Fullone-loop electroweak corrections to $e^+e^- \to \nu\bar{\nu}H$

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Motivation

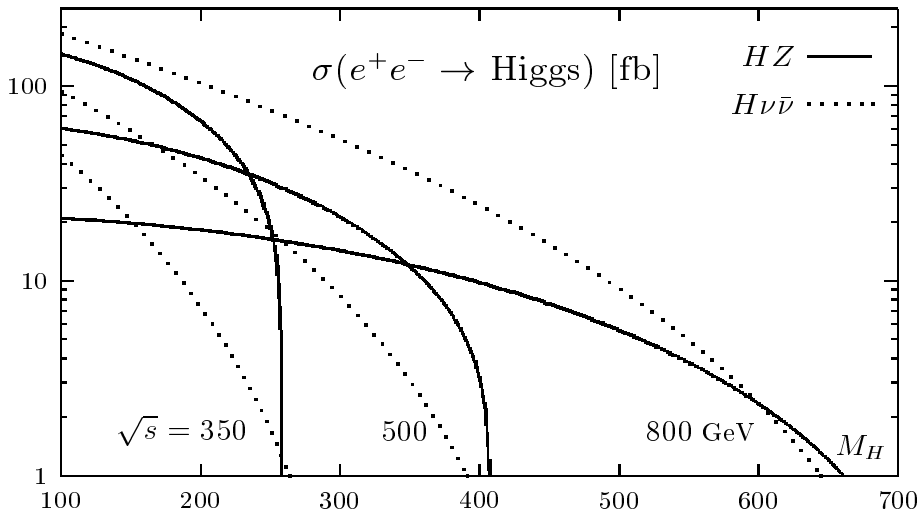
GRACE/1-loop

Checks

Results and comparisons

Higgs production in e^+e^-

Higgstrahlung vs WW fusion



$e^+e^- \to \nu\bar{\nu}h$ dominant process for Higgs production at high energies

Status of one-loop corrections

- Complete $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to Zh$
  A. Denner, J. Kublbeck, R. Mertig, M. Bohm, Z.Phys. C56 (1992) 261.
  B. A. Kniehl Z. Phys. C55 (1992) 605.
- Radiative corrrections to $e^+e^- \to \nu\bar{\nu}h$ in region $\sqrt{s}, m_h \ll 2M_Z$
  B.A. Kniehl, Phys. Rep. 240 (1994) 211.
- Loop corrections to $h \to W^+W^-$ from fermion/sfermion loops in MSSM
  E. Eberl, W. Majerotto, V.C. Spanos, Phys. Lett. B538 (2002) 353
  T. Hahn, S.Heinemeyer, G. Weiglein, hep-ph/0211204.
- Full one-loop electroweak radiative corrections to $e^+e^- \to \nu\bar{\nu}h$
  G.B., Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu, hep-ph/0212261
  Denner, Dittmaier, Roth, Weber, hep-ph/0302198

GRACE/1-loop

- On-mass shell renormalization scheme (Kyoto group) Counterterms are generated automatically
- Use dimensional regularization, $n = 4 - 2\epsilon$
- Infrared divergence regulated with fictitious photon mass
- Symbolic manipulation code based on REDUCE or FORM writes for each diagram

$$T_i^{loop} T_j^{tree\dagger} \tag{1}$$

  Symbolic manipulation performs: Dirac algebra (taking traces for fermion loops), tensor manipulation in $n = 4 - 2\epsilon$, Feynman parametric integrals, shift momentum... Then Fortran code is generated.

- Two libraries for Loop integration: numerical integration with FF package and in-house package for reduction of numerators and numerical integration
- Integration over phase space : BASES an adaptative Monte Carlo

System tested in SM at tree-level and at one-loop

- Internal checks: ultraviolet divergence, infrared finiteness...
- Gauge parameter independence
- Comparison with other calculations $2 \to 2$

Checks on calculation

- Checks performed with
  1. One random phase space point
  2. Full set of diagrams
  3. Quadruple precision

- Ultraviolet Finiteness
  regularise infrared divergence by fictitious photon mass $\lambda = 10^{-15} GeV$
  extract term in $C_{UV} = \frac{1}{\epsilon} - \gamma_E + \log 4\pi$
  stability of result for different values of $C_{UV} \rightarrow$ 30 digits precision)
  repeat this test with non-zero non-linear gauge parameters

- Infrared finiteness
  Regulate infrared divergence in photon exchange diagram with fictitious photon mass λ
  Dependence on λ cancels with the one in soft bremstrahlung
  Soft photon is calculated analytically

$$d\sigma_{soft}(\lambda, E_\gamma, k_c) = d\sigma_0 \delta_{soft}(\lambda, E_\gamma, k_c) \tag{2}$$

  check independence on $\lambda \rightarrow$ 23 digits stability

- Gauge parameter dependence
  Use non-linear gauge fixing (here $C_{UV} = 0$)
  Non-trivial check for:
  Correct input file: Feynman diagrams, counterterms...
  Symbolic manipulation part
  Correctness of reduction formulae for tensor integral (including pentagon)

General Non-linear gauge fixing

$$\begin{aligned}\mathcal{L}_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\ &+\xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\ &-\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2\end{aligned}$$

Choose t'Hooft-Feynman gauge ($\xi_W = \xi_Z = \xi_A = 1$)

1. avoid large numerical cancellations
2. no need for higher order tensor structure in loop integrals

