The LoopVerein: European activities

Higher Order Corrections for standard e^+e^-– Annihilation Processes at High Energies

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ECFA/DESY LCWS St. Malo

··· The goals of the study are:

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- to extend the work of the "LoopVerein", developing new tools and techniques for calculating precise rates for Standard Model and supersymmetric processes that match the expected experimental precision.
- to continue and extend contacts with physicists in North America, Asia and throughout the world.

Traditional EWRC workshops:

Trieste, LEP RC working groups, Brighton,...

$$\left.\begin{array}{c}\text{RADCOR}\\ \text{International}\\ \text{Loops and Legs}\\ \text{DESY Zeuthen}\end{array}\right\}\ \text{RADCOR'02 Banz/Germany}$$

Organized by Karlsruhe University and DESY Zeuthen

(LOC: Blümlein, Hollik, Jegerlehner, Kühn, Riemann)

❶ **The Loop Verein: aims, history**

❷ **The need for progress**

❸ **Radiative corrections in high energy processes**

❹ **The role of $\alpha(E)$ in precision physics**

❺ **Computational physics aspects**

❻ **Status and Outlook**

❶ The Loop–Verein: Aims

- **Motivation**

The main aim are precision calculations of processes within the Standard Model and super symmetric extensions of it

field which got a lot of momentum from precision physics at LEP/SLC

$\Downarrow$

we must keep momentum and improve calculations to meet TESLA requirements of HIGHER ENERGY and HIGHER LUMINOSITY

- **Coordination**

by a theoretical working group, called "The Loop-Verein".

Coordinate activities, identify problems and their solution.

Time scale: a long term project.

- **Invitation**

Everybody is invited to contribute to progress in this field (http://www-zeuthen.desy.de/theory/TLV/TheLoopVerein.html).

Theory coordinators:

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Experimental advice:

Klaus Mönig (moenig@ifh.de)

Invitation to participate in The Loop-Verein

Dear friends,

The high precision which can be reached at TESLA, requires proper control of radiative corrections in the standard model processes. Based on the work which has been done for LEP, the theoretical analyzes must be developed further to cope with the increased energy and luminosity at TESLA. This applies to QED processes like Bhabha scattering for luminosity measurements, the genuine electroweak sector such as WW production, jet production in QCD, and many more other processes. Since LEP operations will terminate soon, it is timely to think about the development of this theoretical program for TESLA in order to ensure a smooth transition to the new field without any interruption. In accordance with Professor D.J.Miller, Chairman of the ECFA/DESY LC Workshop, we therefore propose to form a theoretical working group, briefly called "The Loop-Verein", in which problems in this context are identified and their solution is coordinated. Even though cross-talk is natural, this group will operate independently of the other WG's of the workshop in which theorists cooperate with experimentalists very closely. To discuss all issues, the Loop-Verein will meet for the first time at the Strasbourg-Obernai session of the ECFA/DESY LC Workshop in parallel to the Detector Working Groups on Monday Oct.18. The meeting will be prepared and coordinated by F.Jegerlehner. Experimental advice will be given to the group by Klaus Moenig (moenig@ifh.de). We would like to invite you to this first meeting of the Loop-Verein in order to help us shape the theoretical program and to identify your physics interests in this context. Please tell us whether you are interested in this cooperation on theoretical problems of high-precision standard model physics. Hoping very much to meet you in Strasbourg-Obernai! With kind regards,

F.Jegerlehner fjeger@ifh.de P.M.Zerwas zerwas@desy.de

distributed, August 3, 1999

The Loop–Verein: History

- International Workshop on Linear Colliders: LCWS at Saariselka, Finland, 1991; ...
- Joint ECFA / DESY Study: Physics and Detectors for a Linear Collider 1996
- 2nd Joint ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider 1998/2000
  - Preparation of TESLA TDR:`http://tesla.desy.de/tdr/`
  - COLLOQUIUM, Scientific Perspectives and Technical Realisation of TESLA, March 23/24 2001 at DESY in Hamburg

  The LoopVerein: initiative

  03 Aug 1999: “Invitation to participate in The Loop-Verein” distributed

  03 Aug 1999: 1st member Stefan Dittmaier joins

  10 Nov 1999: G. ’t Hooft and M. Veltman (asked to) join as honorary members

  ..

  Oct 99 to Apr 02: at 5 ECFA/DESY Linear Collider Workshops

  mainly electroweak SM activities; no convener found for QCD part

  ..

