Parton Showering and Higher Orders

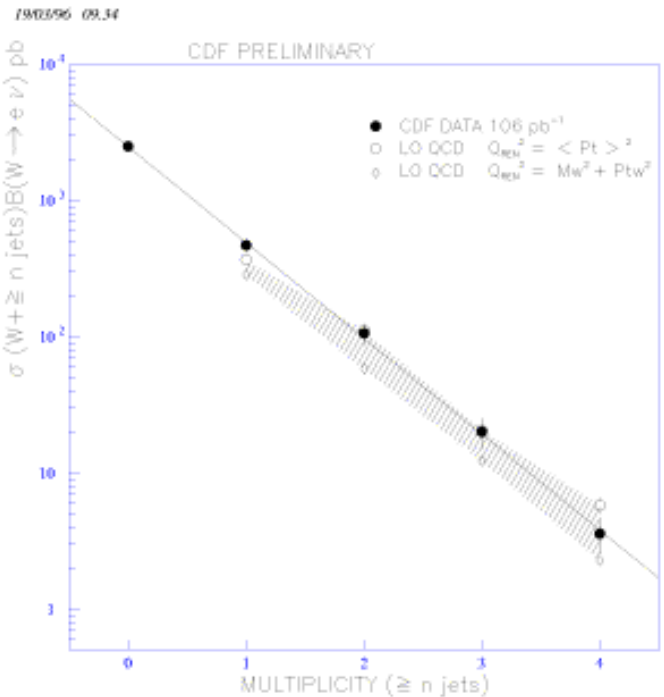
- Experiments rely on Monte Carlo programs
  - *Acceptance, Signatures, Optimize cuts, etc.*
- Many different calculational schemes
  - Tree level (lowest order)
    - Many partons
    - spin correlations
  - (N)NLO
    - Smaller theoretical errors
    - inclusive kinematics
  - "All" orders in towers of logarithms
    - Leading Logarithm, NLL, …
    - Analytic resummation (soft gluons integrated out)
    - Parton showers (soft gluons at leading log)

- Many programs
  - CompHEP / Madgraph / Whizard / …
  - MCFM / DYRad / JetRad / …
  - Pythia / Herwig / Isajet / … / ResBos
- Need a common and systematic comparison
  - Regions of validity (rate and kinematics)
  - Overlap between different schemes
  - Merging between schemes
  - Systematic uncertainty on physical observables

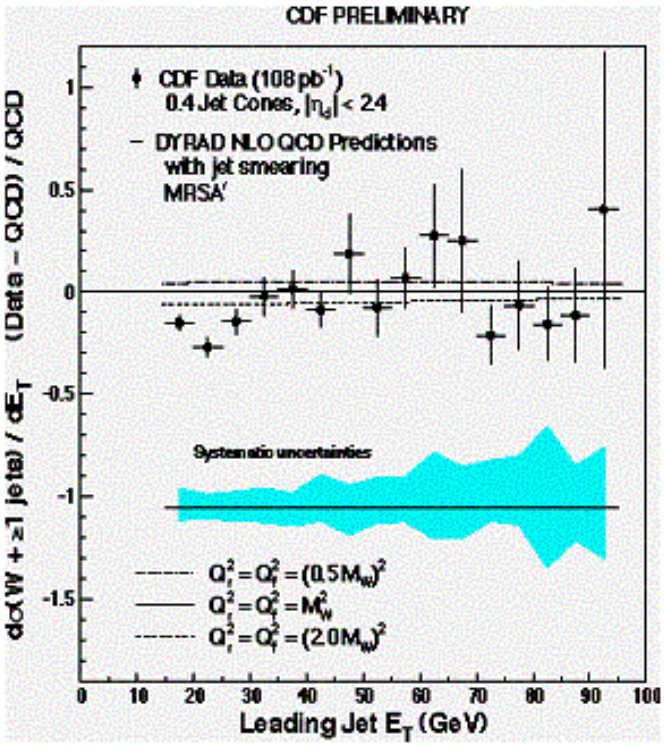
How to make sense of it all?

W + Jet(s) at the Tevatron

- Good testing ground for parton showers, matrix elements, NLO
- Background for new physics
- Large Scale dependence at LO

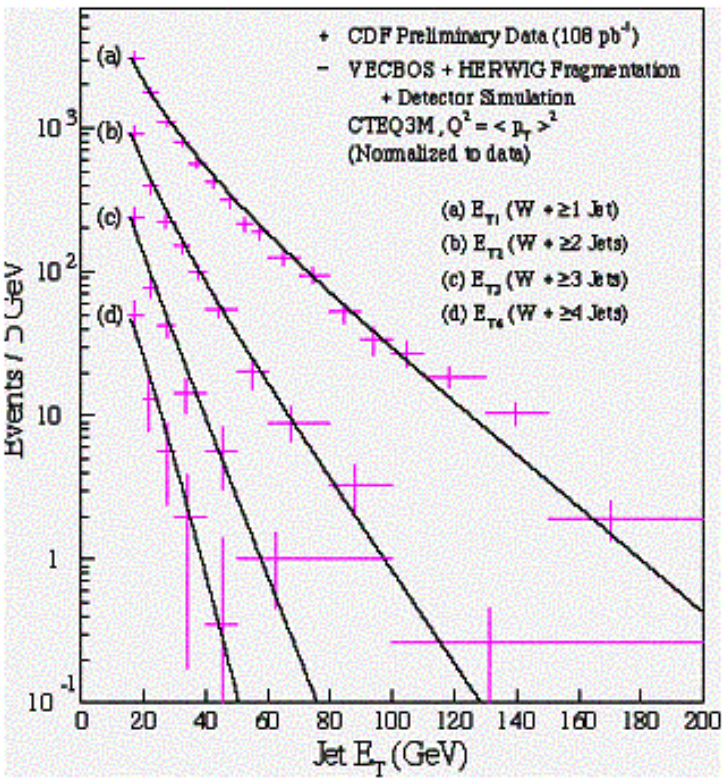


- Good agreement with NLO

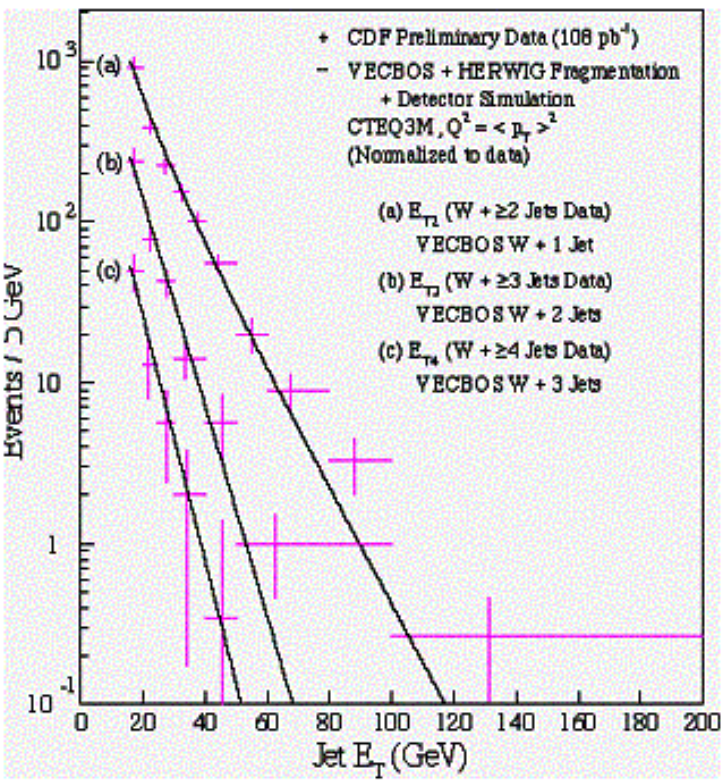


W + jet Rates

- W + ≥ n jet production
- Herwig (Herprt) for additional gluon radiation and for hadronization

