Drell-Yan rapidity distribution at NNLO in QCD

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- **Outline**

  Motivation: NNLO distributions

  New method for differential distributions

  Results: $pp \to \gamma^* + X$

  Conclusions

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LoopFest II

Precision Physics at the LHC

- Unprecedented amount of data will become available at the LHC
- One year at $\mathcal{L} = 10 fb^{-1}$:

  $W \to e\nu$: 10^8 events

  $Z \to e^+e^-$: 10^7 events

  $t\bar{t}$: 10^7 events

  Higgs ($m_H = 700$ GeV): 10^4 events   F. Gianotti, M. Altarelli

- Statistical errors $< 1\%$; measurements limited by experimental systematics, **theoretical uncertainty**
- Need high precision theoretical calculations to fully utilize experimental results!

Is NNLO necessary?

Necessity of NNLO

- W mass: $\Delta M_W \to 15$ MeV

  $d^2\sigma_W/dY\,dp_T^W$, $d^2\sigma_Z/dY\,dp_T^Z$ needed to very high precision

- Extraction of pdfs and parton-parton luminosity

  W, Z rapidity distributions needed to $< 5\%$ M. Dittmar $et.al.$

$\Rightarrow$ Inclusive production of W, Z known Hamberg $et.al$, but insufficient; differential quantities needed to NNLO

- No complete NNLO QCD correction to a differential quantity exists (see Rijken, van Neerven)

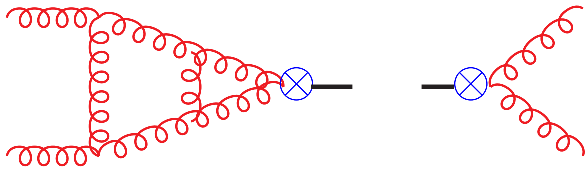
What enters a NNLO calculation?

What is the bottleneck?

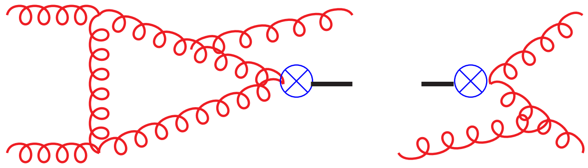
Anatomy of a NNLO calculation

Production of γ^*, H, W, Z at NNLO (schematic):

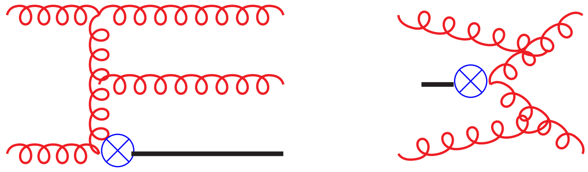
- Double Virtual (V-V)



- Real - Virtual (R-V)



- Double Real (R-R)



- Typically each component is treated differently
  - V-V are reduced to a few independent Master Integrals using recurrence relations satisfied by loop integrals
  - These master integrals then solved using Feynman parameters, Mellin-Barnes, differential equations
  - R-V, R-R components usually treated less systematically; no similar technology developed for their reduction
- Recently a new method proposed for handling these contributions C. Anastasiou, K. Melnikov

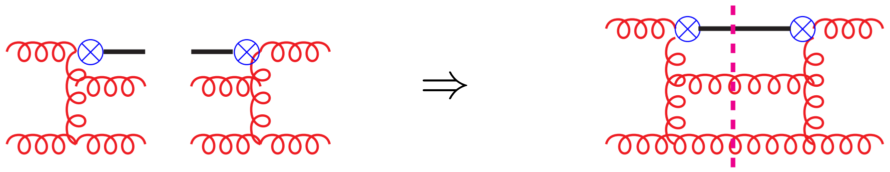
$$\sigma_{\alpha\beta\to 1\ldots n} \propto \int \left[\prod_{i=1}^{n} d^d q_i \delta\left(q_i^2 - m_i^2\right)\right] \delta\left(p_{\alpha\beta} - q_{1\ldots n}\right) \times \left|\mathcal{M}_{\alpha\beta\to 1\ldots n}\right|^2$$

