High precision at a Linear Collider – the need for radiative corrections

W. Hollik, MPI Munich

LoopFest 2002

- Radiative corrections for precision observables in the SM
- Radiative corrections for precision observables in SUSY extensions of the SM
- Radiative corrections for precision Higgs physics
- Radiative corrections for precision SUSY physics

The Standard Model

- the symmetry group $SU(2)_L \times U(1) \times SU(3)_C$
- the principle of local gauge invariance
  $\longrightarrow$ fermion – vector boson interaction
  $\longrightarrow$ vector boson – vector boson interaction
- Higgs mechanism and Yukawa interactions
  $\longrightarrow$ masses M_W, M_Z, m_{fermion}

renormalizable quantum field theory
accurate theoretical predictions

Beyond the Standard Model

- further unification of forces
- hierarchy of interactions
- hierarchy of fermion masses
- concept of Supersymmetry

Standard Model is renormalizable

→ quantum effects are calculable

Nobel Prize 19
→ 't Hooft
Veltman

- lowest order (Born approximation)

$g = O(e)$

- next order (1-loop):

all particles of the SM (virtual states)

top: $\sim m_t^2$, Higgs: $\sim \log M_t$

top

⇓

determination of top mass

Higgs

⇓

constraints on Higgs-boson mass

Higgs mass [GeV]
year
95% limit LEPEW fit
direct search
1993
1994
1995
1996
1997
1998
1999
2000
2001
0
200
400
600
800
$\Delta\chi^2$
Theory uncertainty
$\Delta\alpha^{(5)}_{had}(s)$ =
0.02761±0.00036
0.02738±0.00020
Preliminary
Excluded
m_H [GeV]
10^2

$m_H < 196$ @ 95% C.L.

After LEP:

- Successful era of electroweak precision physics:
  - quantum effects have been established
  - top quark has been discovered where expected
  - Higgs boson is awaiting its discovery

- Even if a Higgs boson will be discovered:
  $\rightarrow$ standard Higgs boson?
  $\rightarrow$ SUSY?
  $\rightarrow \cdots$?

High precision (theory) needed to

- extract parameters (masses, couplings) from e^+e^- processes, e.g.

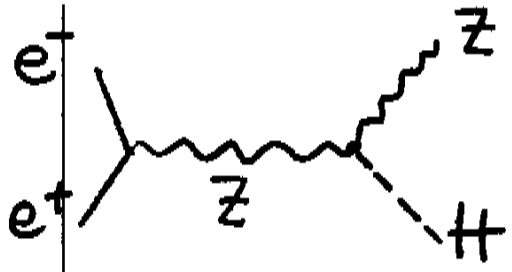
  M_W, m_t, M_H, Yukawa couplings, SUSY(?), ...

- establish model-dependent relations between parameters, e.g.

$$
\begin{aligned}
M_W &\leftrightarrow m_t,\ M_H,\ \ldots \\
m_{\text{fermion}} &\leftrightarrow \text{Yukawa couplings} \\
\text{SUSY masses} &\leftrightarrow \text{SUSY couplings}
\end{aligned}
$$

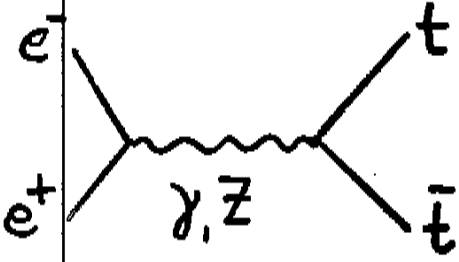
  specific model predictions

  precision tests of SM and beyond,
  especially SUSY models (MSSM)



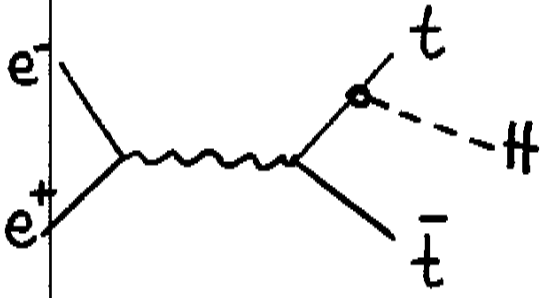
"Higgs - Strahlung"

M_H, Higgs profile

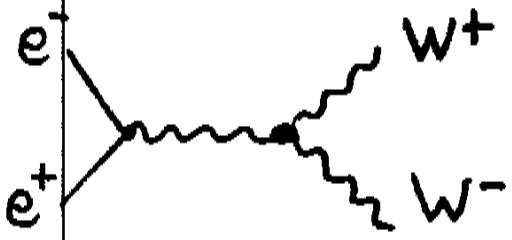


top-quark pairs

m_t, Γ_t, couplings

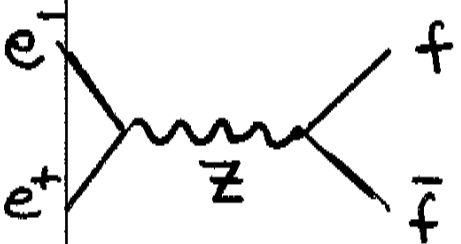


top - Higgs Yukawa coupling



W-pairs

M_W, self interaction



Z resonance [GigaZ]

Z boson couplings

$\sin^2\Theta_{(eff)}$ at M_Z

(expected) experimental precision

error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	6
$\sin^2\theta_{\rm eff}$	0.00017 LEP+SLC	0.00021		0.000013
$m_{\rm top}$ [GeV]	5.1	2	0.2	0.13
$M_{\rm Higgs}$ [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

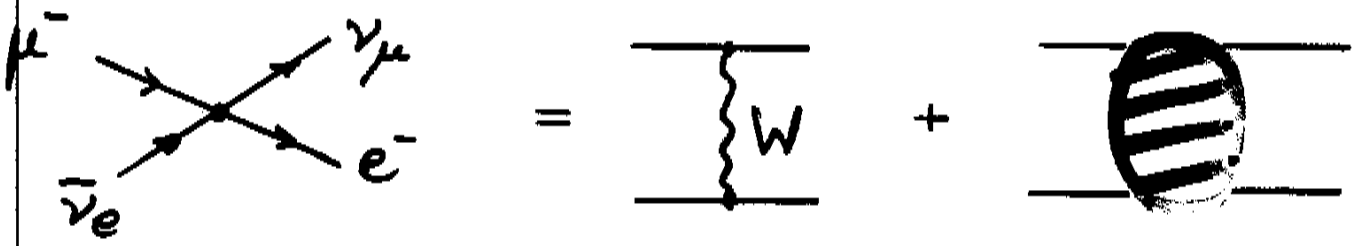
$\delta G_{\rm F}/G_{\rm F} = 1 \cdot 10^{-5}$ (μ lifetime)

Masses of W and Z bosons

correlated via muon lifetime $\leftrightarrow$ Fermi constant G_μ

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1-\frac{8m_e^2}{m_\mu^2}\right)\cdot(1+\delta_{\rm QED})$$

$$G_\mu = 1.16637(1)\cdot 10^{-5}\ \text{GeV}^{-2}$$



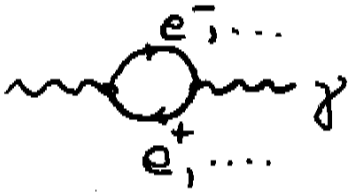
Fermi Model Standard Model

$$G_\mu = \frac{\pi}{\sqrt{2}}\cdot\frac{\alpha}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\cdot\frac{1}{1-\Delta r}$$

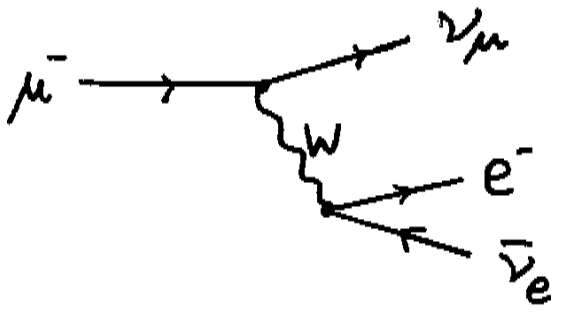
$$\Delta r = \Delta\alpha + \Delta r_W(m_t, M_H)$$

6%

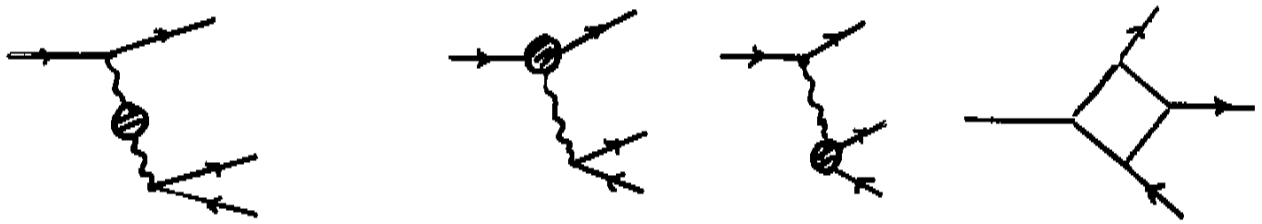
[QED]



μ decay $\longrightarrow$ Δr



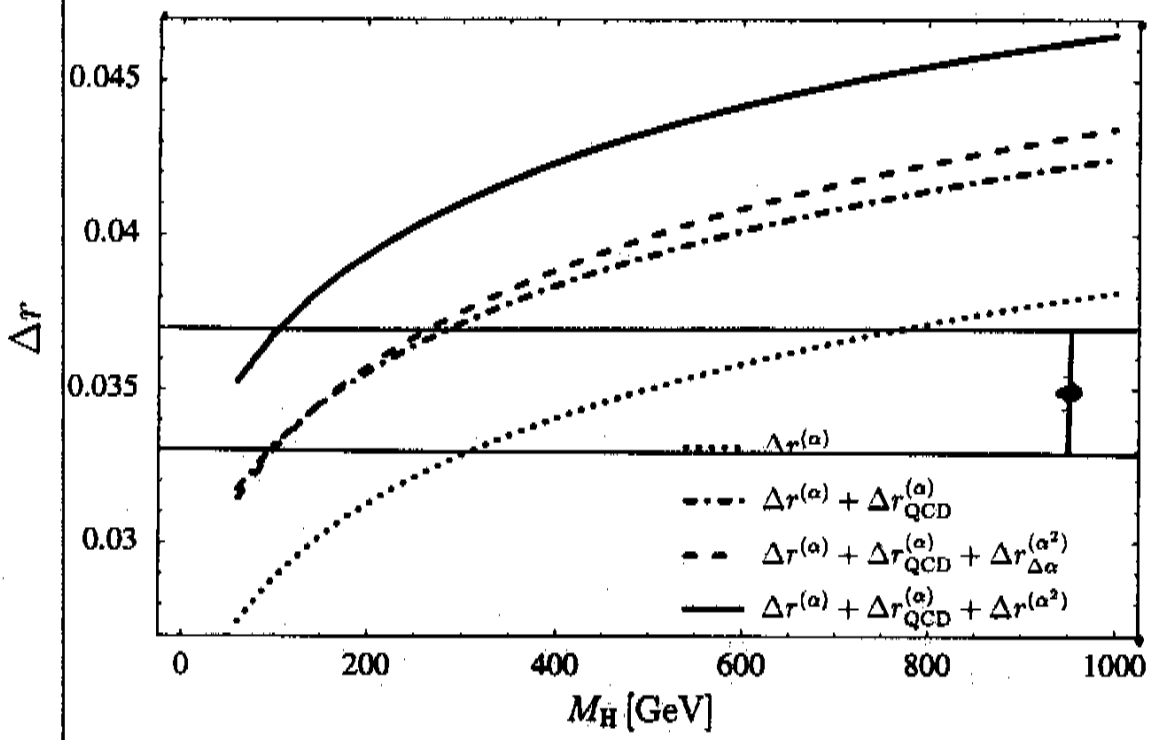
Born



self energy vertex corrections box graph

- 1-loop: complete [Sirlin, Marciano 1980]
- 2-loop: QCD
  EW fermionic loops
  EW bosonic loops: missing
- 3-loop: leading QCD + EW
  (ρ-parameter)