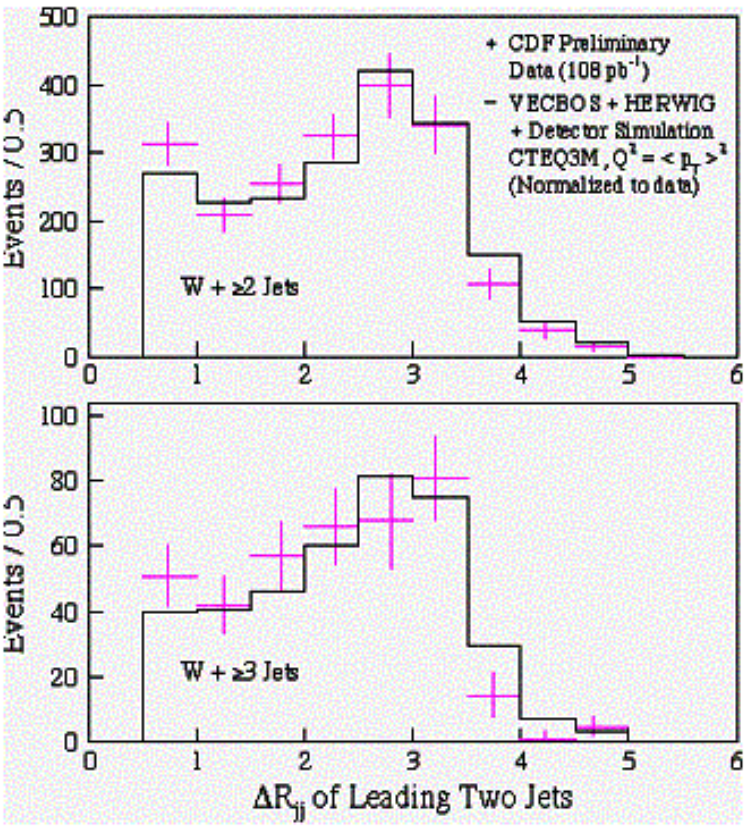


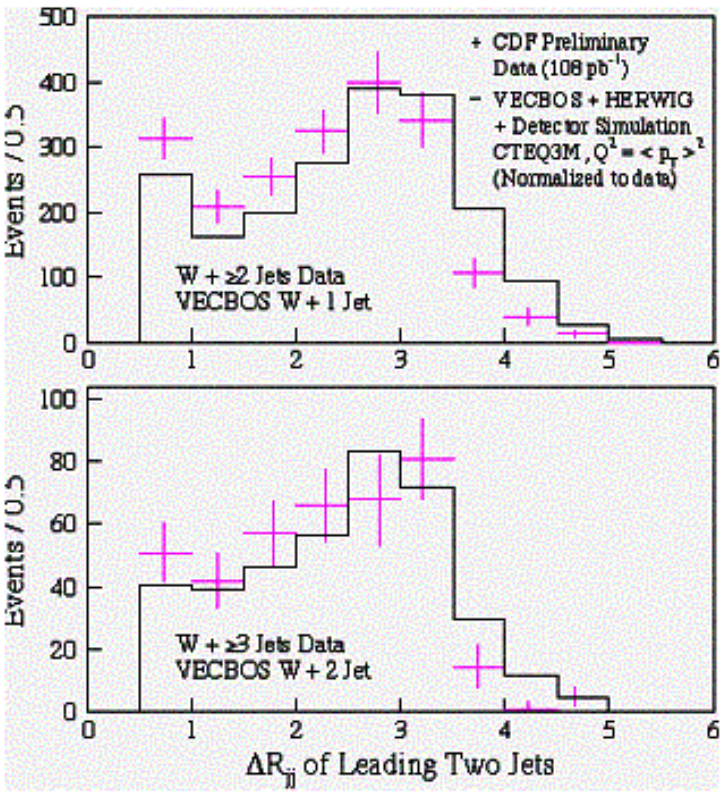
- Can also start off with n–1 jets and generate additional jets using Herwig



VECBOS + HERWIG

- Start with W + n jets from VECBOS
- Start with W + (n−1) jets from VECBOS





When good Monte Carlos go bad

CDF Run0 Data

#events p_T>10 GeV/c	>1 jet	>2 jets	>3 jets	>4 jet
Data	920	213	42	10
VECBOS + HERPRT (Q=<p_T>)				
W + 1jet	920	178	21	1
W + 2jet	-----	213	43	6
W + 3jet	-----	-----	42	10
VECBOS+HERPRT(Q=m_W)				
W + 1jet	920	176	24	2
W + 2jet	----	213	46	6
W + 3jet	----	-----	42	7

VECBOS starting point

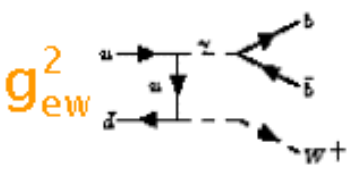
More jets generated by HERWIG parton shower

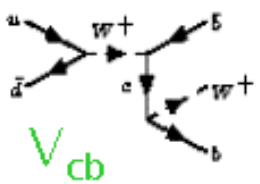
Normalize 1st bin to data

Results are cut dependent

- PS only has collinear part of matrix element
- PS has ordering in angles

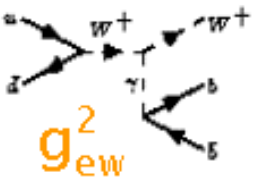
Tree Diagrams for ud→W⁺bb





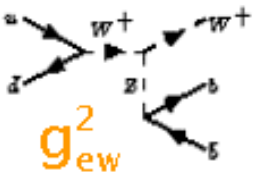
diagr.1

diagr.2

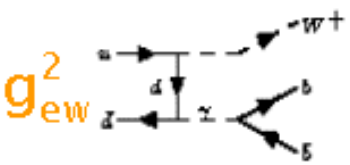


diagr.3

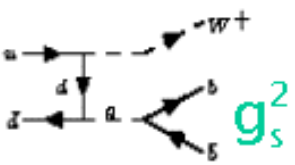
diagr.4



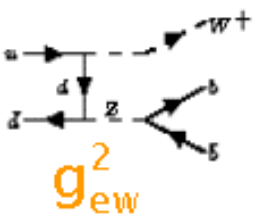
diagr.5



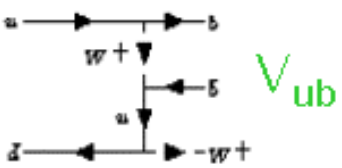
diagr.6



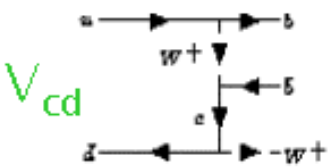
diagr.7



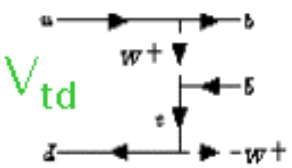
diagr.8



diagr.9



diagr.10



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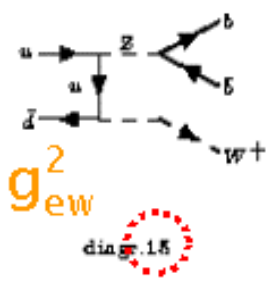


