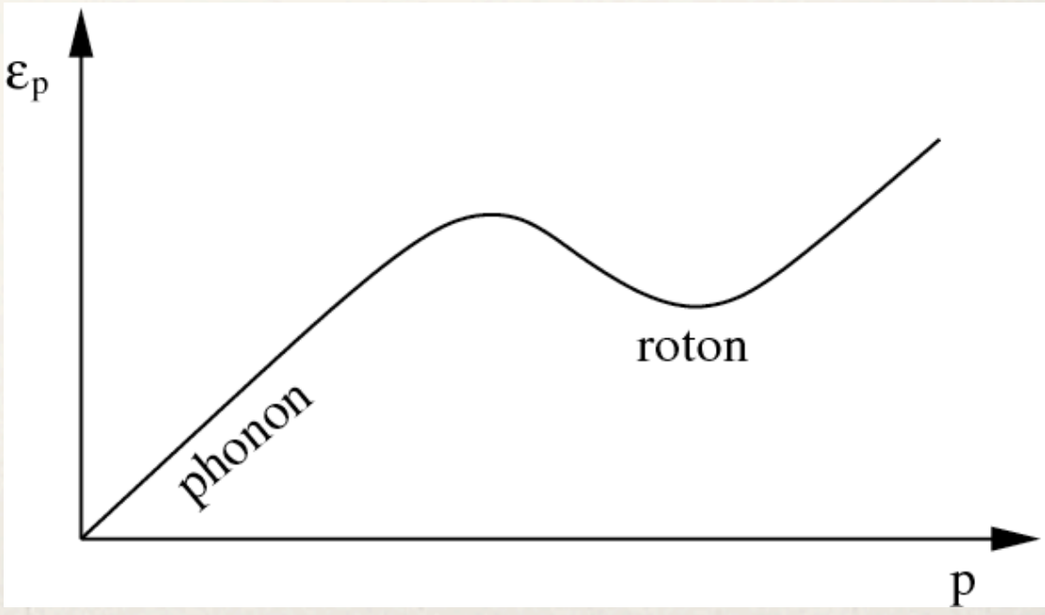
- Macroscopic occupation of a few momentum states
- Dispersion relation of excitations that makes it hard to excite.
- Landau condition



$$E' = E_0 + \epsilon(p) + \vec{p} \cdot \vec{v} + \frac{Mv^2}{2}$$

$$\vec{P}' = \vec{p} + M\vec{v}$$

$$\epsilon(p) + \vec{p} \cdot \vec{v} < 0$$



Static (NR) condensate

$$H = \sum_p \frac{p^2}{2m} a_p^\dagger a_p + \frac{V}{2} \sum_{p_1,p_2,q_1,q_2} a_{p_1}^\dagger a_{p_2}^\dagger a_{q_1} a_{q_2} \delta_{p_1+p_2-q_1-q_2,0}$$

- Bogoliubov transformation
  Separate p=0 from other p $\neq$ 0 operators
  replace $a_0, a_0^\dagger \to \sqrt{N}$

- Diagonalize the Hamiltonian for p $\neq$ 0 states.

$$\epsilon(p) = Cp + \cdots$$

Dynamic (non-uniform) NR condensate

- Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi = \left(\frac{\hbar\nabla^2}{2m} + g|\psi|^2\right)\psi$$

introduction of phase of condensate

$$\psi = \rho e^{i\theta}$$

gradient of phase gives velocity

$$\vec{v}_s = \frac{\hbar\vec{\nabla}\theta}{m}$$

Potential hydrodynamics

$$m\frac{\partial j_i}{\partial t} + \frac{\partial \Pi_{ik}}{\partial x_k} = 0$$

Relativistic scalar condensate

following Alford et. al.

- Charged scalar theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2} m^2 \phi^* \phi - \frac{1}{4}(\phi^* \phi)^2$$

Note: no spontaneous symmetry breaking

- Introduce a large condensate by hand.

$$\phi = e^{i\theta}\left(\rho + \phi_1 + i\phi_2\right)$$

- Separate the Lagrangian into a condensate part and terms quadratic, cubic and quartic in the fluctuation.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$$

- Density and velocity of superfluid: $\rho^2 \qquad v_s^i = \dfrac{\partial^i \theta}{\partial^0 \theta}$

Radial flow of condensate

$$\mathcal{L}^{(0)} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{\rho^2}{2}((\partial_\mu \theta)^2 - m^2) - \frac{\lambda}{4}\rho^4$$

$$\partial_\mu \partial^\mu \rho = \rho[(\partial_\mu \theta)^2 - m^2 - \lambda \rho^2]$$

$$\partial_\mu (\rho^2 \partial^\mu \theta) = 0$$

Solving these equations with

λ=0.01

Boost invariant solutions only, calculations in the transverse plain.

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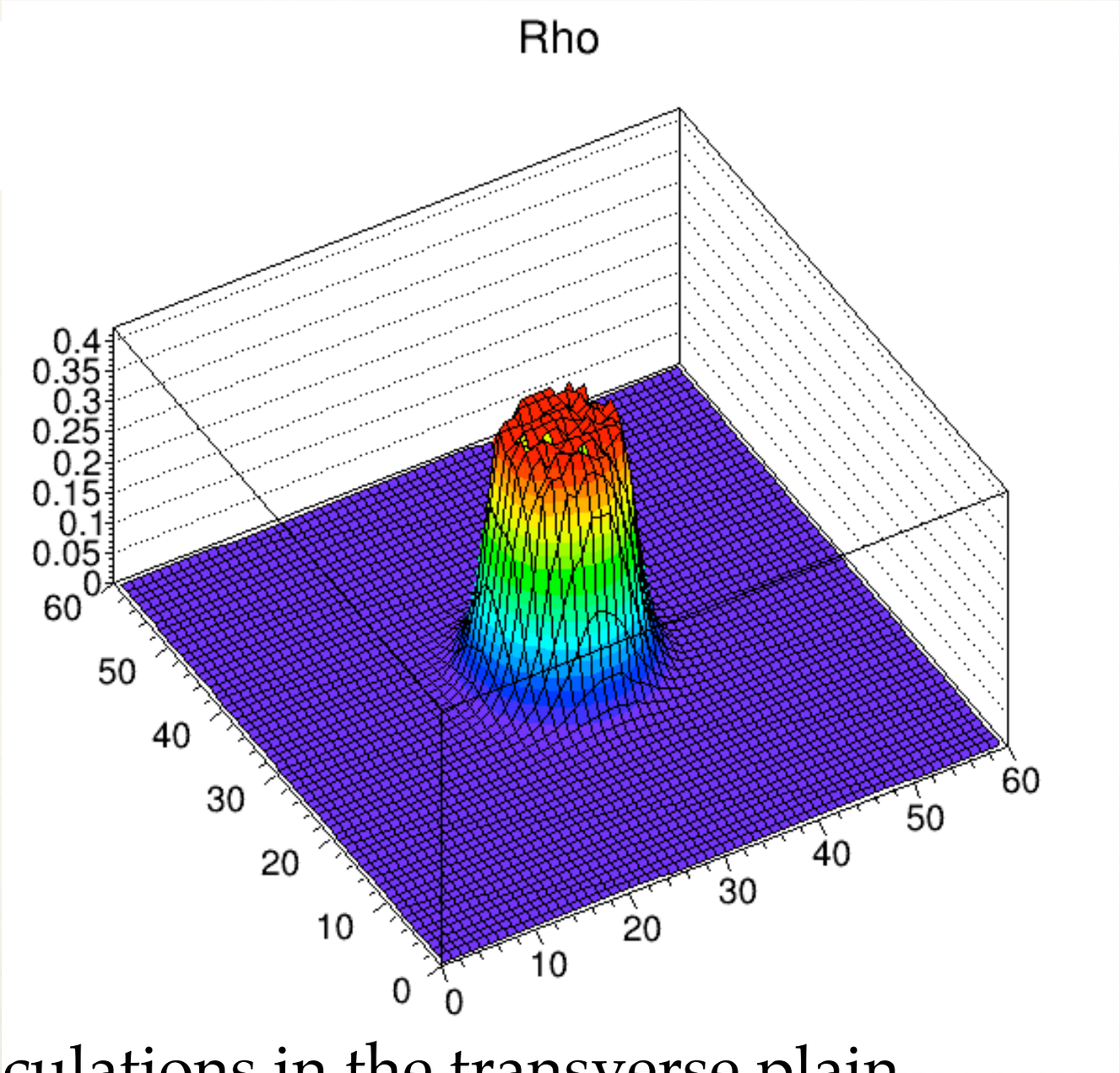
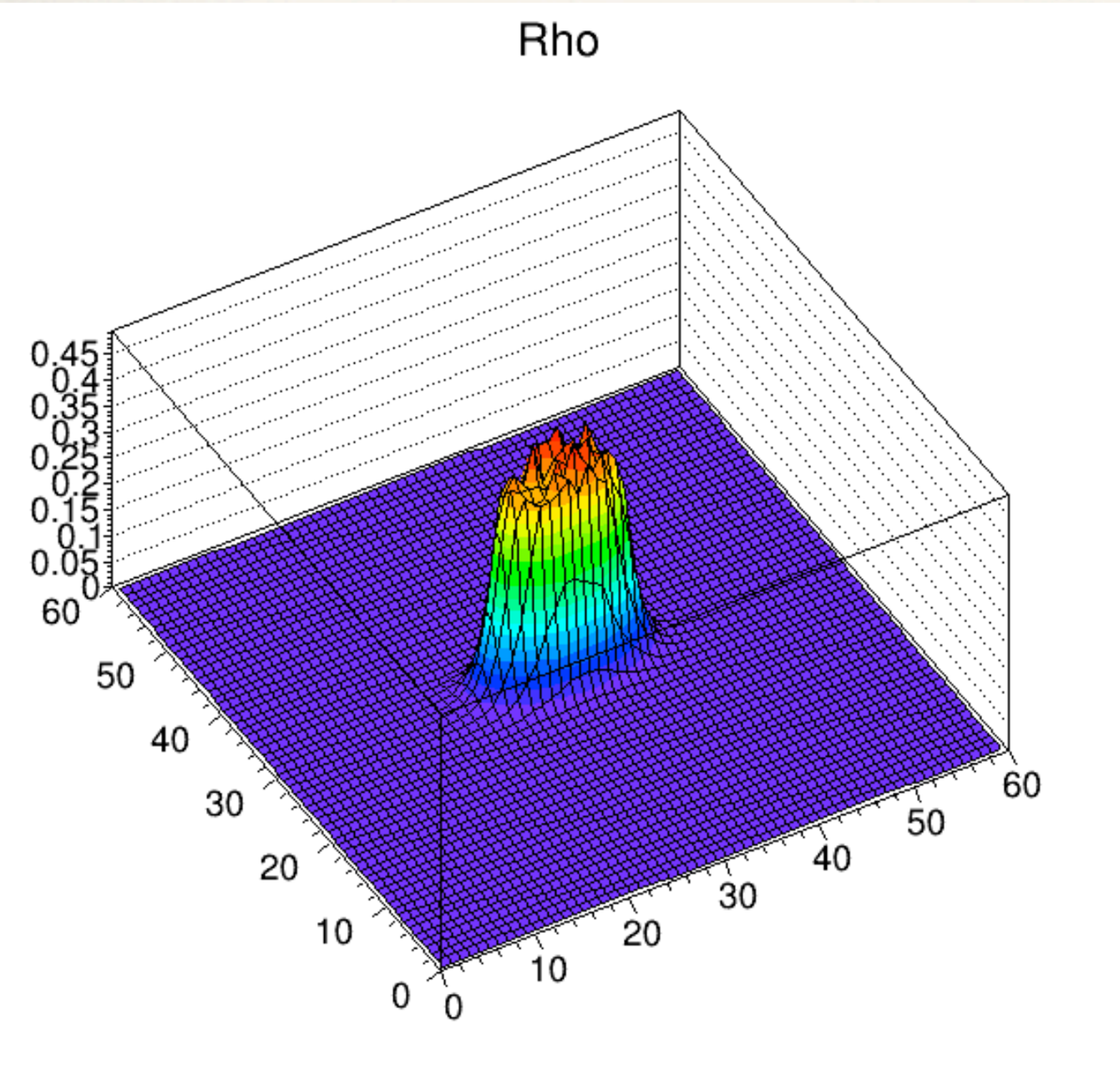


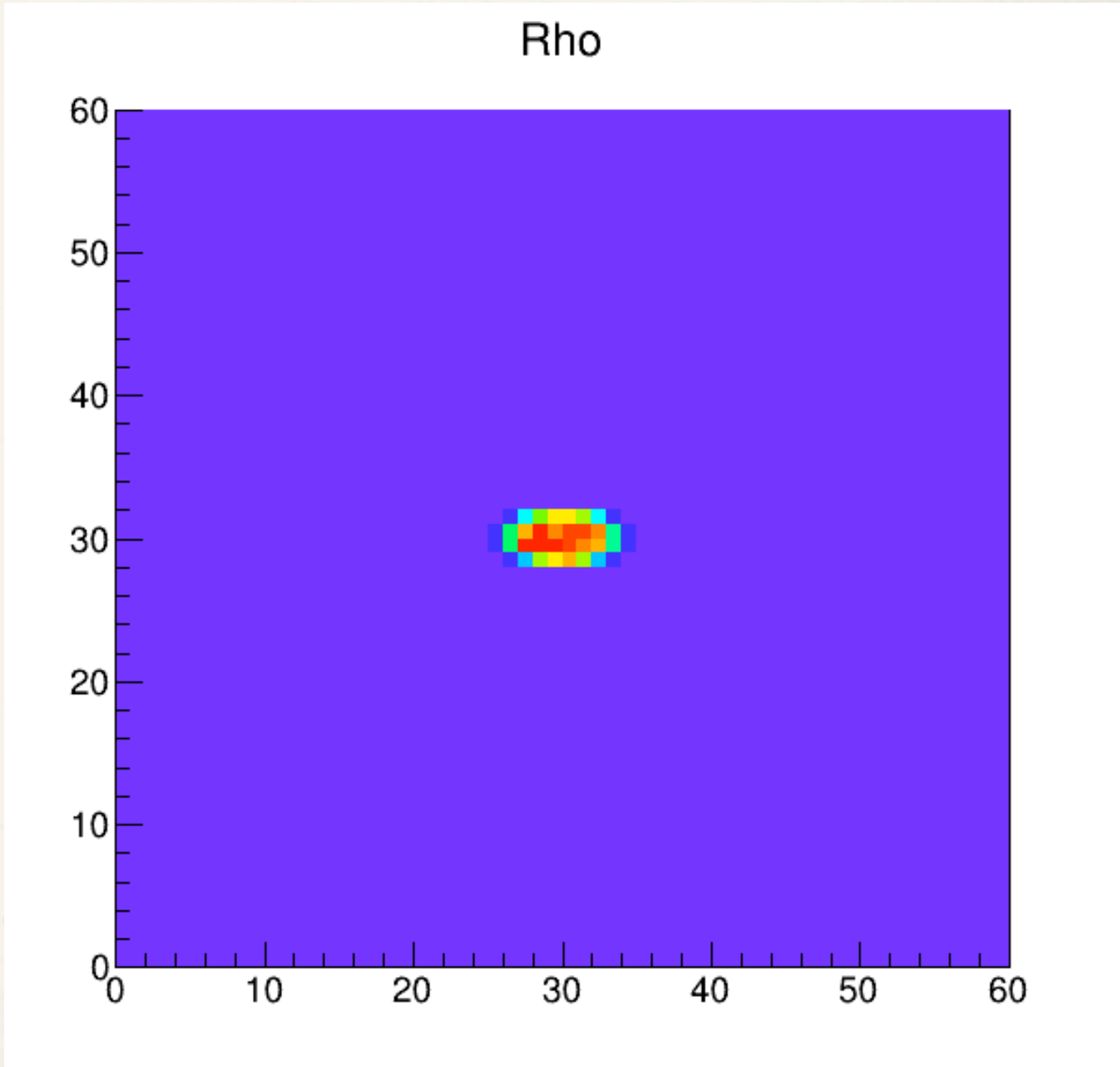
Boost invariant solutions only, calculations in the transverse plain.

Elliptic flow of condensate

Please note: simulations are very very very preliminary

Elliptic flow of condensate





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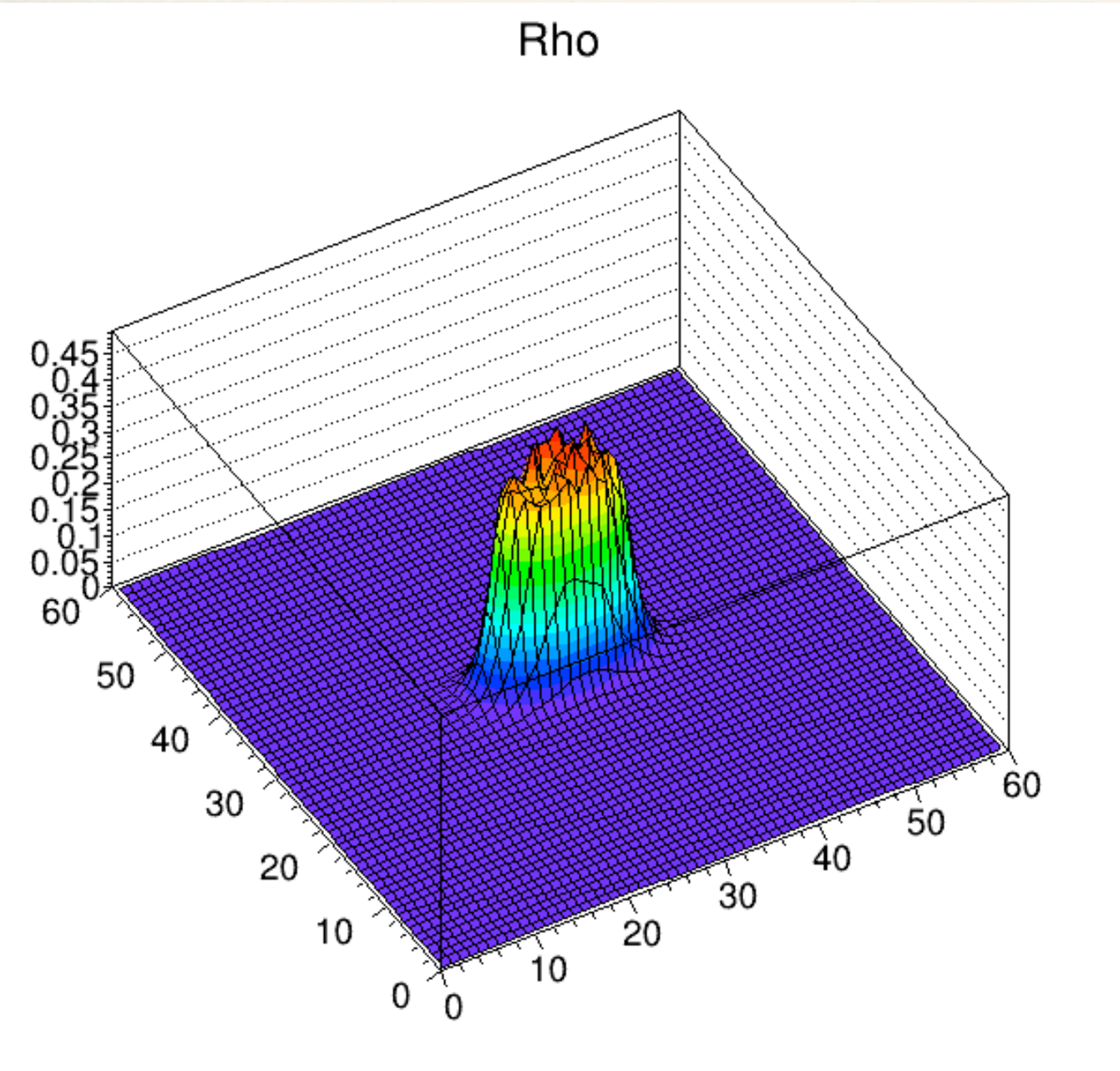
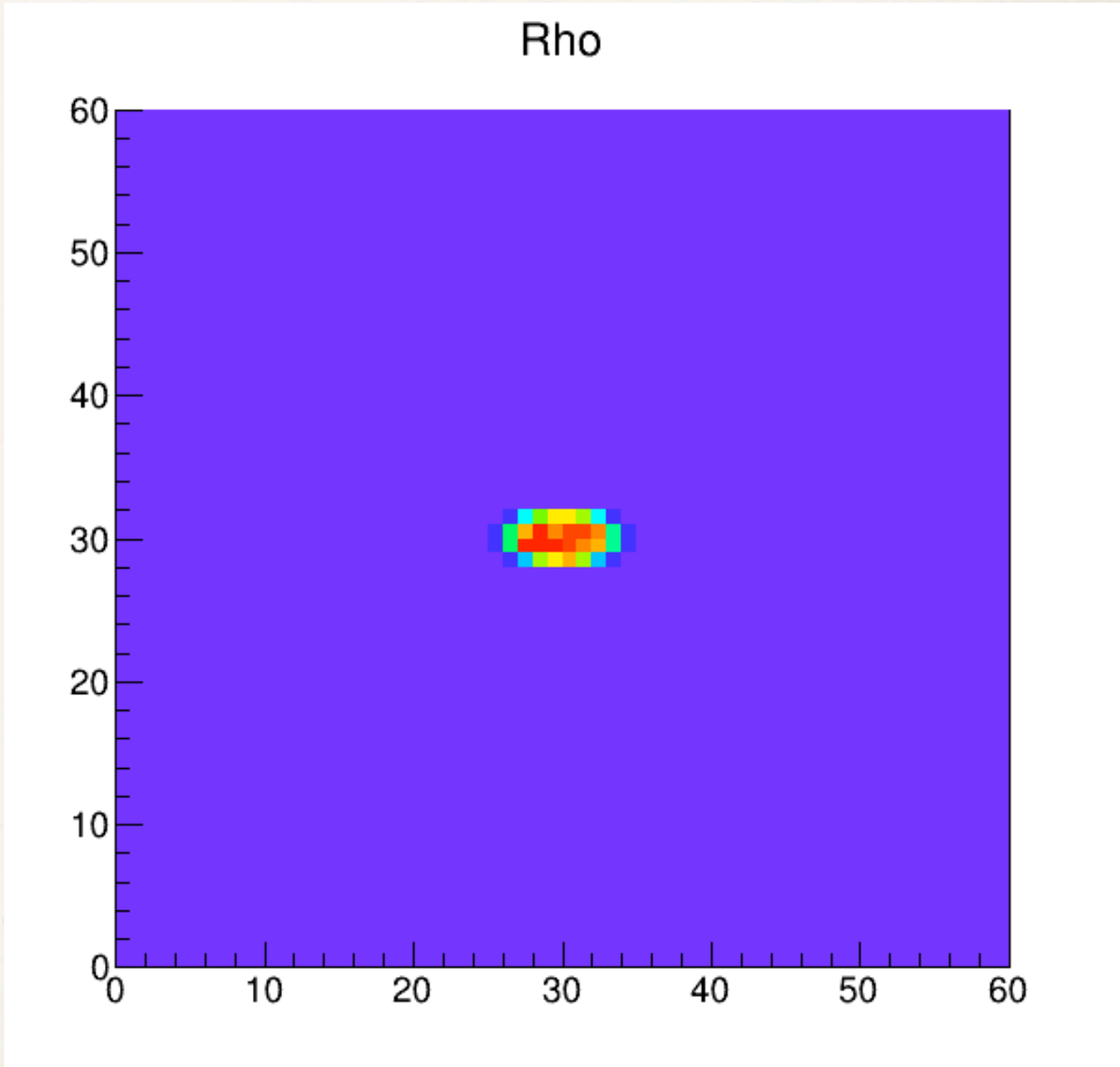
Elliptic flow of condensate





Please note: simulations are very very very preliminary

Fluctuations, interactions, thermalization

- This simple picture is modified by the presence of fluctuations
- There are no walls, but there are many other partons
- Fluctuations can be generated by collision of hard particles with condensate, fluctuations can be generated spontaneously.
- Methodology: condensate is a dense phase, use classical equations of motion
- Fluctuations are a dilute phase, will have to be treated quantally.
- More and fluctuations are generated, they interact and thermalize
- Fluctuations interact with condensate and get modified !****

Mass gap for fluctuations...

Look at part of Lagrangian, quadratic in fluctuations

$$\mathcal{L}^{(2)} = \frac{1}{2}\left\{(\partial_\mu\varphi_1)^2 + (\partial_\mu\varphi_2)^2 + \varphi_1^2[(\partial_\mu\theta)^2 - m^2 - 3\lambda\rho^2] + \varphi_2^2[(\partial_\mu\theta)^2 - m^2 - \lambda\rho^2]\right.$$

Note: we will ignore the effect of pre-pre flow

The generated pre-flow is still there in the simulations above

Interaction with the condensate makes fluctuation massive

$$\Box\varphi_1 = (-m^2 - 3\lambda\rho^2)\varphi_1 \qquad \Box\varphi_2 = (-m^2 - \lambda\rho^2)\varphi_2$$

Mass gaps proportional to ϱ^2 means fluctuations are hard to generate while condensate density is large

SU(2) gauge theory

- Is there a mass gap in excitation/fluctuation spectrum ?
- Separate field into condensate and fluctuation

$$A^a_\mu = A^a_\mu + \mathscr{A}^a_\mu$$

A. Majumder arXiv:1402.1912 [nucl-th]

$$\mathscr{L}_0 = -\frac{F^{\mu\nu}F_{\mu\nu}}{4},$$

$$\mathscr{L}_1 = -\frac{\mathscr{F}^{\mu\nu}\mathscr{F}_{\mu\nu}}{4} + \mathscr{V}$$

here, $\mathscr{L}_1 = \mathscr{L}_2$ in the scalar case ...
just bad notation

$$F^{\mu\nu} = t^a(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + g\epsilon^{abc}A^{b\mu}A^{c\nu})$$

Classical condensate

$$\mathscr{F}^{\mu\nu} = t^a(\partial^\mu \mathscr{A}^{a\nu} - \partial^\nu \mathscr{A}^{a\mu})$$

Quantum fluctuations

SU(2) gauge theory

$$\mathscr{V} = -\frac{g^2}{2}\mathscr{A}^b_\mu \mathcal{S}^{(bc)\mu\nu}\mathscr{A}^c_\nu$$

all factors of the classical field in S. For a real mass S < 0

Assume, diagonalization in color and Lorentz indices

$$\langle A^a_\mu A^b_\nu\rangle \simeq \frac{\delta^{ab}}{N_{c^2}-1}\frac{g_{\mu\nu}}{4}\langle A^a_\alpha A^{a\alpha}\rangle$$

Assume, slowly varying fields and keep, only up to single derivatives

$$A^a_3 = \mathscr{A}^a_3 = 0$$

$$\epsilon^{abc}A^b_\mu \epsilon^{cde}A^{d\mu}\phi^{e0} \simeq 0$$

$$\epsilon^{abc}A^b_\mu \partial_3 A^{c\mu} \simeq 0$$

Condensate in color and spin

$$A^{+j} = \frac{A^{1j} + iA^{2j}}{\sqrt{2}} = \rho^j e^{i\zeta} \qquad (j = 1, 2),$$

$$A^{-j} = \frac{A^{1j} - iA^{2j}}{\sqrt{2}} = \rho^j e^{-i\zeta} \ \& \ A^{3j} = Z^j$$

Following Sannino et al.

$$\rho = \sqrt{\rho_x^2 + \rho_y^2}$$

is the modulus of the condensate

ς is the phase

introduce two random unit vectors, to quantify condensate in x-y

$$\rho^j = \rho \hat{a}^j \text{ and } Z^j = Z\hat{b}^j$$

$$\phi^+ = Ve^{i\eta}\ ,\ \phi^- = Ve^{-i\eta}\ ,\ \phi^3 = U,$$

Solving for mass gap

Solutions for scalar potential

$$V = \rho$$

$$U = \frac{2\rho^2 Z \cos(\zeta - \eta)\, \hat{a} \cdot \hat{b}}{2\rho^2 + Z^2(1 - \hat{a} \cdot \hat{b})}$$

Now substitute back into S and take event average.

$$\mathcal{S} = -Z^2 \left\langle 1 - \frac{4\rho^4 \cos^2(\zeta - \eta)\, (\hat{a} \cdot \hat{b})^2}{\left[2\rho^2 + Z^2(1 - \hat{a} \cdot \hat{b})\right]^2} \right\rangle$$

$$\langle cos^2(\theta) \rangle = \frac{1}{2}$$

$$\mathcal{S} < 0$$

Fluctuations have a mass, so hard to excite...

Conclusions outlook

- p-p, p-A possibly reveal the initial state dynamics of A-A
- There could be a dense highly occupied state of gluons formed in the initial state
- The excitation spectrum seems to have a mass gap, so dense state behaves like a superfluid with very low viscosity
- This allows to generate flow, without full thermalization
- Mass gap means thermalization will happen later,
- System may become strongly coupled by this point
- One could also have instabilities later leading to thermalization

To do list

- Many things left to do:
- Calculate $T^{\mu\nu}$ from the scalar theory and SU(2) and look at time evolution
- Set up a condensate in SU(3) and study the presence of mass gap.
- Calculations need to be renormalized. Need to see range of time where one can apply weak coupling.
- A way out from requiring rapid thermalization.
- Huge implications for low p_T jets, these cannot lose energy as they cannot excite modes in this medium.

Principal Component Analysis and Subleading Flow

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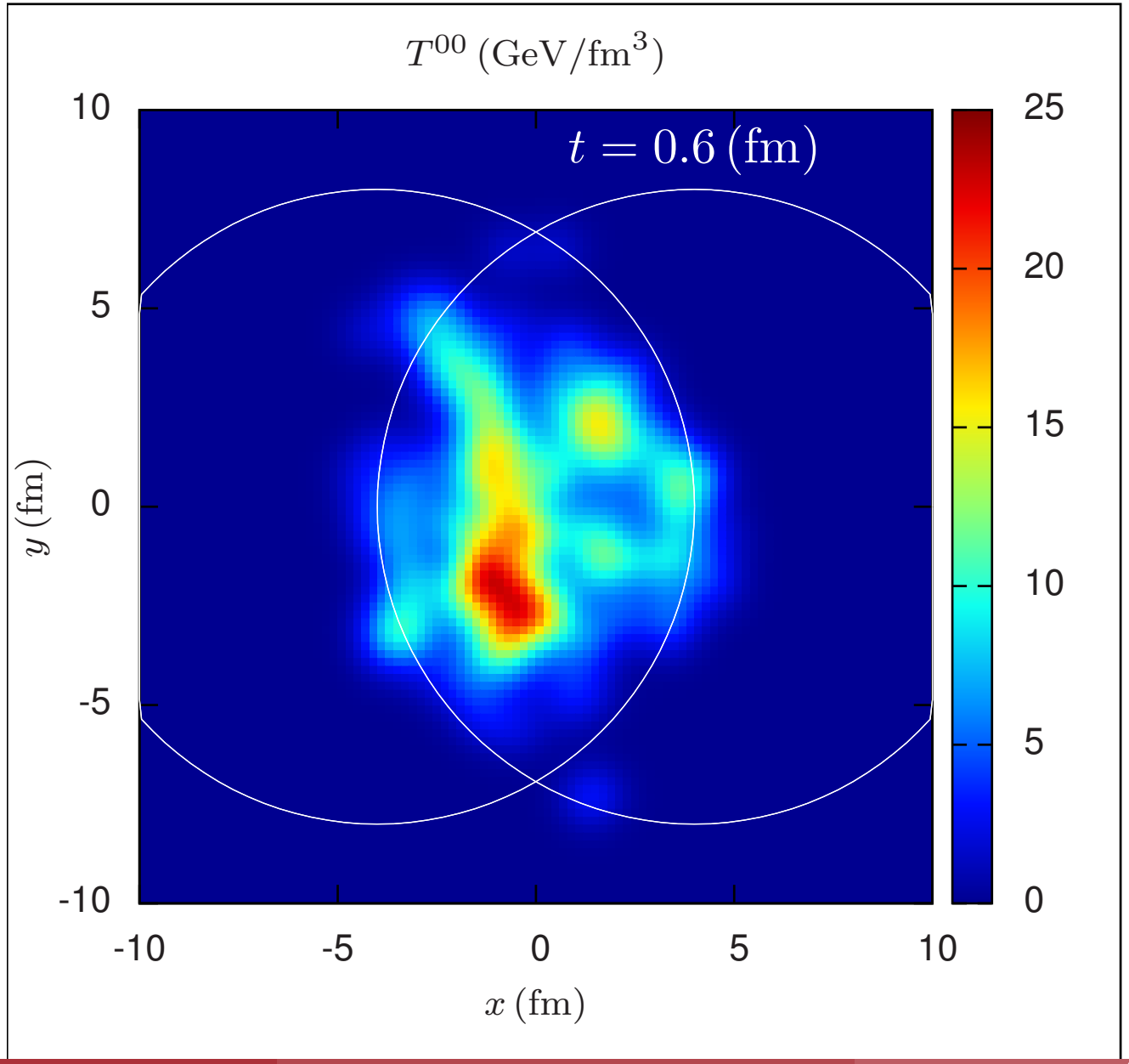
Department of Physics and Astronomy
Stony Brook University

March 6, 2015

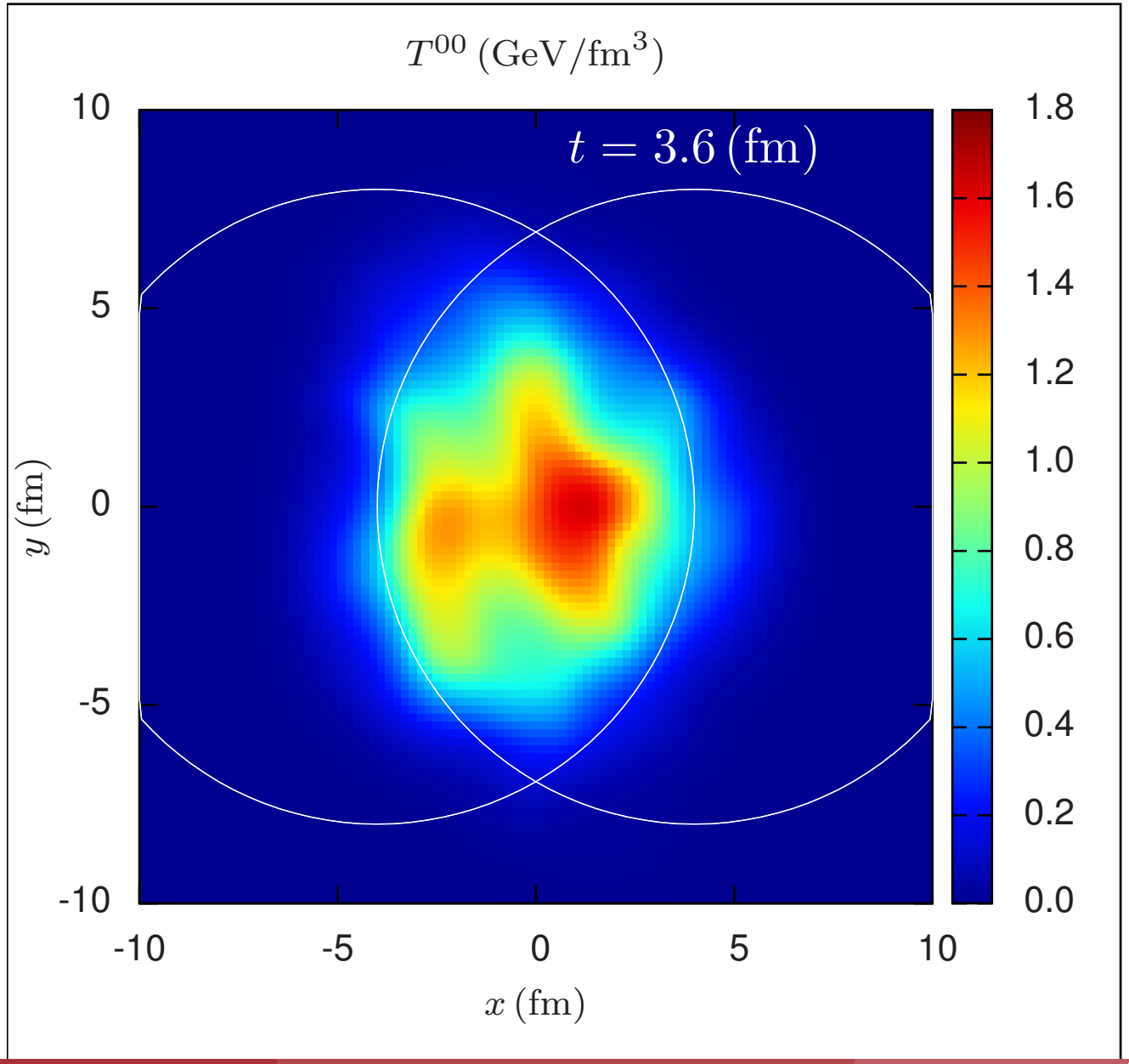
A. Mazeliauskas, and D.Teaney, arXiv:1501.03138
R. Bhalerao, J.Y. Ollitrault, S. Pal, and D.Teaney, arXiv:1410.7739

Stony Brook University

Many sources of flow



Many sources of flow



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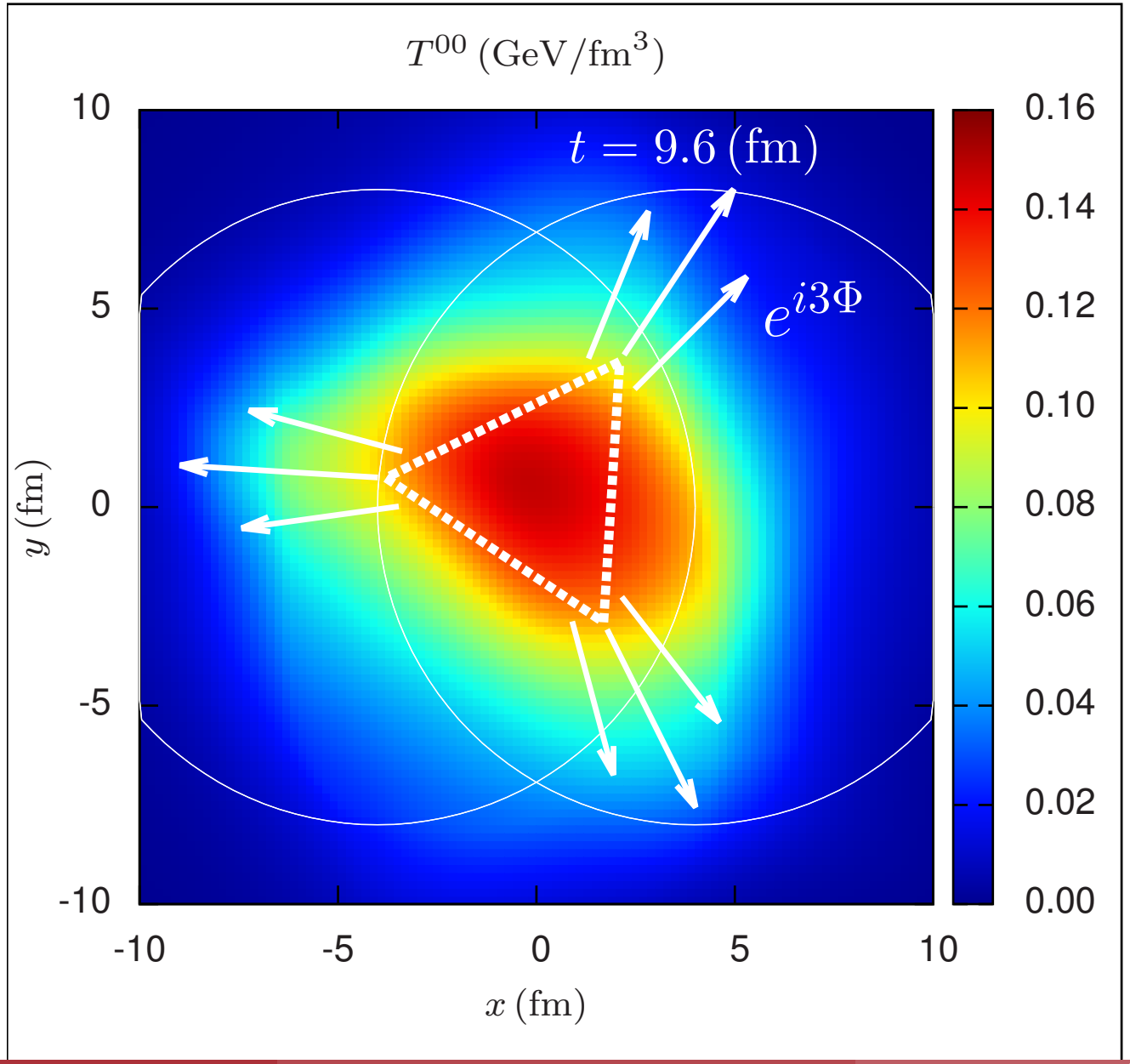


Event by Event Triangular Flow

Event-by-event particle yield expanded in azimuthal angle φ

$$\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}} = V_0(p) + \sum_{n=1}^{\infty} V_n(p) e^{-in\varphi} + \mathrm{h.c.}$$

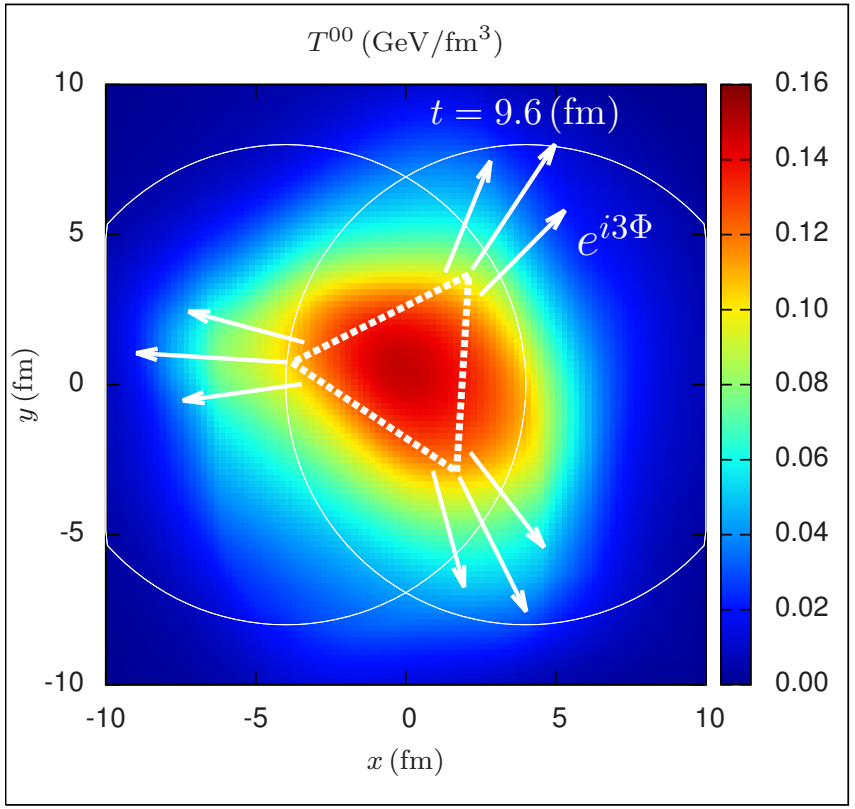
E-by-e triangular flow estimate

$$|V_3(p)| e^{i3\Phi(p)} \equiv \frac{1}{\Delta p} \sum_{j \in \mathrm{bin}(p)} e^{i3\varphi_j}$$

Event averaged correlation matrix

$$\boxed{C_3(p_1, p_2) = \langle V_3(p_1) V_3^*(p_2) \rangle}$$

Good observable – characterize it!



Simple flow model

Single flow

$$|V_3(p)|e^{i3\Phi} = \underbrace{V_3^{(1)}(p)}_{\text{fixed real } p\text{-function}} \times \underbrace{\frac{\varepsilon_3}{\sqrt{\langle \varepsilon_3^2 \rangle}}}_{\text{complex e-by-e orientation}},$$

Single flow $\Rightarrow$ correlation matrix factorizes

$$C_3(p_1, p_2) = \langle V_3(p_1) V_3^*(p_2) \rangle = V_3^{(1)}(p_1) \times V_3^{(1)}(p_2)$$

Measure factorization with

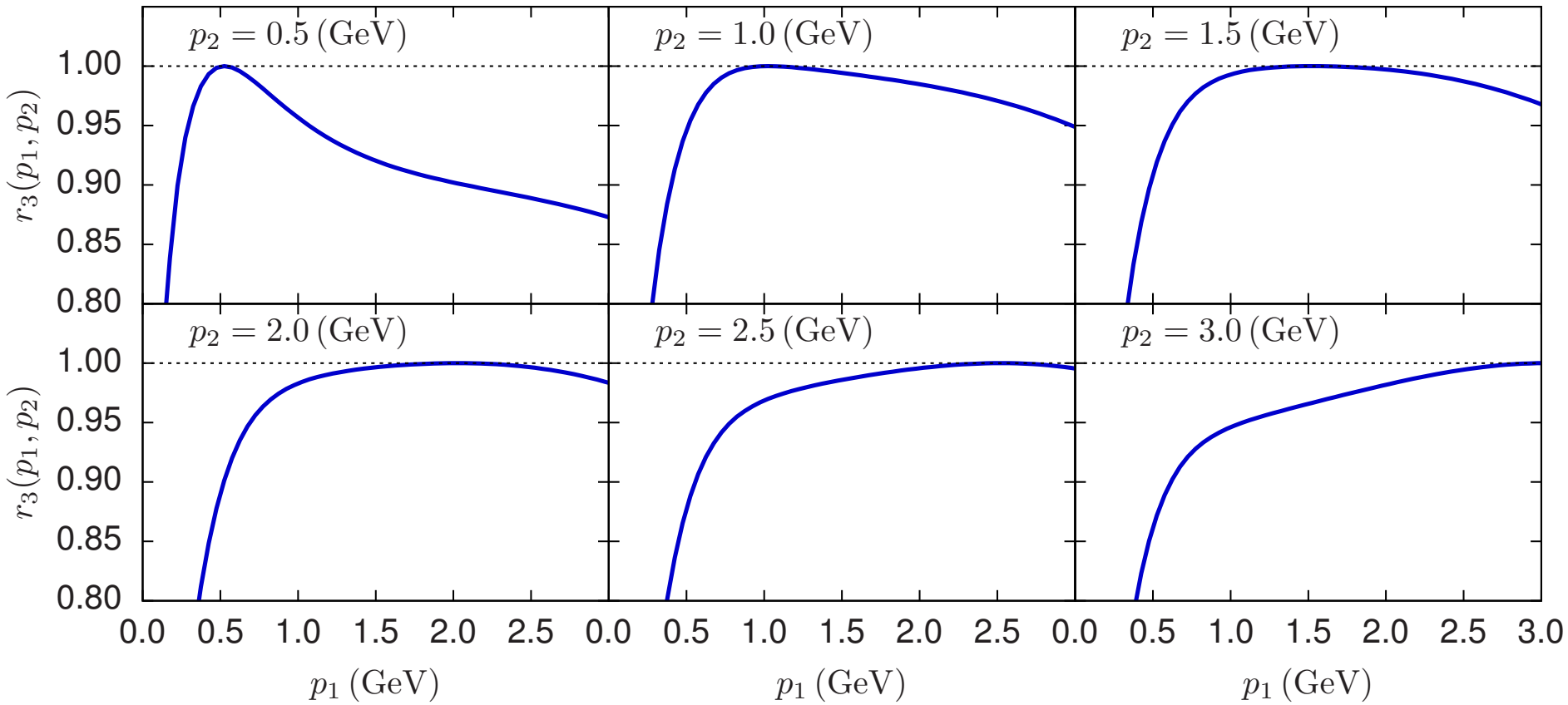
$$r_3(p_1, p_2) = \frac{\langle V_3(p_1) V_3^*(p_2) \rangle}{\sqrt{\langle |V_3(p_1)|^2 \rangle \langle |V_3(p_2)|^2 \rangle}}$$

$r_3(p_1, p_2) = 1$ *for single flow.*

Factorization breaks

Single flow fails. F. Gardim et al, arxiv:1211.0989

$$r_3(p_1, p_2) = \frac{\langle V_3(p_1) V_3^*(p_2) \rangle}{\sqrt{\langle |V_3(p_1)|^2 \rangle \langle |V_3(p_2)|^2 \rangle}} \leq 1$$



See U. Heinz et al, arxiv:1502.0463

Subleading flow

One product does not work!

$$\langle V_3(p_1)V_3^*(p_2)\rangle \neq V_3^{(1)}(p_1)\times V_3^{(1)}(p_2)$$

Try two!

$$\langle V_3(p_1)V_3^*(p_2)\rangle = \underbrace{V_3^{(1)}(p_1)\times V_3^{(1)}(p_2)}_{\text{leading flow}} + \underbrace{V_3^{(2)}(p_1)\times V_3^{(2)}(p_2)}_{\text{subleading flow}}$$

Math: any covariance matrix is a sum of its eigenvectors.

$$\langle V_3(p_1)V_3^*(p_2)\rangle = \sum_a V_3^{(a)}(p_1)\times V_3^{(a)}(p_2)$$

$$V_3^{(a)}(p) = \sqrt{\underbrace{\lambda_a}_{\text{eigenvalue}}} \times \underbrace{\psi_3^{(a)}(p)}_{\text{normalized eigenvectors}}$$

Only need 2-3 terms: $\lambda_1 > \lambda_2 \gg \lambda_3 \gg \ldots$

Principal Component Analysis

Bonus: Eigenvectors $V_3^{(a)}(p)$ is a complete basis for e-by-e $V_3(p)$

$$V_3(p) = \underbrace{\xi_1 V_3^{(1)}(p)}_{\text{leading flow}} + \underbrace{\xi_2 V_3^{(2)}(p)}_{\text{subleading flow}} + \underbrace{\ldots}_{\text{small}} .$$

$\xi_1 \approx \varepsilon_3/\sqrt{\langle \varepsilon_3^2 \rangle}$, but ξ_2 uncorrelated with ξ_1 (by PCA construction)

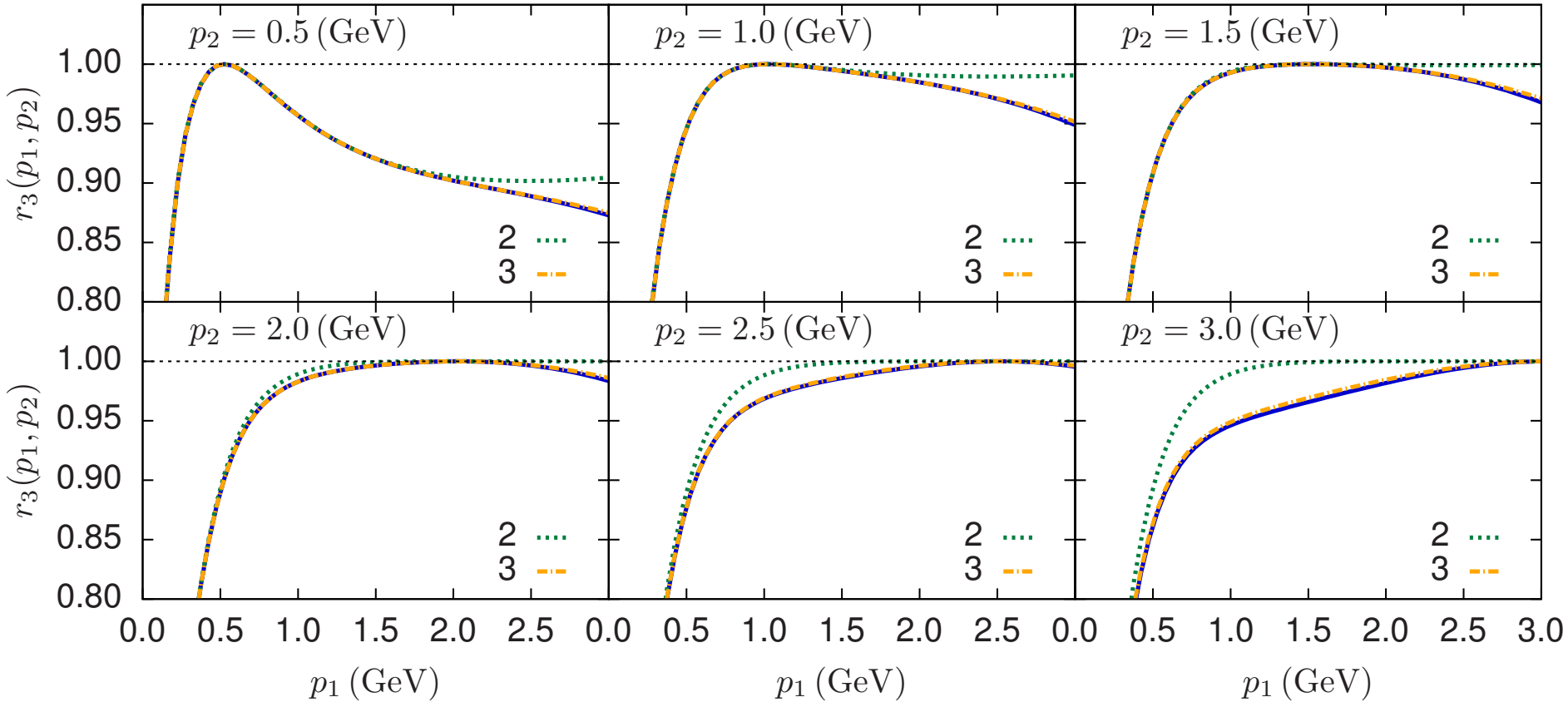
Summary of PCA

- find eigen-values/vectors of $\langle V_3(p_1) V_3^*(p_2) \rangle$
- only 2-3 eigenvectors relevant
- expand e-by-e $V_3(p)$ in leading/subleading flows

Factorization "explained"

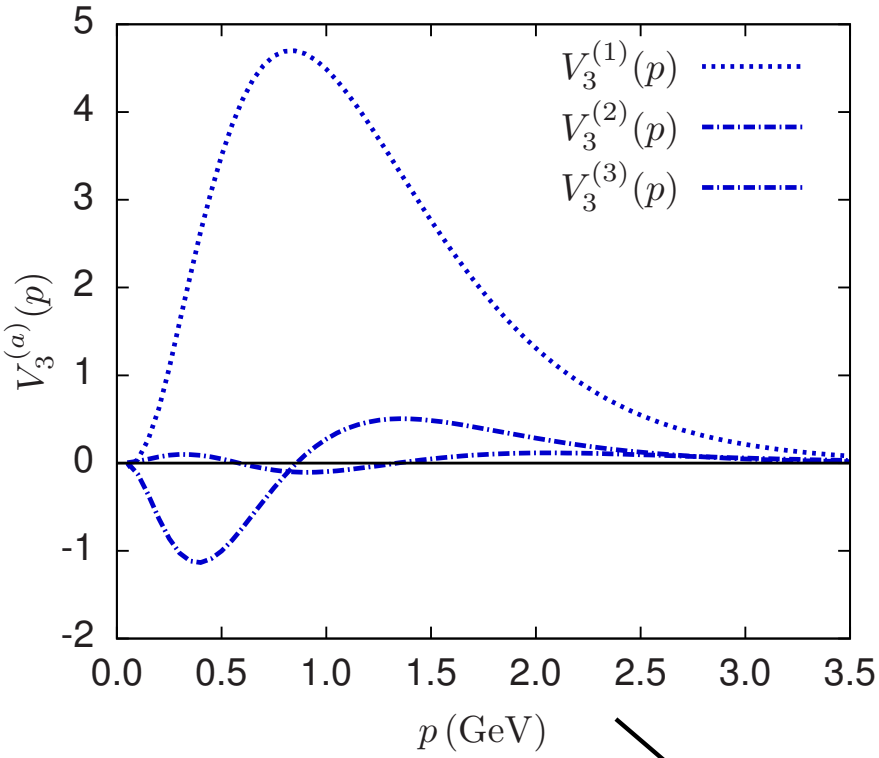
$$\langle V_3(p_1)V_3^*(p_2)\rangle = V_3^{(1)}(p_1)V_3^{(1)}(p_2) + V_3^{(2)}(p_1)V_3^{(2)}(p_2)$$

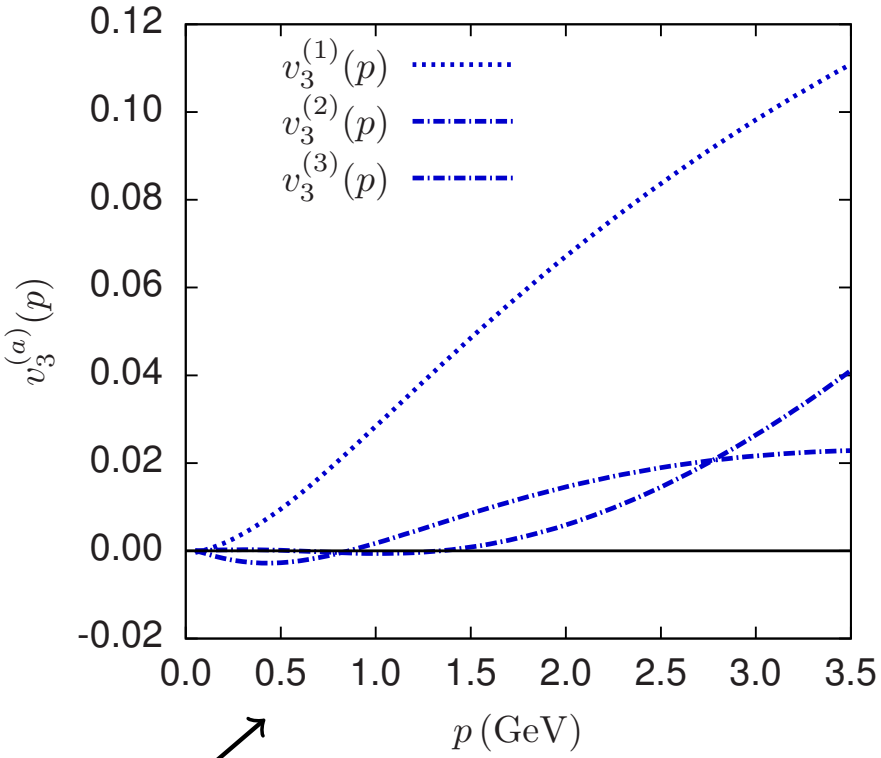
$$r_3(p_1,p_2) = \frac{\langle V_3(p_1)V_3^*(p_2)\rangle}{\sqrt{\langle |V_3(p_1)|^2\rangle \langle |V_3(p_2)|^2\rangle}}$$



Subleading $V_3(p)$ flow

$$V_3(p) = \xi_1 V_3^{(1)}(p) + \xi_2 V_3^{(2)}(p) + \ldots, \quad \xi_a = |\xi_a| e^{i3\Phi_a}$$

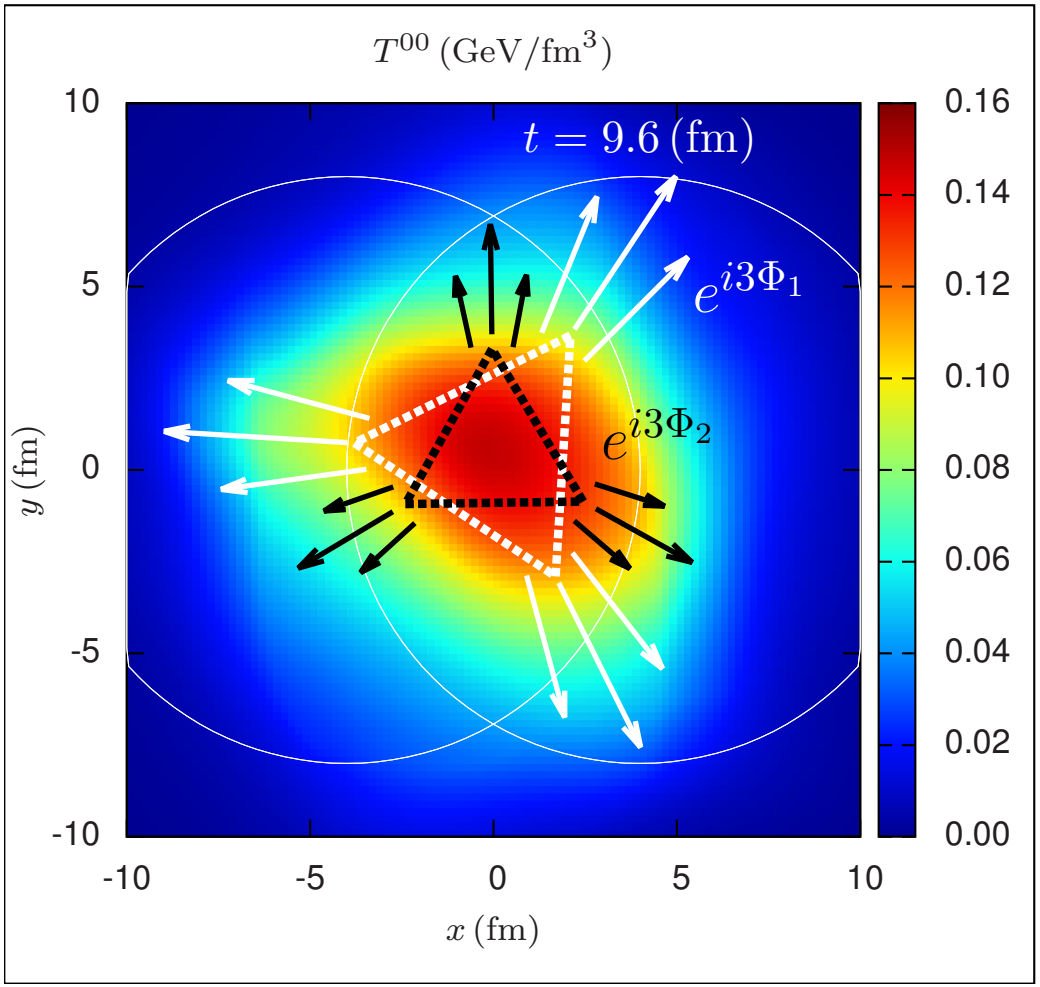




$$v_3^{(a)}(p) = \frac{V_3^{(a)}(p)}{\langle \mathrm{d}N/\mathrm{d}p \rangle}$$

Two flows $\Rightarrow$ two initial geometries

$$V_3(p) = |\xi_1| e^{i3\Phi_1} V_3^{(1)}(p) + |\xi_2| e^{i3\Phi_2} V_3^{(2)}(p)$$

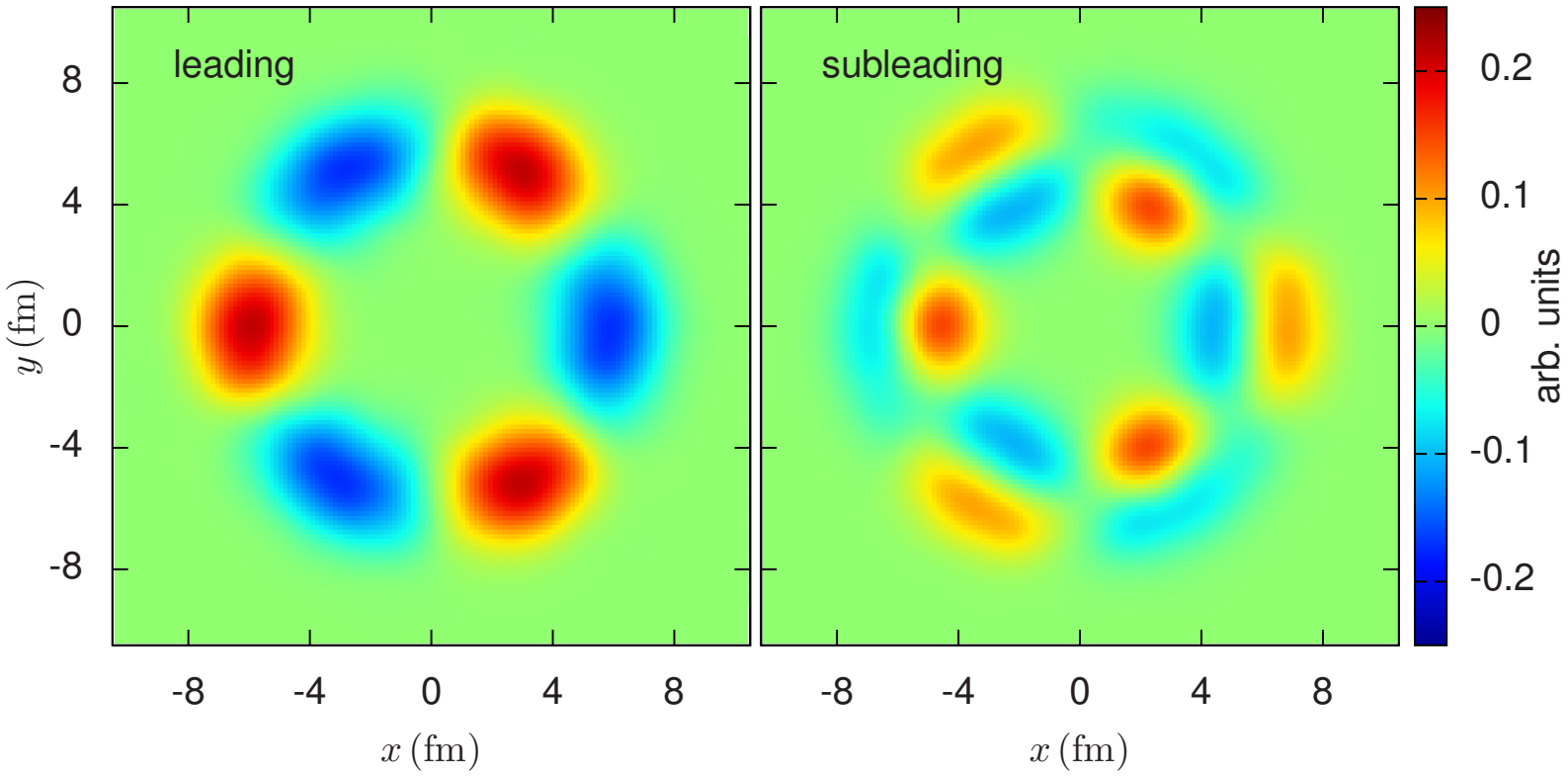


Geometry in subleading flow plane

Correlate entropy density S with subleading flow ξ_2

$$\langle S(r,\phi)\,\xi_2\rangle \quad \text{or} \quad \langle S(r,\phi+\Phi_2)|\xi_2|\rangle$$

Plot $\langle S(r,\phi+\Phi_a)|\xi_a|\rangle \times r^3$ minus background



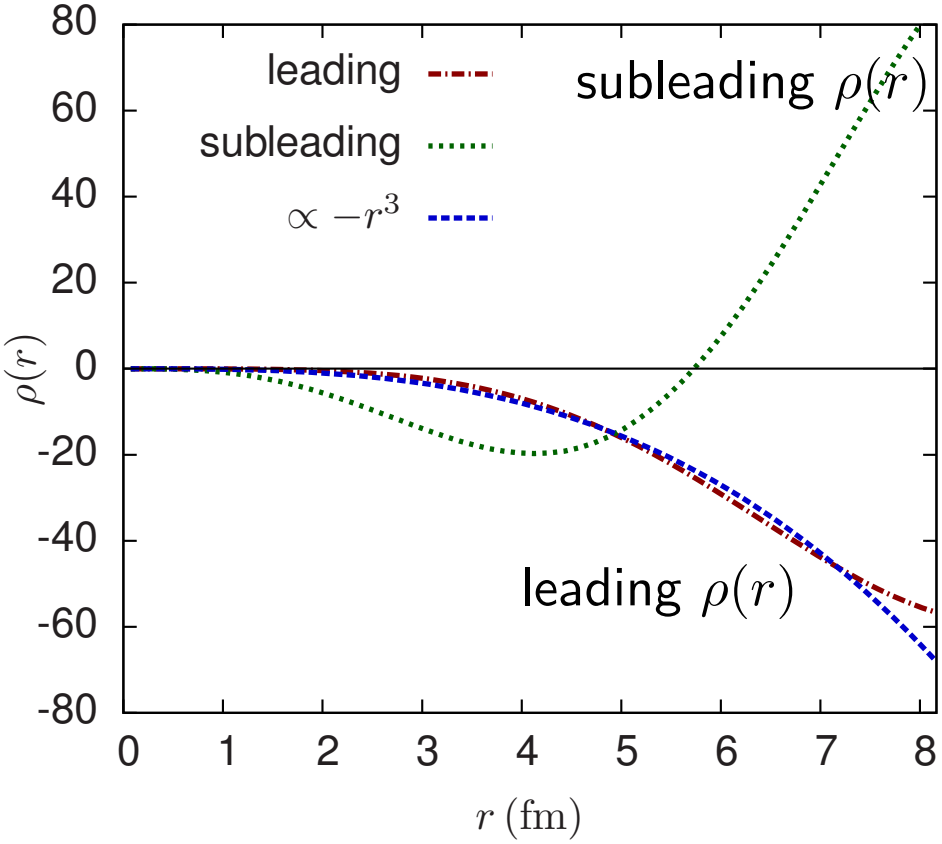
"ε_3" for subleading flow

Leading flow predicted by ε_3

$$\varepsilon_3^{(1)} \propto \langle \underbrace{r^3}_{\text{radial weight}} e^{i3\phi} \rangle$$

Predict subleading flow with

$$\varepsilon_3^{(2)} \propto \langle \rho(r)\, e^{i3\phi} \rangle$$



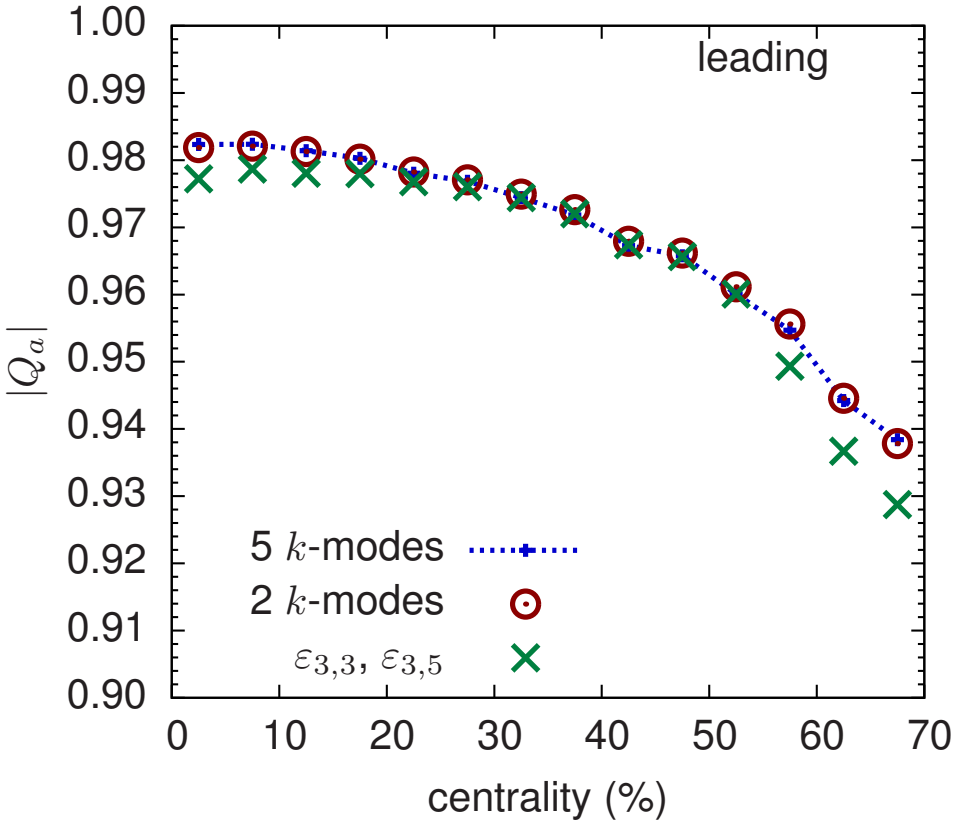
Choose $\rho(r)$ to maximize correlation with subleading flow ξ_2.

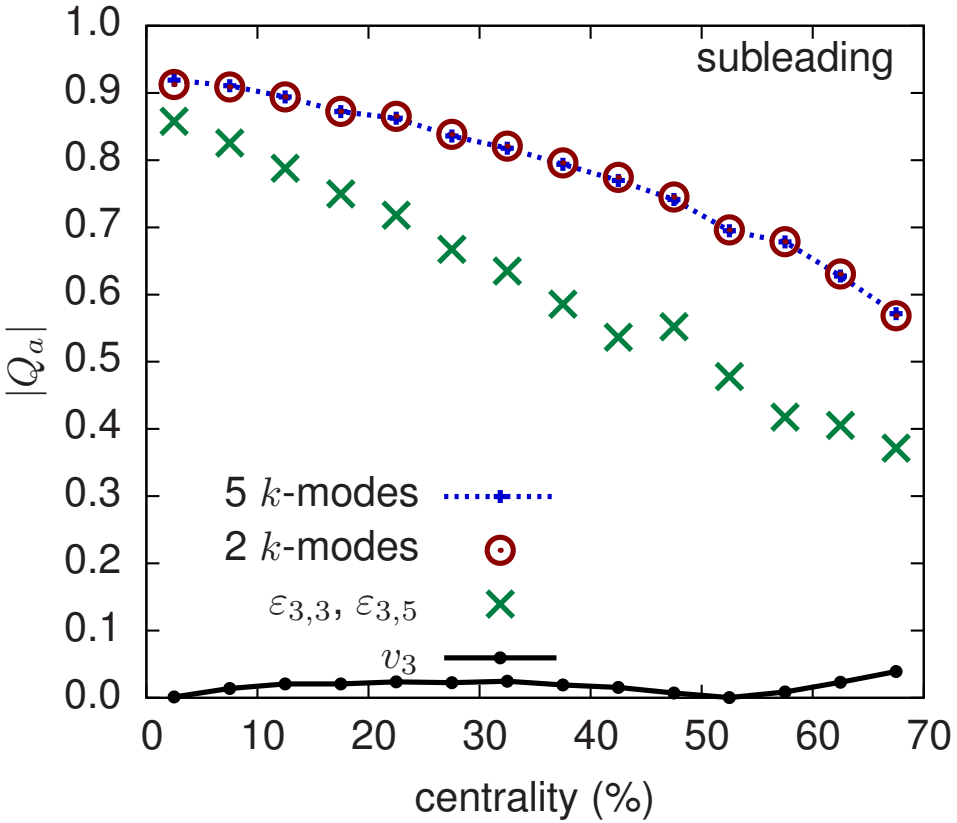
Predicting subleading flow

Stony Brook University

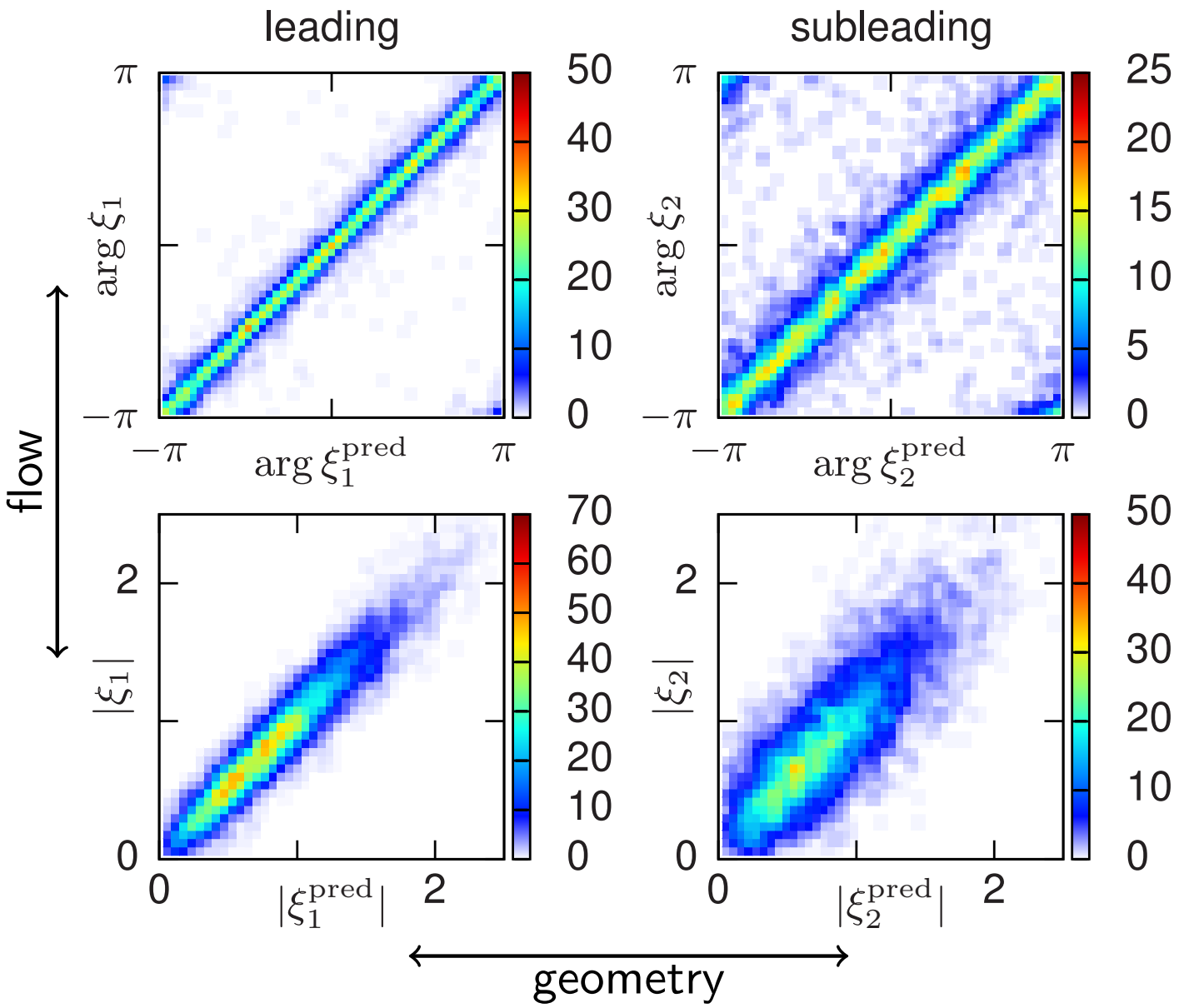
Correlation coefficient between flow and geometry

$$Q = \frac{\langle \varepsilon_3^{(2)} \xi_2^* \rangle}{\sqrt{\langle |\varepsilon_3^{(2)}|^2 \rangle} \sqrt{\langle |\xi_2|^2 \rangle}}$$





Event-by-event correlations



Summary

1. Principal Component Analysis is a systematic way of analyzing two particle correlations.
2. 2-3 principal components contain all information of r_n matrix.
3. Factorization breaking (subleading flow) originates from radial excitation in geometry.

Further work:

- subleading flow for $n = 0, 1, 2, 4, 5 \ldots$,
- principal components in η direction.

Thank you!

Backup

Radial geometry of subleading flow

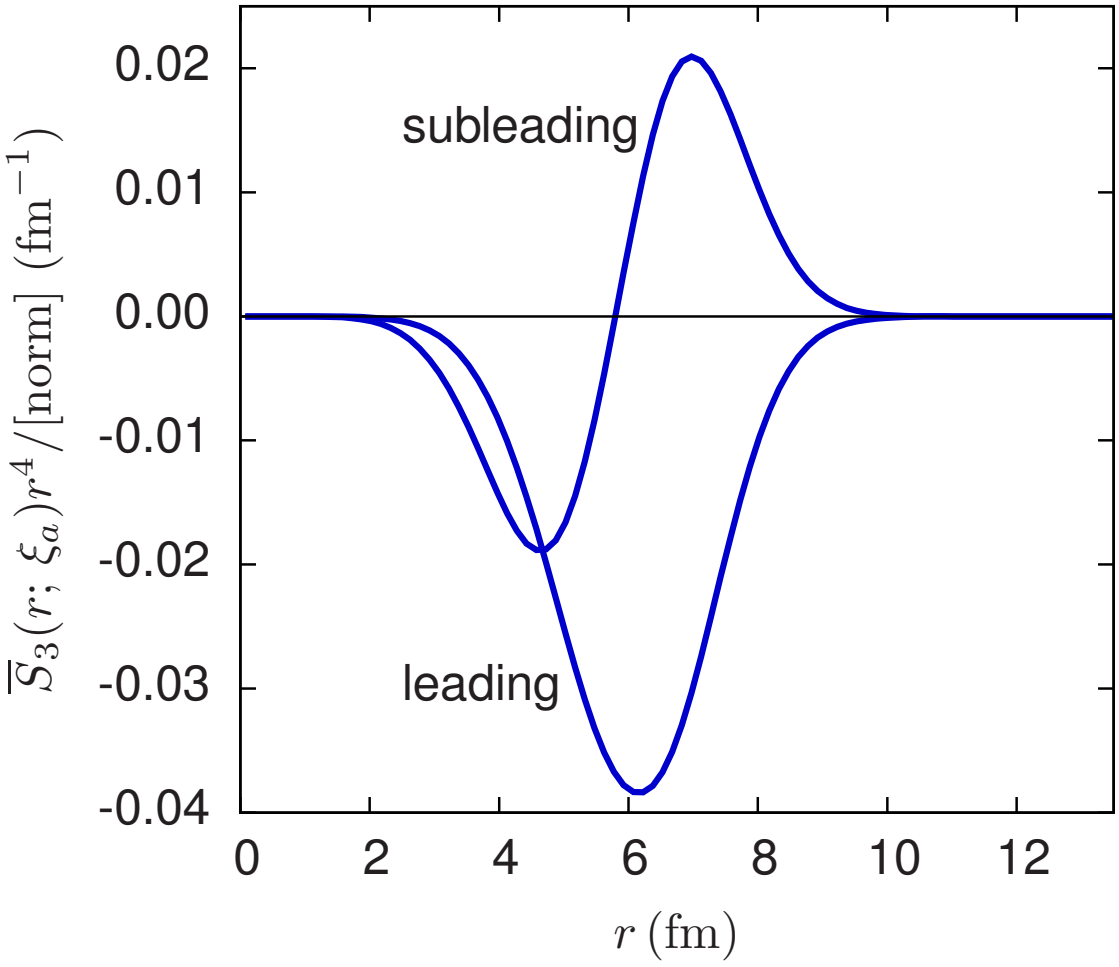
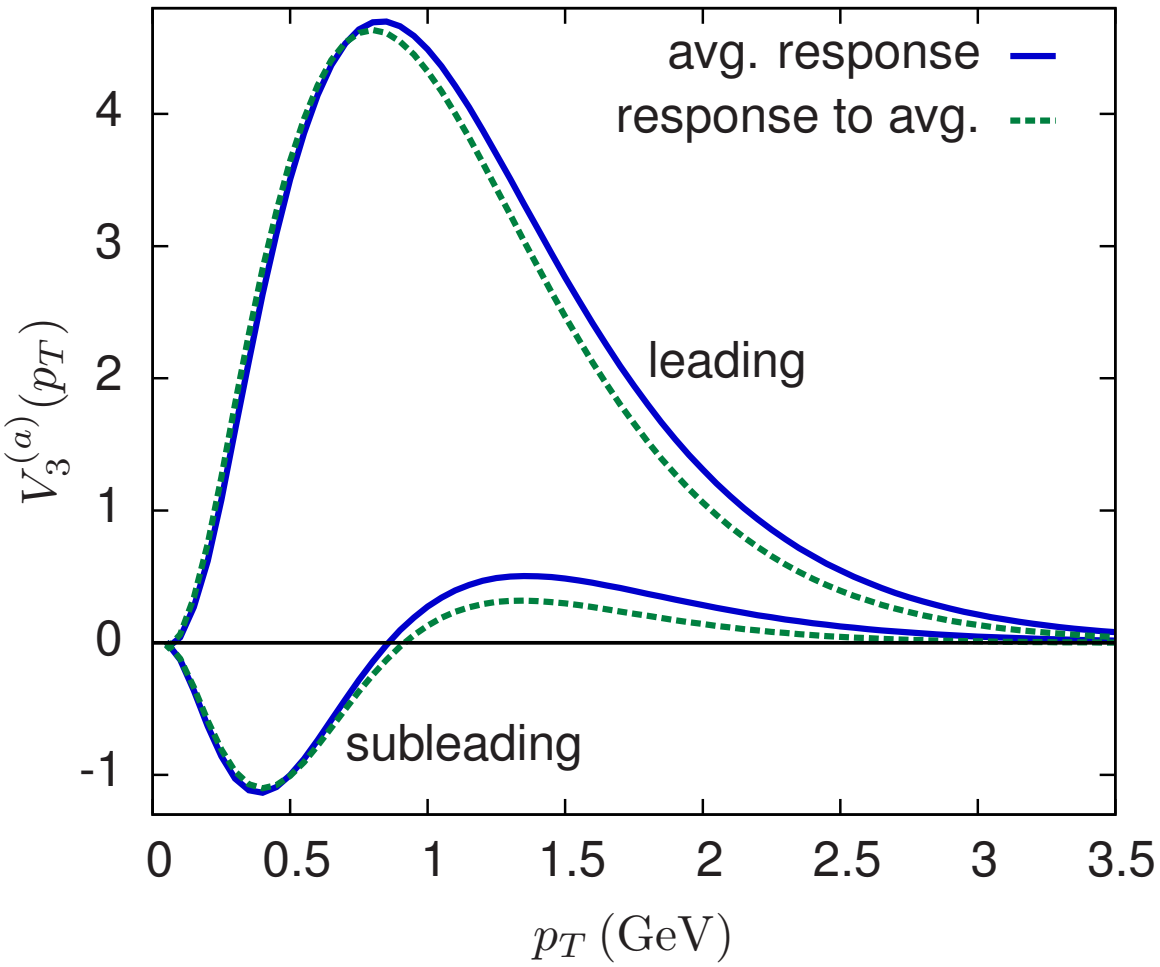


Figure: Third harmonic of averaged entropy – area under the curve $\propto \varepsilon_3$ The result has been multiplied by r^4 and normalized by $\overline{S}_{\mathrm{tot}} R^3_{\mathrm{rms}}$, so that the area under the leading curve is approximately $\varepsilon^{\mathrm{rms}}_{3,3}$.

Single-shot vs event averaged response

- evolve smooth initial geometry with radially excited eccentricity
- compare with event averaged subleading flow

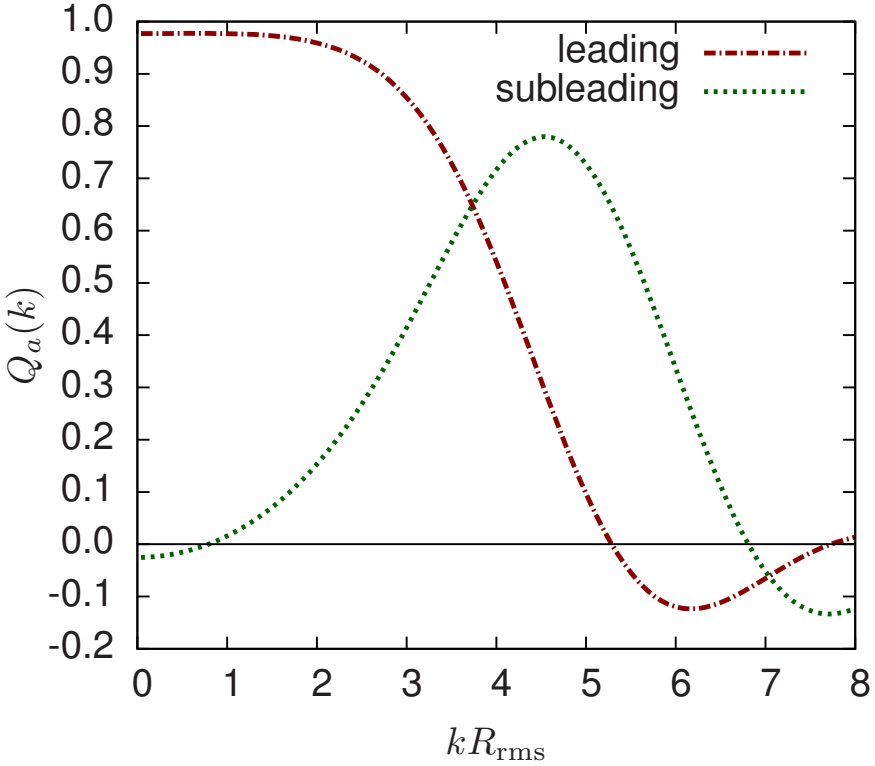


Single term predictor

Correlate flow with Fourier components of geometry

$S_3(k) = \int_0^\infty r\mathrm{d}r J(kr) S_3(r)$

$$Q = \frac{\left\langle S_3(k)\xi_a^* \right\rangle}{\sqrt{\left\langle |S_3(k)|^2 \right\rangle}\sqrt{\left\langle |\xi_a|^2 \right\rangle}}$$



Single-shot evolution subleading flow

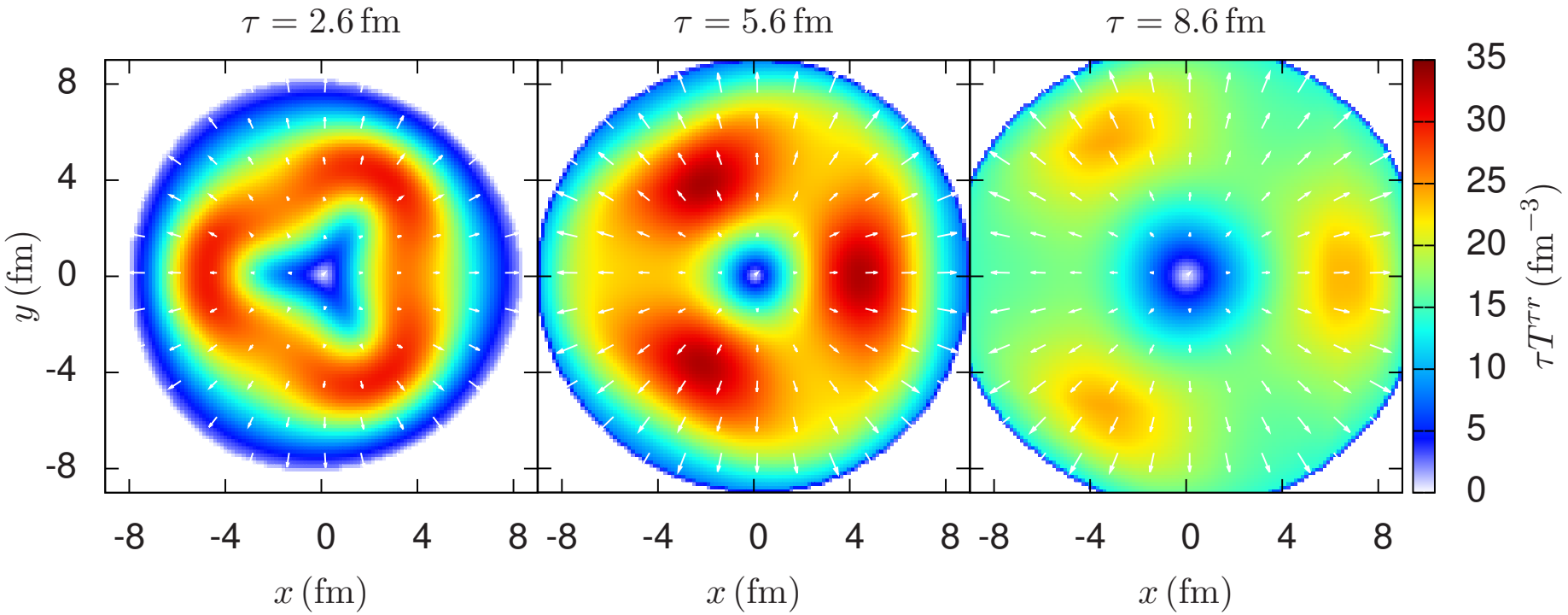


Figure: Hydrodynamic evolution of the subleading triangular flow. The color contours indicate the radial momentum density per rapidity, $T^{\tau r} = \tau(e+p)u^{\tau}u^{r}$, while the arrows indicate the radial flow velocity.

Centrality and viscosity dependence

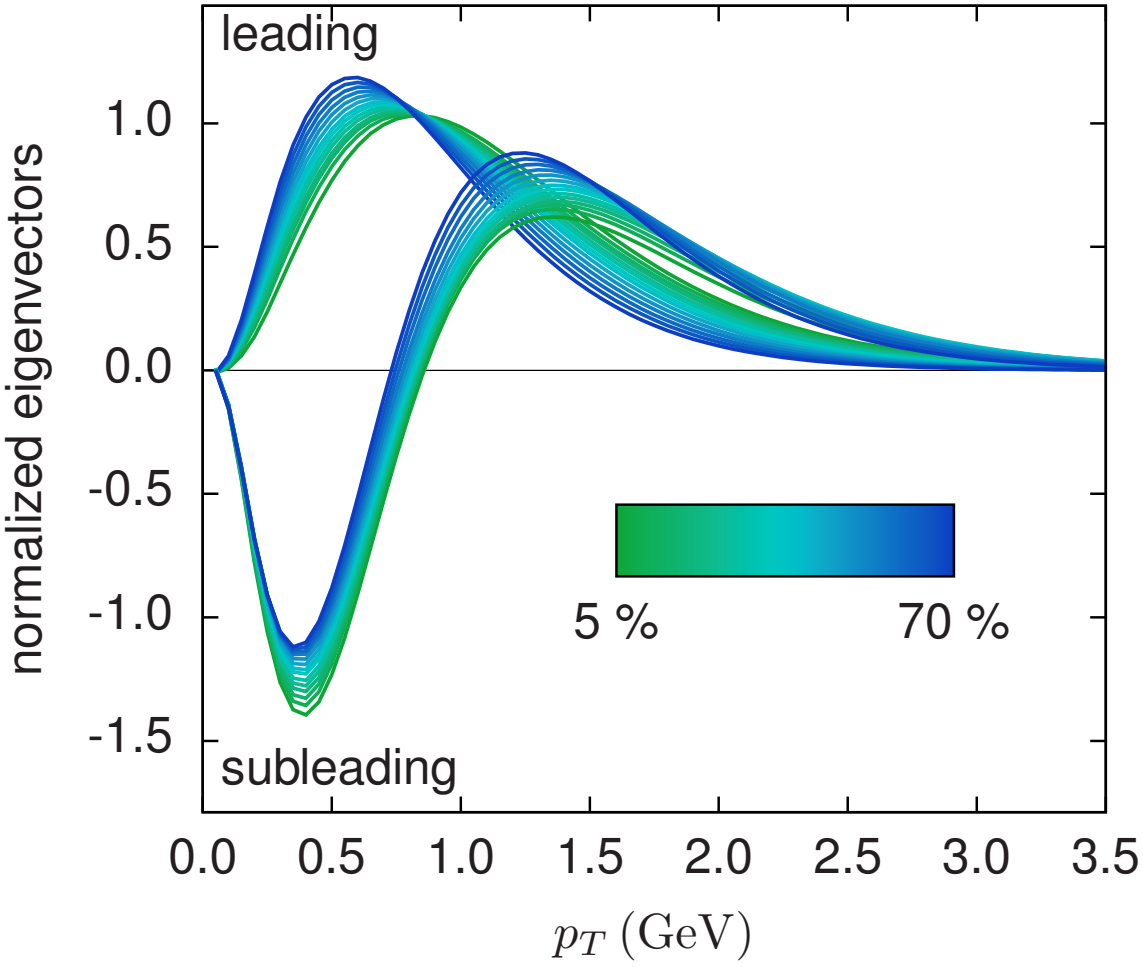


Figure: Centrality dependence of flow eigenvectors $\psi^a(p_T)$.

Centrality and viscosity dependence

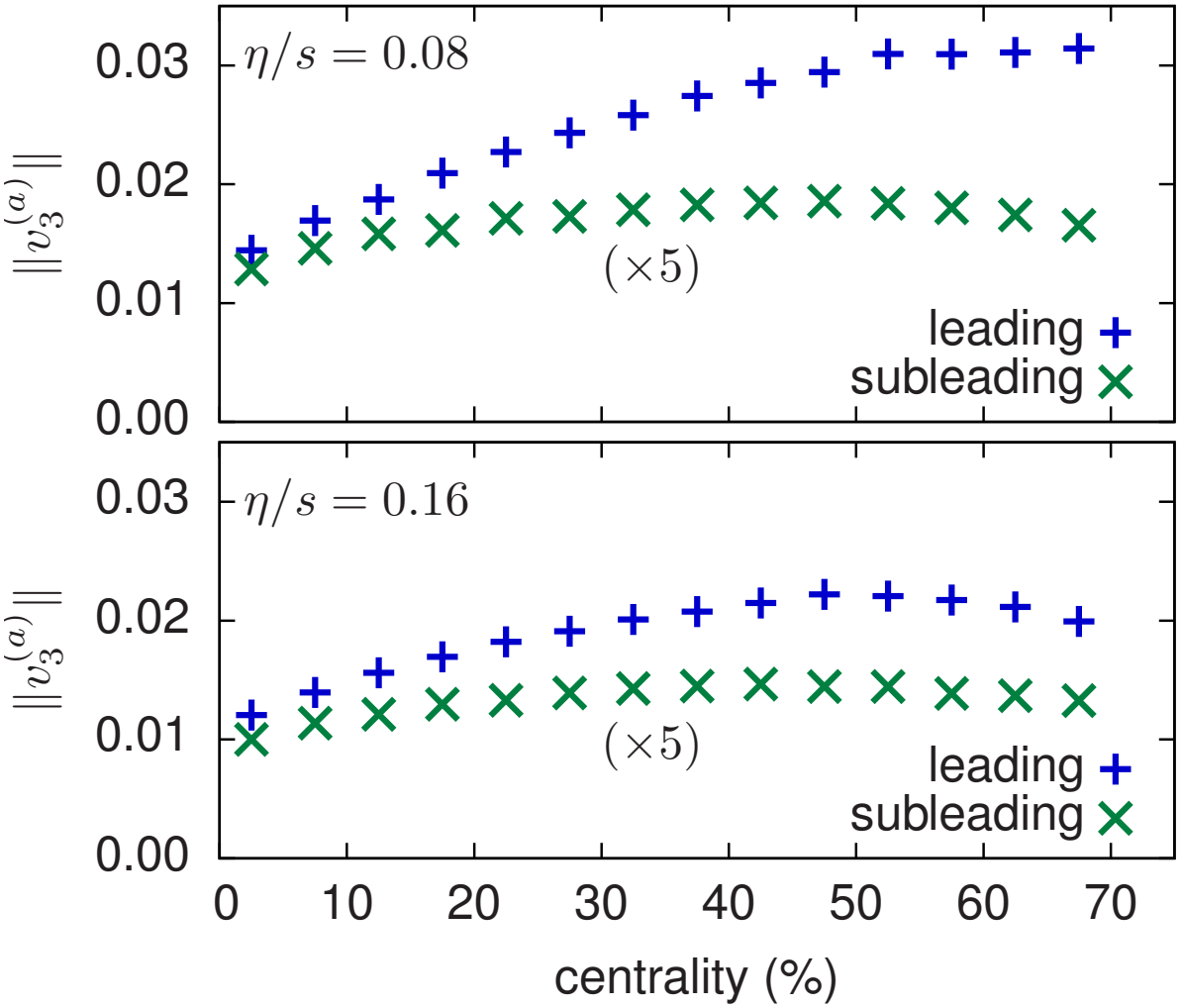
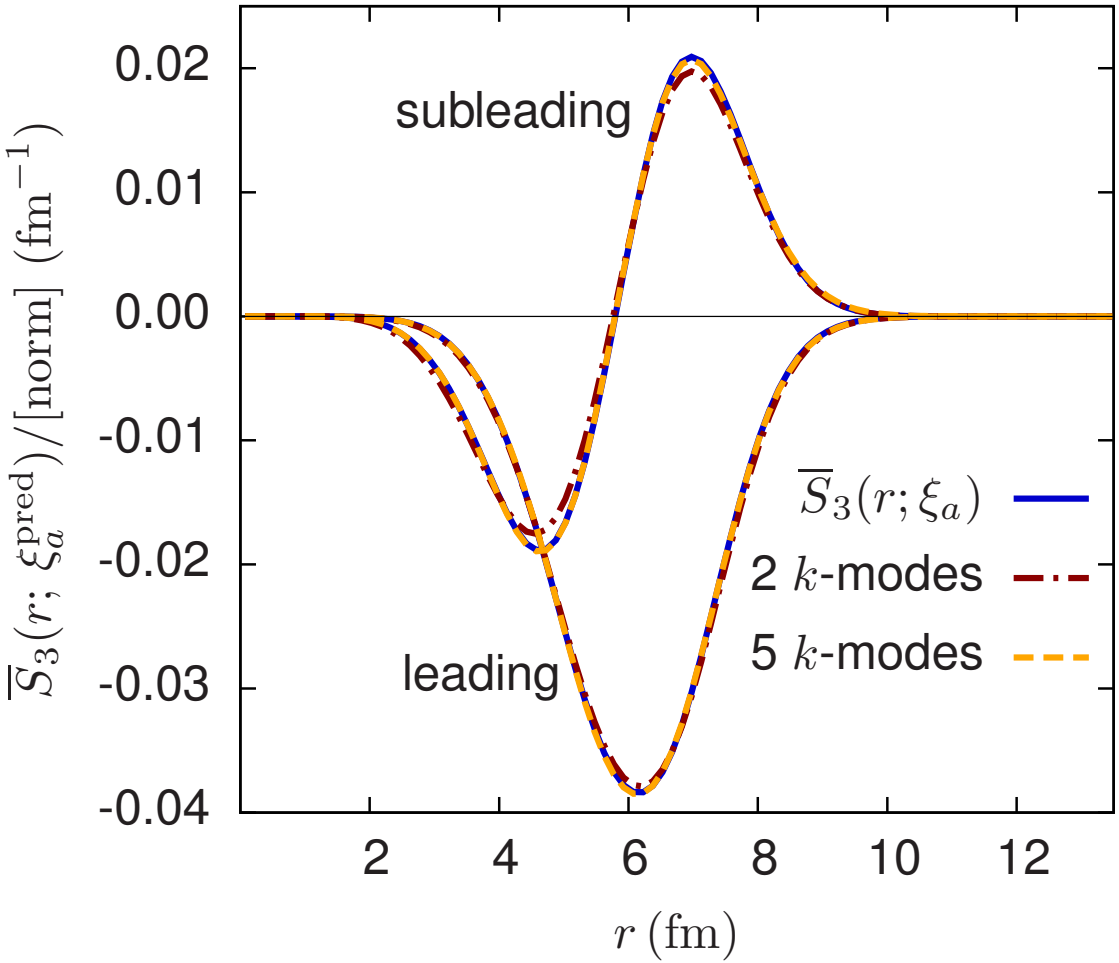


Figure: Centrality and viscosity dependence of scaled eigenvalues $\|v_3^{(a)}\|$. (The subleading flow has been magnified 5 times to bring to scale with leading flow.)

Averaged geometry in predicted event plane



Workshop Summary:
Collectivity in Small Colliding Systems with High Multiplicity

March 4-6 2015

This will not be a summary of results presented here but an attempt to put the various ideas discussed in a simple context.

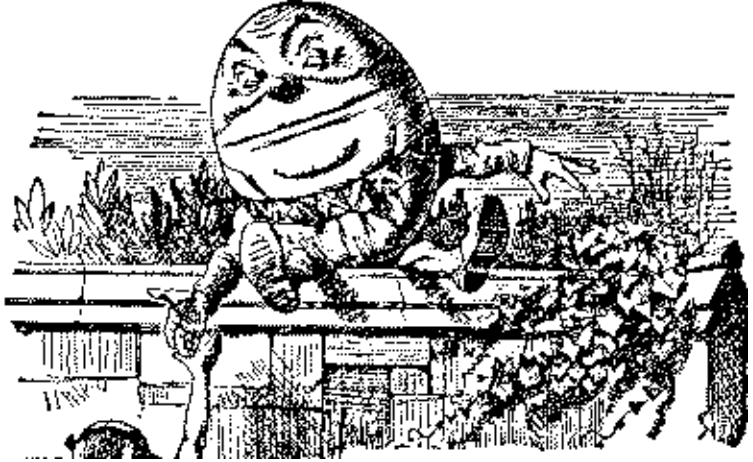
and to tell a few jokes that will get people angry

"If you are that angry, you must not be sure of yourself."
Alice McLerran

"I'm just doing my job."
Larry McLerran

RUPRECHT-KARLS-
UNIVERSITÄT
HEIDELBERG

HUAZHONG UNIVERSITY



"I don't know what you mean by 'glory'," Alice said.

Humpty Dumpty smiled contemptuously. "Of course you don't- till I tell you. I meant 'there's a nice knock-down argument for you!'"

"But 'glory' doesn't mean 'a nice knock-down argument'," Alice objected.

"When I use a word," Humpty Dumpty said, in rather a scornful tone, "it means just what I choose it to mean- neither more nor less."

"The question is," said Alice, "whether you can make words mean so many different things."

"The question is," said Humpty Dumpty, "which is to be master-that's all."

Alice was too much puzzled to say anything; so after a minute Humpty Dumpty began again. "They've a temper some of them- particularly verbs: they're the proudest- adjectives you can do anything with, but not verbs- however, *I* can manage the whole lot of them! Impenetrability! That's what **I** say!"

Through the Looking Glass, Ch. VI

Webster's 3'd International Dictionary:

Collective:

(6) marked by simultaneity, uniformity or similarity (as a response to a stimulus) among all members of a group

High Multiplicity, Thermalization and Proton-Proton or Proton-Antiproton Collisions

Various opinions:

Satz, Rafelski, Shuryak, Bjorken, McLerran, Ruuskanen

All have stated in times past (about 30 years ago) that at sufficiently high multiplicity pp collisions, the mean free path becomes small compared to the size of the system In this limit pp and AA collisions should be generically the same.

Vesa Russkanen and I applied our early hydrodynamical computations proton-antiproton collisions

Bj and friends ran the C0 experiment at the Tevatron studying high multiplicity events, and found p_T enhancements, mass ordering and strangeness enhancement. Features now associated with a hydro-dynamical interpretation

The argument for heavy ions was always the size is much larger so that the assumptions that go into a hydro-dynamical treatment are much better. The energy densities associated with typical events is also large than is that for pp

So if the hydro-dynamical assumptions and the thermal; description is much better in AA than pp or pA, why is there so much excitement about pA and pp in the heavy ion community?

For some it would validate the hydro-dynamical treatment in heavy ion collisions.

For others it provides a laboratory that allows one to test issues poorly understood in heavy ion collisions:

The nature of matter at very early times in such collisions:
CGC, Glasma

The consequences of decoupling

Generically effects such as flow are generated by earlier time phenomena at shorter distance scales in pp or pA collisions than in AA collisions.

The decoupling occurs generically at earlier time and higher temperatures in pp and pA than in AA

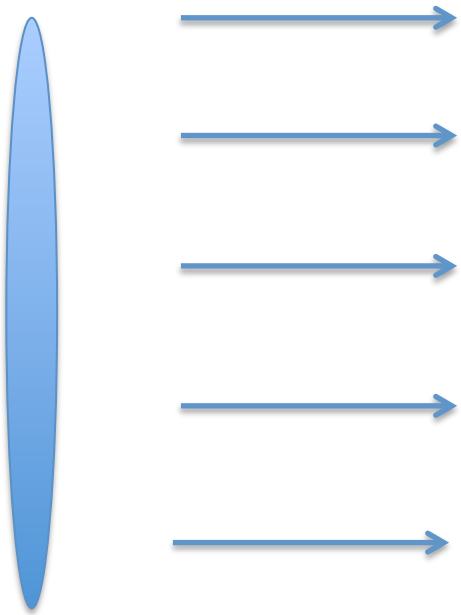
Maybe we can learn something new about QCD matter?

Surprising and exciting result of pA experiments at CERN:

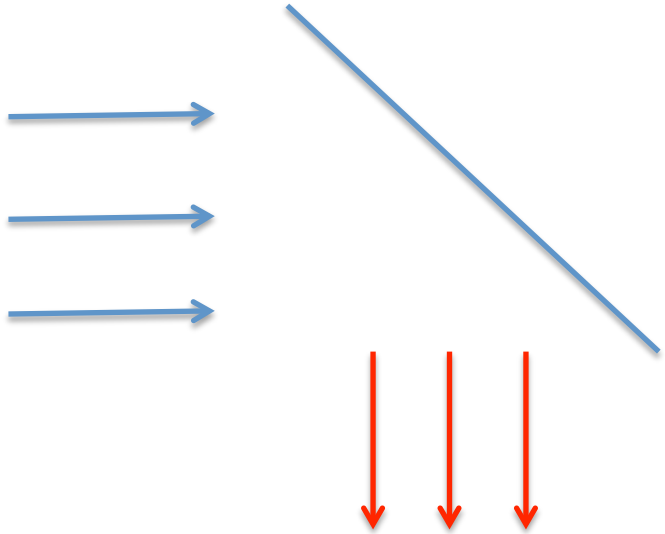
High degree of collective behaviour of particles produced in high multiplicity events

Simple Examples of Collective Behaviour

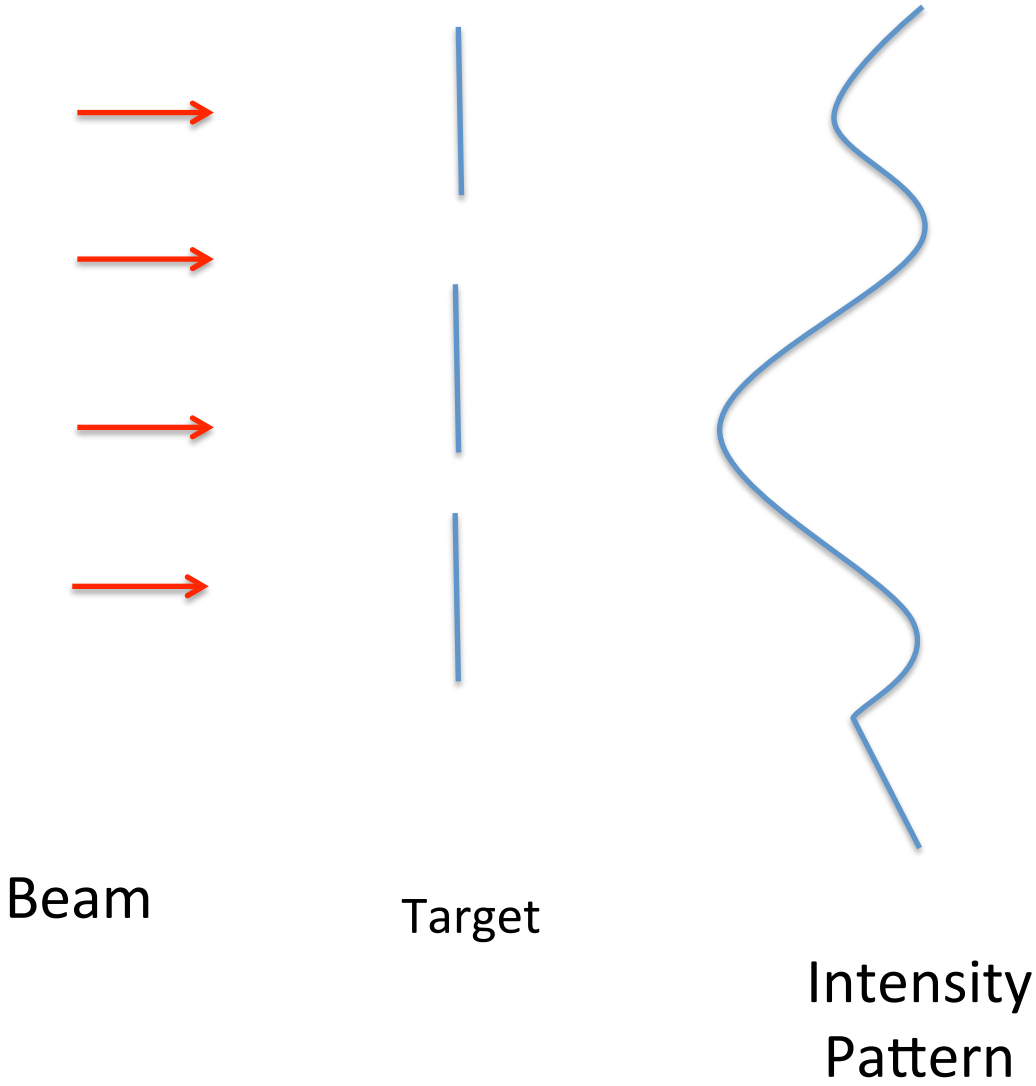
Example 1: Beam of partons going down beam pipe



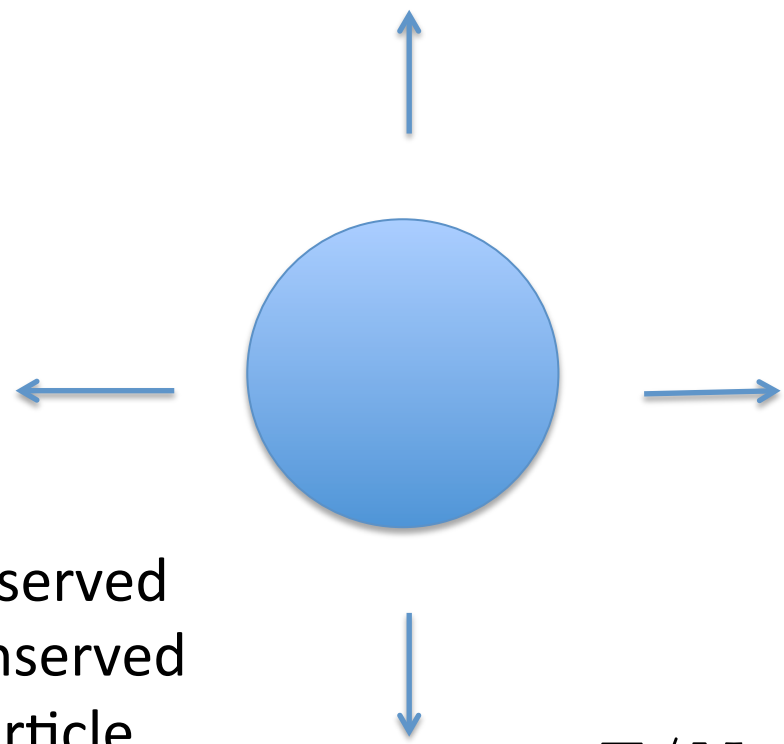
Example 2: Beam of particles hitting target



Example 3: 2 Slit Interference



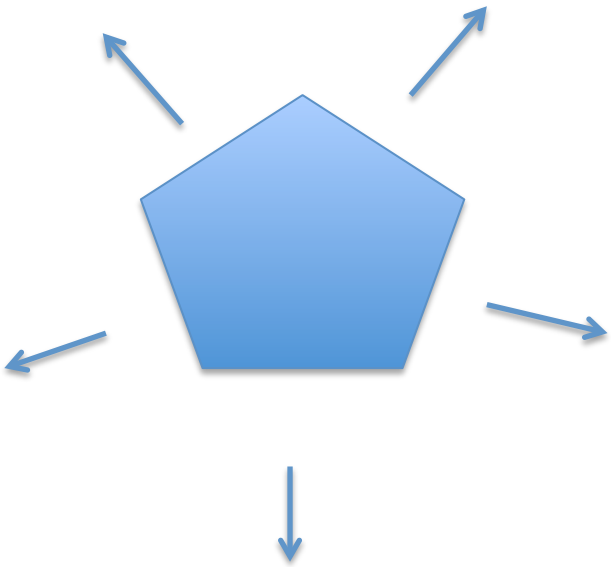
Example 4: An Expanding Gas of Non-Interacting Particles



Energy Conserved
Number Conserved
Energy/Particle Conserved
Radial Flow at late time.

$$\gamma_{radial} \sim E/Nm \sim Q_{sat}/\Lambda_{QCD}$$

Example 5: Opacity



Example 6

Hydrodynamic Flow:

Well motivated theoretically for **Central** AA collisions

Works semi-quantitatively for experimental observables up to uncertainties associated with initial conditions, decoupling and range of applicability

It would be very difficult to give up on such a successful theory of heavy ion collisions in favor of less well motivated model computations:

However:

We need to better understand effect associated with early and late times, and non-hydro physics, such as jet interactions.

We also need to be honest about where when the hydrodynamic description should break down such as peripheral AA and lowish multiplicity pp and pA. **This is not a modeling issue or" I can find some model etc" issue.** You can have an infinite number of wrong theories, and therefore there will always be some wrong theory to describe a finite set of data. **You have to ask deeper questions!**

Something we can all agree on:

Because the odd moments of flow exist, there must be final state interactions.

Is this reasonable: If there is saturation, as almost all of the initial state models incorporate, then if there are strong initial state interactions there should also be strong in the final state

Unsettled questions:

Do we need only one interaction per particle as suggested by Bzdak or do we need many as required by hydro?

Do we need initial state contributions to the moments which might interfere with the final state interactions?

To what degree are fluctuations in the proton probe important for flow measurements in pA?

High pt vs low pt

If there are significant flow effects at some pt, how can it be that there are not significant jet interactions at that same pt? Is there any respectable theoretical consideration which would allow this? Non-respectable that is not obviously wrong?

In CGC and in Biro, Levai, Gyulassy and Vitev:

There is a decoupilng between the coordinate space ellipticities and the induced v_n's.
How do we see this?

Hydrodynamics clearly must break down at early and late times.

How can we see this in an unambiguous way?

Can we measure the effects of P_L/P_T?
If we can isolate such an effect, what does it tell us about the time and size scales involved.

General Comments:

A basic problem is that hydro describes the qualitaitive features of pA and pp collisions. To get better resolution on the issue, one needs better theory which limits parameters. Difficult to do when a fraction of the community is comfortable with fitting data even when the underlying models which are used are either outside their range of validity or have something fundamentally wrong in their implementation. Even for consistent descriptions, very difficult because of large numbers of adjustable parameters.

I'm just doing my job!

Summary of RHIC results on small colliding systems

Dave Morrison (BNL)

Key RHIC data sets for small systems

2003
d+Au
200 GeV
20/nb

2008
d+Au
200 GeV
200/nb

2014
^{3}He+Au
200 GeV
70/nb

2015
p+p
p+Au
p+Al
200 GeV

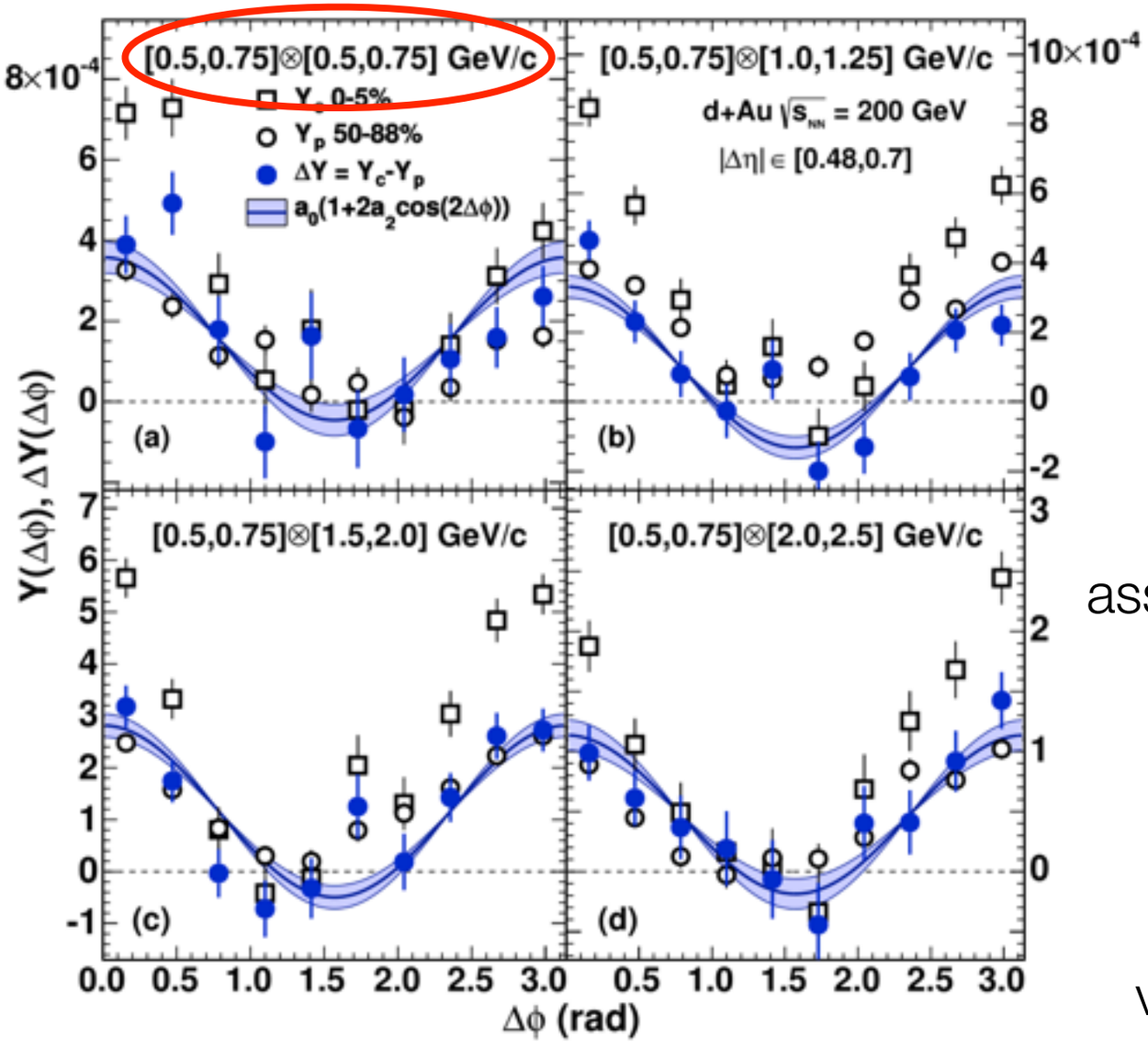
STAR
10M m.b. events

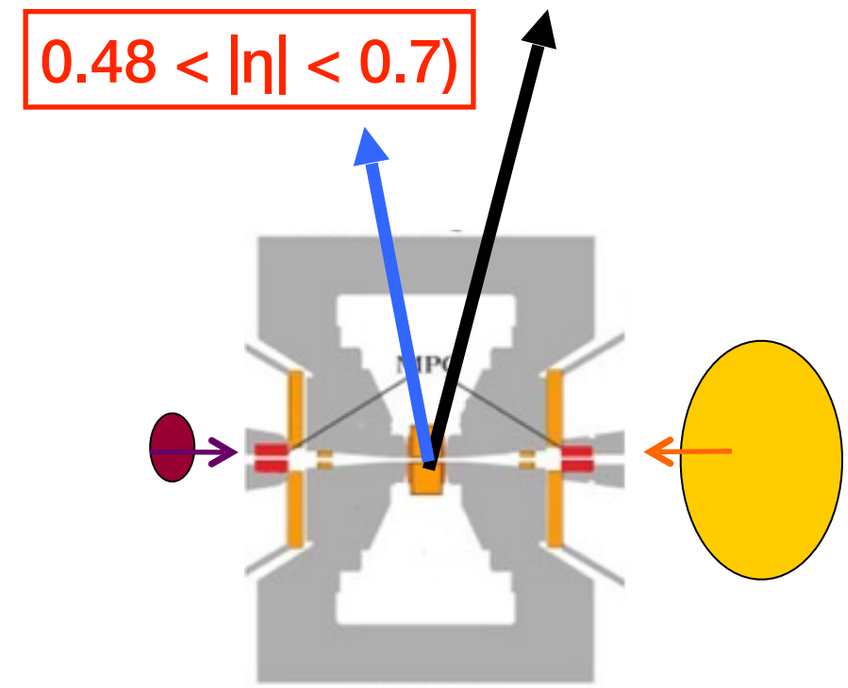
PHENIX
1.6B m.b. events

STAR
70M m.b. events

Two particle correlations in d+Au at RHIC

PHENIX PRL 111, 212301 (2013)





assumption: jet-like contribution to correlations same in peripheral and central d+Au

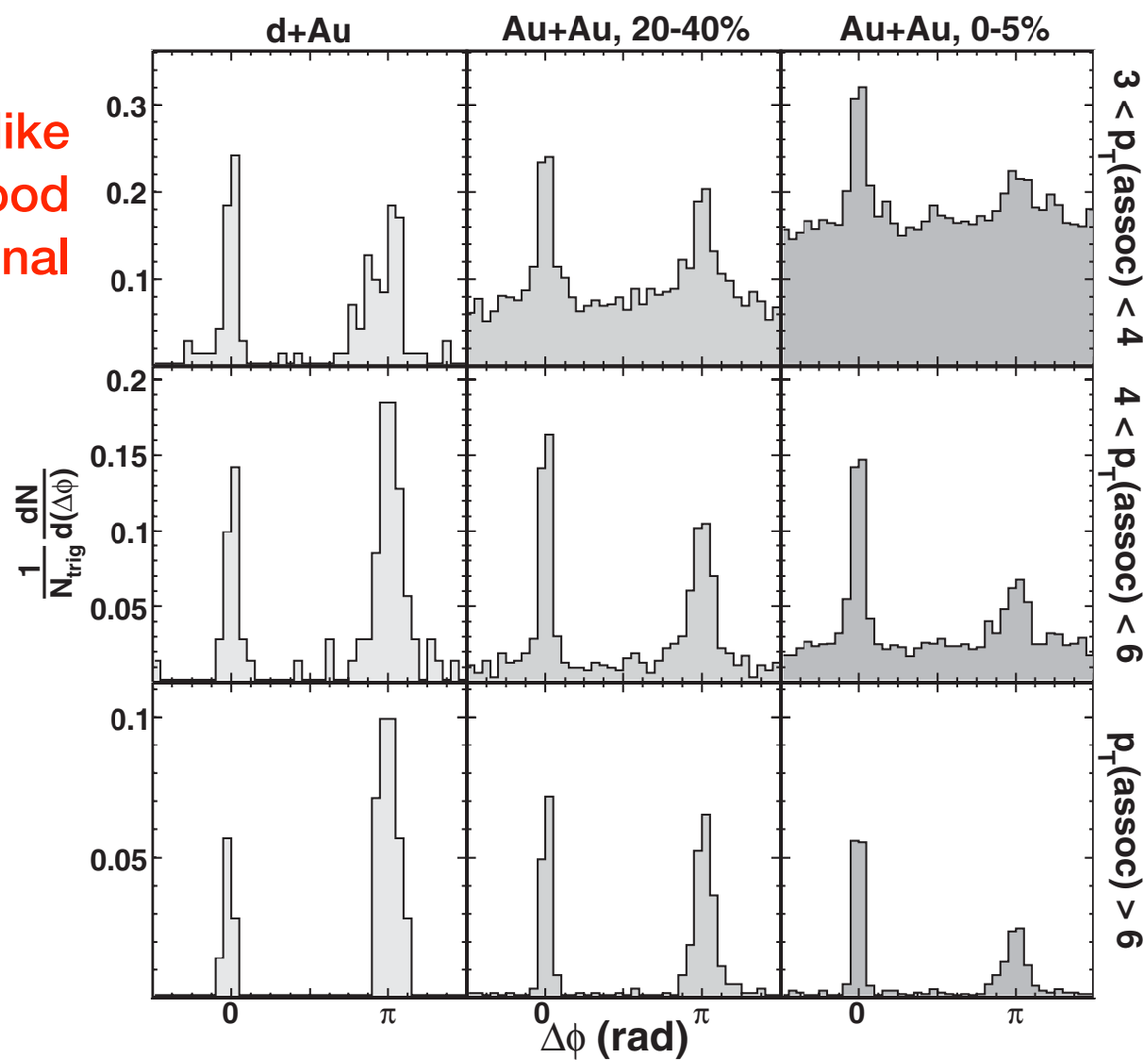
central – peripheral removes centrality independent correlations

varying η selection, like- and opposite-sign correlations used to set systematic uncertainty

One person's signal is another's background

need to determine jet-like correlation well to have good measure of harmonic signal

STAR PRL 97, 162301 (2006)

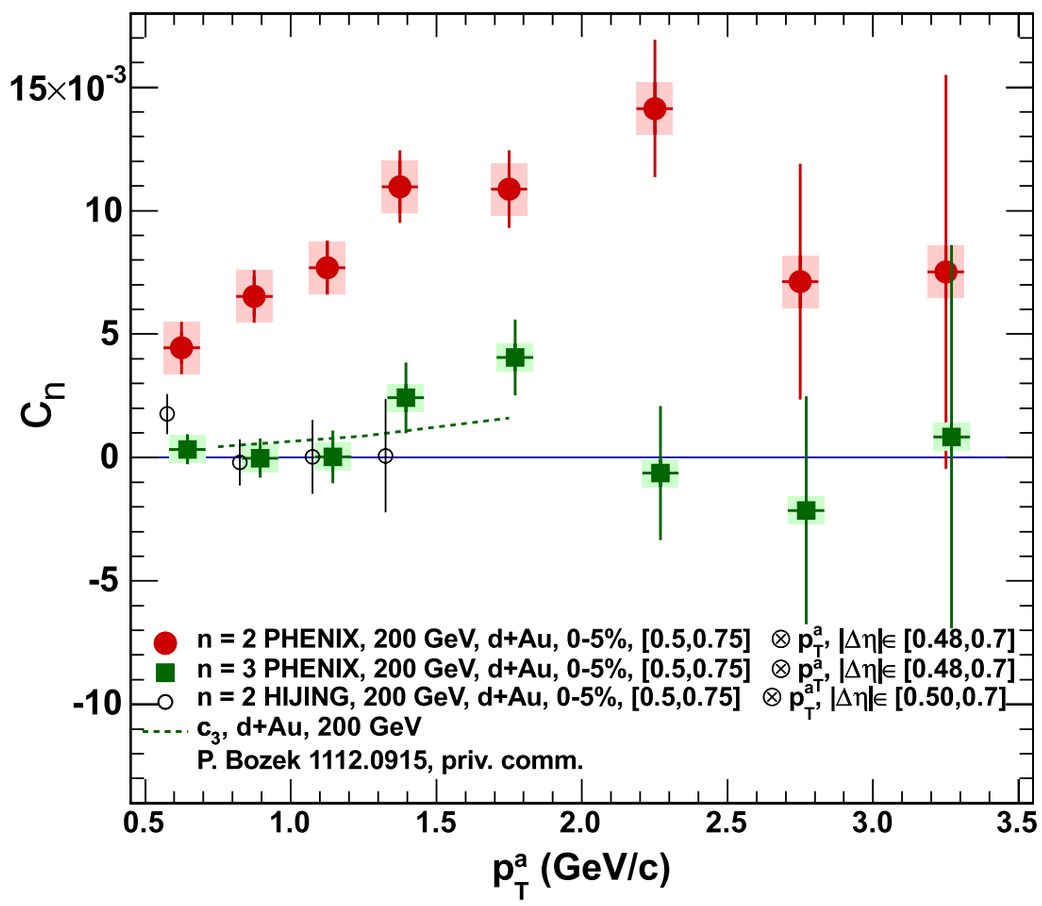


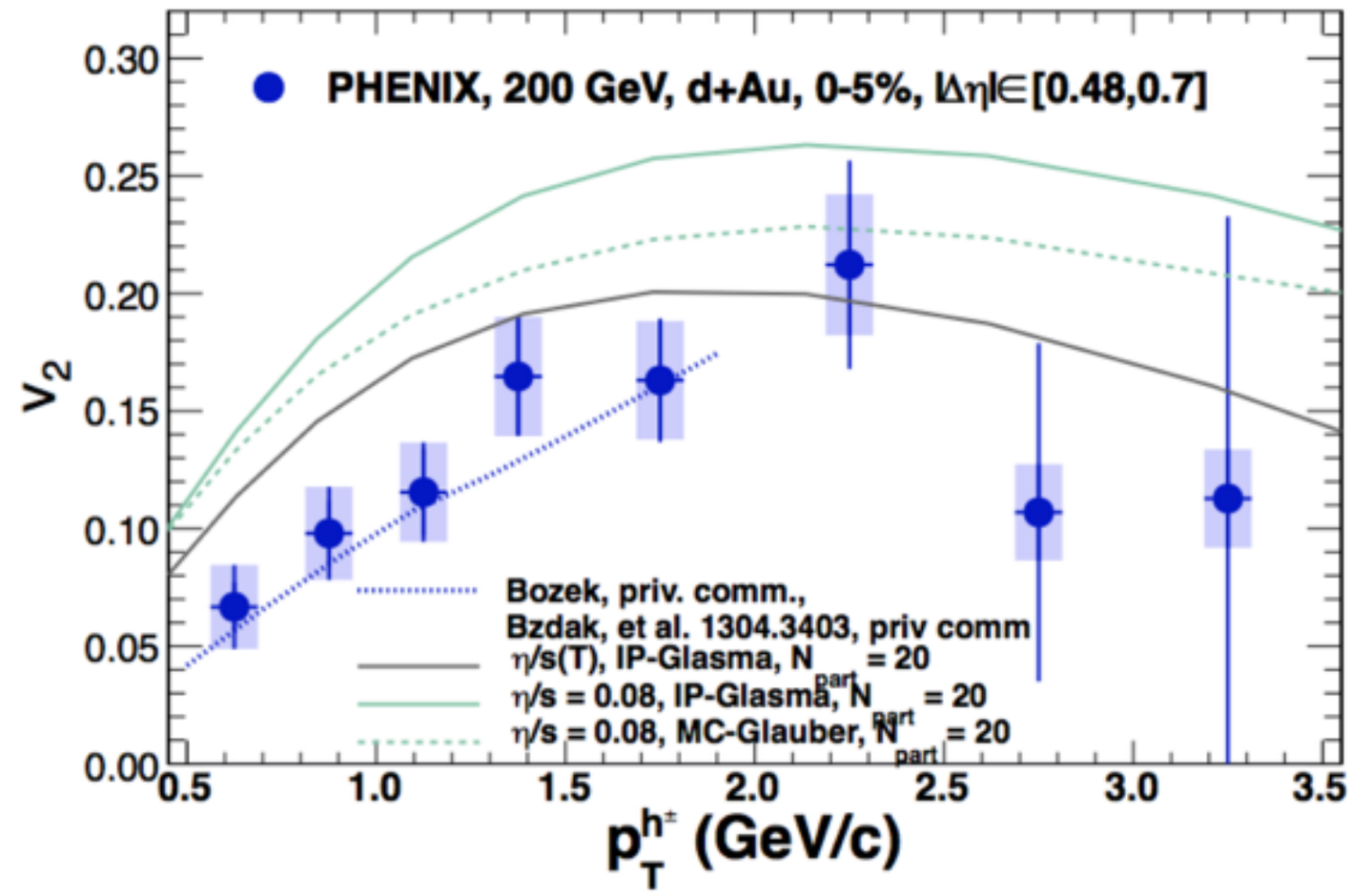
need to determine harmonic background well to have good measure of jet-like signal

Likewise, flow-like signal (1% modulation) not apparent in PHENIX d+Au forward di-hadron analysis.
Phys. Rev. Lett. 107, 172301 (2011)

Two particle correlations in PHENIX central arms

PHENIX PRL 111, 212301 (2013)



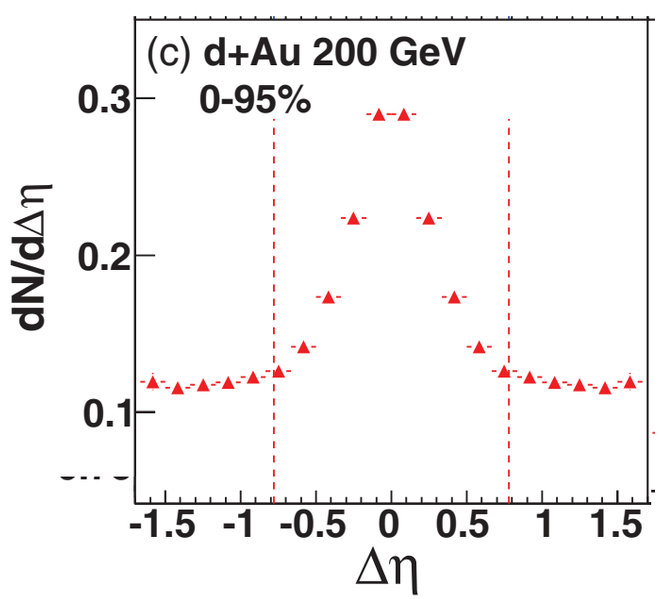


assume factorization $c2(p_T^{trigger}, p_T^{associated}) = v_2(p_T^{trigger}) \times v_2(p_T^{associated})$

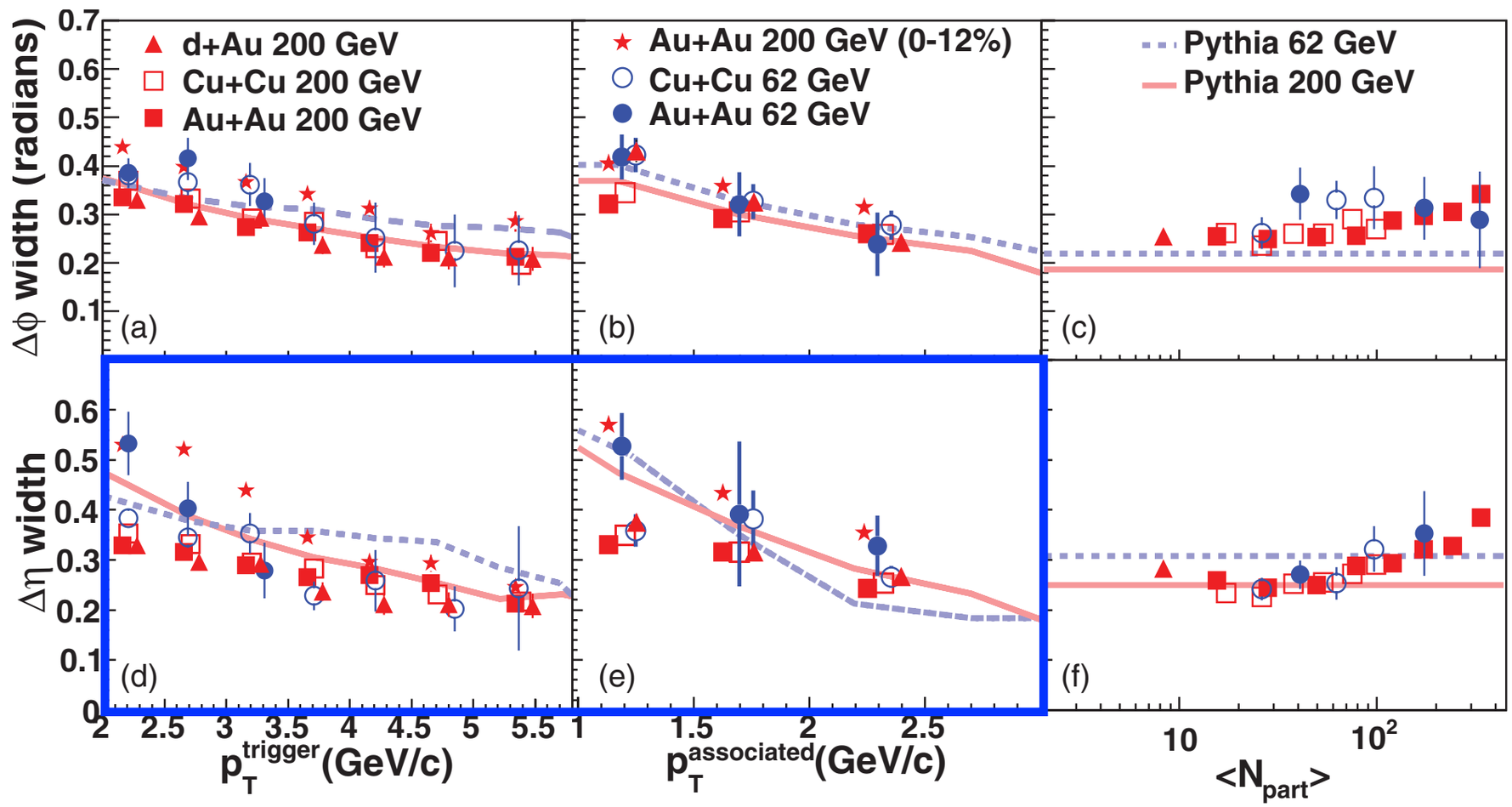
Is there significant residual jet-like contribution?
Hydro interpretation more convincing if broken out by particle type.

η extent of near-side jet-like contribution

$|\Delta\phi| < 0.78$



STAR Phys. Rev. C 85, 014903 (2012)



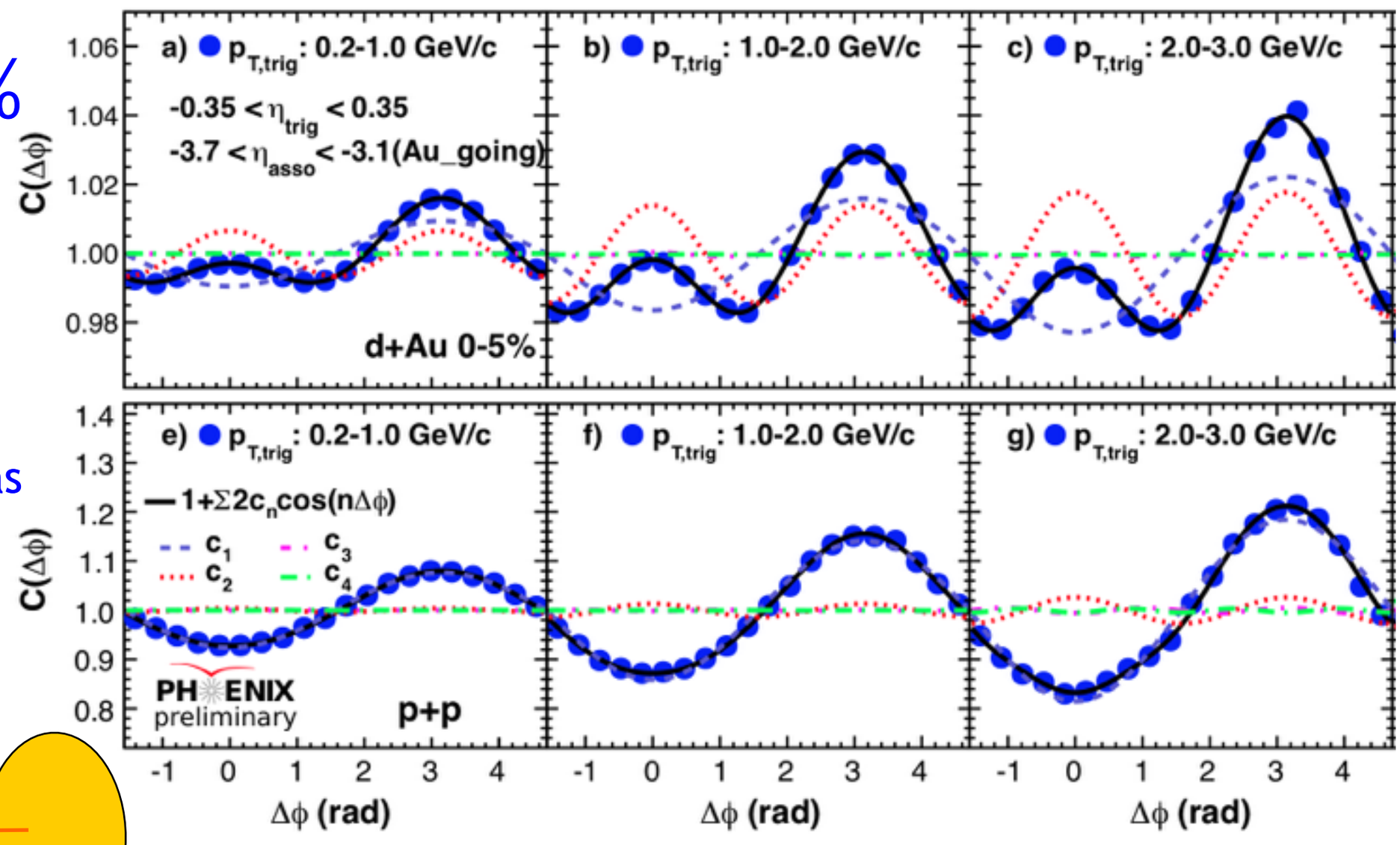
Jet-like component identified using ZYAM subtraction of background.
Widths and trends with $p_T^{trigger,associated}$ in d+Au similar to A+A

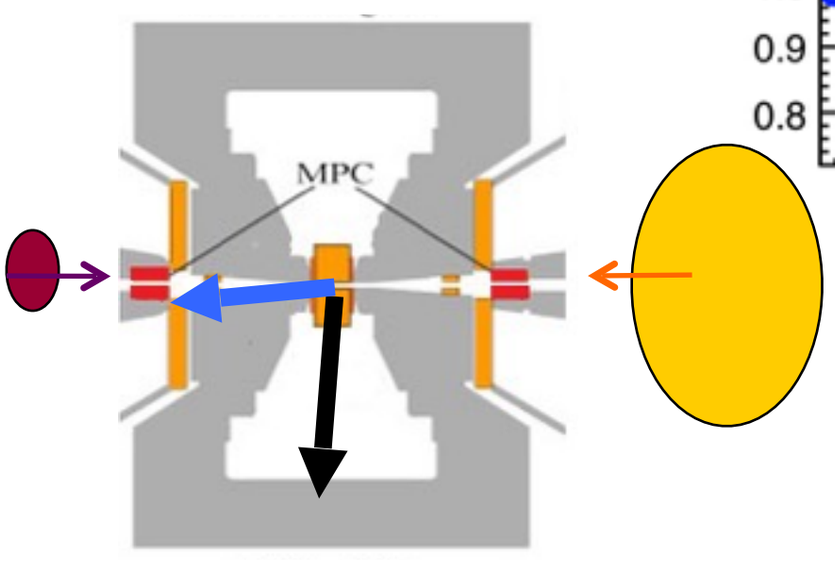
Stretching PHENIX correlation acceptance …

PHENIX arXiv:1404.7461

d+Au 0-5%

p+p min-bias

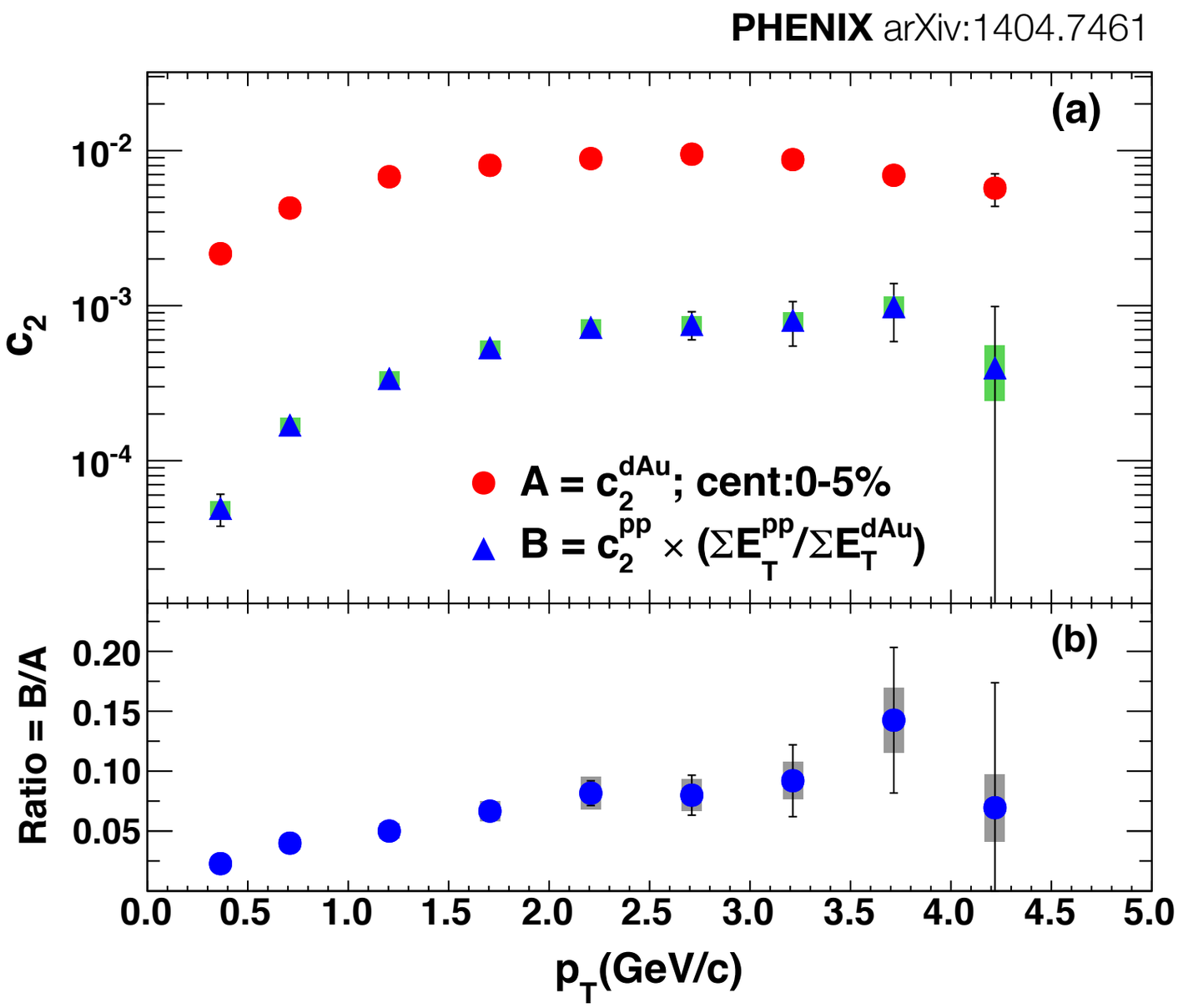




Pair central arm tracks with MPC-S, $\Delta\eta \sim 3.4$

$E = 3$ GeV @ $\eta > 3 \Rightarrow E_T < 300$ MeV

Accounting for jet-like contributions to d+Au c_2



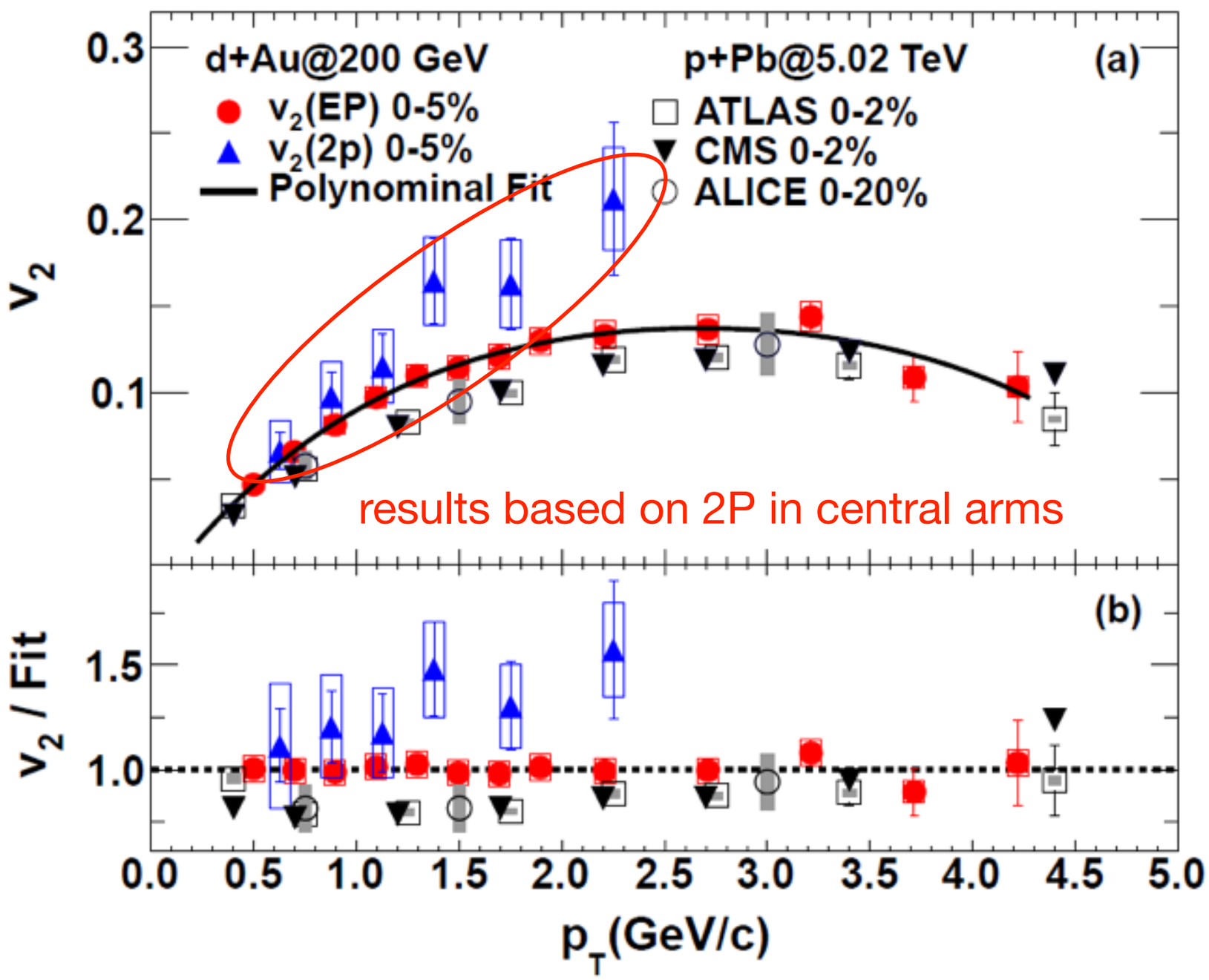
assume jet-like contribution same as in p+p, but effect on correlation diluted by additional multiplicity in d+Au:

$$c_2^{dAu}(p_T) = c_2^{\text{Non-elem.}}(p_T) + c_2^{\text{Elem.}}(p_T)$$
$$\approx c_2^{\text{Non-elem.}}(p_T) + c_2^{pp}(p_T)\frac{\Sigma E_T^{pp}}{\Sigma E_T^{dAu}}$$

if not true, presumably indicates modified jet-like production in d+Au – also interesting.

v_2(EP) of charged hadrons in d+Au

PHENIX arXiv:1404.7461



Use MPC to determine event plane, measure particles in central arms

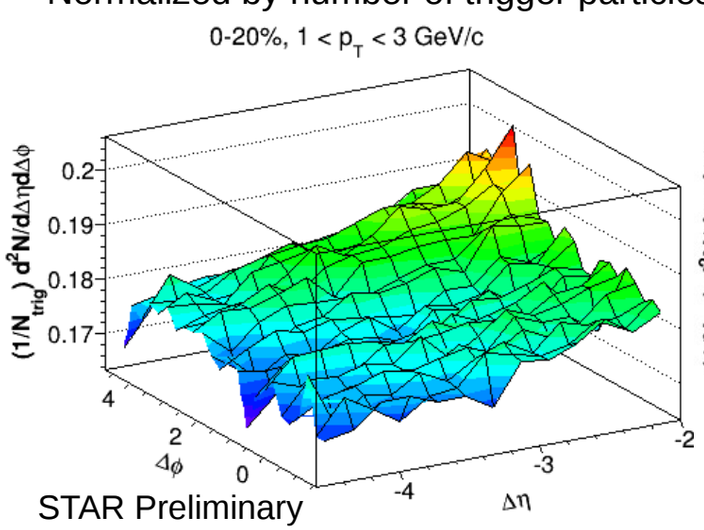
Dihadron $\Delta\eta$–$\Delta\phi$ Correlations

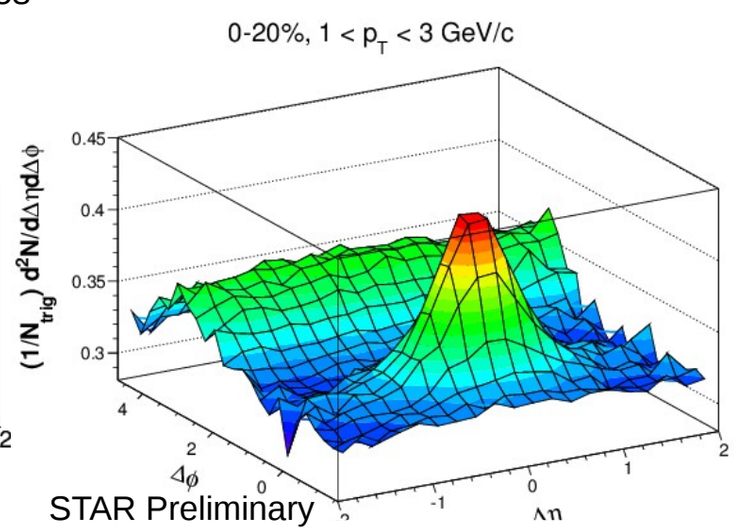
d+Au@200 GeV Run3

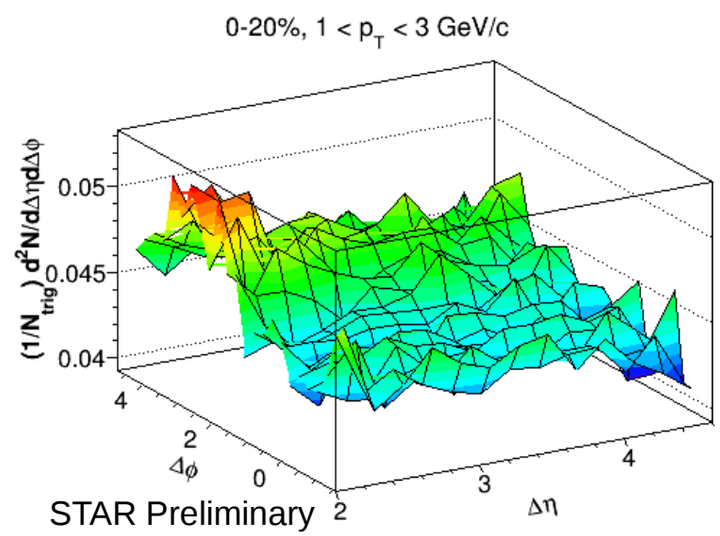
p_T: [1,3]x[1,3] GeV/*c*

Trigger-Associate

Normalized by number of trigger particles







TPC-FTPC (Au-going)	TPC-TPC	TPC-FTPC (d-going)
$-4.5<\Delta\eta<-2$	$-2<\Delta\eta<2$	$2<\Delta\eta<4.5$
ZDC Energy	FTPC Multiplicity or ZDC Energy	ZDC Energy

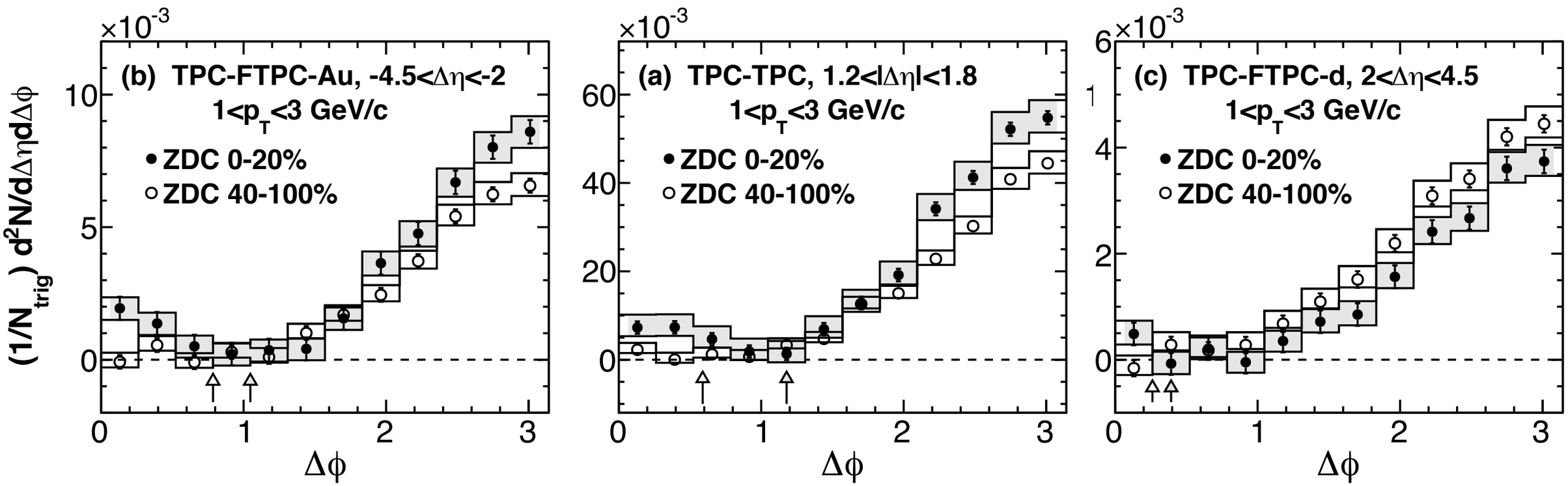
Background estimated by $\Delta\eta$-dependent ZYAM (Zero-Yield-At- Minimum)

STAR long range azimuthal correlations in d+Au

STAR arXiv:1502.07652

high- and low-activity event selections

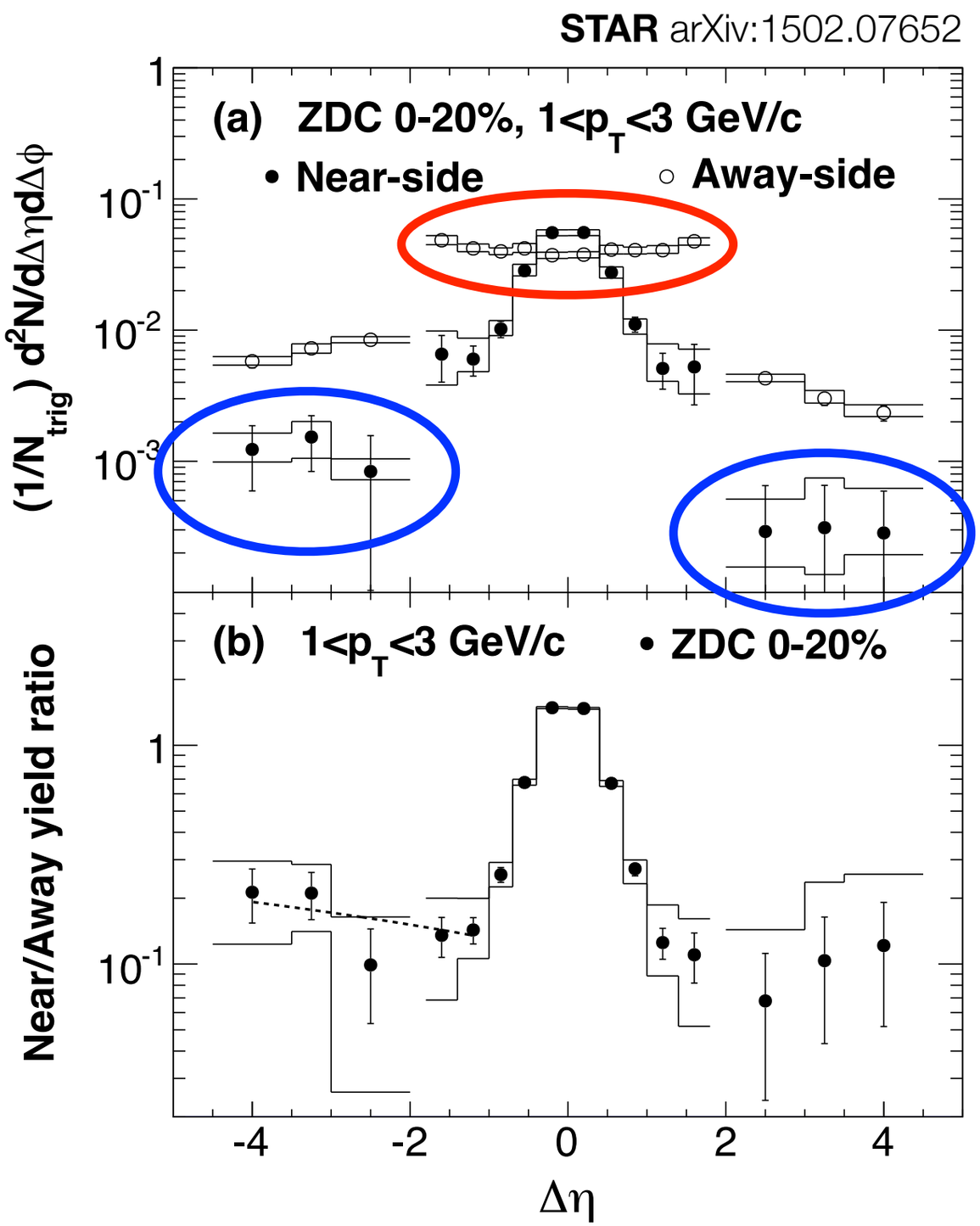
(rearranged order of panels)



mid-backward mid-mid mid-forward

near-side yield apparent in high-activity d+Au over wide η range

Many bins in pseudorapidity

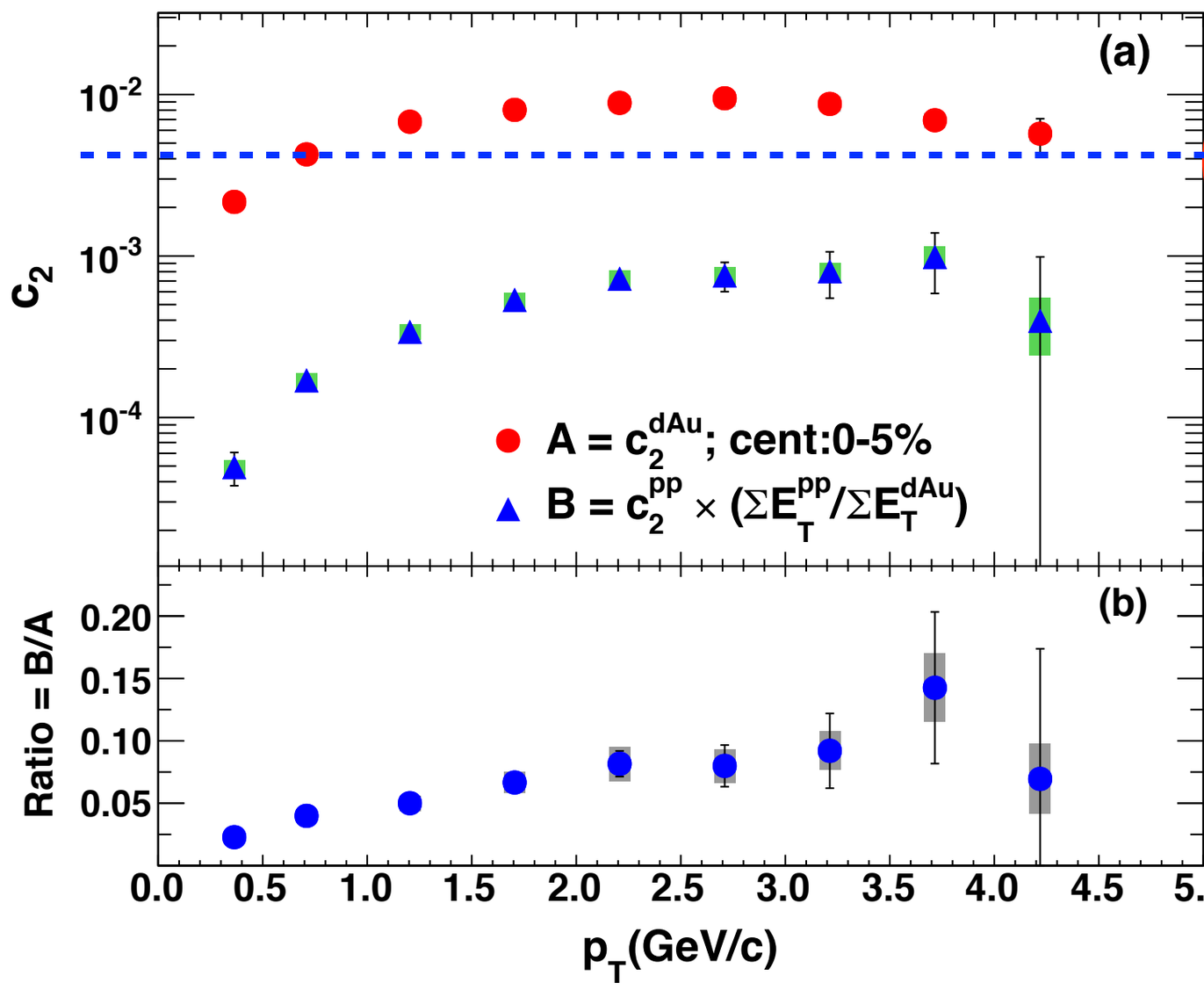


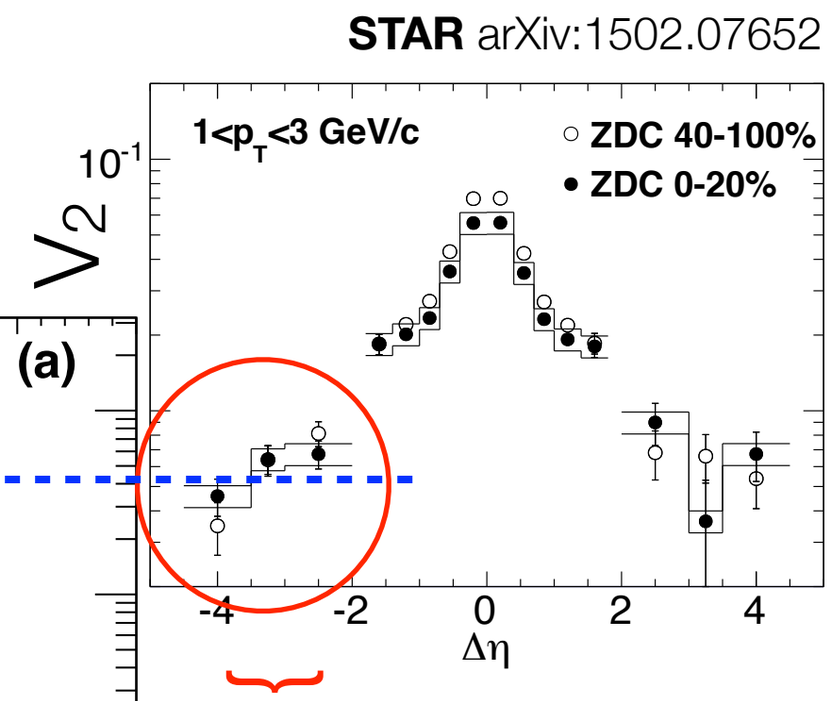
"The away-side correlation shape, noticeably concaved for TPC, is presumably determined by the underlying parton-parton scattering kinematics. On the near side, finite correlated yields are observed at large Δη on the Au-going side in all bins, while the yields are consistent with zero on the d-going side."