  09 May 2002: about 90 members registered

- Extended Joint ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider 2001/2003

The LoopVerein:

03 May 2001: Extended LoopVerein: sLoopVerein or LoopVerino?

Wolfgang Hollik joins as a convener

Precision predictions in SUSY extension of SM

US activities: Snowmass 2001 $\longrightarrow$ LoopFest

World–Wide study: LCWS2002, parallel session

LoopVerein (precision EW and SUSY calculations),

(Conveners : D. Wackeroth, Y. Kurihara,)

Physics Motivation[a]:

Examples:

- Bhabha: workshop "Z Physics at LEP 1" say most urgent complete 2–loop QED calculation;in spite of substantial recent progress (see: Z.Bern, Bhabba scattering at two-loop) no existing complete 2–loop QED MC generator.
- Full electroweak 2–loop $\alpha, G_\mu, M_Z \Rightarrow \sin^2\Theta^\ell_{\rm eff}$ affects indirect Higgs mass bounds from LEP/SLC (see: Graham Wilson, SM, WW, GigaZ (EXP))
- Full 1–loop $e^+e^- \to W^{+*}W^{-*} \to 4\ \mathrm{fermions}$ for LEP 2 (existing double–pole approximation seem O.K., however theoretical uncertainty not really known) (see: Stefan Dittmaier, "Four-fermion production at future e^+e^- linear colliders" and A.Vicini, "Towards a full calculation of one-loop corrections to $e^+e^- \to 4\mathrm{f}$")

New approaches needed:

(see: G.Passarino, "Recent developments on the numerical evaluation of multi-loop Feynman diagrams")

Conceptual problems: finite width, unstable particles

(see: A. Signer, Finite width effects for processes with unstable particles)

our goal meet requirements of TESLA 500/800 GeV incl. GigaZ option

[a] Some of us fear that theory will not be ready when LC starts taking data (after LEP less and less young particle theorists working in the field (postdoc applications droped by more than a factor 2 within a few years). Stanislaw Jadach at Krakow meeting: ask experimentalists to store data for later analysis (estimated 10 years delay, like some issues at LEP)

❷ The need for progress

(What is most interesting for electroweak precision physics)

- threshold scan of WW–threshold region
  (single diagram t–channel dominated)
  $10^4 \to 10^7$ W–pairs **MegaW** $\delta M_W \sim 6$MeV possible

  go from 2 $\longrightarrow$ 2 **LEP 1** to 2 $\longrightarrow$ 4(6) processes **LEP 2/LC**
  (plus γ radiation)
  at 0.2% level or better

- new 2 $\longrightarrow$ 2 scan at Z (A_{LR}, ...)
  $10^6 \to 10^9$ Z's **GigaZ** $\Delta \sin^2 \Theta^\ell_{\mathrm{eff}} \simeq 0.000013$!
  higher statistics:
  model independent scan with 5 scan points!

  go to next order for 2 $\longrightarrow$ 2 processes
  (plus γ radiation)
  increase precision by factor 10 relative to LEP 1

Note: complexity of calculations grows factorial
(a battle against combinatorics)

SM in renormalizable gauge without fermion doublet multiplicity

11 types of lines , 57 types of vertices !

more loops and/or more legs become a true challenge

SUSY extensions even much more complex !

In addition:

the many mass scale complexity

What complicates matters dramatically are the many different mass scales which can be very large and lead to extremely complicated analytic structure of the amplitudes and to dramatic numerical cancellation problems[a].

[a] For 1–loop 2 to 2 processes M. Veltman wrote a CDC assembler routine *FORMF* (which essentially did what *FF* is doing today in a different way in standard double precision FORTRAN) at 128 digits internal precision which allowed him to guarantee 8 digits final precision in such calculations. Alternatively one has to do the cancellations analytically by appropriate series expansions, usually a tedious task.

Compare QCD[a] : 3 types of lines, 4 types of vertices

only mass scales $\alpha_s \Leftrightarrow \Lambda_{\rm QCD},\ m_c,\ m_b,\ m_t$

except for top–production calculations

QCD calculations are mostly done in the massless approximation

[a] Most of what I say concerns electroweak SM calculations, a discussion of QCD is not intended here (see: Top/QCD parallel sessions, M. Tejeda-Yeomans, C. Oleari, A. Czarnecki, M. Macesanu).

- **the high precision challenge: GigaZ:**

  2 $\longrightarrow$ 2 full 2–loop + leading 3–loop

Altogether: new technical challenge

going from a few 100 to several 10000 diagrams !

Example: # of diagrams at tree level WW production and decay:

signal 3 + background

$$\left\{ \begin{array}{ll} e^+e^- \longrightarrow \mu^-\bar{\nu}_\mu\tau^+\nu_\tau & \textbf{6} \\ \vdots & \vdots \\ e^+e^- \longrightarrow e^-\bar{\nu}_e e^+\nu_e & \textbf{53} \end{array} \right.$$