diagr.12

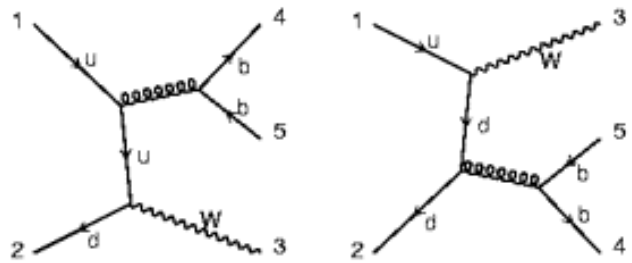
diagr.13

diagr.14

diagr.15

Naïve application of rules leads to many diagrams!

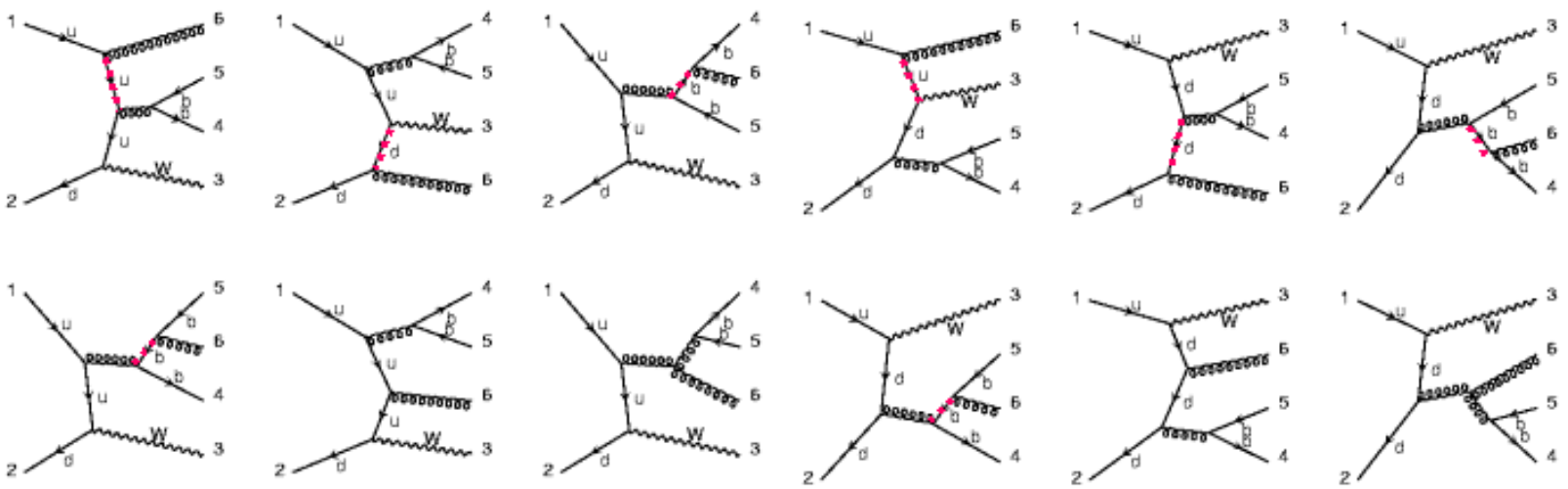
Higher Order: More Topology



Both diagrams have 6 colored lines

How sensitive is M_{bb} to additional gluon radiation?

Amplitudes diverge in soft/collinear limit

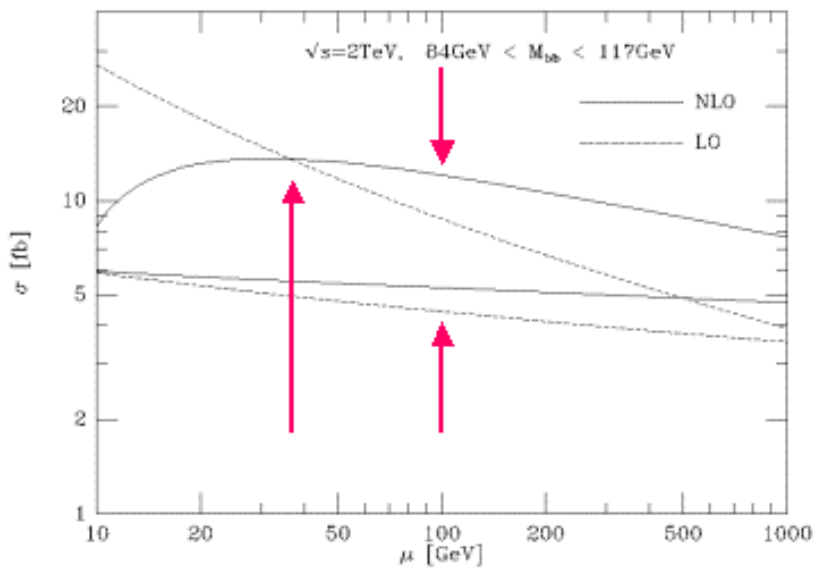


Why Go Beyond Tree Level ?

- Tree level (lowest order) prediction X has a large dependence on the scale μ in couplings
  - "the" hard scale μ is ambiguous
  - Ideally $dX/d\mu=0$, but not possible if $X \sim g(\mu)^N$
    - More likely if $X = a\, g(\mu)^N + b\, g(\mu)^{N+1}$
- No clear way to merge different topologies
  - Some W/Z+N parton events will be reconstructed as W/Z + N−1 jet events
  - Some W/Z+N−1 parton events will be W+N−1 jet events
- No way to avoid soft and/or collinear singularities
  - In fact, multiple gluon emission occurs in these kinematic configurations
- No direct connection to hadronization/fragmentation

Higgs Signal and Background at NLO

- Scale dependence greatly reduced
- Shape of *some* distributions similar up to scale (K) factor
  - *But kinematics are inclusive*
  - Separation of Wbb/Wbbg final states depends on cutoff
    - Multiple, soft-collinear gluon emissions are *not* included



Resummation: Beyond Fixed Order

- Consider W production
  - At LO in pQCD, the rapidity Y and transverse momentum Q_T of the W are fixed by incoming partons
  - At NLO, single gluon emission occurs with $Q_T > 0$