Consistency of STAR and PHENIX correlations?

different notation: $c_2 = V_2$

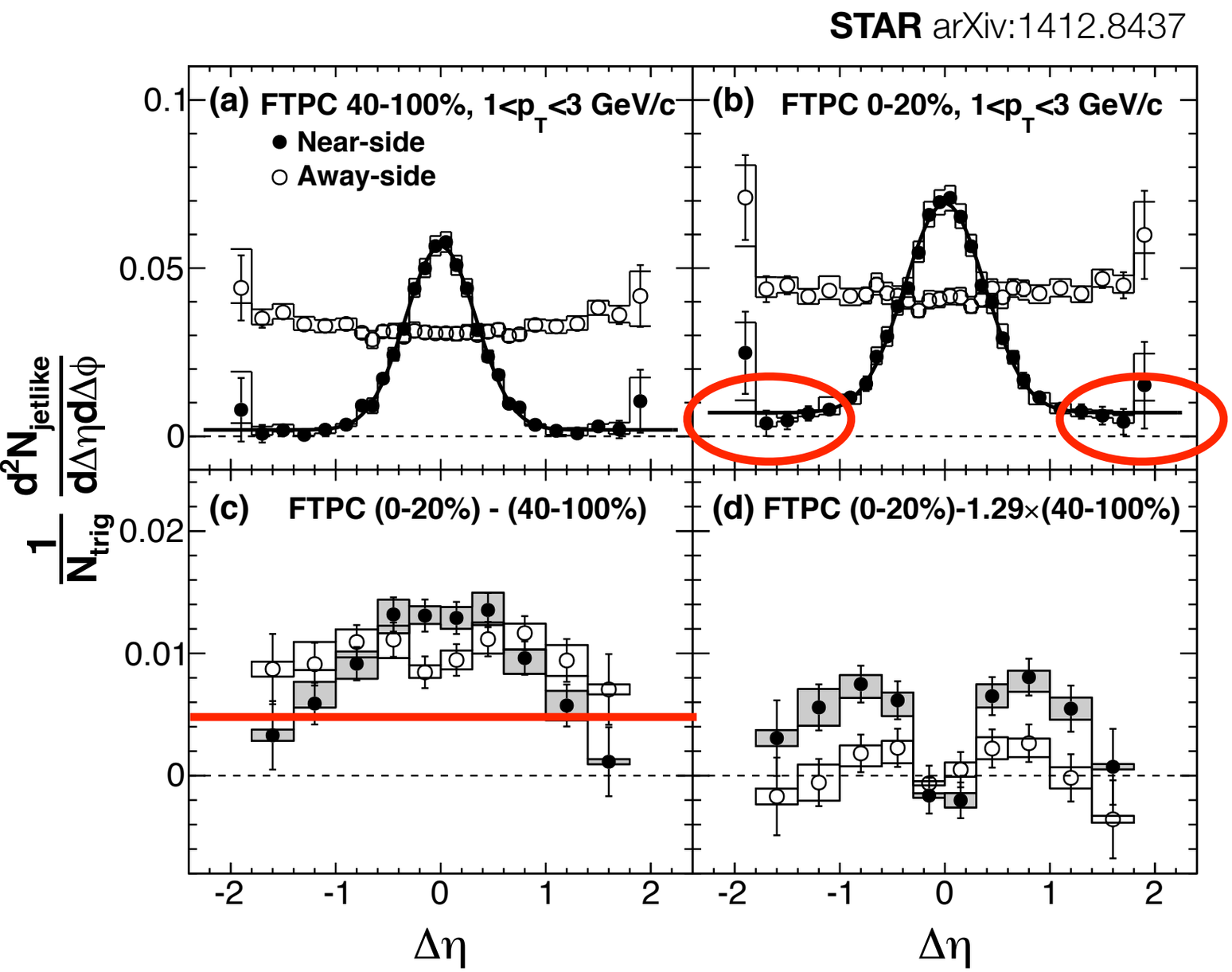
$\langle c_2 \rangle \sim 0.004$





Δη range of PHENIX central arm – MPC correlations on Au-going side

Jet-like yields in high and low activity d+Au events

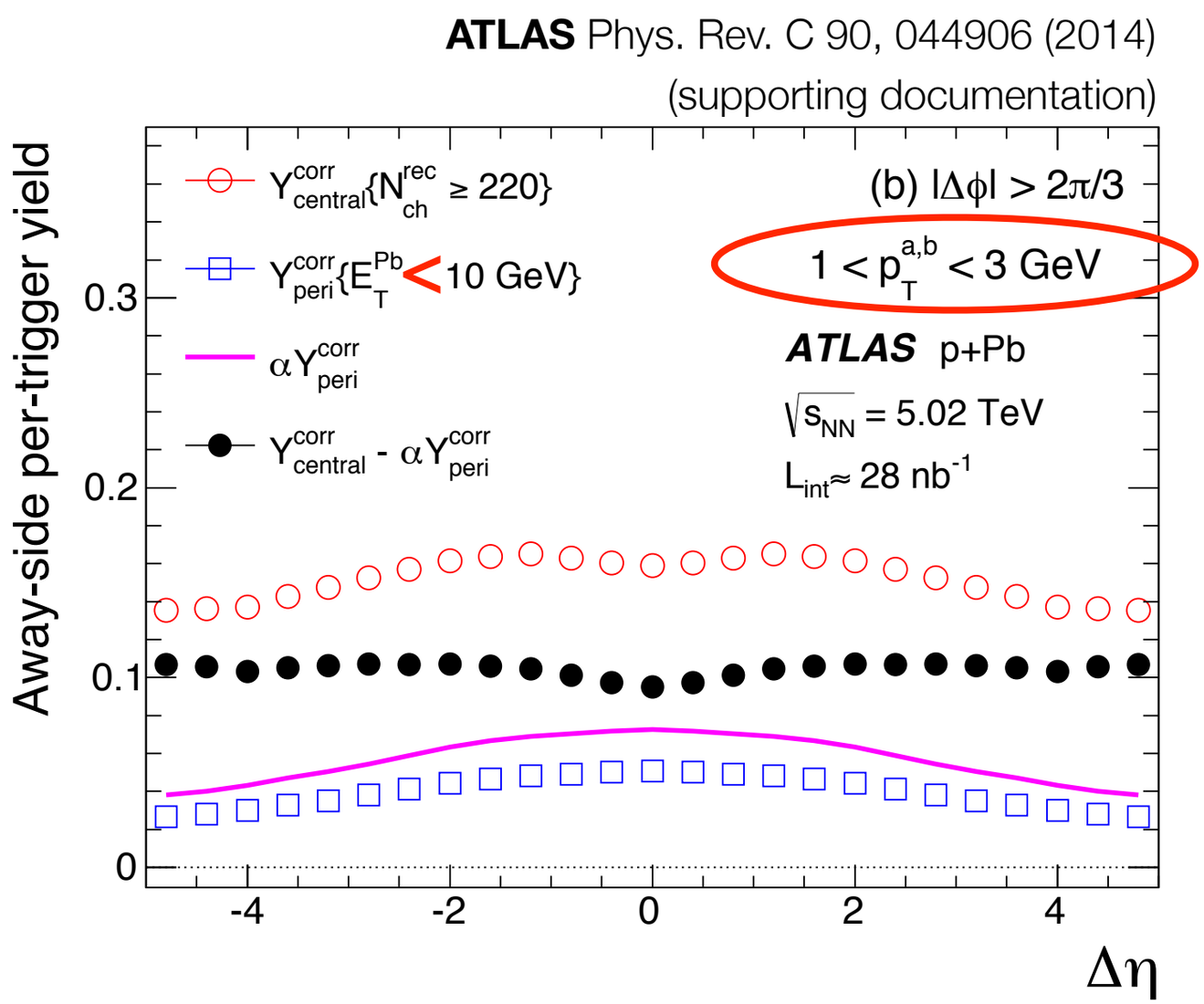


fits to near-side with Gaussian + flat pedestal

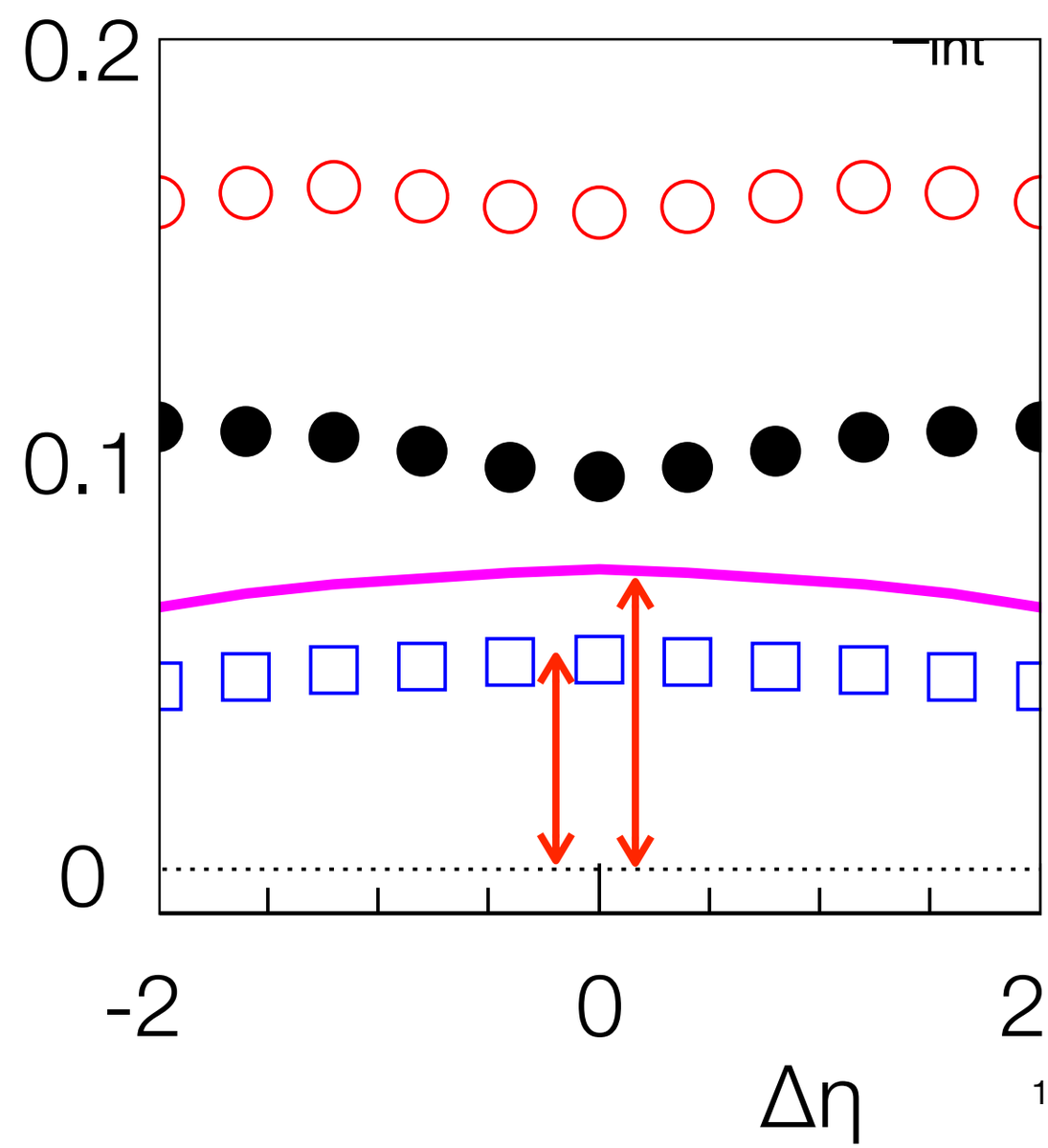
assume ratio of Gaussian yields indicates jet-like yields in high-activity d+Au increase (already an interesting point about modified jet yields in small systems – interesting to test in p+A and ^{3}He+Au)

subtract scaled yield from low-activity d+Au to leave non-jet-like correlation

Similar prescription has also been used by ATLAS

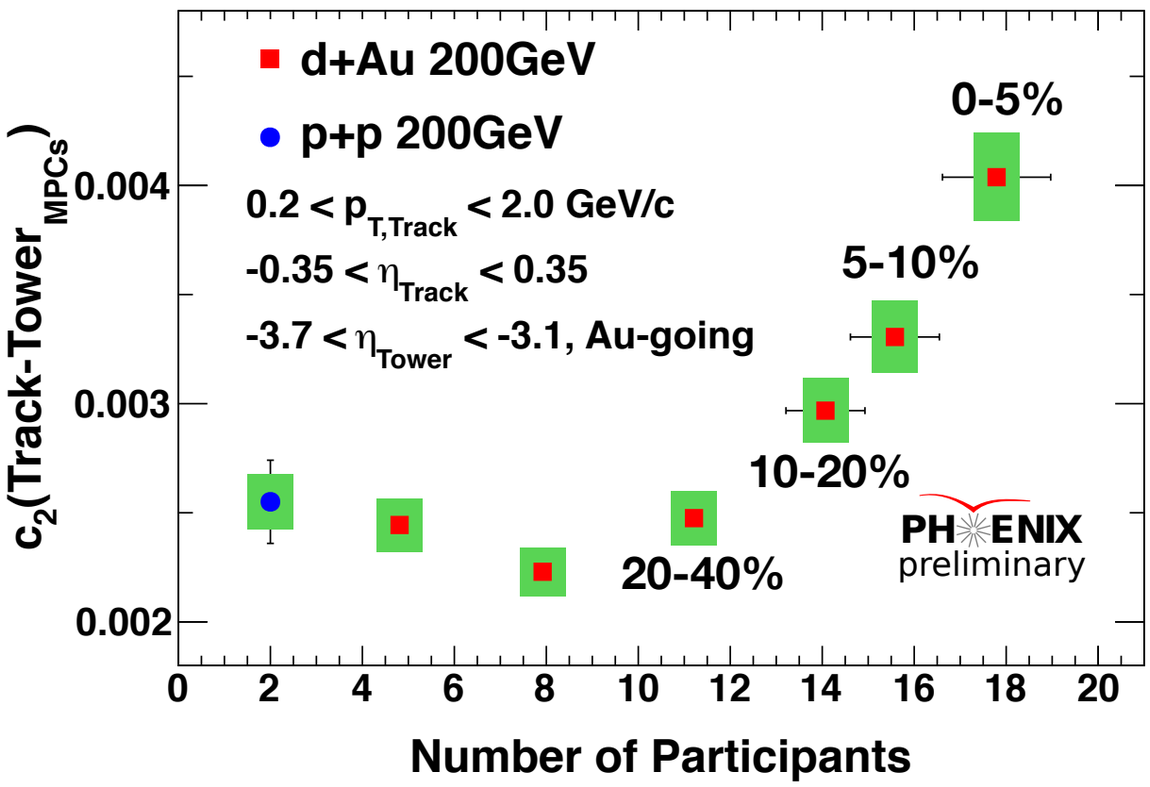


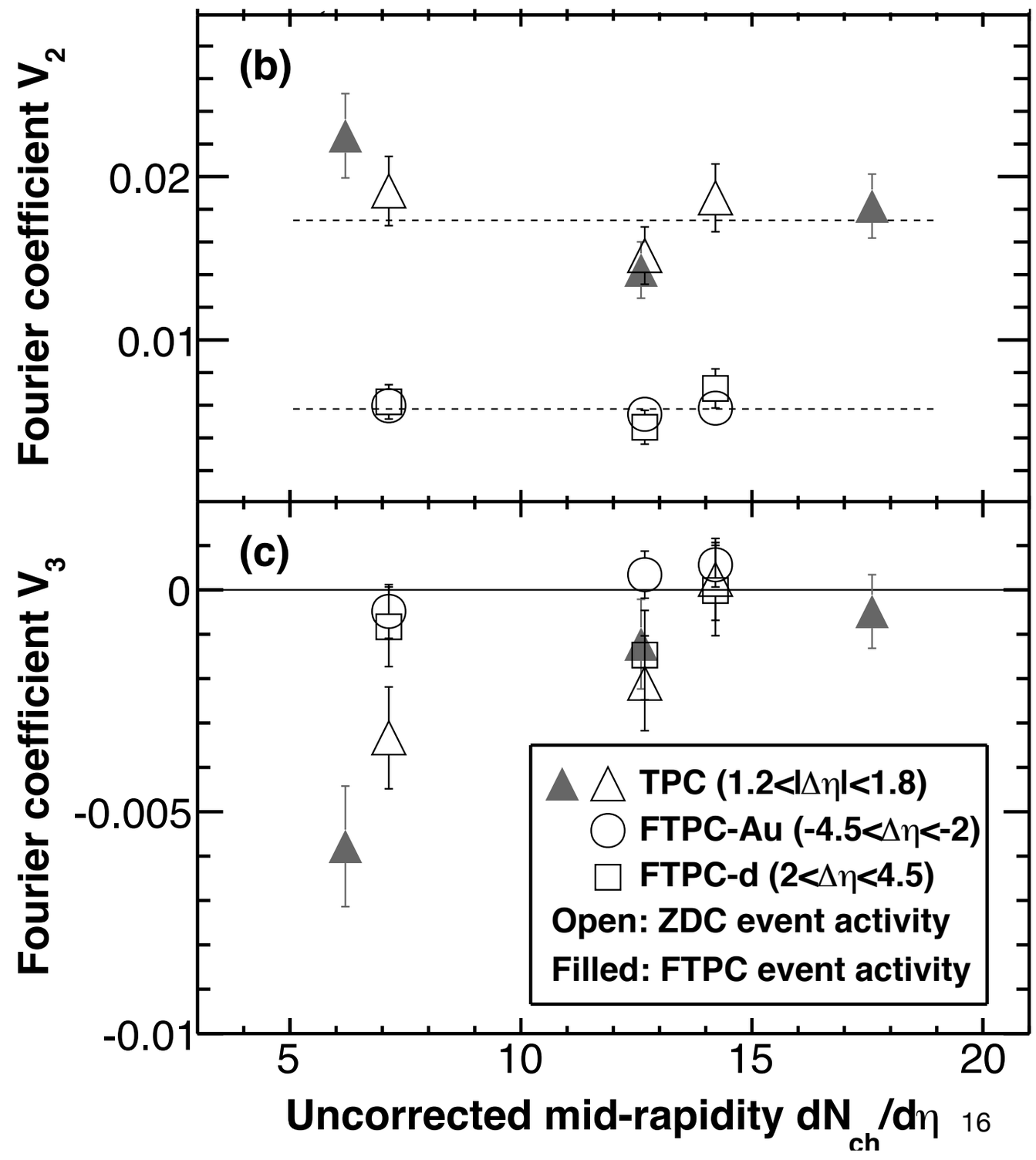
Similar approach
$\alpha \sim 1.4$, compared to
$\alpha = 1.29$ in STAR analysis



Centrality (activity) dependence of d+Au correlation

STAR arXiv:1502.07652

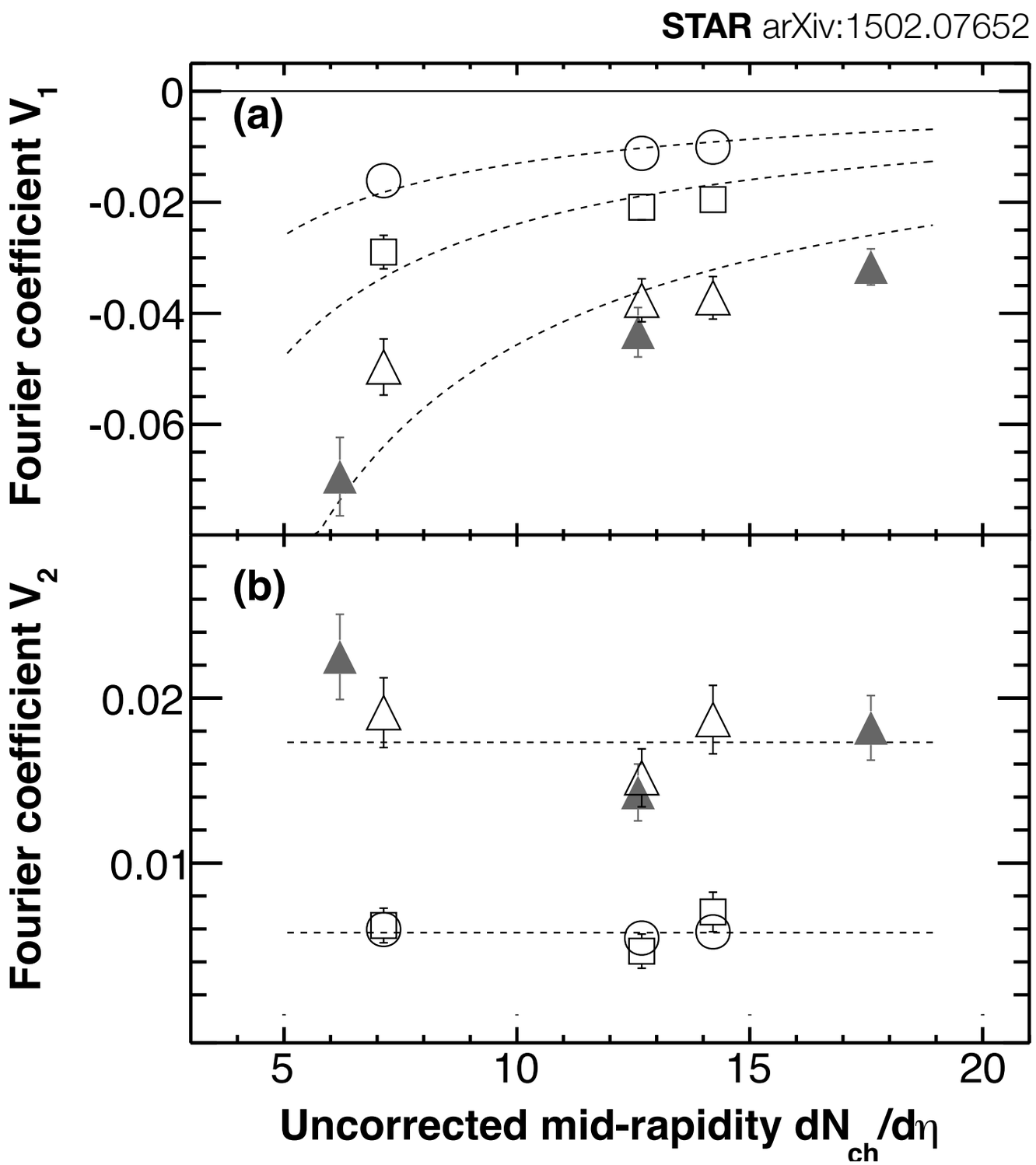




STAR obtains V_2 ~independent of event activity

PHENIX sees a significant increase in c_2 for more central events

Is there interplay between V_2 and V_1?



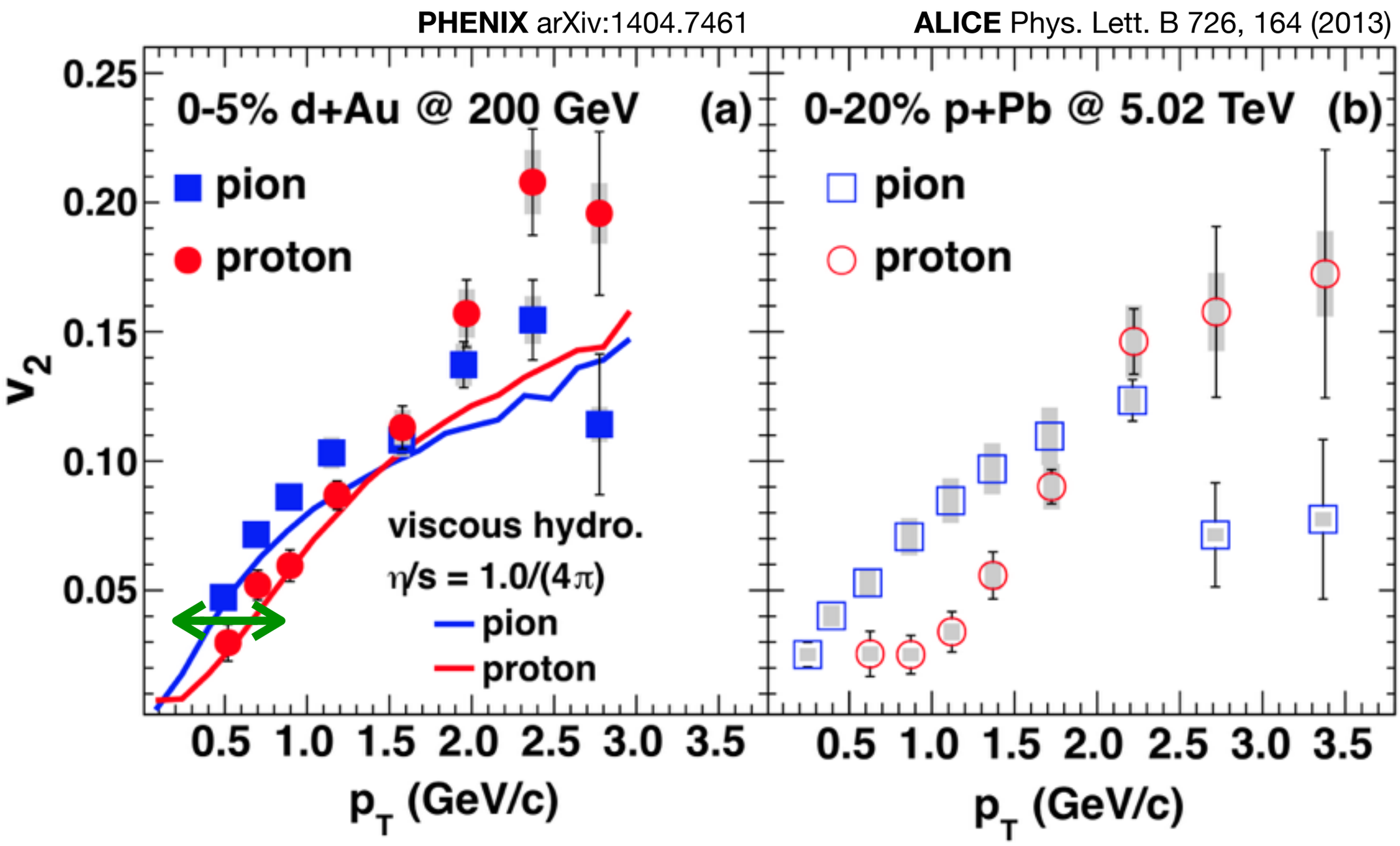
▲ △ TPC (1.2<|Δη|<1.8)
○ FTPC-Au (-4.5<Δη<-2)
□ FTPC-d (2<Δη<4.5)
Open: ZDC event activity
Filled: FTPC event activity

magnitude of V_1 decreases with increasing event activity

magnitude of V_1 is largest for smallest Δη correlations

magnitude of V_1 is larger for correlations in d-going than in Au-going direction

Back to that question of mass ordering ...

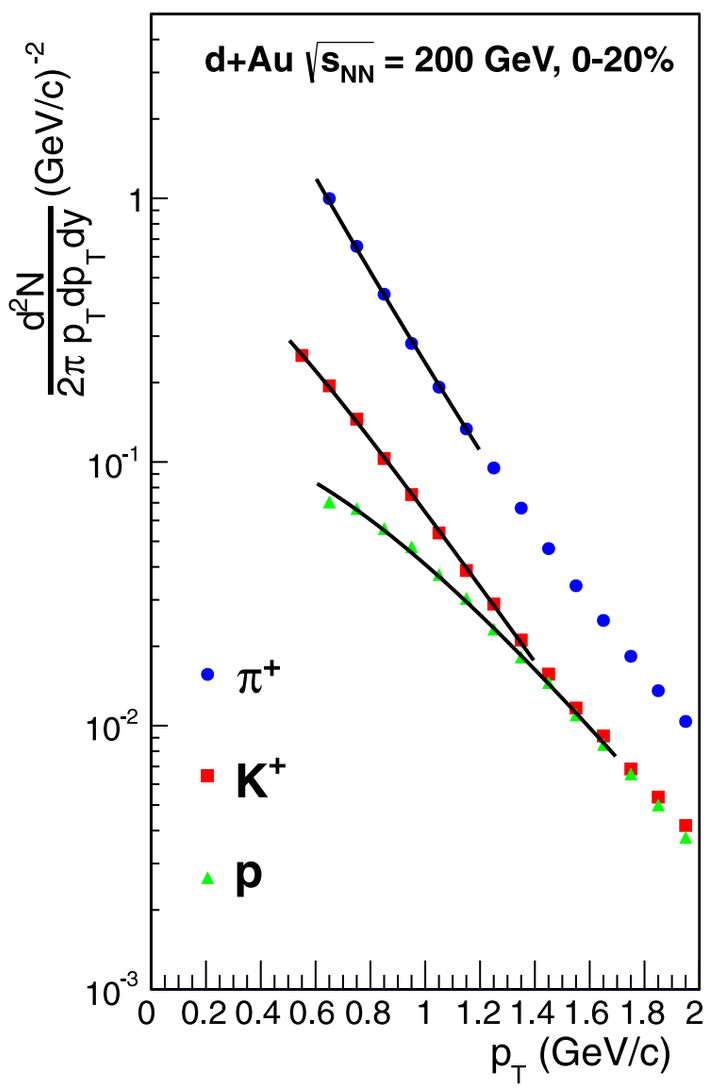


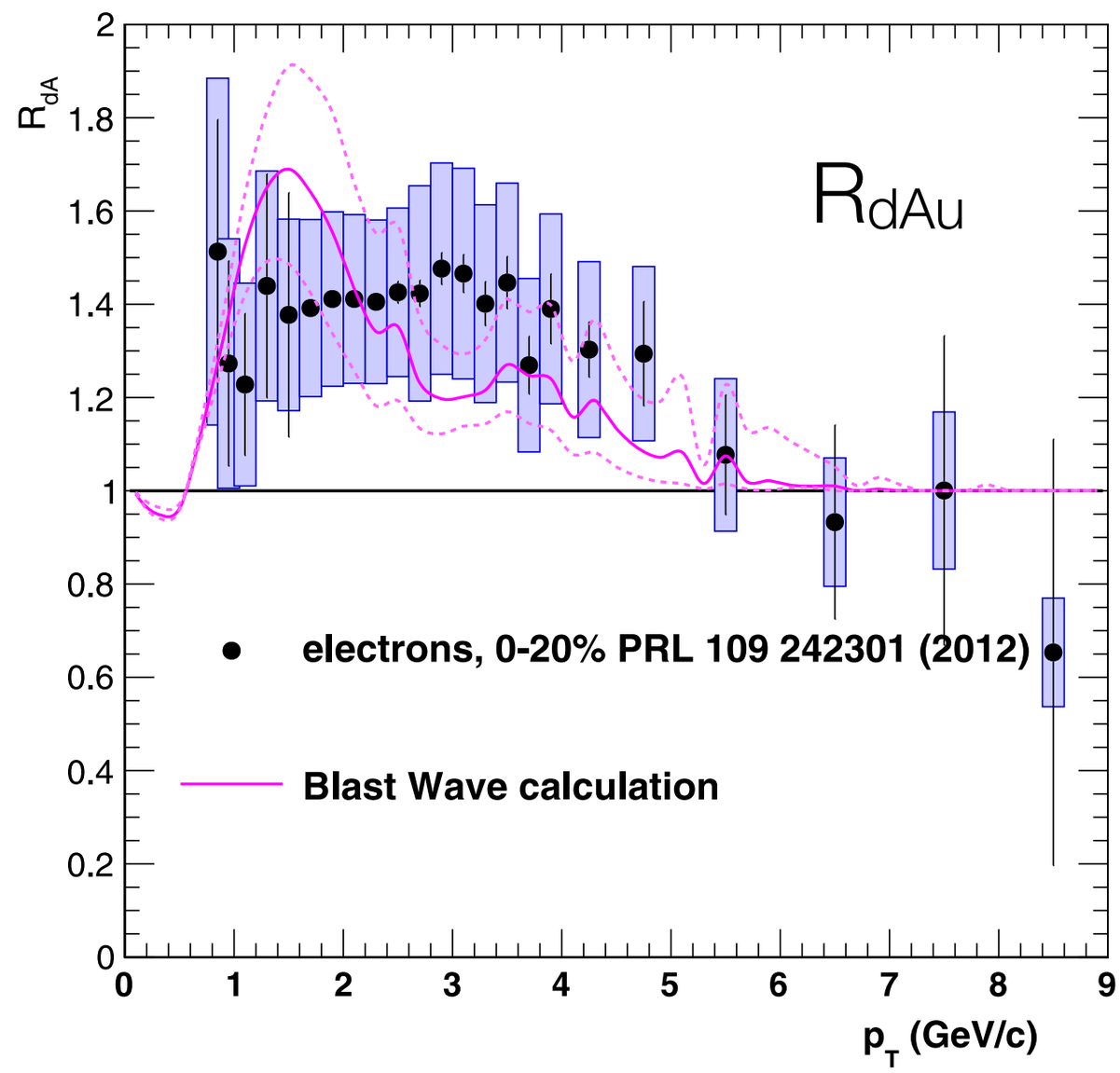
consistent with expectation given a fluid velocity field

Other collective observables in d+Au at RHIC?

Radial flow of heavy flavor in d+Au?

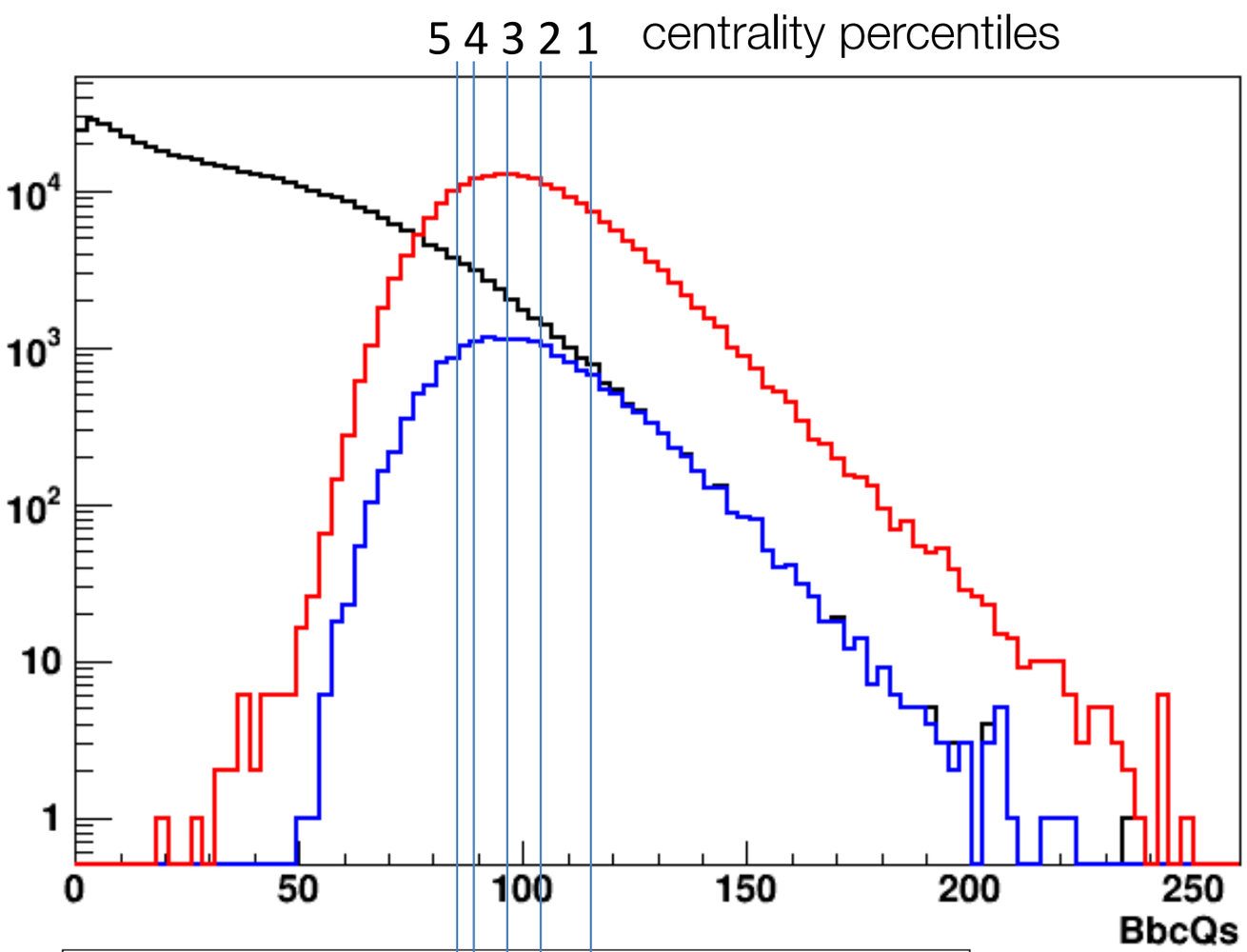
A. Sickles, PLB **731** (2014) 51

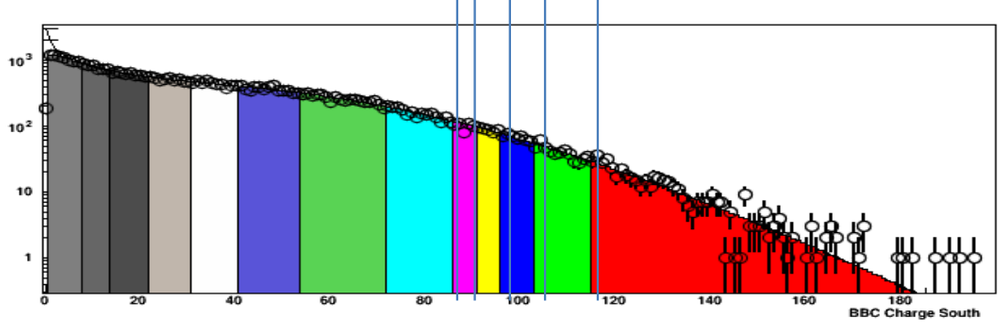




Blast wave fits to measured hadrons + FONLL D,B spectra + decay to electrons

^{3}He+Au in Run-14





Both STAR and PHENIX recorded ^{3}He+Au @ 200 GeV for two weeks at end of Run-14

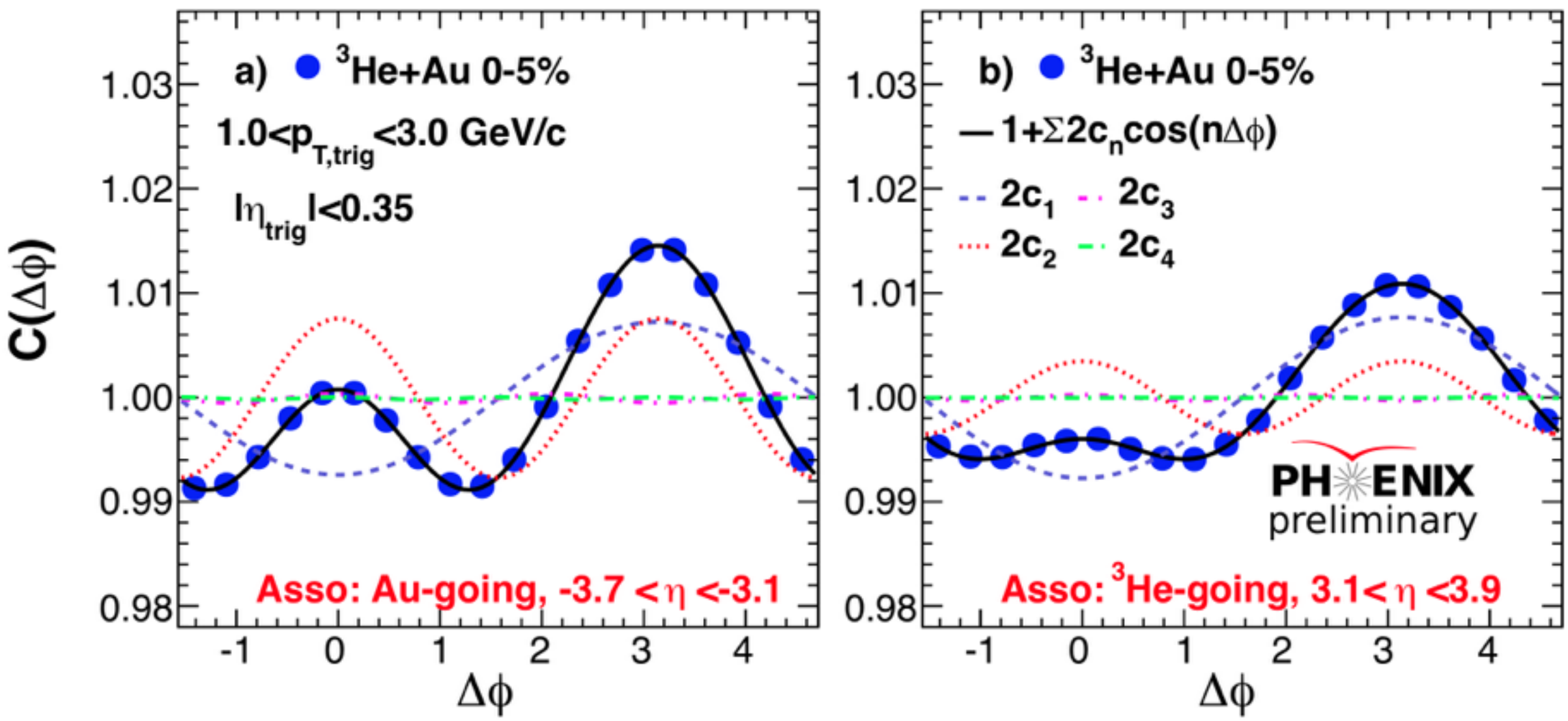
PHENIX – new centrality trigger based on beam-beam counter in Au-going direction

PHENIX – 2.2B MB, 0.8B central

Glauber fit with NBD (J. Nagle)

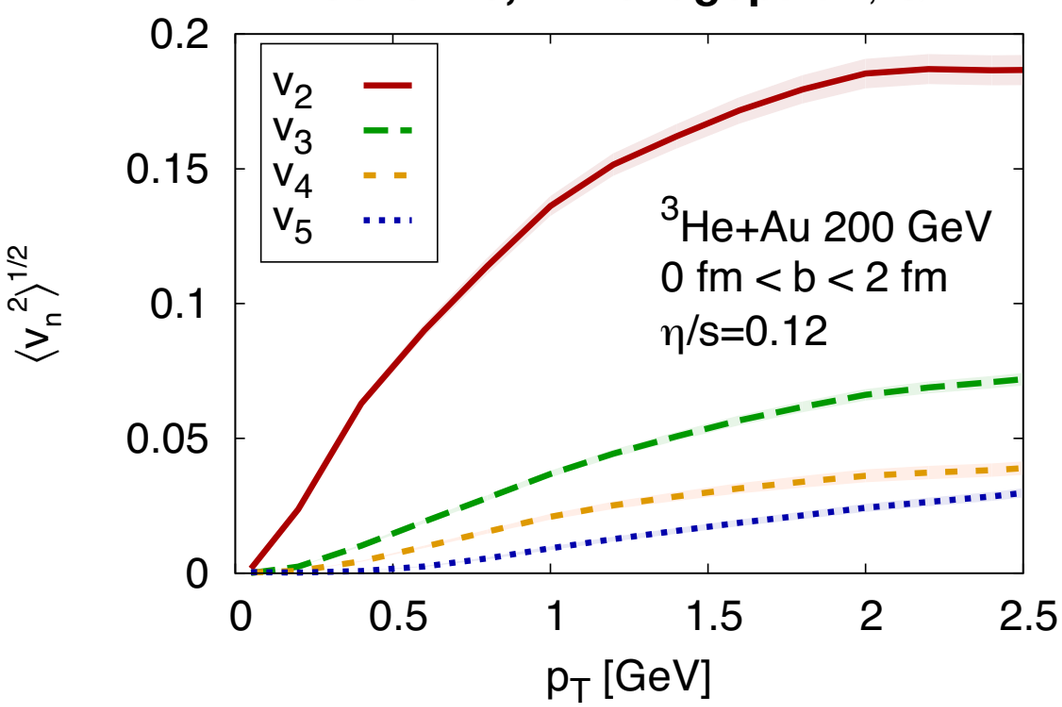
^{3}He wavefunction from Joe Carlson, James Lynn (LANL)

Long-range correlation in ^{3}He+Au

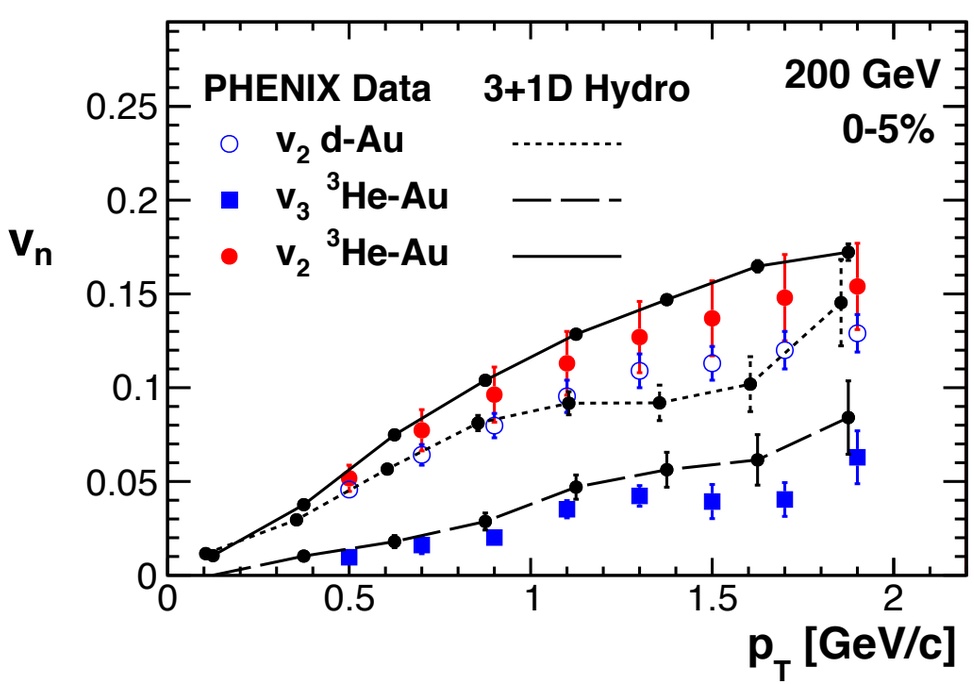


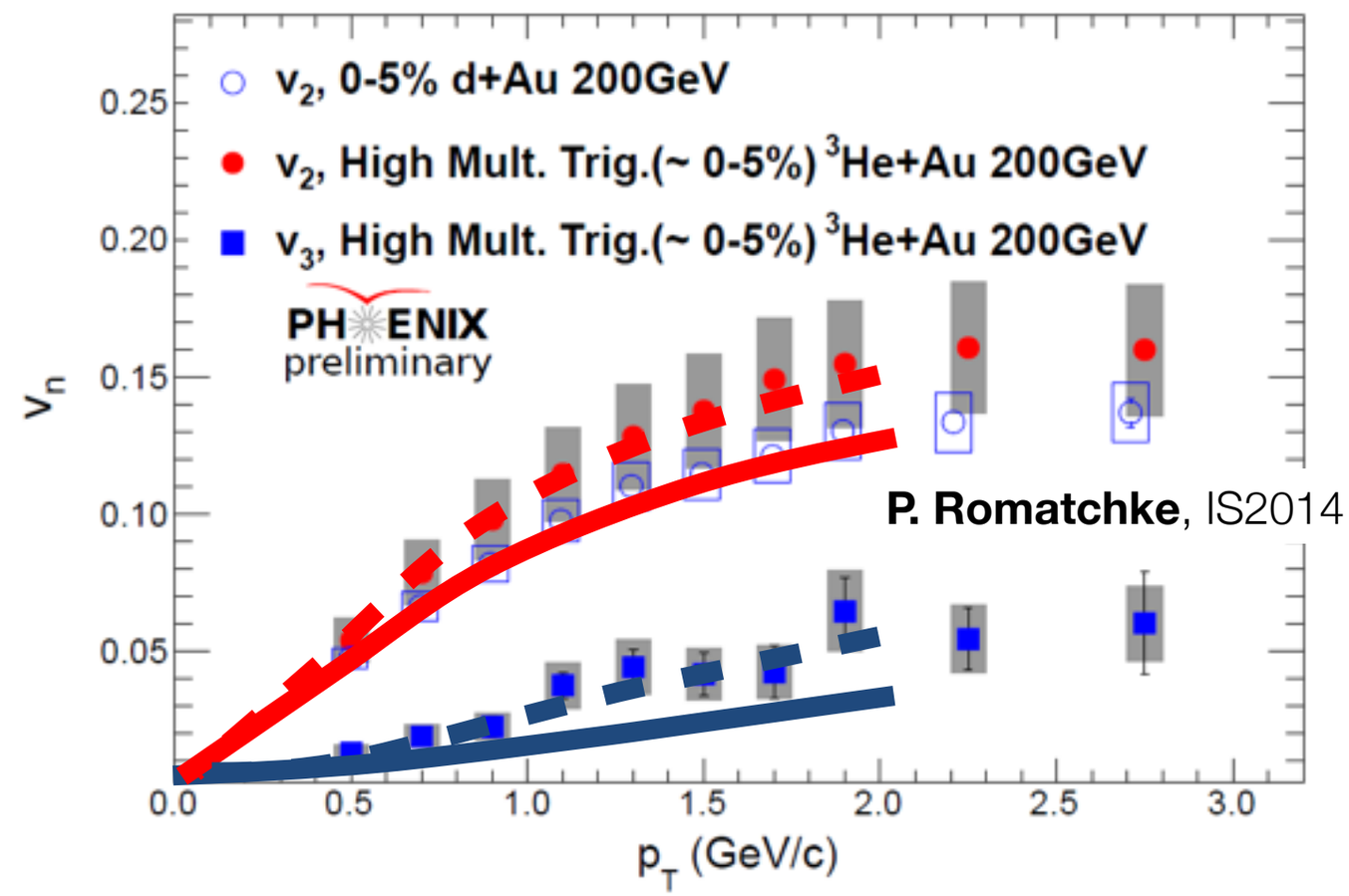
^{3}He+Au results, predictions, and modeling

B. Schenke, R. Venugopalan, QM'14



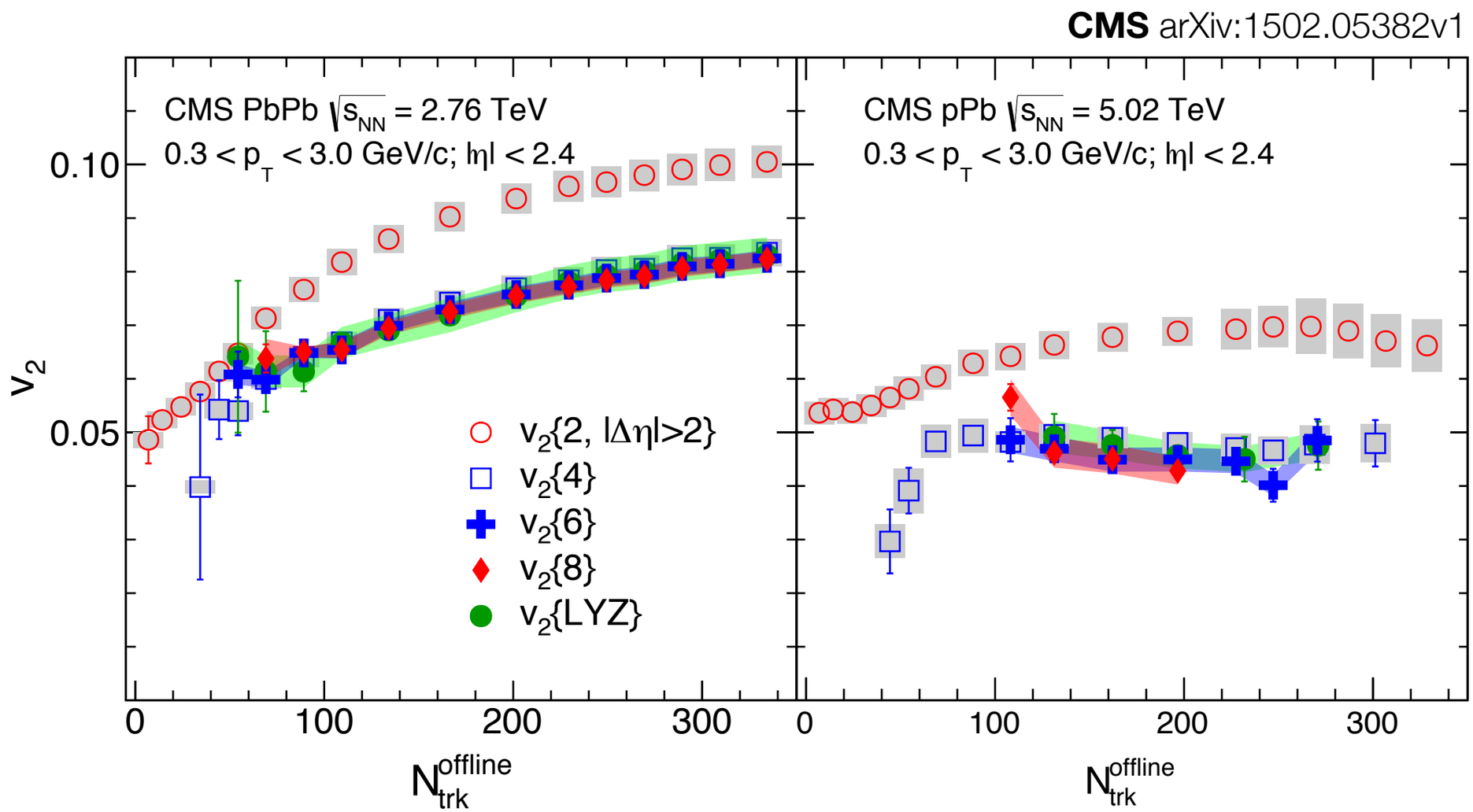
P. Bozek, W. Broniowski arXiv:1503.00468





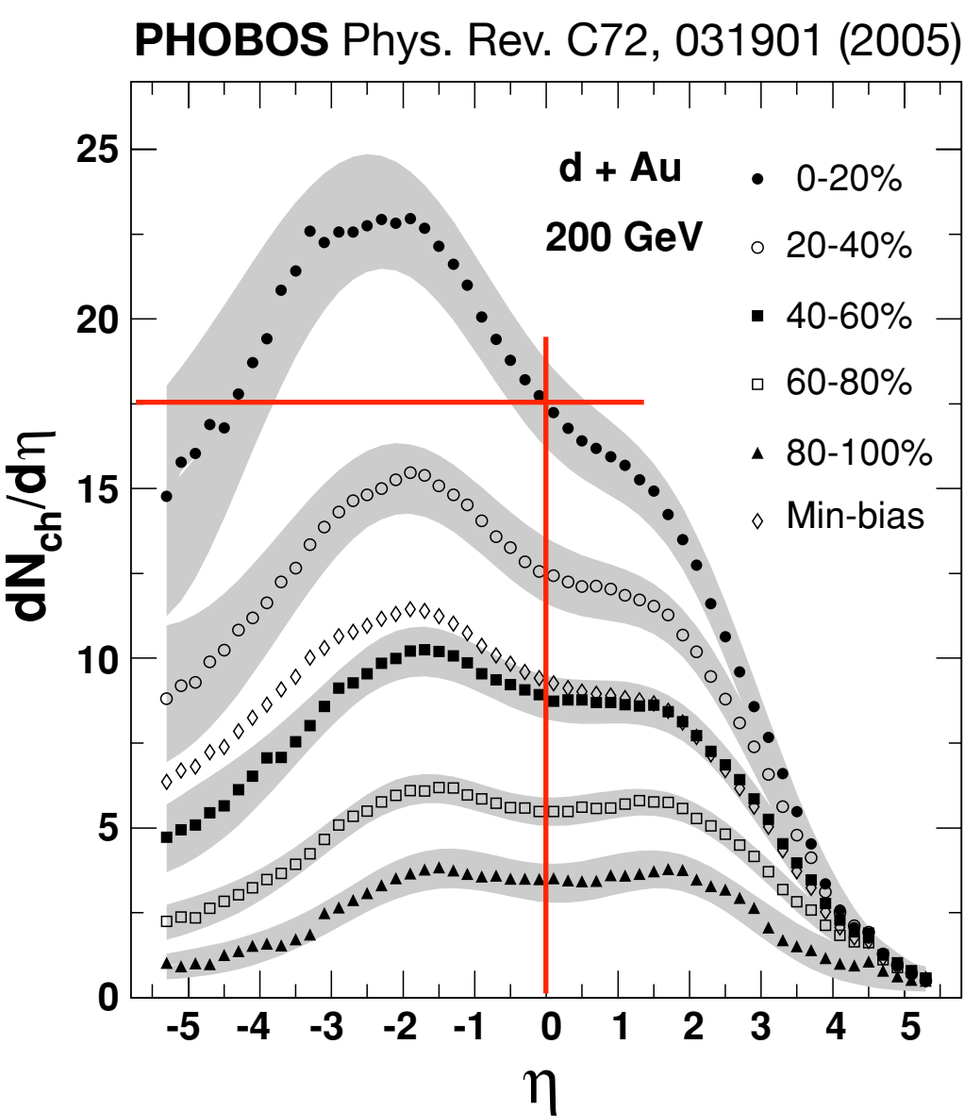
P. Romatchke, IS2014

Multi-particle methods



With debate about how to account for flow and non-flow in d+Au, better to employ known techniques that address this differently – particularly with additional collision geometries.

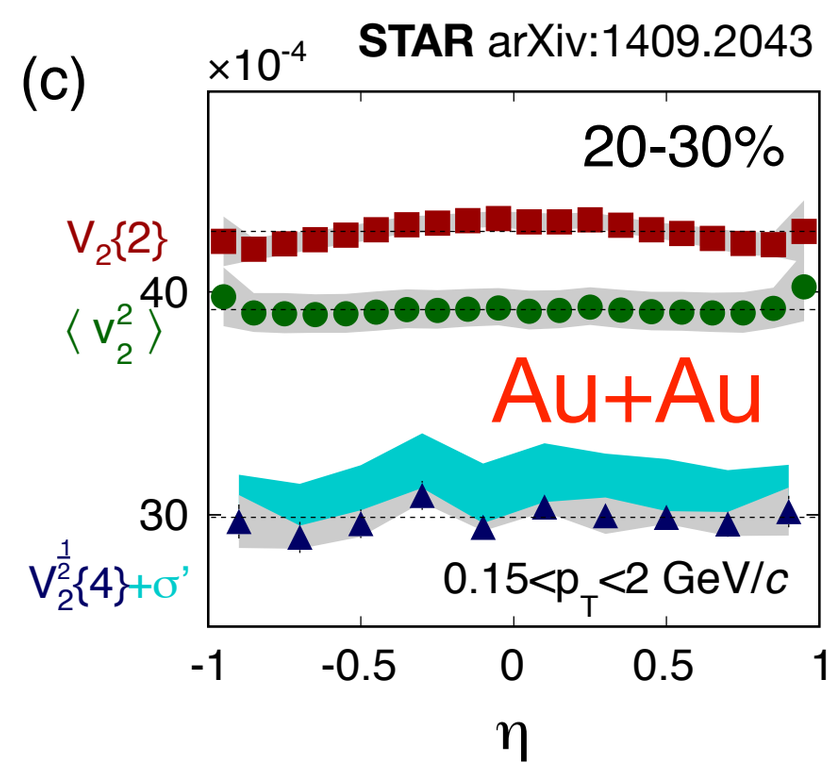
Higher-order cumulant analysis in p,d,^{3}He+A?



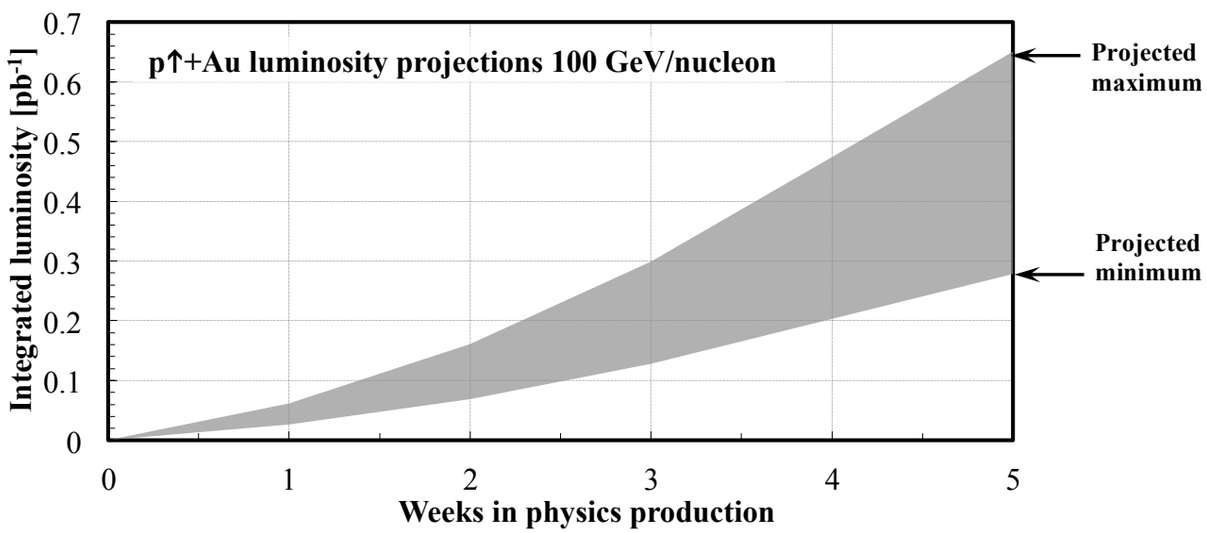
Set scale of track multiplicity in small systems
$dN_{ch}/d\eta \sim 18$ in 0–20% d+Au

Challenging for PHENIX –
⇒ $N_{ch} \sim 6$ in mid-rapidity spectrometers in 0–20% d+Au

Would be great to see STAR cumulant analysis applied to d+Au (and ^{3}He+Au and p+A too!) to help resolve issues of flow, non-flow

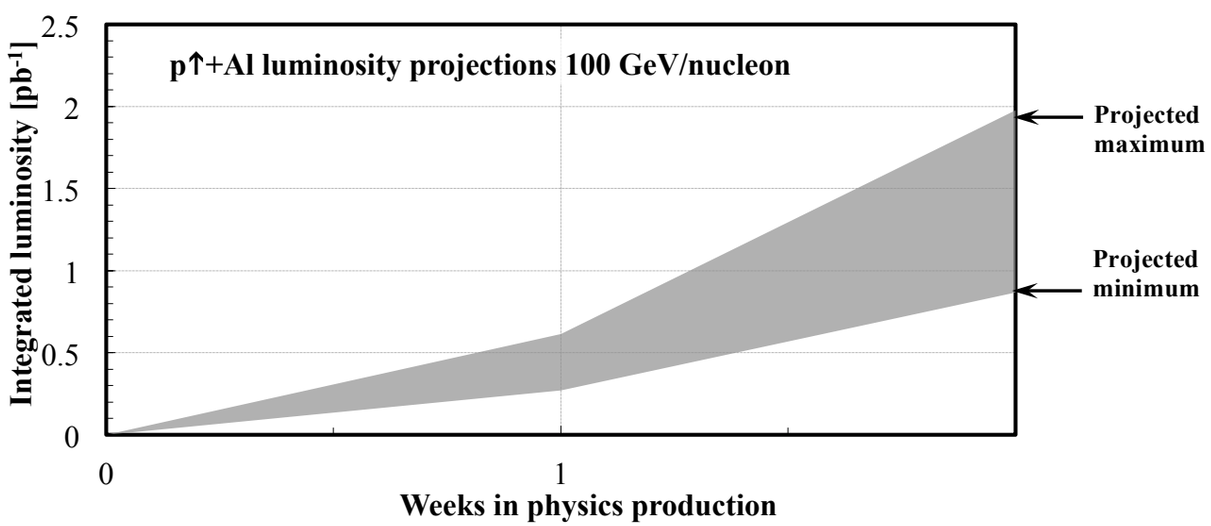


RHIC Run-15



300–650/nb of p+Au
c.f. 200/nb of d+Au in Run-8

- 9 weeks of p+p at $\sqrt{s}$ = 200 GeV
- 5 weeks of p+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV
- 2 weeks of p+Al collisions at $\sqrt{s_{NN}}$ = 200 GeV



1000–2000/nb of p+Al

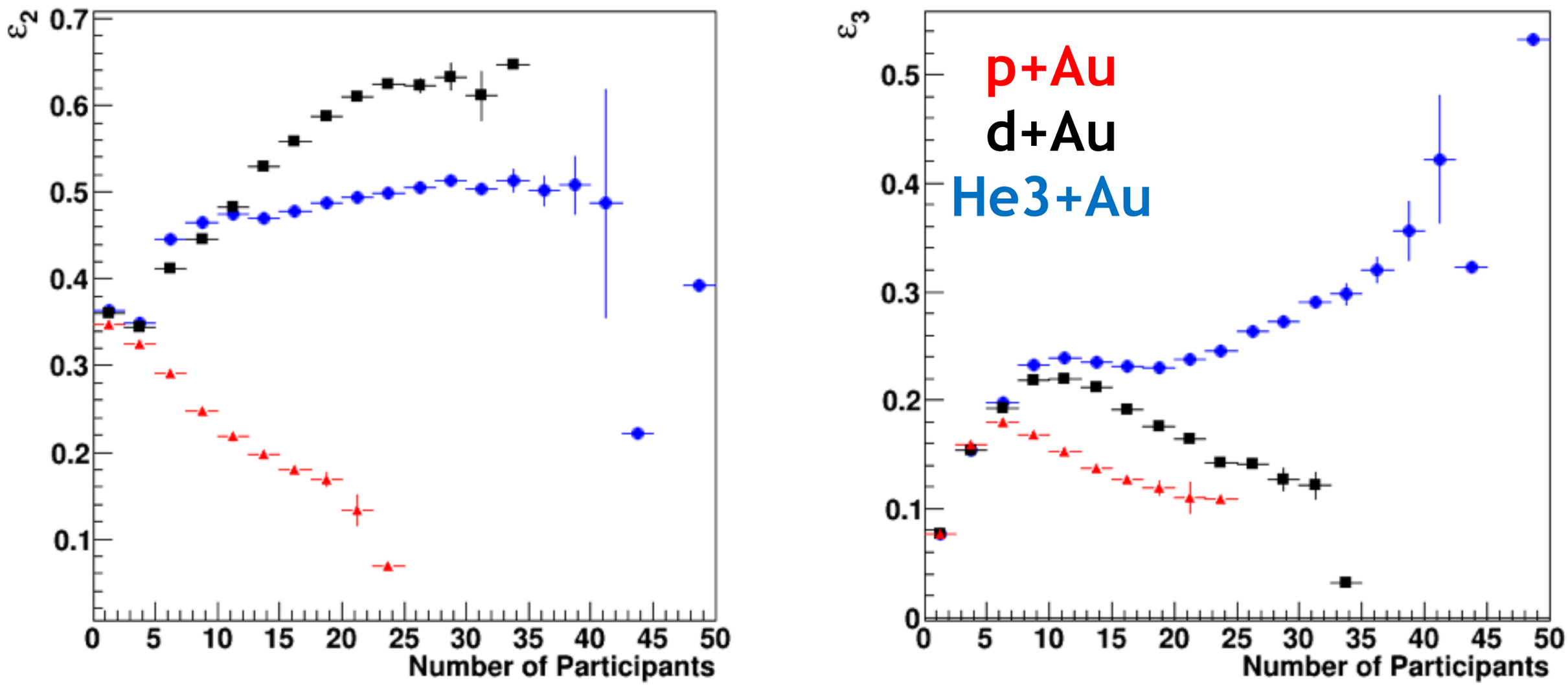
STAR Run-15 request

Run	Energy	Duration	System	Goals	priority	sequence
15	$\sqrt{s_{NN}}$=200GeV	5-week	Transverse p+Au	saturation physics, ridge and reference, $\mathcal{L}$=300 nb^{-1}	1	3
	$\sqrt{s}$=200GeV	12-week	1) p+p	1) HI reference $\mathcal{L}$=90 pb^{-1}, 500M MB	2	
			2) transverse 6 weeks	2) Study transversity, Sivers effects $\mathcal{L}$=40 pb^{-1}, 60% pol.	2	2
			3) longitudinal 6 weeks	3) Study $\Delta g(x)$ $\mathcal{L}$=50 pb^{-1}, 60% pol.	2	1
16	$\sqrt{s}$=200GeV	10-week	Au+Au	Λ_c, D v_2, R_{AA}, Υ 10nb^{-1}, 2billion MB	1	1
	$\sqrt{s}$=510GeV	7-week	Transverse p+p	A_N of $W^{\pm}$, γ, Drell-Yan, $\mathcal{L}$=400 pb^{-1}, 55% pol	2	2

Table 1.1 STAR BUR for run15 and run16. Both requests assume 22 cryo-weeks for RHIC operation, and 17 weeks of physics run with two species in one RHIC year.

"The [Forward Meson Spectrometer], which is sensitive to photons from neutral pions and other particle decays, is situated in the proton direction in p+Au collisions. This will enable us to study the ridge effect from correlation between neutral pions at η~=3 with hadrons at η~=0, and would require an integrated luminosity of 200 nb^{-1}."

Geometry Control

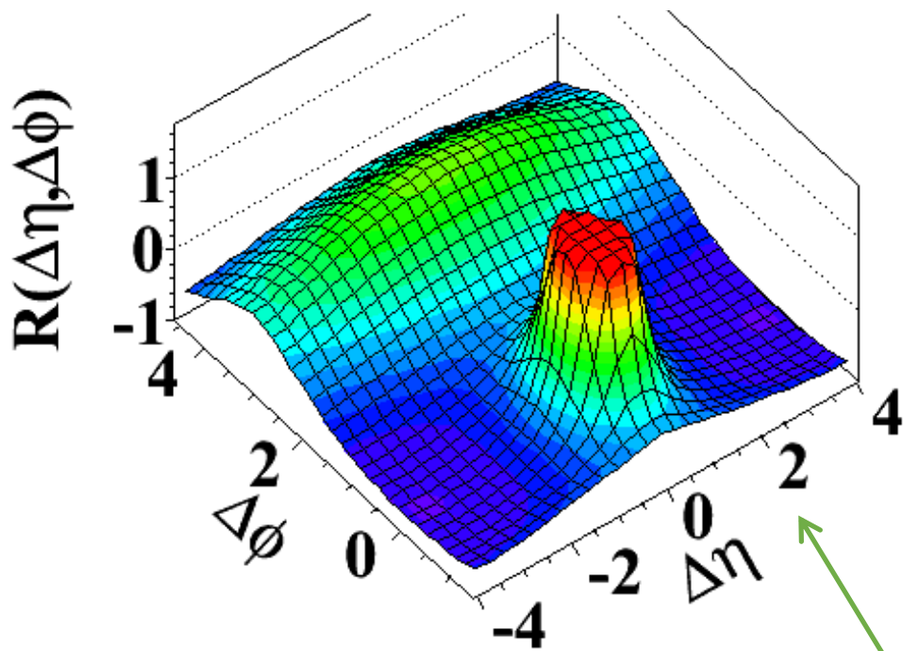


Comparison of different geometries provides key tests of underlying physics (initial conditions, equilibration time, medium properties, etc.)

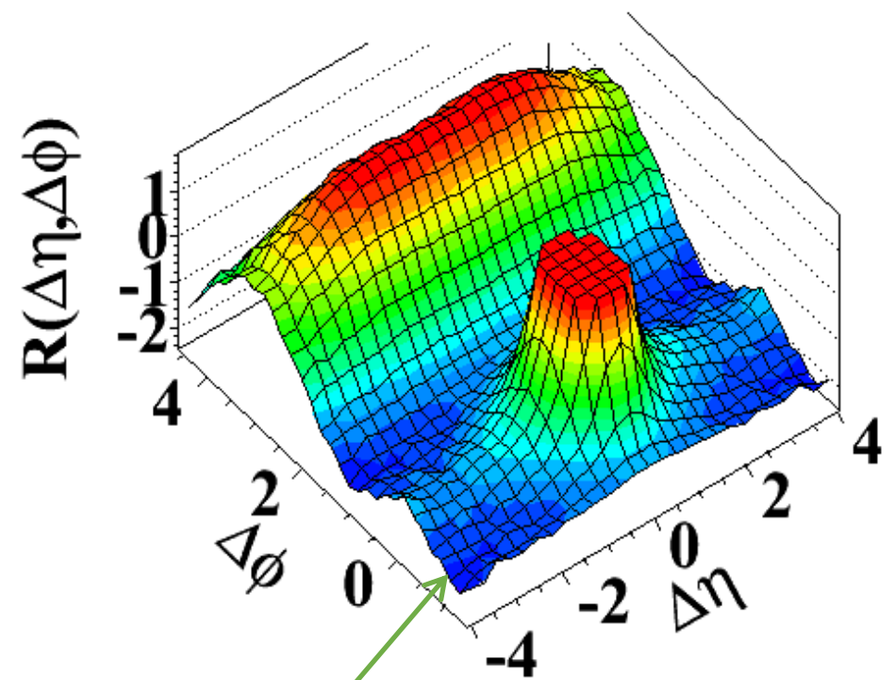
(PAC presentation of PHENIX BUP by J. Nagle)

Two-particle Δη-Δϕ correlation

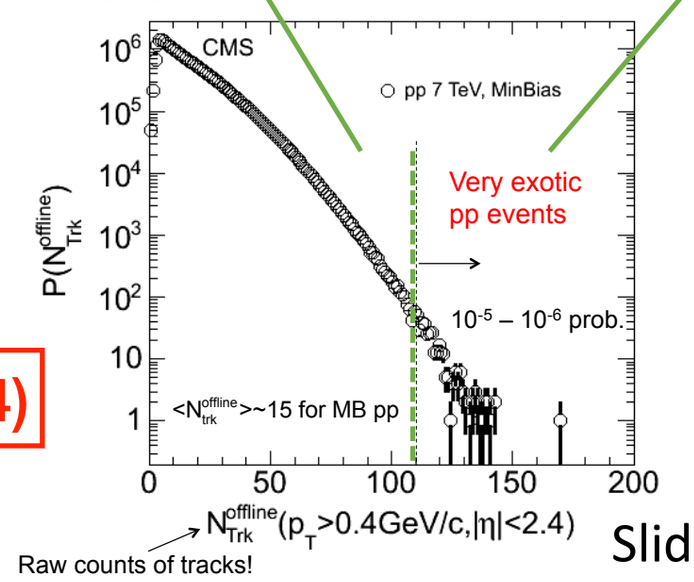
pp <N>~15, 1<p_T<3 GeV/c



pp N>110, 1<p_T<3 GeV/c



Very high-multiplicity pp events are rare in nature



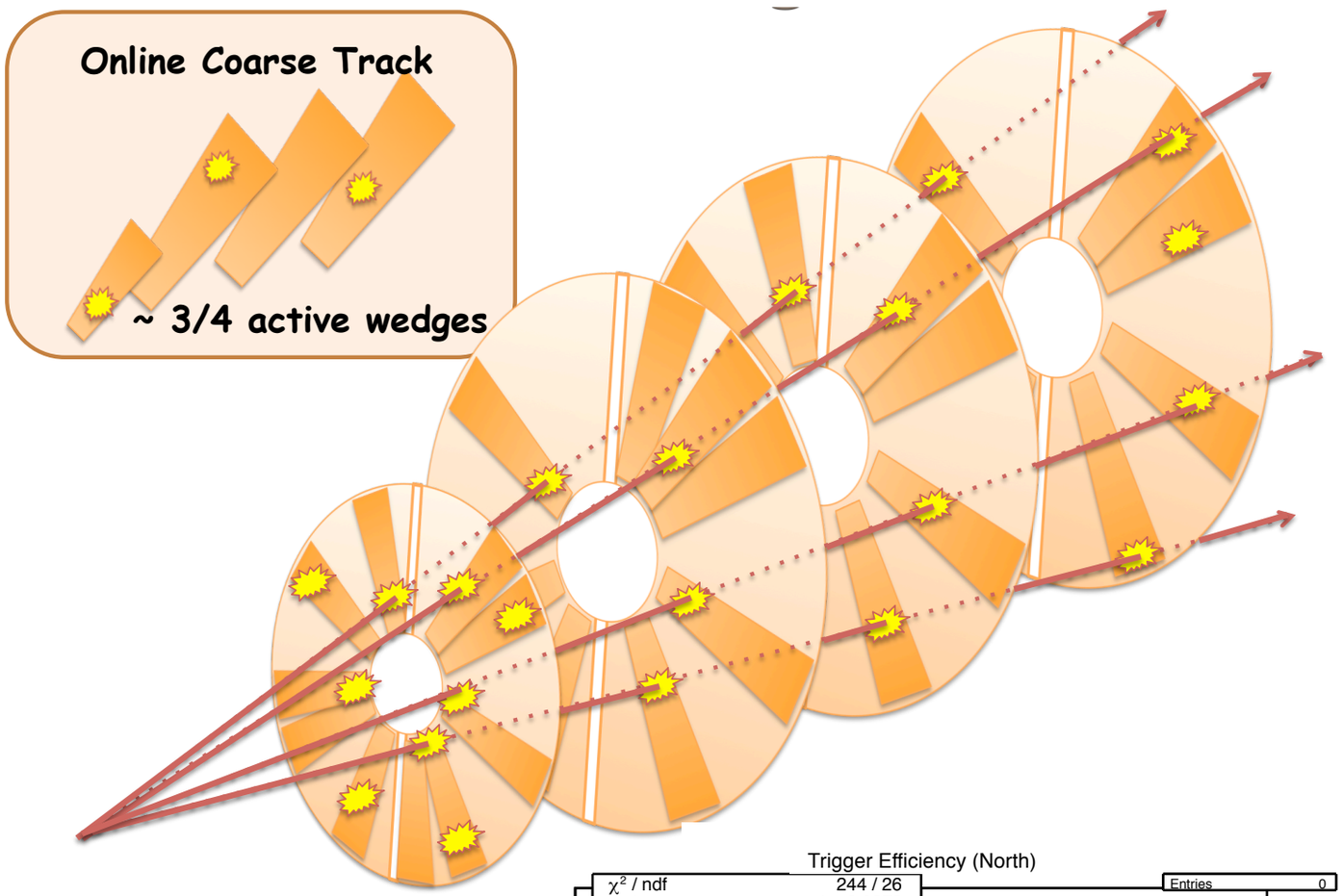
$N_{trk}^{offline}$ (p_T>0.4 GeV/c, |η| < 2.4)

Slide from Wei Lei, Rice University

Triggering on high multiplicity p+p at RHIC

PHENIX forward silicon vertex (FVTX)

N_{trk}^{FVTX} (1.2 < |η| < 2.2) ≥ 12

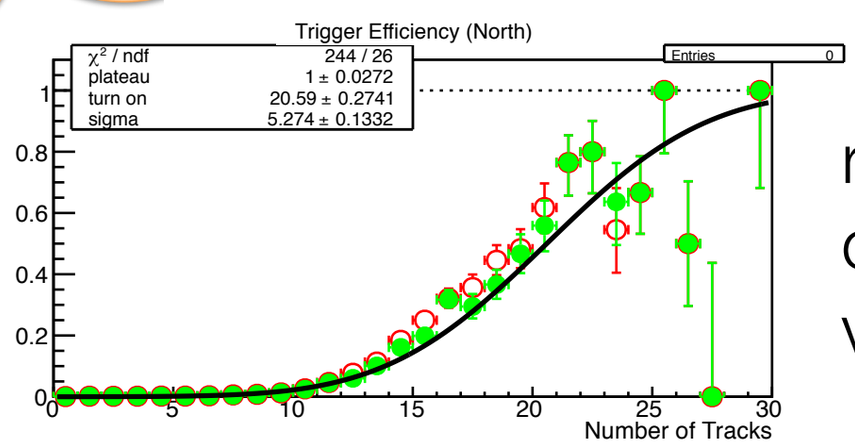


Run15 pp

100 Hz bandwidth for FVTX HighMulti Trigger

9 weeks of pp running assuming 30% uptime

300 Mevents to be accumulated
(~ x4 of Run8 dAu central)



rejection power of ~1500 in combination with a ±10 cm primary vertex trigger (based on BBC)

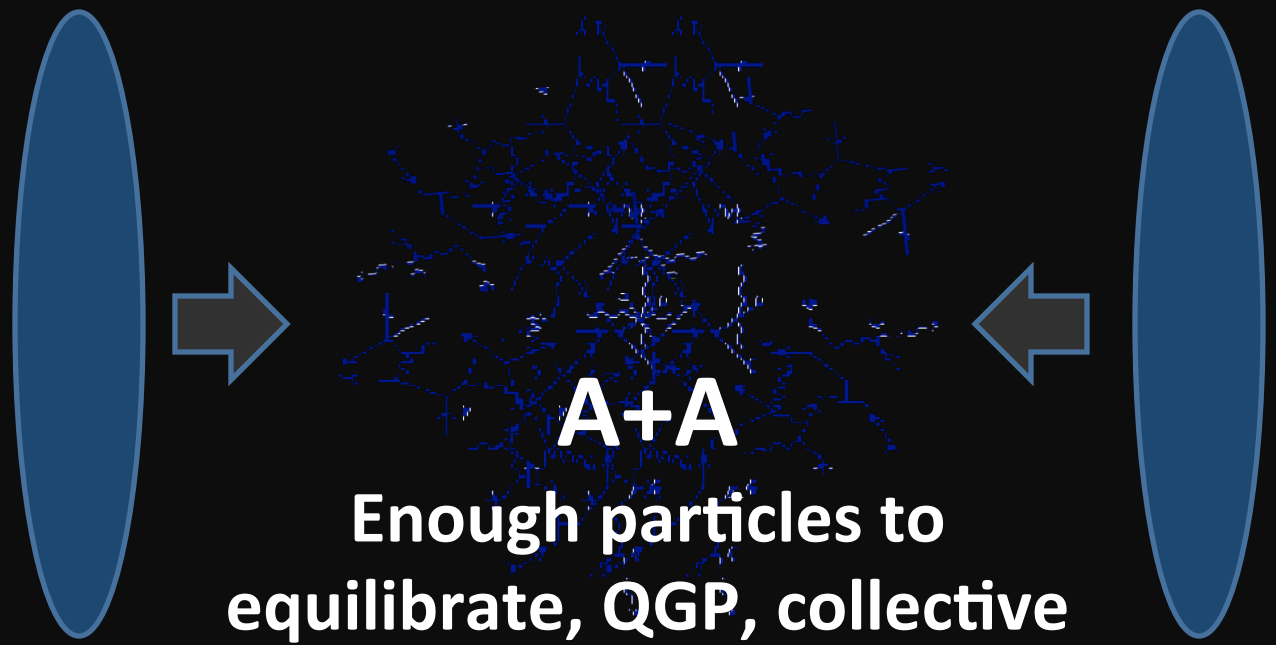
Summary

- long-range near-side correlations in d+Au clearly seen in PHENIX, STAR
  - different prescriptions for accommodating properties of low-activity (peripheral) d+Au or MB p+p in analyzing high-activity (central) d+Au
  - clarify issue of decomposition into jet-like, non-jet-like components
- similar features are seen in LHC p+p, p+Pb correlation data, suggesting common physics, arguing for common approaches
  - e.g., multi-particle cumulant analysis of RHIC d+Au and ^{3}He+Au!
  - anticipate high multiplicity p+p RHIC data from current Run-15
- full modeling to exploit full range of system geometries being provided by RHIC to address physics: high multiplicity p+p, p+Al, p+Au, d+Au, ^{3}He+Au

What can we learn from ^{3}He+A and p(d)+A collisions at RHIC?

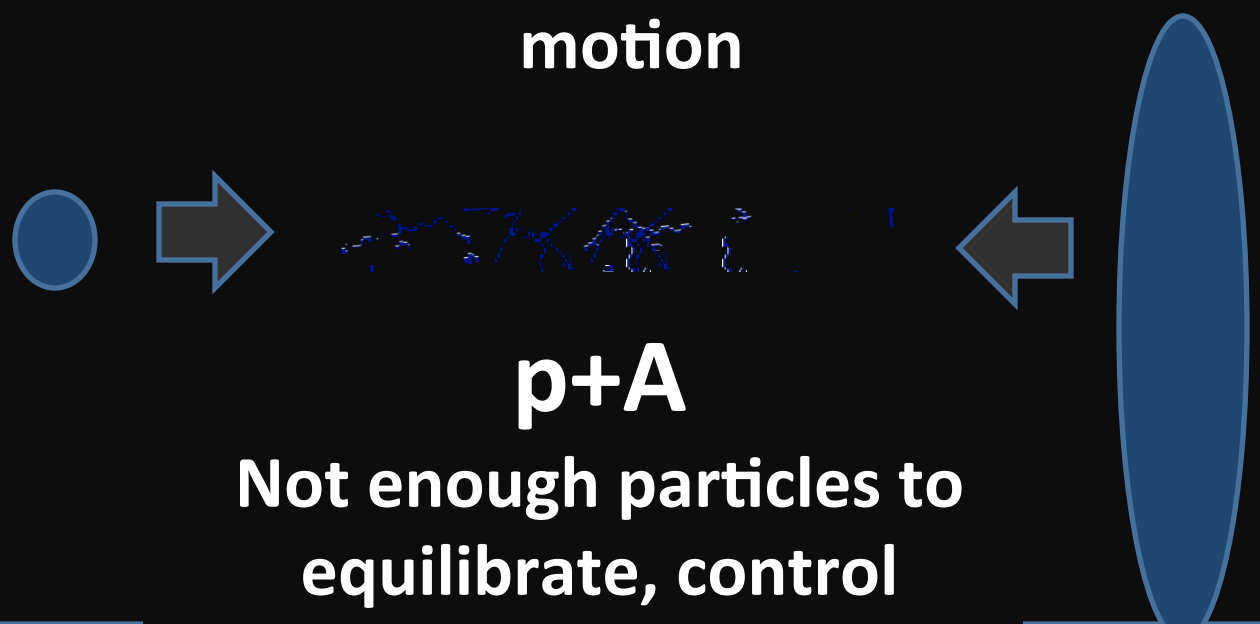
Jamie Nagle
University of Colorado, Boulder, USA

Prejudice



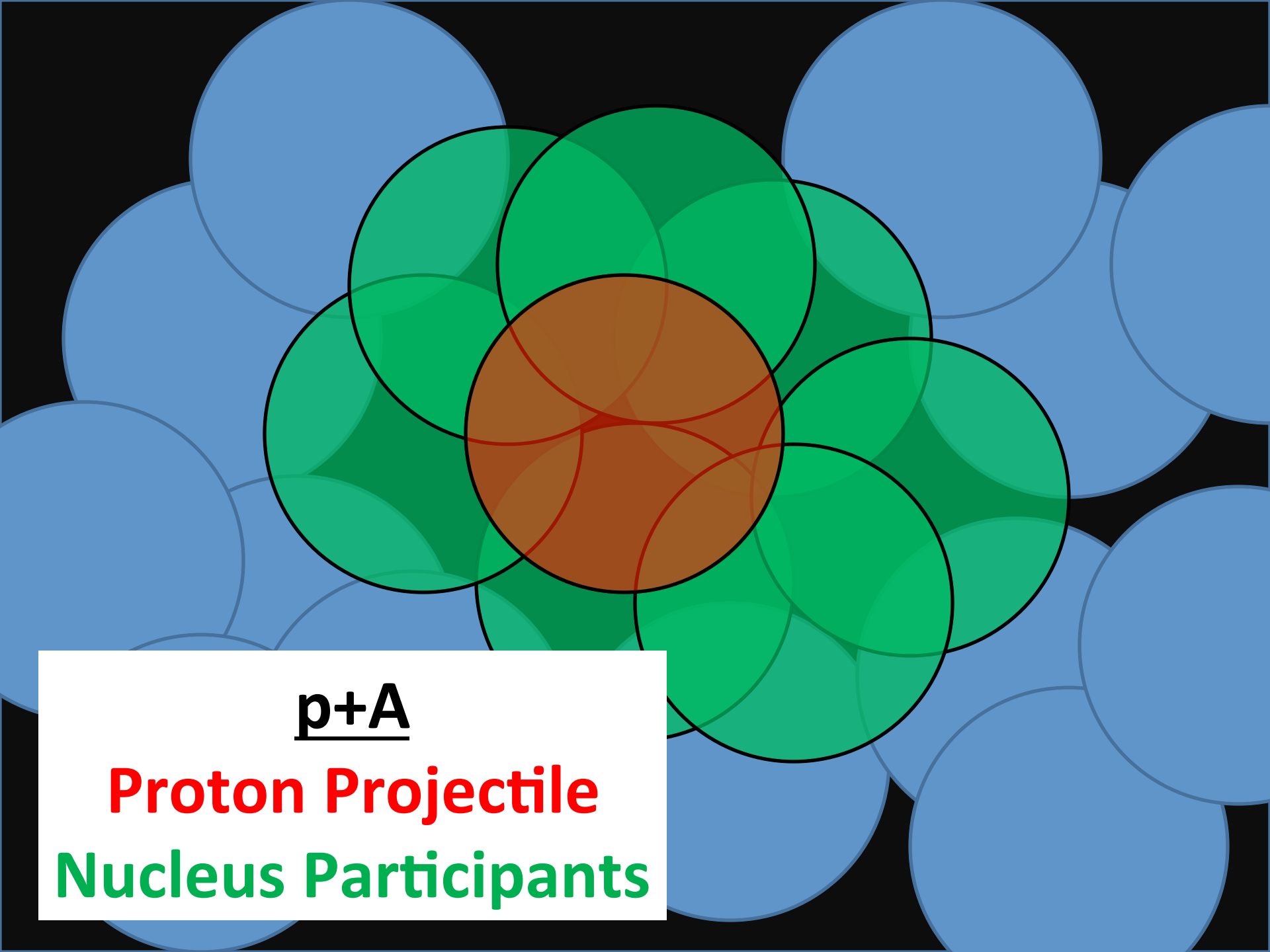
A+A

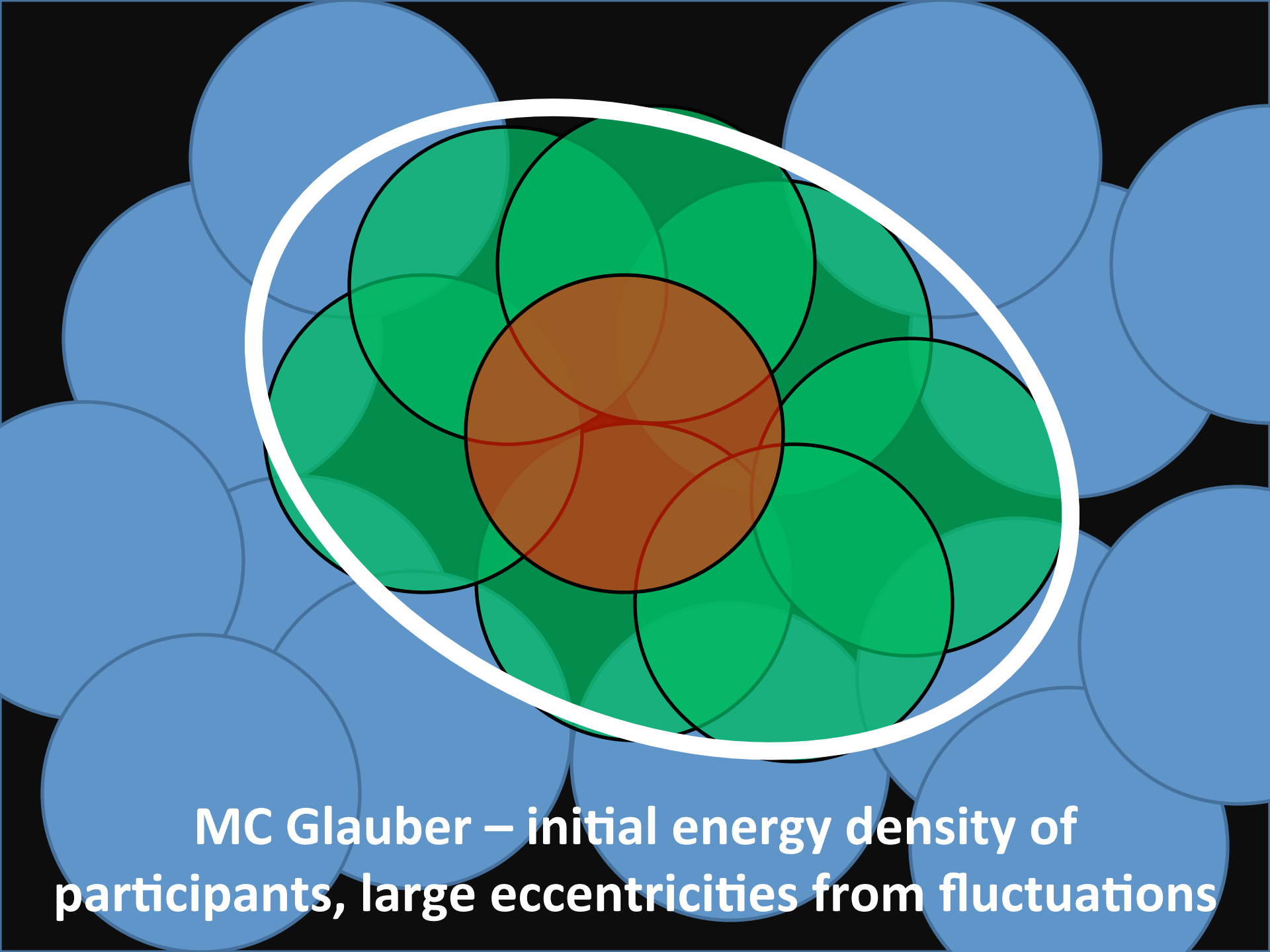
Enough particles to equilibrate, QGP, collective motion



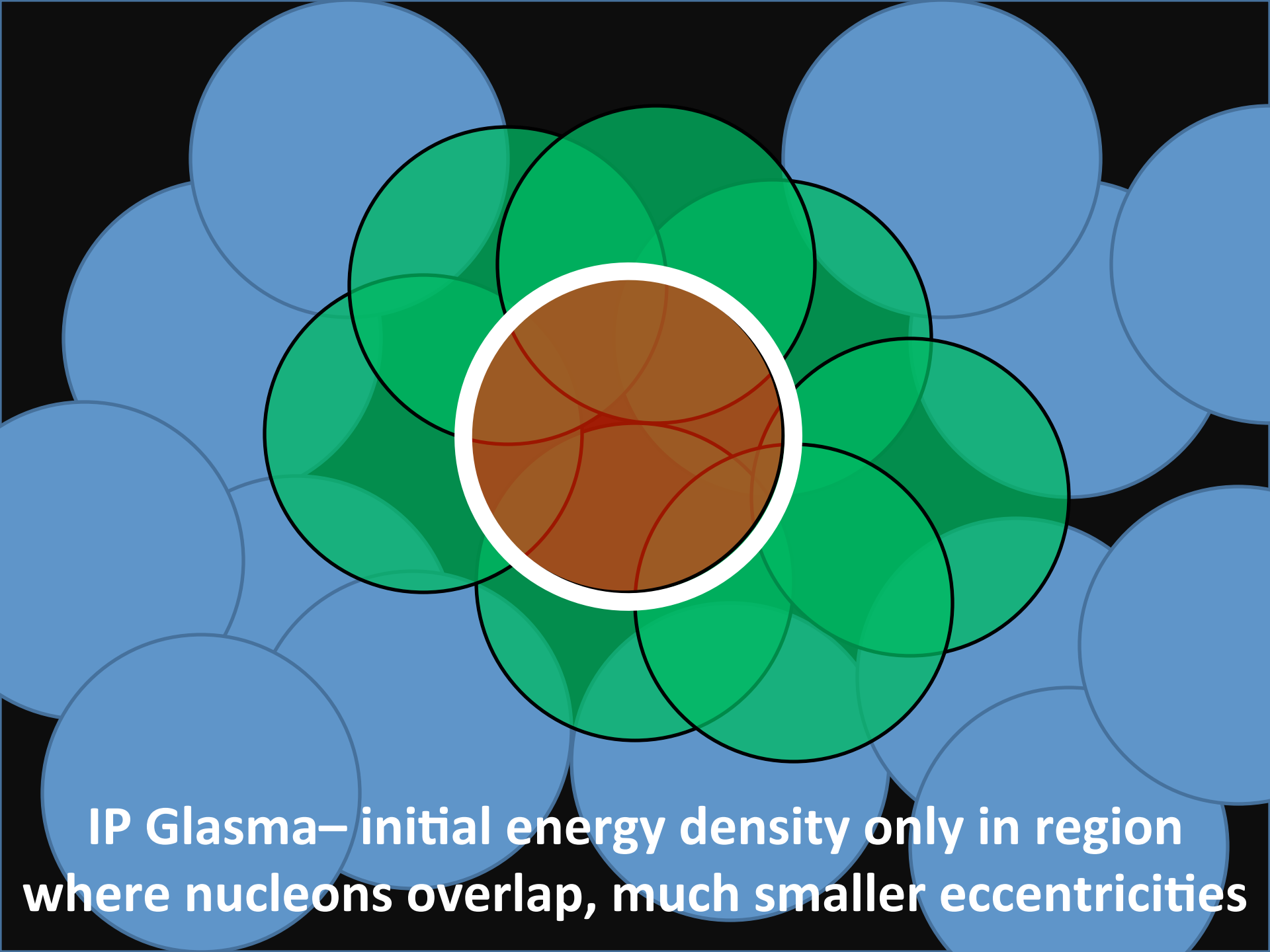
p+A

Not enough particles to equilibrate, control


p+A
Proton Projectile
Nucleus Participants

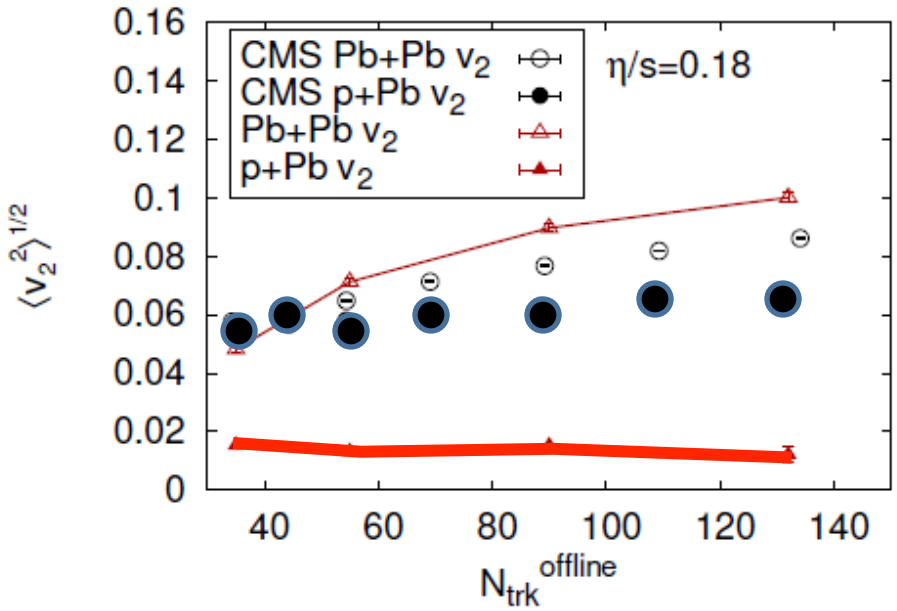


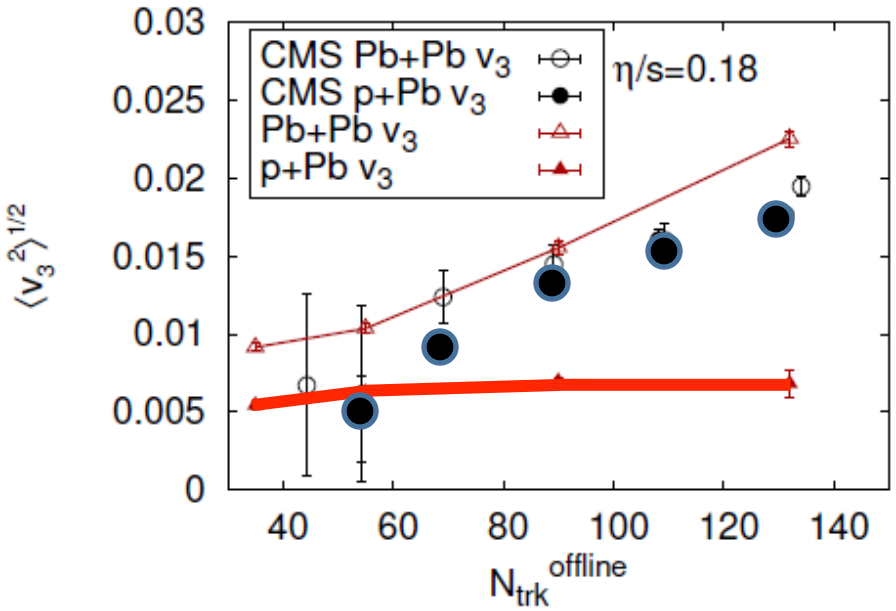
MC Glauber – initial energy density of participants, large eccentricities from fluctuations



IP Glasma– initial energy density only in region where nucleons overlap, much smaller eccentricities

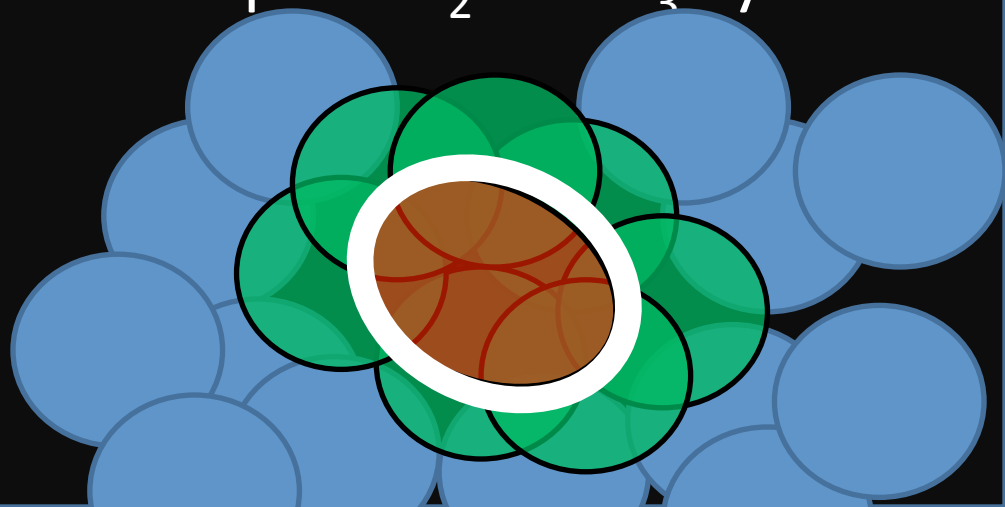
Initial Geometry + Hydrodynamics

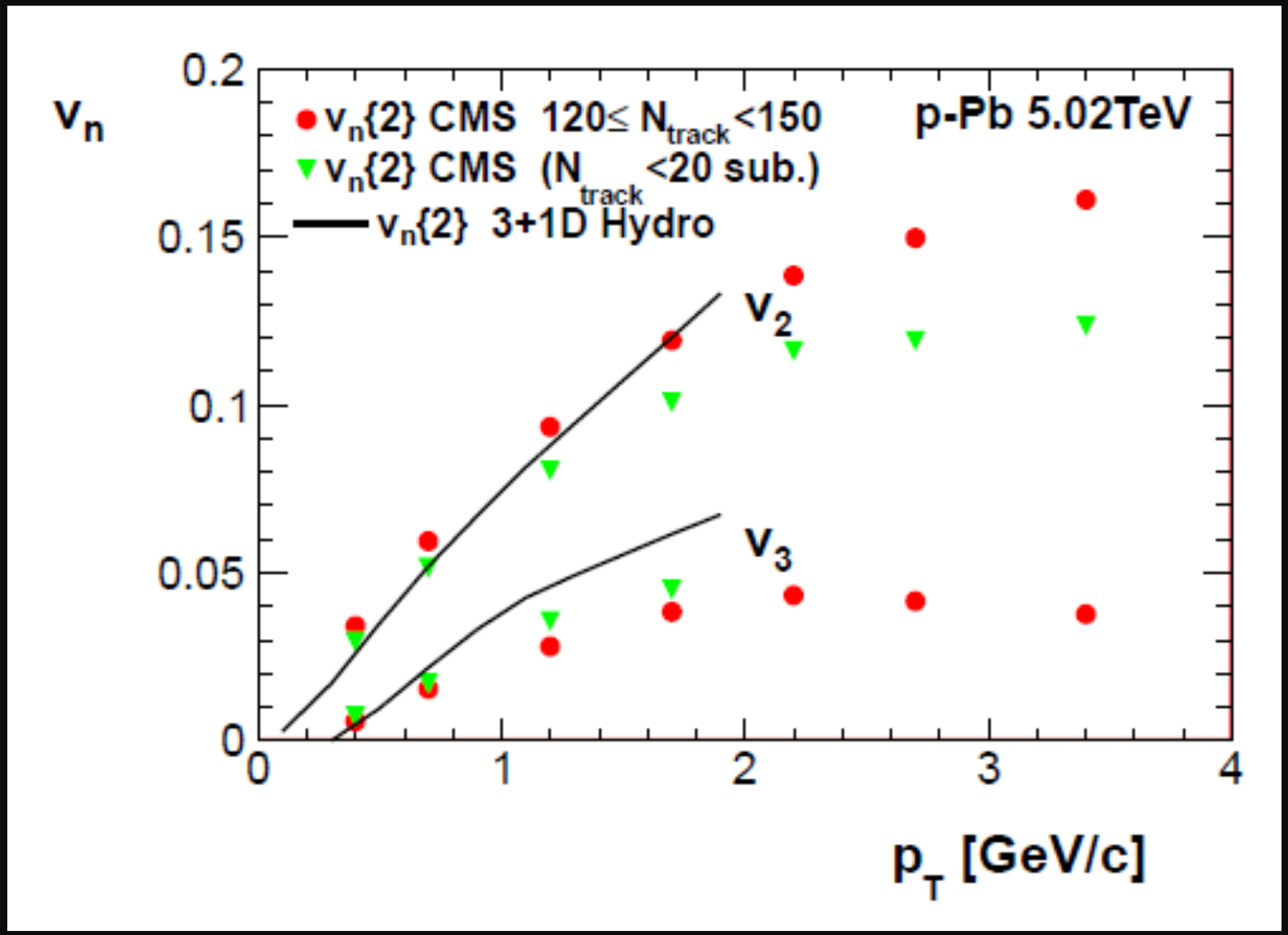




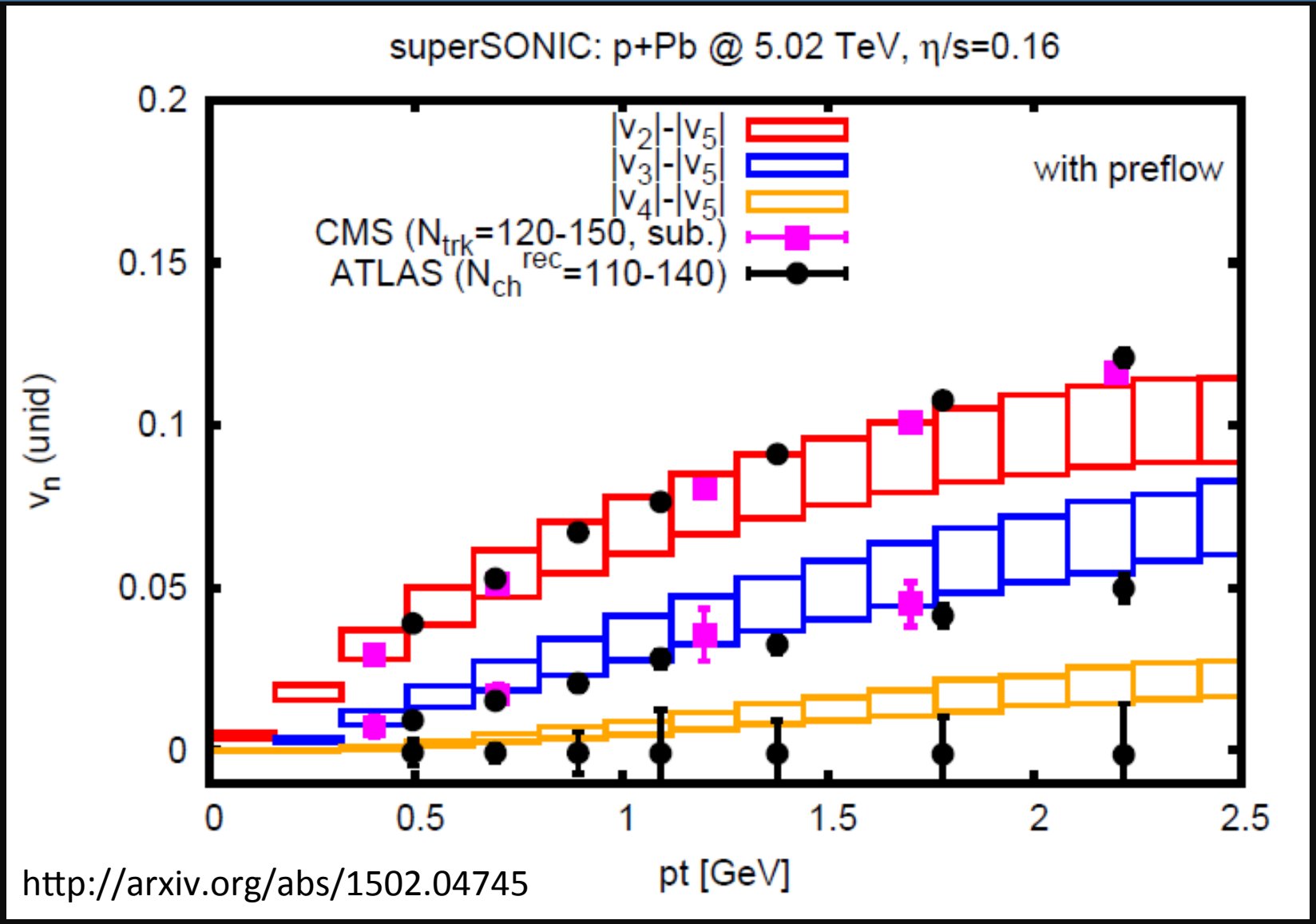
IP Glasma + Hydro – under-predicts p+Pb v_2 and v_3 by x3

One could enhance the effect by including fluctuating proton shape, still smaller initial size



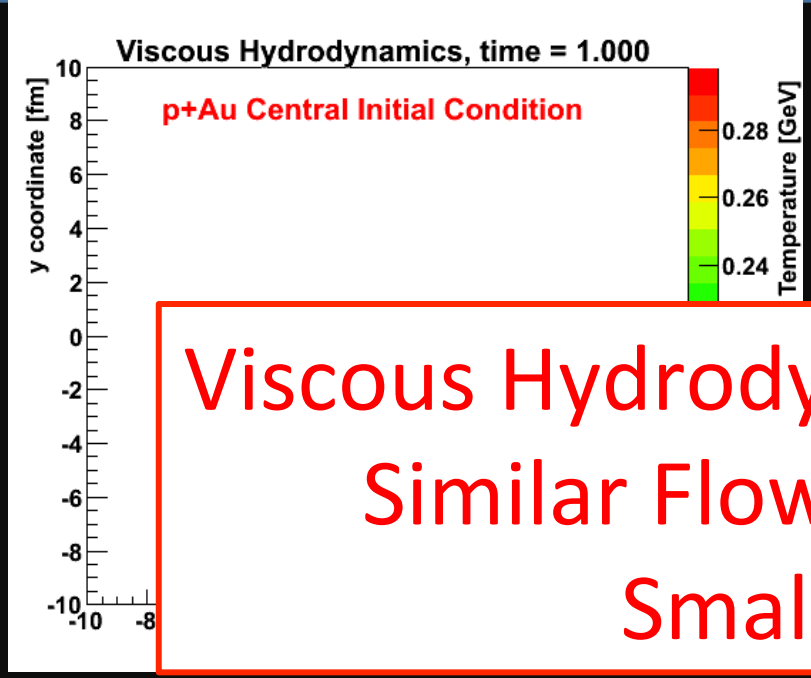


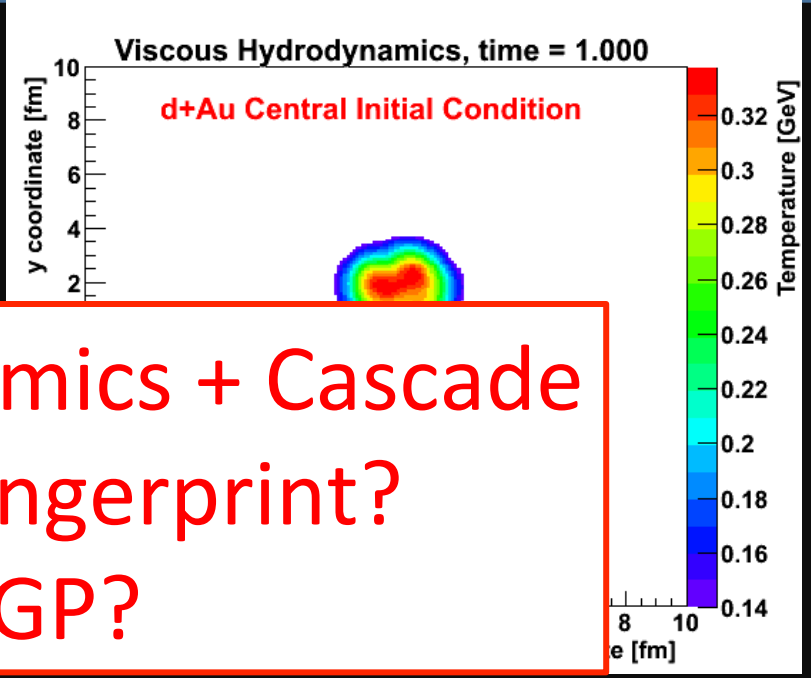
Glauber + Hydro (Bozek) – qualitative description of v_2 and v_3
no pre-equilibrium, no hadronic cascade, 3-dimensional



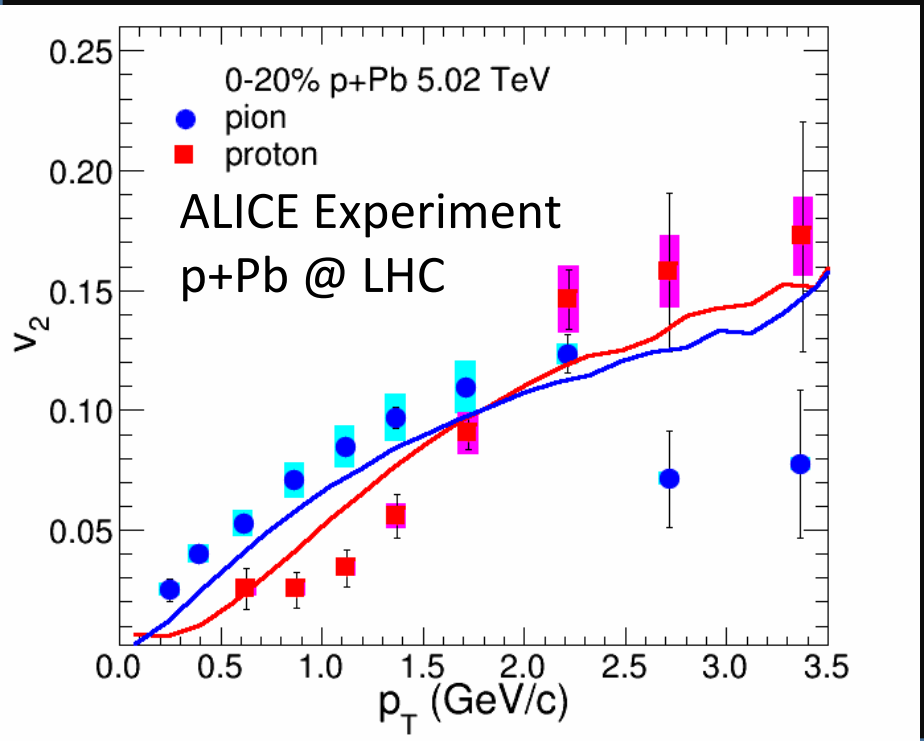
Glauber + PreFlow + Hydro + Hadronic Cascade [superSONIC] (Romatschke)

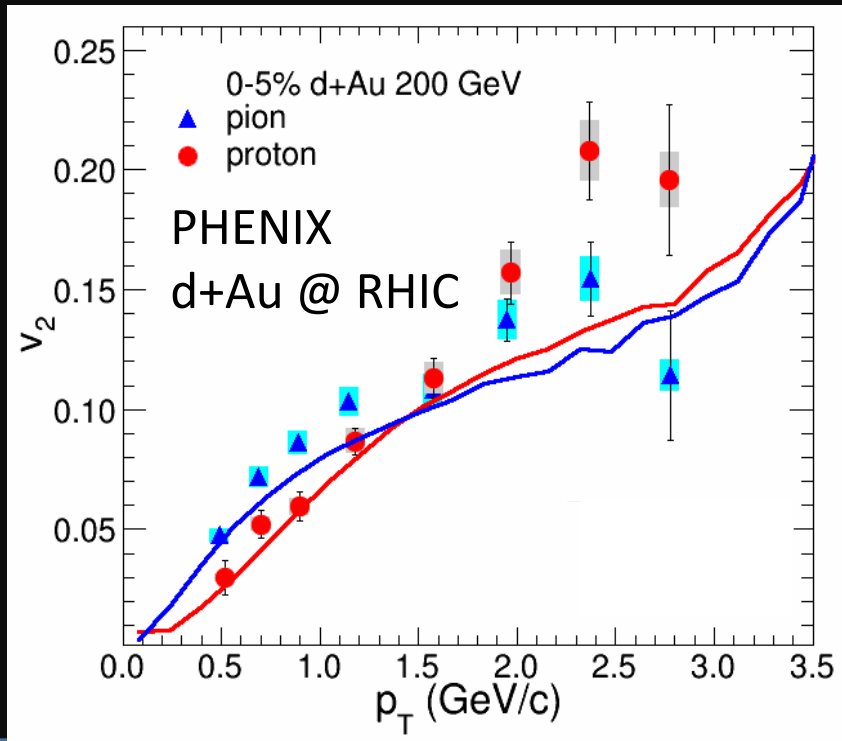
qualitative description of v_2 and v_3



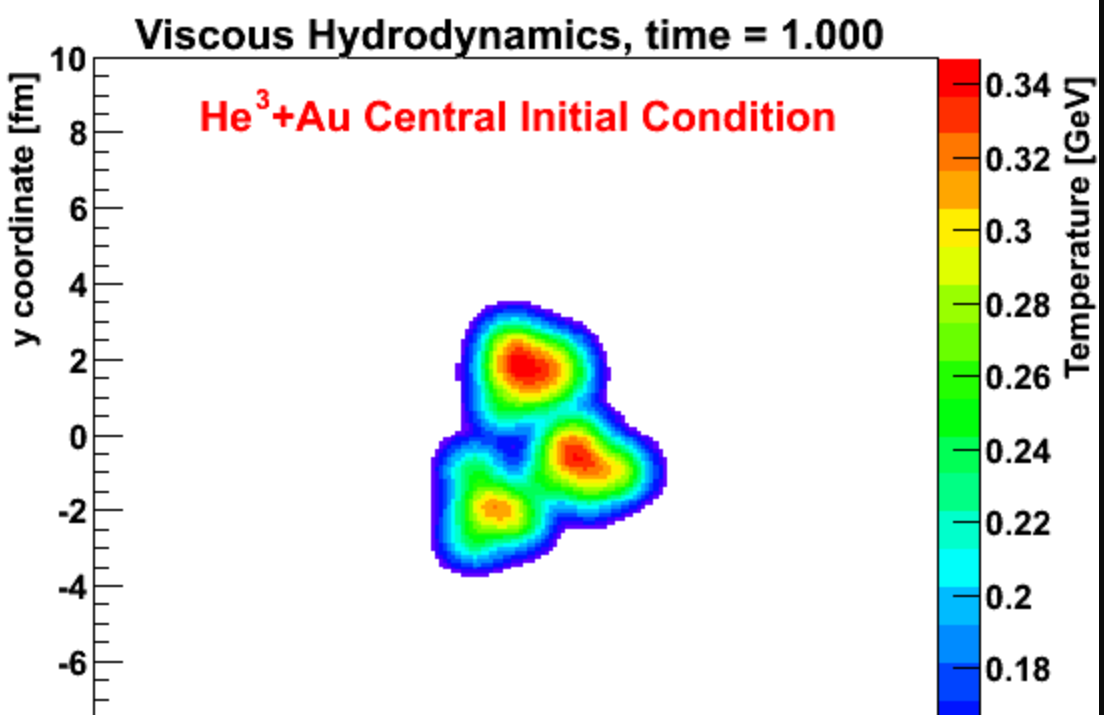


Viscous Hydrodynamics + Cascade
Similar Flow Fingerprint?
Small QGP?



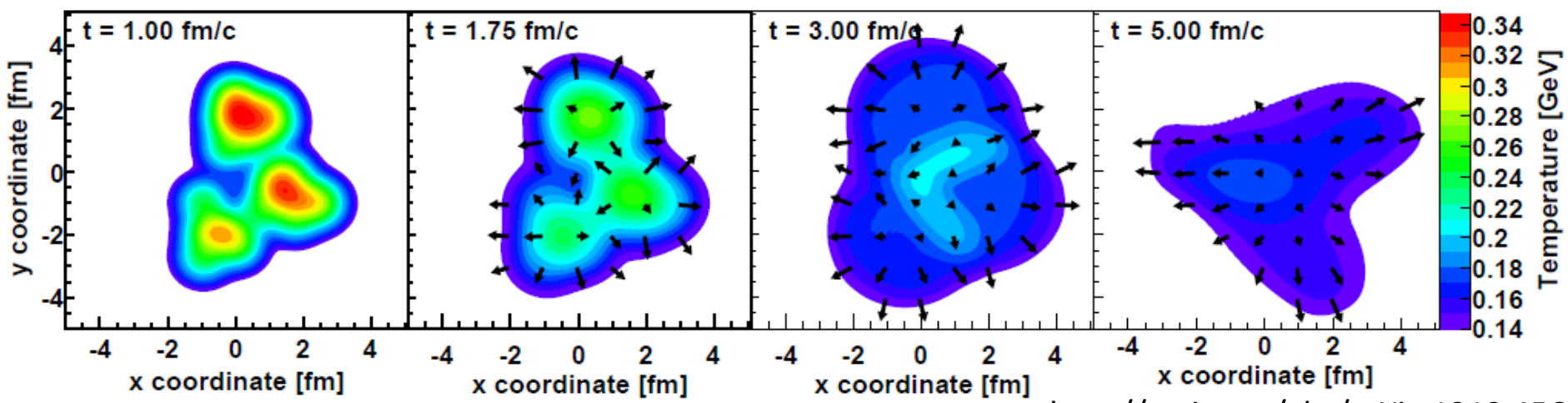


Geometry Engineering



Proposal to run "triangular" system at RHIC

Small systems challenge our picture of required time and size scales



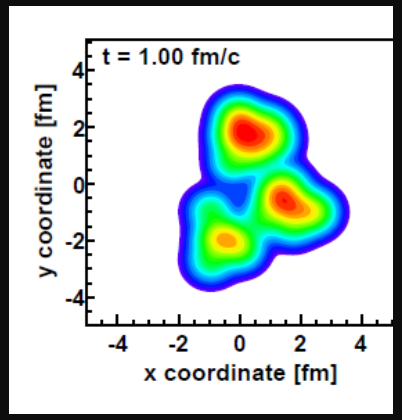
http://arxiv.org/abs/arXiv:1312.4565

Motivation → Ideas → Predictions → Experiment

PHENIX Look Back at d+Au (Christmas 2012)
Idea for ^{3}He+Au (January 2013)
PHENIX Proposal (May 2013)
Theory Predictions (December 2013 - http://arxiv.org/abs/1312.4565)
Running (June 2014)
First Results (December 2014)

One of the fun parts of science… idea, hypothesis, test

http://arxiv.org/abs/1408.2549
Publicly available Helium-3 three-nucleon configurations from Joe Carlson, J. Lynn

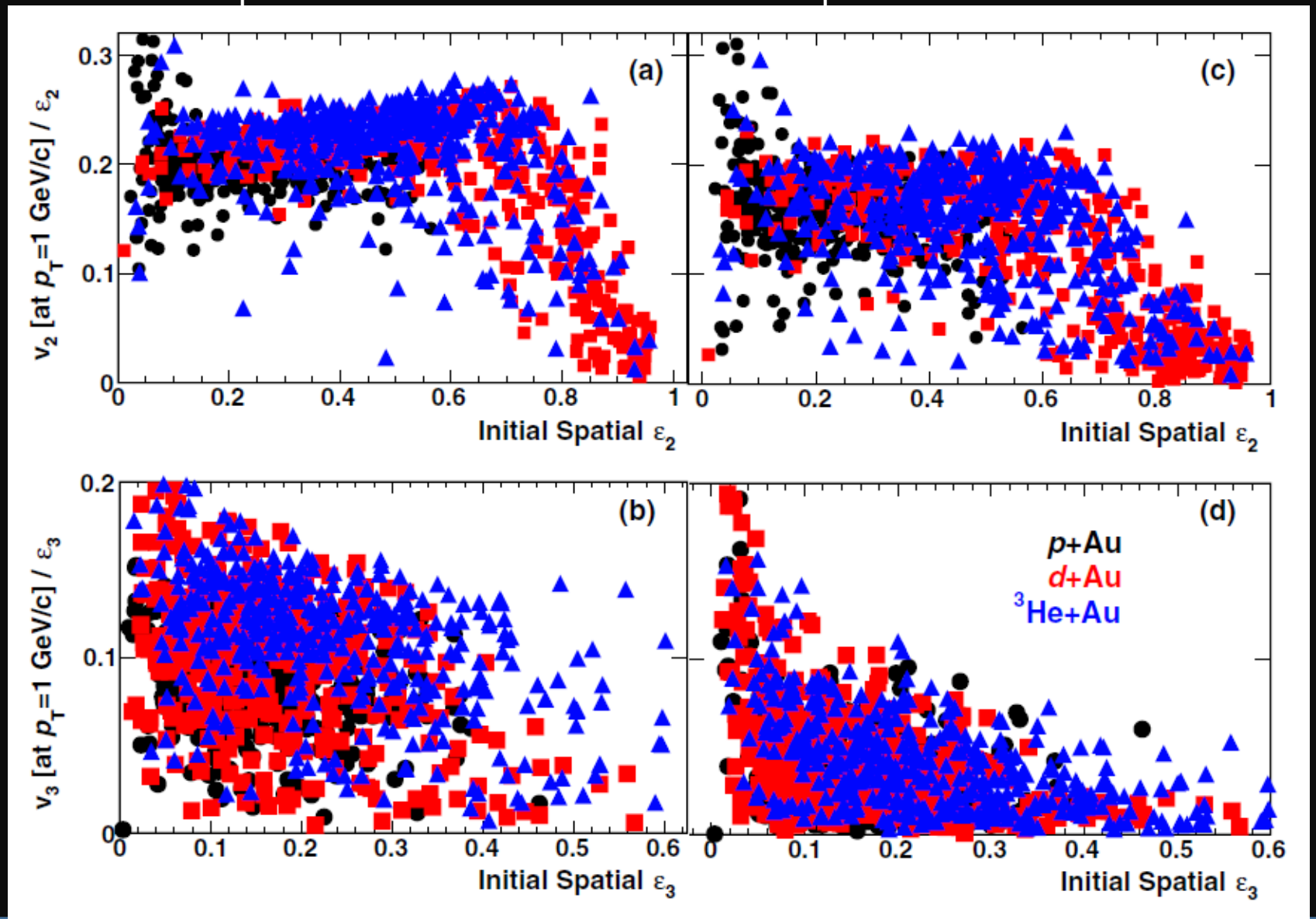


Looking under the hood

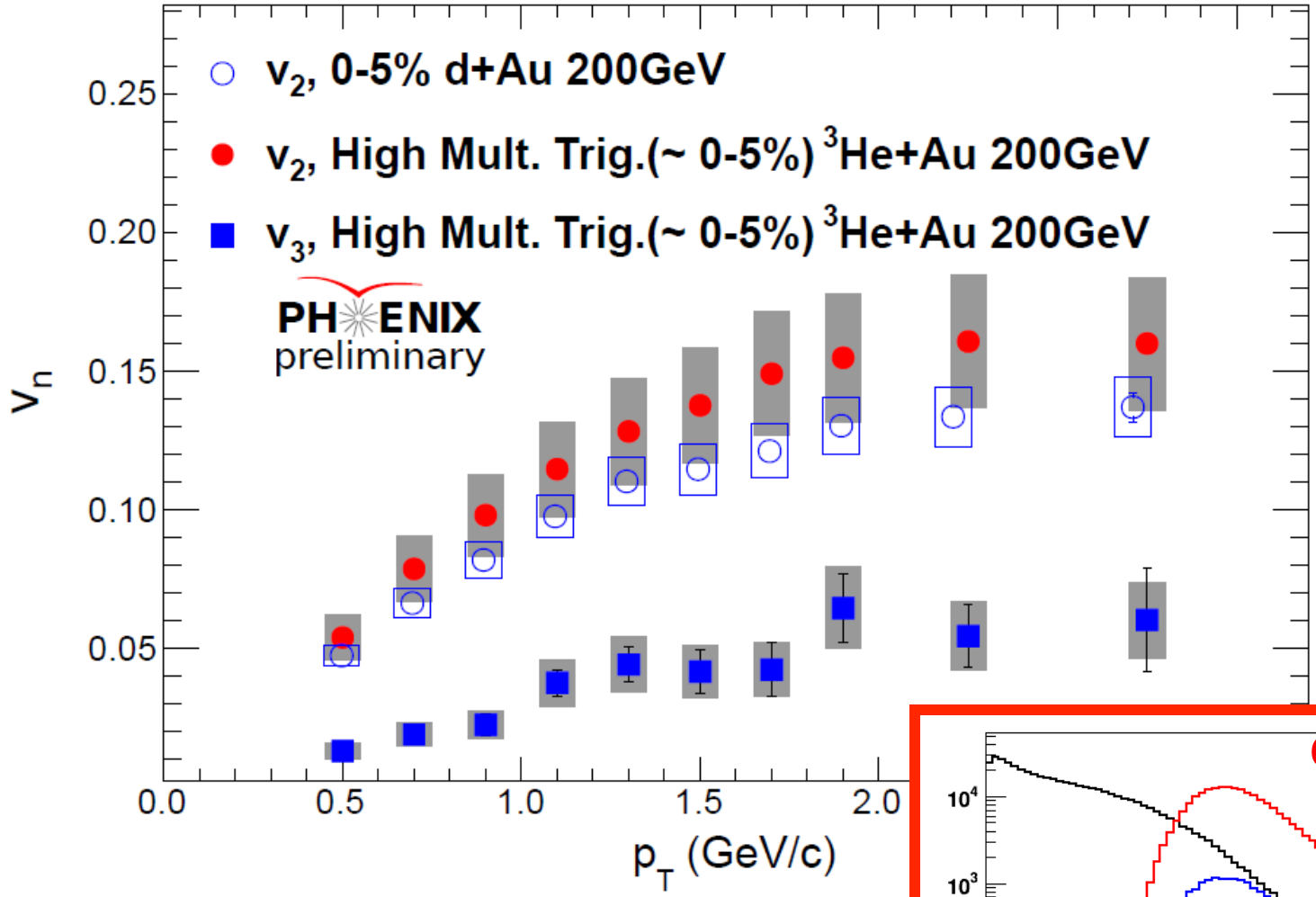
T_f = 150 MeV T_f = 170 MeV

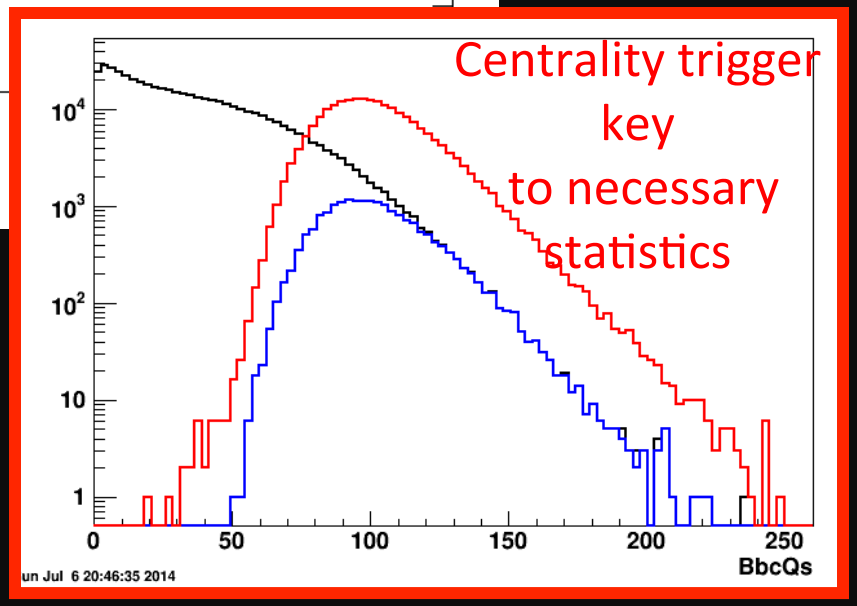
$\frac{v_2}{\varepsilon_2}$

$\frac{v_3}{\varepsilon_3}$



2014 Data



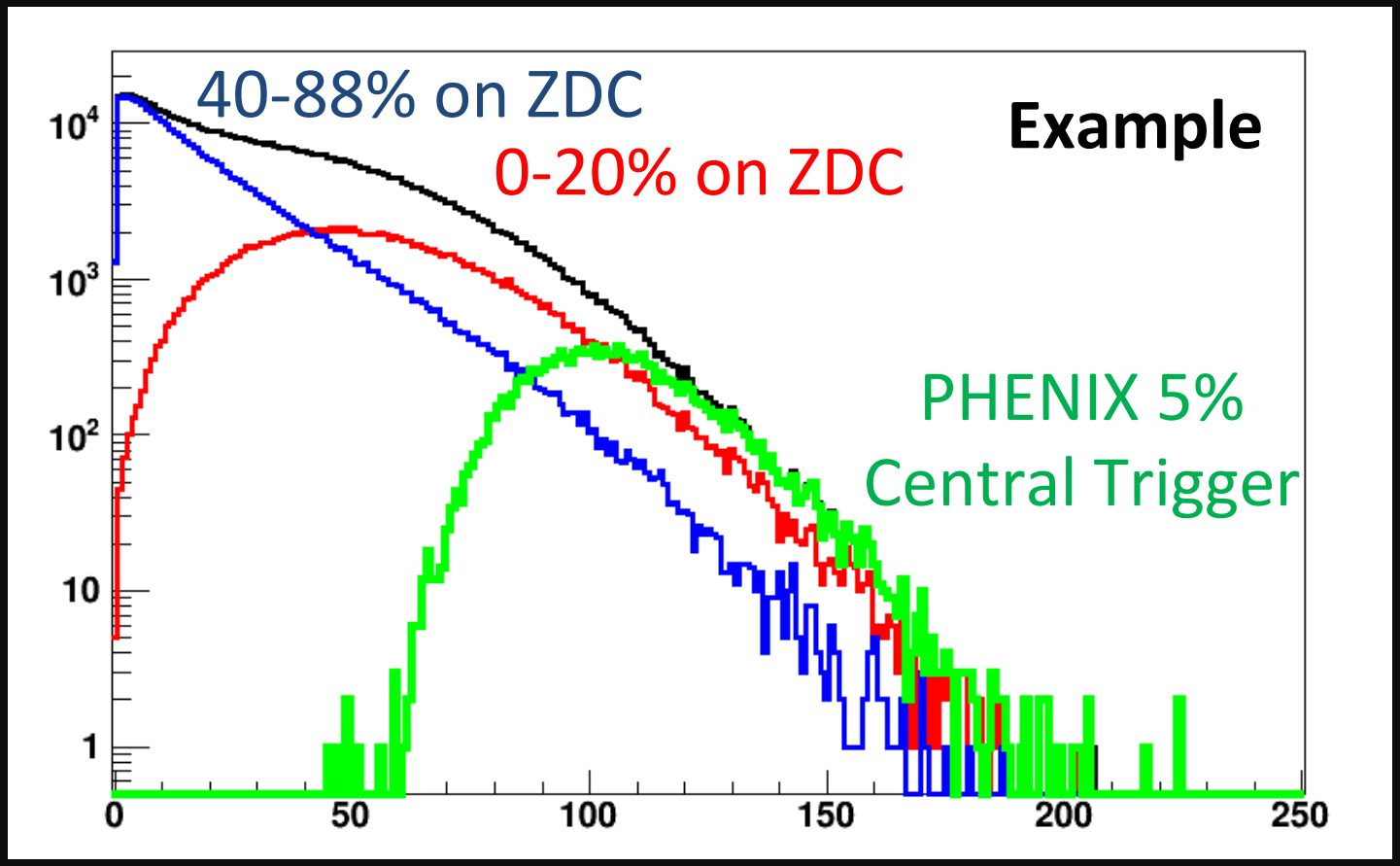


v_2 d+Au ~ v_2 ^{3}He+Au

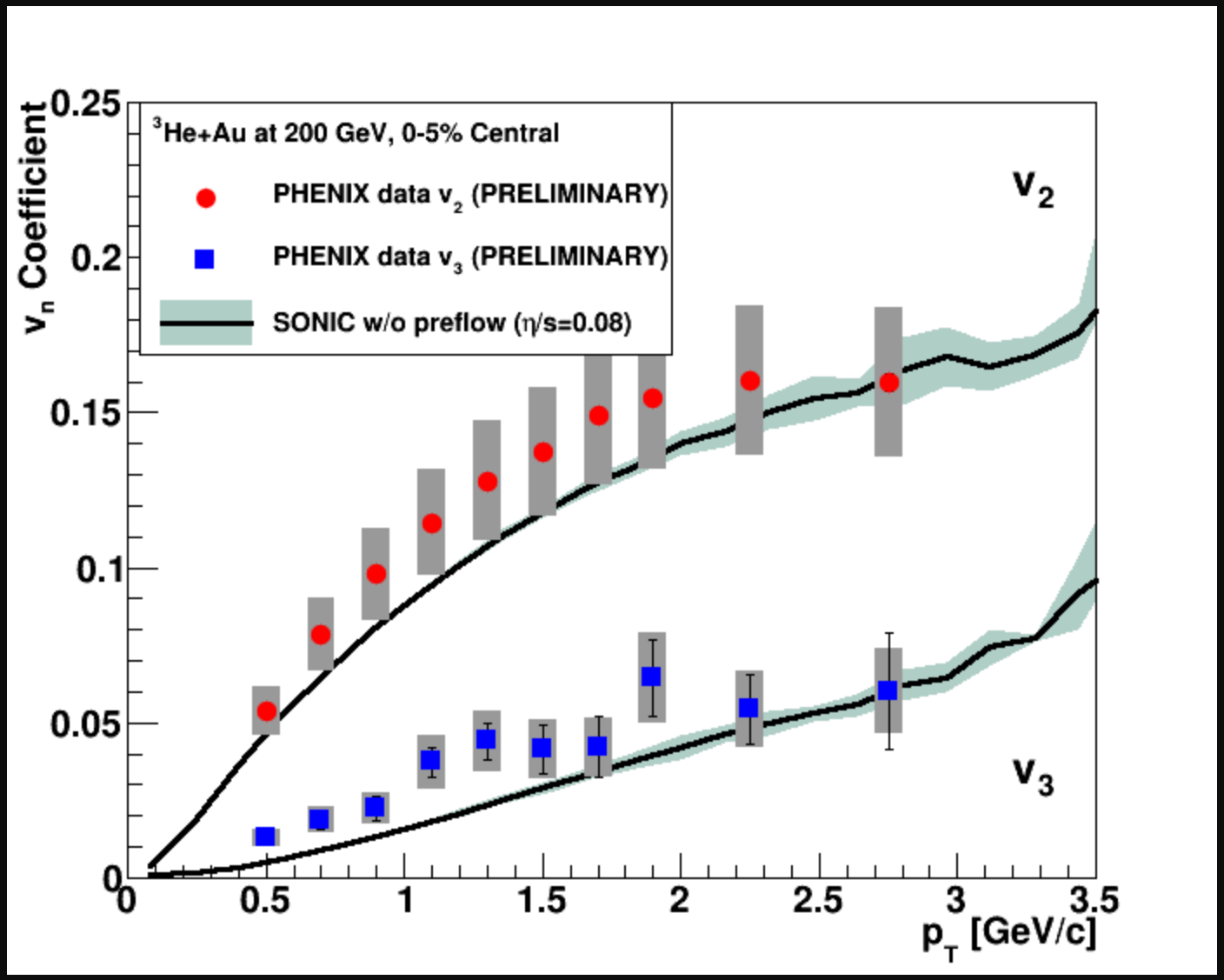
Finite v_3 in ^{3}He+Au

Sincere Request

In addition to exploring new ways to look at the ^{3}He+Au data, it is very important to have some standard measures (2,4 particle cumulants, standardized centrality selections, etc.).
Needed for good quantitative science.



PHENIX BBC Au-Going Charge [3He+Au]

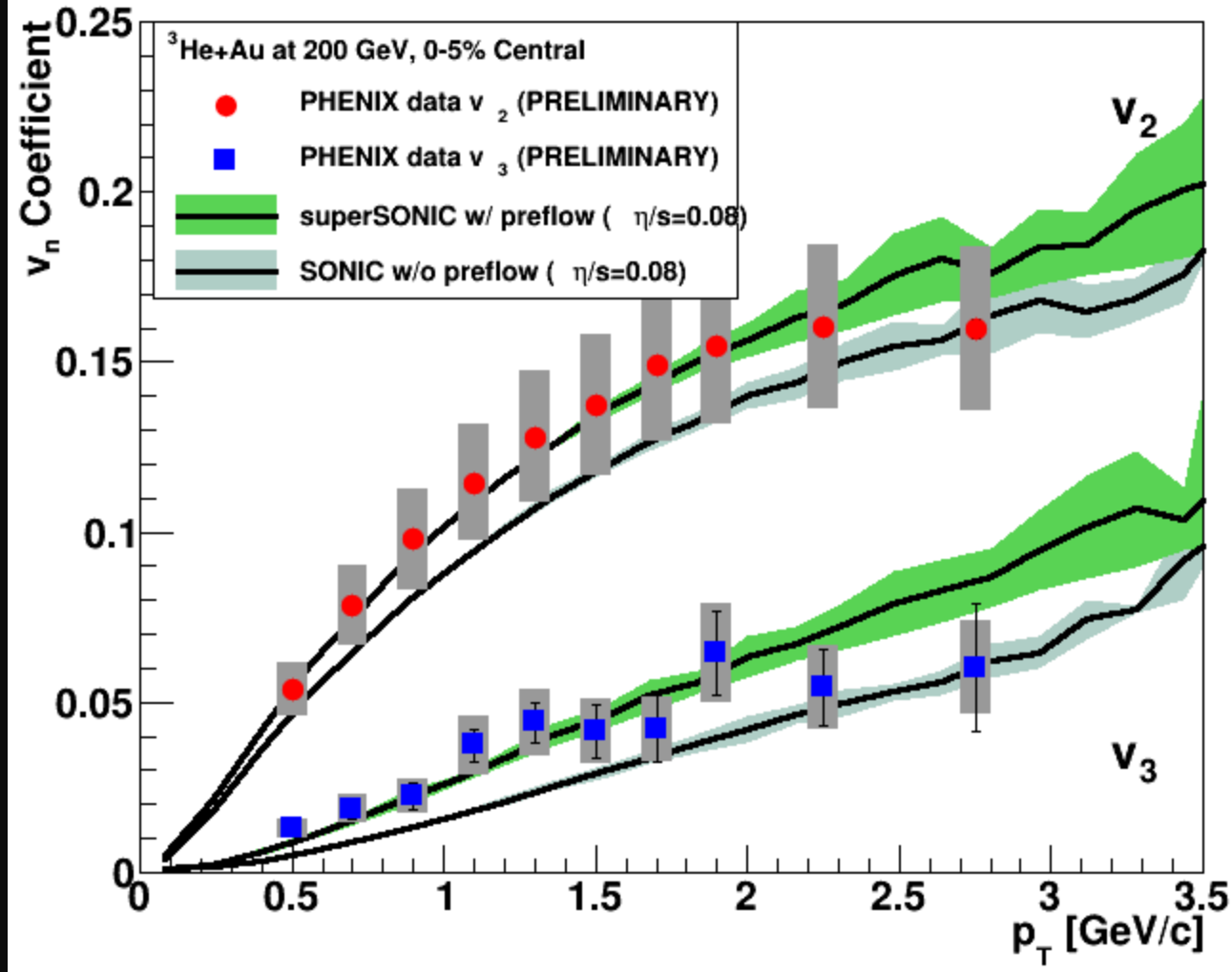


Glauber IC + Hydro ($\eta/s=1/4\pi$) + Hadronic Cascade [SONIC]

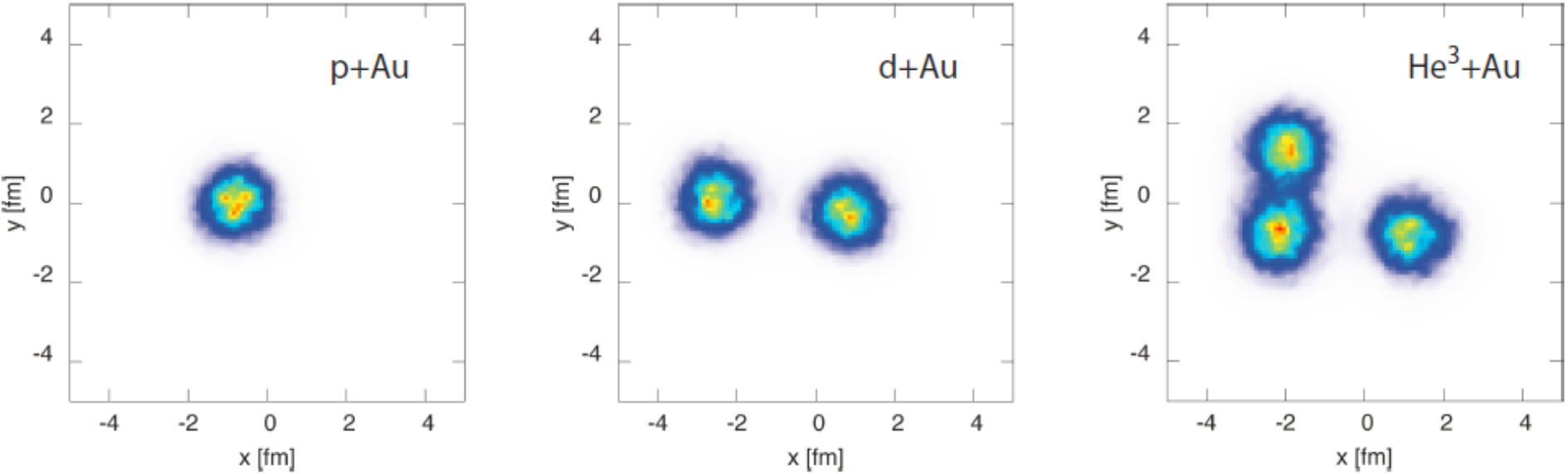
PREDICTION

(JN et al., Phys.Rev.Lett. 113 (2014) 11, 112301)

http://arxiv.org/abs/1502.04745



Glauber IC + PreFlow + Hydro ($\eta/s=1/4\pi$) + Hadronic Cascade [superSONIC]

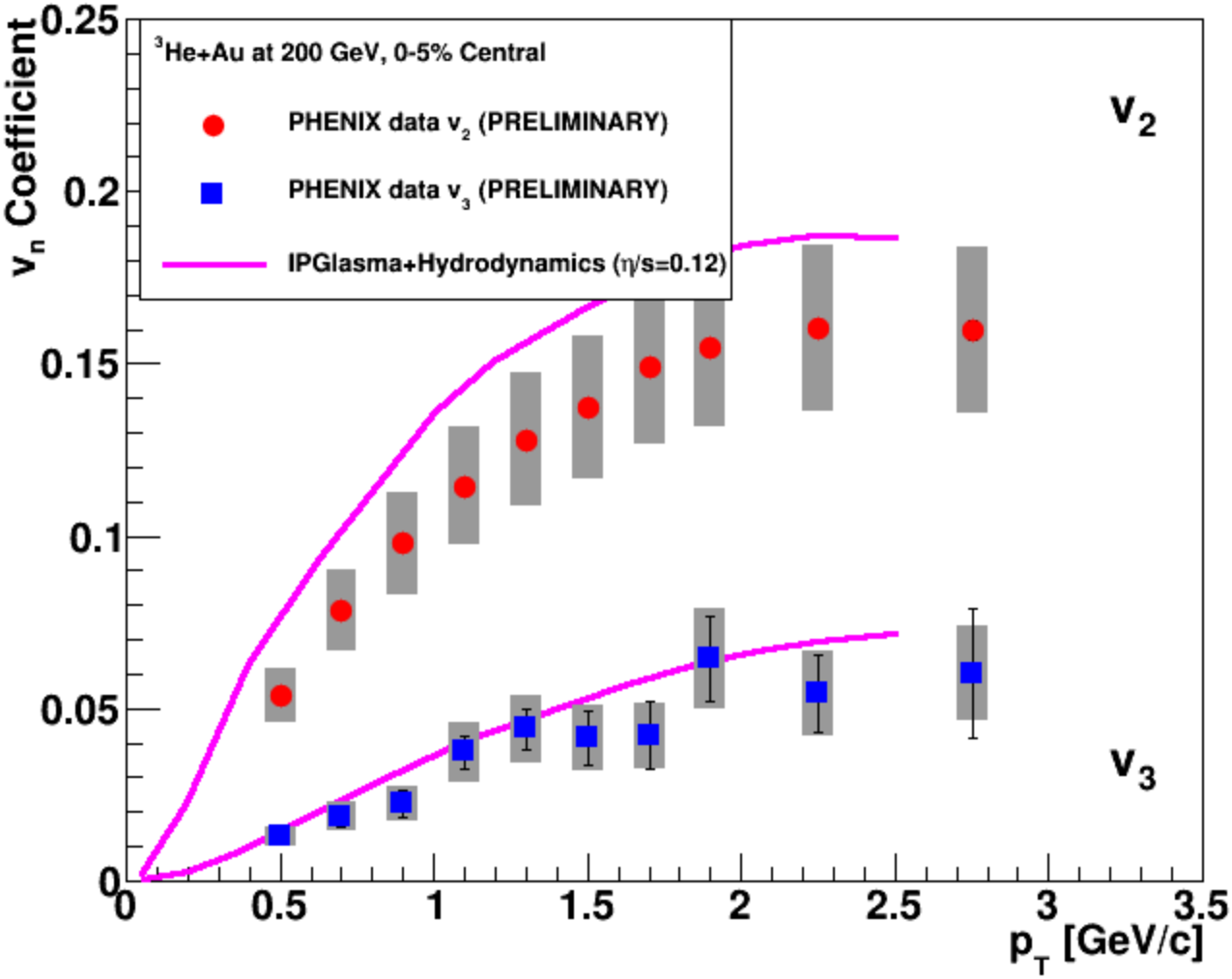


IP Glasma gives very different eccentricity from Glauber in pA

However, in d+Au and ^{3}He+Au,
the eccentricity is driven by the 2 and 3 hot stops,
not their individual geometry

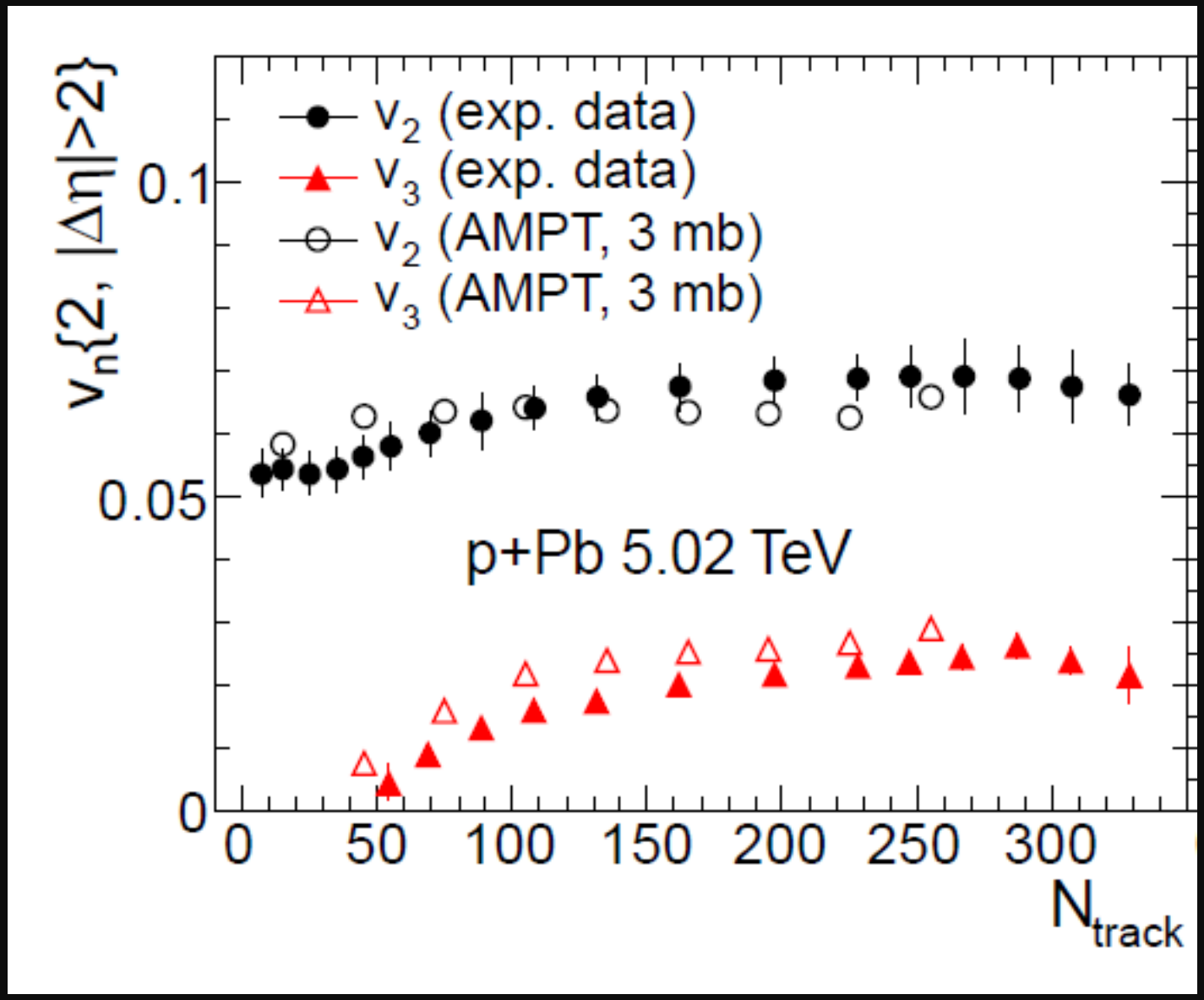
Note though that the IPGlasma has a smaller hot spots
with higher energy density that result in
<u>more rapid radial expansion</u>

Bjoern Schenke, Raju V. http://arxiv.org/abs/1407.7557

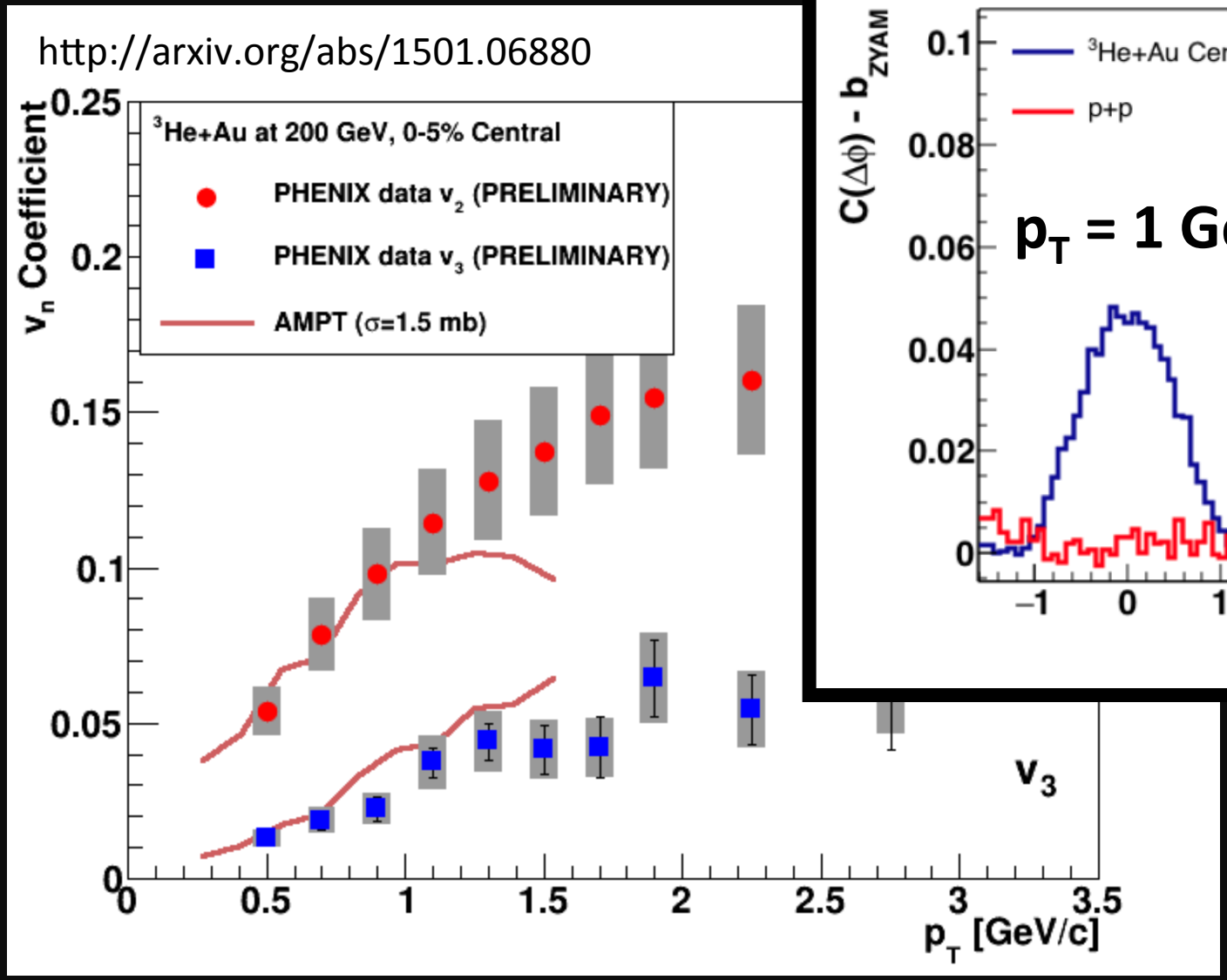


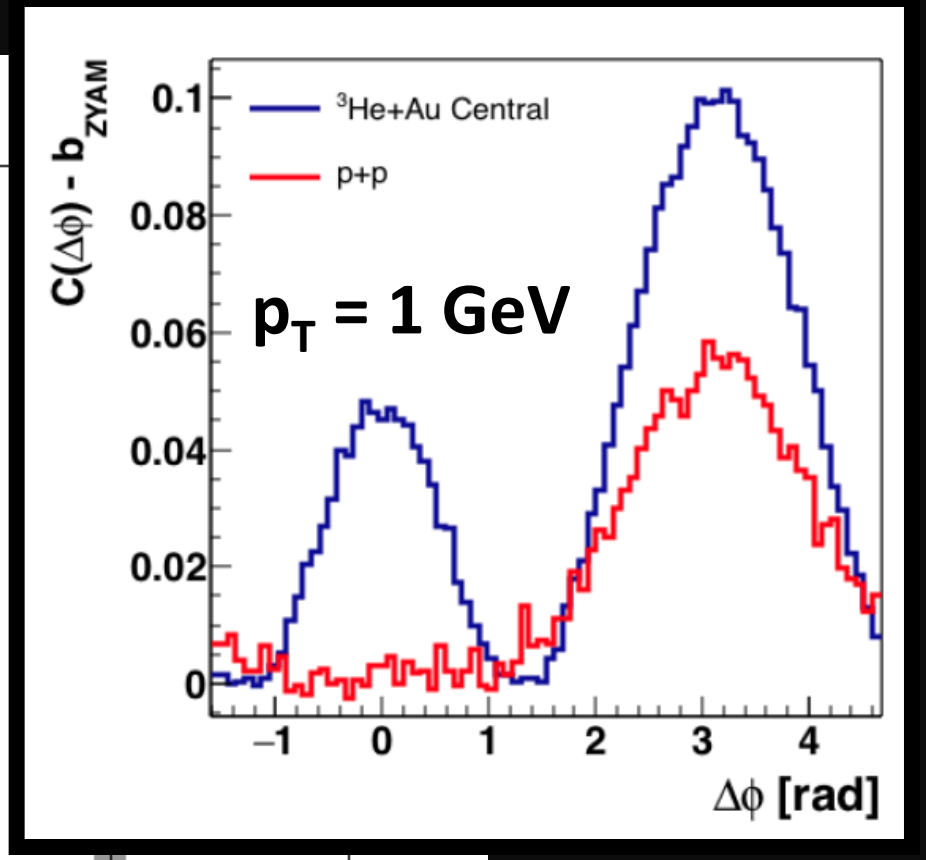
IP Glasma + MUSIC ($\eta/s = 1.5 \times 1/4\pi$)

Much larger initial radial expansion, but also no late cascade

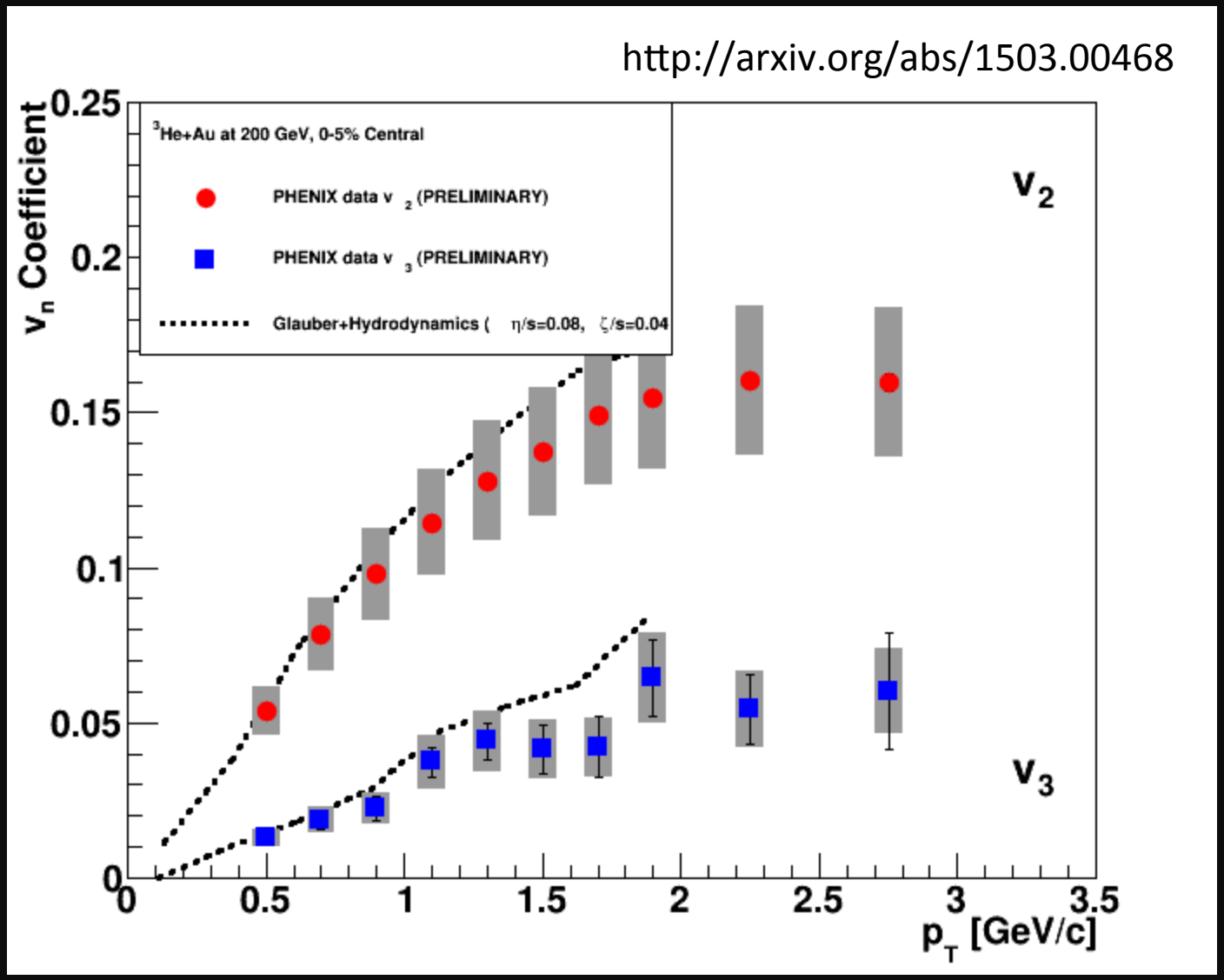


Bzdak, Ma (http://arxiv.org/abs/1406.2804) AMPT and p+Pb



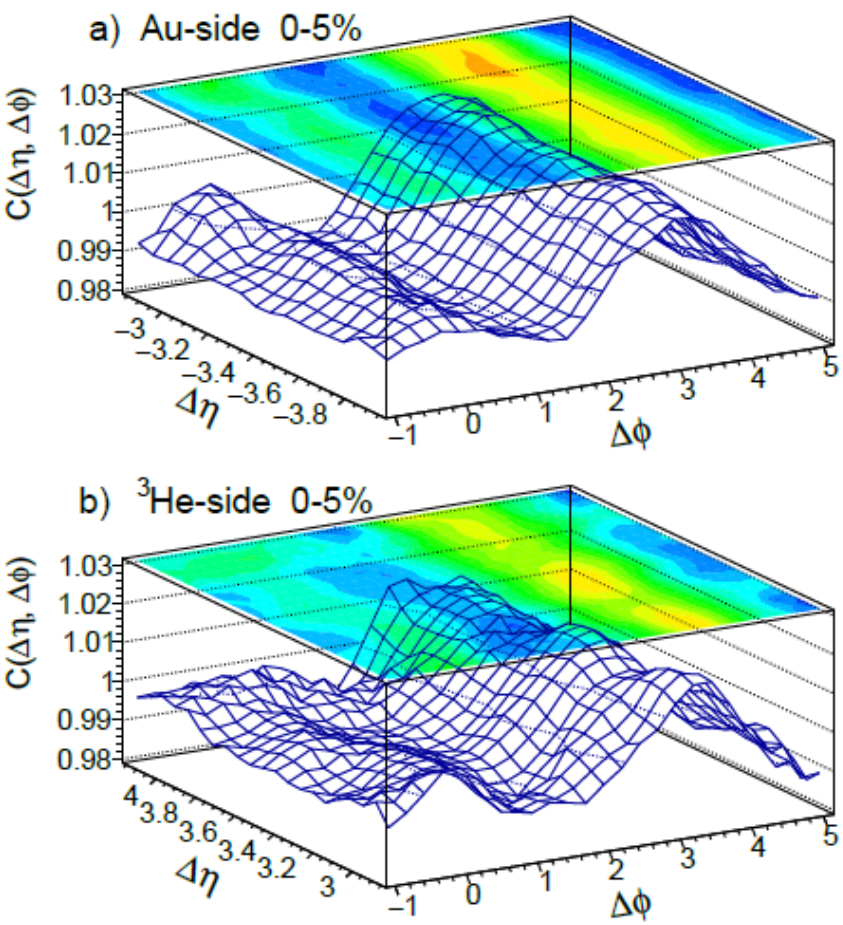


AMPT v2.21 results [Javier Orjuela-Koop et al., arXiv:1501.06880]
String melting on, σ=1.5 mb, only mod. for ^{3}He into Glauber



New paper from Bozek with Glauber IC + 3-d Hydrodynamics
No hadronic cascade

http://arxiv.org/abs/1503.00468



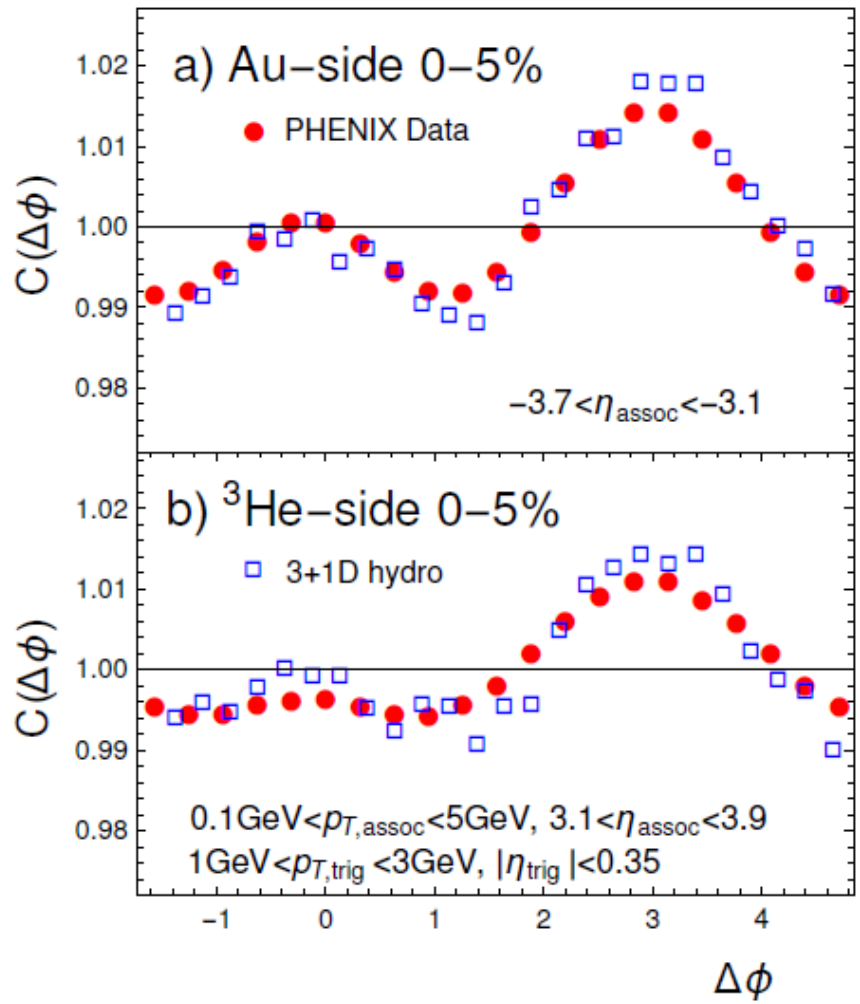
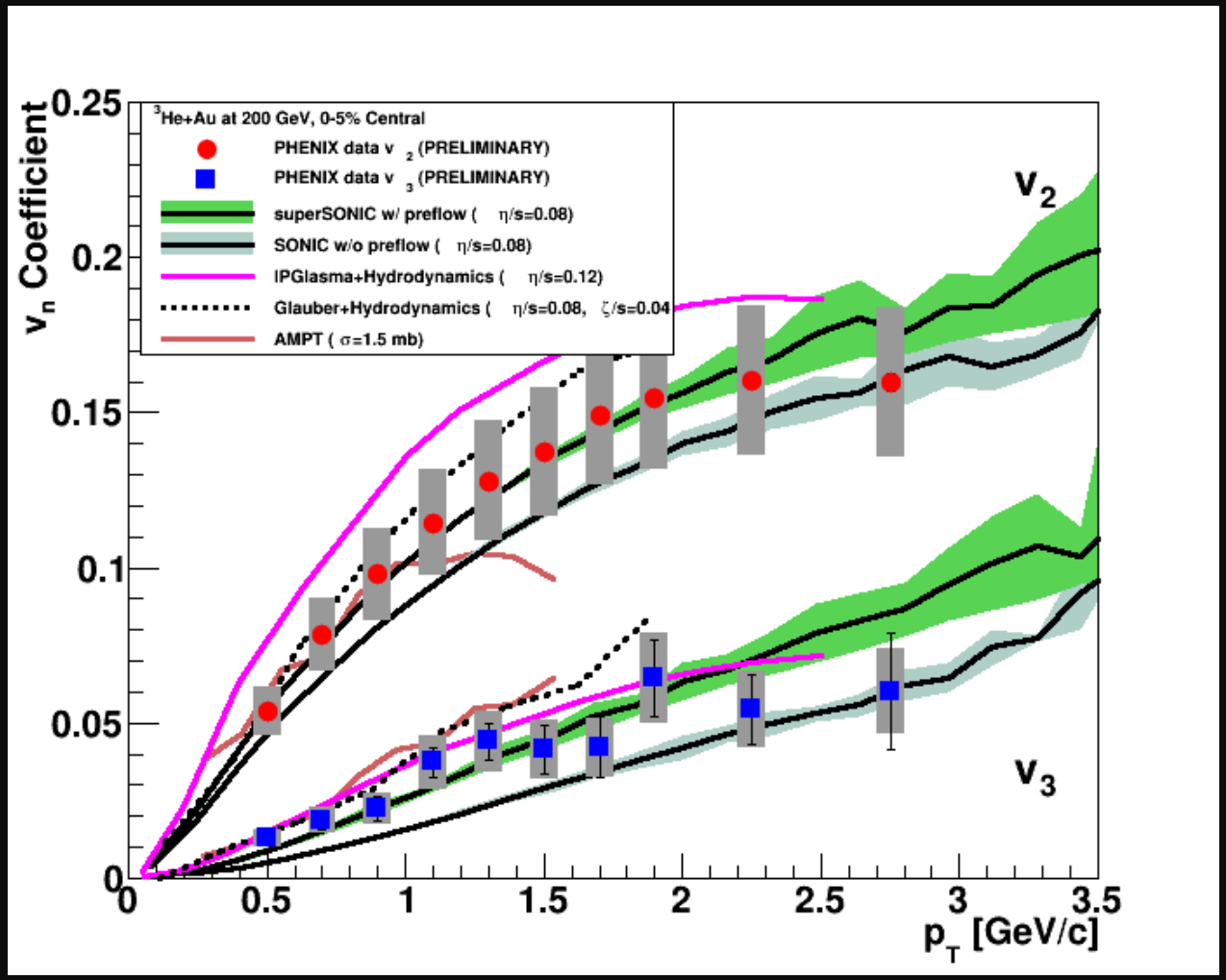
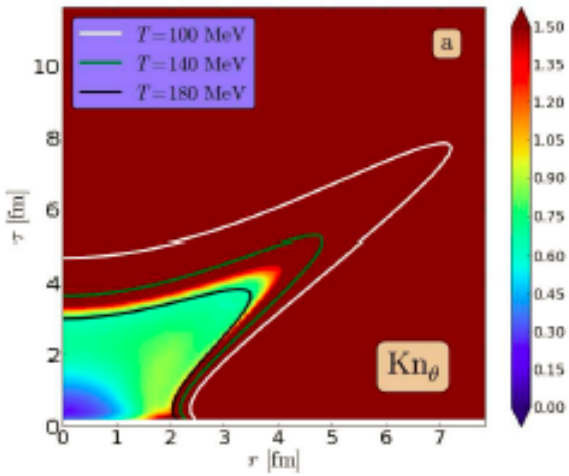


Figure 3: The two-particle correlation function $C(\Delta\eta, \Delta\phi)$ for a) Au-

What have we learned? Need to design control experiments to test different inputs.
e.g. same IC model in p,d,^{3}He+A, same η/s, must have hadronic stage (known to exist)



Applicability of Hydrodynamics?