Other examples: tree unitary gauge, loops linear R_ξ gauge

process		# diagrams	loop level
$e^+e^- \longrightarrow 4\mathrm{f}$	$\Rightarrow$	6 to 144	tree
$e^+e^- \longrightarrow 4\mathrm{f}\gamma$	$\Rightarrow$	14 to 1008	tree
$e^+e^- \longrightarrow \mu^-\bar{\nu}_\mu u\bar{d}$	$\Rightarrow$	1907	1–loop
$e^+e^- \longrightarrow e^+e^-d\bar{d}$	$\Rightarrow$	8522	1–loop
$e^+e^- \longrightarrow e^+e^-e^+e^-$	$\Rightarrow$	21444	1–loop
$V \longrightarrow V$ ($V = W, Z$)	$\Rightarrow$	$\sim$ 50	1–loop
$W \longrightarrow W$	$\Rightarrow$	4084 (792 1pi)	2–loop
$Z \longrightarrow Z$	$\Rightarrow$	2348 (616 1pi	2–loop

Qualitative aspects: LEP1/GigaZ vs. LEP2/LC500

LEP1/GigaZ: simple	LEP2/LC500: complex
(one (2) Born diagram)	(three (10) Born diagrams)
• high X–section	• $\sigma_{\mathrm{WW}} \sim 10^{-3}\,\sigma_{\mathrm{Z}}$
• calculation of RC relatively simple (even at 2–loops)	• RC very difficult even QED at 1–loop missing
• universal ISR	• ISR non–universal due to CC
• almost no background	• substantial background dependent on cuts
• simple way to present data ”pseudo observables” inclusive measurements no cuts needed	• no simple rules known “canonical cuts”
• very precise low systematics	• at least for LC need very precise predictions

Key processes:

- **Bhabha scattering:** for luminosity monitoring,
  full 2–loop needed
  in massless approximation:
  substantial progress the last two years

- all kind of boson production, top production:

$$e^+e^- \to (2f), 4f, 6f, \ldots \text{ plus } \gamma\text{'s}$$

$$\uparrow$$

$$(\gamma,\ Z),\ ZH, WW,\ ZZ,\ Z\gamma,\ \gamma\gamma,\ \cdots, t\bar{t}$$

unstable particles $\longrightarrow$ theoretical problems
(resummation near resonances, order by order perturbation theory breaks down)
problems with unitarity and/or gauge invariance

$\Rightarrow$ masses, widths, couplings, ...

- Alternative initial states

$$\gamma\gamma,\ \gamma e,\ e^-e^- \to \cdots$$

as well as perturbative QCD calculations for hadron–hadron (LHC) and/or lepton–hadron colliders

$$\updownarrow$$

share experience with groups working on hadron collider physics

❸ Radiative corrections in high energy processes

❶ γ radiation $\qquad m_\gamma = 0,\ \ U(1)_{\rm em}$ exact

- IR problem $\Rightarrow$ Bloch–Nordsieck construction
- "inclusive" process (soft γ's)
- virtual + soft (indistinguishable)

$\Rightarrow$ Sudakov double log's cancel

$$\boxed{\left(\frac{\alpha}{\pi}\,\cancel{\log\frac{s}{m_e^2}}\right)^2}$$

❷ W, Z "radiation" at $s \gg M_Z^2$

- M_W, M_Z are physical IR cut-offs
- fixed number of massive gauge bosons
- no need to combine virtual + soft (no unseen W's or Z's)
- virtual corrections alone physical
- nothing to cancel Sudakov log's

$\Rightarrow$ large EWRC (Sudakov double log's)

$$\boxed{\left(\frac{\alpha}{\pi}\log\frac{s}{M_Z^2}\right)^2} \quad ✓$$

Large EWRC at high energies

- radiative corrections grow dramatically with energy

Leading EW Sudakov Logs

$$c_{\mathrm{SD}}^{e^+e^-\to X}\ \frac{\alpha}{\pi}\ \ln^2\left(s/M_W^2\right)$$

coefficient depends strongly on observable !

required talking to experimentalists

- what can experiments measure at what accuracy
- from which observable can we learn what
- jet physics and hadronization problems
- exclusive versus inclusive measurements
- etc.

at the end we need efficient event generators

in order to get a real feeling for the problems

The Sudakov log industry:

Sudakov log's in electroweak processes:

P. Ciafaloni, D. Comelli, F.M. Renard, C. Verzegnassi, J. Kühn, A. Penin, V.S. Fadin, M. Melles, A. Denner, W. Beenakker, A. Werthenbach, V. Smirnov, S. Moch .... and collaborators

Investigate leading RC in HEP processes (since 1998)

Profile of log corrections:

Kühn, Penin, Smirnov, Moch

- large double logarithmic corrections
  e.g. $e^+e^- \to q\bar{q}$ (2 TeV), $M = M_W$ for IR cutoff

-24%	(one–loop)	LL
+3.4%	(two–loop)	LL

- large compensations from sub-leading terms

$\sim$ +35%	(one–loop)	NLL
$\sim$ -9.3%	(two–loop)	NLL

- again alternating sign in NNLL order

$\sim$ -10%	(one–loop)	NNLL
$\sim$ +8.5%	(two–loop)	NNLL

- corresponding pattern in sub-leading N_f terms
- large differences between different flavors

My conclusion: no way to avoid full fixed order calculations !