Vary parameters $\zeta = (\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon})$
For each loop graph, g

$$\mathrm{d}\sigma_g = \mathrm{d}\sigma_g(\zeta) = \Re\left(\mathcal{T}_g^{loop}\cdot\mathcal{T}^{tree\ \dagger}\right)$$

Comprehensive checks of NLG parameters independence, for example for only one non-zero parameter, $d\sigma$ polynomial in ζ

$$\mathrm{d}\sigma_g = \mathrm{d}\sigma_g^{(0)} + \zeta\mathrm{d}\sigma_g^{(1)} + \zeta^2\mathrm{d}\sigma_g^{(2)} + \zeta^3\mathrm{d}\sigma_g^{(3)}$$

can reconstruct $d\sigma_g^{(0,1..4)}$ and verify that $d\sigma$ is independent of ζ

$$\mathrm{d}\sigma = \sum_g \mathrm{d}\sigma_g = \sum_g \mathrm{d}\sigma_g^{(0)}\ , \tag{3}$$

$$\mathrm{sum}_i = \frac{\sum_g \mathrm{d}\sigma_g^{(i)}}{\mathrm{Max}_g(|\mathrm{d}\sigma_g^{(i)}|)} = 0\ \ ,\ \ i = 1,2,3\ , \tag{4}$$

For this, set $\Gamma_Z = 0$ (choose point away from resonance).

NLG checks on $2 \to 2$ processes

Table 1: *Accuracy measured by the number of digits for the gauge-parameter checks on the 26 processes, for all five gauge parameters. An empty entry means that the process does not depend on the gauge parameter. Only one parameter is varied at a time here. The number of diagrams depends on the choice of the gauge parameter, for examples in some gauges some vertices are absent. The number of diagrams that we list corresponds to the gauge which leads to the maximum number of diagrams.*

processes	# of graphs(Loop × Tree)	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\delta}$	$\tilde{\epsilon}$	$\tilde{\kappa}$
$\nu_e\bar{\nu}_e \to \nu_e\bar{\nu}_e$	46×2	–	30	–	–	–
$e^+e^- \to \nu_e\bar{\nu}_e$	112×3	31	31	31	–	32
$e^+e^- \to t\bar{t}$	150×4	32	31	31	31	31
$e^+e^- \to e^+e^-$	288×4	32	30	30	31	31
$e^+e^- \to W^+W^-$	334×4	27	27	30	31	–
$e^+e^- \to Z^0Z^0$	336×3	33	29	31	31	–
$e^+e^- \to H^0Z^0$	341×3	30	30	31	31	30
$\mu\bar{\nu}_\mu \to W^-\gamma$	162×3	27	27	28	–	28
$\mu\bar{\nu}_\mu \to W^-Z^0$	213×4	31	29	30	–	30
$\mu\bar{\nu}_\mu \to W^-H^0$	196×3	30	29	31	31	31
$t\bar{b} \to W^+\gamma$	239×4	22	25	29	–	29
$t\bar{b} \to W^+Z^0$	284×4	31	22	31	–	32
$t\bar{b} \to W^+H^0$	285×4	29	28	21	26	30
$\gamma\gamma \to t\bar{t}$	267×2	24	34	30	–	–
$Z^0Z^0 \to t\bar{t}$	338×3	30	29	31	31	–
$W^+W^- \to t\bar{t}$	354×4	30	26	31	31	–
$Z^0H^0 \to t\bar{t}$	355×4	30	28	29	29	31
$\gamma\gamma \to W^+W^-$	619×5	22	24	32	–	31
$Z^0Z^0 \to Z^0Z^0$	657×3	–	24	31	31	–
$Z^0\gamma \to W^+W^-$	680×5	28	28	31	–	31
$Z^0W^- \to Z^0W^-$	840×6	26	24	29	30	29
$W^+W^- \to W^+W^-$	925×7	27	26	30	31	–
$Z^0H^0 \to W^+W^-$	823×5	29	25	29	26	31
$Z^0H^0 \to Z^0H^0$	830×6	–	23	24	20	31
$H^0W^- \to H^0W^-$	827×6	29	23	22	23	30
$H^0H^0 \to H^0H^0$	805×4	–	–	29	27	–

Comparison with other calculations

$e^+e^- \to ZH$		GRACE-LOOP $\sigma/\sigma_{tree} - 1$	Denner, et al. ZPhys. C56 $\sigma/\sigma_{tree} - 1$
$\sqrt{s} = 500$GeV	$M_H = 100$GeV	4.15239	4.1524
$\sqrt{s} = 500$GeV	$M_H = 300$GeV	6.90166	6.9017
$\sqrt{s} = 1000$GeV	$M_H = 100$GeV	-2.16561	-2.1656
$\sqrt{s} = 1000$GeV	$M_H = 300$GeV	-2.49949	-2.4995
$\sqrt{s} = 1000$GeV	$M_H = 800$GeV	26.10942	26.1094
$\sqrt{s} = 2000$GeV	$M_H = 100$GeV	-11.54131	-11.5414
$\sqrt{s} = 2000$GeV	$M_H = 300$GeV	-12.82256	-12.8226
$\sqrt{s} = 2000$GeV	$M_H = 800$GeV	11.24680	11.2468