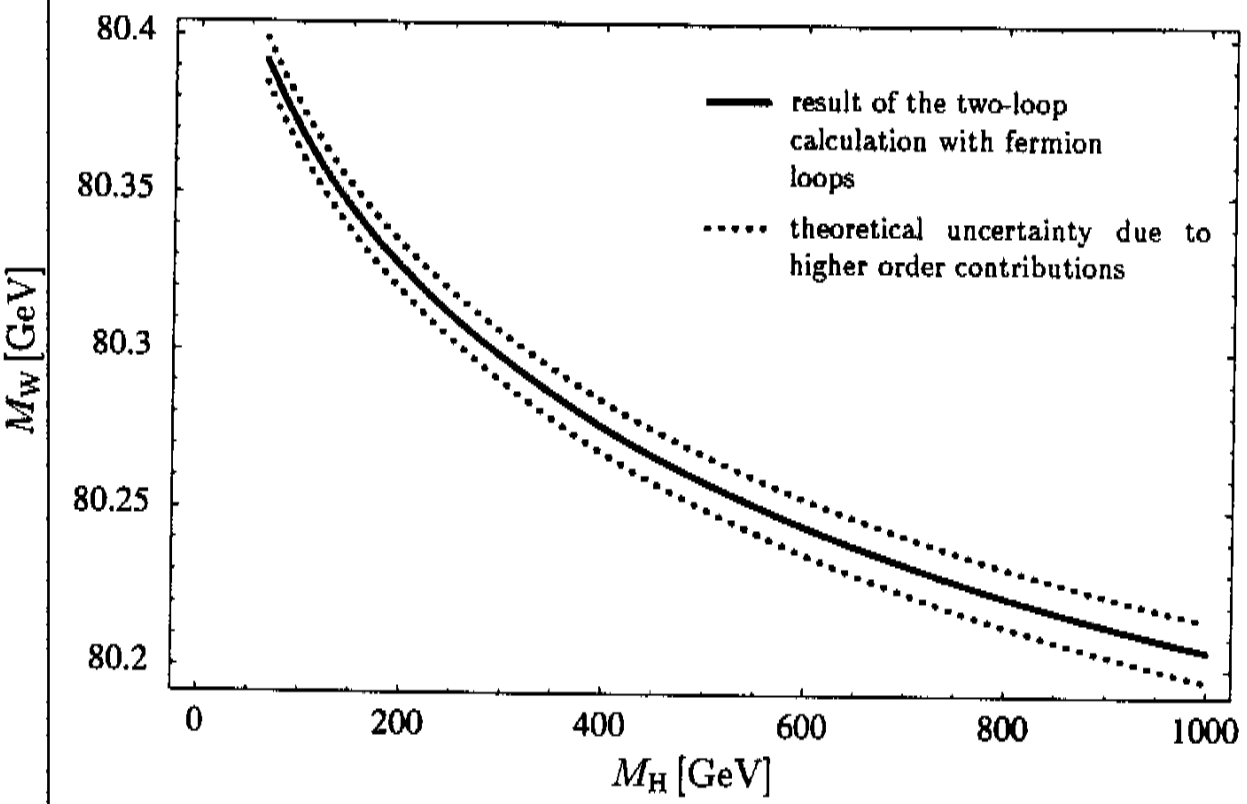
[Freitas, WH, Walter, Weiglein]



Uncertainty from experimental error of input parameters: $\delta m_t = \pm 5.1$ GeV $\Rightarrow \delta M_W \approx \pm 31$ MeV

Estimate of theoretical uncertainty from unknown higher-order corrections:

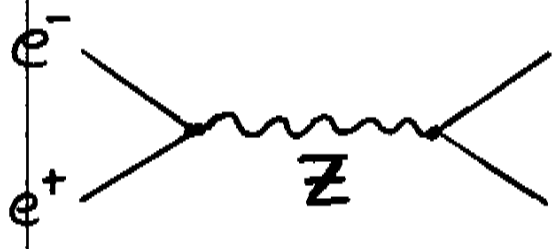
- Purely bosonic $\mathcal{O}(\alpha^2)$ electroweak corrections
- Higher-order electroweak corrections: $\mathcal{O}(\alpha^3)$
- Higher-order QCD corrections: $\mathcal{O}(\alpha\alpha_s^3)$, $\mathcal{O}(\alpha^2\alpha_s)$



For light Higgs masses:

$$\Delta M_W^{\text{theo}} \approx \pm 6\,\text{MeV}$$

Z resonance



~ $16 \cdot 10^6$ at LEP (1989 – 1995)

$$\sigma(s) = 12\pi \frac{\Gamma(Z \to e^+e^-) \cdot \Gamma(Z \to f\bar{f})}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

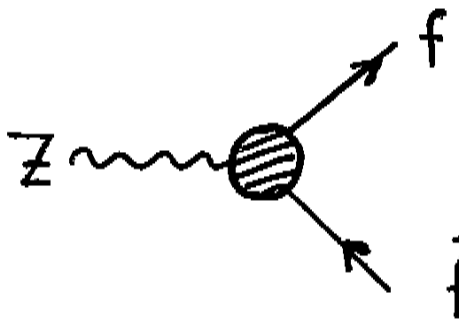
Breit-Wigner

$\uparrow$ E^2_{CMS}

width: $\Gamma_Z = \underbrace{\Gamma(e,\mu,\tau)}_{\text{lept.}} + \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadron.}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu}}_{\text{invisible}}$

- Line shape → M_Z, Γ_Z
- σ_{peak} → partial widths Γ_{lept}, Γ_{had}
- angular distributions, polarization asymmetries → $\sin^2\Theta_W$

Effective Z couplings



$$g_A = \sqrt{\rho_f}\, I_3^f$$

$$g_V = \sqrt{\rho_f}\left(I_3^f - 2 Q_f \sin^2\theta_f\right)$$

$$\rho = \rho(m_t, M_H, \ldots)$$

$$\sin^2\theta_f = \sin^2\theta_f(m_t, M_H, \ldots)$$

f = lepton: $\sin^2\theta_{eff}$

complete at 1-loop

+ 2-loop m_t^4, m_t^2 terms

Prediction for $\sin^2\Theta_{eff}$:

$$\sin^2\Theta_{eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right) \cdot \kappa(m_t, M_H, \alpha_s)$$

↑
complete 2-loop fermionic EW contribution not know

$\left.\begin{matrix} \delta M_W \\ \delta \kappa \end{matrix}\right\} \Rightarrow \quad \delta \sin^2\Theta_{eff}$

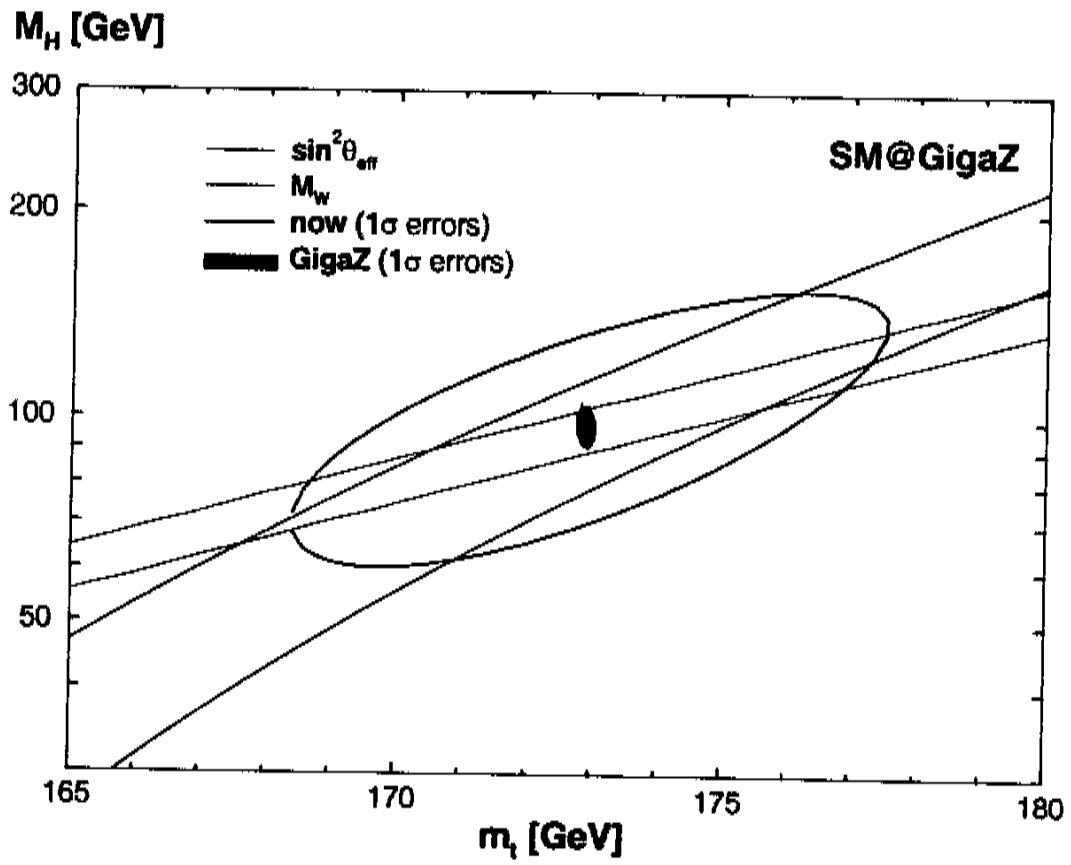
uncertainty of $M_W \rightarrow \quad 8 \cdot 10^{-5}$

compensation (partially) through κ expected

$\Rightarrow$ κ is needed

fermionic + bosonic 2-loop terms

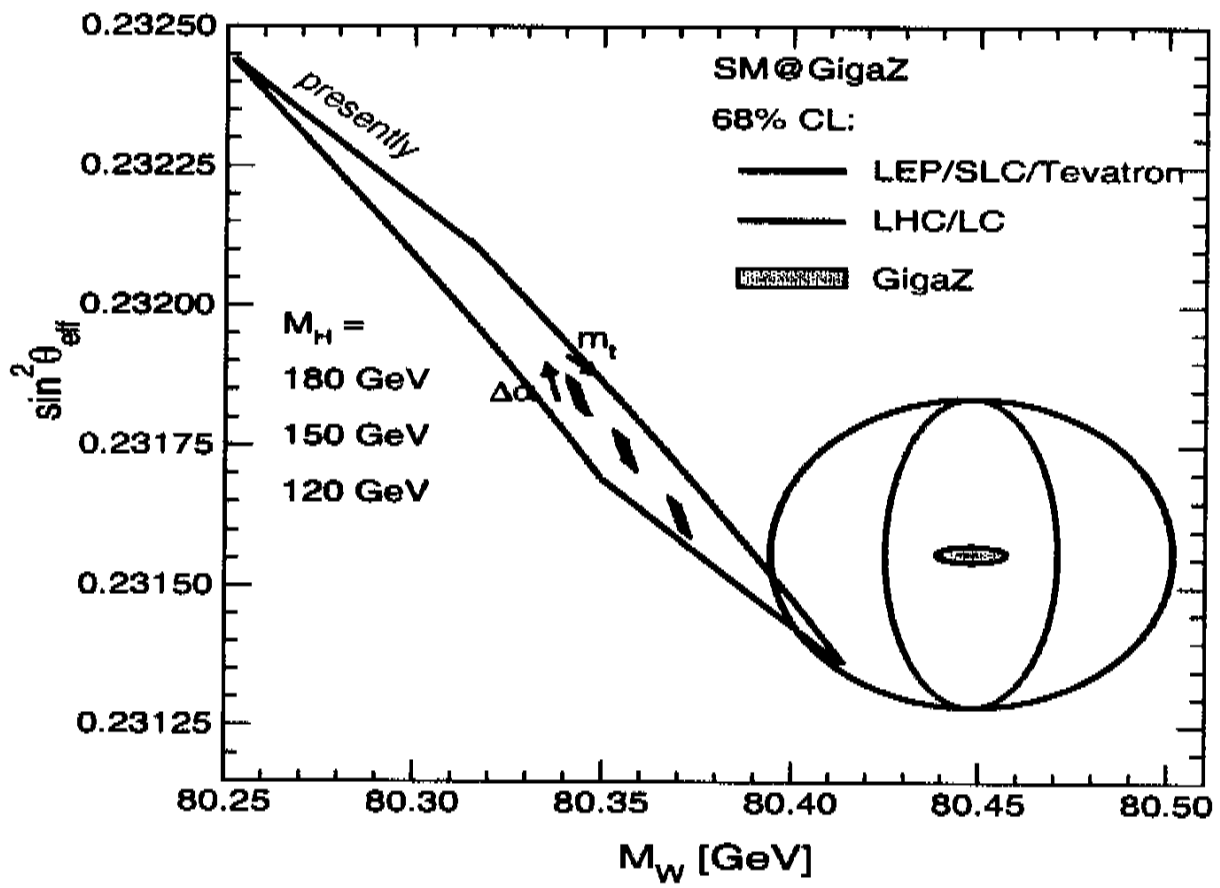
- complete EW 2-loop calculation for Δr and $\sin^2\Theta_{eff}$ necessary for LC / GigaZ



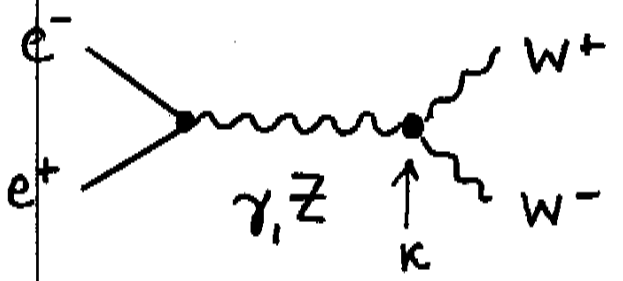
[Erler, Heinemeyer, WH, Weiglein, Zerwas]

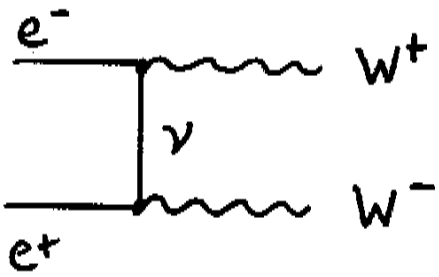
$\delta M_H / M_H \sim 7\,\%$

[*J. Erler, S. Heinemeyer, W. H., G.Weiglein, P. Zerwas '00*]



