SUSY loop corrections to scalar lepton production

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1. Introduction: Gauge and Yukawa couplings

2. SUSY couplings in the electroweak sector

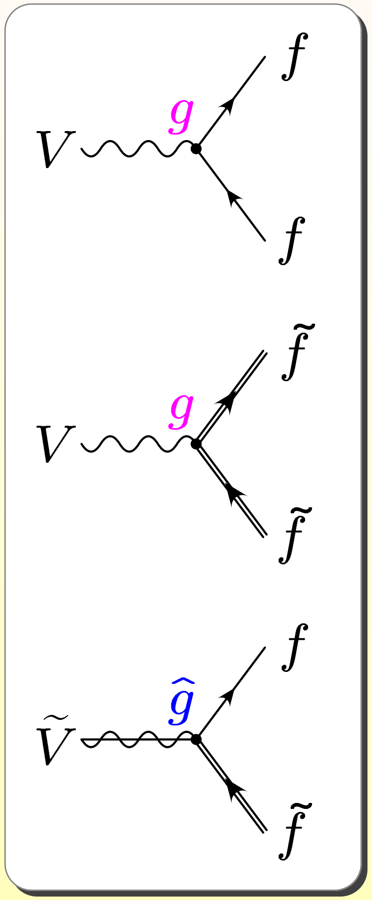
3. NLO corrections to selectron and smuon pair production

1. Introduction: Gauge and Yukawa couplings

Fundamental relation in supersymmetry:

Gauge coupling g = Yukawa coupling $\hat{g}$

$\rightarrow$ required to resolve hierarchy problem



Establish SUSY experimentally:

- Find new particles
- Measure their spin, masses, ...
- **Test SUSY coupling relations**
  $\rightarrow$ compare precise cross-section measurements
  with theoretical predictions

Precision measurements: ideal at high-energy e^+e^- collider

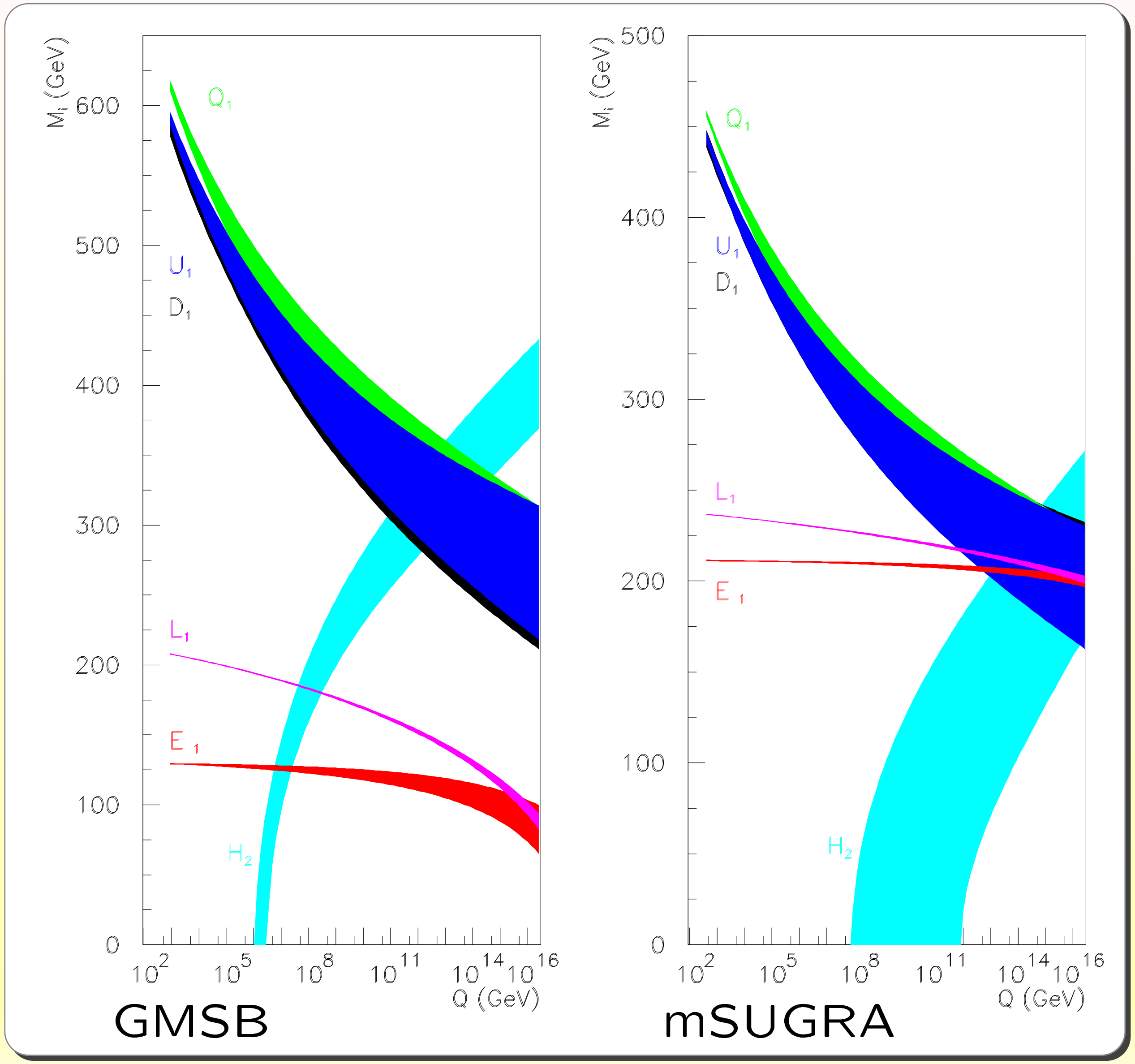
Reconstruction of particle spectrum at higher scales

