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Highlights from BNL and RHIC 2015

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1 Introduction

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) is one of the two remaining operating hadron colliders (the other being the LHC at CERN); and the first and only polarized proton collider. BNL is a multipurpose laboratory, quite different in scope from Fermilab and CERN, with many “cutting edge” major research facilities in addition to RHIC. BNL, which is owned by the U.S. Government but operated by a “management and operating (M&O) contractor” was founded by nine major northeastern universities in 1947 to promote basic research in the physical, chemical, biological and engineering aspects of the atomic sciences and for the purpose of the design, construction and operation of large scientific research facilities that individual institutions could not afford to develop on their own. In addition to RHIC, BNL is now home to the National Synchrotron Light-source-II (NSLS-II), the NASA Space Radiation Laboratory, a Tandem Van de Graaf, an Accelerator Test Facility, a Linac Isotope Producer, the Long Island Solar Farm, a Center for Functional Nanomaterials as well as Radiochemistry, Biological Imaging, Environmental & Climate Sciences groups and a Nonproliferation and National Security Department. BNL has a distinguished history in nuclear & particle physics and accelerator science [1] as well as many other discoveries [2]. So far, Nobel Prizes have been awarded to 12 scientists who were either BNL staff members or performed their Nobel work at BNL [3].

2 News from BNL since ISSP2014

The news this past year was very positive in many ways. The “management and operating (M&O) contract” for BNL, for which there was a solicitation last year [4], was awarded to the present management, BSA, which had run the lab for the past 15 years, so that there was a very smooth transition. The major event was the startup and dedication of the “World’s brightest Synchrotron Light Source”, NSLS-II, by Secretary of Energy Ernest Moniz (who was also an important participant in the USA, France, Germany, UK, China, Russia negotiations with Iran which were ongoing during the ISSP2015 school and reached a

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historic agreement on a Joint Comprehensive Plan of Action on July 14, 2015 to limit Iran’s Nuclear Energy activities to civilian purposes).

In addition to the successful RHIC run, several other activities took place which related to the future. The proposal for a new “Large-Acceptance Jet and Upsilon detector for RHIC” using hadron calorimetry outside a thin-coil superconducting solenoid (originally called sPHENIX) had two important milestones. The laboratory management called a general meeting on June 16, 2015 [5] “to move expeditiously to form a new detector collaboration to take advantage of the physics opportunities” offered by this proposal. The result was the formation of an Institutional Board for the new collaboration, a committee to develop the ByLaws of the new collaboration and to identify spokesperson candidates, working groups to develop a Pre-Conceptual Design Report for a BNL Director’s Cost and Schedule review, Nov 9-10, 2015, and a plan to hold a collaboration meeting December 10-12, 2015.

The second milestone was the mid-winter shipment and safe arrival of the (made in Italy) BABAR superconducting solenoid magnet from SLAC on January 16 to BNL on February 3, 2015, which went smoothly (Fig. 1). The solenoid arrived in mint condition as determined from detailed acceptance tests by the BNL Superconducting Magnet Division.

Figure 1: a)(left) BABAR solenoid leaves SLAC on Air Ride Trailer b) (center) Route across U.S.A. c) (right) Unloading at BNL.

Also important for the future of Nuclear Physics in the U.S. was the completion of the Long Range Planning exercise for the Nuclear Science Advisory Committee (NSAC) in April 2015, with the final version to be submitted to the Department of Energy (DOE) in October 2015 [6], which was unfortunately leaked to the press in May with an article in Nature [7] with the headline “Billion-dollar collider gets thumbs up”. The collider in question is an electron-ion collider, which if located at BNL would be called eRHIC, for which BNL has proposed a highly advanced and energy efficient FFAG electron accelerator/storage ring based on an Energy Recovery Linac that I discussed here last year [4]. The Nature article made two points that I wish to discuss further.

The first involves cost, which a review by an NSAC expert subcommittee [8] put at \$1.5 Billion for either the JLAB or BNL version, which was equal to the JLAB proposal but \$0.5 Billion higher than BNL’s proposed cost because of the committee’s concern of the higher technical risk of the novel advanced accelerator design. Hopefully, ongoing R&D and similar work for the CERN LHeC project [9] will overcome the present technical risk and reduce the cost, because many pundits questioned the feasibility of the U.S. alone building an EIC at the \$1.5 Billion price. The second point made in the article was that “The machine should also solve a puzzle about the proton that has baffled physicists for nearly 30 years” — “strangely the spins of its three constituent quarks add up to only about 1/3 of its own spin.” Evidently, Nature [7] did not know about the latest RHIC-spin results (Fig. 2) which show

Figure 2: a)(left) PHENIX preliminary measurements of A_{LL} of π^0 [11] with DSSV14 fit[12] b) (center) STAR measurement of A_{LL} of jets [11] with DSSV14 fit[12] c)(right) Integral of the helicity asymmetry of gluon structure function $\Delta g(x, Q^2)$ from DSSV14 fit[12] as a function of x_{min} [11].

a significant non-zero gluon spin contribution to the proton from measurements of A_{LL} , the two-spin asymmetry for the scattering of two longitudinally polarized protons: [10]

$$A_{LL} = \frac{1}{P_1 P_2} \frac{(\sigma^{++} + \sigma^{--}) - (\sigma^{+-} + \sigma^{-+})}{(\sigma^{++} + \sigma^{--}) + (\sigma^{+-} + \sigma^{-+})} = \frac{1}{P_1 P_2} \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} \quad \text{if parity is conserved,} \quad (1)$$

where $\sigma^{++} \equiv N^{++}/L^{++}$ is the measured cross section with both beams having ‘+’ helicity, N^{++} is the measured number of events for an integrated luminosity L^{++} , with analogous notation for the other helicity combinations; and P_1 and P_2 are the polarizations of the two beams. The parton helicity asymmetries are related to the proton asymmetries by QCD [10].