Electroweak SUSY Sudakov Log's:

Beccaria, Renard, Verzegnassi, Melles

Calculate asymtotic MSSM Sudakov log's at 1–loop

for $e^+e^- \longrightarrow$ fermions, gauge bosons, Higges,

sfermions, gauginos (in pairs)

Types of large log's:

I) Gauge sector terms: depend on SM couplings

$$\frac{\alpha}{\pi}\, C_X \left[a \ln \frac{q^2}{M^2} \underbrace{- \ln^2 \frac{q^2}{M^2}}_{\text{universal}} \right] ; M = M_W, M_Z$$

MSSM $a = 2$ **final states: fermions, sfermions,**

Higgses, gauginos

SM $a = 3, 4, \cdots$

$a = 0$ **for transversal gauge bosons**

$\ln^2$ **only by virtual W, Z exchange**

$\ln$ **Θ–dependent: in $\xi = 1$ gauge only**

produced by boxes with W, Z exchange

II) Yukawa terms: depend on genuine SUSY couplings

$$\frac{\alpha}{\pi}\, C_Y \left[\ln \frac{q^2}{M^2} \right] \left[\underbrace{m_t^2\,(\cdots) + m_b^2\,(\cdots)} \right]$$

FS + SUSYsensitive

$\tan^2\beta$, $\sin^2\alpha$

❸ The role of $\alpha_{\rm em}(s)$ in precision physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

$\alpha\ ,\ G_\mu, M_Z$ **most precise input parameters**

partially

non-perturbative

relationship

$\Updownarrow$

precision predictions

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters**

for VB physics (Z,W) etc.

$$
\begin{array}{cclcl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta \alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4}
\end{array}
$$

$\Rightarrow$ now a limiting factor

for precision physics

For perturbative QCD contributions very crucial:

precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t$

Hadronic Contributions

General problem in electroweak precision physics:

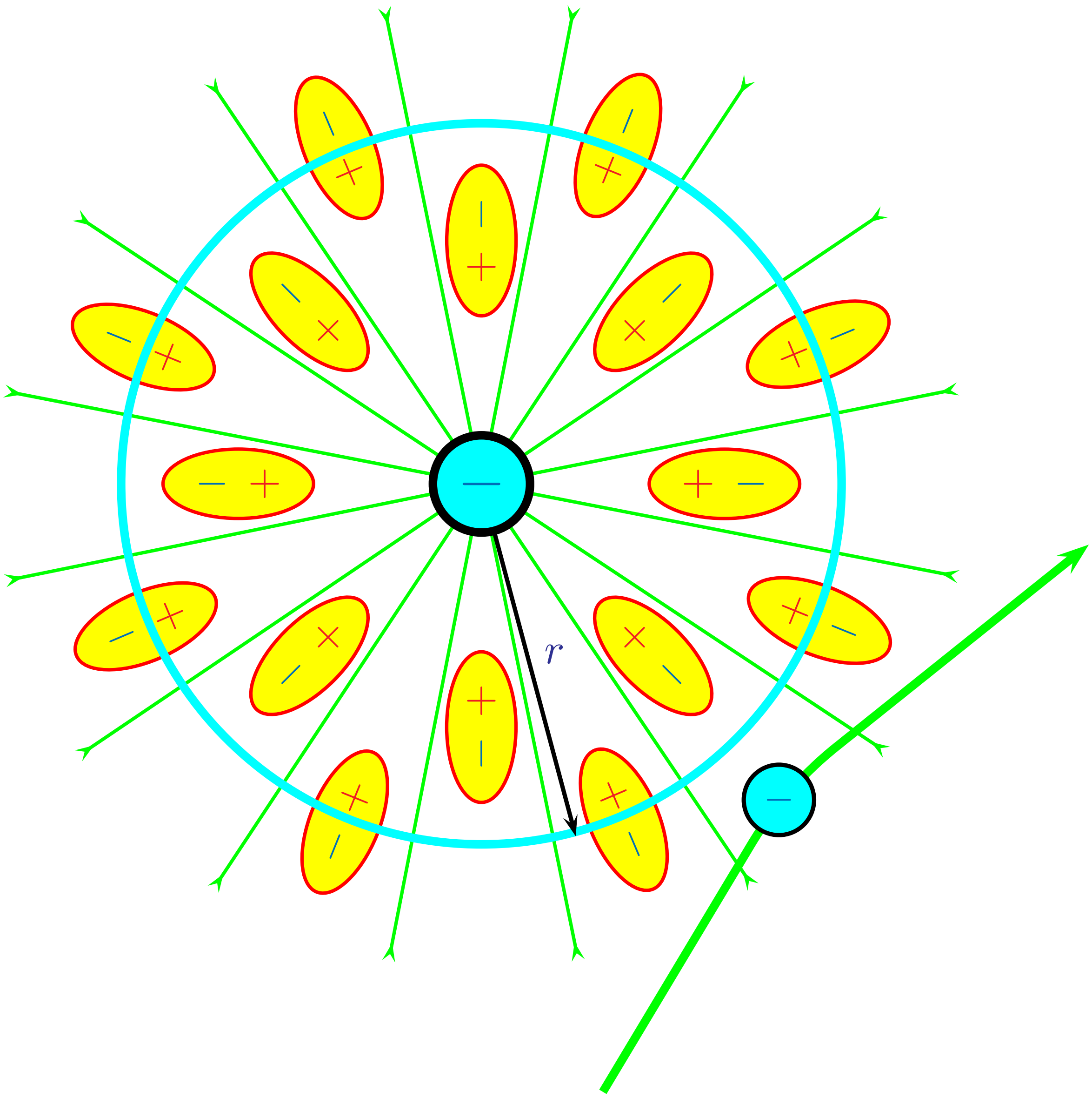
contributions from hadrons (quark loops) at low energy scales