$m_H = 100\text{GeV}, \quad k_c = .05\sqrt{s}$

$W^+W^- \to W^+W^-$		GRACE-LOOP	Denner and Hahn **NPB525** (1998)
2 TeV	tree-level(pb)	77.17067	77.17067
	$\delta(\%)$	-21.0135	-21.0135
5 TeV	tree-level(pb)	14.2443	14.2443
	$\delta(\%)$	-57.1567	-57.1567
10 TeV	tree-level(pb)	3.644573	3.644573
	$\delta(\%)$	-93.9942	-93.9942

$e^+e^- \to t\bar{t}$	GRACE-LOOP	Fleischer, et al. hep-ph/0302259
$\sqrt{s} = 500$GeV		
tree-level (pb)	0.5122751	0.5122744
$\mathcal{O}(\alpha)$	0.526371	0.526337
δ(in%)	2.75163	2.74513
$\sqrt{s} = 1$TeV		
tree-level(pb)	0.1559187	0.1559185
$\mathcal{O}(\alpha)$	0.171931	0.171916
δ(in%)	10.2696	10.2602

$e^+e^- \to \nu\bar{\nu}h$ at one-loop

Number of diagrams

- **12** tree-level diagrams
- **1350** one-loop diagrams
  Among these **98** 5-point functions

Neglecting electron-scalar couplings

- **2** tree-level diagrams: fusion +higgstrahlung
- **249** one-loop diagrams
  Among these **5** 5-point functions

Keep full set of diagrams for checks at one point in phase space
Results for total cross-section based on 249 diagrams (keeping m_e to handle collinear singularities)

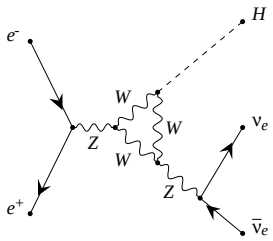
Define s-channel

$$\sigma_s = 3\sigma(e^+e^- \to \nu_\mu\bar{\nu}_\mu h)$$

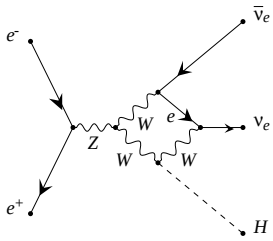
$$\sigma_t = \sigma_{TOTAL} - \sigma_s$$

Selection of loop diagrams

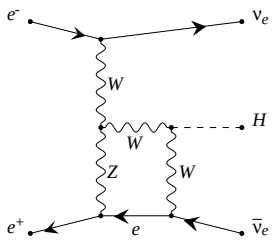
Graph 87



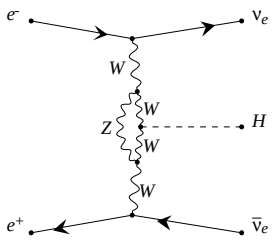
Graph 249



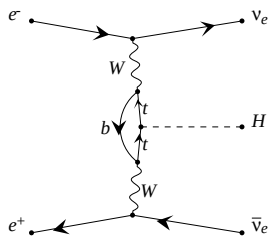
Graph 486



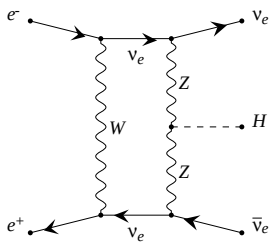
Graph 541



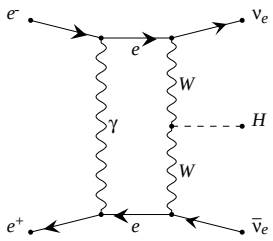
Graph 565



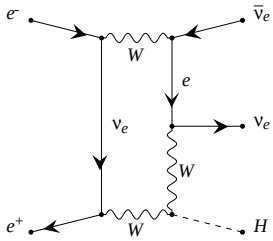
Graph 827



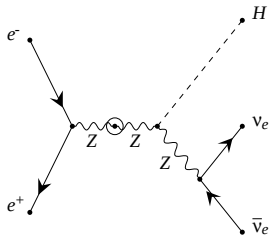
Graph 828



Graph 846



Graph 1312



produced by GRACEFIG

Independence on NLG parameters

	# graphs	sum_3	sum_2	sum_1
$\tilde{\alpha}$	149	–	10^{-28}	10^{-30}
$\tilde{\beta}$	314	–	10^{-31}	10^{-23}
$\tilde{\delta}$	477	10^{-20}	10^{-20}	10^{-26}
$\tilde{\kappa}$	122	–	10^{-23}	10^{-23}
$\tilde{\epsilon}$	128	–	10^{-21}	10^{-30}

Additional check on total cross-section

- Hard bremstrahlung : $e^+e^- \to \nu\bar{\nu}h\gamma$ computed automatically
  Integration over phase space done with BASES, judicious choice of integration variable so MC picks up collinear mass singularity
  Check k_c independence, $k_c = .001 \to .1$, stability within precison of MonteCarlo (4 digits)

Results

Input parameters:
$\alpha^{-1} = 137.0359895$
$M_Z = 91.1876\text{GeV}.$
$m_e = 0.510999\text{MeV}$,$m_\mu = 105.6584\text{MeV}$, $m_\tau = 1.777\text{GeV}$
$M_t = 174\text{GeV}$
$M_u = M_d = 58\text{MeV}$, $M_s = 92\text{MeV}$
$M_c = 1.5\text{GeV}$, $M_b = 4.7\text{GeV}$