Cutkosky rules: $\delta\left(q_i^2 - m_i^2\right) \Rightarrow \frac{1}{q_i^2 - m_i^2 - i\epsilon} - \frac{1}{q_i^2 - m_i^2 + i\epsilon}$

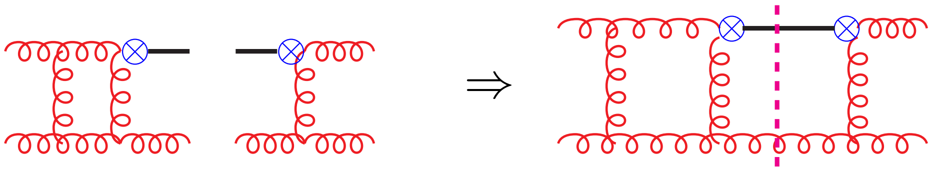
Maps phase space integrals $\Rightarrow$ cut loop integrals

Diagrammatically:

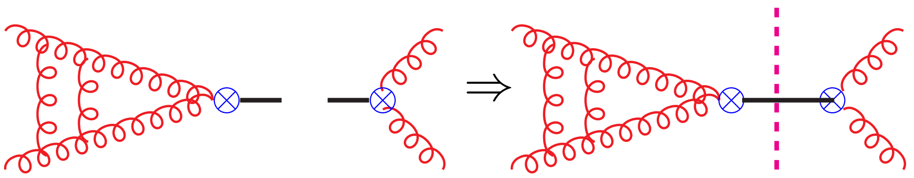
- Double-Real $\Rightarrow$ Triple-Cut



- Real-Virtual $\Rightarrow$ Double-Cut



- Double-Virtual $\Rightarrow$ Single-Cut



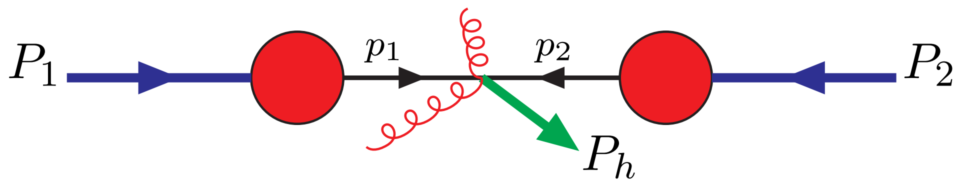
- Can now use the same reduction technology for R-V and R-R as for V-V

- The previous formula was valid for inclusive quantities

Can we extend the method to differential quantities?

NNLO rapidity distributions

$hh \to V$ processes ($h = p, \bar{p}$):



$$P_1 = \tfrac{1}{2}(1, \vec{0}, 1) \qquad p_1 = x_1 P_1$$
$$P_2 = \tfrac{1}{2}(1, \vec{0}, -1) \qquad p_2 = x_2 P_2$$
$$P_h = (E, \vec{p_T}, p_z)$$

Rapidity: $Y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$

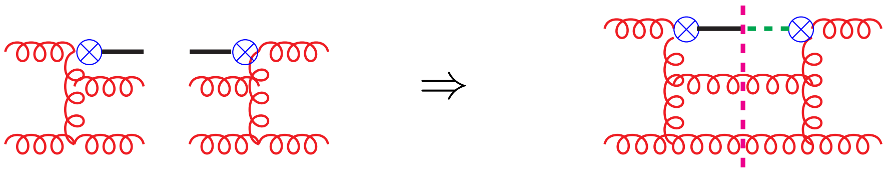
$$u = \frac{x_1}{x_2} e^{2Y}$$

$$\frac{d\sigma}{dY} \propto u \int \left[\prod_{i=1}^{n} d^d q_i \delta\left(q_i^2 - m_i^2\right)\right] \delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right)$$
$$\times \delta\left(p_{\alpha\beta} - q_{1\ldots n}\right) \left|\mathcal{M}_{\alpha\beta \to 1\ldots n}\right|^2$$

Replace the $\delta\left(q_i^2 - m_i^2\right)$ as before; also replace

$$\delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right) \Rightarrow \frac{p_2 \cdot P_h}{(p_1 - u p_2) \cdot P_h - i\epsilon} - (+i\epsilon)$$

Diagrammatically, we have (for the R-R case):



- The cut now also indicates the presence of the rapidity propagator

- Again use the same technology to reduce V-V, R-V, and R-R to a small set of master integrals

$\Rightarrow$ It is clear that this is applicable to many distributions of phenomenological interest: x_F, p_T, doubly-differential quantities, ...