Blair, Porod, Zerwas '00

Extrapolation by RGE evolution

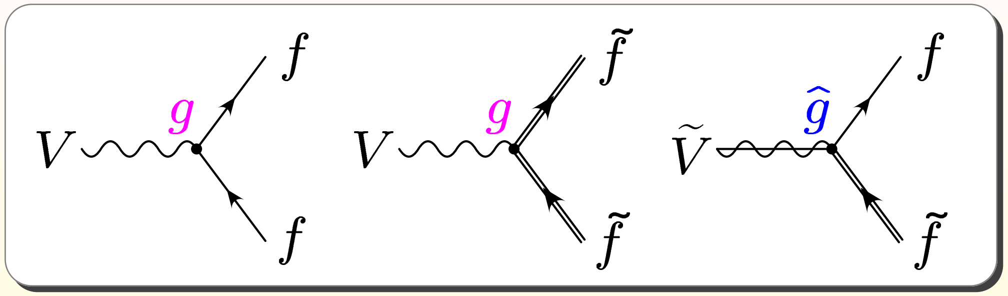
Discrimition between breaking schemes is possible

but requires **high accuracy** of low-energy parameters



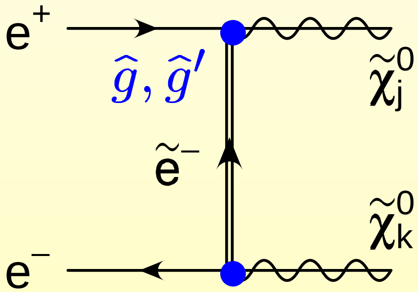
2. SUSY couplings in the electroweak sector

