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Observational study of the relationship between entrainment rate and relative dispersion in deep convective clouds

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Abstract: This study investigates the influence of entrainment rate (λ) on relative dispersion (ε) of cloud droplet size distributions (CDS) in the 99 growing precipitating deep convective clouds during TOGA-COARE. The results show that entrainment suppresses ε , which is opposite to the traditional understanding that entrainment-mixing broadens CDS. To examine how the relationship between ε and λ is affected by droplets with different sizes, CDSs are divided into three portions with droplet radius $< 3.75 \mu\text{m}$ (N_1), radius in the range of $3.75\text{--}12.75 \mu\text{m}$ (N_2) and $12.75\text{--}23.25 \mu\text{m}$ (N_3), respectively. The results indicate that although the droplet concentration at different sizes generally decrease simultaneously as λ increases, the variation of standard deviation (σ) depends mainly on N_3 , while the mean radius (r_m) decreases with decreasing N_3 , but increases with decreasing N_1 . So the influence of entrainment on CDS causes a more dramatical decrease in σ than that in r_m , and further leads to the decrease of ε as entrainment enhances. In addition, a conceptual model of CDS evolution during entrainment-mixing processes is developed to illustrate the possible scenarios entailing different relationships between ε and λ . The number concentration of small droplets and the degree of evaporation of small droplets are found to be key factors that shift the sign (i.e., positive or negative) of the ε - λ relationship.

1. Introduction

Clouds play significant roles in radiative transfer and the earth's radiative energy balance [Lin et al., 2015; Shi and Liu, 2016; Zhang et al., 2017a]. The cloud radiative properties are closely related to cloud droplet size distributions (CDSs) and the corresponding microphysical properties [Zhang et al., 2016, 2017b; Chen et al., 2017]. Relative dispersion (ε) of CDS, the ratio of standard deviation (σ) to mean radius (r_m), is a relative measure of spectral width [Liu et al., 2014] and an important microphysical parameter affecting cloud radiative properties [Martin et al., 1994; Liu and Daum, 2000, 2002; Peng and Lohmann, 2003]. Liu et al., [2008] showed that using an unrealistic ε in climate models, such as assuming a monodisperse cloud, can lead to significant overestimation of cloud albedo and further result in a negative bias of global mean estimates of shortwave cloud radiative forcing. In addition, ε is closely linked to the development of precipitation, and thus is essential to accurate parameterization of precipitation formation [Seifert and Beheng, 2001]. Liu and Daum [2005] introduced ε into the parameterization of autoconversion process and found that autoconversion rate increases with increasing ε . Xie et al., [2013] further applied different parameterizations of ε into the Weather Research and Forecast (WRF) model, and found that surface precipitation was sensitive to ε parameterizations.

Due to the importance of ε , much effort has been devoted to studying the factors affecting ε , such as aerosol loading, vertical velocity and entrainment. For example, some scientists found a positive correlation between ε and aerosol loading [e.g., McFarquhar and Heymsfield, 2001; Liu and Daum, 2002; Wood et al., 2002; Yum and Hudson, 2005], whereas others found a negative correlation [e.g., Ma et al., 2010; Lu et al., 2012a] or no correlation between them [e.g., Zhao et al., 2006; Tas et al., 2015]. Another factor affecting the correlation between ε and aerosol loading is vertical velocity, which was theoretically [Liu et al., 2006] and observationally [Lu et al., 2012a] shown to reduce ε . Recently, Chen et al., [2016] suggested that the relationship of ε to aerosol concentration and vertical velocity is strongly regime-dependent: given an updraft velocity, ε first increases with increasing aerosol loading in the aerosol-limited regime, peaks in the transitional regime and then decreases with further increase in aerosol loading in the updraft-limited regime.