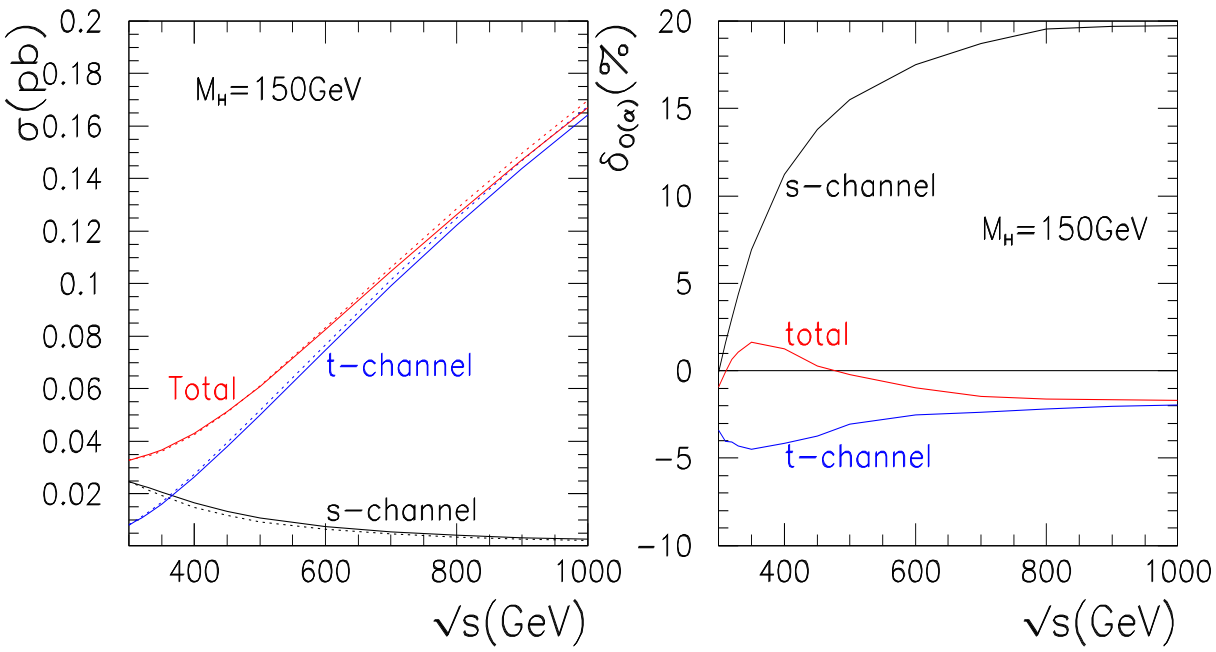
constant fixed Z-width, $\Gamma_Z = 2.4952\text{GeV}$
(required for Higgs-strahlung processes)

Our numerical value of M_W is derived through Δr even though on-shell renormalisation program uses M_W as input parameter,

$$M_W = M_W(M_H)$$

In Δr, we include NLO QCD corrections and two-loop Higgs effects.
We take $\alpha_s(M_Z^2) = .118$, $G_\mu = 1.16639 \times 10^{-5}\text{GeV}^{-2}$)
$\quad \rightarrow \quad \Delta\alpha(M_Z) = .059258$
$M_W = 80.3767\text{GeV}$ for $M_H = 150\text{GeV}$
$M_W = 80.3158\text{GeV}$ for $M_H = 350\text{GeV}.$