Electroweak gauge & Yukawa couplings can be probed in



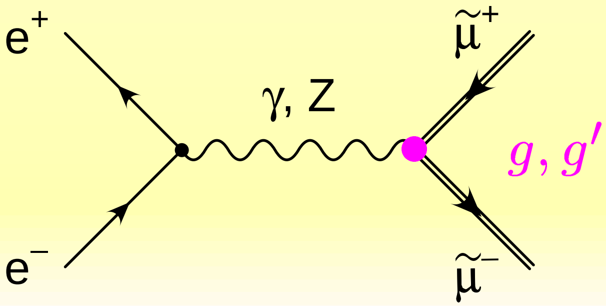
- Neutralino production

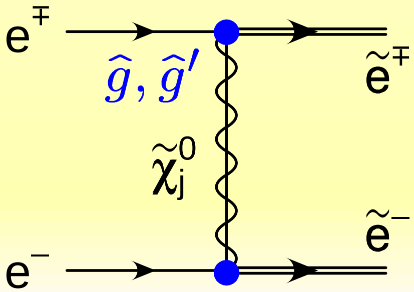
Choi, Kalinowski, Moortgat-Pick, Zerwas '01



- Slepton production

A.F., v.Manteuffel '02





g' U(1) coupl.

g SU(2) coupl.

Masses and decay characteristics

Use different decay modes
to disentangle $\tilde{l}_\mathrm{R}, \tilde{l}_\mathrm{L}$

$\tilde{l}_\mathrm{R} \to l^- \, \tilde{\chi}^0_1$ $\qquad \Gamma_{\tilde{l}_\mathrm{R}} = 210$ MeV

$\tilde{l}_\mathrm{L} \to l^- \, \tilde{\chi}^0_2$ $\qquad \Gamma_{\tilde{l}_\mathrm{L}} = 250$ MeV