In addition to aerosols and vertical velocity, entrainment of dry air into clouds is another important factor known to affect CDS [Lu et al., 2013; Kumar et al., 2014; Yang et al., 2016;

Chen et al., 2017]. Microphysical properties of CDSs in real-world atmospheric clouds are quite different from those of adiabatic CDS [Grabowski and Pawlowska, 1993; Liu and Hallet, 1998; Yum et al., 2015], and are closely related to turbulent entrainment-mixing [Su et al., 1998; Lasher-Trapp et al., 2005; Lu et al., 2013; Tölle and Krueger, 2014; Beals et al., 2015; Xu and Xue, 2015]. Based on numerical simulations, Su et al., [1998] and Burnet and Brenguier [2007] revealed that, without entrainment-mixing process, simulated CDSs were all narrower than in realistic clouds. Burnet and Brenguier [2007] suggested that entrainment-mixing processes produced plenty of droplet trajectory and water vapor supersaturation scenarios, hence leading to broad and multimodal CDS and large ε . The observations of Lu et al., [2013] showed that ε increased in the process of entrainment-mixing, and σ remained almost constant between 0.75 and 1 μm . The roughly constant σ was similar to that found by Pawlowska et al., [2006]. Therefore, Lu et al., [2013] attributed the increase of ε in the entrainment-mixing processes to the decrease of r_m . Prabha et al., [2012] studied the relationships of ε to big and small droplets as influenced by entrainment-mixing processes represented by adiabatic fraction, and found that the relationships were complicated and different between premonsoon and monsoon clouds. Pinsky et al., [2016] analyzed the evolution of ε in the homogeneous entrainment-mixing processes and found that ε increased at first and then approached a constant value.

Despite these efforts devoted to this topic, our understanding of the effect of entrainment on ε is still far from complete, especially for deep convective clouds. Here we examine the relationship between ε and entrainment rate (λ) in the deep convection observed by aircraft during the TOGA-COARE (Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment, 1992-1993) field campaign. For convective clouds, λ quantitatively represents the ratio of the entrained dry air mass (flux) to the cloud mass (flux) per unit height and is an important parameter in convection parameterizations [Del Genio and Wu, 2010; Zhang et al., 2015]. To the authors' knowledge, this is the first observational study on the relationship between ε and λ in deep convective clouds. The rest of the paper is organized as follows. Section 2 briefly describes the TOGA-COARE campaign and the method for estimating λ . Section 3 presents the main results, including the influence of λ on ε and the corresponding mechanisms. Conclusions are presented in Section 4.

2. Data and Approach

2.1. TOGA-COARE

During TOGA-COARE [Webster and Lucas, 1992], Electra research aircraft from NCAR (National Center for Atmospheric Research) carried out 32 observation missions to probe the physical parameters (1 Hz) in deep convective clouds. CDSs were detected by the Forward Scattering Spectrometer Model-100 probe with 15 bins to calculate cloud microphysical parameters including liquid water content (LWC), number concentration (N), ε , σ and r_m . Some other information about measurement ranges and errors of instruments used in this study can be found in Guo et al., [2015]. The following criteria are used to identify individual clouds. CDSs with LWC larger than 0.001 g m^{-3} and N of cloud droplets greater than 10 cm^{-3} are considered to be cloud records [Lu et al., 2012b; Lu et al., 2013]. The percent of updraft in an individual cloud must be larger than 80 % to be selected as growing clouds [Gerber et al., 2008]. Cloud penetration time must exceed 5 s to obtain large clouds [Norgren et al., 2016]. There were a total of 99 precipitating clouds at the $\sim 3000 \text{ m}$ observation height satisfying these criteria and the following analyses are based on these clouds. The mean value, maximum value and standard deviation of the vertical velocities in the 99 clouds are 1.1 m s^{-1} , 6.0 m s^{-1} and 1.1 m s^{-1} , respectively. The number of clouds herein is slightly larger than that (92) in Guo et al., [2015] at the $\sim 3000 \text{ m}$ height under the similar criteria, because the clouds observed in the height range of 2900-3100 m are analyzed in this study, while Guo et al., [2015] only analyzed the clouds in the range of 2950-3050 m. In addition, the mean in-cloud buoyancy is larger than 0 m s^{-2} herein. The purpose is to select more relatively appropriate growing cloud samples.