Outline of calculation

(1) Reduction to master integrals

$\Rightarrow\ \sim 10^6$ integrals $\longrightarrow\ \sim 10$ integrals

(2) Calculation of master integrals

(3) Extraction of soft limits

Reduction to master integrals

- Use integration by parts to derive relations among integrals
- Consider example of one-loop bubble:

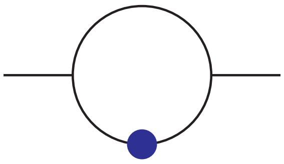
$$I\left[\nu_1, \nu_2\right] = \int d^D k \frac{1}{\left[k^2\right]^{\nu_1}\left[(k+p)^2\right]^{\nu_2}}$$

- Set

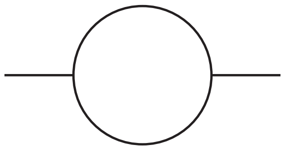
$$\int d^D k \frac{\partial}{\partial k_\mu} \frac{k^\mu}{k^2 (k+p)^2} = 0$$

$\Rightarrow$ Derive

$$I\left[1, 2\right] = -\frac{D-3}{p^2} I\left[1, 1\right]$$



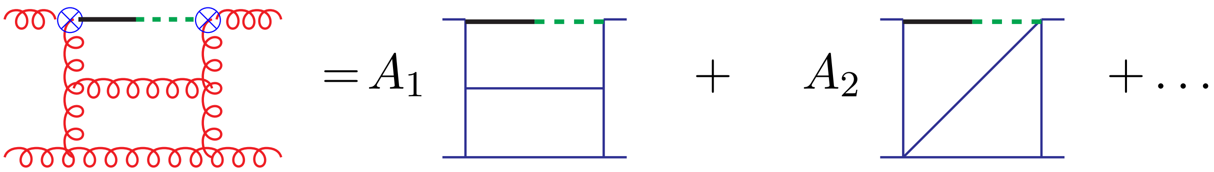
$$= -\frac{D-3}{p^2}$$



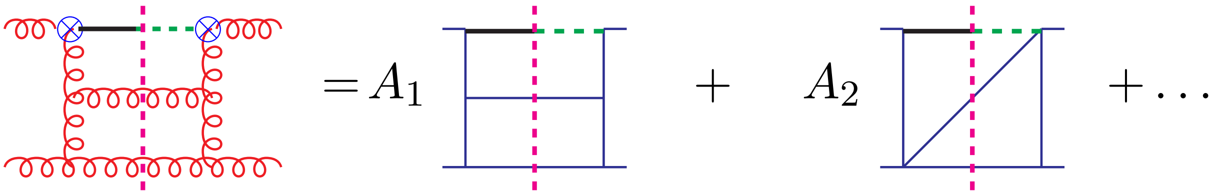
- Reduce integrals to minimal set (master integrals)
- Commutes with taking cuts

  $\Rightarrow$ independent of $\pm i\epsilon$ prescription

- Diagrammatically,
  - Reduce to master integrals with regular propagators



  - Calculate master integrals with the cut present



- System of equations formed by IBP identities can be solved in a completely automated fashion Laporta

$\Rightarrow$ After reductions,
  - 20 master integrals for the R-R contributions
  - 10 master integrals for the V-V and R-V combined

- Same integrals for all processes: γ^*, W, Z, H

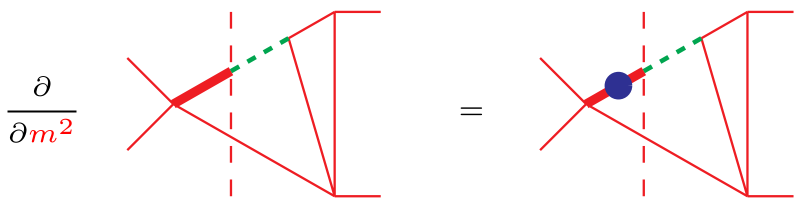
Calculation of master integrals

- For master integrals which can't be directly integrated
  $\Rightarrow$ use differential equations