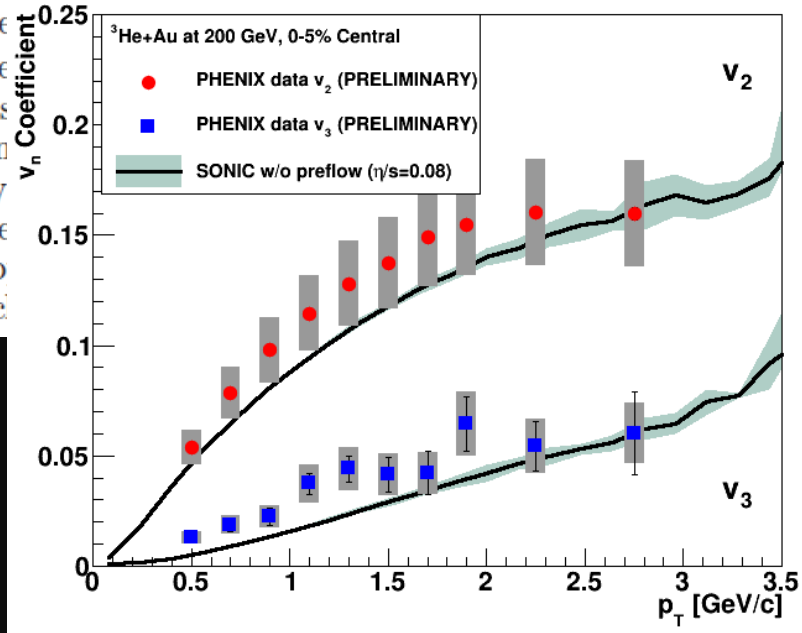


Niemi et al.: http://arxiv.org/abs/1404.7327

- AA collisions, event-averaged initial conditions: If η/s is sufficiently small and τ_0 sufficiently large, second-order hydrodynamics is applicable
- pA collisions: Even for $HH - LQ$ parametrization of η/s, second-order hydrodynamics is barely applicable

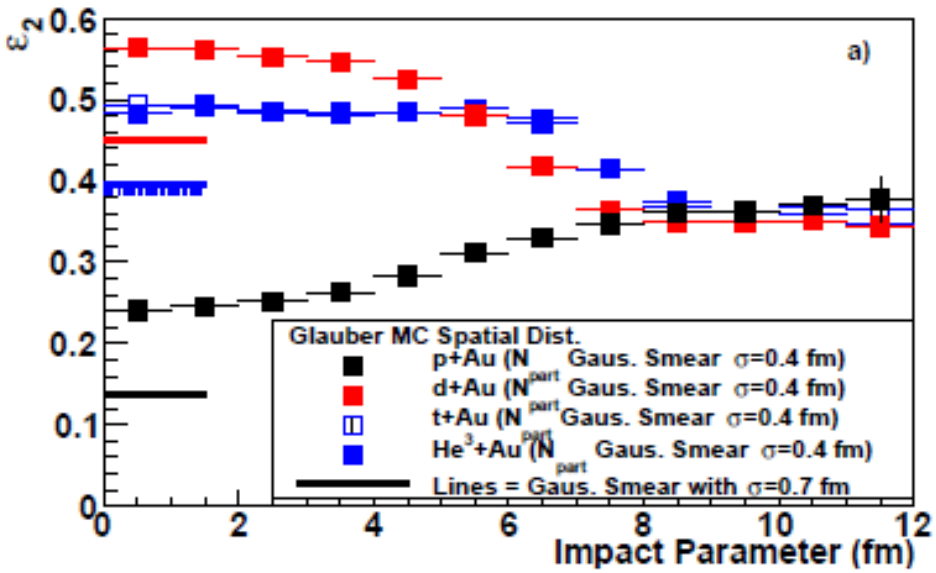
To test if second-order corrections are small compared to fir
ratio $C_\eta = \frac{\tau_\pi(\epsilon+P)}{2\eta}$) is varied by 50 percent around the reference
estimate of the applicability of viscous hydrodynamics. If the
down, one would expect second-order gradient terms to be as
results for particle spectra should vary considerably when chan
of C_η) by 50 percent. Conversely, if final results showed very
for heavy ion collisions, cf. [42]), this could be considered evide
systems. In this sense, the reliability of hydrodynamics as a ap
and expressed as an error band generated by varying C_η, whic

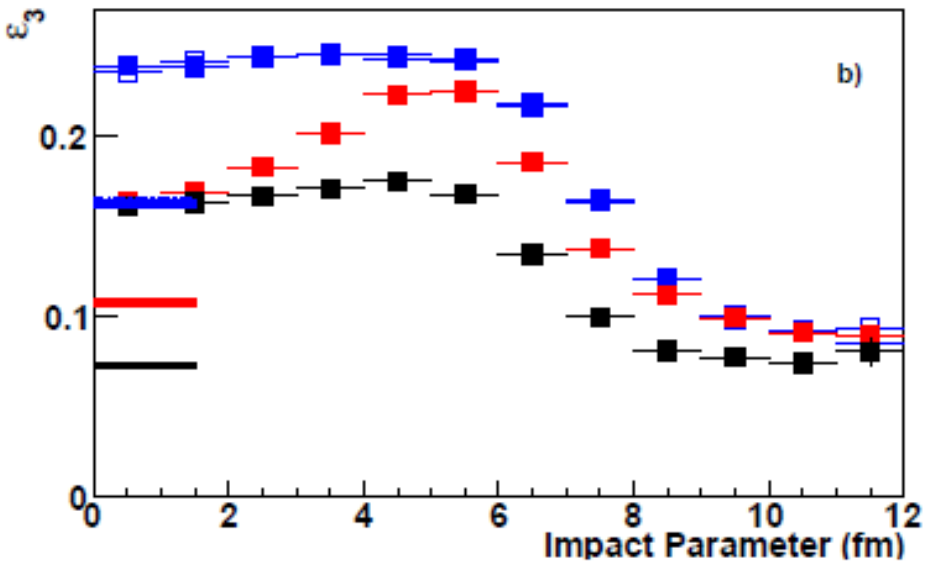
Romatschke: http://arxiv.org/abs/1502.04745

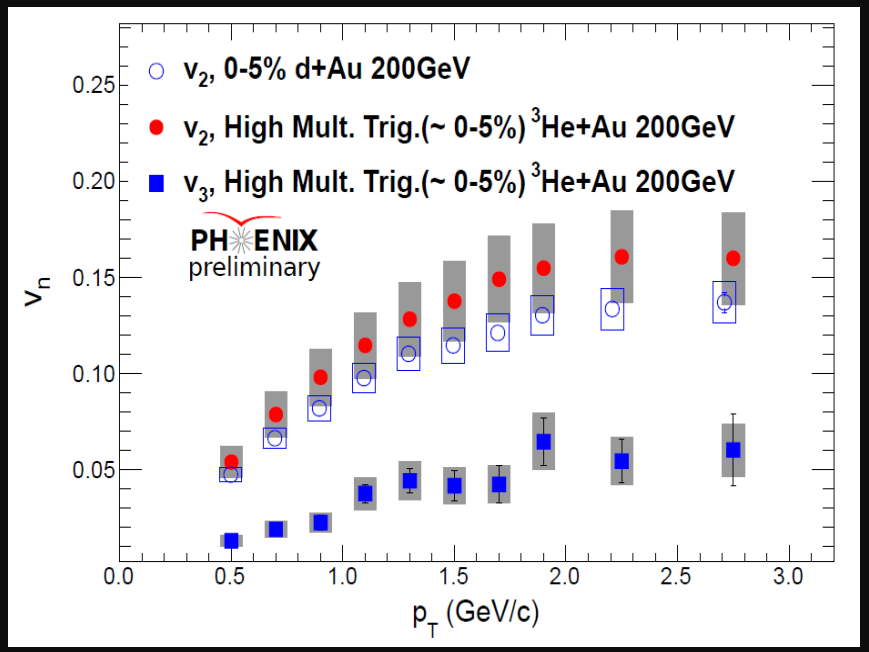


Needs clear resolution

Comparing Systems



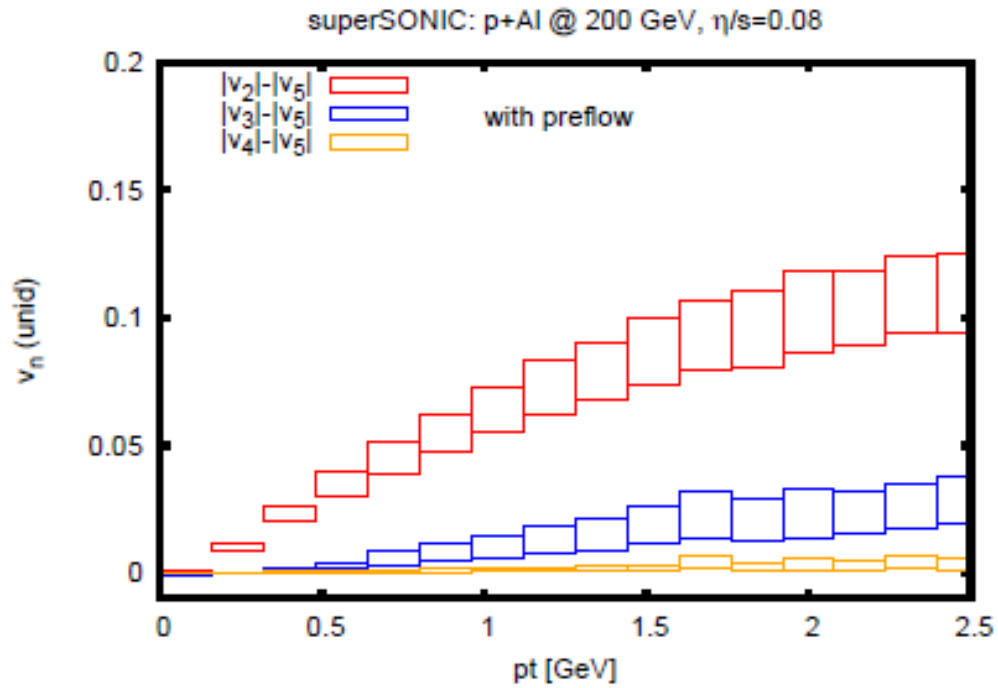


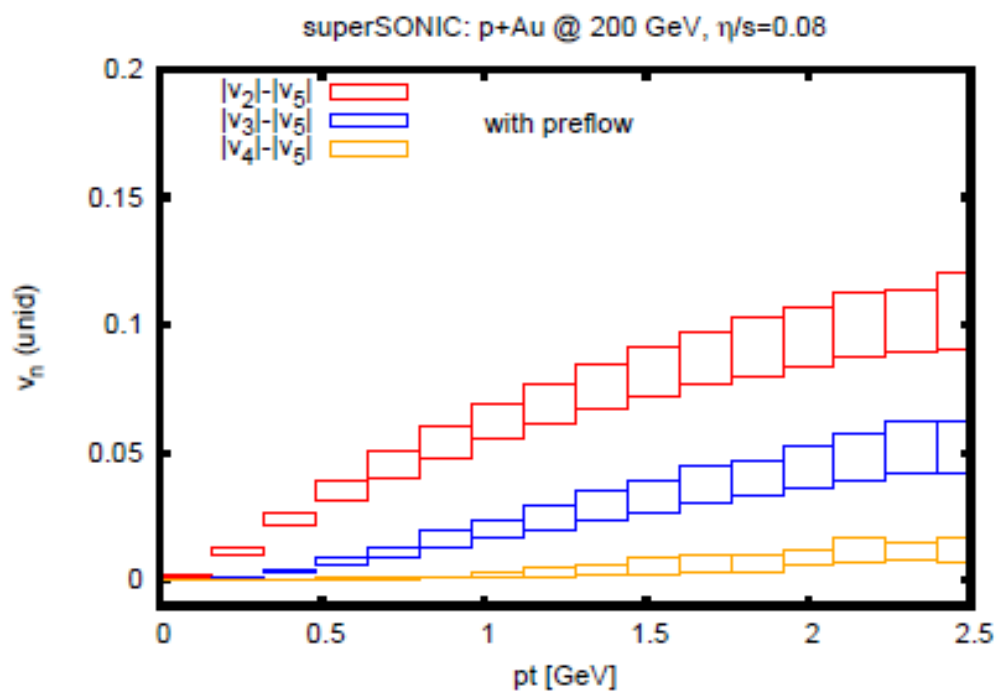


v_2 d+Au ~ v_2 ^{3}He+Au

Expected

v_2 p+Au << v_2 ^{3}He/d+Au

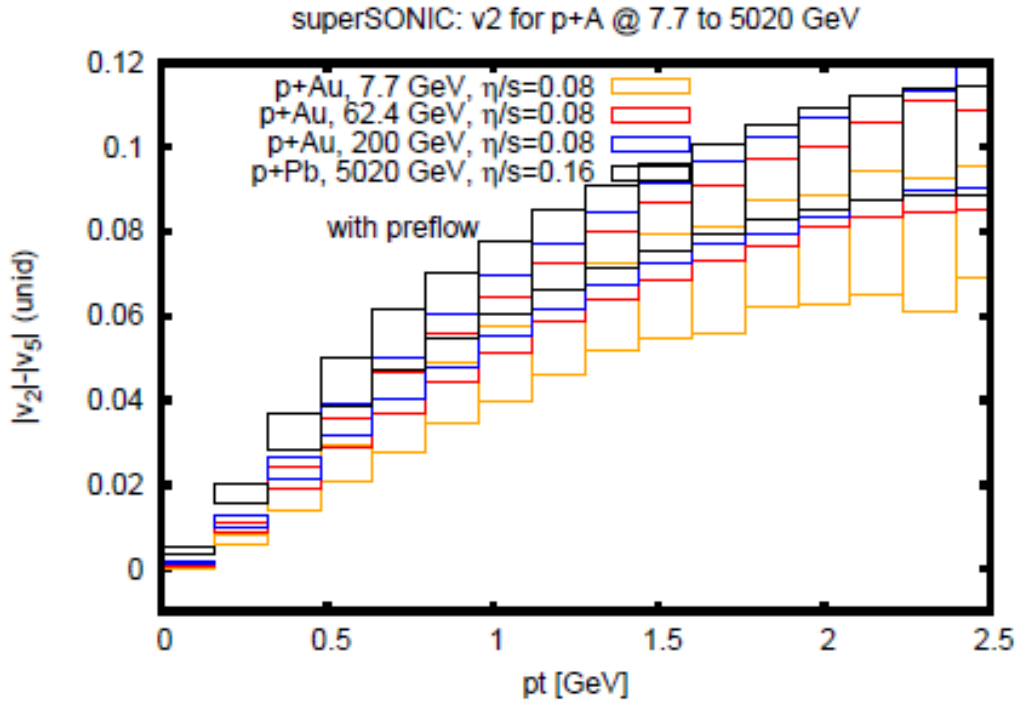


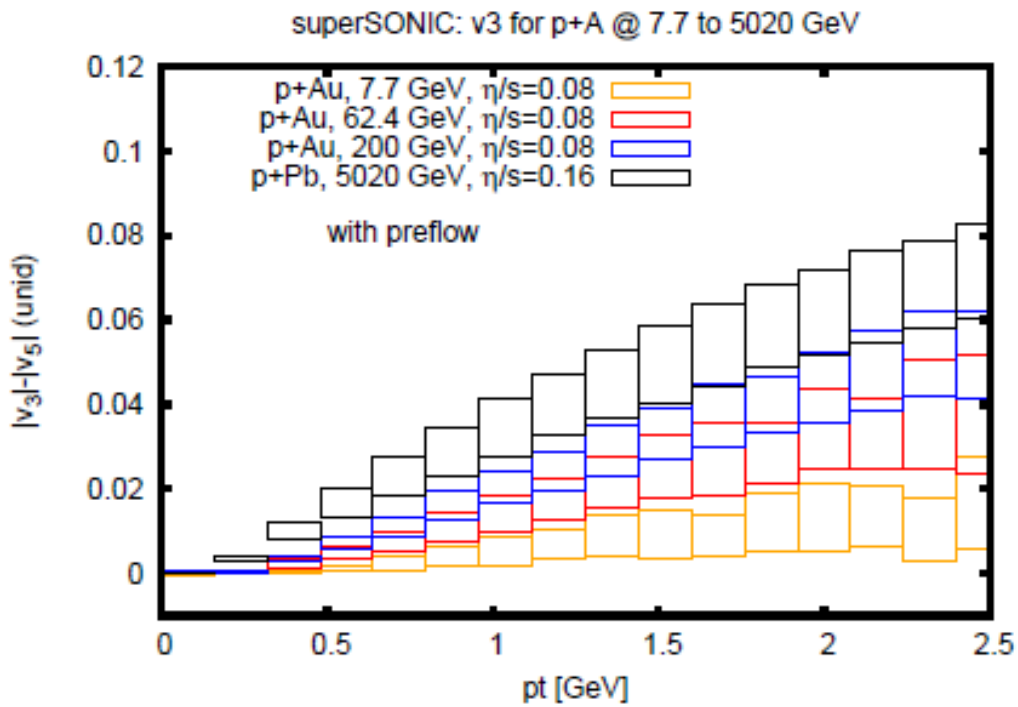


superSONIC predicts for p+Al and p+Au

Data taking for those systems at RHIC in the next couple of months!

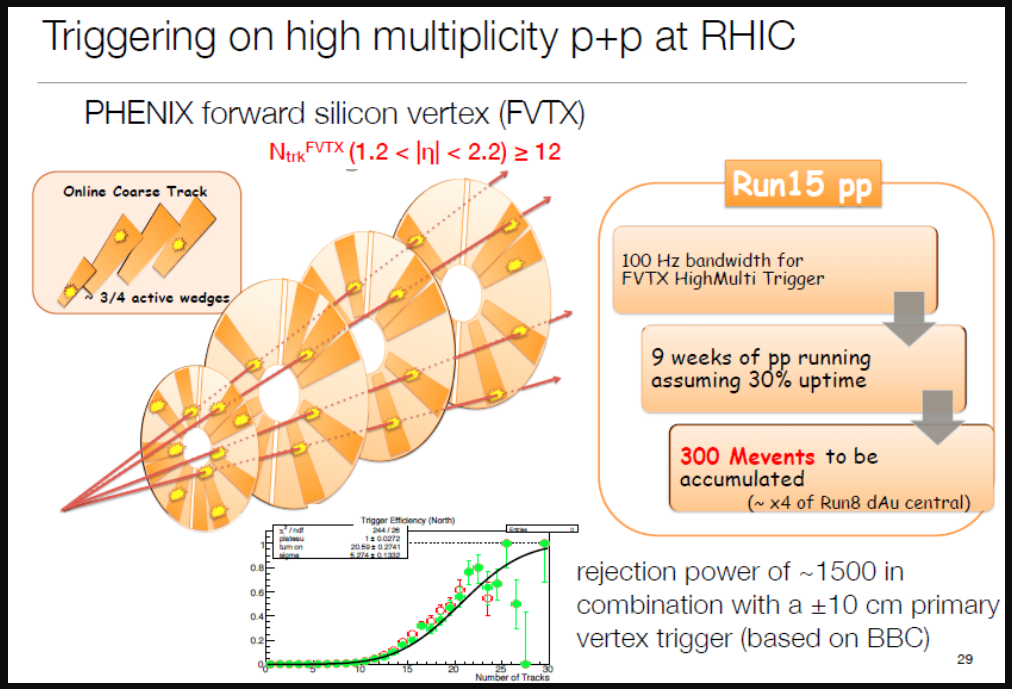
Comparison of geometries at the same energy is key to discriminating competing explanations





Does the signal go away?

Considering proposal of d+A at 10, 39, 62 GeV to really see what happens in nature…



Important to investigate high multiplicity p+p at RHIC energies

What would it really cost to run d/^{3}He+Pb at LHC?
Millions of CHF might be well justified.

LHC d/^{3}He+Pb workshop?

Escape Mechanism

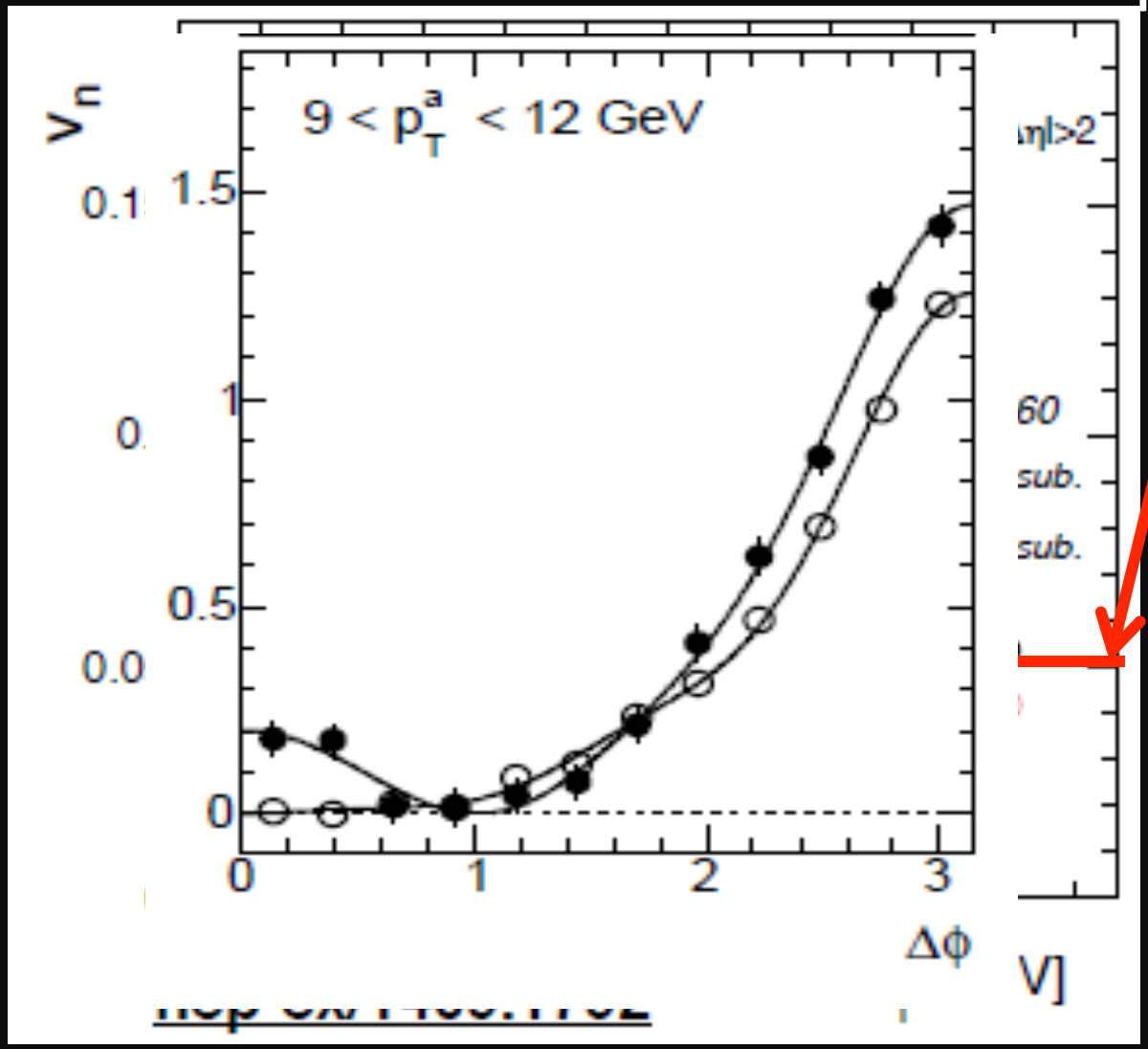
Antenna

Opaque system boiling off particles will have anisotropic emission.
However, … think at least one step ahead and check…

Good to push on alternative ideas, but one snowball does not enable one to conclude that everyone else is wrong.

OTHER SIGNALS?

Makes sense to run all the usual signal codes and ask the same questions!



Autocorrelation?
Quenching?

Theory groups use pre-equilibrium + hydro space-time and calculate jet quenching observables.

What observables are most sensitive

Fluid Probe:

Put a pebble in the stream and watch something out of equilibrium then equilibrate.

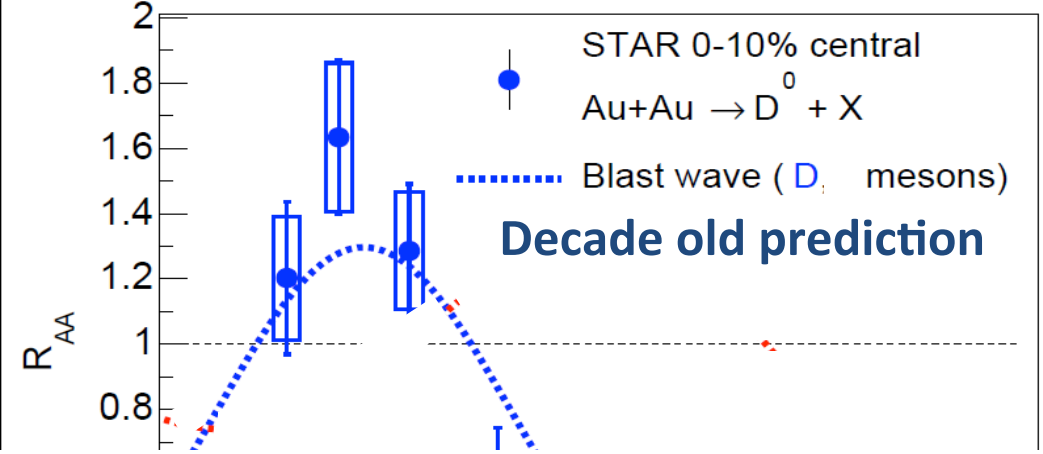
Charm Quark

Beauty Quark

"Does the Charm Flow at RHIC?"

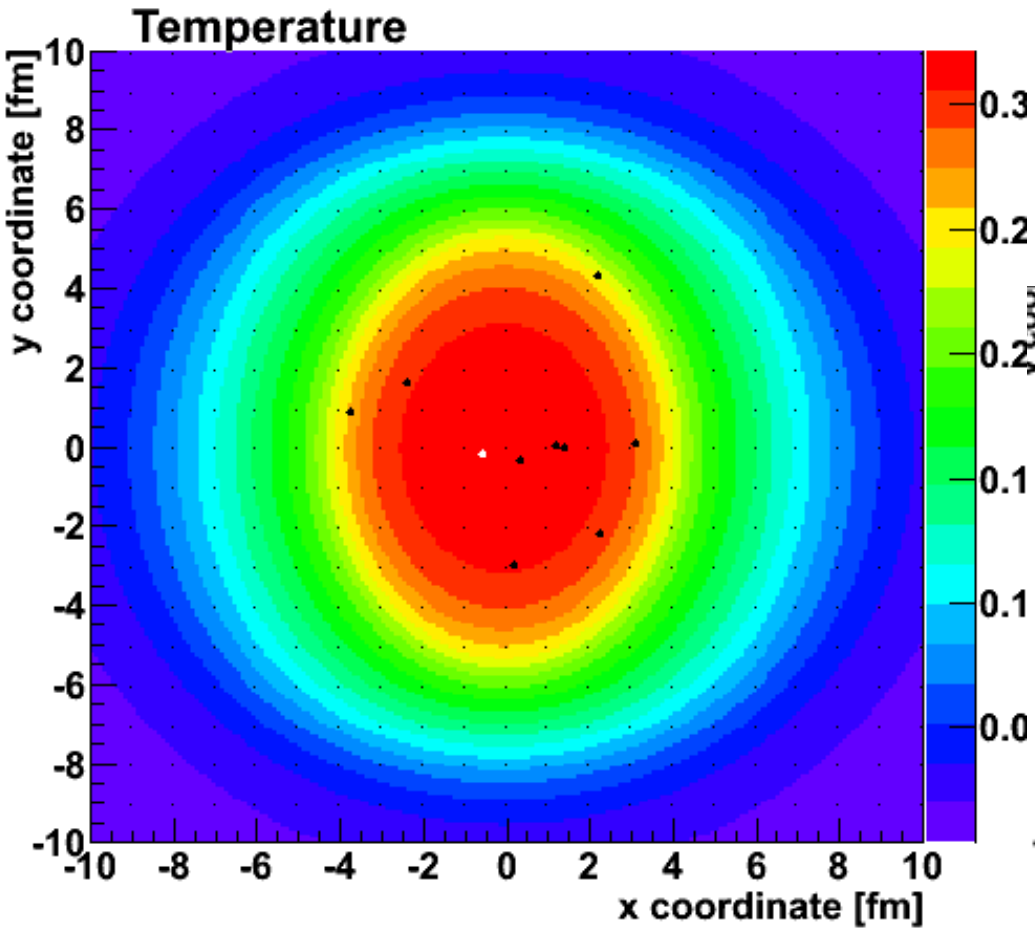
S. Batsouli, S. Kelly, M. Gyulassy (Columbia U.) , J.L. Nagle (Colorado U.) . Dec 2002. 11pp.
Published in **Phys.Lett.B557:26-32,2003**.
e-Print: **nucl-th/0212068**

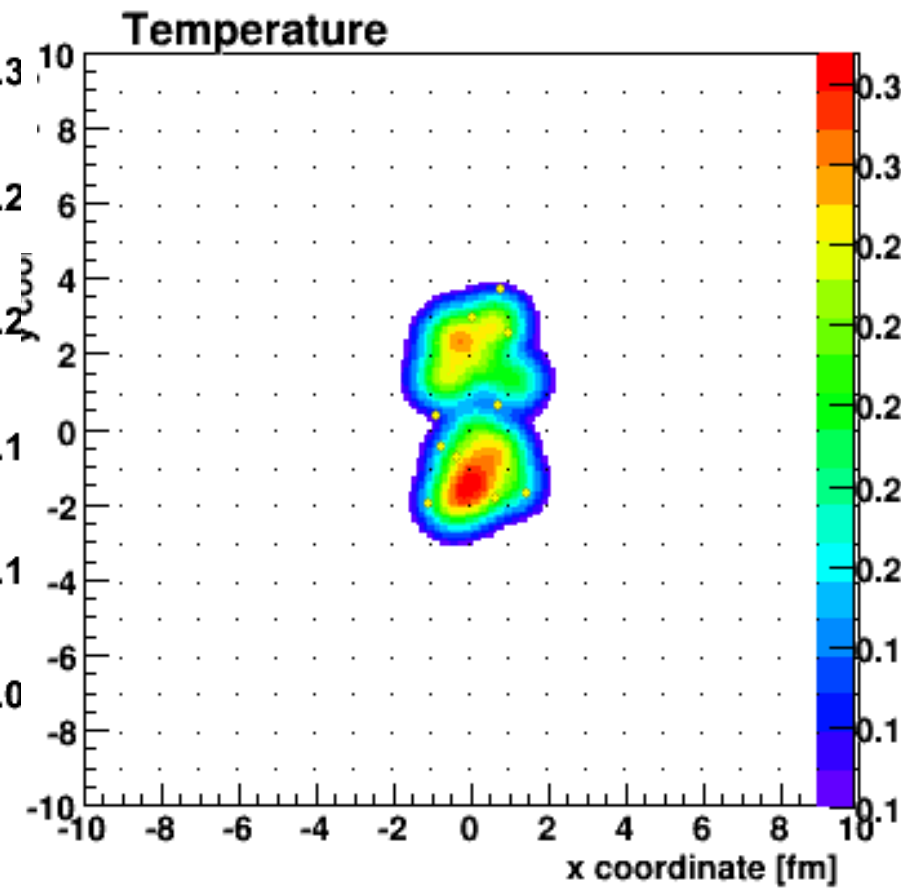
At the time, many said this idea was "ridiculous".



Ratio Au+Au / p+p

Charm quarks pushed out by blast wave of fluid





Summary Of Thoughts

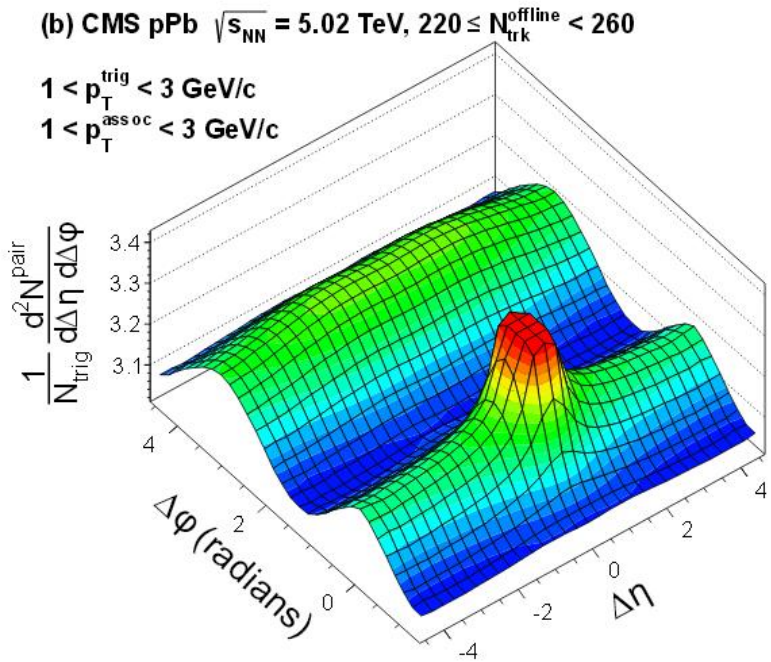
Stony Brook University

Roles of short range correlations and recoil subtraction in ridge measurements

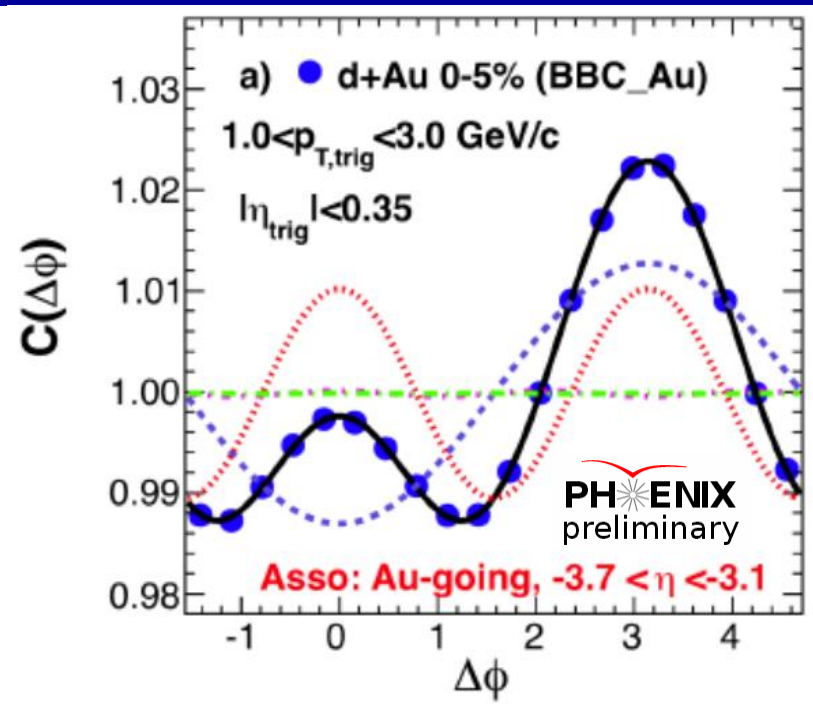
Sooraj Radhakrishnan

RBRC Workshop on Collectivity in Small Colliding Systems with High Multiplicity

Ridge in small systems



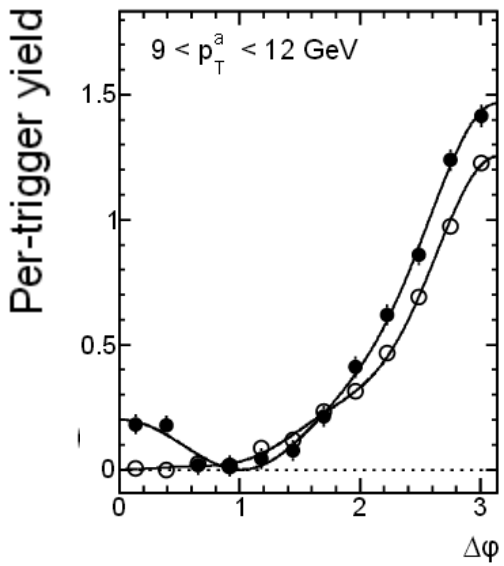
Phys. Lett. B 724

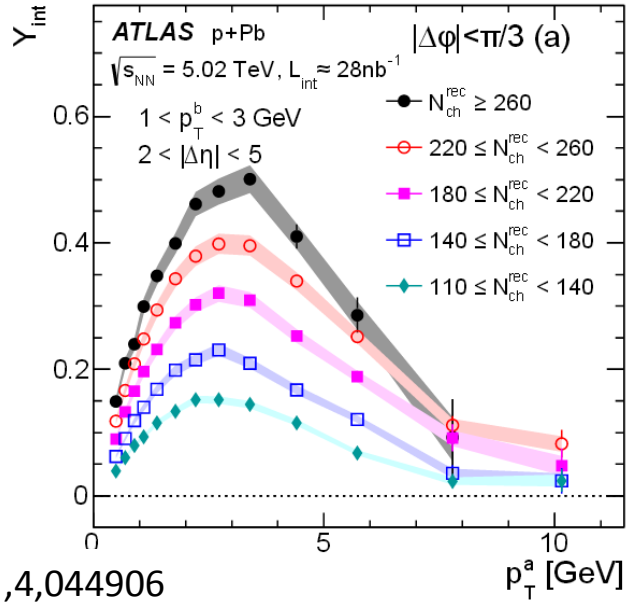


- Correlations extending to large pseudo rapidity separations ('ridge') in high multiplicity p+Pb and d+Au collisions.
- Exists over a large p_T range (up to ~ 10 GeV).
- Many measurements suggest collective origin similar to that in Pb+Pb
  - Multiplarticle cumulants, mass ordering, scaling between p+Pb and Pb+Pb, ..

Measuring the ridge

- Measure yield above flat pedestal.

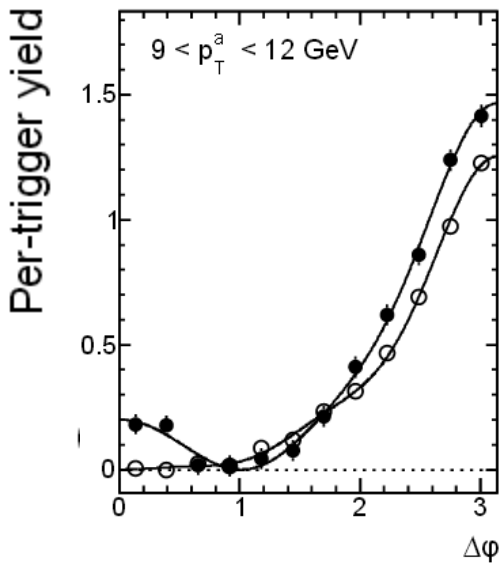


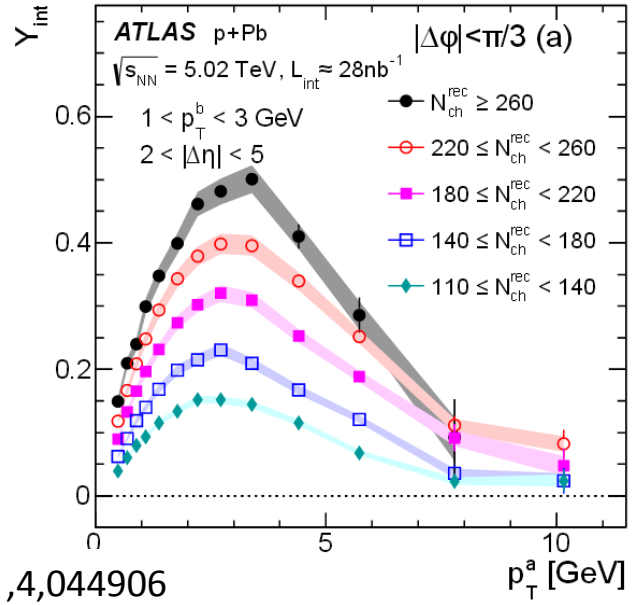


Phys.Rev. C90 ,4,044906

Measuring the ridge

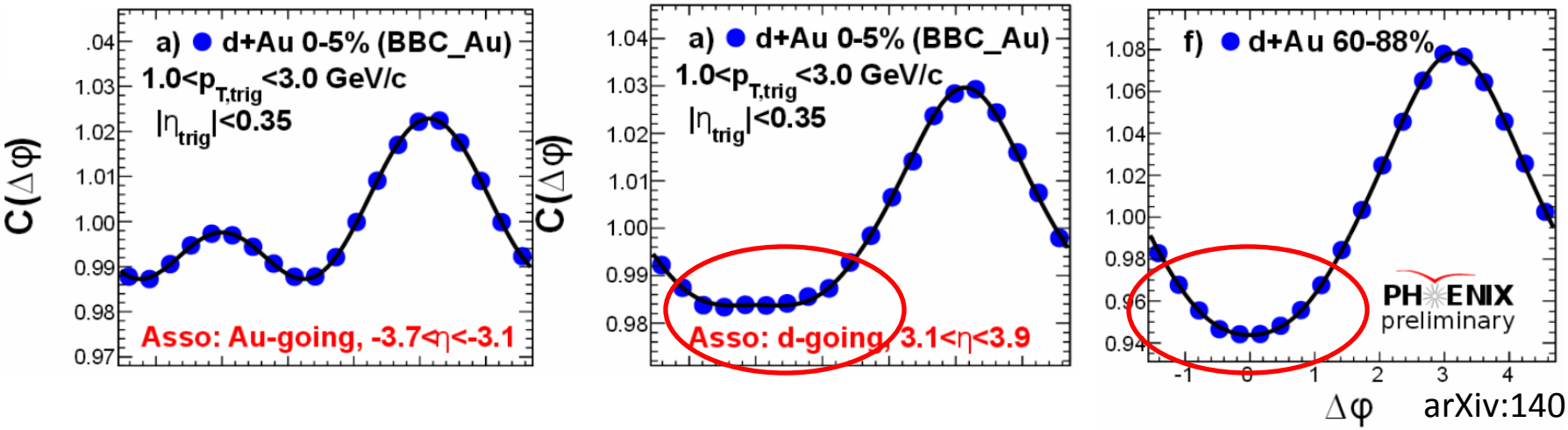
- Measure yield above flat pedestal.





Phys.Rev. C90 ,4,044906

But can be misleading, if ridge amplitude is small.

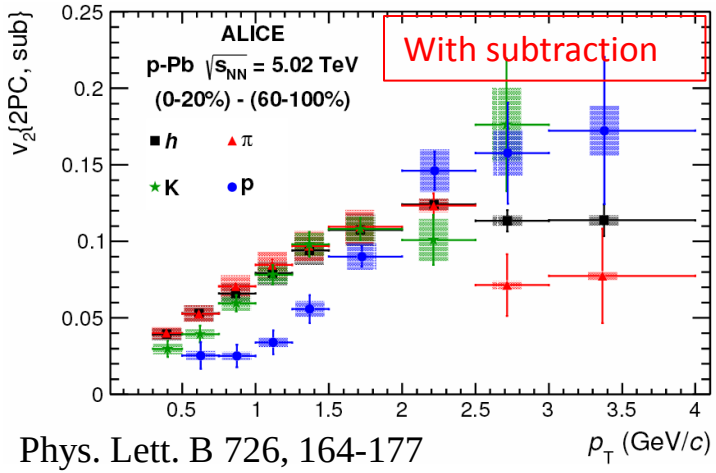


arXiv:1401.2432

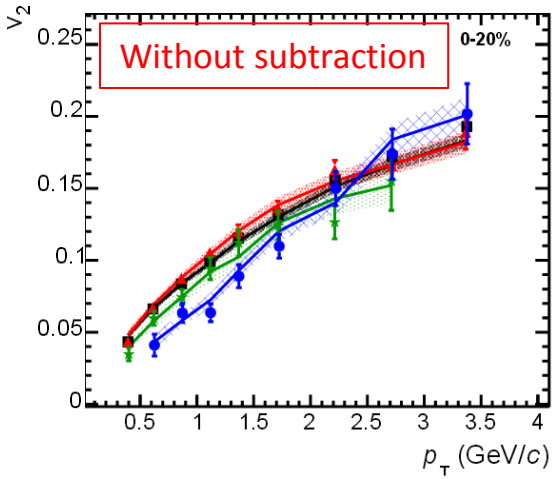
- Clear near side modification on d-going side, but ridge yield is zero!

Measuring the ridge

- Measure v_n from 2PC: with or without recoil subtraction?



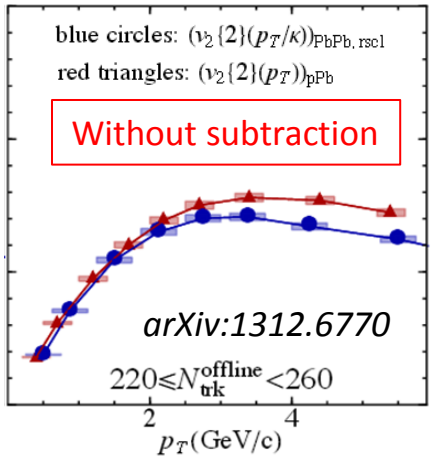
Phys. Lett. B 726, 164-177

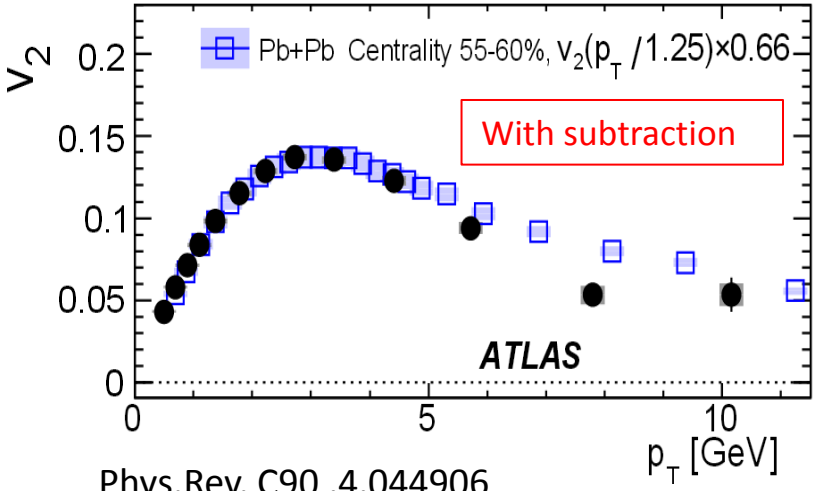


Mass ordering

Scaling between p+Pb and Pb+Pb

- Results differ between subtracted and unsubtracted cases

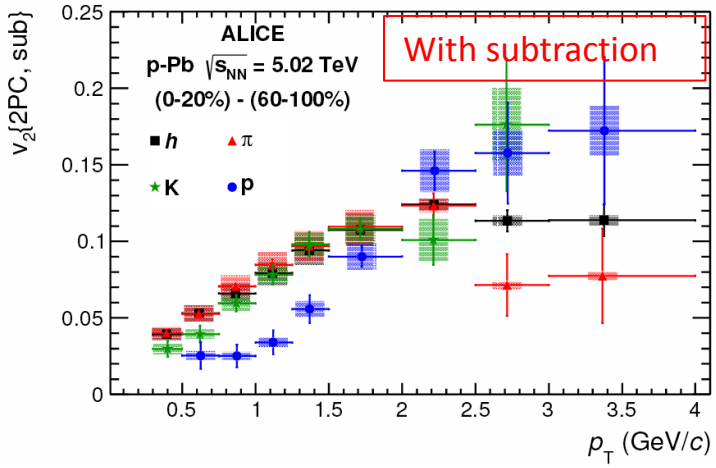


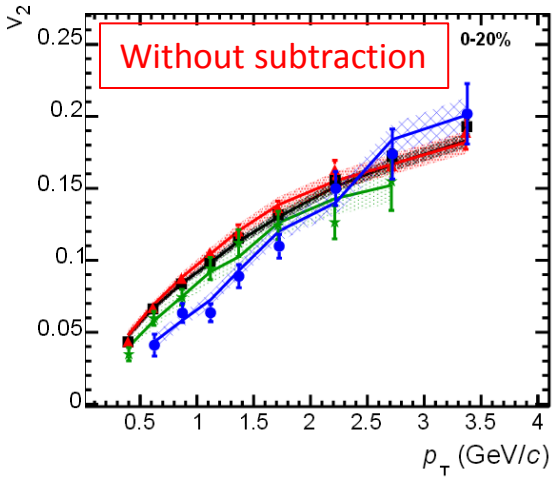


Phys.Rev. C90 ,4,044906

Measuring the ridge

- Measure v_n from 2PC: with or without recoil subtraction?





Mass ordering

Scaling between p+Pb and Pb+Pb

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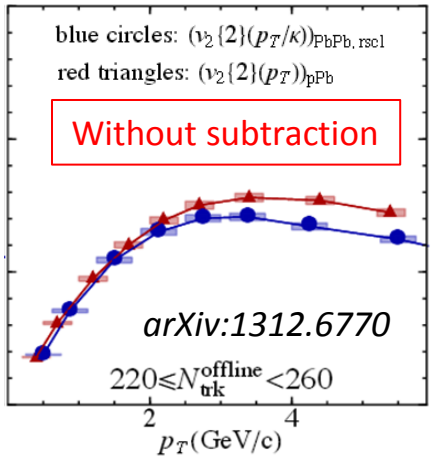
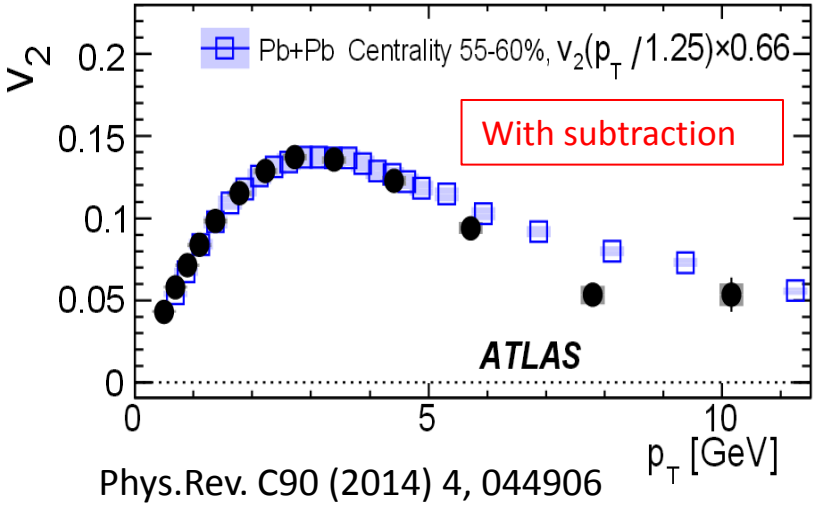
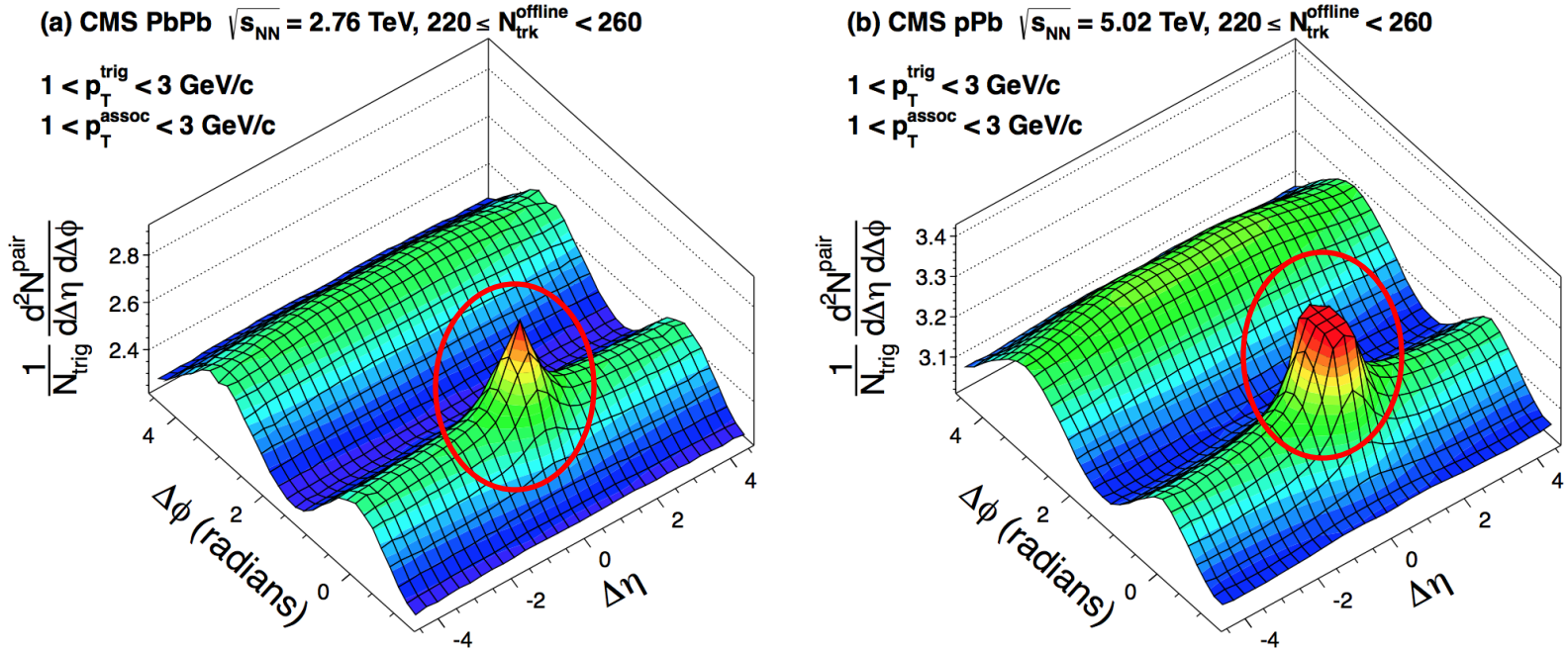




Phys.Rev. C90 (2014) 4, 044906

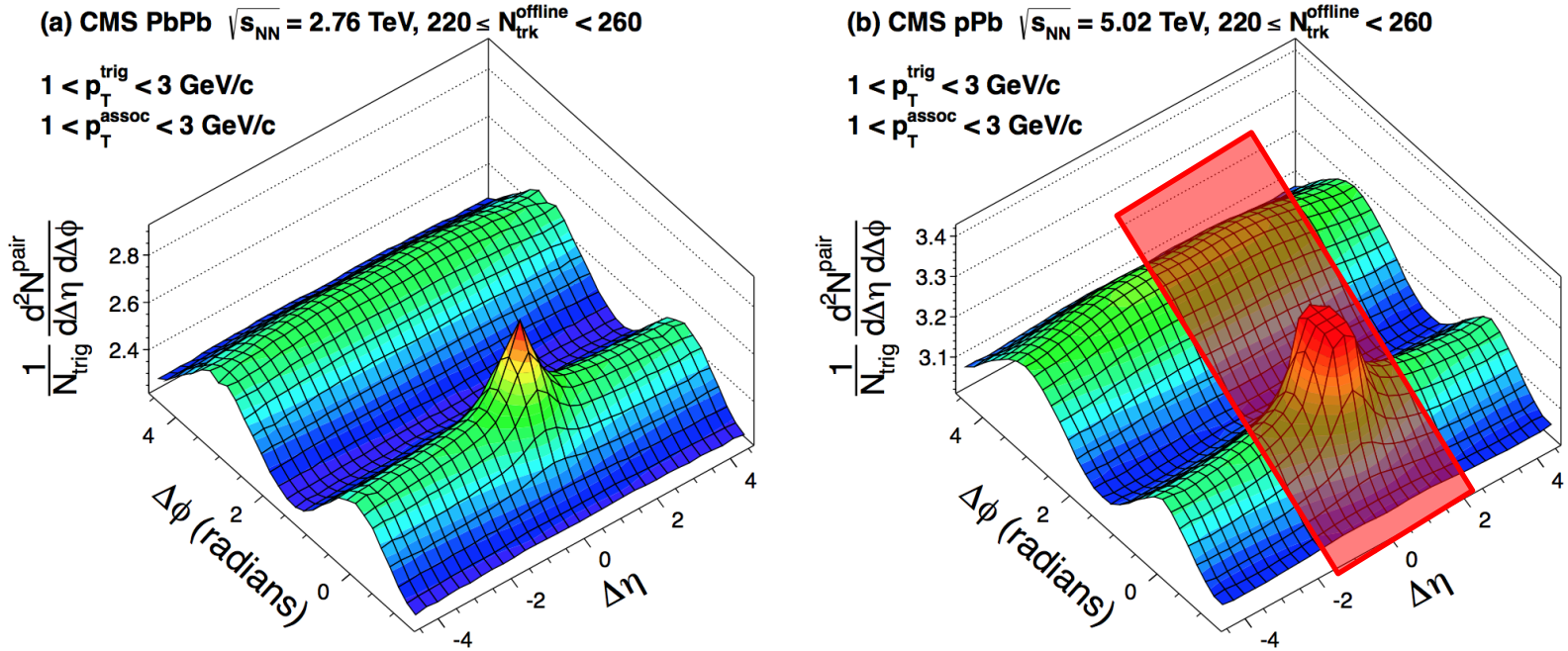
- Measure v_n from multiparticle cumulants
  - But no a priori reason to have non zero $v_2\{4\}$ or $v_3\{4\}$, i.e average geomtery component.

Measuring the ridge



- Larger contribution from non flow correlations in p+Pb compared to Pb+Pb, both on near side and away side
  - Smaller flow signal
  - Events selected using multiplicity triggers could enhance jet correlations.

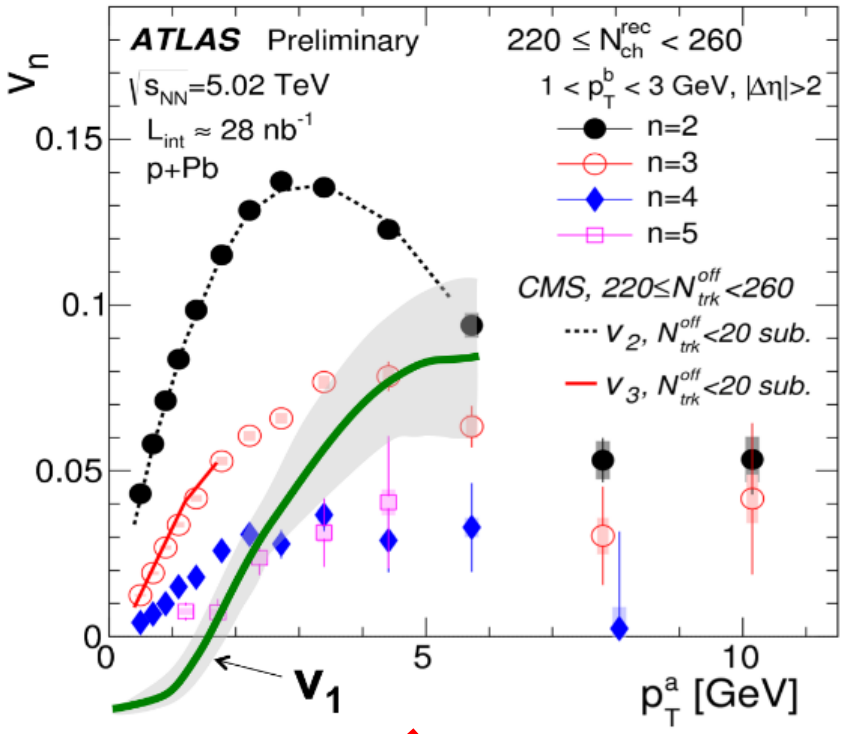
Measuring the ridge



- Much larger contribution from non flow correlations in p+Pb, both on near side and away side
  - Smaller signal
  - Events selected using multiplicity triggers enhance jet correlations.
- Near side jet can be suppressed by a Δη cut
- Away side jet → do recoil subtraction. How good is it?

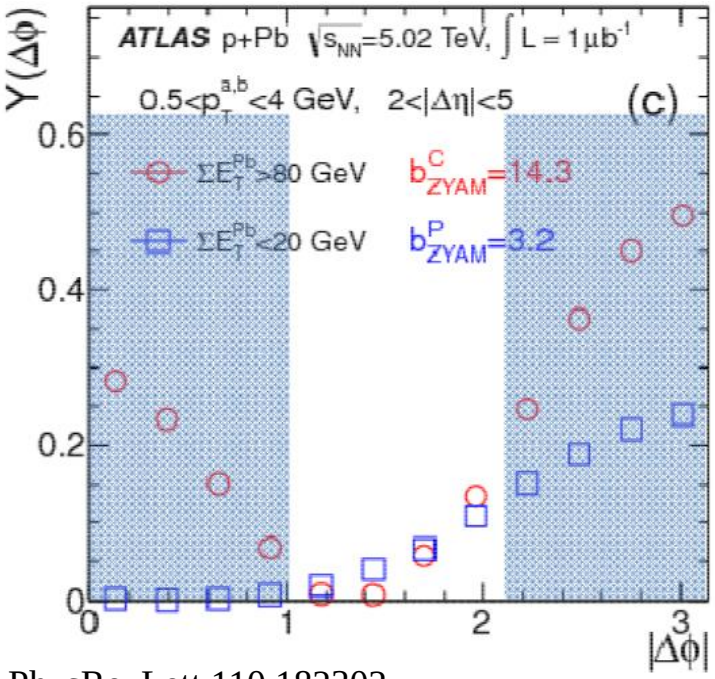
Overview

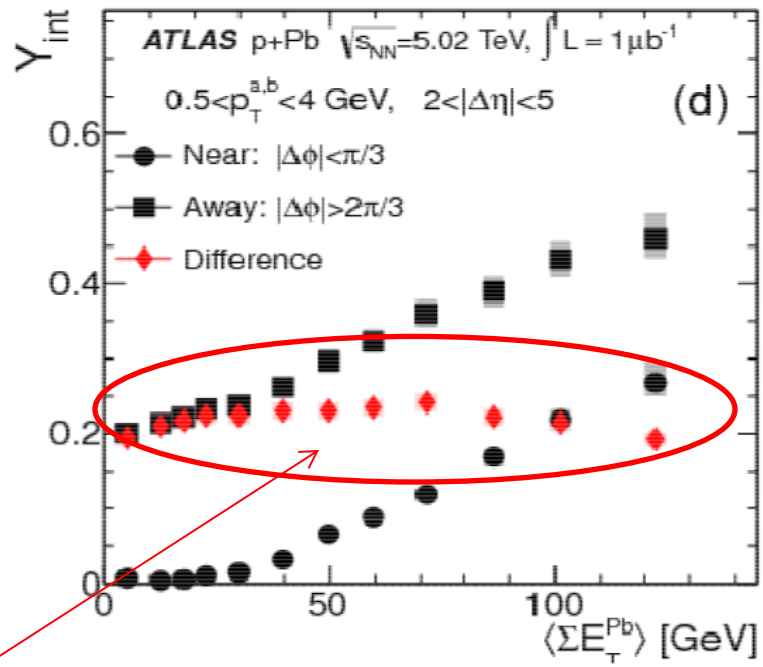
- In general, about recoil subtraction!
  - Reasoning behind and estimation of recoil contribution.
  - Effects on short and long range correlations
  - Consistency of results
  - Dipolar flow (v_1) in p+Pb and recoil subtraction.
  - Some discussion on similar analysis at RHIC



v_1 - v_5 measured by ATLAS using recoil subtraction.

Estimating recoil contribution

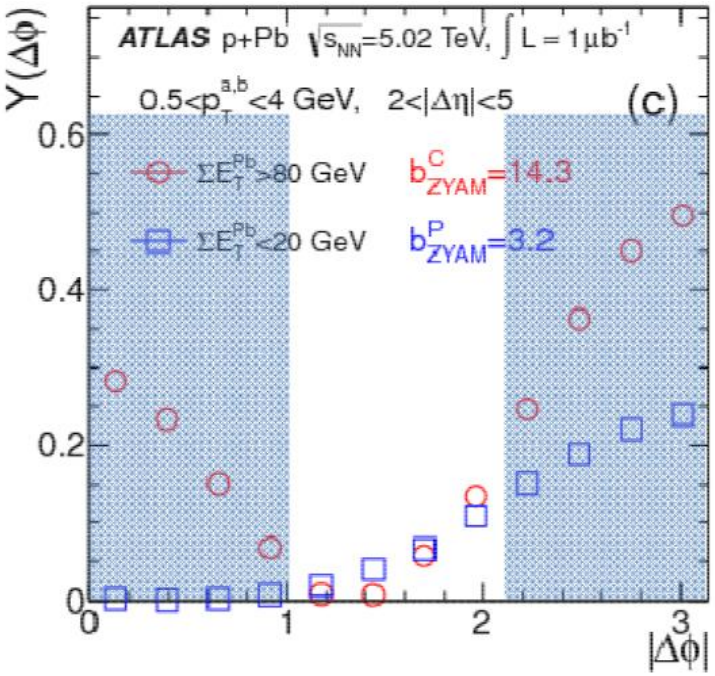


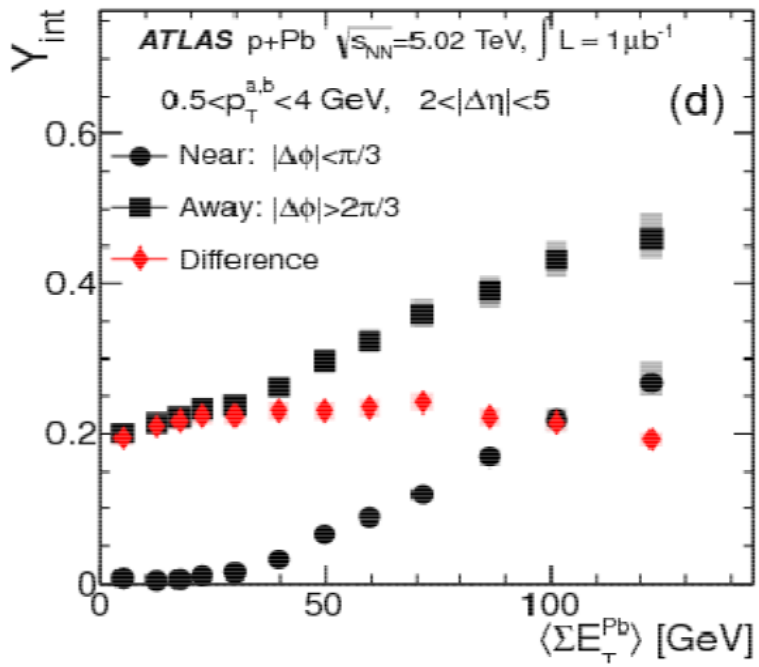


PhysRevLett.110.182302

- Weak centrality dependence for difference of yields between near side and away side.
  - Difference does not have contribution from v_2, v_3, and v_4.

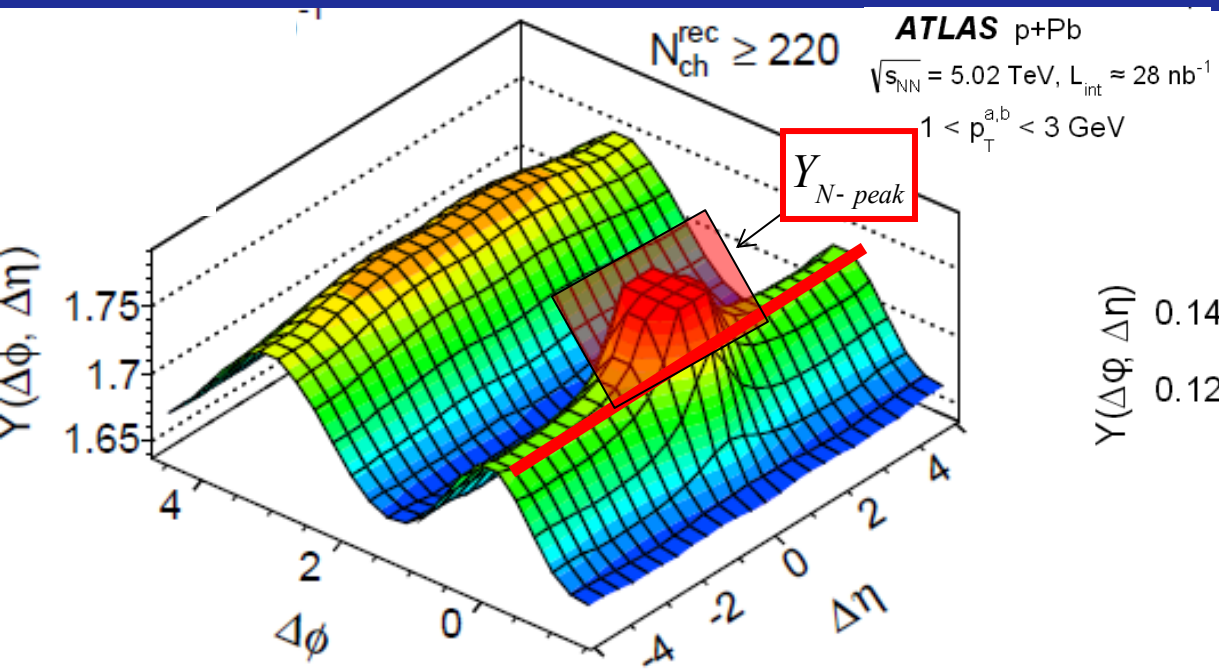
Estimating recoil contribution

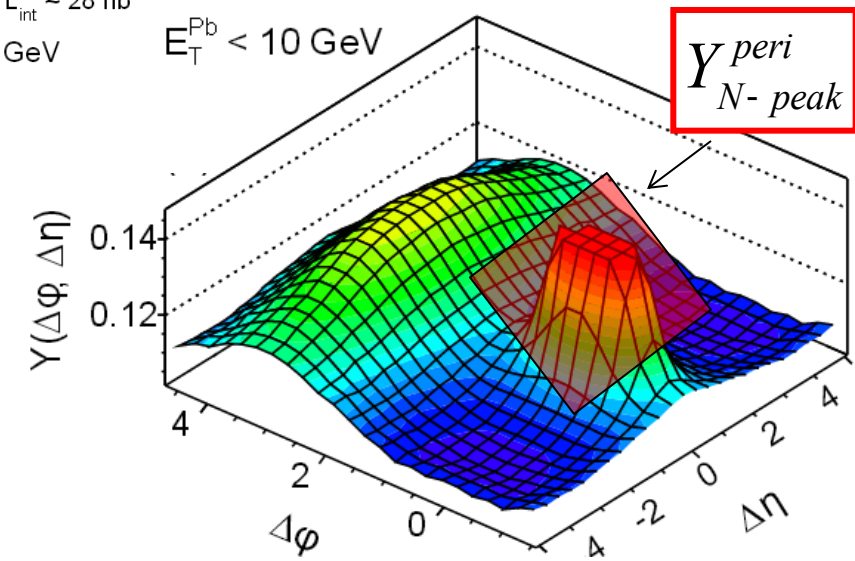




- Weak centrality dependence for difference of yields between near side and away side.
  - Difference does not have contribution from v_2, v_3, and v_4 --> measure of away side jet yield
- Away side jet yield no longer independent of centrality with multiplicity selection
  - But the change can be estimated from the change in near side jet yield

Recoil subtraction





$$Y^{\mathrm{sub}}(\Delta\phi,\Delta\eta) = Y(\Delta\phi,\Delta\eta) - \alpha Y^{\mathrm{corr}}_{\mathrm{peri}}(\Delta\phi,\Delta\eta)$$

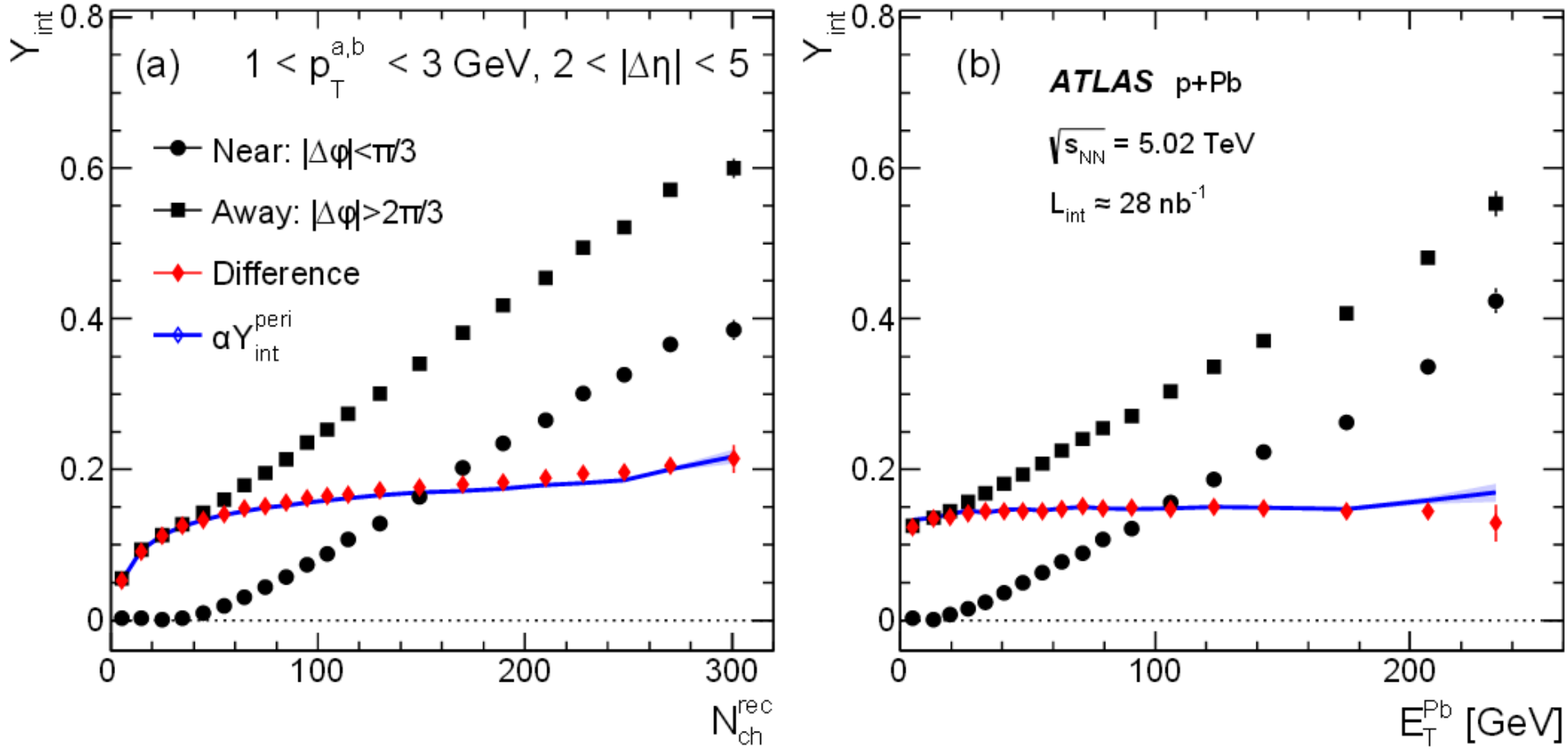
α is chosen such that $\boxed{aY^{peri}_{N\text{-}peak} = Y_{N\text{-}peak}}$

- Peripheral bin is taken to be one with total FCal energy ($E_T{}^{Pb}$) < 10 GeV

$$Y^{\mathrm{N-Peak}} = \int_{|\Delta\eta|<1} Y(\Delta\eta)d\Delta\eta - \frac{1}{3}\int_{2<|\Delta\eta|<5} Y(\Delta\eta)d\Delta\eta$$

$$Y^{\mathrm{corr}}(\Delta\phi,\Delta\eta) = \frac{\int B(\Delta\phi,\Delta\eta)d\Delta\phi d\Delta\eta}{\pi\eta_{\Delta}^{\max}}\left(\frac{S(\Delta\phi,\Delta\eta)}{B(\Delta\phi,\Delta\eta)} - b_{\mathrm{ZYAM}}\right)$$

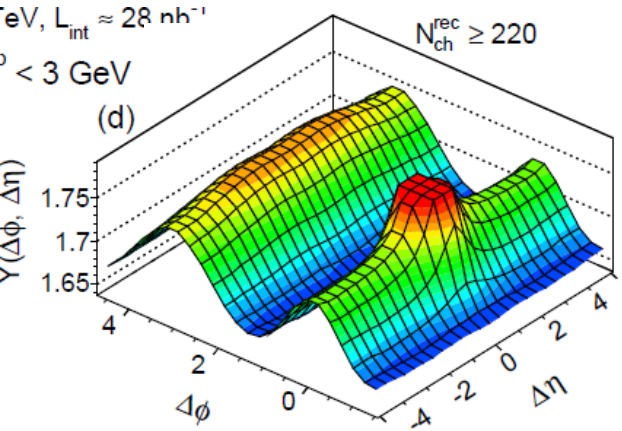
Recoil subtraction



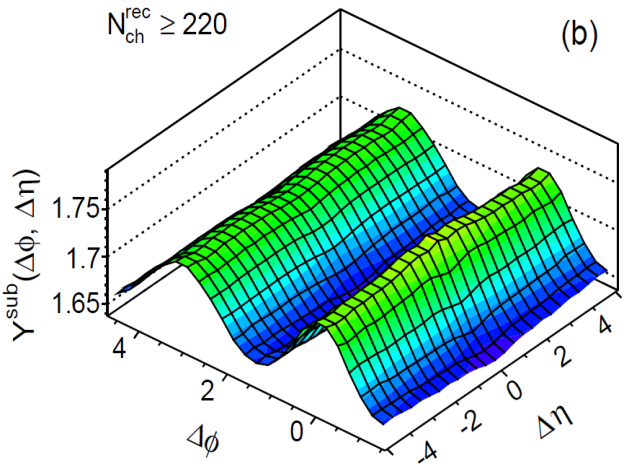
- Estimated recoil contribution gives a good estimate of the yield difference.
  - Difference does not have contribution from v_2, v_3, and v_4.

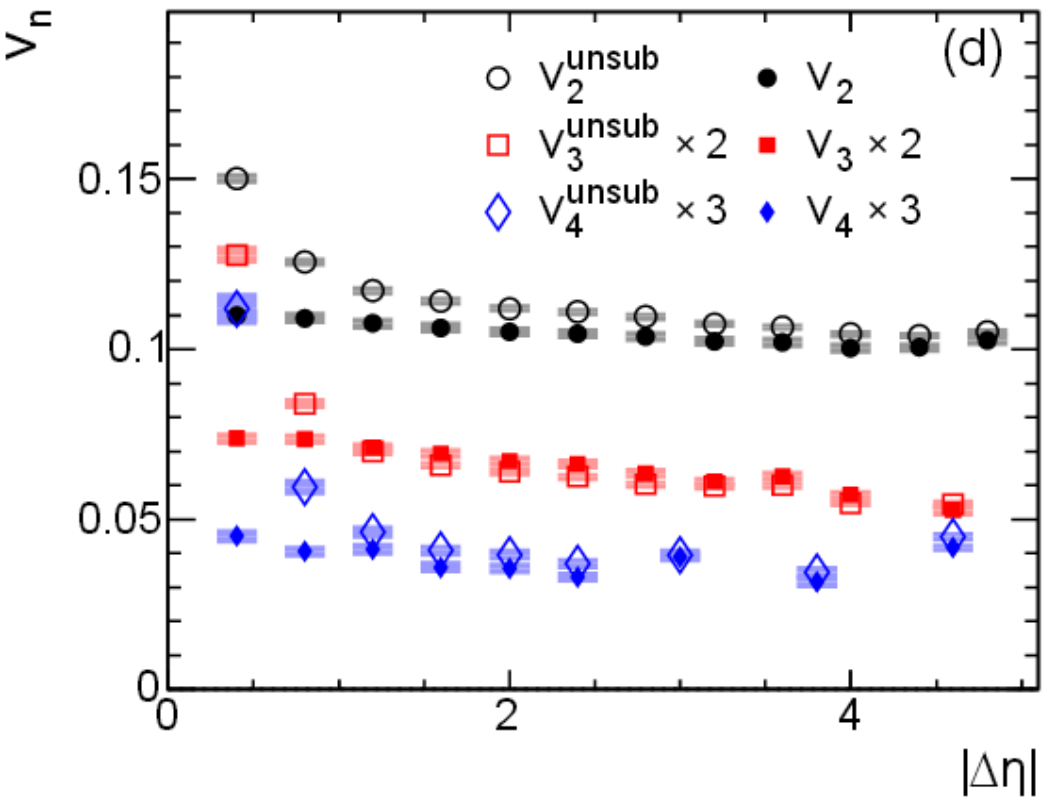
Recoil subtraction and short range

Before subtraction



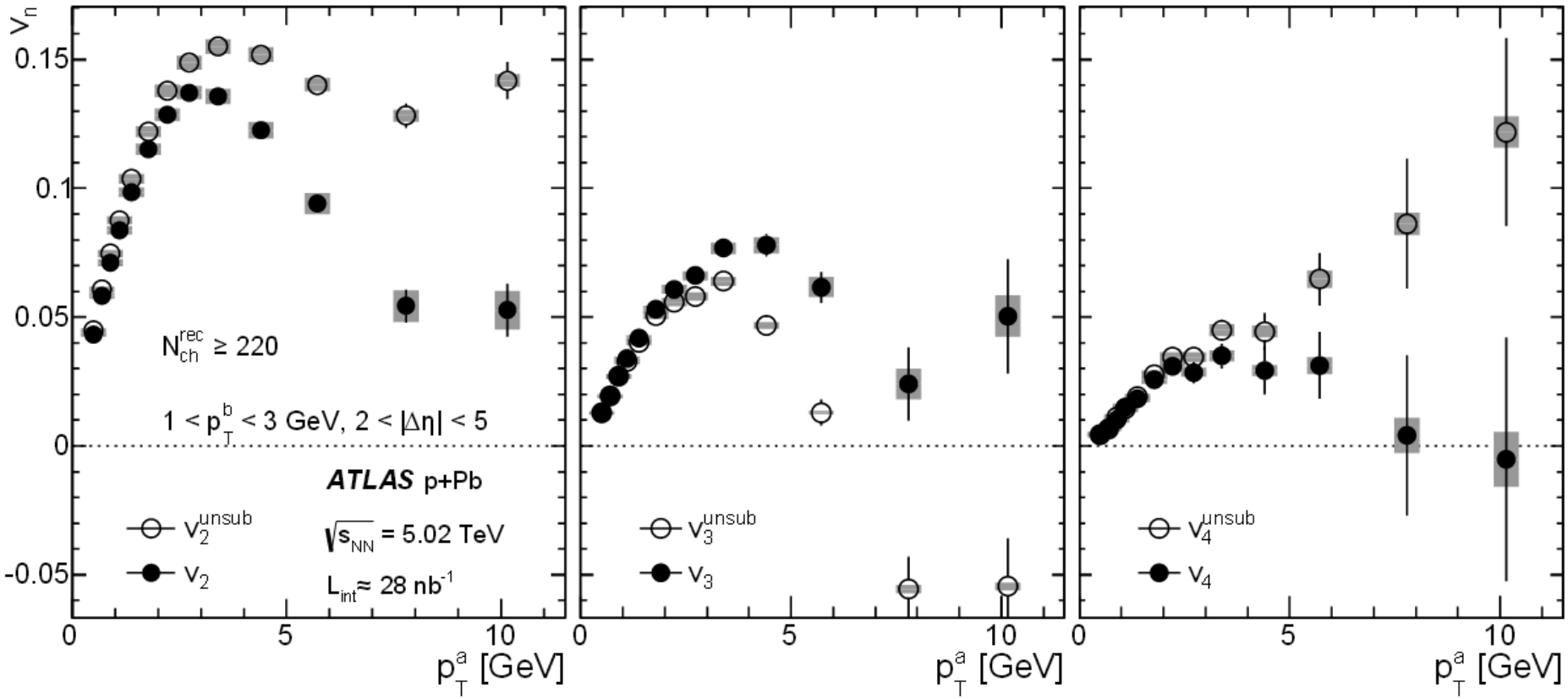
After subtraction





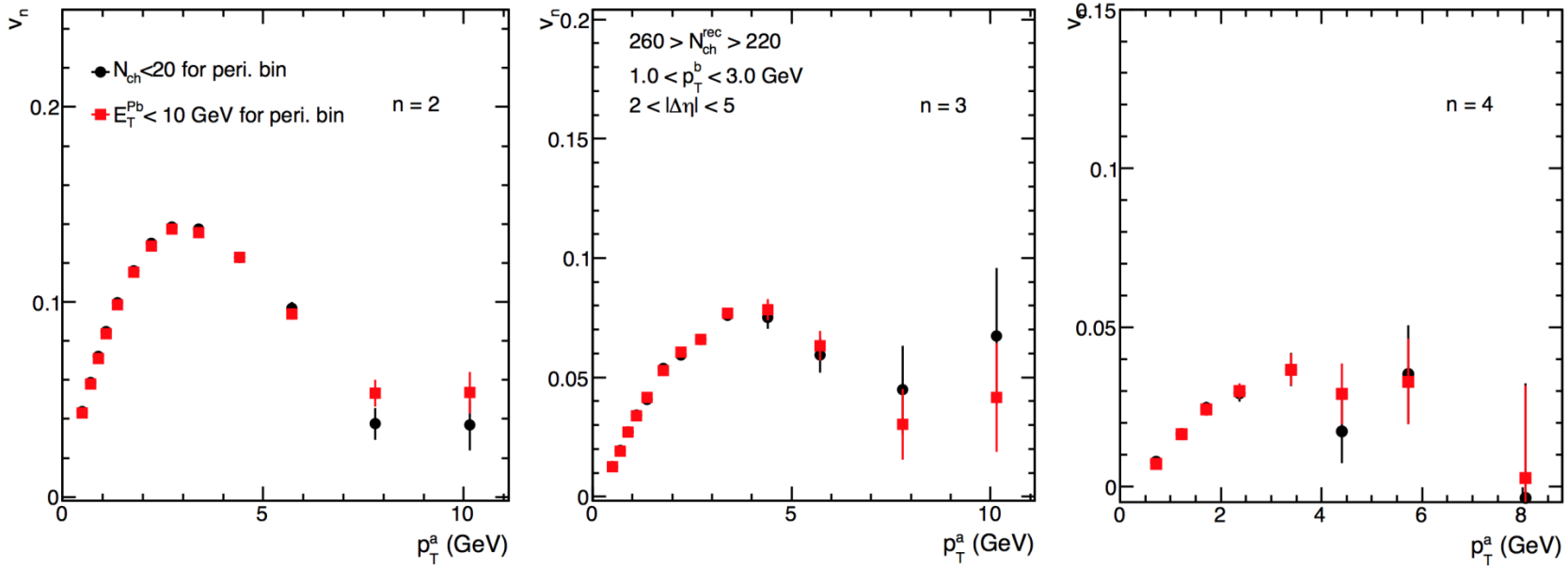
- Short range contribution is minimal beyond $\Delta\eta > 2$
- Subtraction removes most of the short range correlations also.
- Weak $\Delta\eta$ dependence for $v_{n,n}$

Recoil subtraction and v_n



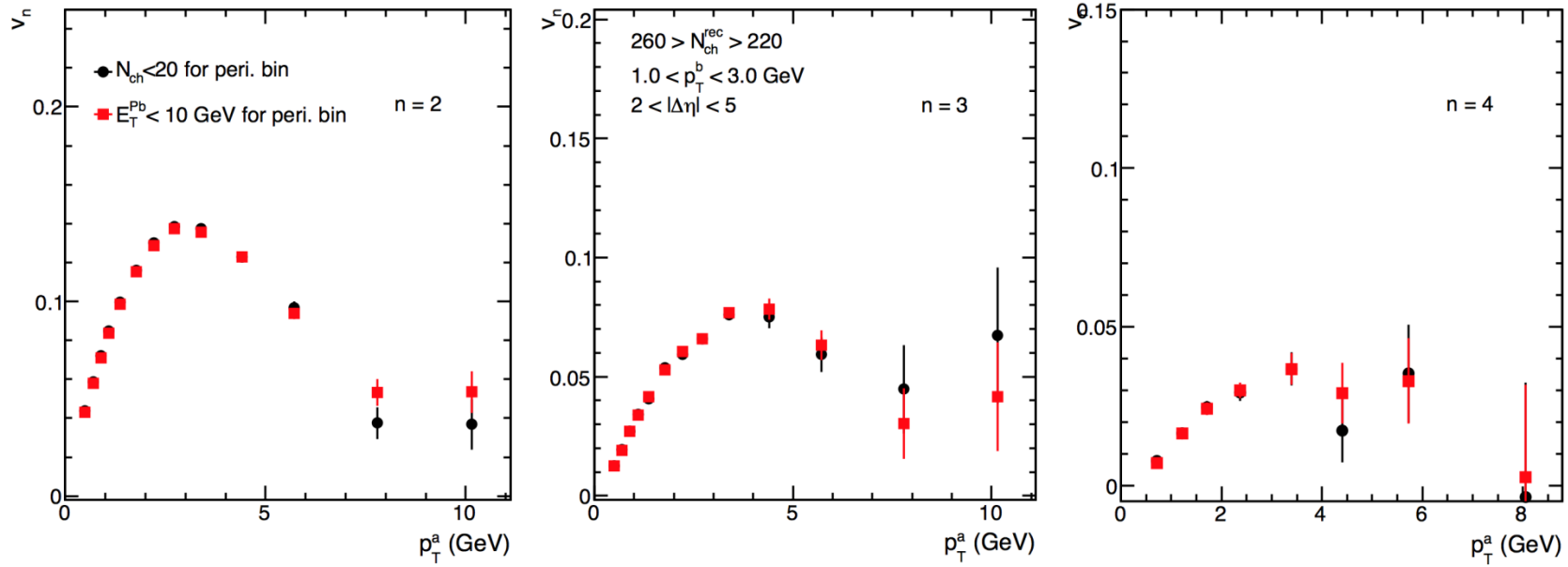
- Away side jet contribution cause even harmonics to increase and odd harmonics to decrease
- Differences exist also at low p_T
- Subtracted values show similar p_T dependence as those in heavy ion collisions.

Choice of peripheral bin

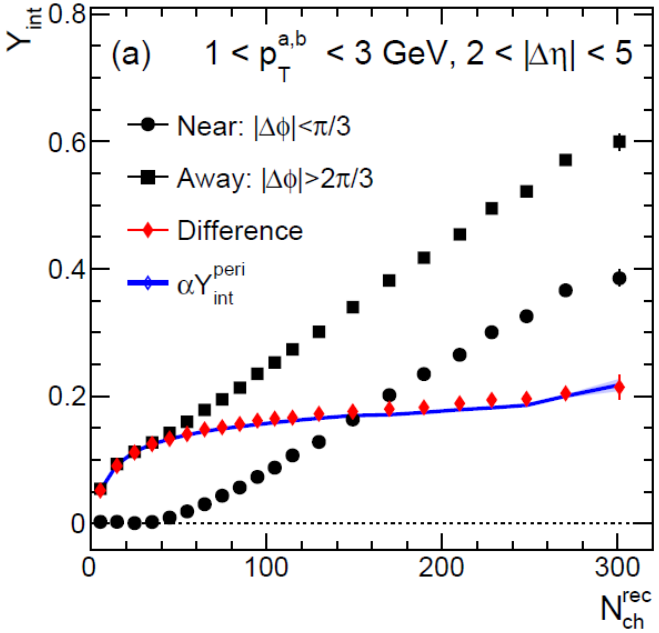


- Performance is similar across the $p_T{}^a$ range if $N_{ch} < 20$ is used as the peripheral bin, instead of $E_T{}^{Pb} < 10$ GeV.

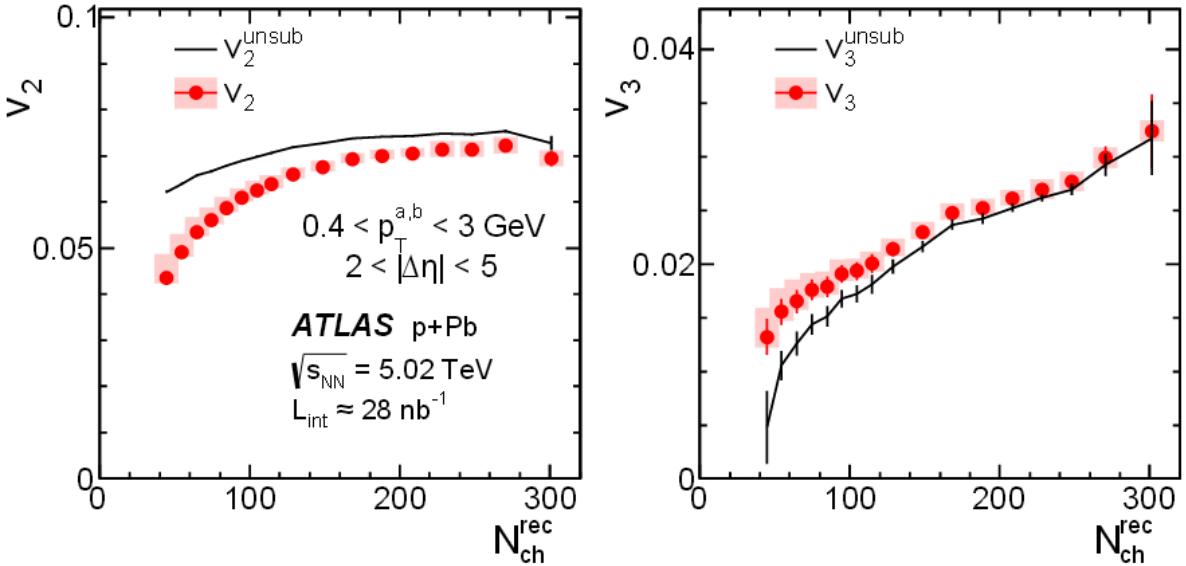
Choice of peripheral bin

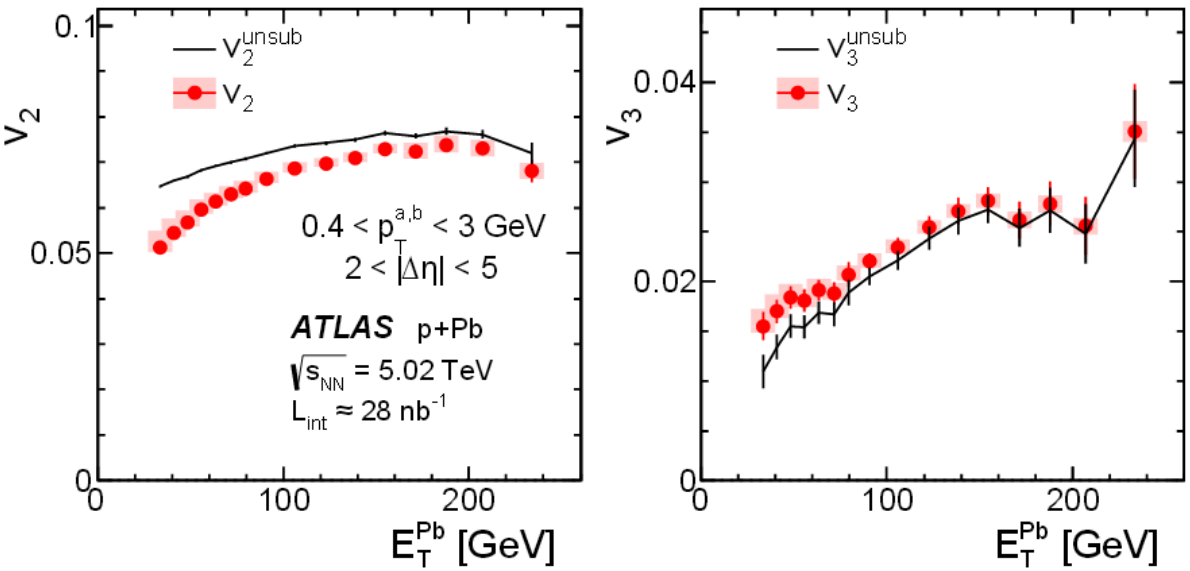


- Performance is similar across the ${p_T}^a$ range if $N_{ch} < 20$ is used as the peripheral bin, instead of ${E_T}^{Pb} < 10$ GeV.
- Expected since recoil estimated using $E_T < 10$ GeV, reproduce the away side jet yield in N_{ch} classes



Centrality bias in vn measurement

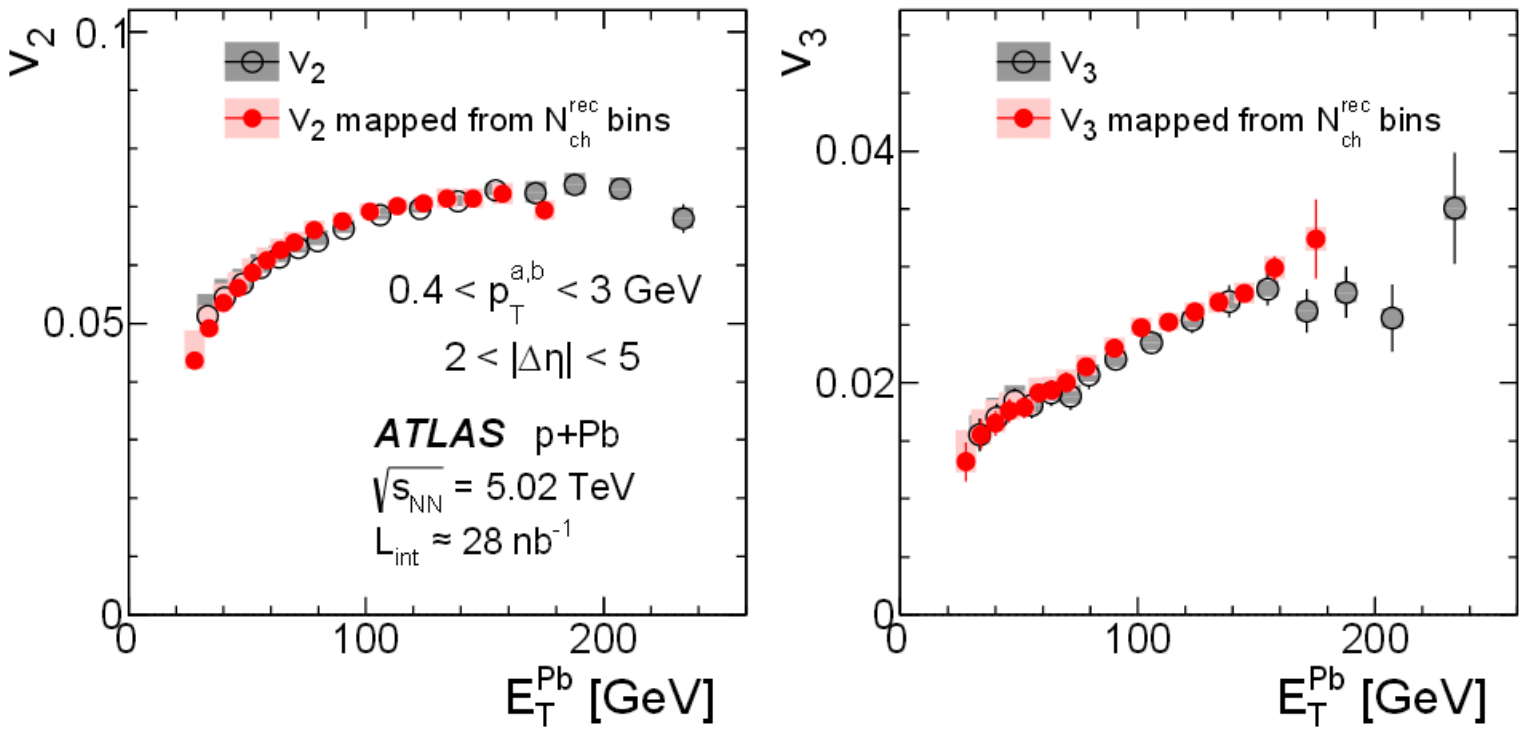




- How do the N_{ch} and $E_T{}^{Pb}$ dependence of v_n compare with each other?

- Mean $E_T{}^{Pb}$ in an N_{ch} class is not biased by autocorrelations
- But v_n can be different from difference in jet contribution in $E_T{}^{Pb}$ and N_{ch} classes.

Centrality bias in vn measurement

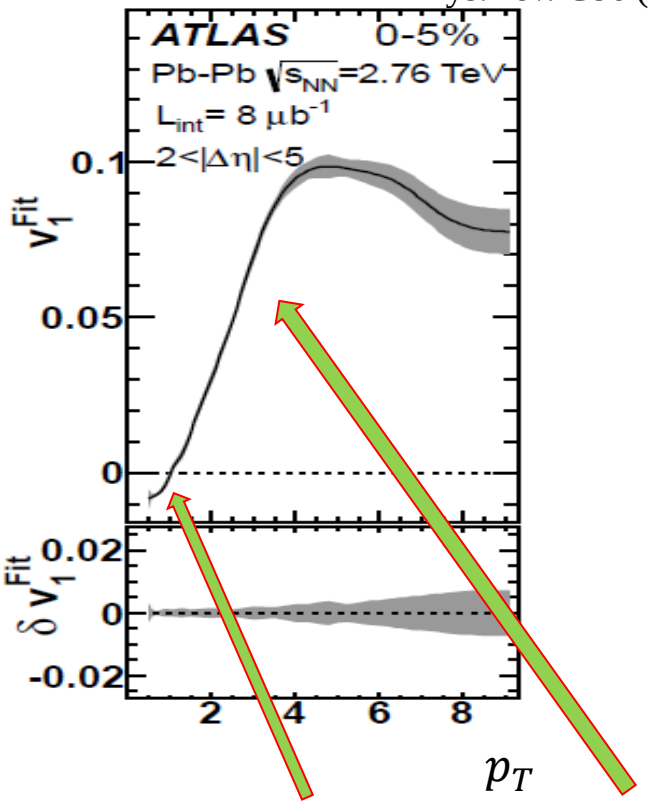


- v_n plot as function of mean E_T in the bin, for events binned in E_T and N_{ch} agree with each other → jet bias to results are small.

Dipolar flow

- In A+A collisions a rapidity even first order flow (v_1) is observed
- Hydrodynamic response to initial dipole asymmetry

Phys.Rev. C86 (2012) 014907



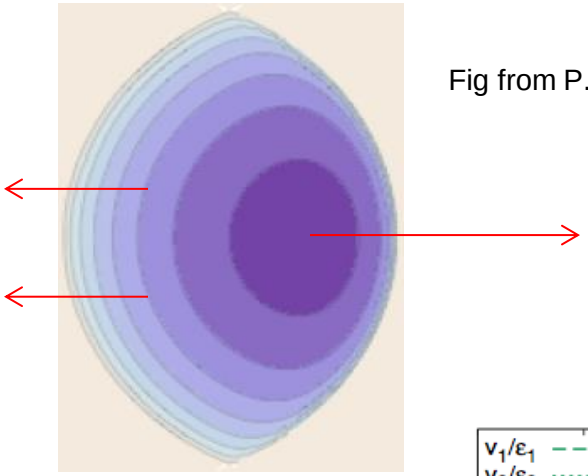
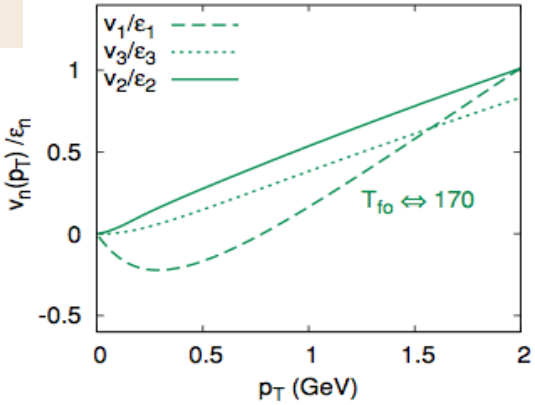


Fig from P. Stankus



- -ve at low p_T and +ve at higher p_T

Dipolar flow

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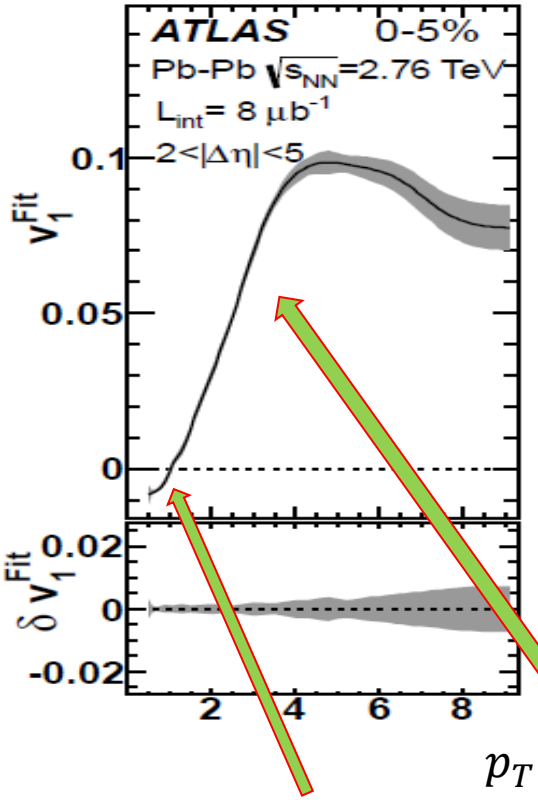
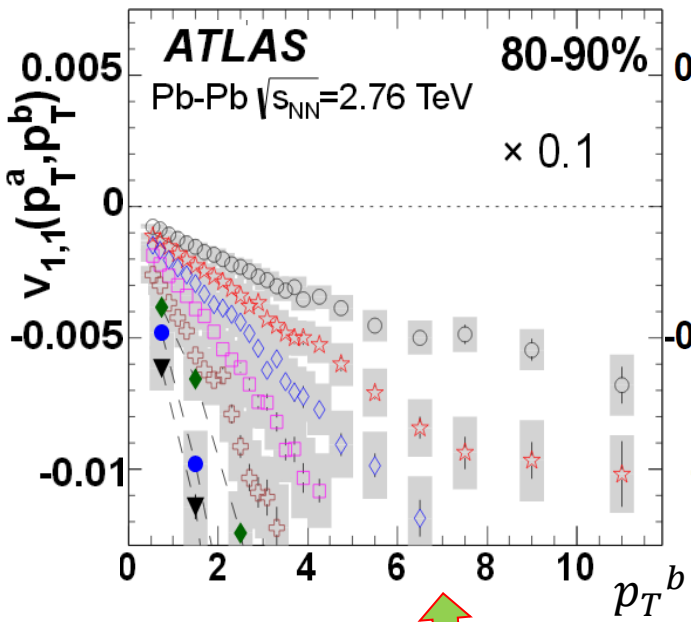
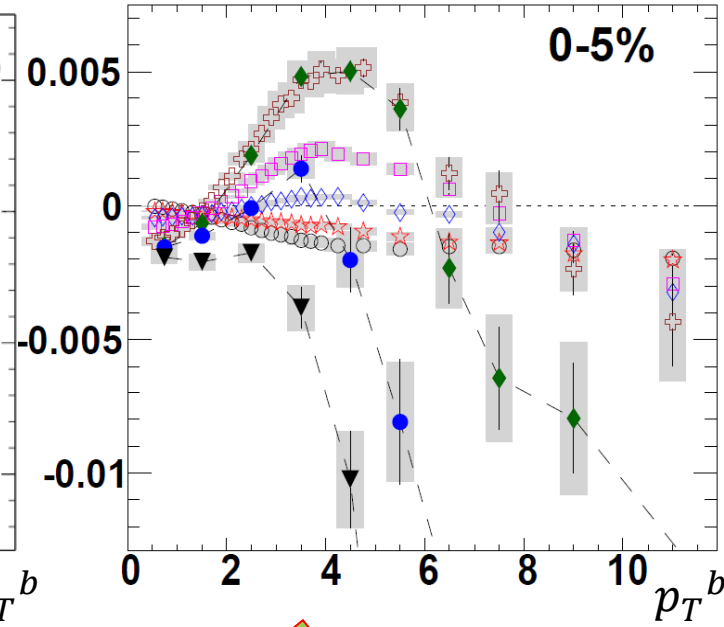
Phys.Rev. C86 (2012) 014907







Mom. conservation

Mom. Conservation + flow

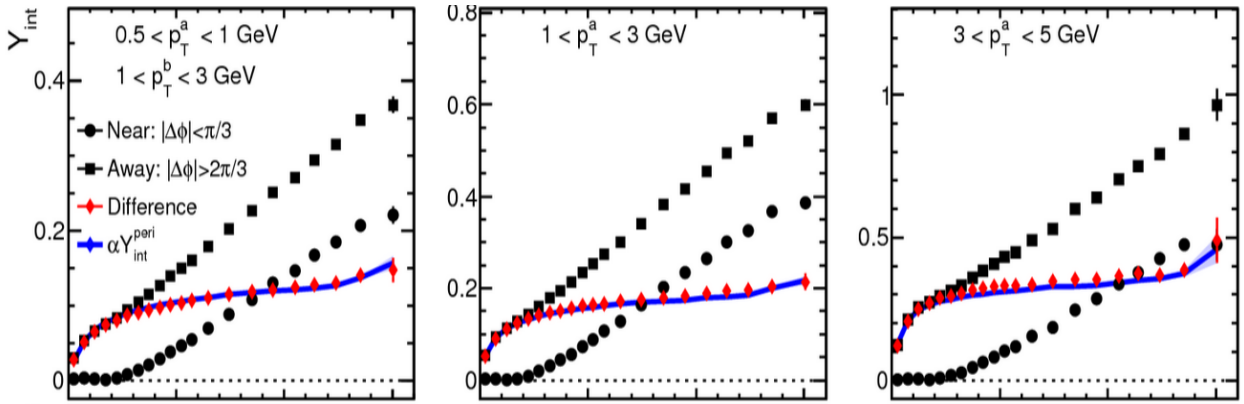
- -ve at low p_T and +ve at higher p_T

- Also has a momentum conservation component at 2PC

$$v_{1,1}(p_T{}^a, p_T{}^b) = v_1(p_T{}^a)v_1(p_T{}^b) - Kp_T{}^a p_T{}^b$$

Dipolar flow in p+Pb

- A similar component can be expected in p+Pb.
- Should show up in yield measurements. $v_{1,1}(p_T{}^a, p_T{}^b) = v_1(p_T{}^a)v_1(p_T{}^b) - Kp_T{}^a p_T{}^b$



- If $v_1(p_T{}^a)$ or $v_1(p_T{}^b)$ = 0, no difference between $\alpha Y_{int}{}^{A,peri}$ and $Y_{int}{}^{diff}$.

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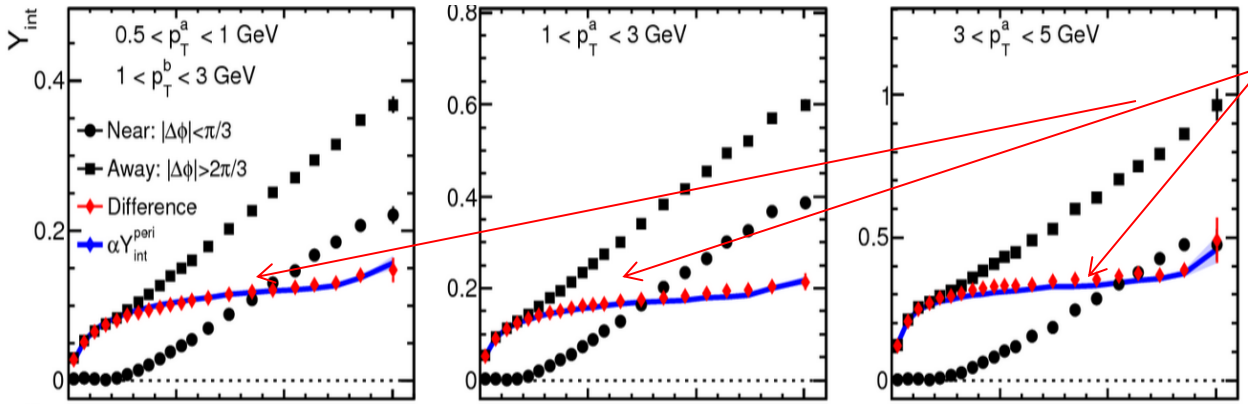

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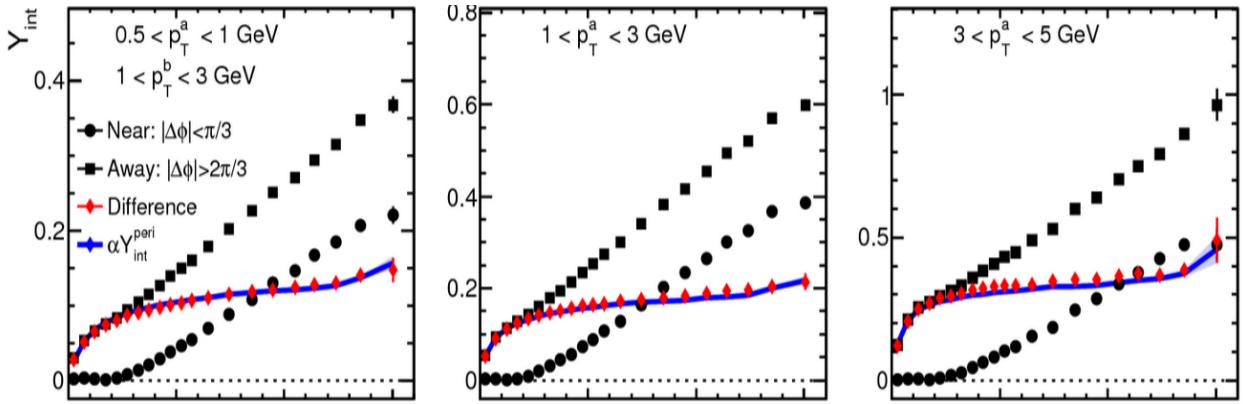

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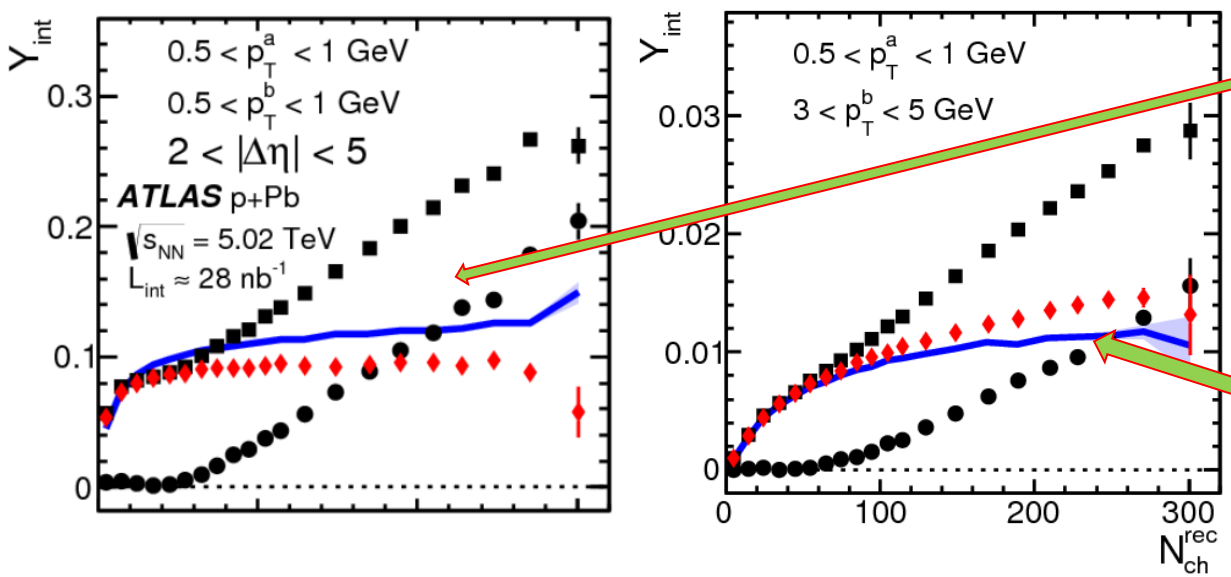
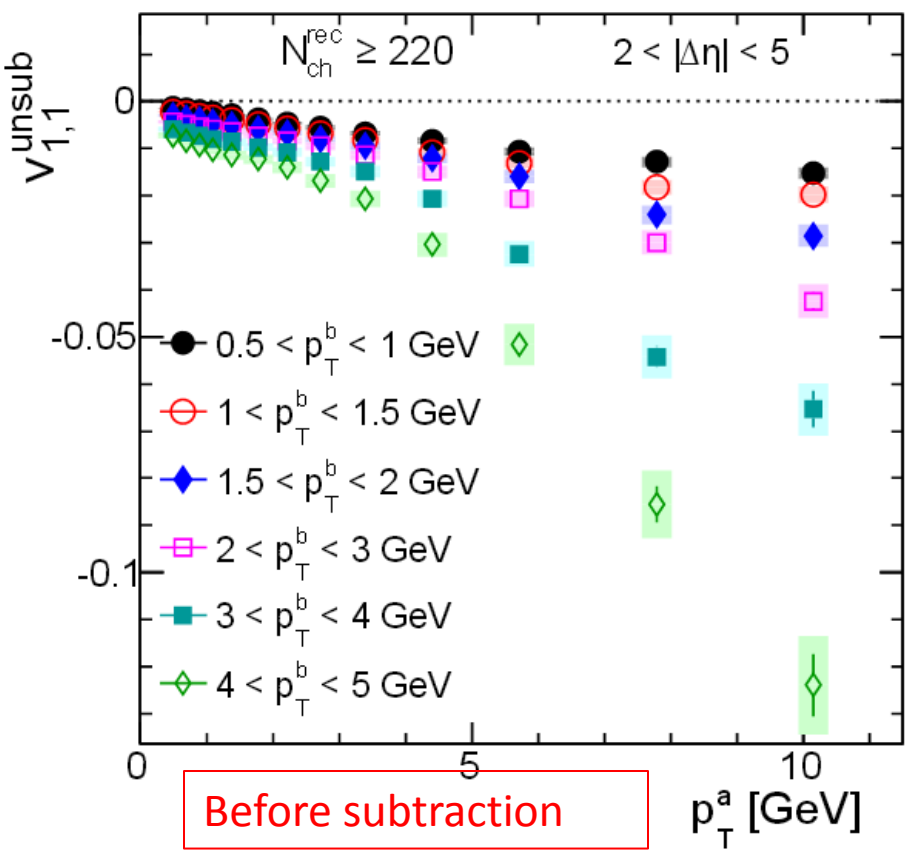
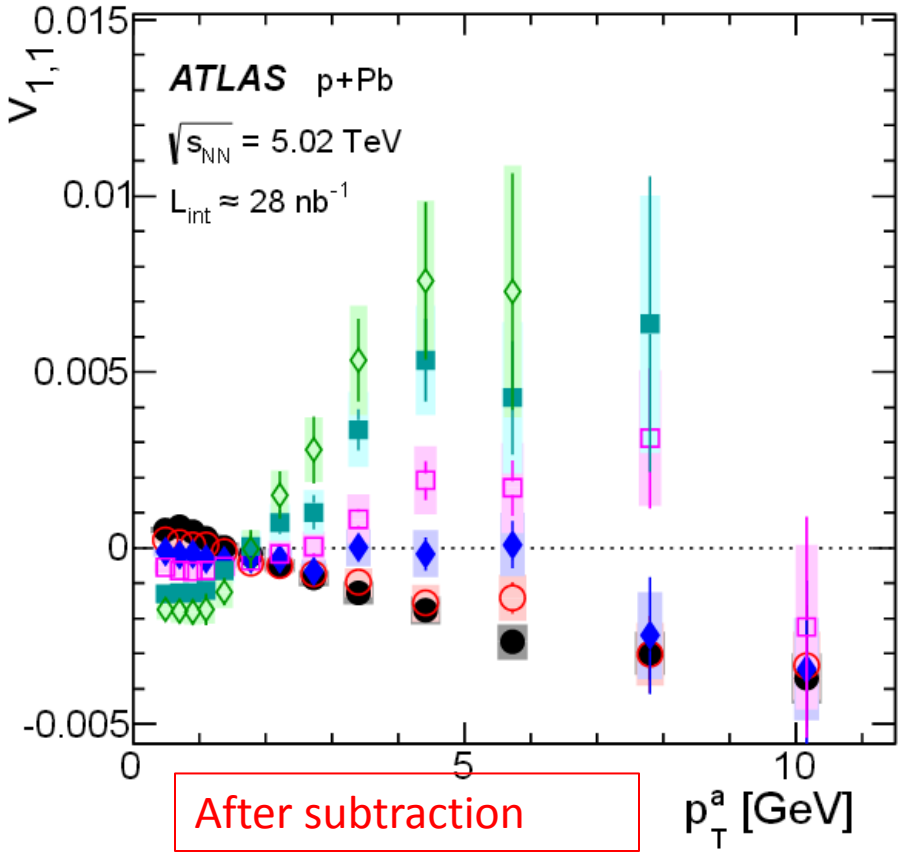


- Posistive difference if both particles at low (high) p_T
- Negative difference if one at low (high) and other at high (low) p_T

$v_{1,1}$ in p+Pb

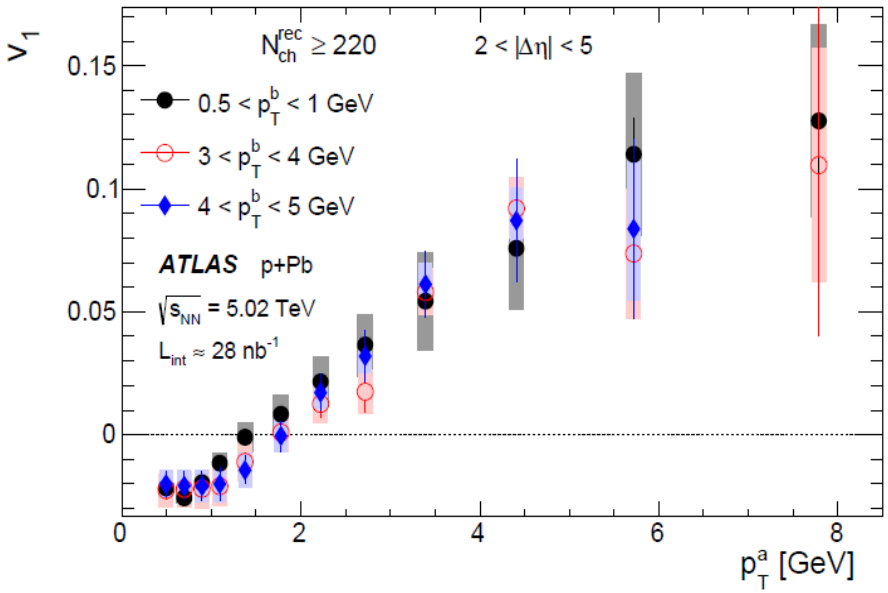
- Can be seen more clearly on the Fourier harmonics.

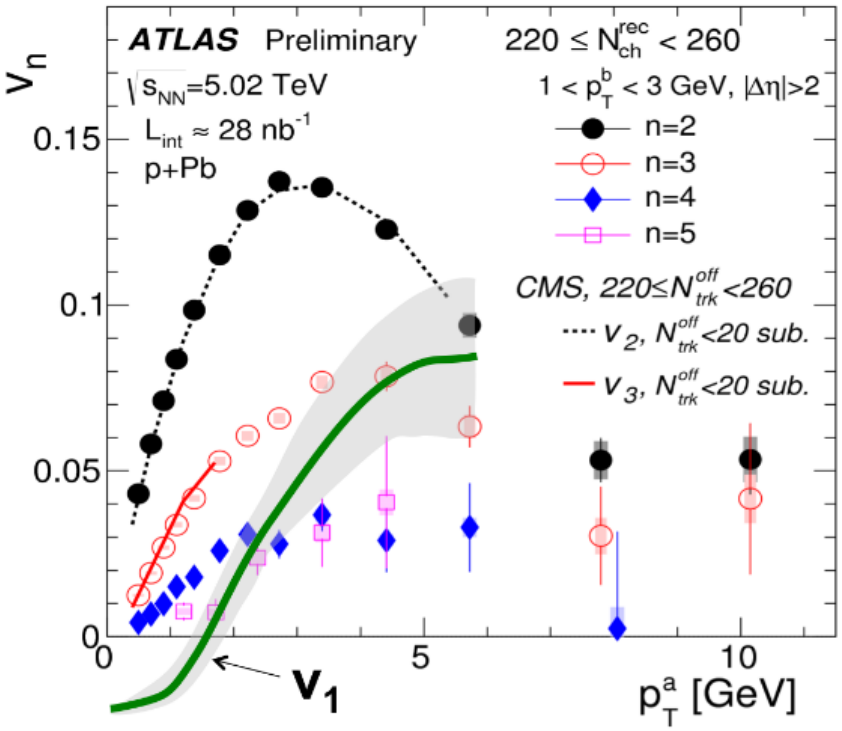




- Before recoil subtraction show mostly a linear dependence on $p_T{}^a$.
- After recoil subtraction, a characteristic $p_T{}^{a,b}$ dependence
  - Similar to that seen in central Pb+Pb collisions

v_1 in p + Pb

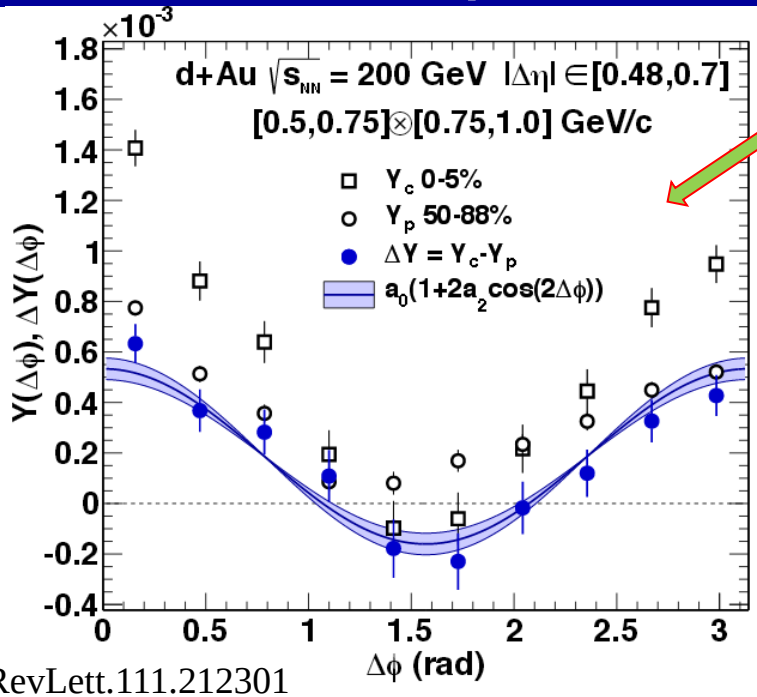




$$v_1(p_T) \equiv \frac{v_{1,1}(p_T, p_T^{ref})}{v_1(p_T^{ref})}$$

- Shows good factorization into single particle v_1
- Similar p_T dependence as in Pb+Pb, crosses zero around 1.5 GeV
- Comparable magnitude as v_3

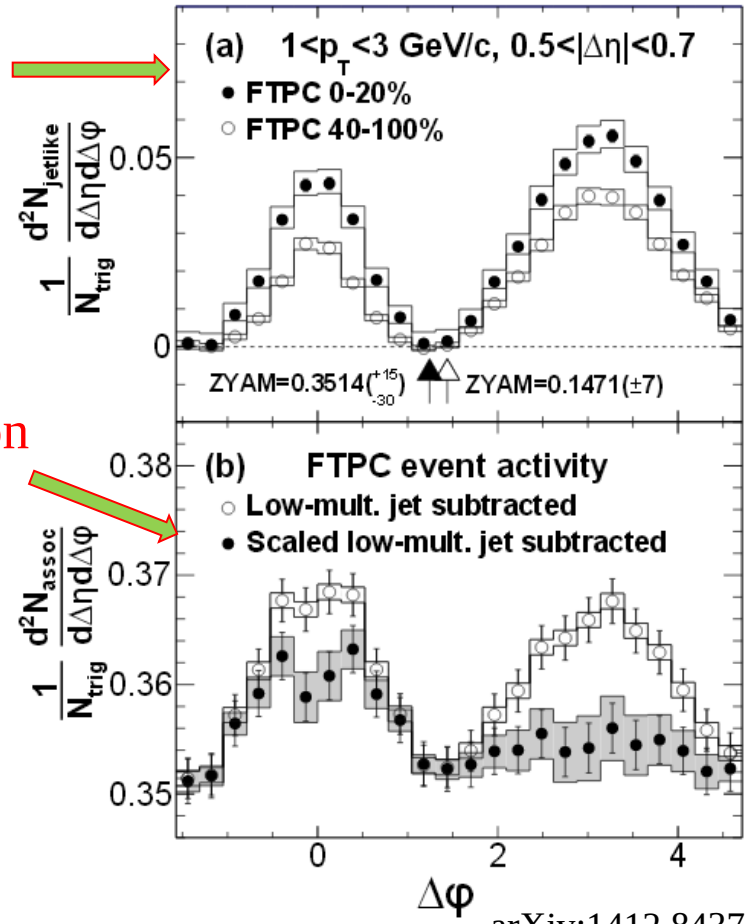
Comparison with RHIC results



PhysRevLett.111.212301

direct subtraction

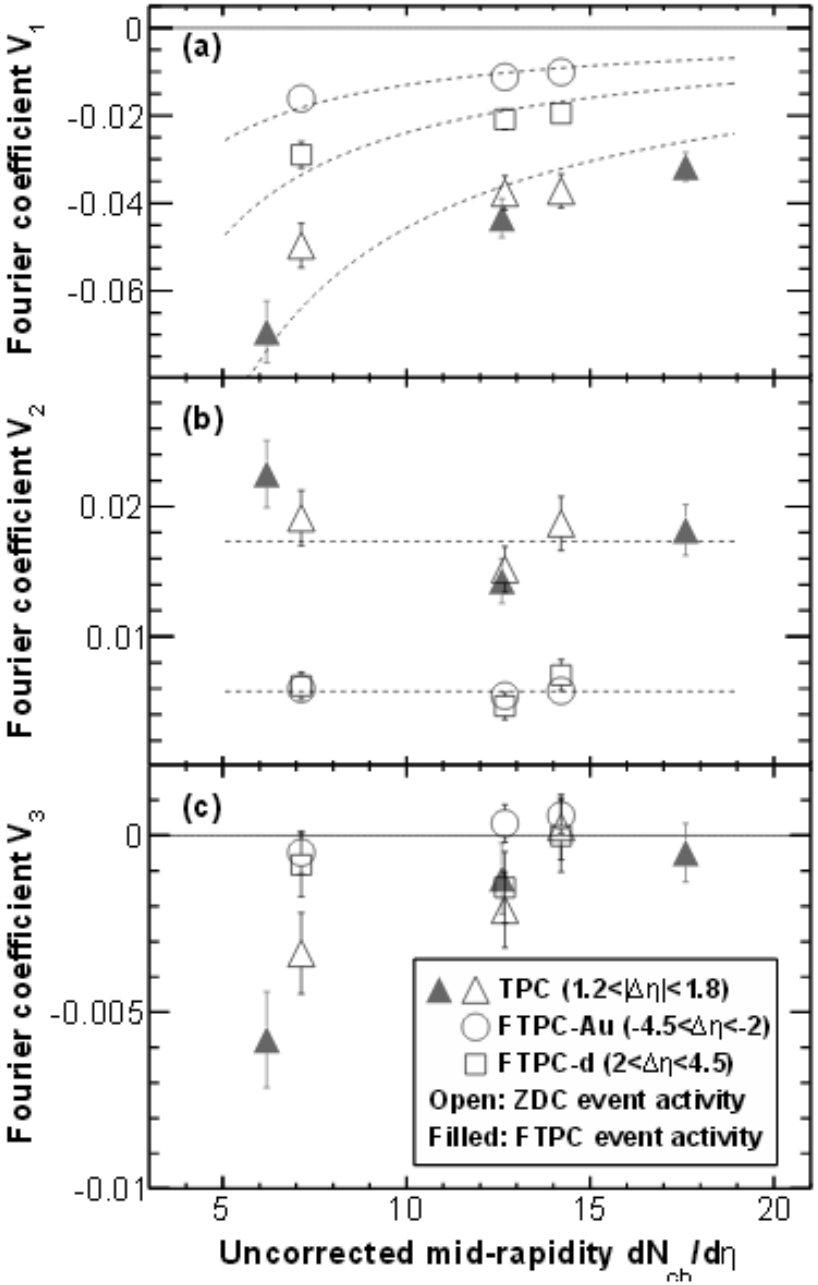
scaled subtraction similar to ATLAS



arXiv:1412.8437

- Double ridge in PHENIX and single ridge at STAR?
  - Multiplicty is smaller at RHIC → recoil component is larger compared to signal
  - Uncertainties in α?
  - STAR uses 40-100% as peripheral, subtracting some flow signal as well?

Comparison with RHIC results



- v_n measured without recoil subtraction (STAR).
- v_1 dominated by momentum conservation component (~1/N).
- v_2 could be biased by non-flow contributions, larger effect at lower multiplicity
- Need some non flow subtraction or multi-particle correlation measurements to make meaningful statements on the (low) multiplicity dependence

arXiv:1502.07652

Summary and Conclusions

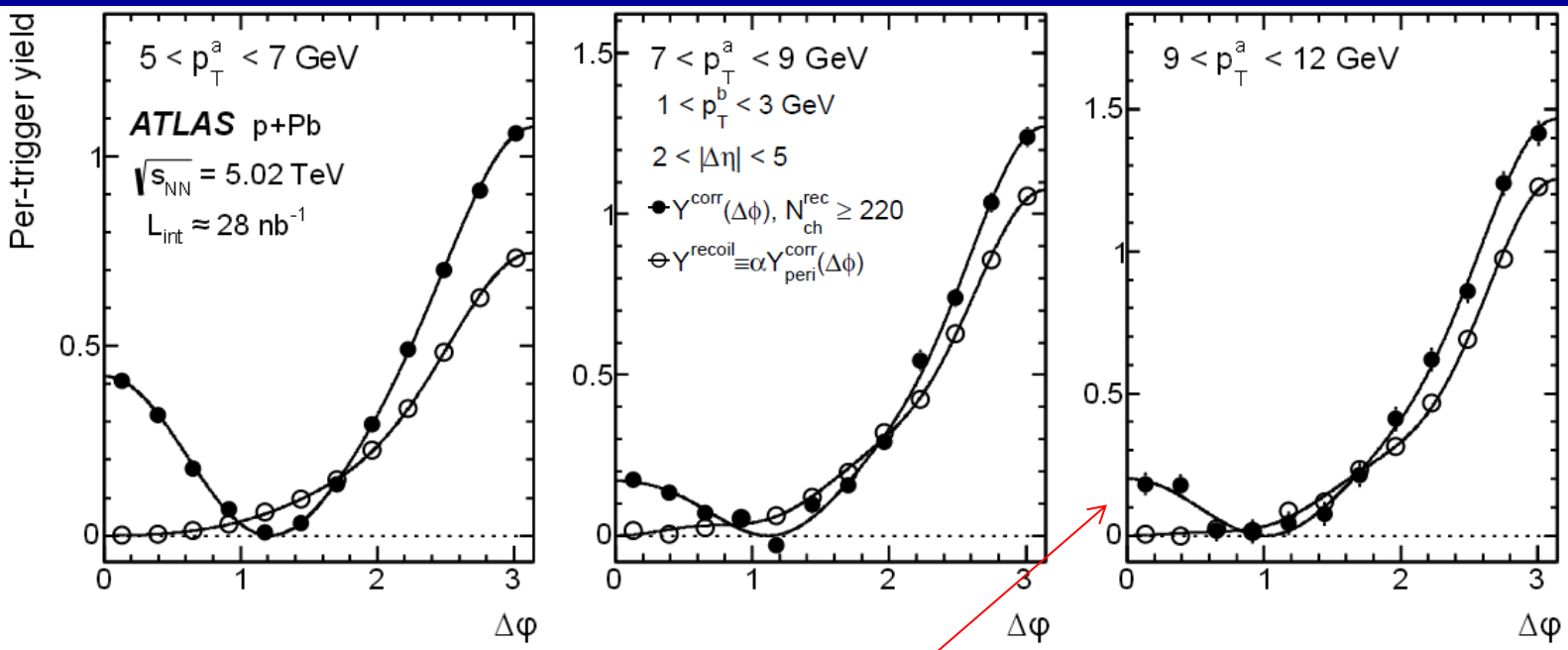
- 2PC in p+Pb have larger contribution (than A+A) from near and away side jets
- Both can be estimated and subtracted out using the recoil subtraction procedure.

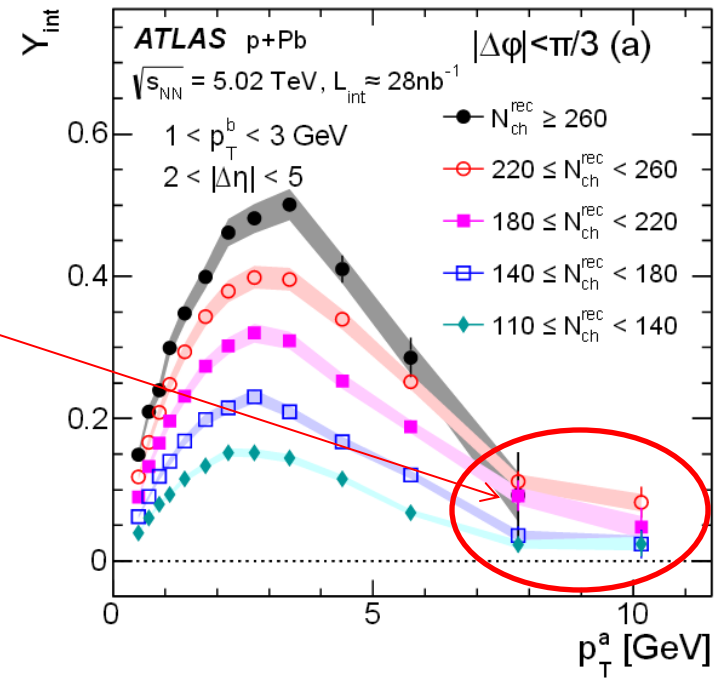
- Results are consistent within a few percent if the peripheral bin used to estimate recoil is changed.
- No residual bias between measurements in ${N_{ch}}^{rec}$ and ${E_T}^{Pb}$ bins
- Recoil subtraction also allows measurement of first order flow, which shows a similar p_T dependence as in Pb+Pb collisions.

- Results from RHIC show large difference between scaled and unscaled subtractions → smaller multiplicity effects?

BACK UP

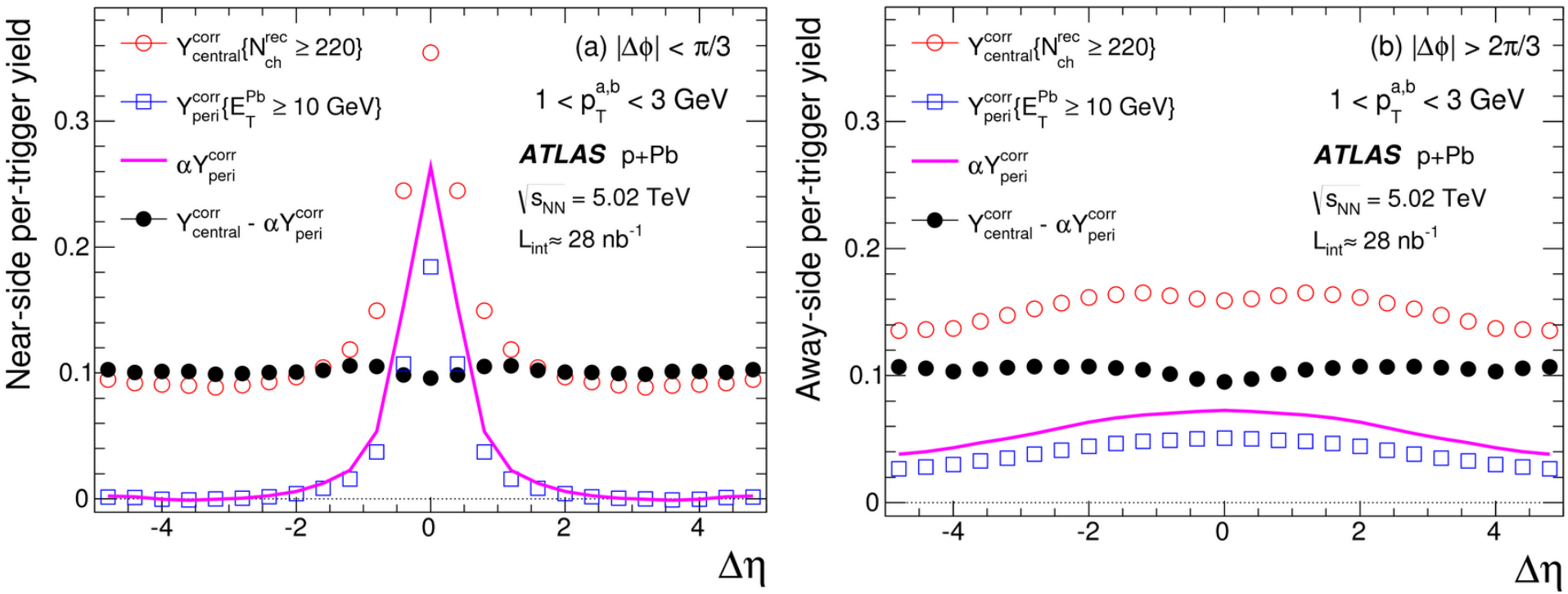
Ridge persists to high p_T





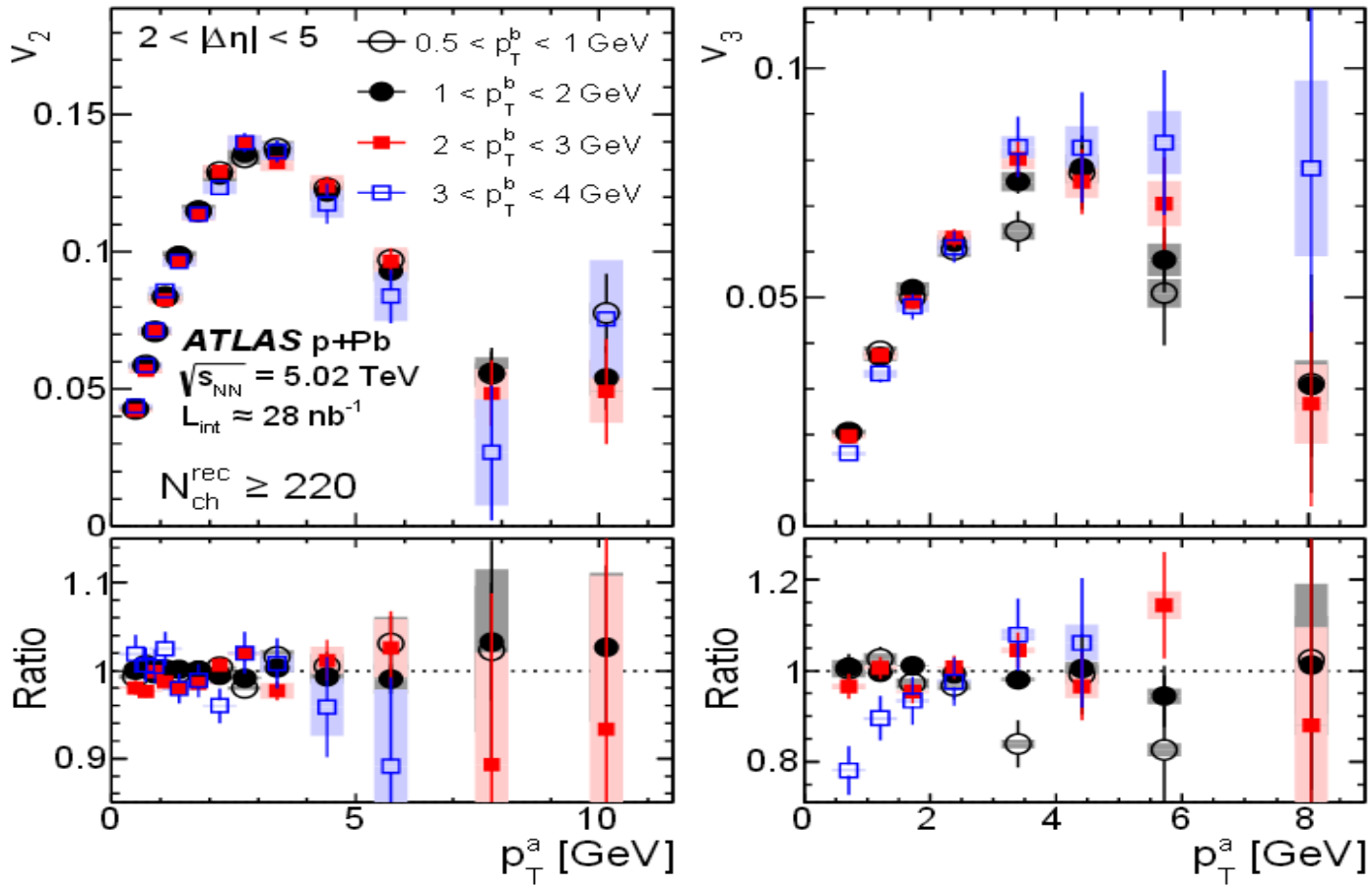
- Near-side ridge visible up to 10 GeV.
- Higher p_T also has larger per trigger jet yields.

Yield in Δη



- Integrated yields in Δη before and after recoil subtraction

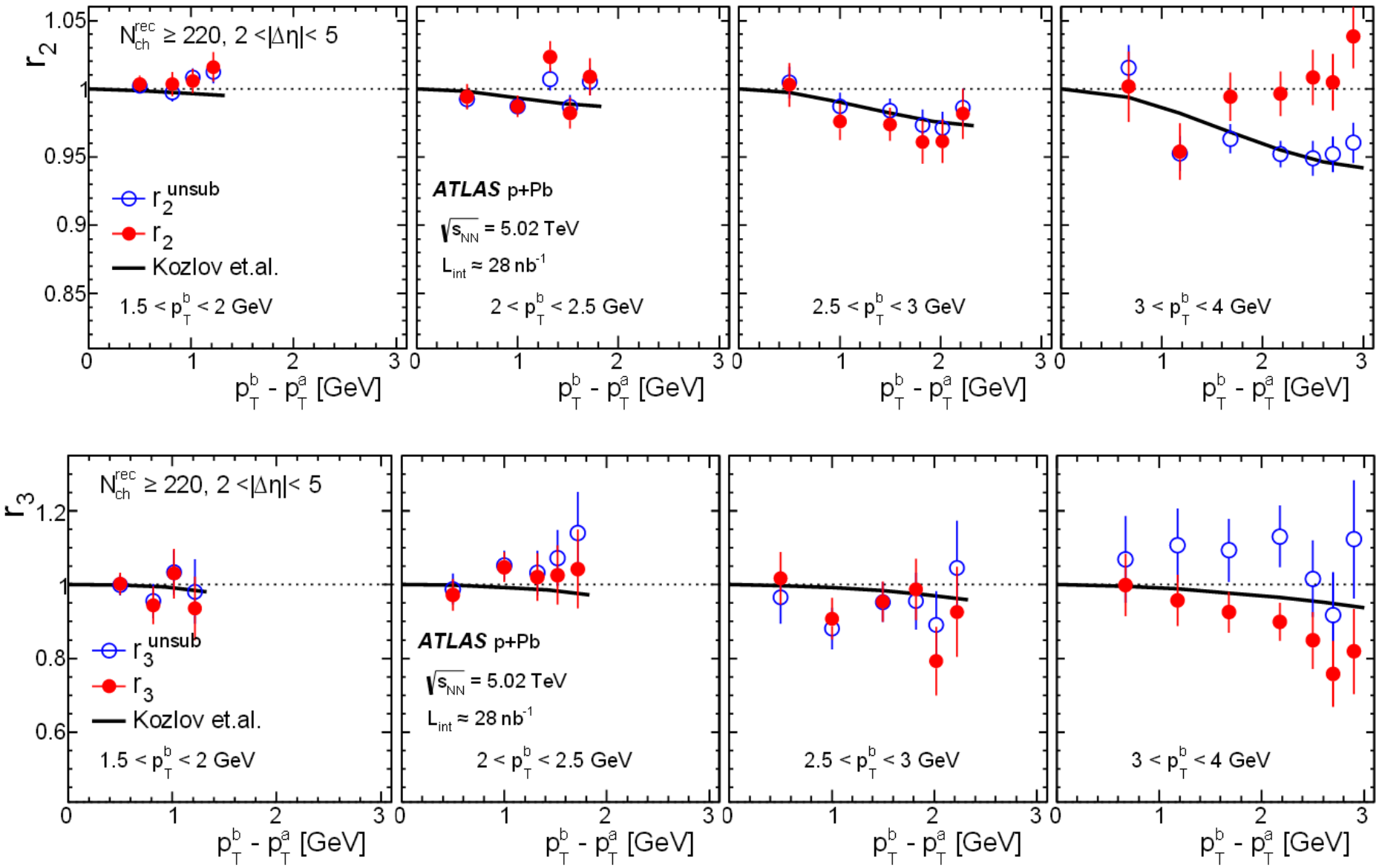
v_n factorization



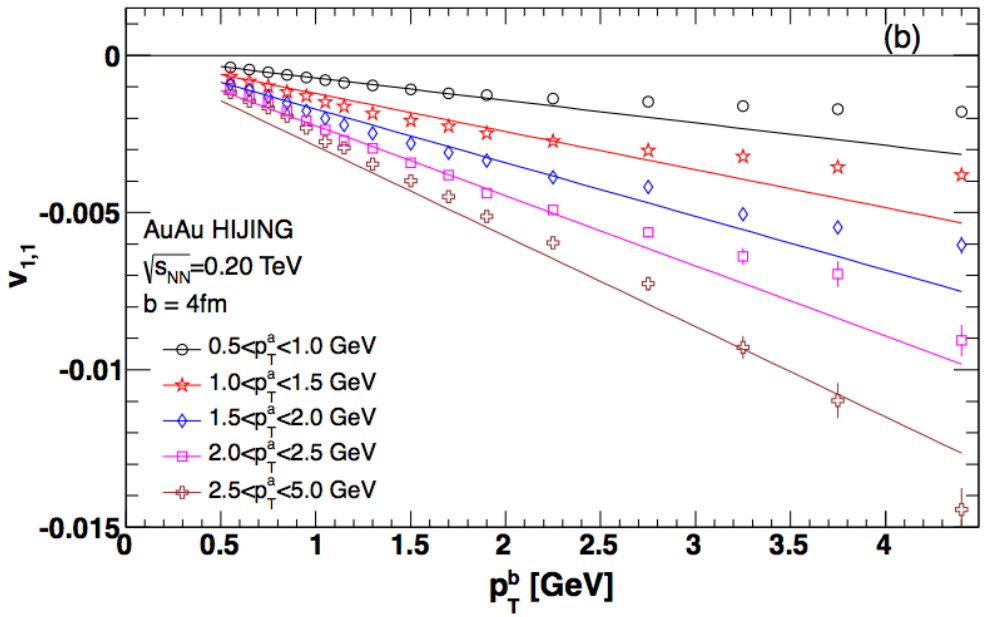
$$v_n(p_T{}^a) = \frac{v_{n,n}\,(p_T{}^a, p_T{}^b)}{\sqrt{v_{n,n}\,(p_T{}^b, p_T{}^b)}}$$

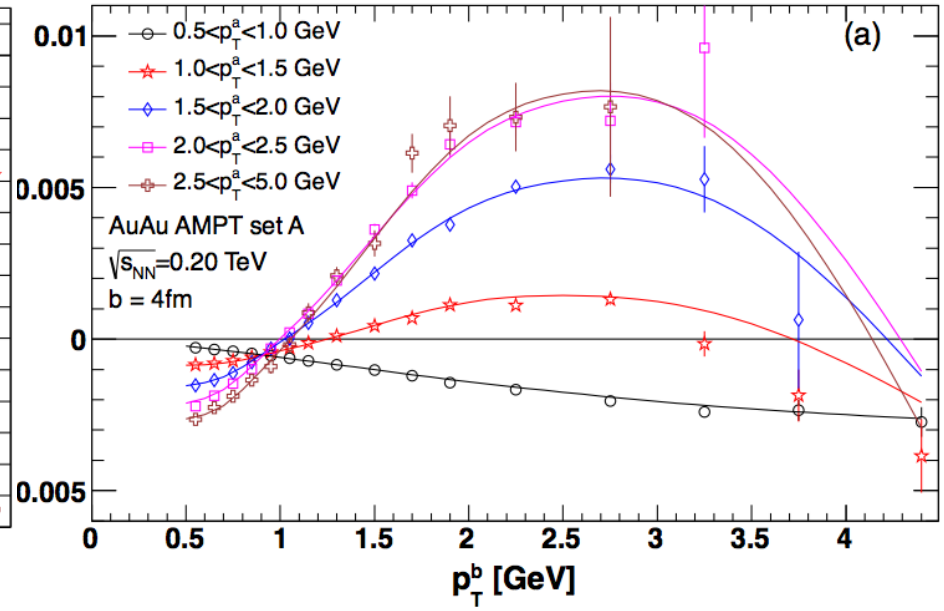
- p_T dependence of v_n consistent within few percent for different choice of $p_T{}^b$ for $p_T{}^b < 3$ GeV.

Factorization



$v_{1,1}$ in HIJING and AMPT





$$v_{1,1}(p_T^a, p_T^b) = -c p_T^a p_T^b$$

$$v_{1,1}(p_T^a, p_T^b) = v_1^{Fit}(p_T^a) v_1^{Fit}(p_T^b) - c p_T^a p_T^b$$

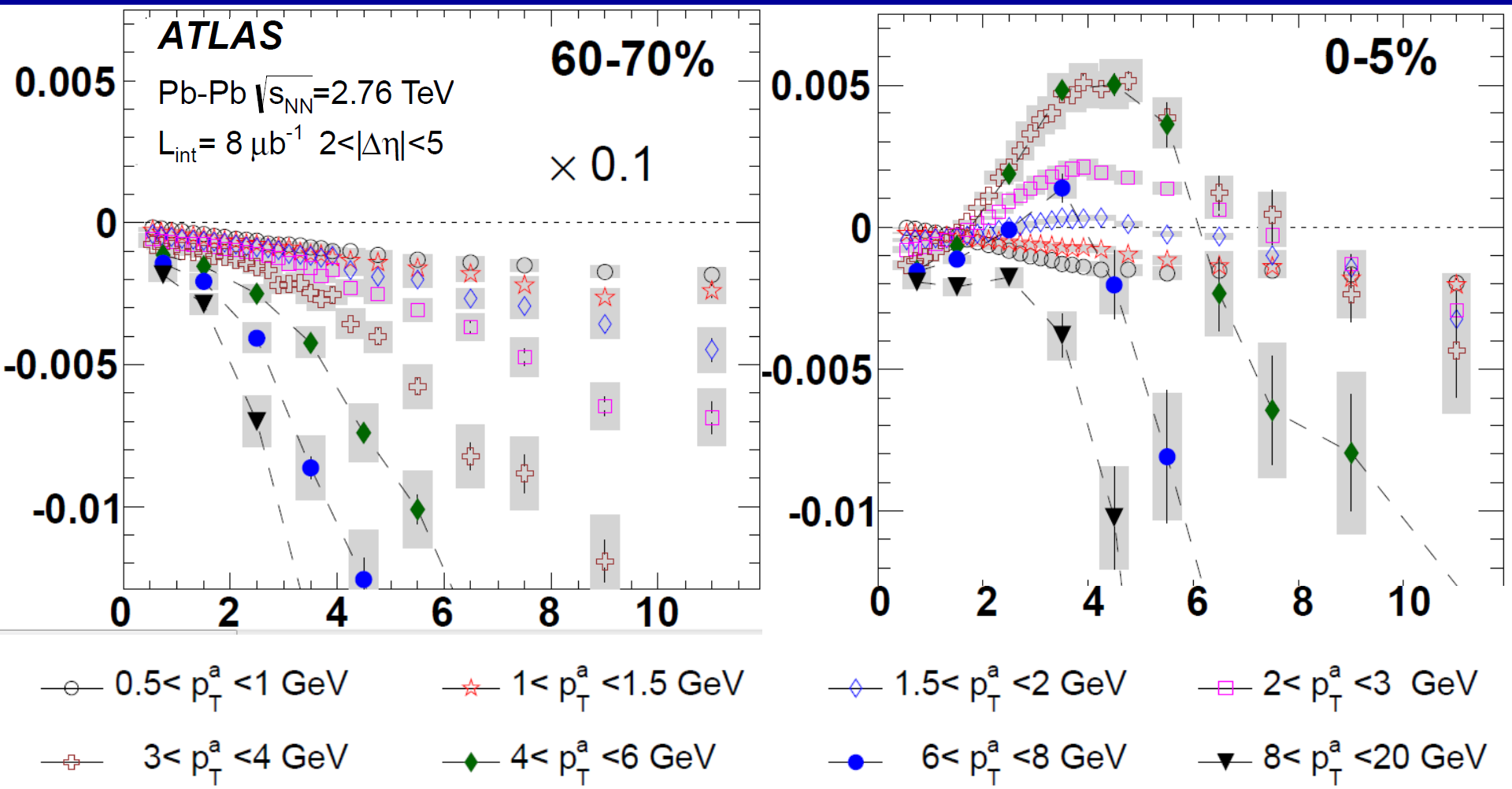
$v_{1,1}$ calculated for pairs with $|\Delta\eta|>1.5$

- AMPT=HIJING +F.S scattering.
  - Interaction strength controlled by α_s and μ.

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha_s^2}{2(t-\mu^2)^2} \qquad \sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$$

- HIJING only need momentum conservation, while AMPT need both
  - The complex p_T dependence of $v_{1,1}$ can naturally be generated from final state interaction

$v_{1,1}$ in peripheral Pb+Pb



- $v_{1,1}({p_T}^a, {p_T}^b)$ in peripheral and central event classes from Pb+Pb collisions.
- In central event class the non flow contributions are suppressed.

Summary of LHC results on small colliding systems

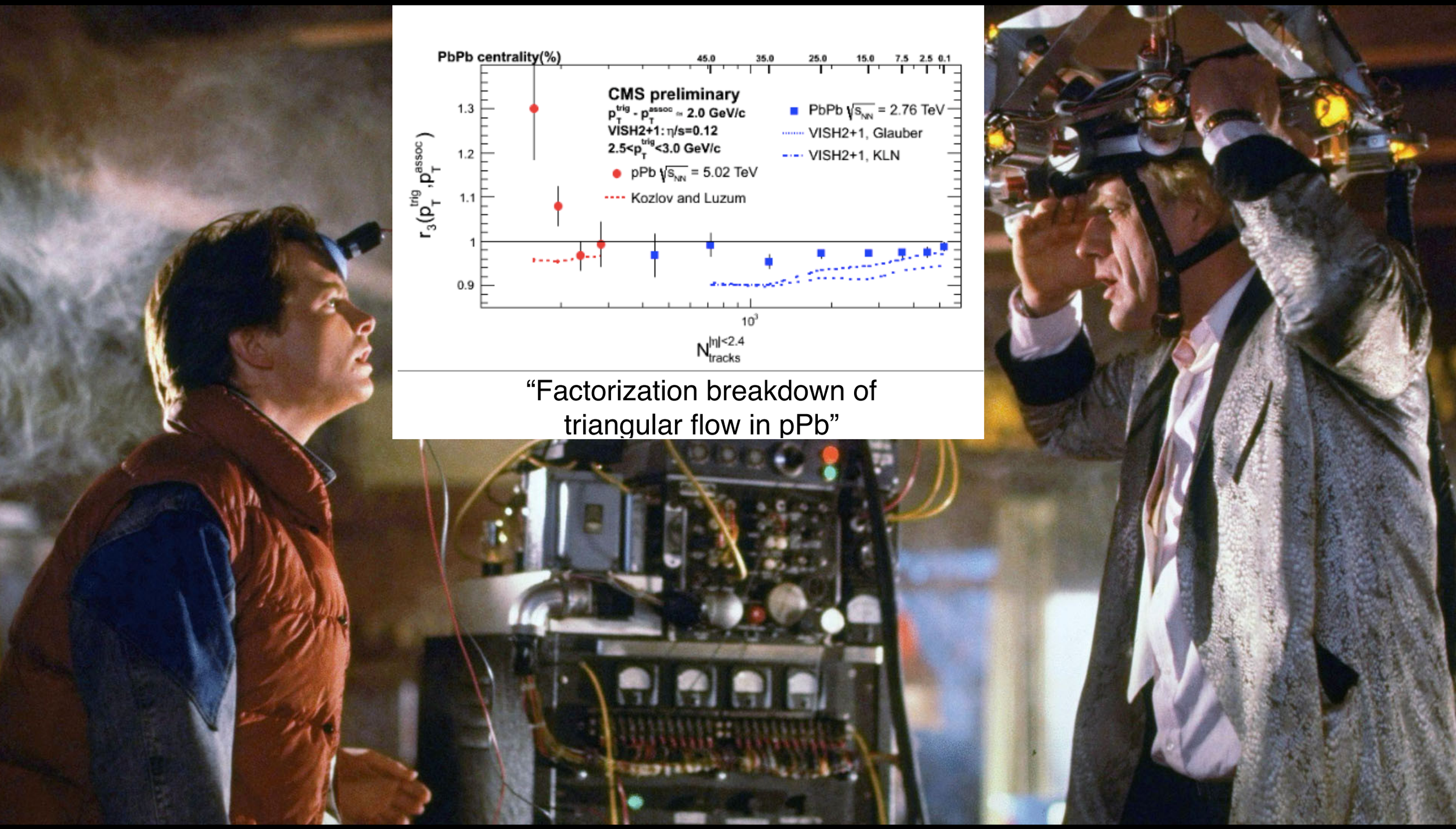
Gunther Roland

BNL, March 4 2015

Collectivity in Small Colliding Systems with High Multiplicity

Back to 2010

"Then tell me, future boy, what are you going to measure in 2015?"



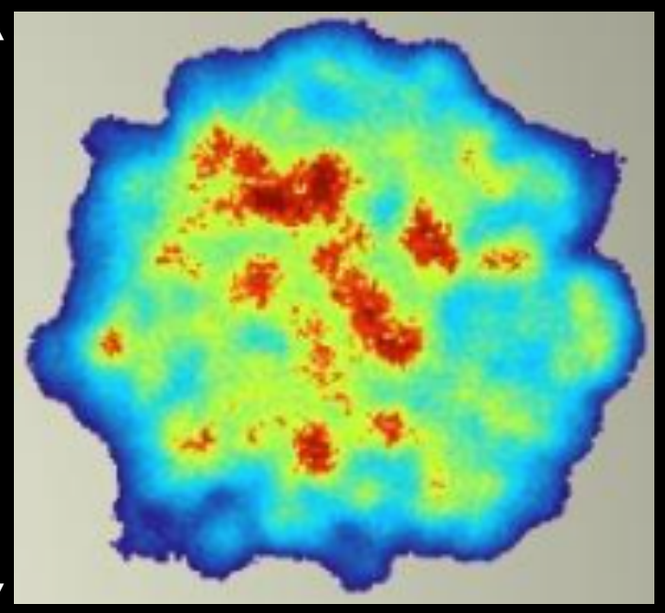
"Factorization breakdown of triangular flow in pPb"

Summary of LHC results on small colliding systems

- Why are we here: "Collectivity" in "small" systems
  - pp and pPb runs were thought to provide "trivial" reference for PbPb physics
  - About 1/4 of LHC "heavy ion" publications are on pPb collisions
  - Vast majority are not "reference" measurements, but studies of "new" physics (or "old" A+A physics) in small systems
- I won't summarize all ~30 pPb papers, but highlight some results of particular interest (to me)

What is “small”?