For those not familiar with the standard nomenclature: the the helicity asymmetry of the structure function $a(x, Q^2)$ for a parton a (where a represents e.g. u -quark, $\bar{u}$ -quark, gluon, etc.) is defined as $\Delta a(x, Q^2) \equiv a^+(x, Q^2) - a^-(x, Q^2)$ and the “+” and “-” refer to partons with the same or opposite helicity as the parent proton. There is a helicity sum rule for the spin 1/2 proton:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g$$

where $\Delta \Sigma \equiv \int_0^1 dx [\Delta u(x, Q^2) + \Delta \bar{u}(x, Q^2) + \Delta d(x, Q^2) + \dots]$ is the combined quark and anti-quark spin contribution, $\Delta G \equiv \int_0^1 dx \Delta g(x, Q^2)$ is the gluon contribution; and L_q and L_g are possible quark and gluon angular momentum contributions. The quark contribution, which has been measured in DIS [13] to be $\Delta \Sigma \approx 0.25$, thus accounts for only $\sim 1/4$ of the proton spin. “The inclusive DIS measurements have, however, very little sensitivity to gluons” [12]. The new results from Fig. 2 are the first evidence of a finite gluon polarization in the proton [12]. Notice that the best fit of DSSV14 in Fig. 2c gives $\Delta G \geq 0.36$ for $x_{min} \leq 10^{-3}$. If improved measurements at $x = 10^{-3}$ by future RHIC runs keep the present best fit of DSSV14 with smaller errors, it is conceivable that $\Delta \Sigma/2 + \Delta G$ could add up to 1/2 with sufficiently small errors to solve the proton spin puzzle.

3 RHIC Operations in 2015 and accelerator future plans

Physics data taking for the 15th run at RHIC started on February 11, 2015 and ended on June 22, all at $\sqrt{s_{NN}} = 200$ GeV. The principal objectives were: i) a comparison p+p

RHIC Run	Year	Species	Energy	Ldt
Run-1	2000	Au+Au	130 GeV	1 μb^{-1}
Run-2	2001-2	Au+Au	200 GeV	24 μb^{-1}
Run-2		Au+Au	19 GeV	0.4 μb^{-1}
		p+p	200 GeV	150 nb $^{-1}$
Run-3	2002/3	d+Au	200 GeV	2.74 nb $^{-1}$
		p+p (L)	200 GeV	0.35 nb $^{-1}$
Run-4	2003/4	Au+Au	200 GeV	241 μb^{-1}
		Au+Au	62.4 GeV	9 μb^{-1}
Run-5	2005	Cu+Cu	200 GeV	3 nb $^{-1}$
		Cu+Cu	62.4 GeV	0.19 nb $^{-1}$
		Cu+Cu	22.4 GeV	2.7 μb^{-1}
		p+p (L)	200 GeV	3.8 pb $^{-1}$
Run-6	2006	p+p (T+L)	200 GeV	10.7 pb $^{-1}$
		p+p	62.4 GeV	100 nb $^{-1}$
Run-7	2007	Au+Au	200 GeV	813 μb^{-1}
Run-8	2007/2008	d+Au	200 GeV	80 nb $^{-1}$
		p+p (T)	200 GeV	5.2 pb $^{-1}$
		Au+Au	9.2 GeV	
Run-9	2009	p+p (L)	200 GeV	16 pb $^{-1}$
		p+p (L)	500 GeV	14 pb $^{-1}$
Run-10	2010	Au+Au	200 GeV	1.3 nb $^{-1}$
		Au+Au	62.4 GeV	100 μb^{-1}
		Au+Au	39 GeV	40 μb^{-1}
		Au+Au	7.7 GeV	260 mb $^{-1}$
Run-11	2011	p+p (L)	500 GeV	27 pb $^{-1}$
		Au+Au	200 GeV	915 μb^{-1}
		Au+Au	27 GeV	5.2 μb^{-1}
		Au+Au	15.6 GeV	13.7 M events
Run-12	2012	p+p (L)	200 GeV	9.2 pb $^{-1}$
		p+p (L)	510 GeV	30 pb $^{-1}$
		U+U	193 GeV	171 μb^{-1}
		Cu+Au	200 GeV	4.96 nb $^{-1}$
Run-13	2013	p+p (L)	510 GeV	156 pb $^{-1}$
Run-14	2014	Au+Au	15 GeV	44.2 μb^{-1}
		Au+Au	200 GeV	2.56 nb $^{-1}$
		He3+Au	200 GeV	134 nb $^{-1}$
Run-15	2015	p+p (L)	200 GeV	59.9 pb $^{-1}$
		p+Au (T)	200 GeV	206.2 nb $^{-1}$
		p+Al (T)	200 GeV	690.8 nb $^{-1}$