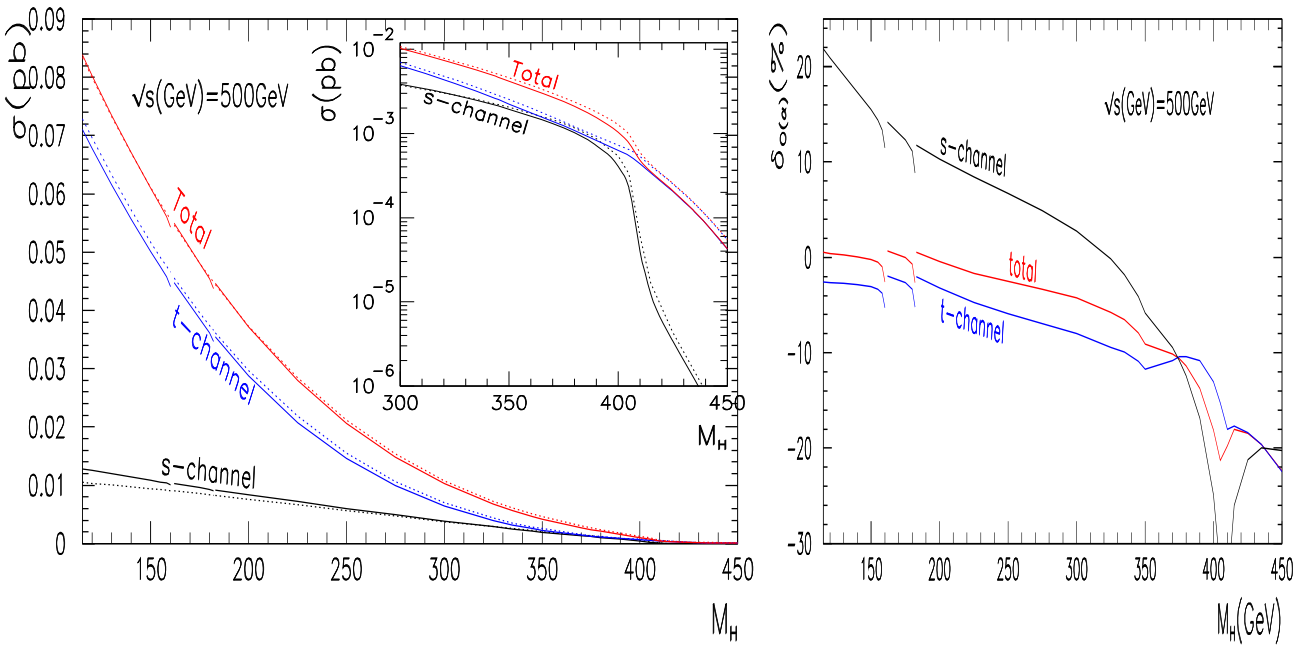
$M_H = 150\text{GeV}$



$$\delta = \frac{\sigma_{\mathcal{O}(\alpha)}}{\sigma_{tree}} - 1$$

- Recall s-channel $\sigma_s = 3\sigma_{\nu_\mu \bar{\nu}_\mu h}$ includes diagrams beyond $e^+e^- \to Zh$
- s-channel: large positive correction mostly virtual+real QED corrections
- Large corrections when Higgstrahlung contribution is small
- Total correction small, lowest around 500GeV, $\delta \approx .2\%$

$\sqrt{s} = 500\text{GeV}$



- spikes at $M_H = 2M_W, 2M_Z$
- Large QED corrections in s-channel

Genuine weak corrections

Subtract QED corrections

s-channel:
correction to $eeZ+$ soft bremstrahlung, universal soft photon factor.

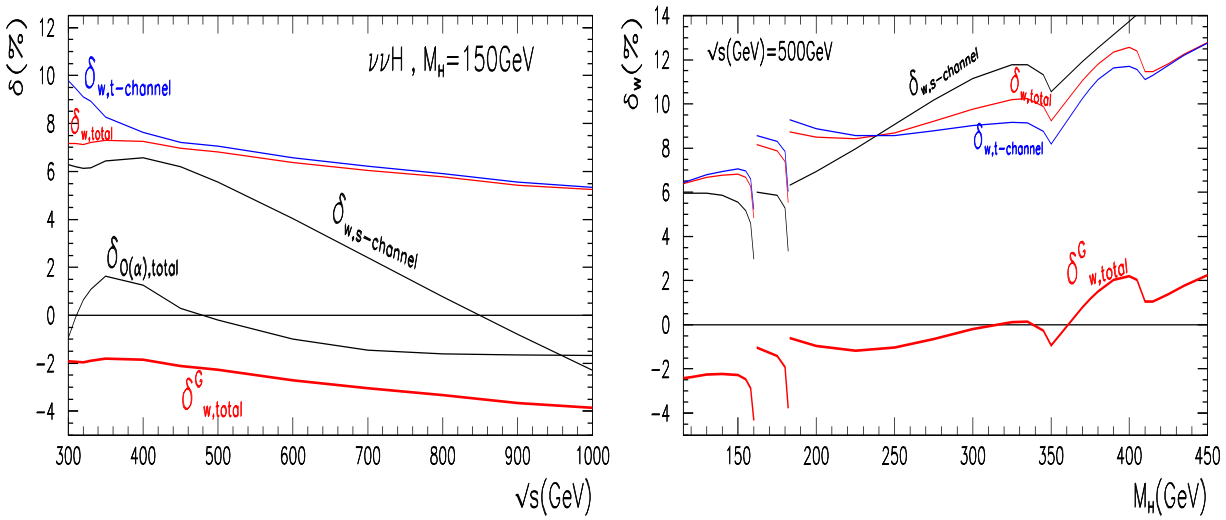
$$\delta_{V+S}^{QED} = \frac{2\alpha}{\pi}\left((L_e - 1)\ln\frac{k_c}{E_b} + \frac{3}{4}L_e + \frac{\pi^2}{6} - 1\right)\ ,\ L_e = \ln(s/m_e^2)$$

E_b the beam energy

Subtracting this contribution from our k_c dependent (numerical) result $\rightarrow$ genuine weak correction
$\rightarrow \delta_{W,s-channel}$

t-channel :
subtract δ_{V+S}^{QED} from full $\mathcal{O}(\alpha)$ since leading (infrared+collinear) contributions are given by universal factor
$\rightarrow \delta_{W,t-channel}$

