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Multi-objective dynamic aperture optimization for storage rings

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We report an efficient dynamic aperture (DA) optimization approach using multi-objective genetic algorithm (MOGA), which is driven by nonlinear driving terms computation. It was found that having small low order driving terms is a necessary but insufficient condition of having a decent DA. Then direct DA tracking simulation is implemented among the last generation candidates to select the best solutions. The approach was demonstrated successfully in optimizing NSLS-II storage ring DA.

Keywords: Multi-objective optimization; dynamic aperture; storage ring.

1. Introduction

In designing storage ring-based X-ray light sources, accelerator physicists are pushed very hard to achieve low emittance pursuing bright photon beam. Low emittance means strong focusing quadrupoles, which requires strong sextupoles (installed at in dispersive region) to compensate chromatic aberrations. These strong sextupoles result in small dynamic aperture (DA), making it difficult to realize efficient off-axis injection, and maintain sufficient Touschek lifetime. Based on modern accelerator theory, improvement of DA can be realized by minimizing various nonlinear driving terms (NDT), which can be computed to a certain order.¹ But there are already several tens terms even we only compute them up to the second-order. In the meantime, the designers have to face the difficulty of specifying weights for numerous NDTs based on their experience, if the conventional optimization method (single merit function) is used.

Recently, people used a new method of simultaneously optimizing dynamic and momentum aperture,² or beam lifetime^{3,4} with multi-objective genetic algorithm (MOGA). This method is very successful. The optimization heavily relies on a powerful and reliable tracking simulator to directly probe the dynamic and momentum apertures (or beam lifetime). Usually the optimization driven by the element-by-element symplectic-tracking simulator⁵ is very time-consuming, especially when ring scale is big. This time-consuming objective function computation significantly degrades optimization efficiency although powerful parallel computers are available.

Based on previous genetic optimization experiences and results, we observed that: (1) the driving terms reduce automatically once DA is set as one of the objectives for optimization;³ (2) the optimal candidates always have small driving terms, while small driving terms do not always imply a good DA.² In another word, having small driving terms is a necessary but insufficient condition for ensuring a good DA. This observation hints an efficient optimization method. First, we can apply MOGA to optimize various driving terms simultaneously to avoid blindly weights on them. Then we implement tracking simulation as a final filter to select the best solutions among the last generation candidates. The reason of why this new strategy becomes efficient is that the optimization driven by analytical formulae computation is much faster than direct tracking simulation. We can further extract the correlation between DA and driving terms from the raw data of simulation to identify which resonances dominate DA. The correlations can help us to better understand the nonlinearity from the view of beam dynamics.

2. Multi-Objective Genetic Algorithm

MOGA⁶ has been widely adopted in storage ring lattice design recently. Ring designer used it to find low emittance lattice for a given magnet layout,^{7,8} or to optimize DA by varying sextupole configuration. The method we are using is an elitist multi-objective evolutionary algorithm proposed by Deb.⁶ It is a population-based evolution algorithm to find the Pareto optimum iteratively. First, a fixed number of candidates is initialized as the first generation, and they are randomly chosen and uniformly distributed in their allowed ranges. Then one pair of them are randomly chosen as parents to cross over to generate two new children according to a certain probability density function. The process of crossover is repeated until the population is doubled. Next, all children mutate randomly also with a certain probability density function. The objective functions and constraints are evaluated for each of these new children. The whole population, including the parents, is then sorted according to their dominance relations. Since parents are included into sorting, elitists are kept once they are found, which can speed up the performance of optimization significantly by preventing the loss of good solutions. Candidates not dominated by anyone are in the first rank. Only half of the better candidates are kept by dropping out the candidates with larger rank. Within same rank, candidates in a high population density region have lower priority to be selected. Up to this point, the population is kept the same but the overall qualities in terms of objective functions and constraints are evolved not worse than the previous population before crossover and mutation. The population is evolved generation by generation until it converges or the maximum number of iterations is reached.

3. Nonlinear Driving Terms

For storage rings, NDTs can be obtained by concatenating individual maps into one-turn-map via the similarity transformation and BCH formula in Lie algebra

Fig. 1. NSLS-II optics in one super-cell.

language.¹ Wang⁹ explicitly derived all second-order driving terms due to sextupoles and chromatic effects of quadrupoles. We combined them with first-order driving terms as our optimization objectives. Of course, h_{11001} and h_{00111} are excluded to control linear chromaticity. In the meantime, three terms, h_{11110} , h_{22000} and h_{00220} are imposed with some extra constraints to reduce tune dependence on amplitude.