2.2. Approach for estimating λ

Guo et al., [2015] estimated λ in growing clouds during TOGA-COARE using the traditional bulk-plume method [Gerber et al., 2008]:

$$\lambda = \frac{\partial \phi_c}{\partial z} \frac{1}{\phi_c - \phi_e}, \quad (1)$$

where z denotes height, ϕ_c and ϕ_e represent conserved physical quantities in cloud and environment, respectively. In our study, moist static energy (MSE) is used as the conserved

quantity when studying entrainment-mixing processes based on many previous studies [e.g., Khairoutdinov and Randall, 2006; Luo et al., 2010; Martin and Paul, 2013; Böing et al., 2014].

In Equation (1), $\partial\phi_c/\partial z$ is the vertical gradient of ϕ_c in cloud. Assuming λ is homogeneously distributed from cloud base to observational level, i.e., the obtained λ value is invariant within this height range, the average MSE on observational height and cloud base is used for ϕ_c . The environmental ϕ_e is taken as the average MSE within the same height range from the vertical profile of environmental MSE that is far away from the cloud edges. See details of the calculations in Guo et al., [2015]. The estimated λ values are used to study the influence of entrainment rate on ε . When λ is estimated assuming entrained air is near cloud edge, the results are similar to Section 3 hereinafter, and therefore are not included in this study.

3. Results

3.1. Influence of entrainment on ε

Entrainment can significantly influence the microphysical characteristics of CDSD. Figure 1 shows the relationship of ε to λ for the total 99 clouds. The correlation is negative with a correlation coefficient (R) of -0.37, suggesting that stronger entrainment reduces ε . This result is opposite to the traditional understanding that entrainment-mixing broadens CDSD [Su et al., 1998; Lasher-Trapp et al., 2005; Lu et al., 2013; Tölle and Krueger, 2014]. It is generally believed that CDSD broadens toward small cloud droplets and thus ε increases in the entrainment-mixing processes. For example, Lu et al., [2013] analyzed the correlation between ε and λ in shallow cumulus and they found that ε increases with increasing λ . But this study shows the inhibition effect of entrainment on ε . As mentioned in the introduction, ε is crucial for precipitation. So the influence of entrainment on ε is also very important. The mechanisms of how entrainment affects CDSD and ε are needed to be examined.

3.2. Influence of entrainment on CDSD

The relationship between ε and λ depends on the influence of entrainment on CDSD. Figure 2 shows the influence of λ on CDSD. Figure 2a is plotted by averaging CDSD for three λ bins with an identical bin width in the range of $\sim 0.10 \text{ km}^{-1}$ to $\sim 0.40 \text{ km}^{-1}$. The numbers of clouds in these three bins are 10, 50 and 39, respectively. With the increase of the whole droplet size

distributions move downward. Figure 2b shows the CSDs averaged for twelve λ bins with an identical bin width. The numbers of clouds in these twelve bins are respectively, 3, 1, 5, 7, 14, 10, 12, 11, 13, 6, 12 and 5. The first two bins are combined into one bin because there are too few samples in the individual bins. The variations of CSDs are quite complicated in Figure 2b compared to Figure 2a. It is interesting to find that the number concentrations of droplets with radius (r) $> \sim 3.75 \mu\text{m}$ decrease when λ increases from $< \sim 0.32 \text{ km}^{-1}$ to $> \sim 0.32 \text{ km}^{-1}$, but the small droplets ($r < \sim 3.75 \mu\text{m}$) increase. This inspires us to study the number concentration variations with λ for different droplet size ranges in more details. Thus CSDs are divided into three portions and the first portion has $r < 3.75 \mu\text{m}$ (N_1 , the first two bins in CSD). The other thirteen bins are divided into two approximately comparable parts (N_2 and N_3), with r between 3.75-12.75 μm (the middle six bins) and 12.75-23.25 μm (the last seven bins), respectively (Figure 2b). The relationships between N_1 , N_2 , N_3 and λ are shown in Figures 3a-3c, respectively.

Figure 3 shows that the correlations between λ and the number concentrations (N_1 , N_2 and N_3) in the three r ranges are all significantly negative as a whole and the correlation coefficients (R) for them are -0.32, -0.49 and -0.57, respectively. This confirms the conclusions in Figure 2a that the entrainment-mixing processes reduce droplet concentrations of all sizes simultaneously. But this is inconsistent with some previous observations [e.g., Lu et al., 2013] and simulations [e.g., Su et al., 1998] which found that CSDs broaden toward small droplets and thus the number concentration of small droplets increases in the entrainment-mixing process. Now the question boils down to how the negative correlations between N_1 , N_2 , N_3 and λ lead to the negative correlation between ε and λ .