LO:

$$\frac{d\hat{\sigma}}{dQ_T^2} \propto \delta(\vec{Q}_T)\sigma_0$$

NLO:

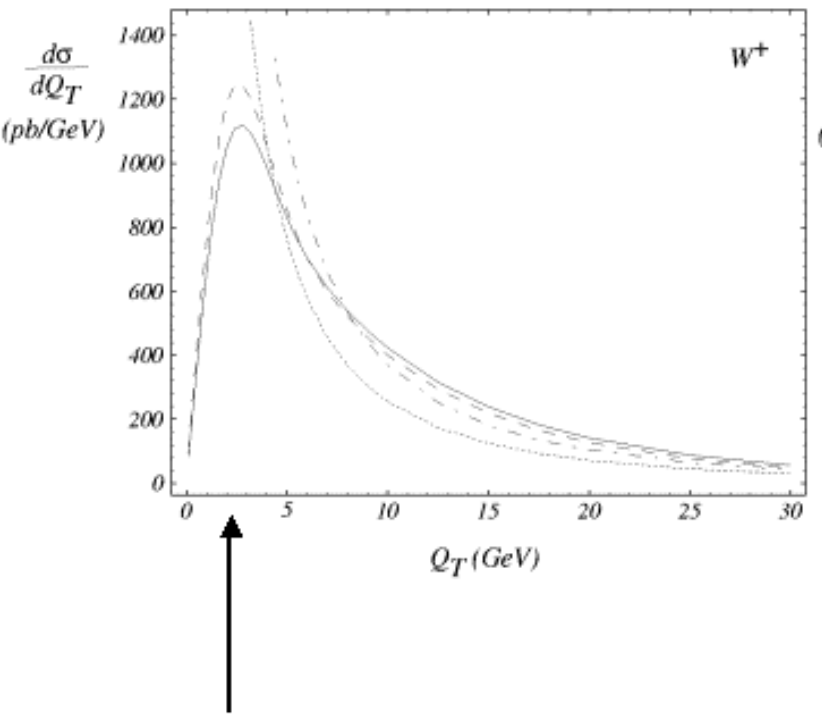
$$\frac{d\hat{\sigma}}{dQ_T^2} \approx \frac{\alpha_S}{Q_T^2} \ln\left(\frac{Q^2}{Q_T^2}\right)\left[c_1 + c_2\alpha_S \ln^2\left(\frac{Q^2}{Q_T^2}\right) + \ldots\right]$$

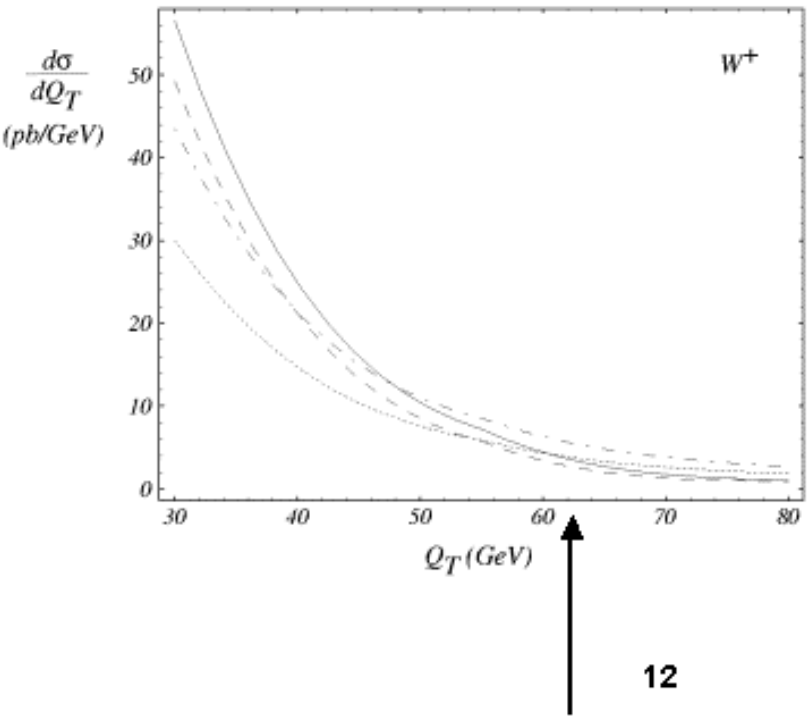
- *Cross sections at large Q_T or Q_T averaged are described well by fixed order in α_S*
- However, some observables are sensitive to region $Q_T << Q$
  - For W/Z production, this is most of the data!
- Solution: Reorganize perturbative expansion
  - $\alpha^N \ln^M(Q^2/Q_T^2)$
  - Sums up infinite series of soft gluon emissions
  - k_T dependent PDF's

Sudakov Effect

- Solid: resummed
  - Superior at lower Q_T
- Dot/Dash-dot: W+1j/W+2j
  - Superior at high Q_T
    - Ln(Q/Q)=0

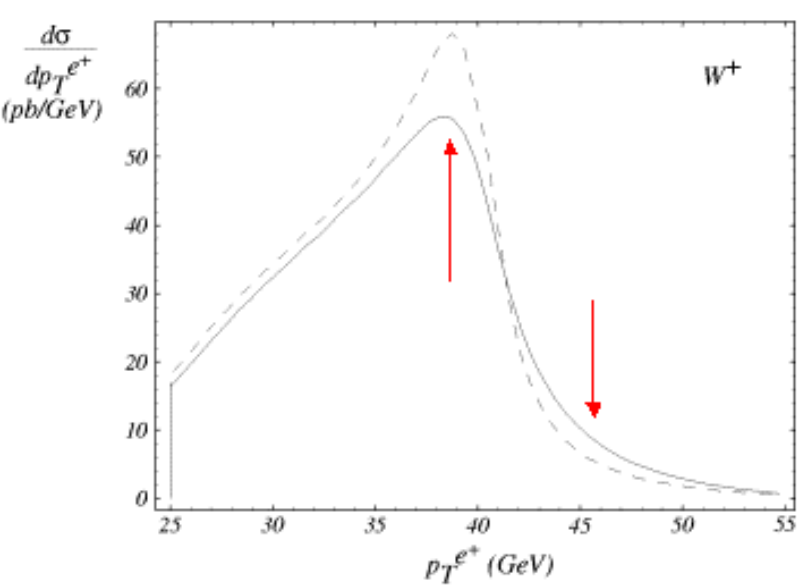
Multiple soft and collinear gluon emissions included, but integrated out

