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Modular focusing ring imaging Cherenkov detector for electron–ion collider experiments[☆]

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ABSTRACT

A powerful new electron–ion collider (EIC) has been recommended in the 2015 Long Range Plan for Nuclear Science for probing the partonic structure inside nucleons and nuclei with unprecedented precision and versatility [1]. EIC detectors are currently under development [2], all of which require hadron identification over a broad kinematic range. A prototype ring imaging Cherenkov detector has been developed for hadron identification in the momentum range from 3 GeV/c to 10 GeV/c. The key feature of this new detector is a compact and modular

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design, achieved by using aerogel as radiator and a Fresnel lens for ring focusing. In this paper, the results from a beam test of a prototype device at Fermilab are reported.

1. Introduction

The ability to identify hadrons is a key requirement for the physics program of the recommended EIC experiments [1]. By tagging the flavor of the struck quark in semi-inclusive deep inelastic scattering (SIDIS), one can obtain the transverse momentum distributions of the strange sea quark, while kaons as decay products from open charm events are important for probing the distribution of gluons in protons and nuclei.

Mesons in the electron beam direction and proton beam direction will carry momenta up to 10 GeV/c and 100 GeV/c, respectively, while the meson momenta in central region are typically less than 10 GeV/c, as shown in Fig. 1 [2,3]. To fulfill different particle identification (PID) requirements associated with the three different kinematic regions of the EIC detectors, an R&D consortium (eRD14) has been formed to study the technology choices of ring imaging Cherenkov (RICH) and time-of-flight detectors. A modular aerogel RICH (mRICH) detector is proposed to provide PID in the momentum range from 3 to 10 GeV/c in the electron beam direction.

In this paper, we present the beam test results of the first mRICH prototype at Fermilab and compare with the results obtained from realistic Geant4-based simulations.

2. Detector design

The current state-of-the-art designs of imaging Cherenkov detectors, such as proximity focusing detectors [4,5], or mirror-based imaging detectors [6–11] require substantial volumes to be able to identify particles close to 10 GeV/c momentum, due to the requirements imposed by the optical elements in the case of mirror-based detectors, or the desired ring separation in the case of proximity focusing devices. In the endcap regions of the proposed EIC detector, such space is not available. A more compact and modular design is required to fit into the available space and still provide hadron PID capability with the momentum coverage from 3 to 10 GeV/c.

The mRICH consists of four components. A block of aerogel serves as the Cherenkov radiator. Immediately following is an acrylic Fresnel lens which focuses the ring image and acts as a UV filter [12]. In the image plane is a pixelated optical sensor, and the gap between the lens and the image plane is bounded by four flat mirrors. The device is shown in Fig. 2. Also shown in Fig. 2 is an event display of a 9 GeV/c pion traveling toward the center of the mRICH detector using the Geant4-based [13,14] GEMC simulation framework.

In the GEMC simulation, the optical properties including the measured transmission, Rayleigh scattering, and the index of refraction of each component are implemented in the detector configuration. The distance from the center of the Fresnel lens and the sensor plane is set to the Fresnel lens focal length. The sensor plane consists of four square sensitive areas with a 1.5 mm thick glass window at the entrance to mimic Hamamatsu multianode PMTs.

Fig. 3 compares the design and performance of the mRICH detector to a proximity-focusing Cherenkov detector. Fig. 4 demonstrates the capability of the mRICH to shift the ring image to the central region of the photon sensor plane for particles which are not incident at the center of the aerogel block. It is clear that much less active photon sensor coverage is needed for the mRICH design in comparison with other RICH detector designs.

The PID performance of a ring imaging Cherenkov detector can be characterized by the separation power (or the number of standard deviations)

$$N_\sigma = \frac{\Delta\theta \sqrt{N_{\text{photon}}}}{\sigma_\theta}, \quad (1)$$

Fig. 1. Meson momentum vs rapidity in the laboratory frame from semi-inclusive deep inelastic scattering between 10 GeV/c electron and 100 GeV/c proton collisions with $Q^2 > 1 \text{ GeV}^2$. The rapidity is mathematically related to the angle of a particle relative to the beam axis. When a particle is traveling perpendicular to the beam axis, its rapidity is equal to zero. For a negative rapidity, the particle is moving toward the electron beam direction.