3.3. Analysis based on σ and r_m

Because ε is the ratio of σ to r_m , so the effect of entrainment on ε should be the balance of the effects of λ on σ and r_m . Inspection of the relationships between σ and λ (Figure 4a) and between r_m and λ (Figure 4b) for the 99 clouds reveals that σ is negatively correlated with λ ($R = -0.50$) and the R of the correlation between r_m and λ is -0.32. However, as λ increases, the decreasing rate of σ (the slope of linear fitting is -9.32) is larger than that of r_m (the slope of linear fitting is -6.12). It is the unbalance of the decreasing rates that leads to the decrease of ε with increasing λ . In addition,

although the correlation between r_m and λ is negative (Figure 4b), the negative correlation is mainly caused by the six clouds with $r_m < \sim 4.90 \mu m$ in the bottom right corner of Figure 4b. If the six clouds were excluded, R for the correlation between r_m and λ would be only -0.07 (93 clouds). The correlation coefficients for the relationships between σ and λ , and between ε and λ in these 93 clouds would be -0.50 and -0.44, respectively. This further confirms that the negative correlation between ε and λ is mainly due to the faster and stronger decrease of σ compared to that of r_m as entrainment enhances if only based on mathematic relation.

As an absolute measure of spectral width of CDS, σ has also been extensively studied. The decrease of σ with increasing λ in Figure 4a also seems in conflict with the traditional understanding that entrainment-mixing increases spectral width (Lasher-Trapp et al., 2005) or the results of approximately constant σ in the process of entrainment-mixing [Lu et al., 2013; Pawlowska et al., 2006]. Recent studies on the correlation between σ and entrainment were full of discrepancies. For example, Prabha et al., [2012] found that σ reached its maximum in the less diluted regions of premonsoon clouds; but this trend was insignificant in monsoon clouds. The theoretical investigations of Pinsky et al., [2016] indicated that in the case of initially narrow CDS and low relative humidity in environment, σ increased and then decreased in the entrainment-mixing processes.

3.4. Comparison of the roles of droplets with different sizes

Detailed analyses are needed to combine the results in Sections 3.2 and 3.3 together and further explain how λ influences CDS, and then σ , r_m and ε for these clouds. According to the influences of λ on CDS and cloud droplets of different sizes discussed in Section 3.2, the variations of σ , r_m and ε with the number concentrations of the three sized droplet populations (N_1 , N_2 and N_3) can provide some clues (Figure 5).

The scatter plots in Figure 5 indicate that, the variation of σ depends mainly on N_3 (Figures 5a-5c). The R value of σ vs. N_3 for these clouds in Figure 5c is 0.88. The increased σ with increasing large droplets was also found in Prabha et al., [2012]. Given the influences of λ on CDS in Figure 2a, the inhibition of λ on N_3 causes a decrease in σ . As expected, r_m decreases with decreasing N_3 , but increases with decreasing N_1 (Figures 5d-5f). The balance of the two

opposite effects between N_1 and N_3 resulting from entrainment leads to the little variation of r_m with increasing λ for the major 93 clouds among 99 clouds (Figure 4b). Based on the above analyses in Section 3.3, the effects of λ on σ and r_m jointly lead to the negative correlation between ε and λ . Furthermore, in view of that entrainment-mixing necessarily reduce N_3 , so the degree of evaporation of small droplets (in size bin of N_1) could be the real factor that regulates and controls the variation of r_m in the process of entrainment-mixing. When the evaporation is incomplete, CDSB broadens toward small size; but when the evaporation is complete, small droplets are removed and r_m tends to increase. Therefore, the incomplete and complete evaporation of small droplets (in size bin of N_1) could lead to the increase and decrease of r_m , respectively and further different variation of ε .