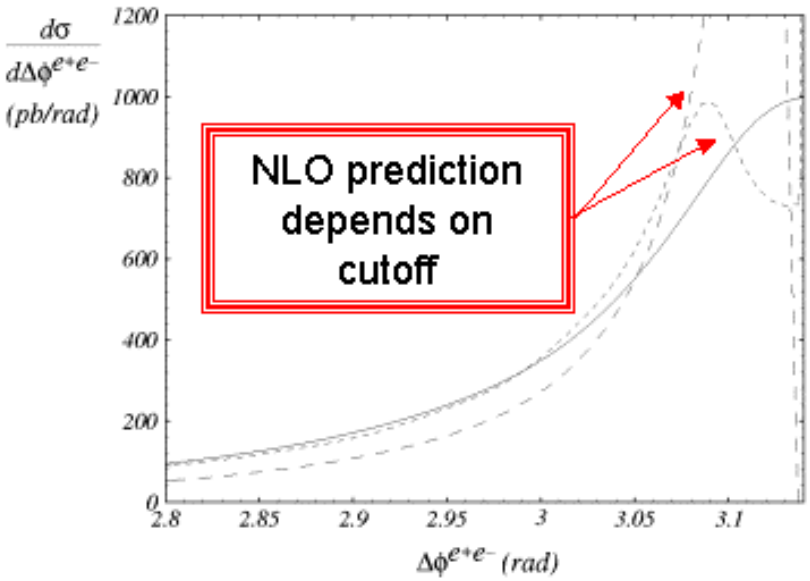




Higher Order vs "All" Order

- In the lower Q_T region, significantly different predictions from NLO
  - Decay products retain information about W production
  - Important for M_W measurement in Run II

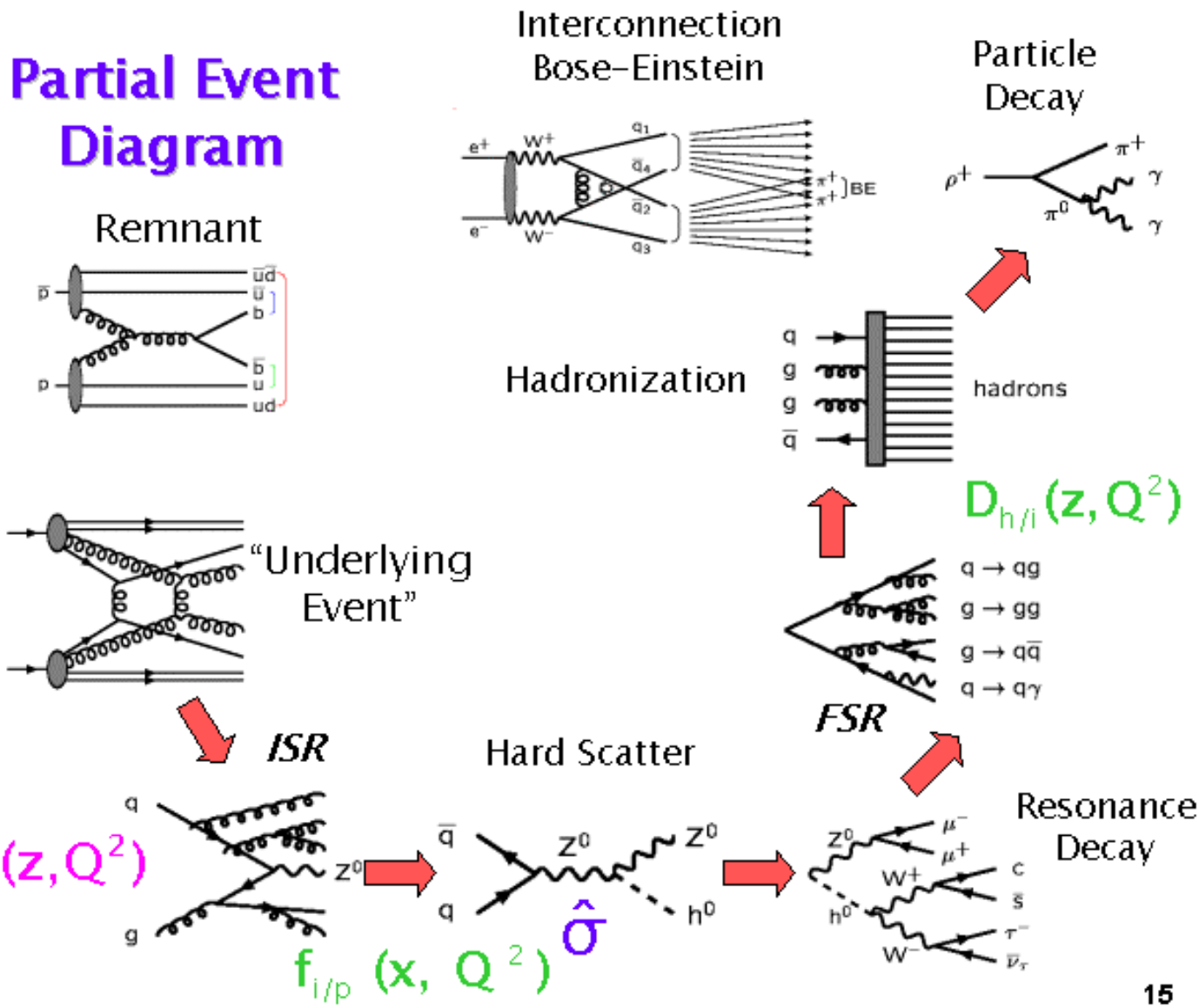




Review of "Hard Scattering"

- Tree Level Predictions
  - Pros
    - Full spin and color correlations
    - "Easy" to calculate
    - Good for large P_T's and angular separations
  - Cons
    - Rate not reliable
    - Not clear how to merge different topologies
- (N)NLO Predictions
  - Pros
    - Reduced scale dependence
    - Merging of topologies
  - Cons
    - Also requires large P_T's and angular separations
- Resummation
  - Pros
    - (N)NLO accuracy with all orders accuracy in kinematics
  - Cons
    - No information on soft gluons
- *All of these approaches ignore details of physics below hard scale*
  - Not yet connected to hadronization
    - Color must be screened

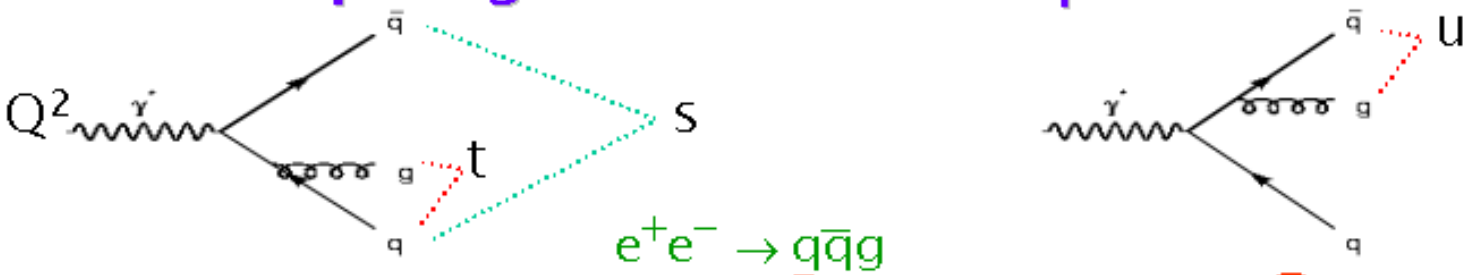
Partial Event Diagram



Parton Shower

- Hard Scattering sets scale Q
- Structure $f(x,Q^2)$ or fragmentation $D(x,Q^2)$ functions, couplings $\alpha_S(Q^2)$, etc. are evaluated at Q
  - Asymptotic states have a scale Q_0~1 GeV
- Incoming/Outgoing partons are highly virtual
  - How do incoming partons acquire mass^2 ~ $-Q^2$?
    - INITIAL STATE RADIATION (ISR)
  - How do outgoing partons approach the mass shell ?
    - FINAL STATE RADIATION (FSR)
- Typically, shedding of virtuality generates many partons with lower virtuality
  - Virtualities on the order of Λ_{QCD} are expected for partons bound in hadrons
- "traditional" calculations based on a small number of Feynman diagrams are incomplete
  - Parton Showering Monte Carlos are an approximation to high-order, perturbative QCD