Years	Beam Species and	Science Goals	New Systems
2014	Au+Au at 15 GeV Au+Au at 200 GeV $^3\text{He}+\text{Au}$ at 200 GeV	Heavy flavor flow, energy loss, thermalization, etc. Quarkonium studies QCD critical point search	Electron lenses 56 MHz SRF STAR HFT STAR MTD
2015-16	$p^\uparrow+p^\uparrow$ at 200 GeV $p^\uparrow+\text{Au}$, $p^\uparrow+\text{Al}$ at 200 GeV High statistics Au+Au Au+Au at 62 GeV ?	Extract $\eta/s(T)$ + constrain initial quantum fluctuations Complete heavy flavor studies Sphaleron tests Parton saturation tests	PHENIX MPC-EX STAR FMS preshower Roman Pots Coherent e-cooling test
2017	$p^\uparrow+p^\uparrow$ at 510 GeV	Transverse spin physics Sign change in Sivers function	
2018	No Run		Low energy e-cooling install. STAR ITPC upgrade
2019-20	Au+Au at 5-20 GeV (BES-2)	Search for QCD critical point and onset of deconfinement	Low energy e-cooling
2021-22	Au+Au at 200 GeV $p^\uparrow+p^\uparrow$, $p^\uparrow+\text{Au}$ at 200 GeV	Jet, di-jet, γ -jet probes of parton transport and energy loss mechanism Color screening for different quarkonia Forward spin & initial state physics	sPHENIX Forward upgrades ?
≥ 2023 ?	No Runs		Transition to eRHIC

Figure 3: a)(left) Year, species and proton polarization (Longitudinal or Transverse), $\sqrt{s_{NN}}$ and integrated luminosity of RHIC runs. b) (right) Future run schedule and new equipment.

measurement for the STAR heavy flavor tracker (HFT) which had its first (Au+Au) run in 2014¹; ii) comparison p+p measurements for the PHENIX central (VTX) and forward micro-vertex (FVTX) detectors and the new MPC-EX detector (a forward charged particle tracker and EM pre-shower detector to complement the forward EM Calorimeters, located at $3.1 \leq |\eta| \leq 3.7$ inside the Muon Pistons); iii) p+Au and p+Al runs for a better baseline of nuclear effects than the previous d+Au measurements. In all cases the protons were polarized: longitudinally for the p+p measurements and transversely for the p+A measurements. Fig. 3a gives the species, c.m. energies ($\sqrt{s_{NN}}$), polarizations and integrated luminosities obtained for all RHIC runs, with new equipment and future plans given in Fig. 3b. The luminosity performance is given in Fig. 4. Note that the p+p integrated luminosity at $\sqrt{s_{NN}}=200$ GeV in 2015 exceeds the sum of all previous p+p integrated luminosity at this $\sqrt{s_{NN}}$.

Figure 4: a)(left) A+B performance, where the nucleon-pair luminosity is defined as $L_{NN} = A \times B \times L$, where L is the luminosity and A, B are the number of nucleons in the colliding species. b) (right) Polarized p+p performance. Note that the thin lines by the years are pointers. Courtesy Wolfram Fischer.

¹The STAR and PHENIX detectors have been operating at RHIC all 15 years [14] with regular upgrades.

3.1 Nobody's perfect—the first asymmetric Z/A run.

Because of the asymmetric p+A running, the final focussing DX magnets near the interaction regions had to be moved. Aperture restrictions were identified and fixed; masks for the protection of the detectors in case of a quench were installed. However since this was the first such run at RHIC, abort kicker prefires were not adequately shielded and the MPC detector was damaged by several prefires and removed from service on June 1 (Fig. 5). The VTX and FVTX detectors were also removed as a precaution. The good news is that only the p+Al measurement was affected by these problems.

Figure 5: MPC damage from yellow-ring abort kicker prefires: top row South MPC, bottom row North MPC. From left to right: Start of run, Feb. 11; after May 11; after May 28; after June 1.

4 10th Anniversary Celebration of the Perfect Liquid

High energy nucleus-nucleus collisions provide the means of creating nuclear matter in conditions of extreme temperature and density where a phase transition is expected from a state of nucleons containing confined quarks and gluons to a state of quarks and gluons, deconfined from their original nucleons, covering a volume that is many units of the confinement length scale (~ 1 fm) in which the q and g , with their color charge fully exposed, freely traverse the medium composed of a large density of similarly exposed color charges. This state of nuclear matter was originally given the name Quark Gluon Plasma (QGP) [15], a plasma being an ionized gas. The QGP was discovered at RHIC, and announced on April 19, 2005. However the results at RHIC [16] indicated that instead of behaving like the anticipated gas of free quarks and gluons, the matter created in heavy ion collisions at nucleon-nucleon c.m. energy $\sqrt{s_{NN}} = 200$ GeV appears to be more like a *liquid*. This matter interacts much more strongly than originally expected, as elaborated in peer reviewed articles by the 4 RHIC experiments [17, 18, 19, 20], which inspired the theorists [21] to give it the new name “sQGP” (strongly interacting QGP). These properties were quite different from the “new state of matter” claimed in a press-conference [22] by the CERN fixed target heavy ion program on February 10, 2000, which was neither peer-reviewed nor published.