$\quad \hookrightarrow \tau^+ \tau^- \, \tilde{\chi}^0_1$

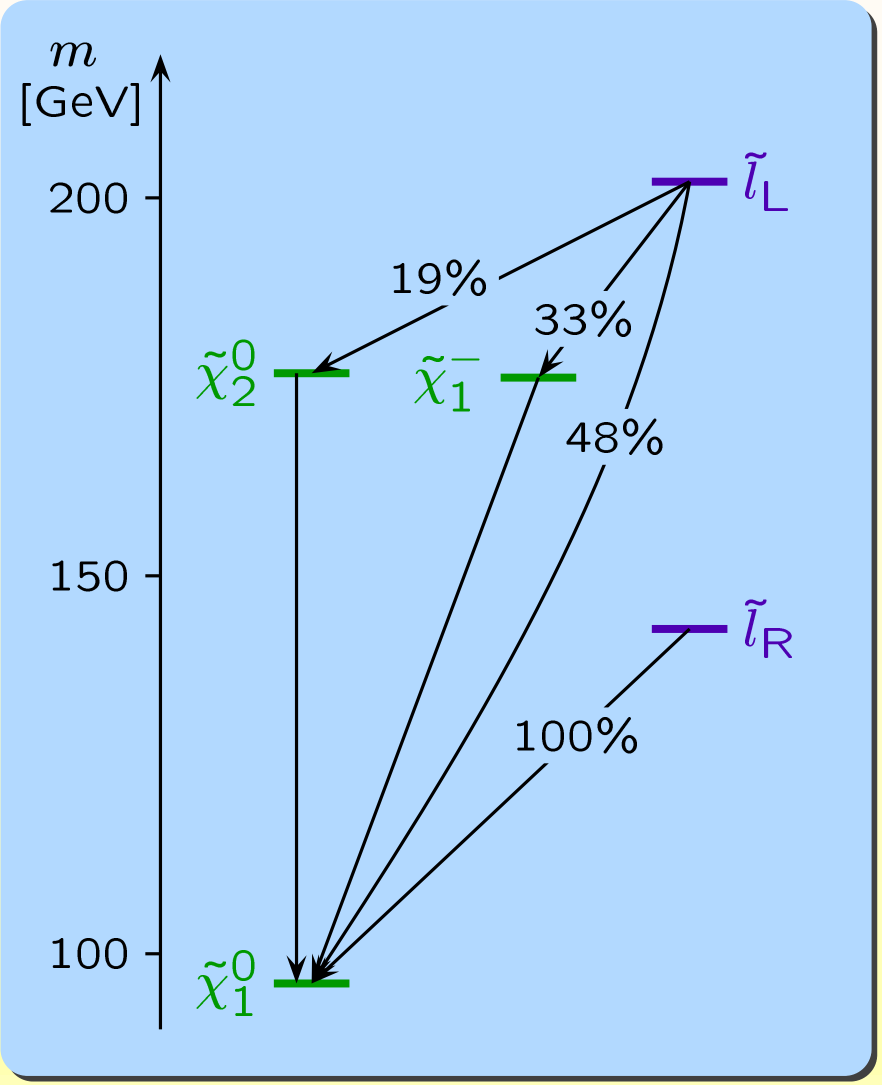
$\to \nu_l \, \tilde{\chi}^-_1$

$\quad \hookrightarrow \tau^- \nu_\tau \, \tilde{\chi}^0_1$

very clean signature:

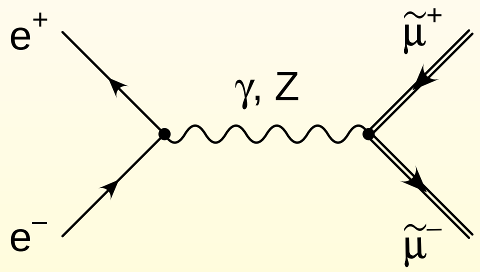
few leptons $+ \not{E}$

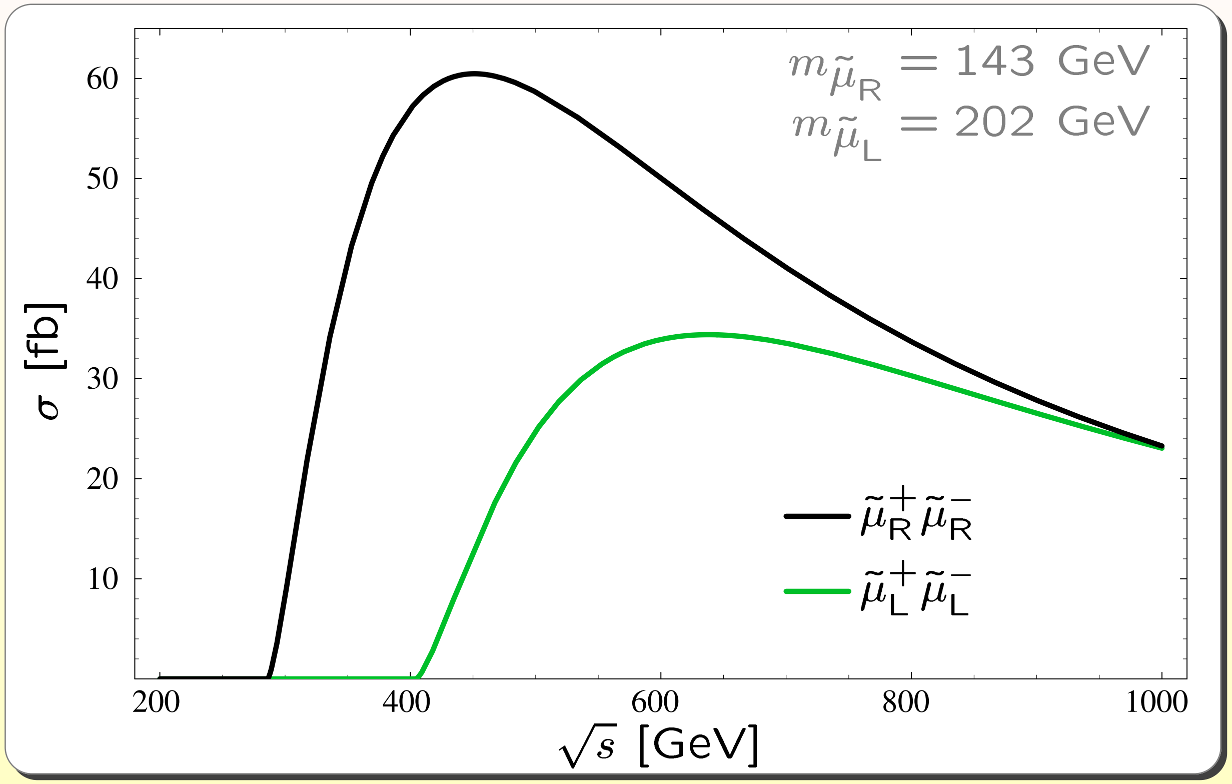
SPS1 scenario



Smuon production

SPS1 scenario





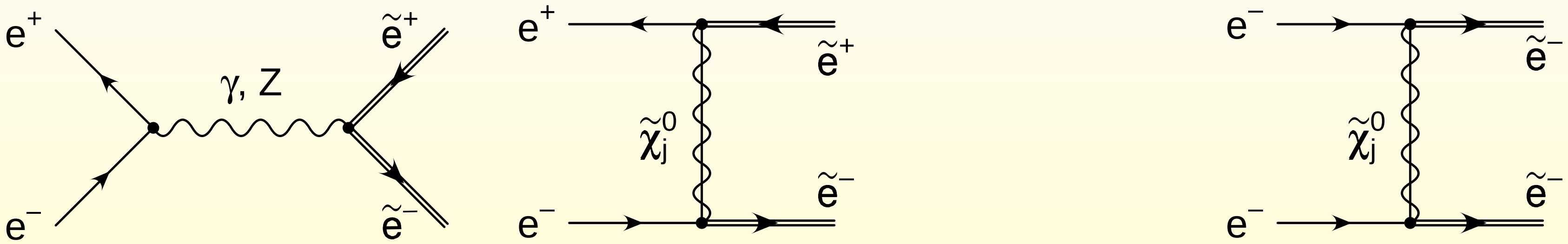
$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

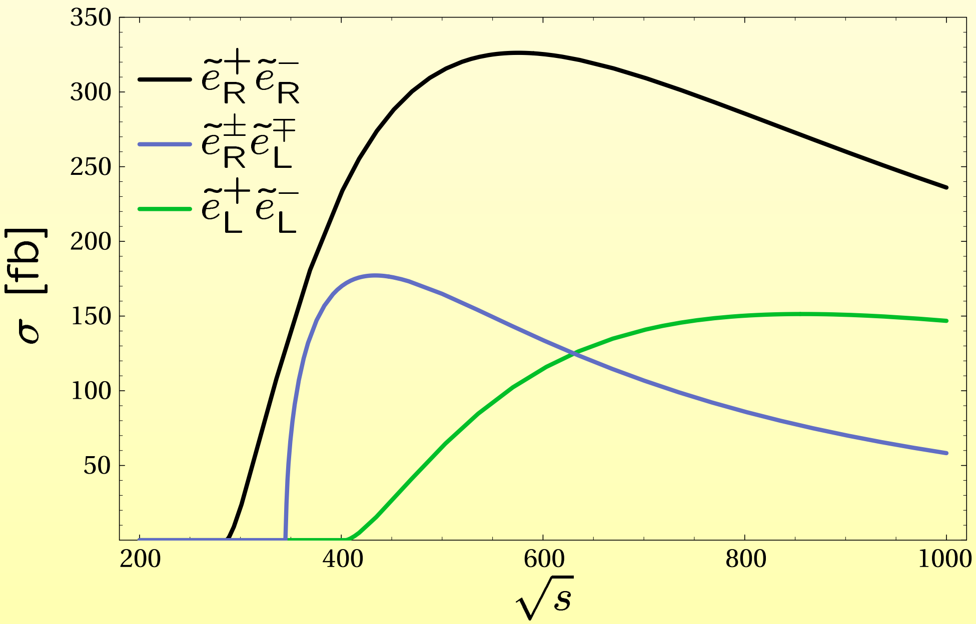
$$\sigma[e^+e^- \to \tilde{\mu}_i^+\tilde{\mu}_i^-]$$