It is also interesting to compare the roles of number concentrations of the droplets with different sizes on ε . Figure 5g shows that ε is positively correlated with N_1 ($R = 0.62$). With the increase of N_1 , σ shows little variation but r_m significantly decreases, and this results in a positive correlation between ε and N_1 . In contrast to N_1 , ε shows little variation with N_2 and N_3 (Figures 5h and 5i). Although ε is still positively correlated with N_3 , the data points are quite scattered. In these clouds, the slope (0.95) of σ vs. N_3 is relatively comparable with that (1.25) of r_m vs. N_3 , which results in the insignificant correlation between ε and N_3 in Figure 5i. Similar reasons are responsible for the marginal correlation between ε and N_2 in these clouds. These results suggest that N_1 is more important than N_2 and N_3 in determining the influence of entrainment on ε . In addition, N_3 is a more important factor affecting σ than N_1 and N_2 .

3.5. Further discussions

Furthermore, the above analyses show that the clouds analyzed in this study exhibit decreasing trends of N_1 with increasing λ . But Lu et al., [2013] observed that the number concentration of small droplets in CDSB increases as entrainment enhances and ε is positively correlated with λ . In fact for the relatively small cloud droplets in CDSB in this study, there are two inconsistent effects of entrainment on them because of different degree of evaporation of small droplets as mentioned above. On one hand, the increase of the number concentration of small droplets is accompanied with the size reduction of large droplets due to evaporation after

entrainment-mixing, consequently leading to the increase of the concentration of small droplets (Figure 6a). Under this circumstance, the evaporation of small droplets is incomplete, σ increases but r_m decreases. This is similar to the traditional understanding about the CSD evolution in the process of entrainment-mixing and this corresponds to the positive correlation between ε and λ . On the other hand, entrainment-induced dilution and evaporation can also reduce the sizes and number concentrations of small droplets. In this case, the evaporation of small droplets is complete, consequently leading to the decrease of small droplets (Figure 6b), and this corresponds to the negative correlation between ε and λ in this study. The mechanism is that because σ mainly reduces with the decrease in of large droplets but shows little variation with small droplets (Figures 5a-5c), thus σ significantly decreases when entrainment-mixing inhibits the concentration of large droplets. As shown in Figure 6b, all droplets with different sizes decrease as entrainment enhances, thus r_m has little change and consequently ε decreases ($\varepsilon=\sigma/r_m$). In a word, in the view of that large droplets necessarily decrease as entrainment enhances, the ultimate effects of entrainment on the number concentration of small droplets and the variation of ε could mainly depend on the degree of evaporation of small cloud droplets. As mentioned above, ε is crucial for precipitation. So the effect of entrainment on CSD leads to the decrease of ε and might reduce the probability of the coexistence of cloud droplets with different sizes, which goes against the collision-coalescence process of cloud droplets and the initiation of precipitation.

The decrease of N_1 with λ also reveals that for these deep convective clouds, there existed some small cloud droplets in CSD when the cloud parcels arrive at the observation levels. It is these already existed small droplets that make it possible for dilution or evaporation to decrease N_1 . These small droplets might be related to secondary mixing events [Lu et al., 2014] or to re-activation of entrained aerosol particles [Su et al., 1998; Lasher-Trapp et al., 2005]. Because of the existence of small droplets, the CSDs are wider than theoretically calculated CSDs. But for narrow CSDs, for example, in an adiabatic condition, the sizes of all droplets gather within a narrow r range. Entrainment-mixing leads to the broadening of CSD toward smaller sizes. This variation of CSD in the entrainment-mixing processes is similar to the variation trend in Figure 6a, i.e., an increase of small droplets. So the traditional understanding for the CSD evolution in the process of entrainment-mixing is mainly based on the theoretical adiabatic CSD and this necessarily results in the positive correlation between ε and λ [Burnet and Brenguier, 2007].

Therefore, it can be speculated that the promoting effect of entrainment on precipitation is also mainly based on the initially narrow CDS (shown in blue in Figure 6a), but not the CDS in Figure 6b. In a word, the effect of entrainment-mixing processes on ε is complicated, depending on the initial CDS shapes and the competition of the aforementioned different effect of entrainment on CDS.