PbPb

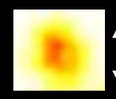


vs

pPb

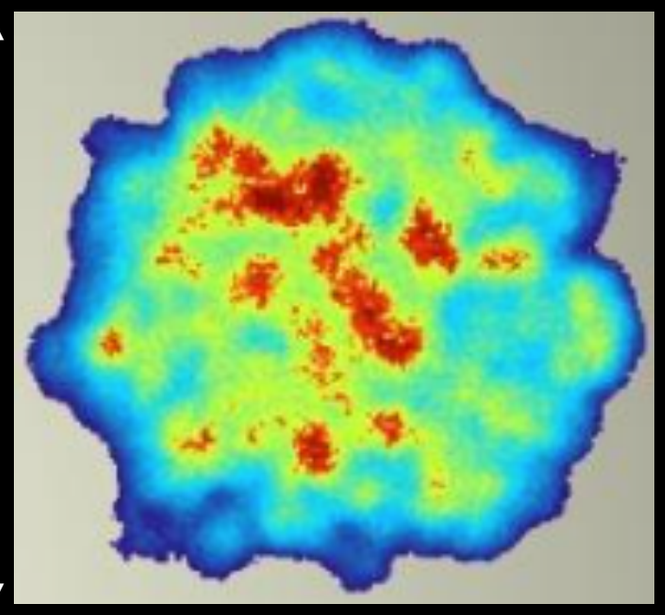
vs

pp



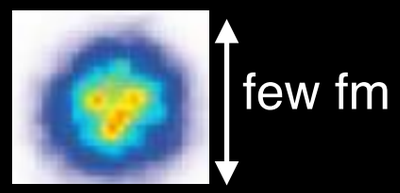
What is “small”?

PbPb



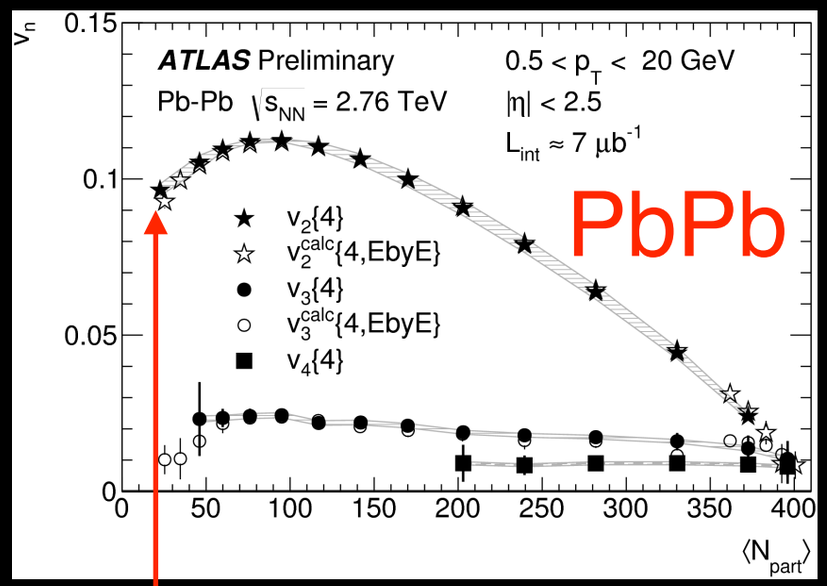
vs

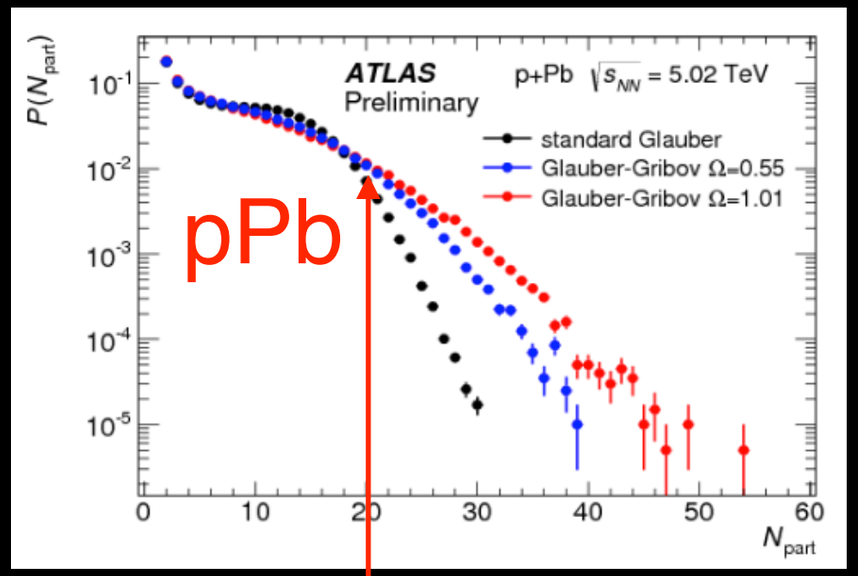
pPb



vs

pp





Large “flow” for $N_{part} \sim 20$

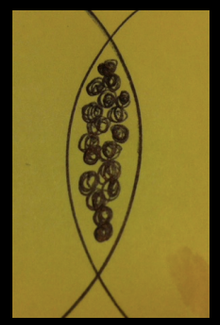
$N_{part} > 20$ reached for few % “central” pPb

$N_{part} = 2$

What is “small”?

PbPb

15fm



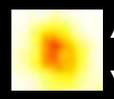
vs

pPb

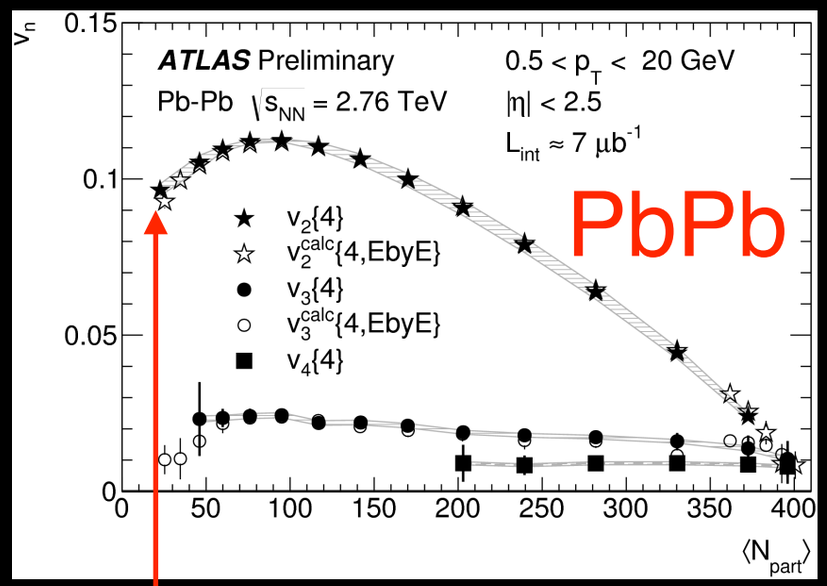
few fm

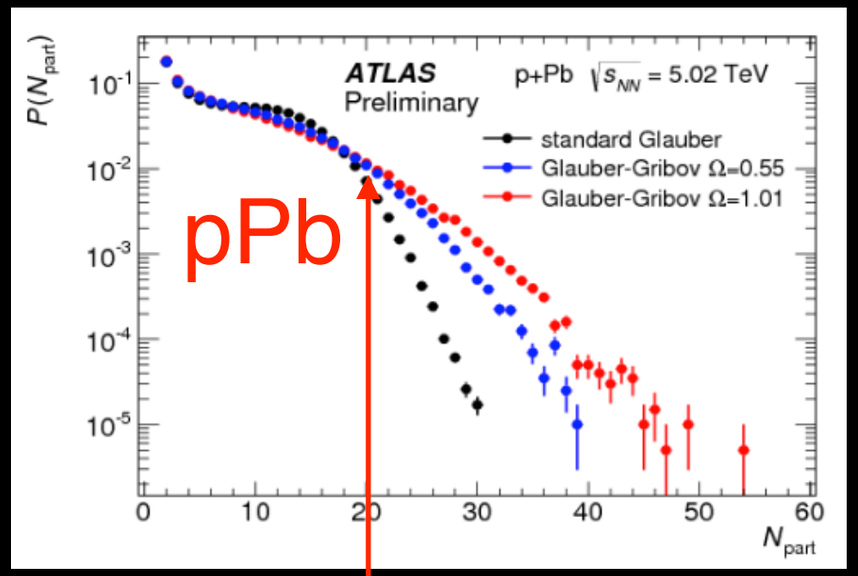
vs

pp



1 fm





Large “flow” for $N_{part} \sim 20$

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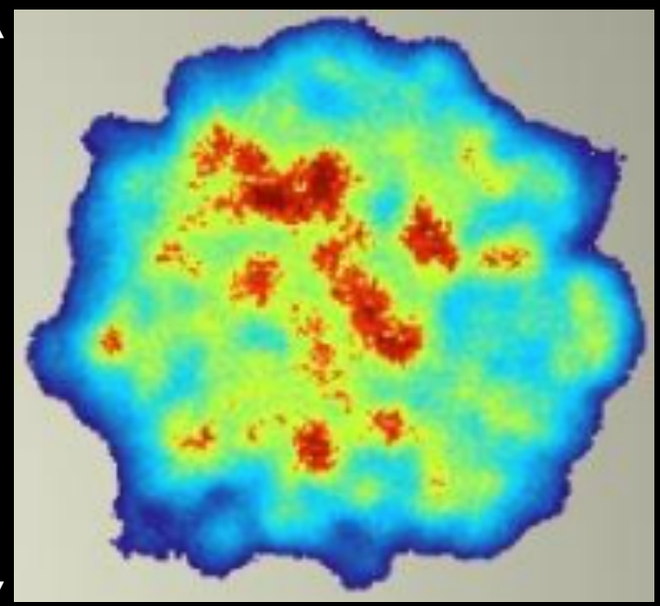
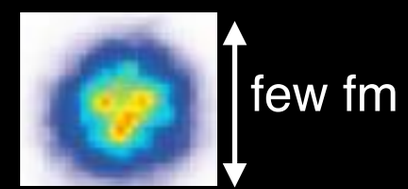
What is "small"?

PbPb



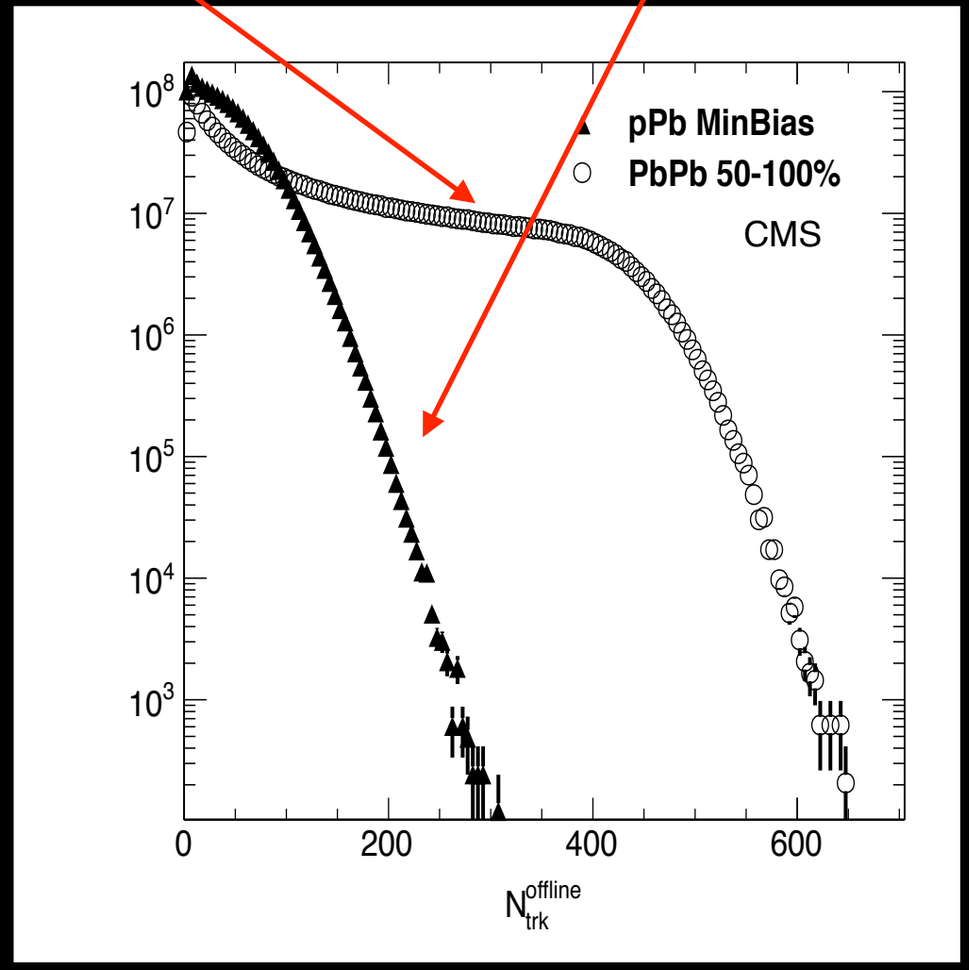
vs

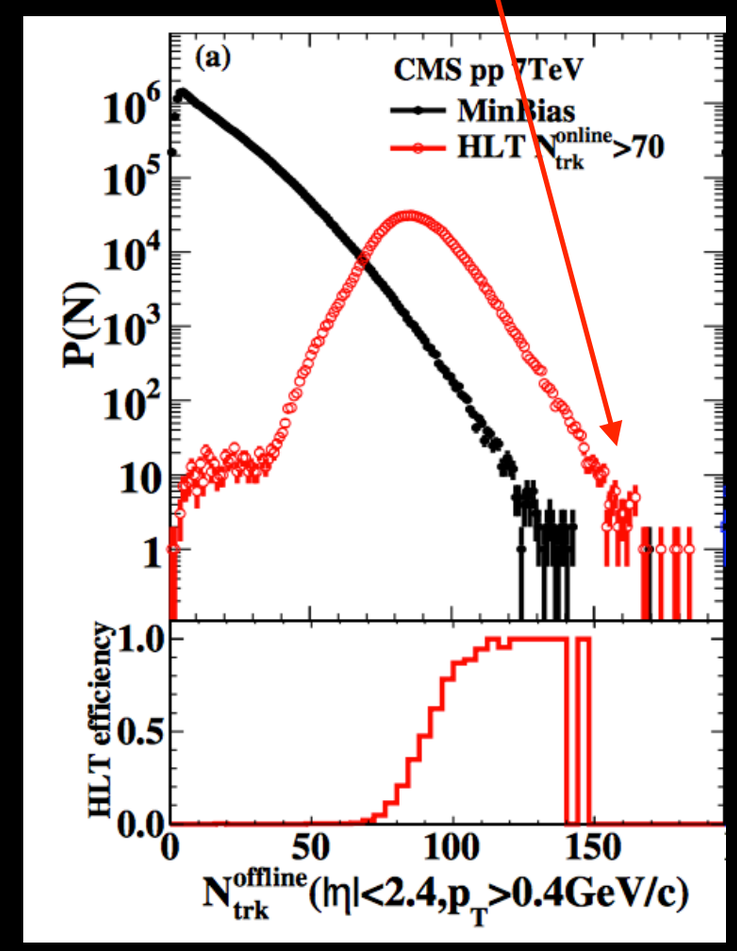
pPb



vs

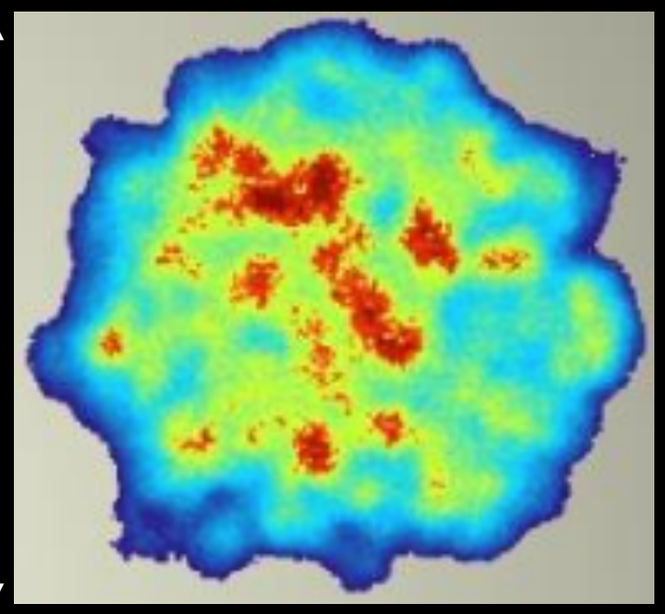
pp





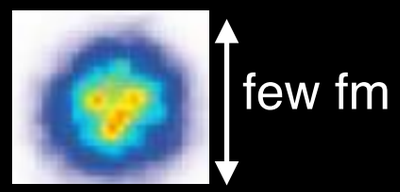
What is "small"?

PbPb



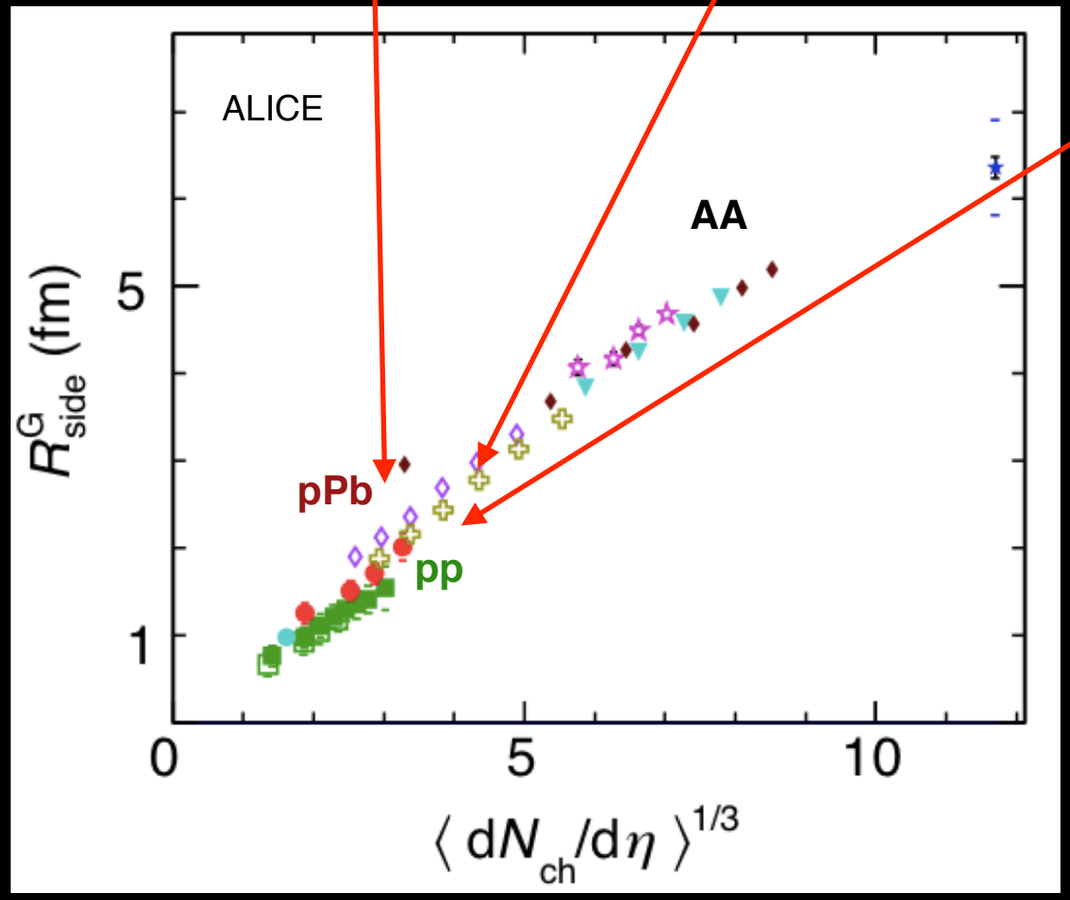
vs

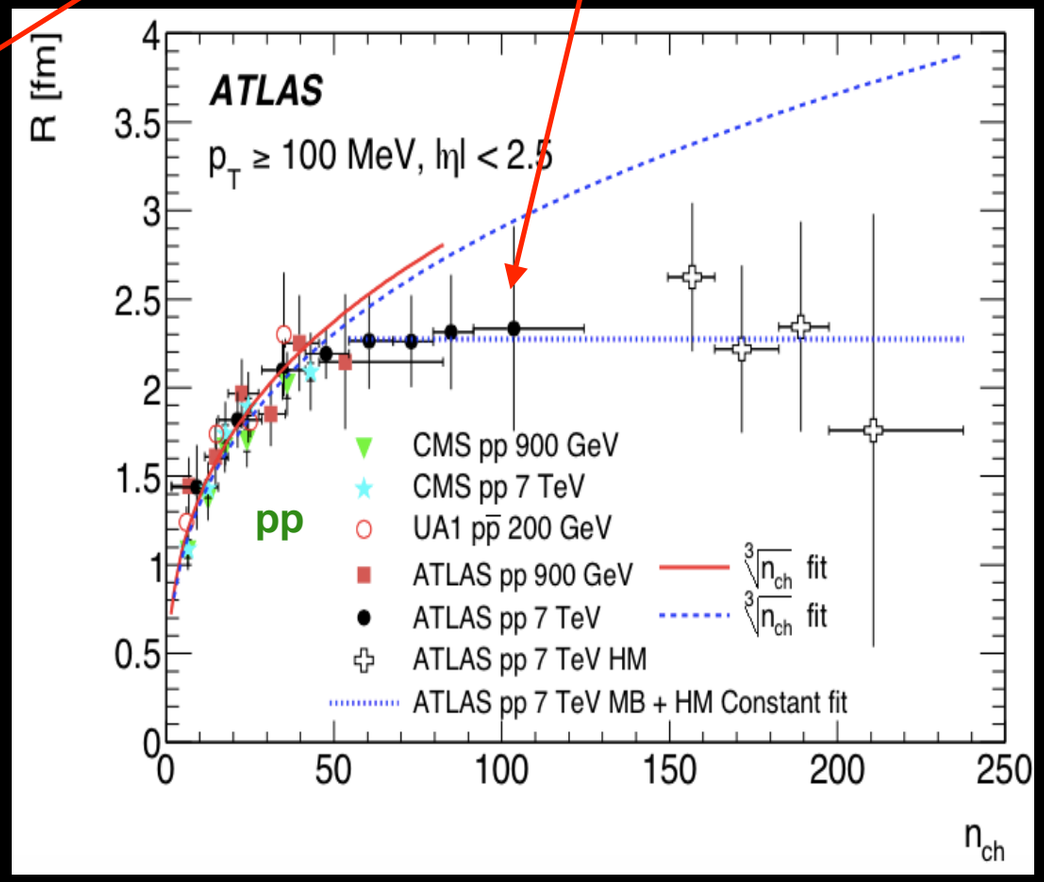
pPb



vs

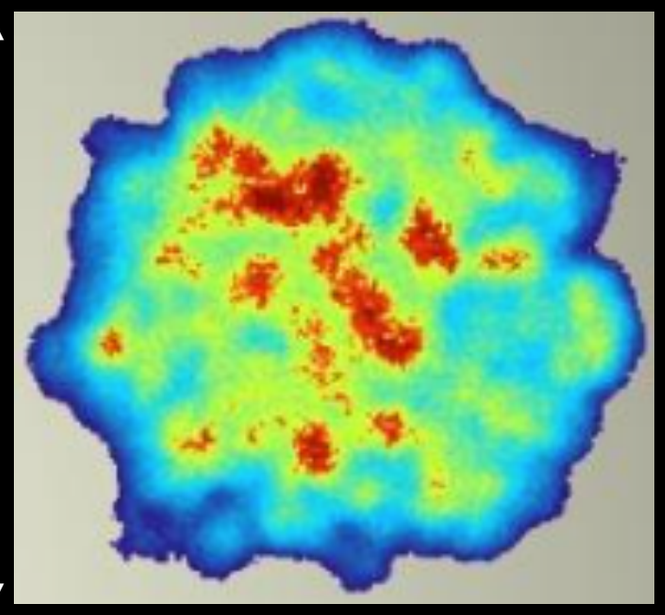
pp





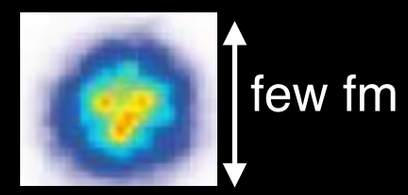
Are pp and pPb "small"?

PbPb



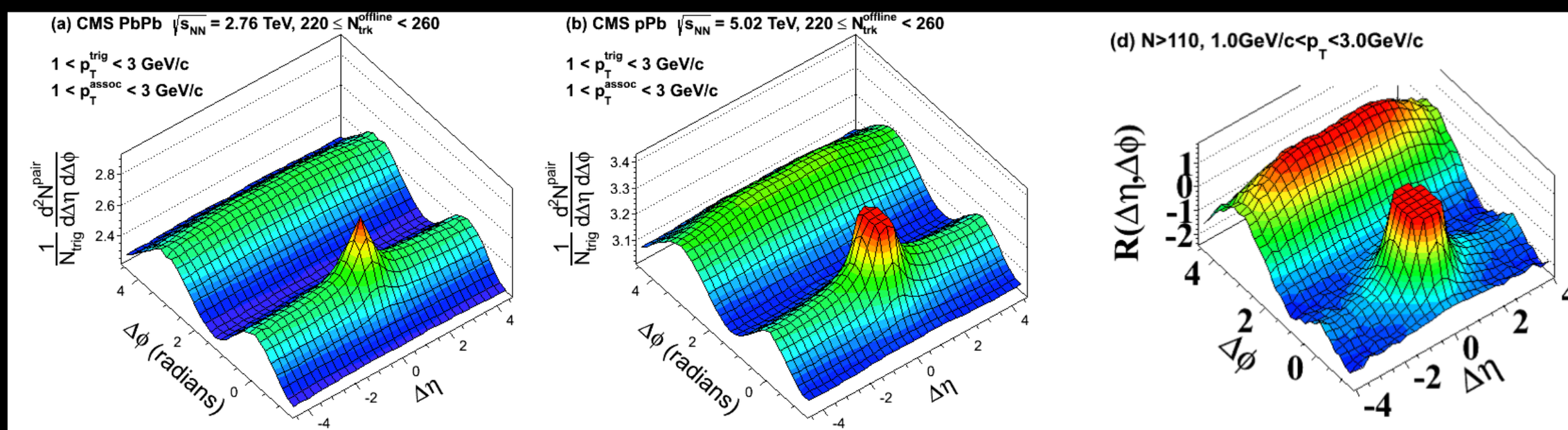
vs

pPb



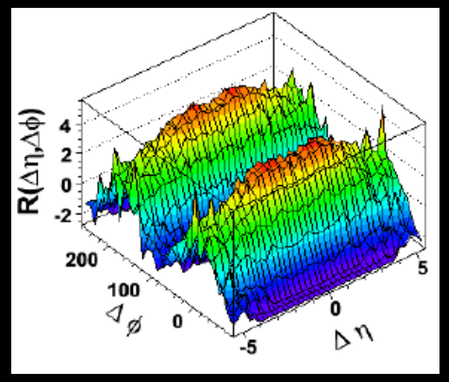
vs

pp



Are pp and pPb "small"?

2D CF can be described as superposition of "jet-like" correlations (intra-jet and jet-jet correlations) + weakly rapidity dependent flow harmonics



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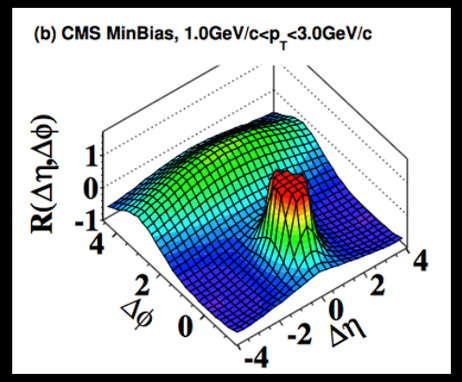
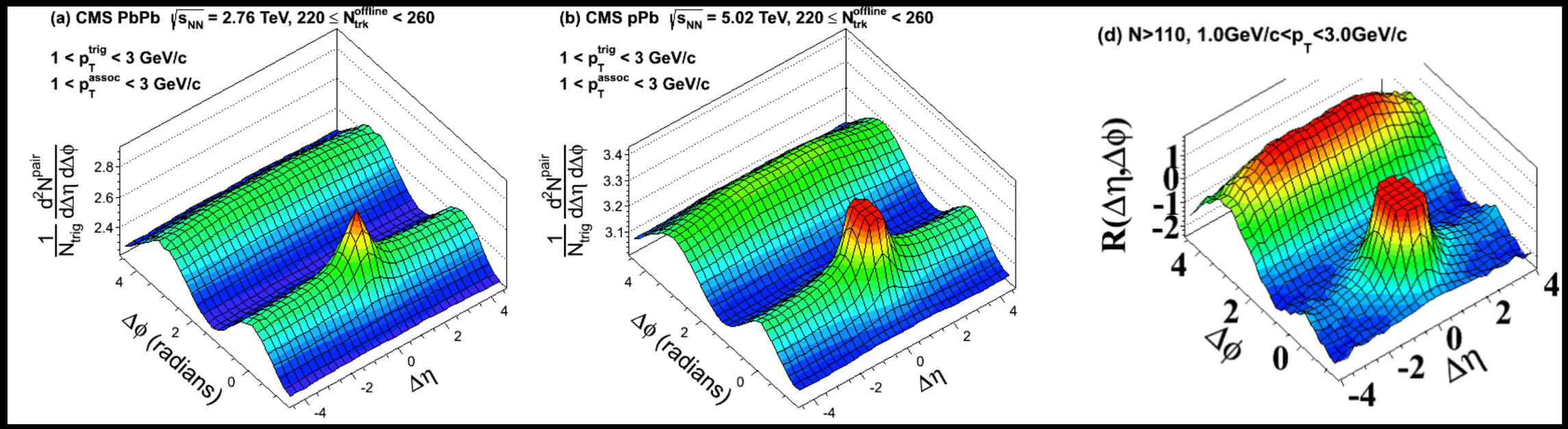


Importance of jet-like correlations drops from pp to pPb to PbPb:

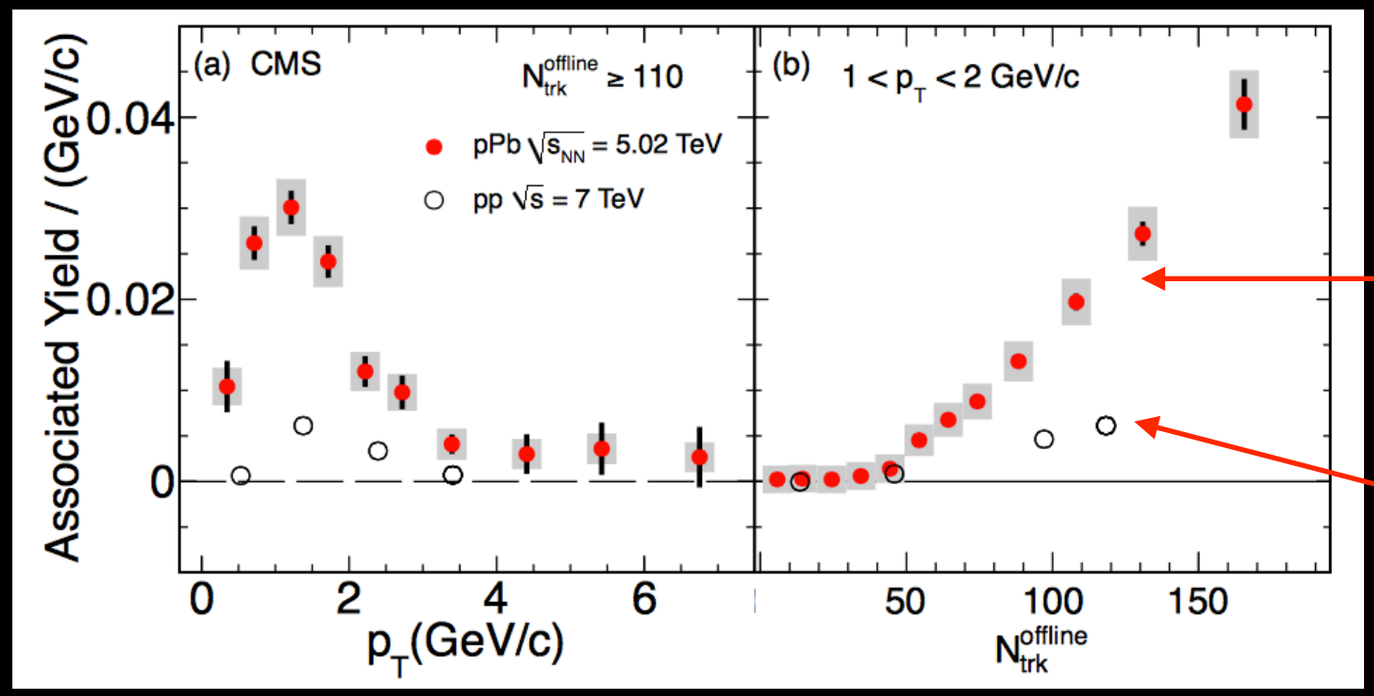
← lower collision energy

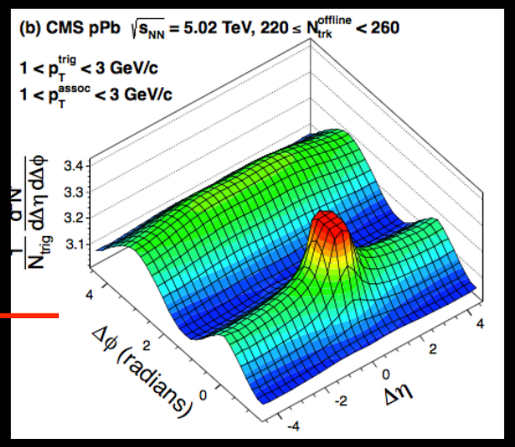
← higher multiplicity

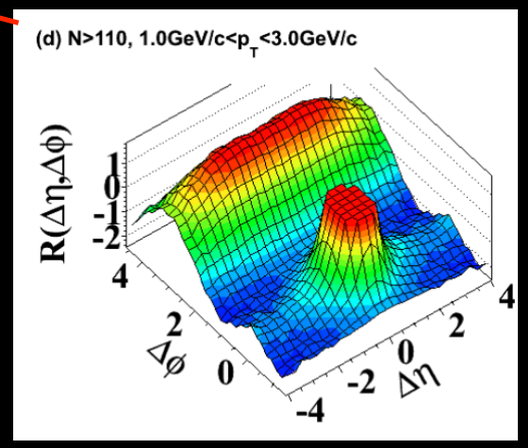


pp vs pPb

Strength of near-side correlations







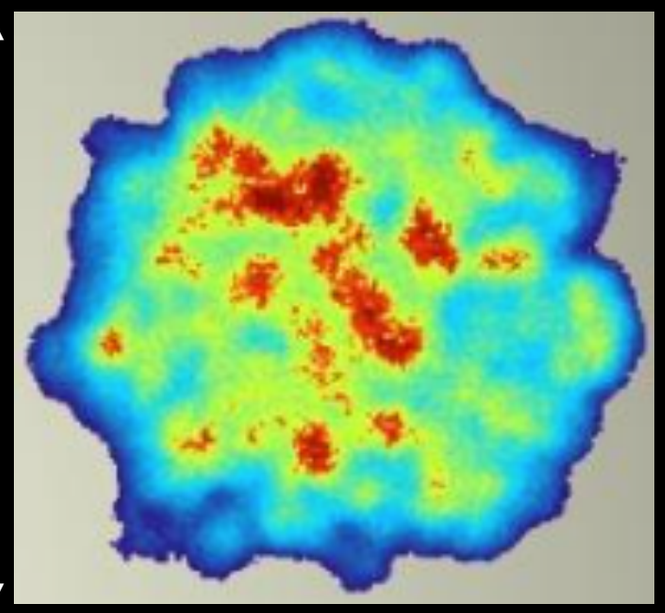
Strength of correlations for same multiplicity much lower in pp than pPb

In pPb (and PbPb), for multiplicity > 50, jet-like correlations are perturbation of flow-like signal

In high multiplicity pp much of final-state multiplicity comes from jet fragmentation
"Flow-like" correlations are perturbations on dominant jet-like structure
Need more data (analyses) to make any judgement on pp

What is “small”?

PbPb

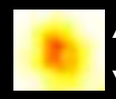


vs

pPb

vs

pp



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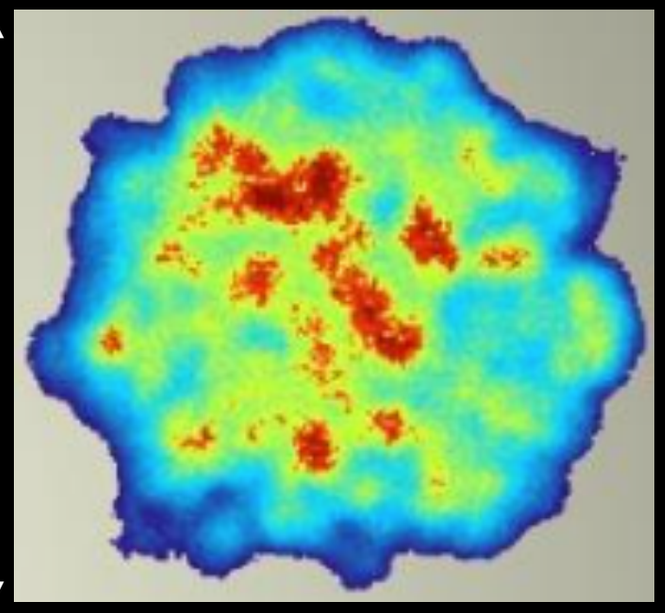
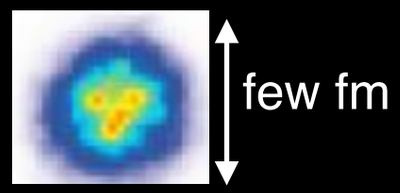
PbPb



vs

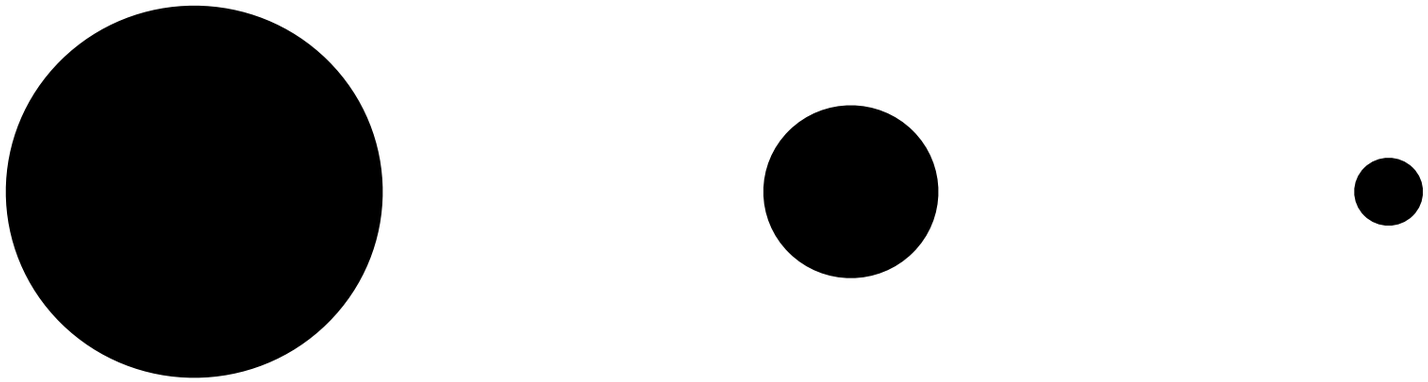
pPb



vs

pp

To decide which circle is “small”, need a length scale



Finite viscosity implies that mean free path is not 0

[provides means to decide when gradients are large, c.f. Rischke at IS2014]

Is there “collectivity” in “small” systems?

What is collectivity?

Is there "collectivity" in "small" systems?

What is collectivity?

A1: I know it when I see it

Is there "collectivity" in "small" systems?

What is collectivity?

A1: I know it when I see it

A2: Correlated production/motion of particles emerging from an expanding medium (matter); not correlated emission of non-interacting particles

I.e. in the context of our field, it relates to the existence of a (near-ideal or not) fluid

Is there "collectivity" in "small" systems?

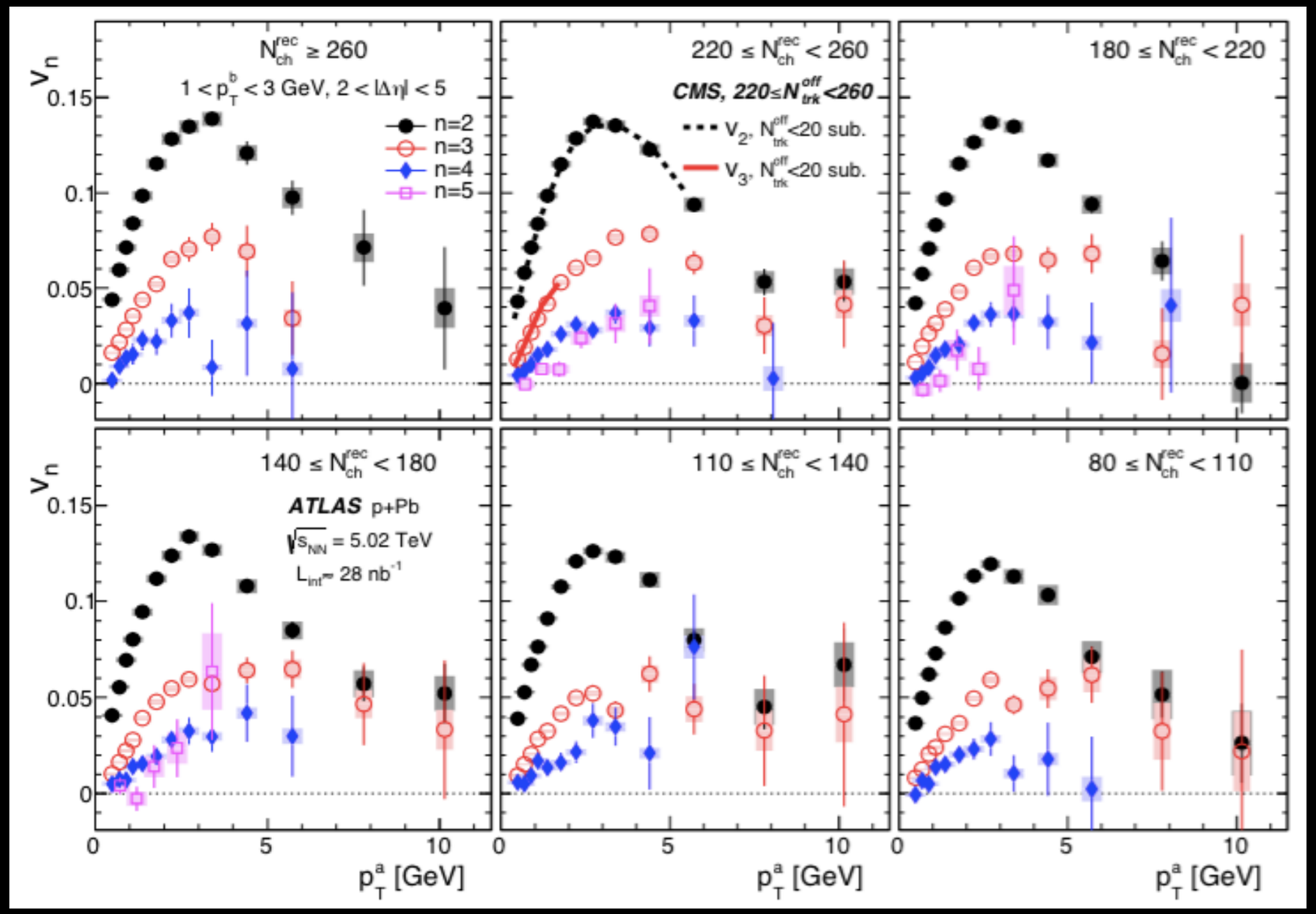
How do we know there is collectivity in "large" collision systems?

A: Wide range of correlation measurements in qualitative/semi-quantitative or quantitative agreement with viscous hydro calculations with sensible initial conditions/transport coefficients

- Azimuthal anisotropies v_n
- Characteristic $v_n(p_T)$ shape
- Mass ordering of $v_n(p_T)$
- Characteristic multiplicity dependence
- Weak rapidity dependence of correlations
- Connection to initial geometry
- Higher order (n>4) correlations
- Factorization breaking
- Mass ordering of p_T spectra
- Event angle correlations

AWOL: Medium response to local energy deposition

$v_n(p_T)$ in pPb



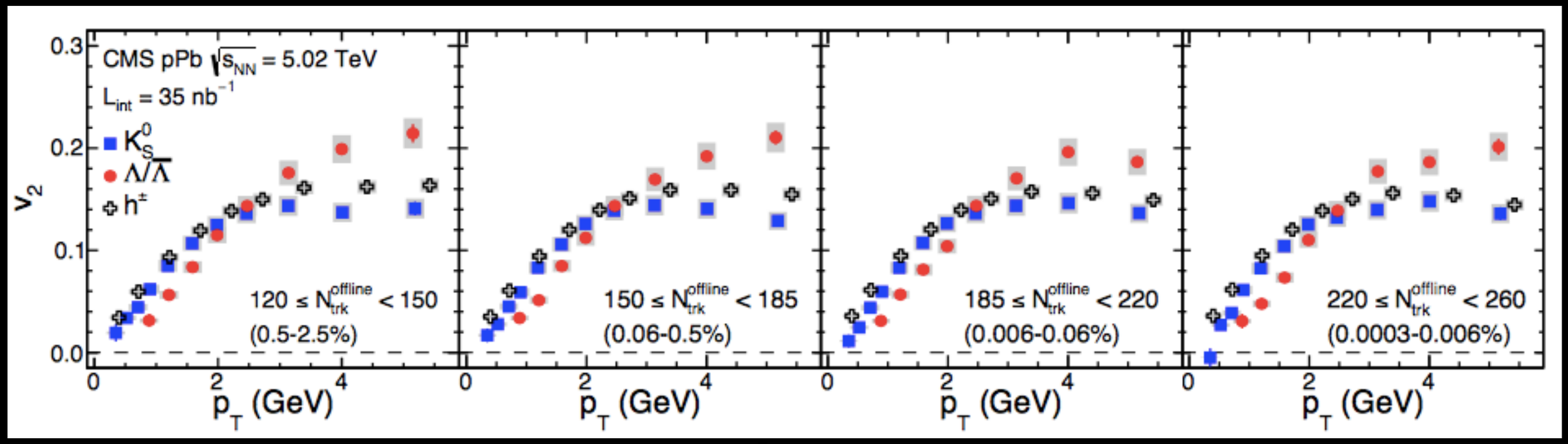
The good:

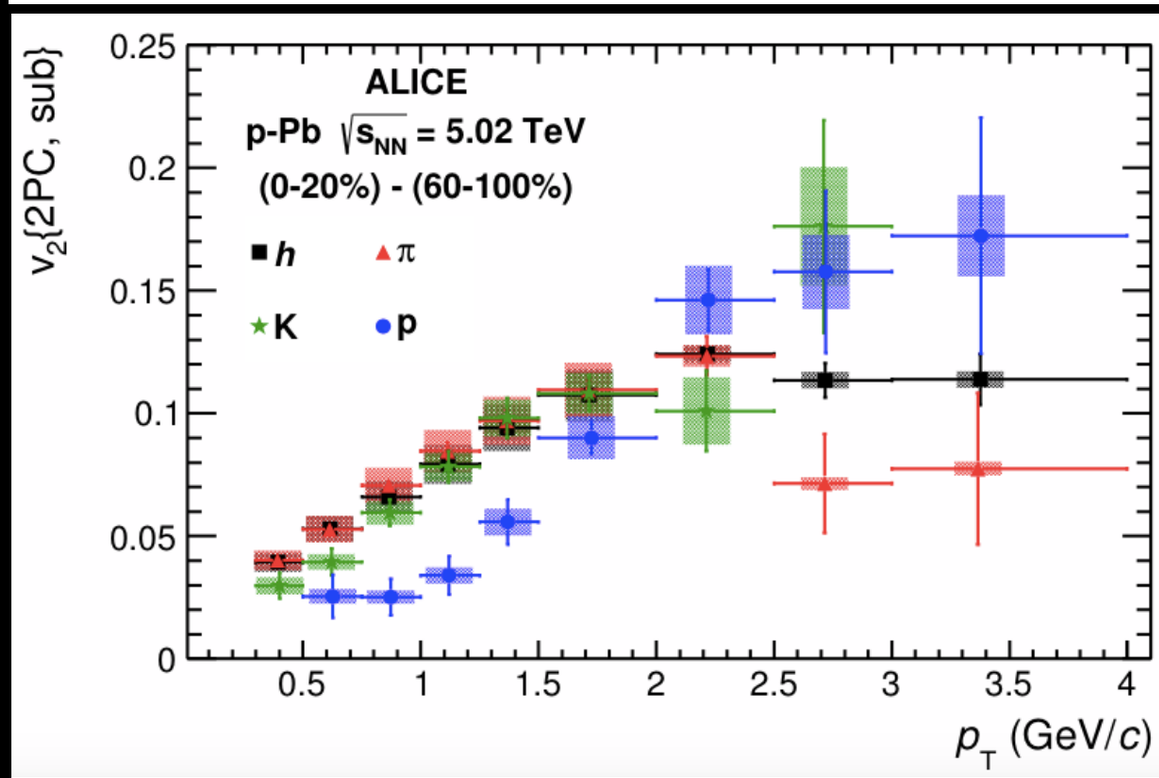
1. Good agreement with CMS
2. Characteristic $v_n(p_T)$ shape as in PbPb
3. Expected “n” ordering as in PbPb

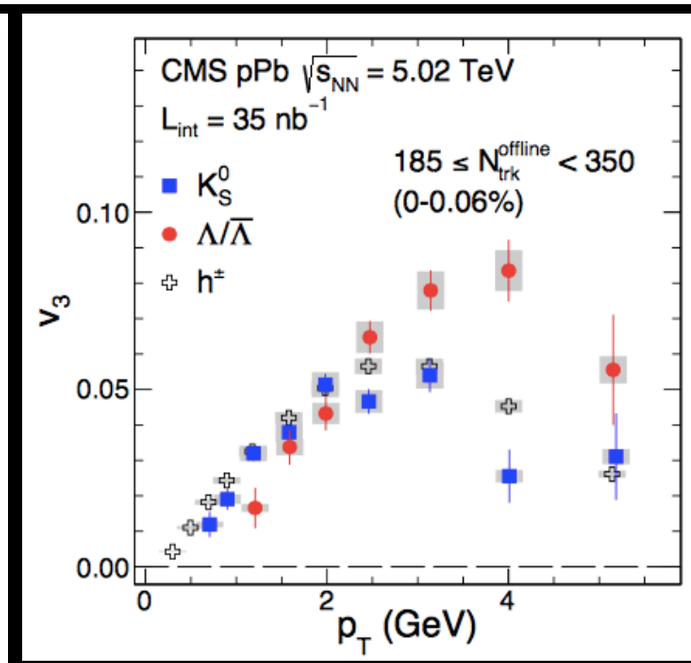
The somewhat confusing:

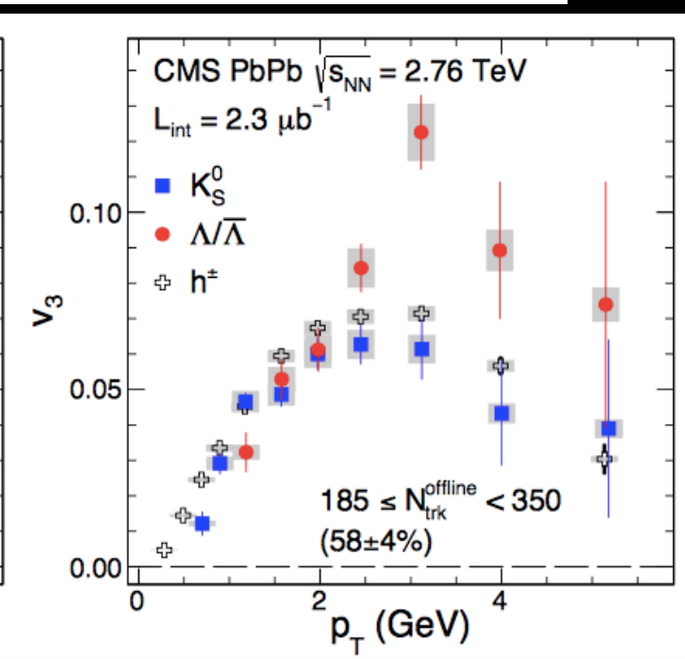
1. $v_2(p_T=10\text{GeV}) \sim 4\%$

Mass ordering of $v_n(p_T)$



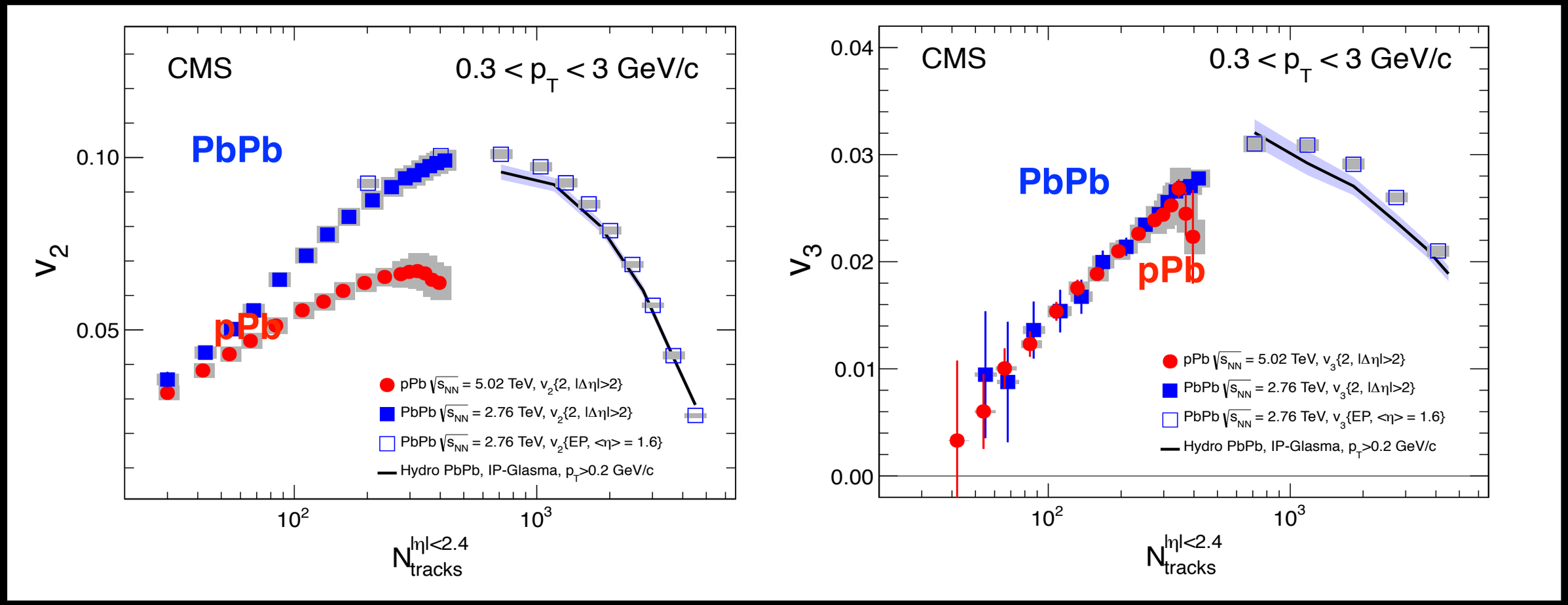




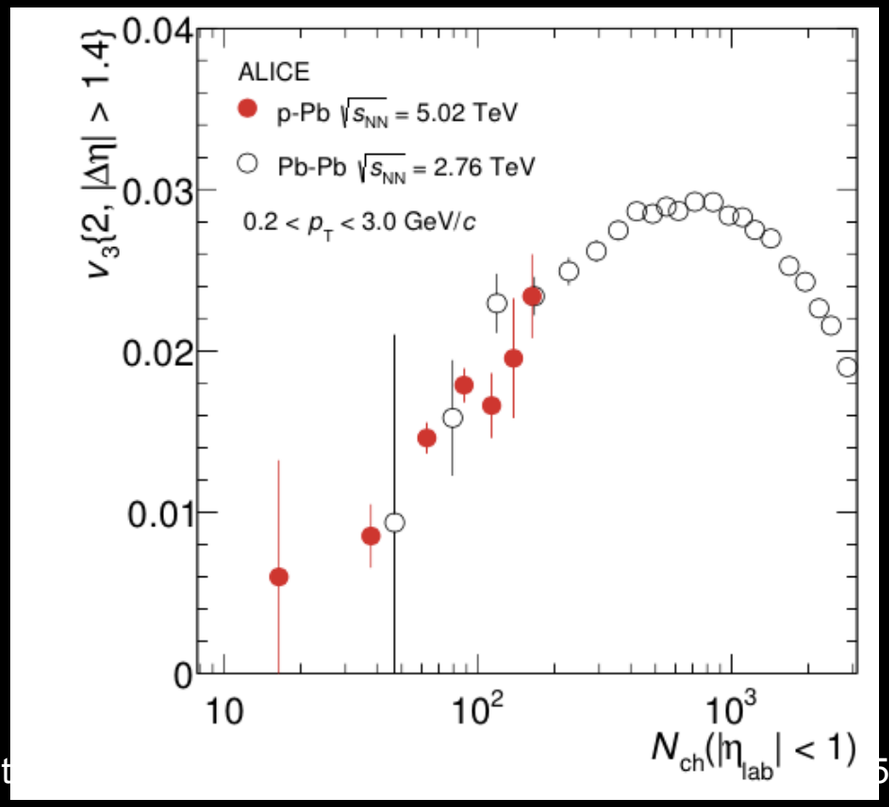


Mass ordering seen in pPb (even stronger than in PbPb at the same multiplicity)

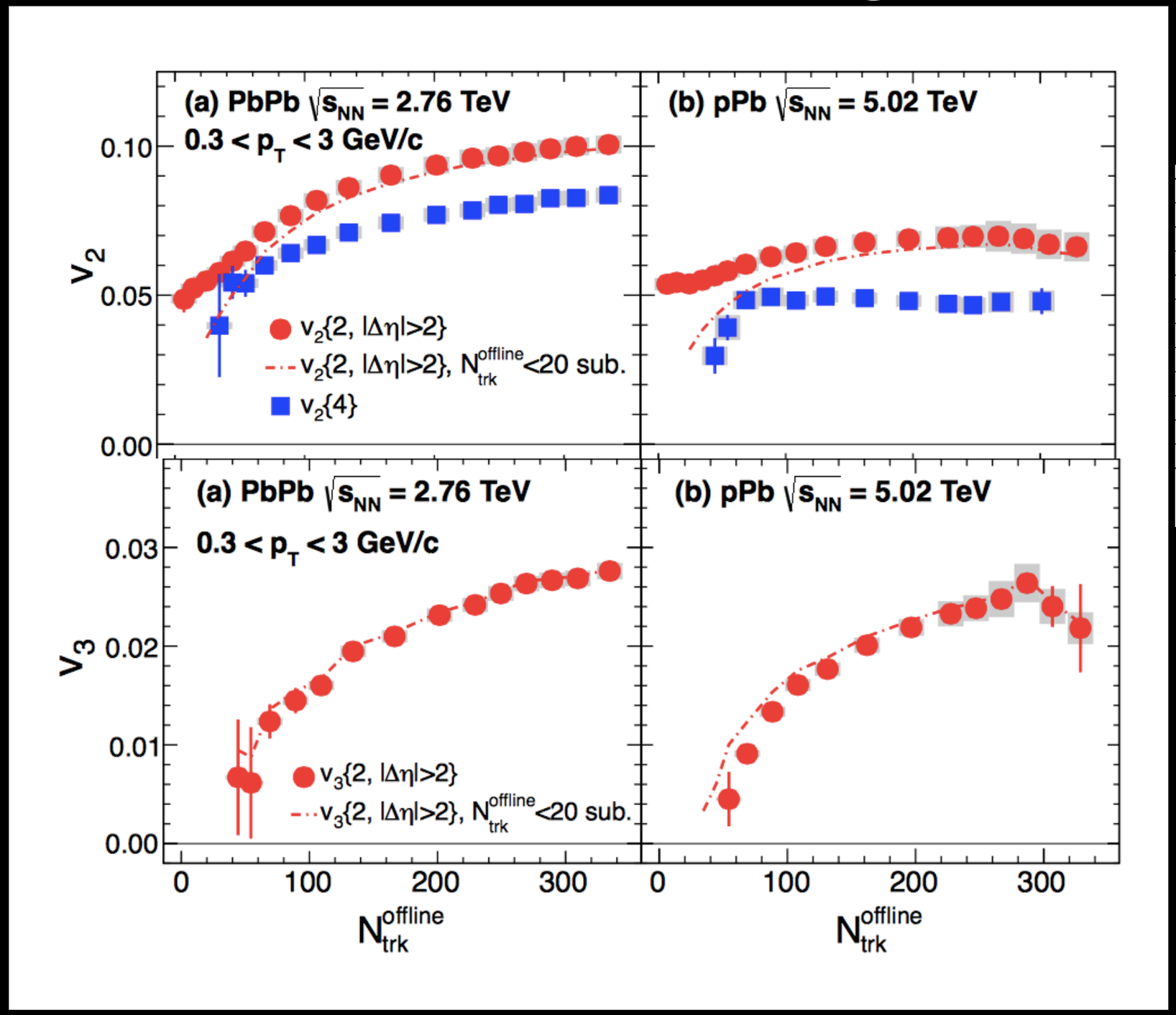
Characteristic multiplicity dependence



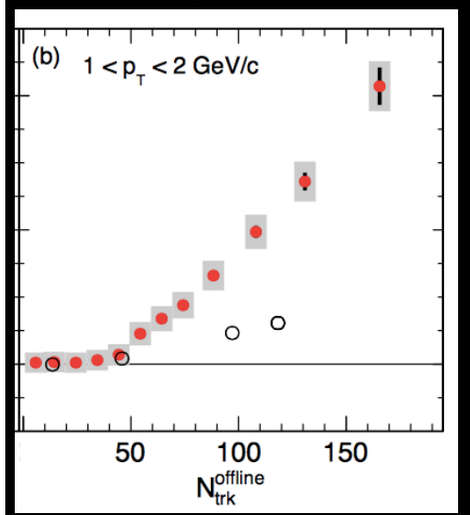
Continuous evolution from “small” (pPb ***and*** PbPb) to “large” (PbPb) system



From small to large



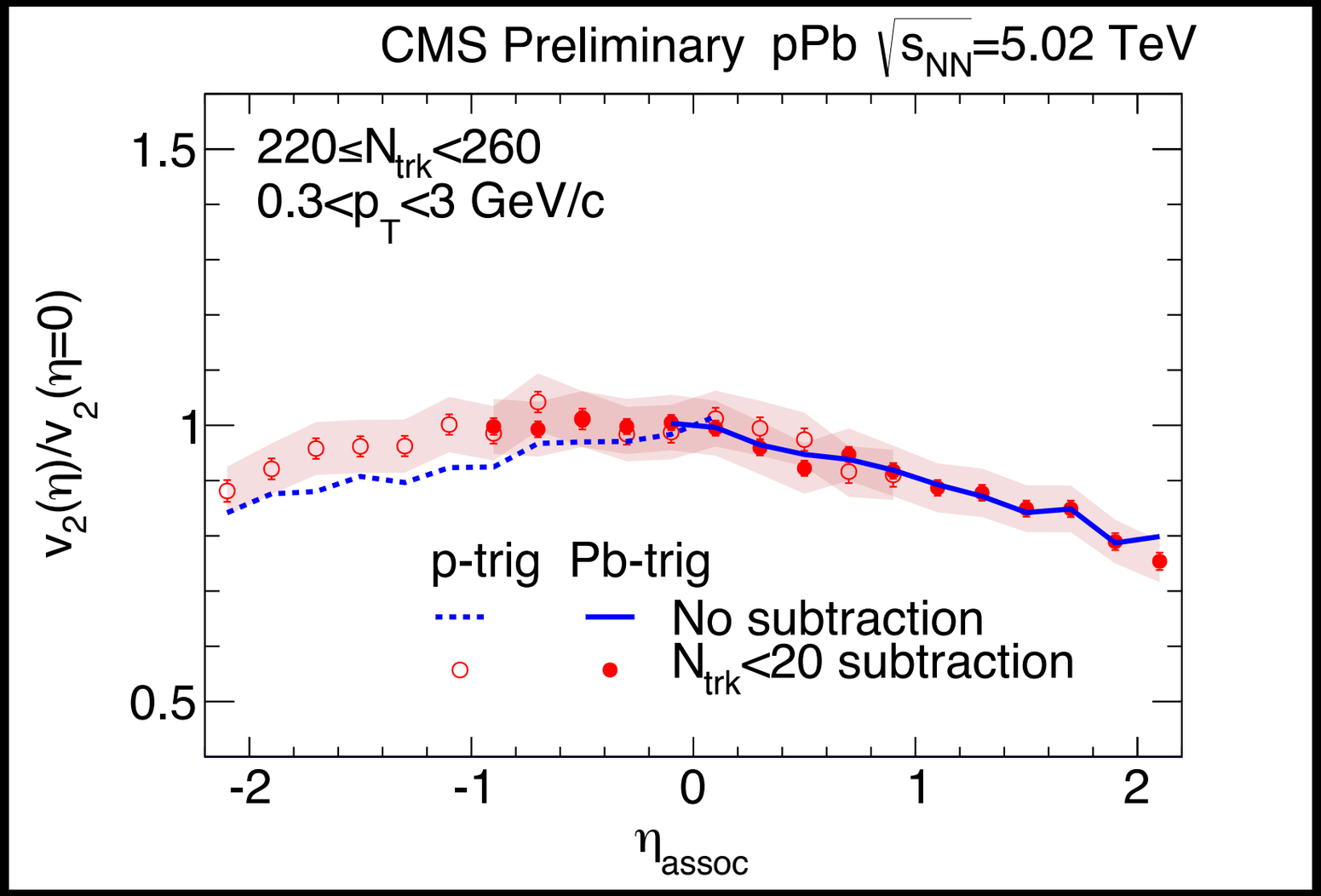
Ridge Yield



No clear evidence for turn-on at low multiplicity

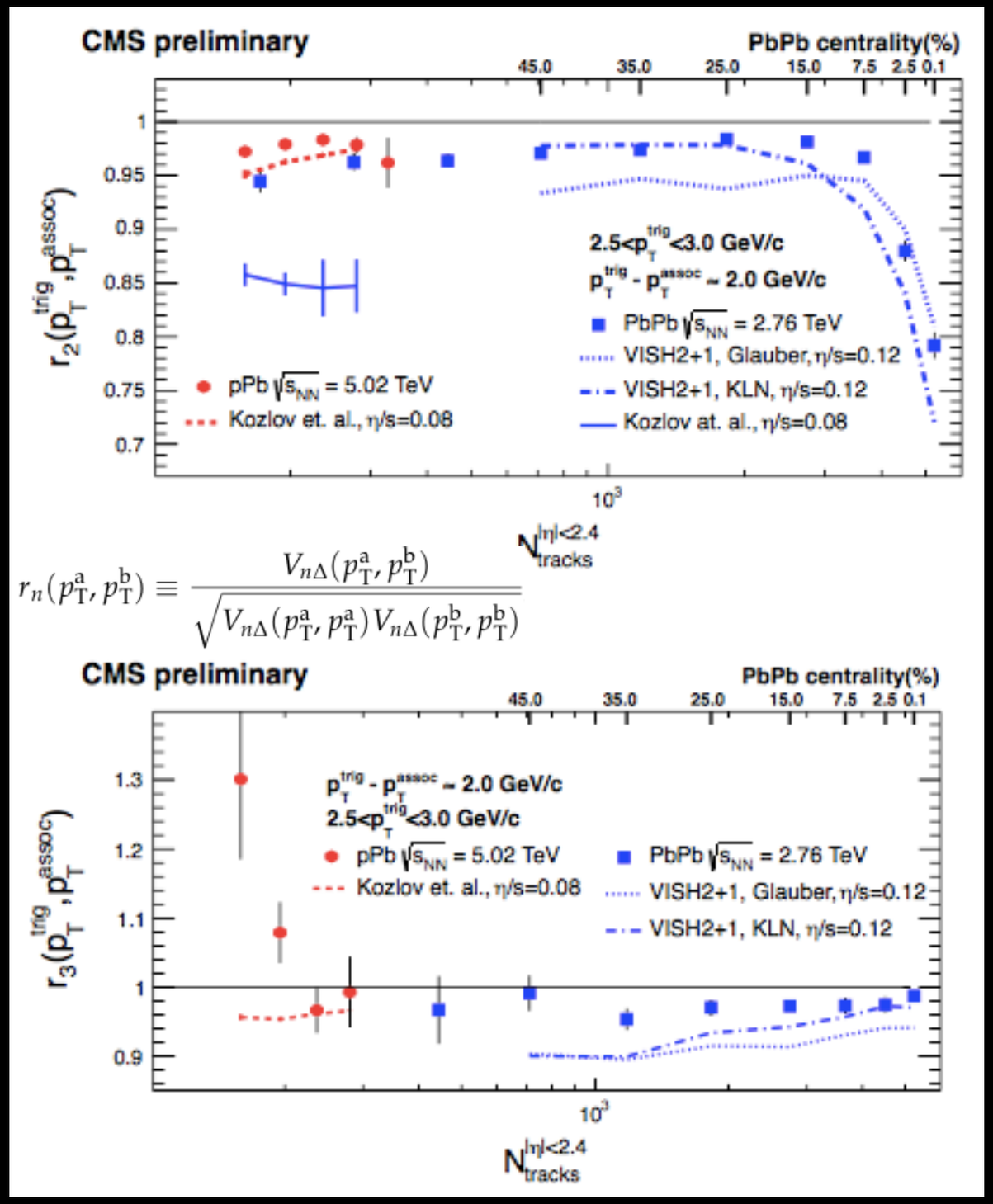
N.b.: at high multiplicity in pPb, “peripheral subtraction” is irrelevant

Rapidity dependence



As for PbPb, weak rapidity dependence with maximum near maximal particle density

Factorization breakdown

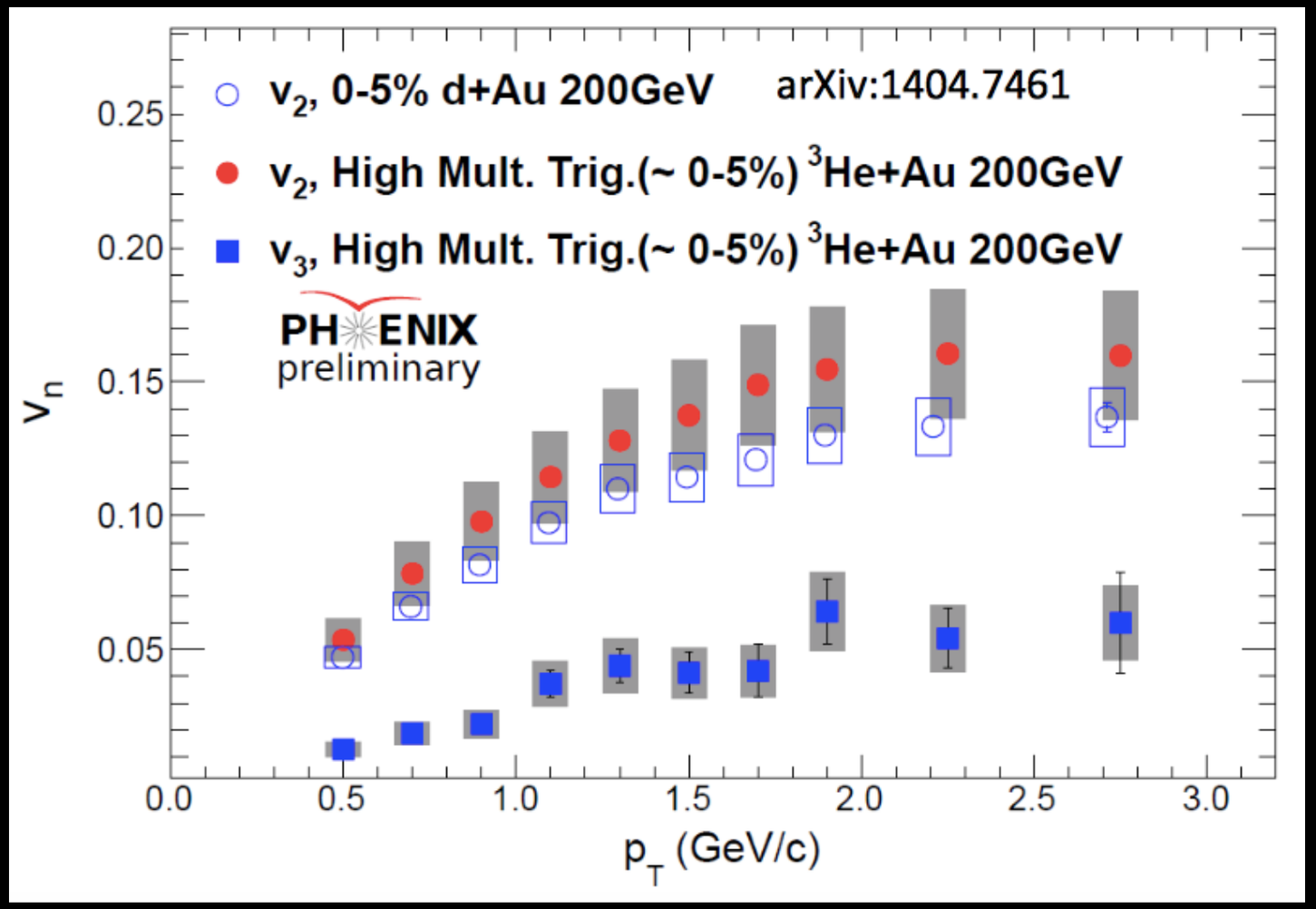


$$r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}}$$

Some tension between (pPb) data and predictions

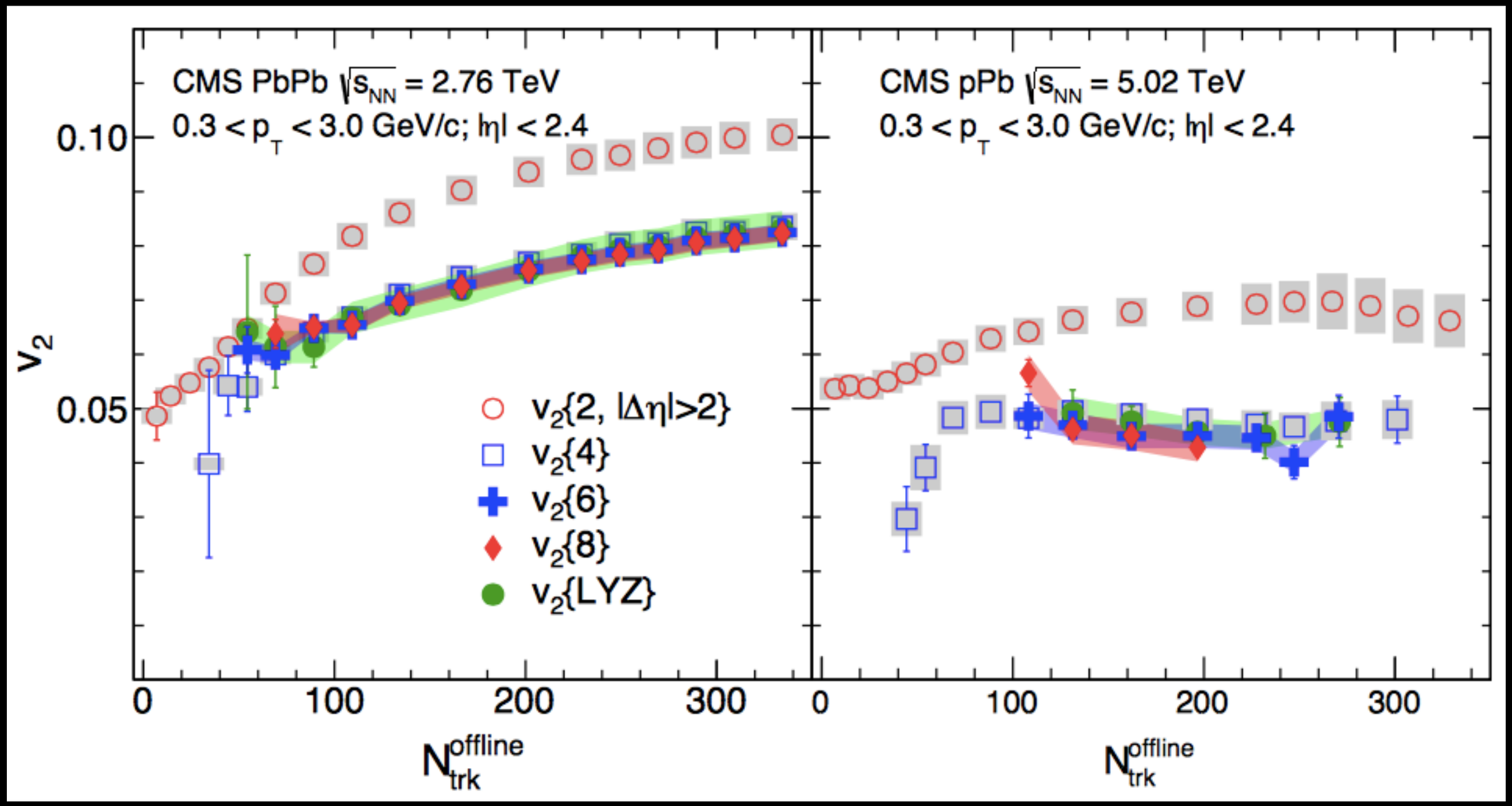
Connection to initial geometry

Connection to initial geometry



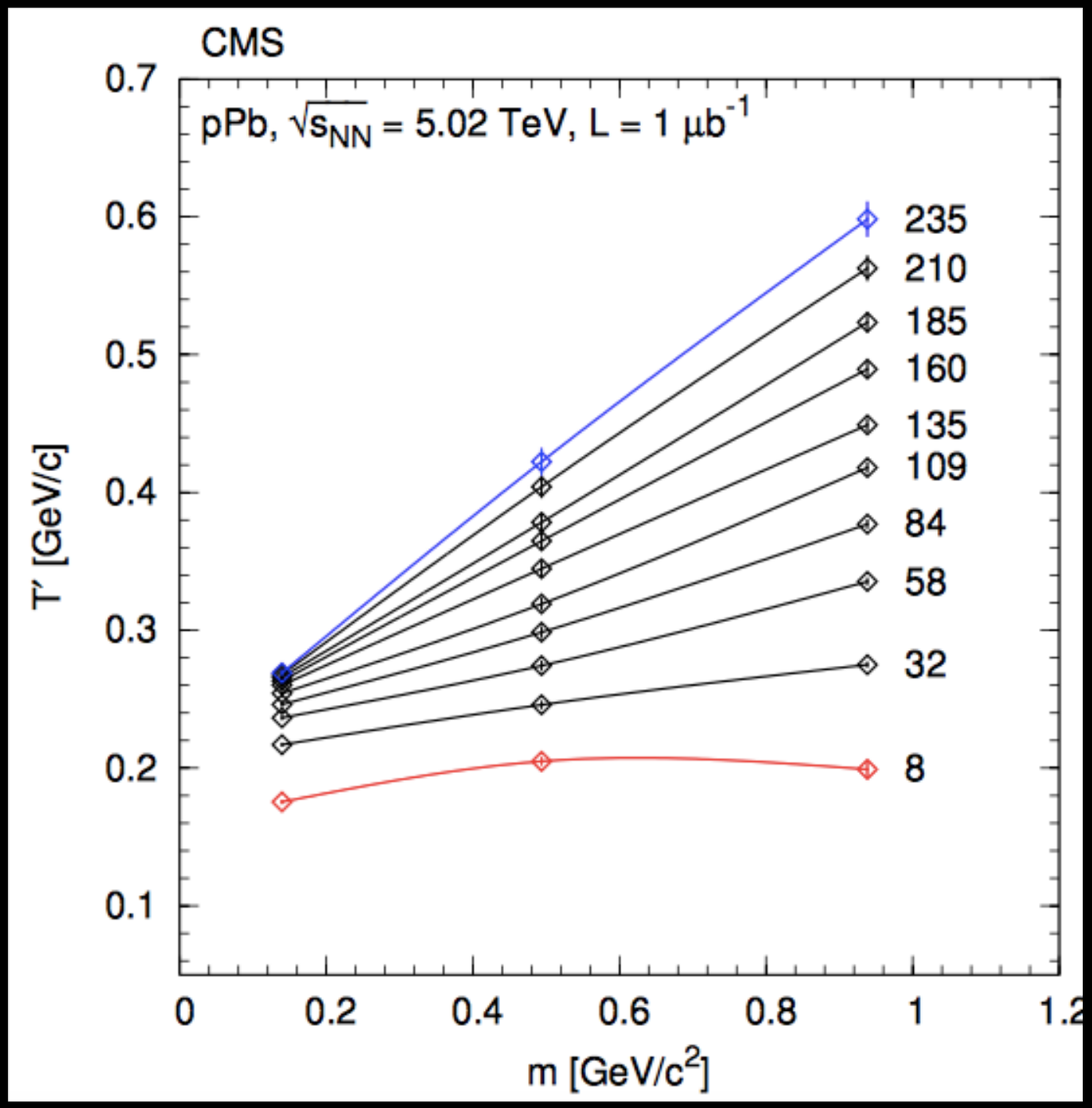
He+Pb at LHC would be nice

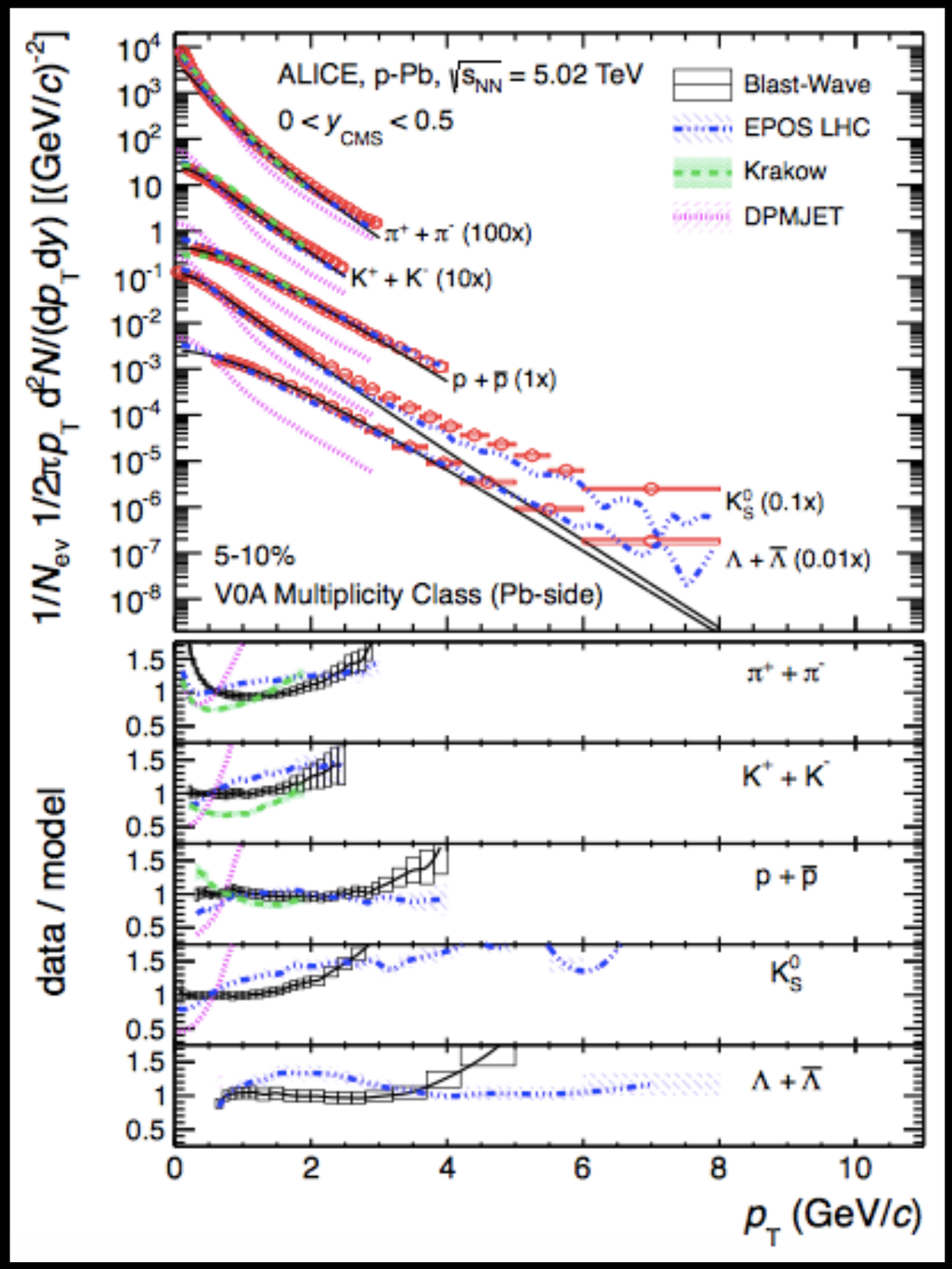
Higher order correlations



v_2 correlations between all particles in the event
Does this define “collectivity”?

“Radial flow”?





For completeness: Expected mass ordering of π, K, p spectra

Is there "collectivity" in "small" systems?

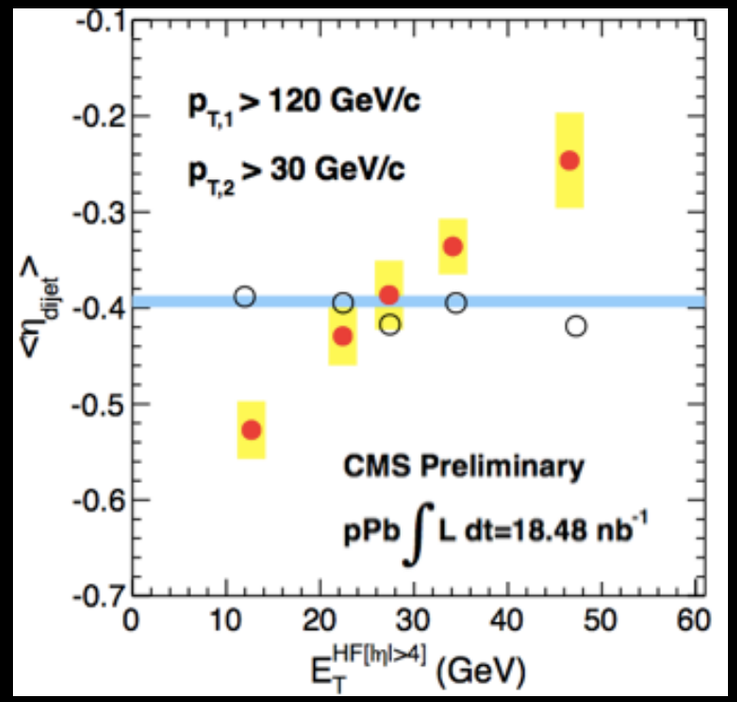
- Azimuthal anisotropies v_n ✔
- Characteristic $v_n(p_T)$ shape ✔
- Mass ordering of $v_n(p_T)$ ✔
- Characteristic multiplicity dependence ✔
- Weak rapidity dependence of correlations ✔
- Connection to initial geometry **LHC?**
- Higher order (n>4) correlations ✔
- Factorization breaking (✔-?)
- Mass ordering of p_T spectra ✔
- Event angle correlations **?**

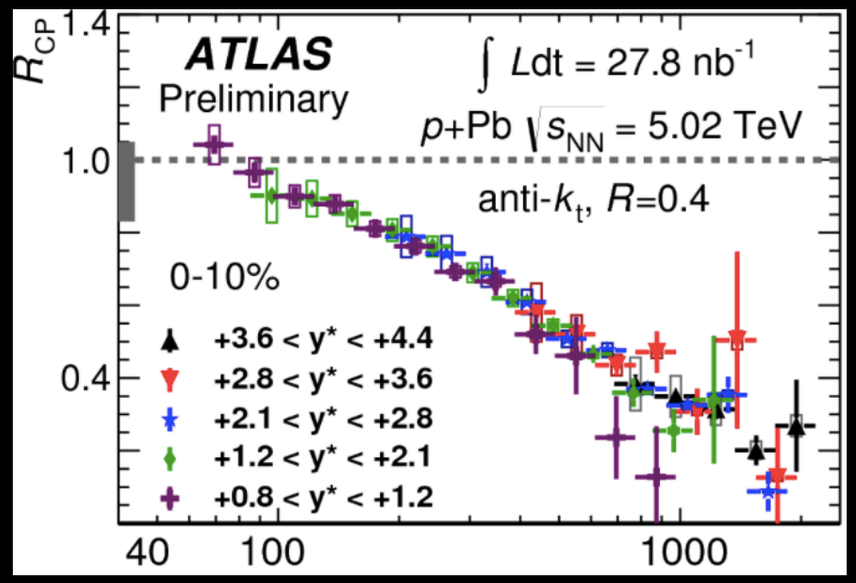
Experimentally, "collectivity" observables show a smooth evolution from "small" to "large" systems

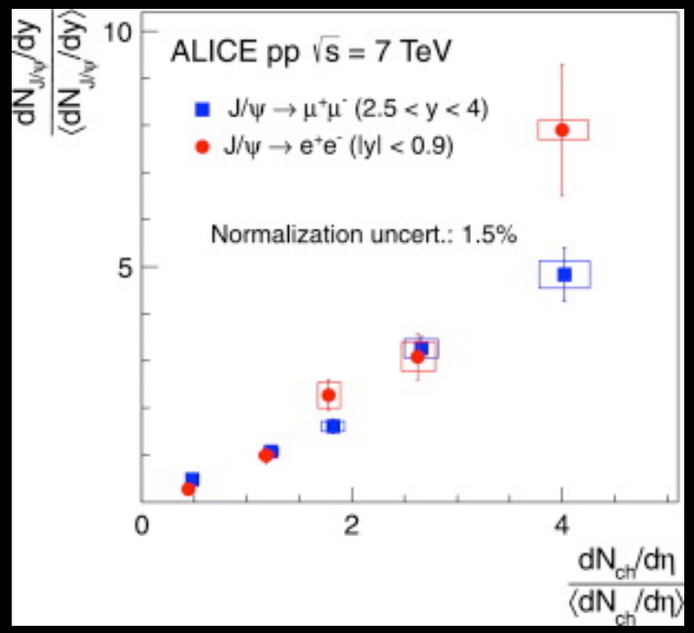
Other signs of "matter" in small systems?

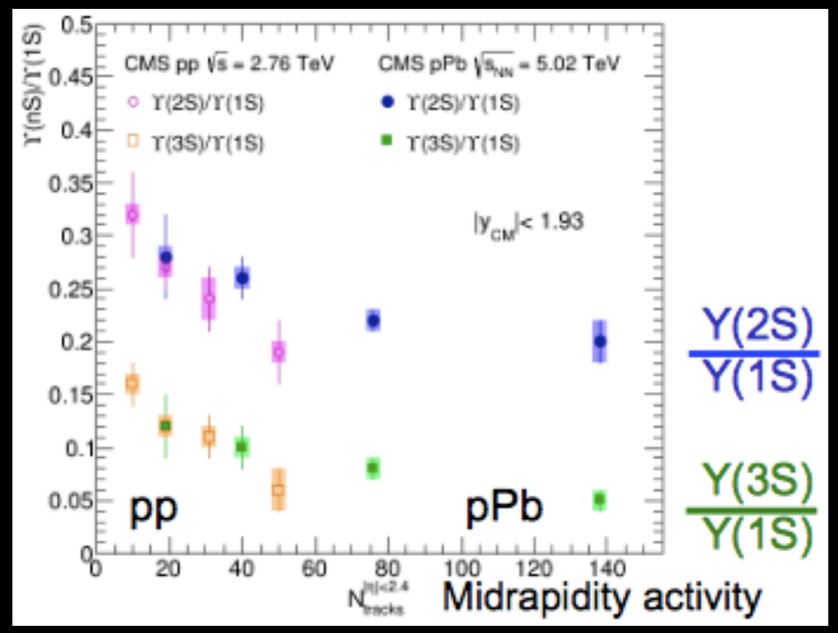
Connection between soft/hard production

Jets vs
“event activity”



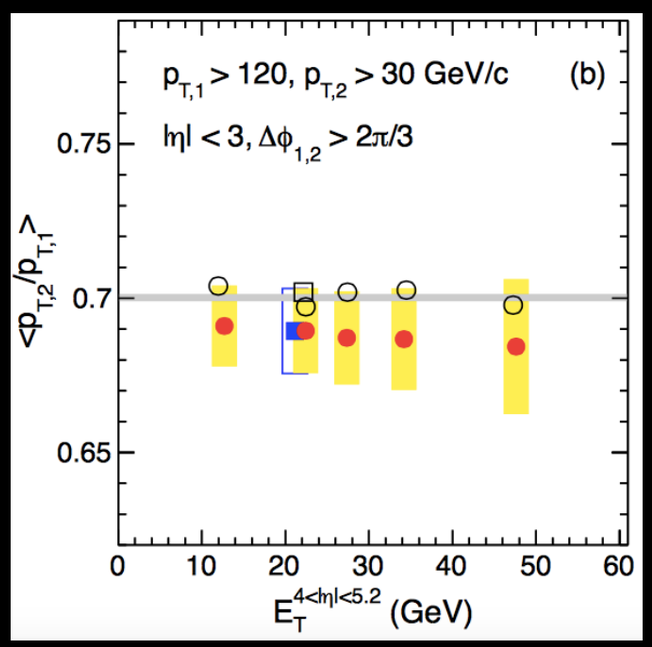




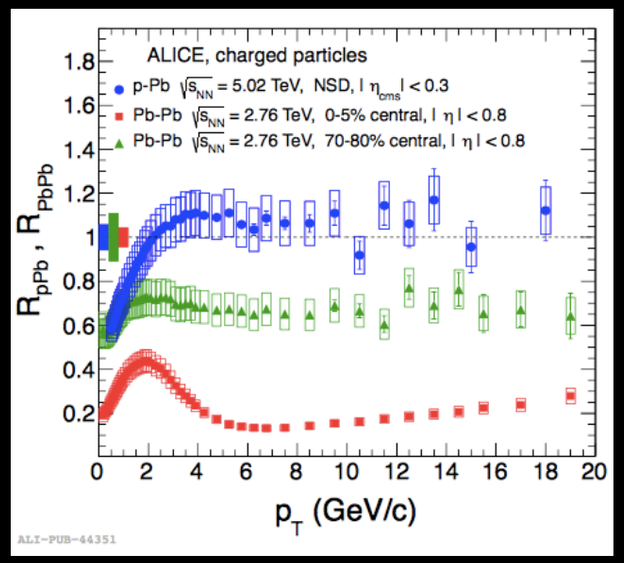


Quarkonia vs
“event activity”

Any signs of jet quenching?

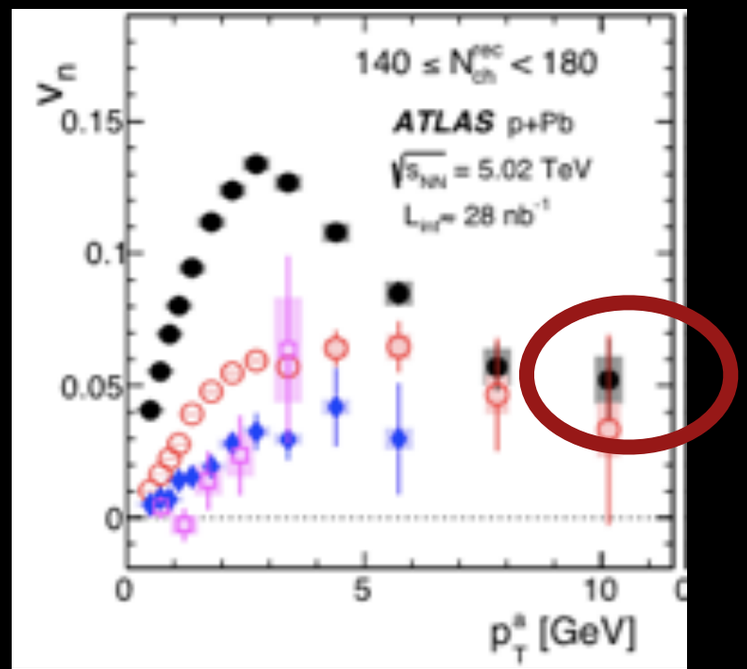


Jets balanced

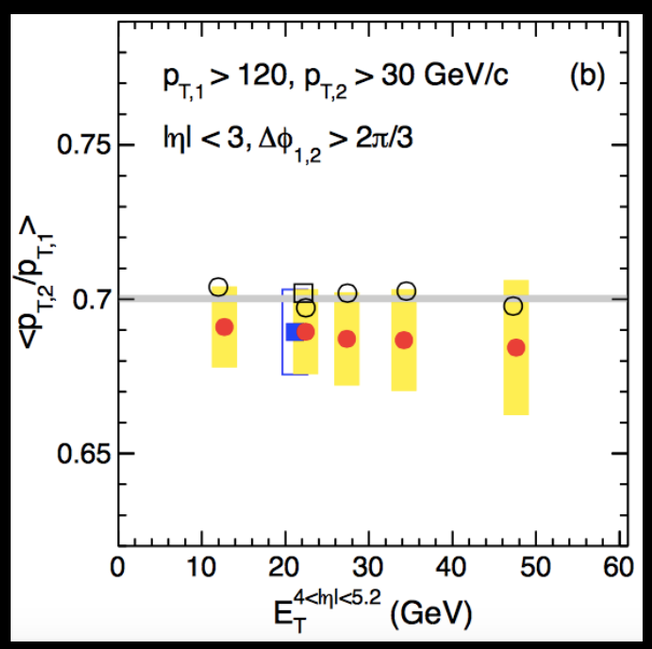


Hadrons unsuppressed

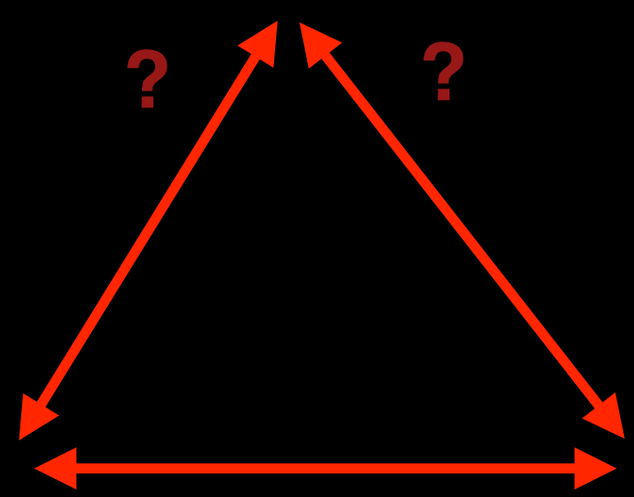
Any signs of jet quenching?

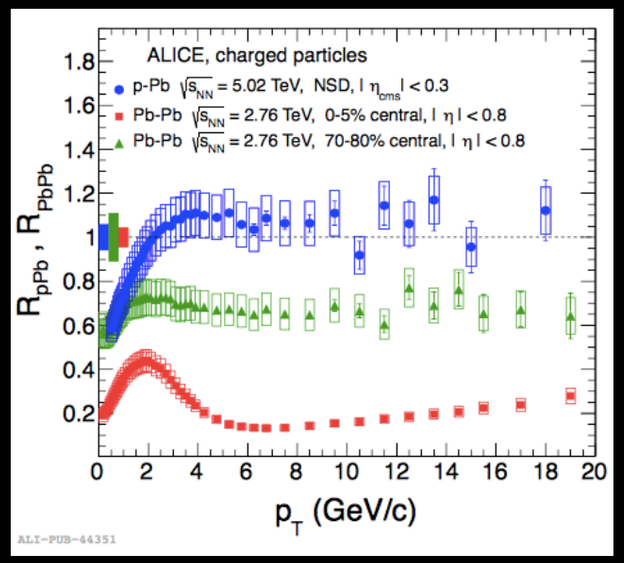


5% v2 at 10GeV



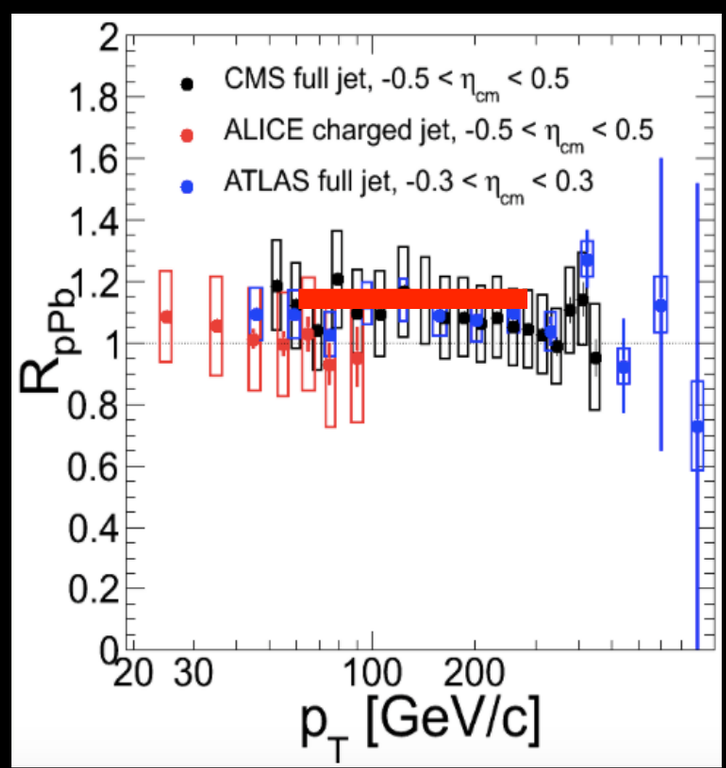
Jets balanced

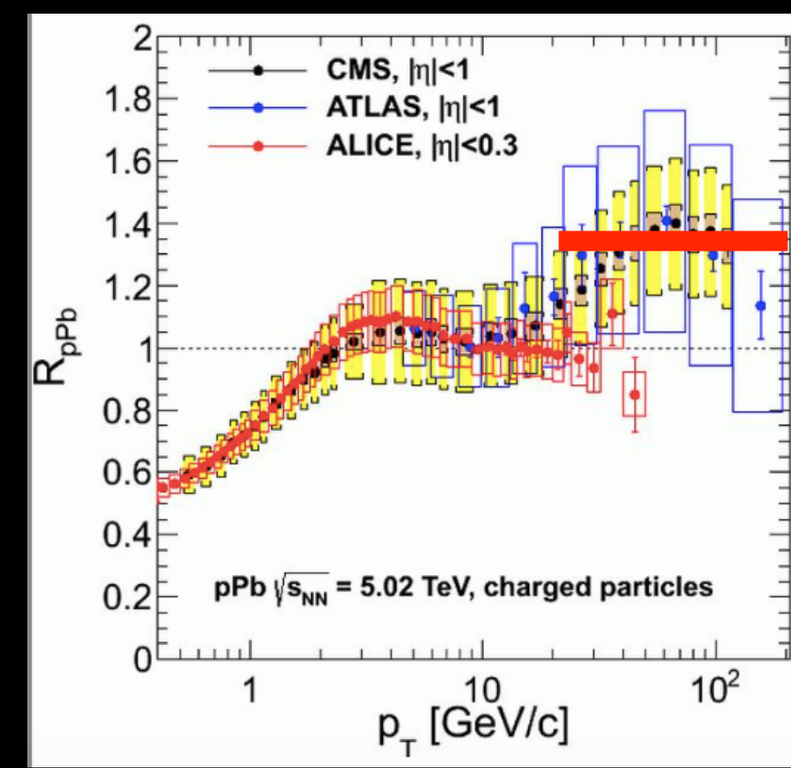




Hadrons unsuppressed

Anything else?

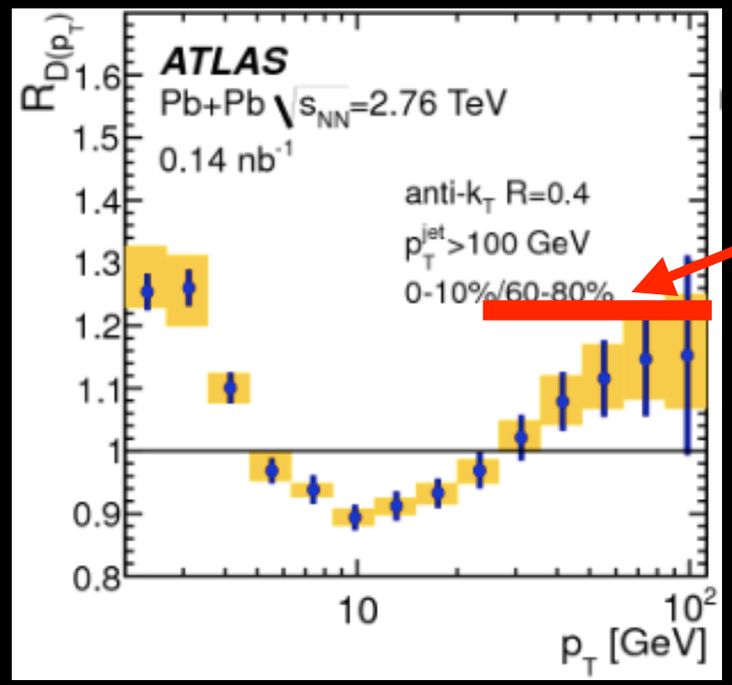




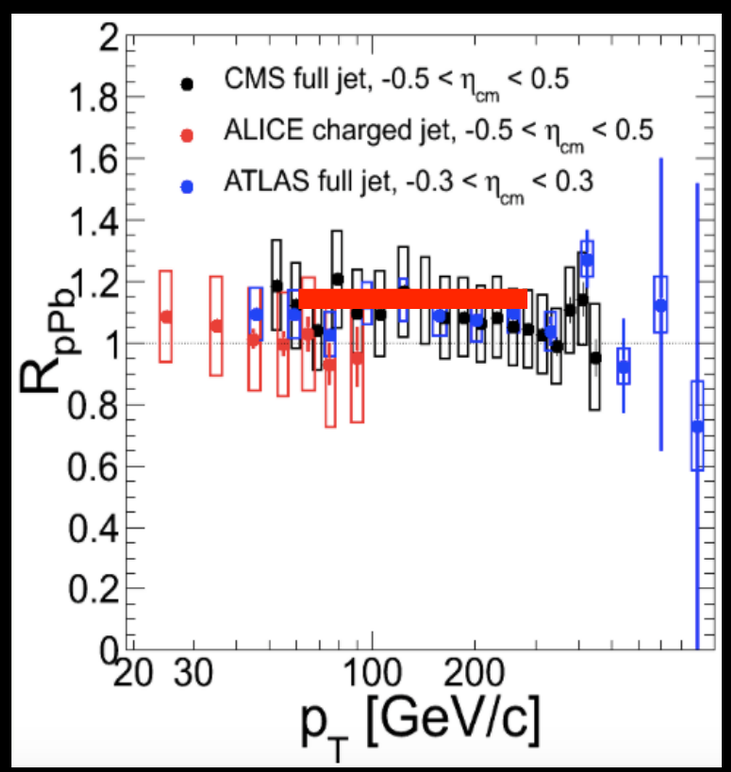
Jets R_{pPb} (100GeV) ~ 1.1

Hadron R_{pPb}(50GeV) ~ 1.35

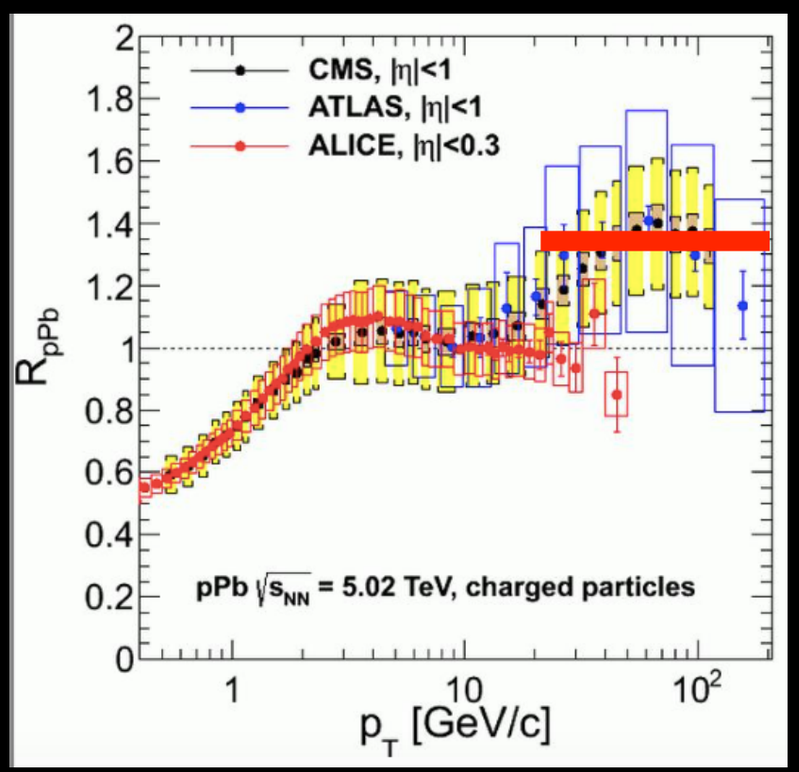
Anything else?



Predicted pPb/pp fragmentation function ratio



Implies $R_{D(pT\sim50GeV)} \sim 1.2$



Jets R_{pPb} (100GeV) ~ 1.1

Hadron R_{pPb}(50GeV) ~ 1.35 (submitted!)

Summary of LHC results on small colliding systems

- Nearly all observables commonly associated with "collectivity" have been studied in small systems at LHC
  - Nearly all results qualitatively show characteristics of "collectivity"
  - Continuous evolution from "small" to "large" systems
- But
  - No clear sign of "matter" in other observables
  - How small is too small?
  - Is there a consistent initial state description with a sensible evolution of transport coefficients that works from small to large?
  - Is there a transition from apparent collectivity to real collectivity?
- Our field turns its lonely eyes to theory

Hydrodynamics in small systems with Glasma initial conditions

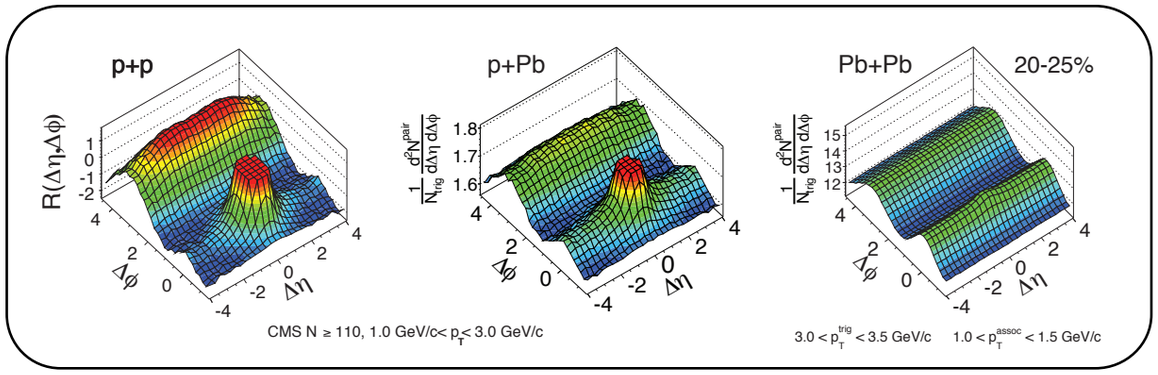
Björn Schenke, Brookhaven National Laboratory

BROOKHAVEN
NATIONAL LABORATORY

Introduction: Small systems p+p, p+A, d+A, ^{3}He+A

- High multiplicity p+p and p+Pb collisions at LHC show similar features as Pb+Pb collisions (ridge, v_n)

- d+Au at RHIC also seems to show similar features



- Interpretation not yet clear:
  - Initial correlations? Theory on this is developing: new insights
  - Initial geometry + collective effects? Fluid dynamics?

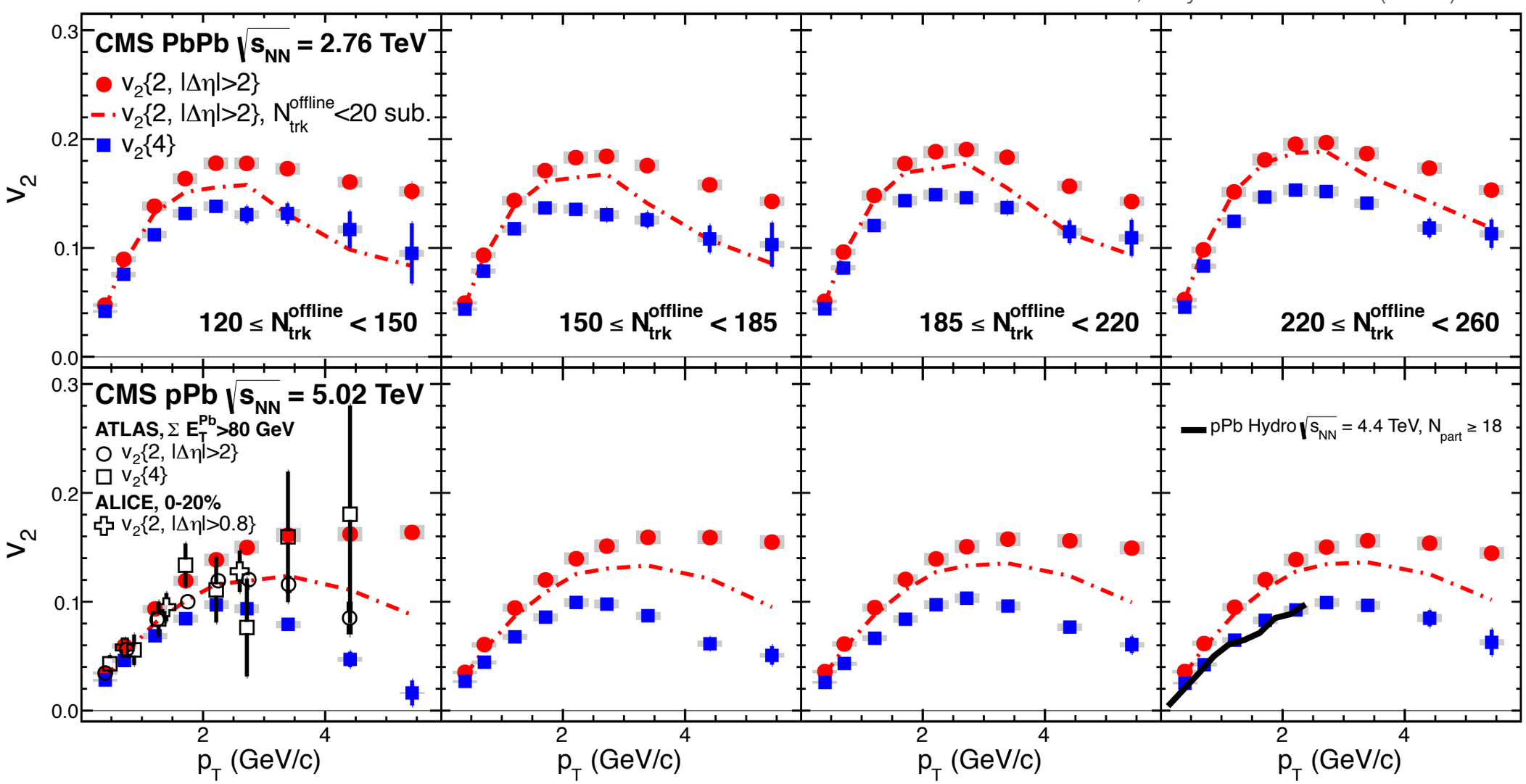
- How well can fluid dynamics do? What are calculations sensitive to? When does hydro break down?

CMS: JHEP 1009 (2010) 091, Phys.Lett. B718 (2013) 795-814, JHEP 1107 (2011) 076 , Eur.Phys.J. C72 (2012)

Why fluid dynamics?

- A motivation for using hydrodynamics in small systems is its success in larger systems and the similarity of experimental data in p+A and A+A collisions

CMS, Phys. Lett. B 724 (2013) 213



Although, the initial states are likely to look quite different

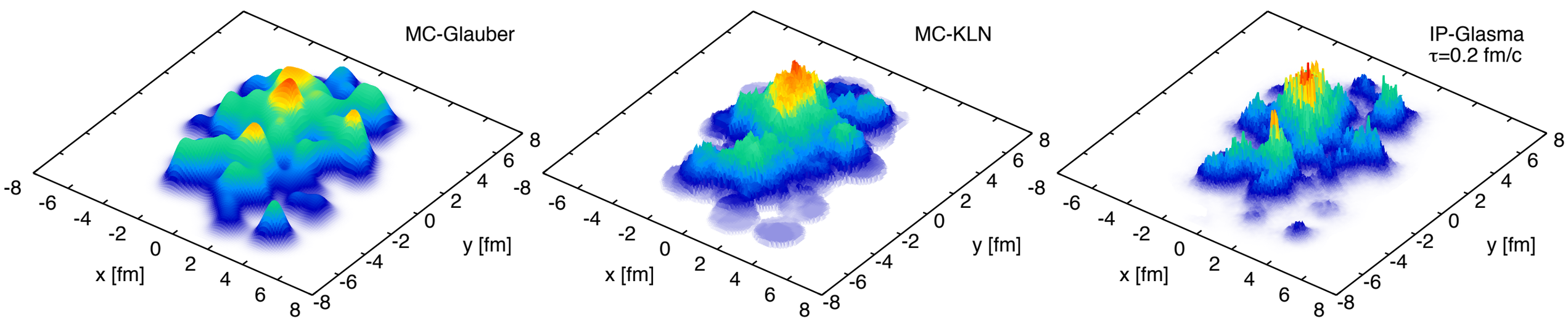
Heavy ions: Initial conditions

- Models need to provide input for fluid dynamic simulations: initial energy density, flow velocities, shear stress tensor

- Initial conditions fluctuate from event to event

- Main source of fluctuations: nucleon positions

- Different models give different *energy density distributions*



C. Gale, S. Jeon, B. Schenke, Int.J.Mod.Phys. A28 (2013) 1340011

Computing the initial state at high energies

B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Gluon number increases with decreasing gluon momentum
- Gluon saturation at $p_T \lesssim Q_s(x, \mathbf{b})$
- Strong fields with occupation $\sim 1/\alpha_s$
  Classical description possible
- IP-Sat model parametrizes $Q_s(x, \mathbf{b})$
  (simple way to include impact parameter dependence)
  KOWALSKI, TEANEY, PHYS.REV. D68 (2003) 114005
- Fit parameters to HERA diffractive data

IP-Glasma initial conditions

B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Sample nucleons from Woods-Saxon distribution
- Sample color charge density $\rho^{A/B}(\mathbf{x}_T)$
- For nucleus A and B compute the path-ordered exponential over its longitudinal extend

$$V_{A/B}(\mathbf{x}_T) = \prod_{k=1}^{N_y} \exp\left(- ig \frac{\rho_k^{A/B}(\mathbf{x}_T)}{\nabla_T^2 + m^2} \right)$$

- m is an infrared cutoff of order Λ_{QCD}
- Wilson lines after the collision are then obtained from V_A and V_B via the Yang-Mills equations

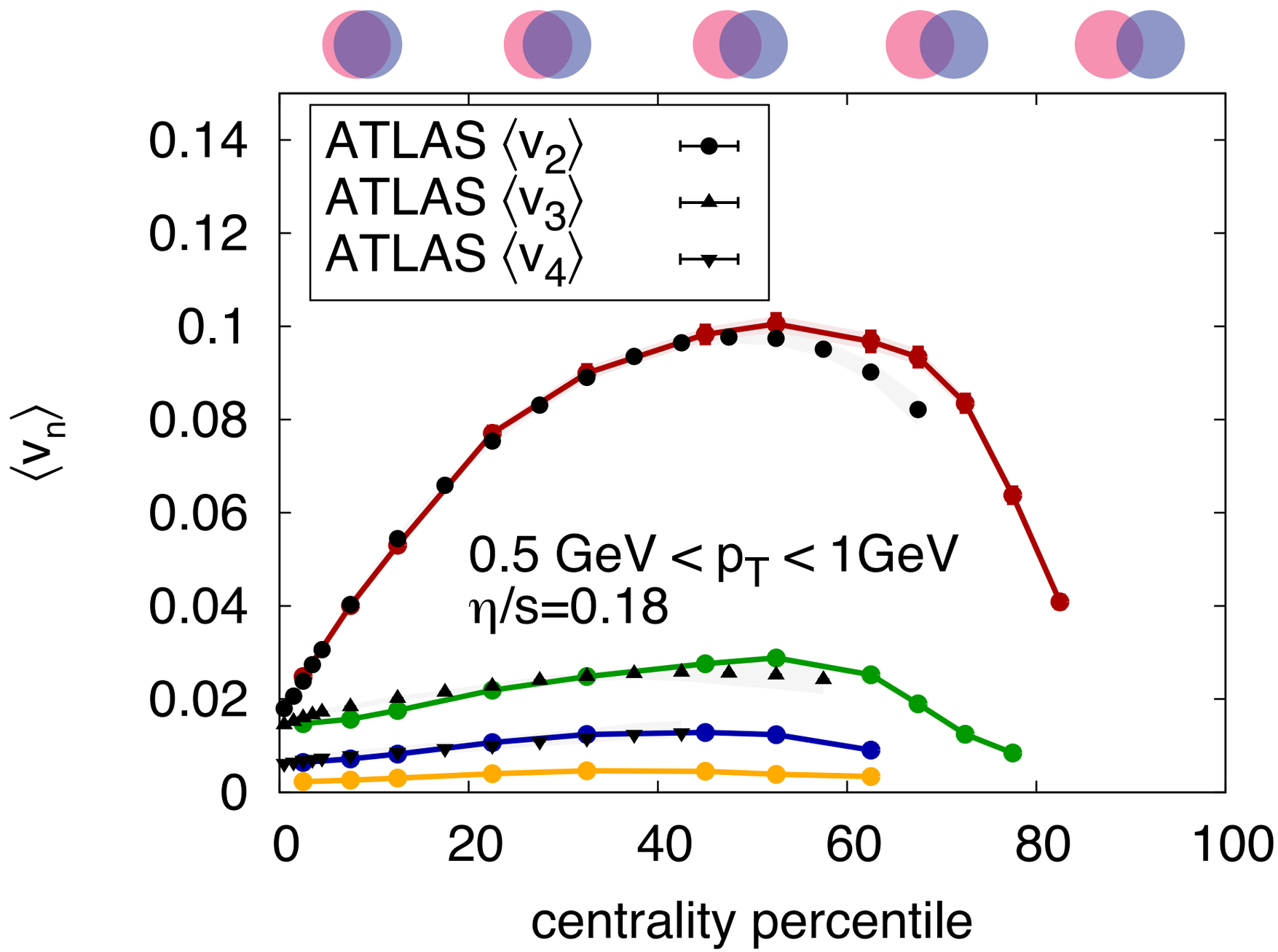
IP-Glasma initial conditions

B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Yang-Mills equations determine:
  - Initial gluon fields from color charges
    KRASNITZ, VENUGOPALAN, NUCL.PHYS. B557 (1999) 237
  - Energy density after the collision
  - Early non-equilibrium time evolution
- Then match fields' $T^{\mu\nu}$ to hydrodynamics by extracting ε and u^{μ}

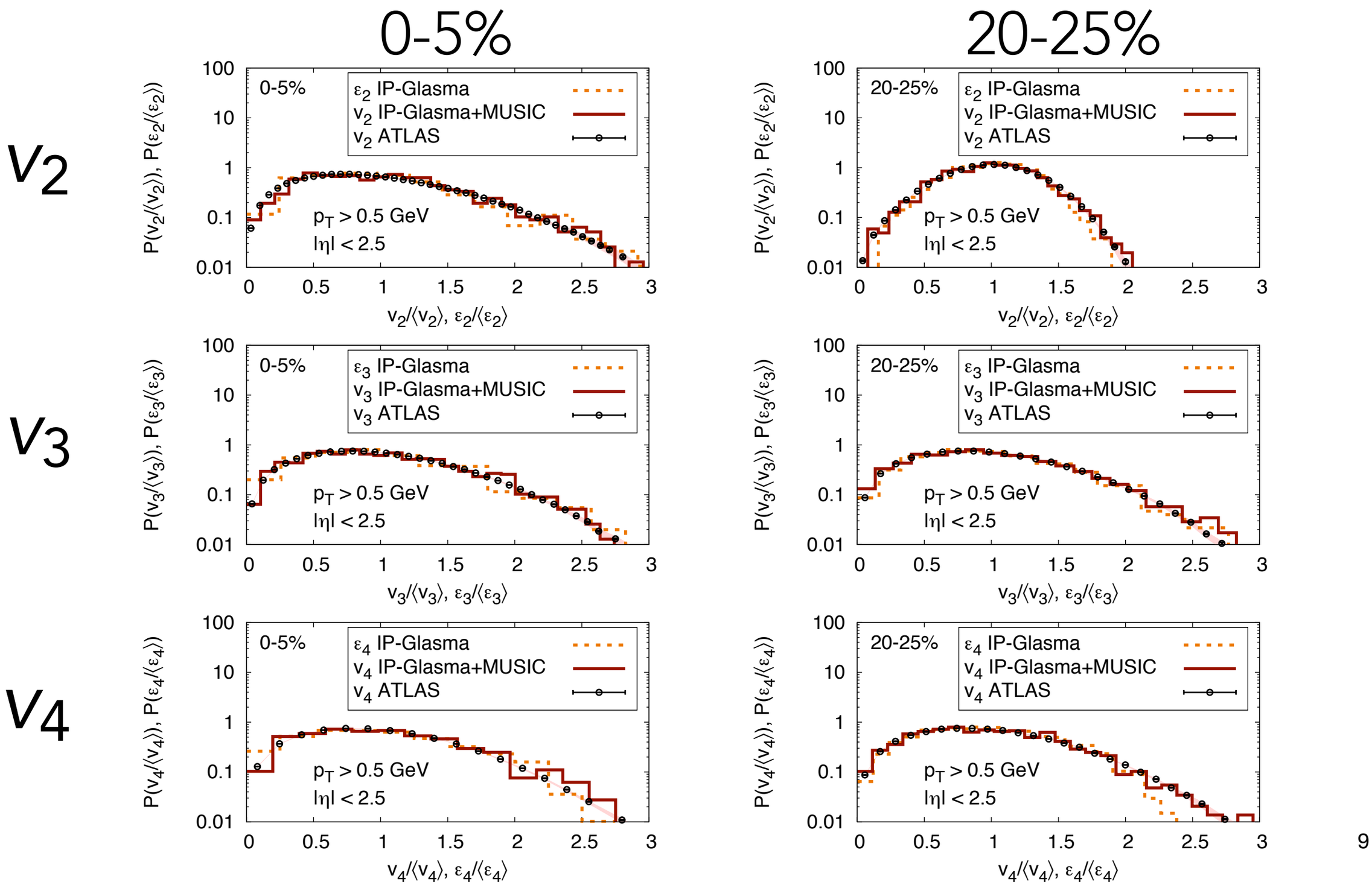
Results in Pb+Pb collisions: Average v_n

EXPERIMENTAL DATA: ATLAS COLLABORATION, JHEP 1311 (2013) 183
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301



Results in Pb+Pb collisions: Event-by-event v_n

ATLAS COLLABORATION, JHEP 1311 (2013) 183
C. GALE, S. JEON, B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PRL110, 012302 (2013)



Results in Pb+Pb collisions: Event-by-event v_n

ATLAS COLLABORATION, JHEP 1311 (2013) 183
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301

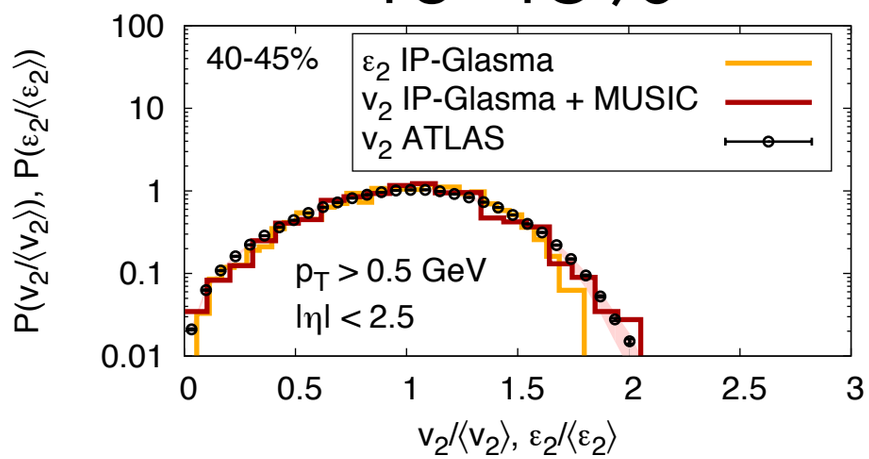
40-45%

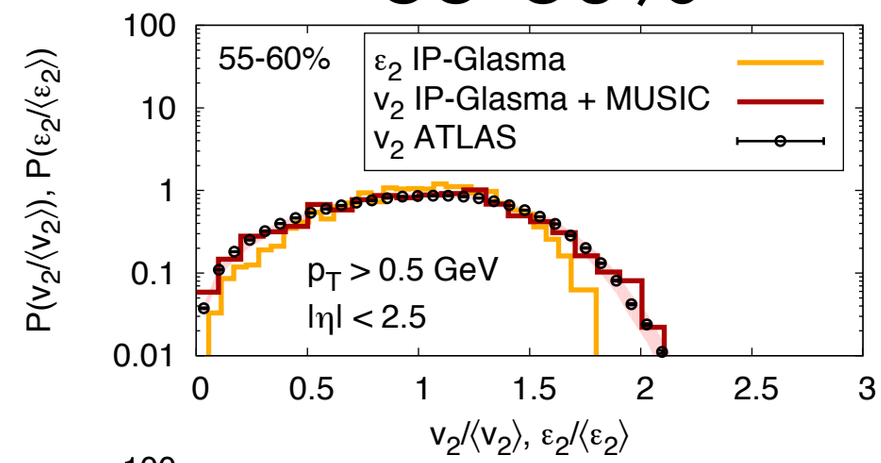
55-60%

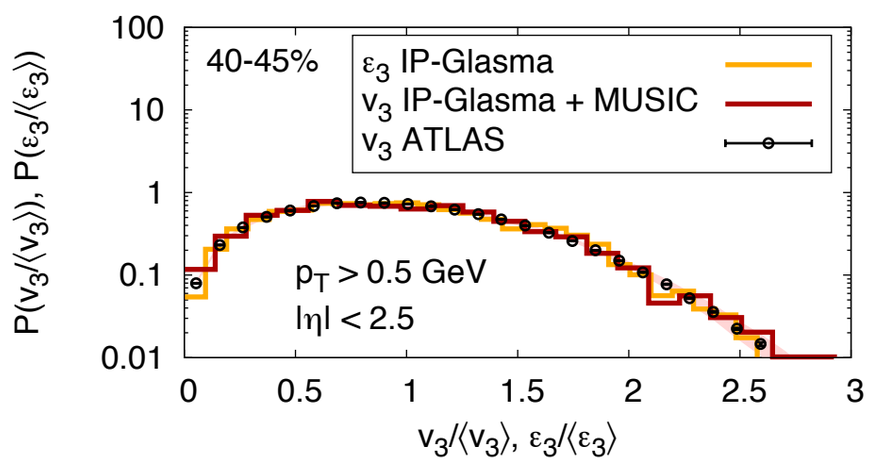
v_2

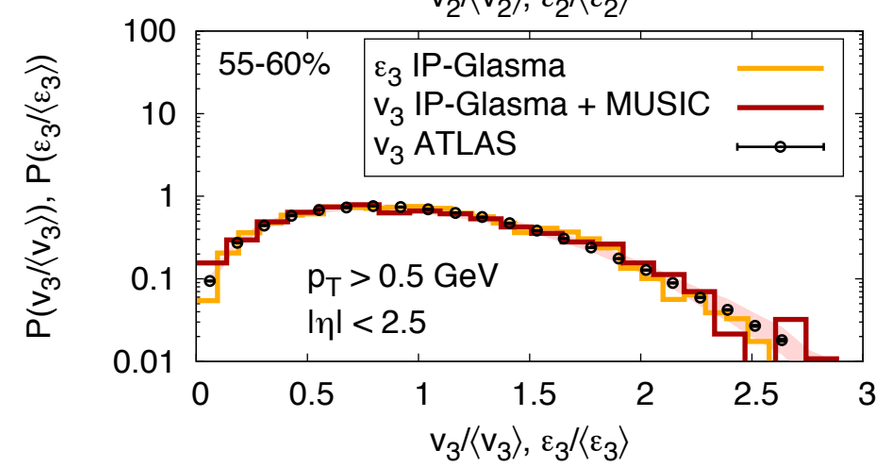
v_3

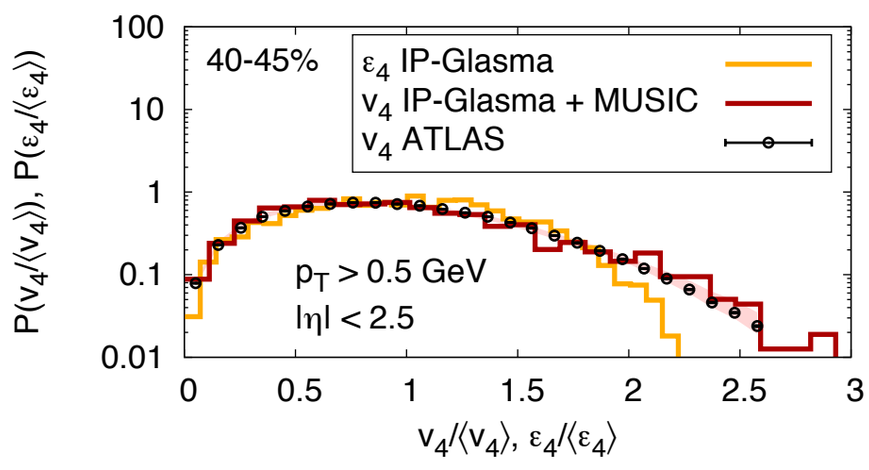
v_4









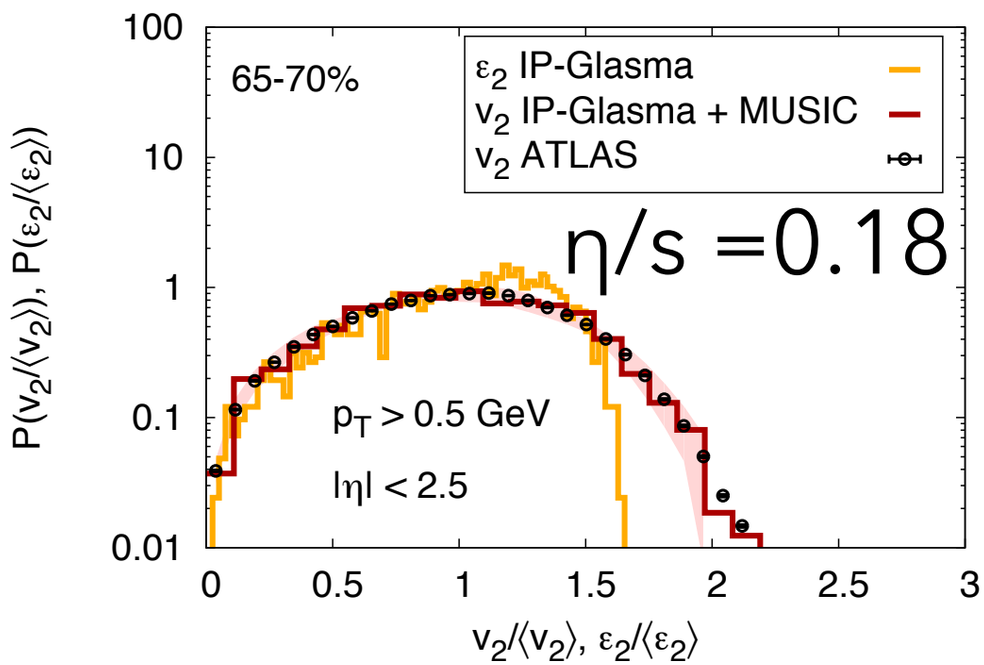


Results in Pb+Pb collisions: Event-by-event v_n

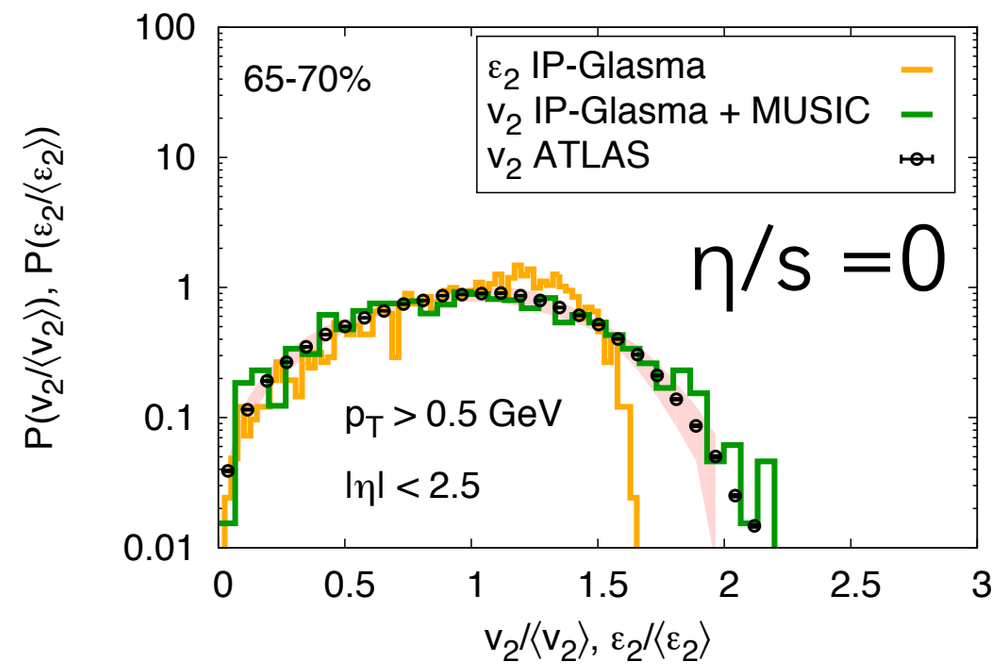
ATLAS COLLABORATION, JHEP 1311 (2013) 183
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301

v_2

65-70%



65-70%



Non-linearities in hydrodynamics are important
Detailed transport properties are not

So how do we do in small systems?

- **System size:**

  Systematics of initial size vs. HBT

- **v_n in p+Pb:**

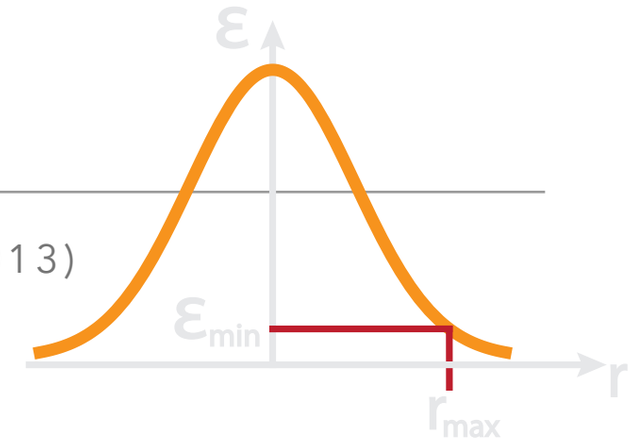
  Energy deposition, effect of proton shape

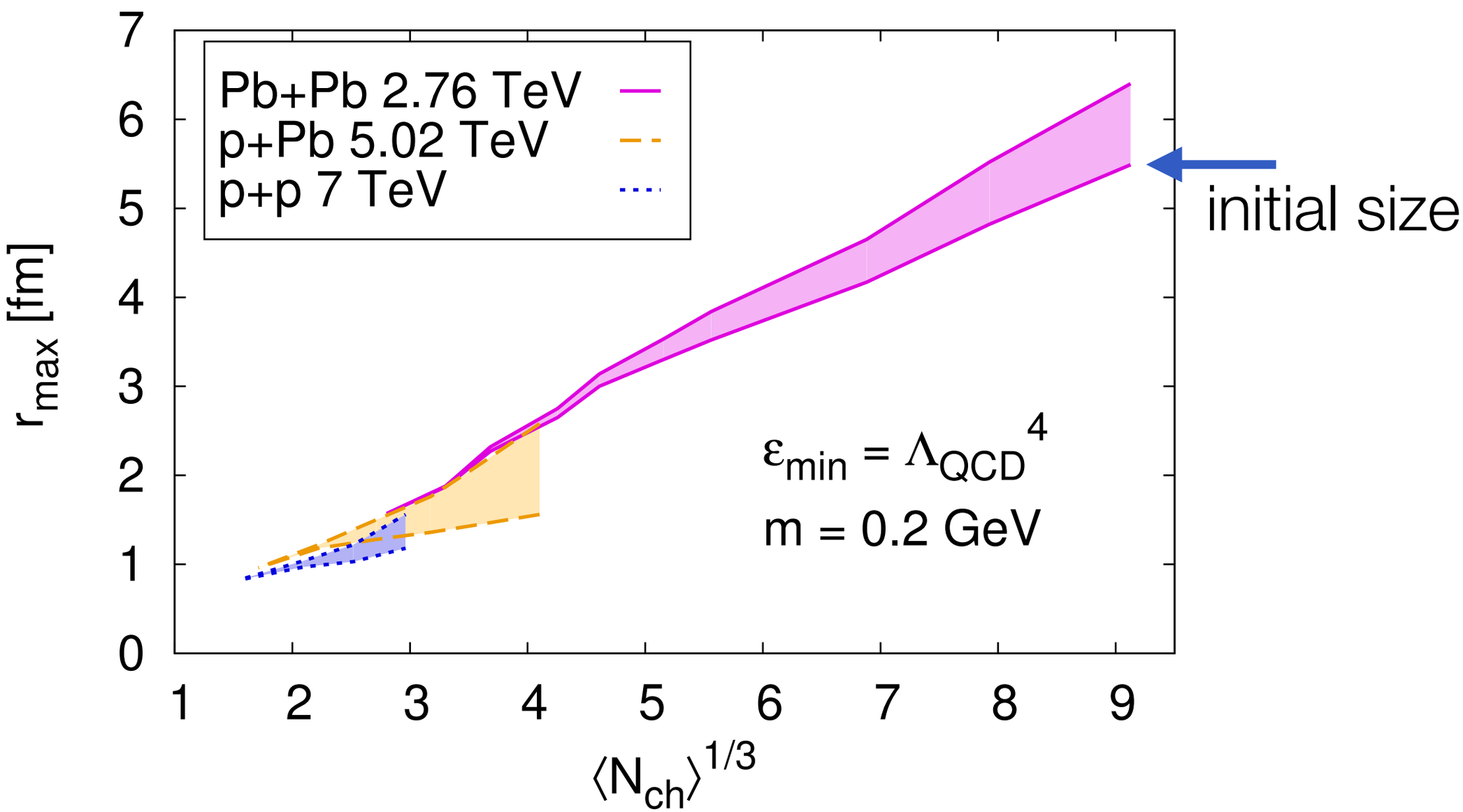
- **v_n in d+Au and ^{3}He+Au:**

  predictions and data comparison

System Size

A. BZDAK, B. SCHENKE, P. TRIBEDY, R. VENUGOPALAN, PRC87, 064906 (2013)
B. SCHENKE, R. VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301

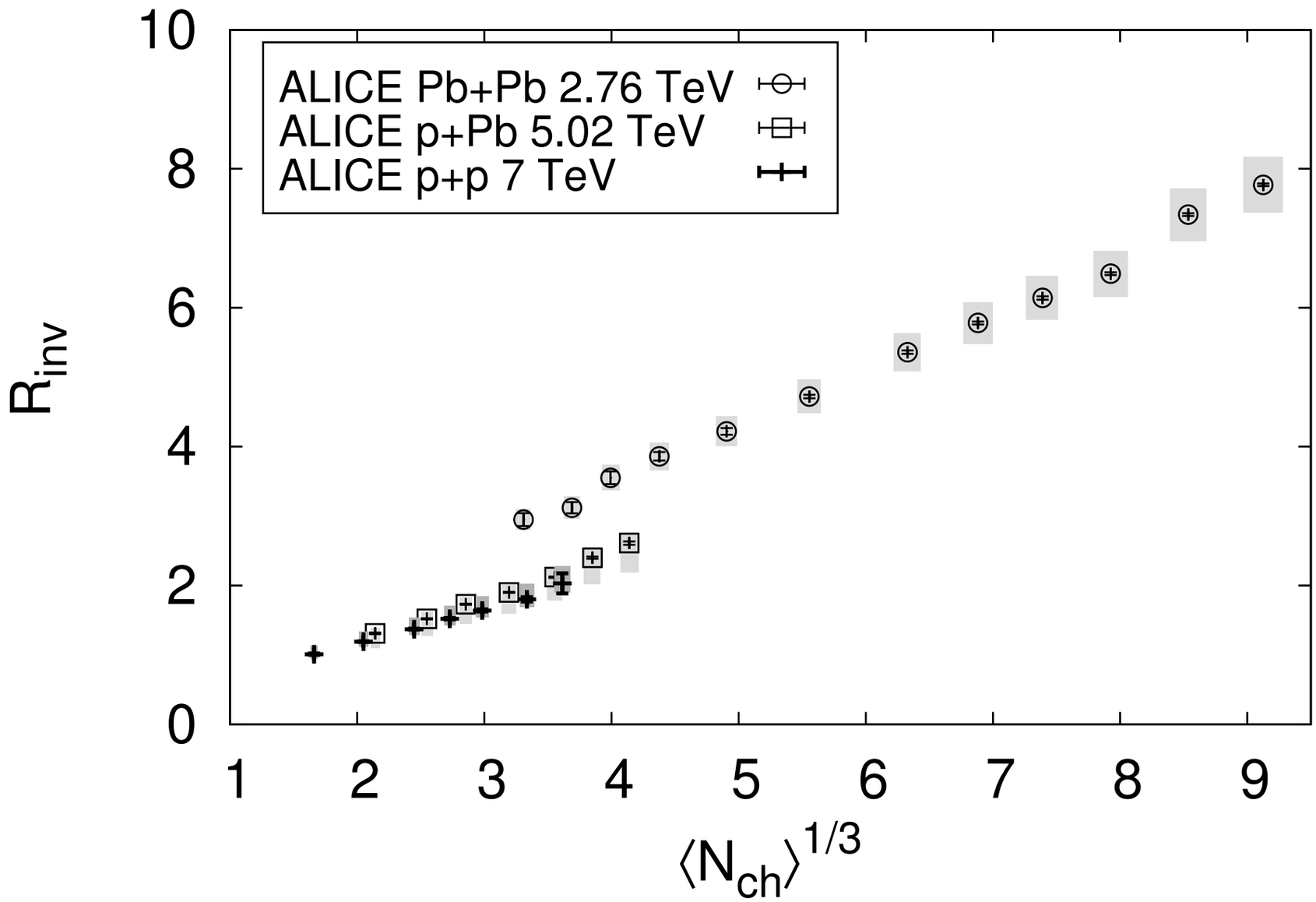




Initial size in p+Pb closer to that in p+p than p+Pb

System Size: Same trend seen in data

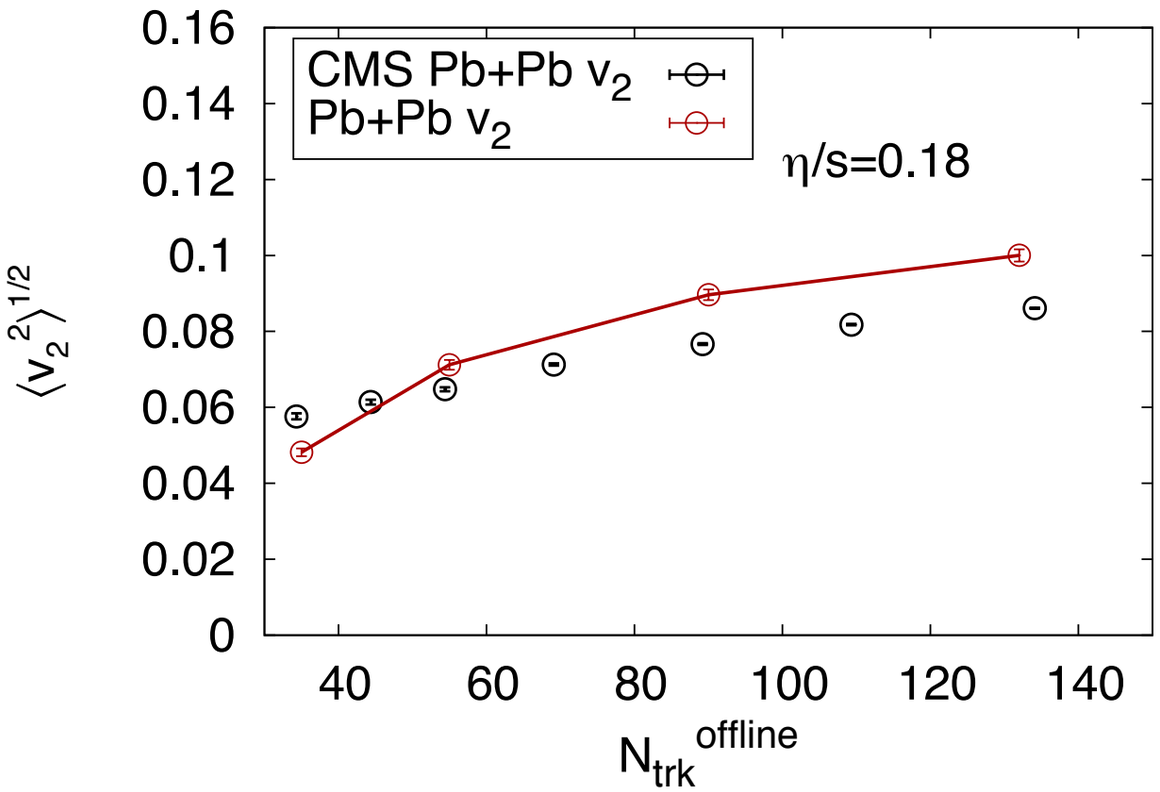
HBT DATA: ALICE COLLABORATION, PHYS.LETT. B739 (2014) 139-151

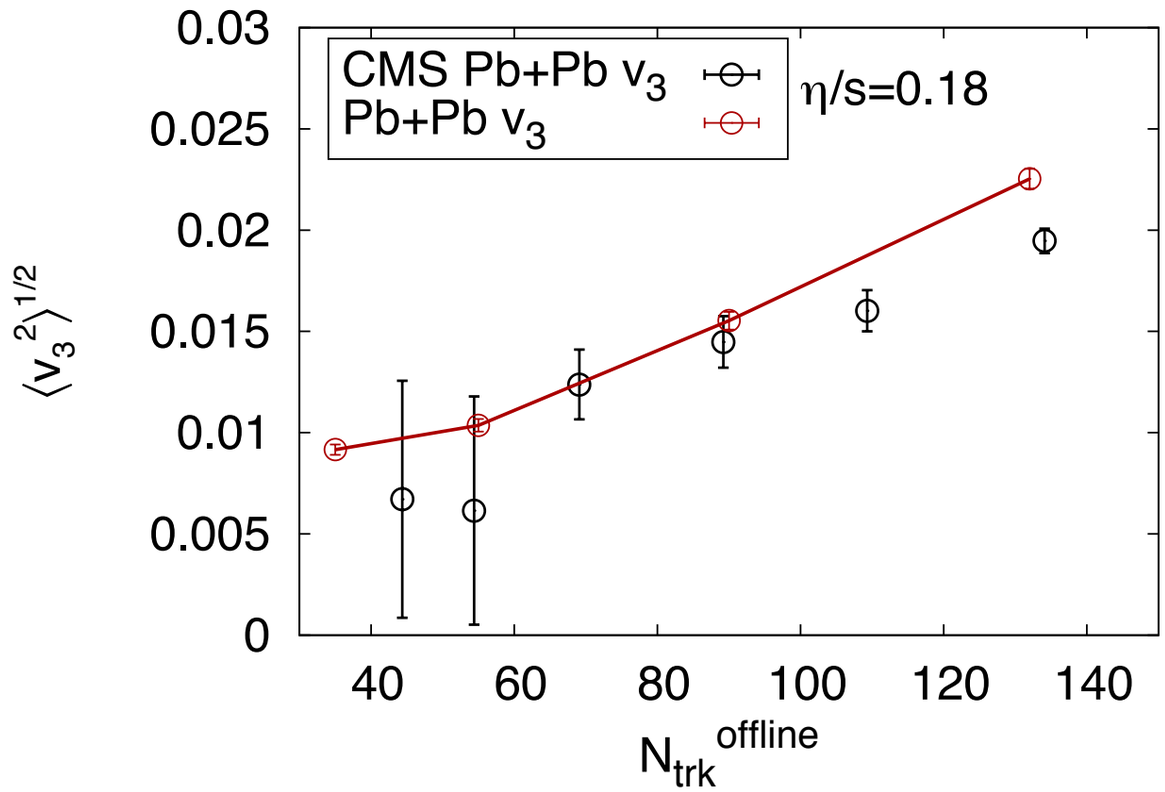


R_{inv} in p+Pb closer to that in p+p than p+Pb

Fourier Harmonics in p+Pb (and Pb+Pb)

CMS COLLABORATION, PHYS.LETT. B724 (2013) 213-240
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301



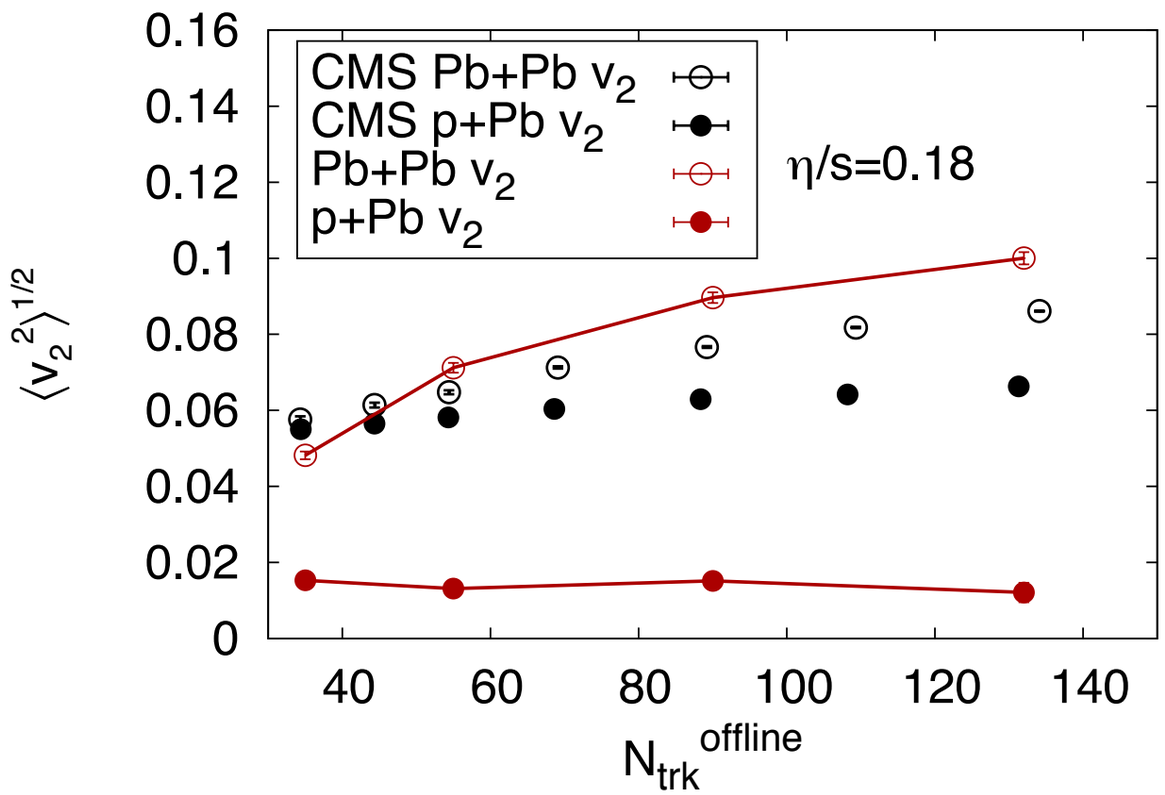


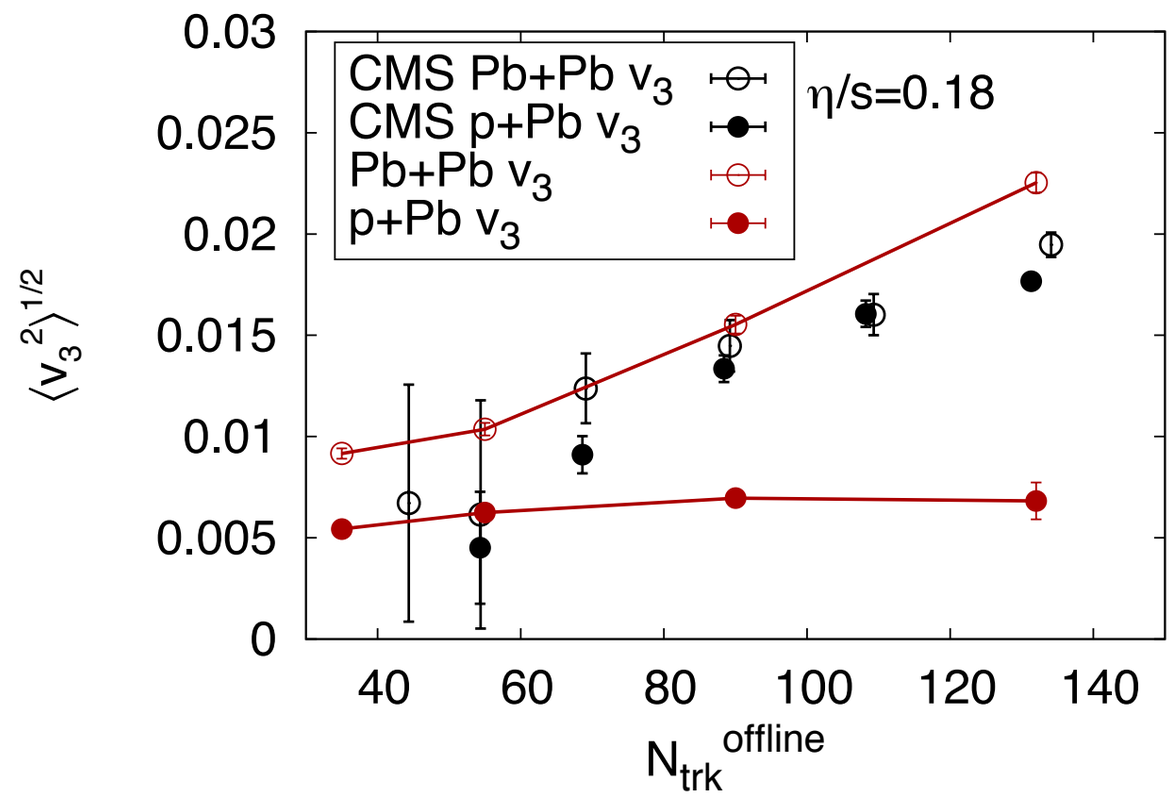
Open symbols: Pb+Pb

Red: IP-Glasma + MUSIC

Fourier Harmonics in p+Pb (and Pb+Pb)

CMS COLLABORATION, PHYS.LETT. B724 (2013) 213-240
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301





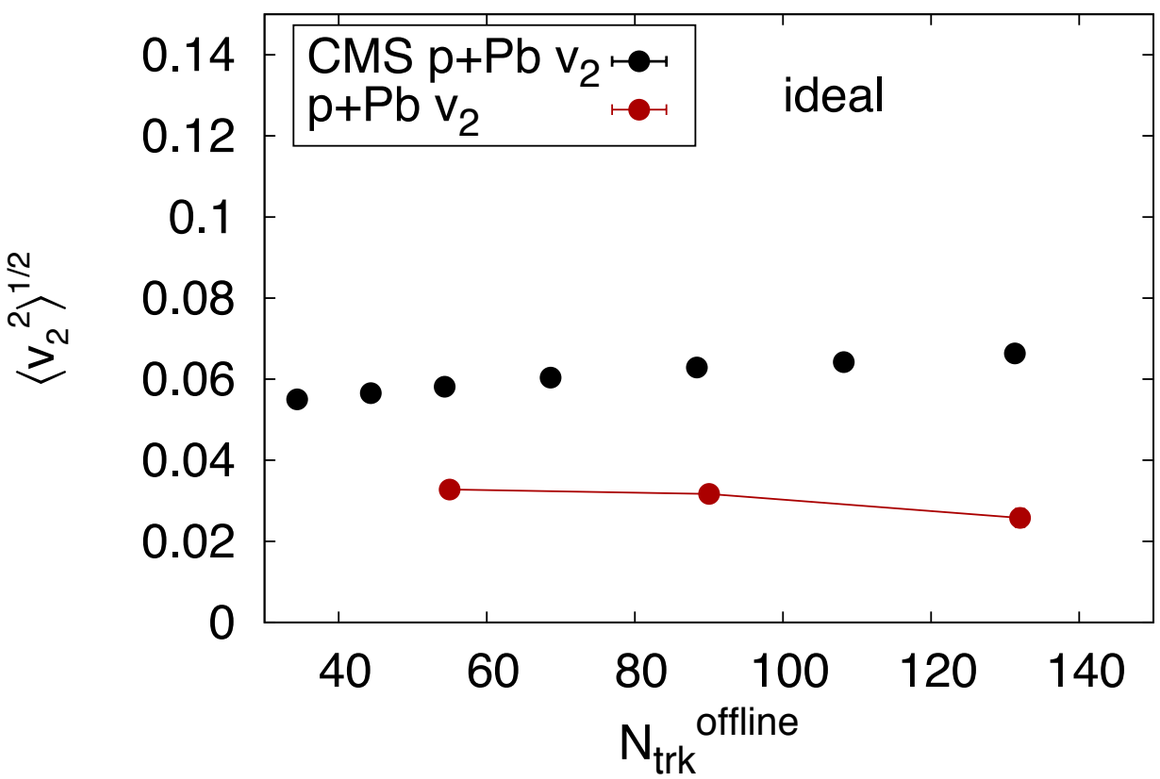
Open symbols: Pb+Pb

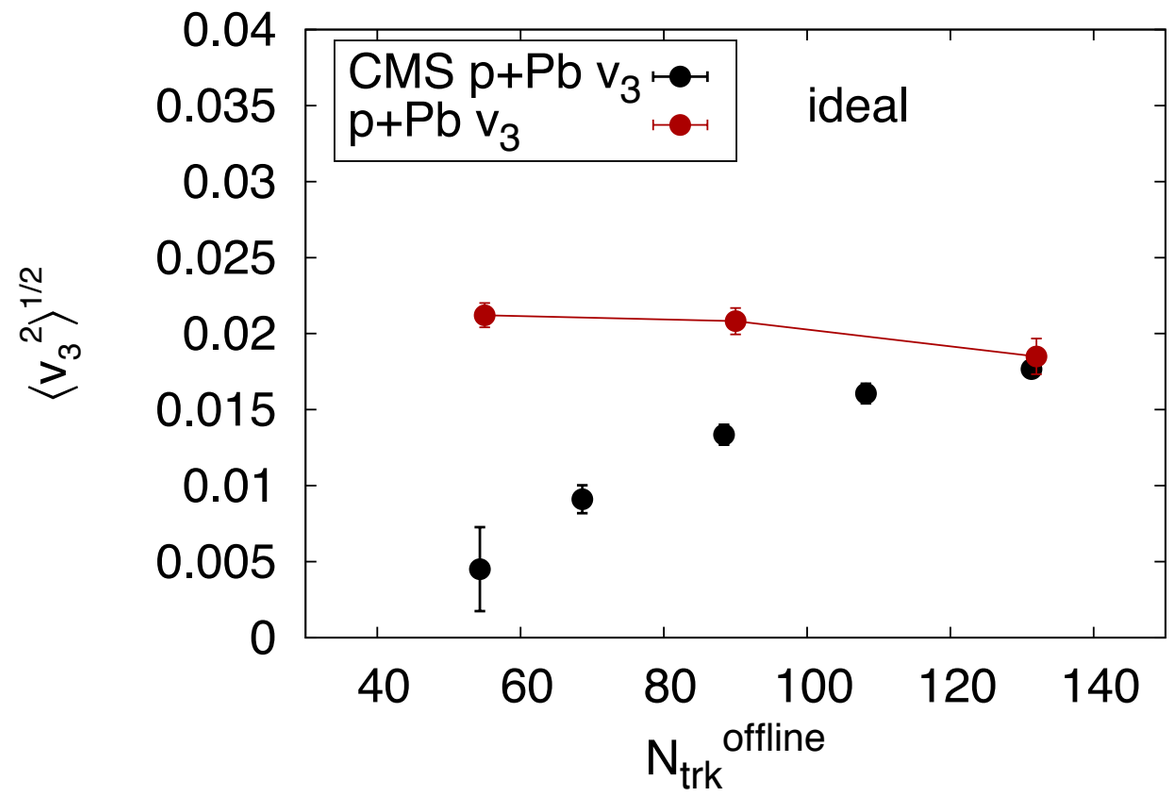
Filled symbols: p+Pb

Red: IP-Glasma + MUSIC

Fourier Harmonics in p+Pb

CMS COLLABORATION, PHYS.LETT. B724 (2013) 213-240
B.SCHENKE, R.VENUGOPALAN, PHYS. REV. LETT. 113 (2014) 102301





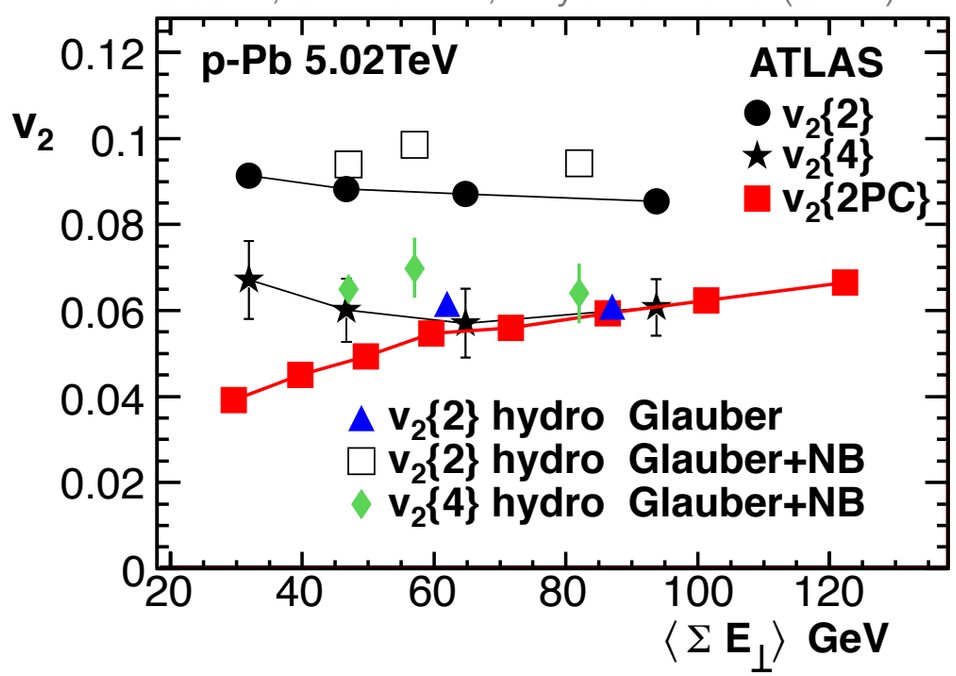
Filled symbols: p+Pb

Red: IP-Glasma + MUSIC

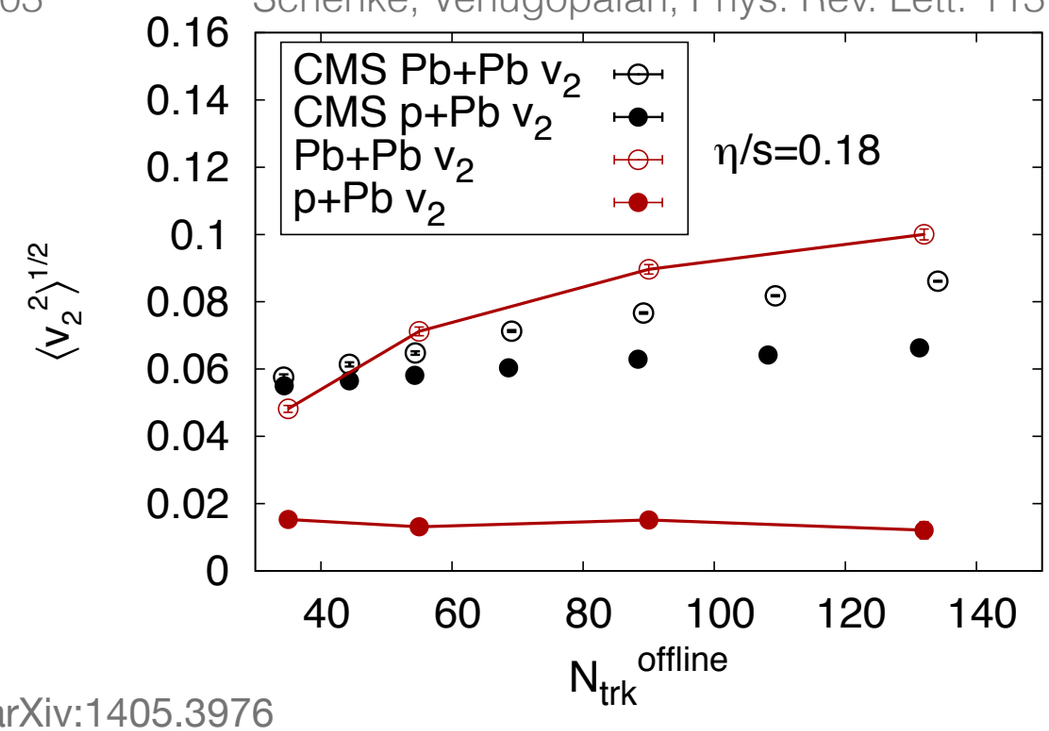
IP-Glasma + hydro does not work at all for p+Pb collisions

Difference to other hydro calculations: Initial state

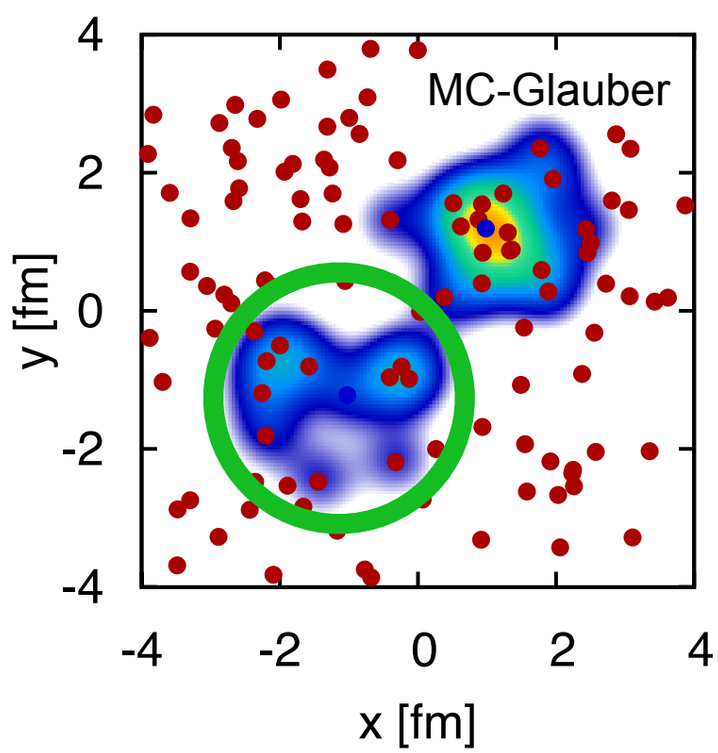
Bozek, Broniowski, Phys.Rev. C88 (2013) 014903

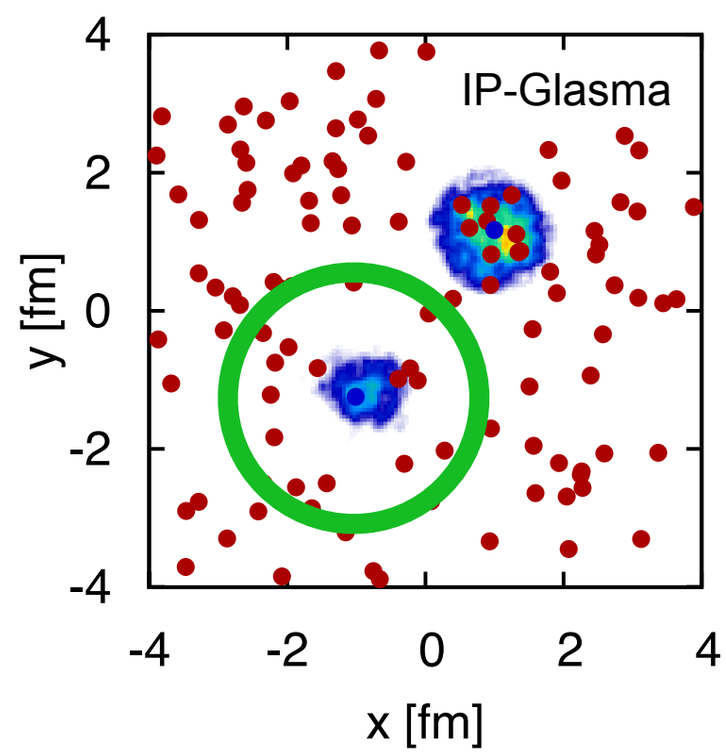


Schenke, Venugopalan, Phys. Rev. Lett. 113 (2014) 102301



see also: Kozlov, Luzum, Denicol, Jeon, Gale, arXiv:1405.3976

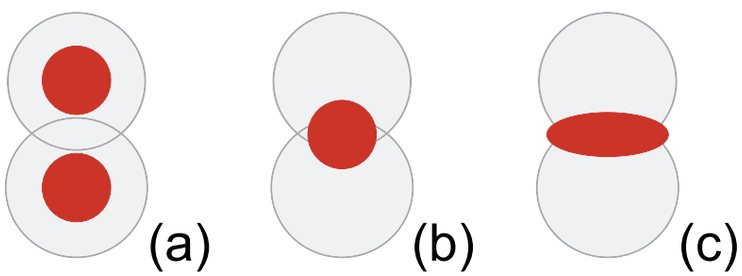




d+Au

Issues in small systems (p+p, p+A)

- MC-Glauber does not constrain energy density dist.