$e^+e^- \longrightarrow W^+W^-$

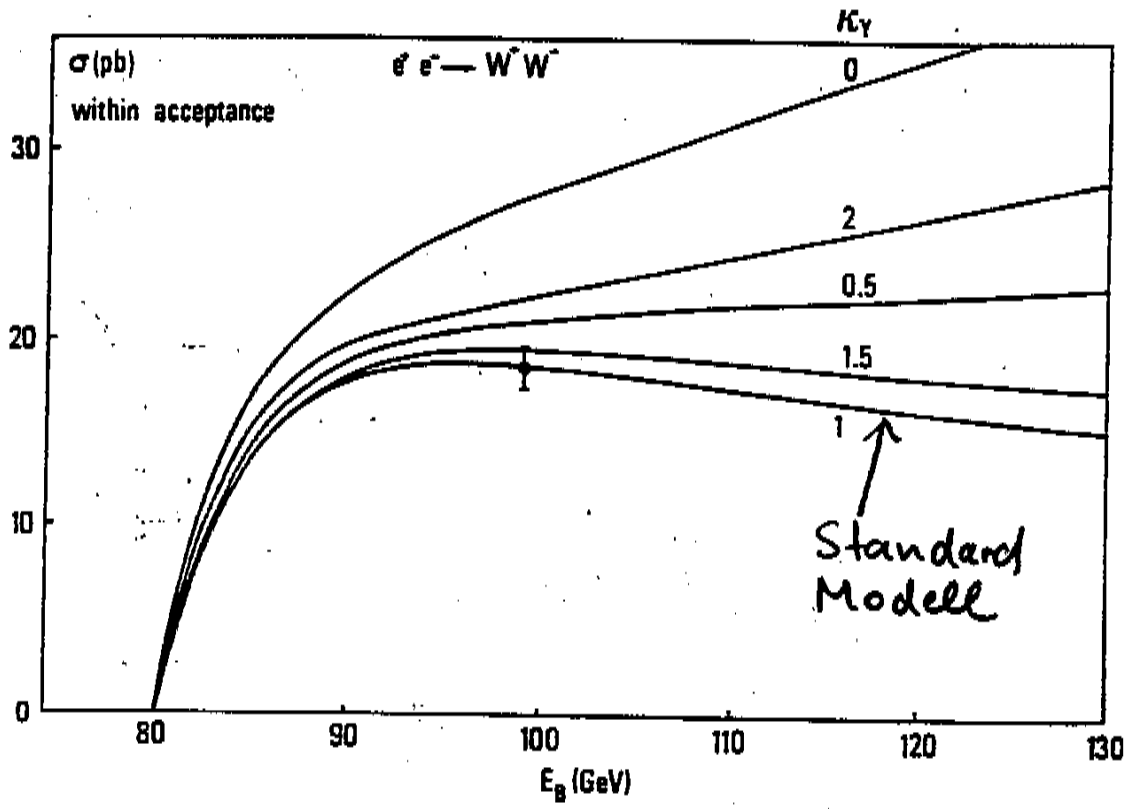


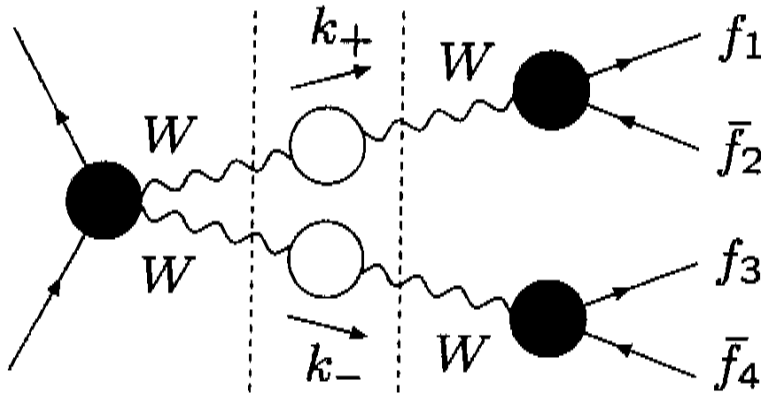


(i) precise measurement of M_W

$\Delta M_W \simeq 40$ MeV at LEP
$\longrightarrow$ 6 MeV at LC / GigaZ

(ii) test of trilinear coupling



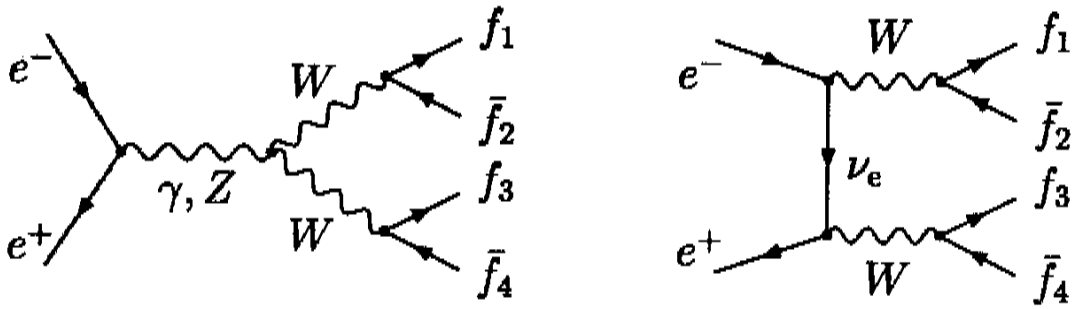


+ radiation of photons

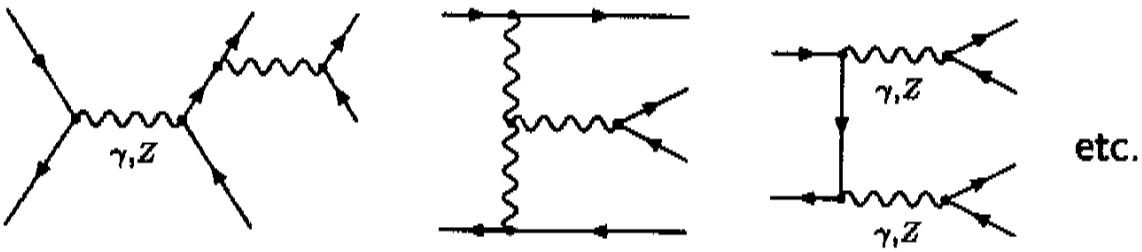
+ background processes

Lowest-order four-fermion cross section

Signal diagrams: two resonant W bosons



Background diagrams: at most one resonant W



etc.

Typical size $\approx \frac{\Gamma_W}{M_W} \approx 2.5\%$

Monte Carlo generator RACOONWW

$\mathcal{O}(\alpha)$ corrections with RACOONWW

Denner, Dittmaier, Roth, Wackeroth '99, '00

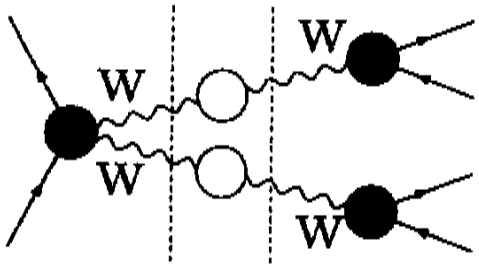
virtual

real

$e^+e^- \to 4f+\gamma$ (tree level):
Full matrix element

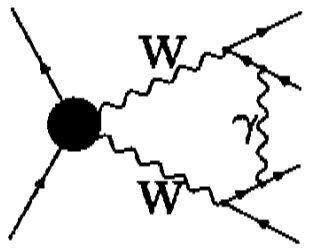
$e^+e^- \to WW \to 4f$ (1 loop):
Double-pole approximation

Factorizable corrections



building blocks:
– W-pair production
– W decay

Non-factorizable corrections



etc.

γ-exchange between production- and decay-subprocesses with $E_\gamma \lesssim \Gamma_W$

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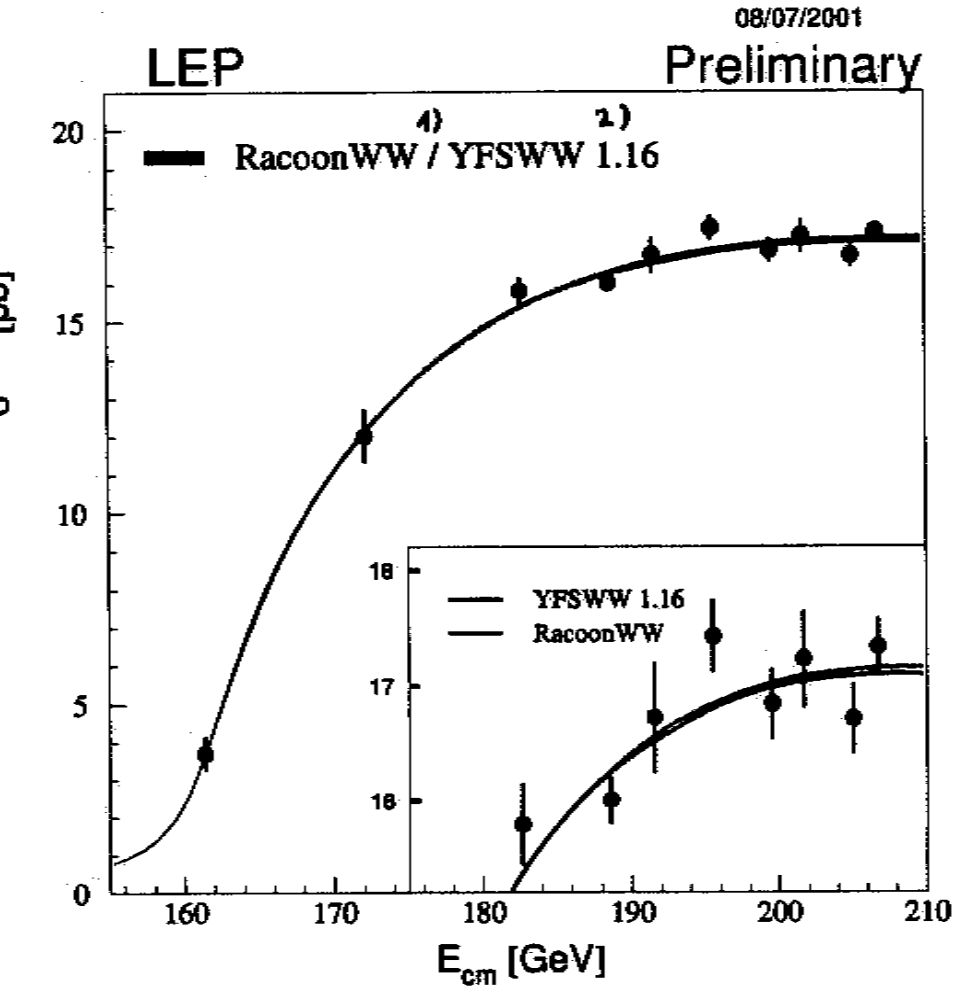
W-pair-production cross section

LEPEWWG '01



agreement confirms
non-universal
electroweak corrections
(2%)

1) Denner et al. 2) Fadach et al.

Radiative corrections

... for the intermediate energy range ($\sqrt{s} \sim 170 - 500\,\mathrm{GeV}$)

- Full electroweak $\mathcal{O}(\alpha)$ corrections in double-pole approximation (DPA)

  Beenakker et al., RacoonWW (Denner et al.), YFSWW (Jadach et al.), Kurihara et al.

  Real radiation ($\mathrm{e^+e^-} \to 4f + \gamma$) known beyond DPA

- Leading logs (LL) of ISR up to $\mathcal{O}(\alpha^3)$
- Theoretical uncertainty (TU): CERN 2000-009
  - total cross section: $\lesssim 0.5\%$
  - distributions: $\sim 0.5 - 1\%$
  - M_W from reconstruction: $\lesssim 15\,\mathrm{MeV}$ Jadach et al. '01

    TU estimate from slope of relative corr. δ on resonance:

    $$\Delta M_\mathrm{W} \sim \tfrac{1}{8}\Gamma_\mathrm{W}^2 \times \Delta\left(\frac{\mathrm{d}\delta}{\mathrm{d}M_\pm}\right)$$

    ($M_\pm$ = reconstructed invariant mass of $\mathrm{W}^\pm$)

- Experimental sensitivity:
  - cross section: $\sim 0.1 - 0.5\%$
  - M_W from reconstruction: $\sim 15\,\mathrm{MeV}$