where $\Delta\theta$ is the difference of the Cherenkov angles produced by two different types of particles, N_{photon} is the number of photon detected, and σ_θ is the single photon Cherenkov angle resolution. Sources of uncertainty of the single photon angle resolution are the emission point σ_{EP} , the chromatic dispersion σ_{Chro} , and the pixel size σ_{Det} [15]. The resulting uncertainty σ_θ for a single photon measurement is the sum in quadrature:

$$\sigma_\theta = \sqrt{\sigma_{EP}^2 + \sigma_{Chro}^2 + \sigma_{Det}^2}. \quad (2)$$

In the lens based mRICH design, σ_{EP} is minimized at the lens focal plane, σ_{Det} can be controlled with the lens focal length, and σ_{Chro} is reduced by selecting the lens transmittance in the near-UV region. As a consequence, the lens insertion improves the RICH performance at high momenta even in a compact device. Preliminary studies show that a 3σ kaon-pion separation is achievable at 10 GeV/c momentum with an aerogel of refractive index $n = 1.03$, a focal length of 6" and a detector pixel of 2 mm.

The first prototype of this device consists of a 3.3 cm-thick aerogel block with index of refraction $n = 1.03$, a Fresnel lens with 76.2 mm (3") focal length from Edmund Optics [16], a mirror set mounted on top, bottom, left and right of the detector interior, and a sensor plane composed of four Hamamatsu H8500 multianode PMTs. The assembled mRICH prototype is shown in Fig. 5 (bottom). The expanded 3D rendering of the detector components is also shown in Fig. 5 (top).

3. Beam test at Fermilab

The first prototype of the mRICH detector was tested at the Fermilab Test Beam Facility (FTBF) under the T-1048 experiment in April 2016. The T-1048 experiment used the MTest beamline which has two modes of operation: primary protons of 120 GeV and a secondary mixed beam consisting primarily of pions, muons, and electrons with energies ranging from 1 to 60 GeV of either positive or negative charge. The beam is delivered as a slow spill with a 4 s duration once per minute with a maximum intensity of $\sim 10^5$ particles per spill. The momentum

Fig. 2. (left) mRICH detector and its components; (right) an event display of a single 9 GeV/c pion traveling toward the center of the mRICH detector.

Fig. 3. Comparison of the ring image characteristics between a proximity-focusing RICH (top, based on the BELLE-2 ARICH detector design) and the mRICH (bottom). The ray diagrams on the left (not to scale, for illustration purposes only) show the optical properties without (top) and with focusing lens (bottom). The ring images are generated with the GEMC simulation from 100 pions of 9 GeV/c traveling in parallel to the principal axis of the lens. The distance from the first aerogel surface to the sensor plane of the proximity RICH is 23 cm. The total thickness of the composite aerogel block is 4 cm. The focal length of the mRICH Fresnel lens is 7.6 cm and the mRICH aerogel block thickness is 3.3 cm. The blank regions in the lower right X-Y hits scatter plot are the physical separations between the photon sensor modules that is only relevant to the mRICH. The orange line in the ray diagram is to indicate the location of the focused ring image produced by the incident particle at a given momentum.

Fig. 4. Comparison of the ring image location on the sensor plane between a proximity-focusing RICH (top, based on the BELLE-2 ARICH detector design) and the mRICH (bottom). The ray diagrams on the left (not to scale, for illustration purposes only) show the optical properties without (top) and with focusing lens (bottom). The ring images are generated with the GEMC simulation from 100 pions of 9 GeV/c traveling in parallel to but off the principal axis of the lens. See Fig. 3 caption for the dimensions of the proximity-focusing RICH and the mRICH.

spread of the beam depends on the beam tune and is $\pm 3\%$. The beam spot size is also dependent on the beam energy and tuning and ranges from ~ 6 mm to several centimeters. Given the short time period, the mRICH detector was tested mainly with the 120 GeV/c primary proton beam at the minimum spot size and parallel to the mRICH axis. The limited data with secondary pion beam at 4 and 8 GeV/c and with light reflected by the lateral mirrors will be the subject of a future publication.

The mRICH prototype detector was mounted at the center of a test stand made from extruded aluminum. Two sets of hodoscopes (each consisting of 4 channels of horizontal 1 cm wide finger scintillator rods and 4 channels of vertical rods) were mounted to the test stand in front and in the back to define the incident particle position. The whole test assembly was put on a X-Y table for aligning the detector with the particle beam. A beam event was triggered using a pair of 7 cm-wide plastic scintillator paddles as shown in Fig. 6.