Where do we put the energy?

What shape does it have?

- IP-Glasma constrains energy density deposition

  However, it does not describe v_n in p+Pb

- Proton substructure should matter

  (if main effect is of collective origin or not)

- Combine constituent quark model with JIMWLK evolution to get proton structure at small *x*

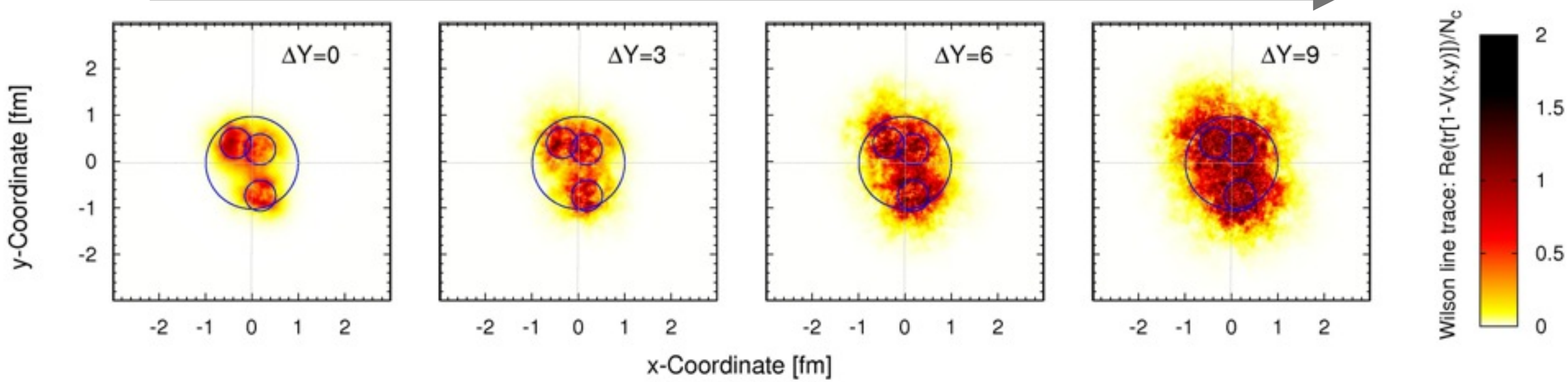
SCHENKE, SCHLICHTING, PHYS. LETT. B739, 313-319 (2014)

Are we sensitive to the shape of the proton?

S. SCHLICHTING, B. SCHENKE, PHYS. LETT. B739, 313-319 (2014)

- Three "Constituent quarks" at large *x*
- JIMWLK evolution with infrared regulator to get gluon distribution at smaller *x*

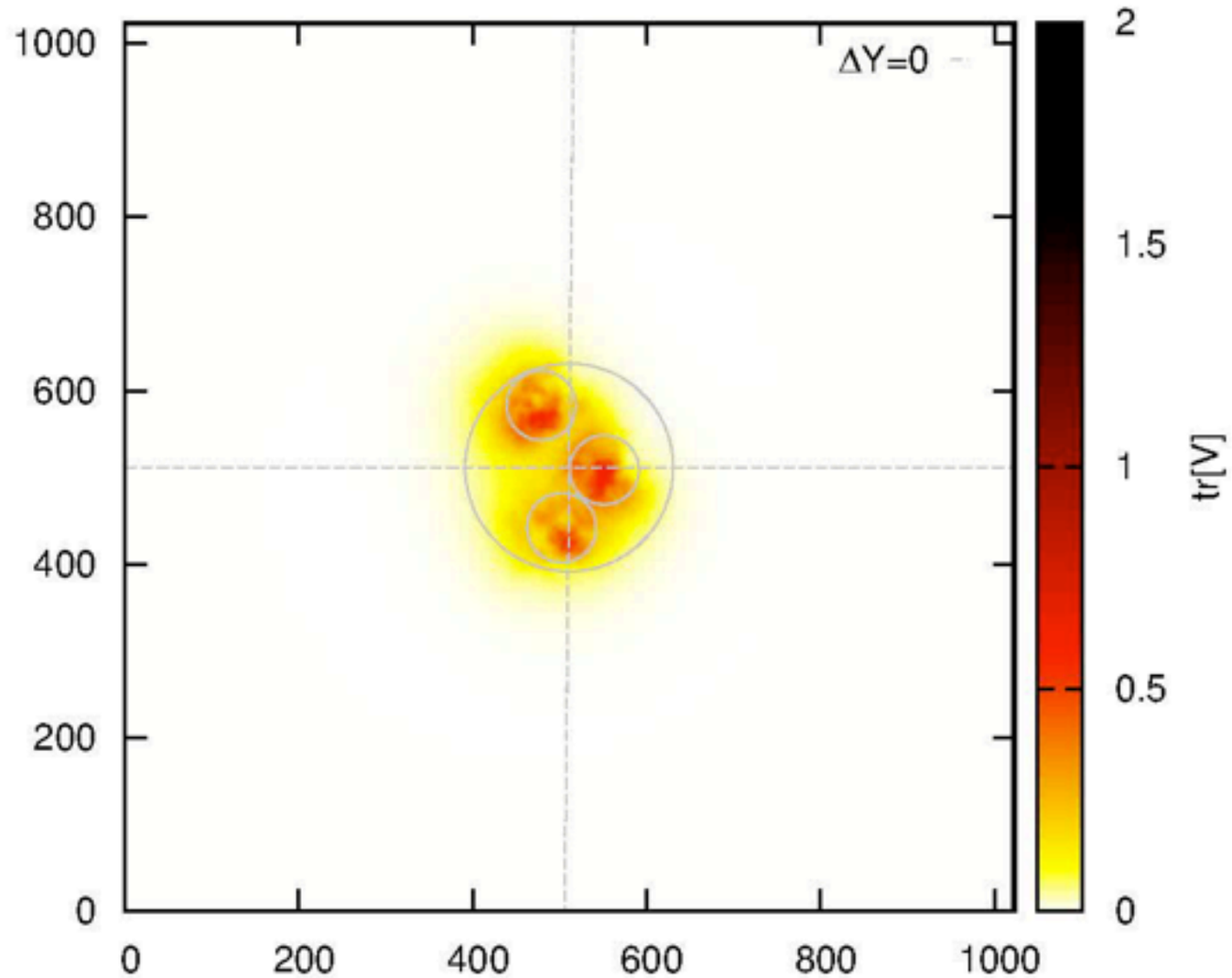
JIMWLK evolution: decreasing *x*



Even at small *x* the proton is not a sphere of gluons

JIMWLK evolution of a “lumpy proton”

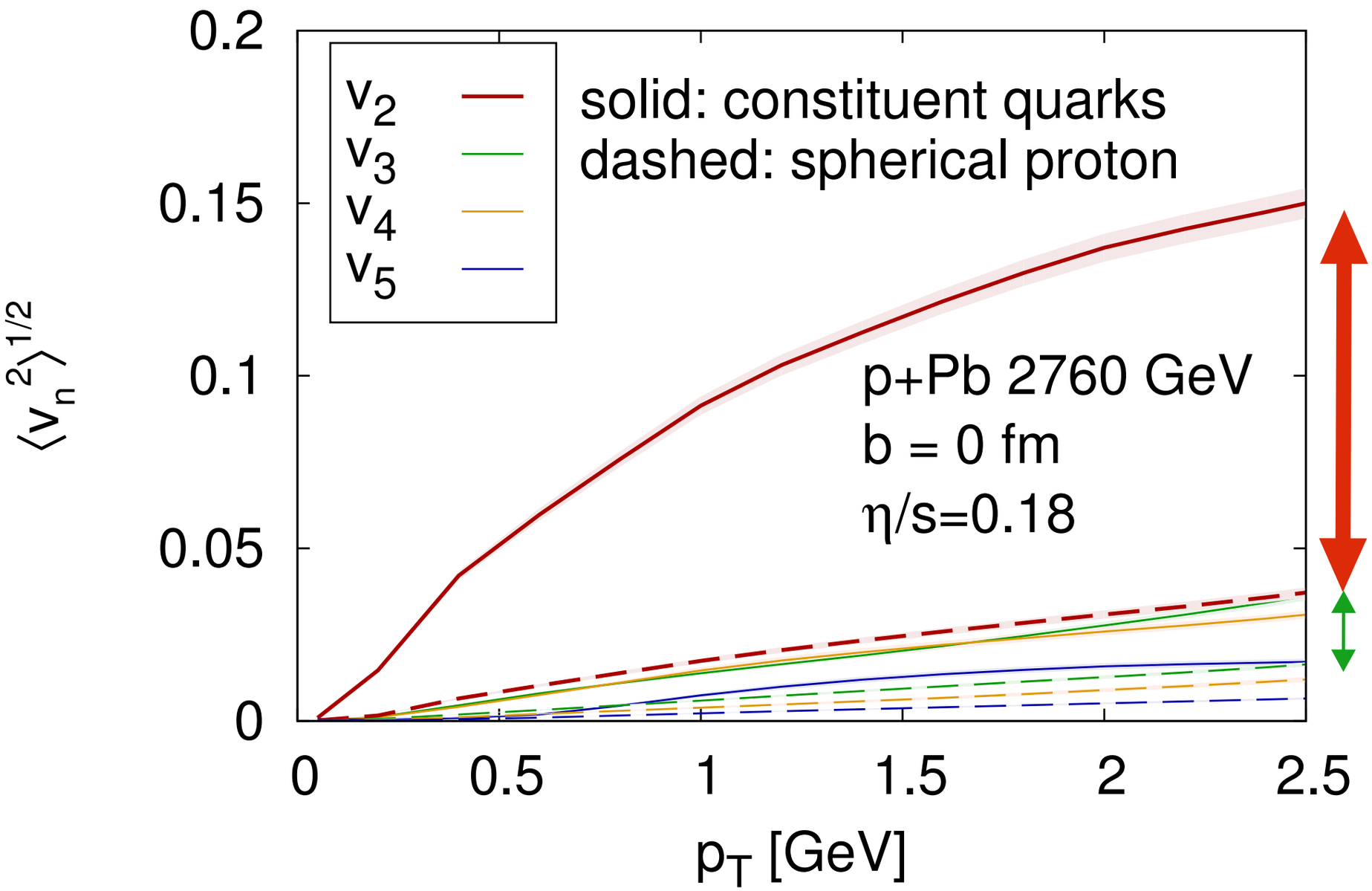
S. SCHLICHTING, B. SCHENKE, PHYS. LETT. B739, 313-319 (2014)



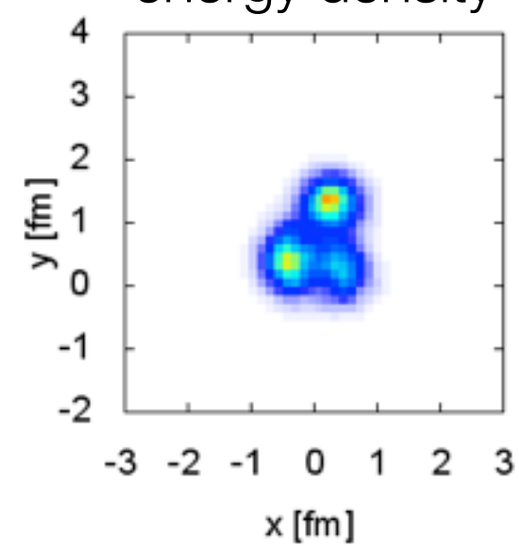
FIXED COUPLING JIMWLK EVOLUTION OF TRACE OF THE WILSON LINE

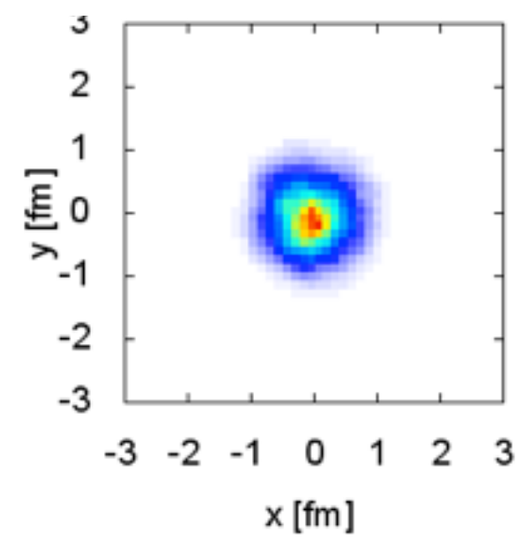
Round vs. structured proton: IP-Glasma + MUSIC

It makes a huge difference!

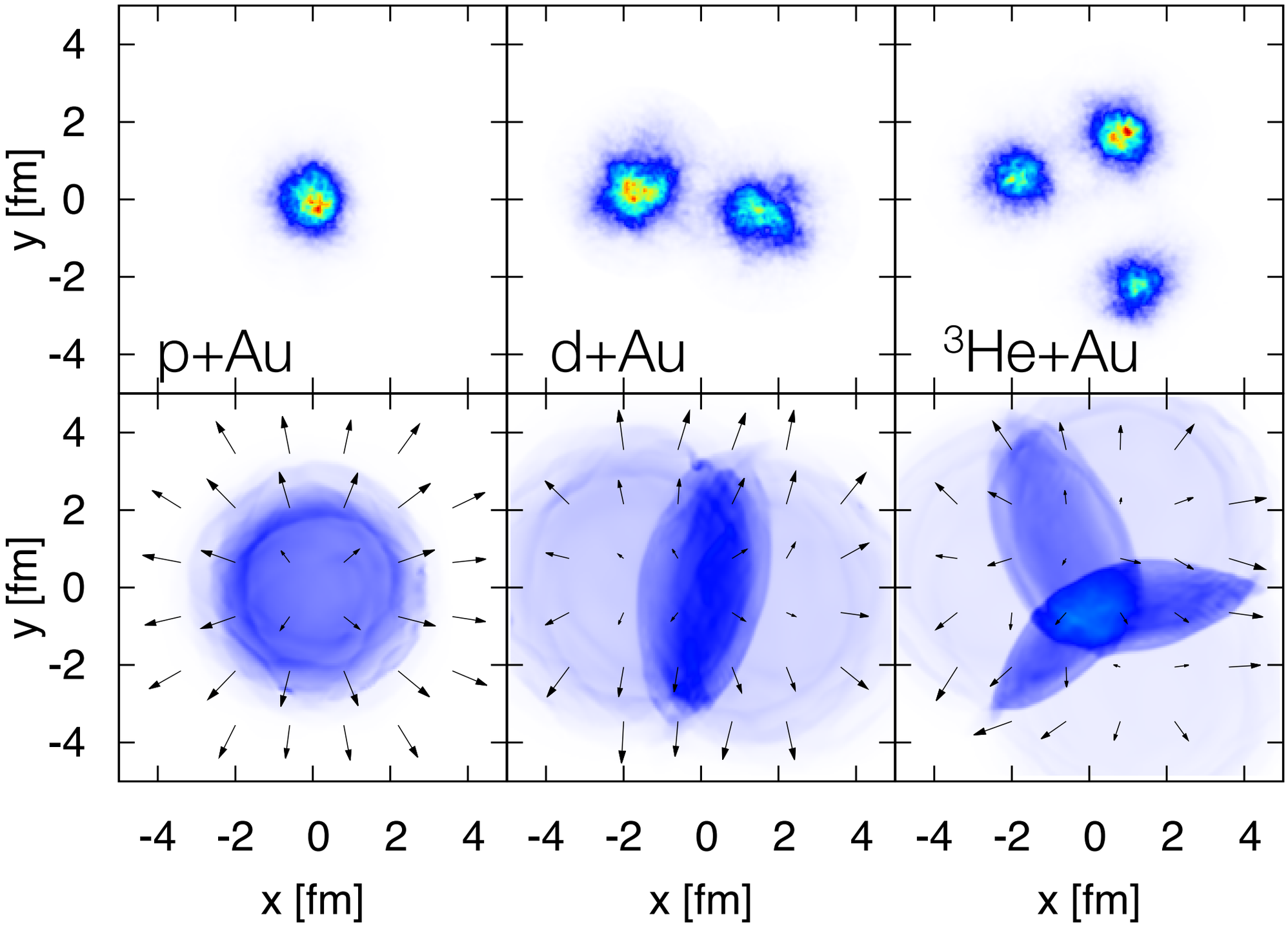


initial energy density





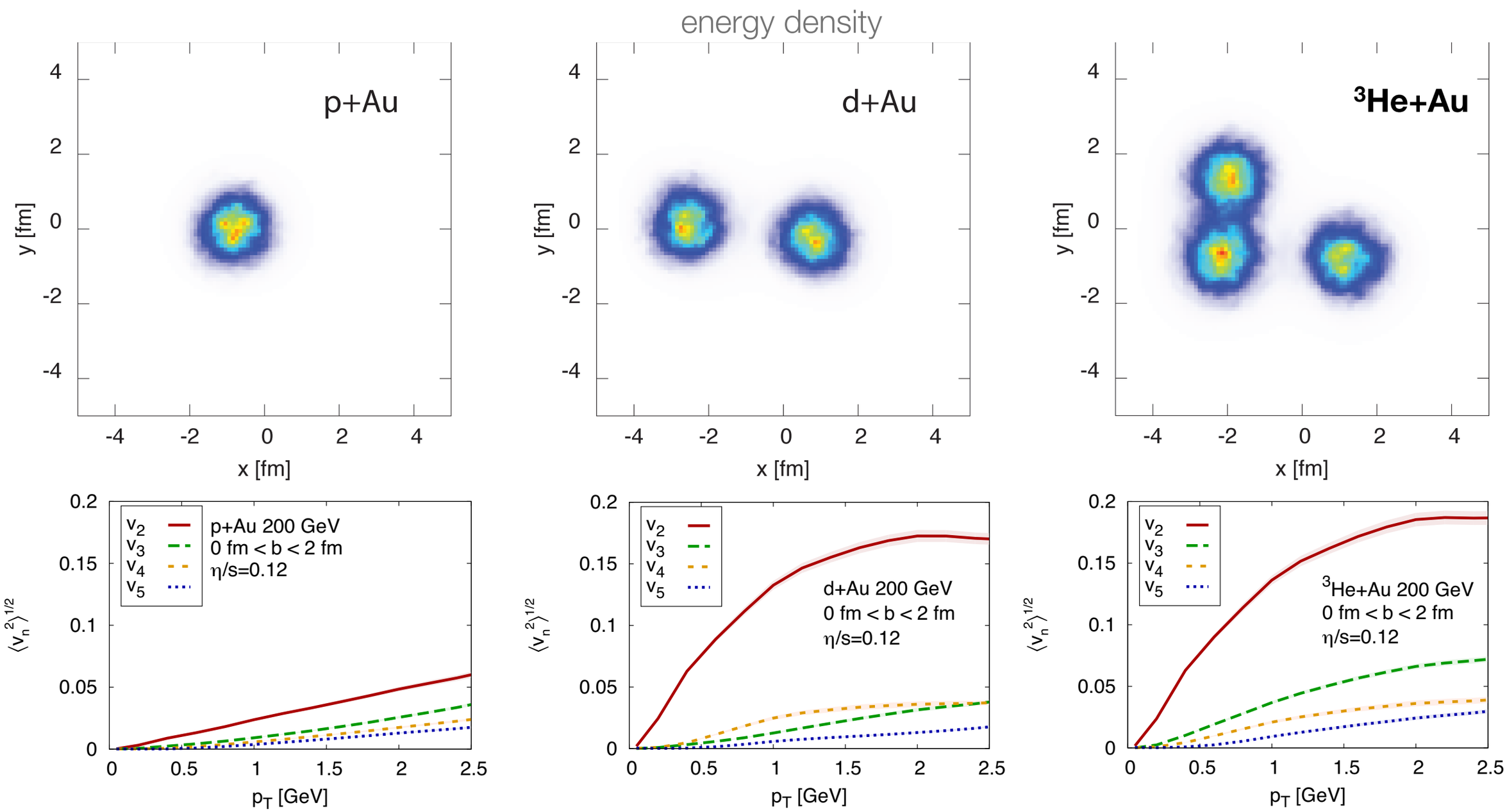
Various small collision systems: IP-Glasma + MUSIC



^{3}He+Au results from IP-Glasma + MUSIC

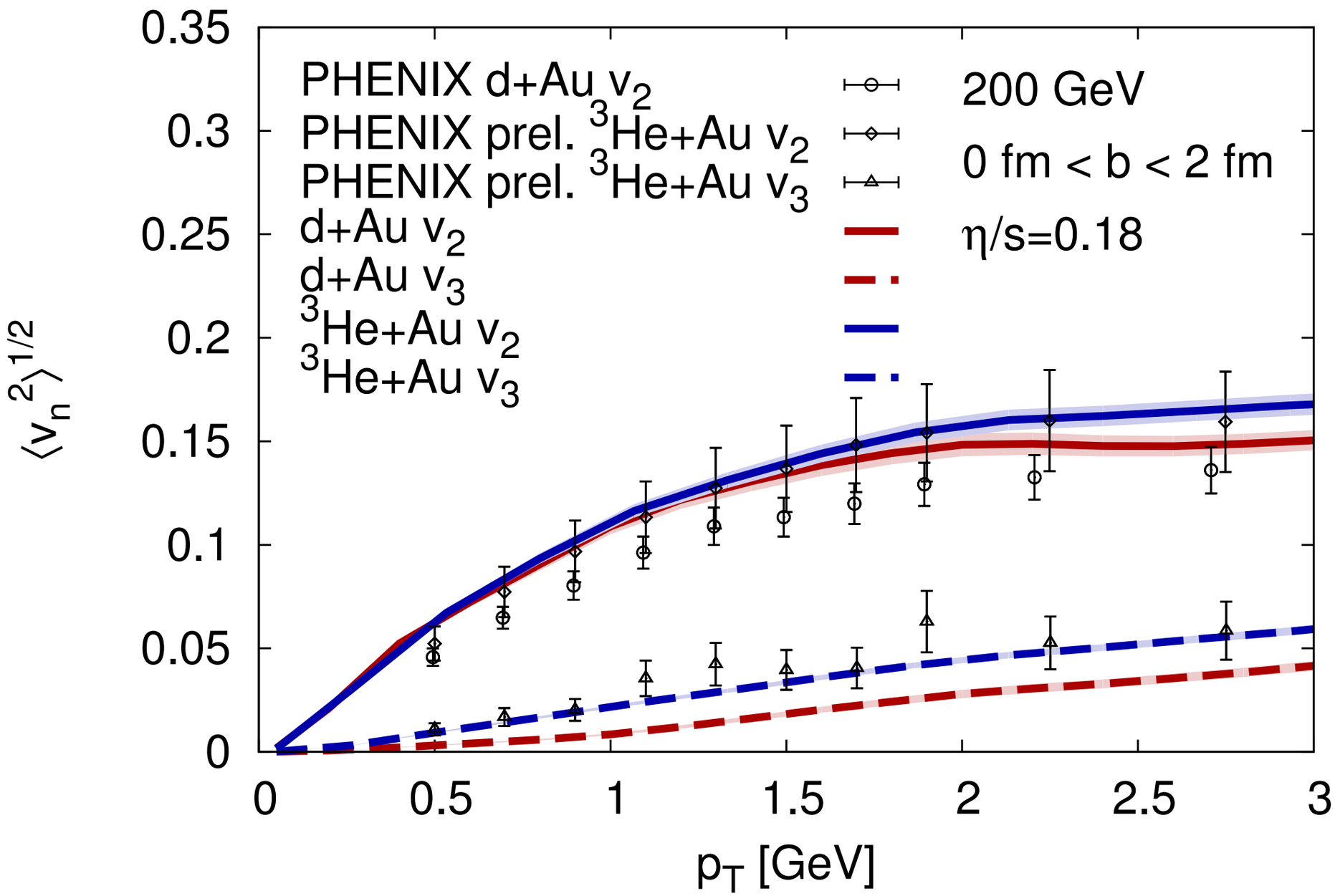
B. SCHENKE, R. VENUGOPALAN, NUCL.PHYS. A931 (2014) 1039-1044

Predictions for η/s = 0.12:



^{3}He+Au results from IP-Glasma + MUSIC

Adjust parameter: η/s = 0.18 - compare d+Au and ^{3}He+Au



Note: No realistic centrality selection

Conclusions

- Relativistic fluid dynamics has been very successful in describing bulk properties of heavy ion collisions
- In p+A collisions hydro results are highly sensitive to the substructure of the proton
- In small systems there are also contributions to anisotropic particle production from the initial state (see S. Schlichting's talk)
- However, they may not survive the final state interactions
- If so, the final state interactions fully account for the v_n's and measurements can provide information on the substructure of the proton

Initial state and azimuthal correlations in p+A

Soeren Schlichting
Brookhaven National Lab

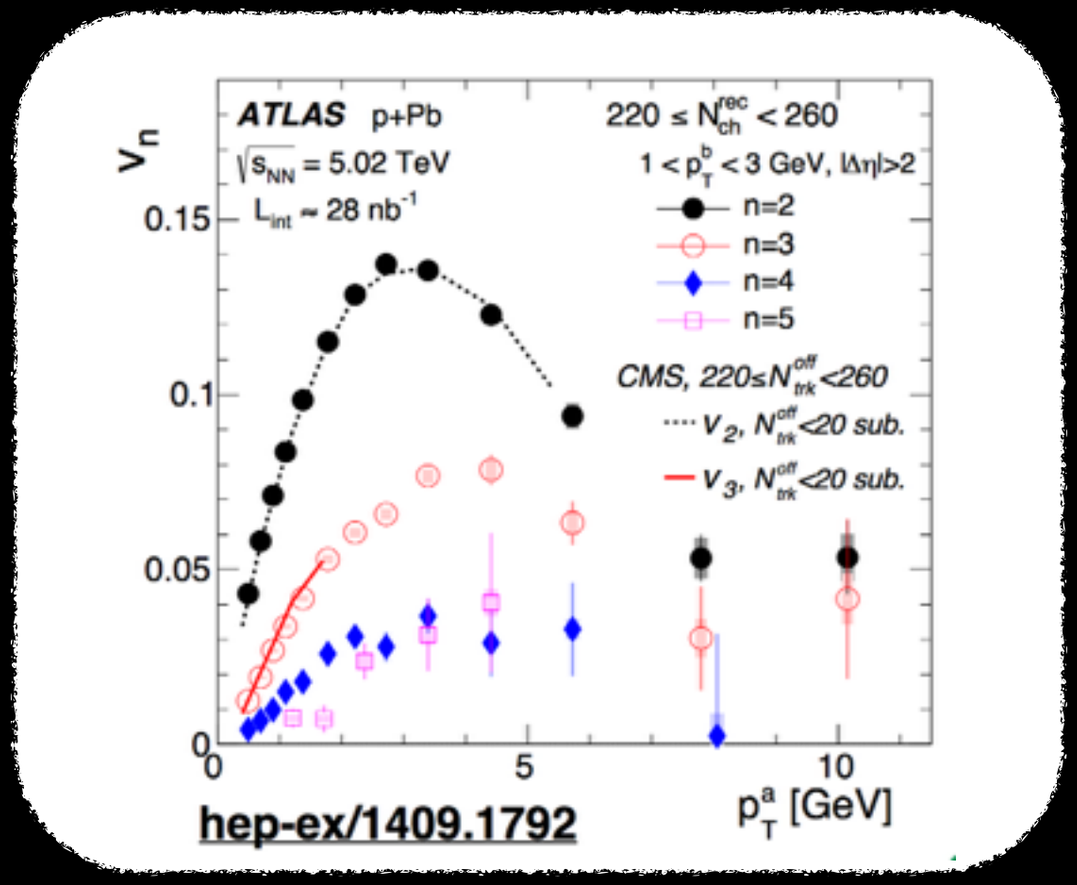
Based on work in collaboration with
B. Schenke,R. Venugopalan
(PLB 739 (2014) 313-319 & arXiv:1502.01331)

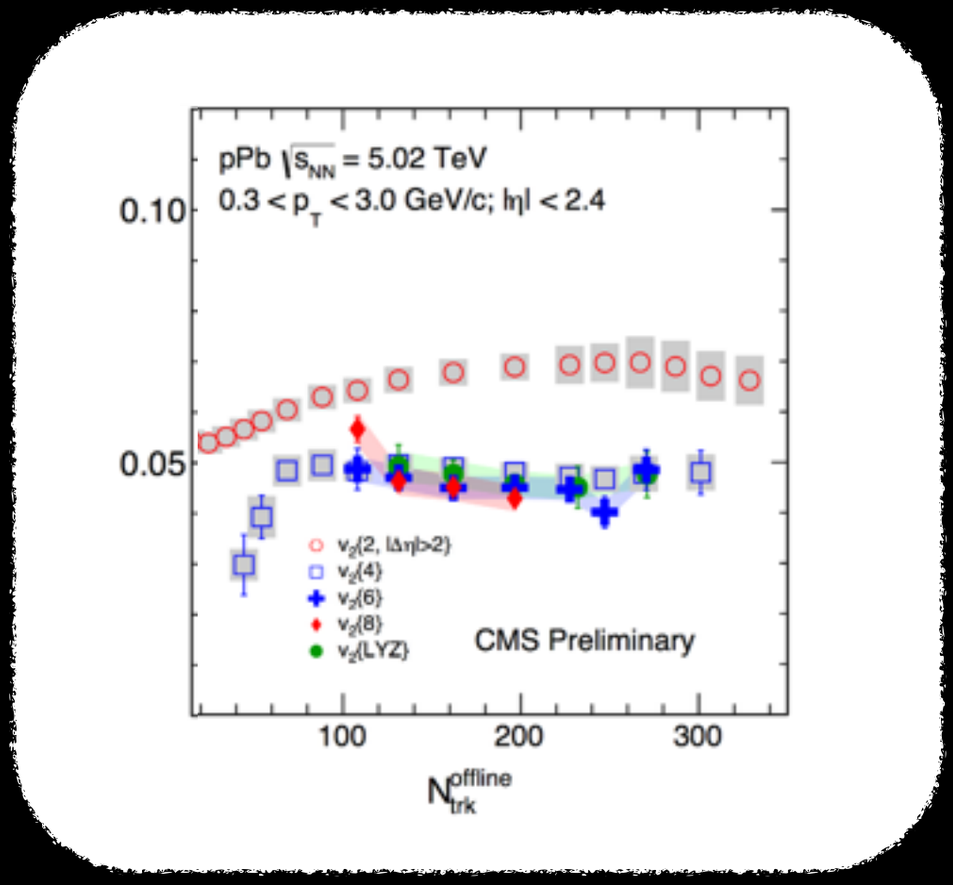
RBRC Workshop Mar 2015, Brookhaven

BROOKHAVEN
NATIONAL LABORATORY

p+A collisions

Strong azimuthal correlations observed in high multiplicity events





- Correlation extends up to high p_T~10GeV
- Correlation between many particles (>8) observed

-> Event-by-event particle production appears to be anisotropic
— Spontaneous breaking of rotational symmetry

Possible explanations for p+A

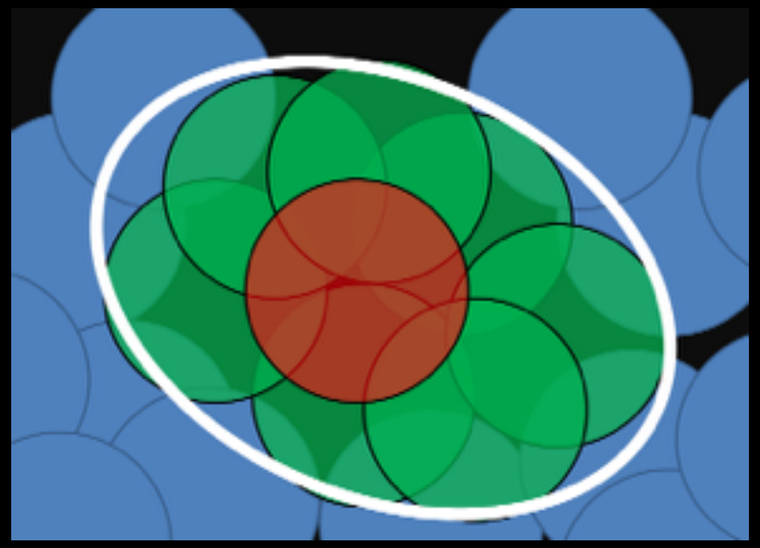
Hydrodynamics?

- Some hydrodynamic calculations are able to reproduce experimental data *(talks by Bozek, Kozlov)*

- *But:* Viscous corrections more problematic for small systems due to larger gradients.

- Strong sensitivity to initial state models. *(talk by Schenke)*

- Problems in describing the high pT data.



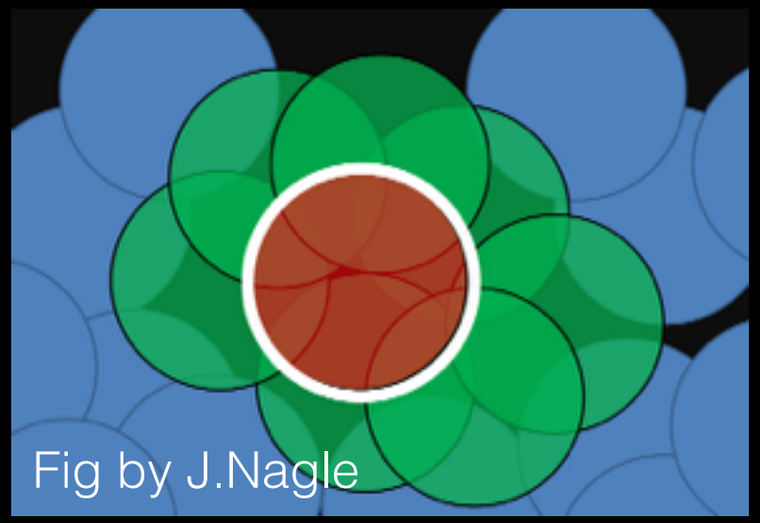


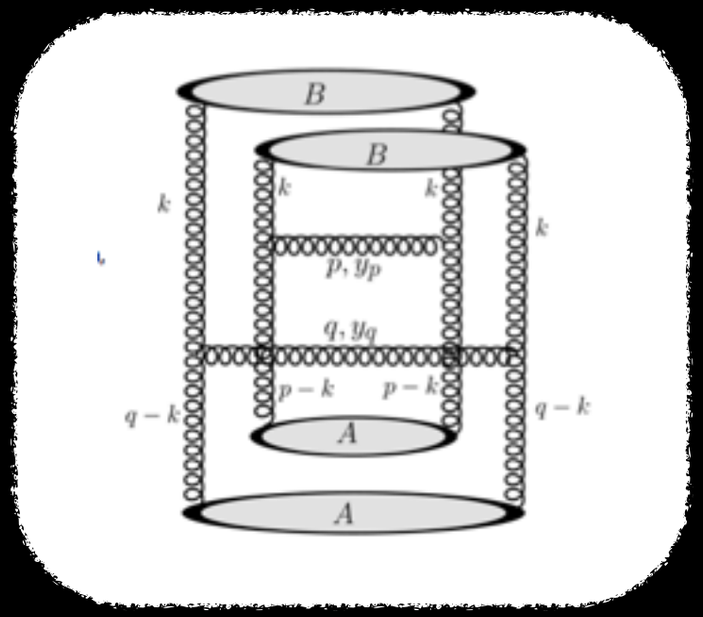
Fig by J.Nagle

Possible explanations for p+A

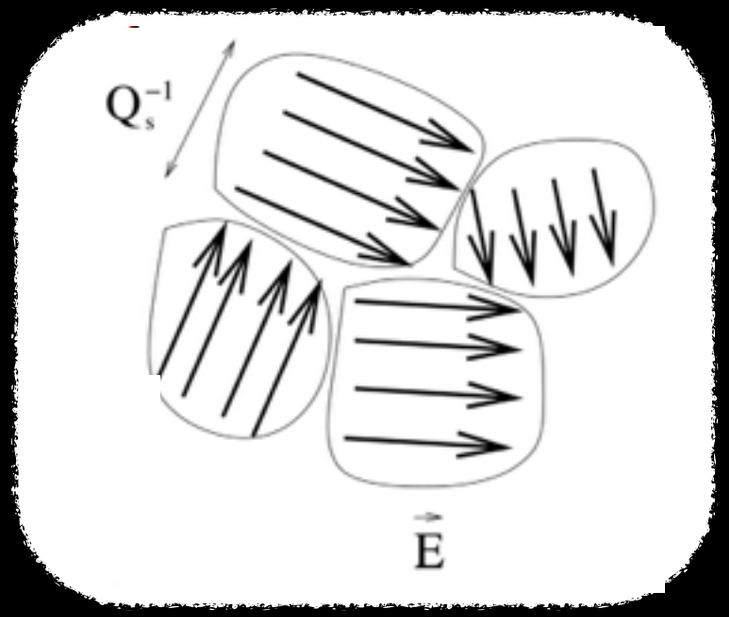
Initial state effects?

- In principle also present in A+A but more pronounced in p+A

  Correlated production mechanism — 'Glasma graphs'?
  (Dusling, Venugopalan,…)



  Breaking of rotational symmetry due to domains of color electric field inside the nucleus?
  (Kovner,Lublinsky; Dumitru, Giannini,…)



- Problems in describing odd harmonics.

Possible explanations for p+A

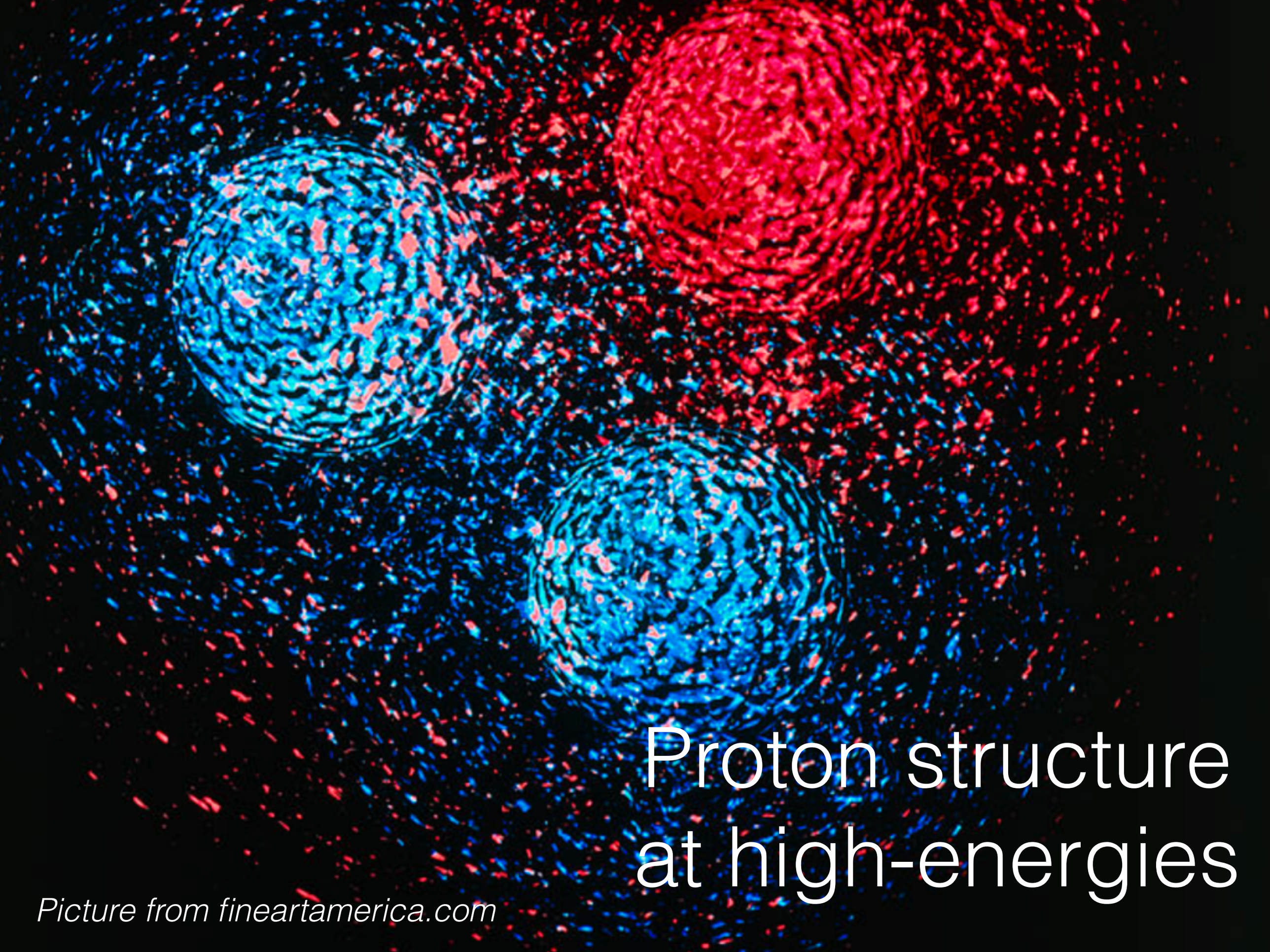
Different mechanisms have been proposed to explain the data

- ***Initial state interactions*** ***and/or***
- ***Initial state geometry + final state interactions***

Irrespective of the mechanism, one needs to understand

I) Nucleon structure and fluctuations on sub-nucleonic scales

II) Initial state dynamics up to τ~1 fm/c



Proton structure at high-energies

Picture from fineartamerica.com

Event-by-event fluctuations

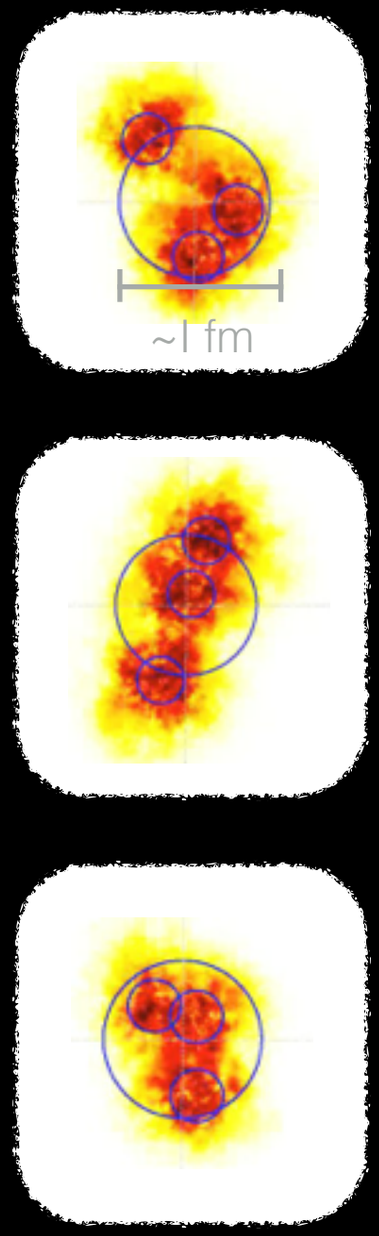
- High-energy scattering of projectile and target is almost instantaneous.

- Semi-classical picture: In each event the projectile resolves a particular realization (snapshot) of the target wave-function.

- Convenient variables to describe nucleon structure are the light-like Wilson lines

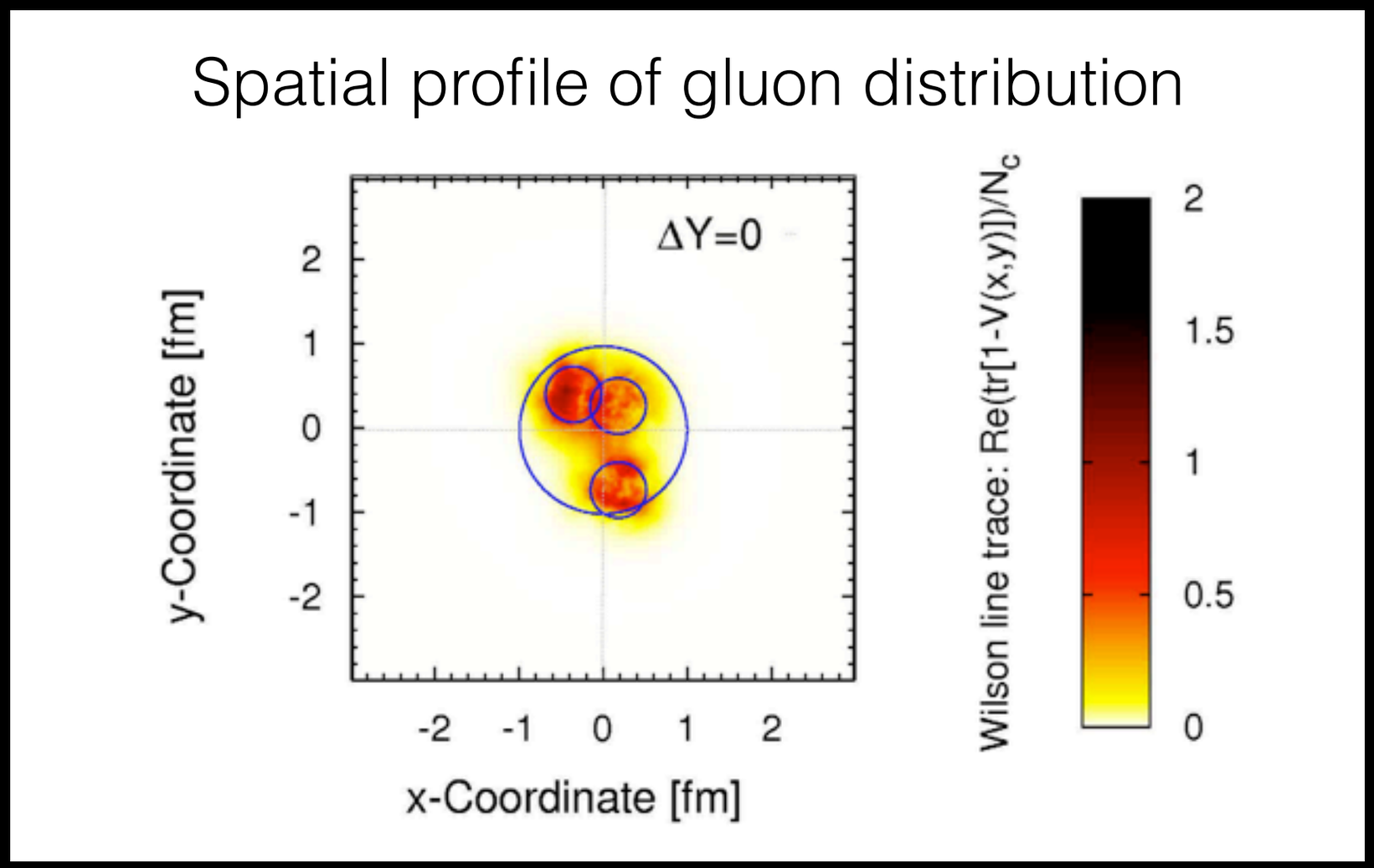
$$V_x = \mathcal{P}\, e^{-ig \int dx^- A^+}$$

(corresponds to forward scattering amplitude of a quark in eikonal approximation)



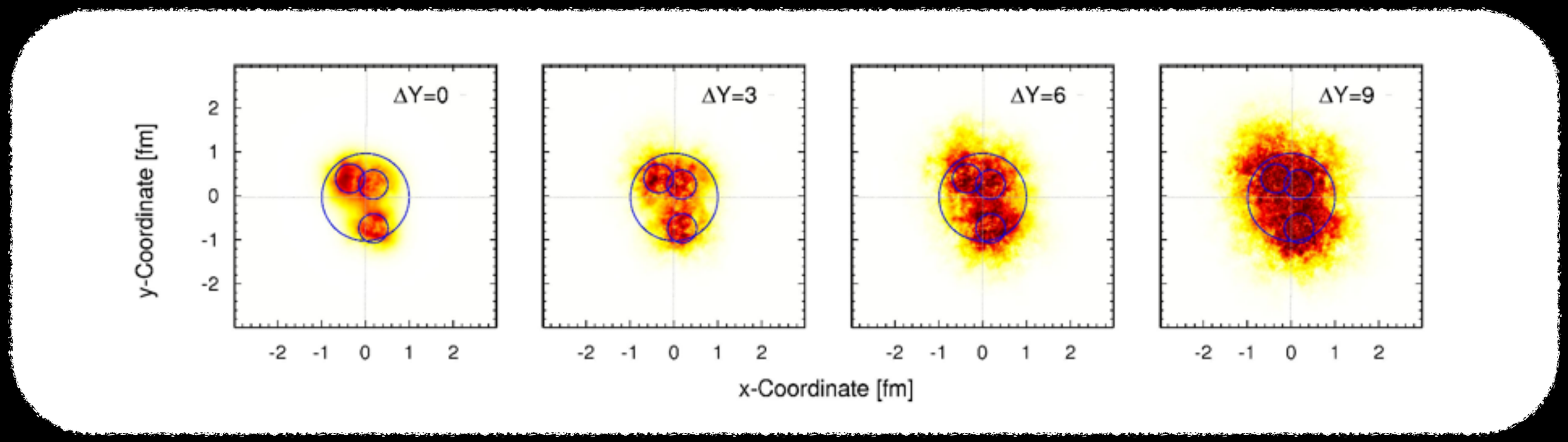
Evolution of fluctuations

Consider fluctuating initial state at moderately small value of x
— inspired by constituent quark models



(SS, Schenke PLB 739 (2014) 313-319)

Evolution of fluctuations

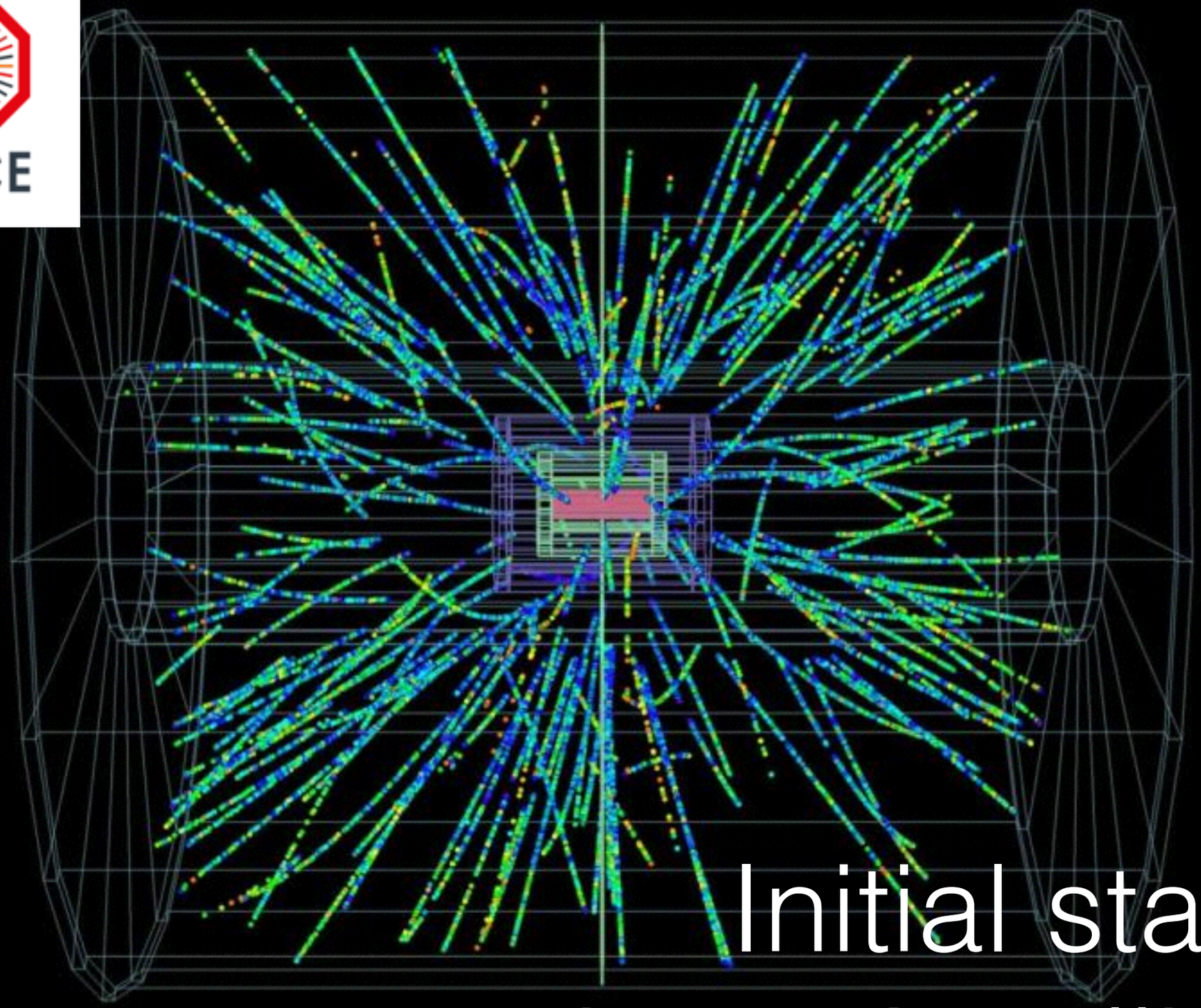


- Hadron radius increases linearly with rapidity — ‘Gribov diffusion’

  -> Nucleon shape remains in tact even after evolution over several units of rapidity

- Small scale fluctuations develop and become finer and finer (Q_s grows) with rapidity

(SS, Schenke PLB 739 (2014) 313-319)



Initial state in p+A collisions

q/g+p collisions

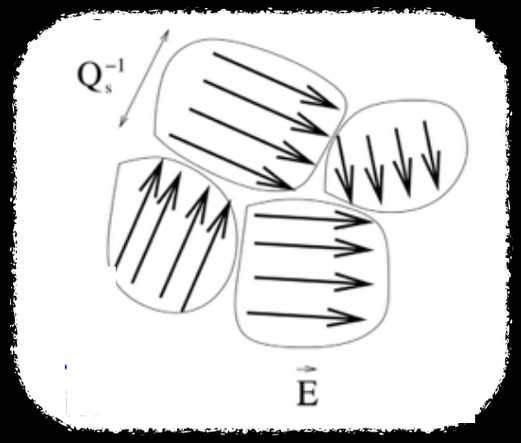
- Description in the dilute-dense framework as scattering of a quark/gluon wave-packet off a high-energy proton target

$$\frac{dN_{q/g}}{dy\, d^2\mathbf{k}} \propto \int_{\mathbf{p},\mathbf{b},\mathbf{r}} W_y^{q/g}(\mathbf{p},\mathbf{b})\, e^{i(\mathbf{k}-\mathbf{p})\mathbf{r}}\, \mathrm{Tr}_{f/a}\Big([V_y(\mathbf{b}+\mathbf{r}/2)-\mathbb{I}]\,[V_y^\dagger(\mathbf{b}-\mathbf{r}/2)-\mathbb{I}]\Big)$$

Single-particle distribution

Incoming q/g wave-packet

Quark/Gluon elastic scattering matrix



- Can be turned into a model for (forward) p+p/A by considering UPD for Wigner function and convolution with fragmentation function (Blaizot,Gelis,Venugopalan; Dumitru,Jalilian-Marian,…)

q/g+p collisions

- Description in the dilute-dense framework as scattering of a quark/gluon wave-packet off a high-energy proton target

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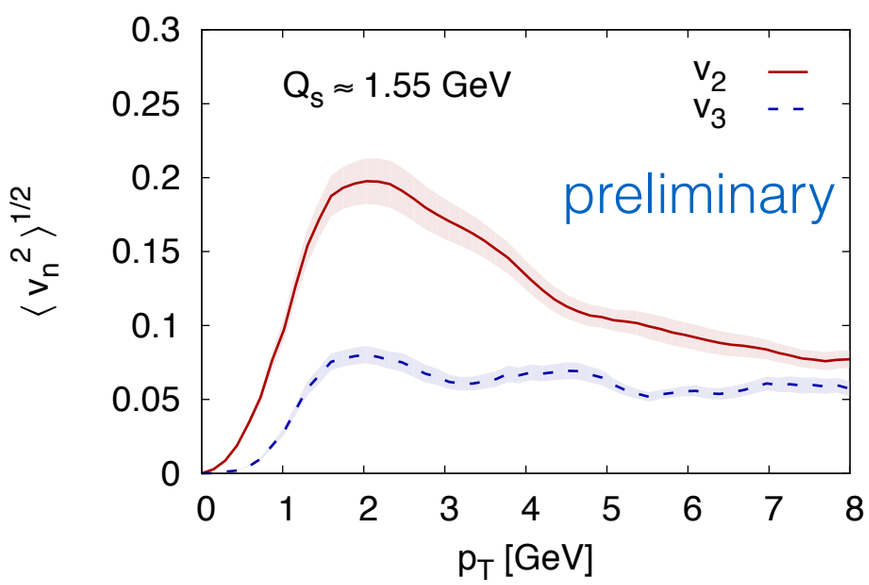
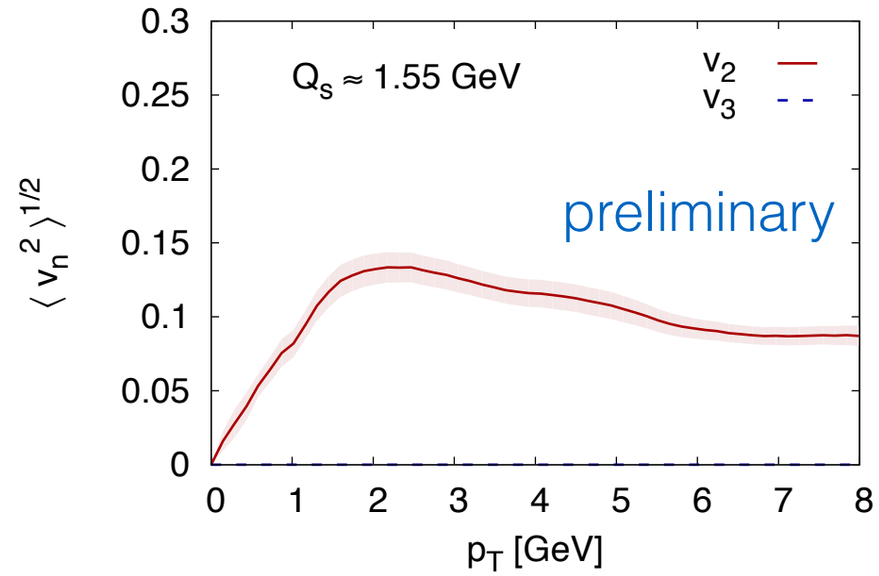
Quarks



Gluons



(SS, Schenke, Venugopalan work in progress)

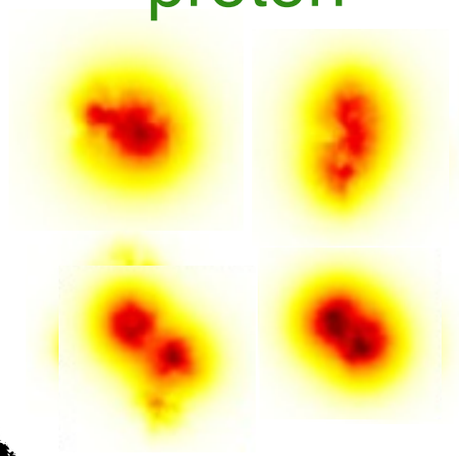
-> Sizeable v_n's up to high p_T — No odd harmonics for gluons.

p+A collisions

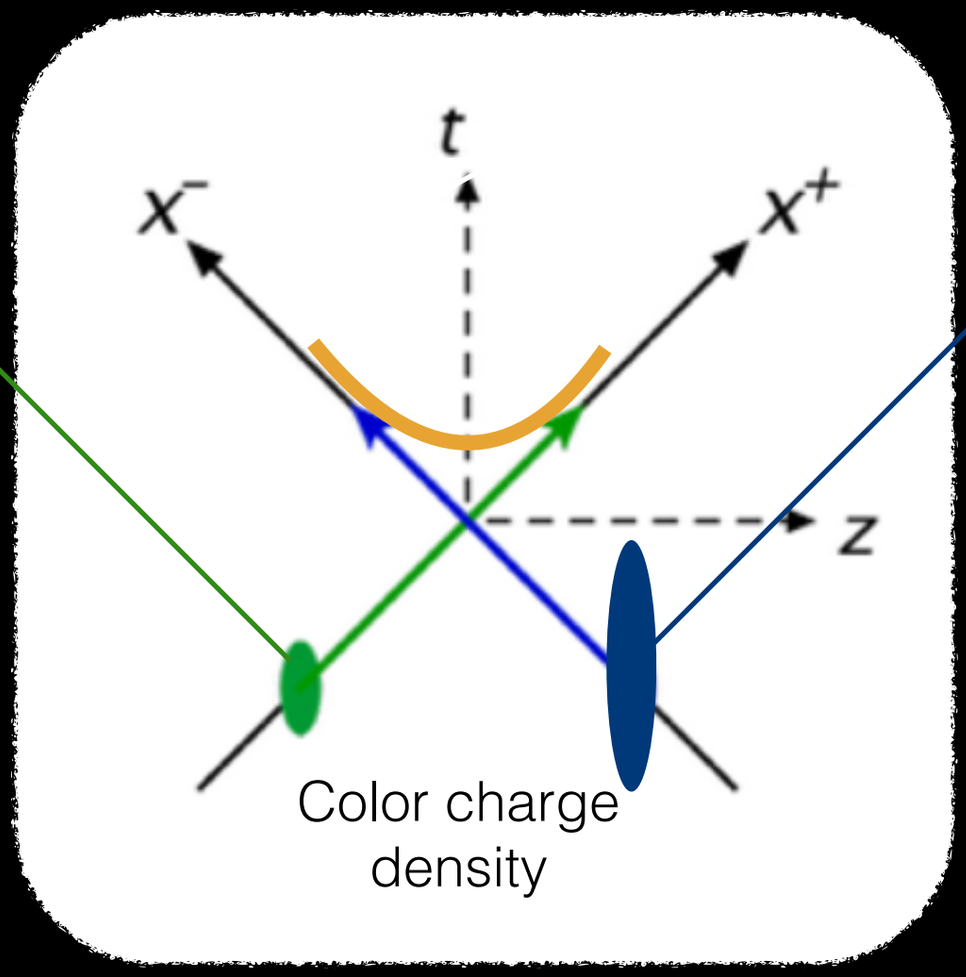
High multiplicity events require that both proton and nucleus are treated as dense objects with high gluon density.

-> Description of initial state and early-time dynamics in the color-glass condensate framework

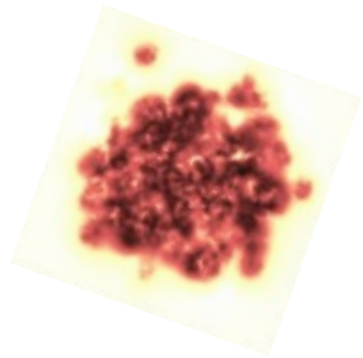
proton



$$J_1^\mu = \delta^{\mu+}\rho_1(x^-, \mathbf{x}_\perp)$$



Nucleus

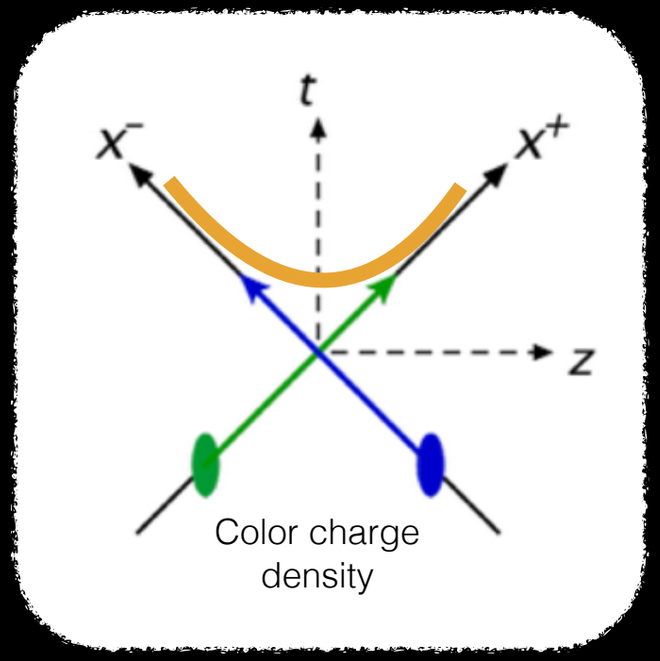


$$J_2^\mu = \delta^{\mu-}\rho_2(x^+, \mathbf{x}_\perp)$$

p+A collision

Initial state (τ=0+) and early time dynamics described by the solution of classical Yang-Mills equations to leading order in α_s
(Kovner,McLerran,Weigert,Krasnitz, Venugopalan,Lappi …)

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

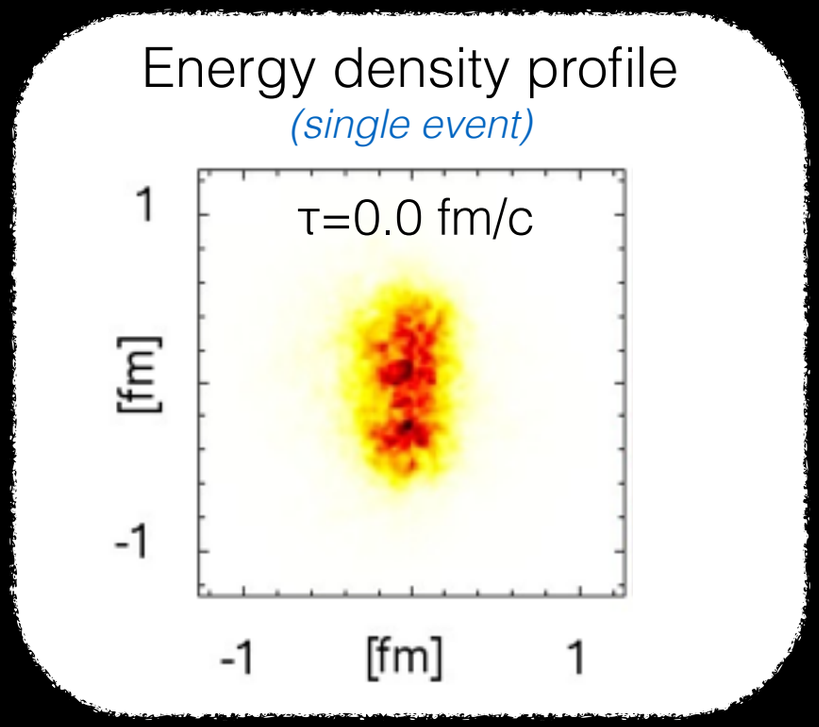


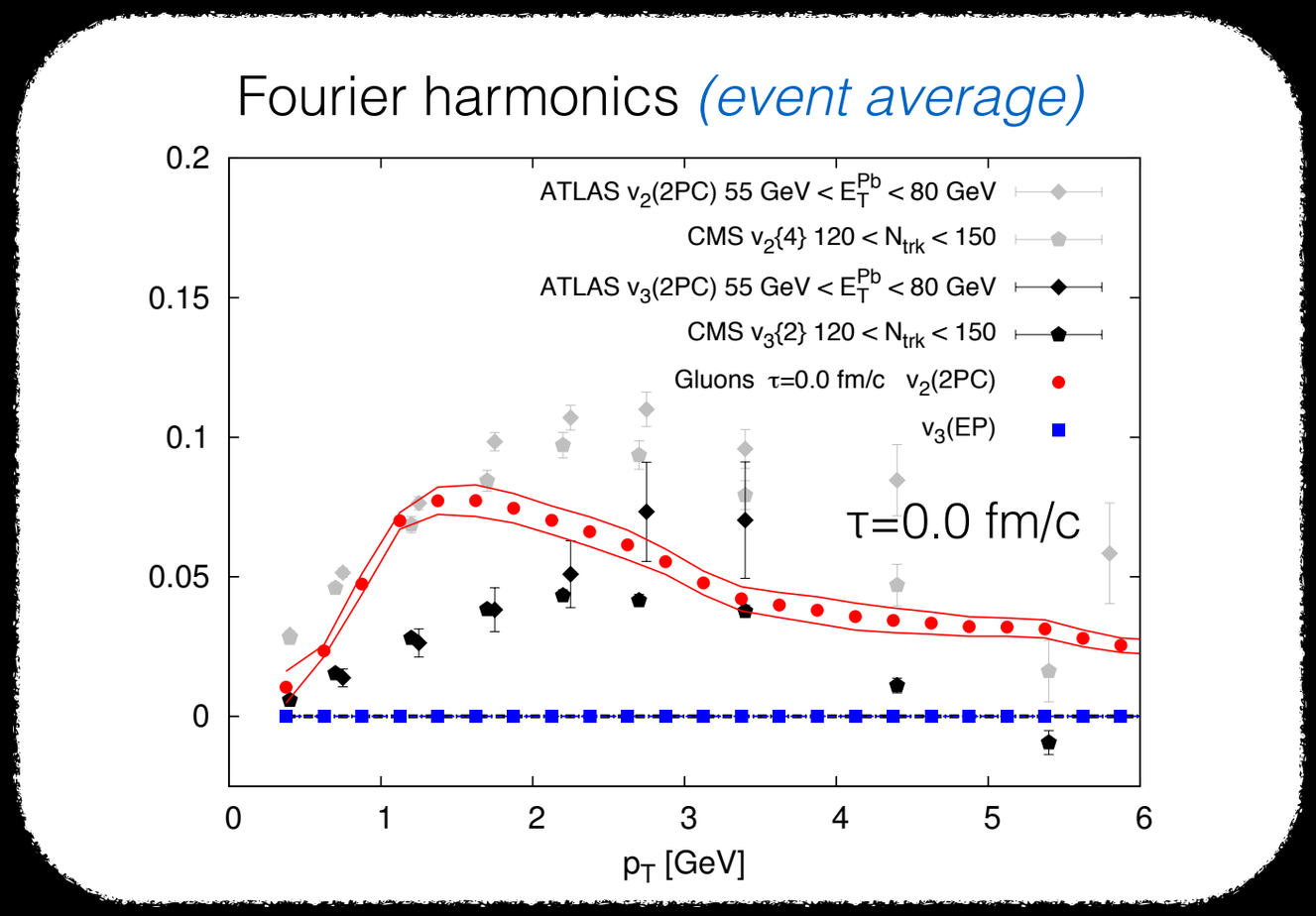
Measure single particle spectrum at different times of the CYM evolution and extract Fourier harmonics $v_n(p_T)$ from event plane and 2PC technique

Includes both connected and disconnected contributions to v_n as well as coherent final state interactions at τ>0

Initial state in p+A

- Initial state properties immediately after the collision ($\tau=0^+$)





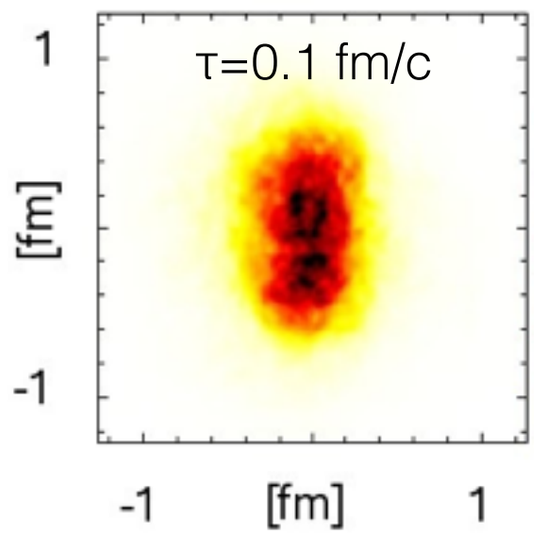
-> Significant momentum space anisotropy v_2 at $\tau=0$ due to production mechanism.

-> No odd harmonics for gluons without final-state interactions. Initial spectrum symmetric under $\mathbf{k}_T$<->$-\mathbf{k}_T$

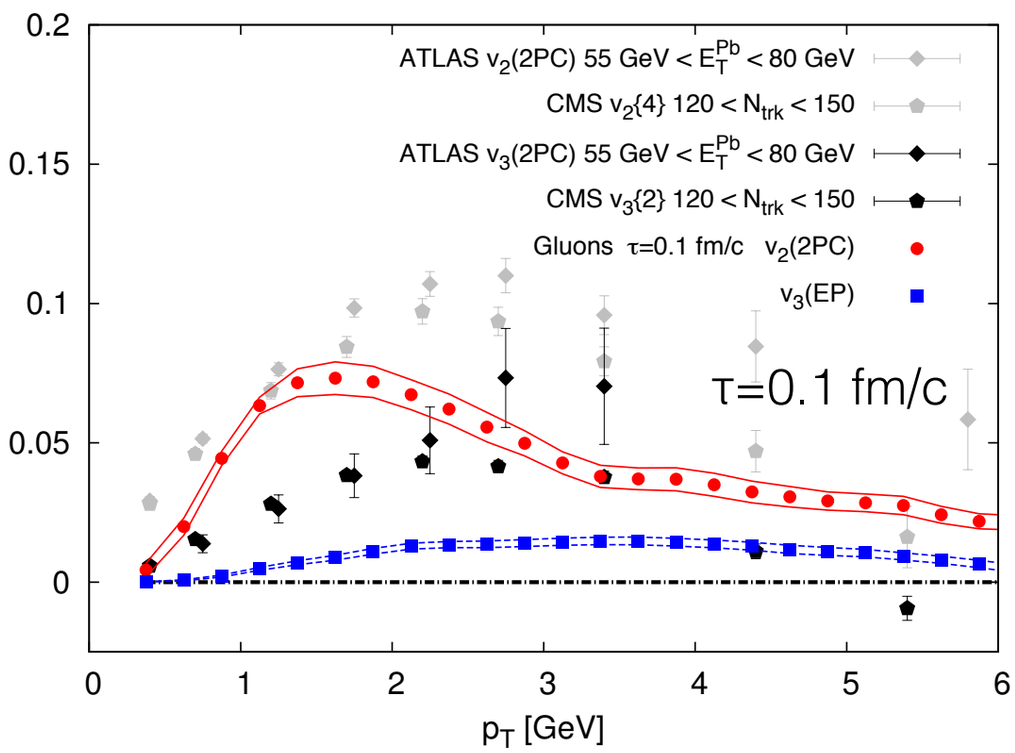
Dynamics in p+A collisions

- Classical (2+1D)Yang-Mills evolution after the collision — includes re-scattering of produced gluons

Energy density profile
(single event)



Fourier harmonics *(event average)*



(SS, Schenke, Venugopalan arXiv:1502.01331)

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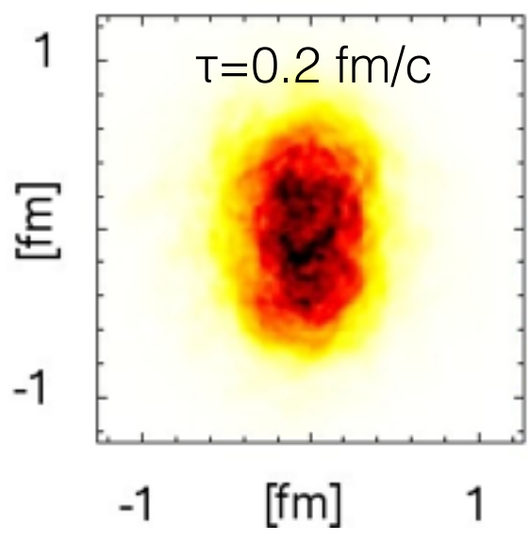
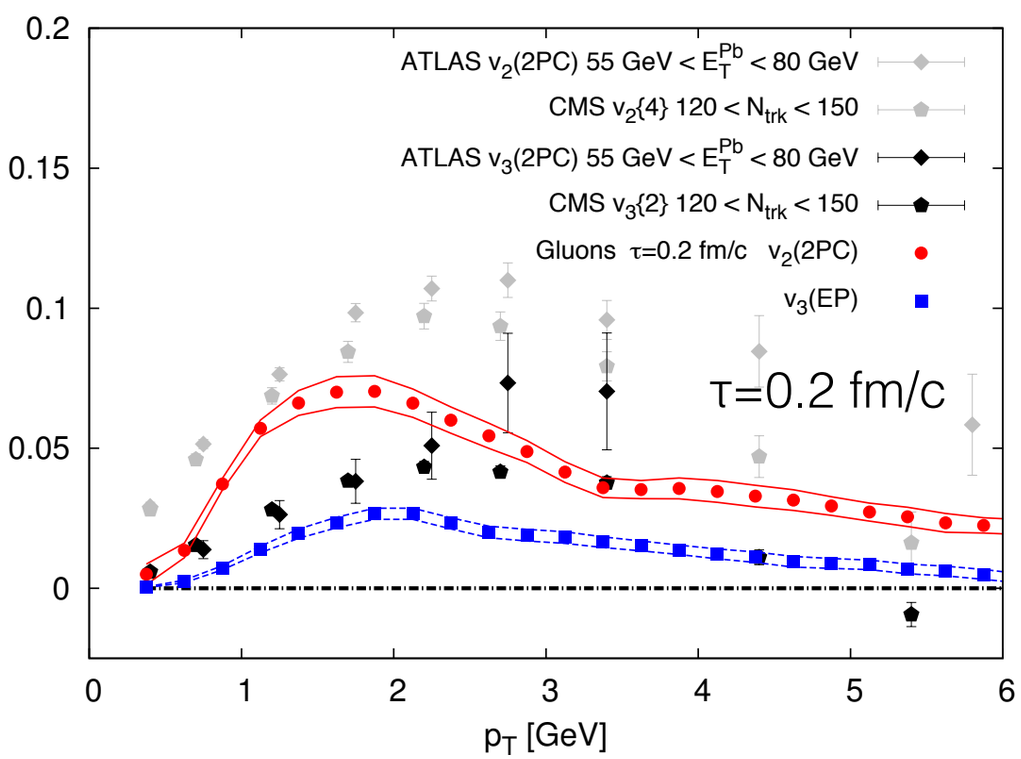


Fourier harmonics *(event average)*



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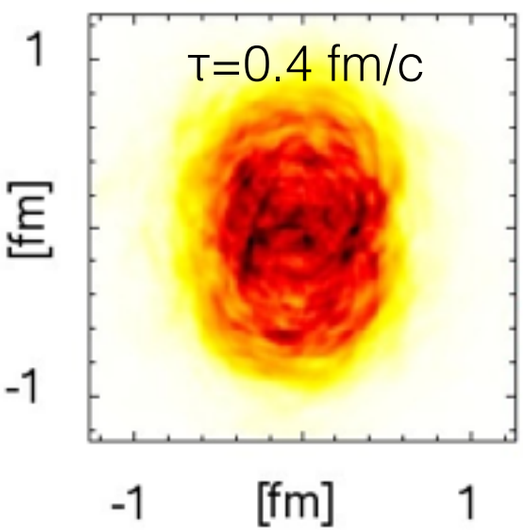
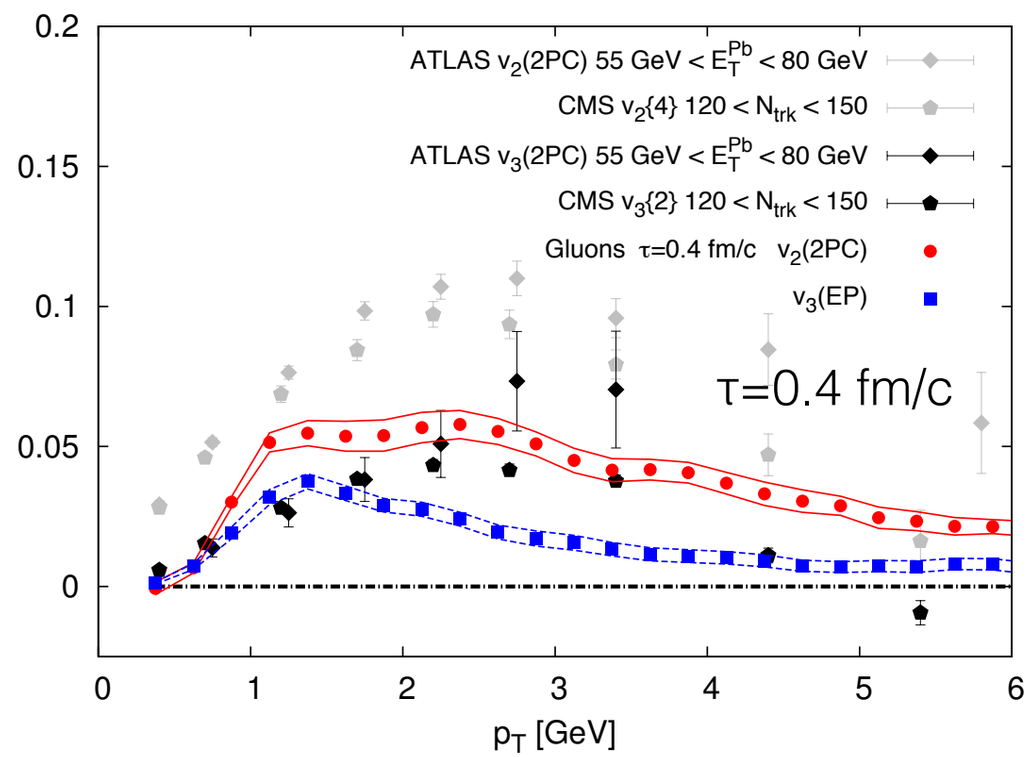


Fourier harmonics *(event average)*

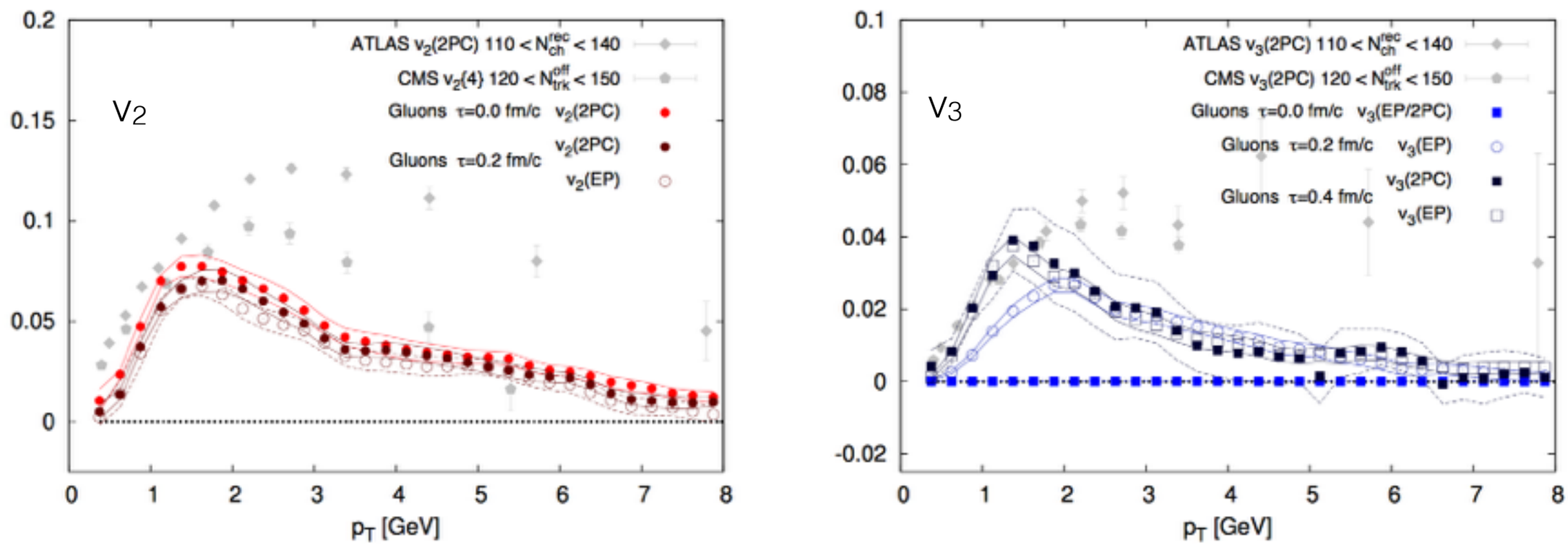


(SS, Schenke, Venugopalan arXiv:1502.01331)

-> Sizeable odd harmonics for gluons generated by pre-equilibrium dynamics on a time scale < 1 fm/c

Effect of CYM evolution

Evolution of v_2 and v_3 in central p+Pb collision

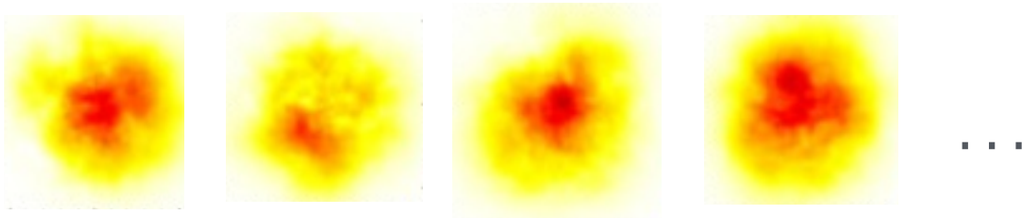


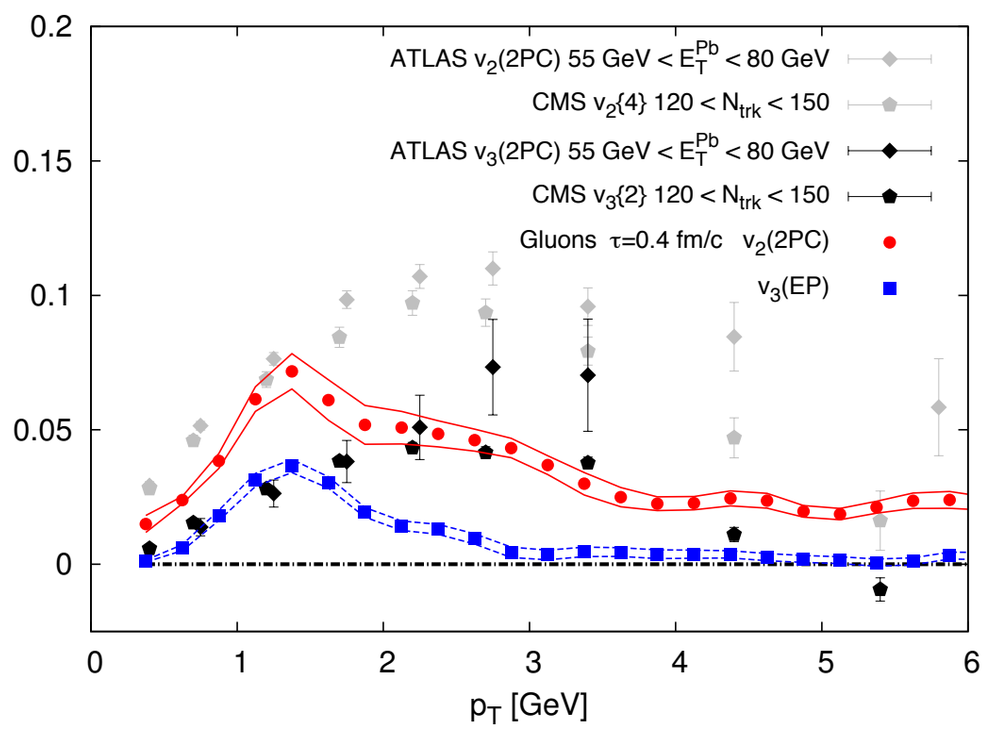
(SS, Schenke, Venugopalan arXiv:1502.01331)

Comparison of event plane measurement vn{EP} and extraction from two particle correlation vn{2PC} yields compatible results

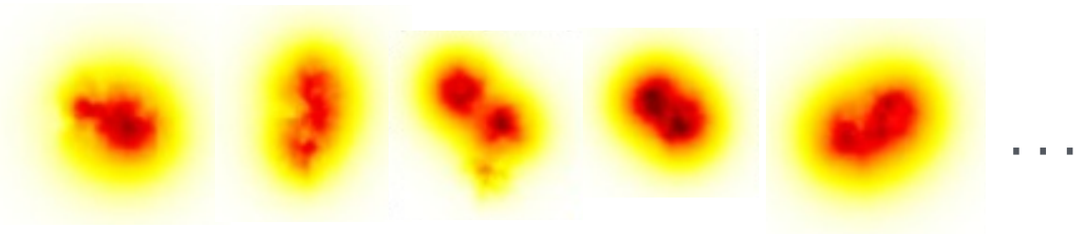
Effect of event geometry

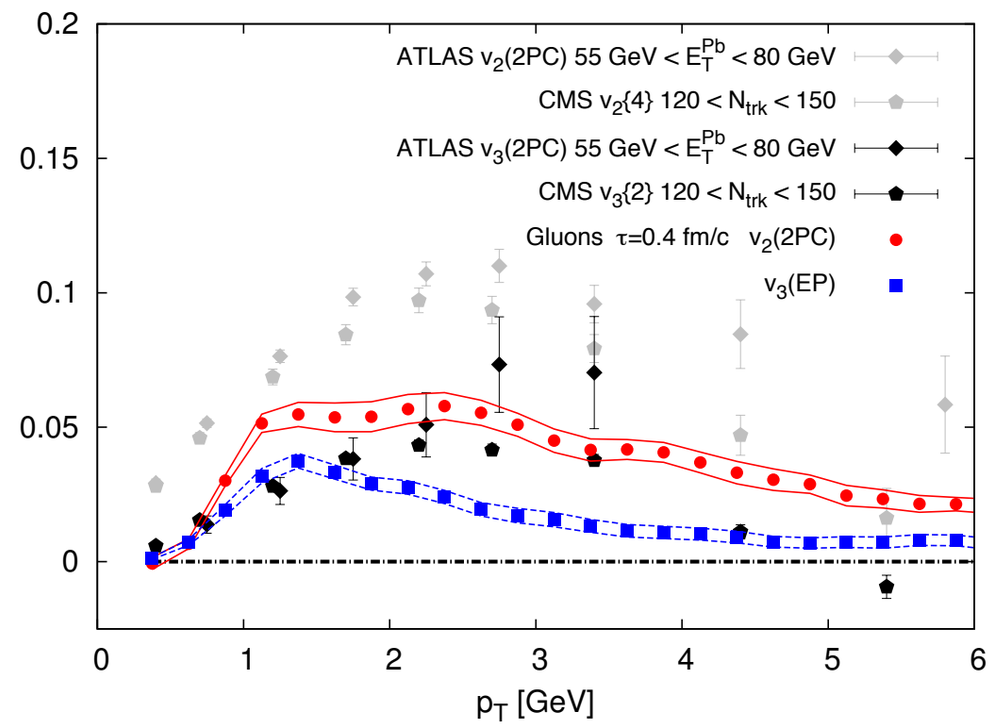
'Spherical' proton





'Eccentric' proton



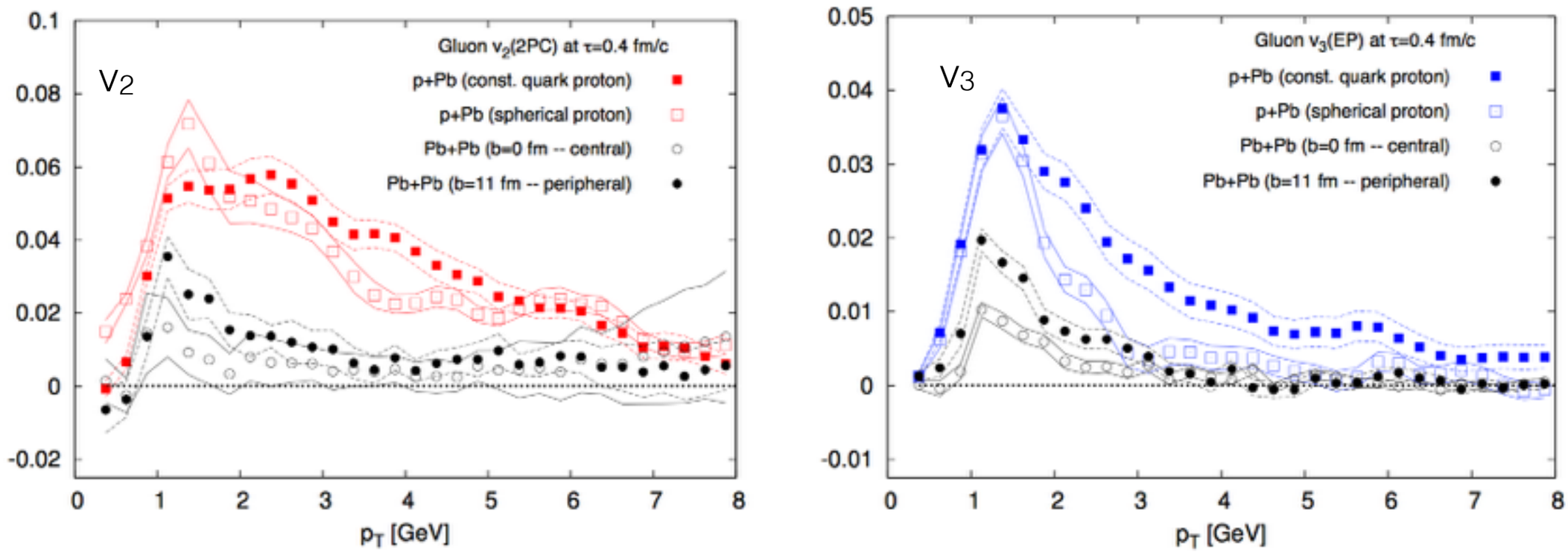


-> Small sensitivity to large scale structure
— very different from hydro (c.f. talk by Schenke)

-> No significant correlation between vn and en observed

System size dependence

Comparison of the effect in p+Pb and Pb+Pb



Significantly smaller effecting Pb+Pb -> points to importance of additional final state interactions to explain Pb+Pb data

Can be attributed to large number of mutually uncorrelated domains probed in Pb+Pb

Conclusions

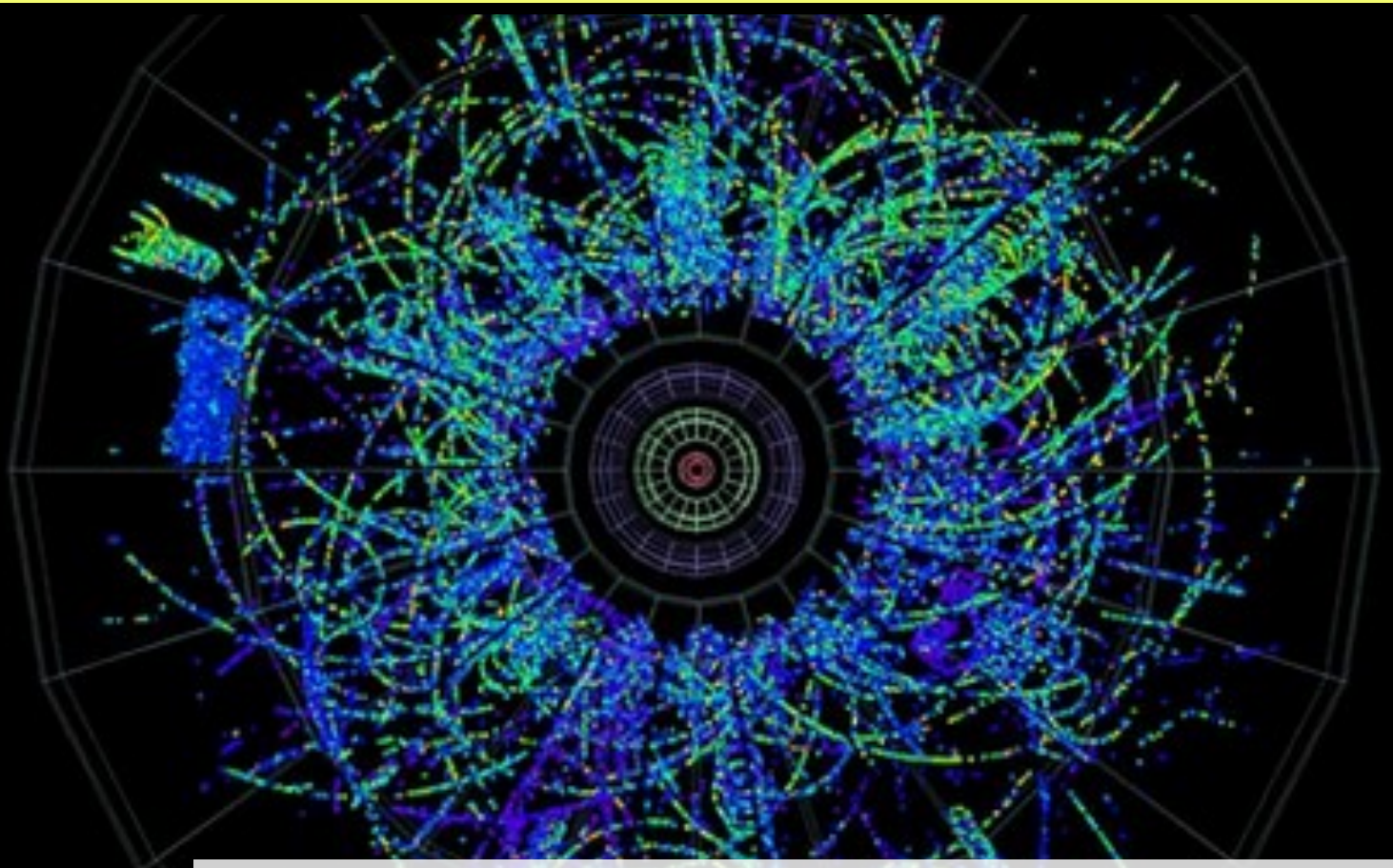
- Event by event fluctuations of the protons sub-nucleonic structure appear on different length scales and may play an important role in our understanding of p+A collisions at the LHC.
- Even large scale fluctuations ~R_p are consistent with small-x evolution

-> Importance to re-evaluate constraints from e+p on proton structure and event-by-event fluctuations

- Initial state effects and early time dynamics (τ<0.4 fm/c) can lead to sizable even ($v_{2,4,...}$) and odd harmonics ($v_{3,5,...}$) up to fairly large p_T.
- Correlations are rather insensitive to global event geometry.
- Substantially larger effect in p+A as compared to A+A.

Initial state effects likely important for p+A but many questions still open — Higher order effects? How to transition to final state?

Collectivity in small systems ?



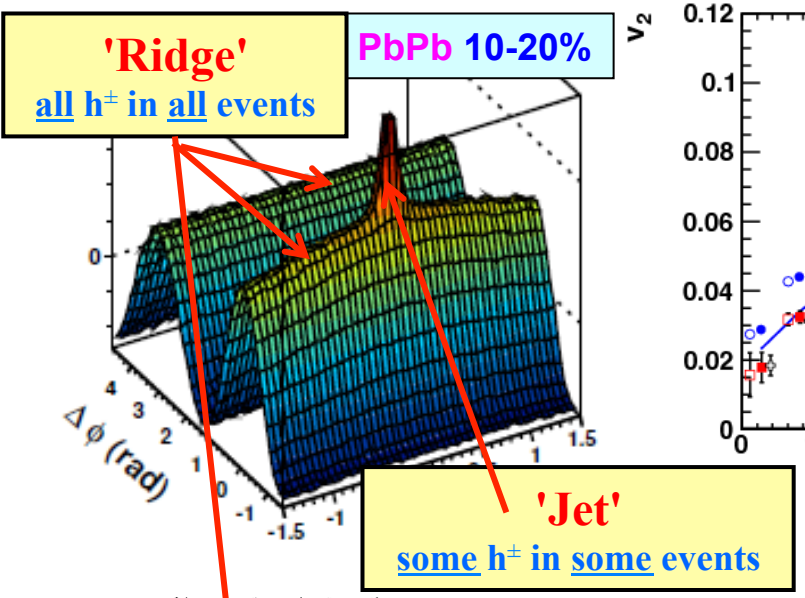
pPb 5.02 TeV
December 2012

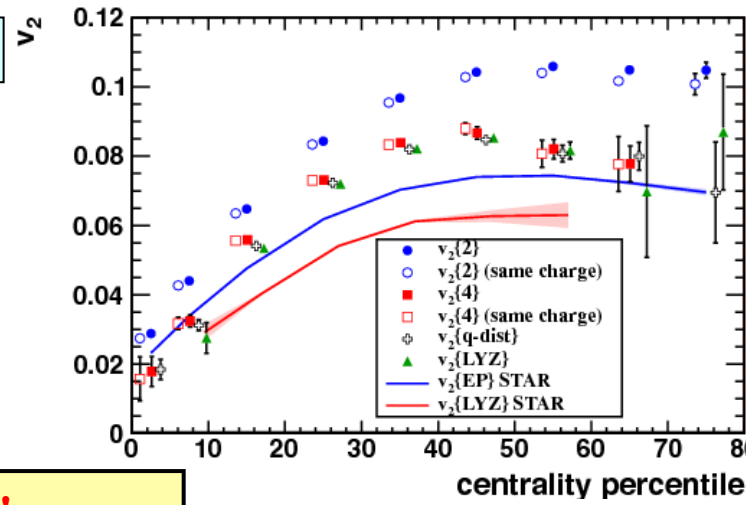
- Is there collectivity in small systems ?
- Why does it matter ?

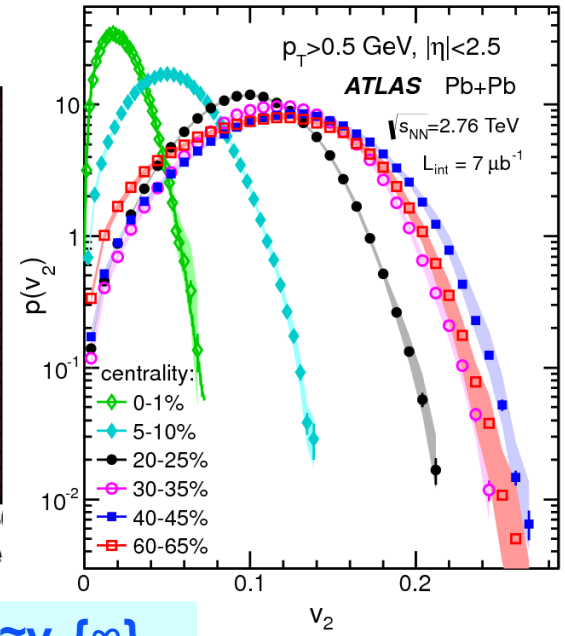
Collectivity

- weak definition:
  - ⇨ **SIMILAR** effect for **ALL** particles (of some kind, say p_T/PID) in **ALL** events
  - ⇨ Collectivity is **experimentally proven** in **AA** (≈ in **pA**)

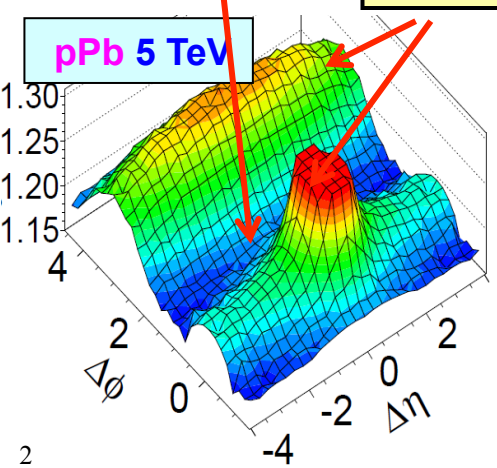
EbE $P(v_2)$

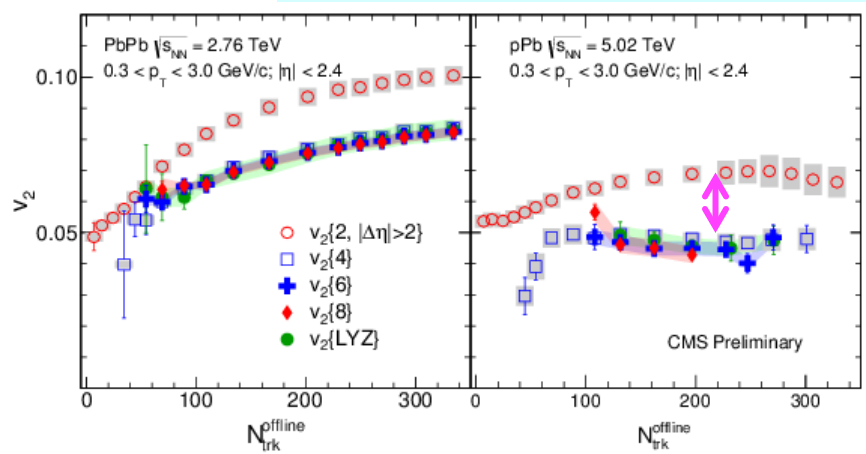






$v_2\{2\} < v_2\{4\} \approx v_2\{6\} \approx \ldots \approx v_2\{\infty\}$





AA definitely coll.
pA practically certain
Teaser:
are jets in e+e-
'weakly collective' ?

Collectivity in large systems: AA

- strong definition: {'thermo' + 'hydro'} - dynamics

  ⇨ **emerging-f(*t*)-** in **strongly interacting, macroscopic** matter with density/pressure **gradients**

- Collectivity is **consistent with ≈ all data in AA** to (very) good accuracy

  ⇨ **thermo-dynamics:**

  - particle ratios (Statistical Model) to 10-30%

  ⇨ **hydro-dynamics:**

  - **LO**: radial (v_0) & elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
  - **NLO**: higher harmonics v_n, PID (*m* dependence) of v_n ('mode mixing' of v_0 & v_2)
  - **NNLO**: non-linear mode mixing ($v_n \neq \varepsilon_n$), factorization violation $r(p_T)$, EbE $P(v_n)$, …

  ⇨ **thermo + hydro:**

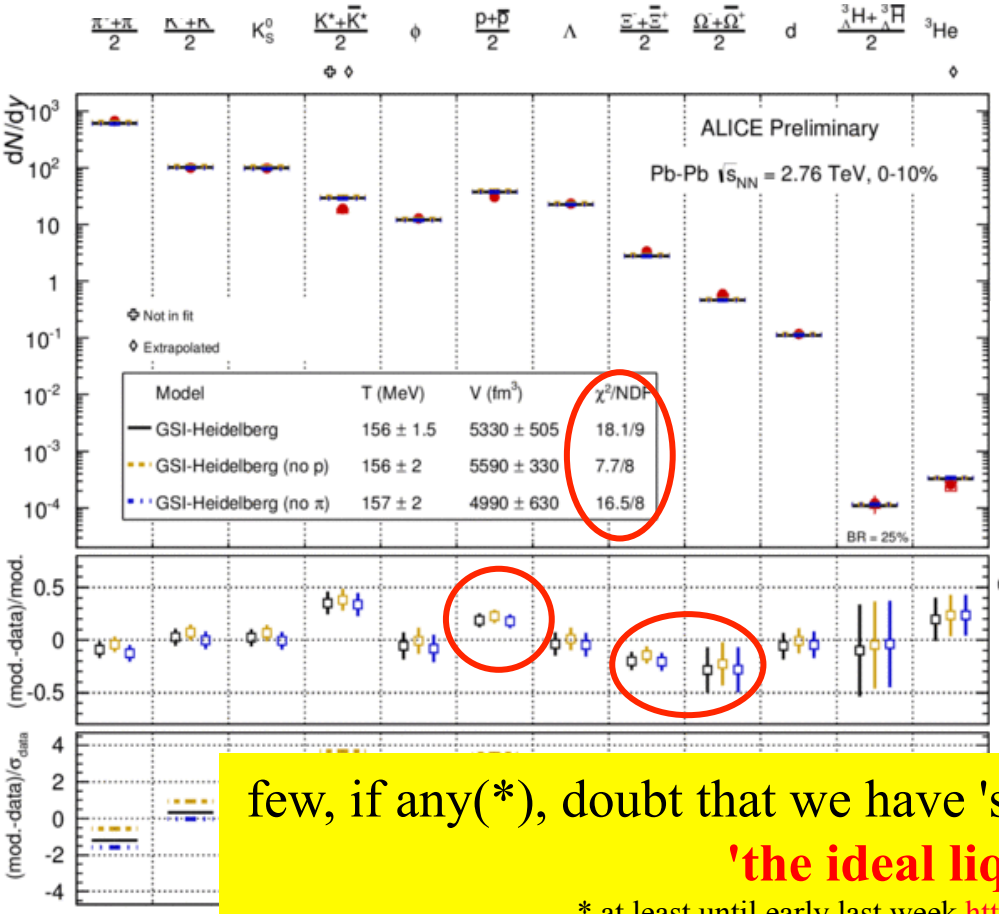
  - HBT $f(T, \beta)$: ($R(m_T)$, $R(N_{ch}^{1/3})$, $R_{out}/R_{side} \approx 1$)

'Standard Model' of heavy ion physics

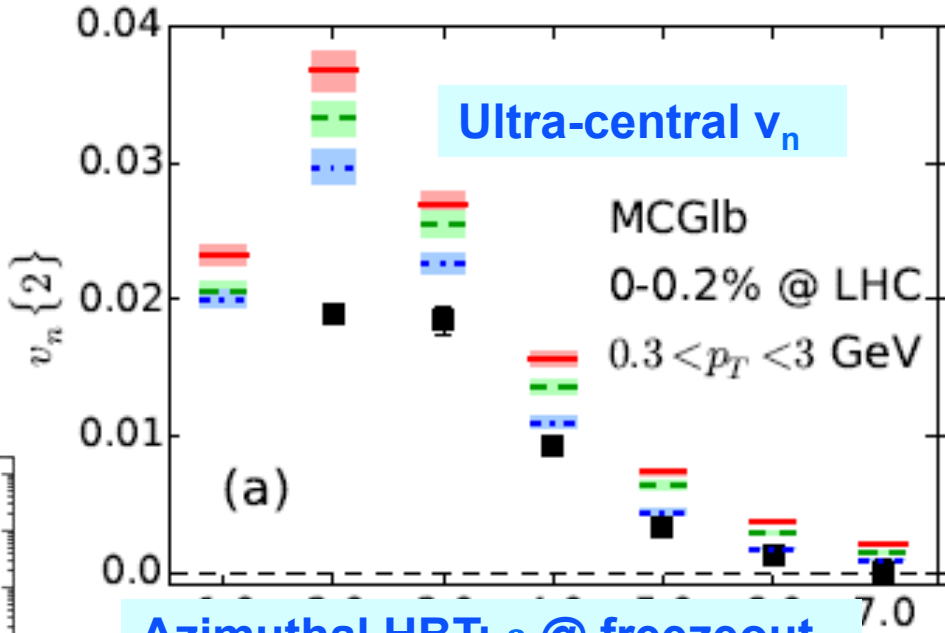
Tensions (presumably work in progress, not cracks)

- p/π ratio, ultra-central v_n, HBT ε_f, …

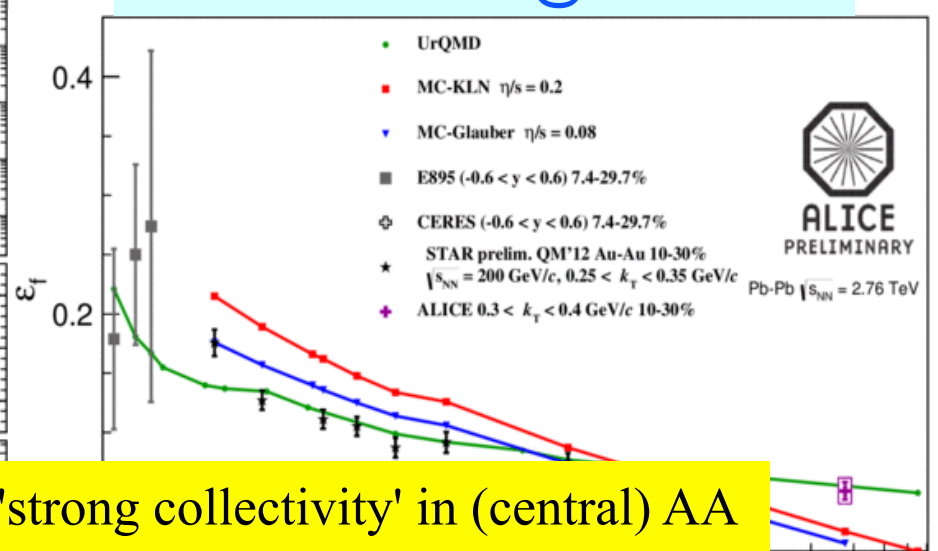
SM fits to particle ratios



Ultra-central v_n



Azimuthal HBT: ε @ freezeout



few, if any(*), doubt that we have 'strong collectivity' in (central) AA

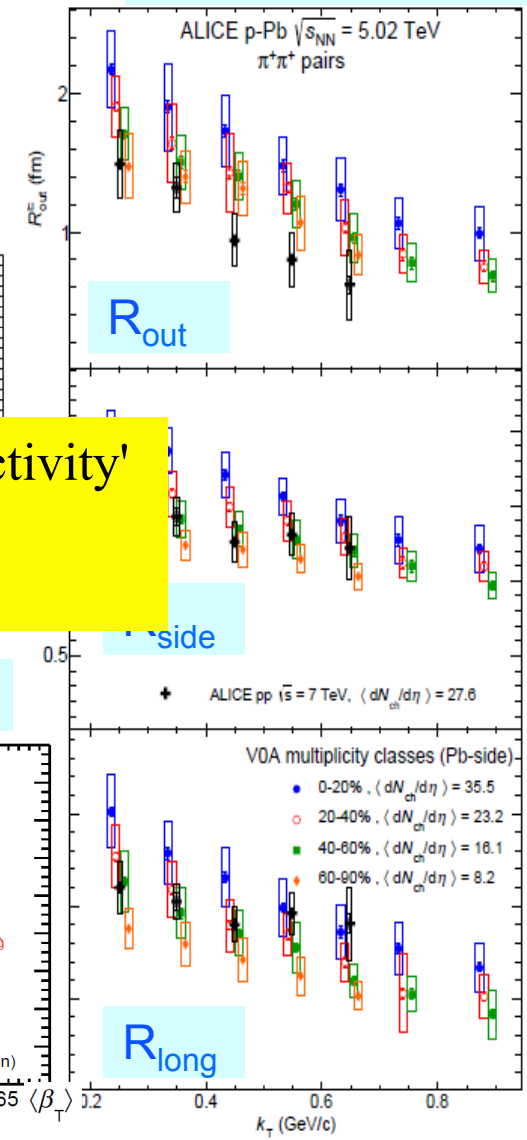
'the ideal liquid sQGP'

* at least until early last week http://arxiv.org/abs/1502.05572 ..

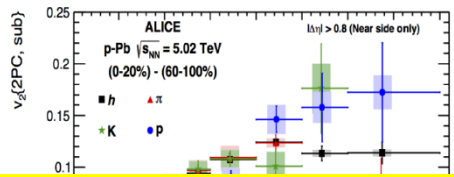
Collectivity in small systems: 'central' pA

- Collectivity **consistent with ≈ all data in central pA** to reasonable accuracy
  - ⇨ **thermo-dynamics:** particle ratios (SM, γ_s=1!) to **≈ 20-30%**
  - ⇨ **hydro-dynamics:** pA, dA, ^{3}He-A
    - ✪ **LO**: radial & elliptic flow
    - ✪ **NLO**: higher harmonics v_n, PID v_n
    - ✪ **NNLO**: not yet tested
  - ⇨ **thermo + hydro:**
    - ✪ HBT ($R(m_T)$, $R(N_{ch}^{1/3})$, $R_{out}/R_{side} \approx 1$)

HBT: R vs K_T

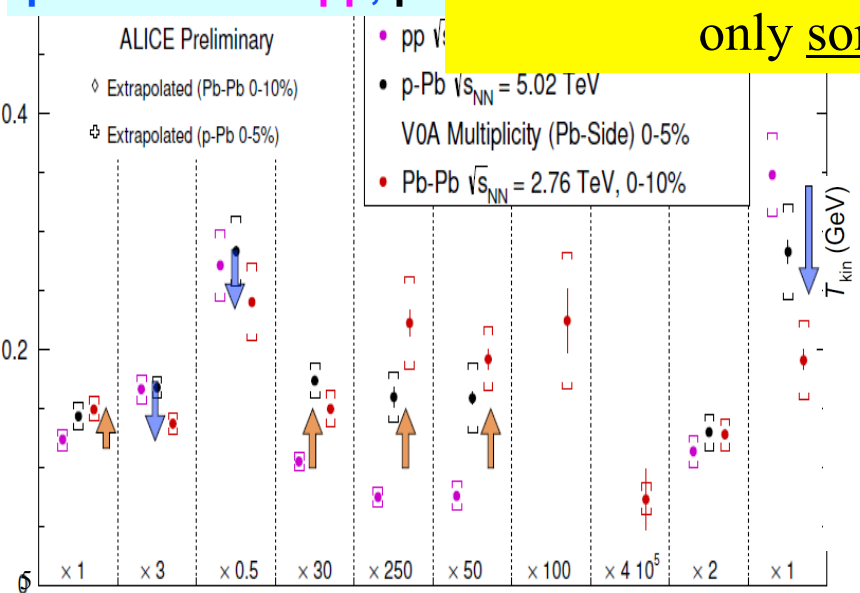


PID v_2

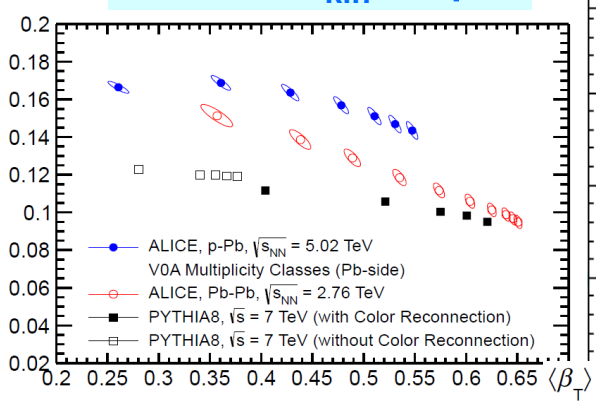


The experimental support for 'strong collectivity' is **not really worse** than AA only somewhat **less tested** ..

particle ratios pp, pA



BW fit: T_{kin} vs β



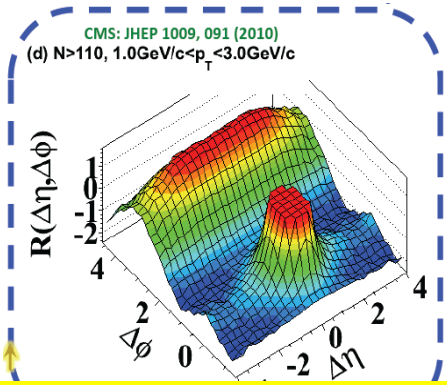
Collectivity in small systems: 'central' pp

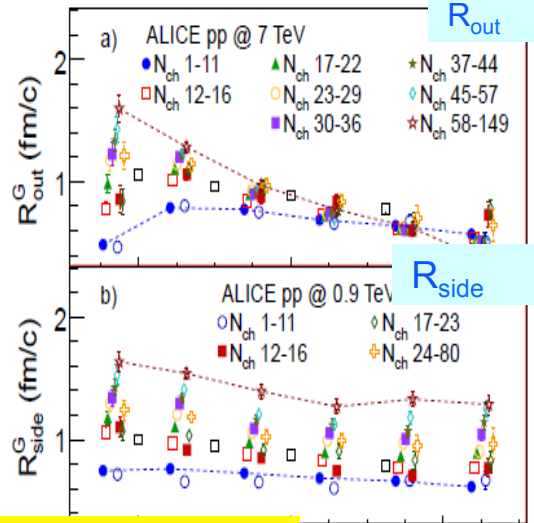
- Collectivity **consistent with data in high N_{ch} pp ??**
  - ⇨ **thermo-dynamics: MB** particle ratios (*ce*-SM) to ≈20-40% γ_s **< 1!**
  - ⇨ **hydro-dynamics:**
    - ✪ **LO**: radial & elliptic flow ?
    - ✪ **NLO**: not yet tested
    - ✪ **NNLO**: not yet tested
  - ⇨ **thermo + hydro:**
    - ✪ HBT ($R(m_T)$, $R(N_{ch}^{1/3})$, $R_{out}/R_{side} \leq 1$)

π	K±	K⁰	K*	φ	p	Λ	Ξ	Ω	d	$^3_\Lambda$H	He

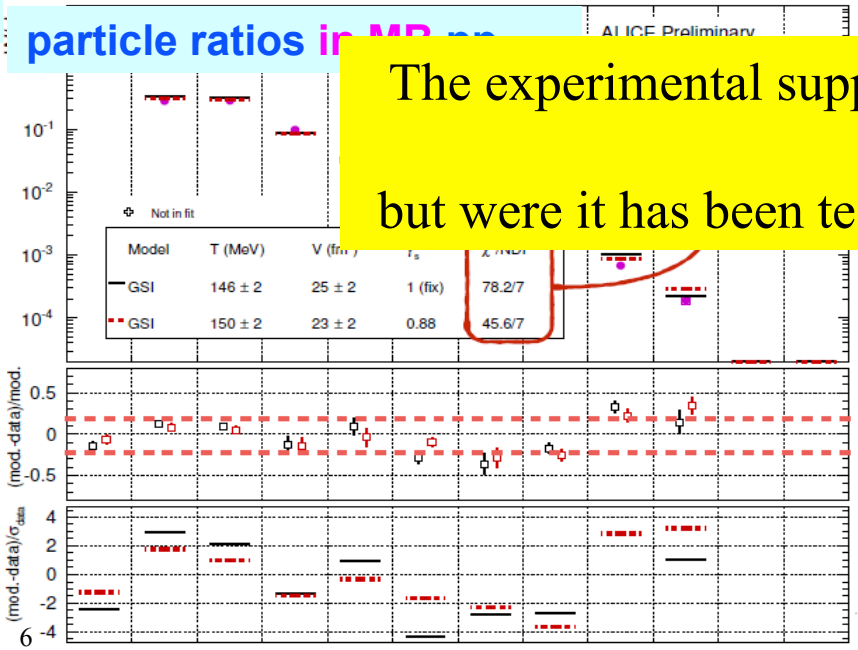
HBT: R vs K_T

The Ridge

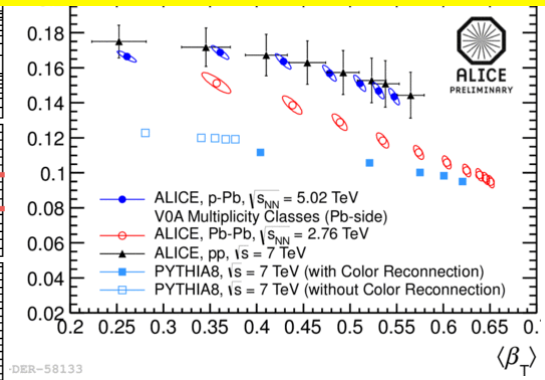


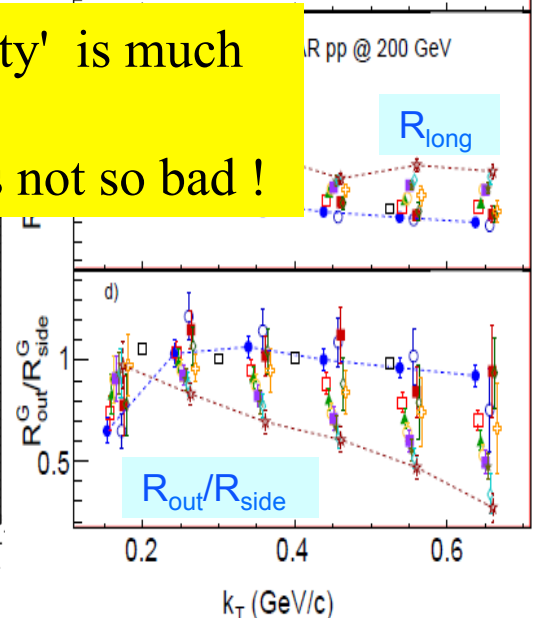


particle ratios in MB pp



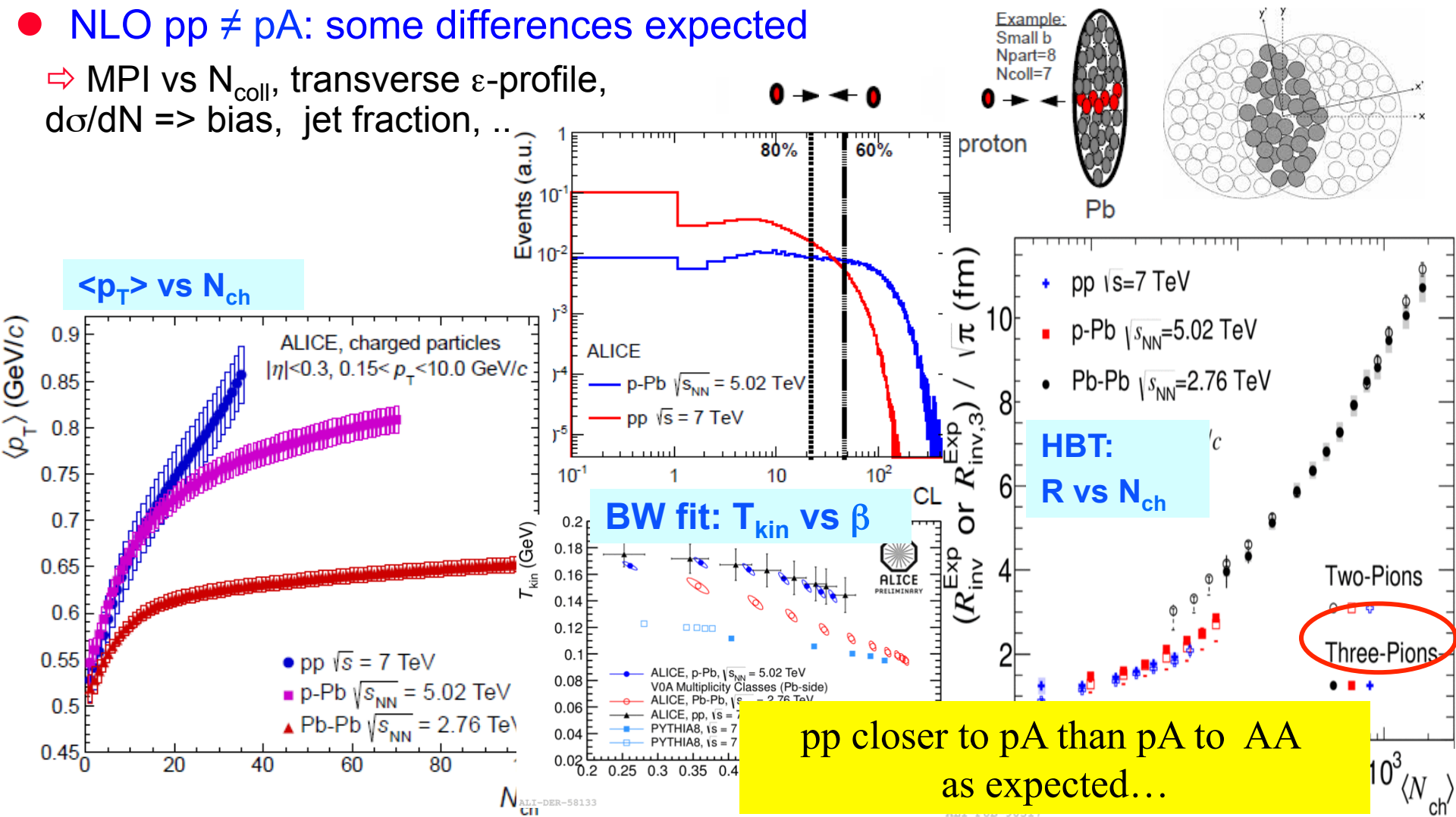
The experimental support for 'strong collectivity' is much **less tested** ..
but were it has been tested, at high N_{ch}, it looks not so bad !





A priori: pp ≈ pA @ same dN/dy

- LO pp = pA: N_{ch}, transverse size & shape => Initial State $\varepsilon(x,y)$:
  - ⇨ **experimentally verified**: R(pp) ≈ R(pA) < R(AA) @ same N_{ch} (≈ N_{part})
  - ⇨ IF there is collectivity in central pA, THEN there is no reason why not in 'central' pp
- NLO pp ≠ pA: some differences expected
  - ⇨ MPI vs N_{coll}, transverse ε-profile, $d\sigma/dN$ => bias, jet fraction, ..



pp closer to pA than pA to AA as expected…

Facts & 'Fiction'

- Experimental facts:
  - ⇨ **weak collectivity** proven in **AA**, essentially proven in **pA**, not known in **pp**
    - ✪ **'all particles in all events'** must be part of any physics model
  - ⇨ strong coll. **(thermo & hydro)** compatible with vast majority of data in **AA & pA**
    - ✪ some areas need work, some tests missing in pA (NNLO)
  - ⇨ very limited data in **pp** at high N_{ch}, but compatible with SC ! **pp ≈ pA @ same N_{ch}**
- Small ≈ LARGE (no 'onset')=> No longer 'Business as usual'
  - ⇨ 1) invent different physics to mimic the same phenomenology (conspiracy theory)
  - ⇨ 2) extend 'heavy ion physics' to include small systems (central pA, MB pp ??)
    - ✪ 2a: heavy ion physics (**sQGP**) in pA/pp
    - ✪ 2b: pp/pA physics (**kinetic non-equilibrium transport**) in AA
- Hypothesis: There **IS** collectivity in small systems at high N_{ch} !
  - ⇨ 1) **pA ≈ AA**: mostly based on measurements
    - ✪ many similar phenomena <=> similar underlying physics
  - ⇨ 2) **pp ≈ pA**: mostly based on (my) a priori expectations & limited exp. results
    - ✪ mind the jet-bias in pp !
  - ⇨ 3) **≈ ≠ =**: differences are interesting and important to study !
    - ✪ finite size/finite time/non-equilibrium effects teach us about dynamics

pA ≈ AA: Objections

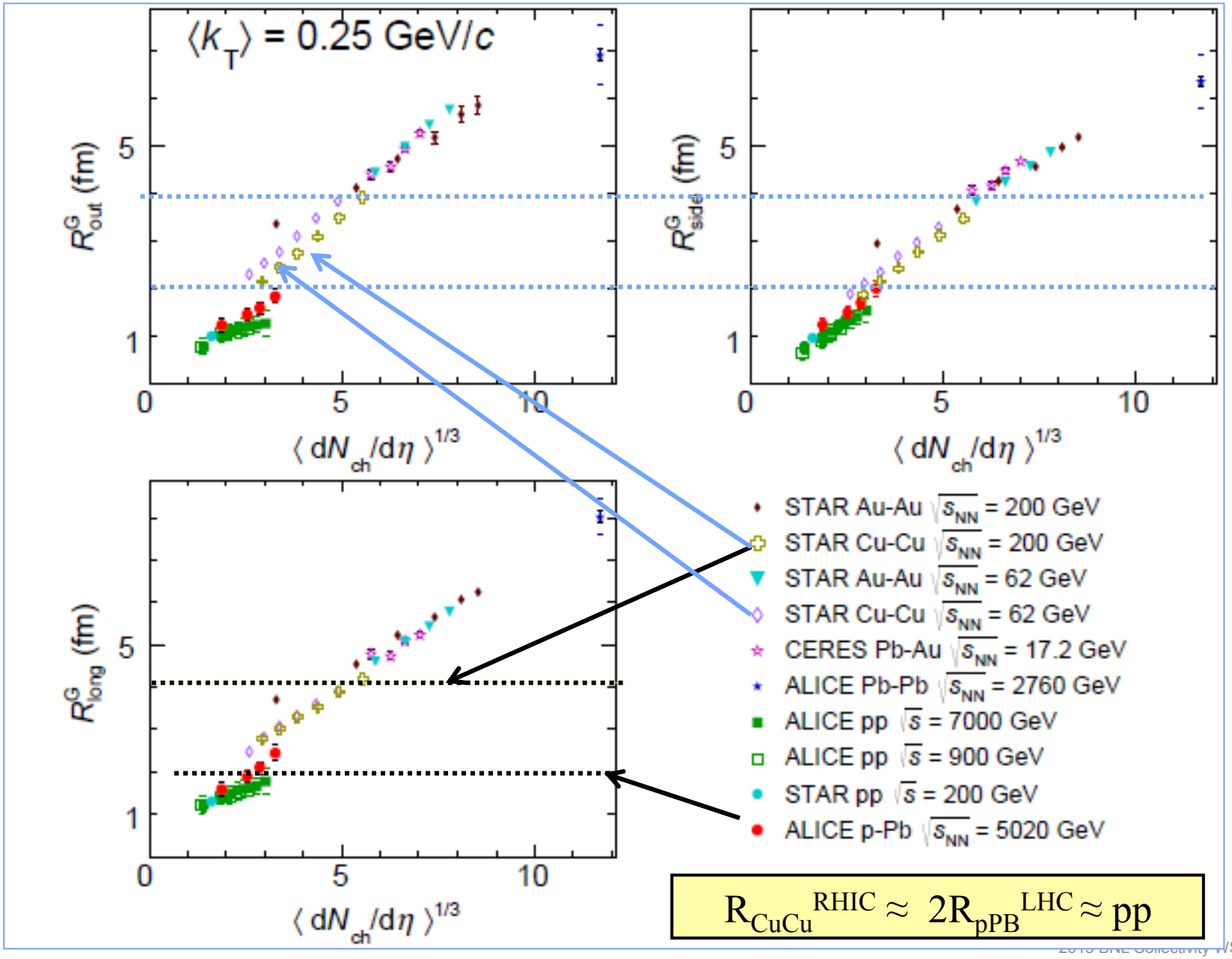
- sQGP: no ideal liquid w/o jet-quenching (energy loss)
  - ⇨ hydro is dimensionless **($K_n = \lambda/R << 1$)**,
    - ✪ R = 1-2 fm may be big: τ_0(<0.2 fm/c?), $\eta/S \approx 0 \Rightarrow \lambda \approx 0$; $1/\langle p_t \rangle \approx 0.3$ fm
  - ⇨ energy loss has dimension: **$\Delta E \sim (\Delta x - a)^n$**, Δx = path length, a = formation length
    - ✪ $\Delta E_{pA} > 1/5 \times \Delta E_{AA}$ (n = 1, a = 0), and possibly much more
    - ✪ is measured **R_{pPb} (in)compatible** with <u>quantitative</u> expectations for ΔE ?

- hydro not applicable (large K_n, large gradient/viscosity corrections, ..)
  - ⇨ we may need different TH tools, not necessarily different physics ?
    - ✪ 'hydro' : physics driven by gradients in SI matter
    - ✪ smoothly extends into 'partial hydro/decoupling/non-equilibrium' regime ?

- difference is size is significant, but not huge !
  - ⇨ made up (largely ?) by higher density ?
  - ⇨ ES: the conformal QGP: pA may be smaller (R ↓), but also much denser (λ ↓)

$$R_{CuCu}^{RHIC} \approx 2R_{pPB}^{LHC} \approx pp$$

HBT radii pp, pA, AA



$$R_{CuCu}^{RHIC} \approx 2R_{pPB}^{LHC} \approx pp$$

pA ≈ AA: Objections

- similar phenomena ≠> same physics
  - ⇨ counter examples:
    - ✪ elliptic flow: v_2(< 2 GeV) = **hydro** ($\Delta\rho/\Delta x$), v_2(> 10 GeV) **quenching** (ρ)
      linked by geometry and SI matter, but driven by distinct physics
    - ✪ radial flow: AA **hydro**, pp: **Color Reconnection** ?
      'hydrodynamics' similar (emission from boosted system T, β), dynamics different

  - ⇨ **likelihood** argument: better if many independent phenomena are involved
    - ✪ 'razor': **What looks more natural & simple:** different or similar physics ?

Is it a Rose ?

- If it looks like a rose

- + smells, + feels,

+ pricks, + tastes, + .. like a rose

<u>Summary Part I:</u>

similar phenomenology (measured or expected) in
pp, pA and AA
overwhelmingly likely based on similar physics:
inhomogeneous, dense, strongly interacting matter
SM: drop of **ideal liquid sQGP: thermo + hydro dynamics**

B-SM: cloud of grey/black mist (**sMOG**): **non-eq. kinetics** 1502.05572

II) Why is it interesting ?

- 'Looking under the hood': what makes the sQGP tick ?
  - ⇨ stat. mechanics (thermo & hydro) hide very well the details: **d.o.f & dynamics**
    - ✪ **strength & limitation:** same results for different underlying 'stuff' (theories/models)
    - ✪ thermal system knows nothing about how and why it arrived in equilibrium
  - ⇨ go towards and **beyond the limits** of thermo & hydro
    - ✪ study deviations due to **finite size/finite time** (=> small systems)

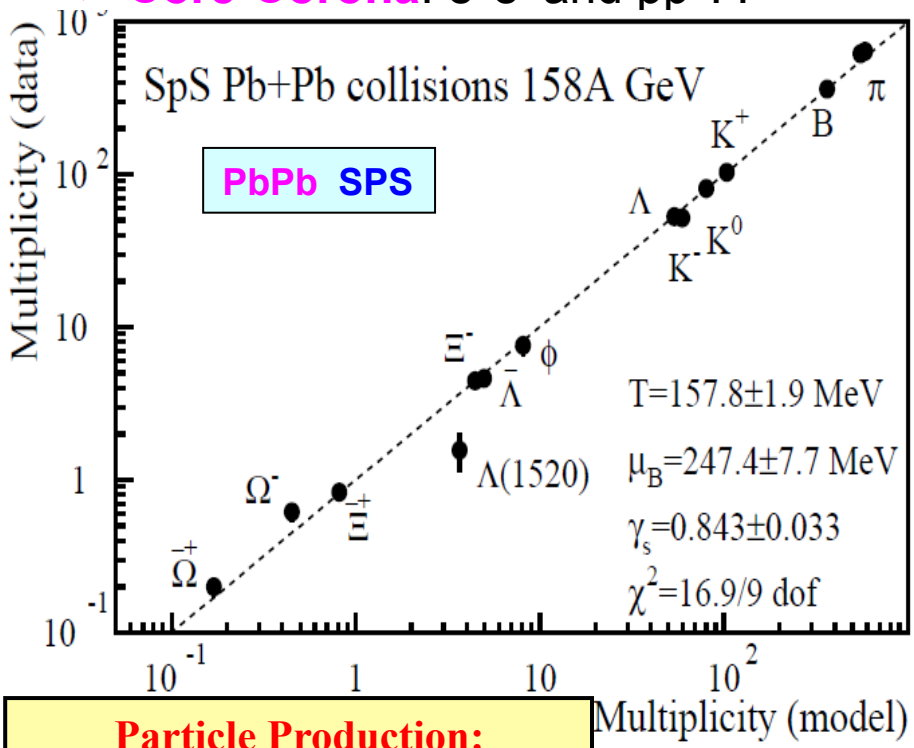
- 'Looking for the transition': how does collectivity emerge f(*r, t*) ?
  - ⇨ change <u>size & lifetime & density</u> (**pp -> pA -> AA**)

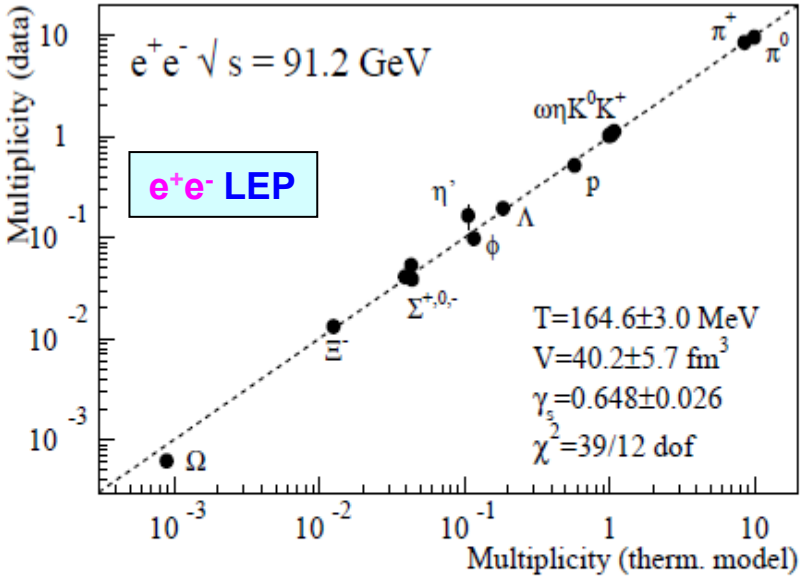
- 'Looking for the beginning': universal aspects of soft QCQ ?
  - ⇨ connection to soft QCD in ‘non-collective’ systems like MB pp or even e^+e^- ?

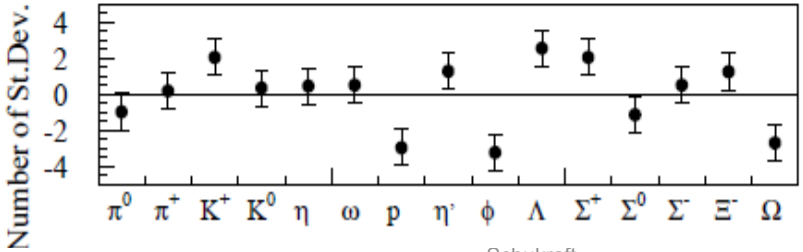
pp-pA-AA: Similarities & Differences

- Similar Particle Production
  - ⇨ Striking & very non-trivial similarities between pp(e+e-) and AA
  - ⇨ Striking & very non-trivial difference ('strangeness enhancement):

- Different Explanations & no connection (in general):
  - ⇨ **Born** (pp) <-> **Evolving** (AA) into equilibrium
  - ⇨ γ_s(GC) or r_c (SC) are fudge factors , i.e. not predicted/calculable as f(√(s),dn/dy, ..)
  - ⇨ **Core-Corona**: e^+e^- and pp ??







Particle Production: Data versus Thermal Model

Known, but often ignored

- No quantitative interpretation which smoothly describes small & large
  - despite evident relevance for understanding HOW we reach thermal ratios..

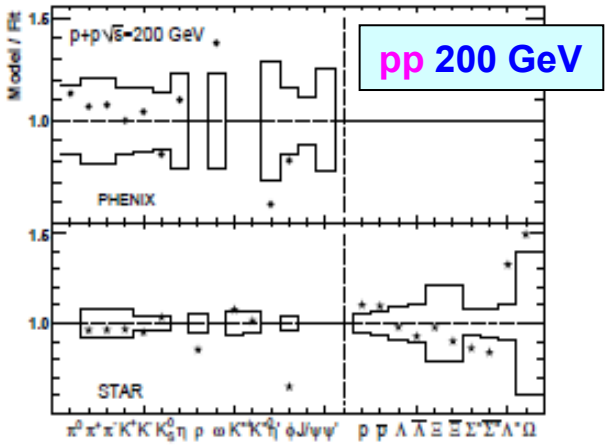


FIG. 18: Ratio of integrated yields predicted by the statistical model [40] to those of the constrained Tsallis fits for various particles. Results for fits to PHENIX data are shown in the upper panel and STAR data in the lower panel. The band reflects the uncertainty of the Tsallis fit results and includes the trigger uncertainty of 9.7% for PHENIX. The lower panel ... rediction was ... IX. There are ... l dashed line

Particle production: Data /Thermal Model

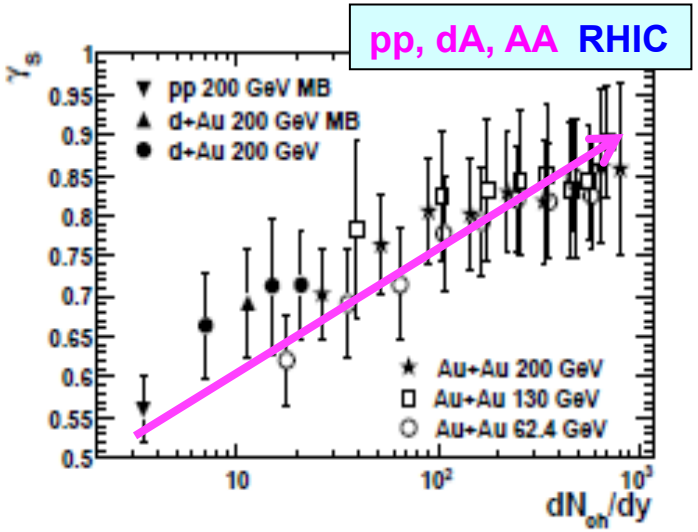


FIG. 35: Strangeness suppression factor extracted from chemical equilibrium model fit to pp and d+Au data at 200 GeV, and Au+Au data at 62.4 GeV, 130 GeV, and 200 GeV. Errors shown are the total statistic ... ties. The 200 GeV pp and Au+ ... Ref. [17].

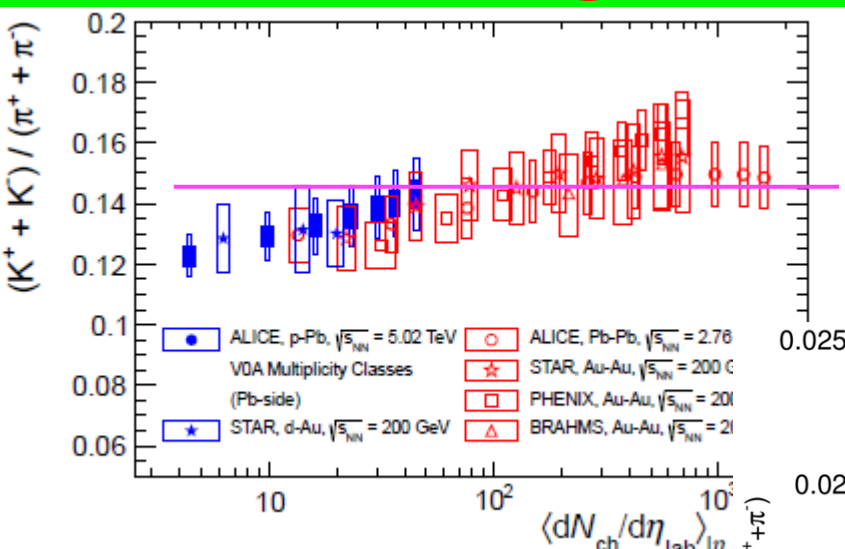
Particle production: γ_s vs dN_{ch}/dy

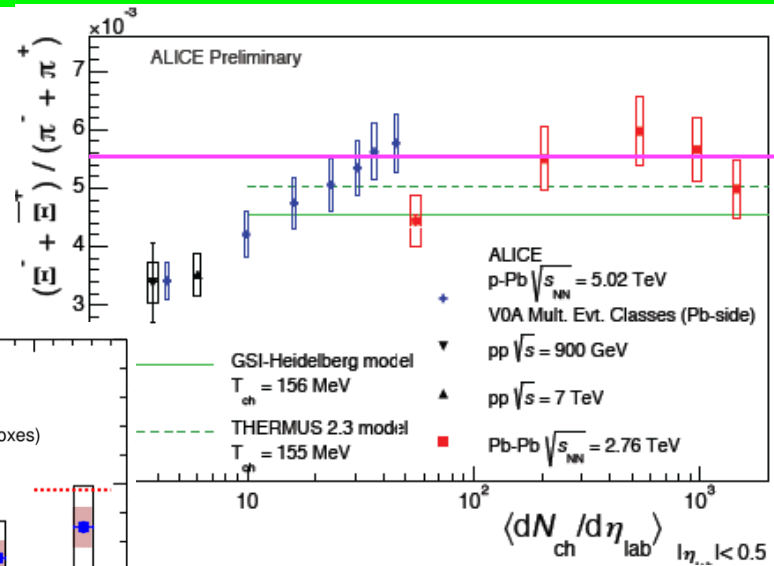
Although statistical models are not commonly used to describe $p+p$ data the agreement of the statistical model calculation with the STAR results was found to be accurate for most particles except for the ρ, ϕ, and Λ^* [40]. Leaving aside baryons, for which the calcula-

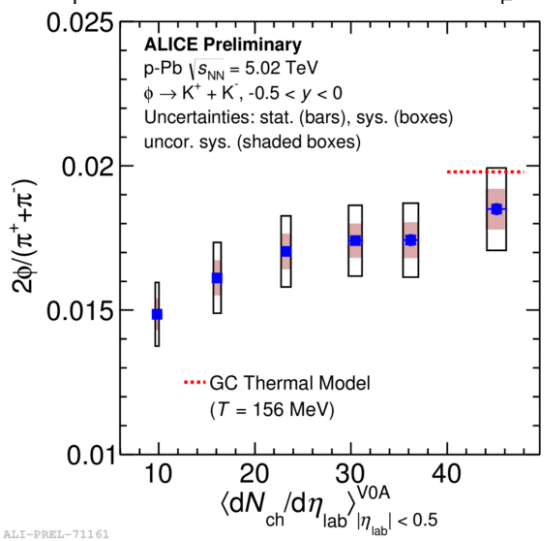
nificantly suppressed in these collisions. The strangeness suppression factor in medium-central to central Au+Au collisions is not much below unity; the strangeness and light flavor are nearly equilibrated, which may suggest a fundamental change from peripheral to central collisions.

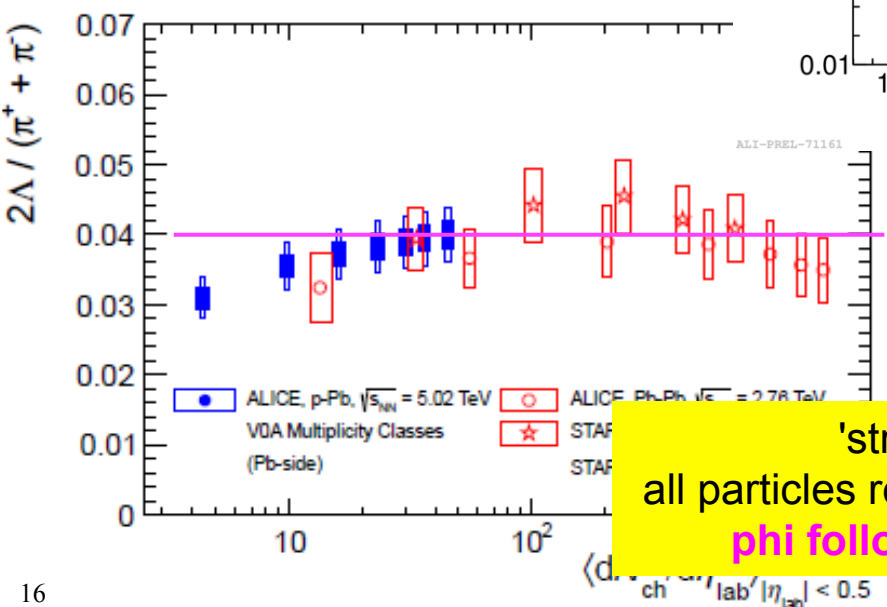
Star http://arxiv.org/abs/0808.2041 Phenix http://arxiv.org/abs/1005.3674 STAR http://arxiv.org/abs/nucl-ex/0607033

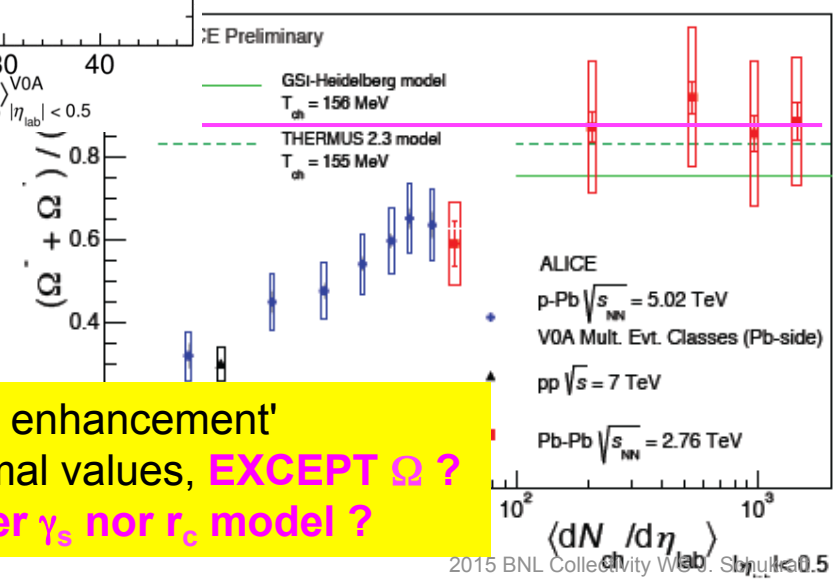
Strange Developments in pA









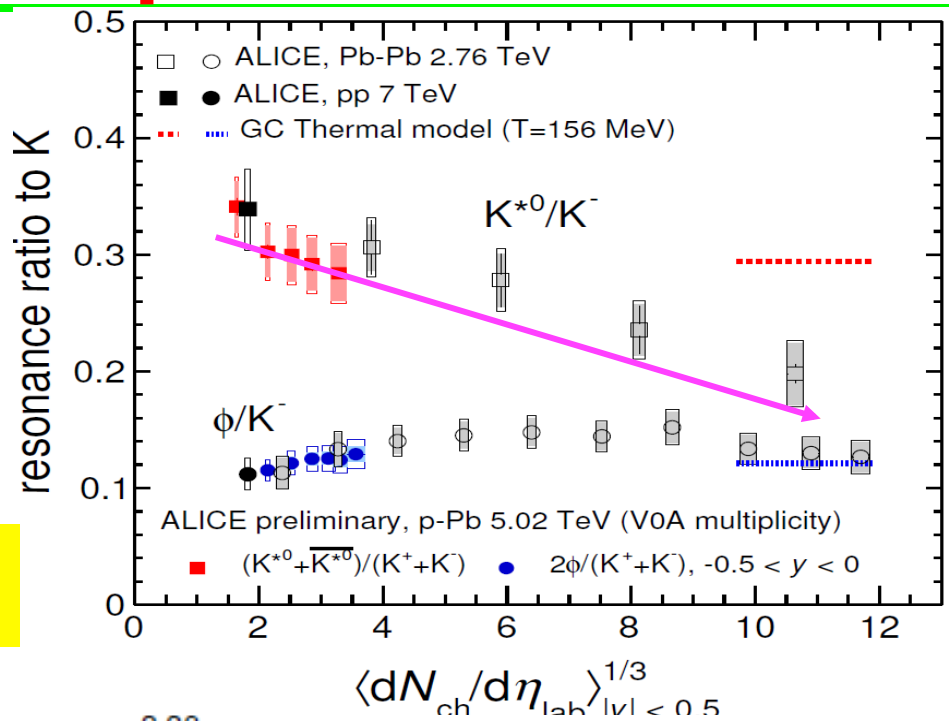


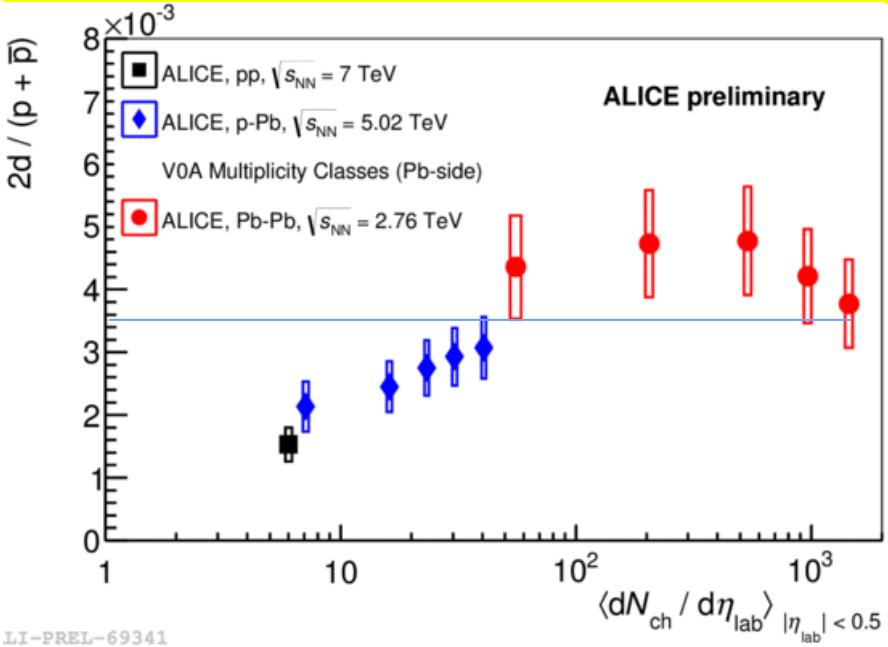
'strangeness enhancement'
all particles reach thermal values, **EXCEPT Ω ?**
phi follows neither γ_s nor r_c model ?

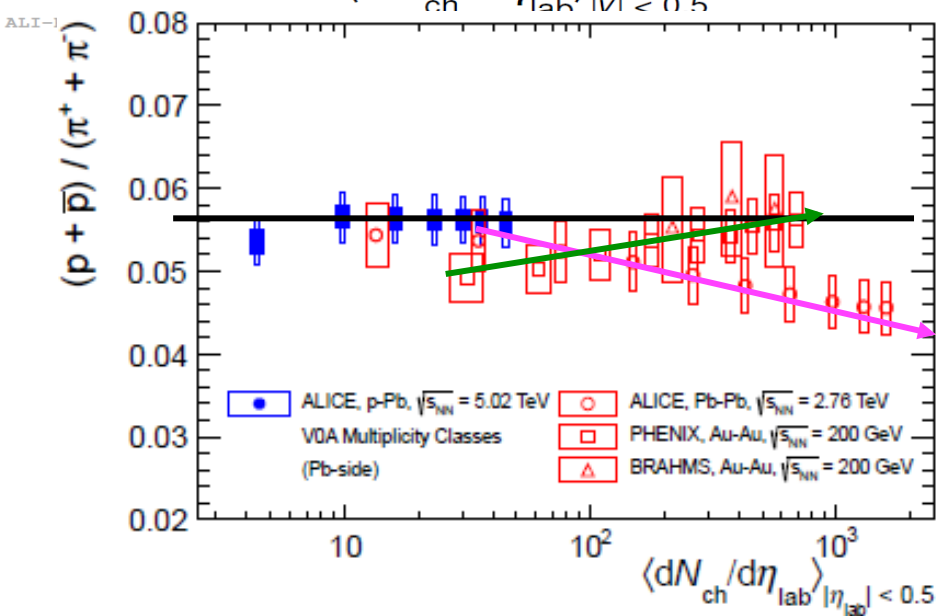
Other Developments

- Final state chemistry ?
  - ⇨ K* : decay + scattering
    - ✪ or sequential freeze-out (lower T_{chem})
  - ⇨ p/π: B annihilation (= seq. freezeout)
- light nuclei d,(^{3}He)
  - ⇨ pp x 2, reaches ≈ SM value
  - ⇨ canonical suppression ? $B_2(N_{ch}) \neq c$?

do we see dynamics at work
while reaching (leaving) thermodynamics ??

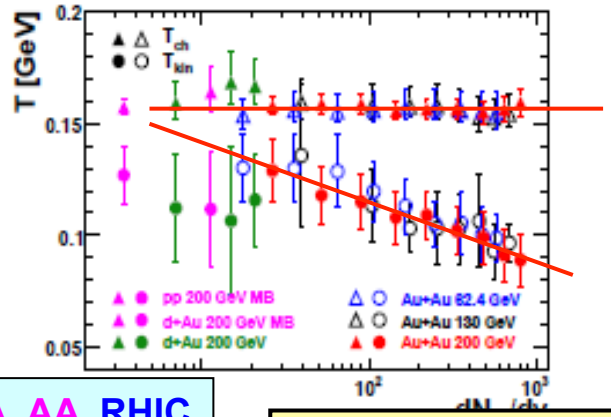






Momentum Spectra

- Known. but not really understood



pp, dA, AA RHIC

Radial Flow fit (BW): T & <β> vs dN_{ch}/dy

FIG. 36: (color online) Chem peratures as a function of Errors shown are the total tainties. The 200 GeV pp and Au+Au data are taken from Ref. [17].

The interpretation of the fit parameters is difficult in the context of a $p+p$ collision where the system is not expected to thermalize and the volume is small. It is important to note that in a pure thermal model, all emitted particles would be expected to reflect the same temperature. Non-thermal effects such as flow would modify this result. In $p+p$ collisions, the particle spectra clearly show different slopes and those slopes are not in agreement with the T parameter that results from the statistical model fit to the particle ratios. As no flow is thought to be present in the $p+p$ system and the results of Section IV B support that conclusion, this result is a further indication of contributions to the particle spectra from non-thermal processes like mini-jets.

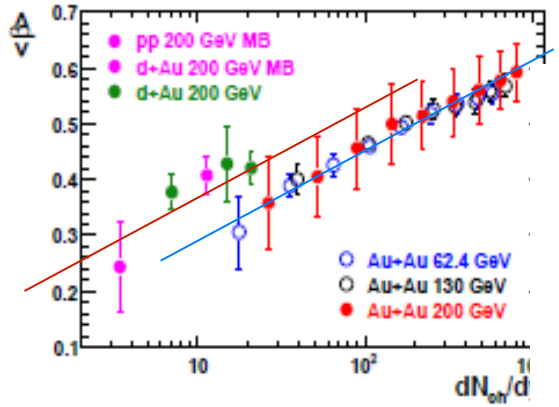


FIG. 37: (color online) Average transverse radial flow v ity extracted from blast-wave model fit to pp and d+A 200 GeV, and to Au+Au collisions at 62.4 GeV, 130 and 200 GeV as a function of the charged hadron multipl Errors shown are the total statistical and systematic u tainties. The 200 GeV pp and Au+Au data are taken Ref. [17].

pp 200 GeV

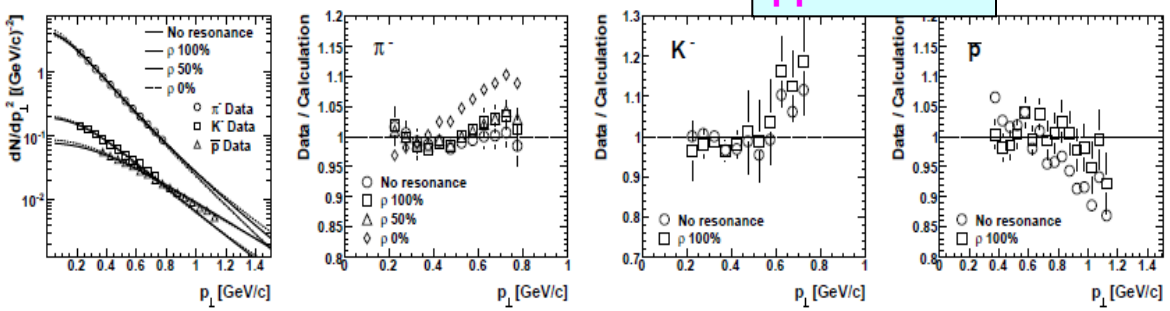


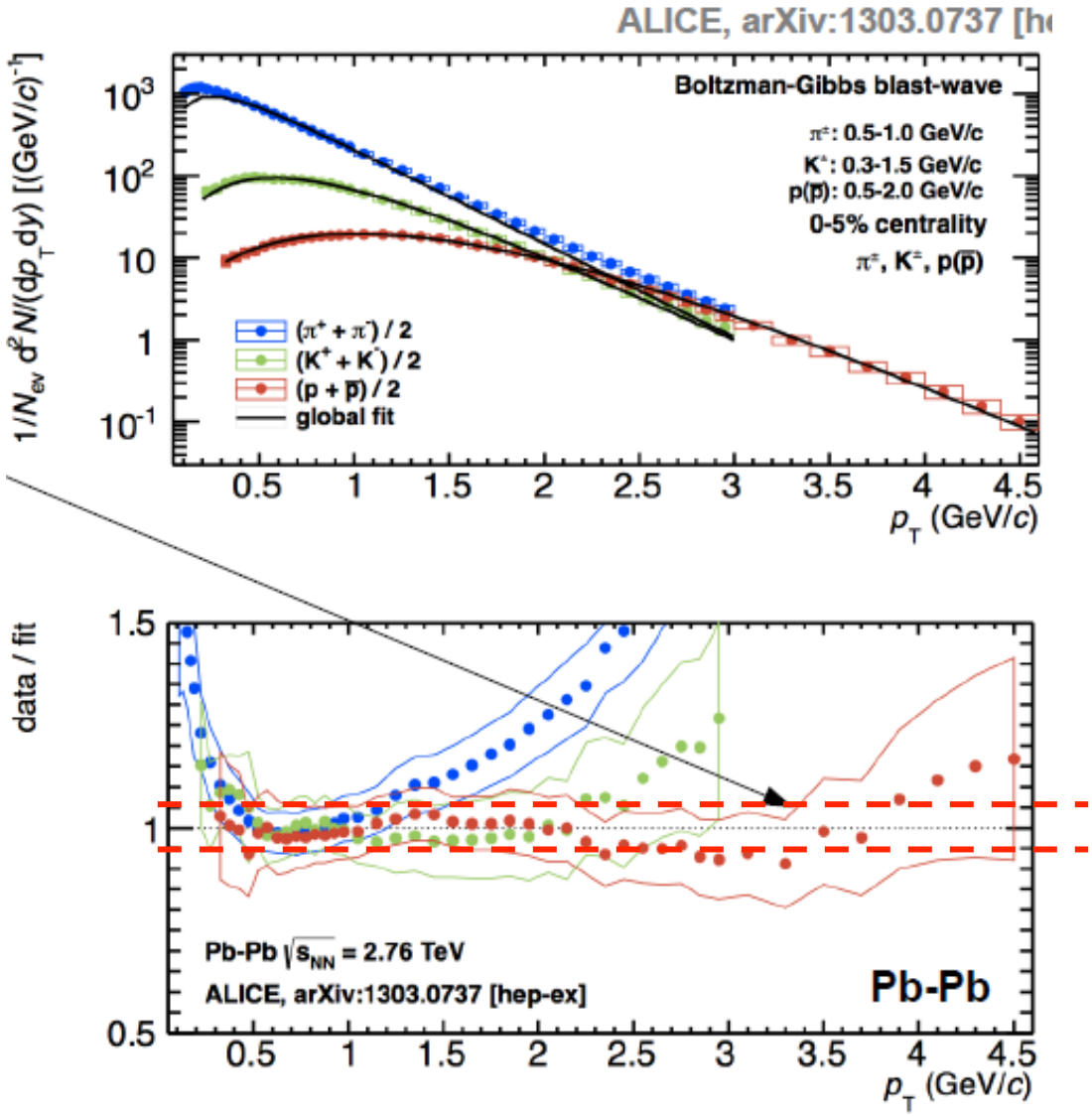
FIG. 45: Left panel: Fit of the calculated spectra (curves) to the measured ones (data points) in pp collisions at 200 GeV [17]. Four calculated spectra are shown for π^- (upper curves): including resonances with three different ρ contributions and excluding resonances. Only two calculated curves are shown for K^- (middle curves) and $\bar{p}$ (lower curves): including resonances with nels: Ratios of data spectrum to calculations. Two calculations are shown for for π^-. Error bars are the quadratic sum of the statistical and point point for two sets of the data points for π^- and only one set for K^- and $\bar{p}$.