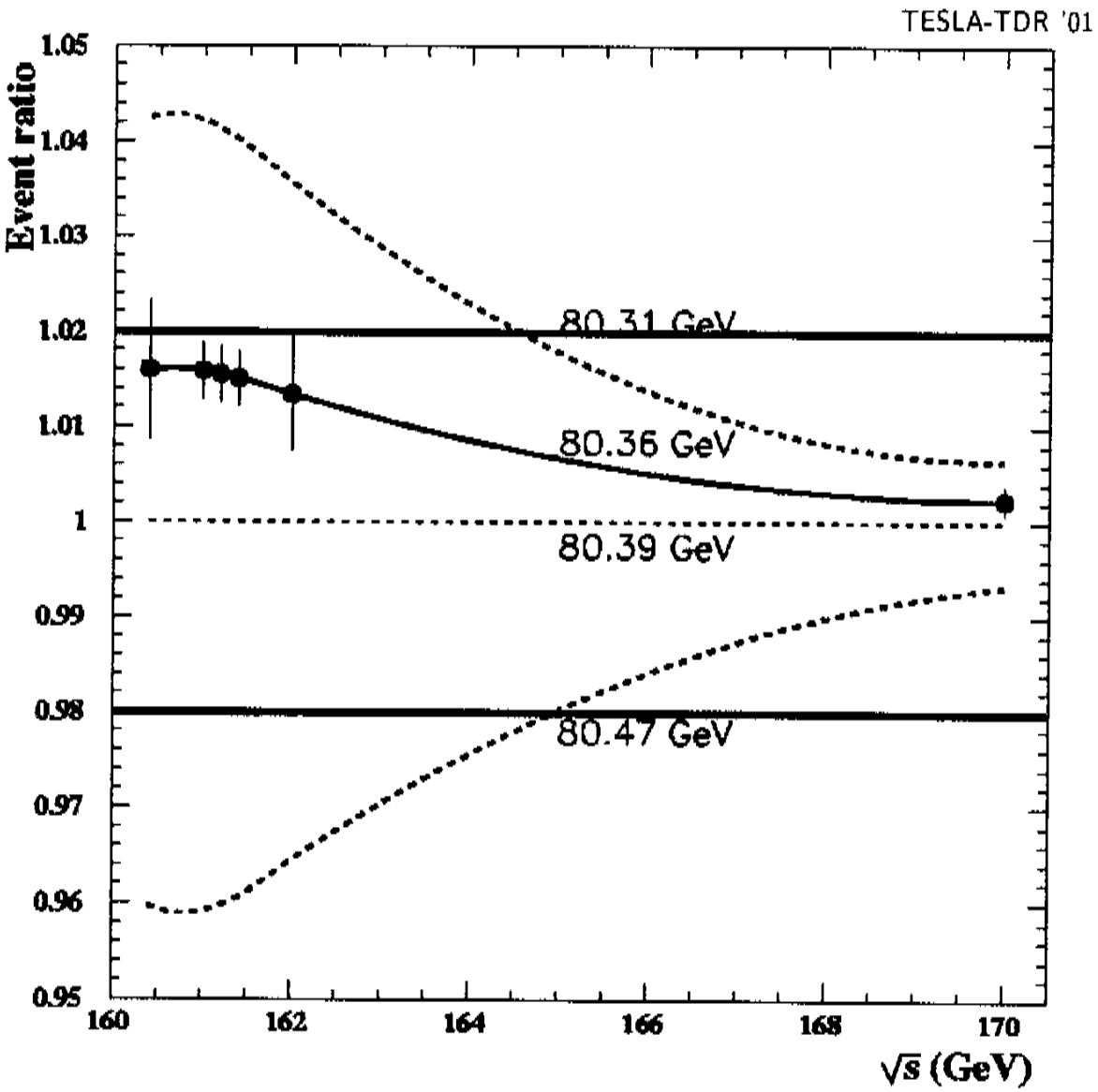
Radiative corrections

...in the threshold region ($\sqrt{s} \lesssim 170\,\mathrm{GeV}$)

- DPA not reliable
- Only approximation by leading universal corrections available (LL ISR, Coulomb singularity, effective couplings)
- TU:
  - cross section: $\sim 2\%$
  - distributions: $\gtrsim 2\%$

M_W measurement from threshold scan:

Experimental sensitivity: $\Delta M_W(\mathrm{exp}) \sim 6\,\mathrm{MeV}$



—— ±2% TU on cross section (no better estimate available)

⇒ Precise prediction of threshold slope necessary (with reliable TU)

Towards future improvements

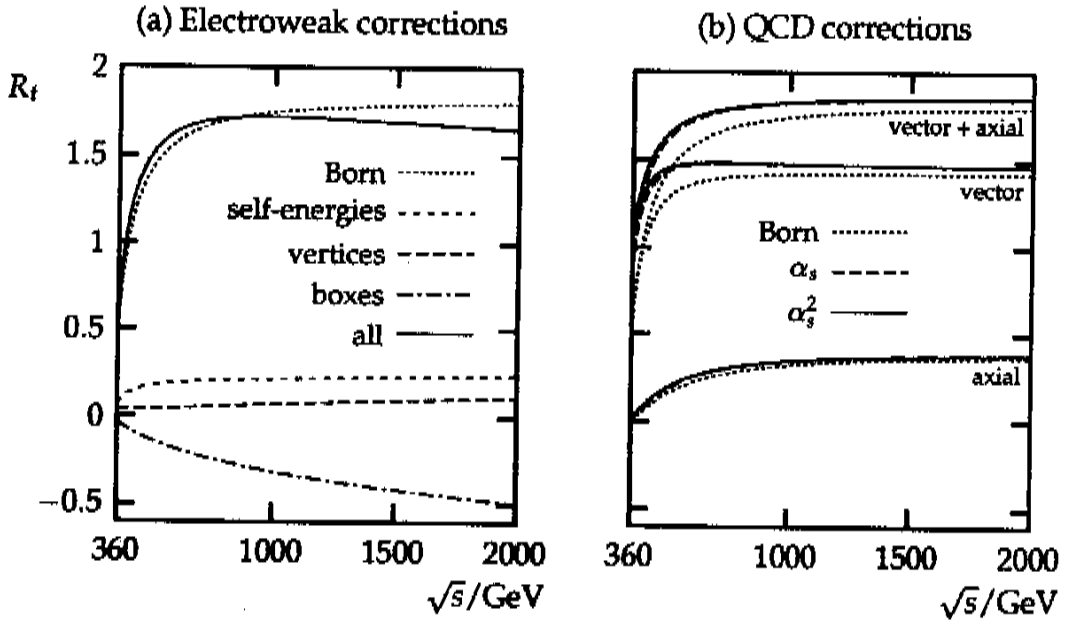
Full $\mathcal{O}(\alpha)$ correction to $e^+e^- \to 4f$

- $e^+e^- \to 4f + \gamma$ known for all final states

  Denner et al. '99, Montagna et al. '00, Papadopoulos '00

- Virtual corrections not known $e^+e^- \to 4f$ at $\mathcal{O}(\alpha)$
  - $\sim 10^3 - 10^4$ complicated one-loop diagrams
  - potential numerical instabilities (5-/6-point functions)
  - many non-equivalent final states
  - problems with gauge invariance in connection with instability of W and Z bosons
    constant-width approach reasonable first attempt (not for high energies)

- $\sigma(e^+e^- \to t\bar{t}) = R_t \cdot \sigma_0, \qquad \sigma_0 = \dfrac{4\pi\alpha^2}{3s}$



[Kühn, Hahn, Harlander]

approximative treatment of $O(\alpha\alpha_s)$,
complete $O(\alpha\alpha_s)$ contribution missing

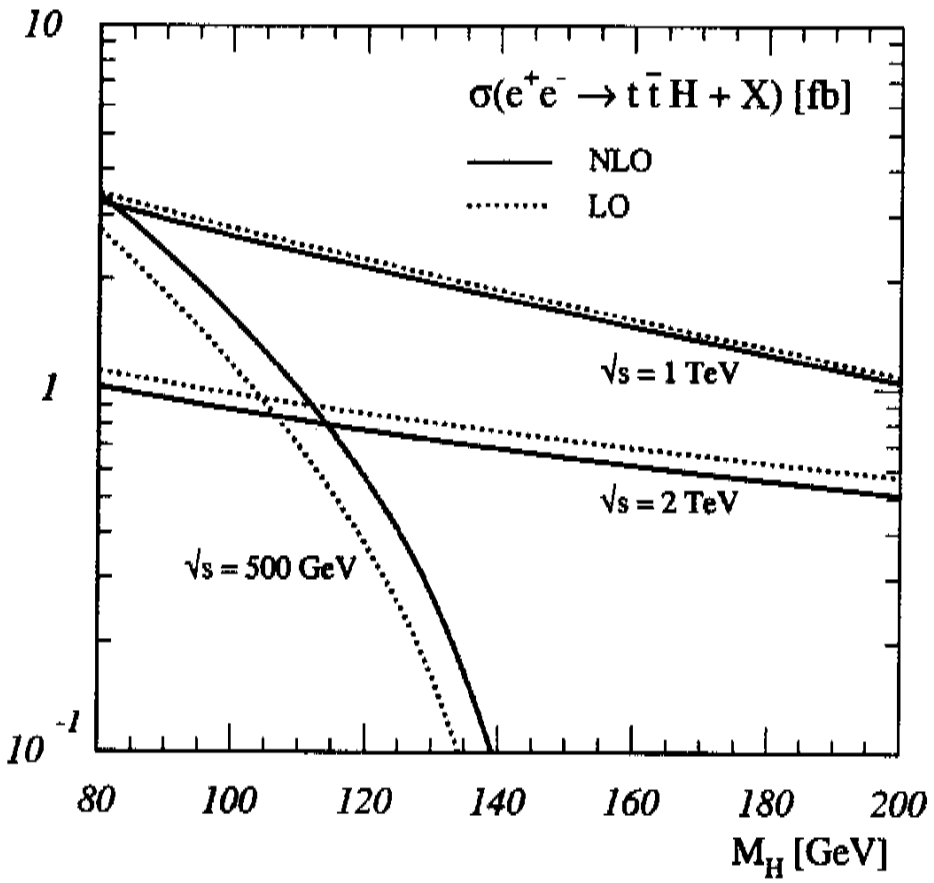
threshold region $\longrightarrow$ m_{t} and α_s
discussion at $O(\alpha_s^2)$ [Hoang et al.]

top quarks unstable
$t\bar{t} \to (bW^+)(\bar{b}W^-) \to$ 6 fermions
$e^+e^- \to$ 6 fermion background required

Measurement of the top–Higgs Yukawa coupling

$$e^+e^- \to t\bar{t}\,H$$

Effect of QCD corrections:



[Dittmaier, Krämer, Liao, Spira, Zerwas]

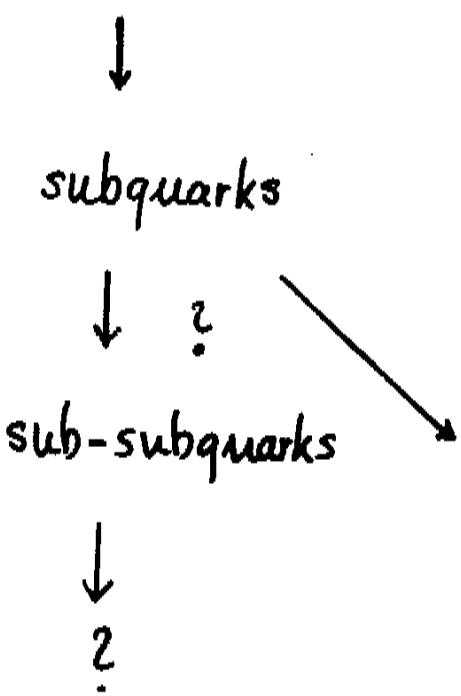
[Dawson, Reina]

Many open questions ...

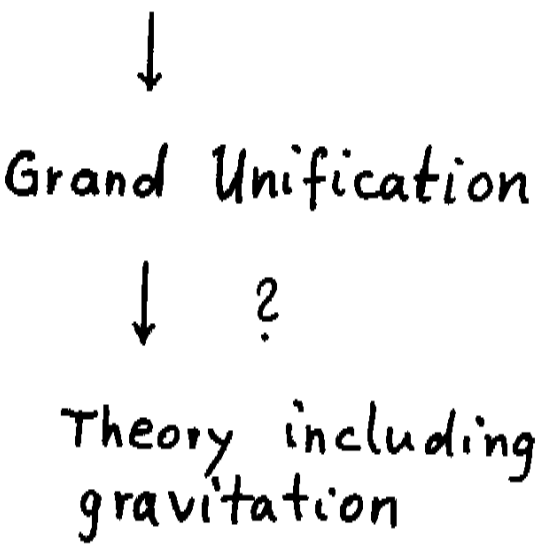
- Is the Higgs mechanism the origin of all masses?
- Why are the fermion masses so different?
  Why is the top quark so heavy?
- Is there a new structure?
  Another symmetry?