4. Data analysis and results

Since the main goal of this prototype study is to test the working principles of the detector design and its performance, the analysis focused on the 120 GeV primary proton beam test data set, because of its small

beam size, in order to clearly identify the saturated ring image formed on the sensor plane. Fig. 7 shows the accumulated photon hits generated from the 120 GeV proton beam incident at the center of the mRICH (left panel) and the hit displays from the Geant4 simulation (right panel) for one thousand events. The simulation matches reasonably well the data on the rings, but not the noise hits observed in the data.

Typical noise sources include photons from Rayleigh scattering inside the aerogel block, Cherenkov photons from the Fresnel lens and from the sensor glass windows, and possibly from the back-scattered particles from the sensor electronics produced by the incident beam particle. The noise hits from the Rayleigh scattering inside the aerogel block and the Cherenkov photons from the Fresnel lens generate random and uniform hits on the sensor plane. Cherenkov photons generated inside the PMT glass window and the noise hits from the back-scattered particles tend to be close to the spot where the beam particles hit because of the short distance among these components. Except for the noise hits from the back-scattered particles, we have implemented these noise sources in the simulation.

In order to quantify the test results, a ring finding algorithm, using a circular Hough transform [17], was developed for determining the radius of the Cherenkov ring image and the number of the Cherenkov

Fig. 5. (top) 3D rendering of the mRICH detector components; (bottom) a photo of the assembled mRICH prototype detector without the top cover.

photons on an event-by-event basis. In each event, the most likely ring is extracted from a probability density distribution, which is built by all possible rings found in that event. Fig. 8 shows the reconstructed rings from two typical events from the test beam data.

We also made analytical estimates of the ring radius and the number of the detected Cherenkov photons and compared them with the beam

test data and with the simulation results. The analytical equations are given in Appendix. In the calculation of the total number of photons, the transmission of each component and the quantum efficiency of the Hamamatsu H8500 multianode PMT are included. The analysis results are summarized in Table 1. Both the Cherenkov ring radius and the number of photons found from the test beam data and from simulations agree very well with the analytical calculations.

For verifying the shifted ring image feature of the mRICH, we show the accumulated photon hits in Fig. 9 from the 120 GeV/c proton beam incident at the lower-left quadrant of the mRICH (left panel) and the hit displays from the corresponding simulation (right panel). The simulated proton beam is uniformly distributed within a circular area of 1 cm in diameter in order to mimic the realistic beam size. Because of the small primary proton beam size, we are able to get an excellent agreement between the results from the beam data and the simulated results as shown in Fig. 9.

In order to determine the radius of the shifted ring image in the central region of the photon sensor plane, we applied in this case the same ring finding algorithm described in [17] and summarized the results in Table 2. Also shown in Table 2 is the total number of photons which includes the background photons emitted from the glass window of the PMT. We observed about 55% more photons in this case than when the beam hits the center of the mRICH without traversing any of the PMT glass windows. The number of photons on the ring, however, is reduced by 1.5 photons per event on average. A remarkable agreement is seen between the data and the simulated results.

5. Summary and conclusion

A prototype of a modular RICH detector has been successfully tested at Fermilab. The design features of this detector were well demonstrated with consistent results between test data, the detector simulation using Geant4, and the analytical calculations. After completing this initial validation, a new prototype is now under construction. This includes a 6-inch focal-length Fresnel lens and new Hamamatsu multi-anode PMTs with a pixel size of 3 mm × 3 mm, which should provide the resolution needed to demonstrate that the mRICH can satisfy the particle identification requirements of a future EIC detector.

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Fig. 6. The mRICH test setup at the Fermilab Test Beam Facility.

Table 1

The radius of the Cherenkov ring image and the number of Cherenkov photons at the sensor plane from the 120-GeV proton beam incident at the center of the mRICH. The total number of photons is the number of photons which reach the sensor plane. The number of photons on the ring is the number of photons counted along the reconstructed ring image.

	Analytical calculation	Test beam data	Simulation
Radius (mm)	19.4	19.0 ± 1.3	18.9 ± 1.0
Total number of photons	10.4	11.0 ± 2.9	11.1 ± 2.9
Number of photons on the ring		5.9 ± 1.8	5.8 ± 1.5

Fig. 7. Accumulated ring image displays: (left) from 120 GeV proton beam incident at the center of the mRICH; (right) from the matching Geant4 simulations. The blank regions are the gaps between the four Hamamatsu H8500 modules.