$$\Downarrow$$

Digression

Most important example: photon vacuum polarization $\Rightarrow$ electro-magnetic charge screening (effective or running charge)

Vacuum polarization and charge screening

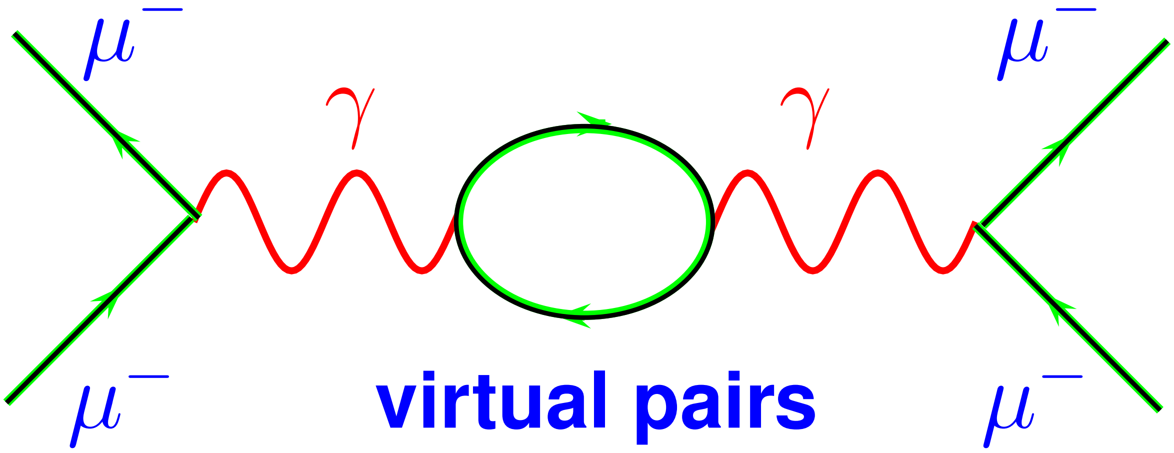


In a collision of impact energy E the effective charge is the charge contained in the sphere of radius $r \simeq 1/E$, which due to vacuum polarization is larger than the classical charge seen in a large sphere ($r \to \infty$)

$\Downarrow$

charge screening

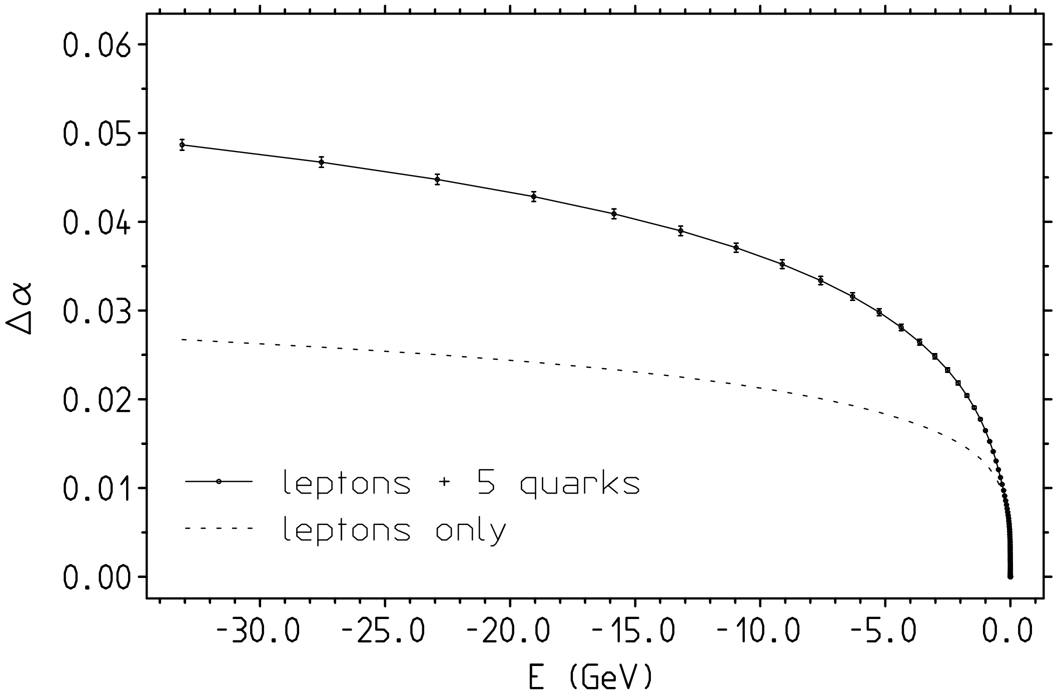
(charge renormalization).