Two ways:

(A)
further substructure



(B)
SM particles fundamental, but:
there is a further symmetry



Supersymmetry (SUSY)

maximum symmetry of space-time
(non-trivial augmentation of Poincare symmetry)

additional symmetry **fermions – bosons**

fermions (spin 1/2)	$\leftrightarrow$	s-fermions (spin 0)
gauge bosons	$\leftrightarrow$	gauginos (spin $\frac{1}{2}$)
Higgs bosons	$\leftrightarrow$	Higgsinos (spin $\frac{1}{2}$)
		SUSY partners

masses of SUSY partners $> 100\,\text{GeV}$ (experimentally)

lightest Higgs boson $< 135\,\text{GeV}$ (theoretically)

SUSY makes further unification of forces possible

$\rightarrow$ extrapolation to high energies (GUT scale)

- gauge coupling unification
- Yukawa coupling unification
- radiative ew symmetry breaking

The Minimal Supersymmetric Standard Model (MSSM)

⇒ each SM multiplet is enlarged to its double size

⇒ number of particles are doubled in the MSSM

Superpartners for Standard Model particles

$[u, d, c, s, t, b]_{L,R}$	$[e, \mu, \tau]_{L,R}$	$[\nu_{e,\mu,\tau}]_L$	Spin $\frac{1}{2}$
$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R}$	$[\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R}$	$[\tilde{\nu}_{e,\mu,\tau}]_L$	Spin 0

g	$\underbrace{W^{\pm}, H^{\pm}}$	$\underbrace{\gamma, Z, H_1^0, H_2^0}$	Spin 1 / Spin 0
$\tilde{g}$	$\tilde{\chi}_{1,2}^{\pm}$	$\tilde{\chi}_{1,2,3,4}^{0}$	Spin $\frac{1}{2}$

Enlarged Higgs sector: Two Higgs doublets

physical states: $h^0, H^0, A^0, H^{\pm}$

goldstones: G^0, $G^{\pm}$

Systematic studies of higher-order terms in the MSSM required for

- Indirect tests of the MSSM
  → virtual SUSY effects in precision observables

- Precision studies for SUSY particles
  → determination of masses & couplings
  → reconstruction of model parameters

- Direct versus indirect tests
  → precision observables for precisely measured SUSY parameters
  → consistency check

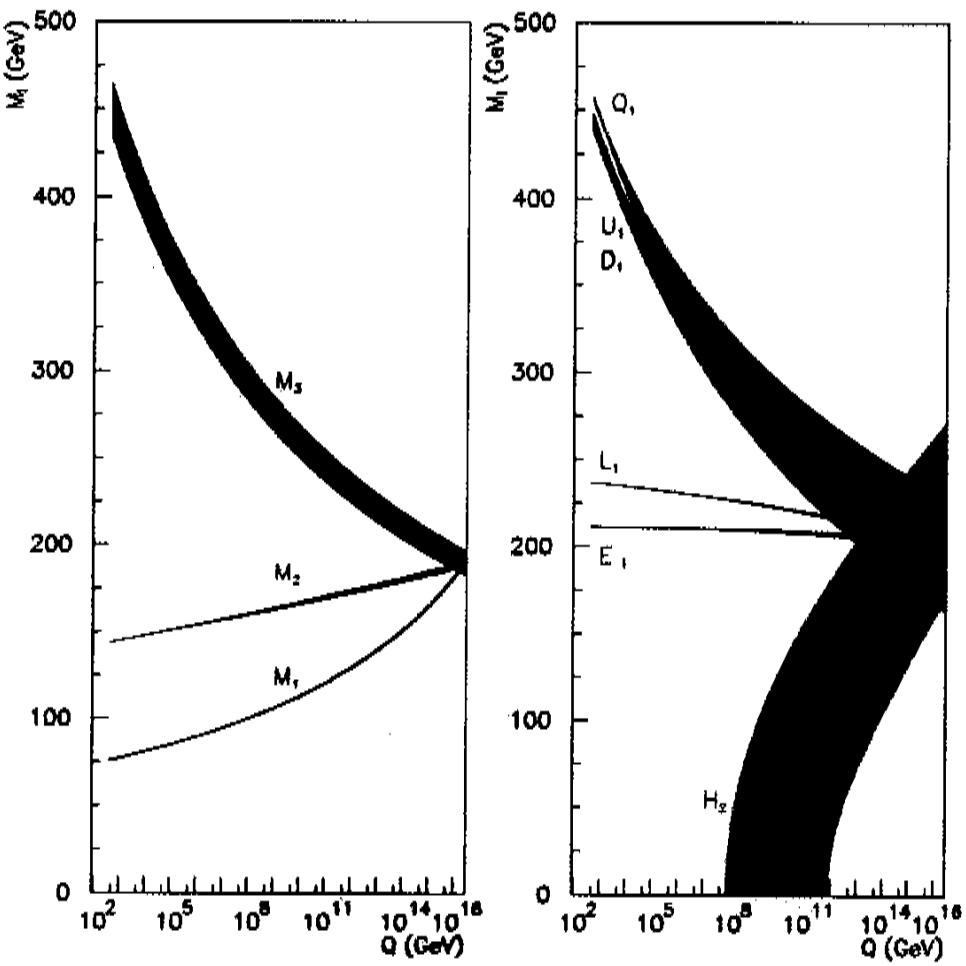
Processses with external

(i) standard particles

(ii) Higgs bosons, especially light Higgs h^0

(iii) SUSY particles

TESLA-TDR (2001)

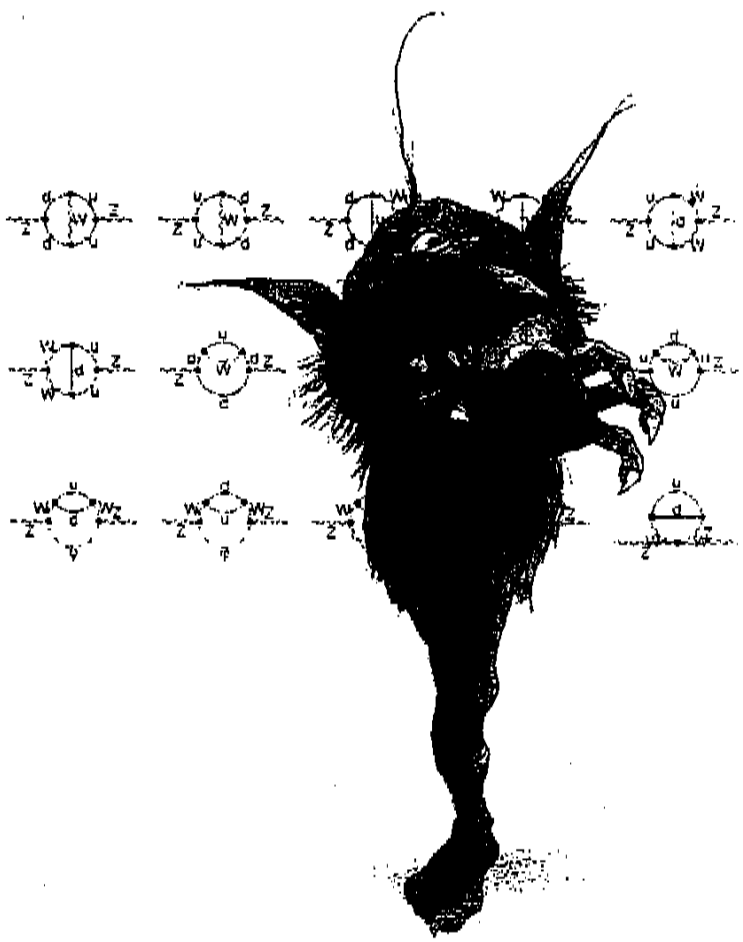


determine mass parameters in "low-energy" measurements of s-particle masses and X sections

→ test of GUT models

Minimal Supersymmetric Standard Model (MSSM)

- renormalizable quantum field theory, precision calculations possible $\longrightarrow$ FeynArts
- predictions $\longleftrightarrow$ experiments



FeynArts 3 User's Guide

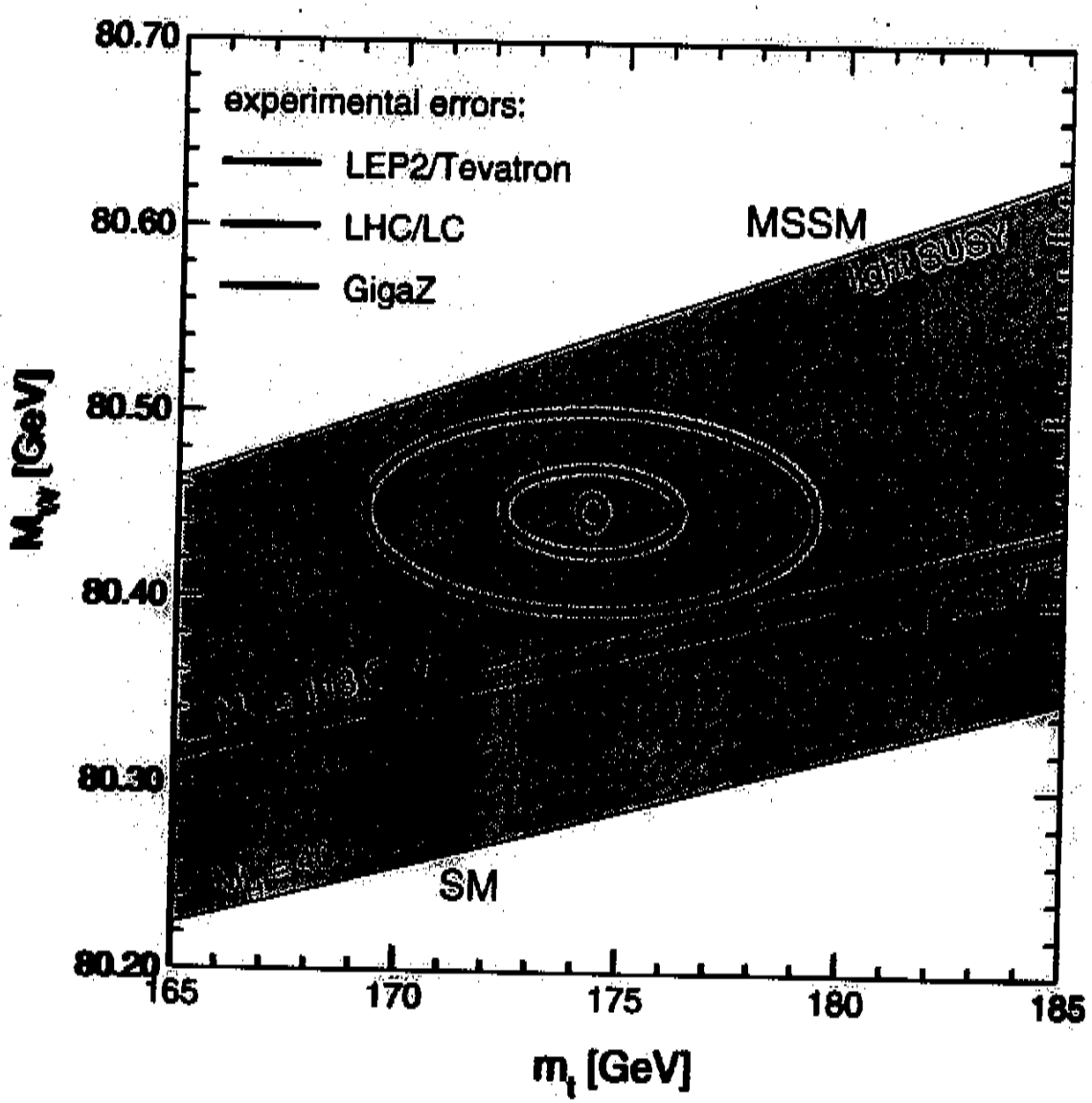
September 23, 2001 Thomas Hahn

http://www.feynarts.de

Prediction for M_W in the SM and the MSSM:

[*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. Weiglein '97*]

[*S. Heinemeyer, W. Hollik, G. Weiglein '98*]

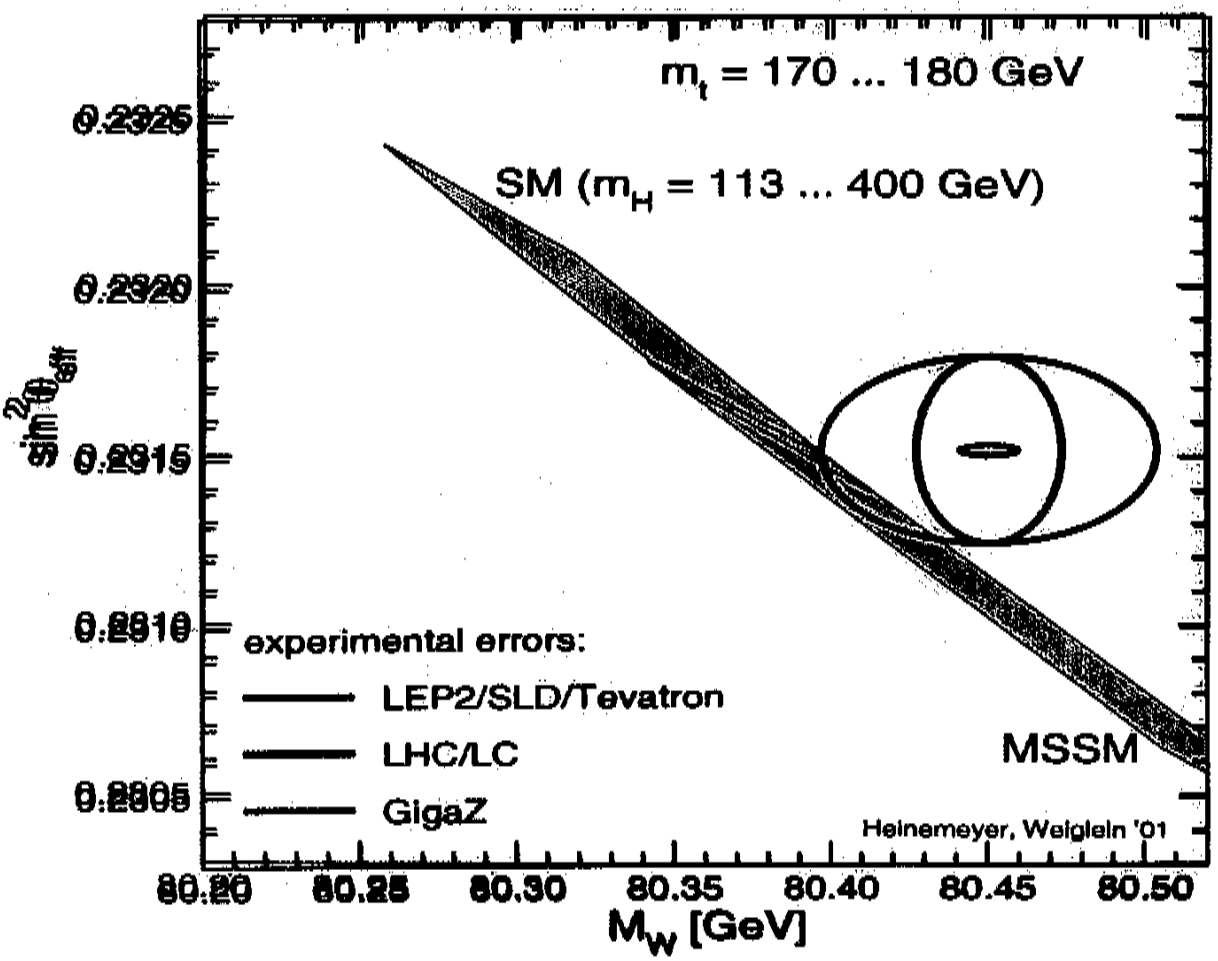


SM: M_H varied

MSSM: SUSY parameters varied

MSSM consistency check:

Prediction for M_W, $\sin^2\theta_{\rm eff}$ in the SM and MSSM vs. prospective accuracies at LEP2/SLD/Tevatron , LHC/LC, GigaZ:



⇒ Highly non-trivial check of the MSSM with GigaZ

Higgs sector of the MSSM:

MSSM: Enlarged Higgs sector:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

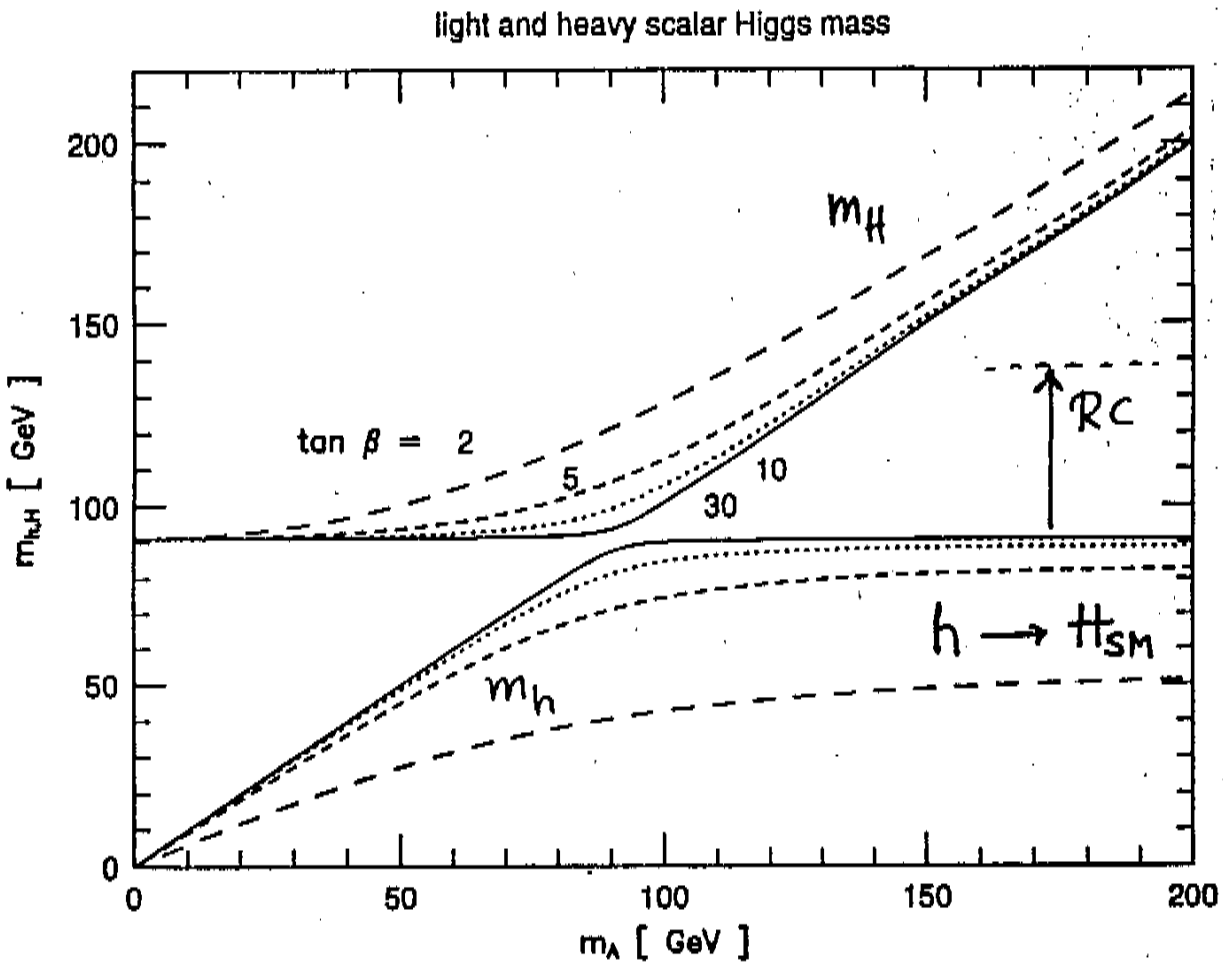
Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

Physical states: $h^0, H^0, A^0, H^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$



lowest order

$$m_h < m_Z$$

$$m_h < m_A$$

- Maximum value of m_h:

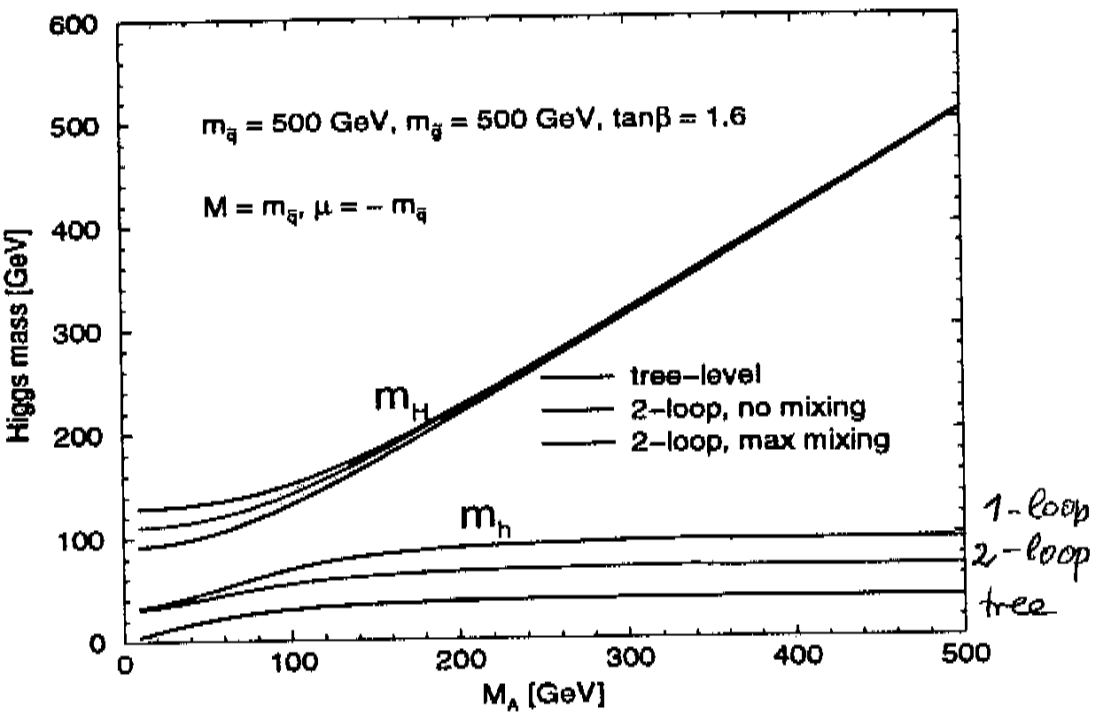
$$m_A \to \infty$$

$$\tan\beta \to \infty$$

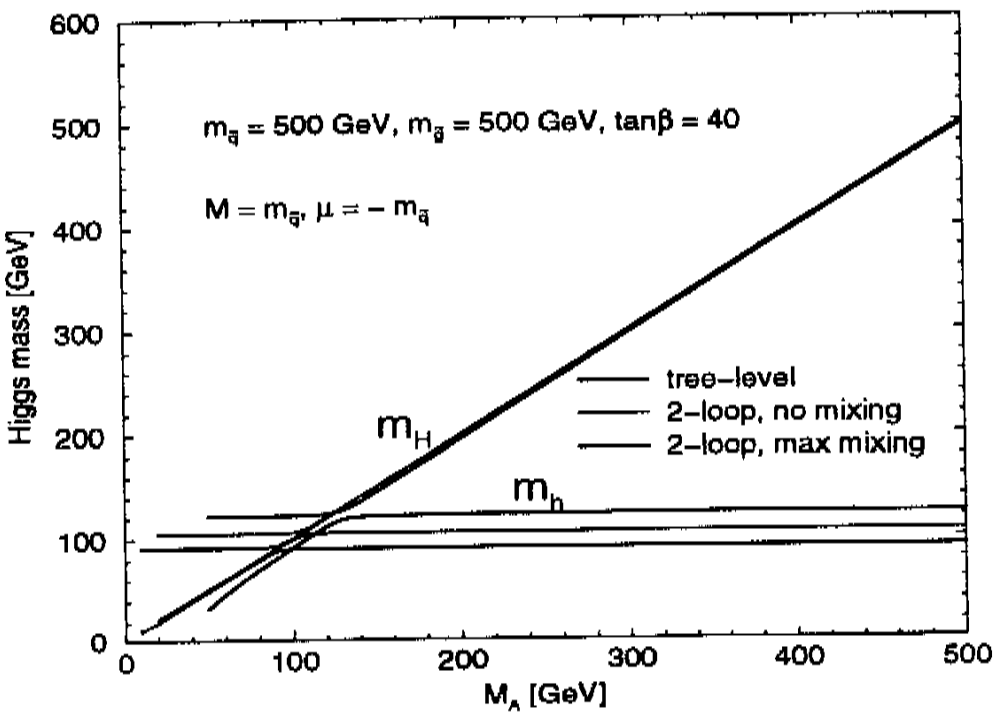
- large radiative corrections

Dependence of m_h, m_H on M_A :

For $\tan\beta = 1.6$:



For $\tan\beta = 40$:



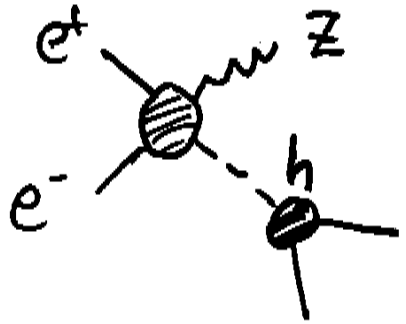
Status of m_h prediction in the MSSM:

- Complete one-loop result:
  $\delta m_h \lesssim 50 - 100$ GeV
  [*P. Chankowski, S. Pokorski, J. Rosiek '94*]
  [*A. Dabelstein '95*]
- Leading logs at two-loop, $\mathcal{O}(\alpha\alpha_s)$:
  $\delta m_h \lesssim 10 - 20$ GeV
  [*J. Casas, J. Espinosa, M. Quirós, A. Riotto '95*]
  [*M. Carena, M. Quirós, C. Wagner '95*]
  [*H. Haber, R. Hempfling, A. Hoang '97*]
  [*S.Heinemeyer, W. H., G. Weiglein '98*]
- Leading non-log corrections at $\mathcal{O}(\alpha\alpha_s)$:
  $\delta m_h \lesssim \sim 5$ GeV
  [*S.Heinemeyer, W. H., G. Weiglein '98*]
- Leading $\mathcal{O}\left(G_F^2 m_t^6\right)$ terms:
  $\delta m_h \lesssim \sim 3$ GeV
  [*M. Carena, M. Quirós, C. Wagner '95*]
  [*H. Haber, R. Hempfling, A. Hoang '97*]
- Subleading $\mathcal{O}\left(G_F^2 m_t^6\right)$ terms:
  $\delta m_h \lesssim \sim 3$ GeV
  [*J. Espinosa, R. Zhang '00*]
  [*A. Brignole, G. Degrassi, P. Slavich, F. Zwirner '01*]

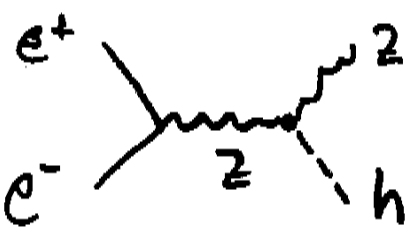
→ FeynHiggs 1.2 [Frank, Heinemeyer, Hollik, Weiglein]

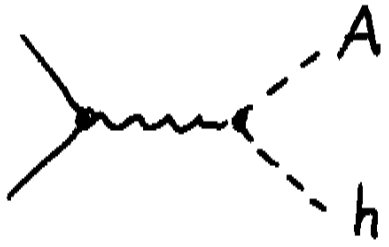
theoretical uncertainty of $m_h \sim 2$ GeV

Production and Decay:

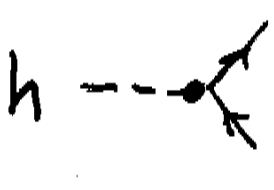


- Cross sections





- Decay widths



(τ, c, ...)