Example: gluon emission in γ^* events



$e^+e^- \to q\bar{q}g$

$$d\sigma(q\bar{q}g) = \sigma_0 \left\{ \frac{\alpha_s}{2\pi}\frac{dt}{t}dz\left[P_{qq}(z) - \frac{t}{Q^2}\right] + \frac{\alpha_s}{2\pi}\frac{du}{u}dz\left[P_{qq}(z) - \frac{u}{Q^2}\right] \right\}$$

$t \to 0$ when gluon is Soft, collinear or both

$z \to 1$ when gluon is Soft, collinear or both

$$z = \frac{s}{Q^2}, P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$s = 2p_q \bullet p_{\bar{q}}, t = 2p_q \bullet p_g, u = 2p_{\bar{q}} \bullet p_g$$

- **Factorization of Mass Singularities**
- **Probability of one additional soft emission proportional to rate without emission**
  - $d\sigma_{N+1} = \sigma_N\ \alpha_S/2\pi\ dt/t\ dz\ P(z)$

Tower of emissions described by Sudakov Form Factor

- Series of subsequent showers "*exponentiate*"
- Shower of resolvable emissions $q^*(p) \rightarrow q(zp) + g([1-z]p)$
  - Emission RESOLVED if $z_C < z < 1 - z_C$
- Probability of ***no*** resolvable emission for small δt ($\ln Q^2$)

$$1 - \sum_{b,c} \int_{z_-(t)}^{z_+(t)} dz \, \frac{\alpha_S(t)}{2\pi} P_{a\rightarrow bc}(z) \delta t$$

Sum over all numbers of irresolvable emissions :

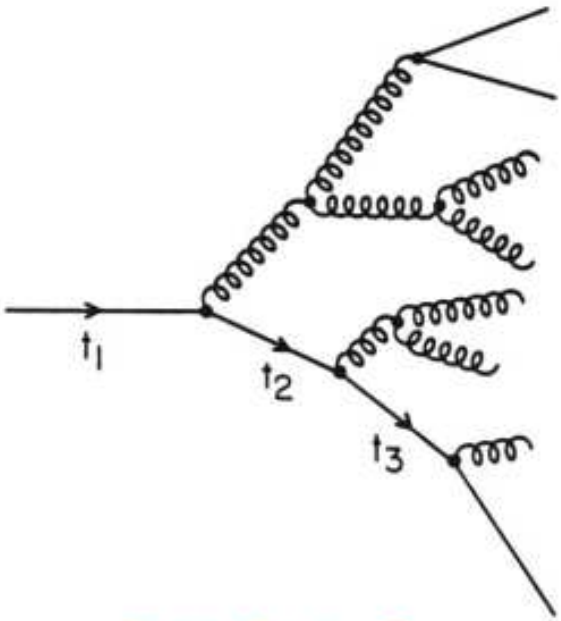
$$S(t) = \exp\left\{ - \int_{t_0}^{t} dt' \sum_{b,c} \int_{z_-(t')}^{z_+(t')} dz \, \frac{\alpha_S(t)}{2\pi} P_{a\rightarrow bc}(z) \right\} \Leftrightarrow \Delta(t)$$

$$z_+ \sim 1 - z_C, z_- \sim z_C$$

- $\text{Prob}(t_{start}, t) = S(t_{start})/S(t)$
  - Pick random r and solve for new t
  - Resolvable emission at the end of "nothing"
  - Continue picking new t's down to $t_{end} \sim \Lambda_{QCD}$
  - Stop shower & begin hadronization

Virtuality-Ordered PS

Highly virtual



Nearly on-shell

$$t_1 > t_2 > t_3$$

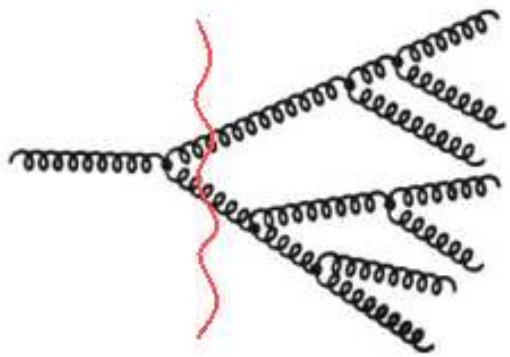
$$t_i = \ln(Q_i^2)$$

Parton Shower is a Resummation

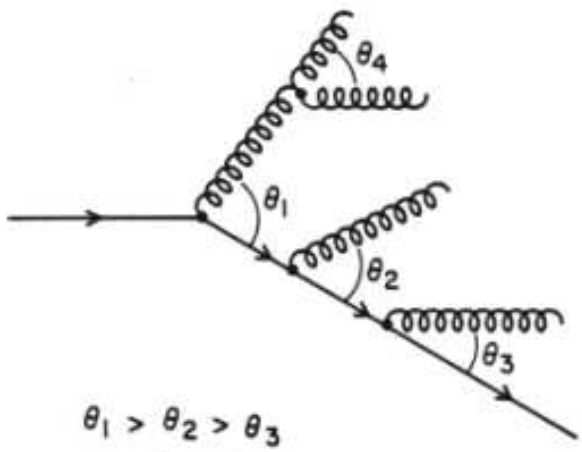
- Analytic Resummation
  - soft gluon emissions exponeniate into Sudakov form factor
  - k_T conserved
  - Total rate at (N)NLO
    - modified PDF's
  - corrections for hard emission
  - soft gluons are integrated out
    - Predicts observables for a theoretical W
  - Needs modelling of non-perturbative physics
- Parton Showering
  - DGLAP evolution generates a shower of partons
    - LL with some N-LL
  - Exact gluon kinematics
  - LO event rates
  - underestimates single, hard emissions
  - Explicit history of PS
    - More closely related to object identified with a W
  - natural transition to hadronization models
    - Follow color flow down to small scales

Angular-Ordered PS

- Showers should be Angular-Ordered
  - $\zeta = p_i \cdot p_j / E_i E_j = (1 - \cos\theta_{ij}) \sim \theta_{ij}^2/2$
  - $\zeta_1 > \zeta_2 > \zeta_3$ ...
- α depends on $k_T^2 \Rightarrow z(1-z)Q^2$



- Dead Cone for Emissions
  - $Q^2 = E^2 \zeta < Q^2_{max}$
  - $Q^2_{max} = z^2 E^2$
  - $\zeta < 1$ [not 2]
  - $\theta < \pi/2$
    - No emissions in back hemisphere