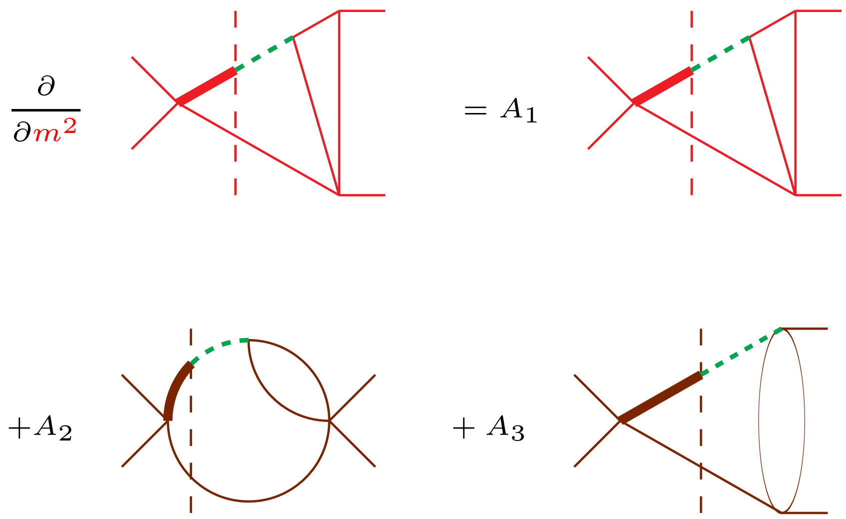
- Two kinematic factors: u, m^2

$$\frac{\partial}{\partial m^2}\left\{\int d^D k \frac{1}{(k^2-m^2)(k+p)^2\cdots}\right\} = \int d^D k \frac{1}{(k^2-m^2)^2(k+p)^2\cdots}$$

- Diagramatically,



- Can reduce r.h.s. using IBPs; introduce cuts to evaluate master integrals



- Obtain a similar diff. eq. for u

- Inhomogeneous term contains simpler boundary integrals; solve for these first

- Solve as series in ϵ

Extraction of soft limits

- Matrix elements contains terms which behave as

$$|\mathcal{M}|^2 \propto \frac{1}{u-z}\,, \quad |\mathcal{M}|^2 \propto \frac{1}{1-uz}$$

with $z = m^2/\hat{s}$

- Produce singularities when integrated over phase space

$\Rightarrow$ extract these explicitly

- Three singular regions of phase space:

(1) $u = z$

(2) $u = 1/z$

(3) $u = z = 1$

- Need master integrals in these kinematic limits to higher orders in ϵ

- Example: $u \to z$ limit:

$$|\mathcal{M}|^2 \sim \frac{A}{u-z} + \ldots$$

- Regulate singularity with ϵ
- Extract singularities through expansion of the form

$$(u-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(u-z) + \frac{1}{[u-z]_+} + \ldots$$

$\Rightarrow$ $(u-z)^{-2\epsilon}$ behavior must come from master integrals

- Near $u = z$, can write

$$MI = (u-z)^{-1-2\epsilon} f(u,z)$$

$$f(u,z) = c_1(z) + c_2(z)(u-z) + \ldots$$

- Master integral now contains the term

$$MI \sim \frac{1}{\epsilon}\delta(u-z)\, c_1(z) + \ldots$$

$\Rightarrow$ Need the function $c_1(z)$ to 1 higher order in ϵ

- Similar situation in $u = 1/z$ limit; need 2 higher orders in ϵ for $u = z = 1$, but here $c_1 = c_1(1)$ is a pure number

Status

- All reductions completed
- All master integrals calculated

- First application: $pp \to \gamma^* + X$
  - Components combined, singularities cancelled, analytic expressions obtained for $q\bar{q}$ intitial state