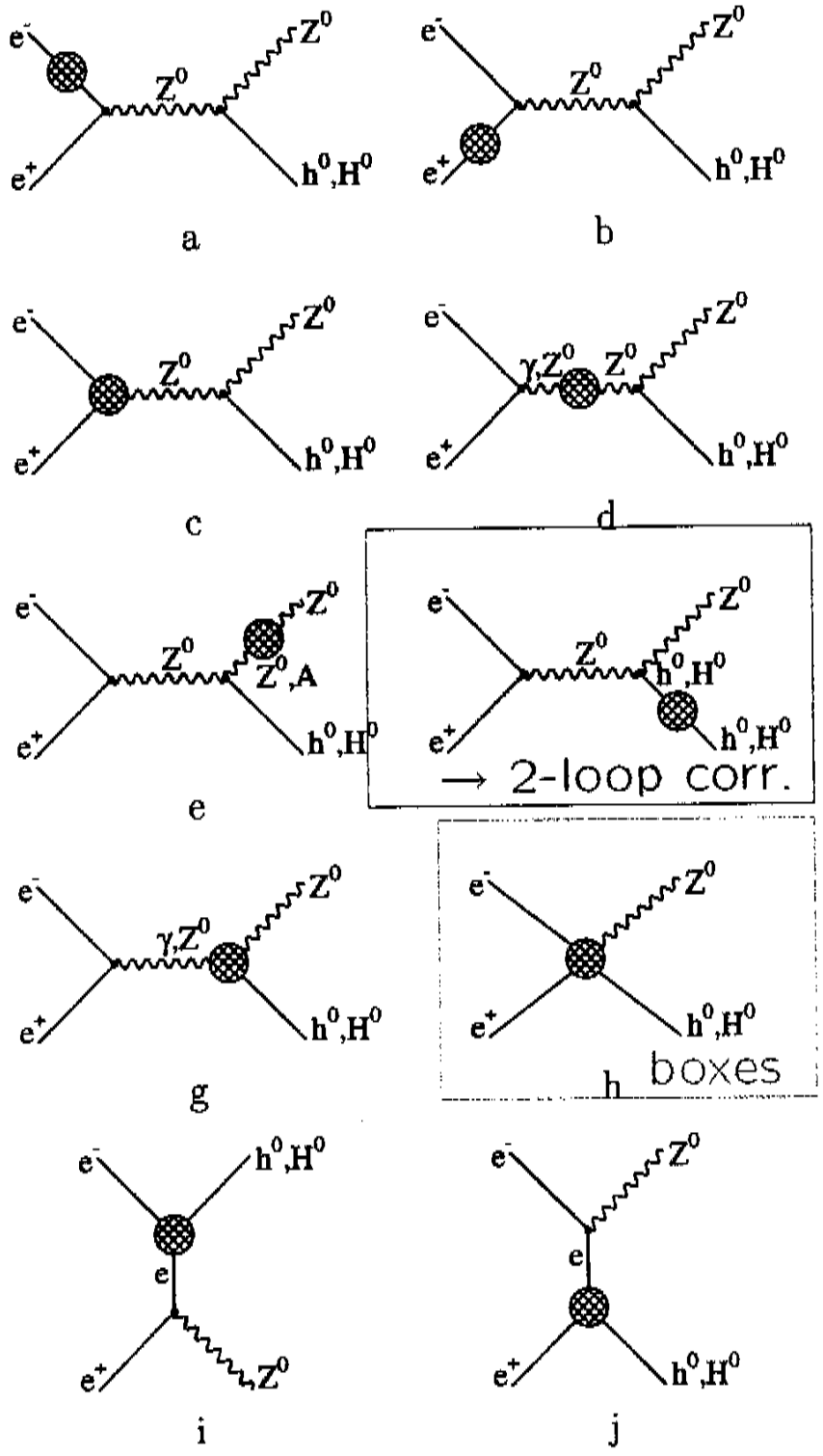
are complete at 1-loop,

→ implement higher order terms

Higgs production at a LC: $e^+e^- \to hZ$

Complete Feynman-diagrammatic one-loop result + dominant two-loop corrections (*FeynHiggs*)

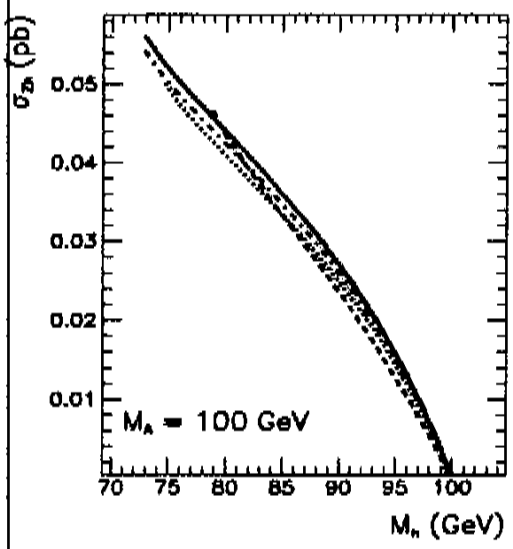
[*S. Heinemeyer, W. Hollik, J. Rosiek, G. Weiglein '01*]

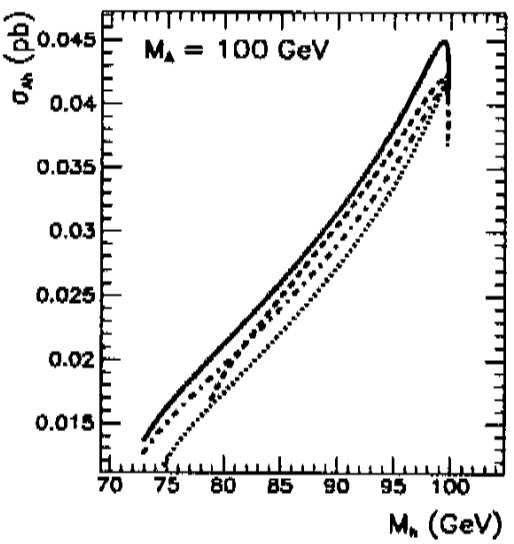


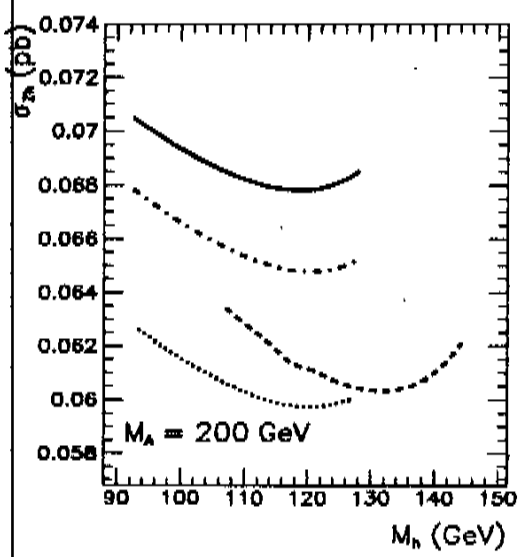
$e^+e^- \to hZ, hA$, $\sqrt{s} = 500$ GeV, maximal $\tilde{t}$ mixing:

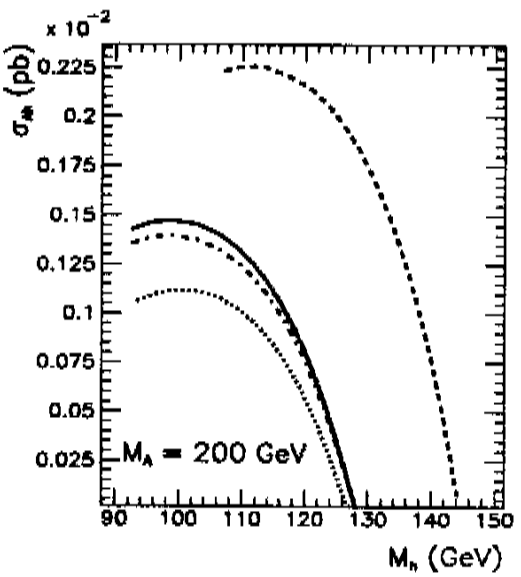
$M_{\tilde{q}} = 1$ TeV, $M_{\tilde{l}} = 300$ GeV, $M_2 = \mu = 200$ GeV

solid: FD 2L (with box), **dot-dashed**: FD 2L (no box)
dashed: FD 1L, **dotted**: α_{eff} RG approximation









Diff. FD 2L − α_{eff} RG: $\sigma_{Zh} \approx 10$–15%, $\sigma_{Ah} \approx 25\%$

QCD corrections [$g \to \gamma$: QED corrections]

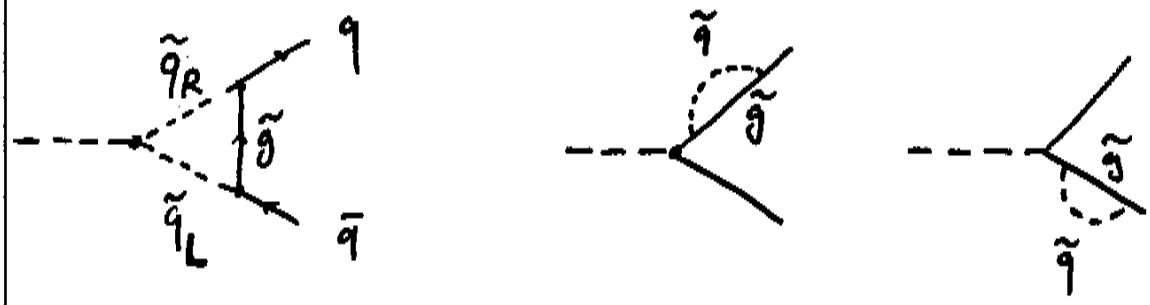
dominant : $m_{pole} \to \overline{m}(M_{h/H})$

large effects (- 40%)

QED corrections are small (< 0.4% for $b\bar{b}$)

h decay

SUSY - QCD corrections:



large effects $\sim m_{\tilde{g}}\, \mu \tan\beta \cdot \alpha_s$

Dabelstein

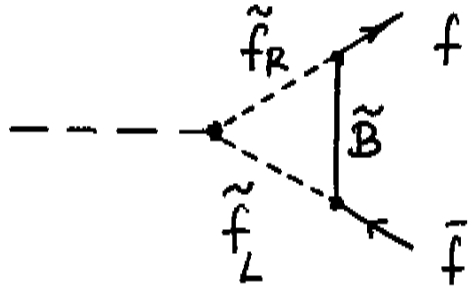
Coarasa, Jiménez, Solà

Haber et al.

non-universal [b] ≠ [τ]

electroweak corrections

large effects $\sim M_1 \mu \tan\beta \cdot (Y_{\tilde{f}_L} Y_{\tilde{f}_R})$



non-universal

[b] + [τ]

Resummation of leading α_s and EW contributi for large $\tan\beta$,

absorption into effective tree-level coupling

$$m_f \rightarrow \overline{m}_f (M_h) \cdot [1 + \Delta_f]$$

Carena, Mrenna, Wagner

Carena, Garcia, Nierste, Wagner

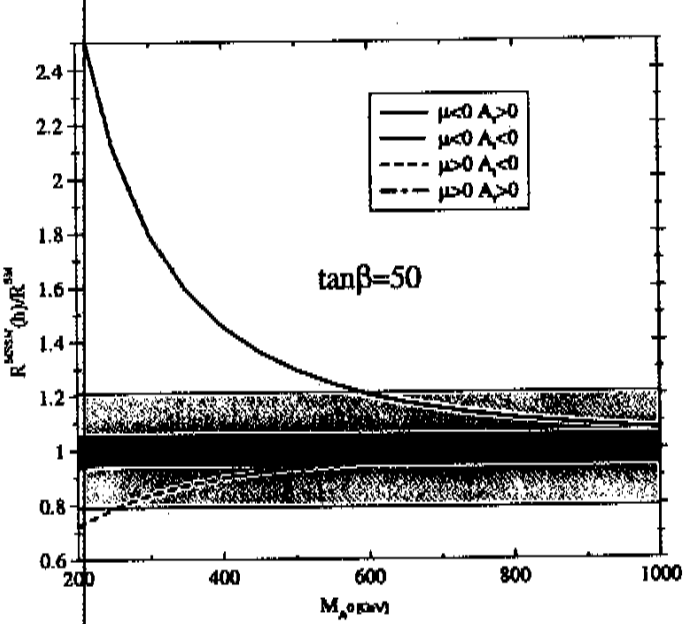
Eberl, Hidaka, Kraml, Majerotto, Yamada

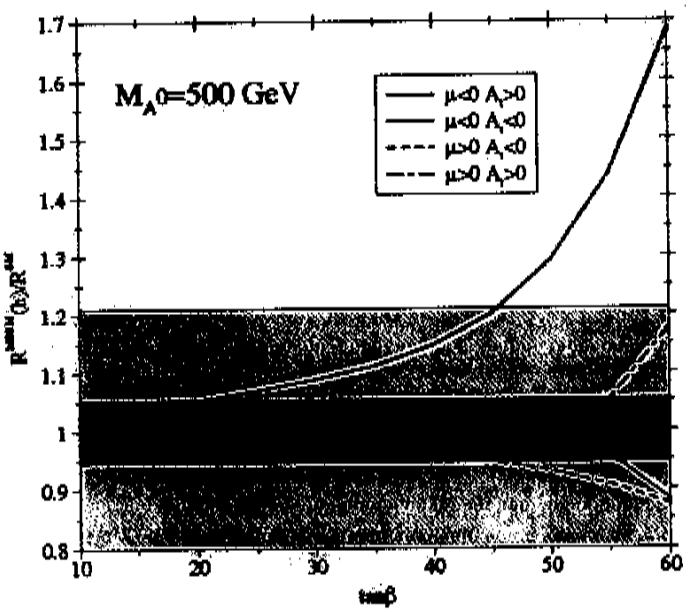
For fixed M_A: SUSY non-decoupling

$$\Delta_b \simeq \tan\beta \left\{ \frac{\alpha_s}{3\pi} - \frac{\alpha_2}{16\pi} \left(3 + \frac{7}{9} \frac{s_W^2}{c_W^2} \right) \right\}$$

$$\Delta_\tau \simeq - \tan\beta \cdot \frac{\alpha_2}{16\pi} \left(1 + \frac{s_W^2}{c_W^2} \right)$$

$$R = \frac{\Gamma(h \to b\bar{b})}{\Gamma(h \to \tau^+\tau^-)}$$





LC

[Guasch, WH, S. Peñaranda]