I had a few additional quips in my discussion of this discovery in previous ISSP talks and proceedings [23]. Perhaps I was too harsh because in an older previous publication [16] I noted that it was not unreasonable to expect surprises in the search for the QGP, i.e. “Many of the predicted properties will be found but will not be the QGP; the QGP will have a few unpredicted or unexpected properties; the search will uncover many unexpected backgrounds and new properties of p+p and A+A collisions, some of which may be very interesting phenomena in their own right.” In fact this is what has happened² and is still happening this year.

At the time of the proposals for experiments at RHIC (also for the LHC [25]) in 1990-91, the “gold-plated” signature for deconfinement in the QGP was the suppression of J/Ψ production caused by the Debye screening of the QCD potential in the QGP so that the c and $\bar{c}$ quarks could not bind [26]. However, as predicted a year later, in 1987 [27], enhanced production of c and $\bar{c}$ quarks in A+A collisions and recombination of $c\bar{c}$ into J/Ψ might (and did [23]) “hinder” J/Ψ suppression as evidence for the QGP.

In 1997, an additional signal of QGP formation was proposed [28], Jet Quenching—energy loss from coherent LPM radiation of hard-scattered partons exiting the QGP, which would result in an attenuation of the jet energy and broadening (enhanced acoplanarity) of the di-jets formed by fragmentation of the outgoing partons. This has been a robust signature of QGP formation (although more complicated than the original proposal) and is one of the few remaining signals without an alternate explanation.

In my opinion, the key QGP discoveries at RHIC, so far, are the following:

- Suppression of high p_T hadrons from hard-scattering of initial state partons [29]; also modification of the away-side jet [30].
- Elliptic flow at the Hydrodynamic limit as a near ideal fluid [31] with shear viscosity/entropy density, η/s , at or near the quantum lower bound $\eta/s \approx \frac{1}{4\pi}$ [32].
- Elliptic flow of particles proportional to the number of constituent quarks [33, 34, 35].
- Mid-rapidity transverse energy, $dE_T/d\eta$ and charged particle multiplicity density $dN_{ch}/d\eta$ proportional to the number of constituent quark participants N_{qp} . [36, 37].
- Higher order flow moments proportional to density fluctuations of the initial colliding nuclei [38, 39].
- Suppression and flow of heavy quarks roughly the same as that of light quarks [40].
- QCD hard prompt photons not suppressed [41], do not flow [42].
- Production [43] and flow [42] of thermal soft photons.

4.1 A few illustrations

4.1.1 $dE_T/d\eta$ and $dN_{ch}/d\eta$ proportional to constituent quark participants N_{qp} .

The closely related charged particle density $dN_{ch}/d\eta$ and transverse energy density $dE_T/d\eta$ at mid-rapidity are composed of soft particles with low transverse momenta and a steeply falling exponential spectrum, $d\sigma/p_T dp_T \propto \exp -6p_T$ in p+p collisions. The first measurements [44] of the total charged multiplicity N_{ch} in p+A collisions at nucleon-nucleon c.m. energies $\sqrt{s_{NN}}=10\text{-}20$ GeV showed that N_{ch} was proportional to the number of nucleons, N_{part} , that participate (i.e are struck) in the collision of the nuclei.³ N_{part} scaling was originally expected

²It may be interesting for some readers to look back at some of the early predictions for the QGP [24].

³ N_{part} is an estimate of the centrality of the collision, the overlap of the two colliding nuclei [16].

to be valid at RHIC energies but was not observed at mid-rapidity (Fig. 6a). The correct scaling found at mid-rapidity (including $\sqrt{s_{NN}}=10\text{-}20$ GeV) is the number of (massive) constituent quark participants N_{qp} [45] (Fig. 6b,c).

Figure 6: PHENIX[45] mid-rapidity (a) $dN_{ch}/d\eta/(0.5N_{part})$ as a function of N_{part} , (b) $dN_{ch}/d\eta/(0.5N_{qp})$ (c) $dE_T/d\eta/(0.5N_{qp})$ as a function of N_{qp} , in Au+Au collisions at the $\sqrt{s_{NN}}$ indicated.

4.1.2 Collective Flow—anisotropic production of particles.

Collective flow (anisotropic production of particles) [46], or simply flow, is a collective effect which can not be obtained from a superposition of independent N+N collisions. “It does not assume necessarily the hydrodynamic flow, which in particular would require a thermalization of the system” [33] but this distinction is not generally discussed. The almond shaped overlap region of the A+A collision causes the particles to be emitted more favorably in the reaction plane (see Fig. 7a). The semi-inclusive single particle spectrum is modified by an expansion in harmonics of the azimuthal angle of the particle with respect to the reaction plane, $\phi - \Phi_R$:

$$\frac{Ed^3N}{dp^3} = \frac{d^3N}{p_T dp_T dy d\phi} = \frac{d^3N}{2\pi p_T dp_T dy} \left[1 + \sum_n 2v_n \cos n(\phi - \Phi_R) \right]. \quad (2)$$

The Fourier coefficient v_2 , called elliptic flow, is predominant at mid-rapidity. Odd harmonics were thought to be forbidden by the symmetry $\phi \rightarrow \phi + \pi$ of the almond. Only in 2010 was it realized that the nuclear geometry fluctuated from event to event and did not respect the average symmetry [38].