In addition, it is also interesting to relate entrainment-mixing mechanisms, i.e., homogeneous and inhomogeneous mixing [Baker and Latham. 1979; Baker et al., 1980] to ε . Usually the relationship between volume-mean radius (r_v) and LWC can be used to judge the entrainment-mixing types. When mixing is homogeneous, all droplets evaporate at the same time with the dilution of the entrained air. So r_v reduces with the decreasing LWC. But for extreme inhomogeneous mixing, some droplets evaporate completely but the others are not influenced at all. Thus r_v does not change with the decreasing LWC. As analyzed in Guo et al., [2015], homogeneous mixing dominates in the clouds analyzed. However, Lu et al., [2013] found an increase of ε with increasing λ in the shallow cumuli wherein homogeneous mixing dominated. The theoretical study of Pinsky et al., [2016] indicated that the variation of ε in the process of homogeneous mixing was uncertain and depended on the environmental relative humidity and the initial properties of CDS. There is a need to consider ε in investigation of entrainment-mixing processes [Yum et al., 2015], but it is unclear now how ε changes in homogeneous entrainment-mixing processes. This topic is still in its infancy and deserves further studies in the future to examine.

4. Conclusions

Entrainment can significantly influence the microphysical characteristics of cloud droplet size distribution (CDS). Relative dispersion (ε) is a crucial microphysical parameter affecting cloud properties. So the influence of entrainment on ε of CDS in the 99 identified growing deep convective clouds observed at the ~ 3000 m height from the TOGA-COARE field campaign is examined in this study.

The results indicate that in these clouds, the correlation between ε and λ is negative, i.e., entrainment suppresses the ε of CDS. This is opposite to the traditional understanding that

entrainment broadens CDS toward smaller droplets, i.e., ε increases in the entrainment-mixing processes. In order to study the cloud droplet number concentration variations with λ for different droplet size ranges in more detail, CDSs are divided into three portions with droplet radius $< 3.75 \mu\text{m}$ (N_1), radius in the range of $3.75\text{-}12.75 \mu\text{m}$ (N_2) and $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively. The further analyses indicate that the number concentrations of all cloud droplets with different sizes decrease simultaneously as λ increases. This is also inconsistent to some previous observations [e.g., Lu et al., 2013] and simulations [e.g., Su et al., 1998] which found that CDS broaden toward small droplets and thus the number concentration of small droplets increase in the entrainment-mixing process.

In these 99 clouds, the variation of σ depends mainly on N_3 , while r_m reduces with the decrease of N_3 , but increases with decreasing N_1 , so the influence of entrainment on CDS causes a decrease in σ but little variation in r_m ; thus ε generally reduces as λ increases for these clouds. Furthermore, the comparison of the roles of droplets with different sizes reveals that N_1 is more important than N_2 and N_3 in determining the influence of entrainment on ε . But N_3 is a more important factor affecting σ than N_1 and N_2 . In a word, the effect of entrainment on CDS leads to the decrease of ε and might go against the collision-coalescence process of cloud droplets and precipitation. Finally, a conceptual model of CDS evolutions during entrainment-mixing processes is developed to show the scenarios where the relationship between ε and λ is positive or negative; the conceptual model further indicates that the small droplet and the degree of evaporation of small droplets are key factors shifting the positive or negative relationship. It is speculated that the promoting effect of entrainment on precipitation is mainly based on initially narrow CDS.

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Caption Lists

Figure 1. Scatter plot of entrainment rate λ vs. relative deviation ϵ of CDSD in the total 99 clouds. Legend provides the correlation coefficient (R) and p values for correlation.

Figure 2. Average CDSDs correspond to different entrainment rate (λ) for the total 99 clouds when CDSDs are averaged in (a) three bins and (b) twelve bins. The abscissas and ordinates represent cloud droplet radius (r) and dN/dr , respectively. The color bars represent λ . These CDSDs are divided into three portions (N_1 , N_2 and N_3 , based on droplet radius $< 3.75 \mu\text{m}$ (N_1), radius in the range of $3.75\text{-}12.75 \mu\text{m}$ (N_2) and $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively) by two dashed vertical lines in (b). See text for more details.