SUSY masses ~ 1.5 TeV

Input MSSM

$\alpha, M_Z, M_W,$ $M_A, \tan\beta,$ M_1 M_2, μ $M_{\tilde{Q},\tilde{L}}, A_f$

gauge Higgs charginos neutralinos sfermio

Renormalization on-shell scheme

Conditions:

α, M_W, M_Z: on-shell, as in SM

M_2, μ: on-shell $M_{\chi_1^+}, M_{\chi_2^+}$

$\tan\beta$: as in Higgs-calculations
[Dabelstein; Heinemeyer, WH, Weiglein; Chankowski, Pokorski, Rosiek]

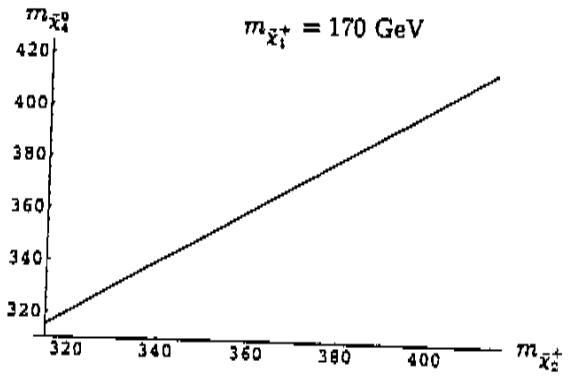
[other parameters enter only at 1-loop]

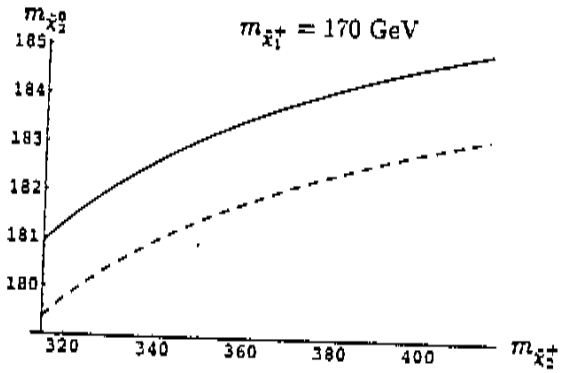
+ field renormalization

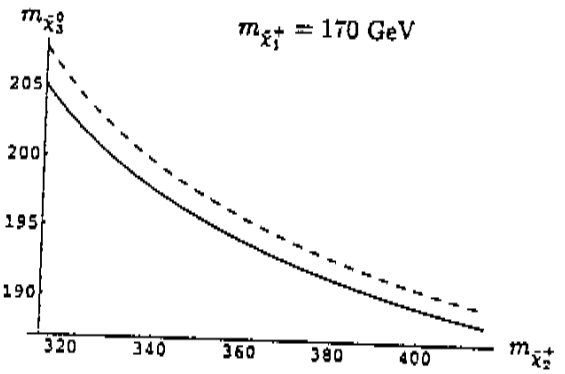
$M_A = 150$ GeV, $M_{\tilde{Q}} = M_{\tilde{L}} = 300$ GeV, $A_f = 900$ GeV

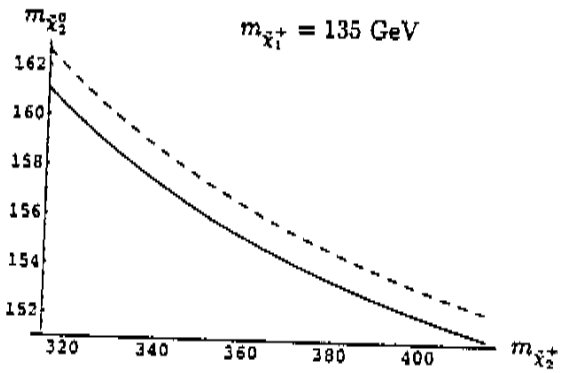
$\tan\beta = 10$

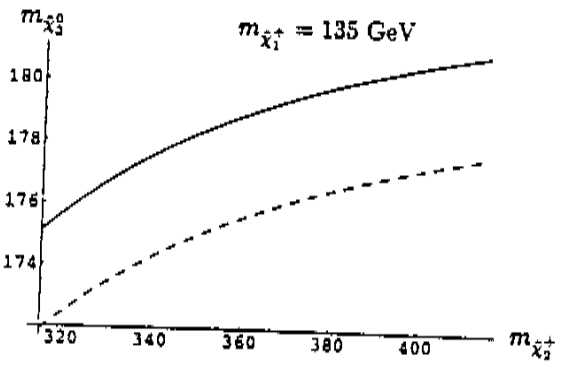
$m_{\chi_1^0} = 110$ GeV

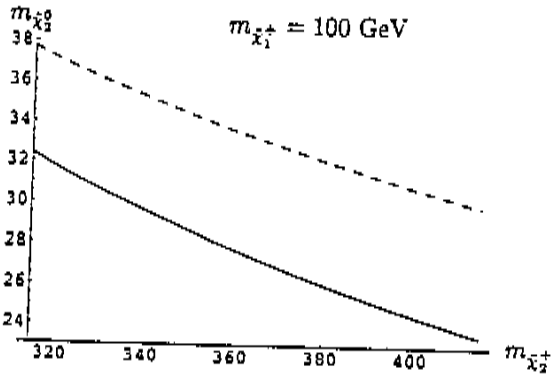


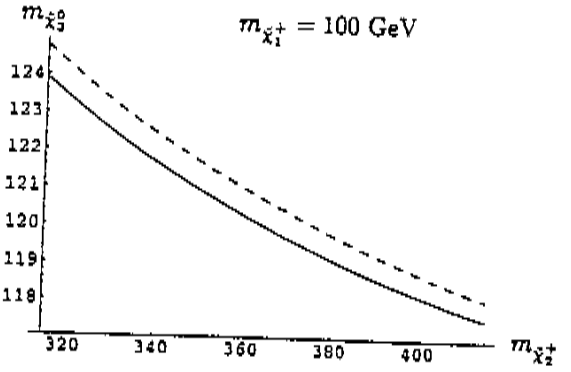










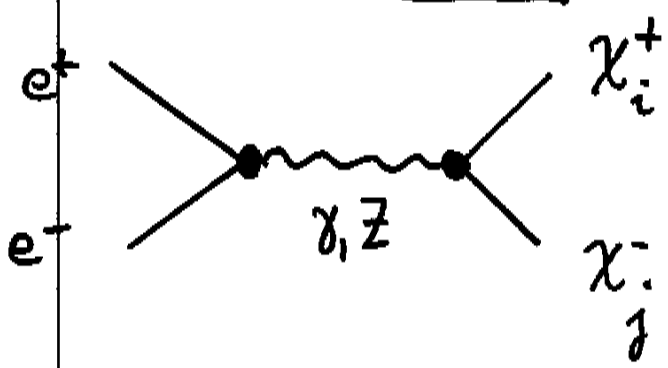


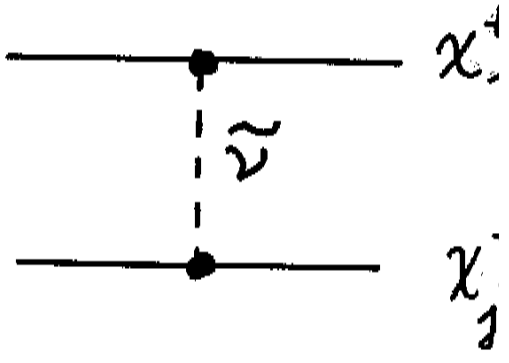
— — — Born order

——— 1-loop order

[Fritzsche, Wtf]

- **Lowest order:**





determined by gauge couplings, chargino masses $m_{\tilde{\nu}}$.

- **next order:** dependent on all SUSY parameters, full MSSM spectrum

- previous work:
  - Diaz, King, Ross: (t,b), $(\tilde{t}\,\tilde{b})$ loops
  - Kiyoura, Nojiri, Pierce, Yamada: all $q, \tilde{q}$ loops

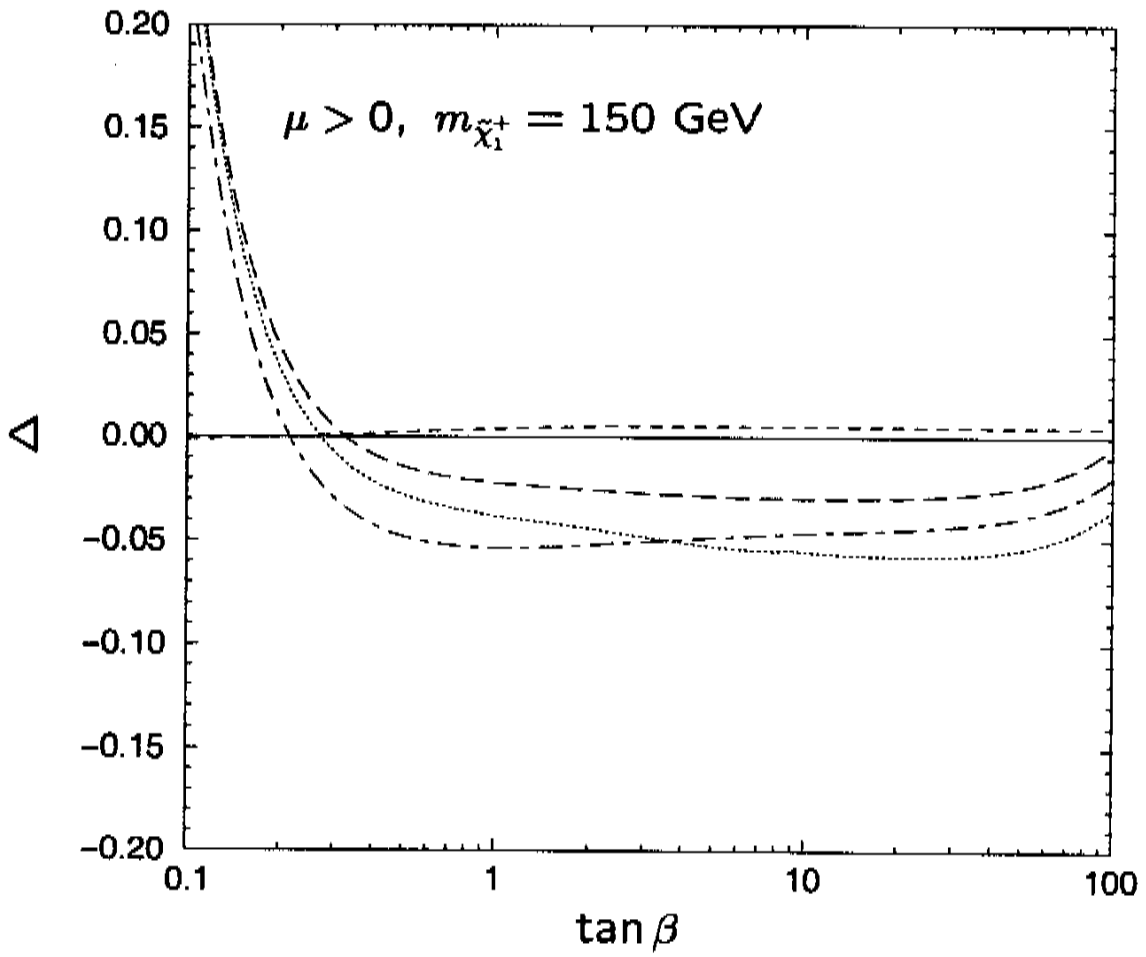
- complete 1-loop: T. Blank, WH

$$e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^+$$

[*T. Blank, W. Hollik, '00*]

(s)top/(s)bottom loops — SM fermion loops

(s)fermion loops — complete 1loop



$$\Delta = (\sigma - \sigma_0)/\sigma_0$$

Conclusions

- Linear Collider with high luminosity and higher energies $\rightarrow$ high-precision experiments
  verify SM or identify signals from "beyond the SM"

- theoretical effort for high-precision calculations, new complexity of calculations:
  $2 \rightarrow 2$ processes at 2-loop + leading 3-loop
  $2 \rightarrow 4\,(6)$ processes at complete 1-loop

- Specific ideas beyond the SM (SUSY, GUT) require radiative corrections
  – indirect: test precision observables
  – Higgs sector: identify origin of Higgs bosons
  – SUSY particles: reconstruct breaking parameters

- Challenge for theoreticians
  world-wide collaborations ?