Flow measurements contributed two of the most important results about the properties of the QGP: i) the scaling of v_2 of identified particles at mid-rapidity with the number of constituent-quarks n_q in the particle— v_2/n_q scales with the transverse kinetic energy per constituent-quark, KE_T/n_q , because particles have not formed at the time flow develops (Fig. 7b); ii) the persistence of flow for $p_T > 1$ GeV/c which implied that the viscosity is small [31], perhaps as small as a quantum viscosity bound from string theory [32], $\eta/s = 1/(4\pi)$, where η is the shear viscosity and s the entropy density per unit volume. This led to the description of the “sQGP” produced at RHIC as “the perfect fluid”.

Figure 7: (a) (left) Almond shaped overlap zone generated just after an A+A collision where the incident nuclei are moving along the $\pm z$ axis. The reaction plane by definition contains the impact parameter vector (along the x axis) [47]. (b) (right) v_2/n_q vs. KE_T/n_q for baryons and mesons [48].

4.1.3 More surprises this year, flow in three small systems.

Last year [4], I noted that results in 2013 observed what looked very much like collective flow in p+Pb at LHC and d+Au at RHIC. These systems were believed to be too small for collective effects which inspired the He^3 +Au run at RHIC in 2014 to see whether triangular flow (v_3) would be more prominent with a 3 nucleon projectile (Fig. 8a). The He^3 +Au result

Figure 8: a) (left) (above) Simulated initial energy depositions for p, d and He^3 on a larger nucleus; (below) the QGP geometry resulting from hydrodynamic flow [6]. b) (center) PHENIX [49] v_2 and v_3 from He^3 +Au with 0-5% centrality at $\sqrt{s_{NN}} = 200$ GeV. c) (right) v_2 from p,d, He^3 +Au at $\sqrt{s_{NN}} = 200$ GeV [50].

(Fig. 8b) [49] did indeed show a strong v_3 as well a v_2 comparable to the d+Au (Fig. 8c). Perhaps no longer surprising, the p+Au data from this year's run [50] also showed a comparable v_2 , but smaller, consistent with the more spherical initial geometry. Of special interest is how well the large v_3/v_2 ratio in He^3 +Au corresponds to the ratio of the eccentricities in the initial geometry because translation of the geometrical anisotropy into the flow anisotropy depends on the viscosity and dissipates with larger viscosities for larger v_n . The issue of whether this is really hydrodynamic flow is still one of the very hot topics in RHI physics.

4.1.4 Jet quenching at RHIC — Suppression of high p_T particles

The discovery at RHIC that π^0 's produced at large transverse momenta are suppressed in central Au+Au collisions by a factor of ~ 5 compared to binary-collision scaling from p+p

collisions (Fig. 9a) [29] is arguably *the* major discovery in Relativistic Heavy Ion Physics. At RHIC energies, the steeply falling exponential spectrum at low p_T becomes negligible relative to the power-law dependence of hard-scattering for $p_T \geq 3$ GeV/c [51]. The suppression in A+A collisions is presented as $R_{AA}(p_T)$, the ratio of the yield of e.g. π^0 per A+A collision of a given centrality to the binary-scaled p+p cross section at the same p_T , where $\langle T_{AA} \rangle$ is the average overlap integral of the nuclear thickness functions for that centrality

$$R_{AA}(p_T) = \frac{(1/N_{AA}) d^2 N_{AA}^{\pi}/dp_T dy}{\langle T_{AA} \rangle d^2 \sigma_{pp}^{\pi}/dp_T dy} \quad (3)$$

Fig. 9a illustrates another advantage of high p_T suppression as a QGP probe: at $\sqrt{s_{NN}}$ of CERN SpS fixed target and ISR A+A (i.e. $\lesssim 31$ GeV), high p_T production is enhanced.

A related but possibly greater discovery, the opposite of what was predicted [52], is that heavy quarks observed using direct-single- $e^\pm$ from c and b -quark decay are suppressed comparably to light quarks (π^0) for $p_T \gtrsim 4$ GeV/c (Fig 9b) and also exhibit flow (Fig 9c) [40]. The important impact of this discovery was that it provided a demonstration that heavy quarks were strongly coupled to the medium with viscosity/entropy density $\eta/s \approx (1.3 - 2)/4\pi$, close to the quantum lower bound [32], reinforcing the ‘perfect fluid’, and stimulating a broad spectrum of possible explanations (e.g see Ref. [23]).

Figure 9: (a) (left) PHENIX [29] discovery of suppression ($R_{AA} \sim 0.3$) of charged hadrons and π^0 in Au+Au central collisions at $\sqrt{s_{NN}} = 130$ GeV compared to enhancement ($R_{AA} > 1.0$) at lower $\sqrt{s_{NN}}$. (right) PHENIX direct-single- $e^\pm$ in Au+Au at $\sqrt{s_{NN}} = 200$ GeV [40]: (b) R_{AA} (central) (c) v_2 (minimum bias) vs. p_T , compared to π^0 .