Figure 3. Scatter plots of entrainment rate (λ) vs. number concentration of cloud droplets with radius (a) $< 3.75 \mu\text{m}$ (N_1), in the range of (b) $3.75\text{-}12.75 \mu\text{m}$ (N_2) and (c) $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively. Each legend also provides the correlation coefficient (R) and p values for each correlation.

Figure 4. Scatter plots of entrainment rate λ vs. (a) standard deviation (σ) and (b) mean radius (r_m) of CDSD in the total 99 clouds. Each legend provides the correlation coefficient (R) and p values for each correlation.

Figure 5. Scatter plots of standard deviation (σ) vs. number concentration of cloud droplets with radius (a) $< 3.75 \mu\text{m}$ (N_1), in the ranges of (b) $3.75\text{-}12.75 \mu\text{m}$ (N_2) and (c) $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively for the total 99 clouds. (d)-(f) and (g)-(i) are the same as (a)-(c) but for mean radius (r_m) and relative dispersion (ϵ) vs. number concentration, respectively. Each legend also provides the correlation coefficient (R) and p values for each correlation.

Figure 6. Conceptual evolutions of CDSDs (from the blue to the red CDSD) after entrainment-mixing processes for (a) initially narrow CDSD, and (b) the 99 clouds analyzed in this study.

Figure 1. Scatter plot of entrainment rate (λ) vs. relative deviation (ϵ) of CDSD in the total 99 clouds. Legend provides the correlation coefficient (R) and p values for correlation.

Figure 2. Average CDSDs correspond to different entrainment rate (λ) for the total 99 clouds when CDSDs are averaged in (a) three bins and (b) twelve bins. The abscissas and ordinates represent cloud droplet radius (r) and dN/dr , respectively. The color bars represent λ . These CDSDs are divided into three portions (N_1 , N_2 and N_3 , based on droplet radius $< 3.75 \mu\text{m}$ (N_1), radius in the range of $3.75\text{-}12.75 \mu\text{m}$ (N_2) and $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively) by two dashed vertical lines in (b). See text for more details.

Figure 3. Scatter plots of entrainment rate (λ) vs. number concentration of cloud droplets with radius (a) $< 3.75 \mu\text{m}$ (N_1), in the range of (b) 3.75 – $12.75 \mu\text{m}$ (N_2) and (c) 12.75 – $23.25 \mu\text{m}$ (N_3), respectively. Each legend also provides the correlation coefficient (R) and p values for each correlation.

Figure 4. Scatter plots of entrainment rate (λ) vs. (a) standard deviation (σ) and (b) mean radius (r_m) of CDS in the total 99 clouds. Each legend provides the correlation coefficient (R) and p values for each correlation.

Figure 5. Scatter plots of standard deviation (σ) vs. number concentration of cloud droplets with radius (a) $< 3.75 \mu\text{m}$ (N_1), in the ranges of (b) $3.75\text{-}12.75 \mu\text{m}$ (N_2) and (c) $12.75\text{-}23.25 \mu\text{m}$ (N_3), respectively for the total 99 clouds. (d)-(f) and (g)-(i) are the same as (a)-(c) but for mean radius (r_m) and relative dispersion (ε) vs. number concentration, respectively. Each legend also provides the correlation coefficient (R) and p values for each correlation.

Figure 6. Conceptual evolutions of CDSs (from the blue to the red CDS) after entrainment-mixing processes for (a) initially narrow CDS, and (b) the 99 clouds analyzed in this study.

This study investigates the influence of entrainment rate on relative dispersion of cloud droplet size distribution in 99 clouds. To the authors' knowledge, this is the first observational study on the relationship between relative dispersion and entrainment rate in deep convective clouds. It is found that relative dispersion is negatively correlated with entrainment rate, and this is opposite to the traditional understanding that entrainment-mixing broadens cloud droplet size distribution. Herein we examine the mechanisms for the negative correlation between relative dispersion and entrainment rate. A conceptual model of cloud droplet size distribution evolutions during entrainment-mixing processes is developed to illustrate the possible scenarios entailing different relationships between relative dispersion and entrainment rate. The correlation between entrainment-mixing mechanism and relative dispersion is also discussed in paper.