Large QED corrections require higher order treatment



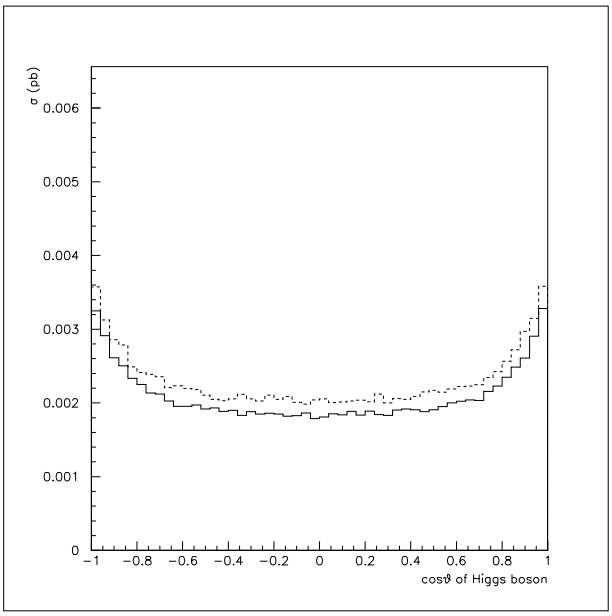
Weak corrections in term of G_μ

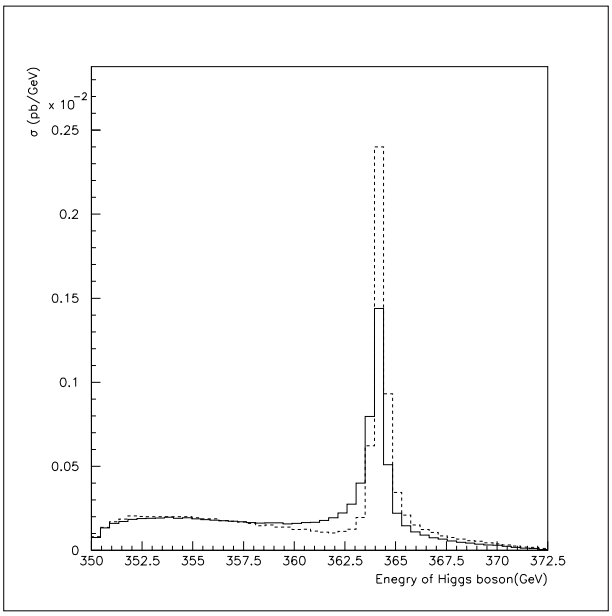
Write tree-level in term of G_μ $\rightarrow$ subtract universal weak corrections (fermionic contribution) to two-point function
Define non-universal weak correction

$$\delta_W^G = \delta_W - 3\Delta r$$

- Note $\Delta r \approx 3\%$
- $\delta_W^G \approx -(2-4)\%$, given the precision in e^+e^- collider must be taken into account
- $\delta_{W,s-channel}^G \approx -12\%$ at 1TeV , similar to $e^+e^- \rightarrow Zh$

Distributions





- Angular distribution $\cos\theta_H$
- Energy distribution E_H

Comparison with other result

M_H [GeV]	M_W [GeV]	σ_{tree} [fb]	σ [fb]	$\sigma/\sigma_{tree} - 1$ [%]	
150	80.3767	61.074(7)	60.98(2)	−0.2	*GRACE*
		61.076(5)	60.80(2)	−0.44(3)	*DDRW*
200	80.3571	37.294(4)	37.16(4)	−0.4	*GRACE*
		37.293(3)	37.09(2)	−0.56(4)	*DDRW*
250	80.3411	21.135(2)	20.63(2)	−2.5	*GRACE*
		21.134(1)	20.60(1)	−2.53(3)	*DDRW*
300	80.3275	10.758(1)	10.30(1)	−4.2	*GRACE*
		10.7552(7)	10.282(4)	−4.40(3)	*DDRW*
350	80.3158	4.6079(5)	4.184(4)	−9.1	*GRACE*
		4.6077(2)	4.181(1)	−9.27(3)	*DDRW*

DDRW: Denner, Dittmaier, Roth, Weber, hep-ph/0302198

Good agreement:
Total corrected cross-section agree within 0.3%.
Corrections relative to the lowest-order cross section agree within 0.2%.

$e^+e^- \to t\bar{t}h$ at one-loop

QCD+EW one-loop corrections

Number of diagrams

- **12** tree-level diagrams
- **2327** one-loop diagrams
  Among these **164** 5-point functions

Neglecting electron-scalar couplings

- **6** tree-level diagrams
- **758** one-loop diagrams
  Among these **29** 5-point functions

Keep full set of diagrams for checks at one point in phase space
Results for total cross-section based on 758 diagrams (keeping m_e to handle collinear singularities)

One-loop QCD corrections:
System passes succesfully the usual checks:
ultraviolet finitenesss, infrared finiteness (regularise with gluon mass).
Note in this process no triple gluon vertex and no NLG tests

Preliminary results show full agreement with

S. Dawson, L. Reina, hep-ph/9808443 (only photon exchange contribution)

S. Dittmaier, M. Kramer, Y. Liao, M. Spira, P. Zerwas, hep-ph/9808433

$\sqrt{s}$	M_H	Tree(fb)	NLO/QCD(fb)	K	DKLSZ
500 GeV	100	.975649	1.319	1.35	1.35
$\alpha_s = .09579063$	140	.036965	.07498	2.03	2.01
	100	2.30979	2.196	0.95	0.95
1 TeV	140	1.56222	1.4906	0.95	0.95
$\alpha_s = .08902483$	180	1.08686	1.0423	0.96	0.96
	100	.810497	.7345	0.90	0.90
2 TeV	140	.639267	.57597	0.90	0.90
$\alpha_s = .083161506$	180	.525385	.46993	0.89	0.89

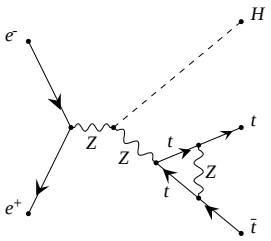
For EW part:

System passes succesfully the usual checks:

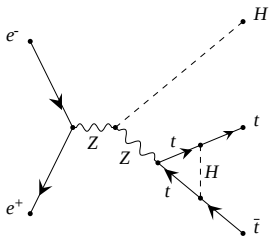
- ultraviolet finiteness (better than 20 digits)
- infrared finitenesss (better than 20 digits)
- NLG dependence (better than 20 digits)
- k_c dependence consistent with MonteCarlo statistical error (0.02%)