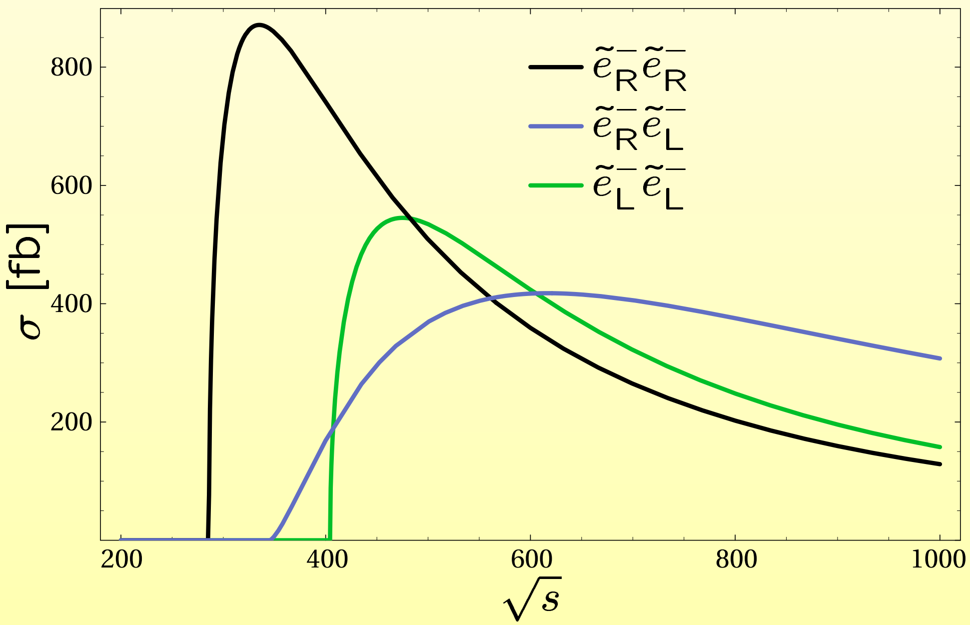
$$= \frac{\pi\alpha^2}{3s}\beta^3\left[1 - g_i\,\frac{1-4s_W^2}{s_W c_W}\,\frac{s}{s-M_Z^2} + g_i^2\,\frac{1+(1-4s_W^2)^2}{16 s_W^2 c_W^2}\left(\frac{s}{s-M_Z^2}\right)^2\right]$$

Selectron production

Additional t-channel neutralino exchange







Determination of SUSY couplings

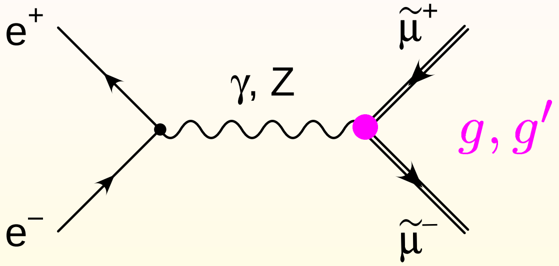
From cross-sections of

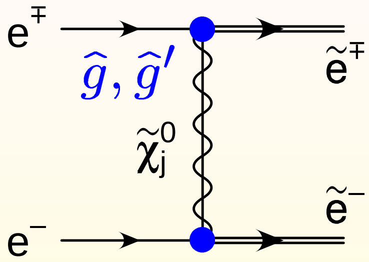
$e^+e^- \to \tilde{\mu}^+_R \tilde{\mu}^-_R \to \mu^+\mu^- \tilde{\chi}^0_1 \tilde{\chi}^0_1$