4. Applications

4.1. NSLS-II lattice

NSLS-II¹⁰ lattice has 30 double bend achromatic (DBA) cells. Two DBA cells with mirror symmetry have low- and high-beta functions at short and long straight sections. The linear optics for one cell is illustrated in Fig. 1. The whole bare lattice has 15-fold symmetry. Each DBA cell has nine sextupoles. Three of them are sitting in the dispersive region for chromaticity correction. The other six sextupoles are geometric sextupoles.

4.2. Chromaticity +7/+7 lattice

To effectively suppress beam transverse instabilities at high beam current, high positive linear chromaticity is preferable. High positive linear chromaticity means stronger sextupoles, and then a smaller DA. The DA optimization is therefore demanding. In this case, the linear chromaticity is corrected to +7/+7 with three chromatic sextupoles (Fig. 2). Then the free parameters left for DA optimization are the strengths of 6 families of geometric sextupoles in the nondispersive straights. Our objective functions are 30 low order NDTs totally. The searching space of

Fig. 2. Tune versus energy deviation for linear chromaticity +7/+7.

free variables (i.e. sextupole strengths) is limited within the ranges limited by the engineering specification and power supply polarities. Three tune-dependency-on-amplitude terms were imposed with some extra constraints to boost the convergence. We use a population of 4000, and run for 100 generations. With 100 Xeon 2.33 GHz CPUs in a Sun Grid Engine cluster, it takes 6–12 hours to finish the optimization. Finally, we use a symplectic tracking code to check the DAs for all these 4000 candidates in the last generation. Based on tracking results, we can choose some of the best solutions among them.

The frequency map analysis was carried out for one of solutions as shown in Figs. 3 and 4. This sextupole configuration has been successfully tested at the NSLS-II ring. DA is proved to be sufficient for a decent injection efficient (80–90%) and up to 18 h lifetime was observed with 10 mA stored beam in 100 bunches.

4.3. Preliminary study for low-alpha lattice

Low-alpha lattice with short bunch longitudinal length can provide coherent synchrotron radiation in the THz range. Such kind of lattice has been tested on most of modern synchrotron radiation facilities. This preliminary study focuses on the feasibility of low-alpha mode on the NSLS-II storage ring. The DA optimization is more difficult than the previous case, because we need to control linear chromaticity, DA and longitudinal stability simultaneously with sextupole configuration. There is no dispersion-free section, thus we cannot separate chromaticity correction and DA optimization. Once the zeroth-order momentum compactor is small enough, the higher order terms will play important roles in the longitudinal motion stability.

Fig. 3. Dynamic aperture in x - y plane.

Fig. 4. Energy acceptance in x - δ plane.

And finally, we still need sufficient DA and energy acceptance to achieve decent injection efficiency and lifetime. All these optimization objectives need to be satisfied simultaneously with a given sextupole configuration. The preliminary results on linear optics, DA, energy acceptance and stable RF bucket are shown in Figs. 5–8, which are positive and promising.

Fig. 5. Linear optics for low-alpha configuration.

Fig. 6. Dynamic aperture in x - y plane.

4.4. Correlation between DA and NDTs

MOGA produces numerous raw data, which can be used to study the correlations between DA and NDTs. Usually, inside the last generation, none of candidate in the first rank is dominated by any others. From the view of multi-objective optimization, they are equally good if only considering NDTs. But from the aspect of DA, they are quite different. We use DA tracking simulator to compute their DAs for all 4000 candidates, and illustrate their correlation with the summation of NDTs in Fig. 9. By observing the correlation, we can conclude that having small low order NDTs is necessary but insufficient condition for having a good DA (as marked with red A). The necessity is demonstrated by observing that all candidates with large DAs must have small NDTs without exception.

Fig. 7. Energy acceptance in x - δ plane.

Fig. 8. Stable bucket for low alpha configuration.

Fig. 9. (Color online) Correlation between DA and NDTs.

5. Summary

An efficient approach of using MOGA driven by NDTs computation to optimize storage ring DA is discussed in this paper. The optimization produces numerous raw data, which can satisfy the necessary condition to have a good DA at the first step. Among them we can select some good solutions with the DA tracking code. Applying this approach on the NSLS-II bare lattice to achieve a high positive chromaticity was demonstrated experimentally. And a preliminary study on low-alpha lattice is quite positive also. So far, we applied this method to a ring-based light source. But it will be interesting to explore how to implement this algorithm on high-energy ring collider design.

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