$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \to \gamma^*$$

Effective finestructure "constant" $\alpha(E)$

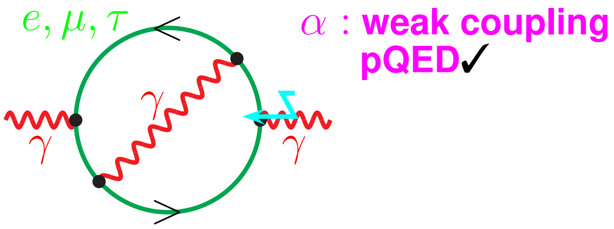
appears in many places in physics! **large EWRC !**



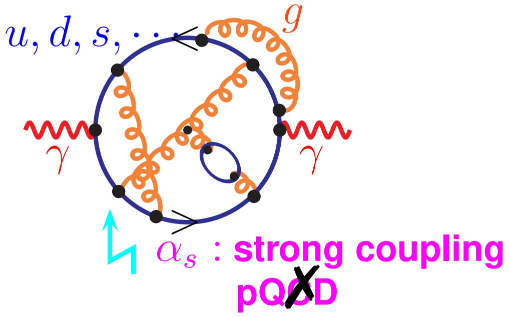
- The fine structure constant $\boxed{\alpha}$ of Quantum Electrodynamics and its extension to the electroweak Standard Model is one of the fundamental constant of nature.

- Due to charge screening effects (vacuum polarization) it is in fact a function $\boxed{\alpha(E)}$ of the energy scale E typical for a given process ($\alpha(0) = \alpha$).

- The contribution of the leptons to the vacuum polarization can be reliably calculated in perturbation theory, since $\alpha(0) = 1/137.036..$ is small.



- In contrast the quark/hadron contributions cannot be reliably calculated in perturbation theory because they exhibit strong interaction and the strong coupling $\alpha_s(E)$ increases at low energies (due to the asymptotic freedom=anti-screening) such that the perturbation expansion fails to "converge".



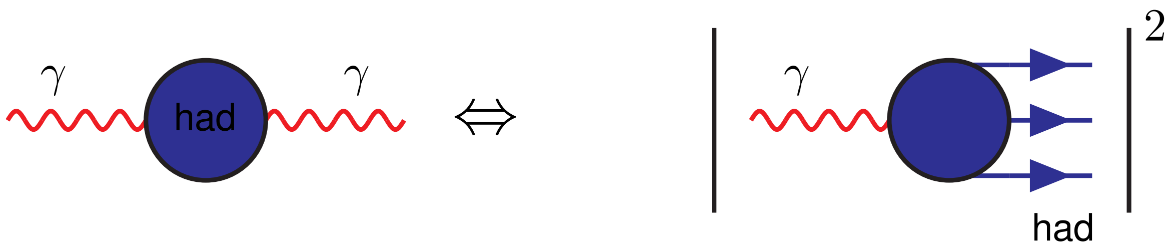
❑ **The way out:**

non–perturbative approach by dispersion relation

The total cross section

$$\sigma(e^+e^- \to hadrons)$$

for hadro-production in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$



$\Pi'_\gamma(q^2)$ $\quad\quad\quad\quad$ $\sim \sigma_{\rm tot}^{\rm had}(q^2)$

causality (analyticity) + unitarity (optical theorem)