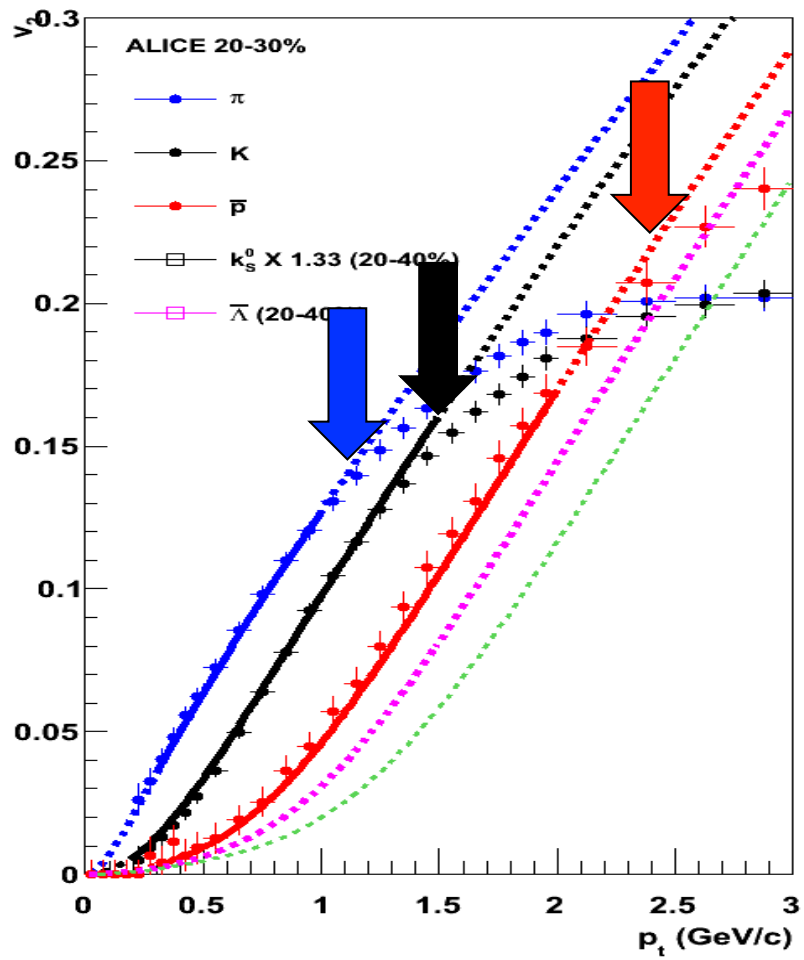
Radial Flow fit (BW): Data/Fit (π,K,p)

Star http://arxiv.org/abs/0808.2041

Hydro in PbPb

- ideal (BW) hydro ≈ ok < 1-2 GeV(p, K), > 3-4 GeV (p, Λ, ..) in both p_T and v_2
  - ⇨ deviations above: 'smooth decoupling ?'

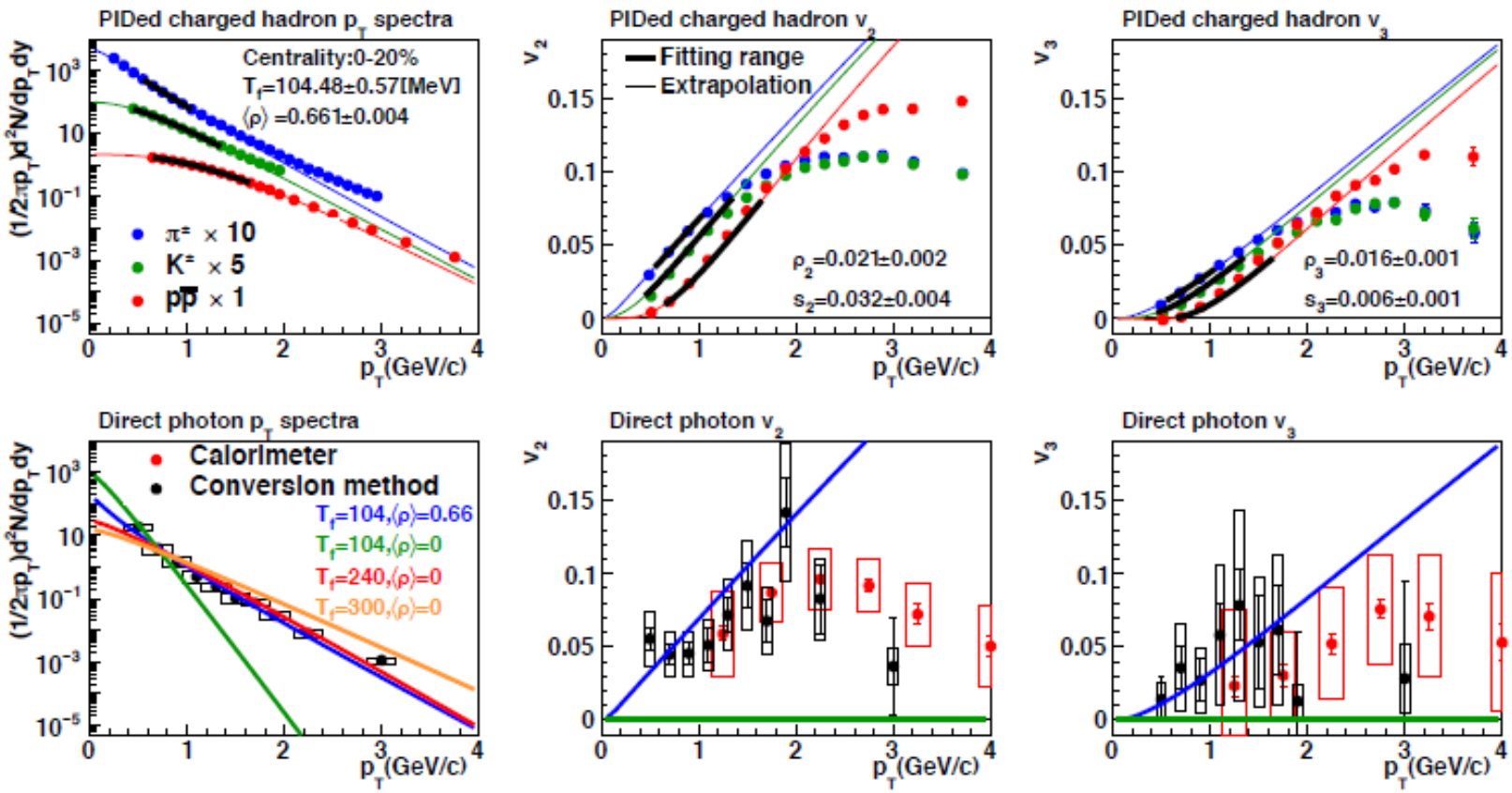




https://n-ext.inha.ac.kr/indico/event/13/contribution/14/material/slides/0.pdf

Blast-wave fit

Sanshiro Mizuno, D-thesis

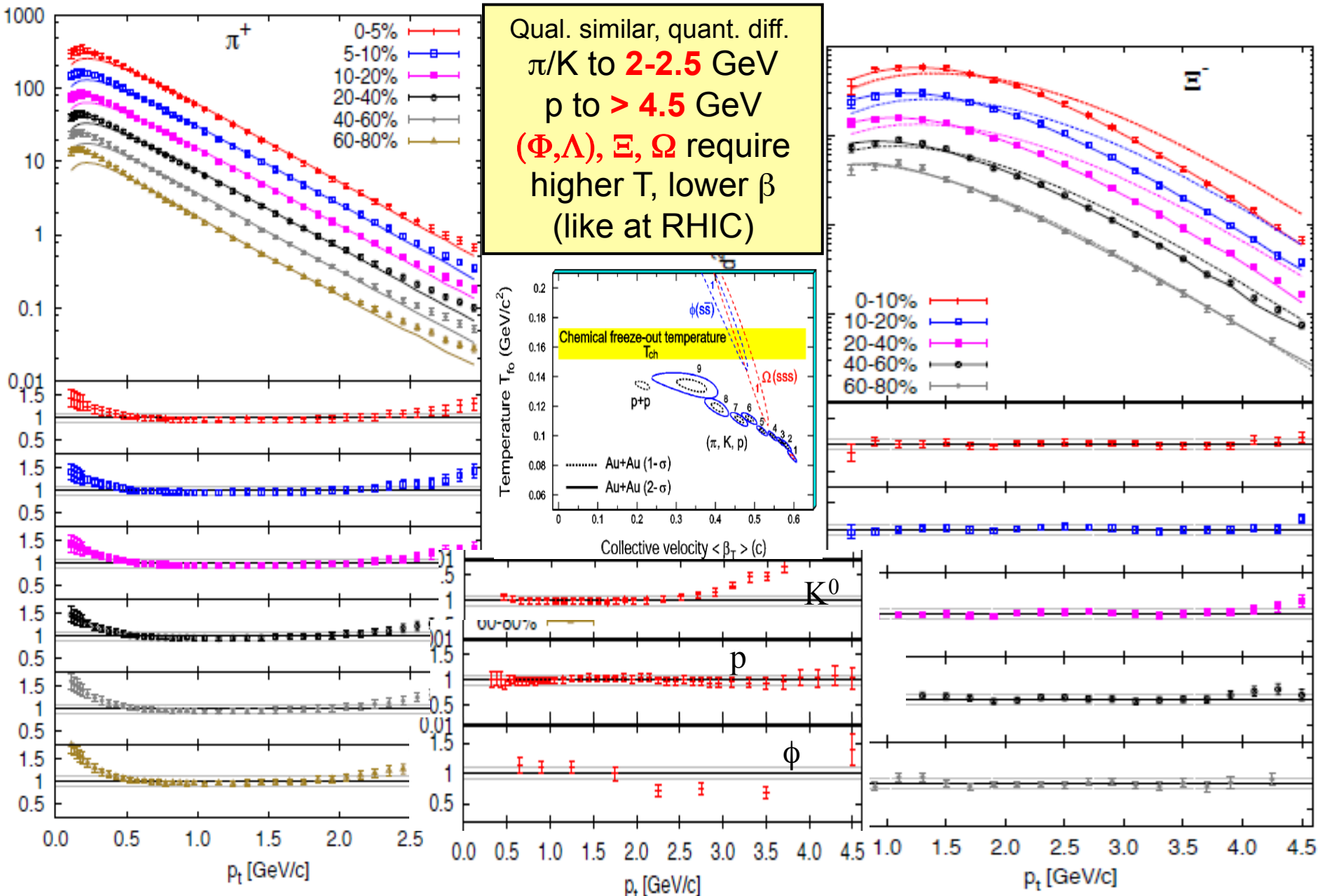


- Blast wave fit to direct photon p_T spectra & v_n
  - p_T spectra can be fitted with the same T for hadrons
  - Reasonable description for v_2 & v_3

'Decoupling' at RHIC at somewhat lower p_T

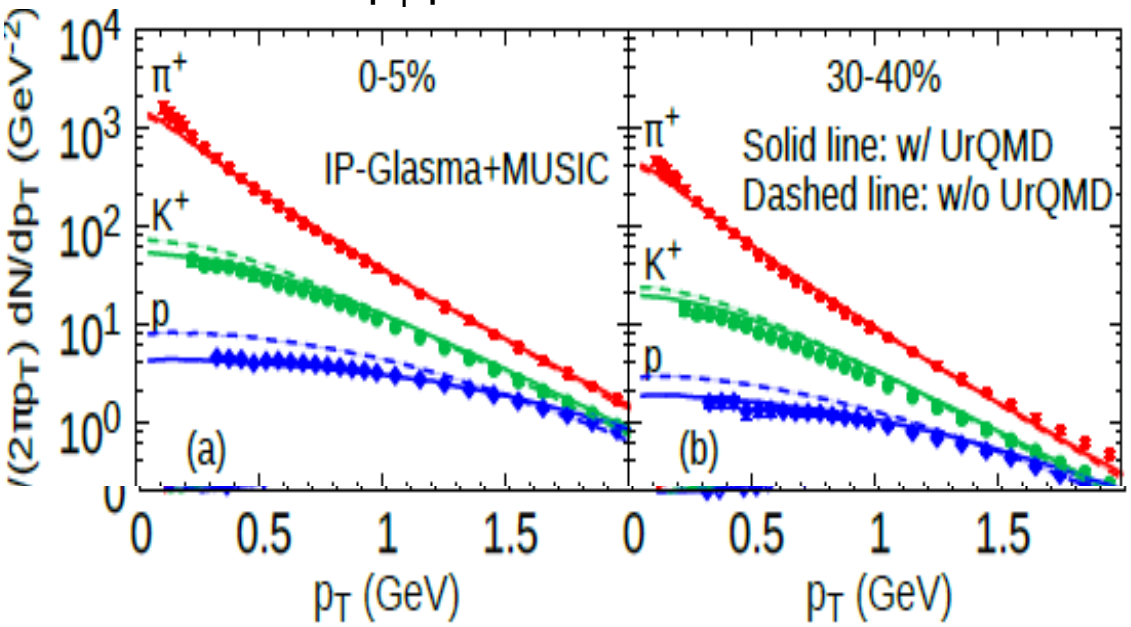
BW + Resonances (DRAGON 1502.01247)

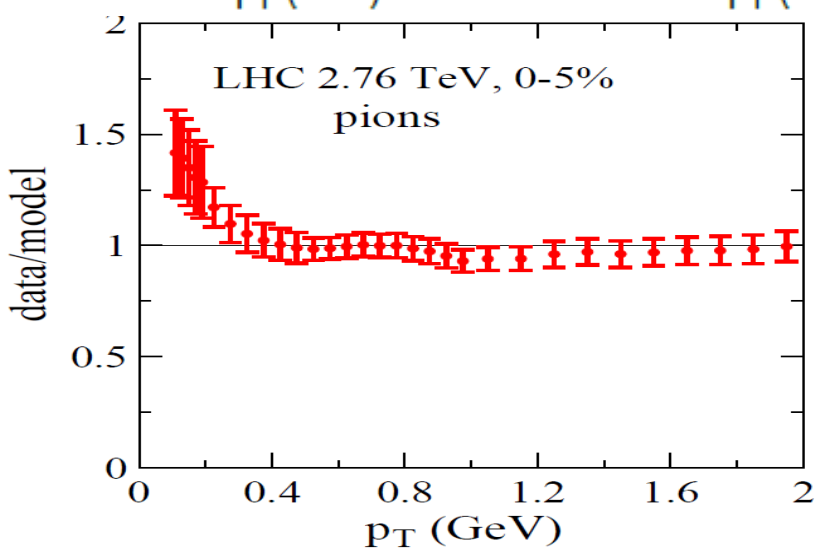
Qual. similar, quant. diff.
π/K to **2-2.5** GeV
p to **> 4.5** GeV
(Φ,Λ), Ξ, Ω require
higher T, lower β
(like at RHIC)

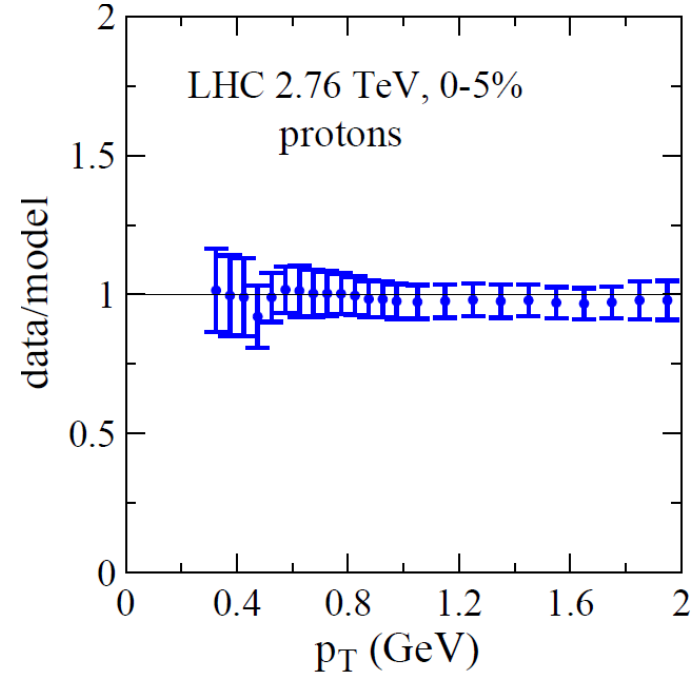


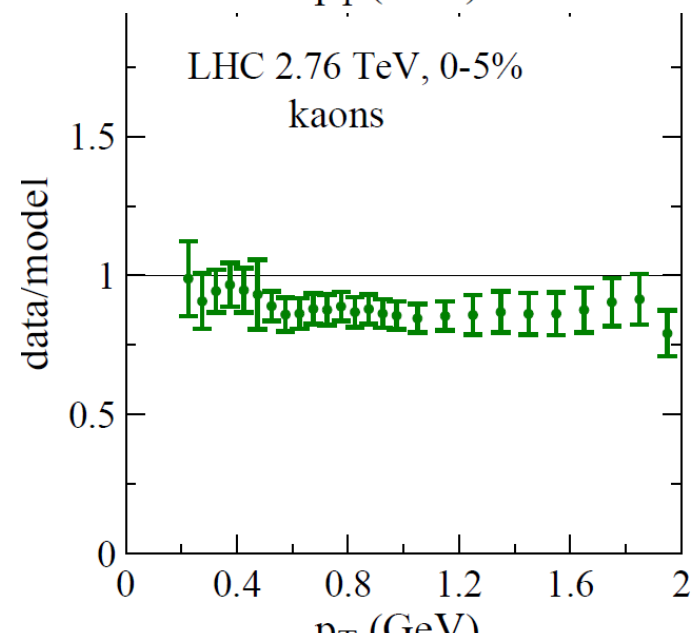
Real Hydro: IP-Glasma + Music (1502.016750)

- Data/Model (almost too perfect !)
  - ⇨ note that low p_T pion excess !!





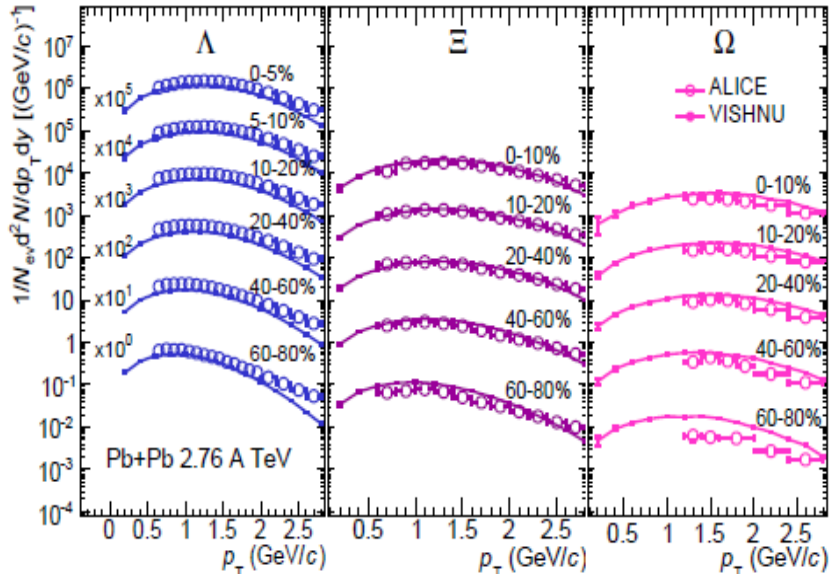




Hydro + AB does very well
(T, β, v_n)

Real Hydro Comparison

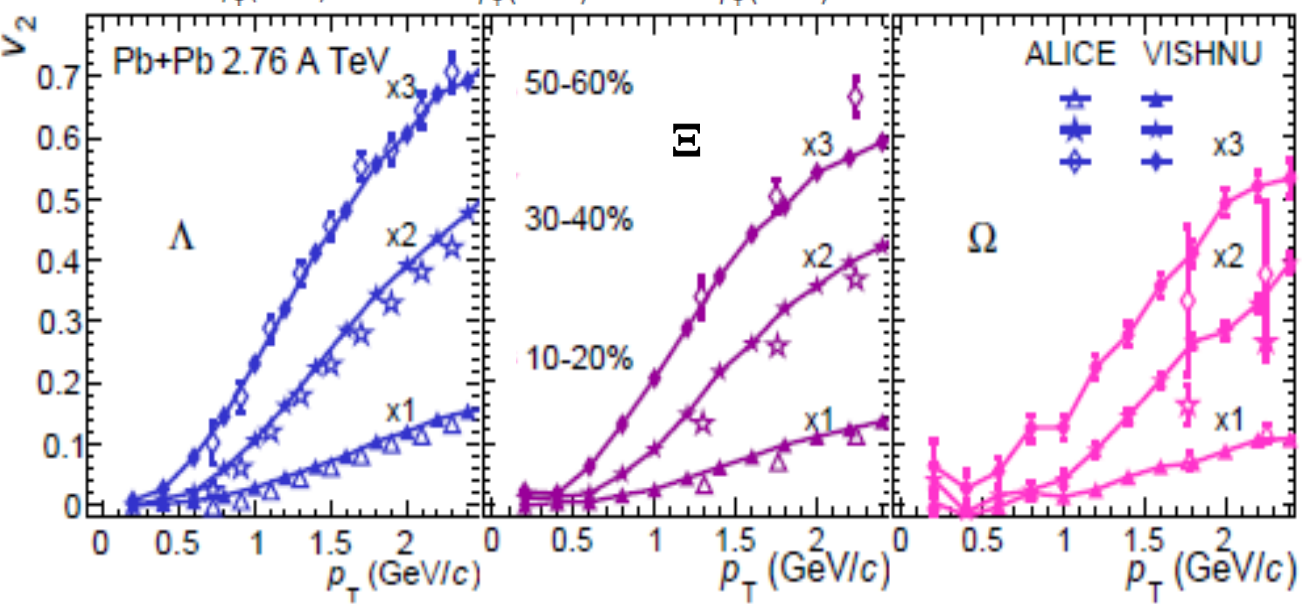
- VISHNU(Hydro + URQMD)1501.03286

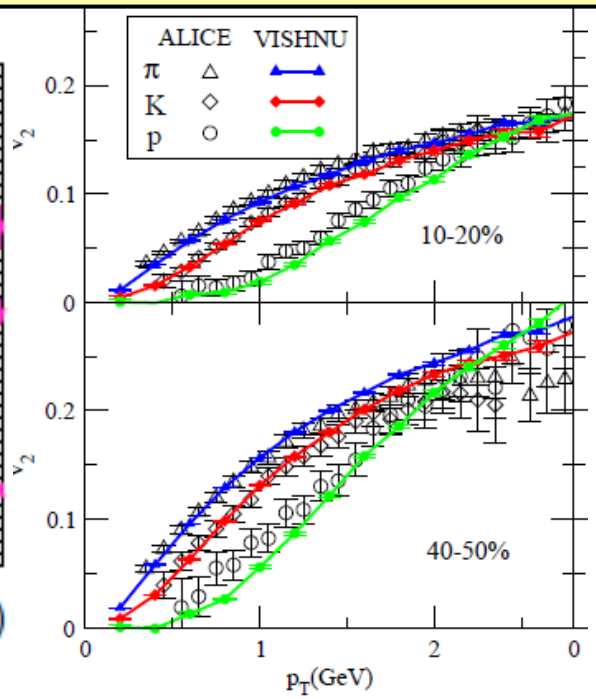


AfterBurner important:

incr. radial flow in hadron phase
=> explain differential freeze-out T_{kin} ?
=> meson-baryon crossing in v_2 ?

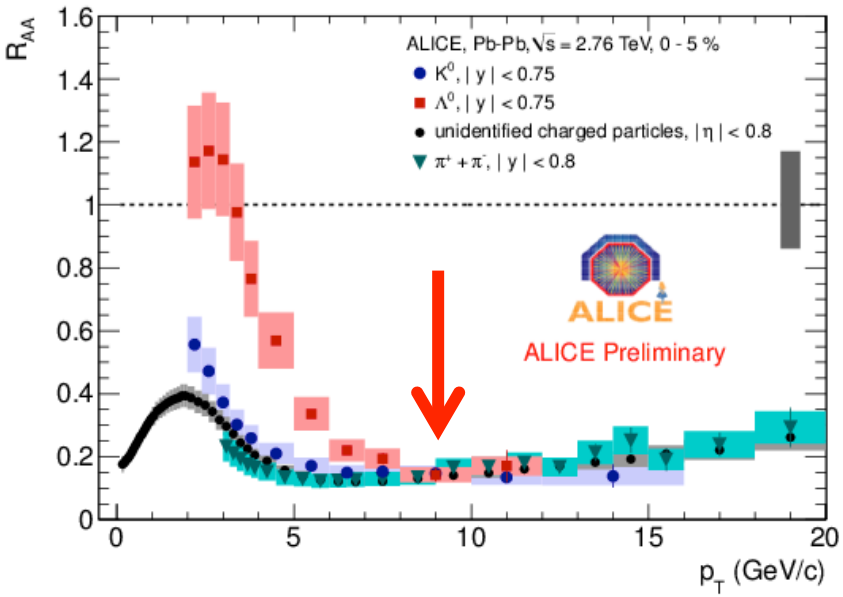
Would be interesting to see where data & (hydro + AB) deviate:
- Does it coincide in p_T & v_n
- Higher decoupling p_T for large mass ?

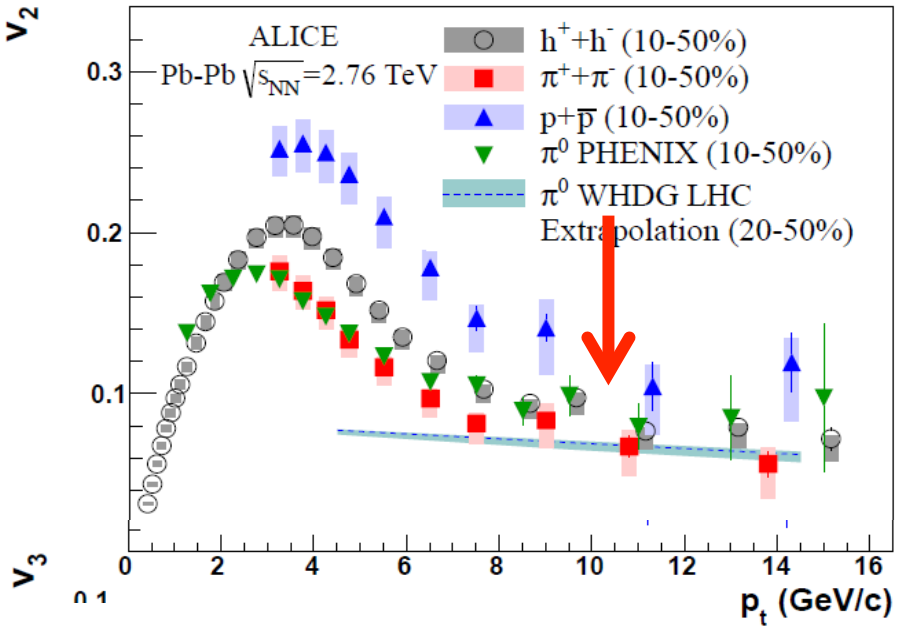




What happens after Hydro ?

- mass matters, up to a point (again, $p_T \approx v_2$)..
  - ⇨ presumably, 'falling out of hydro' is a smooth process (over large p_T range)
    - ✪ do we need **new physics** (eg coalescence) at intermediate p_T (4-10 GeV) ?
    - ✪ or a **smooth transition** between hydro and jets ?

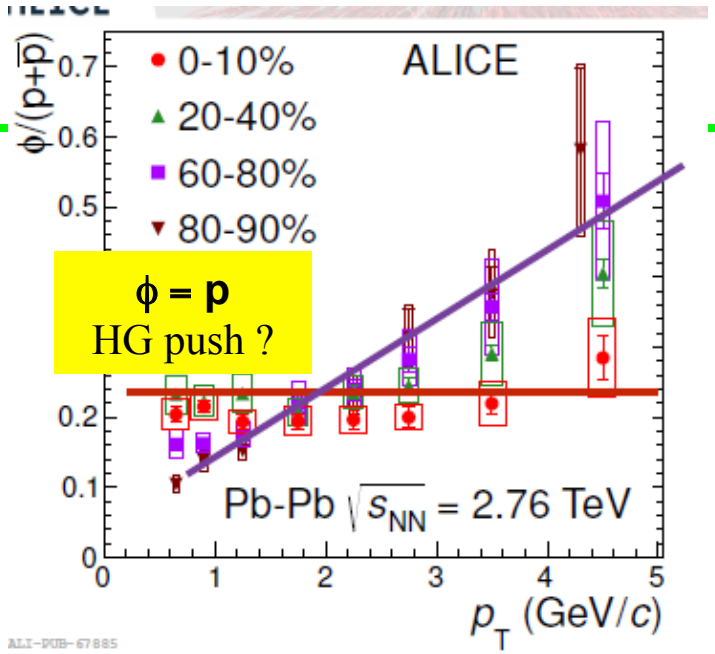


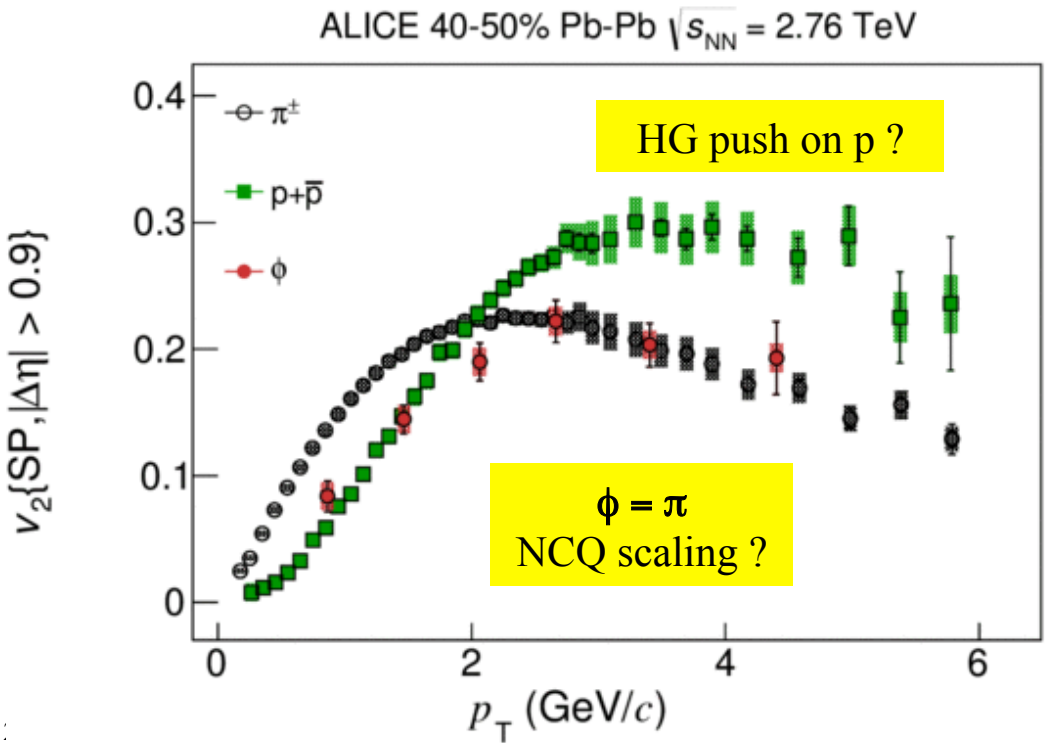


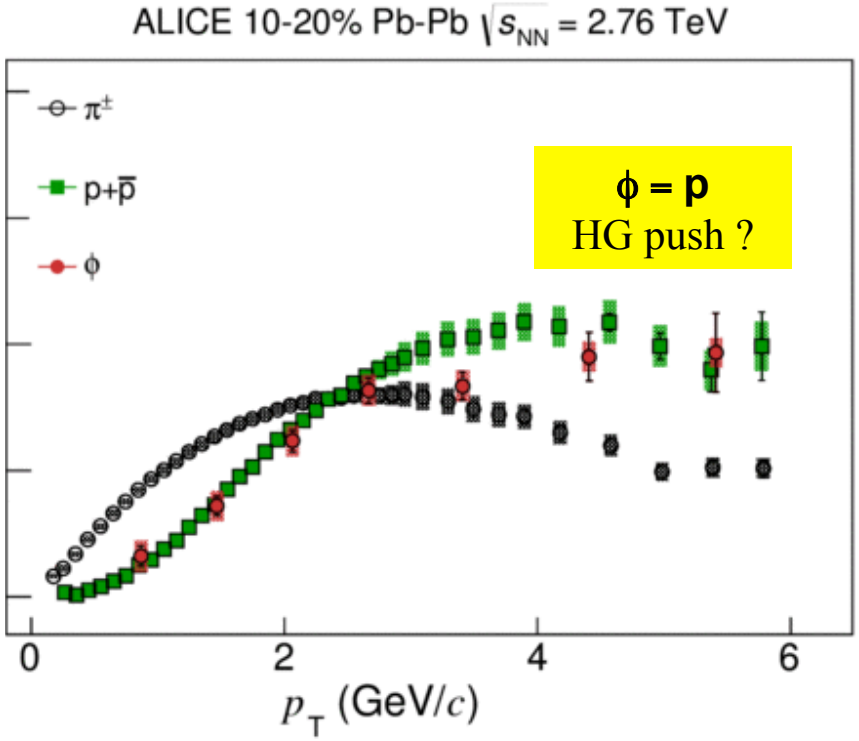
Will the ϕ tell ?

NCQ scaling in 40-50%
or
it's breaking in 10-20%
could be a (m, σ) effect in the HG !

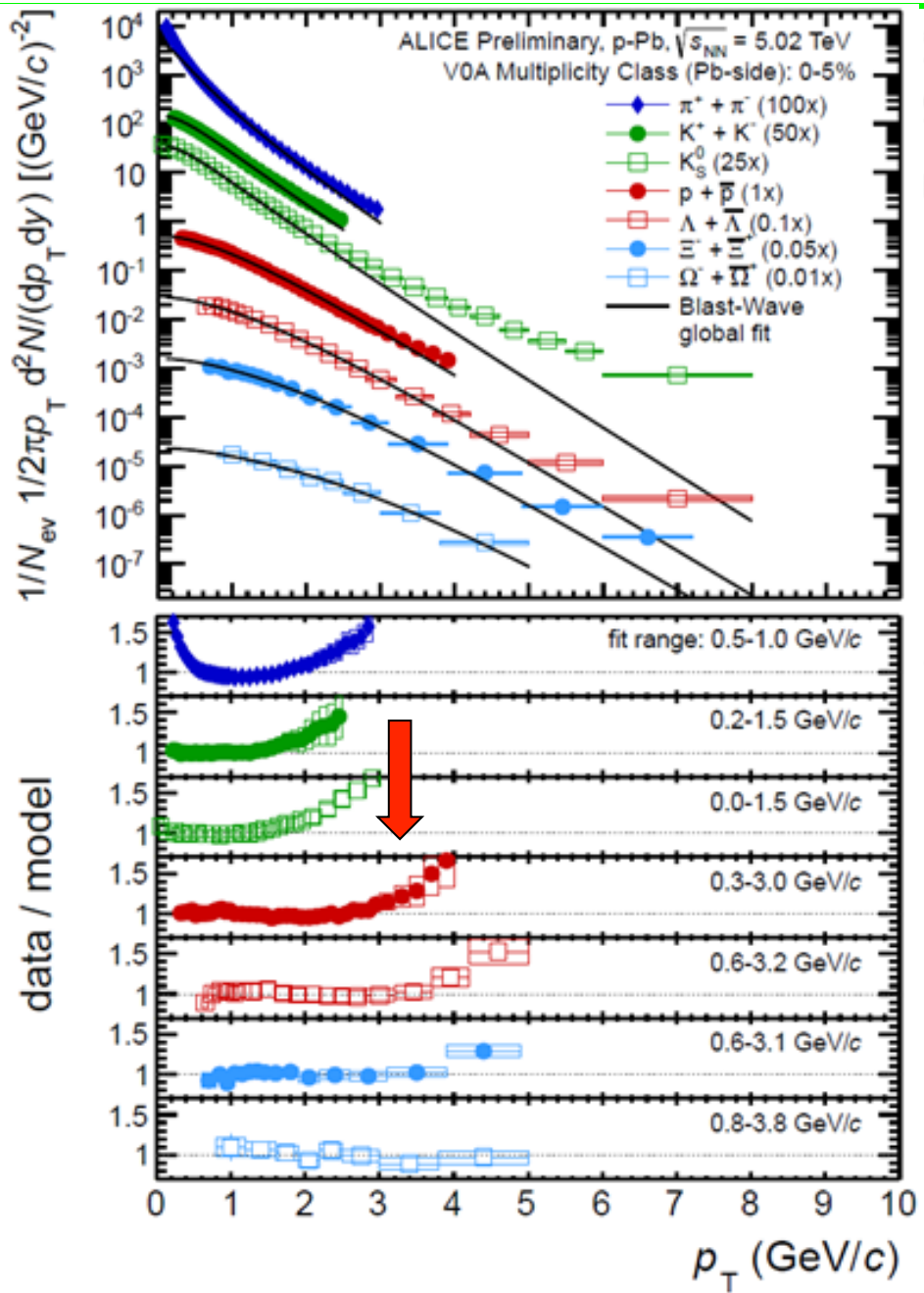
If only we could switch off the HG..

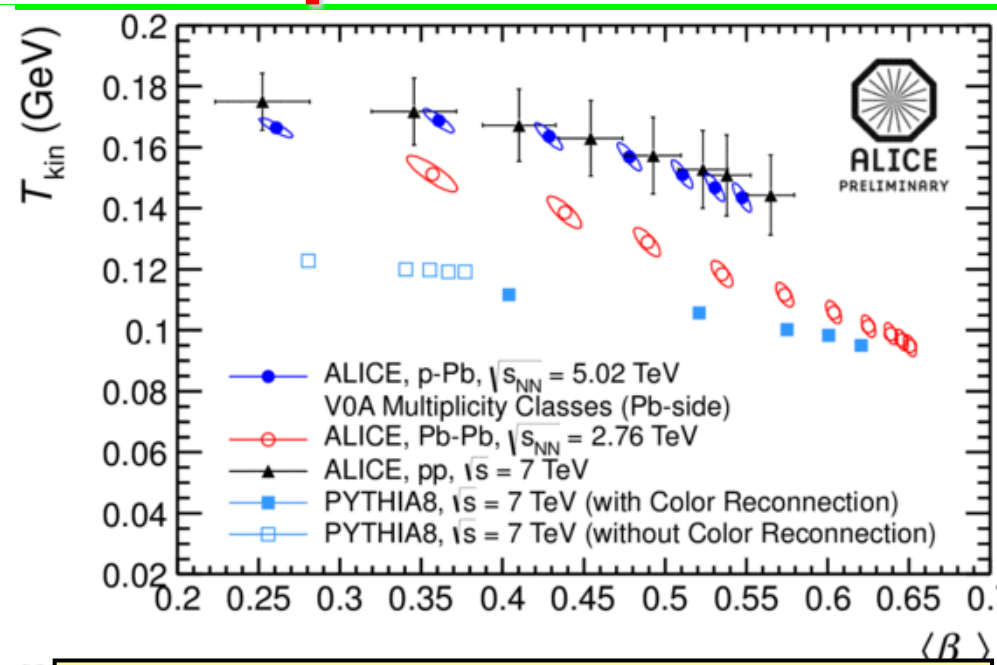






Radial Flow in pPb





earlier 'decoupling'

$T_{kin} \approx T_{chem}$

$\beta, T_{kin}(\pi, K, p) \approx \beta, T_{kin}(\Xi, \Omega)$

(**d, 3He**) in PbPb compatible with **flow**
d in pPb compatible with **coalescence**

no (or little) hadronic rescattering ?

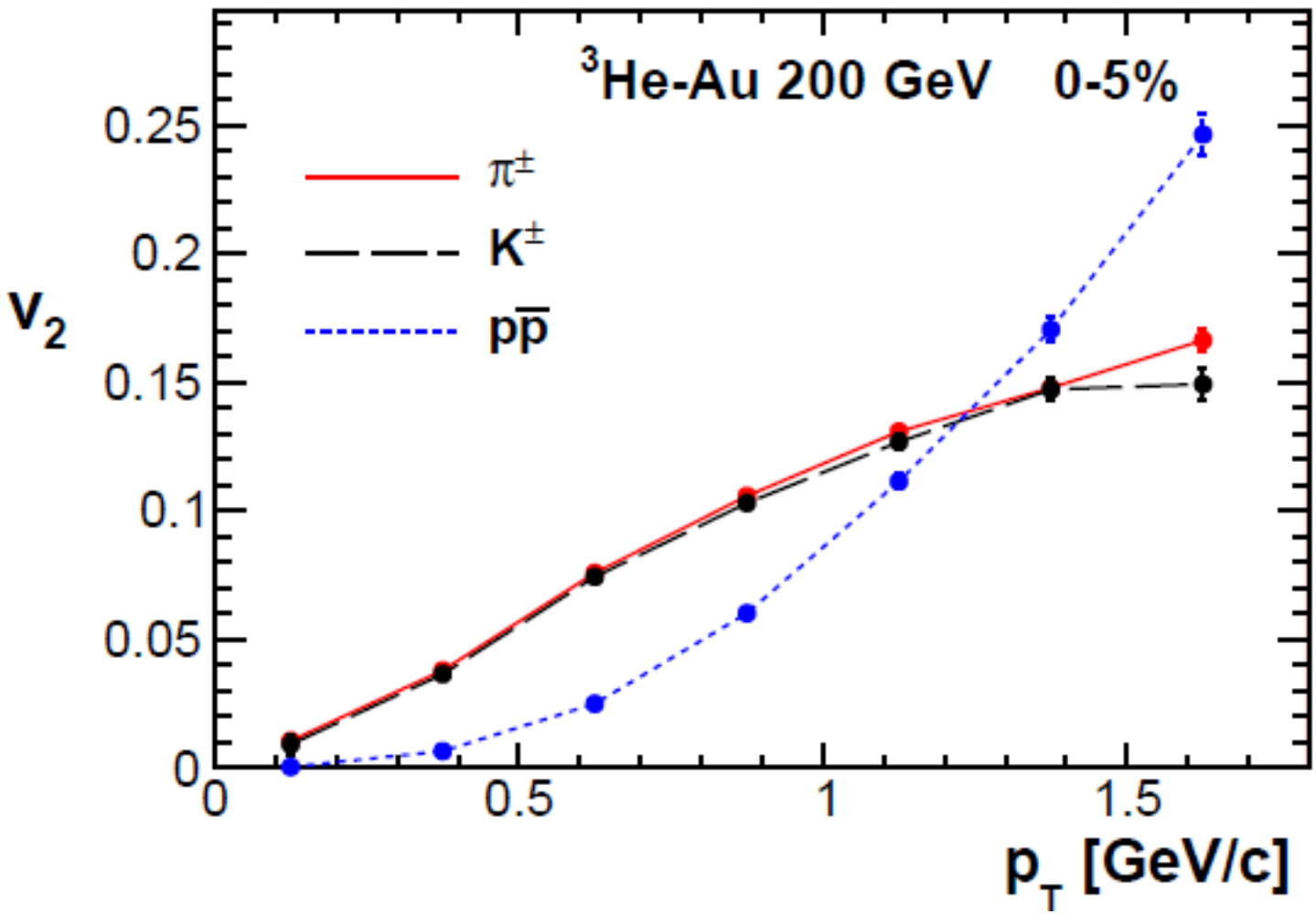
clear & direct view on the sQGP ??

Hypothesis: The physics underlying 'collectivity' signals is the same in AA, pA, and pp

- **MANY similar/identical observations**(@ similar N_{ch})**, no inconsistency** (?), **..**
  - ⇨ **1)** particle ratios (γ_s -> 1)
  - ⇨ **2)** p_T-spectra (radial flow),
  - ⇨ **3)** anisotropic flow: $v_n \sim \varepsilon_n$, v_n(p, d, ^{3}He), v_n(b), $v_n(p_T)$, v_2(LYZ), v_n(PID)
  - ⇨ **4)** HBT $r(N_{ch}, m_T)$
- **.. make the hypothesis increasingly more likely**
  - ⇨ a most 'natural' assumption, to <u>be proven wrong rather than right</u>
  - ⇨ subtle is the lord, but malicious he is not
- **What is is the 'underlying physics' ?**
  - ⇨ **sQGP: thermo + hydro** dynamics ('at the edge') ?
  - ⇨ **sMOG**: strongly **interacting FS matter** with **density gradients** (<u>1502.05572</u>)
  - ⇨ **CGC:** weakly int. **dense IS matter** + some conspiracies (in AA !!)
  - ⇨ ???
- **Why should we care ?**
  - ⇨ **leave the comfort zone** of infinite size equilibrium to study **dynamics**
    - ✪ from small & dilute -> large & dense: **emergence & limits of 'collectivity'**
  - ⇨ looking 'under the hood' is mandatory, whatever we may find !

Hydrodynamic modeling of ^{3}He-Au collisions at $\sqrt{s_{NN}} = 200$ GeV

Piotr Bożek[a], Wojciech Broniowski[b,c]



http://arxiv.org/abs/1503.00468

Collectivity in small systems with large multiplicity, **pp and pA**

Edward Shuryak
Stony Brook

Workshop "Collectivity in Small Colliding Systems with High Multiplicity", BNL, March 4-6,2015

- 2010: ridge in pp *(high mult.events P=10^-6 or M$ per event!)* then seen in central pA *(P=few percents)*

- ***Femtoscopy (ALICE) in pp shows signs of flow in radii (k_t)***

- *central-peripheral => **antiridge**=elliptic flow, consistent with hydro (Bozek et al) and "sound systematics" from AA (Lacey).*

- *v2 and v3 scaling pA/AA* (arXiv:1408.3411 *Basar Teaney)*

- *PHENIX does **d Au*** and finds about twice larger v2, again in agreement with Glauber+*hydro (Bozek et al) => He3 run results (to be reported here)*

- *(2013) strong radial flow suggested* (arXiv:1301.4470, *ES,Zahed)* with surprisingly small viscous corrections even for very small sizes. QGP is conformal: **small system and high T compensate each other**

- *(2013)* ***strong radial flow* seen** in identified particle spectra (by CMS and ALICE) fitted by hydro and blast wave: it is indeed **stronger than in AA**

- QM 2014 v2{4,6,8} are the same , from CMS => **collectivity**

- **femtoscopy radii in pp (Hirono+ES,arXive:1412.0063) and spectra (Kalaydzhyan+ES) +hydro => (surprisingly) small initial size 0.6-1 fm**

- **pA: many wounded nucleons and strings. "Spaghetti collapse" scenario (1404.1888, Kalaydzhyan+ES), now also established in AdS/QCD (Iatrakis, Ramamurti,ES)**

THE SMALLEST DROPS OF QGP

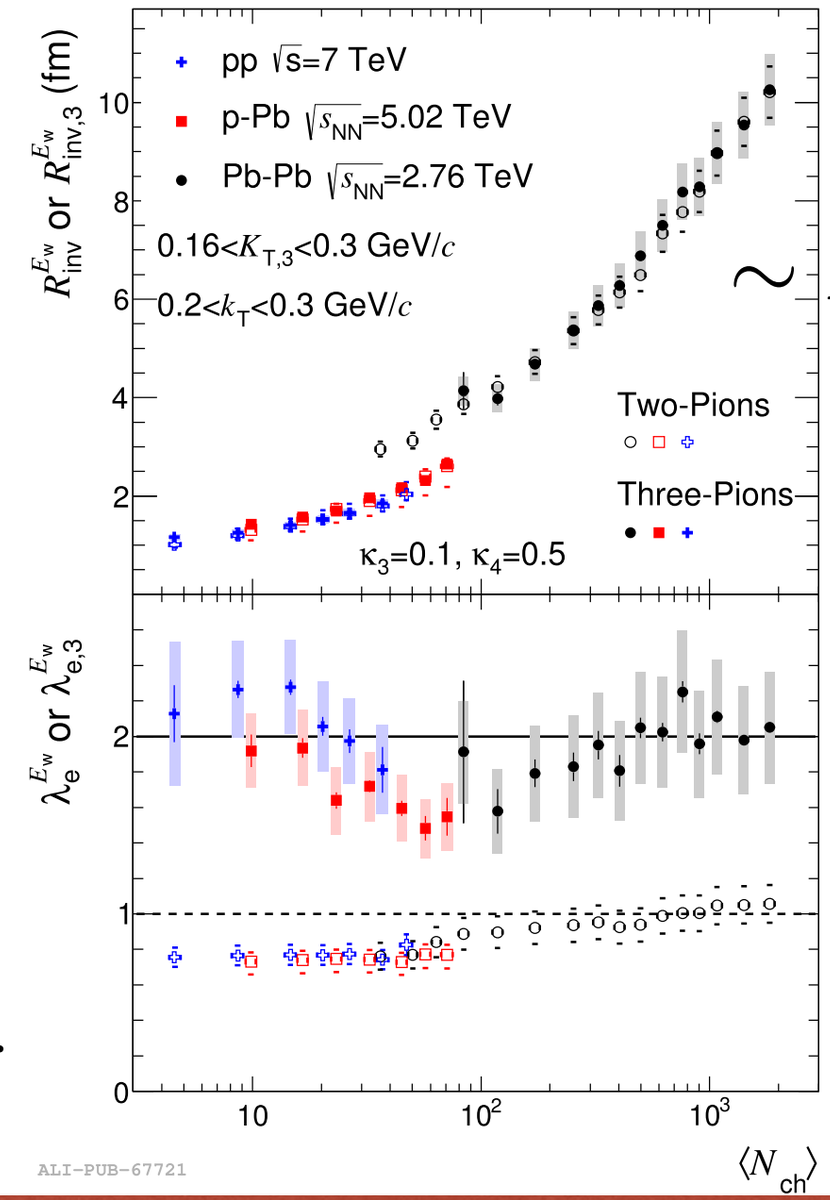
AA data follow N^1/3 curve => fixed freeze out density

Yet the pp, pA data apparently fall on a different line

Why do those systems get frozen at higher density, than those produced in AA?
(hint #1)

$$< n\sigma v >= \tau_{coll}^{-1}(n) \sim \tau_{expansion}^{-1} = \frac{dn(\tau)/d\tau}{n(\tau)}$$

So, more "explosive" systems, with larger expansion rate, freezeout earlier, at higher density.



$\sim N_{ch}^{1/3}$

(up-
unc-

Where is the room for that, people usually ask, given that even the final size of these objects is not large but even smaller than in peripheral AA, which has weak radial flow.
Well, the only space left is at the beginning: those systems must start accelerating earlier, from even smaller size,

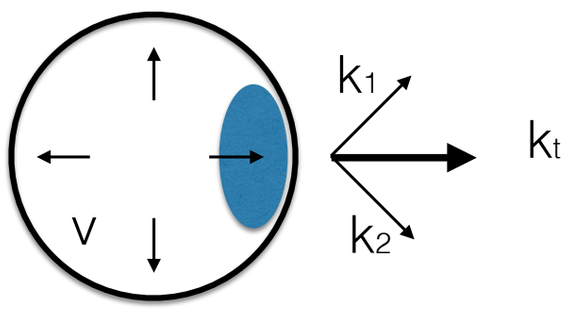
arXiv:1412.0063v1 [hep-ph] 29 Nov 2014

Femtoscopic Signature of Strong Radial Flow in High-multiplicity *pp* Collisions

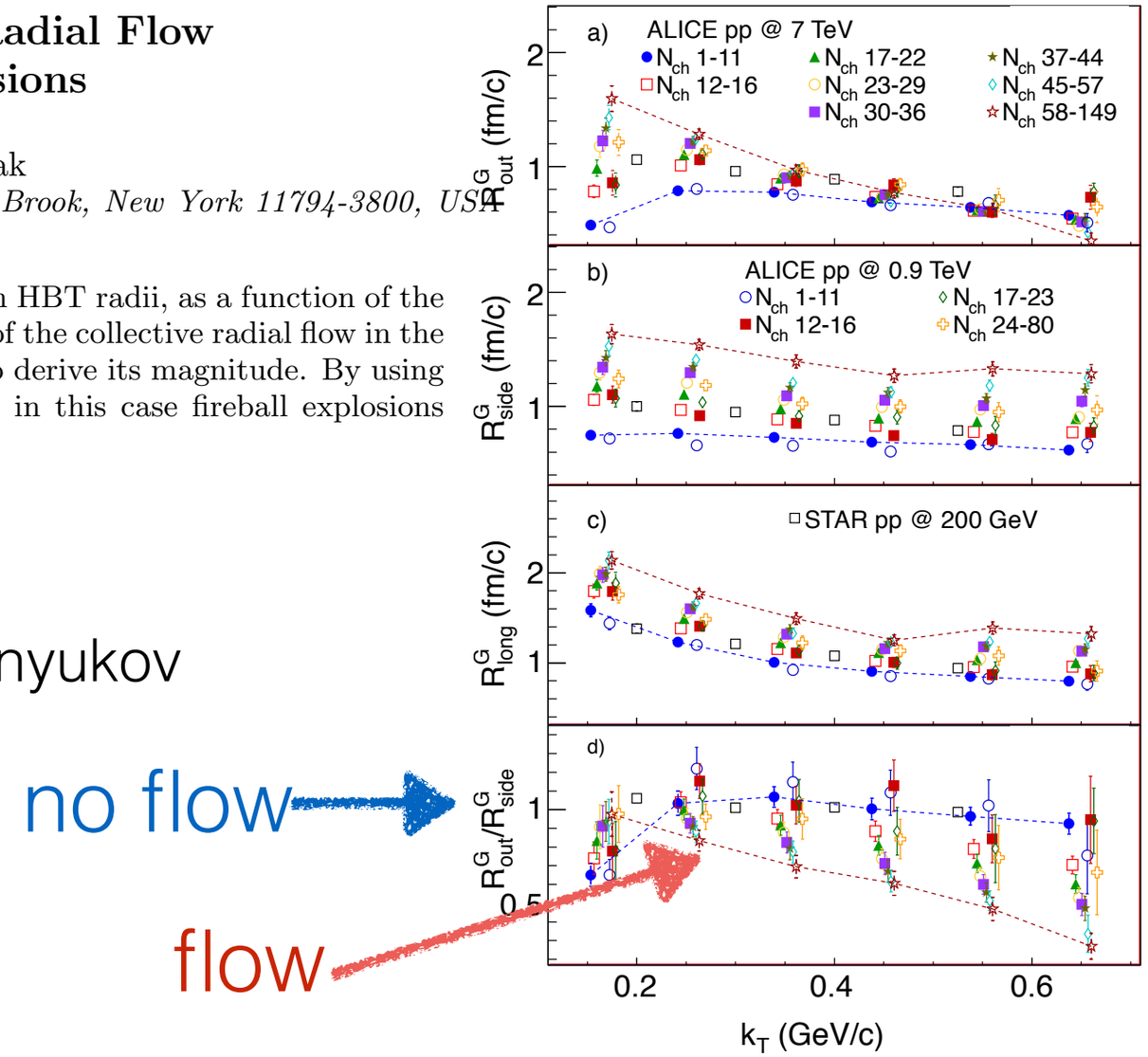
Yuji Hirono* and Edward Shuryak
Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA

(Dated: December 2, 2014)

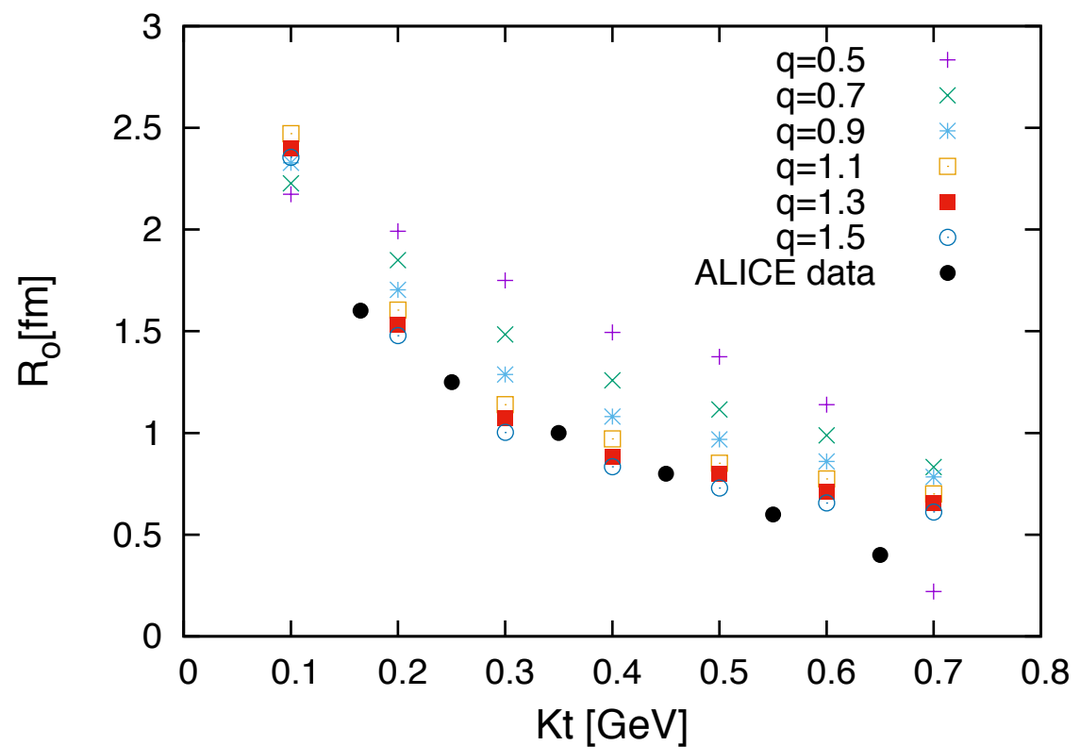
Hydrodynamic simulations are used to calculate the identical pion HBT radii, as a function of the pair momentum k_T. This dependence is sensitive to the magnitude of the collective radial flow in the transverse plane, and thus comparison to ALICE data enables us to derive its magnitude. By using hydro solutions with variable initial parameters we conclude that in this case fireball explosions starts with a very small initial size, well below 1 fm.



Makhlin,Sinyukov



For most multiplicity bins the radii
do not depend on kt of the pair,
but the largest multiplicity one shows strong reduction:
this is a signature of the radial flow



Gubser solution
at early time
+numerical hydro
at later stages

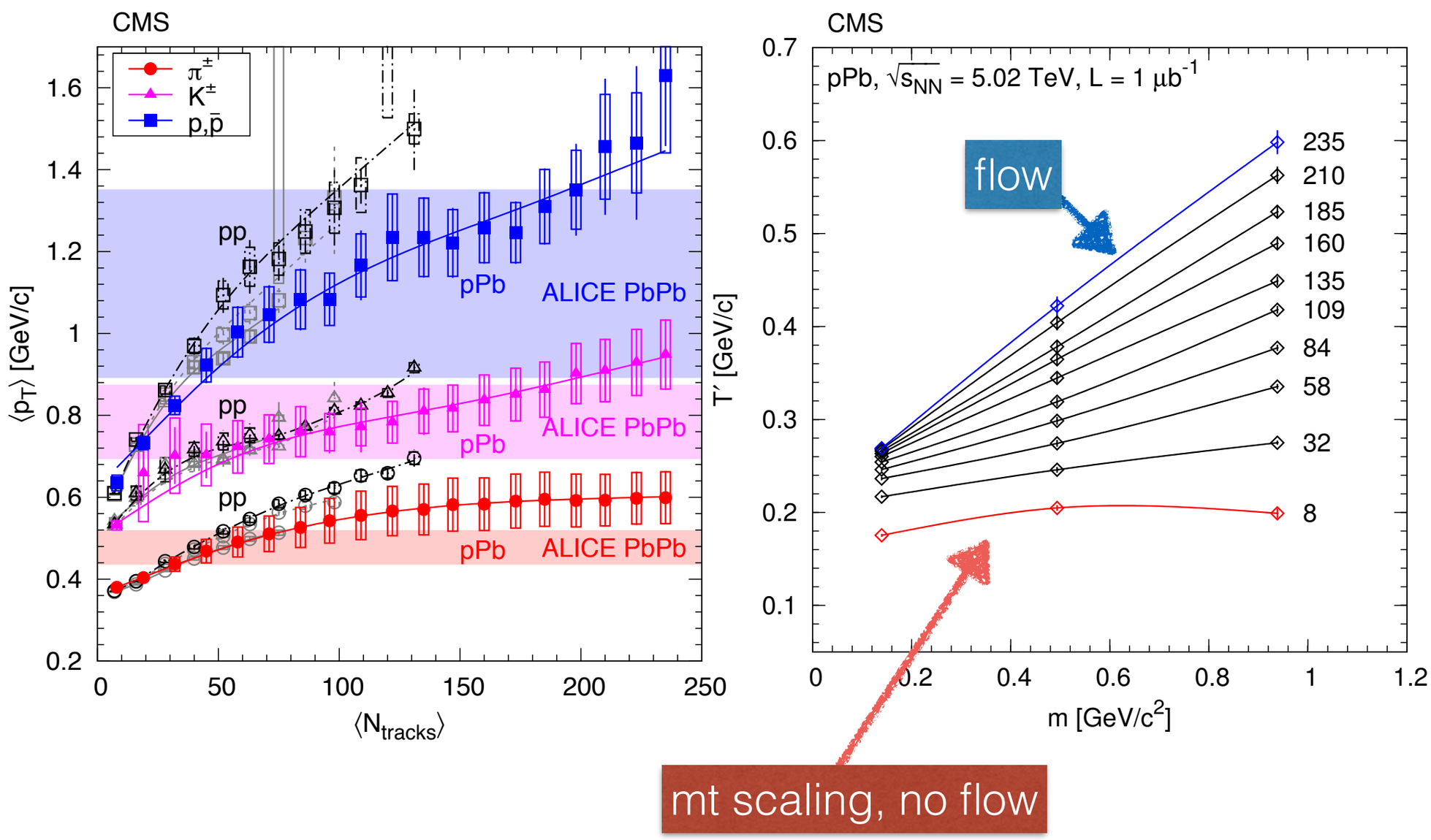
$$t = q\bar{\tau}, \quad r = q\bar{r}$$

$$\frac{\epsilon}{q^4} = \frac{\hat{\epsilon}_0 2^{8/3}}{t^{4/3}\left[1 + 2(t^2 + r^2) + (t^2 - r^2)^2\right]^{4/3}}$$

$$v_\perp(t,r) = tanh(y_\perp) = \frac{2tr}{1 + t^2 + r^2}$$

conclusion: in order to describe
ALICE femtoscopy pp data
one needs very strong flow
=> surprisingly small initial size1/q=2/3 fm

the radial flow



PHYSICAL REVIEW C **88**, 044915 (2013)

High-multiplicity *pp* and *pA* collisions: Hydrodynamics at its edge

Edward Shuryak and Ismail Zahed

We predicted the radial flow in pp/pA to be even stronger than in central AA

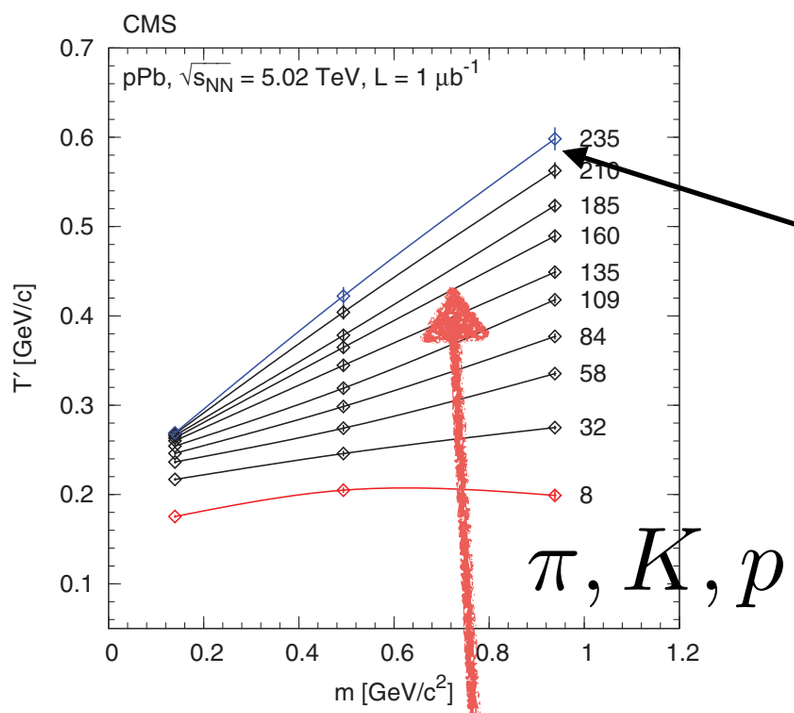


FIG. 8. (Color online) The slopes of the $m_\perp$ distribution T' (GeV) as a function of the particle mass, from [13]. The numbers on the right

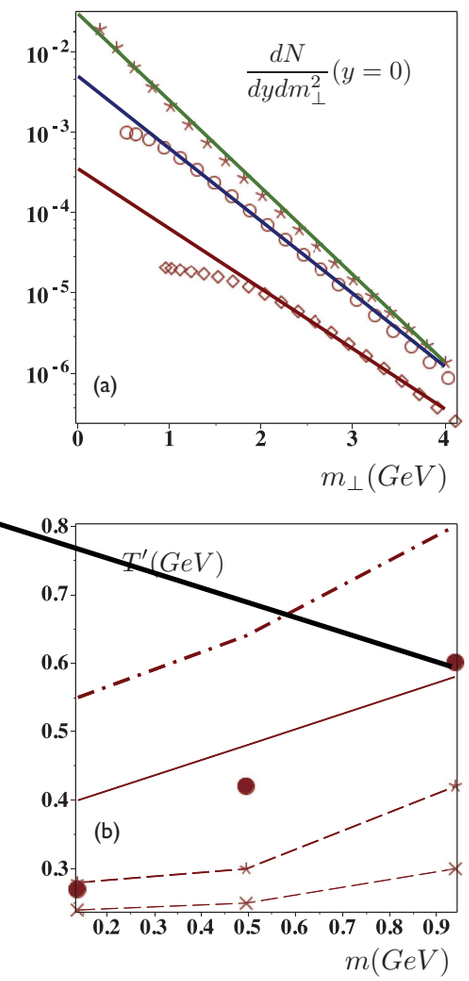


FIG. 9. (Color online) (a) A sample of spectra calculated for π, K, p, top-to-bottom, versus $m_\perp$ (GeV), together with fitted exponents.(b) Comparison of the experimental slopes $T'(m)$ versus the particle mass m (GeV). The solid circles are from the highest multiplicity bin data of Fig. 8, compared to the theoretical predictions. The solid and dash-dotted lines are our calculations for freeze-out temperatures $T_f = 0.17, 0.12$ GeV, respectively. The asterisk-marked dashed lines are for Epos LHC model, diagonal crosses on the dashed line are for AMTP model.

some hydro underpredict the radial flow

Not the Mt scaling at large Ntr => not a large Qs but a collective flow: p=m v

Collective flow in high-multiplicity proton-proton collisions

Tigran Kalaydzhyan* and Edward Shuryak†
Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA
(Dated: February 27, 2015)

We present an evidence of strong radial flow in high-multiplicity *pp* collisions. We analyze the CMS data on the inclusive spectra of the charged pions, kaons and protons in the LHC $\sqrt{s} = 7\,\text{TeV}$ collisions. For $\langle N_{\text{tracks}} \rangle \gtrsim 75$ we demonstrate the consistency of the hydrodynamic description with the (idealized) Gubser's flow. Using a one parameter fit of the model to experimental data, we obtain the initial fireball size to be smaller and of the order of 1 fm. At smaller multiplicities, the fit cannot be performed which shows a limitation o
with falsifiability of our theory.

how does flow depend on the initial size of the fireball?

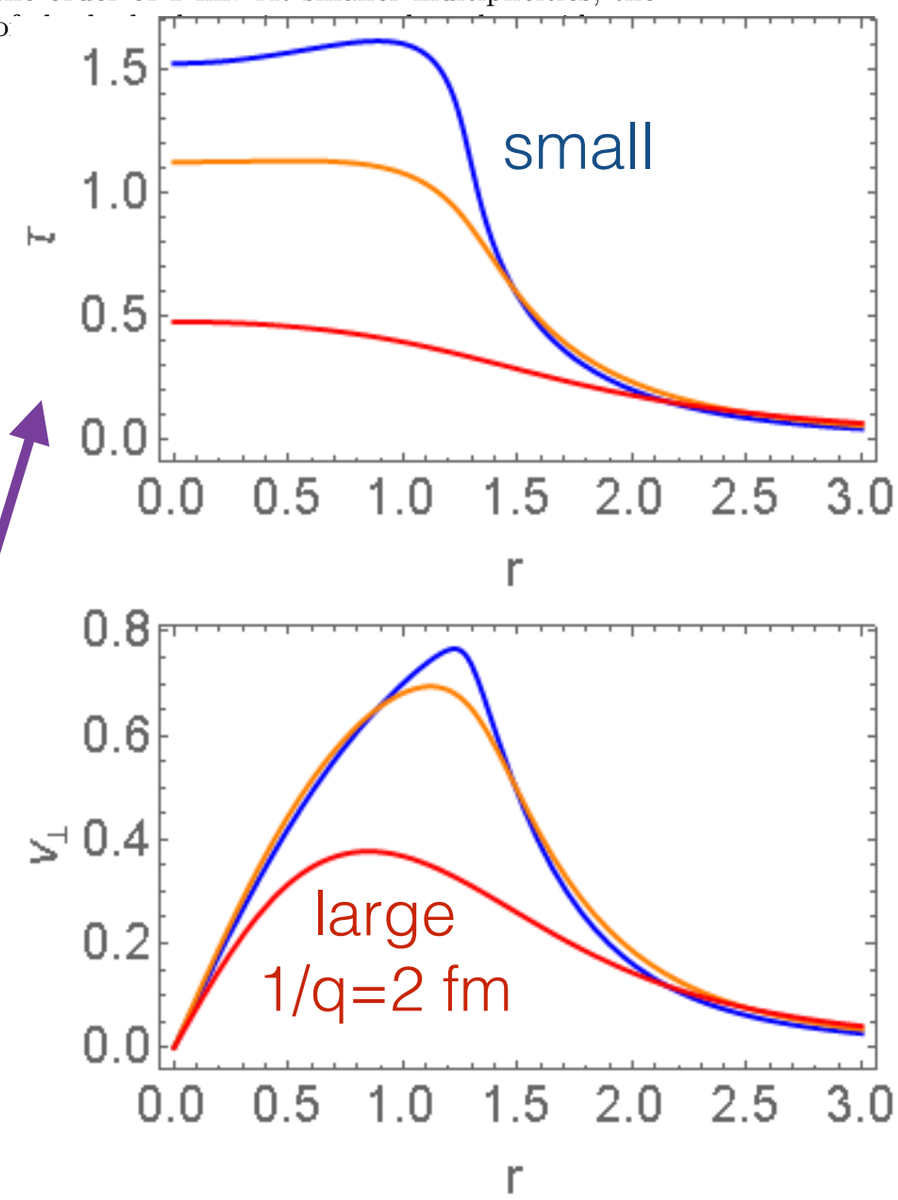
$$\epsilon(\bar{\tau}, \bar{r}) = \frac{\epsilon_0 (2q)^{8/3}}{\bar{\tau}^{4/3}[1 + 2q^2(\bar{\tau}^2 + \bar{r}^2) + q^4(\bar{\tau}^2 - \bar{r}^2)^2]^{4/3}},$$

$$v_\perp(\bar{\tau}, \bar{r}) \equiv \tanh \kappa(\bar{\tau}, \bar{r}) = \frac{2q^2 \bar{\tau}\bar{r}}{1 + q^2\bar{\tau}^2 + q^2\bar{r}^2},$$

Gubser's flow

$$\epsilon_0 = f_*^{-1/3} \left(\frac{3}{16\pi} \frac{dS}{d\eta} \right)^{4/3};$$

one parameter is fixed by the multiplicity, so only one is left, q
dimensionless size qr,and time q tau



lesson: the smaller the system, the more explosive it becomes

Spectra of identified particles compared with the data

$$\frac{dN_i}{dy\, m_T dm_T} = \frac{g_i}{\pi^2}\int_0^{r_{\rm cut}} r_\perp dr_\perp\, \tau\, e^{\mu_i(r_\perp)/T_f} \times$$
$$\times \left[m_T \mathrm{K}_1\Big(\frac{m_T \cosh\kappa(r_\perp)}{T_f}\Big)\mathrm{I}_0\Big(\frac{p_T \sinh\kappa(r_\perp)}{T_f}\Big) \right. \qquad (9$$
$$\left. - p_T \frac{\partial\tau}{\partial r_\perp}\mathrm{K}_0\Big(\frac{m_T \cosh\kappa(r_\perp)}{T_f}\Big)\mathrm{I}_1\Big(\frac{p_T \sinh\kappa(r_\perp)}{T_f}\Big)\right],$$

The solution defines the Cooper-Fry surface and the transverse rapidity on it

the spectra versus m_T, for the highest multiplicity bins: note that they are not parallel => NO M_T SCALING !

Smaller Nh<70 cannot be fitted by hydro, M_t scaling

Note that K,P spectra are fitted better than that for pions: this is because of the feed down from the resonance decay

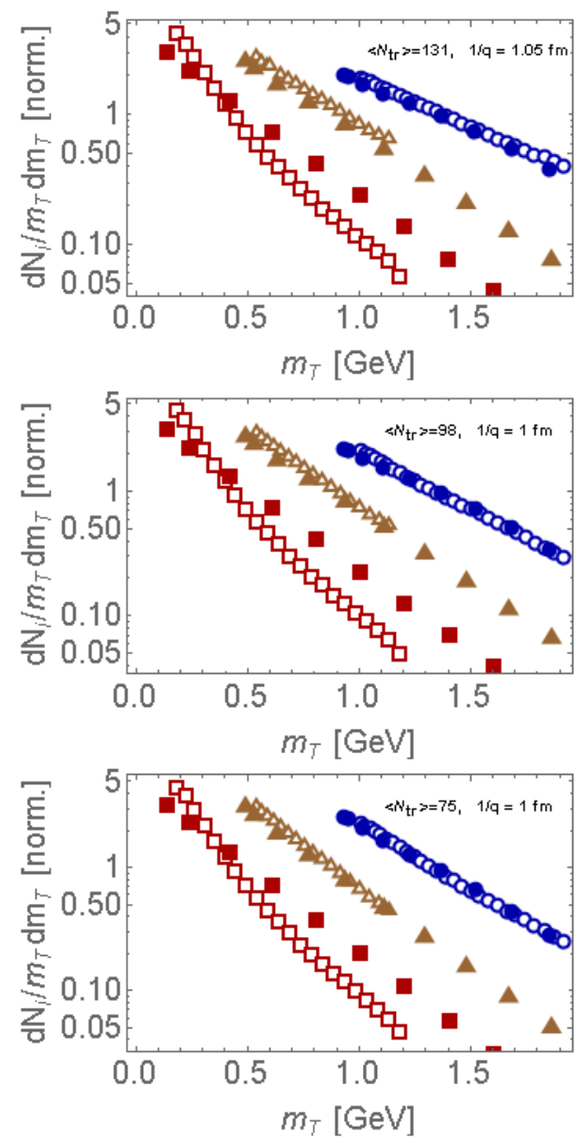
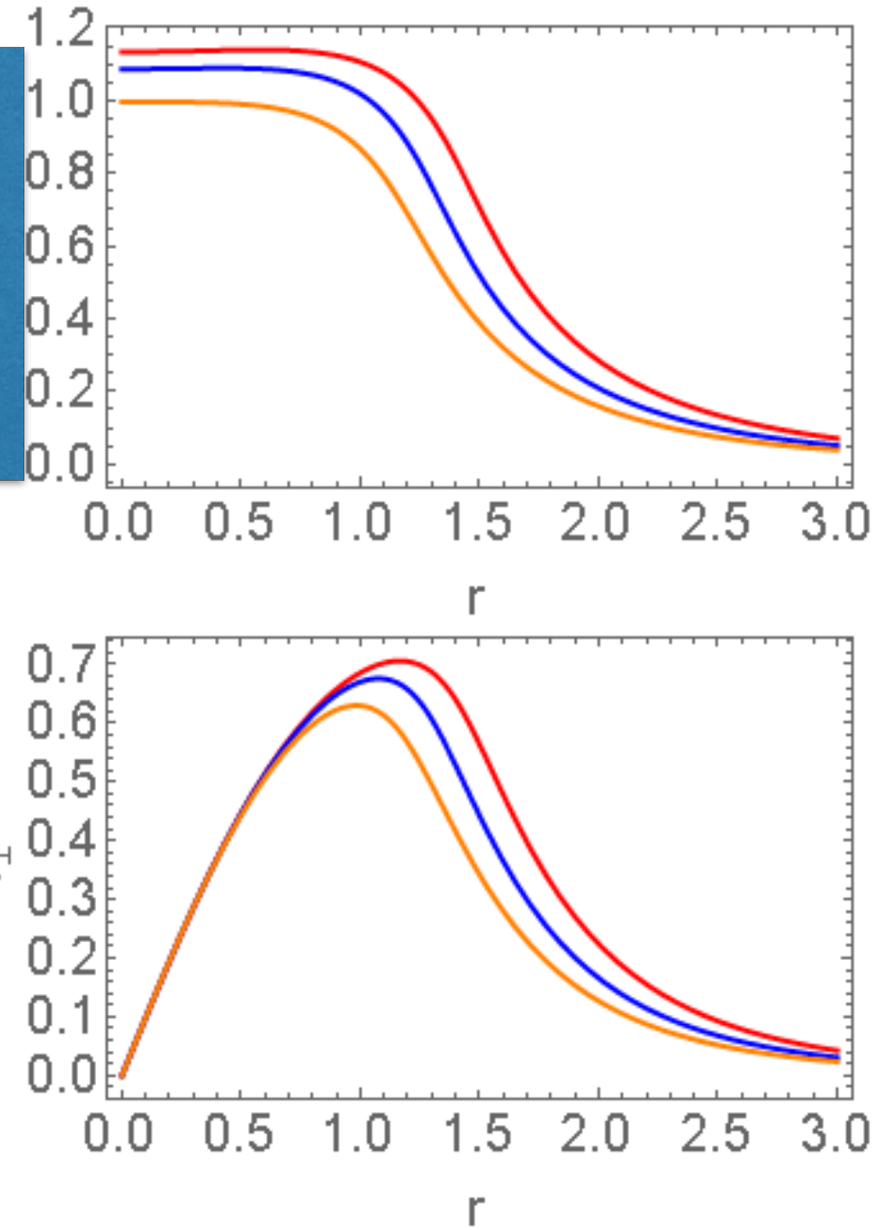


FIG. 2: (color online) Normalized spectra of pions (squares), kaons (triangles) and protons (discs) at different multiplicities. Open symbols correspond to the CMS data [6] for $|\eta| < 2.4$ and $\sqrt{s} = 7$ TeV, while the solid ones are obtained from the best one-parameter fit of the Gubser's flow.

What we learned from this
s the value of the fitted flow velocity
implies the value of the parameter
1/q=1 fm (+- few percent)



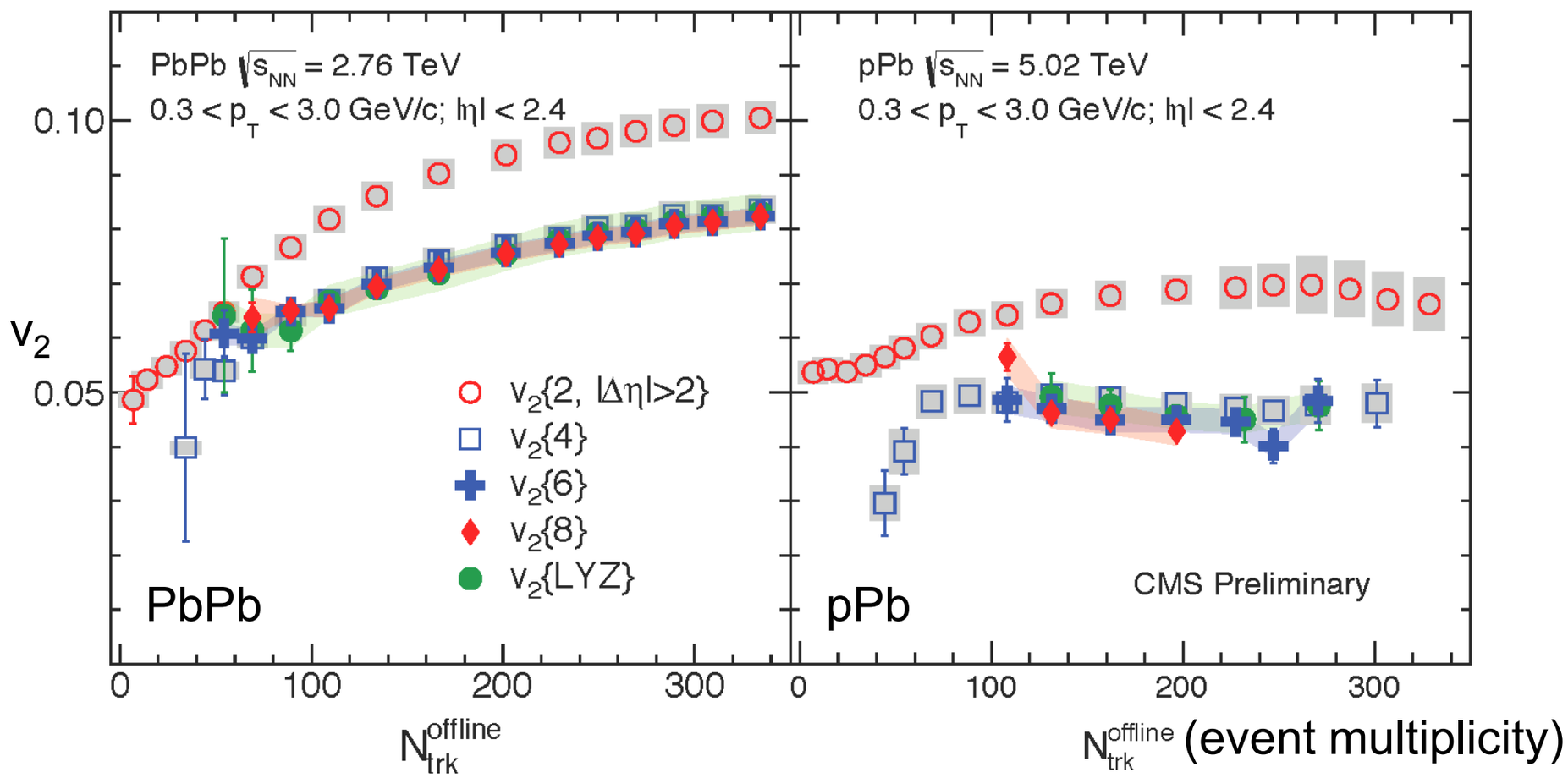
As time τ goes to zero, one sees that the corresponding energy density (2) becomes very large, and it is physically obvious that at some "initial time" τ_i the hydrodynamical desciption should break down. We do not know, of course, know from our analysis what its value can be, since the final observable does not depend on it.

We also do not know, and do not even speculate, what physical process is responsible for the system formation.

pp

FIG. 3: (Color online) Profiles of the freezeout surfaces (left) and velocities (right) for the Gubser solutions at various values of the parameters $(1/q\,[\mathrm{fm}], \epsilon_0)$ used in our calculations. Upper red - (1.05, 12.7), middle blue - (1.0, 8.6), lower orange - (1.0, 6.0).

Collectivity of v2 in central pA: the case is closed by these beautiful CMS “killer plot”



The v2 magnitude tells us about fluctuations in the initial state in AA it is Glauber wounded nucleons: what is it in pA and pp?

Schenke, Venugopalan

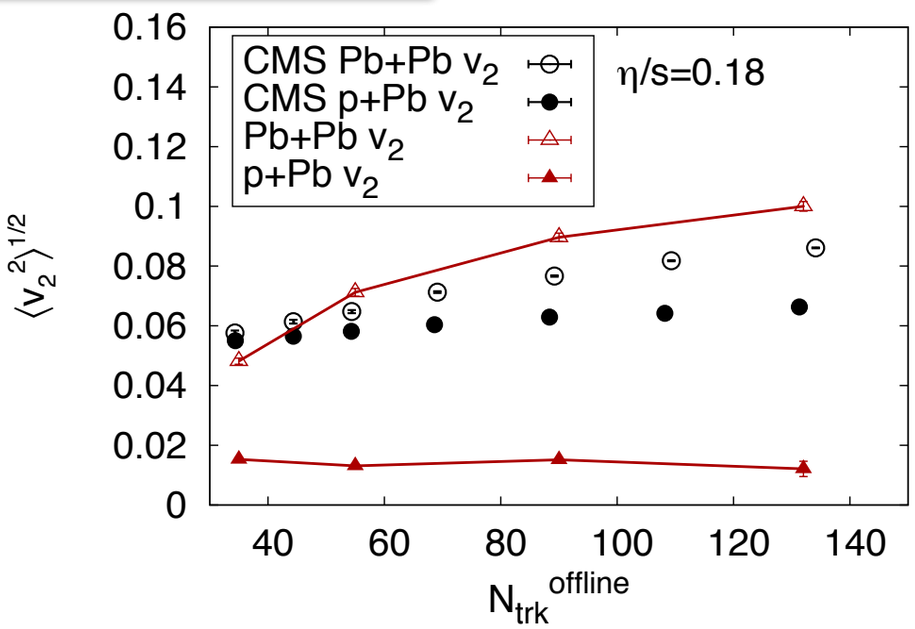
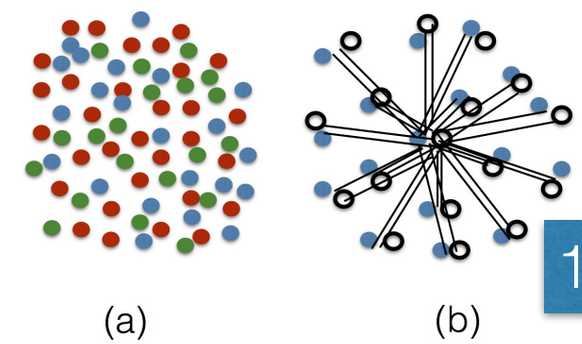


FIG. 25: (Color online) Multiplicity dependence of the root-mean-square elliptic flow coefficient v2 in Pb+Pb (open symbols) and p+Pb collisions (filled symbols) from the IP-Glasma+music model (connected triangles) compared to experimental data by the CMS collaboration.

IP glasma



16 Pomerons

FIG. 27: Sketch of the initial state in central pA collisions. The plot (a) corresponds to IP-glasma model, with colored circles representing multiple gluons. Fig.(b) is for $N_p = 16$ Pomerons, each represented by a pair of cold strings. The open circles are quarks and filled blue circles are diquarks.

where $N = N_p N_g$ for (a) and only $N = N_p$ for (b).

$$\epsilon_n \sim \frac{1}{\sqrt{N}} \qquad \frac{\epsilon_n^{(b)}}{\epsilon_n^{(a)}} \sim \frac{1}{\sqrt{N_g}} \sim 4$$

conclusion: no glasma in pA but Pomerons/strings instead

the strength of flow does not follow the (naive) ordering of the entropy density

Let us summarize those (naive) estimates: in terms of the initial entropy density one expects the following order of the densities

strings

$$\frac{dN^{pA}_{maximal}}{dA_\perp} \sim \frac{dN^{AA}_{peripheral}}{dA_\perp} \ll \frac{dN^{AA}_{central}}{dA_\perp} \ll \frac{dN^{pp}_{maximal}}{dA_\perp} \tag{27}$$

glasma

and may thus expect that the radial flow follows the same pattern. The data however show it is *not* the case.

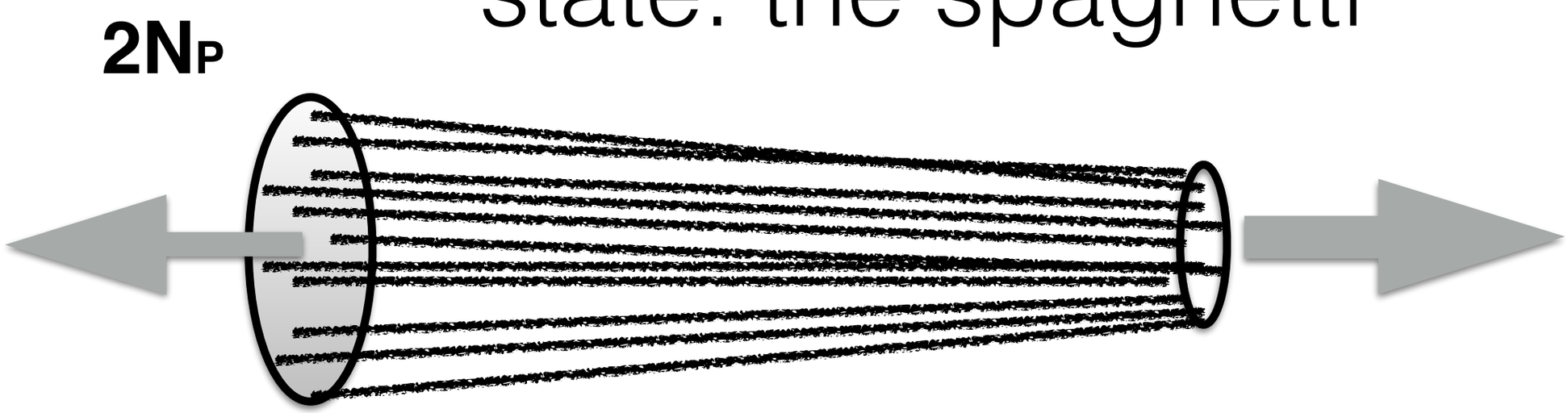
The "radial flow puzzle" for central pA

case. If so, the size of the system is reduced and density increased by a significant factor, leading to a different order of the densities:

$$\frac{dN^{AA}_{peripheral}}{dA_\perp} < \frac{dN^{AA}_{central}}{dA_\perp} < \frac{dN^{pA}_{maximal}}{dA_\perp} < \frac{dN^{pp}_{maximal}}{dA_\perp} \tag{3}$$

corresponding to the strength of the flow. Another notable consequence of this collapse is that the combined field of all strings become strong enough to restore chiral symmetry and thus create the QGP fireball, needed for an explosion.

the simplest multi-string state: the spaghetti



N(strings)=2N(Pomerons)

in small multiplicity bins strings are broken independently (the Lund model),

but one should obviously think about their interaction if their number grows

arXiv:1404.1888v1 [hep-ph] 7 Apr 2014

string interaction via sigma meson exchange

T. Iritani, G. Cossu and S. Hashimoto, arXiv:1311.0218

our fit uses
the sigma mass
600 MeV

$$\frac{\langle \sigma(r_\perp) W \rangle}{\langle W \rangle \langle \sigma \rangle} = 1 - C K_0(m_\sigma \tilde{r}_\perp)$$

$$\tilde{r}_\perp = \sqrt{r_\perp^2 + s_{string}^2}$$

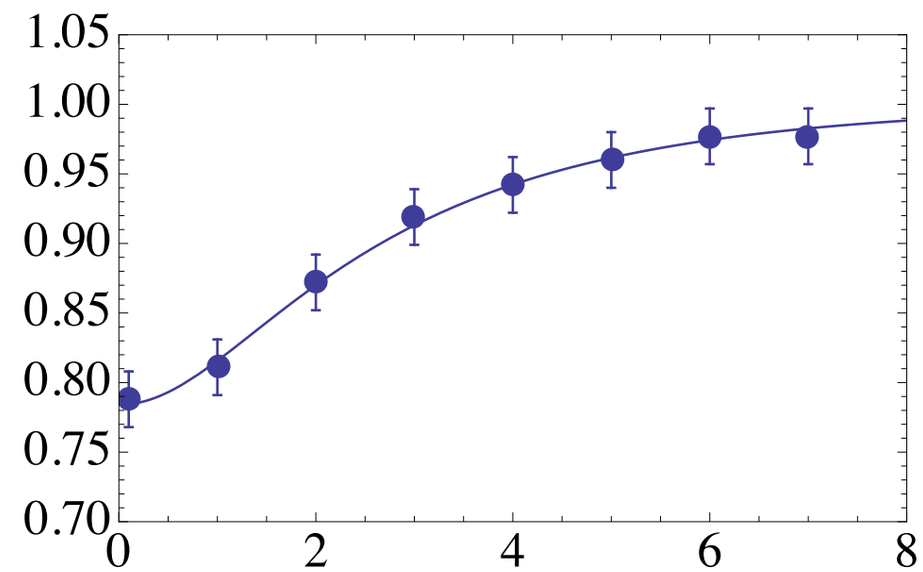


FIG. 2. (Color online). Points are lattice data from [12], the curve is expression (8) with $C = 0.26$, $s_{string} = 0.176\,\mathrm{fm}$.

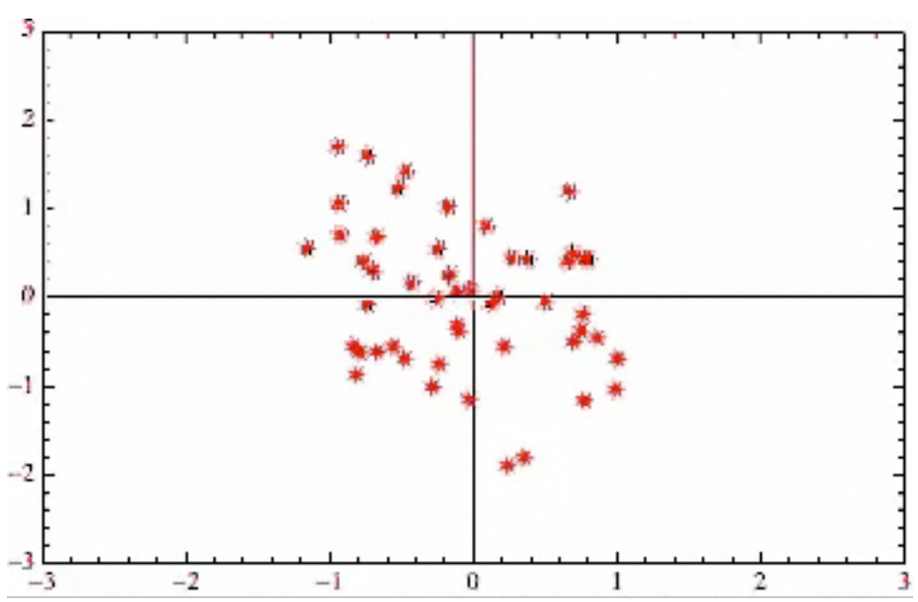
So the sigma cloud around a string is there!

2d spaghetti collapse

Basically strings can be viewed as a 2-d gas of particles with unit mass and forces between them are given by the derivative of the energy (8) , and so

$$\ddot{\vec{r}}_i = \vec{f}_{ij} = \frac{\vec{r}_{ij}}{\tilde{r}_{ij}}(g_N\sigma_T)m_\sigma 2K_1(m_\sigma\tilde{r}_{ij}) \qquad (19)$$

with $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ and "regularized" $\tilde{r}$ (9).



$$E_{tot} = \sum_i \frac{v_i^2}{2} - 2g_N\sigma_T \sum_{i>j} K_0(m_\sigma r_{ij})$$

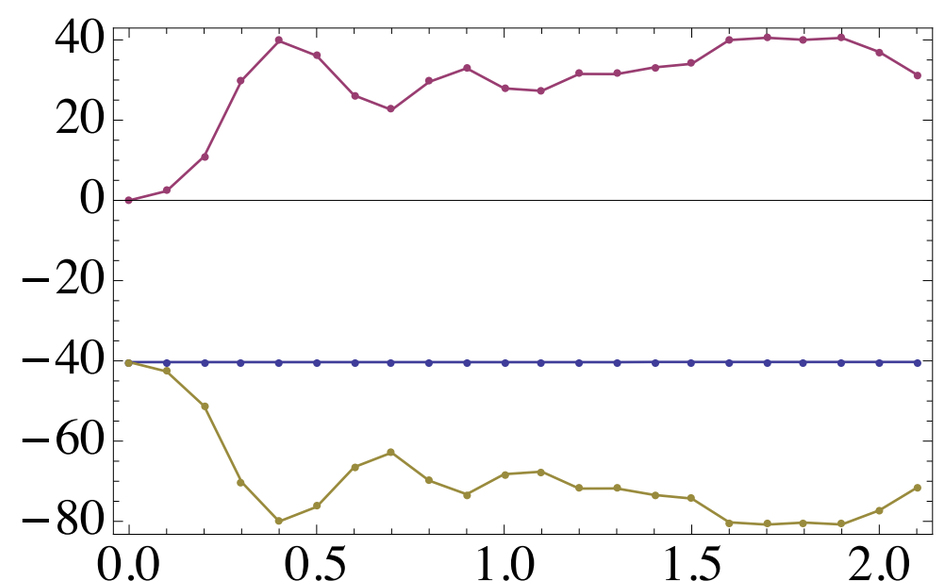


FIG. 4. (Color online) The (dimensionless) kinetic and potential energy of the system (upper and lower curves) for the same example as shown in Fig. 6, as a function of time t(fm). The horizontal line with dots is their sum, namely E_{tot}, which is conserved.

Collective string interactions in AdS/QCD and high multiplicity pA collisions

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(Dated: March 2, 2015)

Recently there appeared interest in collective interaction of QCD strings. Intrinsic attractive interaction of strings in the context of holographic models of the AdS/QCD type, or σ exchanges for QCD strings – can significantly affect properties of the multi-string systems. The high multiplicity pA collisions are the simplest example of the kind, producing "spaghetti" of many strings extended in the longitudinal (beam) direction. We study their collective field

Since we will be dealing with strings, let us remind the reader that those live in the "bulk" of the AdS_5 space, carrying electric color fluxes from charges on its boundary (that is, "our" or gauge theory world). The Maldacena calculation [1] for two static charges derived the shape of bending (geodesic) string. As the setting is scale invariant, the modified strong coupling Coulomb law remains $V \sim 1/r$, with only the coefficient modified. First, the calculation for the charges moving with fixed velocity $\pm v$ away from each other has been performed by Lin and Shuryak [25], who found that for small enough v there is a scaling solution for the falling string, generalizing Maldacena case into a "generalized Ampère law". Yet, above certain critical velocity this becomes unstable, and the stable solution in that range has been found numerically. The hologram of that string calculated in [3] showed a near-spherical, non-hydrodynamical explosion,

The background and fields are defined in papers by Kiritsis et al. [6, 7]. The specific calculation we follow includes back reaction of the quarks in V-QCD with Potential I [11].

The action for gravity and the dilaton Φ is

$$S = M^3 N_c^2 \int d^5x \sqrt{-g}[R - \frac{4}{3} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi)] \quad (5)$$

The overall setting includes background with a (conformal) gravity metric of the form

$$g_{\mu\nu} = \exp(2A(z))[dz^2 + \eta_{ij} dx^i dx^j] \quad (6)$$

where $\eta_{ij} = \mathrm{diag}(-,+,+,+)$ is the Minkowski metric.

The t' Hooft λ coupling is directly related to the dilaton: $\lambda = \exp(\Phi)$.

strings in the bulk and on the boundary

$$\partial_t \left(e^{2A_s(Z)} \frac{\dot{Z}}{\sqrt{1-\dot{Z}^2}} \right) = \sqrt{1-\dot{Z}^2}\,\partial_z \left(e^{2A_s(Z)} \right)$$

$$A_s(z) = A(z) + \frac{2}{3}\Phi(z)$$

gravity and antigravity allows the strings to levitate

A holographic image of the bulk string is the QCD string

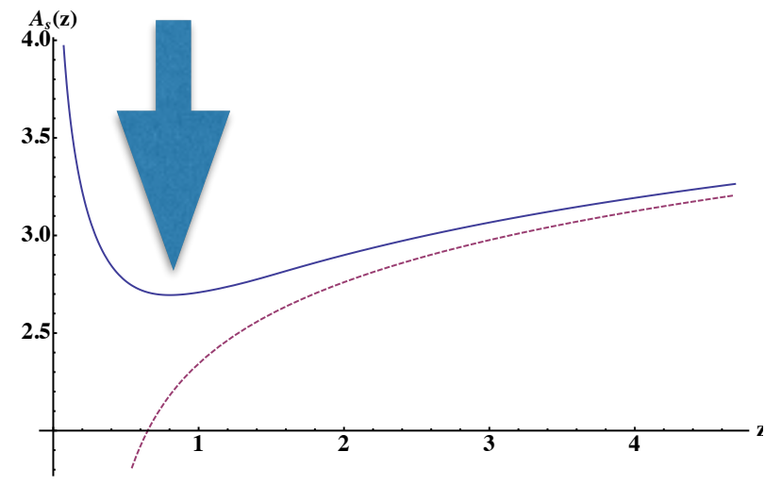


FIG. 2: The combination $A_s(z)$ as a function of the holographic coordinate z (solid) compared to it's IR (large z) asymptotics (dashed). It is noticed that $A_s(z)$ has a minimum corresponding to the equilibrium scale of the QCD string.

Hadronic spectroscopy in AdS/QCD in Veneziano limit

Kiritsis et al

$N_c, N_f \to \infty \quad N_f/N_c = x = const$

Next come a close pair of the second and third states, with mass ratios to the first one $m_3/m_1 \approx 2.6$. Since in the calculation the strange quark is as light as u, d, there should not be a separate $f_0(1710)$ state, and this pair can be identified with a close pair $f_0(1370), f_0(1500)$; at $x = N_f/N_c = 1$ their splitting is also correct. Different x-dependence of the third state from others hints that it is indeed mostly a glueball, but this feature is not robust, as it depends on the details of the potential.

The five lowest masses in units of the UV scale of the model are

$$\frac{m_1}{\Lambda_{UV}} = 1.53\,, \quad \frac{m_2}{\Lambda_{UV}} = 3.54\,, \quad \frac{m_3}{\Lambda_{UV}} = 3.94\,,$$
$$\frac{m_4}{\Lambda_{UV}} = 4.86\,, \quad \frac{m_5}{\Lambda_{UV}} = 5.45 \qquad (17)$$

Selecting the numerical value of Λ_{UV} from the second and third state masses, which are narrow and therefore well mapped to phenomenology, we fix the absolute units in our model to be

$$\Lambda_{UV} = 387\,\text{MeV}, \quad m_\sigma = 592\,\text{MeV} \qquad (18)$$

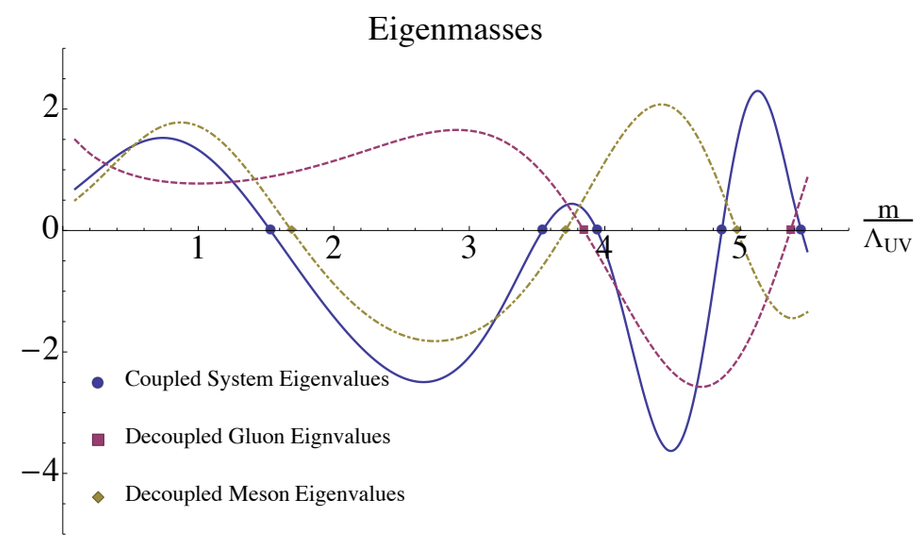
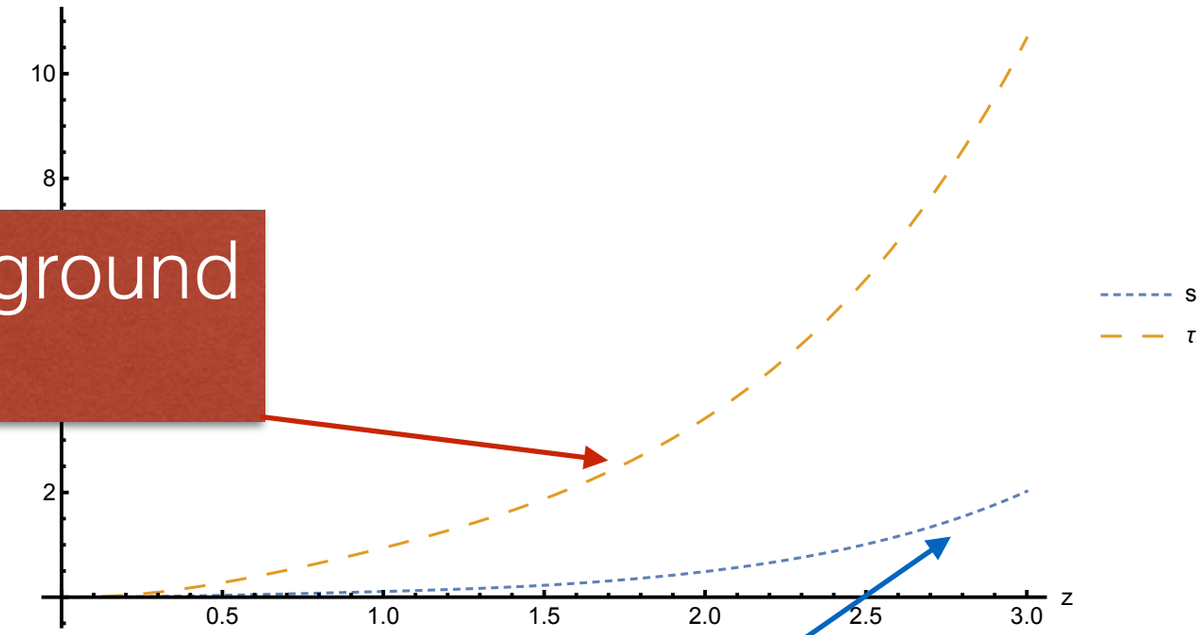


FIG. 3: The determinant of the UV boundary value of two linearly independent solutions of the scalar fluctuation equations, versus the mass parameter in Λ_{UV} units. It's five zeros (red points) indicate the normalizable solutions and the corresponding masses are those of the lowest five scalars. Two other curves correspond to unmixed equations as explained in the text.

the bottom-line is that the model does a very good job even on the most complicated part of hadronic spectroscopy: the 0++scalars and their mixing

so a string has 2 holograms: thin, via glueballs, and thick via sigma meson

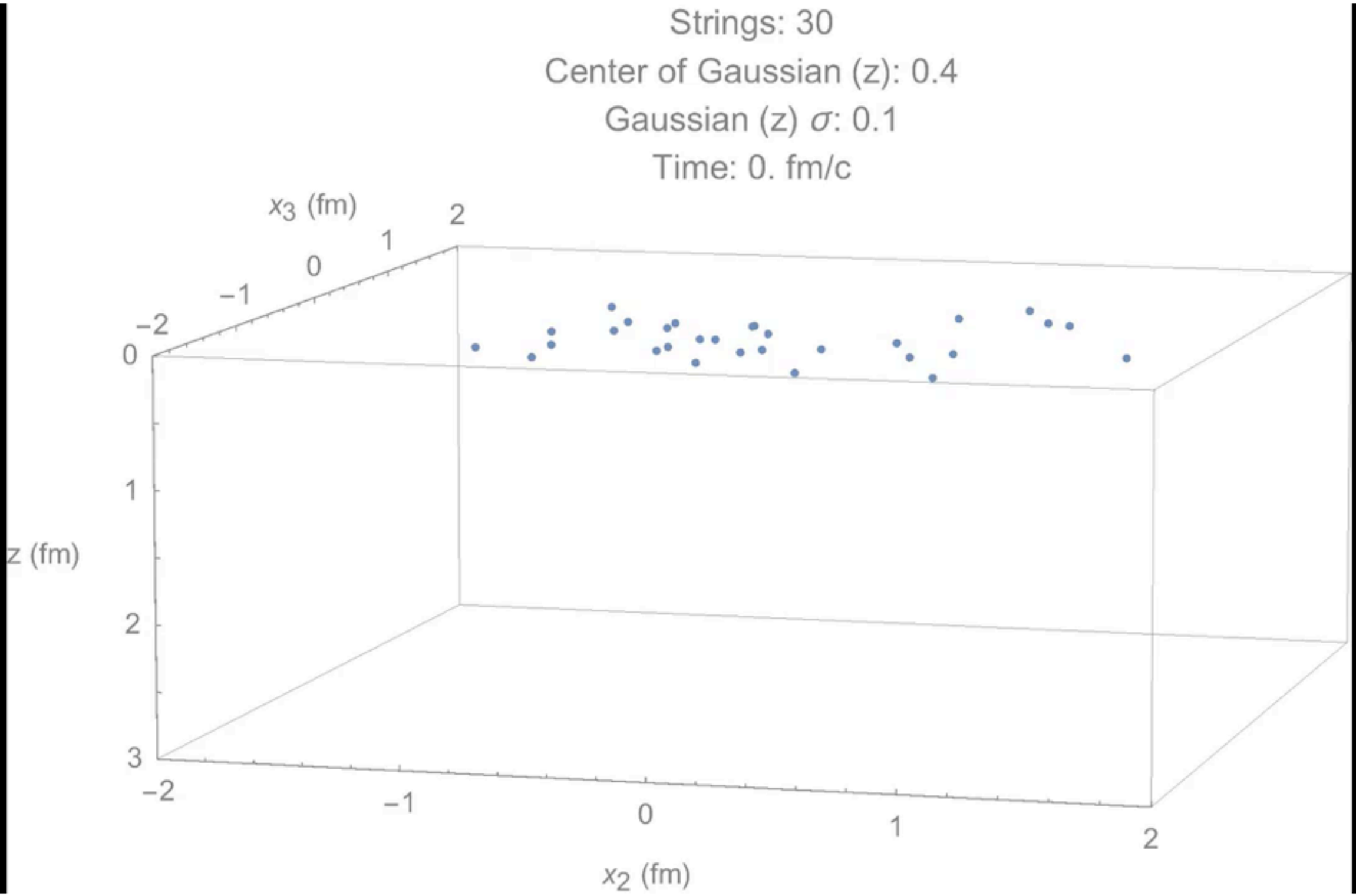
string interaction and collectivization



red dashed curve is the background quark condensate

Lattice (Iritani et al), need 5-6 strings to do so

blue is (with the opposite sign) the sigma field from 1 string: thus about coinciding strings restore chiral symmetry


Strings: 30
Center of Gaussian (z): 0.4
Gaussian (z) σ: 0.1
Time: 0. fm/c
x3 (fm)
−2
−1
0
1
2
z (fm)
0
1
2
3
−2
−1
0
1
2
x2 (fm)

summary

- “small system” show collective flow, with magnitude even stronger than in central AA, via **femtoscopy and spectra of k,p,Lambda’s**

- this may only happen if they **start their life being very small (1/q=0.7-1 fm)**

- for pA: we know the size, it is supposed to be a multi string “spaghetti” (=> small hydro flow)

- **unless multi string “spaghetti” collapses**. We now see it holographically to happen

- the high multiplicity pp starts very small and dense: glasma?

outline

- small system femtoscopy: why are they different from AA? *(yes, they are more explosive)*
- high multiplicity pp and HBT radii: the flow => initial size *(paper with Yuji Hirono, see his talk later)*
- spectra of identified secondaries and flow *(paper with Tigran Kalaydzhyan)*
- what do fluctuations/harmonics tell us about the initial state?
- pA: "spaghetti collapse" => sQGP explosion *(paper with Tigran Kalaydzhyan)*
- *how the ``fundamental strings" in AdS/QCD models generate the QCD string dynamics (paper with Ioannis Iatrakis, Adith Ramamurti)*

Initial asymmetries and emergent collectivity in high energy pA collisions

Vladimir Skokov

Western Michigan University

March 5, 2015

Disclaimer

- Only high transverse momentum, $k_\perp > 3$ GeV, corresponding to small resolution scale for conjugate variable $r_\perp < .07$ fm.
  Recent ATLAS results: anisotropy persists up to $k_\perp \approx 10$ GeV.

  Imperative to apply short-scale QCD dynamics.
- Dilute-dense limit (relevant for pA collisions). Scattering of projectile (p) partons off target (A).
- No comparison to experiment:
  we are not there yet and without transparent microscopical computations there is no point to try to describe data.
  - Our computations are for large $|\vec{k}|_\perp$
  - No correlation with low $|\vec{k}|_\perp$
  - No normalization by the total number of particles.

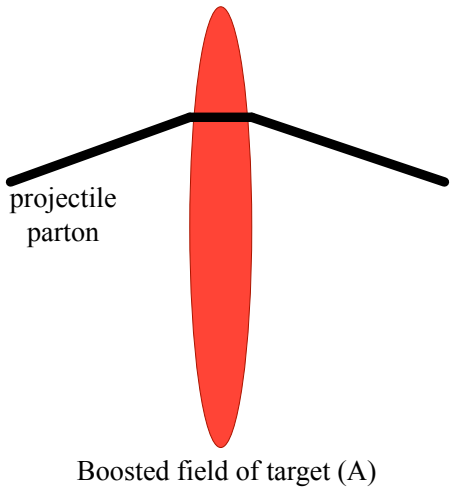
Outline

- Motivation: Azimuthal asymmetry from connected diagrams in dilute-dense limit and high order cumulants; $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$?
- Single-particle azimuthal asymmetry from initial state
- Azimuthal anisotropy in MV model: numerical results
- JIMWLK evolution: fate of anisotropy at small x
- A more realistic approach by T. Lappi
- Conclusions

Scattering cross section for dilute-dense limit I

Goal: to find $v_2\{n\}$ in dilute-dense limit at high $k_\perp$

- In eikonal approximation, parton propagation is described by light-like Wilson line $V(\vec{x}) = \mathcal{P}\exp\left(ig\int dx^- A^+(x^-,\vec{x})\right)$



- S-matrix in momentum space: $\langle S_1(\vec{k}_\perp)\rangle = \frac{1}{d_\mathcal{R}}\left\langle \mathrm{tr}_\mathcal{R} V(\vec{k}_\perp)V^\dagger(\vec{k}_\perp)\right\rangle$ or can be obtained from

$$\langle S_1(\vec{k}_\perp)\rangle = \int d^2b\, d^2r\, e^{i\vec{r}_\perp\vec{k}_\perp}\, S_1(\vec{x}_\perp,\vec{y}_\perp),$$

where $\vec{x}_\perp = \vec{b}_\perp + \vec{r}_\perp/2$ and $\vec{y}_\perp = \vec{b}_\perp - \vec{r}_\perp/2$ and
$\langle S_1\rangle = \frac{1}{d_\mathcal{R}}\left\langle \mathrm{tr}_\mathcal{R} V(\vec{x}_\perp)V^\dagger(\vec{y}_\perp)\right\rangle$; $\quad k_\perp \propto 1/r_\perp$; $\quad \vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$
$d_\mathcal{R}$ is dimension of representation $\mathcal{R}$

$$S_A(\vec{r}) = \frac{N_c^2|S_F(\vec{r})|^2 - 1}{N_c^2 - 1}$$

$S_F(\vec{r})$ can be complex, while $S_A(\vec{r})$ is real.

Scattering cross section for dilute-dense limit II

- Scattering to high transverse momentum corresponds to small $|\vec{r}\,| \propto 1/k_\perp$. Gradient expansion of vector potential $A^+(x^-, \vec{x})$ gives (fundamental representation only)

$$\langle S_1(\vec{r}, \vec{b})\rangle - 1 = \left\langle \frac{(ig)^2}{2N_c}\,\mathrm{tr}\left(\vec{r}\cdot\vec{E}(\vec{b})\right)^2 + \frac{1}{2}\left[\frac{(ig)^2}{2N_c}\,\mathrm{tr}\left(\vec{r}\cdot\vec{E}(\vec{b})\right)^2\right]^2 + O(r^6)\right\rangle$$

  Light-cone electric field of target in covariant gauge $E^i(\vec{b}) = \int dx^- F^{+i} = -\partial^i \int dx^- A^+(x^-, \vec{b})$.

- For m-quarks (only leading order is shown)

$$\langle S_m\rangle - 1 = \left(\frac{(ig)^2}{2N_c}\right)^m \left\langle \mathrm{tr}(\vec{r}_{1,\perp}\vec{E}_1)^2 \mathrm{tr}(\vec{r}_{2,\perp}\vec{E}_2)^2 \cdots \mathrm{tr}(\vec{r}_{m,\perp}\vec{E}_m)^2\right\rangle; \quad \vec{E}_i = \vec{E}(\vec{b}_{i,\perp})$$

- By knowing $\langle \vec{E}(\vec{b}_1)\vec{E}(\vec{b}_2)\rangle$, one can compute S_m, cumulants, $c_n\{m\}$ and harmonics $v_n\{m\}$ of azimuthal anisotropy.

Scattering cross section for dilute-dense limit III

Event averaging corresponds to averaging over target ensemble $\langle \vec{E}(\vec{b}_1)\vec{E}(\vec{b}_2)\rangle$.

In McLerran-Venugopalan model

$$\frac{g^2}{N_c}\langle E_i^a(\vec{b}_1)E_j^b(\vec{b}_2)\rangle = \frac{1}{N_c^2-1}\delta^{ab}\delta_{ij}\,Q_s^2\,\Delta(\vec{b}_1-\vec{b}_2)$$

Cumualnts of azimuthal anisotropy I

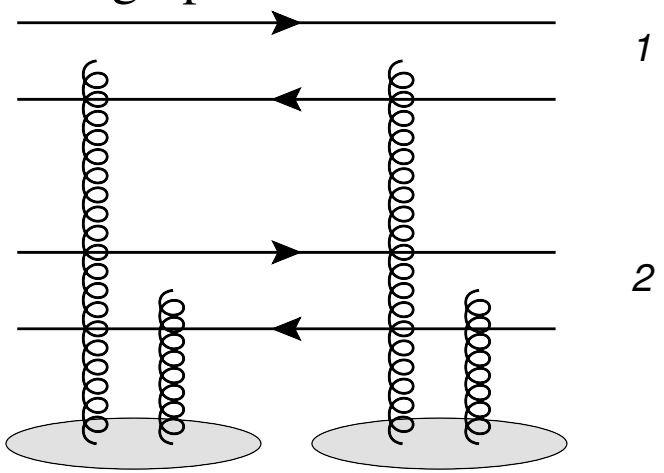
- Cumulants of azimuthal anisotropy can be readily computed. Cumulants are defined in such a way as to cancel disconnected pieces not associated with **single** particle azimuthal anisotropy. For example

$$c_n\{4\} = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle_{\text{conn}} = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle - 2\langle e^{in(\phi_1-\phi_3)}\rangle^2$$

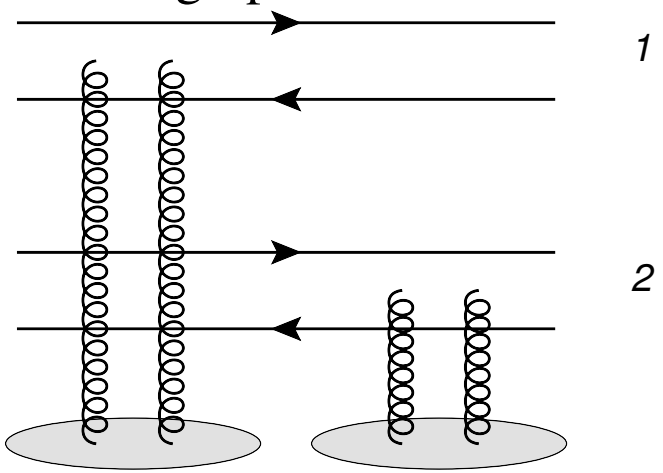
.

- In field theory, this corresponds to considering fully connected diagrams only

Connected graph



Disconnected graph



V. S. 1412.5191

Cumualnts of azimuthal anisotropy II

- There are $(2m-2)!!$ ways to contract $S_m(\vec{r}_1, \vec{b}_1, \ldots, \vec{r}_m, \vec{b}_m)$ in fully connected way:

$$\langle S_m(\vec{r}_1, \vec{b}_1, \ldots, \vec{r}_m, \vec{b}_m) - 1\rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4}\right)^m \frac{1}{(N_c^2-1)^{m-1}}$$
$$\Delta(\vec{b}_1 - \vec{b}_2)\Delta(\vec{b}_2 - \vec{b}_1)\cdots\Delta(\vec{b}_{m-1} - \vec{b}_m)\Delta(\vec{b}_m - \vec{b}_1)$$
$$(\vec{r}_1\vec{r}_2)(\vec{r}_2\vec{r}_3)\cdots(\vec{r}_{m-1}\vec{r}_m)(\vec{r}_m\vec{r}_1) + \text{permutations.}$$

- Averaging with respect to impact parameters (for Gaussian $\Delta(\vec{b})$):

$$\langle S_m(\vec{r}_1, \ldots, \vec{r}_m) - 1\rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4}\right)^m \frac{1}{(N_c^2-1)^{m-1}} \frac{\xi^{m-1}}{m}$$
$$(\vec{r}_1\vec{r}_2)(\vec{r}_2\vec{r}_3)\cdots(\vec{r}_{m-1}\vec{r}_m)(\vec{r}_m\vec{r}_1) + \text{permutations.}$$

$\xi = S_c/S_p = 1/N_D$.

- In large N_c, normalization of angular averages is defined by disconnected contribution

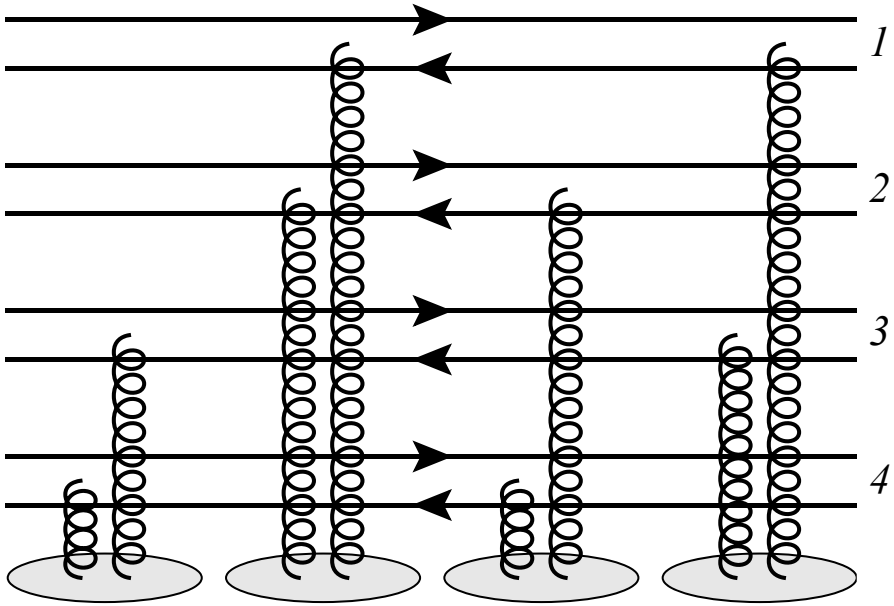
$$\langle S_m(\vec{r}_1, \ldots, \vec{r}_m) - 1\rangle^{\text{disc.}} \approx \left(-\frac{Q_s^2}{4}\right)^m \prod_{i=1}^{m} r_i^2.$$

V. S. 1412.5191

Cumualnts of azimuthal anisotropy III

- Not all $(2m-2)!!$ terms contribute to cumulants. $m!!(m-2)!!$ nonzero terms are defined by all possible contractions of terms entering with opposite signs before ϕ's in $e^{2i(\phi_1+\phi_2+\cdots+\phi_n-\phi_{n+1}-\phi_{n+2}-\cdots-\phi_{2n})}$.
- For 4 particles, $e^{2i(\phi_1+\phi_2-\phi_3-\phi_4)}$

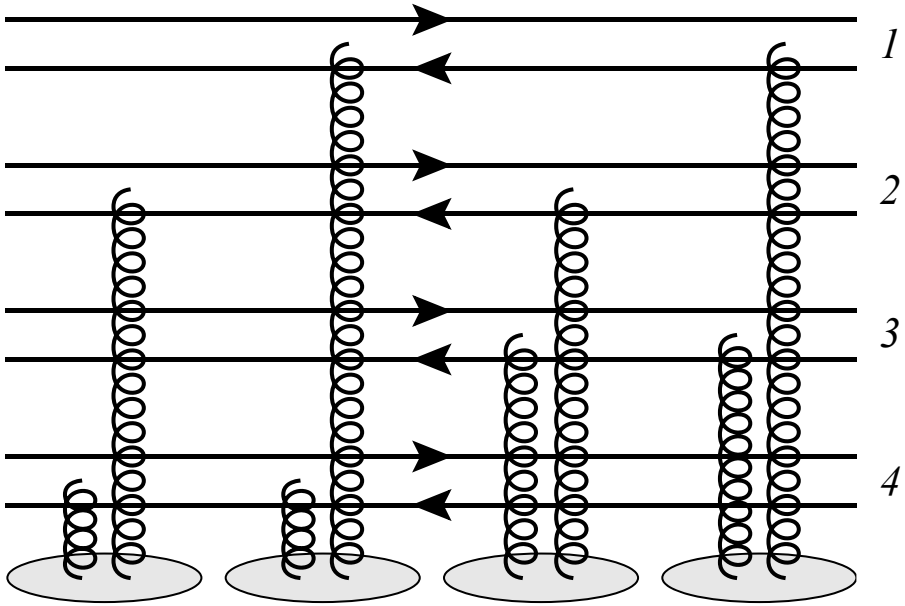
Graph that does not contribute



$\propto (\vec{r}_1\vec{r}_3)(\vec{r}_2\vec{r}_4)(\vec{r}_1\vec{r}_2)(\vec{r}_3\vec{r}_4)$

V. S. 1412.5191

Graph that does contribute



$\propto (\vec{r}_1\vec{r}_3)(\vec{r}_1\vec{r}_4)(\vec{r}_2\vec{r}_3)(\vec{r}_2\vec{r}_4)$

$c_2\{m\}$ FROM CONNECTED DIAGRAMS

- Final result

$$c_2\{m\} = \frac{m!!(m-2)!!}{m\, 2^m} \left(\frac{\xi}{N_c^2 - 1} \right)^{m-1}$$

- Suppressed by powers of $1/N_c^2$ and $\xi = S_c/S_p$.

- $c_2\{m\}$ are manifestly positive for any m.

- Same result remains true for adjoint representation (Casimir operators cancel in normalized observables).

$v_2\{m\}$ FROM CONNECTED DIAGRAMS I

- Harmonics are related to cumulants. Relation of flow coefficients to cumulants (see N. Borghini, P. M. Dinh and J. Y. Ollitrault 0105040)

$$v_2^{2k}\{2k\} = (-1)^{k+1} \times (\text{Numerical coefficient}) \times \mathrm{c}_2\{\mathrm{m} = 2\mathrm{k}\} = \kappa_{2\mathrm{k}}\mathrm{c}_2\{\mathrm{m} = 2\mathrm{k}\}$$

- First few κ_{2m} (also in the talk by Jiangyong Jia "a_{2m}"):

Order, $2m$	2	4	6	8	10	12	14
$1/\kappa_{2m}$	1	-1	-4	33	-456	9460	-274800

- Idea behind these numbers: if we have dominating **single** particle azimuthal anisotropy $v_2\{1\}$ then

$$v_2^m\{m\} = v_2^m\{1\} + \text{corrections}$$

Purpose for hydrodynamics: extract genuine $v_2\{1\}$ and suppress "non-flow".

$v_2\{m\}$ FROM CONNECTED DIAGRAMS II

Skipping the details (see Appendix of `V. S. 1412.5191`)

$$\kappa_{2m} = \frac{v_n^{2m}\{2m\}}{c_n\{2m\}} = (-1)^{m+1}\left[m!(m-1)!\sum_{k=1}^{\infty}\left(\frac{2}{j_{0,k}}\right)^{2m}\right]^{-1}.$$

At large orders the sum can be approximated by the first term.

$v_2\{m\}$ FROM CONNECTED DIAGRAMS III

- Harmonics:

$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m}\left(\frac{\xi}{N_c^2-1}\right)^{m-1}; \qquad \beta_m = 2\sum_{k=1}^{\infty}\left(\frac{2}{j_{0,k}}\right)^m \approx 2\left(\frac{2}{j_{0,1}}\right)^m$$

- Explicitly for second and fourth order:

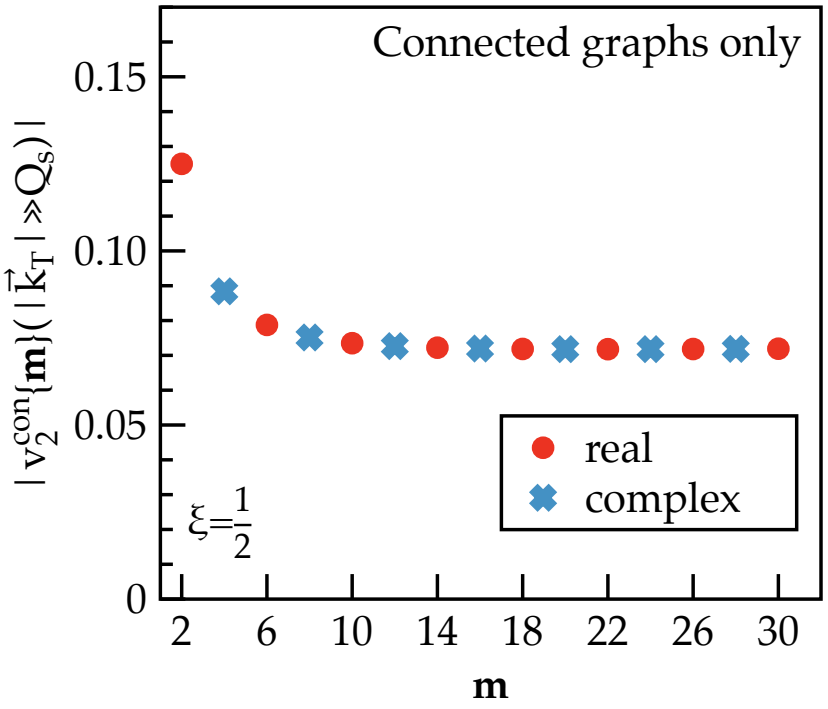
$$v_2^2\{2\} = \frac{1}{4}\frac{\xi}{N_c^2-1}; \qquad v_2^4\{4\} = -\frac{1}{4}\left(\frac{\xi}{N_c^2-1}\right)^3$$

- $v_2\{4\}$ is complex!
- $m \to \infty$:

$$\lim_{m\to\infty}|v_2\{m\}| = \frac{\xi}{N_c^2-1}\frac{j_{0,1}}{2}; \qquad j_{0,1} = 2.40483$$

(similar to LYZ, see talk by Nicolas Borghini)

$v_2\{m\}$ FROM CONNECTED DIAGRAMS: ILLUSTRATION



V. S. 1412.5191

A. Dumitru, L. McLerran, V. S. 1410.4844

- Hierarchy of $|v_2\{m\}|$.
- **Complex $v_2\{4k\}$, $k \in \mathcal{Z}$; including $v_2\{4\}$ and $v_2\{8\}$**
- Experiment: high multiplicity pA $c_2\{4\} < 0 \rightsquigarrow v_2\{4\} \in \mathcal{R}$
- **Theory: connected graph only** $c_2\{4\} > 0 \rightsquigarrow v_2\{4\} \in \mathcal{C}$
- In order to describe high $k_\perp$ with IS effects, one needs disconnected graphs with azimuthal anisotropy
- I believe that similar conclusion is valid for dense-dense limit and "glasma" graph

- A hydro practitioner: "non-flow". However, very different from conventional non-flow contributions (i.e. resonance decay): long-range in rapidity, approximate equality of high order harmonics $|v_2\{m\}|$.

Single particle anisotropy

- **Kovner&Lublinsky**: Re $S_\rho - 1 = \frac{(ig)^2}{2N_c}\mathrm{tr}(\vec{r}\vec{E})^2$; If projectile partons scatter off same electric field, they pick up same transverse momentum (independent on rapidity of partons). E plays similar role to Q in Miklos Gyulassy talk?!
- To take this into account, let modify $E - E$ correlator:

$$\frac{g^2}{2N_c}\left\langle \mathrm{tr}\, E_i(\vec{b}_{1\perp})E_j(\vec{b}_{2\perp})\right\rangle = \frac{1}{4}Q_s^2\,\Delta(\vec{b}_{1\perp}-\vec{b}_{2\perp})\left(\delta^{ij} + 2\,\mathcal{A}\left(\hat{a}^i\hat{a}^j - \frac{1}{2}\delta^{ij}\right)\right)$$

- Angular distribution for scattering of single quark, for fixed $\hat{a}$.

$$\left(\frac{1}{\pi}\frac{dN}{dk^2}\right)^{-1}\frac{dN}{d^2k} = 1 - 2\mathcal{A} + 4\mathcal{A}(\hat{k}\cdot\hat{a})^2\,.$$

Consequently, elliptic harmonic of single-particle distribution: $v_2 \equiv \left\langle e^{2i(\phi_k-\phi_a)}\right\rangle_{\hat{a}} = \mathcal{A}$.

- Repeating calculations, see details in †:

$$v_2^2\{2\} = c_2\{2\} = \xi\left(\mathcal{A}^2 + \frac{1}{4(N_c^2-1)}\right)$$

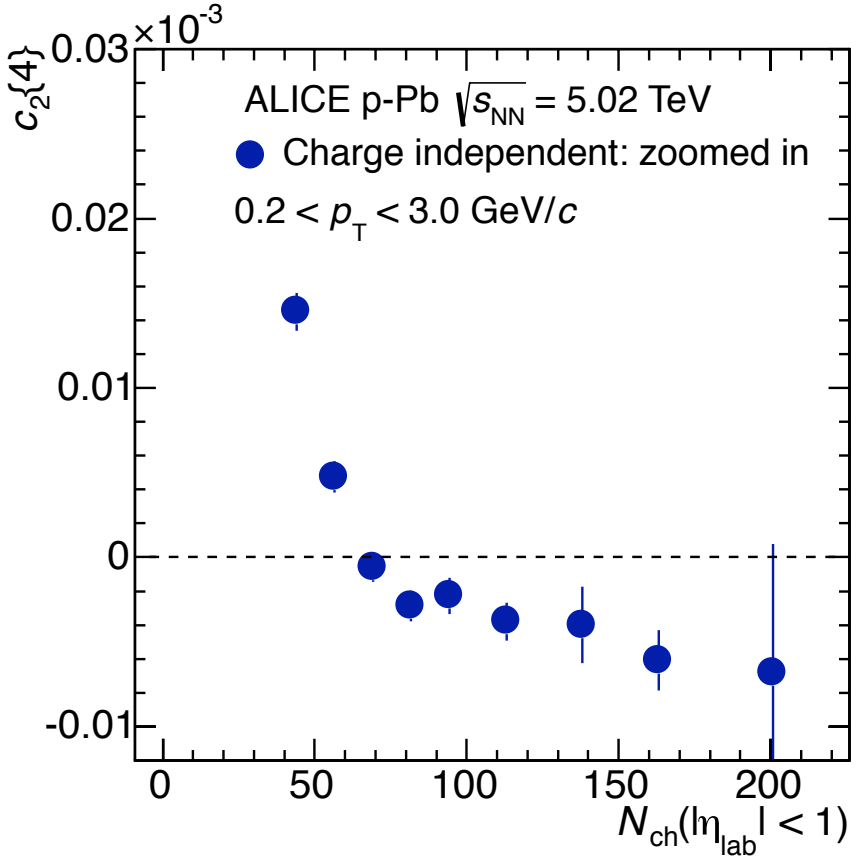
$$v_2^4\{4\} = -c_2\{4\} = \xi^3\left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right)$$

Factors of $\xi = 1/N_D$: partons scatter off domain with same $\vec{E}$ orientation.
$c_2\{2\}$: change of regime when $\mathcal{A} \sim 1/N_c$
$c_2\{4\}$: change of regime when $\mathcal{A} \sim 1/N_c^{3/2}$

A. Kovner, M. Lublinsky,1109.0347, 1211.1928;
† A. Dumitru, L. McLerran, V.S. 1410.4844;

A. Dumitru, A. Giannini 1406.5781
A. Dumitru, V. S. 1411.6630

Single particle anisotropy



ALICE Coll. 1406.2474

similar by CMS Coll 1305.0609

- $$c_2\{4\} = -\xi^3 \left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3} \right)$$
- Interpretation in terms of IS:
  - $N_{ch} < 60$: dominated by **connected** contribution to $c_2\{4\}$
  - $N_{ch} > 60$: dominated by single particle disconnected contribution to $c_2\{4\}$
  - Warning: current result cannot be directly compared to experimental data

Single particle anisotropy: illustration of hierarchy

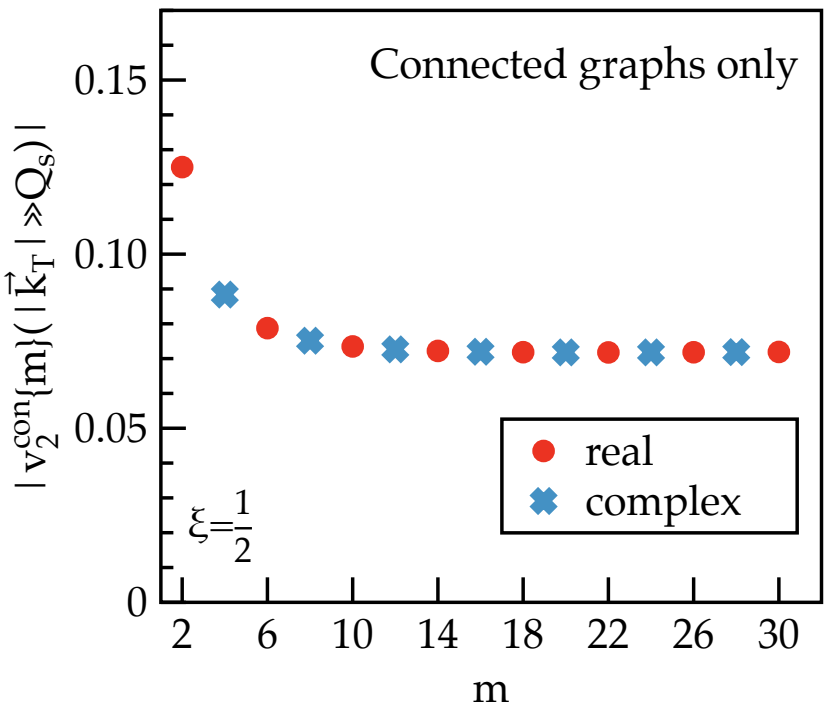
- In general it is difficult to derive general expression for $v_2\{m\}$.
- Limiting cases are easy:

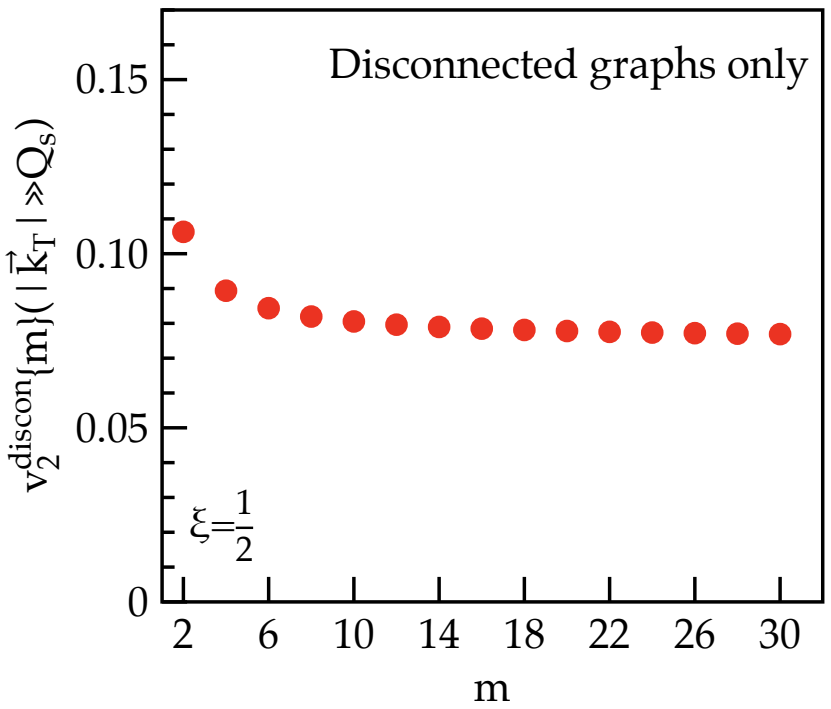
- Connected graphs dominate:

$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m}\left(\frac{\xi}{N_c^2-1}\right)^{m-1};$$

- $\mathcal{A}$(single particle anisotropy) dominates:

$$v_2\{m\} = \xi^{1-1/m}\mathcal{A}; \quad \text{at large } m \quad v_2\{m\} = \xi\mathcal{A}$$





Probability distribution?! Can be reconstructed from the cumulants.

MV model for high energy

- Large-x valence partons are modeled by random, recoilless color charges $\rho^a(\vec{x}_\perp)$ creating semi-classical small-x gluon fields $A^a(\vec{x}_\perp)$.
- Gaussian distribution of sources

$$S_{\rm eff}[\rho^a] = \int dx^- d^2x_\perp \, \frac{\rho^a(x^-,\vec{x}_\perp)\,\rho^a(x^-,\vec{x}_\perp)}{2\mu^2}$$

- Weizsäcker-Williams fields:

$$A^{\mu a}(x^-,\vec{x}_\perp) = -\delta^{\mu +}\frac{g}{\nabla_\perp^2}\rho^a(x^-,\vec{x}_\perp)\,.$$

- Propagation of fundamental charge in this field

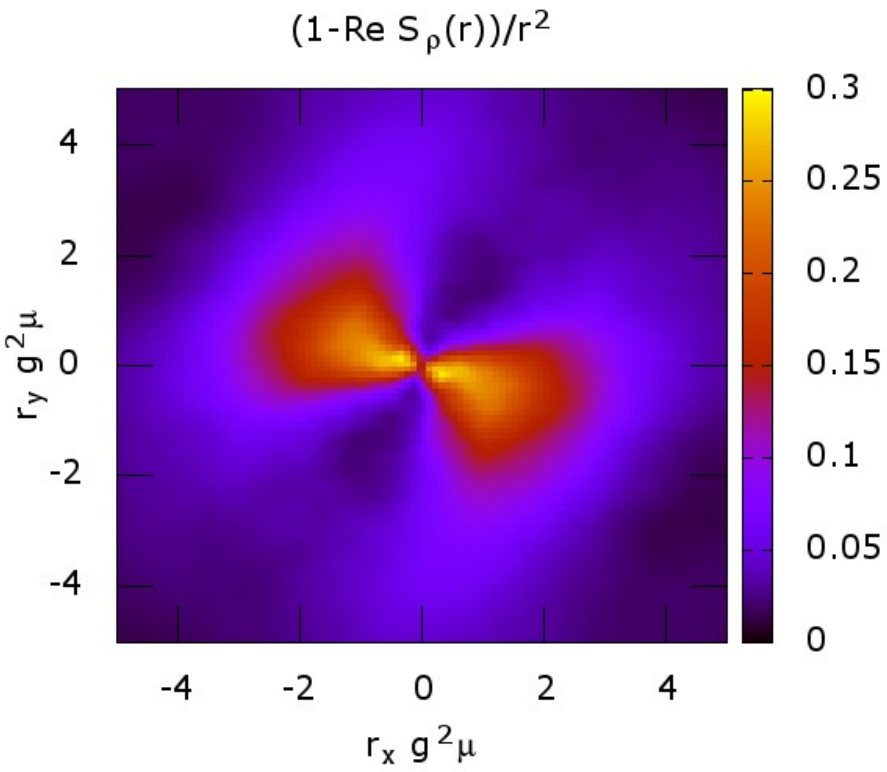
$$V(\vec{x}_\perp) = \mathbb{P}\exp\left\{-ig\int dx^- t^a A^{+a}(x^-,\vec{x}_\perp)\right\}$$

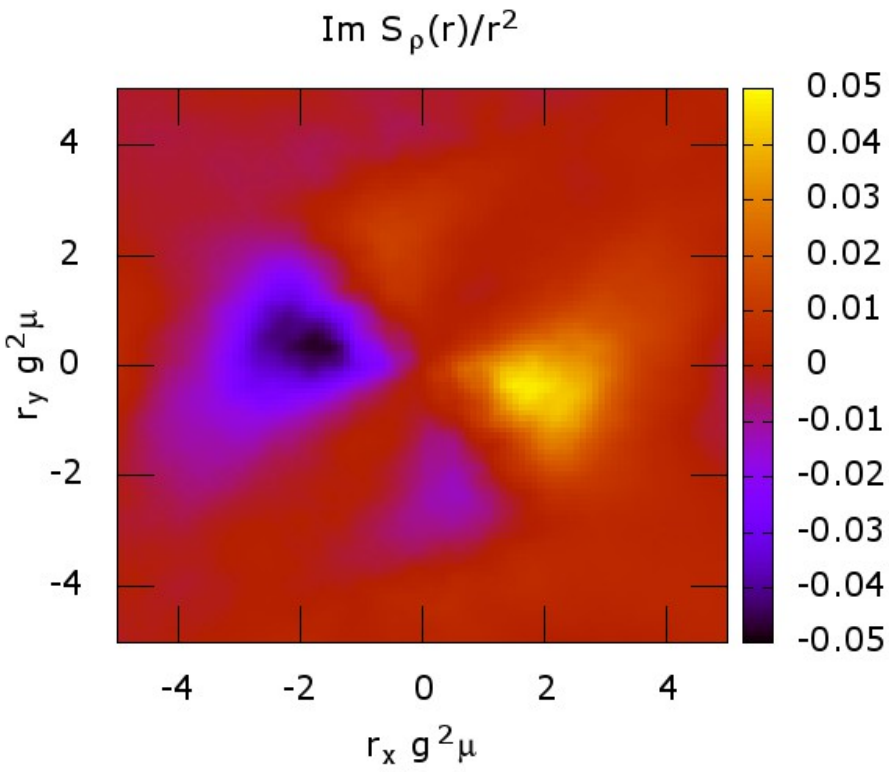
- S-matrix for scattering charge off given target field configuration

$$S_\rho(\vec{r}_\perp,\vec{b}_\perp) \equiv \frac{1}{N_c}{\rm tr}V^\dagger(\vec{x}_\perp)\,V(\vec{y}_\perp), \quad \vec{r}_\perp \equiv \vec{x}_\perp - \vec{y}_\perp\,, \;\; 2\vec{b}_\perp \equiv \vec{x}_\perp + \vec{y}_\perp$$

Details on numerical implementation:
A. Dumitru, V. S. 1411.6630
T. Lappi 0711.3039

Single configuration I





- Normalization $1/r^2$ mimics LO of isotropic part of cross-section
- Real part is dominated by $\cos 2\phi_r$
- Imaginary part is dominated by $\cos \phi_r$

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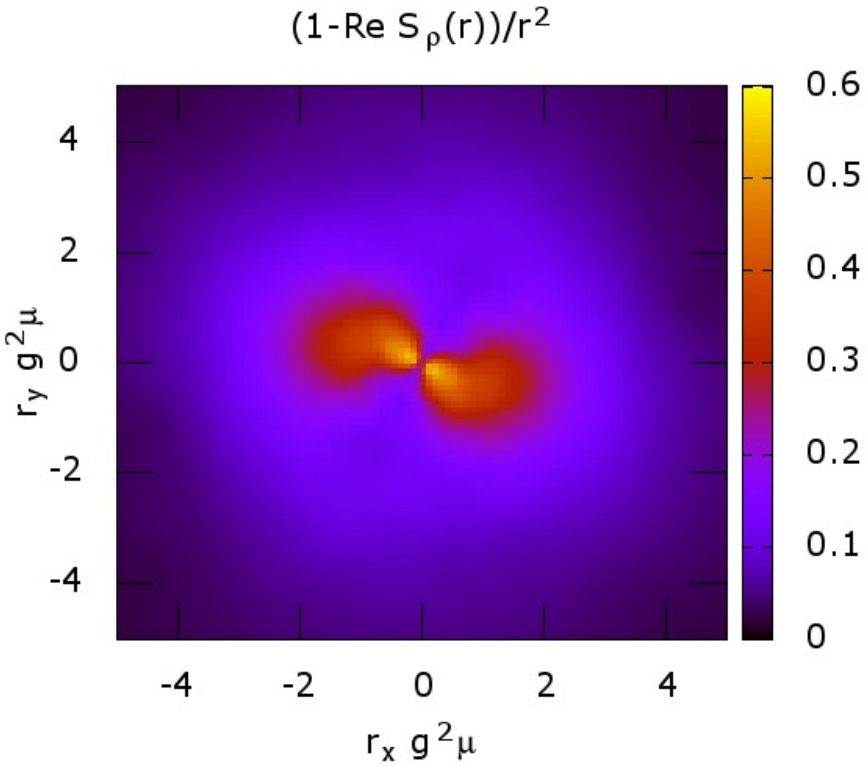
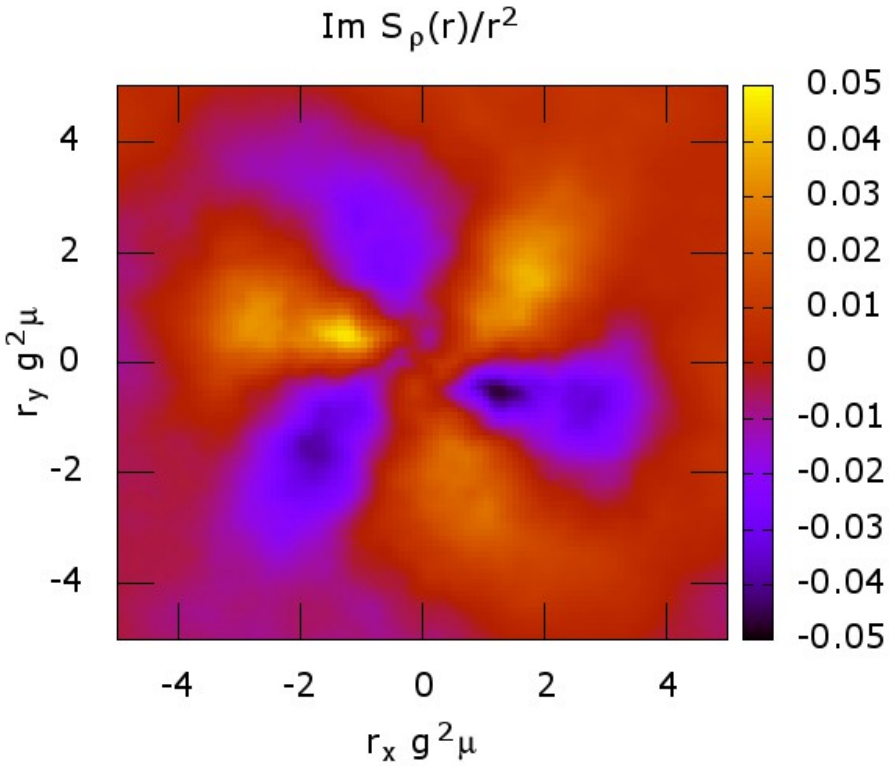
Single configuration II





- Higher orders are seen in real part
- Imaginary part is dominated by $\cos 3\phi_r$

A. Dumitru, V. S. 1411.6630

Amplitudes: definition

- Goal is to extract amplitudes of azimuthal anisotropy
- In each event Fourier decomposition is performed:

$$1 - \mathrm{Re}\, S_\rho(\vec{r}_\perp) \equiv D_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp)\left(1 + \sum_{n=1}^{\infty} A'_{2n}(r_\perp)\cos(2n\phi_r)\right)$$

$$\mathrm{Im}\, S_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp)\sum_{n=0}^{\infty} A'_{2n+1}(r_\perp)\cos((2n+1)\phi_r)$$

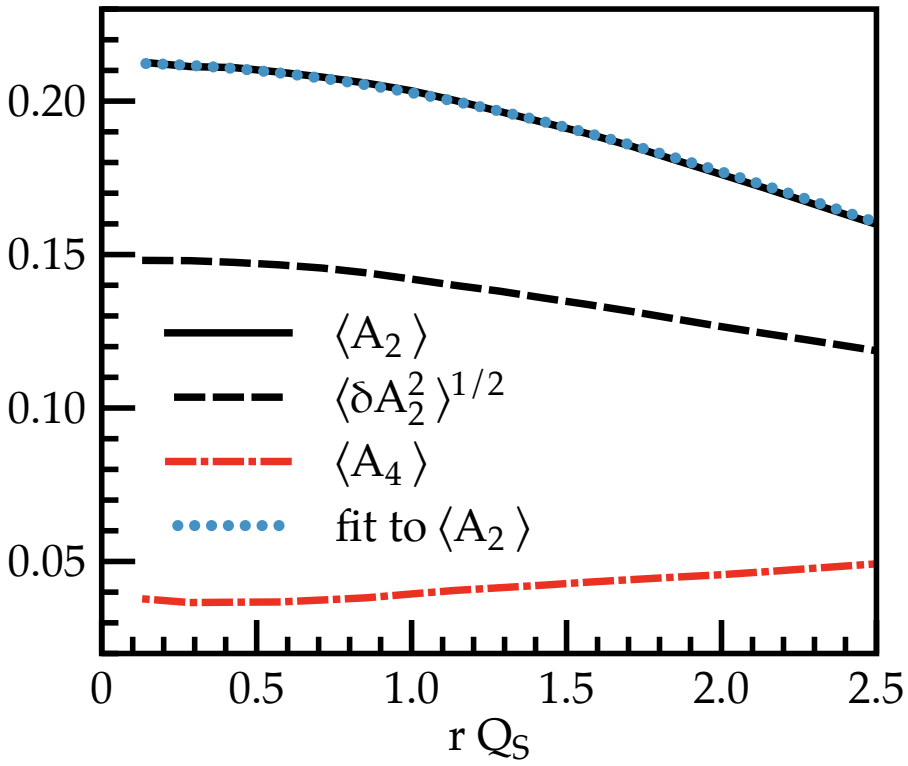
- To cancel trivial random phase:

$$A_n(r_\perp) = \frac{\pi}{2}|A'_{2n}(r_\perp)|$$

- Results are presented in terms of Q_s defined by $\left\langle \mathcal{S}_\rho \right\rangle (r_s = \sqrt{2}/Q_s) \stackrel{!}{=} e^{-1/2}$

A. Dumitru, V. S. 1411.6630

Amplitudes: even harmonics



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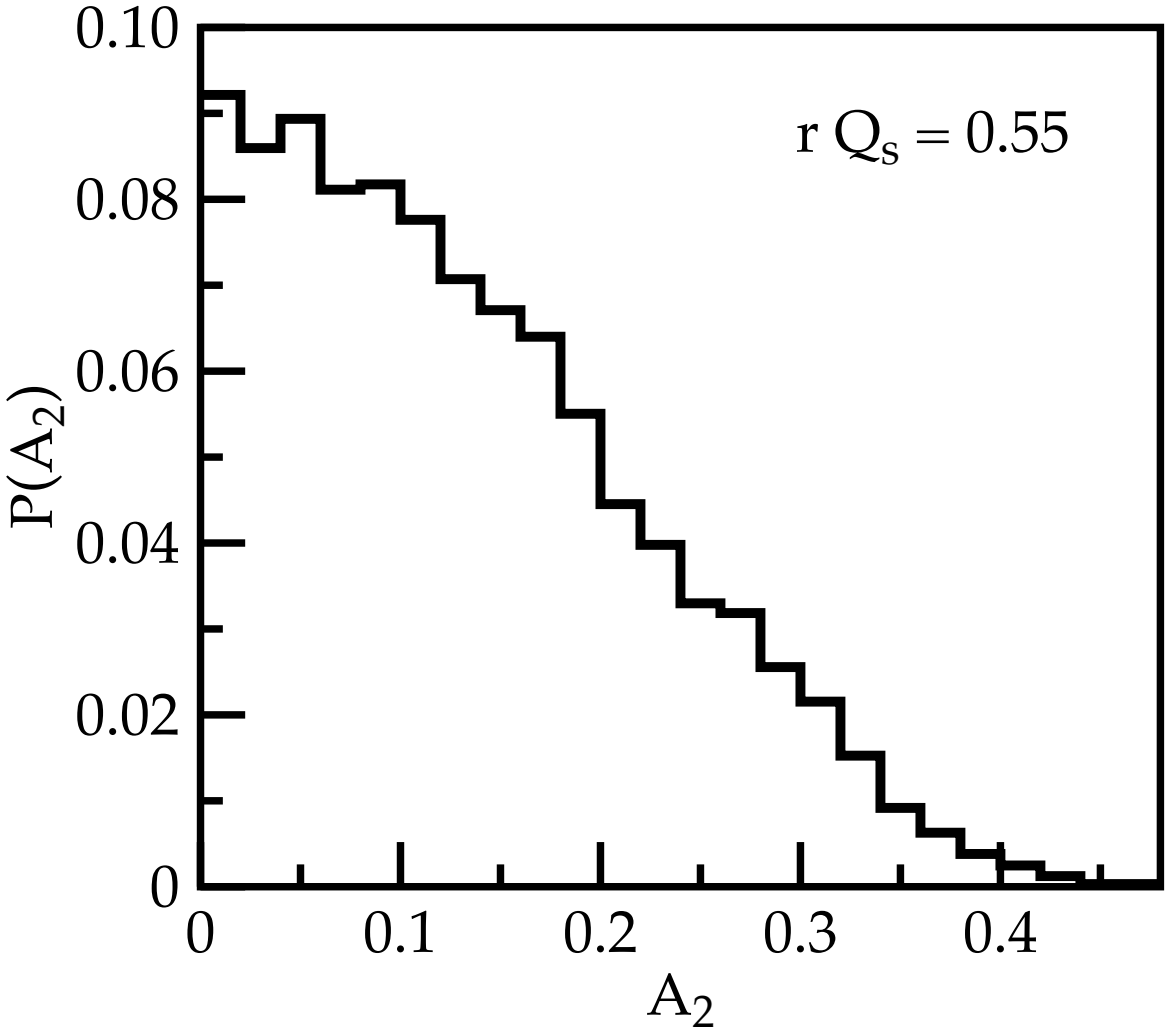
- Largest amplitude: $\langle A_2 \rangle$
- Finite $\langle A_2 \rangle$ at $r \to 0$; $\langle A_2 \rangle$ is approximately constant for $r < 1/Q_s$
- $\langle \delta A_2^2 \rangle^{1/2}$ is comparable to $\langle A_2 \rangle$: fluctuations are rather high
- $\langle A_4 \rangle$ is significantly smaller
- Fit of $\langle A_2 \rangle$ motivated by h_1^g of distribution of linearly polarized gluons (for an unpolarized target) introduced in TMD factorization

$$\delta^{ij} f_1^g(x, \vec{k}^2) + \left(\hat{k}^i \hat{k}^j - \frac{1}{2}\delta^{ij}\right) h_1^{\perp g}(x, \vec{k}^2) \, .$$

In MV (A. Metz and J. Zhou, 1105.1991):

$$h_1^{\perp g}(x, \vec{r}^2) \propto \frac{1}{r^2 Q_s^2}\left[1 - \exp\left(-\frac{r^2 Q_s^2}{4}\right)\right]$$

Fluctuations of A_2



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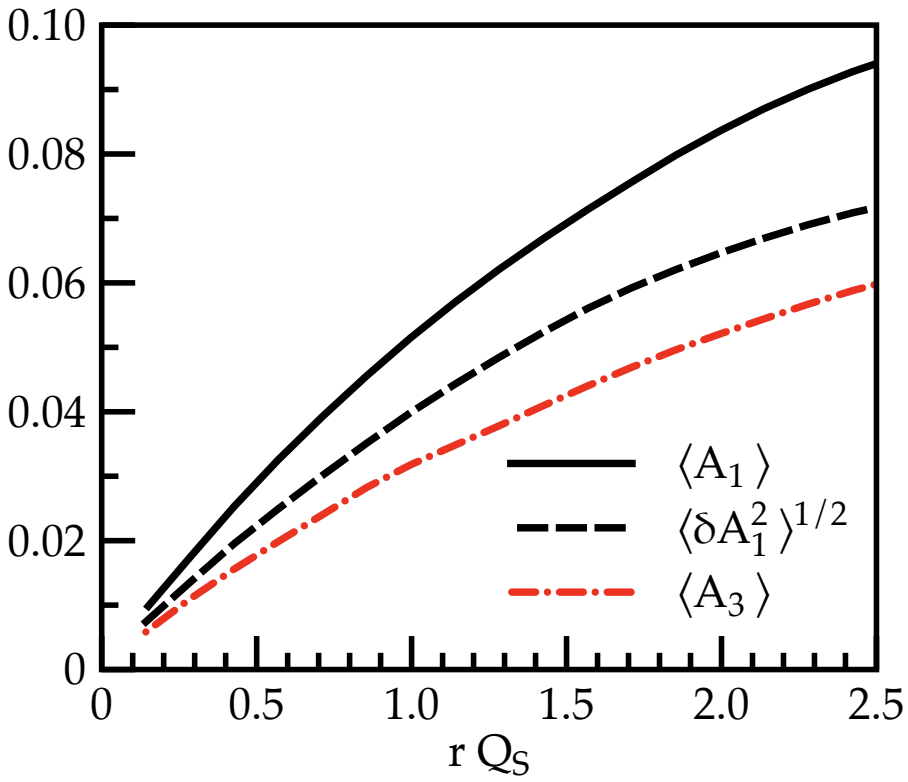
Amplitudes: odd harmonics



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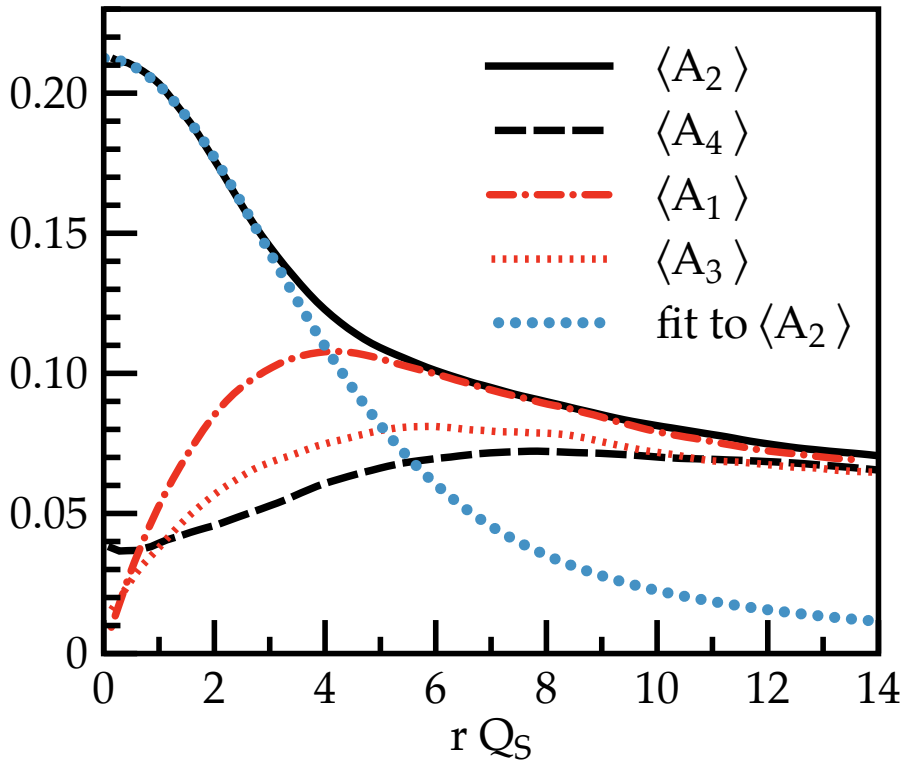
- Expectation value of Im S_ρ is 0
- However, odd harmonics are non-zero!
- At small r, $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero, as expected from analytic arguments

(A. Dumitru and A. Giannini, 1406.5781)

$$\mathrm{Im}\, S_\rho \propto \alpha_s r^3 \cos\phi_r$$

Amplitudes: large r



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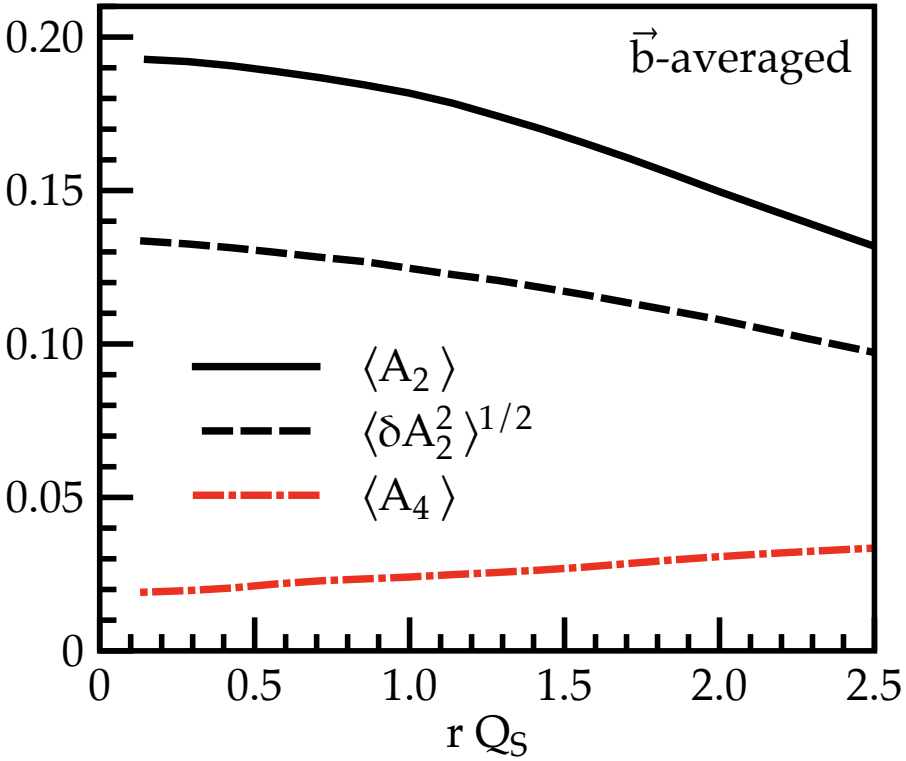
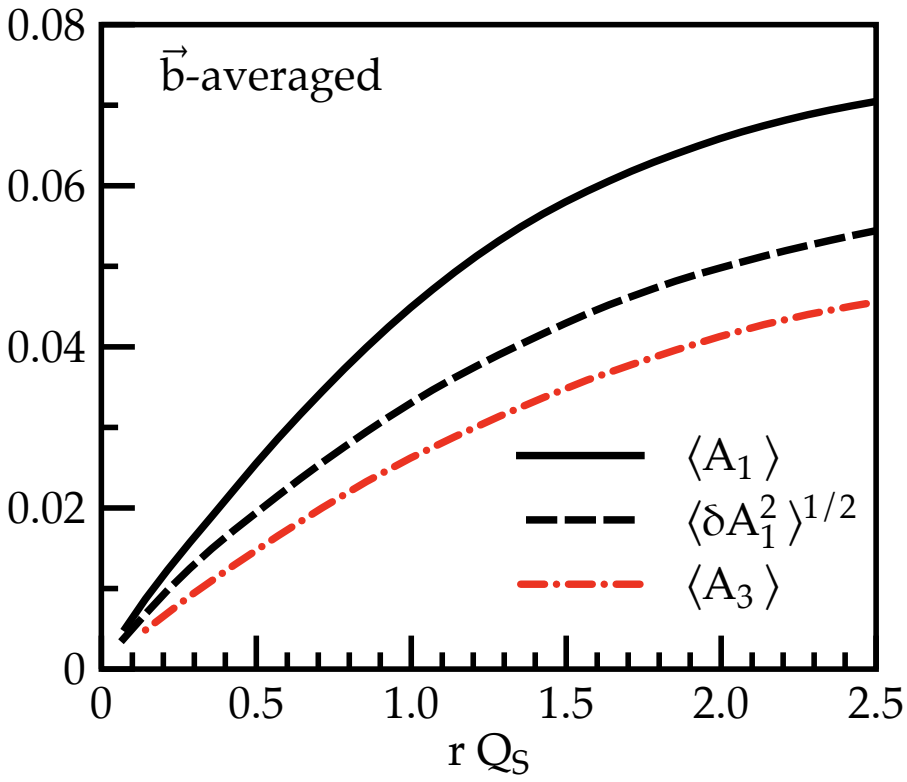
- Fit breaks down at large $r > 3Q_s$: analytical derivation involves adhoc IR cut-offs introduced arbitrary
- Amplitudes approach common non-zero function at large r: expected universal scale invariance of fluctuations of azimuthal dependence of S-matrix

Amplitudes: averaging over finite $\vec{b}_\perp$

$$\overline{D}_\rho(\vec{r}_\perp, \vec{b}_\perp) = \int \frac{d^2\vec{b}'_\perp}{\pi r_\perp^2}\, \Theta\left(r_\perp - |\vec{b}_\perp - \vec{b}'_\perp|\right) D_\rho(\vec{r}_\perp, \vec{b}_\perp)$$





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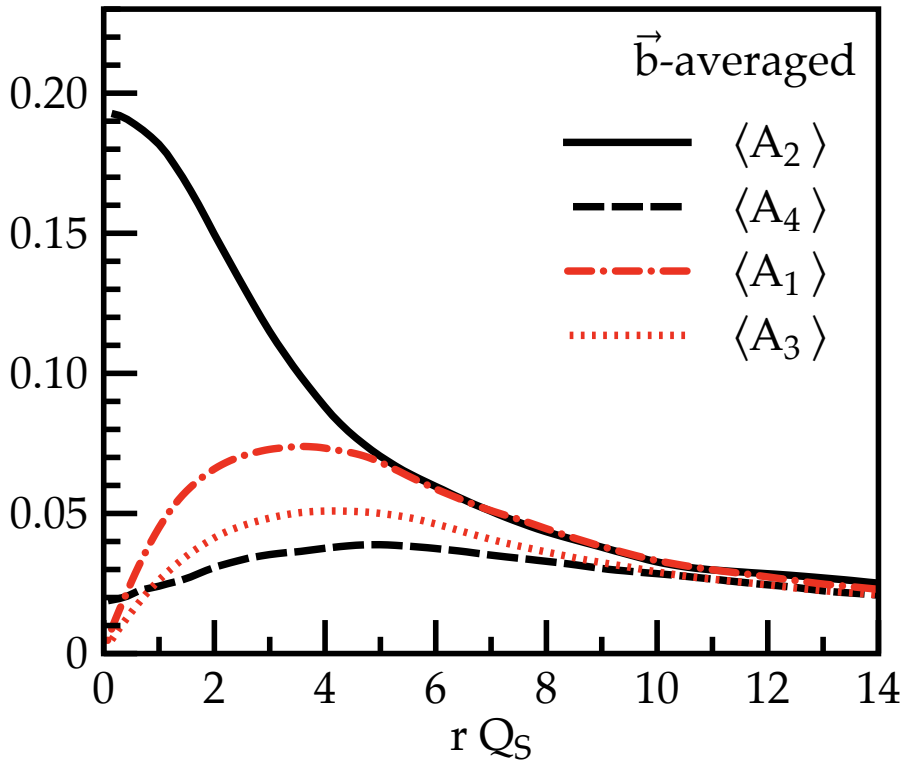
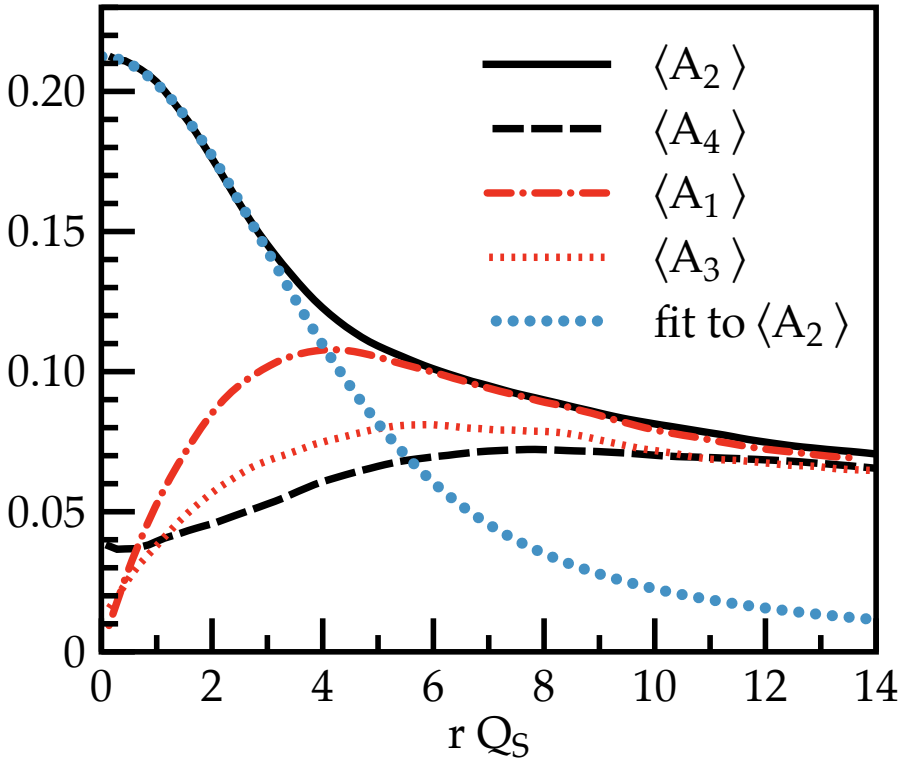
Amplitudes: averaging over finite $\vec{b}_\perp$





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Beyond MV

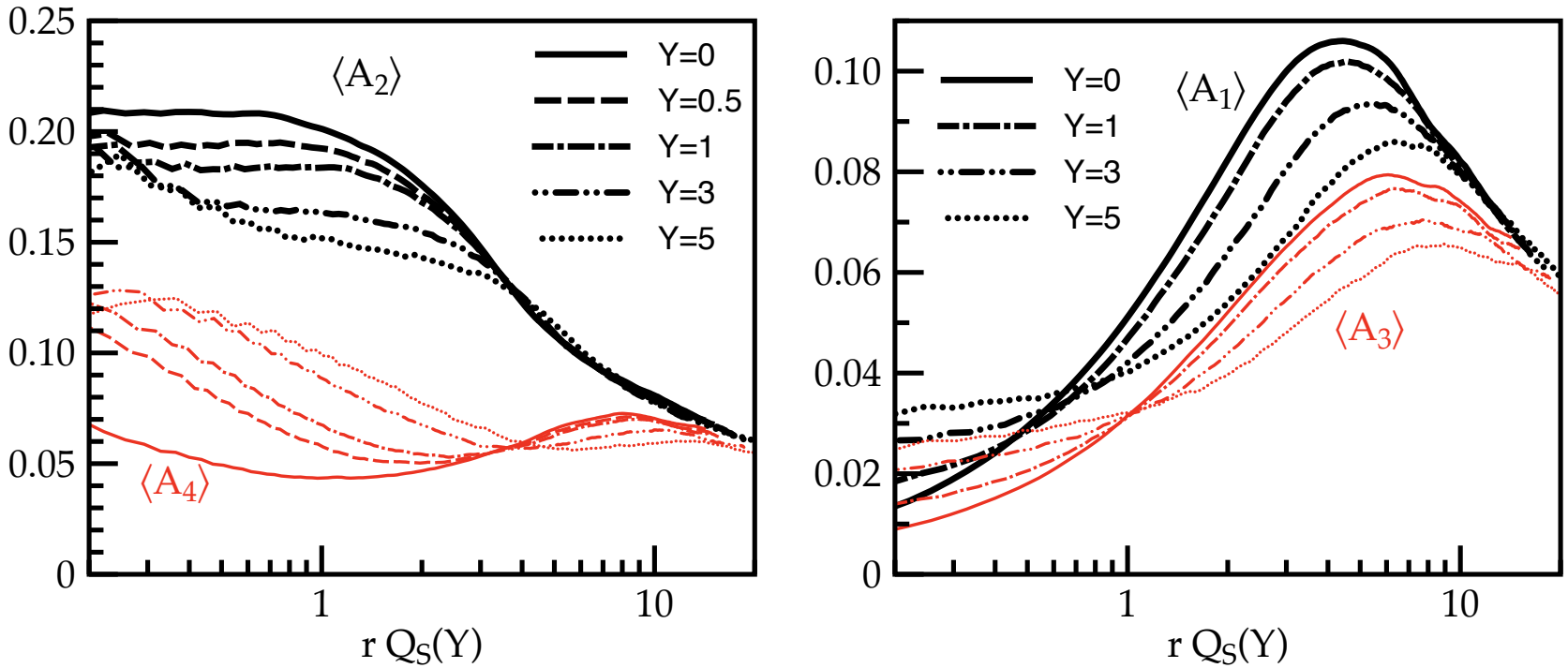
- Beyond MV: inclusion of quantum fluctuations.
- Previous **"mean-field" BK studies**
  showed that azimuthal anisotropy decreases exponentially in Y.
  `See A. Kovner & M. Lublinsky 1211.1928.`
  - Anisotropic initial conditions $1 - S \propto e^{-1/4 Q_S^2 r^2 (1 + \# \cos(2\phi))}$
  - BK equation assuming uniform distribution in impact parameter space

$$\partial_Y N(\vec{r}) = \frac{C_F \alpha(r^2)}{2\pi} \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1) \left(N(\vec{r}_1) + N(\vec{r}_2) - N(\vec{r}) - N(\vec{r}_1) N(\vec{r}_2) \right)$$

  - In this particular set up I was able to reproduce their conclusion: azimuthal anisotropy decays very fast with $Y = \ln(x_0/x)$.
  - Crucial assumption: uniformity in impact parameter space
  - Even at level of initial condition (given by MV model), azimuthal anisotropy of $\mathcal{S}(\vec{r}, \vec{b})$ arises due to fluctuations of soft fields in transverse impact parameter plane.
- **JIMWLK** equation with both $\vec{r}$ and $\vec{b}$.