Fig. 8. Two examples of the reconstructed rings using the circular Hough transform algorithm.

Table 2

The radius of the Cherenkov ring image and the number of Cherenkov photons at the sensor plane from 120 GeV proton beam incident at the lower-left quadrant of the mRICH.

	Test beam data	Simulation
Radius (mm)	20.8 ± 1.5	21.6 ± 0.9
Total number of photons	17.6 ± 3.5	17.0 ± 3.4
Number of photons on the ring	4.4 ± 1.0	4.4 ± 1.4

constructing the prototype detector. This work was partially supported with the EIC R&D funding from the Office of Nuclear Physics in the Office of Science of the Department of Energy.

Appendix. Analytical calculation

In this appendix, we provide equations of the analytical calculations of the ring image radius and the number of Cherenkov photons at the sensor plane. See Fig. 3 for variable notations in the ray diagram.

The emission angle of Cherenkov radiation θ_c is given by

$$\theta_c = \cos^{-1} \frac{1}{\beta n}, \quad (\text{A.1})$$

where $\beta = \frac{v}{c}$ is the ratio of incident particle velocity to the speed of light inside the radiator, and n is the index of refraction of the aerogel. Considering the refraction at the interface between the aerogel and the air, the refracted angle θ of a Cherenkov photon is

$$\theta = \tan^{-1} \sqrt{\frac{n^2 \beta^2 - 1}{1 - (n^2 - 1) \beta^2}}, \quad (\text{A.2})$$

and the Cherenkov ring radius is

$$\begin{aligned} r &= f \cdot \tan \theta \\ &= f \cdot \sqrt{\frac{n^2 \beta^2 - 1}{1 - (n^2 - 1) \beta^2}}. \end{aligned} \quad (\text{A.3})$$

Fig. 9. Accumulated ring image displays: (left) from 120 GeV proton beam incident at the lower-left quadrant of the mRICH; (right) from the corresponding Geant4 simulations. The blank regions are the gaps between the four Hamamatsu H8500 modules.

The number of photons emitted in the aerogel block is

$$N_\gamma = 2\pi\alpha L \left(1 - \frac{1}{\beta^2 n^2}\right) \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2}, \quad (\text{A.4})$$

where L is the block thickness and α is the hyperfine structure constant. However, not all photons survive to reach to the photon sensors. Therefore, one should include the transmissions of each component. The number of surviving photons is

$$N_\gamma = 2\pi\alpha L \left(1 - \frac{1}{\beta^2 n^2}\right) \times \int_{\lambda_1}^{\lambda_2} QE(\lambda) \cdot T_{\text{aerogel}}(\lambda) \cdot T_{\text{lens}}(\lambda) \cdot T_{\text{glass window}}(\lambda) \frac{d\lambda}{\lambda^2}, \quad (\text{A.5})$$

where $QE(\lambda)$ is the quantum efficiency of the H8500 photon sensor. $T_{\text{aerogel}}(\lambda)$, $T_{\text{lens}}(\lambda)$, and $T_{\text{glass window}}(\lambda)$ are the transmissions of the aerogel block, Fresnel lens, and glass entrance window of the photon sensor, respectively.