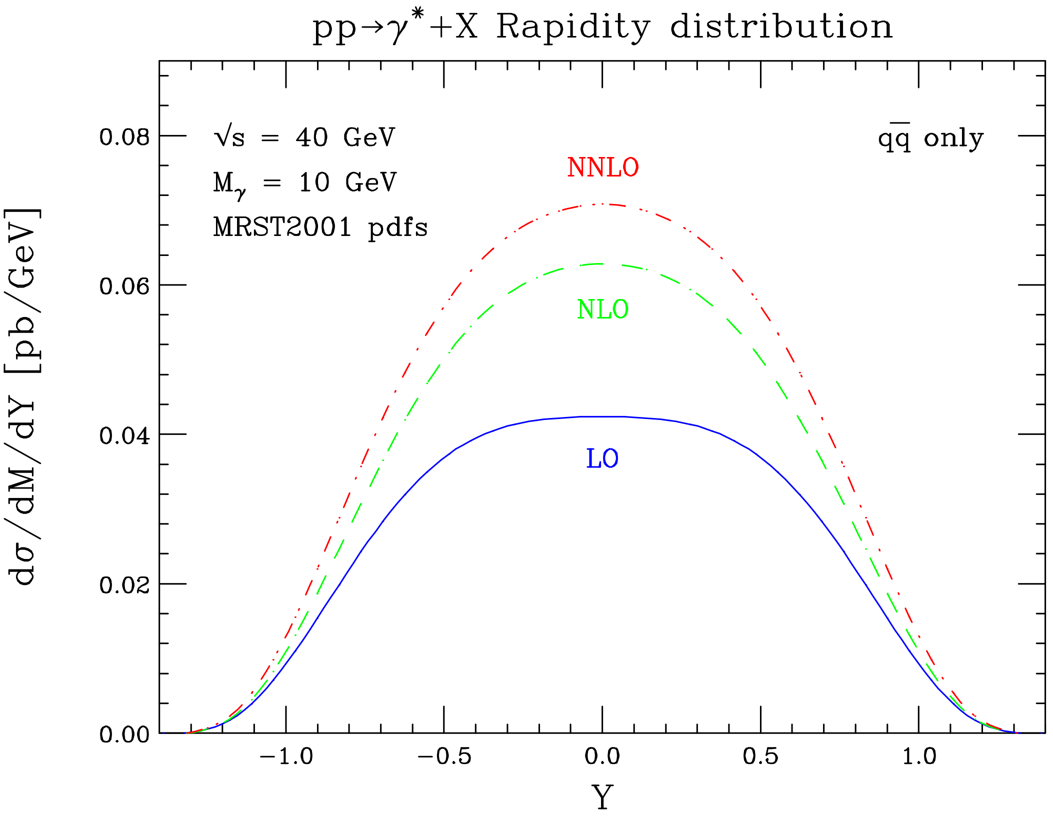
  $\Rightarrow$ Too large to include here

  - Non-trivial test of the method NLO: Altarelli $et.al.$, 1979
  - For $m_{\gamma^*} = 4-16$ GeV, relevant for the fixed target experiment E866 at FNAL: measures

    proton's $\bar{d}(x)/\bar{u}(x)$

    $x \to 1$ q_{val} distributions

Numerics (**Preliminary!**)



- Use MRST approximate NNLO pdfs
- NNLO corrections increase NLO result by $\approx 10-20\%$

$$K_{NNLO}(Y) = \frac{d\sigma_{NNLO}/dY}{d\sigma_{LO}/dY}$$

$$K_{NNLO}(0) \approx 1.67, \quad K_{NNLO}(1) \approx 1.40$$

$$K'_{NNLO}(Y) = \frac{d\sigma_{NNLO}/dY}{d\sigma_{NLO}/dY}$$

$$K'_{NNLO}(0) \approx 1.13, \quad K'_{NNLO}(1) \approx 1.19$$

Only $q\bar{q}$ initial state computed: remainder due soon ...

Conclusions

- Have described a new method for the calculation of differential distributions at higher orders in perturbation theory

- Map phase space constraints $\Rightarrow$ effective propagators; can then treat phase space integrals and multi-loop integrals identically

- Applied to $pp \to \gamma^* + X$; preliminary results indicate a non-negligible NNLO rapidity dependence

- Coming soon: W, Z, H rapidity distributions