JIMWLK I

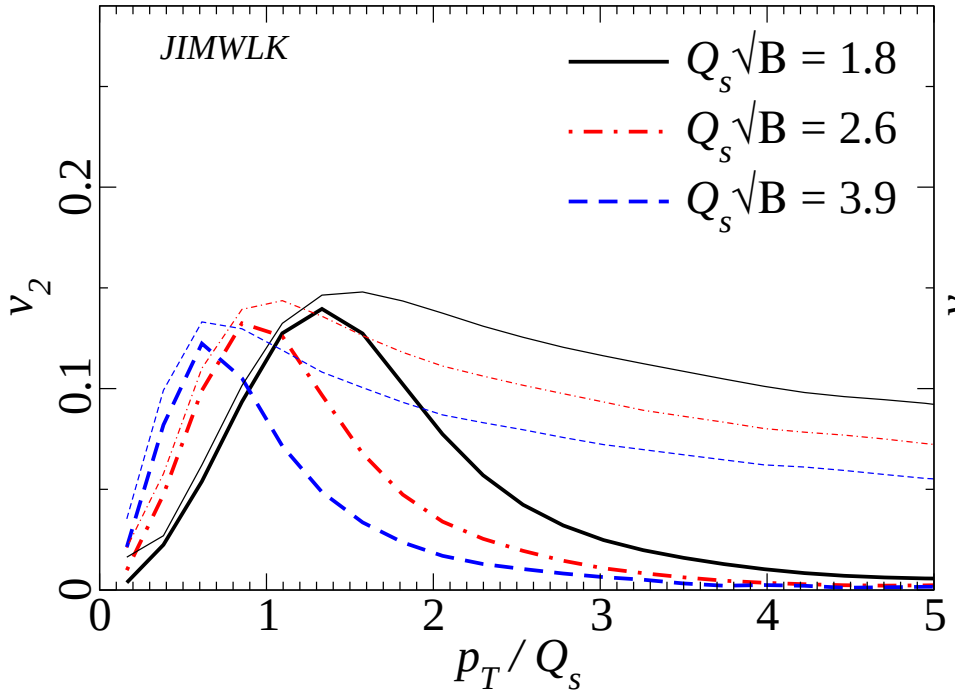
Evolution over a step ΔY in rapidity opens up phase space for radiation of gluons and modifies classical action. This is taken into account by functional renormalization group equation, JIMWLK.
In terms of `random walk" in space of Wilson lines
(see e.g. `T. Lappi and H. Mäntysaari, 1212.4825`):

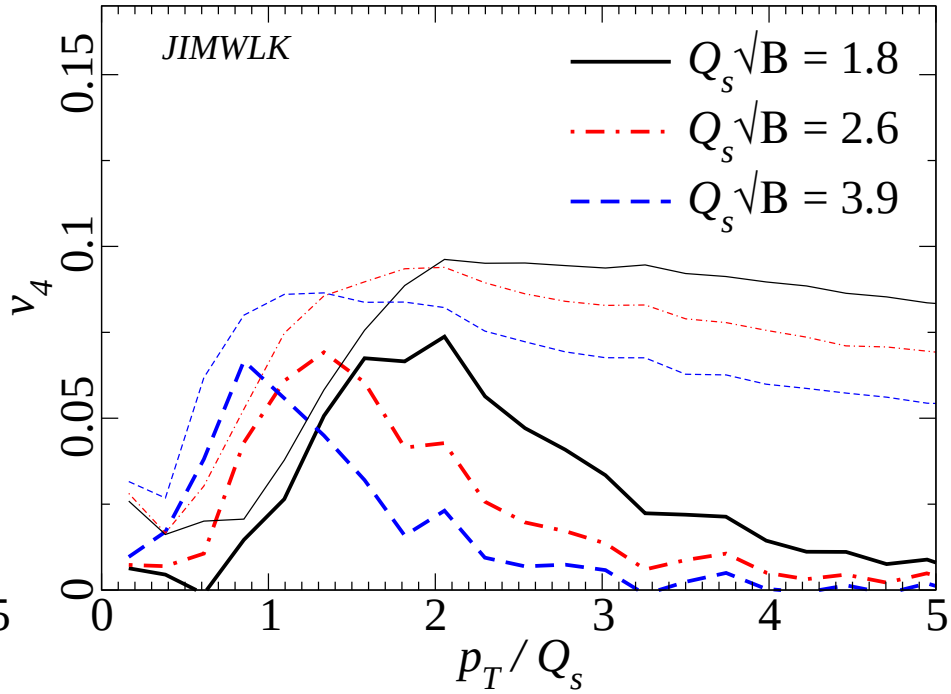


`A. Dumitru, V. S. 1411.6630`

JIMWLK II

Thick lines: Calculations are performed following the experimental analysis (including correlations between soft and high $k_\perp$ particles)
Thin lines: only high $k_\perp$





B is a typical size of a proton in hard particle production at small x.

T. Lappi arXiv:1501.05505

Odd harmonics

- Single particle anisotropy:
  •[A] for fundamental representation S_F has odd harmonics, e.g. $\cos(3\phi)$;
  •[B] for adjoint representation S_A is manifestly real and thus can have only even harmonics (S_A)
- Two particle azimuthal anisotropy: [A] does not help much if there is an approximate quark—anti-quark symmetry of projectile wave function at small x. Indeed, two particle correlation function summed over qq, $q\bar{q}$, $\bar{q}q$ and $\bar{q}\bar{q}$ channels is C-even

$$S_2 \propto \Big(\mathrm{tr}\, V^\dagger(\vec{x}_1)\, V(\vec{y}_1) + \mathrm{tr}\, V(\vec{x}_1)\, V^\dagger(\vec{y}_1) \Big) \Big(\mathrm{tr}\, V^\dagger(\vec{x}_2)\, V(\vec{y}_2) + \mathrm{tr}\, V(\vec{x}_2)\, V^\dagger(\vec{y}_2) \Big)$$

  is real, and so has even cumulants only.
- Obtaining non-zero $c_1\{2\}$ and $c_3\{2\}$ may require to account for (at least) one additional soft rescattering of (anti-) quarks besides their hard scattering from target shockwave.

Conclusions

- Neglecting initial state one-particle azimuthal anisotropy: complex $v_2\{4\}$
- Initial state one-particle azimuthal anisotropy is present due to fluctuating valence quarks. This disconnected contribution to $c_2\{4\}$ results in real $v_2\{4\}$
- MV model:

  even amplitudes $\langle A_2 \rangle$ and $\langle A_4 \rangle$ of azimuthal anisotropy are approximately constant for $k_\perp > Q_s$ ($r_\perp < 1/Q_s$);

  odd amplitudes $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero at $k_\perp \to \infty$ ($r_\perp \to 0$)

  $\langle A_2(k_\perp \sim Q_s) \rangle \approx 20\%$ $\quad \langle A_4(k_\perp \sim Q_s) \rangle \approx 4.7\%$
- Small-x evolution does not significantly modify anisotoropy
- Does $v_2\{m\}$ has a direct connection to TMD h_1^g?!
- No multiplicity bias were imposed in our studies.
- Gunter Roland: collectivity as "correlated emission of interacting particles". In this talk, collectivity is correlated emission of non-interacting particles. Gluon saturation is essentially multi-particle collective phenomenon.

Meanings of Ridge measurements

Fuqiang Wang
Purdue University

Outline

- Brief introduction

- **STAR data (arXiv:1412.8437, 1502.07652)**
- Uncertainty principle v_n? (arXiv:1404.4119)
- **AMPT escape anisotropy (arXiv:1502.05572)**

- Summary/open questions

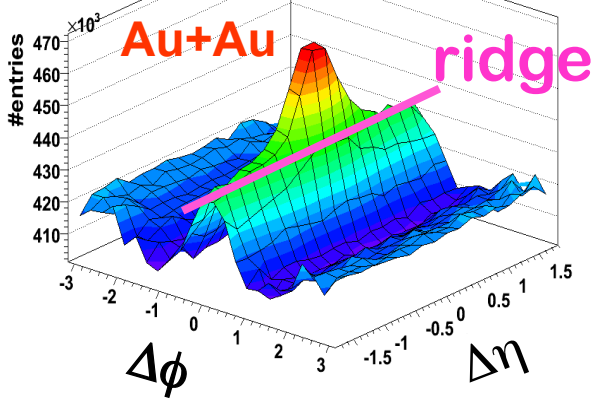
The Ridge

Two-particle angular correlations

$$dN/d\phi \sim 1 + 2v_2\cos2(\phi-\psi_2) + 2v_3\cos2(\phi-\psi_3)$$

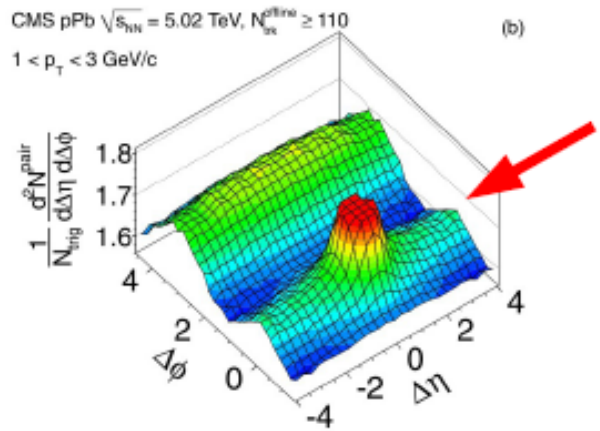
$$dN/d\Delta\phi \sim 1 + 2v_2^2\cos2\Delta\phi + 2v_3^2\cos2\Delta\phi$$

STAR, PRC 80 (2009)

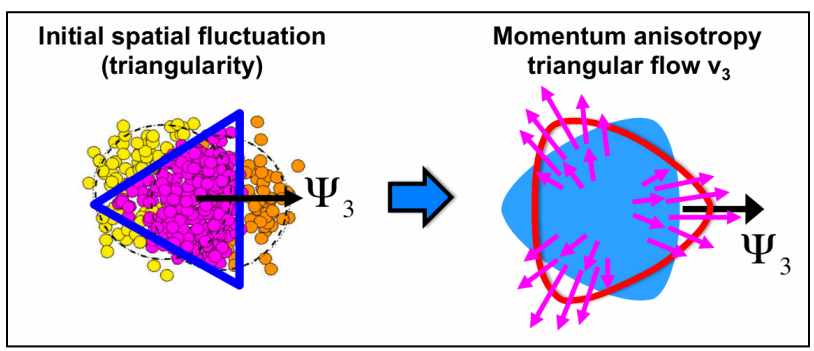


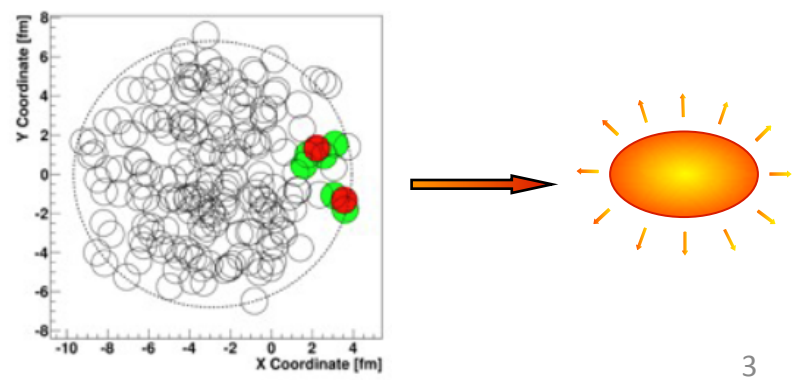
Elliptic flow subtracted

CMS pPb PLB 718 (2013) 795

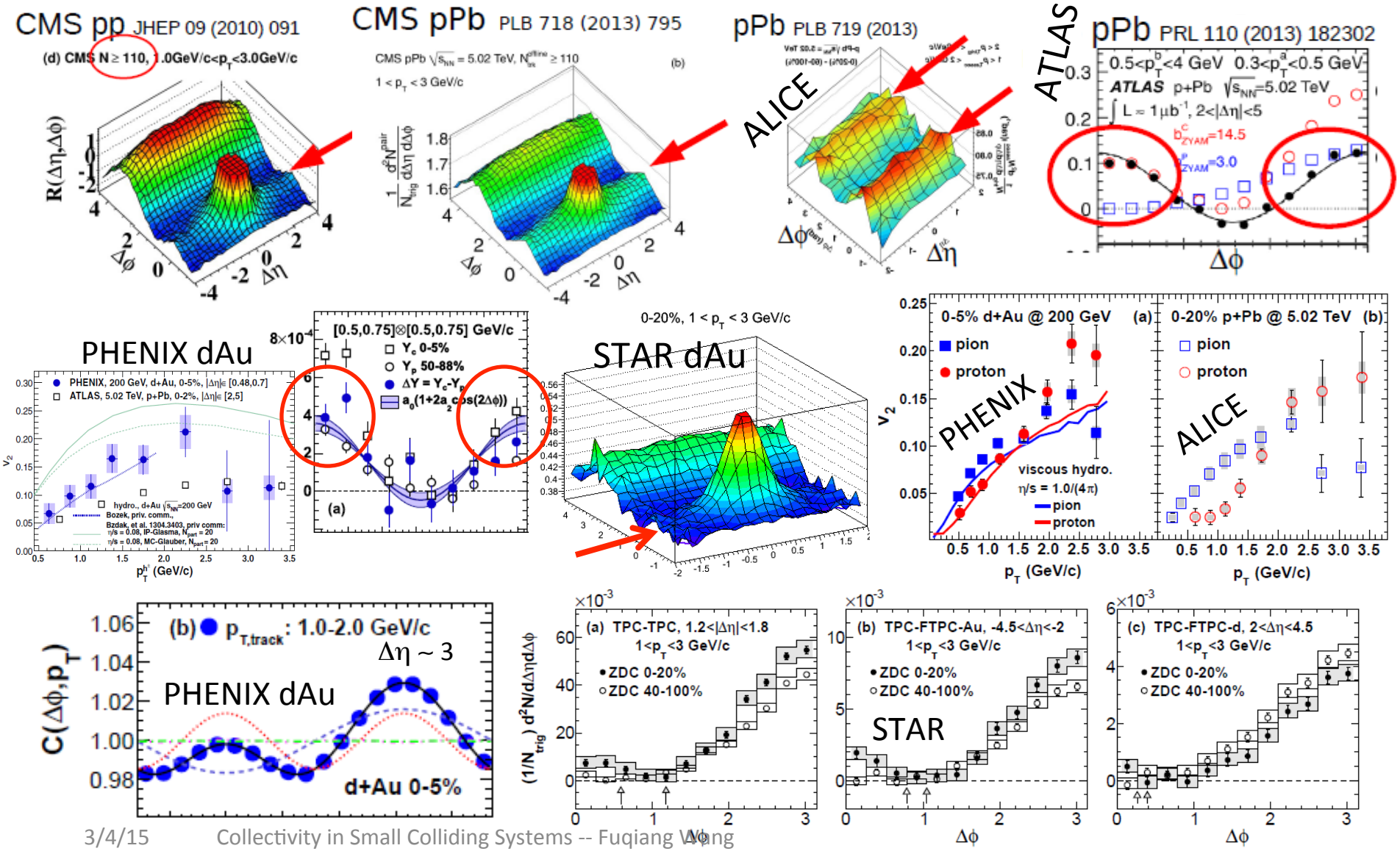


Above uniform pedestal





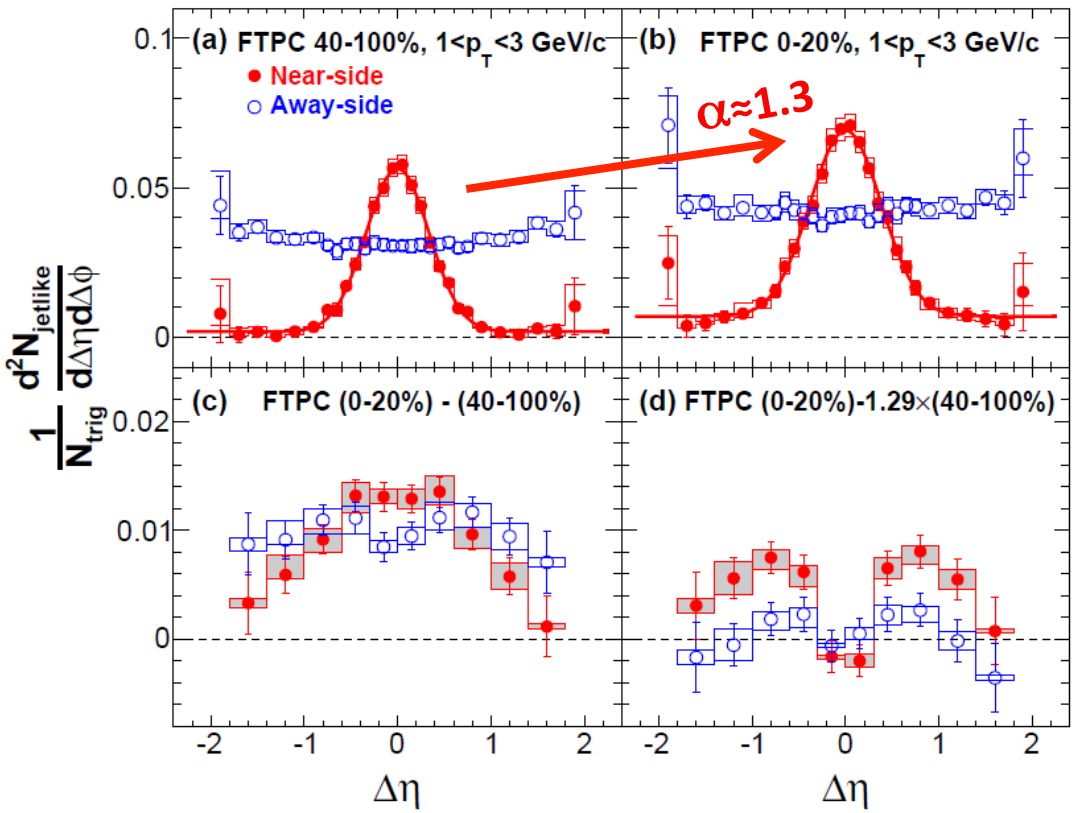
Ridge in small systems, and everywhere



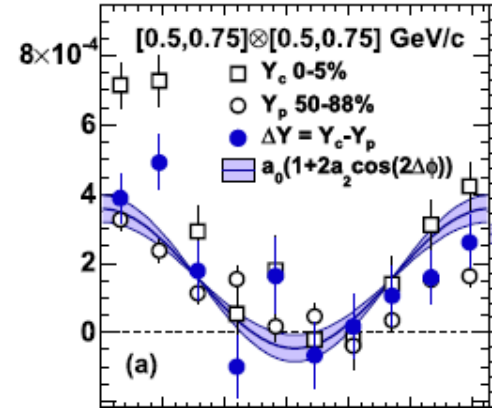
Careful about central – peripheral

Central – peripheral in attempt to subtract jets

STAR, arXiv:1412.8437, accepted by PLB

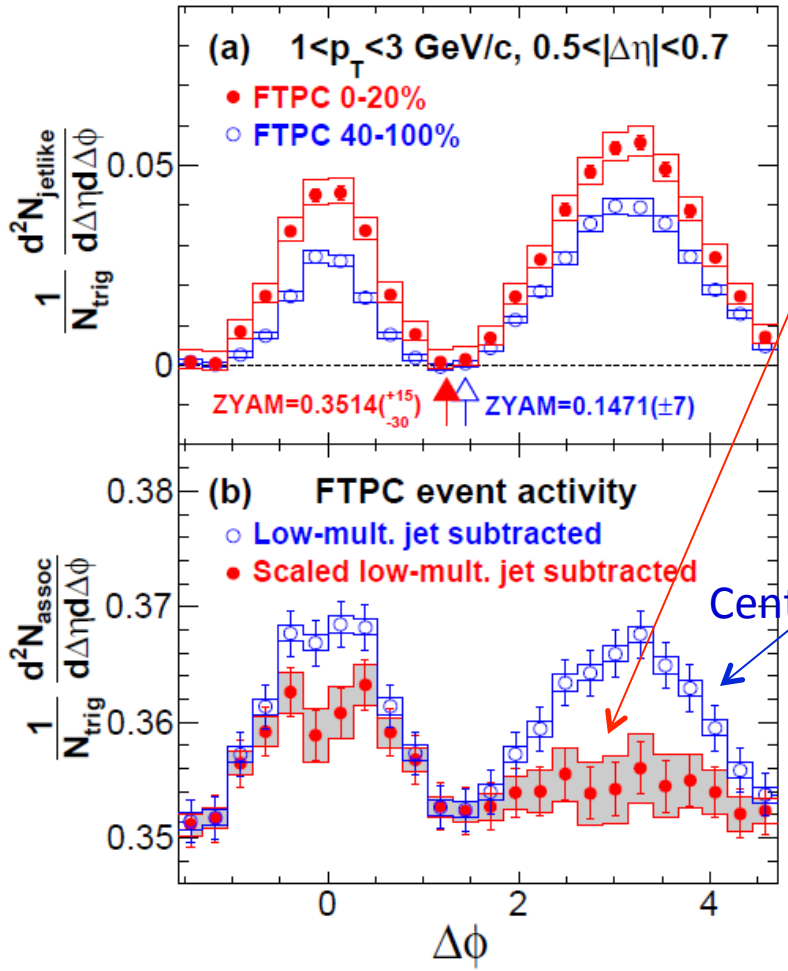


PHENIX, PRL 111 (2013)



- Central and peripheral jets differ
- central – peripheral has caused confusions
- Experimentally we should do a better job

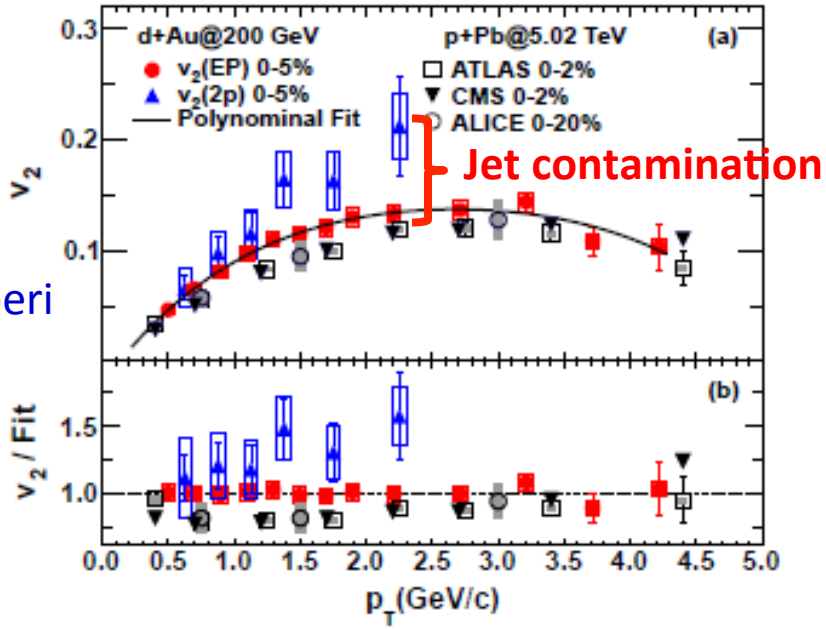
Double ridge is vulnerable



Cent – α*peri

After a simple "correction" the double ridge is gone.

Cent – peri



Remarks

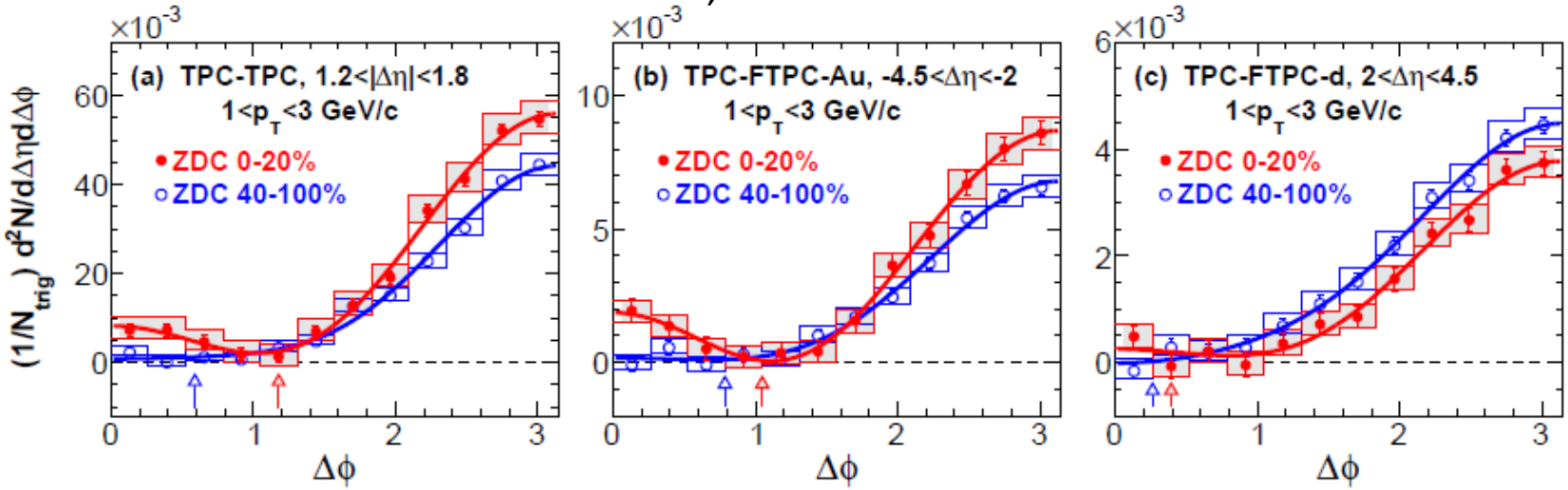
- With peripheral subtraction, jet contributions may be reduced, but results can be uninterpretable.
- Central – peripheral may be OK at LHC as jet residual is relatively small with the large event multiplicity.

- Wise to move away from central – peripheral, including PHENIX.
- Go to large $\Delta\eta$, so near-side jet contribution is minimal.

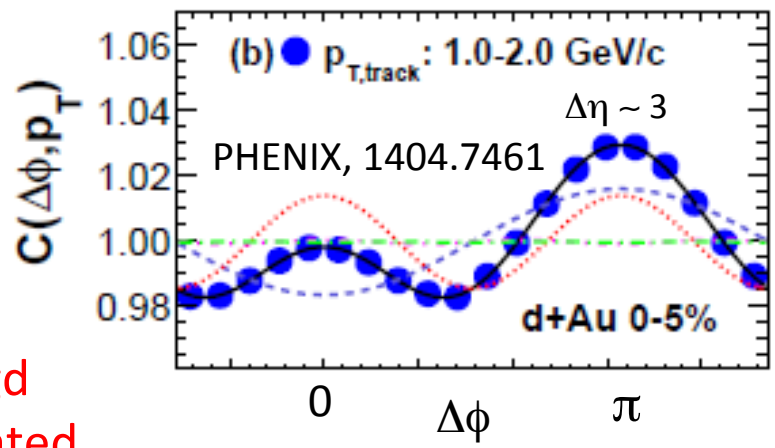
- Without peripheral subtraction, the away-side is dominated by jets. But the results are cleaner, and effect of jets is easier to comprehend.

Long-range correlations in d+Au

STAR, arXiv:1502.07652



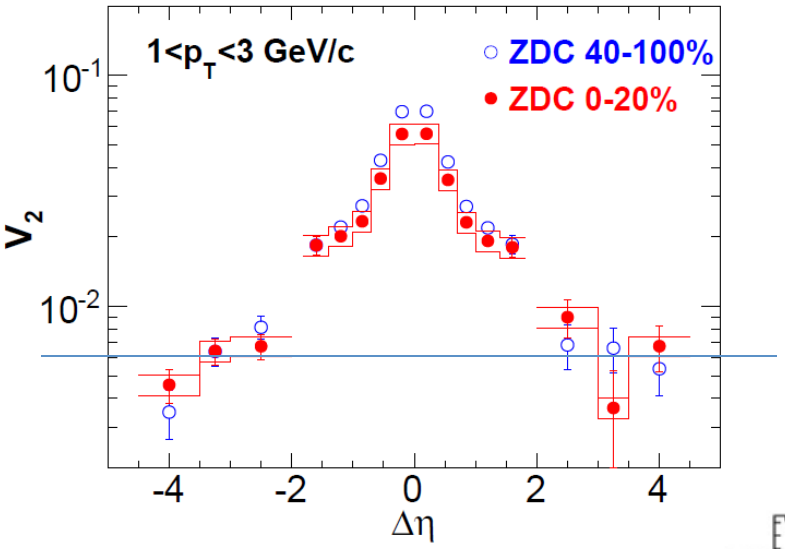
- ZYAM uniform background subtracted
- Near-side long-range finite ridge, unlikely from near-side jet
- Away-side Au- vs. d-side behavior differ



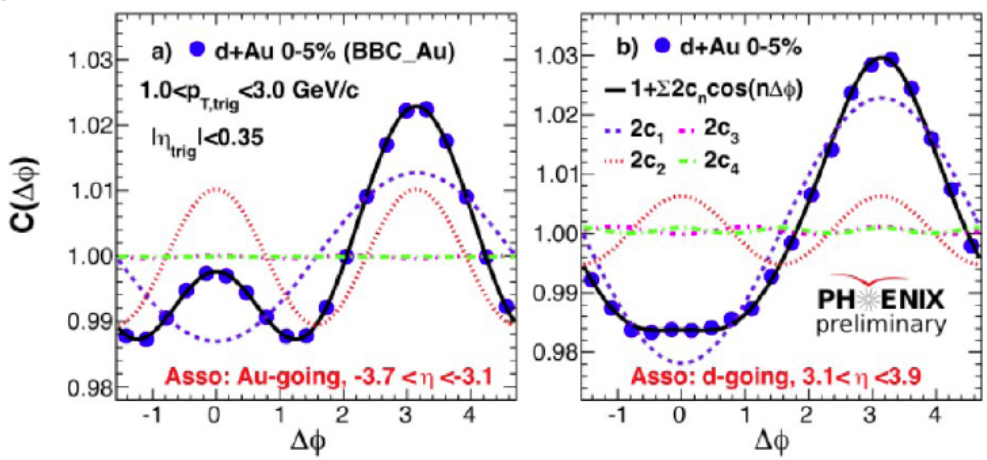
ZYAM: Correlated signal + uncorrelated bkgd
Fourier coefficients: All particles are correlated

Fourier coefficients: hydro?

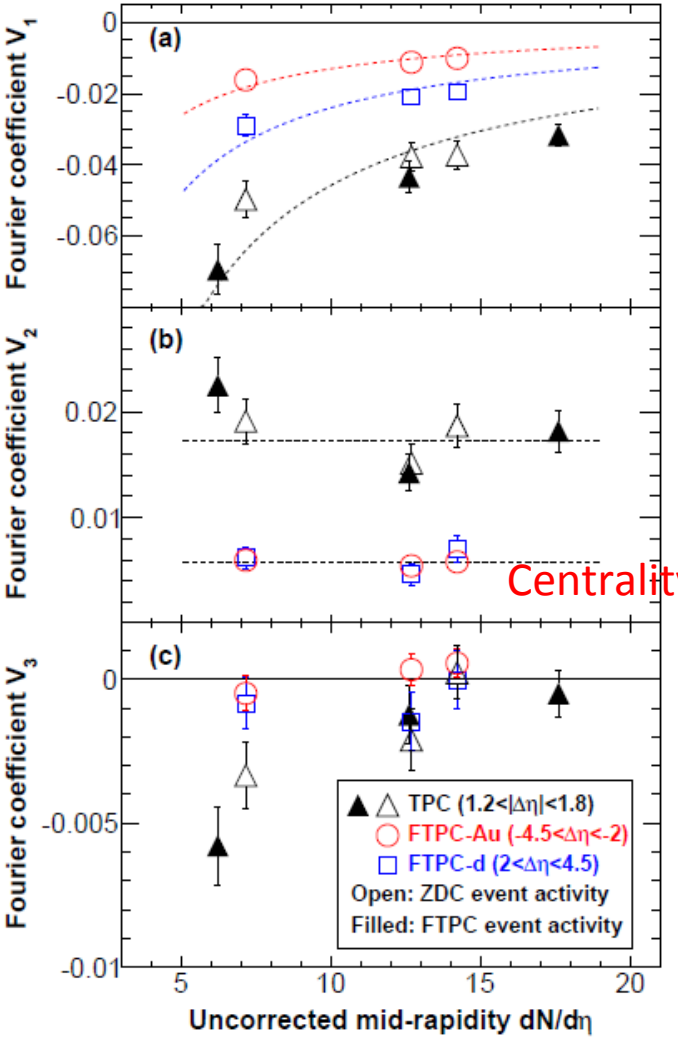
STAR, arXiv:1502.07652



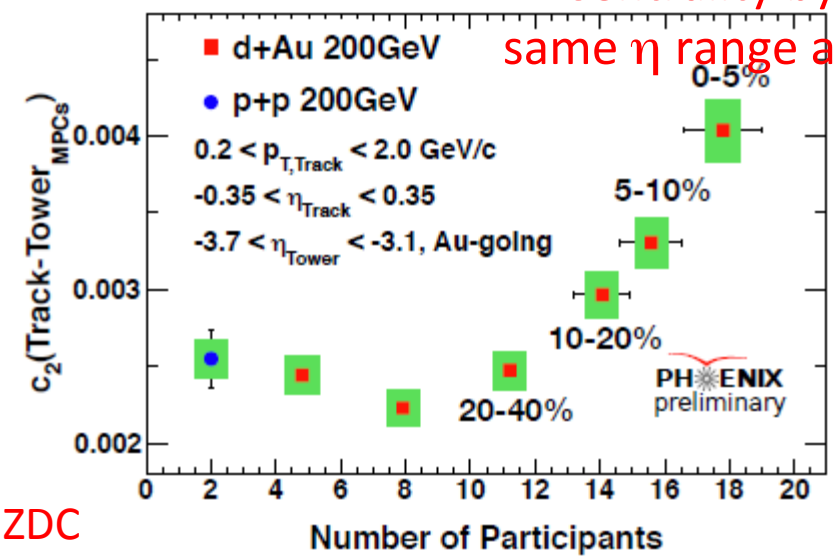
- No v_2 difference: central vs. peripheral
- No v_2 difference: Au-going vs. d-going
- These features are not normally expected in hydro



Multiplicity dependence



Centrality by ZDC

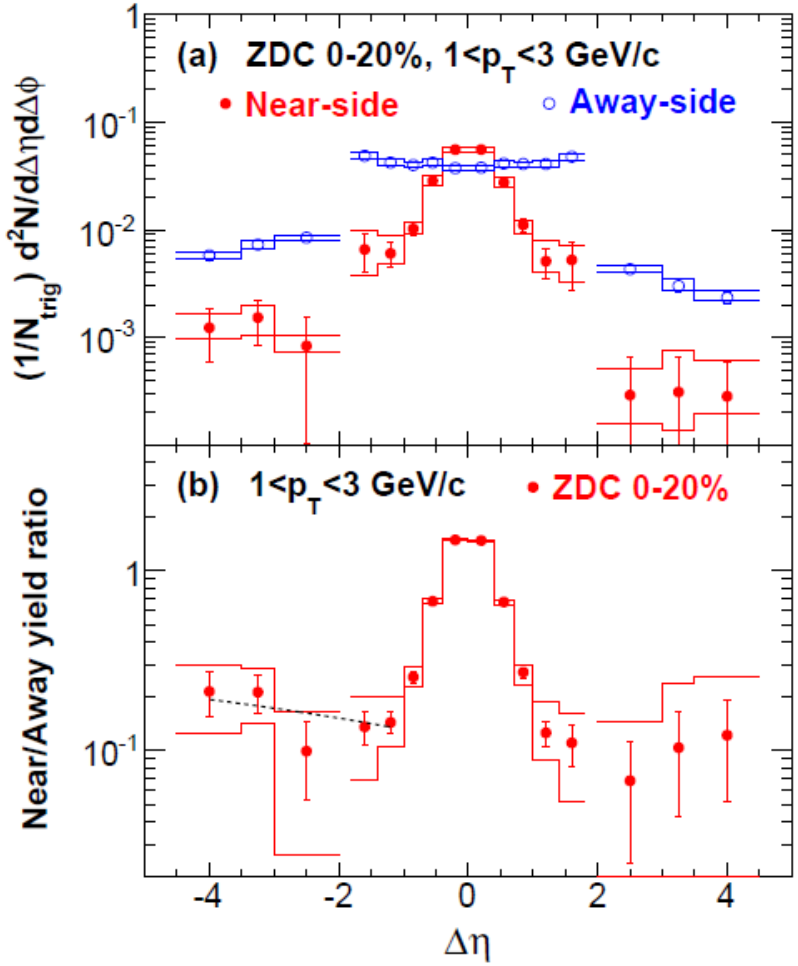


Centrality by BBC
same η range as signal

x2

- V_2 nearly constant over multiplicity: coincident compensation of jet + flow?
- Hydro gives same v_2 over multiplicity?

Or something else?



- The ridge does not scale with ZYAM (underlying background): Fourier coefficients decrease drastically (x5) with $\Delta\eta$
- But, the ridge is proportional to the away-side jet at given $\Delta\eta$
- Is ridge somehow related to jet?
- Color reconnection between beam jet and midrapidity jet?

Remarks

- Central vs. peripheral comparison is complex:
  - on Au-going side, central > peripheral on away side, but
  - on d-going side, central < peripheral.

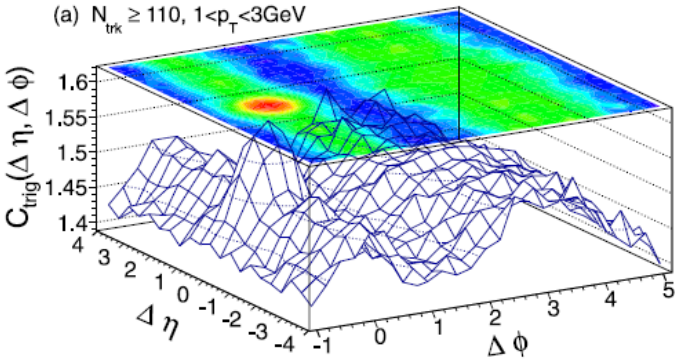
  It must have something to do with the underlying partonic kinematics.

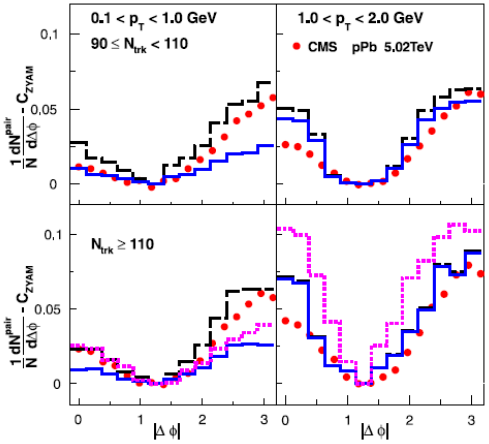
- And yet, V_2's are similar between Au- and d-side, unexpected from hydro.

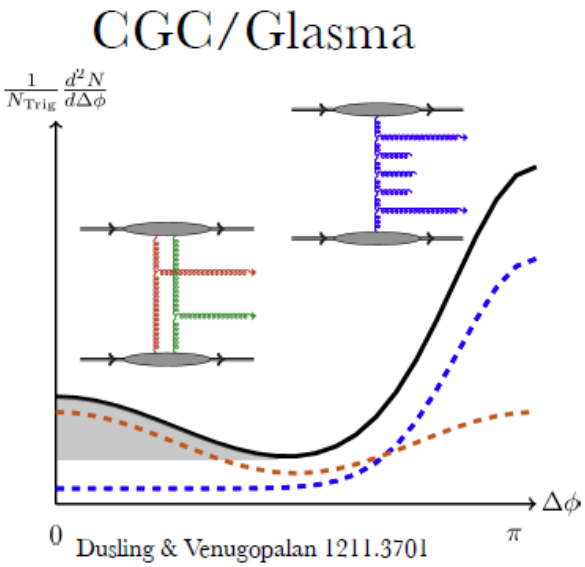
- V_2 is relatively insensitive to multiplicity.
  If peripheral V_2 is completely from jets, it will be reduced by multiplicity.
  Non-jet components coincidentally compensate the reduced jet effect?

- Can it be possible that there is a 2nd order harmonic in all pairs from peripheral to central?

Excitement is the possibility of hydro or CGC in small systems

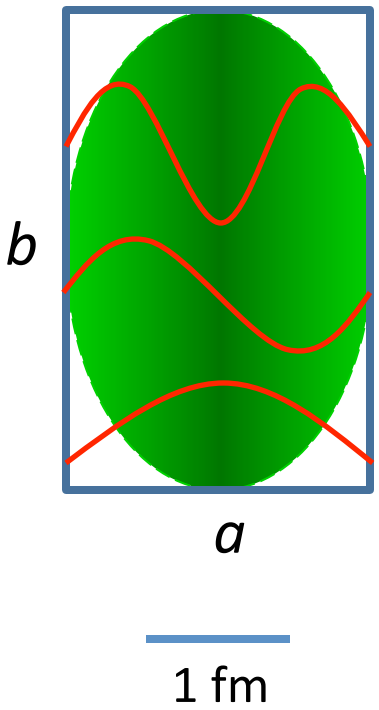






- Hydro is classical
- Is hydro applicable to such small systems?
- Mean free path may be small relative to the size, ~ fm?

Why should classical physics work?



$$-\frac{\mathrm{h}^2}{2m}\nabla^2\psi = E\psi \quad \Rightarrow \quad \psi \propto \begin{cases} \cos\frac{n_{odd}\pi}{a}x \\ \sin\frac{n_{even}\pi}{a}x \end{cases}$$

Take even mode for example:

$$\langle p_x^2 \rangle = \mathrm{h}^2 k^2 \;; \quad \langle x^2 \rangle = \frac{a^2}{4} - \frac{2}{k^2} \;; \quad k = \frac{n_{odd}\pi}{a}$$

$$\sqrt{\langle p_x^2 \rangle \cdot \langle x^2 \rangle} = \mathrm{h}\sqrt{\frac{k^2 a^2}{4} - 2} = \mathrm{h}\sqrt{\frac{\pi^2}{4} n_{odd}^2 - 2} > \mathrm{h}/2$$

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{b^2 - a^2}{b^2 + a^2} = \varepsilon \quad \text{for all } n.$$

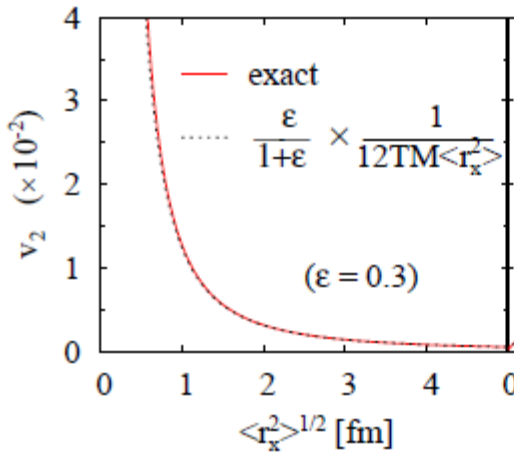
Single state anisotropy

Initial v_2 from Quantum Mechanics

D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

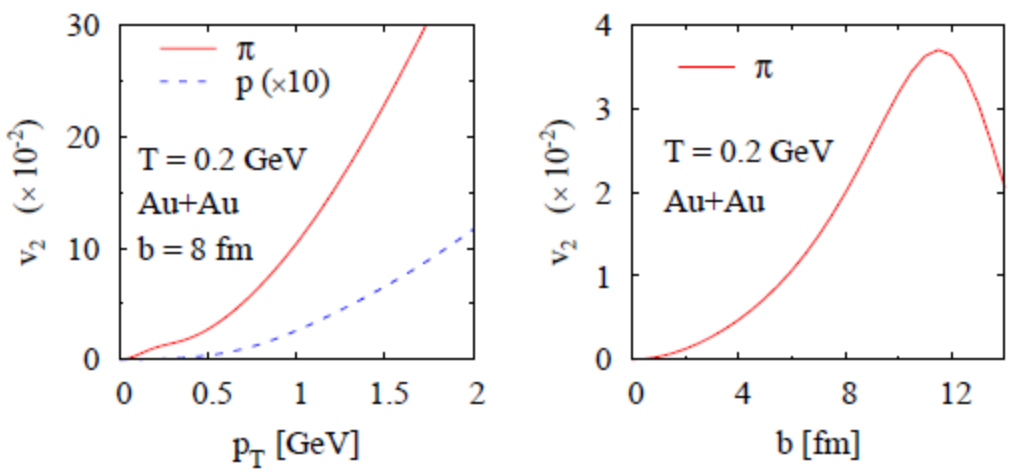
$$\bar{v}_2 \approx \frac{\hbar^2}{12 k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1+\varepsilon}$$

Thermal weighting of all states



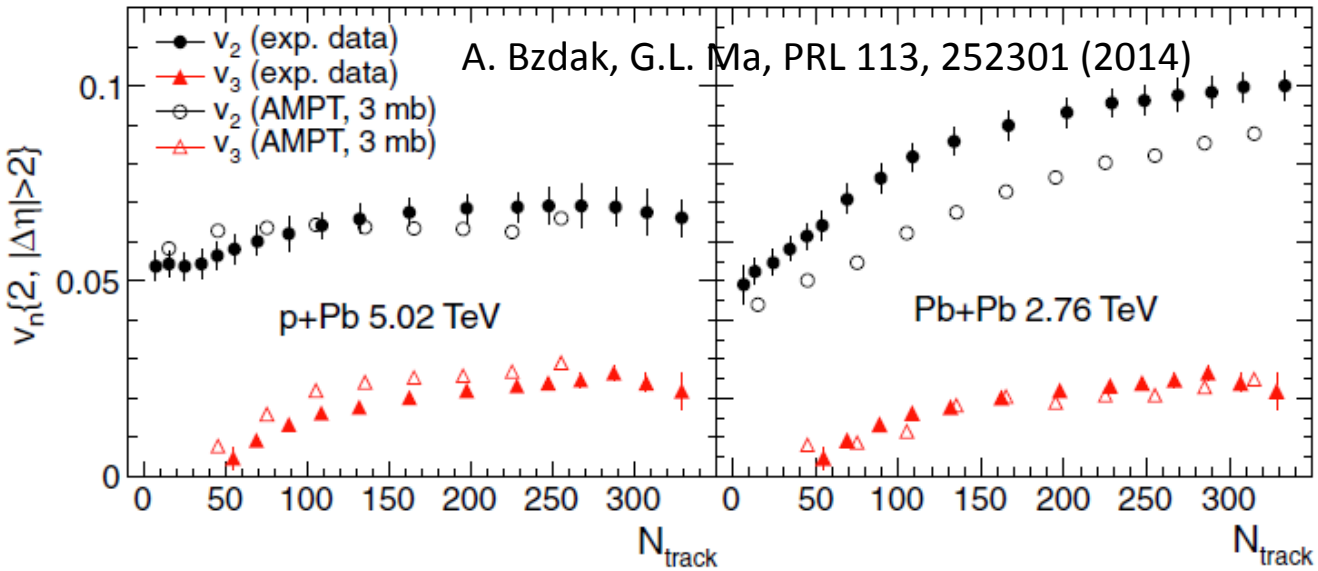
Typical Au+Au collisions

$b = 8$ fm: $\langle r_x^2 \rangle^{1/2} = 1.5$ fm and $\langle r_y^2 \rangle^{1/2} = 2.2$ fm

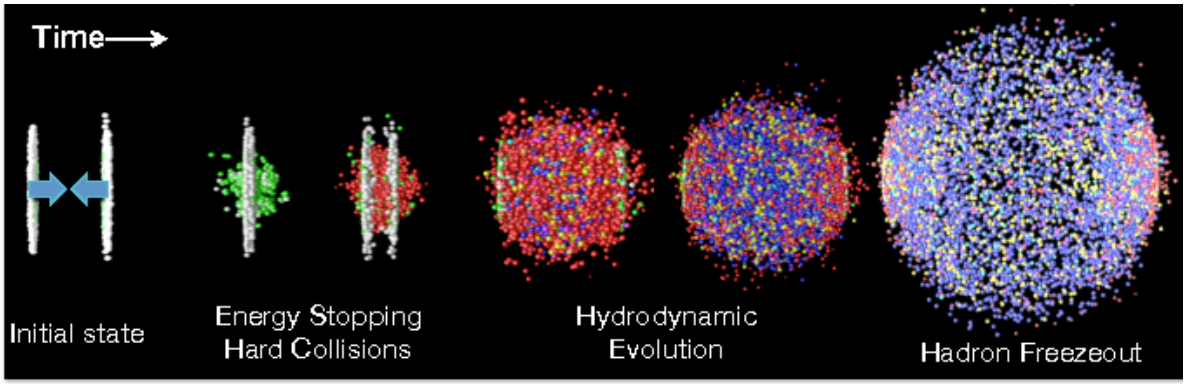


AMPT can describe the ridge too…

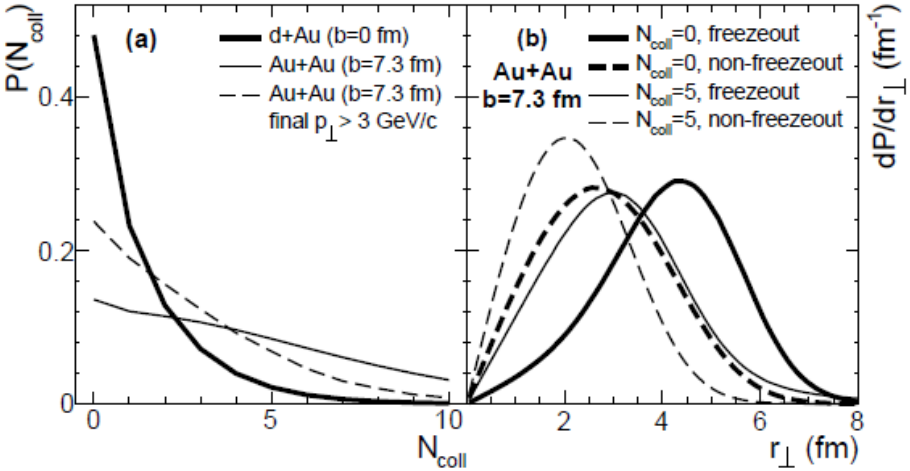
- AMPT: A Multi-Phase Transport (string melting turned on)
- Partons (quarks) liberated from nucleons and strings
- Parton cascading: elastic scattering with 3 mb cross-section
- Partons cease to interact: freeze-out, coalescence into hadrons



Parton cascade history

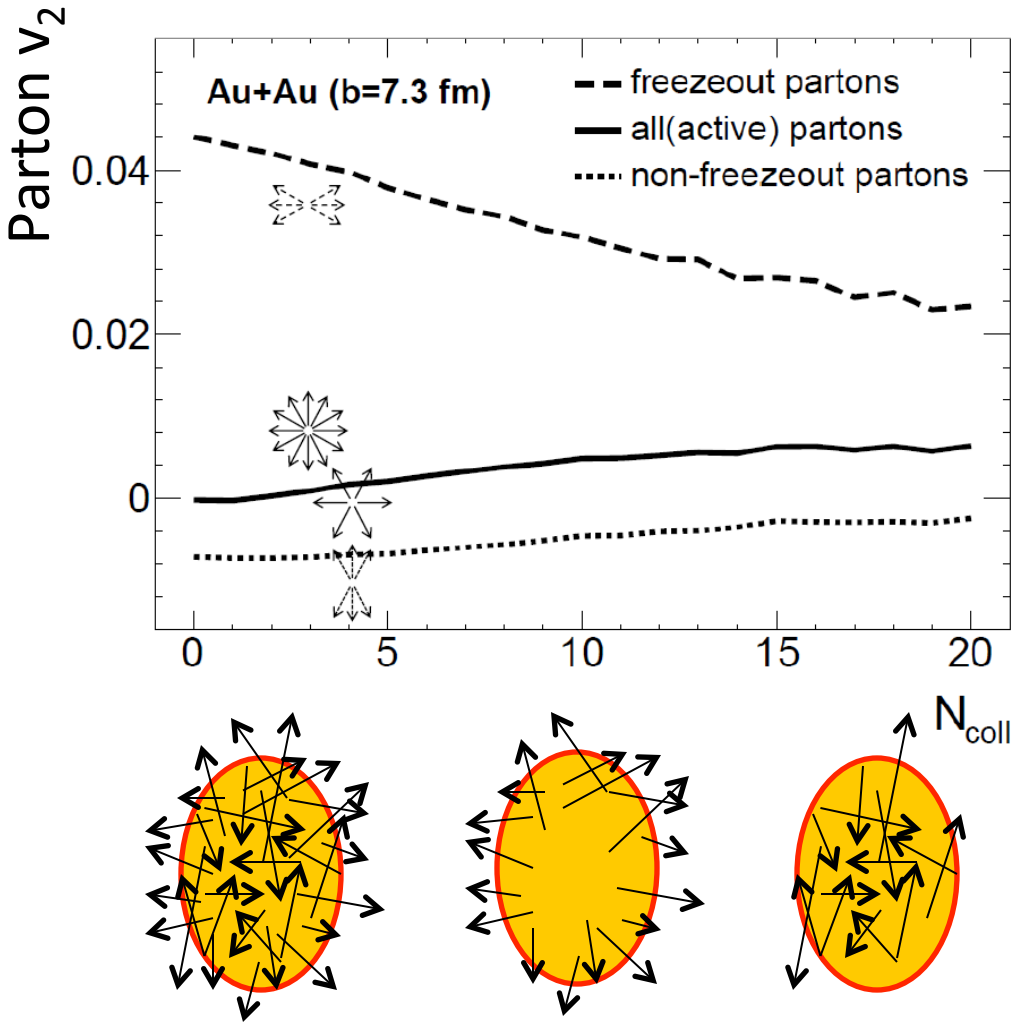


- Get into transport code
- Follow cascading history, microscopic interactions
- Investigate how parton v_n is generated



How is anisotropy developed?

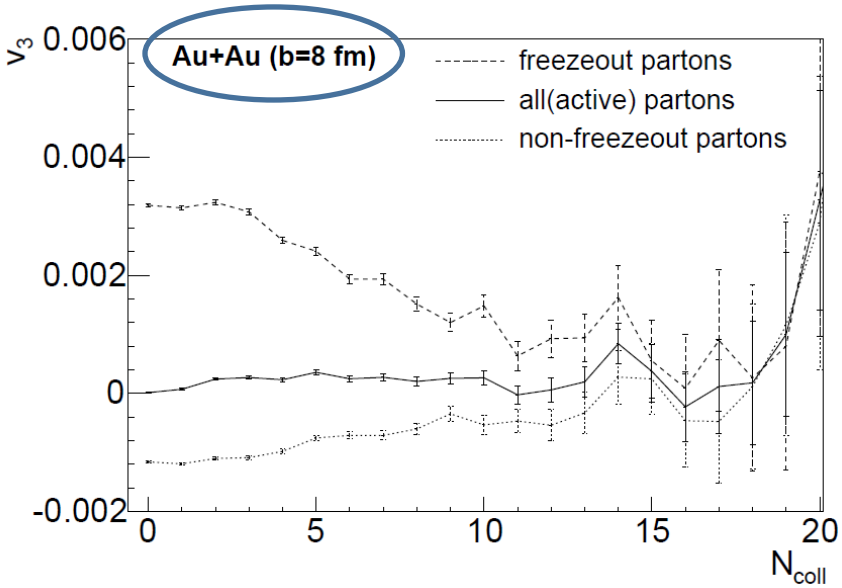
L. He, T. Edmonds, Z.-W. Lin, F. Liu, D. Molnar, FW, arXiv:1502.05572



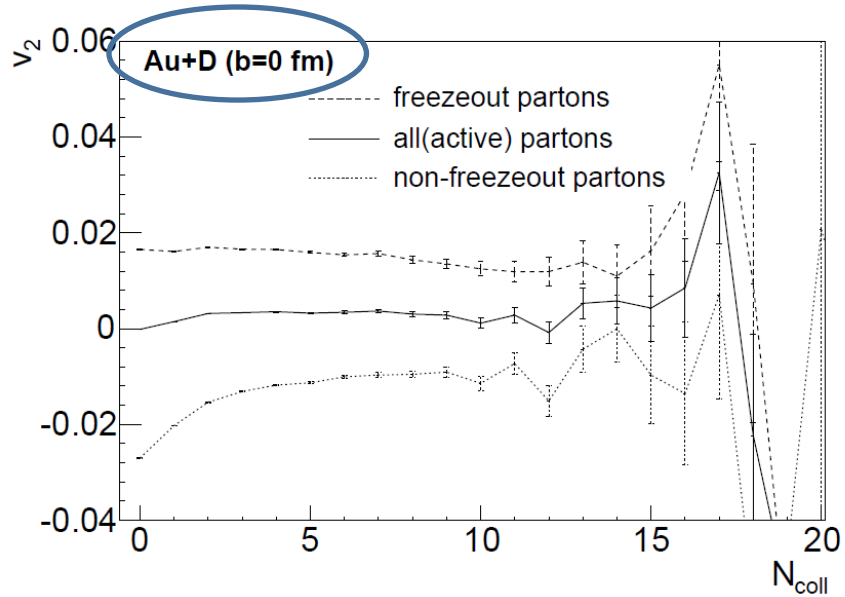
- Partons freeze out with large positive v_2, even when they do not interact at all.
- This is due to larger escape probability along x than y.
- Remaining partons start off with negative v_2, and become ~isotropic (v_2~0) after one more collision.
- Process repeats itself.
- Similar for v_3.
- Similar for d+Au collisions.

Similar for v_3, and d+Au

Similar for v_3



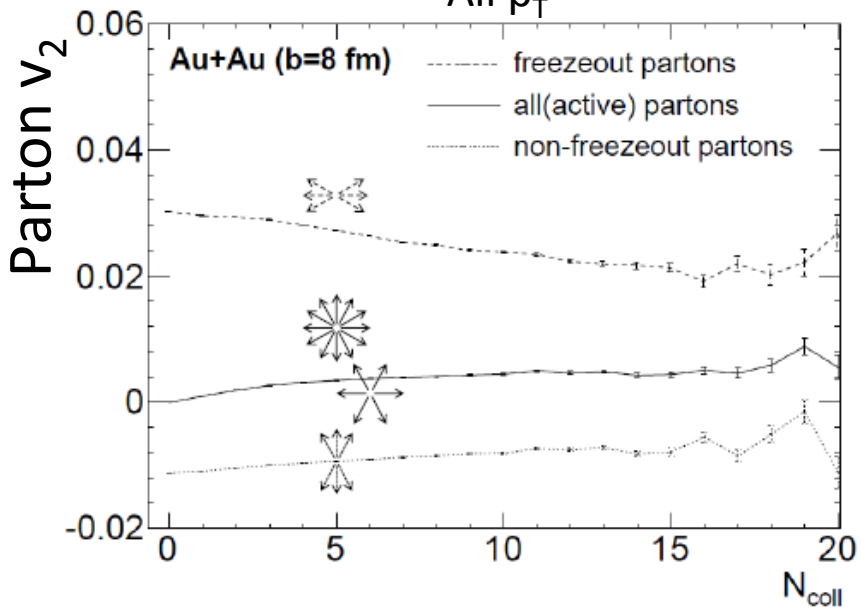
Similar for d+Au



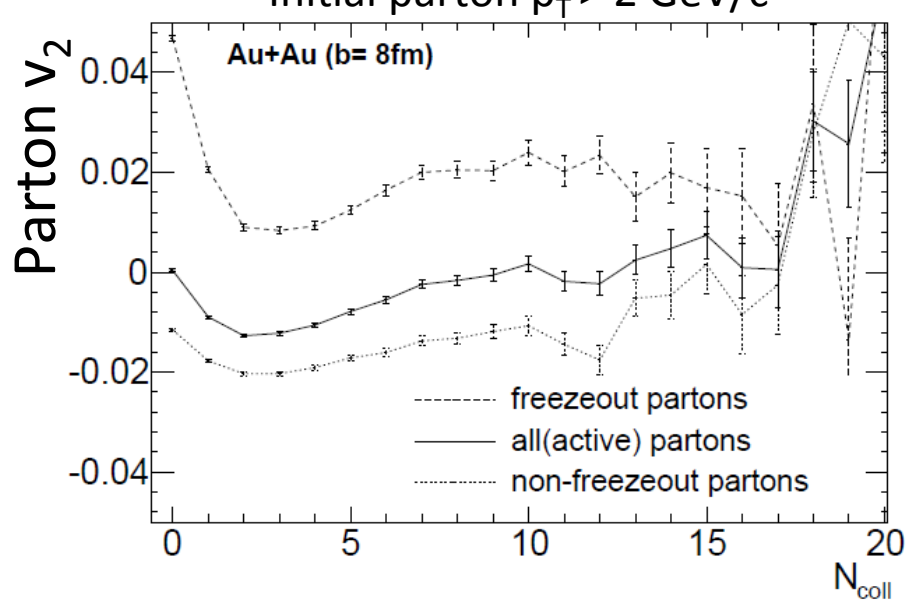
“Escape” mechanism

- Majority of v_n comes from the “escape” mechanism (“surface” emission).
- Considered to be responsible only for high-p_T v_n.
- Results suggest no fundamental difference between high-p_T and low-p_T.

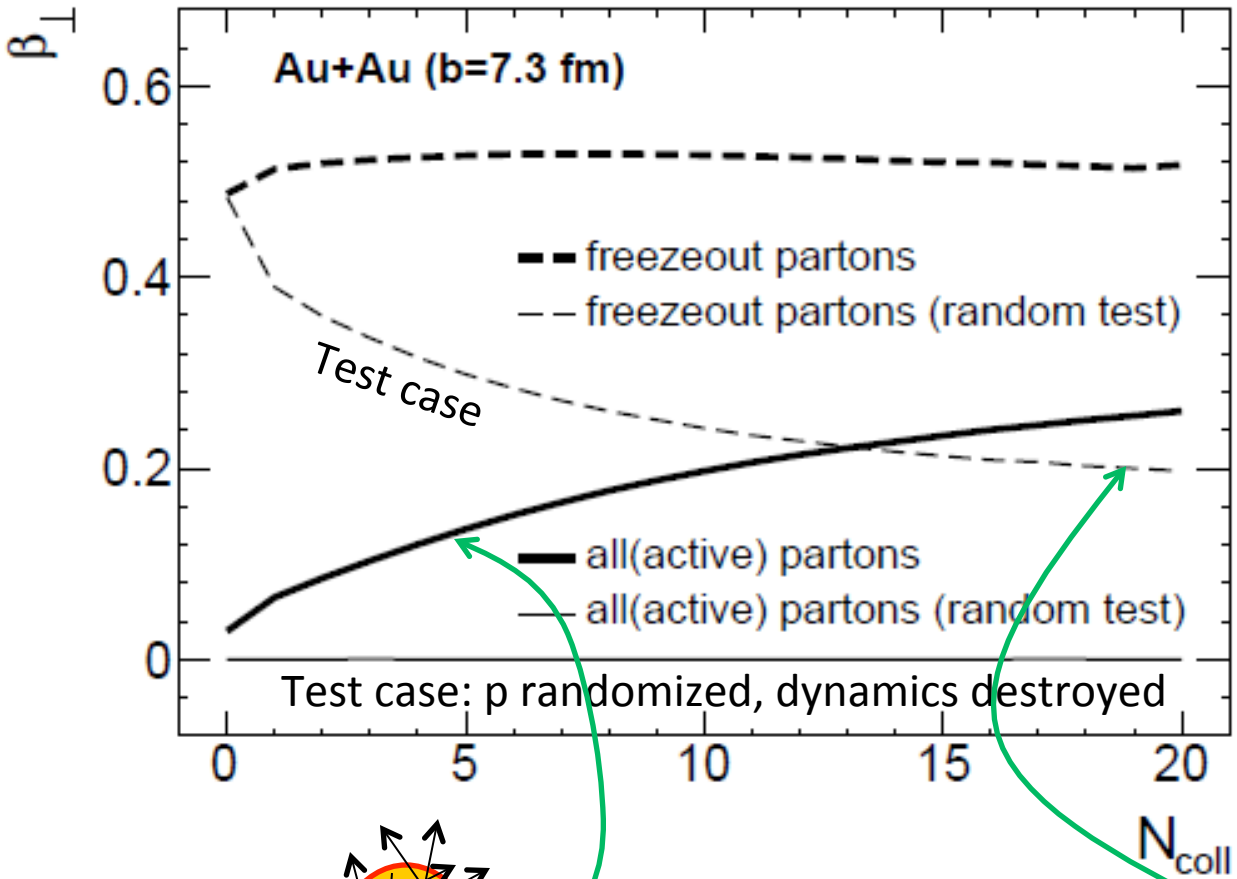
All p_T



Initial parton $p_T > 2$ GeV/c



Where are the dynamics?

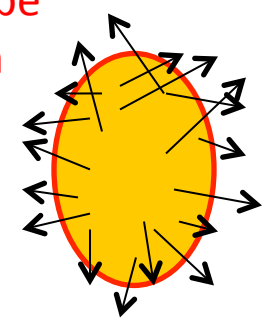


Strong r-p correlation

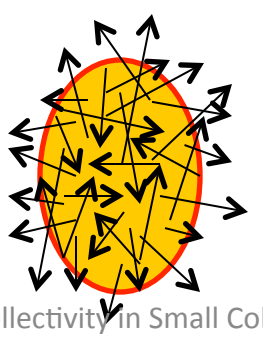
Increase purely from "escape" mechanism

Finite r-p correlation

r-p correlation purely from "escape" mechanism ("Surface" emission)



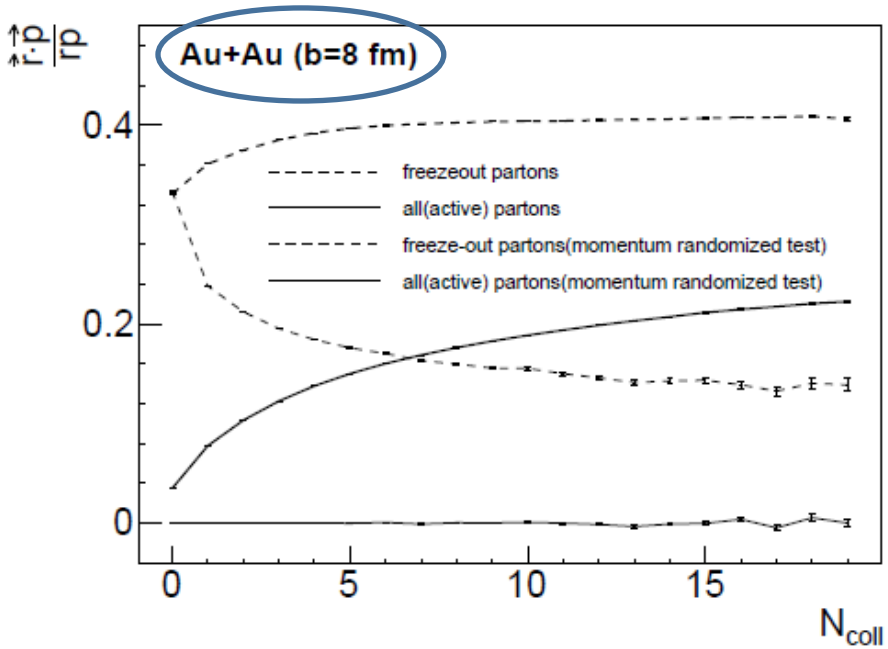
Collective motion, but not collectivity

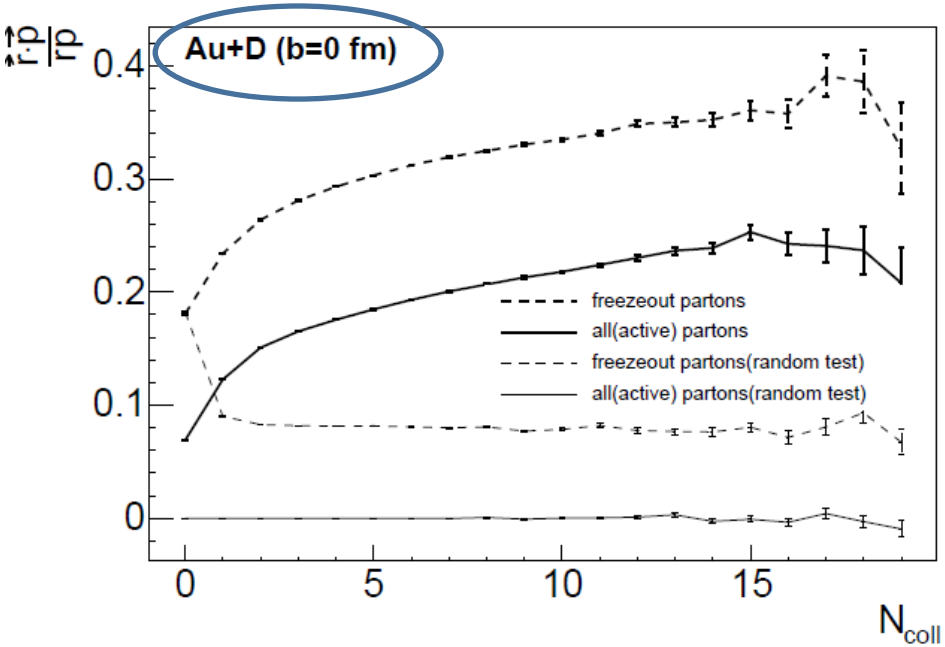


Collectivity

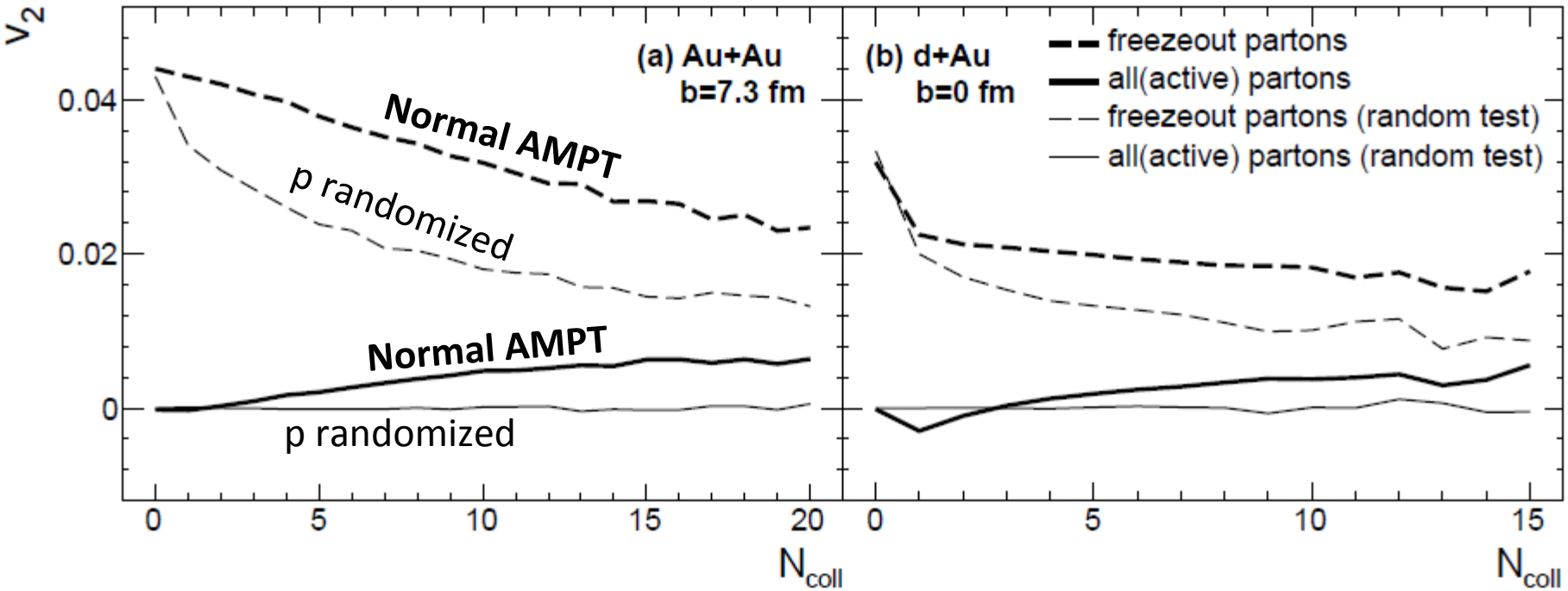
Collective motion ≠ Collectivity

Similar for d+Au



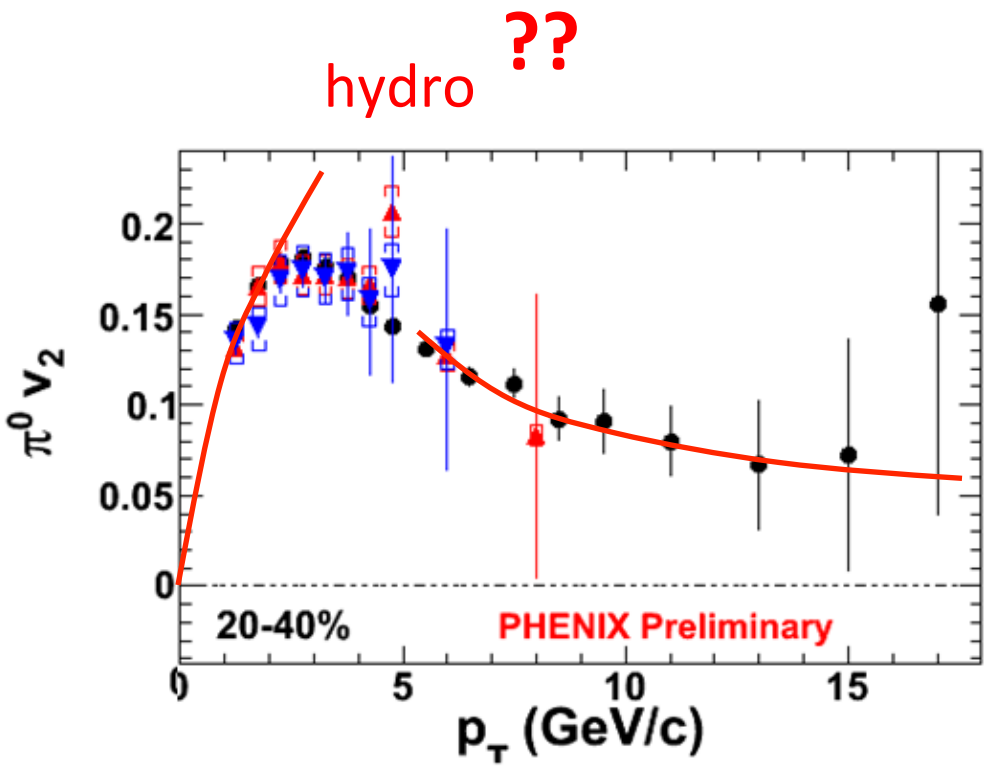


Majority anisotropy from escape



- Majority of anisotropy comes from the final-step "escape" mechanism.
- Escape yields a slightly larger v_2 in normal AMPT than in random case. The escape probability (parton sees) differs in these two cases.
- The partons start with small v_2 before escape (freezeout).
- This small v_2 is due to dynamics, result of hydrodynamic pressure push. It is this flow that is most relevant. However it plays a minor role.

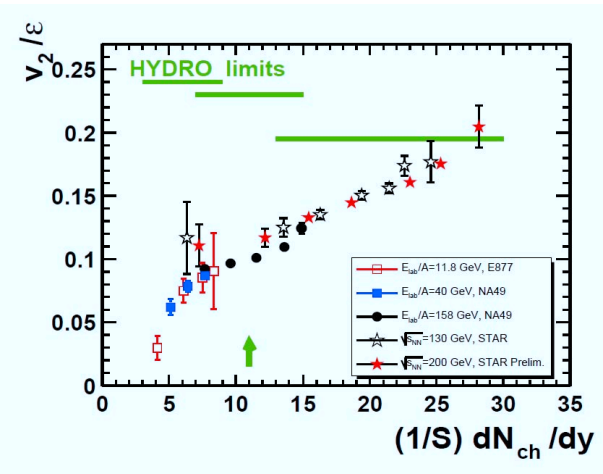
Our flow paradigm?

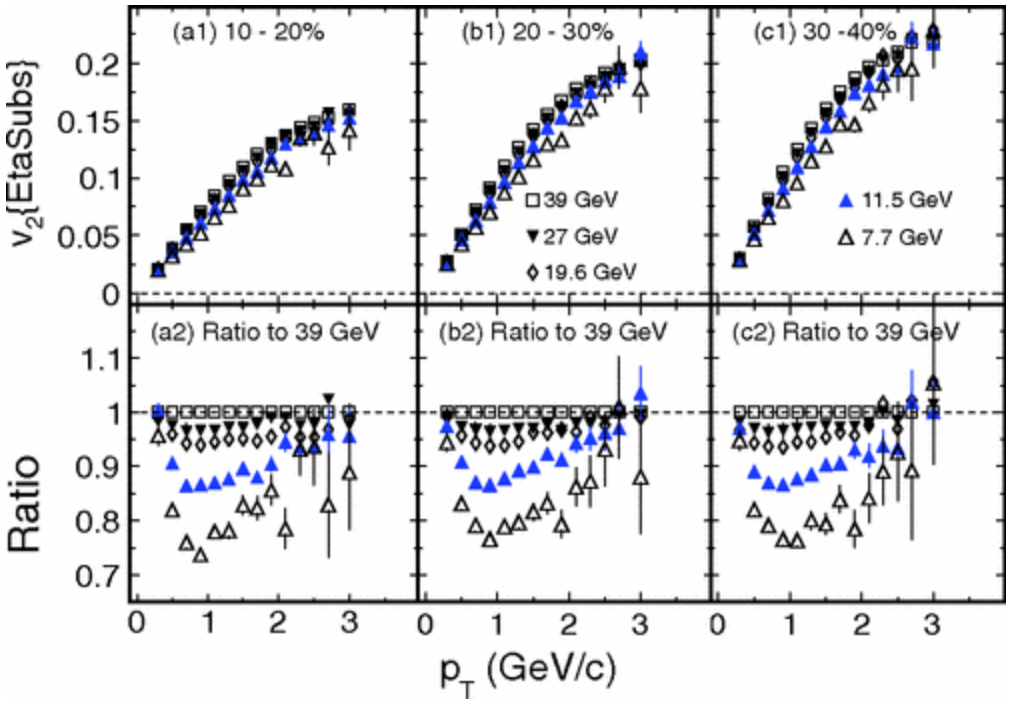


pathlength-dep.
energy loss

Hydrodynamics

- Hydrodynamics have pressure driven evolution only.
- Energy-momentum cell freeze-out controlled by local temperature/density.
- The escape mechanism is not obviously present in hydrodynamics.





Which is real?

- Escape anisotropy has all characteristics of "flow" so multi-particle correlations etc. It's just not hydro flow.

- Hydrodynamics describe data well.
  AMPT also describes data well.

- Which is more real? How to distinguish?
  - Pressure push generates radial flow
  - Escape mechanism does not generate radial flow

Many open questions

- Many interesting features in small system data. Can hydro describe them all?
- Is hydro simply a model with many parameters too?
- Do we not have to worry about quantum physics?
- Are we really right that the majority of the measured anisotropy is indeed hydro flow?
- Is the "nearly perfect liquid" actually far from perfect?
- …
- …
-

From pp to eA: what do we have? what do we expect to do?

Zhangbu Xu

- Other than correlations;
  How pp/dA is different from AA
- Statistics and radial flow
- Future Measurements for collectivity study
- Summary and Outlook

BROOKHAVEN NATIONAL LABORATORY
a passion for discovery
Office of Science
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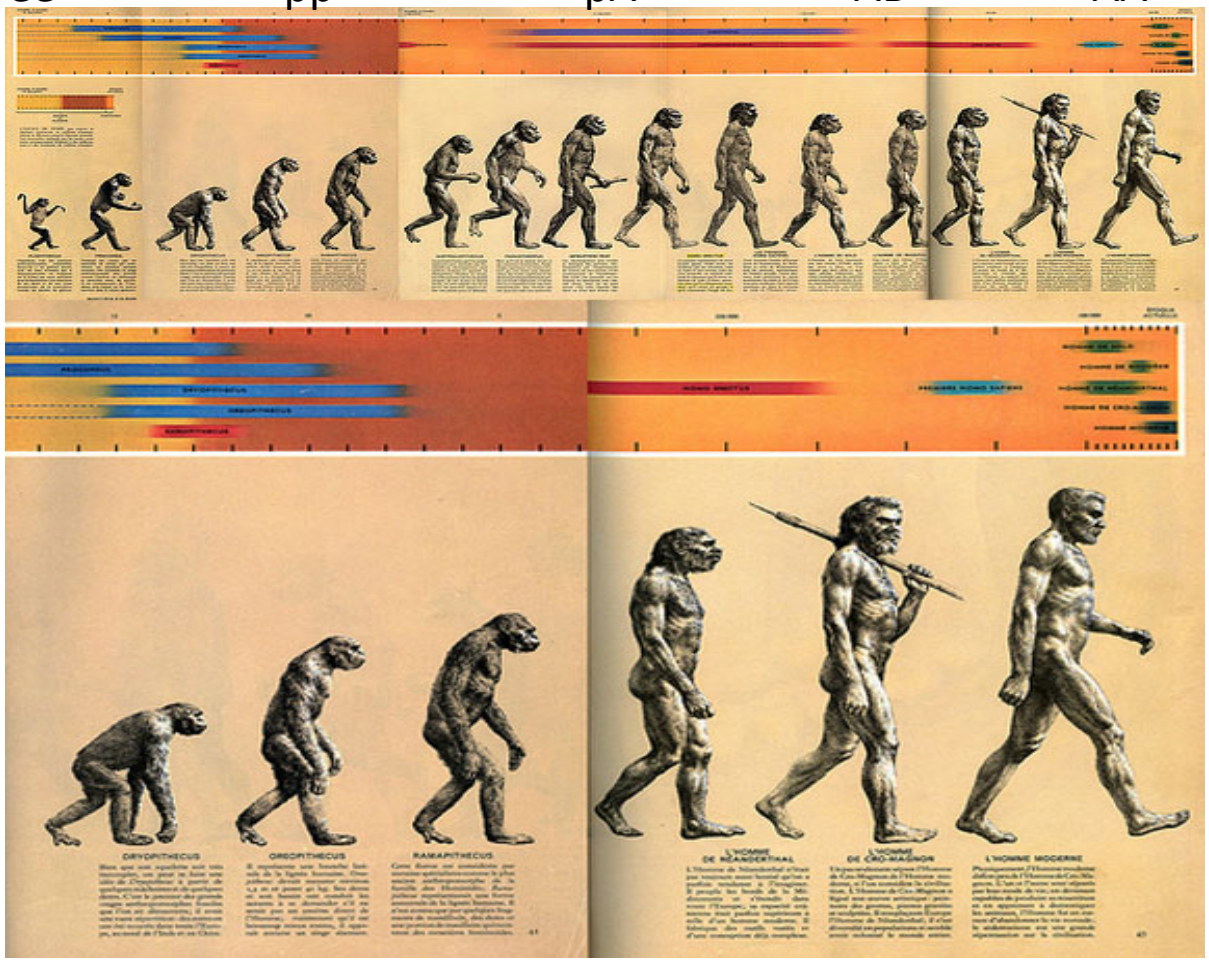
March of Progress in (QGP)

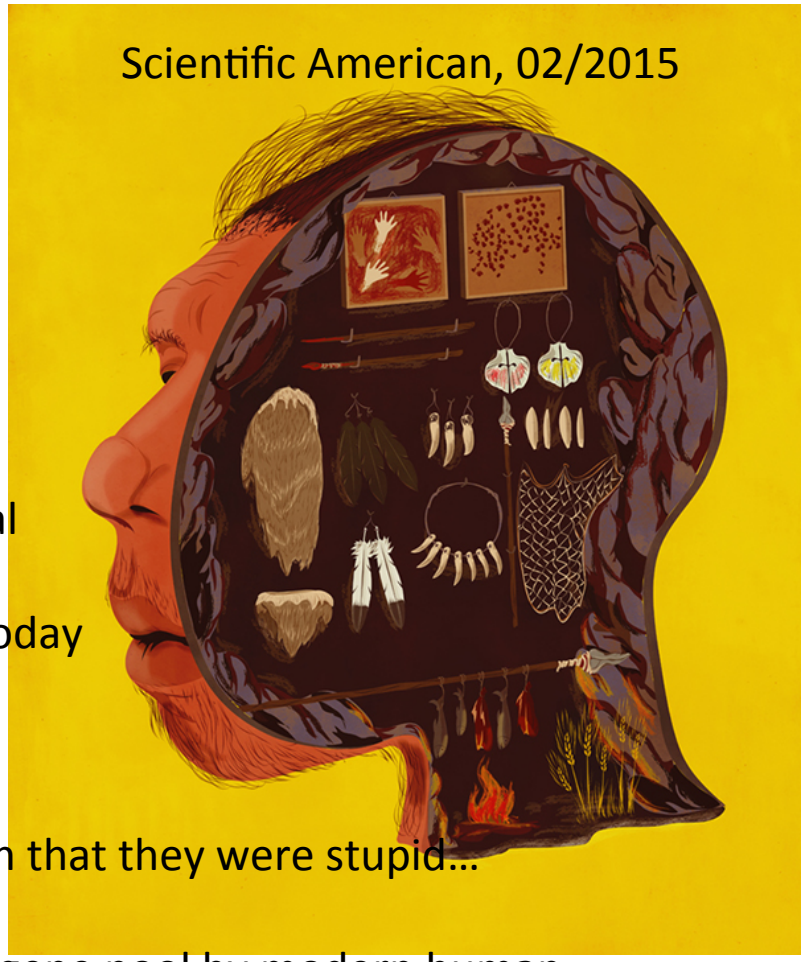
ee pp pA AB AA pA pp



Bjorken:
What takes you so long?

Neandertal vs Modern Human

Neandertals lack:
Language,
social skill,
technological ingenuity,
foraging savvy



Small systems lack:
Flow,
Thermalization,
Experimental signals,
Volume and time

Gene:
1.5-2% DNA from Neandertal
35—70% of Neandertal
genome persists in people today
Total difference 0.12%

Signatures:
V2 flow correlations
Hydrodynamic model
V2 mass ordering
High Multiplicity

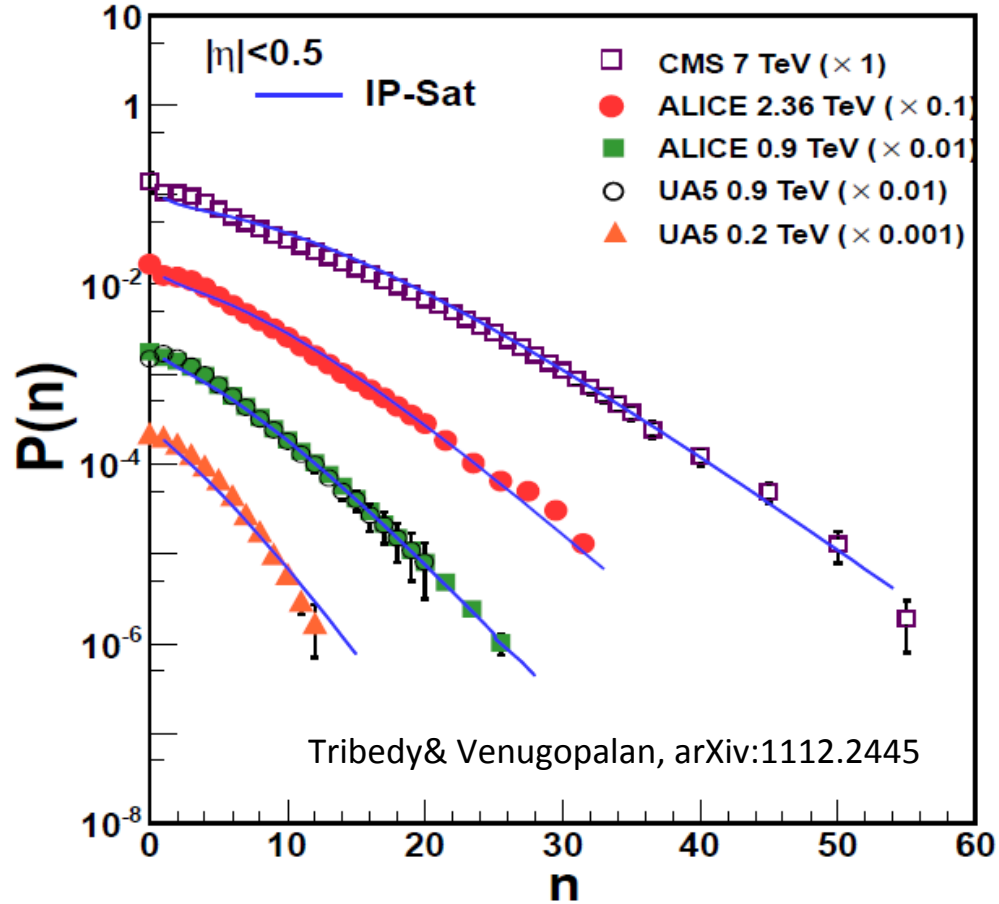
Reason of extinction:
"That's not necessarily a sign that they were stupid…
It's just what happens."
What happens is swamping gene pool by modern human.
It is just **statistics**.

Maybe the important
difference is just
statistics

What statistics should be used?

Nonextensive:
Fluctuation is larger with bigger system
Data clearly shows NO Boltzmann-Gibbs
Can be explained in theory
with multiple gluon interactions

an **ensemble** (also **statistical ensemble**)
...is a probability distribution for the state of the system.



where, in the Glasma flux tube framework [22], we can derive the negative binomial distribution

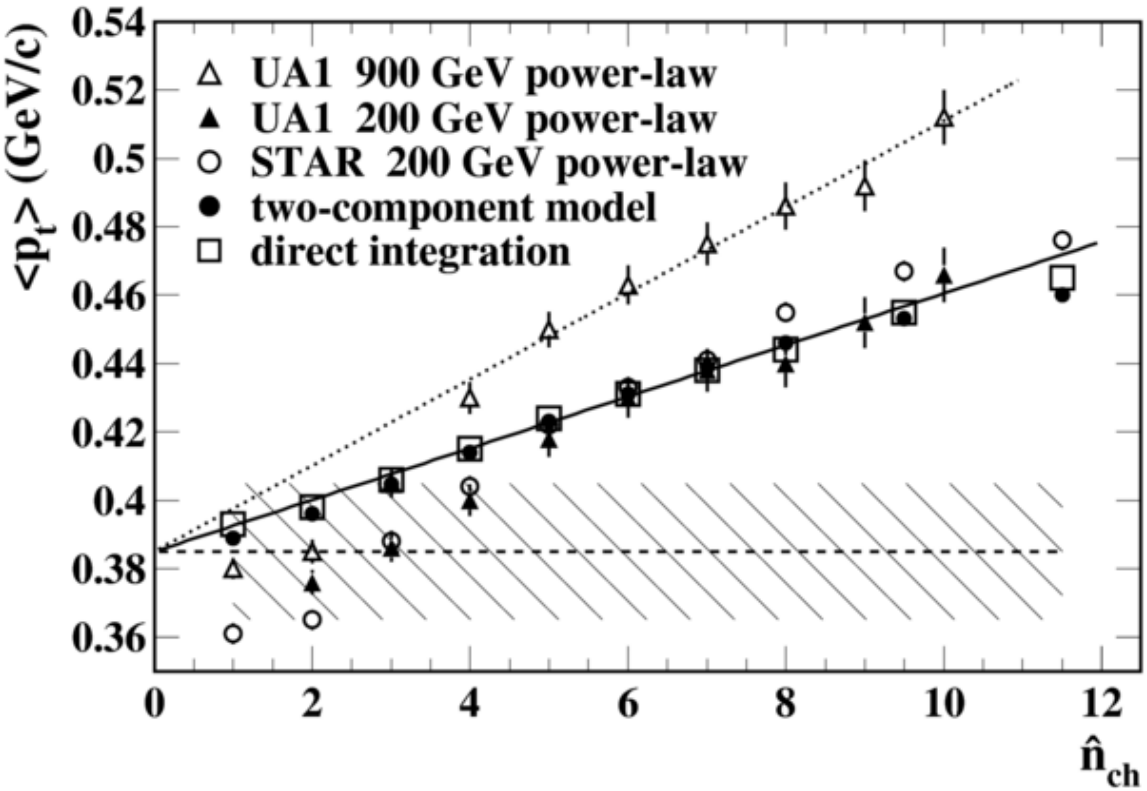
$$P_n^{\mathrm{NB}}(\bar{n},k) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}} .$$

with the parameter k defined specifically to be

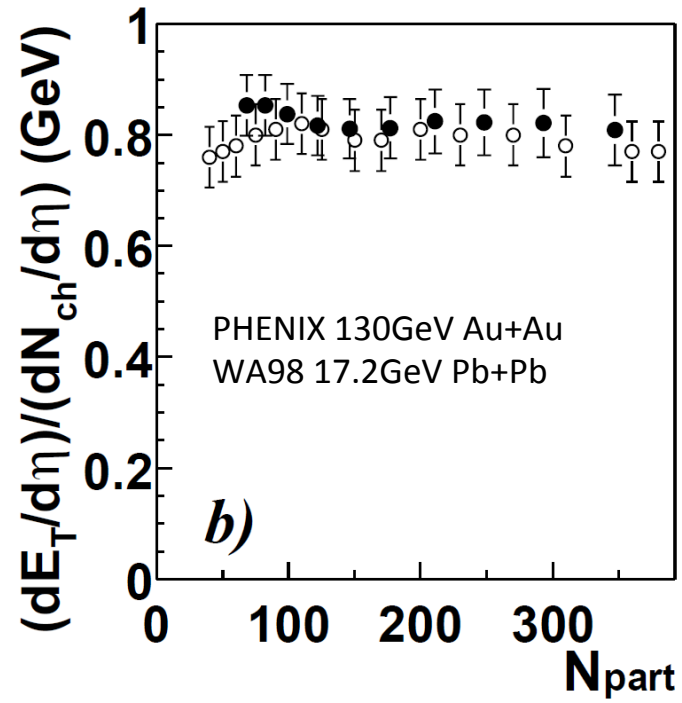
$$k(\mathbf{b}_\perp) = \zeta \frac{(N_c^2-1)Q_S^2 S_\perp}{2\pi} ,$$

Intensive parameters

STAR Collaboration, Phys. Rev. D **74** (2006) 32006

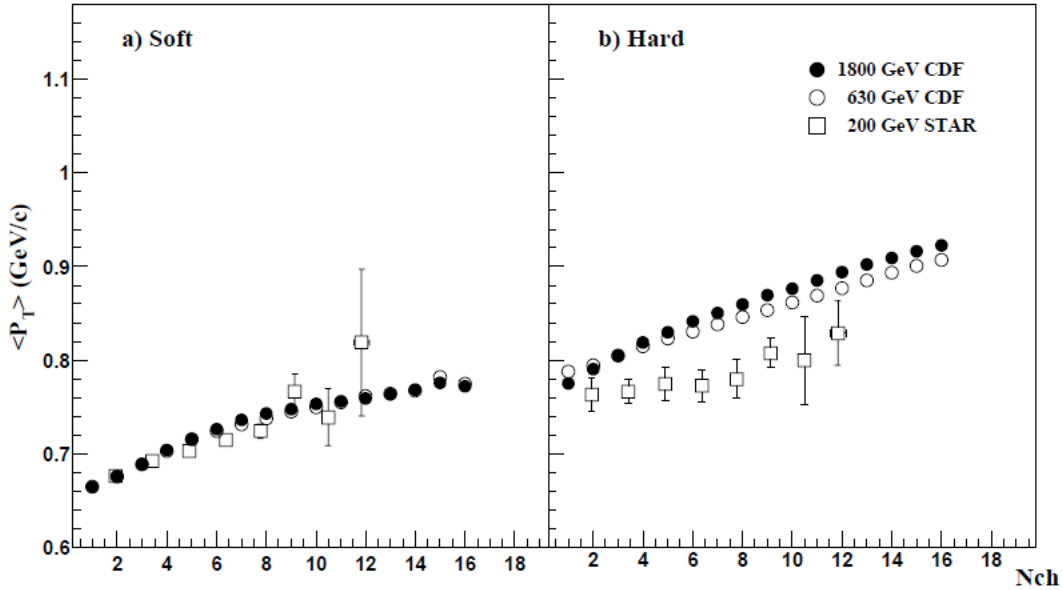


PHENIX, PRL (2001)



Intensive parameters (energy or momentum per particle) depends strongly on extensive parameters (multiplicity, volume).
On the contrary, $\langle E_T \rangle$ extremely flat vs energy and centrality in A+A

Classes of events within multiplicity



CDF Collaboration, PRD 65 (2002) 072005
Shengli Huang, PHD (2004) USTC

Fig5.7 The multiplicity (Nch) dependence of mean transverse momentum $<P_T>$ in "soft" and "hard" events. The errors bars are statistical errors only. The STAR

Soft event class slower multiplicity dependence
Hard event class strong multiplicity and energy dependence

Characteristics of Radial flow

Because of azimuthal symmetry we can integrate over ϕ making use of the modified Bessel function $I_0(z) = (2\pi)^{-1}\int_0^{2\pi} e^{z\cos\phi}d\phi$:

$$E\frac{d^3n}{d^3p} = \frac{g}{(2\pi)^2}\int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta\left[m_T\cosh y\frac{\partial z}{\partial\zeta} - m_T\sinh y\frac{\partial t}{\partial\zeta}\right] \times \int_0^R r\,dr\,\exp\left(-\frac{m_T\cosh\rho\cosh(y-\eta)-\mu}{T}\right)I_0\left(\frac{p_T\sinh\rho}{T}\right) \quad (14)$$

For the transverse mass spectrum we integrate with the help of another modified Bessel function $K_1(z) = \int_0^\infty \cosh y\, e^{-z\cosh y}dy$:

$$\frac{dn}{m_T dm_T} = \frac{g}{\pi}m_T\int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta\left[\cosh\eta\frac{\partial z}{\partial\zeta} - \sinh\eta\frac{\partial t}{\partial\zeta}\right]\int_0^R r\,dr\,K_1\left(\frac{m_T\cosh\rho}{T}\right)I_0\left(\frac{p_T\sinh\rho}{T}\right) = \frac{2g}{\pi}m_T Z_{\tilde{t}}\int_0^R r\,dr\,K_1\left(\frac{m_T\cosh\rho}{T}\right)I_0\left(\frac{p_T\sinh\rho}{T}\right) \quad (15)$$

E. Schnedermann, J. Sollfrank, U. Heinz, nucl-th/9307020, PRC48 (cited 364)

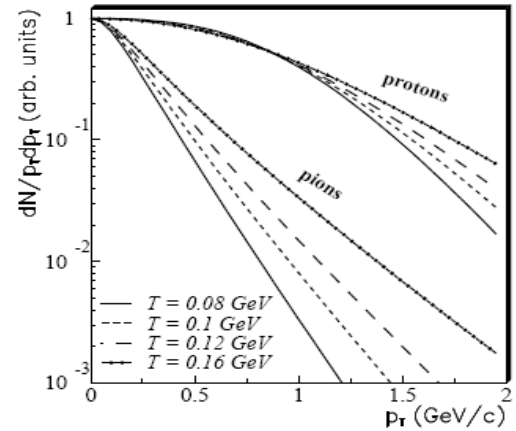


FIG. 4: Transverse momentum spectra for protons (upper curves) and pions (lower curves), as calculated by Equation 19, for several values of the temperature parameter T. Other parameters follow the "round" source defaults of Table I. All spectra are arbitrarily normalized to unity at $p_T = 0$.

F. Retiere, M. Lisa, PRC70

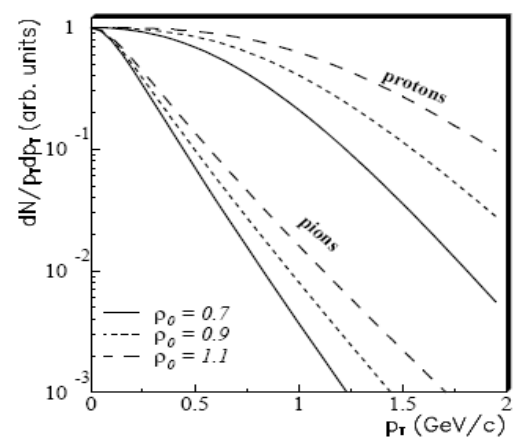


FIG. 5: Transverse momentum spectra for protons (upper curves) and pions (lower curves), as calculated by Equation 19, for several values of the radial flow parameter ρ_0. Other parameters follow the "round" source defaults of Table I. All spectra are arbitrarily normalized to unity at $p_T = 0$.

Assumptions:

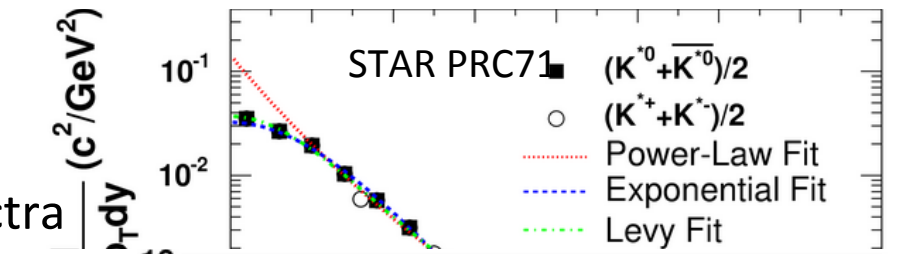
1) Local thermal equilibrium ➔ Boltzmann distribution
2) Longitudinal and transverse expansions (1+2)
3) Radial flow profile $\rho(r) \propto A\tanh(\beta_m(r/R)^n)$, (n=1)
4) Temperature and $\langle\beta\rangle$ are global quantities

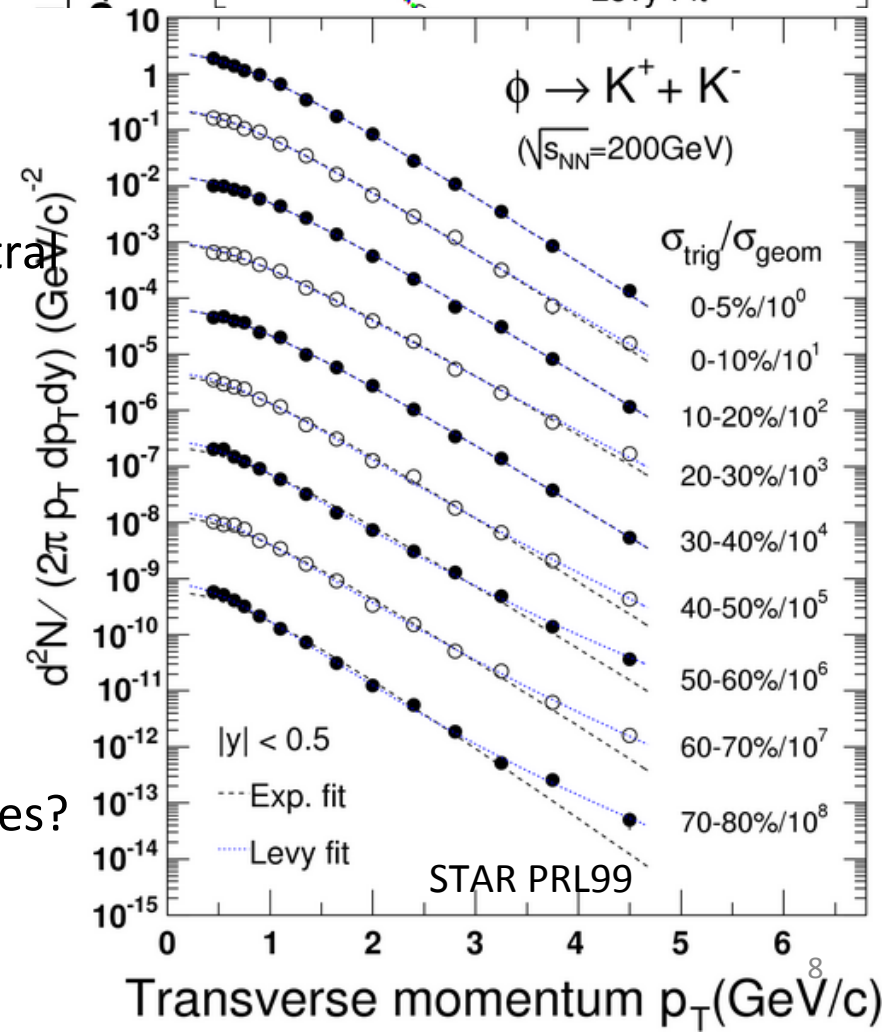
Radial flow: Convex Shape,
experimental spectra: Concave Shape

Limitations extended to small system

- Strong assumption on local thermal equilibrium
- Arbitrary choice of p_T range of the spectra (low and high cuts)
- Flow velocity $<\beta>=0.2$ in p+p
- Lack of non-extensive quantities to describe the evolution from p+p to central A+A collisions
- example in chemical fits: canonical to grand canonical ensemble
- m_T spectra in p+p collisions: Levy function or m_T power-law
- m_T spectra in A+A collisions: Boltzmann or m_T exponential
- What function can capture these features?

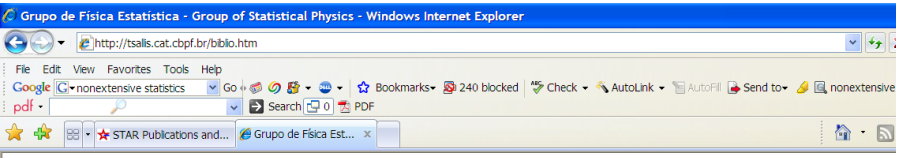
Radial flow: Convex Shape,
experimental spectra: Concave Shape





Tsallis Statistics

- **Nice web based notebooks: Tsallis Statistics, Statistical Mechai for Non-extensive Systems and Long-Range Interactions**
  http://www.cscs.umich.edu/~crshalizi/notabene/tsallis.html
- http://tsallis.cat.cbpf.br/biblio.htm



Nonextensive Statistical Mechanics and Thermodynam

E-Mail: tsallis@cbpf.br

BIBLIOGRAPHY

FURTHER INFORMATION

AVAILABLE BOOKS

TABLE OF FOUNDATIONS

SET OF MINI-REVIEWS (Special issue of Europhysics News, European Physical S
- Europhysics News 36 (6) (Nov-Dec 2005)
- Erratum: Europhysics News 37 (1) (Jan-Feb 2006), page 25

MEXICO PRIZE FOR SCIENCE AND TECHNOLOGY
- Speeches and photos

INSTITUTO NACIONAL DE SISTEMAS COMPLEXOS

Number of visits since 14 March 1997: 40527

Last updating: 18 Sep 2008

Group of Statistical Physics (CBPF). CBPF

ENTROPY AND STATISTICAL MECHANICS: FOUNDATIONS

	BG (thermal equilibrium)	q ≠ 1 (thermal metaequilibrium, nonequilibrium)
Distribution of velocities at equilibrium	Maxwell 1860	R. Silva, A.R. Plastino, J.A.S. Lima Phys Lett A 249, 401 (1998) R.S. Mendes and C. Tsallis Phys Lett A 285, 273 (2001)
Kinetic equation Molecular chaos hypothesis	Boltzmann 1872	J.A.S. Lima, R. Silva, A.R. Plastino Phys Rev Lett 86, 2938 (2001)
	Khinchin 1949	cond-mat/0606038 and /0606040 S. Abe and A.K. Rajagopal Europhys Lett 52, 610 (2000)

Let us thus concentrate on the other class, systems with fluctuations. Consider a system of ordinary statistical mechanics with Hamiltonian H. Tsallis statistics with $q > 1$ can arise from this ordinary Hamiltonian if one assumes that the temperature β^{-1} is locally fluctuating. From the integral representation of the gamma-function one can easily derive the formula [7,15]

$$\left(1+(q-1)\beta_0 H\right)^{-\frac{1}{q-1}} = \int_0^\infty e^{-\beta H} f(\beta)\,\mathrm{d}\beta, \tag{3}$$

where

$$f(\beta) = \frac{1}{\Gamma\left(\frac{1}{q-1}\right)} \left\{\frac{1}{(q-1)\beta_0}\right\}^{\frac{1}{q-1}} \beta^{\frac{1}{q-1}-1} \exp\left[-\frac{\beta}{(q-1)\beta_0}\right] \tag{4}$$

is the probability density of the χ^2 distribution. The above formula is valid for arbitrary Hamiltonians H and thus of great significance. The left-hand side of eq. (3) is just the generalized Boltzmann factor emerging out of nonextensive statistical mechanics. It can directly be obtained by extremizing S_q. The right-hand side is a weighted average over Boltzmann factors of ordinary statistical mechanics. In other words, if we consider a nonequilibrim system (formally described by a fluctuating β), then the generalized distribution functions of nonextensive statistical mechanics are a consequence of integrating over all possible fluctuating inverse temperatures β, provided β is χ^2 distributed.

Negative Binomial Distribution: κ=1/(q-1

Temperature fluctuation: $\frac{\langle 1/T^2\rangle - \langle 1/T\rangle^2}{\langle 1/T\rangle^2} = 1-q$

G. Wilk: arXiv: 0810.2939; C. Beck, EPL57(2002)3

It is all about the q-statistics

BASIC QUANTITIES
q-exponential : $\exp_q(x) \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \longrightarrow_{q\to 1} e^x$
q-logarithm : $\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q} \longrightarrow_{q\to 1} \ln x$
Boltzmann-Gibbs entropy : $S_{BG} \equiv -k\sum_{i=1}^{W} p_i \ln p_i$
q-entropy : $S_q \equiv k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = k\sum_{i=1}^{W} p_i \ln_q(1/p_i) = -k\sum_{i=1}^{W} p_i^q \ln_q p_i \longrightarrow_{q\to 1} S_{BG}$
Escort distribution : $P_i \equiv p_i^q / \sum_{j=1}^{W} p_j^q$
Ensemble q-average : $\langle A\rangle_q \equiv \sum_{i=1}^{W} A_i P_i = \sum_{i=1}^{W} A_i\, p_i^q / \sum_{j=1}^{W} p_j^q$

◄ **Box:** The two basic functions that appear in Nonextensive Statistical Mechanics are the q-exponential and the q- logarithm with $\ln_q(\exp_q x) = \exp_q(\ln_q x) = x$. They are simple generalizations of the usual exponential and logarithmic functions which are retrieved by performing a $|1 - q| << 1$ expansion. Similarly the q-entropy generalizes the standard Boltzmann-Gibbs entropy. The escort distribution is a generalization of the usual ensemble averaging function to which it reduces for $q = 1$.

europhysics news NOVEMBER/DECEMBER 2005 **185**

- Why is this relevant to us (Heavy-ion physics)?
  - We have dealt with Boltzmann distribution
    But the spectra are clearly non-Boltzmann
  - It is easy to make a change
  - It is easy to compare
  - Change m_T exponential to m_T power law

$$(1+\frac{q-1}{T}m_T)^{-1/(q-1)}$$

Tsallis statistics in Blast Wave model

Because of azimuthal symmetry we can integrate over ϕ making use of the modified Bessel function $I_0(z) = (2\pi)^{-1} \int_0^{2\pi} e^{z\cos\phi} d\phi$:

$$E\frac{d^3n}{d^3p} = \frac{g}{(2\pi)^2}\int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta \left[m_T \cosh y \frac{\partial z}{\partial \zeta} - m_T \sinh y \frac{\partial t}{\partial \zeta}\right] \times \int_0^R r dr \exp\left(-\frac{m_T \cosh\rho \cosh(y-\eta) - \mu}{T}\right) I_0\left(\frac{p_T \sinh\rho}{T}\right) \quad (14)$$

For the transverse mass spectrum we integrate with the help of another modified Bessel function $K_1(z) = \int_0^\infty \cosh y\, e^{-z\cosh y} dy$:

$$\frac{dn}{m_T dm_T} = \frac{g}{\pi} m_T \int_{-\mathcal{Z}}^{\mathcal{Z}} d\zeta \left[\cosh\eta \frac{\partial z}{\partial \zeta} - \sinh\eta \frac{\partial t}{\partial \zeta}\right] \int_0^R r dr\, K_1\left(\frac{m_T \cosh\rho}{T}\right) I_0\left(\frac{p_T \sinh\rho}{T}\right)$$
$$= \frac{2g}{\pi} m_T Z_{\tilde{t}} \int_0^R r dr\, K_1\left(\frac{m_T \cosh\rho}{T}\right) I_0\left(\frac{p_T \sinh\rho}{T}\right) \quad (15)$$

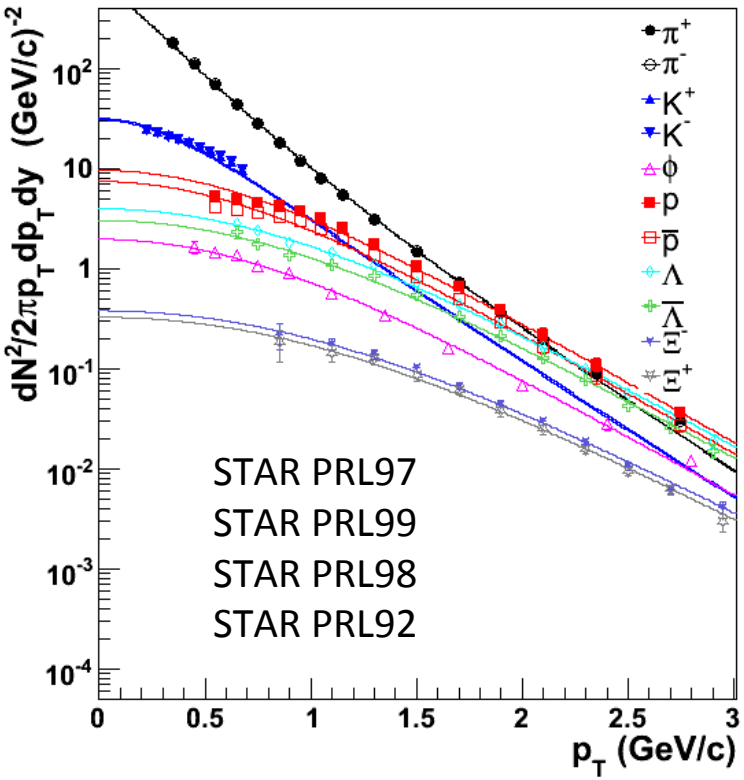
With Tsallis distribution, the BlastWave equation is:

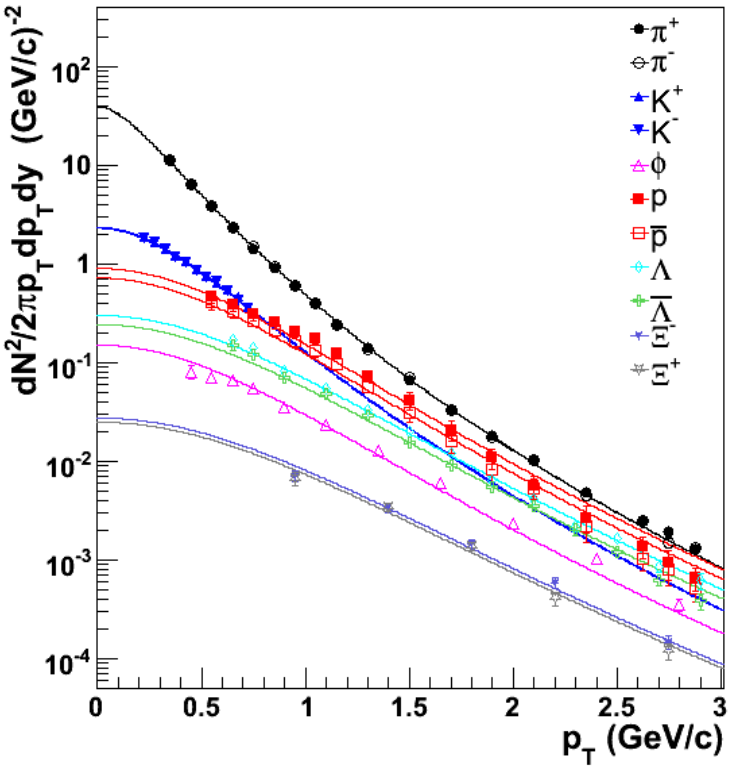
$$\frac{dN}{m_T dm_T} \propto m_T \int_{-Y}^{+Y} \cosh(y) dy \int_{-\pi}^{+\pi} d\phi \int_0^R r dr \left(1 + \frac{q-1}{T}(m_T \cosh(y)\cosh(\rho) - p_T \sinh(\rho)\cos(\phi))\right)^{-1/(q-1)}$$

Where $\rho = \mathrm{Atanh}(\beta_m (r/R)^n)$, n=1 ; any of the three integrals is HypergeometryF1

β: flow velocity

Fit results in Au+Au collisions





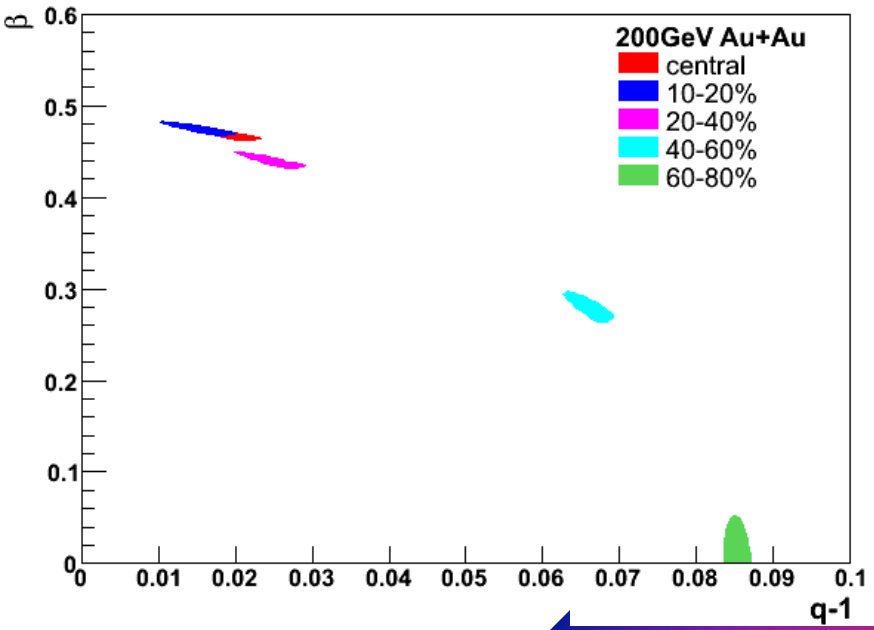
Phys.Rev. C79 (2009) 051901

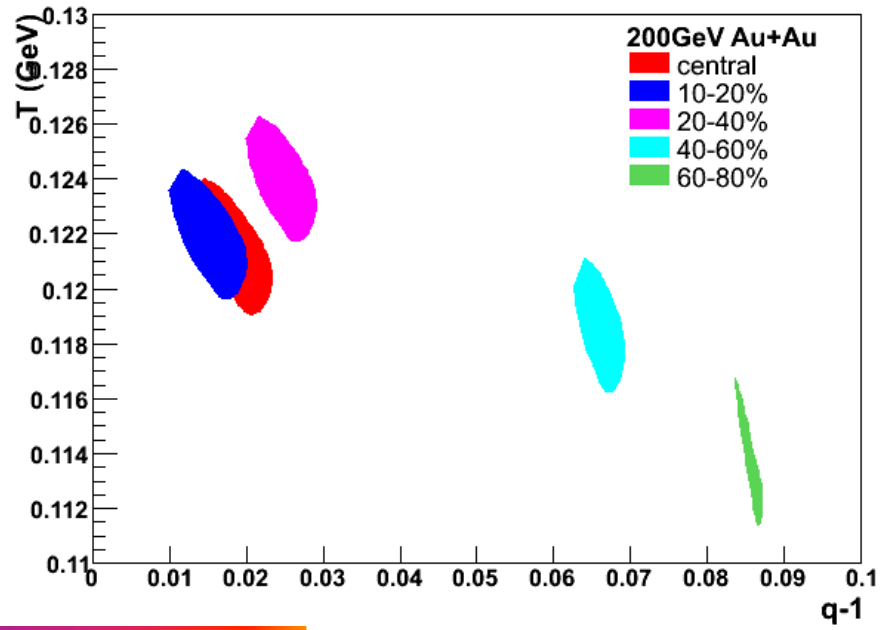
Au+Au 0—10%:
<β> = 0.470+- 0.009
T = 0.122 +- 0.002
q = 1.018 +- 0.005
chi^2/nDof = 130 / 125

Au+Au 60—80%:
<β>=0
T = 0.114 +- 0.003
q = 1.086 +- 0.002
chi^2/nDof = 138/ 123

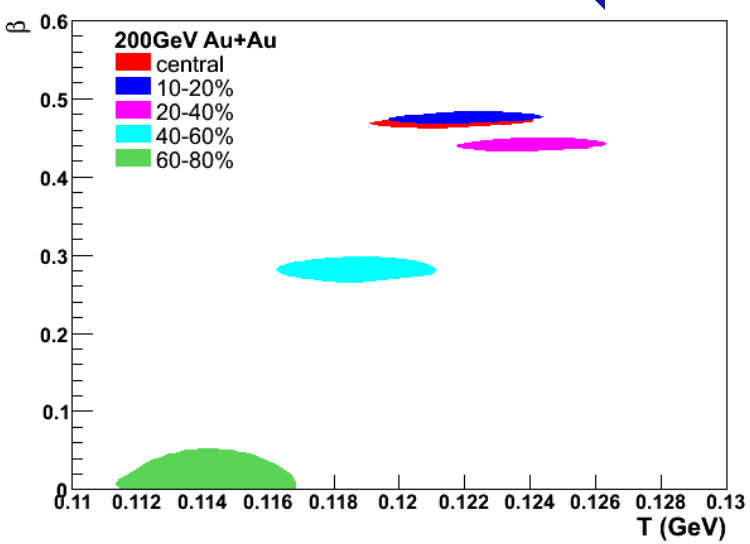
All baryons and mesons in one consistent fit

Dissipative energy into flow and heat





More thermalized

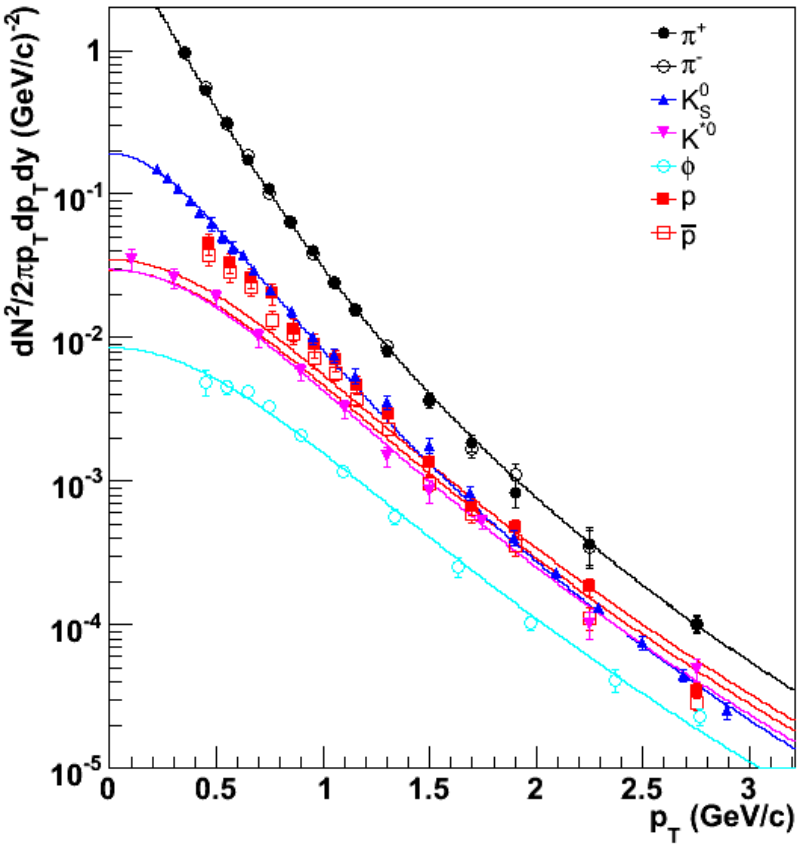


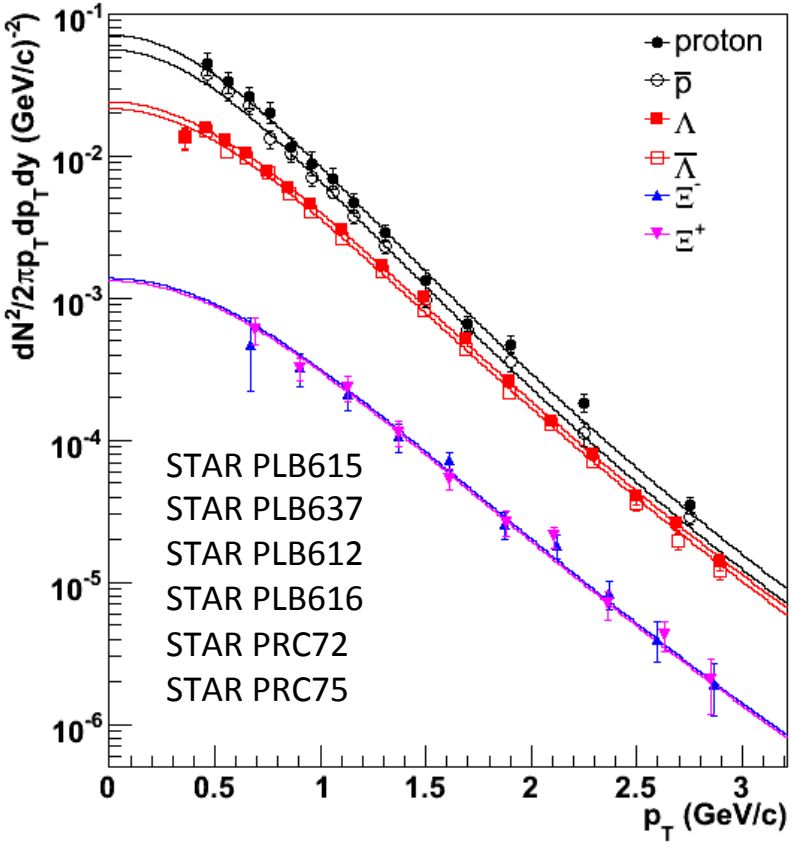
1. Decrease of q➔1, closer to Boltzmann
2. Increase of radial flow (0➔0.5)
3. Increase of temperature
4. T, $\beta \propto (q-1)^2$, NOT linear (q-1)

Phys.Rev. C79 (2009) 051901

Results in p+p collisioins

Phys.Rev. C79 (2009) 051901



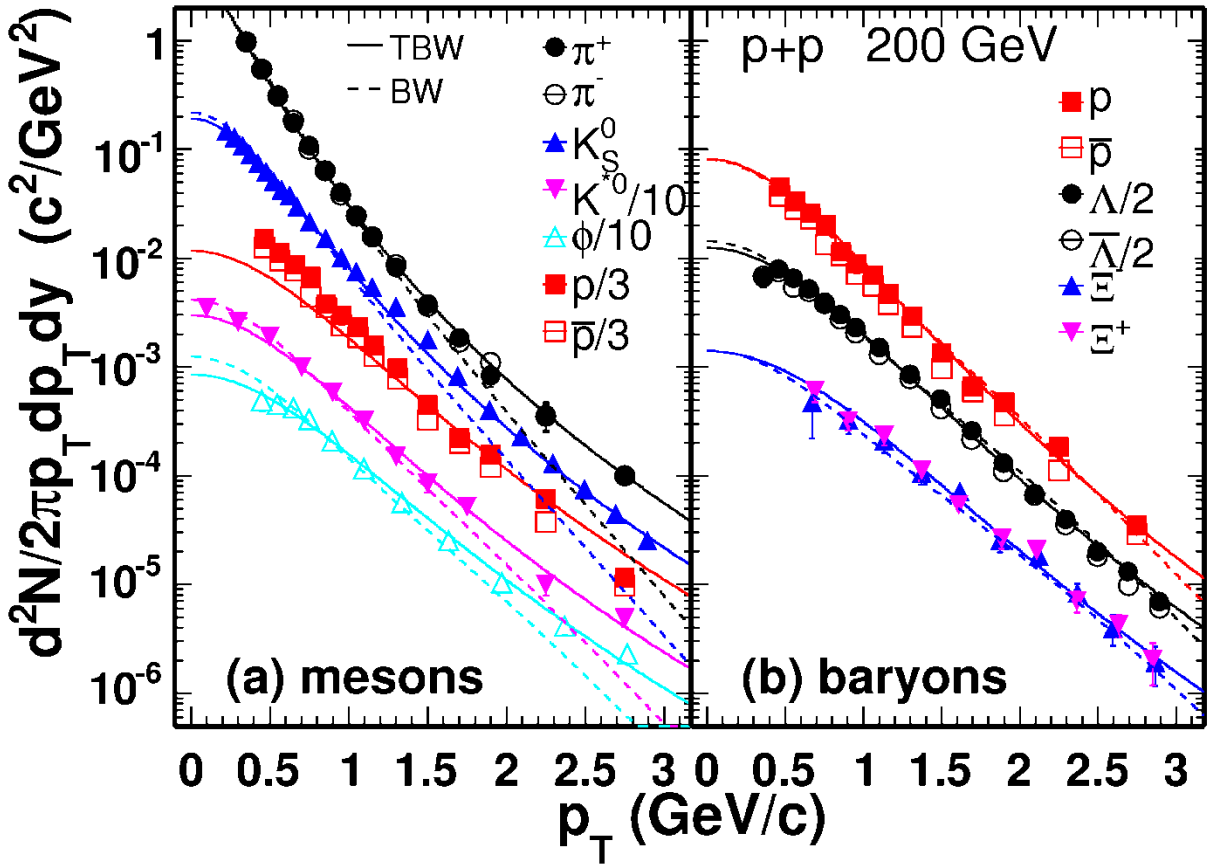


<β> = 0
T = 0.0889+- 0.004
q = 1.100 +- 0.003
chi^2/nDof = 53 / 66

<β> = 0
T = 0.097+- 0.010
q = 1.073 +- 0.005
chi^2/nDof = 55 / 73

baryons and mesons are incompatible in one fit

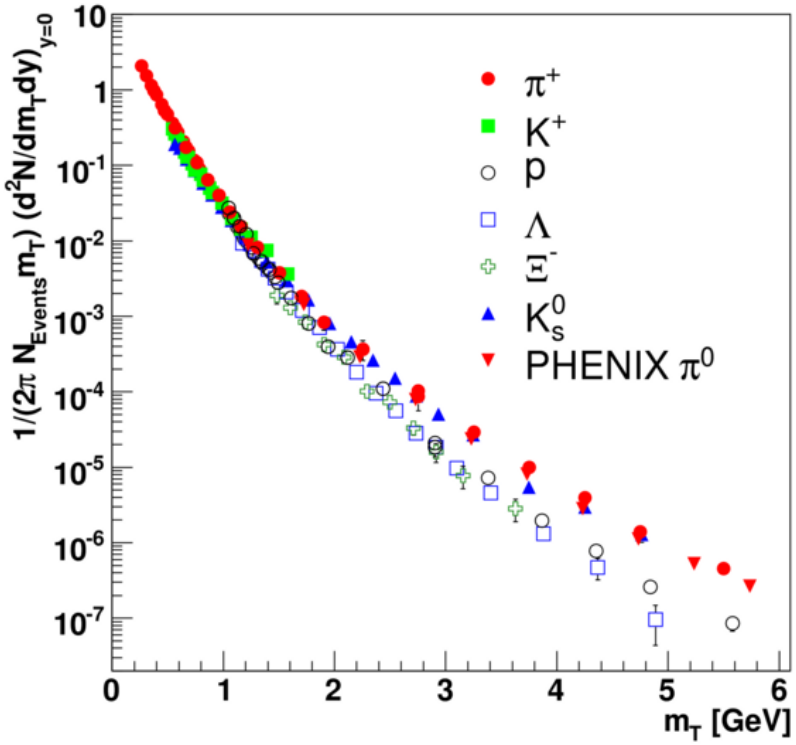
How is result different from BGBW?

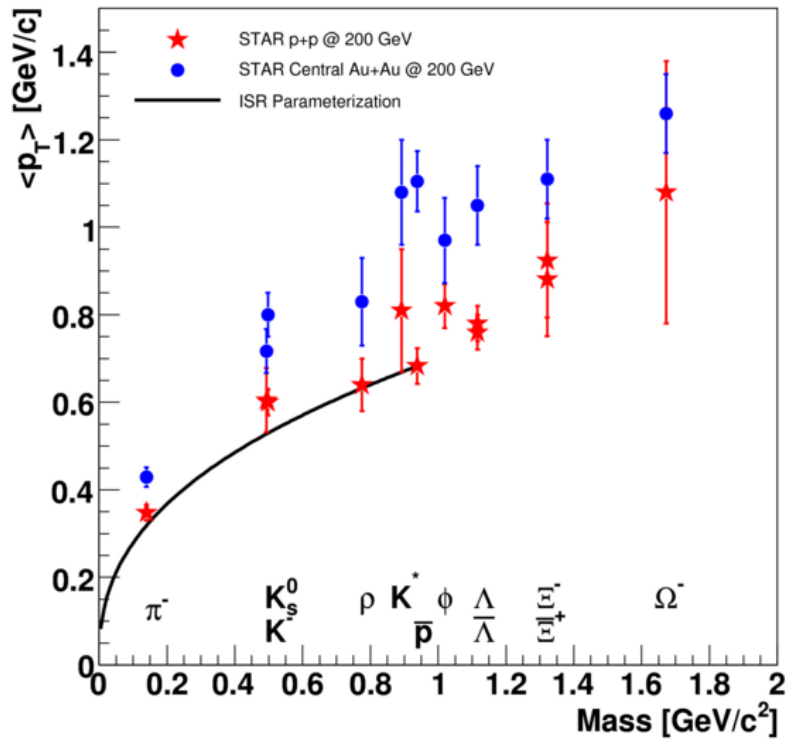


BGBW: underpredicts higher pt yields for all mesons in p+p
Baryons and mesons are created differently in p+p:
baryons from gluons and popcorn model?

Baryon and meson are different classes

Radial flow: Convex Shape,
experimental spectra: Concave Shape

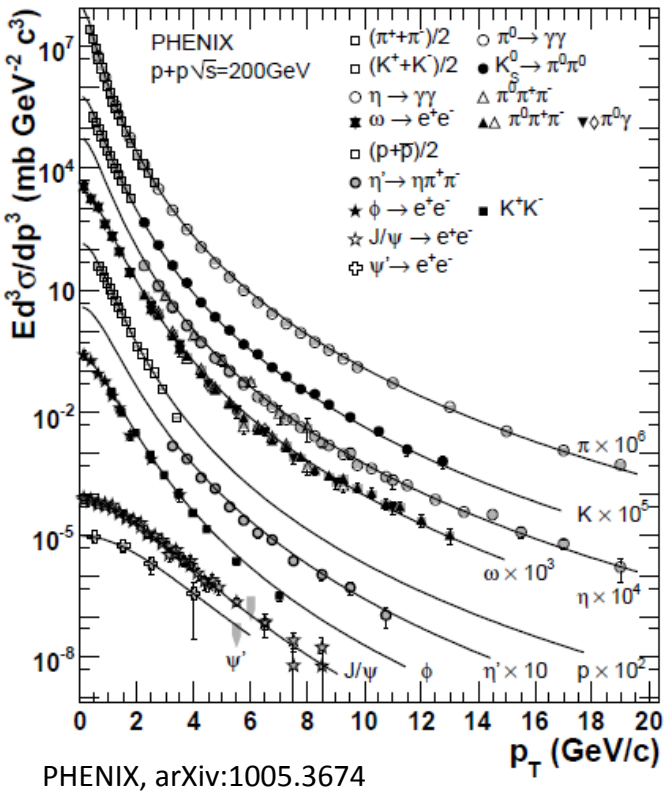




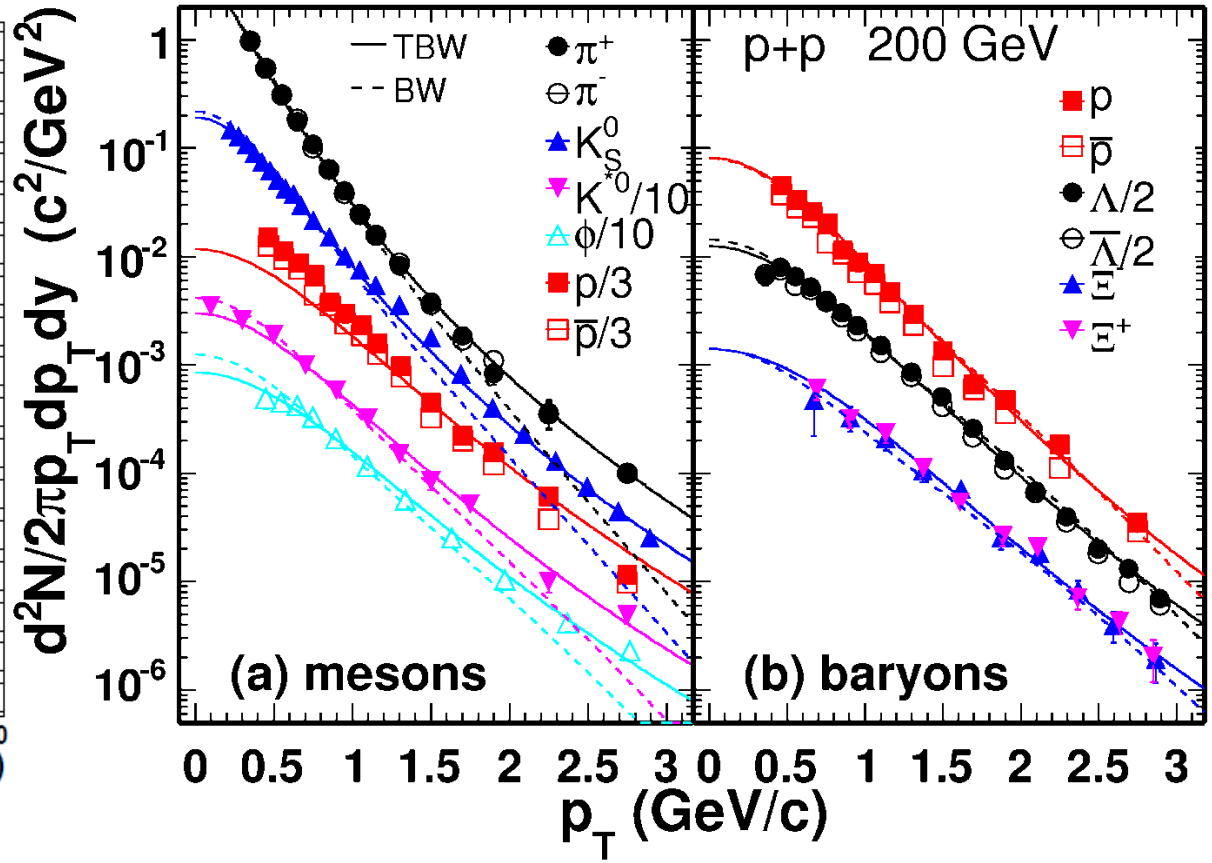
STAR PRC75

In p+p collisions, the m_T spectra of baryons and mesons are in two groups
Maybe we should not call p+p system as a whole global system
However, equilibrated toward more central Au+Au collisions

Extend to high p_T



PHENIX, arXiv:1005.3674



$$(1+\frac{q-1}{T}m_T)^{-1/(q-1)}$$

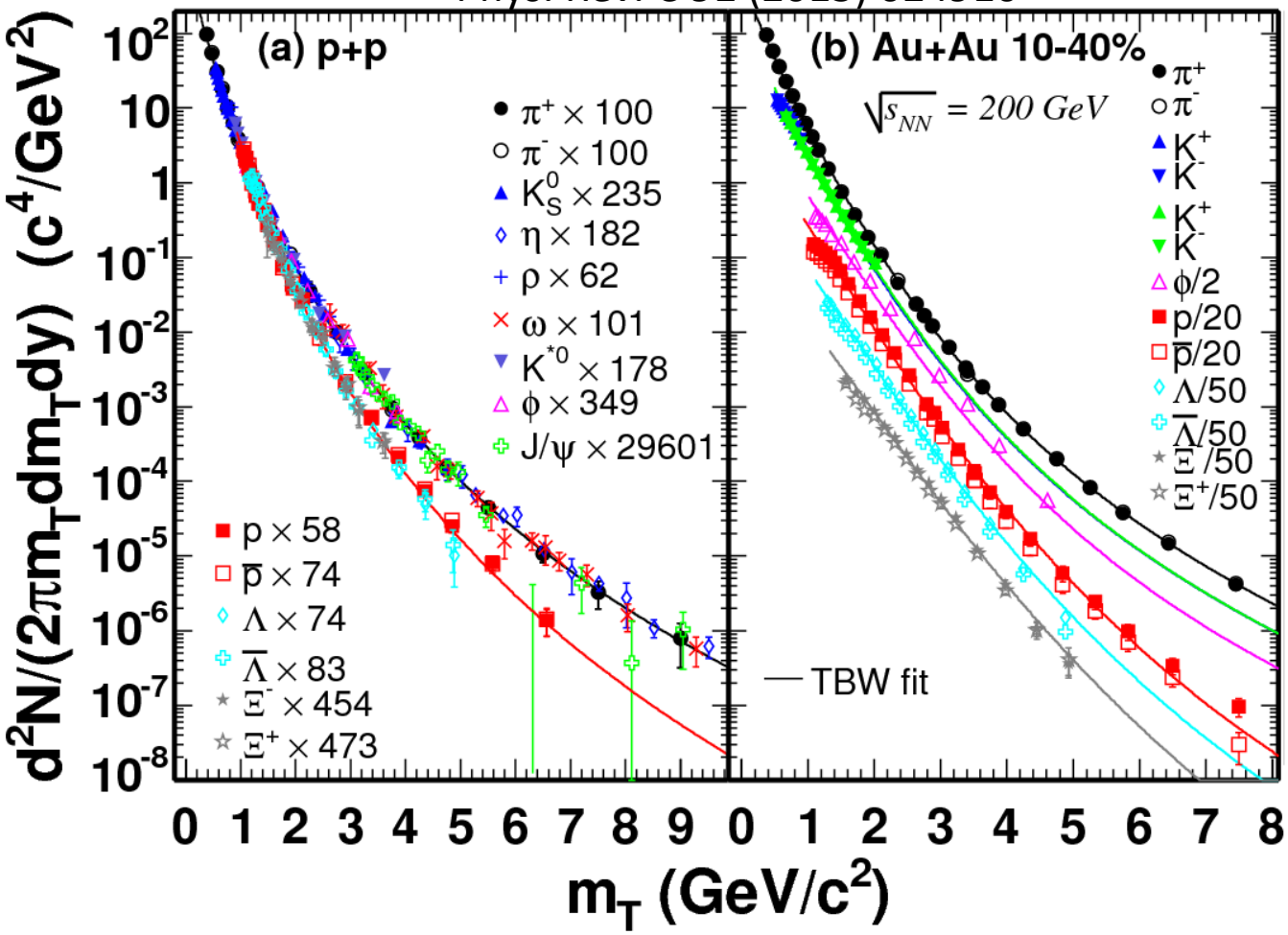
$$\xrightarrow{q\to 1} e^{-m_T/T}$$

q-1=0.10+-(~0.01)
Velocity β =0

Blast-Wave with Tsallis Statistics
Z. Tang et al., PRC 79 (2009) 051901

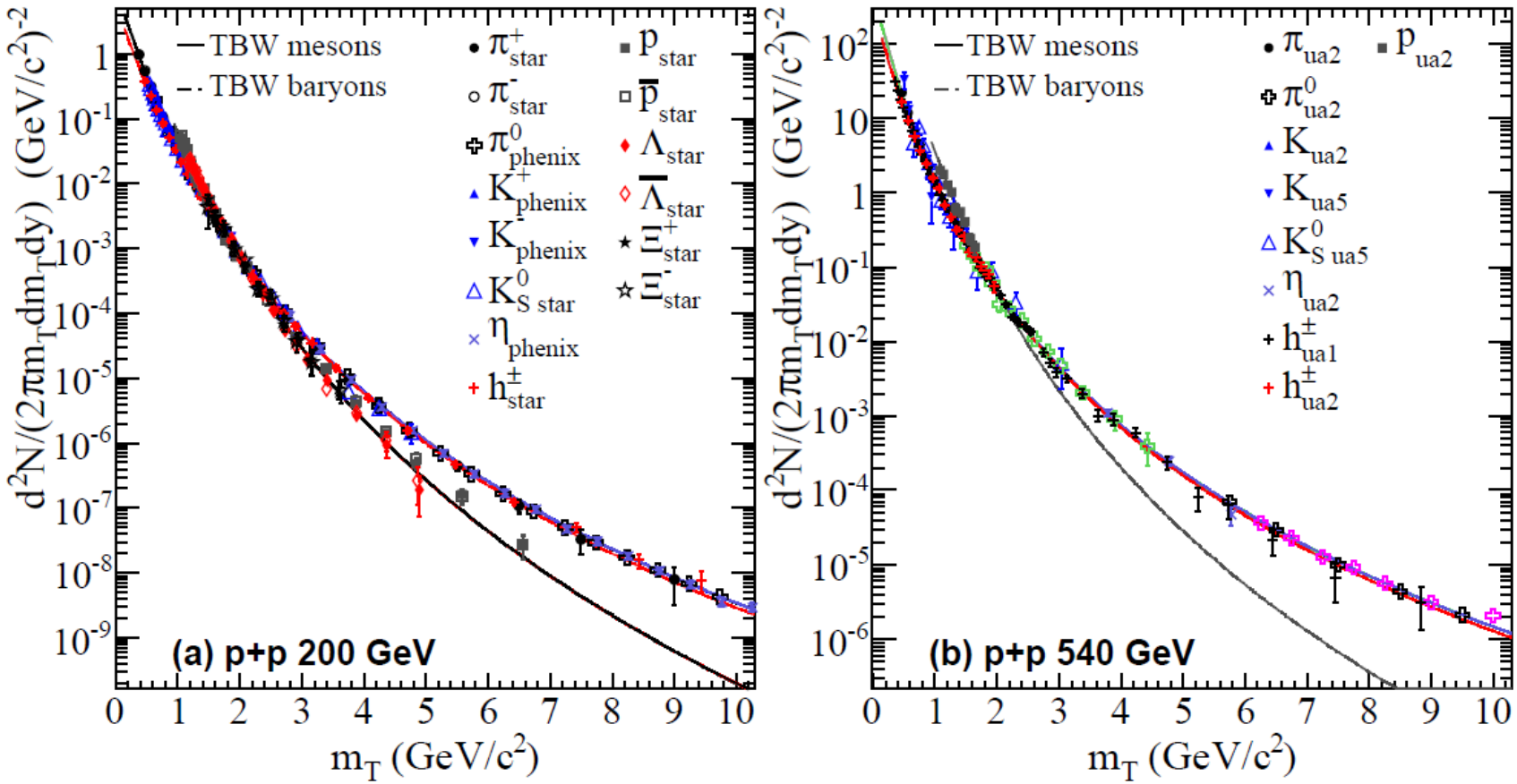
Fit to p+p Spectra in 200 GeV

Phys. Rev. C **91** (2015) 024910



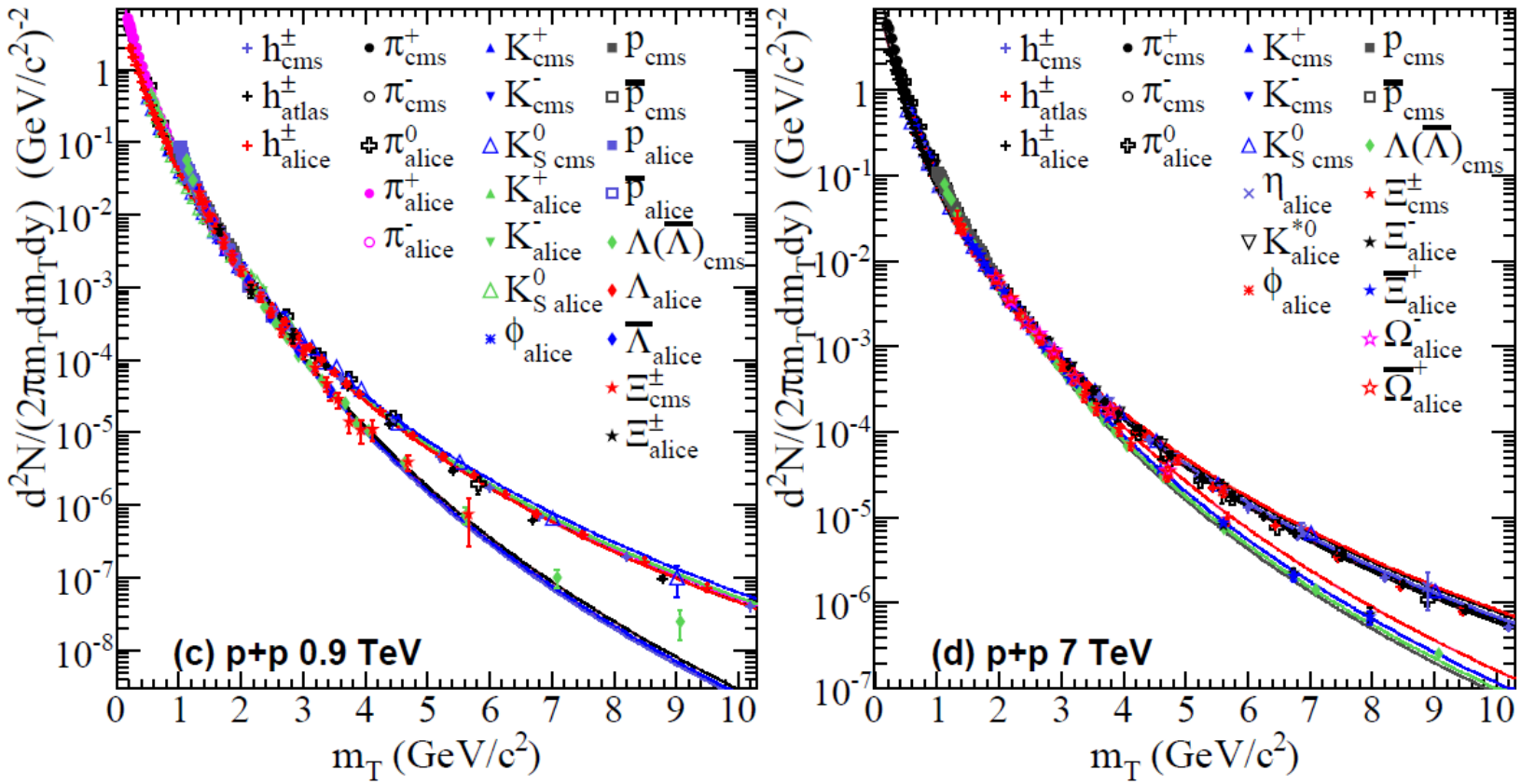
Steeper slope for baryons at high pt (more sensitive to spectral change)
If V2 are the same, actual elliptic flows depend on spectra

Beam Energy Dependence (i)



$\sqrt{s}$	$\langle\beta\rangle$	T (MeV)	$q_M - 1$	$q_B - 1$	χ^2/ndf
7 TeV	0.320 ± 0.005	70.3 ± 0.8	0.1314 ± 0.0003	0.1035 ± 0.0008	490/431
900 GeV	0.264 ± 0.005	74.6 ± 0.5	0.1127 ± 0.0003	0.0827 ± 0.0008	545/501
540 GeV	$0.000^{+0.105}_{-0.000}$	81.8 ± 0.6	0.1158 ± 0.0007	0.0841 ± 0.0036	205/168
200 GeV	$0.000^{+0.124}_{-0.000}$	92.3 ± 2.7	0.0946 ± 0.0006	0.0743 ± 0.0015	268/268

Beam Energy Dependence (ii)



$\sqrt{s}$	$\langle\beta\rangle$	T (MeV)	$q_M - 1$	$q_B - 1$	χ^2/ndf
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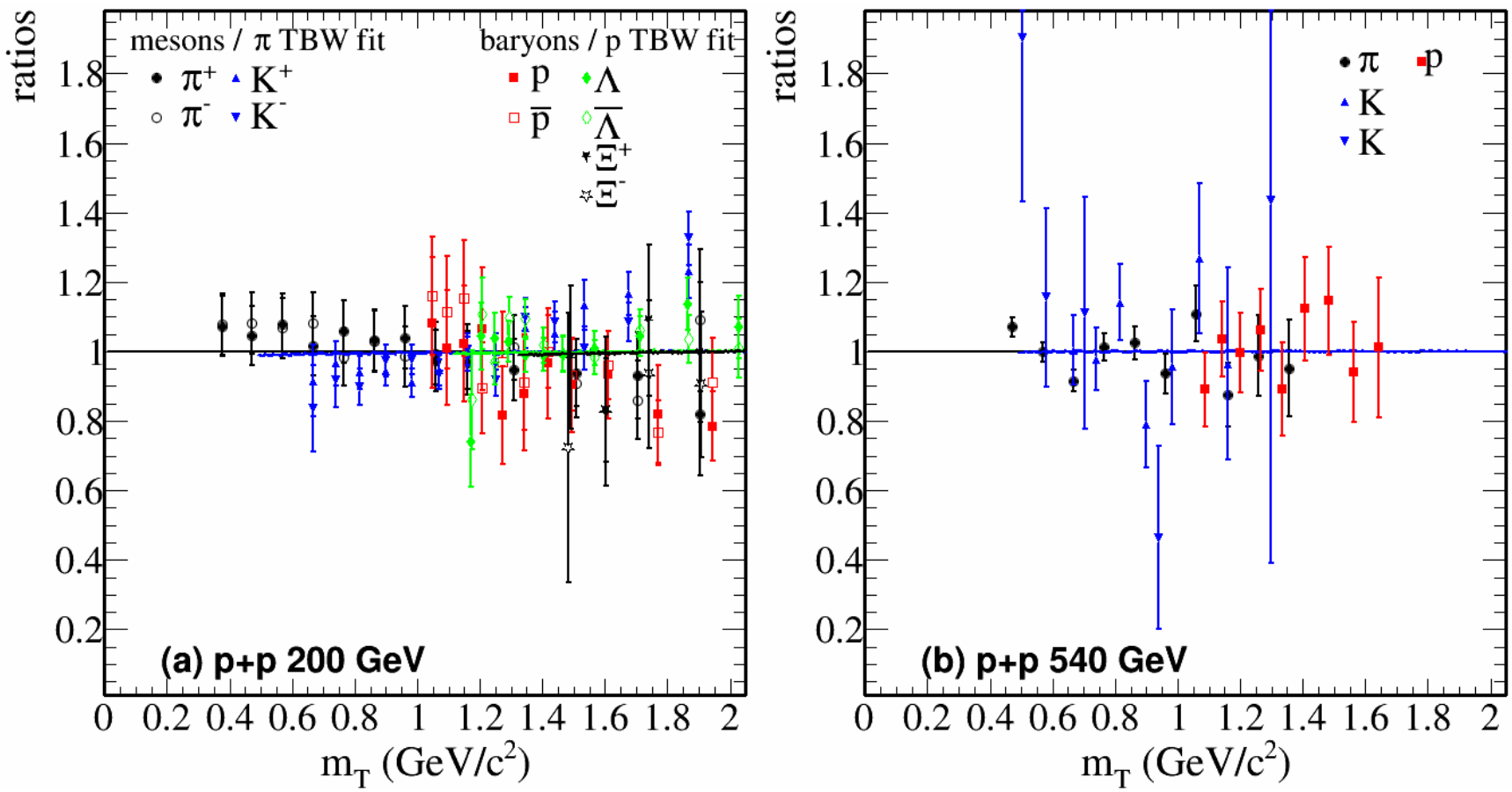
200 GeV	$0.000^{+0.124}_{-0.000}$	92.3 ± 2.7	0.0946 ± 0.0006	0.0743 ± 0.0015	268/268

m_T scaling

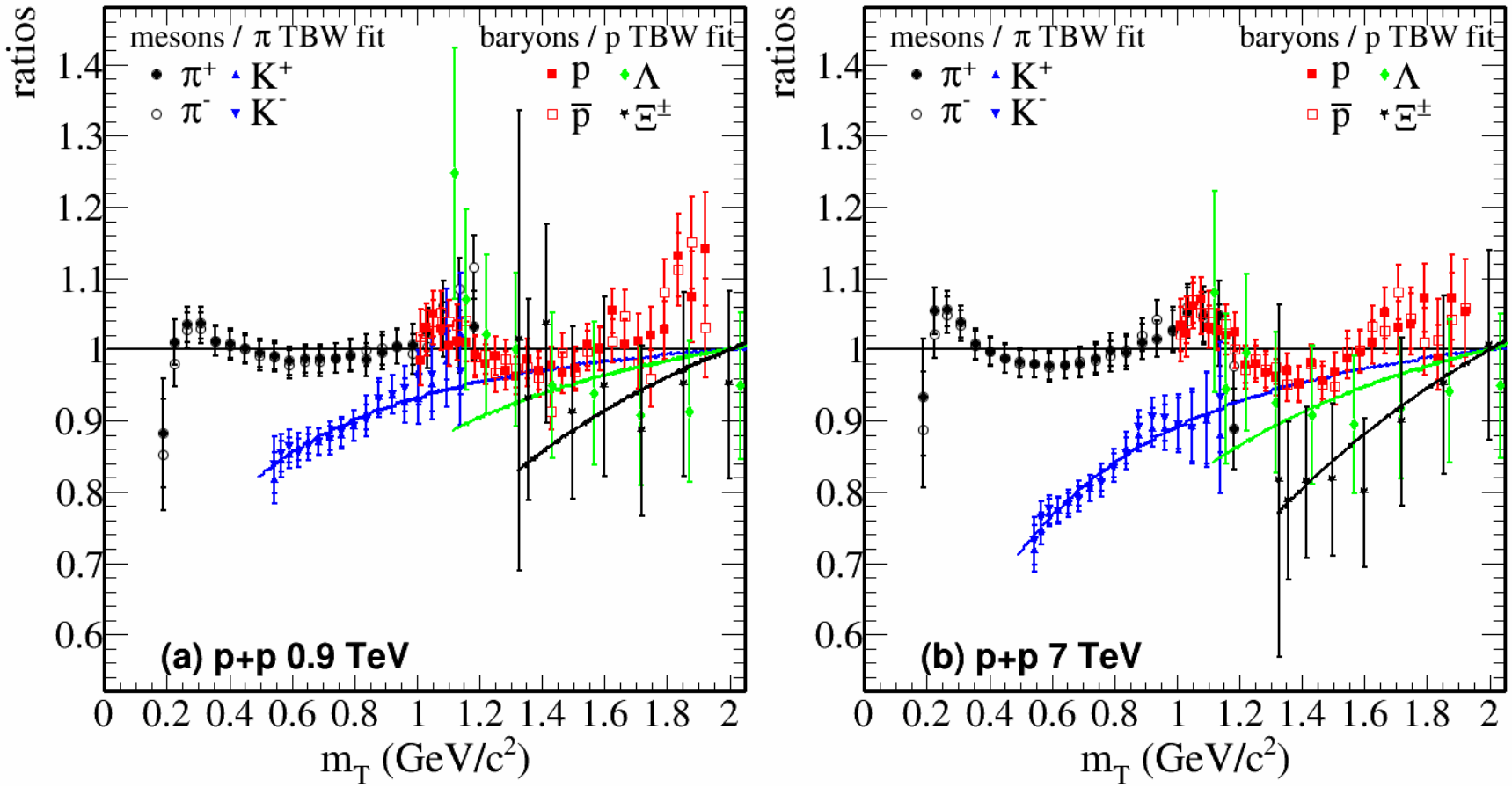
Phys. Rev. C **91** (2015) 024910



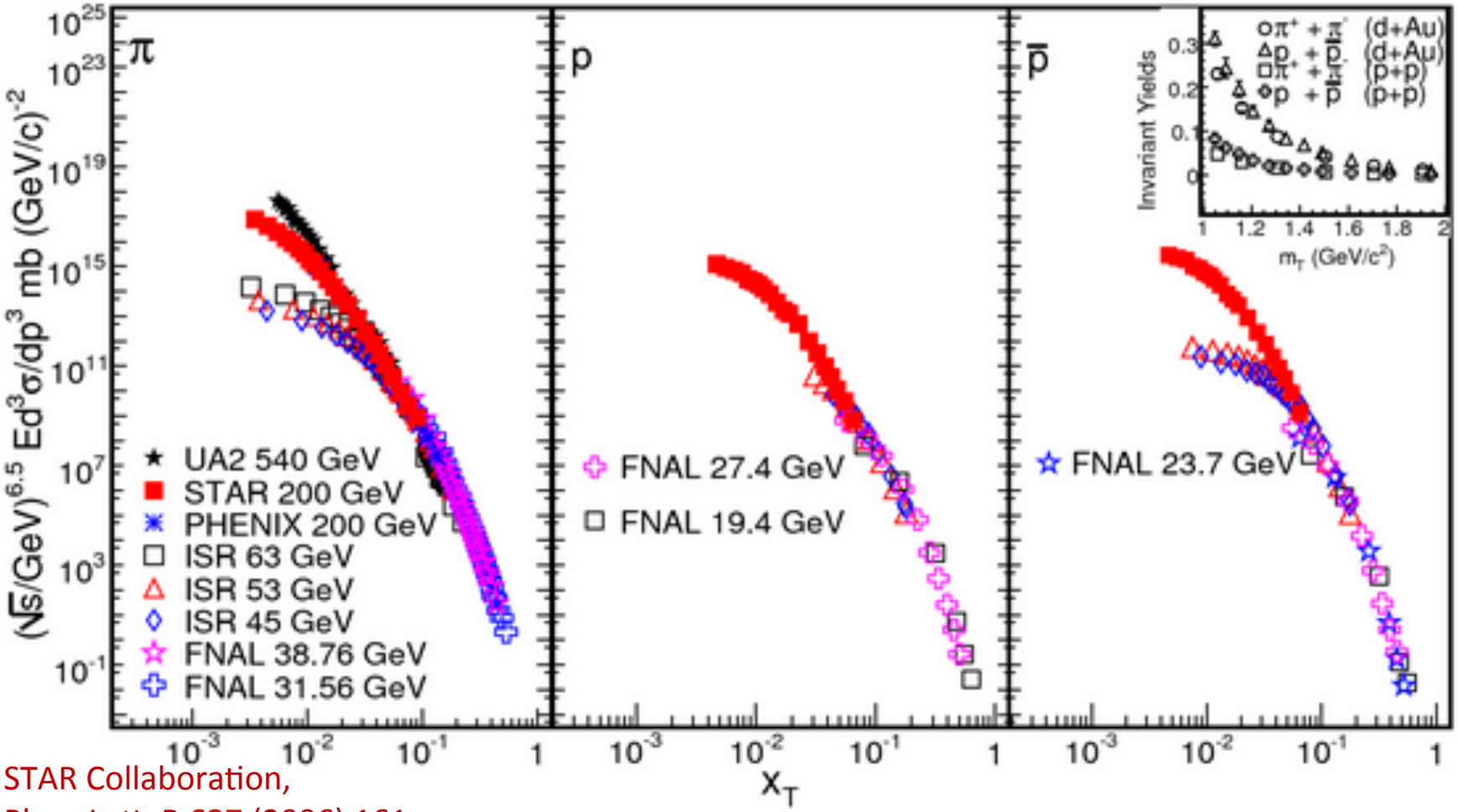
Higher multiplicity may be different, but what ensemble should we use?
Are we biasing sample in a microscopic way?