5 RHIC Beam Energy Scan (BES) in search of a critical point—aided by Lattice QCD

This subject has remained one of my main interests since I got sandbagged by a press release from LBL during ISSP2011 [23] claiming that “By comparing theory with data from STAR, Berkeley Lab scientists and their colleagues map phase changes in the QGP” after I had criticized a different laboratory for physics by press release. It turned out that the LBL press release and publication were later found to be “not useful” [53] (i.e. “not correct” [23]). The good news is that this year my colleagues and I at PHENIX have actually made such a comparison and were able to use our measurements of net-charge fluctuations [54] together with

a Lattice QCD calculation [55] to find both the freezeout temperature, T_f , and the Baryon Chemical Potential μ_B at several values of $\sqrt{s_{NN}}$ at RHIC, without particle identification!

To set the stage for the new measurement, Fig. 10a shows a proposed phase diagram for nuclear matter (which I emphasized in last year's proceedings [4] had an incorrect μ_B scale) as corrected by Frithjof Karsch; and Fig. 10b shows measurements of the total charge multiplicity $N_{ch} = N_{ch}^+ + N_{ch}^-$ as a function of centrality in Au+Au collisions at RHIC together with fits to Negative Binomial Distributions (NBD) [56], $P(m) = \frac{(m+k-1)!}{m!(k-1)!} \frac{(\mu/k)^m}{(1+\mu/k)^{m+k}}$, where $P(m)$ is normalized from $0 \leq m \leq \infty$ and $\mu \equiv \langle N_{ch} \rangle = \langle m \rangle$.

Figure 10: (a) Figure 20a from Ref. [4] corrected by F. Karsch. (b) PHENIX [56] fits of Negative Binomial Distributions to $N_{ch}/\langle N_{ch} \rangle$ measurements from $\sqrt{s_{NN}} = 200$ GeV Au+Au for the centralities indicated.

Figure 11 shows the new PHENIX $\Delta N_{ch} = N_{ch}^+ - N_{ch}^-$ distributions from Au+Au collisions, not yet corrected for the reconstruction efficiency $\epsilon \approx 0.69$ within the acceptance [54]. These distributions can not be compared directly to Lattice QCD predictions which use

Figure 11: PHENIX [54] $\Delta N_{ch} = N_{ch}^+ - N_{ch}^-$ distributions uncorrected for the efficiency within the acceptance, $|\eta| < 0.35$, $\delta\phi = \pi$, $0.3 < p_T < 2.0$ GeV/c, in Au+Au at two centralities, for all $\sqrt{s_{NN}}$ measured.

cumulants from a Taylor expansion of the free energy $F = -T \ln Z$ around the critical temperature T_c , where Z is the partition function (or sum over states) which is of the form $Z \propto e^{-(E - \sum_i \mu_i Q_i)/kT}$ and μ_i are chemical potentials associated with conserved charges. The terms of the Taylor expansion, which are obtained by differentiation, are called susceptibilities, denoted χ . The connection of this method to the measurements is that the cumulant

generating function in mathematical statistics is also a Taylor expansion of the $\ln$ of an exponential, where the n -th cumulant κ_n represents the n -th moment of a distribution with all n -fold combinations of the lower order moments subtracted. One of the most important properties of cumulants is that they are additive for the sum of statistically independent random variables, x and y where $\langle xy \rangle = \langle x \rangle \langle y \rangle$: $\kappa_n(x+y) = \kappa_n(x) + \kappa_n(y)$.

A recent important theorem from (of all places) Quantitative Finance [57] for “integer value Levy processes” (notably Poisson and Negative Binomial Distributions) has proved that the cumulants κ_j for the distribution $P(x-y)$ of the difference of samples from two such distributions, $P^+(x)$ and $P^-(y)$, with cumulants κ_j^+ and κ_j^- , respectively, are $\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$ so long as the distributions are not 100% correlated. The individual N_{ch}^+ and N_{ch}^- distributions are in fact NBD [54] so this theorem would allow a great simplification of the analysis but is presently used only as a check of the very complicated standard method which starts with the calculation of the cumulants of the $\Delta N_{\text{ch}} = N_{\text{ch}}^+ - N_{\text{ch}}^-$ distributions from Fig. 11.