$\theta_1 > \theta_2 > \theta_3$

$\theta_1 > \theta_4$

The Programs

- ISAJET
  - Q^2 ordering with no coherence
  - large range of hard processes
- PYTHIA
  - Q^2 ordering with veto of non-ordered emissions
  - large range of hard processes
- HERWIG
  - complete color coherence & NLO evolution for large x
  - smaller range of hard processes
- ARIADNE
  - complete color dipole model (best fit to HERA data)
  - interfaced to PYTHIA/LEPTO for hard processes

Parton Shower Summary

- Accelerated color charges radiate gluons
  - Gluons are also charged
  - Showers of partons develop
    - IMPORTANT effect for experiments
- Showering is a Markov process and is added to the hard scattering with P=1
  - Derived from factorization theorems of full gauge theory
  - Performed to LL and some NLL accuracy with exact kinematics
  - Color coherence leads to angular ordering
- Modern PS models are very sophisticated implementations of perturbative QCD
  - Still need *hadronization models* to connect with data
  - Shower evolves virtualities of partons to a low enough values where this connection is possible

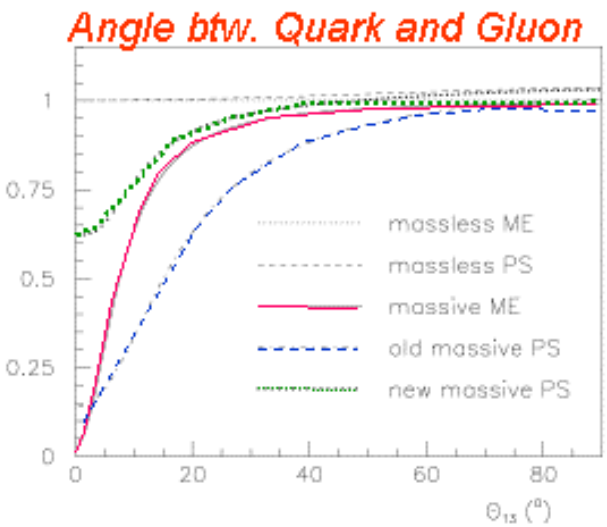
Correcting the Parton Shower

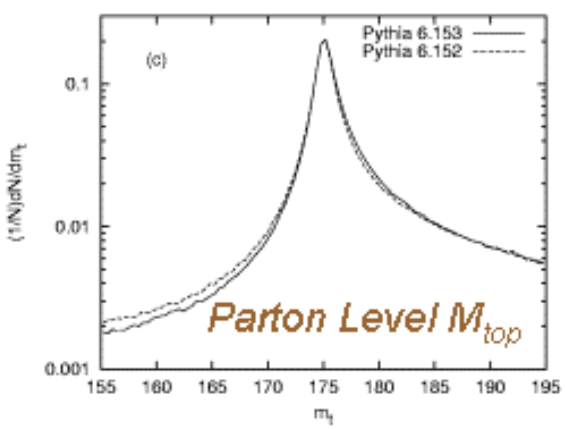
- PS is an accurate description for soft/collinear kinematics
  - Most of the data for a given process!
- Underestimates wide angle emissions
  - Also, no $1/N_C$ suppressed color flows
  - Tails of kinematic distributions are often *most interesting*
- PS with a single emission can be reweighted to behave like fixed-order result
  - Correct all or hardest-so-far emissions this way
- Populate kinematic regions not included in PS
  - Delicate matching between different regions
- Actual correction is generator dependent
- No attempt to generate NLO rate

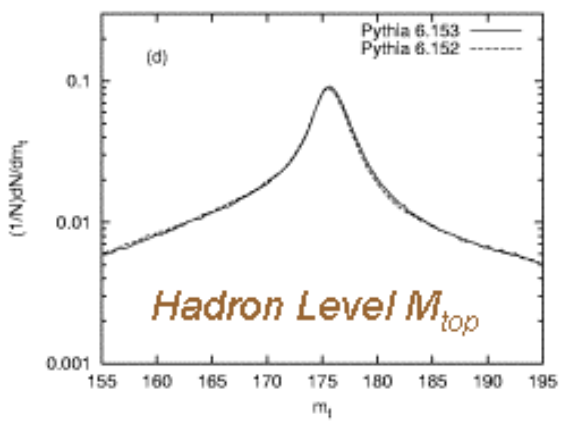
Sjostrand/Miu/Seymour/Gorcella …

Pythia Corrections to Top Decay

- Relatively easy for Q^2 ordered showering
- Rewrite Parton Shower weight in terms of Matrix Element kinematics
- Modify PS probability by W_{ME} / W_{PS}







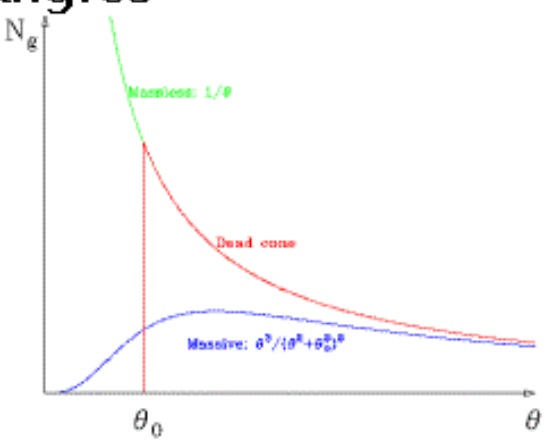
Parton Showering and Heavy Quarks

- Heavy Quarks look like light quarks at large angles but are sterile at small angles

Eikonal expression for soft gluon emission:

$$\frac{d\sigma(q\bar{q}g)}{d\sigma(q\bar{q})} \propto -\left(\frac{p_1}{p_1 \bullet p_3} - \frac{p_2}{p_2 \bullet p_3}\right)^2 \frac{d^3p_3}{E_3}$$

$$\Rightarrow \frac{d\sigma(x_3, \theta_{13}, r = m_Q/E_Q)}{d\sigma(x_3, \theta_{13}, 0)} = \left(\frac{\theta_{13}^2}{\theta_{13}^2 + r^2}\right)^2$$



- Naïve θ–ordered shower has a cutoff
  - $\theta > \theta_0 = m_Q/E_Q = r$
  - Creates 'dead cone'
- Virtuality–ordered shower also needs a special treatment

Top decay in HERWIG

$t \to bW \qquad dP = \frac{d\Gamma(t\to bWg)}{\Gamma_0} \qquad \Gamma_0 = \Gamma(t \to bW)$