Breaking of m_T scaling

Phys. Rev. C **91** (2015) 024910



Spectra in d+Au collisions

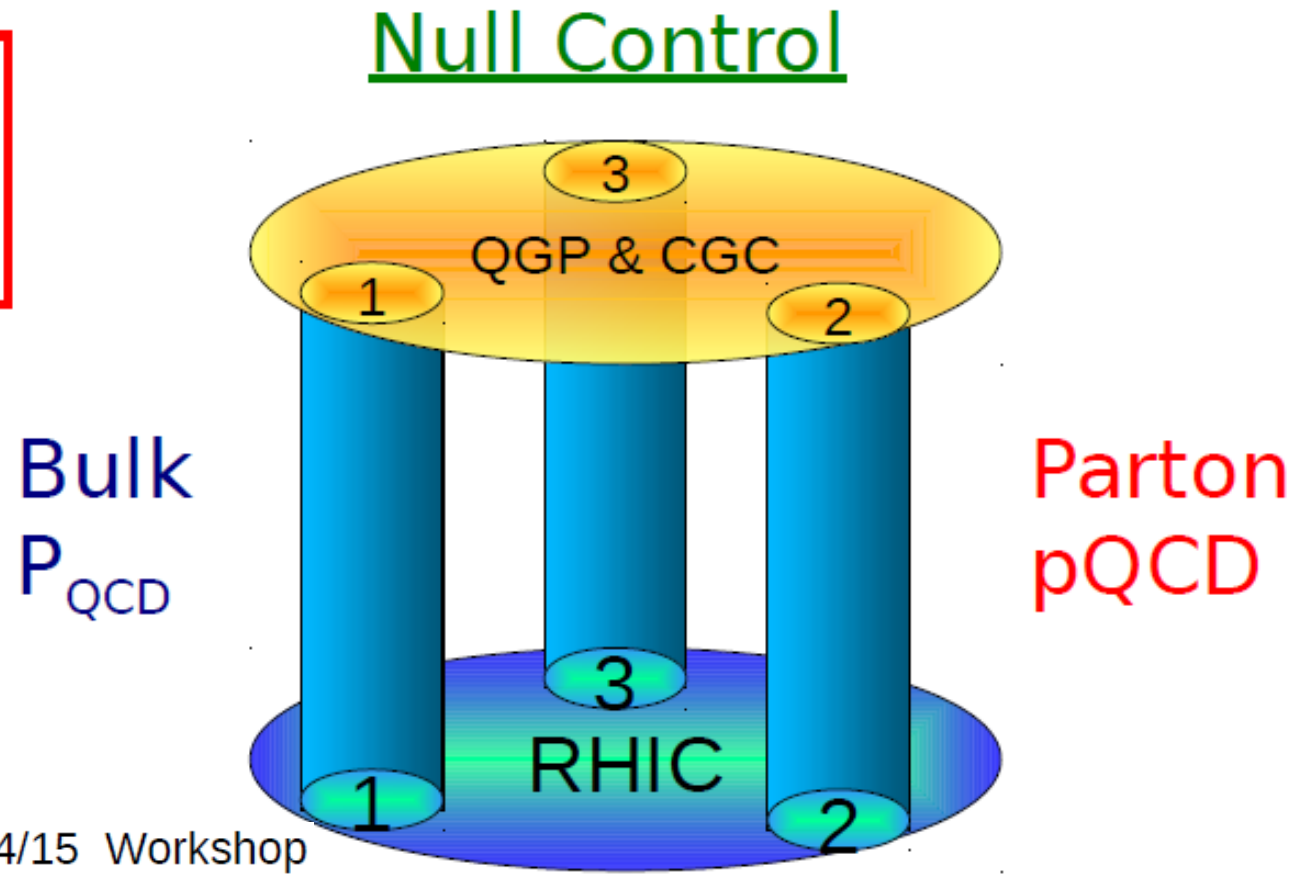


STAR Collaboration,
Phys. Lett. B **637** (2006) 161

The proton and pion spectra in p+p collisions follow xT-scaling with a beam-energy dependent factor of 6.5 above pT~ 2 GeV/c. The pion and proton spectra follow transverse mass scaling for mT< 2 GeV/c2 in both p+p and d+Au collisions, suggesting the transition Region from soft to hard process domination occurs at pT~ 2 GeV/c in these collision systems.

Last time, I checked, they are still there!

Three
Stable legs



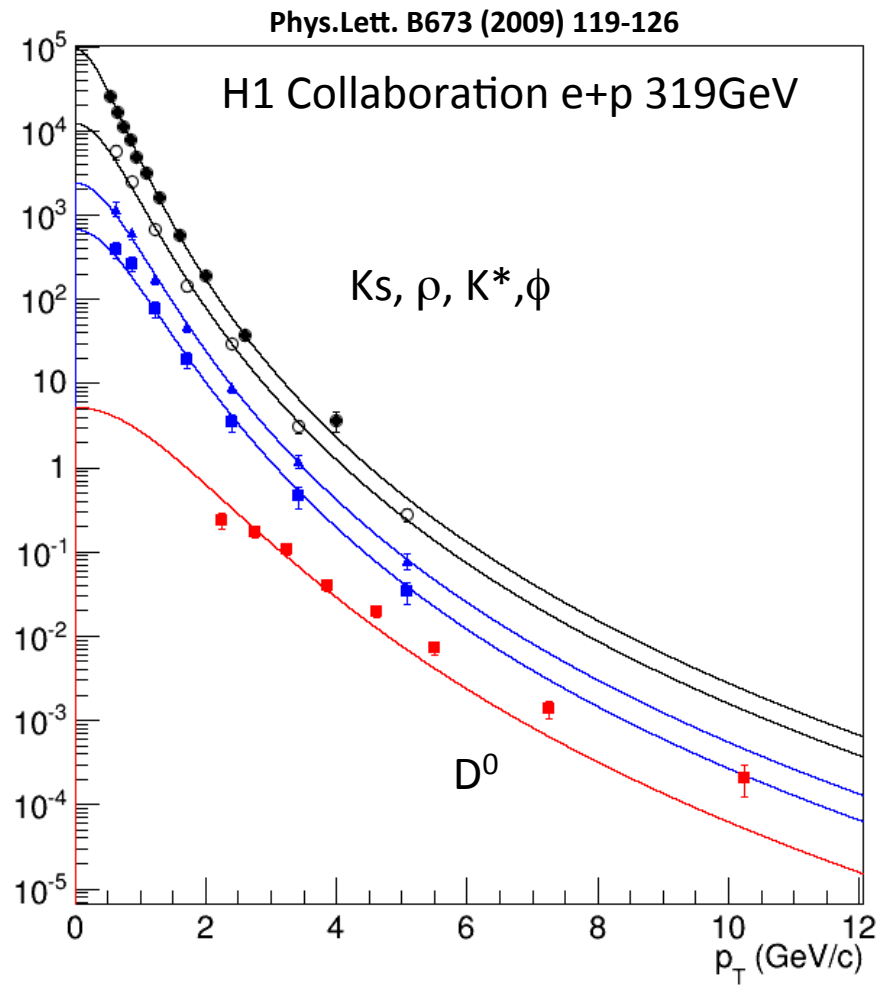
M. Gyulassy @ RBRC 3/4/15 Workshop

Likely optical illusion: it is impossible to see both top and bottom sides at the same time.
We cannot claim flow and no flow at the same time!

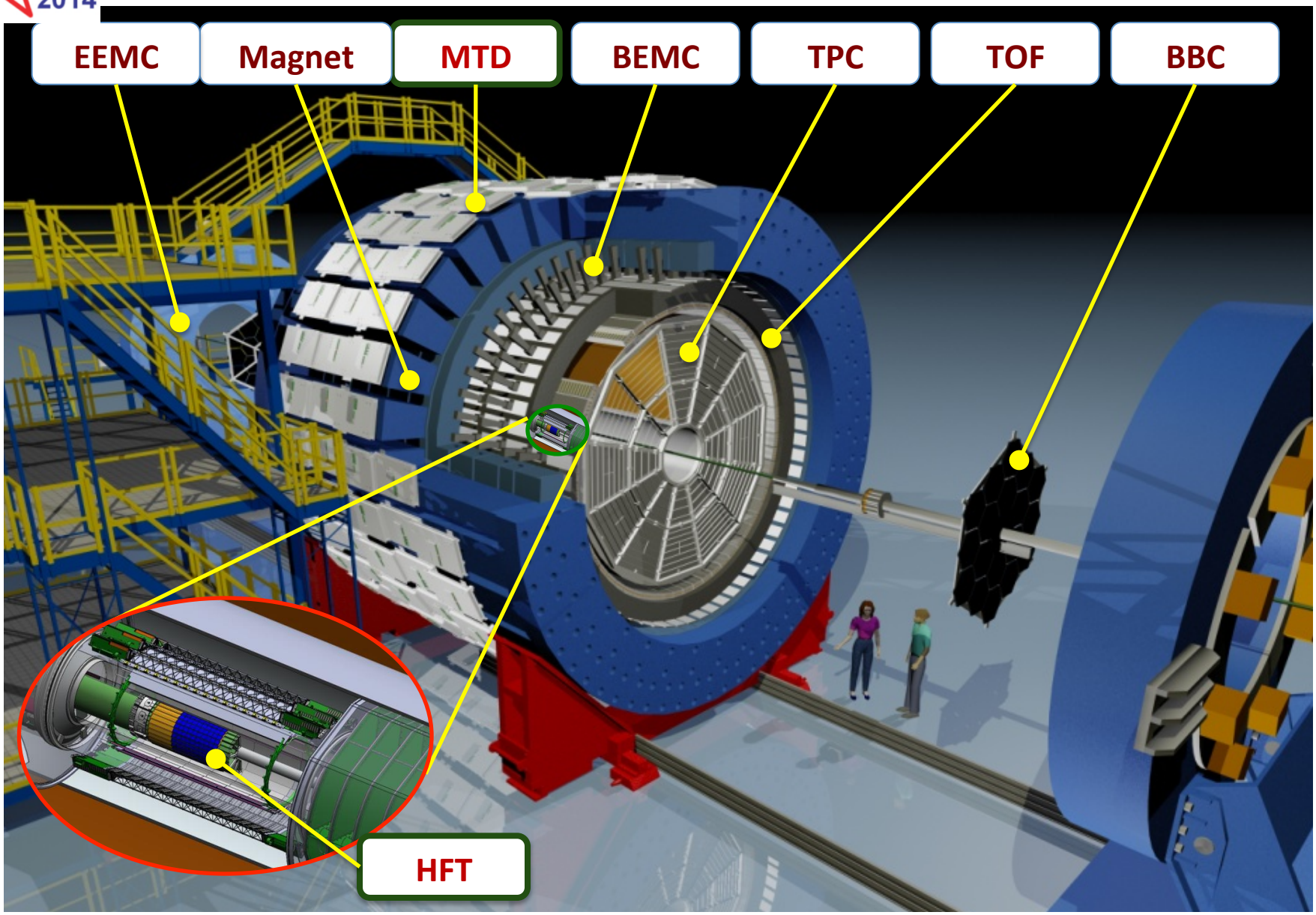
e+p collisions at 319GeV

e+p 319GeV
betaS = 0.0001 +- 0.6946
<beta> = 0.0001 +- 0.4631
T = 0.0873 +- 0.0073
q_M = 1.1060 +- 0.0039
chi^2/nDof = 18.54 / 24

p+p 200 GeV:
<beta> = 0.000 +0.124 - 0.000
T = 0.0923 +- 0.0027
q_M = 1.0946 +- 0.0006
chi^2/nDof = 268/268



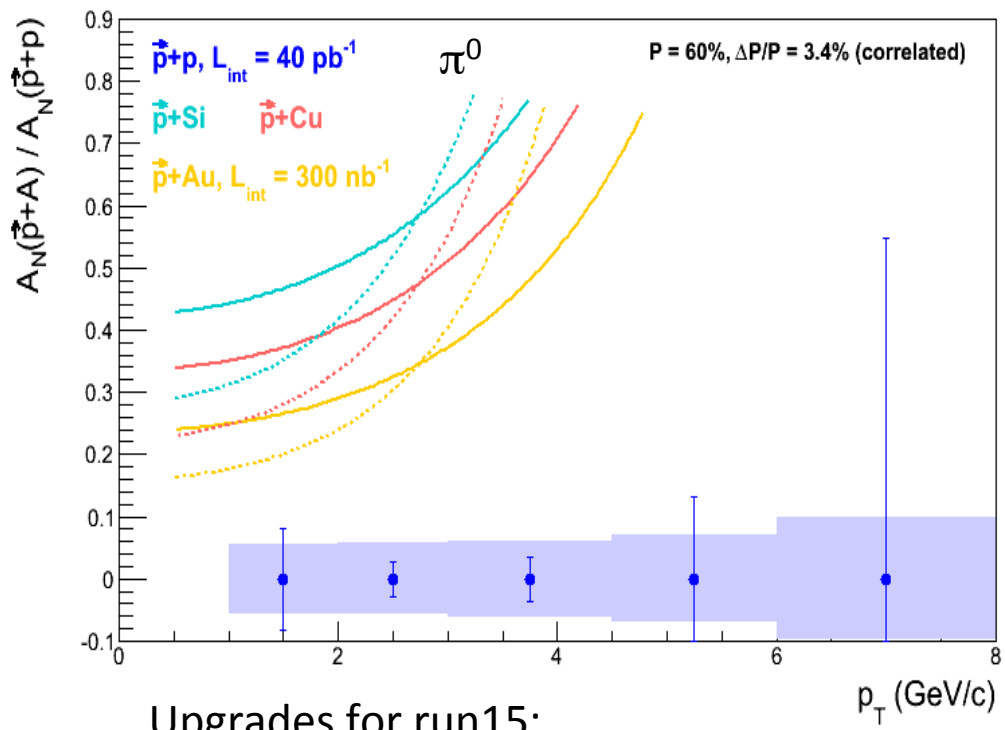
STAR Detector System

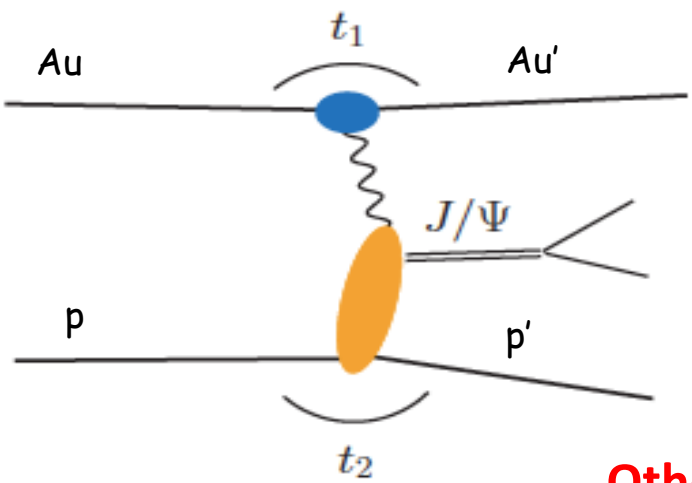


X10^3 increases in DAQ rate since 2000

Heavy-Ion meets Spin (run15)

- 5 weeks
  Polarized **p+Au**
  L=300nb^{-1}
- Saturation physics,
- pA-ridge
- Cold Nuclear Effect
- GPD gluon (pre-EIC)





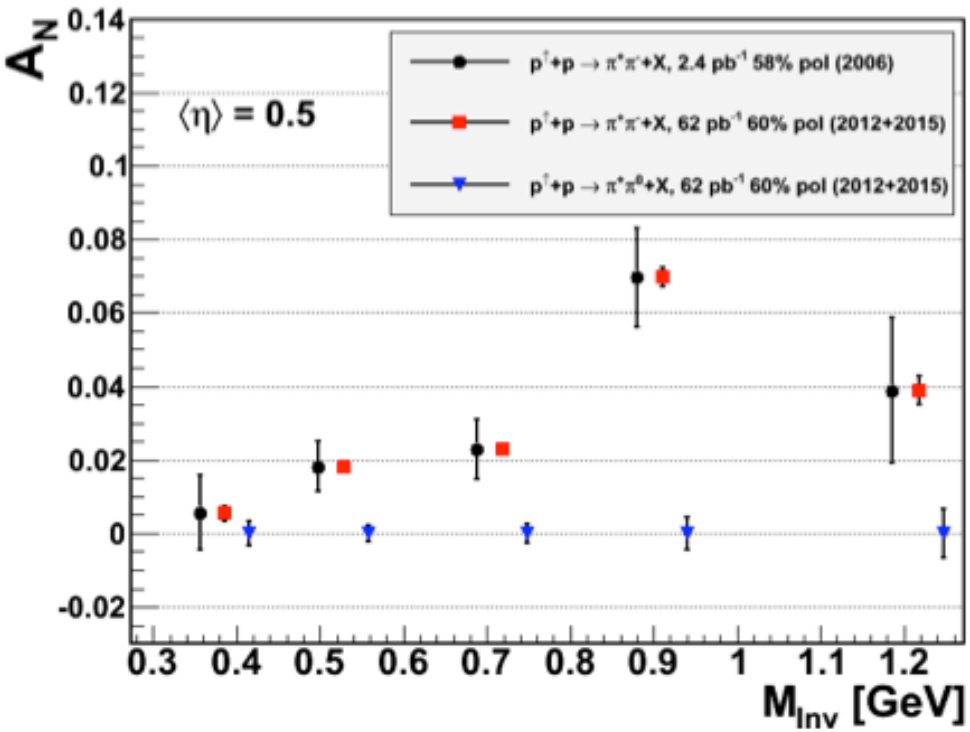
Upgrades for run15:

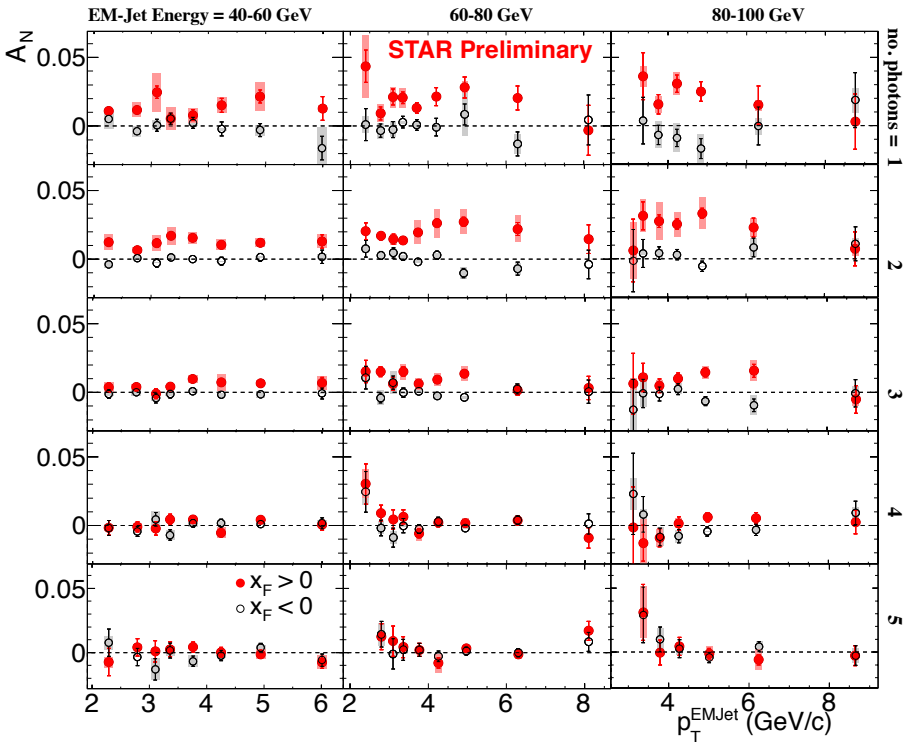
1. Roman-Pot Phase II
   (move RP closer)
2. Refurbish FMS
3. FMS pre-shower
4. Forward Instrumentation
   (proton in **BLUE**)

Other Extreme Geometry: Cu+Au, ^{3}He+Au

Transverse Polarization Measurements

kinematic and dynamical effects that are directly sensitive to quark transversity and partonic motion within the proton.





The only non-zero A_N measurements at mid-rapidity
Interference fragmentation Function

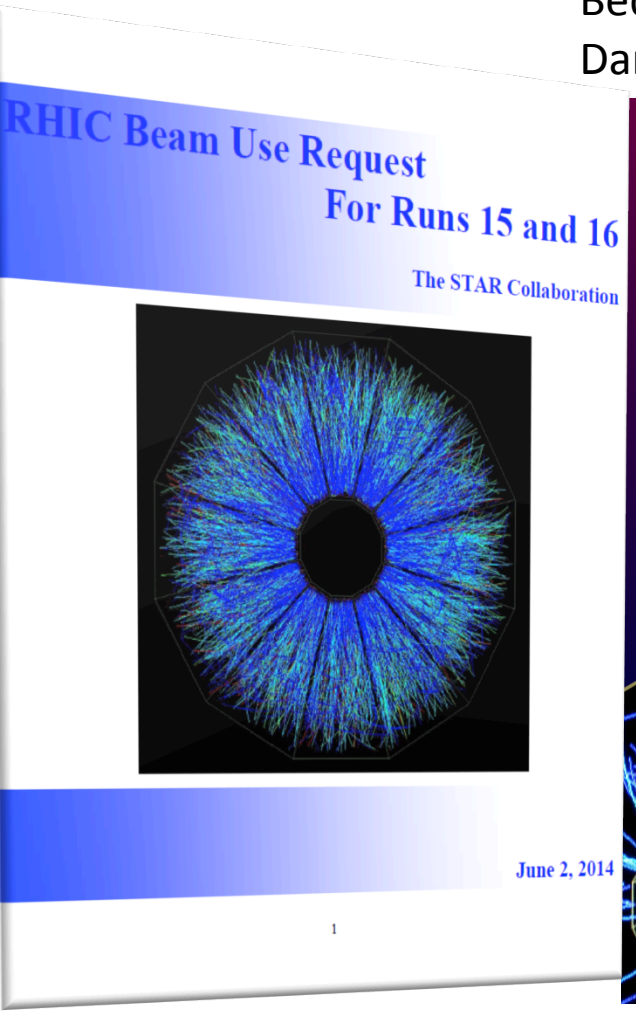
A_N decreases with increased jettiness of the event; Possible diffractive nature.

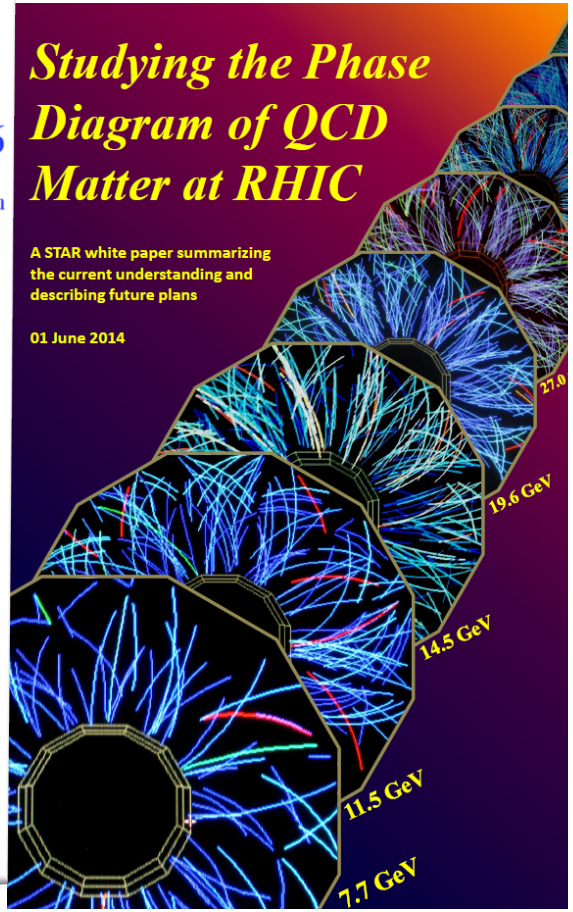
run15 Roman Pot Upgrade critical to the program

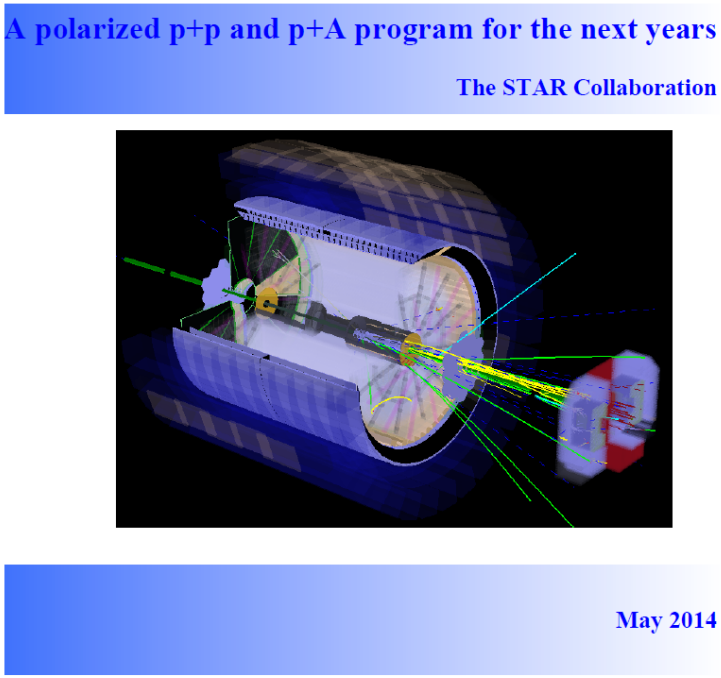
STAR Plan on BUR, BES II and pp/pA LoI

Committee co-chairs:
Bedanga Mohanty (NISER)
Dan Cebra (UC Davis)

Committee co-chairs:
Elke Aschenauer (BNL)
Ernst Sichtermann (LBL)
Huan Huang (UCLA)







https://drupal.star.bnl.gov/STAR/starnotes/public/sn0605

https://drupal.star.bnl.gov/STAR/starnotes/public/sn0598

https://drupal.star.bnl.gov/STAR/starnotes/public/sn0606

STAR Forward Upgrade: Tracking and Calorimetry

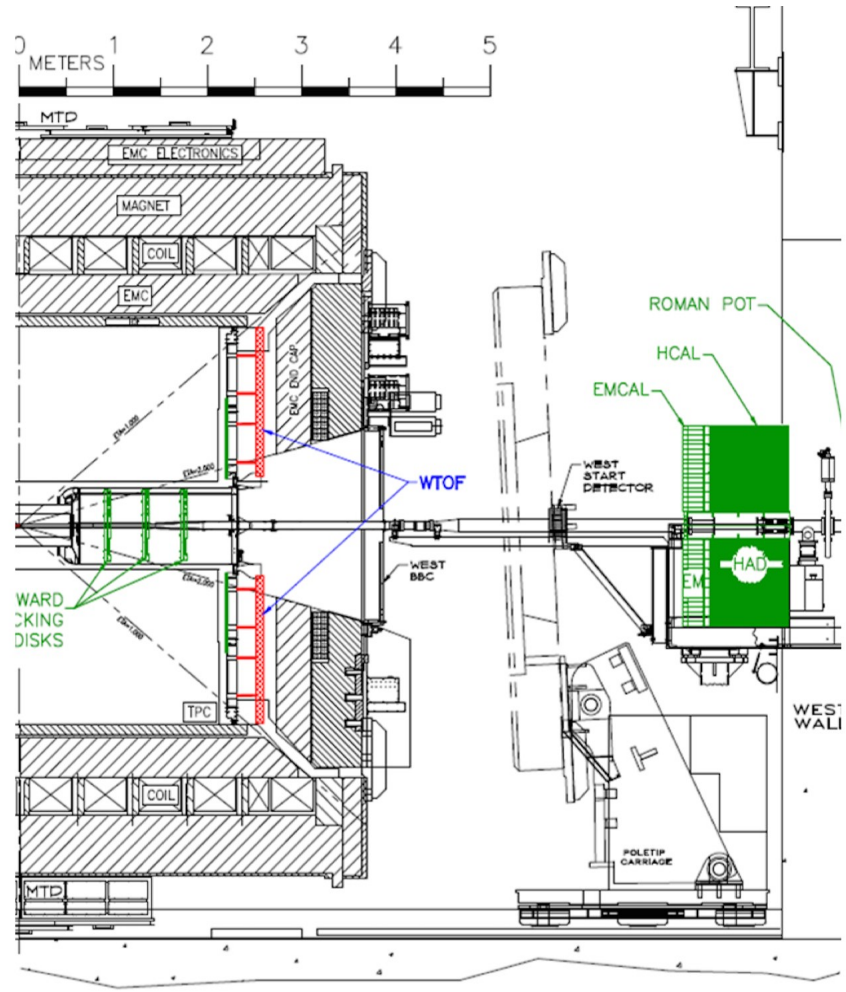
Fourier expansion of FCal ET distribution:

$$2\pi \frac{dE_T}{d\phi} = \Sigma E_T \left[1 + 2 \sum_n q_n \cos n \left(\phi - \Phi_n^{\text{obs}} \right) \right]$$

(ATLAS, CMS method)

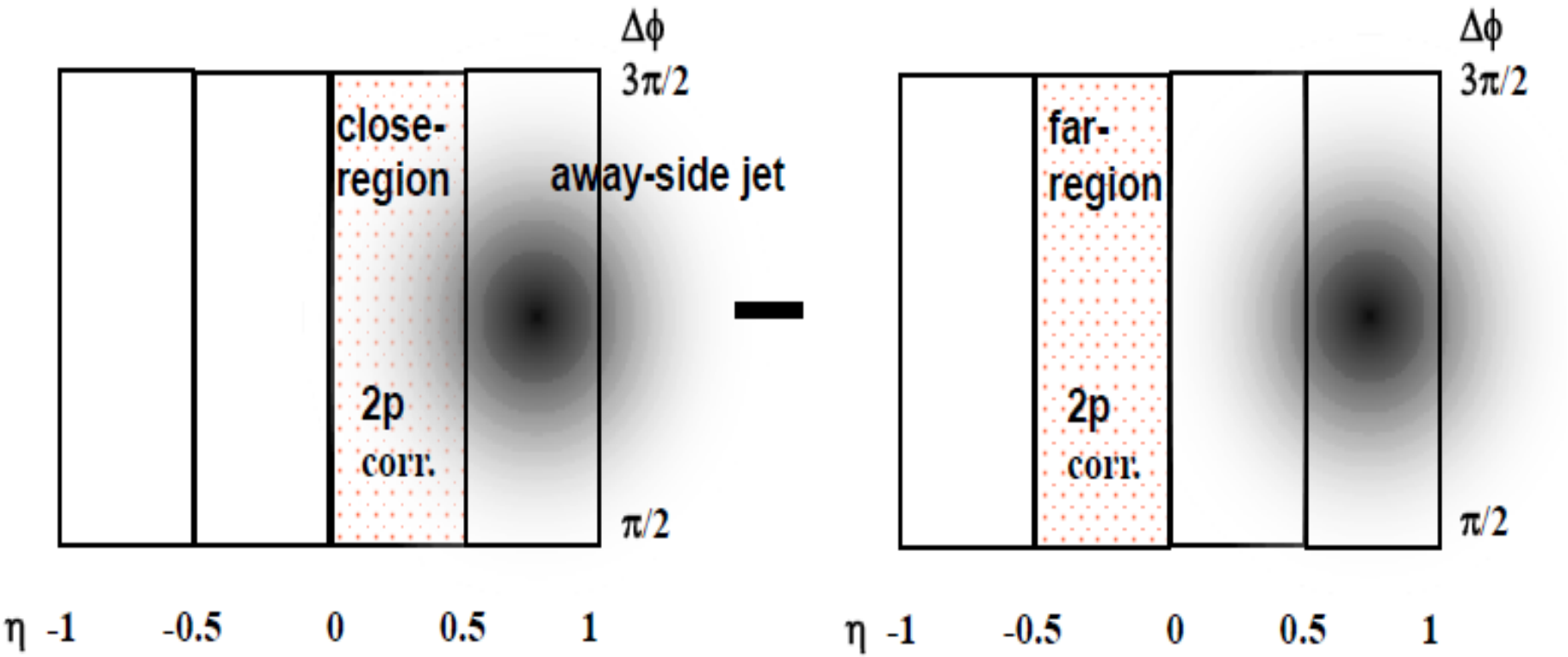
Event shape selection

- 0th order event-shape selection: Centrality by ΣE_T (system size)
- 2nd order event-shape selection: ellipticity by q2 (system shape)
- 3rd order event-shape selection: triangularity by q3 (system shape)



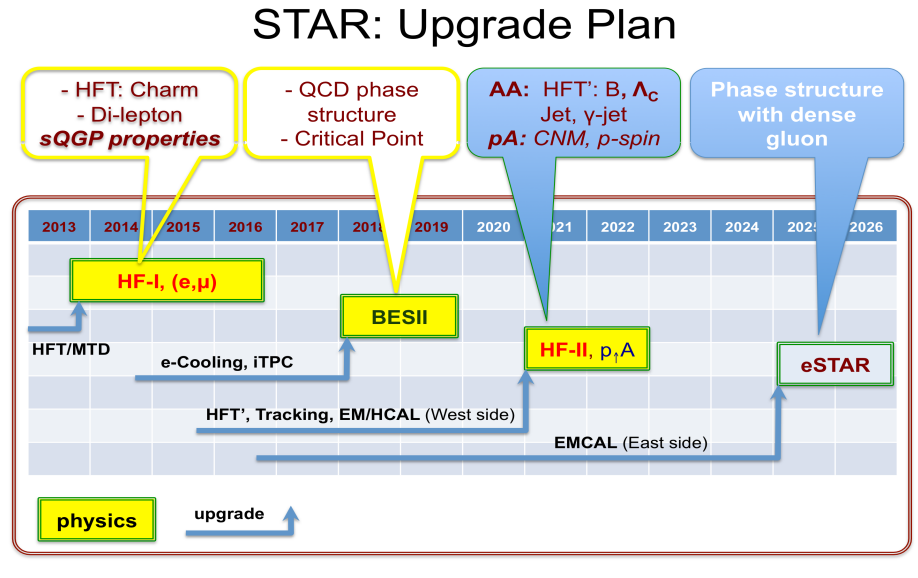
Extreme Geometry: d+Au Collisions

diharon in close Pseudo-rapidity range

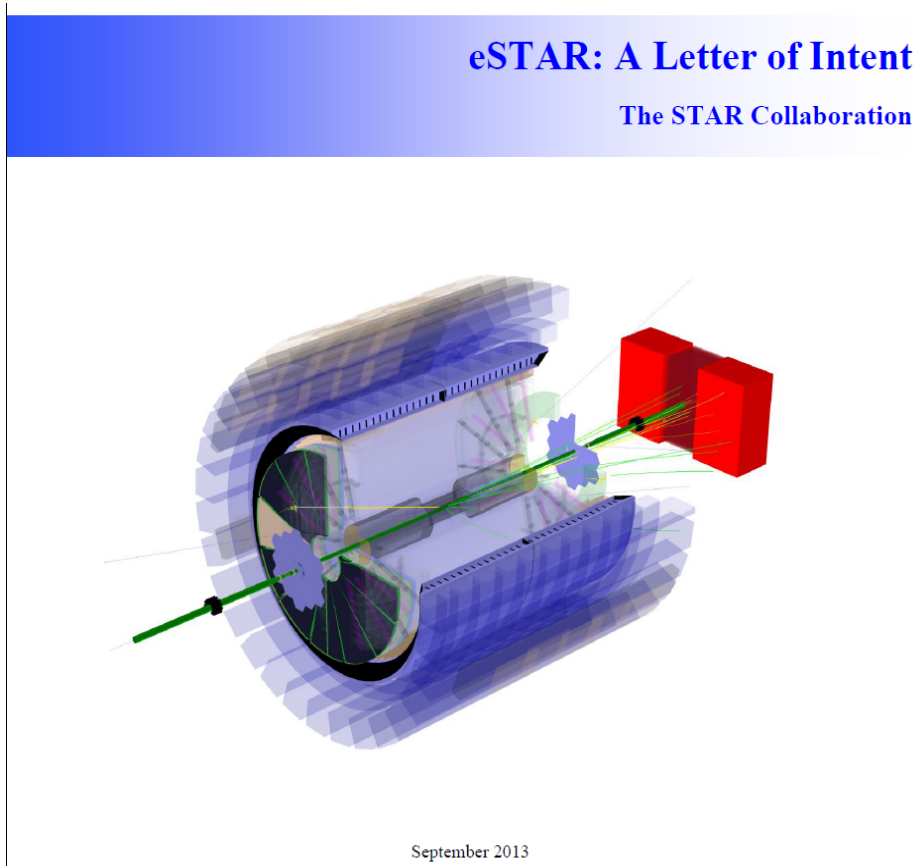


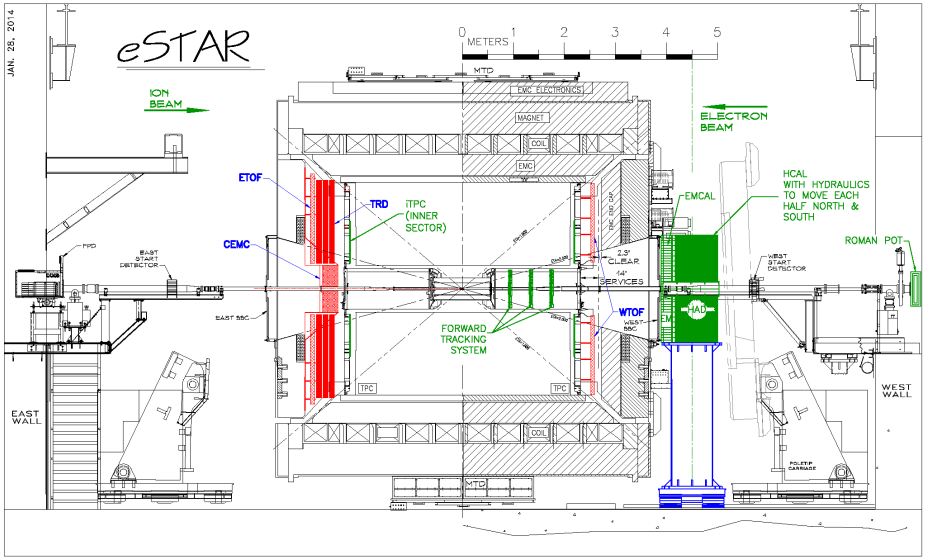
- High-mult. (before ZYAM) – jet in low-mult.
  → Double Ridge
- High-mult. (before ZYAM)– scaled jet in low-mult.
  → Away-side diminished

STAR Long-Term Plan



https://drupal.star.bnl.gov/STAR/starnotes/public/sn0592





Semi-inclusive Measurements

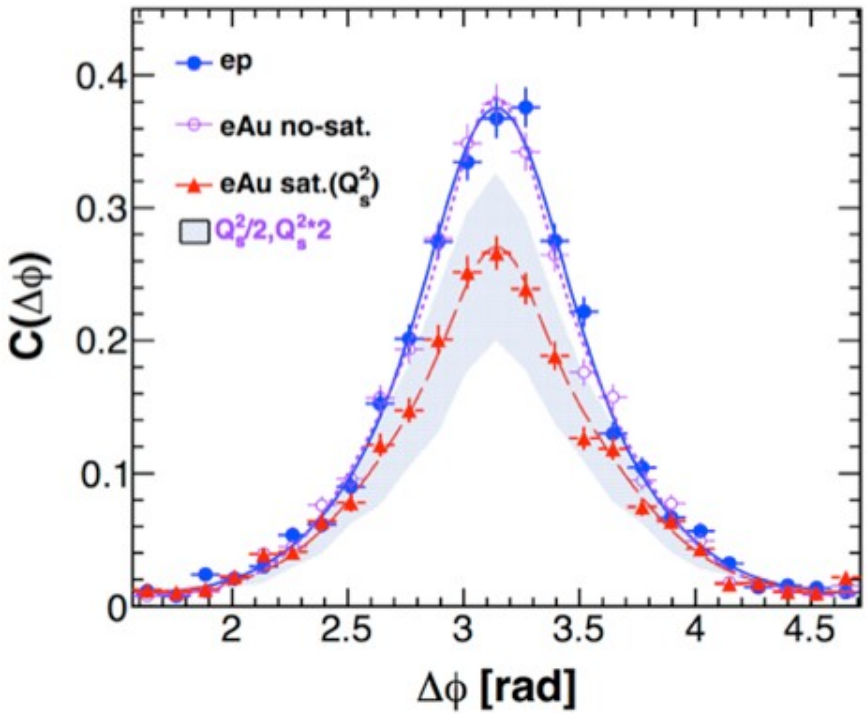
Azimuthal correlations in di-hadron (semi-inclusive deep-inelastic scattering) measurements,

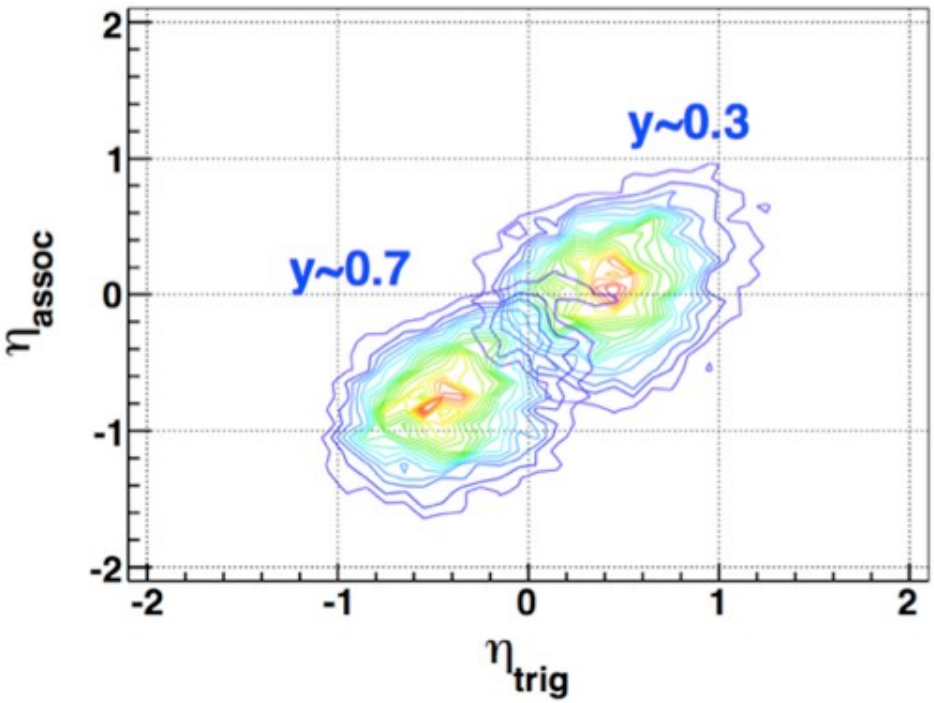
$$e + \mathrm{Au} \rightarrow e' + \mathrm{Au} + h_1 + h_2 + X$$

provide sensitivity to gluons and have been proposed as a robust probe of saturation:

Fig.2.6

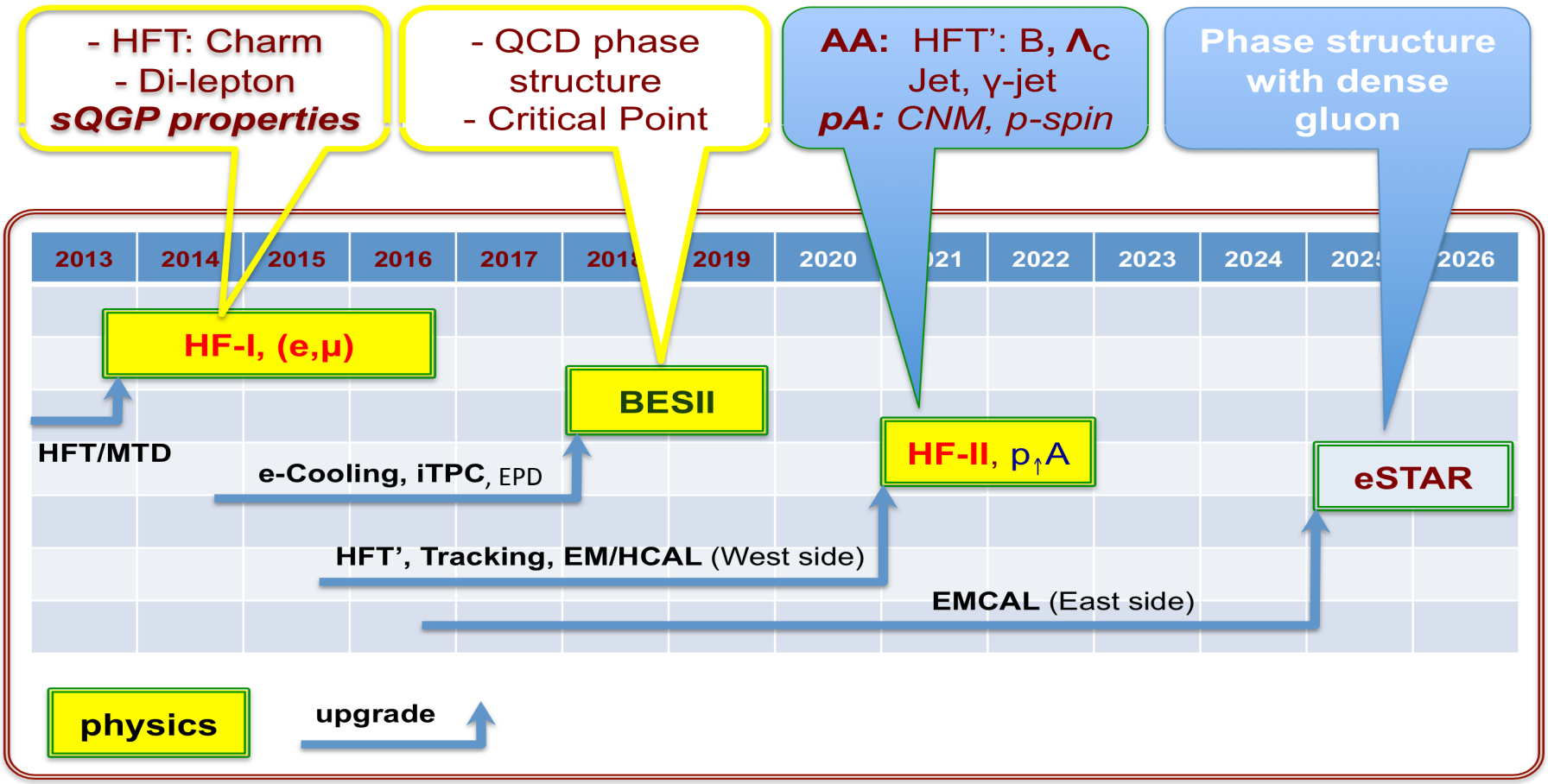
Why not flow, flux tube and multiplicity dependence? – E. Shuryak





eSTAR projections for 10 GeV electrons scattering off 100 GeV/nucleon Au beams, 1 fb^{-1}.

STAR: Upgrade Plan



Conclusions and Outlook

- Non-equilibrium effects are needed to describe the spectra in p+p collisions
- Are we manipulating the ensemble microscopically?
- Non-zero radial flow (breaking of mT scaling) is observed at 900GeV and 7TeV p+p collisions
- Study of d+Au and p+Pb, and high multiplicity p +p events are on-going
- New detectors and EIC simplify and isolate the physics origins

Data references (p+p)

- 200 GeV RHIC data (STAR+PHENIX)
- charged pions, protons and anti-protons(STAR) (Phys.Lett.B637:161-169,2006)
- K0S, lambda, lambdabar, xiMinus, xiPlus(STAR) (Phys. Rev. C 75, 064901(2007))
- piZero(Phenix) (Phys. Rev. D 76, 051106 (2007))
- KPlus, KMinus(Phenix) (Phys. Rev. C 74, 024904 (2006))

- 540 GeV data (UA2, UA1)
- pion, kaon, proton, charged particles(UA2, 1983) (Banner M, Bloch P, Bonaudi F, et al. Inclusive charged particle production at the CERN pp collider[J]. Physics Letters B, 1983, 122(3): 322-328.)
- kaon, K0S (Alner G J, Alpg?rd K, Anderer P, et al. Kaon production in pp reactions at a centre-of-mass energy of 540 GeV[J]. Nuclear Physics B, 1985, 258: 505-539.)
- pionZero(pT<=40 GeV/c, 1.0<|eta|<1.8), pionZero(pT<=15 GeV/c, |eta|<=0.85), eta(3<=pT<=6 GeV/c, |eta|<=0.85) (Banner M, Bloch P, Bonaudi F, et al. Inclusive particle production in the transverse momentum range between 0.25 and 40 GeV/c at the CERNSp\ bar pS collider[J]. Zeitschrift f¨¹r Physik C Particles and Fields, 1985, 27(3): 329-339.)
- pionZero(1.5<pT<4.5 GeV/c, <|eta|>=0) (Banner M, Bloch P, Bonaudi F, et al. Inclusive< i> ¦Ð</i>< sup> 0</sup> production at the CERN p-p? collider[J]. Physics Letters B, 1982, 115(1): 59-64.)
- K/pion, pbar/pion (Alexopoulos T, Allen C, Anderson E W, et al. Mass-identified particle production in proton-antiproton collisions at sqrt [s]= 300, 540, 1000, and 1800 GeV[J]. Physical Review D, 1993, 48(3): 984.)
- charged particles(UA1,1982) (Arnison G, Astbury A, Aubert B, et al. Transverse momentum spectra for charged particles at the cern proton-antiproton collider[J]. Physics Letters B, 1982, 118(1): 167-172.)

- 0.9 TeV pp collisions at LHC data:
- pionPlus, pionMinus, KPlus, KMinus, proton, pbar(CMS) (Chatrchyan S, Khachatryan V, Sirunyan A M, et al. Study of the inclusive production of charged pions, kaons, and protons in pp collisions at\ sqrt {s}= 0.9, 2.76,\ mbox {and} 7~\ mbox {TeV}[J]. The European Physical Journal C, 2012, 72(10): 1-37.)
- K0S, lambda, Xi(CMS) (Khachatryan V, Sirunyan A M, Tumasyan A, et al. Strange particle production in pp collisions at\ sqrt {s}= 0.9 and 7 TeV[J]. Journal of High Energy Physics, 2011, 2011(5): 1-40.)
- pionZero(ALiCE) (Abelev B, Abrahantes Quintana A, Adamov¨¢ D, et al. Neutral pion and ¦Ç meson production in proton¨Cproton collisions at and[J]. Physics Letters B, 2012.)
- charged particles(CMS) (Charchyan S. Charged particle transverse momentum spectra in pp collisions at\ sqrt {s}= 0.9 and 7 TeV[J]. 2011.)

- 7 TeV pp collisions at LHC data:
- pionPlus, pionMinus, KPlus, KMinus, proton, pbar(CMS) (Chatrchyan S, Khachatryan V, Sirunyan A M, et al. Study of the inclusive production of charged pions, kaons, and protons in pp collisions at\ sqrt {s}= 0.9, 2.76,\ mbox {and} 7~\ mbox {TeV}[J]. The European Physical Journal C, 2012, 72(10): 1-37.)
- K0S, lambda, Xi(CMS) (Khachatryan V, Sirunyan A M, Tumasyan A, et al. Strange particle production in pp collisions at\ sqrt {s}= 0.9 and 7 TeV[J]. Journal of High Energy Physics, 2011, 2011(5): 1-40.)
- pionZero, Eta(ALiCE) (Abelev B, Abrahantes Quintana A, Adamov¨¢ D, et al. Neutral pion and ¦Ç meson production in proton¨Cproton collisions at and[J]. Physics Letters B, 2012.)
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Eccentricity fluctuations and correlations in pp, pA and AA

Li Yan

CNRS, Institut de Physique Théorique, Saclay

6 March 2015, BNL, RBRC workshop

Collectivity in Small Colliding Systems with High Multiplicity

Art Poskanzer and Jean-Yves Ollitrault

- PLB742 (2015) 290-295, PRC (2014) 024903, PRL 112 (2014) 082301
- arXiv:1503.00880

Outline of this talk

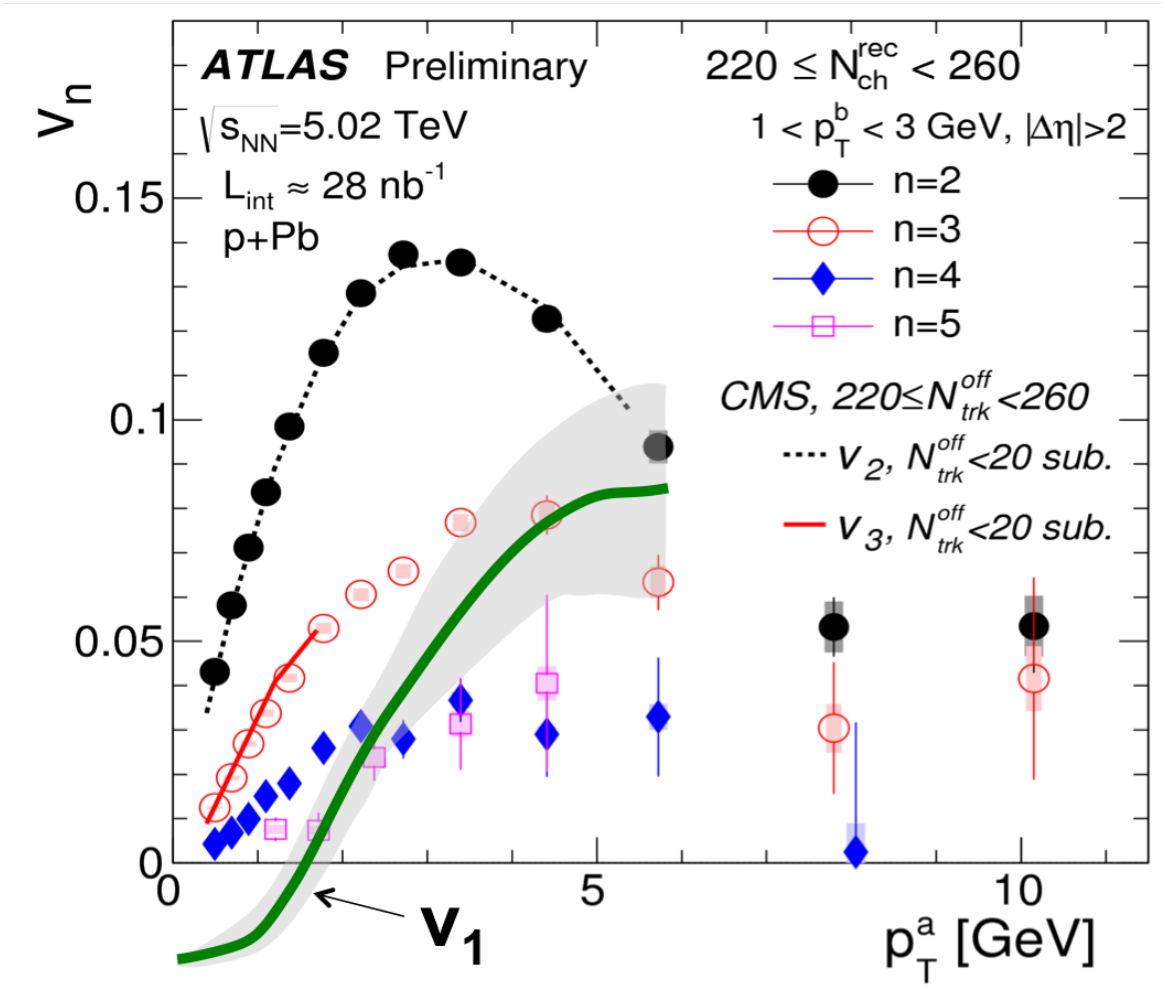
- Fluctuations of initial eccentricity and harmonic flow.
  - Parameterizations of anisotropy fluctuations: power and elliptic power
  - Applications to fluctuations of harmonic flow: AA and pA

$$\text{Pb-Pb} \rightarrow \text{extraction of } \frac{\eta}{s} \quad \text{and} \quad \text{p-Pb} \rightarrow \text{ patterns of } v_2\{m\}$$

- Correlations of initial eccentricity and harmonic flow.
  - Correlations between v_2 and v_3: p-Pb, d-Au and ^{3}He-Au

Motivation: collectivity in small colliding systems?

Harmonic flow from p-Pb high-multiplicity events: $V_n = v_n \exp(in\Psi_n)$



J. Jia, Nucl.Phys. A (2014) arXiv:1408.0066

Anisotropies in the particle spectrum from pre-equilibrium or late stage?

IF medium collective expansion in small colliding systems

Collective expansion (hydro) scenario for small colliding systems:

- Harmonic flow is associated to initial anisotropies by collective expansion
- Assuming linear eccentricity scaling (hydro): for $n \leq 3$.

$$\underline{V_n = \kappa_n \mathcal{E}_n} \quad \text{with } \mathcal{E}_n = \varepsilon_n \exp\left(in\Phi_n\right)$$

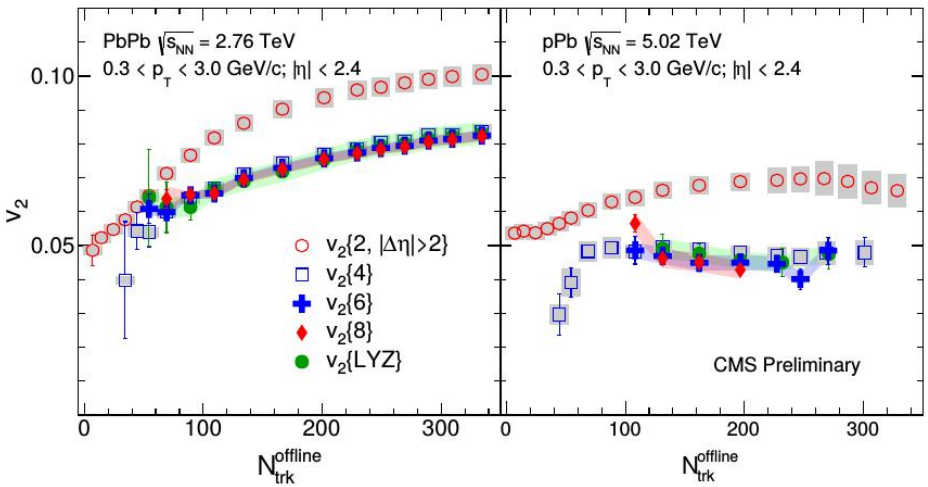
- Detailed patterns of the observed harmonic flow:
  - Fluctuations of V_n $\Leftrightarrow$ Fluctuations of $\mathcal{E}_n$ $\sim \varepsilon_n$
  - Correlations of V_n $\Leftrightarrow$ Correlations of $\mathcal{E}_n$ $\sim \Phi_n$

Require measurements of multi-particle correlations $\sim \langle V_n^a (V_m^*)^b \rangle$.

Event-by-event fluctuations of harmonic flow

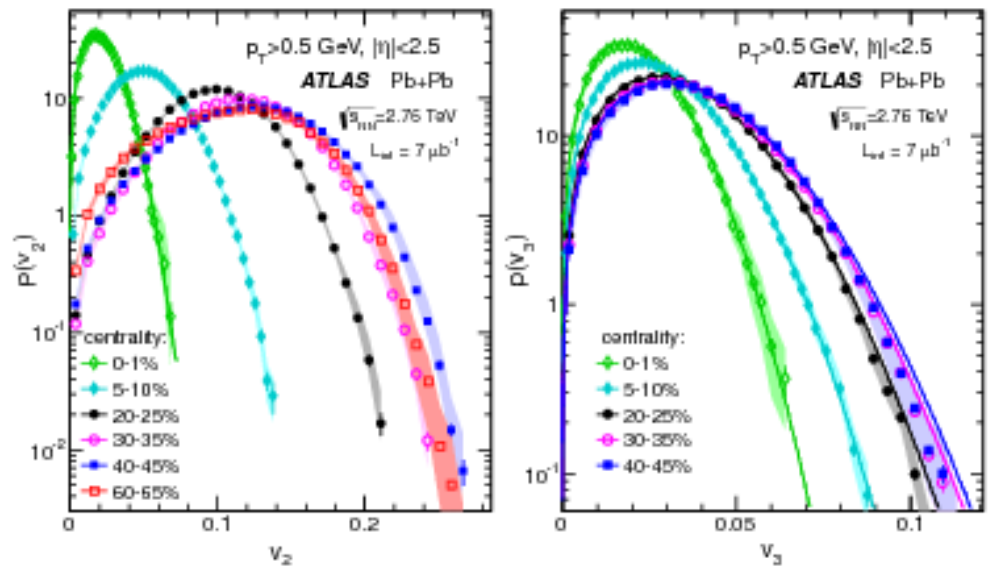
In experiments: p-Pb and Pb-Pb

(CMS)PLB724 (2013)213-240



Cumulants: $v_2\{m\}$

(ATLAS) JHEP 1311(2013)183



EbE distribution: $P(v_n)$

Cumulants and EbE distribution:

$$v_n\{m\} \quad \sim \quad \langle v_n^m \rangle = \langle V_n^{m/2} (V_n^*)^{m/2} \rangle = \int dv_n P(v_n) v_n^m$$

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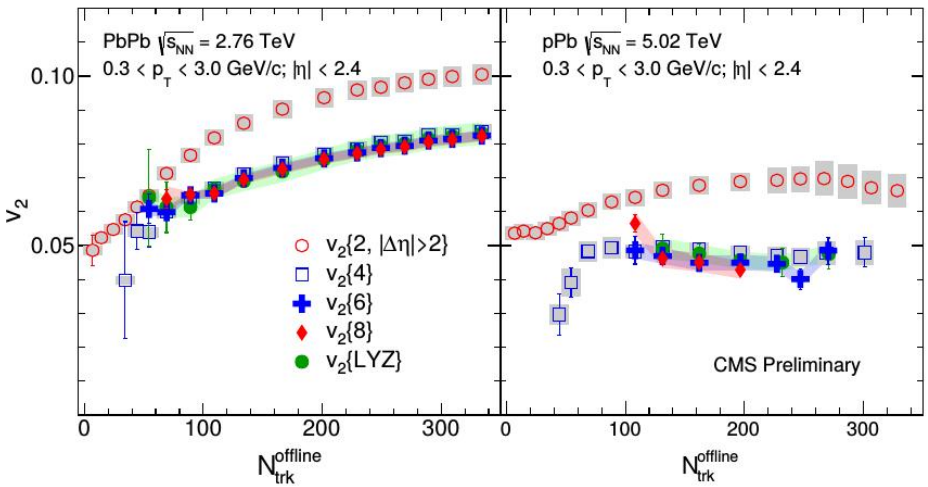
In experiments: p-Pb and Pb-Pb

(CMS)PLB724 (2013)213-240



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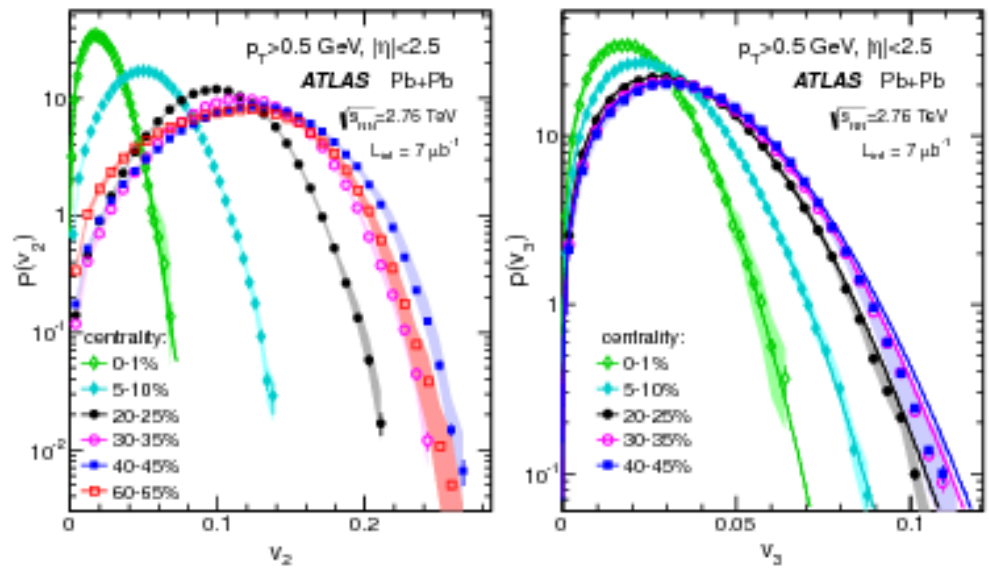

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$$\varepsilon_n\{m\} \quad \sim \quad \langle \varepsilon_n^m \rangle = \langle \mathcal{E}_n^{m/2} (\mathcal{E}_n^*)^{m/2} \rangle = \int d\varepsilon_n P(\varepsilon_n) \varepsilon_n^m$$

Elliptic Power distribution and Power distribution

Two dimensional distribution of $\mathcal{E}$:

$$\mathcal{E} = \varepsilon_x + i\varepsilon_y$$

- **Elliptic Power distribution** : fluctuation + initrinsic geometry

$$P_{\rm EP}(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}} \frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}}, \qquad \text{with } |\mathcal{E}| < 1$$

$\alpha \sim N \Rightarrow$ fluctuations, $\quad \varepsilon_0 \Rightarrow$ average RP eccentricity (roughly)

- **Power distribution** : pure fluctuation-driven ($\varepsilon_0 = 0$)

$$P_{\rm Power}(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1} \qquad \Leftarrow \qquad P_{\rm EP}(\varepsilon_0 \to 0)$$

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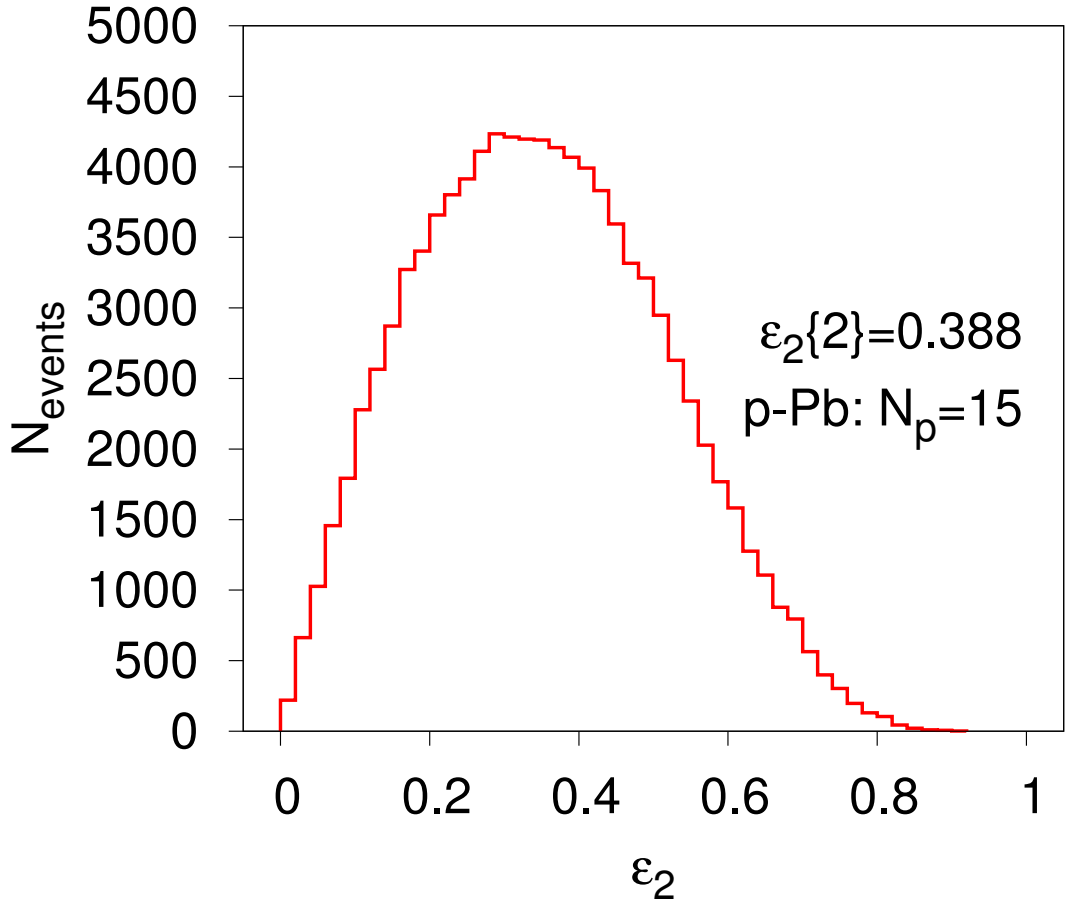
1D distributions are obtained after integration over azimuthal angle.

Event-by-event fluctuations of ε_2 in p-Pb: $P(\varepsilon_2)$

- Event-by-event distribution of ε_2 with PHOBOS Monte Carlo Glauber:



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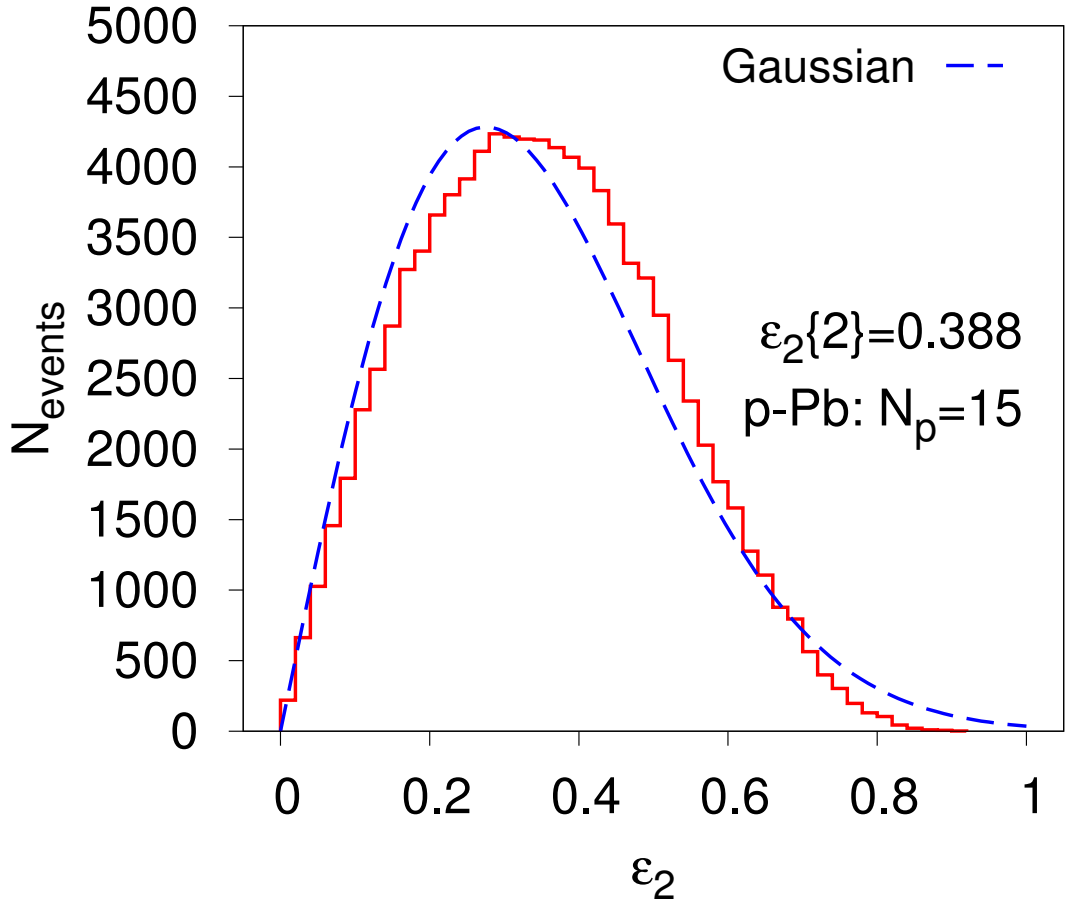

Gaussian distribution:

$$P(\varepsilon_2) = \frac{2\varepsilon_2}{\sigma^2} \exp\left(-\frac{\varepsilon_2^2}{\sigma^2}\right)$$

Event-by-event fluctuations of ε_2 in p-Pb: $P(\varepsilon_2)$

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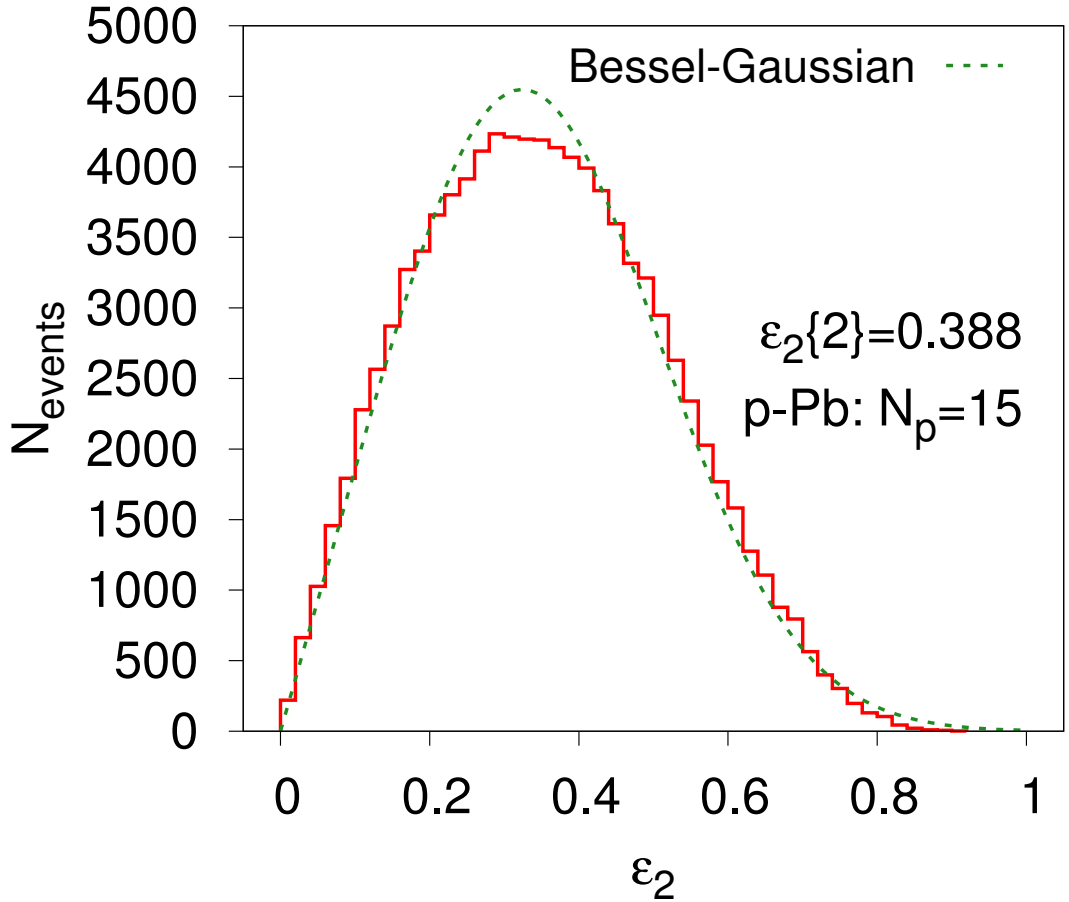
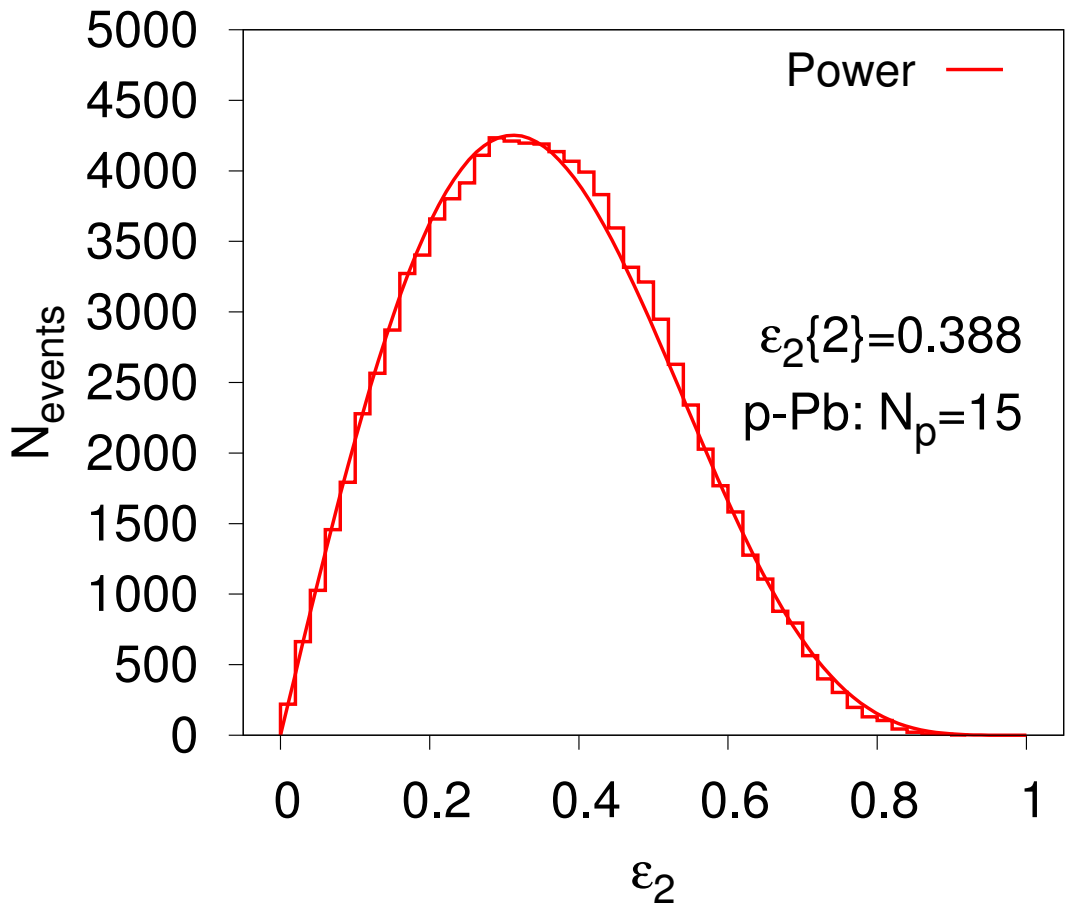


Bessel-Gaussian distribution:

$$P(\varepsilon_2) = \frac{2\varepsilon_2}{\sigma^2} \mathrm{I}_o \left(\frac{2\varepsilon_2 \bar{\varepsilon}_2}{\sigma^2} \right) \exp \left(-\frac{\varepsilon_2^2 + \bar{\varepsilon}_2^2}{\sigma^2} \right)$$

Event-by-event fluctuations of ε_2 in p-Pb: $P(\varepsilon_2)$

- Event-by-event distribution of ε_2 with PHOBOS Monte Carlo Glauber:



Power distribution:

$$P(\varepsilon_2) = 2\alpha\varepsilon_2(1-\varepsilon_2^2)^{\alpha-1}$$

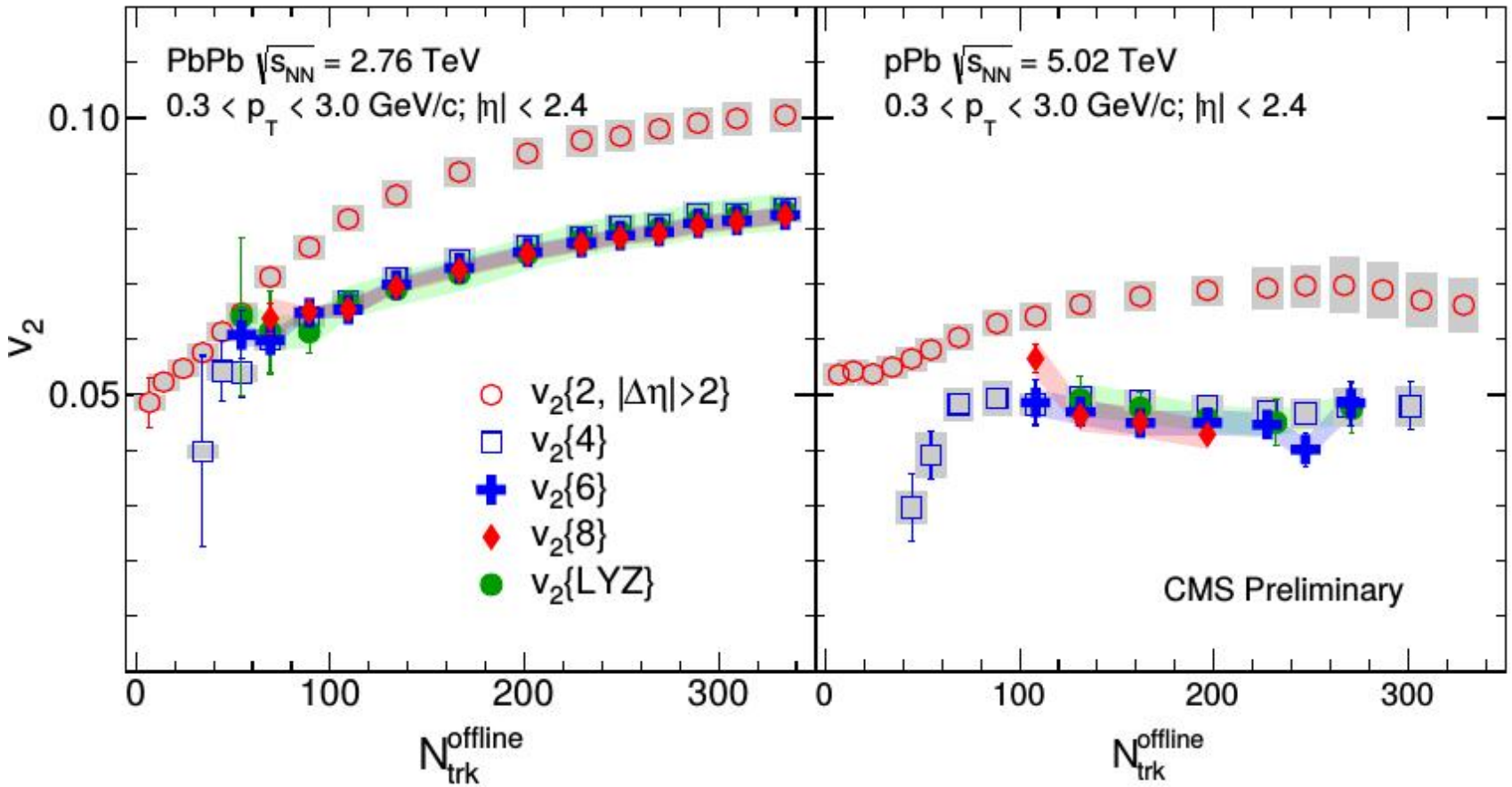
Remarks:

- EbE distribution of ε is not trivial! (Non-Gaussian)
- Elliptic power and power universally describe EbE ε in pp, pA and AA:
  - ε_2, ε_3,
  - for different models: MC-Glb, MC-KLN, IP-Glasma
  - for different centrality classes.

A good understanding of the statistical property of ε.

Fluctuations in p-Pb system: cumulants of v_2

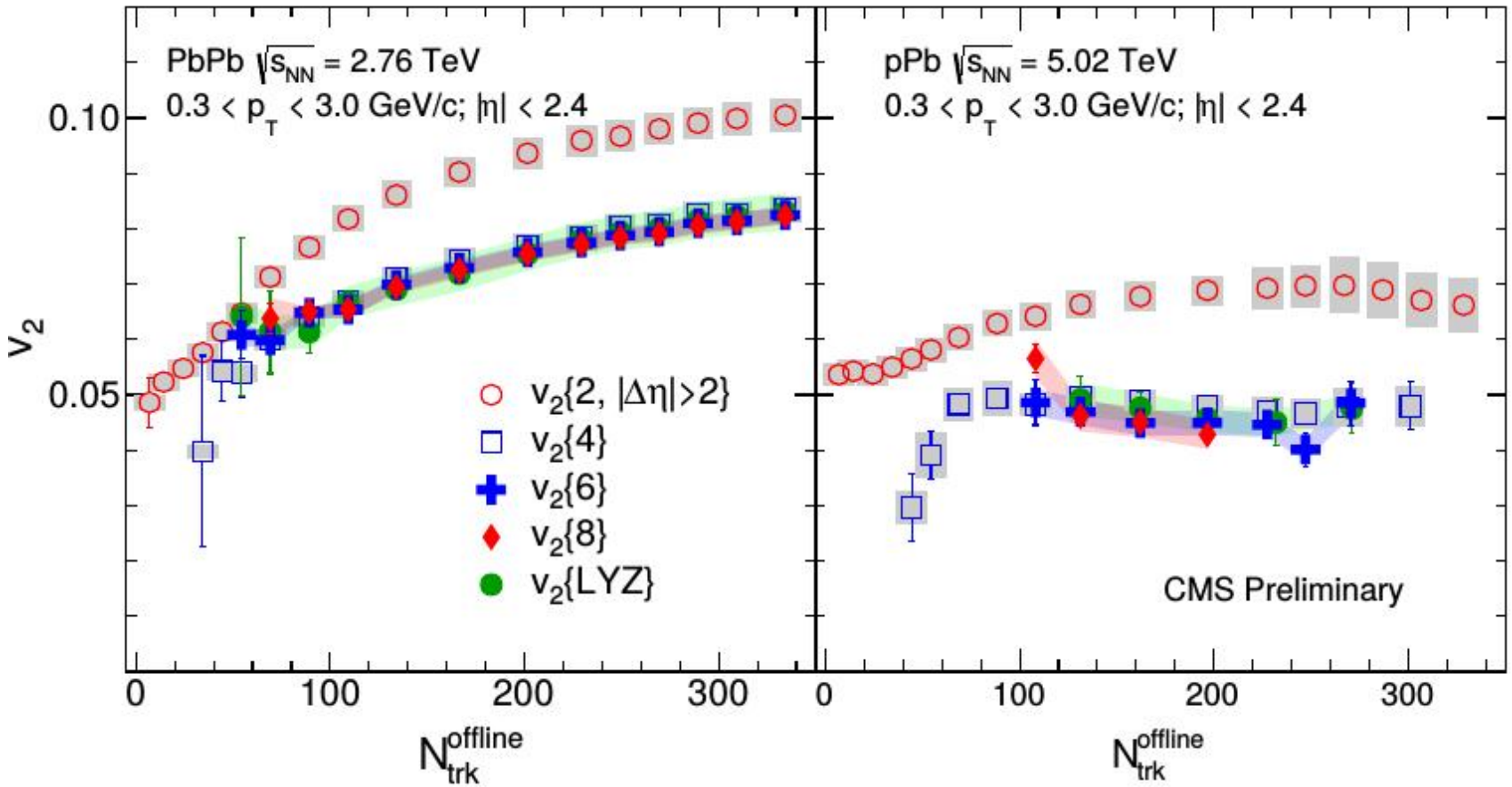
CMS collaboration, CMS-PAS-HIN-14-006



$$0 < v_2\{8\} \approx v_2\{6\} \approx v_2\{4\} < v_2\{2\}$$

Fluctuations in p-Pb system: cumulants of v_2

CMS collaboration, CMS-PAS-HIN-14-006



$$0 < v_2\{8\} \approx v_2\{6\} \approx v_2\{4\} < v_2\{2\}$$

power distribution: $\Leftrightarrow$ $0 < \varepsilon_2\{8\} \approx \varepsilon_2\{6\} \approx \varepsilon_2\{4\} < \varepsilon_2\{2\}$

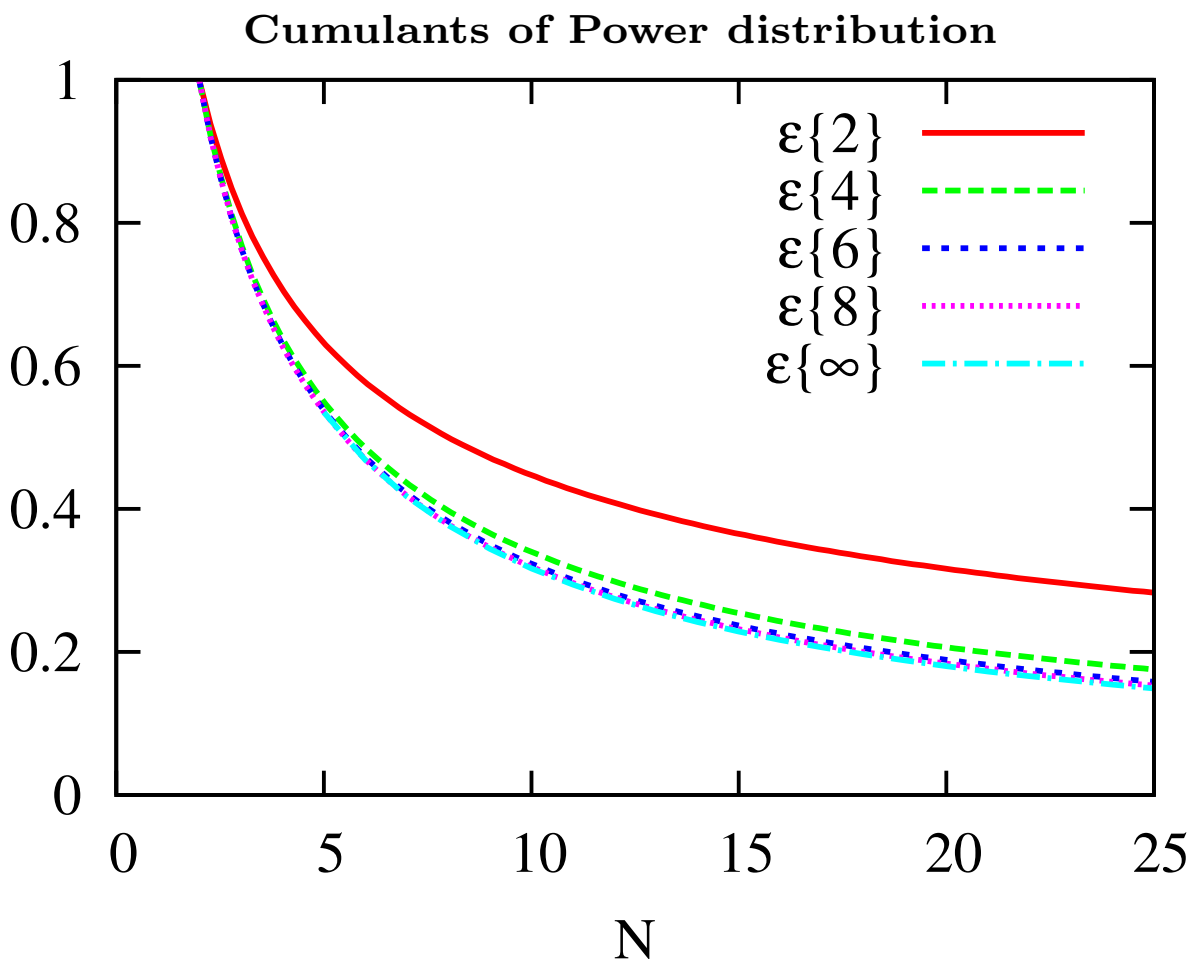
Cumulant of ε_2 in p-Pb

- Model predictions of cumulants of $\varepsilon\{m\}$:

	Gauss	BG	Power
$\varepsilon\{2\}$	σ	$\sqrt{\sigma^2+\bar{\varepsilon}^2}$	$\frac{1}{\sqrt{1+\alpha}}$
$\varepsilon\{4\}$	0	$\bar{\varepsilon}$	$\left[\frac{2}{(1+\alpha)^2(2+\alpha)}\right]^{1/4}$
$\varepsilon\{6\}$	0	$\bar{\varepsilon}$	$\left[\frac{6}{(1+\alpha)^3(2+\alpha)(3+\alpha)}\right]^{1/6}$
$\varepsilon\{8\}$	0	$\bar{\varepsilon}$	$\left[\frac{48\left(1+\frac{5\alpha}{11}\right)}{(1+\alpha)^4(2+\alpha)^2(3+\alpha)(4+\alpha)}\right]^{1/8}$

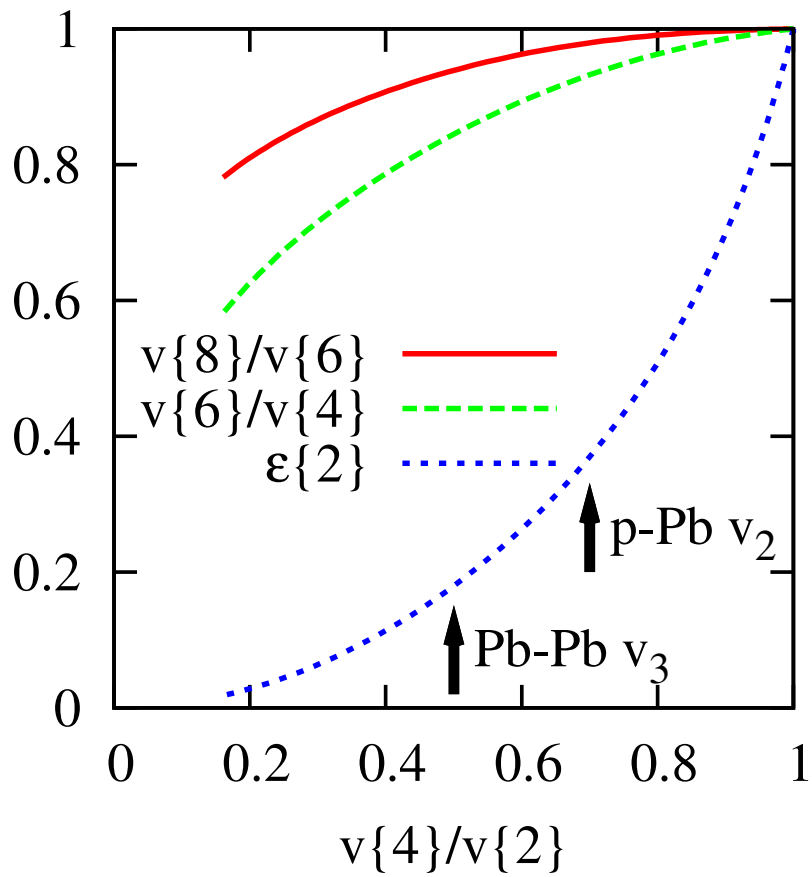
$\Rightarrow$ Analytical relations between cumulants $\varepsilon\{m\}$.

- Qualitative pattern from Power: $0 < \varepsilon_2\{8\} \approx \varepsilon_2\{6\} \approx \varepsilon_2\{4\} < \varepsilon_2\{2\}$



(See also Bozek et al. arXiv:1304.3044 and Bzdak et al. arXiv:1311.7325)

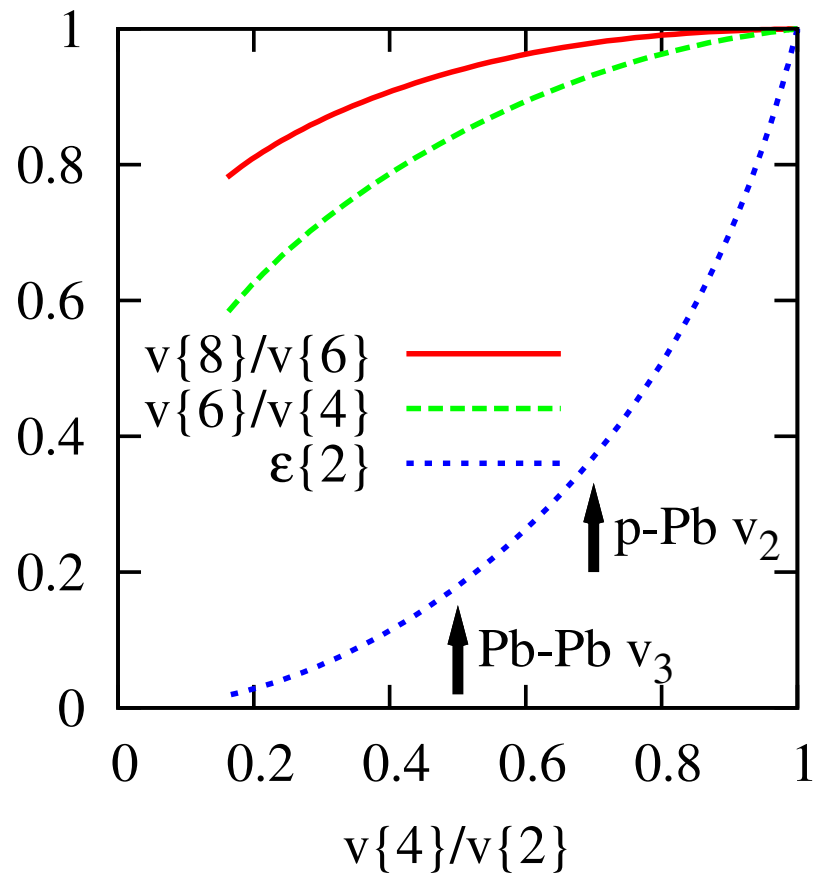
- Quantitatively from Power:



By solving α and:

$$\frac{\varepsilon\{4\}}{\varepsilon\{2\}} = \frac{v\{4\}}{v\{2\}}, \ldots$$

- Quantitatively from Power:



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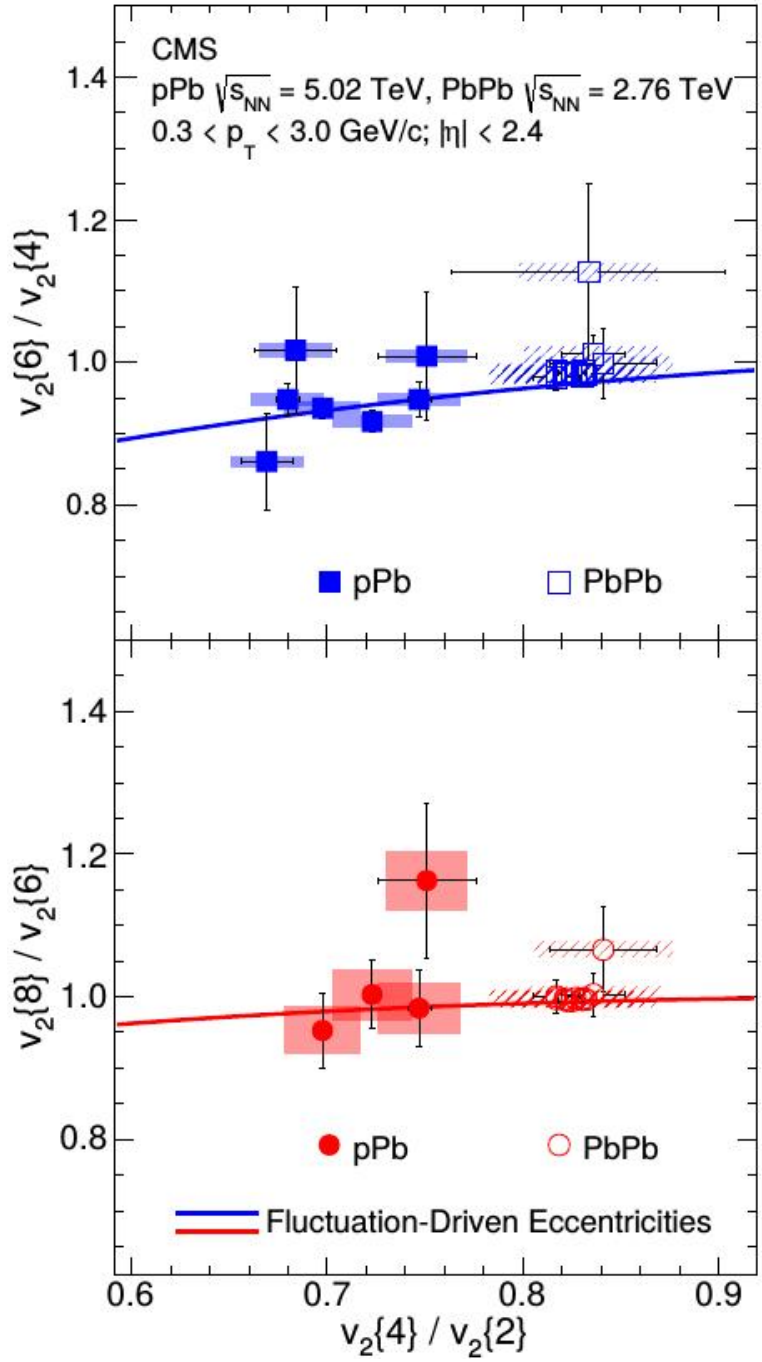

CMS collaboration, CMS-PAS-HIN-14-006

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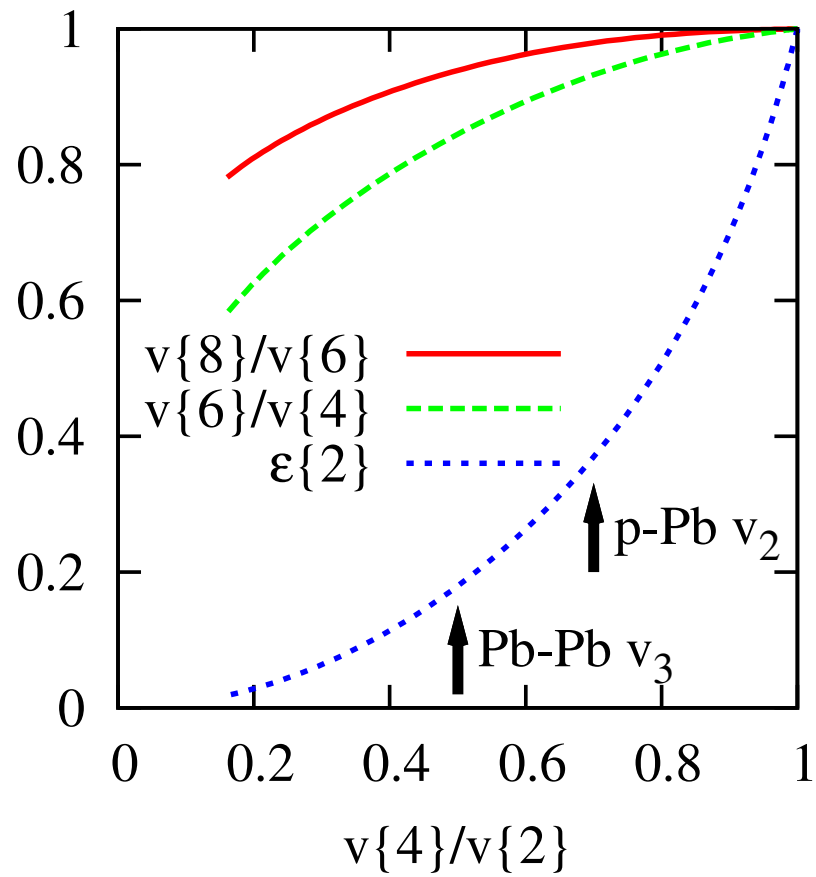

By solving α and:

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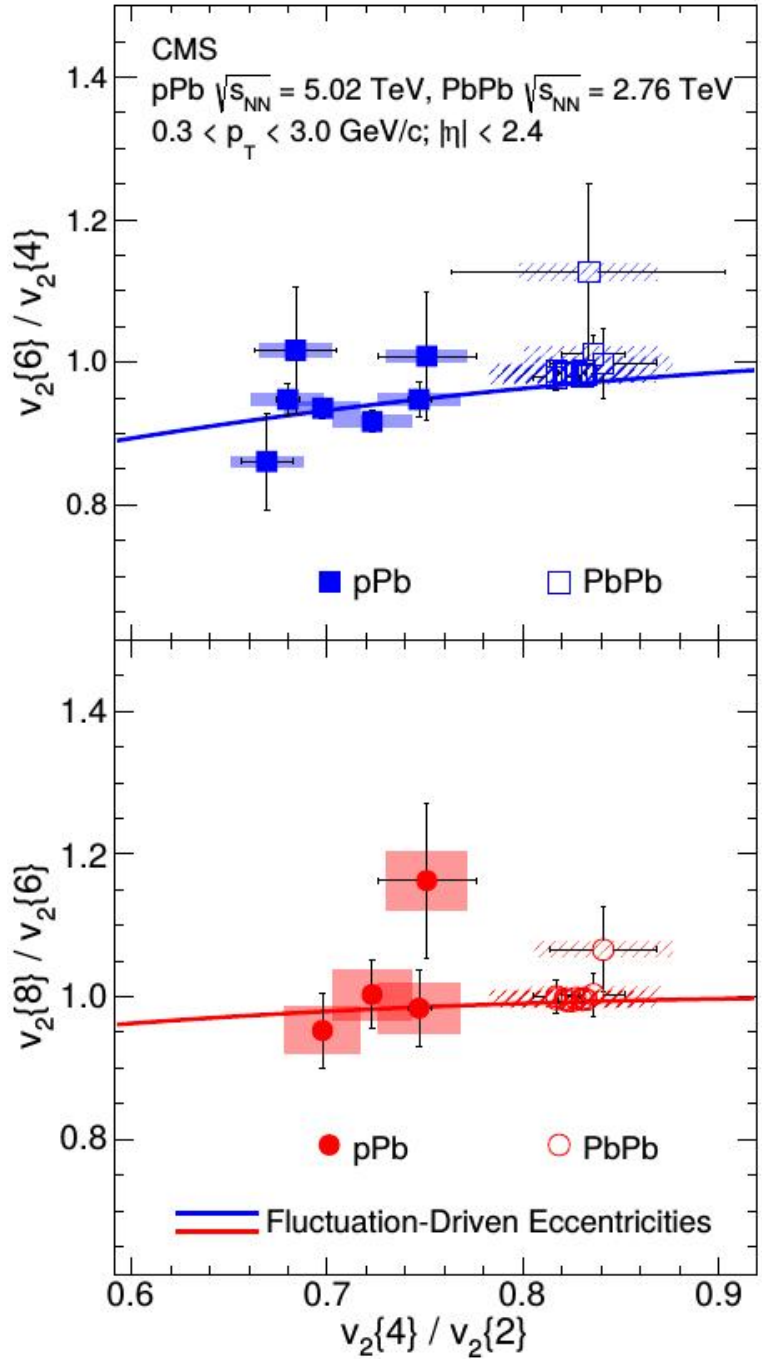
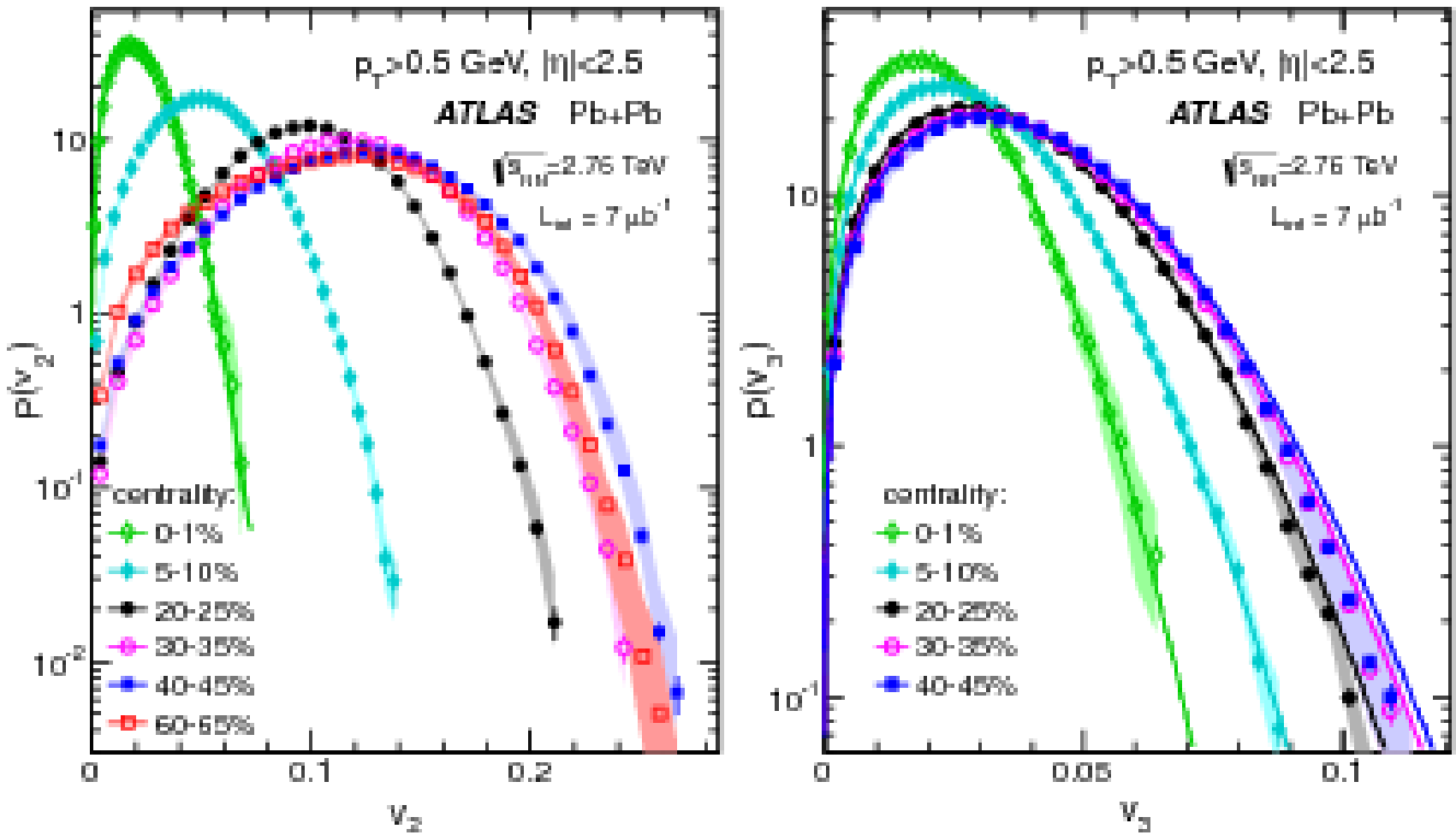
Signals of collective expansion?



CMS collaboration, CMS-PAS-HIN-14-006

Pb-Pb EbE V_n distribution:

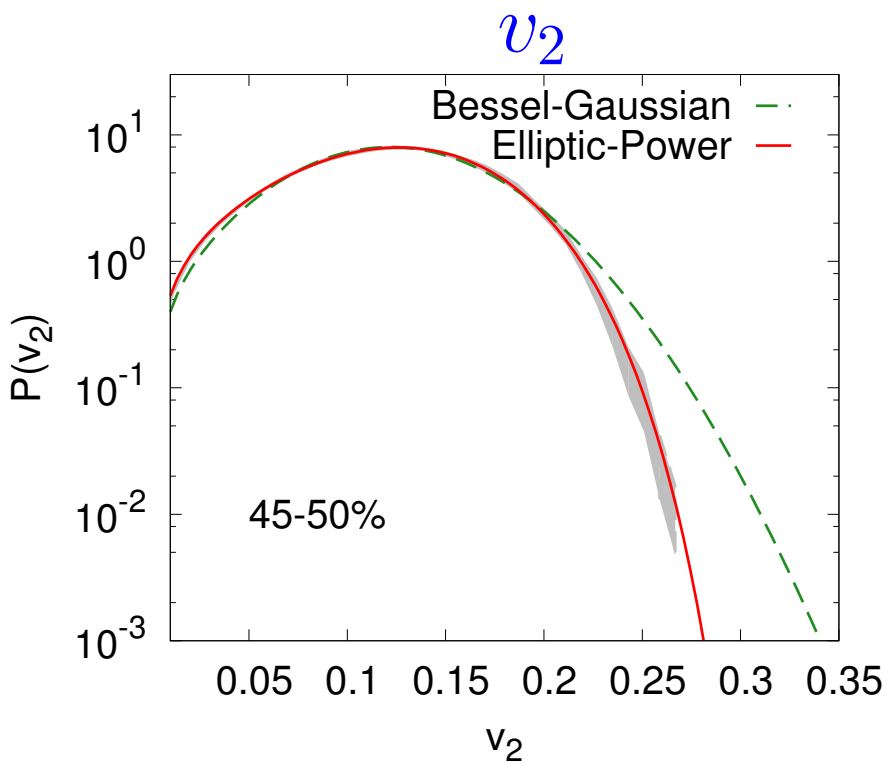


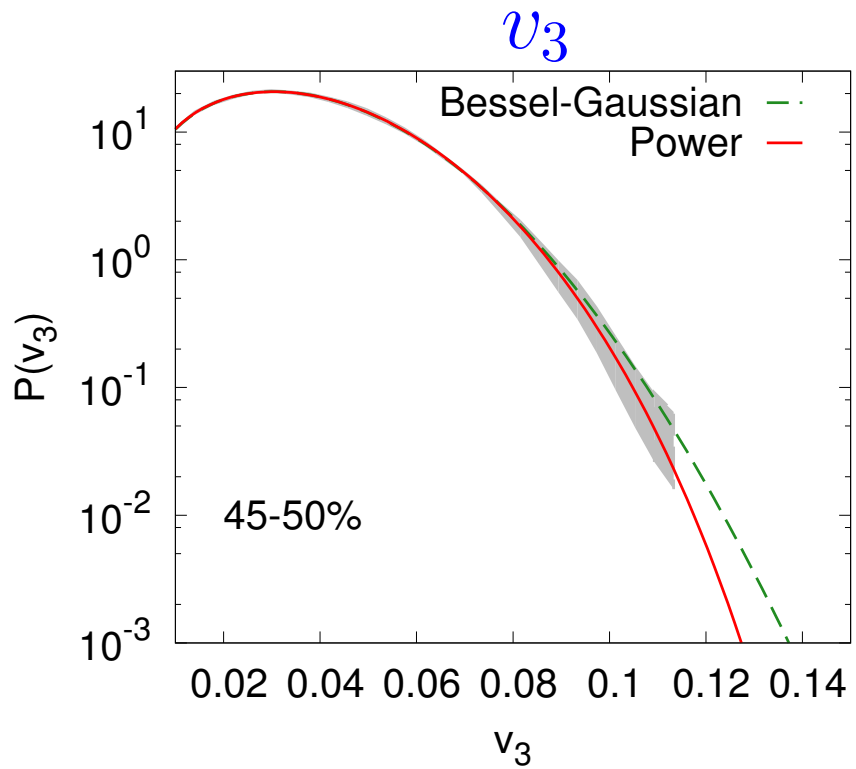
Rescaled (Elliptic) Power: fit ATLAS EbyE v_n distribution

- Distribution of v_n is 'rescaled' Elliptic Power or Power distribution.

$$P(\varepsilon_n)d\varepsilon_n = P(\varepsilon_n(v_n,\kappa_n))\left|\frac{\partial\varepsilon_n}{\partial v_n}\right| dv_n \to \underline{P(v_n/\kappa_n)/\kappa_n}dv_n \quad \to \text{fit } v_n \text{ distribution}$$

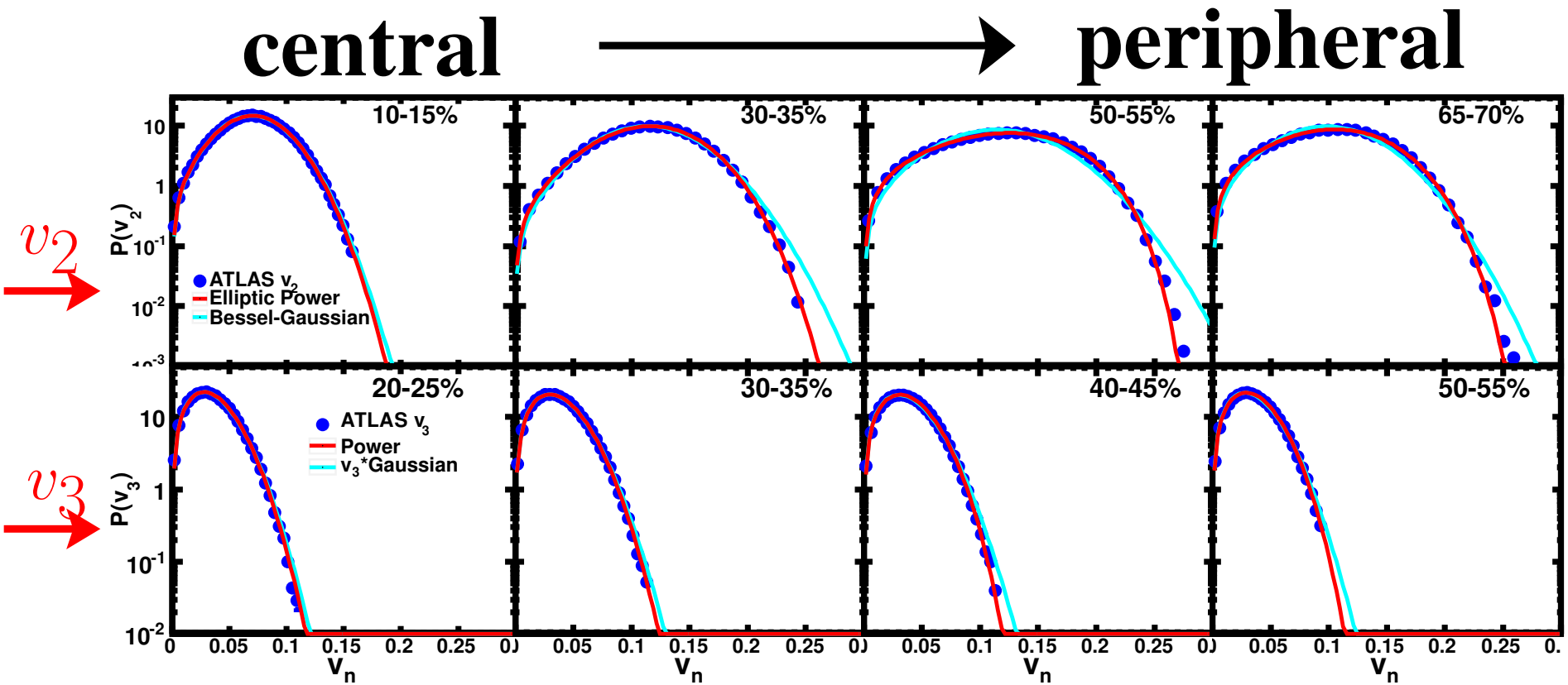
- Rescaled (Elliptic) Power parameterization: ATLAS v_2 and v_3 at 45-50%.





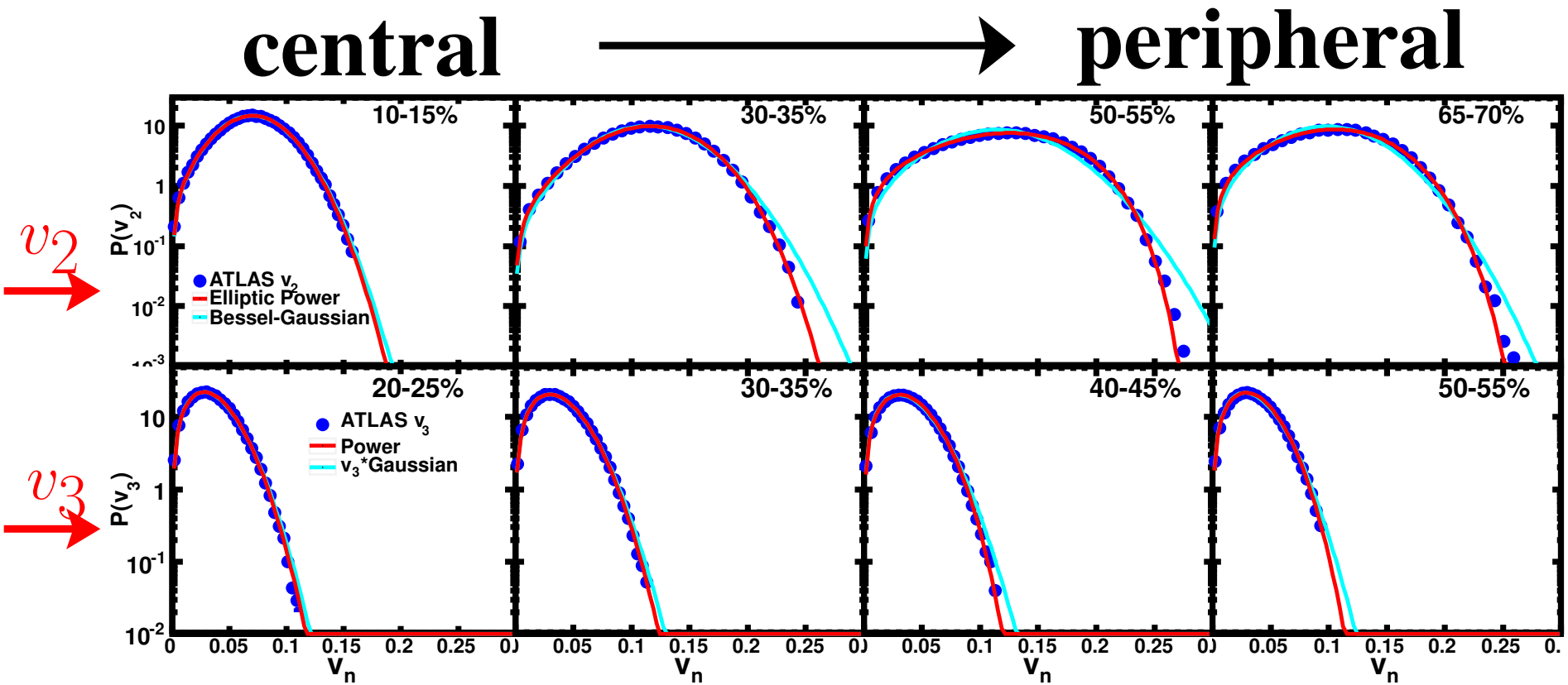
- Non-Gaussianity: $\kappa_n \Rightarrow$ flow resp., and $(\varepsilon_0, \alpha) \Rightarrow$ information of initial state.

ATLAS EbyE v_n distribution



- Universal fit with Elliptic Power and Power parameterizations.
- *Disentangled* parameters can be read off from fit: κ_n, (ε_0, α)

ATLAS EbyE v_n distribution

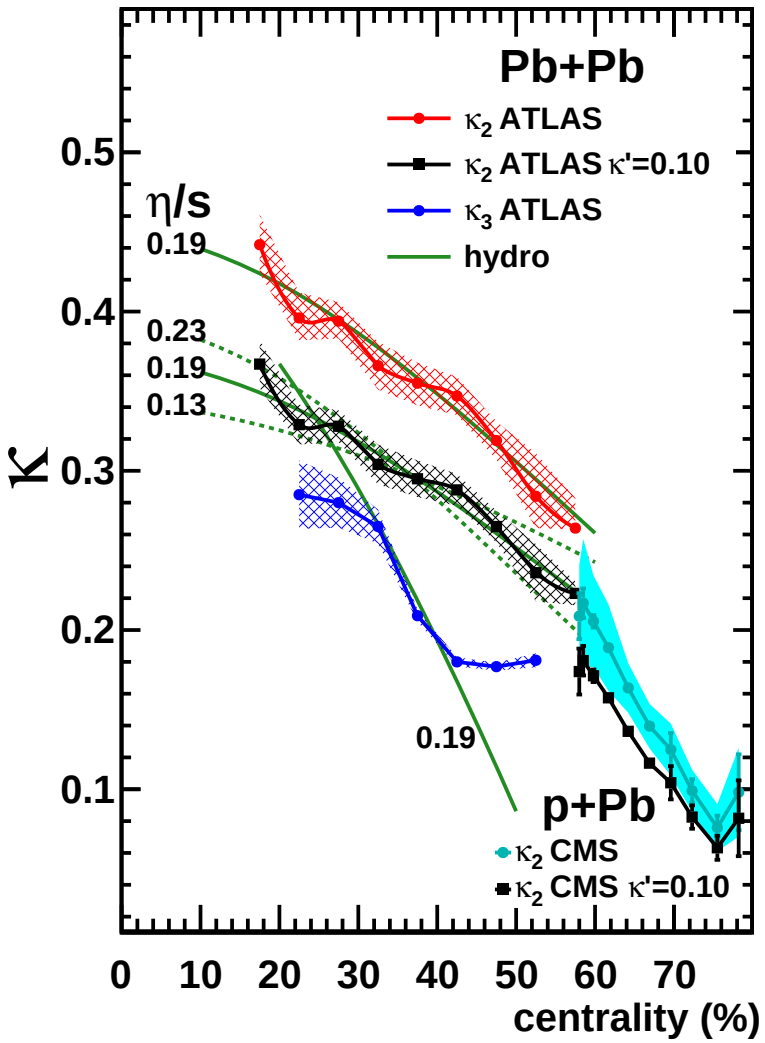


- Universal fit with Elliptic Power and Power parameterizations.
- *Disentangled* parameters can be read off from fit: κ_n, (ε_0, α)
- Beyond linear eccentricity scaling:

$$v_n = \kappa_n \varepsilon_n + \kappa_n \kappa_n' \varepsilon_n^3 + \ldots \quad \rightarrow \quad \left| \frac{dv_n}{d\varepsilon_n} \right|$$

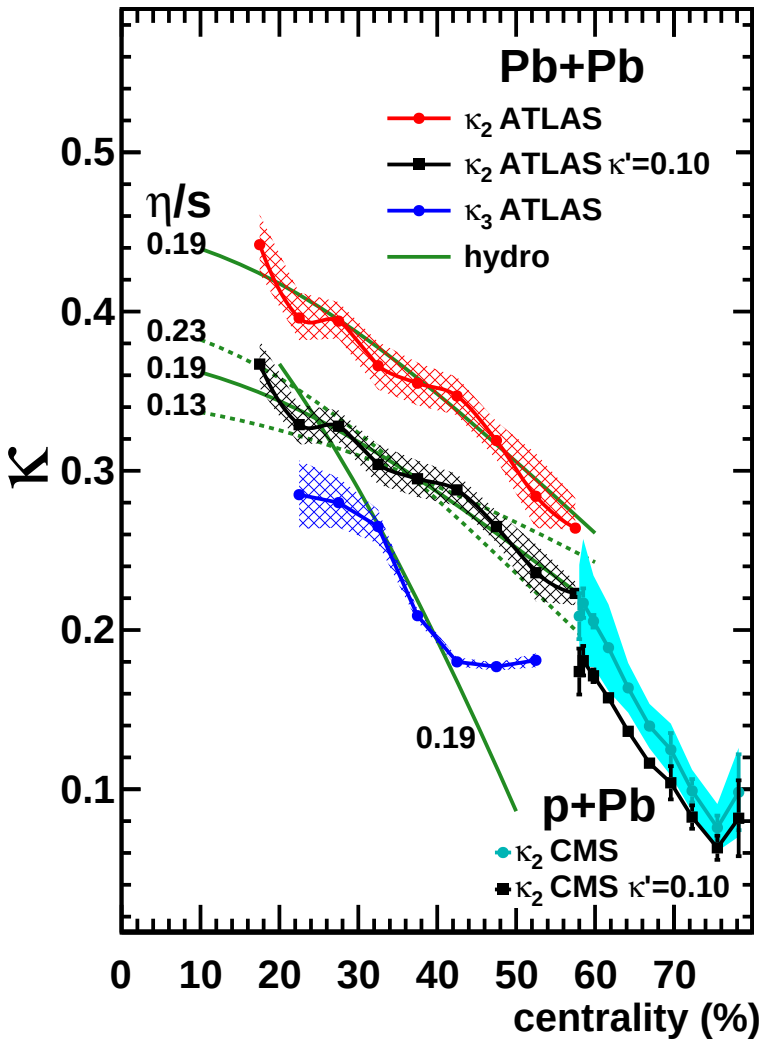
κ_n and extracting η/s in hydrodynamic response

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ from fit



κ_n and extracting η/s in hydrodynamic response

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ from fit



- Fit by hydro. leads to estimate of η/s

Event-Plane correlations:

- Initial state plane correlations + medium expansion (for higher harmonics).
- Initial state plane correlations: <u>fluctuation</u> + <u>background geometry</u>
- Correlation between V_2 and V_3: ATLAS scalar-product method

  $\langle \cos 6(\Phi_2 - \Phi_3) \rangle_w$

Event-Plane correlations:

- Initial state plane correlations + medium expansion (for higher harmonics).
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- Correlation between V_2 and V_3: ATLAS scalar-product method

$$\langle \cos 6(\Phi_2 - \Phi_3) \rangle_w \Leftrightarrow \quad \mathcal{P}_{23}^{vv} = \frac{\langle V_2^3 (V_3^*)^2 \rangle}{\langle |V_2|^6 \rangle^{1/2} \langle |V_3|^4 \rangle^{1/2}}$$

Event-Plane correlations:

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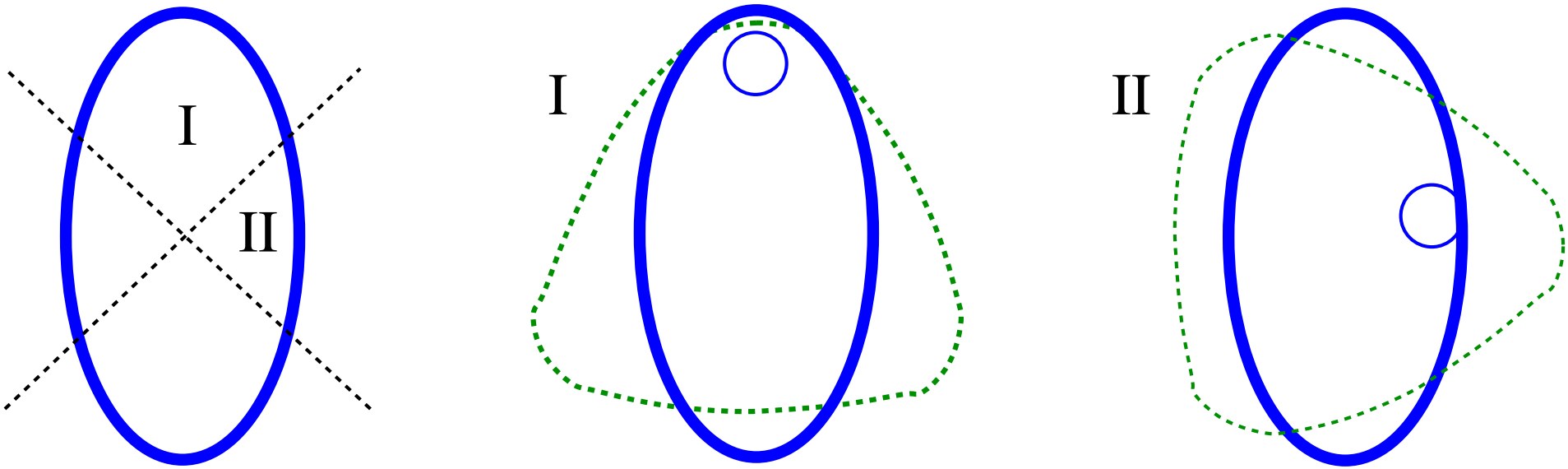
$$\langle \cos 6(\Phi_2 - \Phi_3) \rangle_w \Leftrightarrow \quad \mathcal{P}_{23}^{vv} = \frac{\langle V_2^3 (V_3^*)^2 \rangle}{\langle |V_2|^6 \rangle^{1/2} \langle |V_3|^4 \rangle^{1/2}} \Leftrightarrow \quad \mathcal{P}_{23}^{\varepsilon\varepsilon} = \frac{\langle \mathcal{E}_2^3 (\mathcal{E}_3^*)^2 \rangle}{\langle |\mathcal{E}_2|^6 \rangle^{1/2} \langle |\mathcal{E}_3|^4 \rangle^{1/2}}$$

- Qualitatively, (anti-correlation between $\mathcal{E}_2$ and $\mathcal{E}_3$ w.r.t. an elliptic shape)



$\mathcal{P}_{23}^{\varepsilon\varepsilon} = -1$ $\qquad$ $\mathcal{P}_{23}^{\varepsilon\varepsilon} = 1$

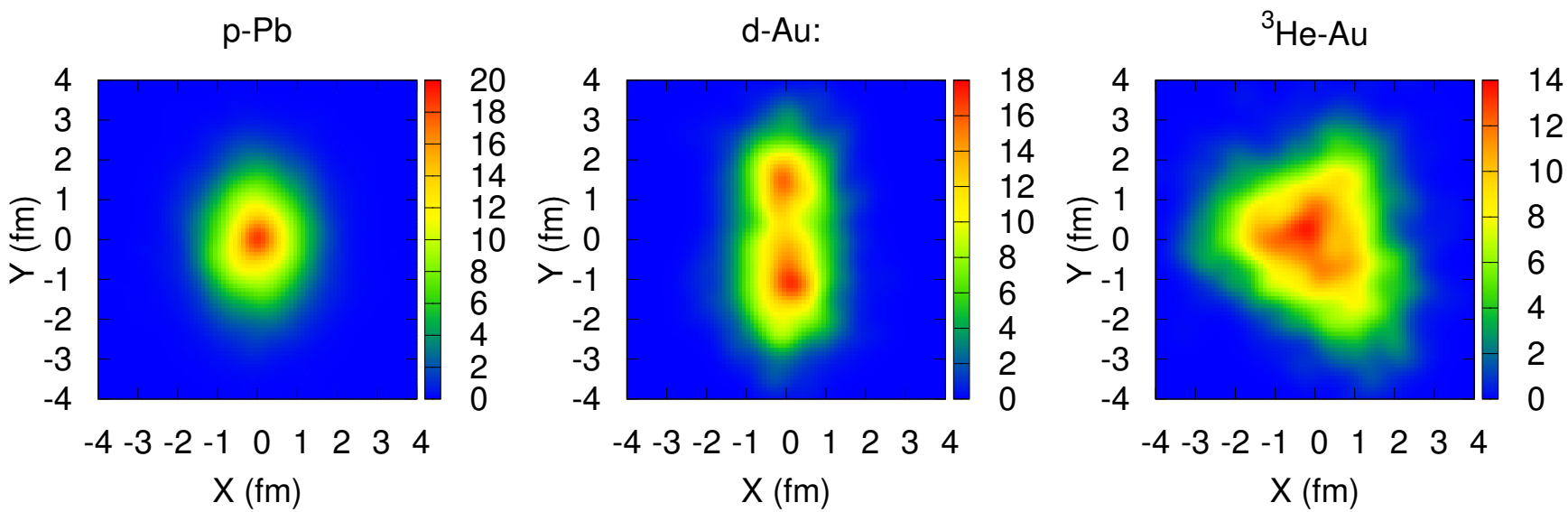
Anti-correlations between $\mathcal{E}_2$ and $\mathcal{E}_3$ w.r.t. a triangular shape.

- Quantitatively, from N-independent sources:

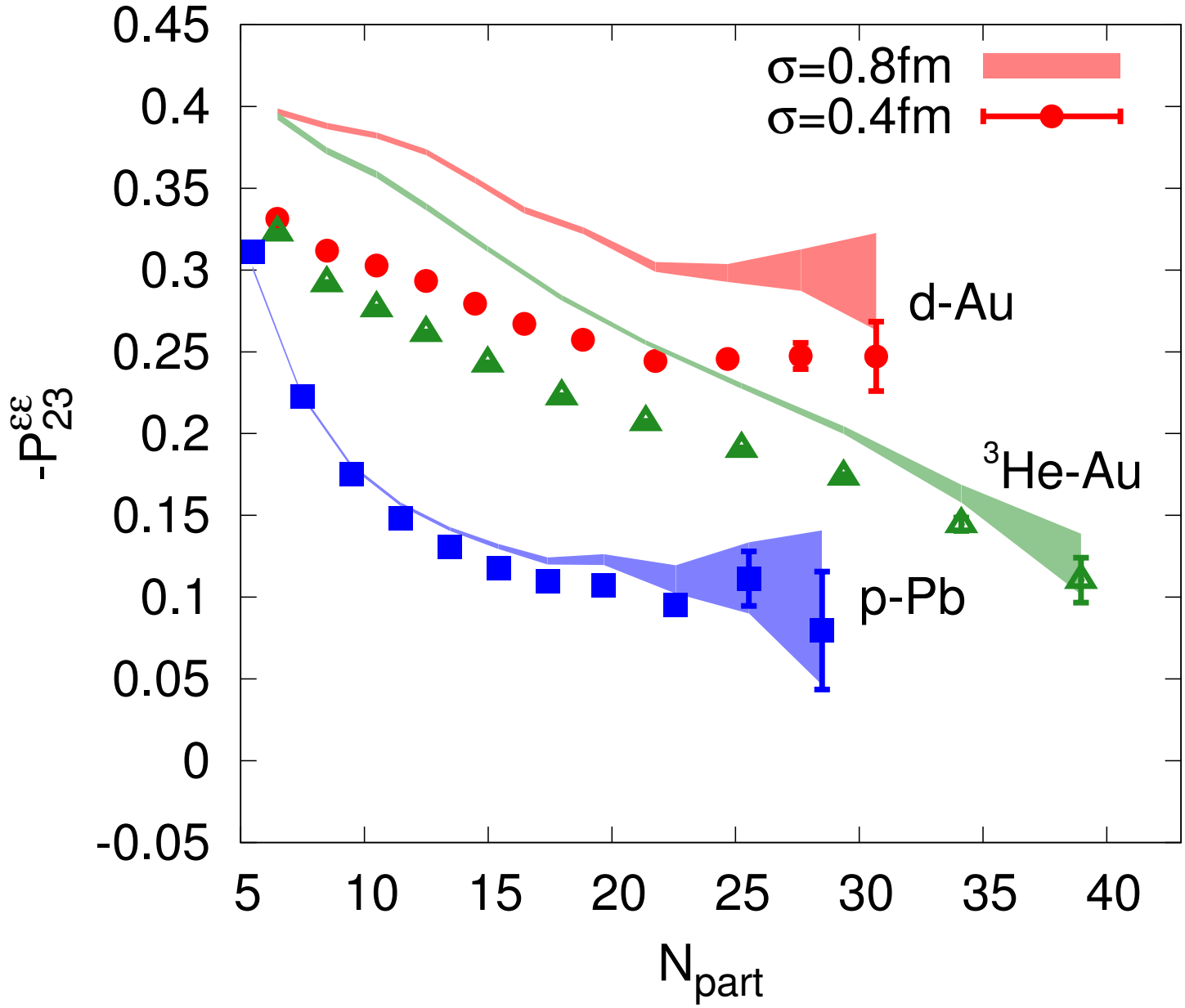
$$|\mathcal{P}_{23}^{\varepsilon\varepsilon}(\bar{\mathcal{E}} = 0)| \sim O\left(\frac{1}{N^{3/2}}\right), \quad |\mathcal{P}_{23}^{\varepsilon\varepsilon}(\bar{\mathcal{E}}_3 \neq 0)| \sim O\left(\frac{1}{N^{1/2}}\right), \quad |\mathcal{P}_{23}^{\varepsilon\varepsilon}(\bar{\mathcal{E}}_2 \neq 0)| \sim O(1)$$

Central collision events of p-Pb, d-Au, ^{3}He-Au

Averaged density profile of central events: (PHOBOS Monte Carlo Glauber)

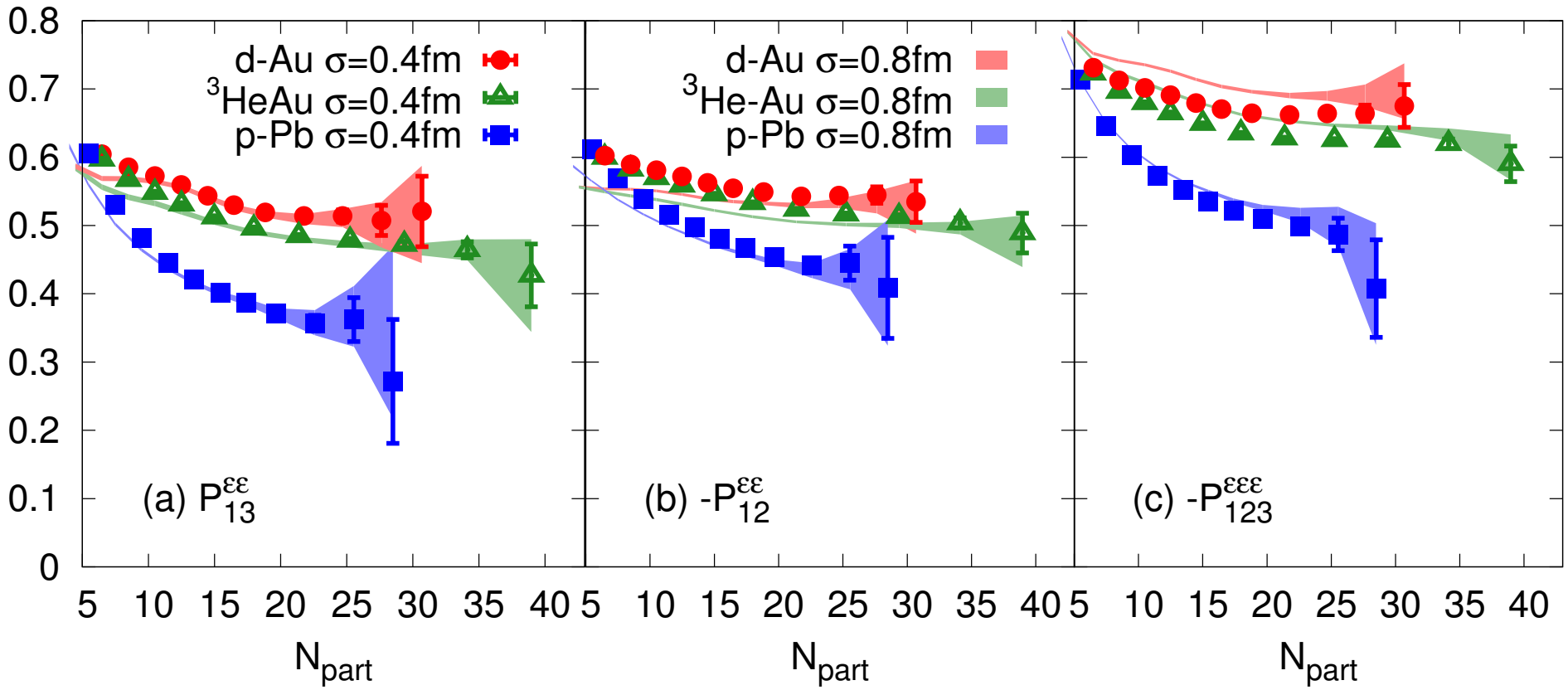


- Medium collective expansion?
- Intrinsic geometry due to the structure of proton, deuteron and ^{3}He



PHOBOS MC-Glb, with Negative Binomial weighting

Correlations involving V_1:



PHOBOS MC-Glb, with Negative Binomial weighting

Summary

- Fluctuations of harmonics: parameterizations with Elliptic Power and Power
  1. Applications to p-Pb: cumulants of harmonic flow in p-Pb
  2. Applications to Pb-Pb (also p-Pb): disentangle initial state and medium exp.:

$$\eta/s \qquad \text{and} \qquad (\alpha, \varepsilon_0)$$

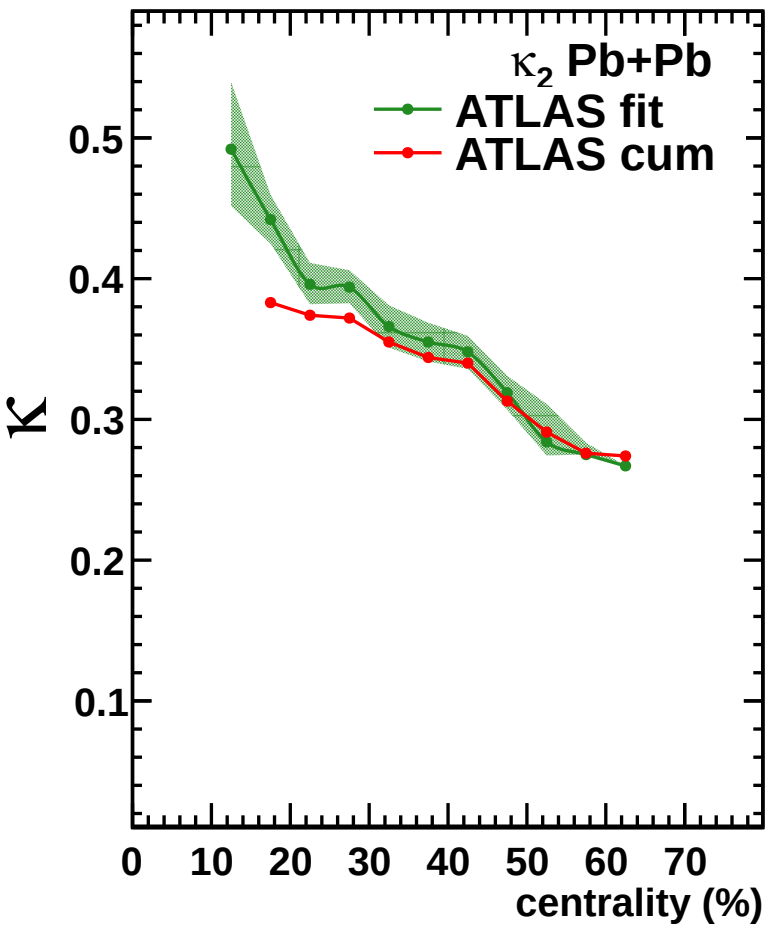
- Correlations of harmonics in small colliding systems:
  1. Anti-correlations between V_2 & V_3 in d-Au, ^{3}He-Au: geometry + fluctuations
  2. Anti-correlation between V_2 & V_3 in p-Pb: fluctuations.
  3. Correlation patterns involving V_1.

<u>Alternative test of collectivity in small colliding systems.</u>

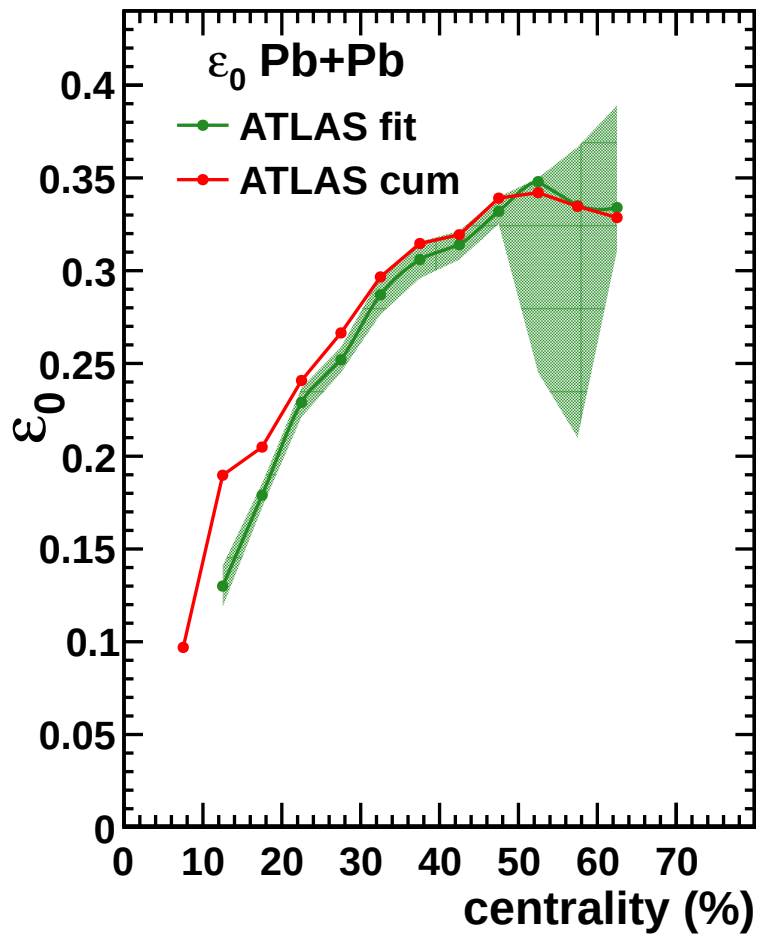
Back-up slides

Solving cumulants of v_n

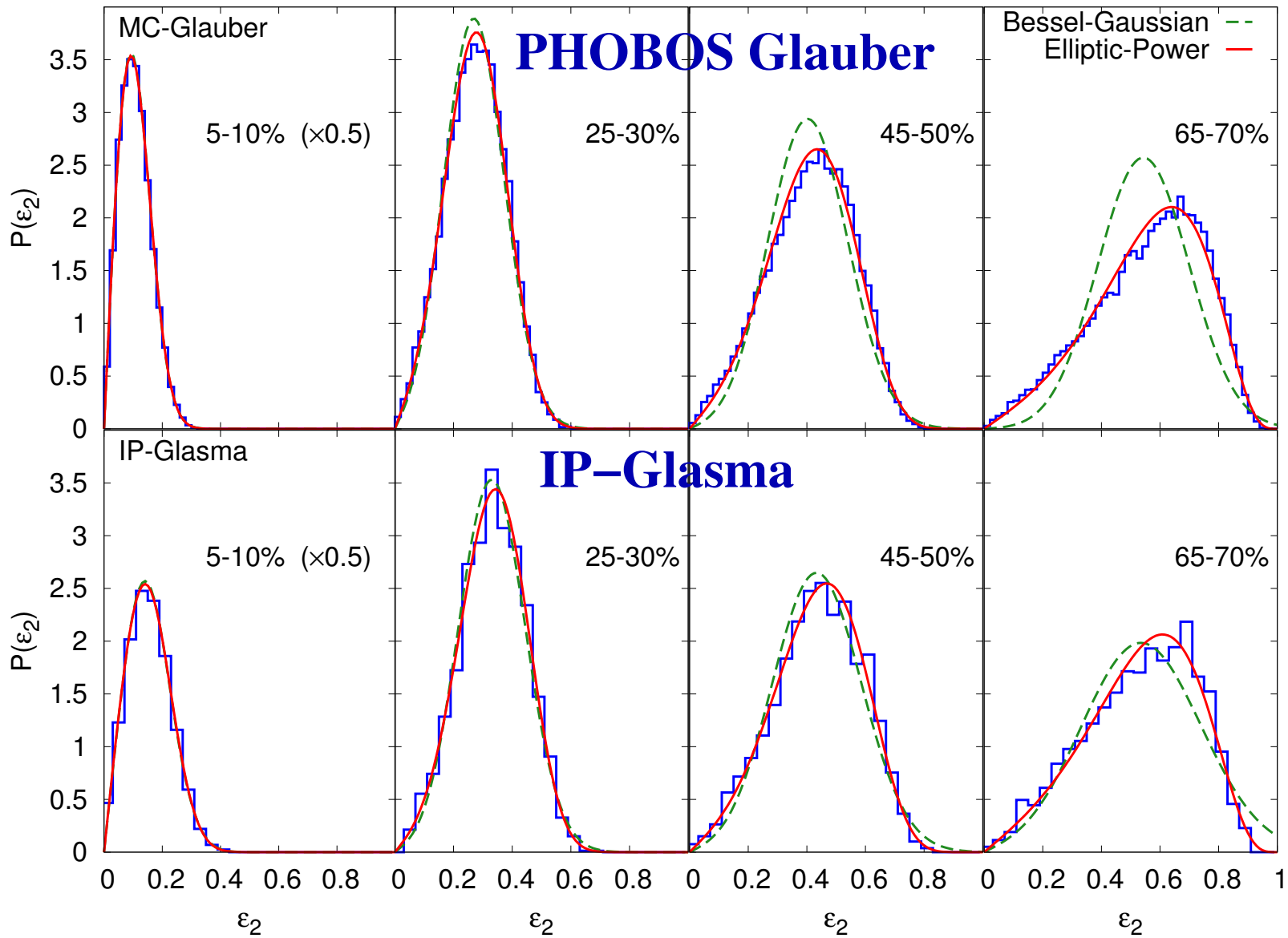
Resp. coefficient



Average RP eccentricity

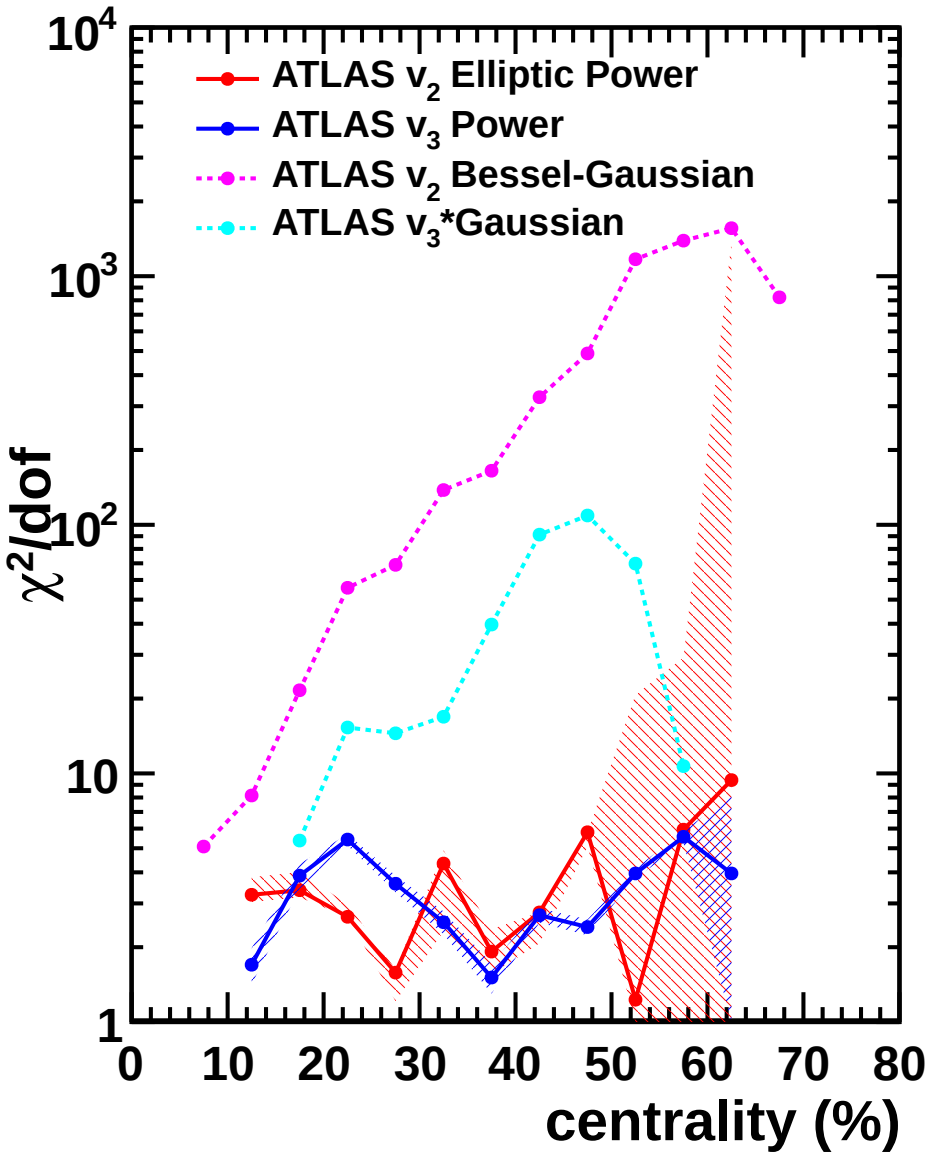


Elliptic Power distribution: test of universality



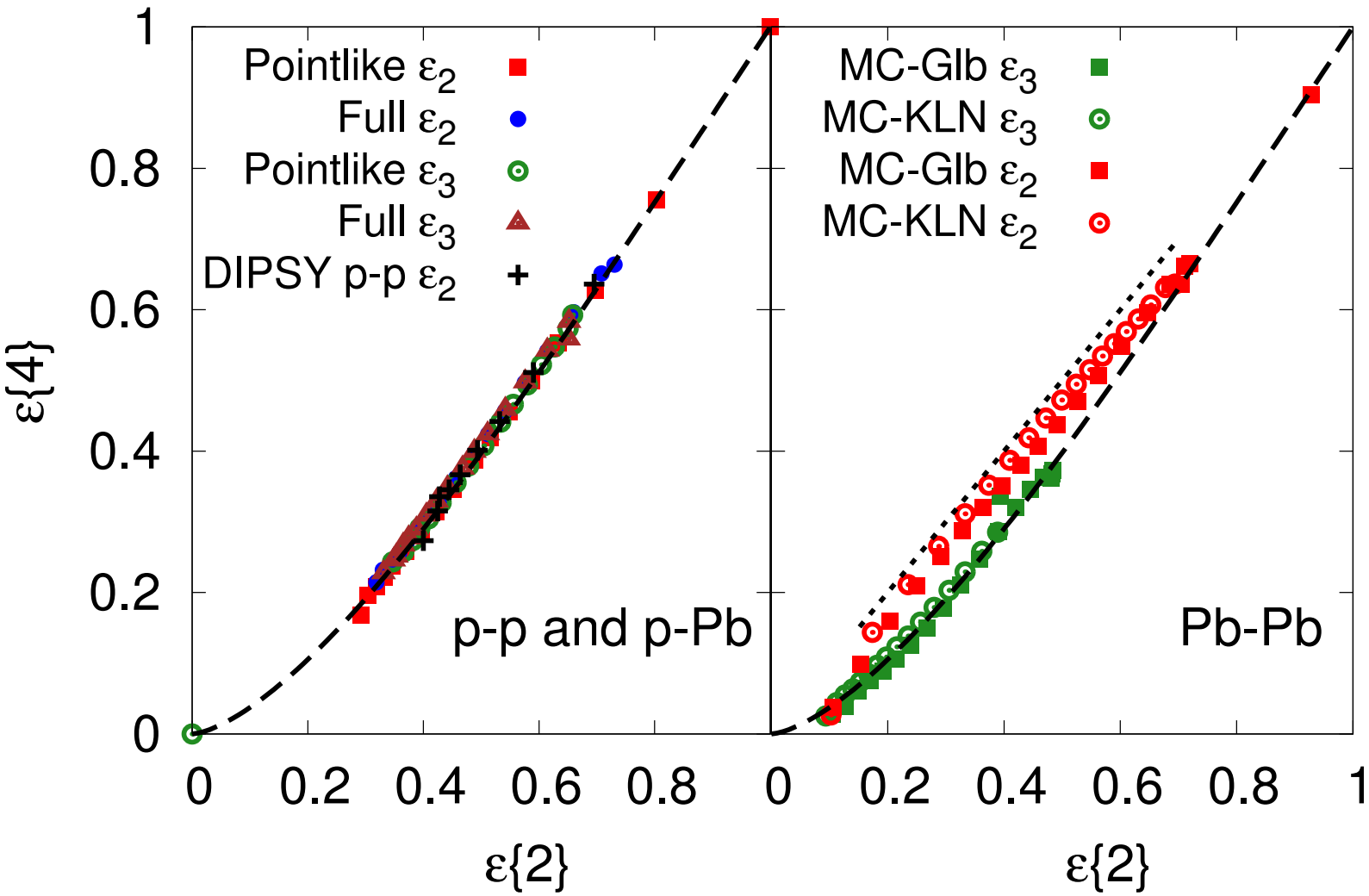
PHOBOS Glauber (B.Alver et al, arXiv:0805.4411) *IP-Glasma (B.Schenke et al, arXiv:1312.5588)*

Goodness of fit



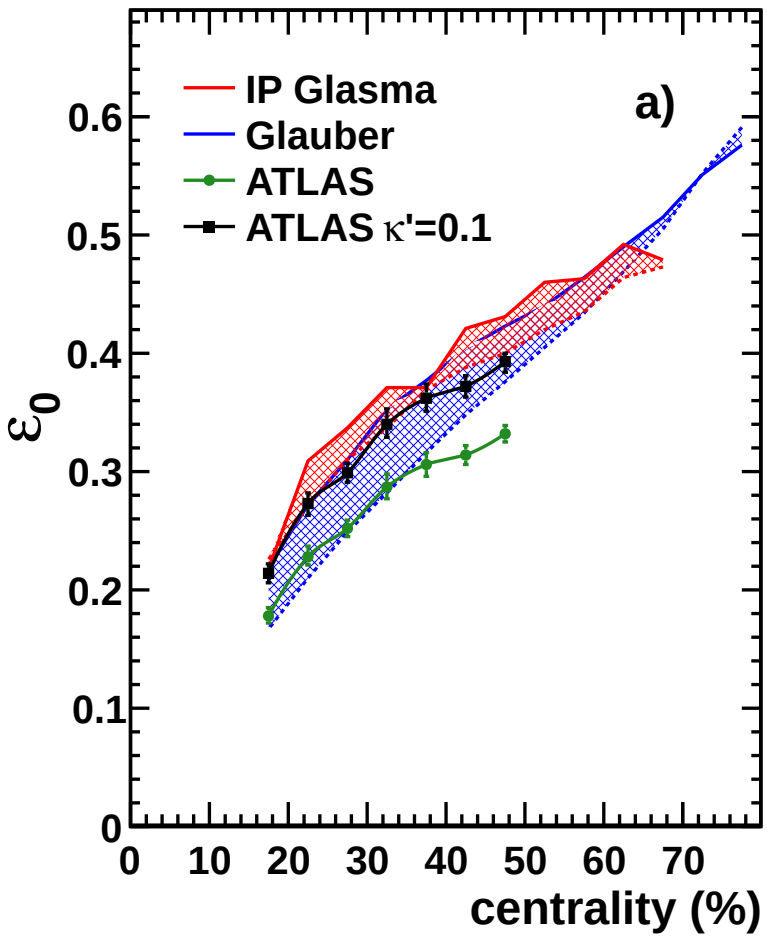
Power distribution: test of universality

Analytical relation: $\varepsilon\{4\} = \varepsilon\{2\}^{3/2} \left(\frac{2}{1+\varepsilon\{2\}^2}\right)^{1/4}$

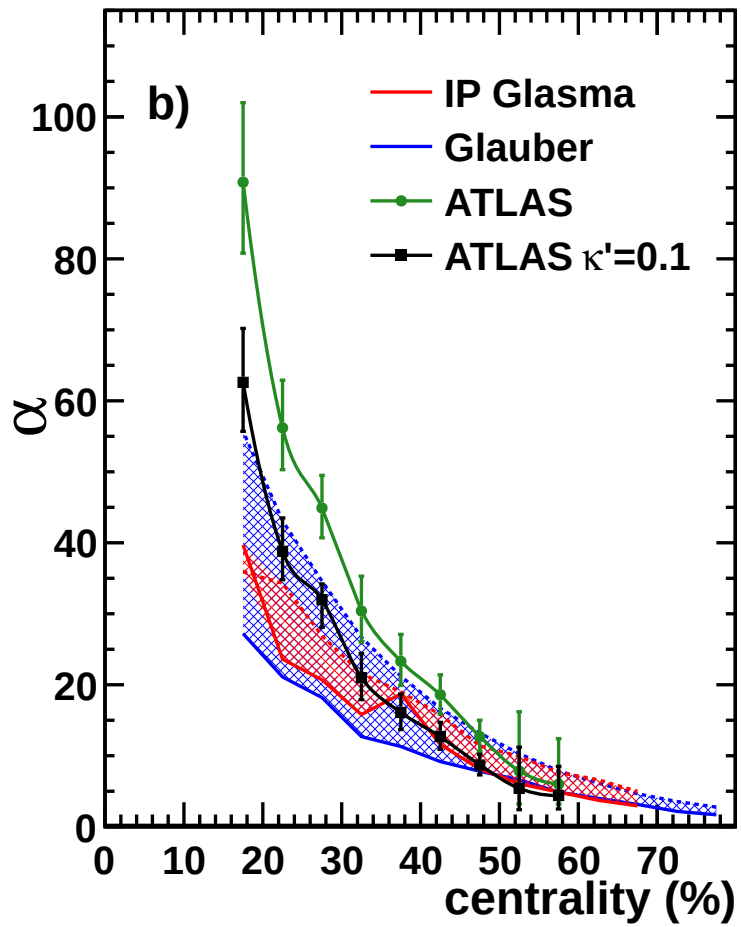


Extract information of initial state from the fit

Average RP eccentricity



Fluctuations



- Fluctuations become stronger for peripheral collisions.
- ε_0 grows with centrality percentage.

Collectivity in Small Colliding Systems with High Multiplicity

March 4-6, 2015
Large Seminar Room, Physics, Bldg. 510

WEDNESDAY, MARCH 4

08:30-09:00 **REGISTRATION**

09:00-12:20 Session I. Chair: Larry McLerran

09:00-09:10 Robert Pisarski.....*Welcome*
09:10-09:50 David Morrison.....*Summary of RHIC results on small colliding systems*
09:50-10:30 Gunther Roland.....*Summary of LHC results on small colliding systems*

10:30-11:00 **COFFEE BREAK**

11:00-11:40 Miklos Gyulassy .....*Theoretical perspectives on small colliding systems*
11:40-12:20 Edward Shuryak.....*What's the "matter" created in high multiplicity pp/pA collisions?*

12:20-14:00 **LUNCH**

14:00-17:50 Session II. Chair: Tetsufumi Hirano

14:00-14:40 Nicolas Borghini .....*Review of cumulant framework and experimental techniques*
14:40-15:20 Jiangyong Jia .....*Meanings of cumulant measurements*
15:20-16:00 Fuqiang Wang.....*Meanings of Ridge measurements*

16:00-16:30 **COFFEE BREAK**

16:30-17:10 Constantin Loizides .....*Event selection and centrality bias in pA collisions*
17:10-17:50 Sooraj Radhakrishnan .....*Roles of short range correlations and recoil subtraction*

THURSDAY, MARCH 5

09:00-12:30 Session III. Chair: Jiangyong Jia

09:00-09:40 Jurgen Schukraft .....*A comparative perspective on the small versus the large*
09:40-10:20 Jamie Nagle .....*What can we learn from He3-Au and proton-Au collisions at RHIC?*
10:20-11:00 Vladimir Skokov.....*Initial asymmetries and "emergent collectivity" in high energy pA collisions*

11:00-11:30 **COFFEE BREAK**

11:30-12:00 Piotr Bozek .....*Hydrodynamic simulations of pp/pA collisions*
12:00-12:30 Igor Kozlov.....*Collective behavior in small colliding systems*

12:30-14:00 **LUNCH**

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14:00-18:00 Session IV. Chair: Jinfeng Liao

14:00-14:30 Kevin Dusling .....*Ridge from glasma production in pp and pA collisions*
14:30-15:00 Soeren Schlichting .....*Gluon v_n's from classical Yang-Mills dynamics*
15:00-15:30 Abhijit Majumder .....*Coherent color fields and high multiplicity pp/pA collisions*

15:30-16:00 **COFFEE BREAK**

16:00-16:30 Roy Lacey .....*Acoustic scaling analysis from AA to pA/dA collisions*
16:30-17:00 Gabriel Denicol.....*Dissipative limit and Knudsen number for small colliding systems*
17:00-18:00 Discussion Session.....*Is there collectivity? What's behind such collectivity?(Moderator: Jinfeng Liao)*

18:30-20:00 **WORKSHOP DINNER** @ Berkner Hall Lobby

FRIDAY, MARCH 6

09:00-12:30 Session V. Chair: Constantin Loizides

09:00-09:30 Bjoern Schenke .....*Hydrodynamics in small systems with glasma initial conditions*
09:30-10:00 Adam Bzdak .....*Cumulants in small system & Ridge from AMPT*
10:00-10:30 Li Yan .....*Fluctuating eccentricities in pp, pA and AA collisions*

10:30-10:50 **COFFEE BREAK**

10:50-11:15 Aleksas Mazeliauskas .....*Subleading harmonic flow in heavy ion collisions*
11:15-11:40 Yin Jiang.....*Size, shape, and dissipative length in small colliding systems*
11:40-12:05 Yuji Hirono.....*HBT study of the pp collisions*
12:05-12:30 Angelo Esposito.....*Effect of hadronization on v_n from gluon bremsstrahlung in pA*

12:30-14:00 **LUNCH**

14:00-16:40 Session VI. Chair: Rob Pisarski

14:00-14:40 Tetsufumi Hirano .....*Hydrodynamic noise in small systems*
14:40-15:20 Zhangbu Xu .....*From pA to eA: what we know and what we expect to learn?*
15:20-16:00 Wei Li .....*Experimental status and future measurements for pp/pA collisions*
16:00-16:40 Larry McLerran .....*Workshop summary*

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Collectivity in Small Colliding Systems with High Multiplicity

March 4-6, 2105
Physics, Bldg. 510, Large Seminar Room

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Li Keran

Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T.D. Lee

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