For each distribution the n -th order moment can be calculated: $\mu'_n \equiv \langle x^n \rangle = \sum_i x_i^n P(x_i)$, where $\mu'_1 \equiv \mu = \langle x \rangle$, $\mu_n \equiv \langle (x - \mu)^n \rangle$ and x_i represents a bin in the ΔN_{ch} plot with a fraction $P(x_i)$ of the total number of events. Then one computes the cumulants κ_m from the moments, where presently the analyses compute only the first four cumulants:

$$\begin{aligned} \mu = \kappa_1 = \mu'_1 \quad \sigma^2 = \mu_2 \equiv \langle (x - \mu)^2 \rangle = \kappa_2 = \mu'_2 - \mu'^2_1 \quad \mu_3 = \kappa_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1 \\ \mu_4 - 3\mu'^2_2 = \kappa_4 = \mu'_4 - 4\mu'_3\mu'_1 - 3\mu'^2_2 + 12\mu'_2\mu'^2_1 - 6\mu'^4_1 \end{aligned} \quad (4)$$

The cumulants are then corrected for reconstruction efficiency. Ratios of the corrected cumulants are used to compare to the calculated ratios of susceptibilities because the volume dependences of the calculations cancel: $\mu/\sigma^2 = \kappa_1/\kappa_2 \approx \chi_1/\chi_2$; $S\sigma^3/\mu = \kappa_3/\kappa_1 \approx \chi_3/\chi_1$; $S\sigma = \kappa_3/\kappa_2 \approx \chi_3/\chi_2$; $\kappa\sigma^2 = \kappa_4/\kappa_2 \approx \chi_4/\chi_2$; where $S \equiv \kappa_3/\sigma^3$ and $\kappa \equiv \kappa_4/\kappa_2^2$ (Fig. 12).

Figure 12: $\sqrt{s_{NN}}$ dependence of net-charge cumulant ratios in Au+Au central (0-5%) collisions. PHENIX [54]: (a) κ_1/κ_2 , (a') κ_3/κ_1 , (b) κ_3/κ_2 , (b') κ_4/κ_2 . STAR [58]: (c) κ_3/κ_2 , (c') κ_4/κ_2 . The error bars are statistical and the caps are systematic uncertainties.

The new PHENIX net-charge results [54] for the corrected cumulant ratios shown in Fig. 12a,b are compared to the previous STAR net-charge measurements [58] for $S\sigma$ and $\kappa\sigma^2$ in Fig. 12c. There are two important observations from this comparison: i) for PHENIX the error on all the corrected cumulant ratios is 20-30%, while for STAR the error on $\kappa_1/\kappa_2 = \mu/\sigma^2$ is $< 1\%$ but for $\kappa_3/\kappa_2 = S\sigma$ is $\sim 40\%$ and for $\kappa_4/\kappa_2 = \kappa\sigma^2$ is $> 100\%$; ii) the PHENIX standard cumulant ratios and errors (circles) are checked by NBD fits to the N_{ch}^+ and N_{ch}^- distributions plus the cumulant theorem [57] (triangles) and agree—which proves that the errors are correct and that the cumulant theorem works.

As illustrated in Fig. 13, the PHENIX ($\blacklozenge$) errors allow both μ_B and T_f to be determined from the Lattice QCD calculations [55] while the large STAR ($\star$) error on $R_{31} = \kappa_3/\kappa_1$ does not give a solution for T_f .

Figure 13: Lattice QCD calculations of cumulant ratios (lines) [55] (a) $R_{12} = \kappa_1/\kappa_2$ (b) $R_{31} = \kappa_3/\kappa_1$. The vertical lines in the boxes represent the error of the R_{12} and R_{31} measurements at $\sqrt{s_{NN}} = 27$ GeV and the horizontal lines the error of the best fit values of T_f and μ_B . The STAR measurement of R_{31} has a such a huge error that the data point ($\star$) could go anywhere in the dashed box. The horizontal dashes and position of the $\star$ for R_{31} are an assumption [59] based on the STAR net-proton measurement [60].

Figure 14 shows that the PHENIX + Lattice results for T_f and μ_B from net-charge fluctuations, with no particle identification, are in excellent agreement with the best accepted analysis of T_f and μ_B from baryon/anti-baryon ratios. [61]. I believe that this is a first!

Figure 14: (a) $\sqrt{s_{NN}}$ dependence of μ_B from PHENIX+Lattice [54] net-charge results. Open squares are from STAR net-charge together with net-protons [59]. Dashed line and other data points are from (b), the best accepted analysis of T_f and μ_B vs $\sqrt{s_{NN}}$ from baryon/anti-baryon ratios [61].

5.1 Why are the STAR errors on R_{31} so large? Corrections?

A fraction of the tracks that fall on the detector acceptance are not reconstructed, a clearly random, thus Binomial, effect. This is further complicated if the efficiencies are different for N_{ch}^+ (ϵ^+) and N_{ch}^- (ϵ^-). The standard Koch-Bzdak [62] Binomial efficiency correction, e.g. to correct κ_3 of an arbitrary net-charge distribution, is:

$$\kappa_3 = \kappa_1 + 2\kappa_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30} - 3\kappa_1(N + F_{02} - 2F_{11} + F_{20}) \quad (5)$$

where the corrected values are $\langle N^+ \rangle = \langle N_{\text{ch}}^+ \rangle / \epsilon^+$, $\langle N^- \rangle = \langle N_{\text{ch}}^- \rangle / \epsilon^-$, $\kappa_1 = \langle N^+ \rangle - \langle N^- \rangle$, $N \equiv \langle N^+ \rangle + \langle N^- \rangle$. Also, a new quantity, the double Factorial Moment has been introduced:

$F_{ik} = \sum_{N_1=i}^{\infty} \sum_{N_2=k}^{\infty} P(N_1, N_2) \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!}$, which looks simple, but is very difficult to compute because the joint probability $P(N_1, N_2)$, e.g. $P(13^+, 11^-)$ etc., is required. This means that to make the computation one must know not only the ΔN_{ch} distribution but also both the N^+ , the N^- distributions and their correlation. $P(N^+, N^-)$ can be obtained from the data by making a 3d lego plot with base axes N^+ and N^- and height equal to the normalized number of events with (N^+, N^-) , but this costs a huge dilution of the statistical errors, so approximations or random-sampling of the lego plot are used. My guess is that this dilution plus the huge complication in e.g. the calculation of κ_3 in Eq. 5 are one reason for the huge STAR errors in $S\sigma = \kappa_3/\kappa_2$ and $\kappa\sigma^2 = \kappa_4/\kappa_2$ but I haven't verified it with them.