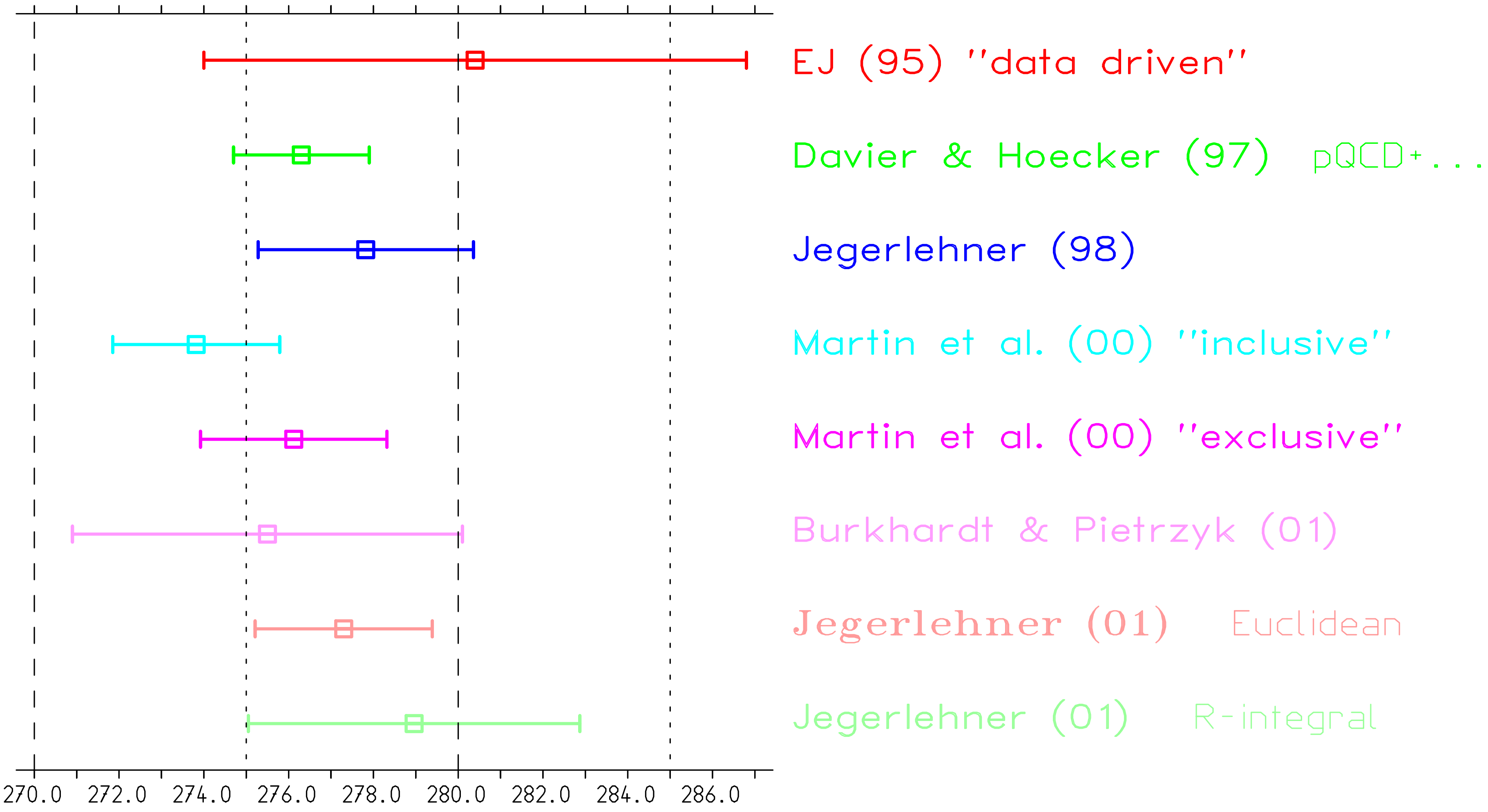
$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm QCD}(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

Recent evaluations of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$



A second error comes from the variation of the QCD parameters in the perturbative QCD part. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

(see: F.J. hep-ph/0104304, hep-ph/0105283)

Status and Perspectives:

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\;\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta \sin^2\theta$	Ref.	Method
0.00065	0.000232	[EJ95]	data $< 12.$ GeV
0.00016	0.000057	[DH98]	data < 1.7 GeV
0.00025	0.000089	[FJ98]	Euclidean > 2.5 GeV
0.000197	0.000070	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	0.000078	[MOR00]	same but “exclusive”
0.000460	0.000164	[BP00]	data
0.000391	0.000139	[FJ00]	as EJ95 + new data
0.00007	0.000025	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.000018	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	0.000160	LEP+SLD	

❺ Computational physics aspects:

How to beat factorial growth in complexity of calculations?

Memory problems, performance problems, software problems

Levels: analytic (formulae manipulating programs, memory and speed problems), numerical (stability and efficiency problems)

- **parallelization**
- **new algorithms**
- **automatic calculations**
- **standardization**

❑ **need flexible software packages which can easily accommodate new physics: SM $\longrightarrow$ MSSM, SSM,..., GUTs,... general: new fields, new vertices, anomalous couplings, extra dimensions, etc.**

❻ Status and Outlook:

"Technical" Status[a]:

❏ $2 \to 2$–processes (plus γ–radiation):

Prototype fermion–pair–production $e^+e^- \to \bar{f}f$ (LEP-1/TESLA-GigaZ).

full 1–loop

For such processes (together with the normalizing processes for renormalization (Thomson-scattering, μ–decay,...)) full SM as well as MSSM calculations at the 1–loop level may be performed in a canonical and efficient way. Tools are the basic scalar 1-loop integrals [1], tensor-integral reduction [2] (see also the Mathematica program *LERG-I* [3]) and the numerical implementation in the Fortran–package *FF* [4]. *TRACER* [5] is a Mathematica–program for the D-dimensional Dirac–algebra. All this is largely automatized:

- based on Mathematica/FORM [6] are *FeynArts, FeynCalc, FormCalc und LoopTools* of the Würzburg/Karlsruhe-group [7]– [9] (1–loop– with 2–loop extensions),
- the package *GRACE* [10] (1–loop) of the Japanese KEK-group [11],
- new a project *CalcPHEP* (1–loop) of the Dubna/Torino-Collaboration [12],
- finally our Bielefeld/Dubna/DESY-Zeuthen packages based on FORM and MAPLE: Feynman diagrams are generated with the help of *QGRAPH* [13] and processed into FORM code by our DIagram ANAlyzer *DIANA* [14] (1– up to 3–loops).

[a] For a more extensive physics oriented status report see: W. Hollik's talk

2–loop self-energies

At the 2–loop level a substantial set of leading corrections have been calculated. For no channel a complete 2–loop calculation exists! This, in spite of the fact, that such a calculation would have been very important for the interpretation of the Higgs mass bound from LEP/SLC.

Again there exist a number of programs, which calculate the "bubble–diagrams" (analytical), obtained by low energy expansions, but also arbitrary self-energy diagrams (analytical and 1-dimensional integral-representations):

- a class of massive 2–loop–integrals [15], which depend on one scale only have been implemented in ***ONSHELL2*** [16],
- in general exact analytic results are not known and one has to resort to series expansions at low or high energies [17]– [19], which may be combined with methods of conformal mapping and Padé–resummation,
- a combined analytical-numerical program for 2–loop self-energy functions has been developed by the Würzburg/Leiden–Collaboration [20, 21],
- for the reduction of integrals to a basis of standard-integrals there exist packages which solve the systems of recurrence-relations [22]. The procedure has been implemented in the Mathematica–program ***TARCER*** [23]. Utilizing relations between integrals in different dimensions D the problem of irreducible numerators could be solved [24].

Form factors for arbitrary momenta can only be calculated analytically in the massless approximation or by expansion about low or high momenta. For double–box diagrams more or less only massless results are available (see below).

The most advanced calculations in this context are the recent complete 2–loop fermion–loop contributions to Δr [25] and the complete 2–loop bosonic contributions to the W and Z mass counter-terms in the on-shell renormalization scheme [26].

massless 2–loop QED

In e^+e^--physics, the QED corrections are particularly important as they grow logarithmically with energy (at even higher energies electroweak Sudakov log's may be even more important, however). A very important pioneering calculation has been performed 15 years ago by the Leiden-group [27] for $e^+e^- \to \bar{f}f$ where $f \neq e$. A particular problem here is Bhabha–scattering (with $f = e$). Only recently we came closer to a complete 2–loop QED–calculation. Needless to mention that such a calculation would have been crucial for LEP, now it comes to late, but will be indispensable for a LC. A complete calculation for the virtual corrections in the massless approximation has been obtained recently by the SLAC/CALTECH-group [28] (see also related results for $q\bar{q} \to Q\bar{Q}$ in QCD from the Durham-group).

The result is based on a recent breakthrough in analytically calculating the 2–loop box–diagrams [29]– [33] for massless leptons on the mass–shell (one line may be off-shell which is important for the $e^+e^- \to 3\ \mathrm{Jets}$ calculations in QCD). For massive leptons a first result has been obtained [34].

In should be stressed that before the results can be applied to physics, one has to include mass effects appropriately and combine the virtual with the real photon emission, still, a highly non–trivial task.

❑ $2 \to n$–processes ($n = 4, 6$ plus γ–radiation)

1–loop

Such reactions with more complicated final states are the leading channels at energies beyond the W–pair threshold at LEP 2 and future linear colliders. A precise theoretical understanding is indispensable for the determination of the properties of unstable particles like the Higgs and the gauge bosons[a]. Here a theoretical precision at the level 0t 0.2% or better must be required. So far not a single complete 1–loop calculation exist. There are, however, serious attempts in this direction [36, 37].

The status here is the following:

- the most precise calculation concerns W–pair production in the double–pole approximation [38] (see also [39, 40]) with an estimated accuracy of about 1%;
- in addition new algorithms have been developed [41–43], which will be important for performing complete 1–loop calculations;
- there exist new ideas [44, 45], to tackle the problems numerically.

The corresponding bremsstrahlung processes have been calculated at tree level in [46] for massless fermions and in [47] for massive ones.

At high energies QED–corrections are adequately described be the structure function approach known from QCD. Leading-, next-to-leading-, etc. corrections obey evolution equations, which allow one to systematically resum the most important terms and thereby improve the numerical precision.

we need collective effort in larger collaborations !

[a] See [35] for a discussion of the problems related to the proper treatment of unstable particles

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