$Q_{t\max} Q_{b\max} = p_t \cdot p_b$

Top rest frame: $Q_{t\max} = m_t \qquad Q_{b\max} = p_t \cdot p_b / m_t$

No soft-gluon radiation from the top quark

b quark radiates soft gluons for $\xi < 1$, i.e. $0 < \theta < \pi/2$

Total energy loss is roughly right

Error neglecting backward radiation:

$$\Delta = \frac{\alpha_S}{2\pi}(1 - \log 2)$$

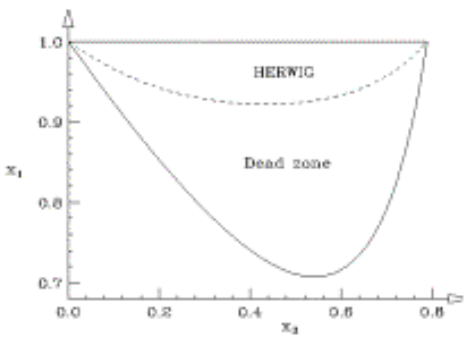
Problems with angular distributions

$\pi/2 < \theta < \pi$: dead zone

Matrix-element corrections to top decays

$t(q) \to W(p_1) b(p_2) g(p_3)$

$x_1 = \frac{2p_1\cdot q}{m_t^2} - a \qquad x_3 = \frac{2p_3\cdot q}{m_t^2} \qquad a = \frac{m_W^2}{m_t^2}$



$$\frac{1}{\Gamma_0}\frac{d^2\Gamma}{dx_1\,dx_3} = \frac{1}{(1-a)\left(1+\frac{1}{a}-2a\right)} 2\frac{\alpha_s}{2\pi} C_F \frac{1}{x_3^2(1-z_1)} \left\{ (1+\tfrac{1}{a}-2a)\left[(1-a)x_3-(1-x_1)(1-x_3)-x\right.\right.$$
$$\left.\left. +(1+\tfrac{1}{2a})x_3(x_1+x_3-1)^2+2x_3^2(1-x_3)\right\}\right.$$

Hard correction: $d^2\Gamma$ in the dead zone

Soft correction: $d^2\Gamma$ in the populated region for every hardest-so-far emission

Cutoff to avoid the soft divergence: $E_g > 2$ GeV

Correcting the NLO Calculation

- Total rate is more stable to hard scale variation
- Single wide-angle emission from onset
  - "models" soft-collinear region by single gluon with an inclusive subtraction of singularities
- Desire:
  - $\sigma(N+1)\times PS + \sigma(N)\times PS - \text{Overlap}$
  - $\text{Overlap} \sim \sigma(N)\times PS$ (all orders) truncated to fixed order
- Implementations
  - Phase space slicing with δ chosen to remove $\sigma(N)$
    - Only one configuration to shower
    - Positive definite weights
    - Sensitivity to large logarithms of δ must arise in some distributions
    - Expansion to fixed order will not agree with NLO result

Baer/Reno/Dobbs/Potter/ …

- Subtraction
  - No dependence on a cutoff
  - Suitably modified subtraction expands to NLO result
  - NLO calculation fixes initial kinematics for PS and relative weights of (modified) N and N+1 body contributions
  - Negative weights are generated
    - Can be treated practically using positive definite integrands but keeping track of sign
  - Relies on unmodified PS algorithm
    - Consistent?
    - Power law dependence on some showering parameters
      - Should be small

Frixione/Webber

Parton Shower and Factorization

- Standard Result
  - Some freedom in defining PDF/Fragmentation function
  - Differences not observable for inclusive calculations
- Parton Shower needs a special treatment *[Collins]*
  - Exact kinematics throughout shower
    - NLO predicts gauge boson at rapidity Y
    - PS starts from Y_0 and generates Y from emission/boost
    - Mismatch involving evaluation of PDFs at different x
    - Affects Overlap computed in previous approaches
  - PDFs for PS depend on process and showering algorithm
    - Demonstrated large effect for small parton x
- Same conclusion from considering the analytic resummation methods ala Collins/Soper/Sterman *[Mrenna]*

PS, Factorization, and Resummation

- b/Q_T–space resummation yields (N)NLO rate and "all orders" kinematics

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy}(h_1 h_2 \to W + X) = \frac{d}{dQ_T^2}\tilde{W}(Q_T, Q, x_1, x_2) + Y$$

$$\tilde{W} = (C \otimes f)(C \otimes f)\exp(-T) \quad H(Q)$$

$$T(Q_T, Q) = \int_{Q_T^2}^{Q^2} \frac{dm}{m}\left[A \ln\left(\frac{Q^2}{m}\right) + B\right]$$

$$(C \otimes f)[x] = \int_x^1 \frac{dz}{z} C(x/z) f(z)$$

At LO, $$\frac{d\sigma}{dQ_T^2} = \sigma_0 \frac{d}{dQ_T^2}\left\{\frac{\exp\left[-\frac{1}{2}T(Q_0, Q)\right] f(x, Q_T)}{\exp\left[-\frac{1}{2}T(Q_0, Q_T)\right] f(x, Q)}\right\}$$

- **MW PS algorithm**
  - **Sudakov determines P_T^W**
- **Soft gluon emissions are integrated out**
  - **Sudakov contains soft pieces not in DGLAP**
- **Total rate can be calculated to any given order in α_S**

- Y = Fixed order–Asymptotic = Overlap!
  - Contains negative weights – just aesthetics?

Summary of NLO Showering

- Correct the PS for emissions that are not soft/collinear enhanced
  - Reweight by $(ME^2_{PS})/(ME^2_{Exact})$ [PYTHIA]
  - Fill out Dead Cone and correct shower [HERWIG]
  - Still have to normalize to NLO rate
- Add PS to (N)–(N+1) body configurations of full NLO calculation with suitable subtractions
  - Phase Space Slicing with δ chosen to eliminate (N) body configurations
    - Approximation to kinematics sensitive to $\log(\delta)$
  - Subtraction method with modifications to (N) and (N+1) body "Kernels"
    - NLO predictions recovered upon expansion to $O(\alpha^2)$
    - Negative weight events are generated
  - Inconsistency in using "inclusive" definitions of PDFs/Fragmentation functions

Summary (cont)

- Process dependent PS
  - Factorization must be re-evaluated when exact kinematics are treated
  - Integration over K_T and virtualities implicit to usual PDFs/Fragmentation functions
  - Feature also seen in analytic resummation (Collins/Soper/Sterman)
- Analytic Resummation has essential ingredients
  - Process dependent PDFs recover NLO rate and weight Sudakov
  - Y-piece subtraction corrects for soft-gluon approximation inside Sudakov
    - Introduces negative weights

Related Ideas

- QCD Matrix Elements + Parton Showers
  - Catani/Kraus/Kuhn/Webber & Lönnblad
  - Attempts to piece together many tree level processes using PS
    - Generate all $Z \to N$ parton (tree level) processes simultaneously
    - For a given N, select a given topology
    - Match topology to a PS history
    - Reject if event would have been generated by $M<N$ with a parton shower attached
  - Not clear how to match to higher-order rate
- Fully numerical NLO in Coulomb gauge
  - Soper and Kramer
  - Claims a more natural connection to PS

Conclusions

- Great activity amongst relatively few practitioners to develop NLO parton showering
  - Several good ideas which need to be synthesized
- Hadron collider phenomenology has greatest priority now
  - Naively, effects will be larger with initial state radiation
- Precision physics at the NLC will be sensitive to our understanding of the parton shower
    - Already a large "systematic" uncertainty to Run II measurements (M_W and m_t)
    - Perhaps a larger "systematic" at LEP than we think
      - b–quark asymmetries
      - "4–jet" events
      - ...