References

- [1] A. Aprahamian, et al. Reaching for the horizon: The 2015 long range plan for nuclear science, 2015.
- [2] A. Accardi, et al., Electron-Ion Collider: The next QCD frontier, *Eur. Phys. J. A* 52 (9) (2016) 268 URL <http://dx.doi.org/10.1140/epja/i2016-16268-9>.
- [3] eRD14 PID Consortium, An integrated program for particle identification (PID) for a future electron-ion collider (EIC) detector, 2016. URL https://wiki.bnl.gov/conferences/images/d/d2/ERD14_FY17.v2.pdf (accessed 07.06.16).
- [4] T. Iijima, et al. A novel type of proximity focusing RICH counter with multiple refractive index aerogel radiator, *Nucl. Instrum. Methods Phys. Res. Sect. A*, (3) 383, <http://dx.doi.org/10.1016/j.nima.2005.05.030> ISSN:0168-9002.
- [5] R. Pestotnik, et al., Aerogel RICH for forward PID at Belle II, *Nucl. Instrum. Methods Phys. Res. Sect. A* 732 (2013) 371. <http://dx.doi.org/10.1016/j.nima.2013.06.080>. URL <http://www.sciencedirect.com/science/article/pii/S0168900213009261> Vienna Conference on Instrumentation 2013.
- [6] M. Adinolfi, et al., Performance of the LHCb RICH detector at the LHC, *Eur. Phys. J. C* 73 (5) (2013) 2431 URL <http://dx.doi.org/10.1140/epjc/s10052-013-2431-9>.
- [7] A.A. Augusto Jr, et al., The LHCb detector at the LHC, *J. Instrum.* 3 (08) (2008) S08005 URL <http://stacks.iop.org/1748-0221/3/i=08/a=S08005>.
- [8] J. Engelfried, et al., SELEX RICH performance and physics results, *Nucl. Instrum. Methods Phys. Res. Sect. A* 502 (1) (2003) 285–288. [http://dx.doi.org/10.1016/S0168-9002\(03\)00289-4](http://dx.doi.org/10.1016/S0168-9002(03)00289-4). URL <http://www.sciencedirect.com/science/article/pii/S0168900203002894> Experimental Techniques of Cherenkov Light Imaging. Proceedings of the Fourth International Workshop on Ring Imaging Cherenkov Detectors.
- [9] J. Engelfried, et al., The SELEX phototube RICH detector, *Nucl. Instrum. Methods Phys. Res. Sect. A* 431 (1–2) (1999) 53– 69. [http://dx.doi.org/10.1016/S0168-9002\(99\)00043-1](http://dx.doi.org/10.1016/S0168-9002(99)00043-1). URL <http://www.sciencedirect.com/science/article/pii/S0168900299000431>.
- [10] M. Contalbrigo, E. Cisbani, P. Rossi, The CLAS12 large area RICH detector, *Nucl. Instrum. Methods Phys. Res. Sect. A* 639 (1) (2011) 302–306. <http://dx.doi.org/10.1016/j.nima.2010.10.047>. URL <http://www.sciencedirect.com/science/article/pii/S0168900210022862> Proceedings of the Seventh International Workshop on Ring Imaging Cherenkov Detectors.
- [11] R.A. Montgomery, A ring imaging Cherenkov detector for CLAS12, *Nucl. Instrum. Methods Phys. Res. Sect. A* 732 (2013) 366–370. <http://dx.doi.org/10.1016/j.nima.2013.08.012>. URL <http://www.sciencedirect.com/science/article/pii/S0168900213011388> Vienna Conference on Instrumentation 2013.
- [12] D. Fields, H. van Hecke, J. Boissevain, B. Jacak, W. Sondheim, J. Sullivan, W. Willis, K. Wolf, E. Noteboom, P. Peters, R. Burke, Use of aerogel for imaging Cherenkov counters, *Nucl. Instrum. Methods Phys. Res. Sect. A* 349 (2) (1994) 431–437. [http://dx.doi.org/10.1016/0168-9002\(94\)91207-6](http://dx.doi.org/10.1016/0168-9002(94)91207-6). URL <http://www.sciencedirect.com/science/article/pii/0168900294912076>.
- [13] J. Allison, et al., Recent developments in Geant4, *Nucl. Instrum. Methods Phys. Res. Sect. A* 835 (2016) 186–225. <http://dx.doi.org/10.1016/j.nima.2016.06.125>. URL <http://www.sciencedirect.com/science/article/pii/S0168900216306957>.
- [14] GEant4 Monte-Carlo (GEMC). URL <https://gemc.jlab.org/gemc/html/index.html> (accessed 07.31.2017).
- [15] C. Lippmann, Particle identification, *Nucl. Instrum. Methods Phys. Res. Sect. A* 666 (2012) 148–172. <http://dx.doi.org/10.1016/j.nima.2011.03.009>. URL <http://www.sciencedirect.com/science/article/pii/S0168900211005419> Advanced Instrumentation.
- [16] Edmund Optics, Specification of Fresnel Lens (stock no. 32–593). URL <http://www.edmundoptics.com/optics/optical-lenses/fresnel-lenses/fresnel-lenses/32593/>.
- [17] E.E. Zelniker, I.V.L. Clarkson, Maximum-likelihood estimation of circle parameters via convolution, *IEEE Trans. Image Process.* 15 (4) (2006) 865–876.