Selection of loop diagrams

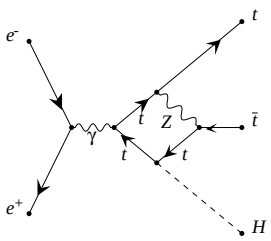
Graph 22



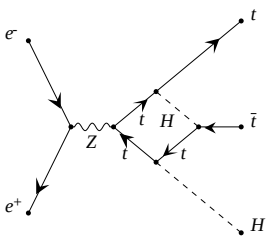
Graph 23



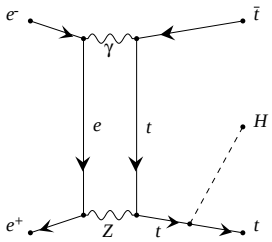
Graph 780



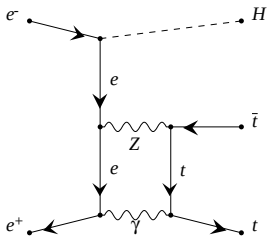
Graph 801



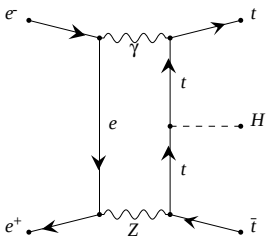
Graph 1586



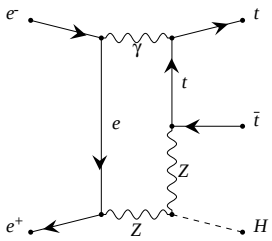
Graph 1202



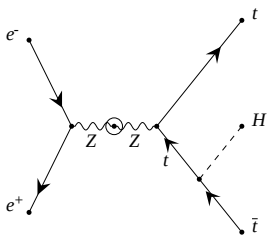
Graph 1566



Graph 1546

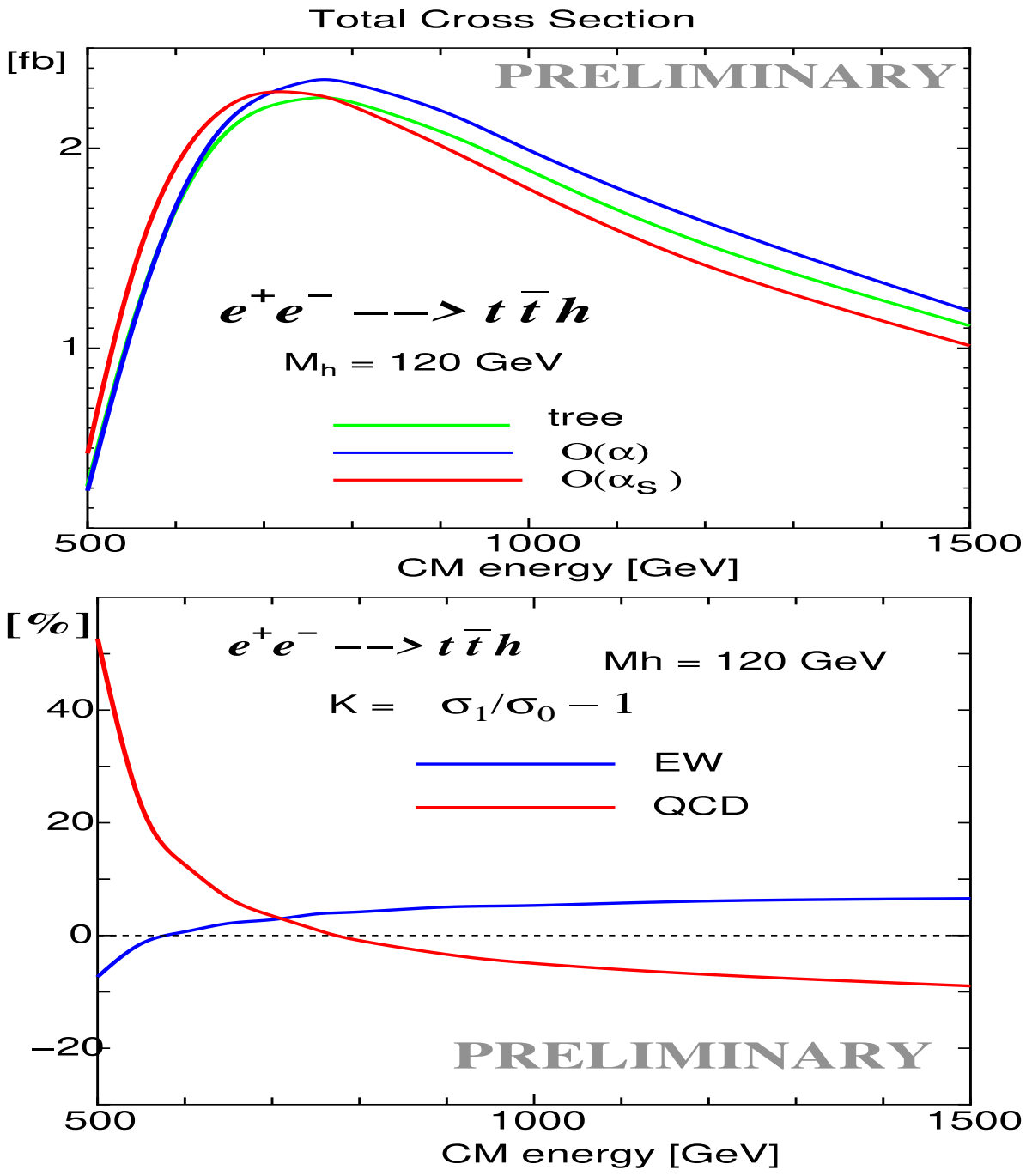


Graph 2284



produced by GRACEFIG

$M_H = 120\text{GeV}$

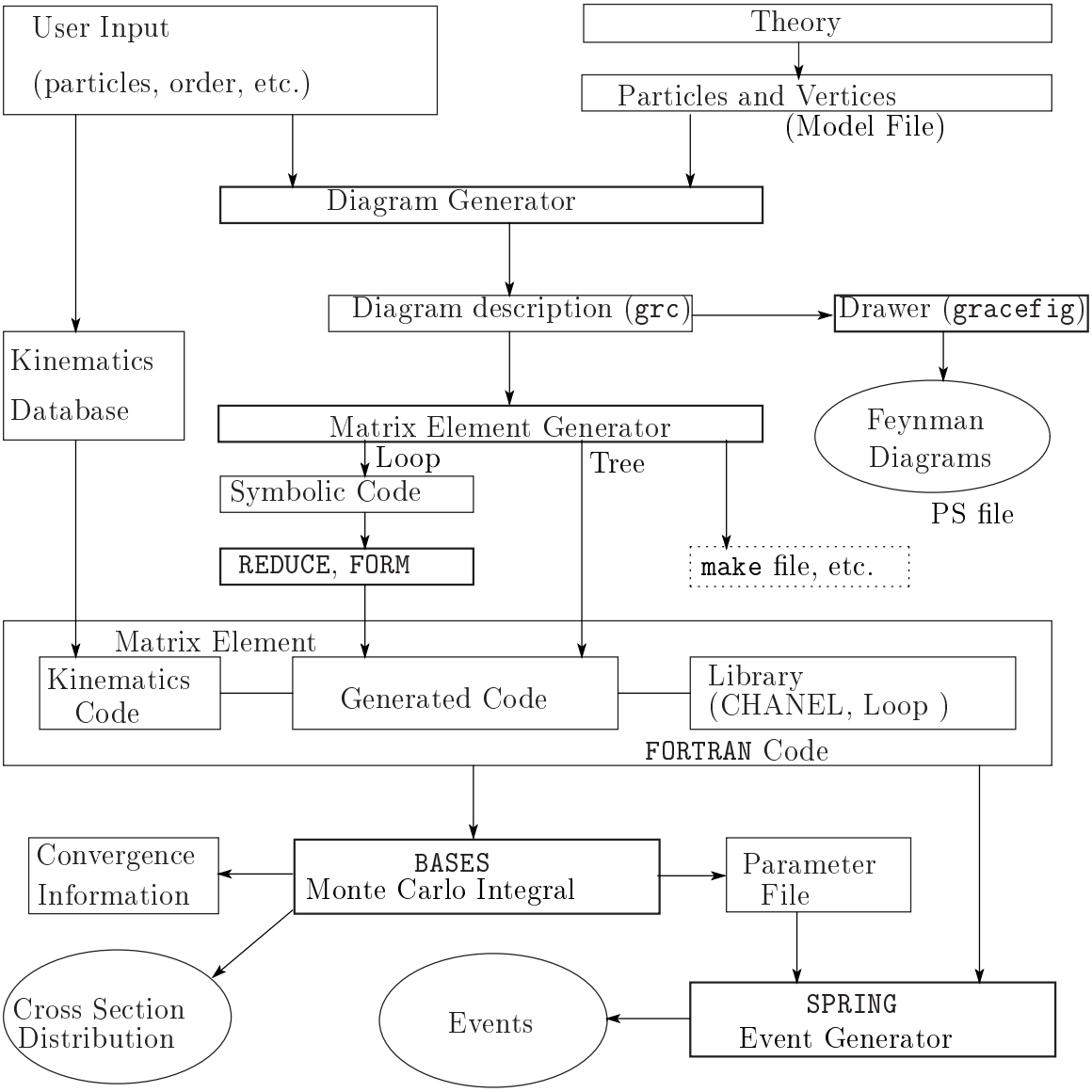


Summary

- Full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to \nu\bar{\nu}h$ with Grace/Loop
- Consistency of results checked with NLG fixing (full 5 parameters)
- Result in agreement with Denner et al.
- For $M_H = 150$GeV modest $\delta_{\mathcal{O}(\alpha)} = \pm 2\%$ (very small at 500GeV). Genuine weak corrections for $M_H = 150$GeV, range 5-7%
- For heavier Higgs, corrections can be larger, $\delta_{\mathcal{O}(\alpha)} = \pm 10\%$, $M_H = 350$GeV
- Need to include multiple photon emission

Conclusion

GRACE/Loop system is working well for $2 \to 2$, $2 \to 3$ processes in the SM.


User Input
(particles, order, etc.)
Theory
Particles and Vertices
(Model File)
Diagram Generator
Diagram description (grc)
Drawer (gracefig)
Kinematics
Database
Matrix Element Generator
Feynman
Diagrams
PS file
Loop
Tree
Symbolic Code
REDUCE, FORM
make file, etc.
Matrix Element
Kinematics
Code
Generated Code
Library
(CHANEL, Loop)
FORTRAN Code
Convergence
Information
BASES
Monte Carlo Integral
Parameter
File
Cross Section
Distribution
Events
SPRING
Event Generator