5.1.1 If you know the distribution, you know all the moments and cumulants

When I first saw net-charge distribution plots like Fig. 11, e.g. see Ref. [23], my thought was to fit them to NBD so that the title of this section would apply. However, they are not NBD, they are the difference between two NBD. This is why the cumulant theorem [57] is so important: once the cumulants, κ_j^+ and κ_j^- , of NBD fits to the individual N_{ch}^+ and N_{ch}^- are known, the cumulants of the ΔN_{ch} distribution are simply $\kappa_j = \kappa_j^+ + (-1)^j \kappa_j^-$. Perhaps equally important, the correction for efficiency to the measured N_{ch}^+ and N_{ch}^- is a “binomial split” of an NBD [63]—if a population is $\text{NBD}(\mu_t, k)$ and divided randomly with probability p onto a smaller region, the distribution on the region is $\text{NBD}(p\mu_t, k)$, i.e. the mean changes but the k parameter remains constant. Thus if we measure $\mu^+ = \langle N_{\text{ch}}^+ \rangle$ in the detector with efficiency $p = \epsilon^+$, the efficiency corrected mean, μ_t^+ equals μ^+/ϵ^+ and e.g. the efficiency corrected κ_3/κ_1 is:

$$\frac{S\sigma^3}{\mu} = \frac{\kappa_3}{\kappa_1} = \frac{\kappa_3^+ - \kappa_3^-}{\kappa_1^+ - \kappa_1^-} = \frac{\mu_t^+[1 + 3(\frac{\mu_t^+}{k^+}) + 2(\frac{\mu_t^+}{k^+})^2] - \mu_t^-[1 + 3(\frac{\mu_t^-}{k^-}) + 2(\frac{\mu_t^-}{k^-})^2]}{\mu_t^+ - \mu_t^-} \quad , \quad (6)$$

which is a lot simpler than Eq. 5 and dramatically simpler than for $\kappa_4/\kappa_2 = \kappa\sigma^2$, which the reader can figure out from Table 1 compared to Ref. [62].

5.2 The End?

Of course this is still not the whole story. Since there can be no fluctuations of conserved quantities such as net-charge or net-baryon number in the full phase space, one must go to “locally conserved quantities” [64] in small rapidity intervals to detect a fraction which then fluctuates, i.e. varies from event to event. Then the question becomes what the required acceptance is for an experimental result to compare with the Lattice QCD calculations.

Table 1: Cumulants for Poisson, Negative Binomial Distributions
Measured with efficiency ϵ corrected to the true value

Measured Cumulant	Corrected Poisson	Corrected Negative Binomial
$\kappa_1 = \mu$	$\mu_t \equiv \mu/\epsilon$	$\mu_t \equiv \mu/\epsilon$
$\kappa_2 = \mu_2 = \sigma^2$	μ_t	$\mu_t(1 + \mu_t/k) \equiv \sigma_t^2$
$\kappa_3 = \mu_3$	μ_t	$\sigma_t^2(1 + 2\mu_t/k)$
$\kappa_4 = \mu_4 - 3\kappa_2^2$	μ_t	$\sigma_t^2(1 + 6\mu_t/k + 6\mu_t^2/k^2)$
$S \equiv \kappa_3/\sigma^3$	$1/\sqrt{\mu_t}$	$(1 + 2\mu_t/k)/\sigma_t$
$\kappa \equiv \kappa_4/\kappa_2^2$	$1/\mu_t$	$(1 + 6\mu_t/k + 6\mu_t^2/k^2)/\sigma_t^2$
$S\sigma = \kappa_3/\kappa_2$	1	$(1 + 2\mu_t/k)$
$\kappa\sigma^2 = \kappa_4/\kappa_2$	1	$(1 + 6\mu_t/k + 6\mu_t^2/k^2)$
$\mu/\sigma^2 = \kappa_1/\kappa_2$	1	$1/(1 + \mu_t/k)$
$S\sigma^3/\mu = \kappa_3/\kappa_1$	1	$(1 + 3\mu_t/k + 2\mu_t^2/k^2)$

Another issue is whether “net-protons” are equivalent to net-baryons since the neutrons are not measured. Further complicating this, the experiments measure in a fixed $\delta\eta$ interval but the rapidity distributions change (dramatically for protons because of stopping [65]) as the $\sqrt{s_{NN}}$ is reduced. Also Lattice QCD calculations are not now available for $\mu_B > 0.2$ GeV... There are many unresolved issues—stay tuned!!!

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