$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_R \to e^+e^- \tilde{\chi}^0_1 \tilde{\chi}^0_1$

$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_L \to e^+e^- \tilde{\chi}^0_1 \tilde{\chi}^0_2$

$\hookrightarrow \tau^+\tau^- \tilde{\chi}^0_1$



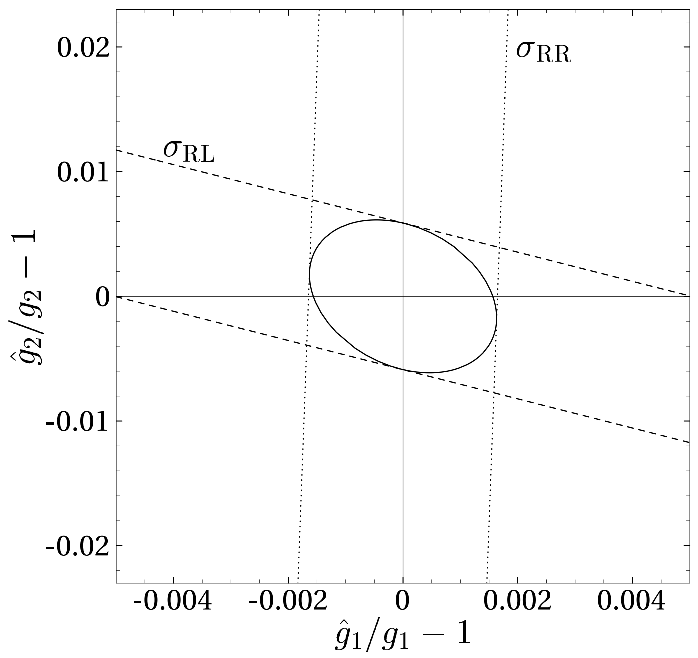


- Backgrounds from SM and SUSY, reduce with cuts and beam pol.
- $P(e^-) = 80\%$ $\quad P(e^+) = 50\%$
- Dependence on other MSSM parameters

Tesla TDR '01

$\frac{\delta g'}{g'} \approx 1\%$ $\qquad \frac{\delta \hat{g}'}{\hat{g}'} \approx 0.2\%$ $\qquad \frac{\delta \hat{g}}{\hat{g}} \approx 0.7\%$

$\sqrt{s} = 500$ GeV, $\int L = 500$ fb^{-1}



3. NLO corrections to $\tilde{e}$ and $\tilde{\mu}$ pair production

- SUSY couplings can be extracted at LC with accuracy of $\lesssim \mathbf{1\%}$
  $\rightarrow$ Radiative corrections necessary

$\mathcal{O}(\alpha)$ corrections to slepton production

$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$, $e^\pm e^- \rightarrow \tilde{e}^\pm\tilde{e}^-$ A.F., v.Manteuffel '02

- **On-shell renormalization** for sparticle masses and mixings
  $\rightarrow$ J. Guasch

- **Dimensional reduction**
  preserves SUSY and gauge symmetries to 1-loop

- $\overline{\text{MS}}$ scheme for renomalization of $\tan\beta$

Neutralino/chargino renormalization

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix} \qquad Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

$\rightarrow$ 3 parameters (M_1, M_2, μ) and 6 physical particles ($\tilde{\chi}^\pm_{1,2}$, $\tilde{\chi}^0_{1\ldots4}$)

1. On-shell conditions for all particles
2. Determine counterterms for M_1, M_2, μ from on-shell conditions for e.g. $\tilde{\chi}^\pm_{1,2}$, $\tilde{\chi}^0_1$
3. Calculate other mass counterterms ($\tilde{\chi}^0_{2,3,4}$) $\rightarrow$ shift in $m_{\tilde{\chi}^0_{2,3,4}}$ predicted

Pierce Papadopoulos '94

Eberl, Majerotto, Kincel, Yamada '01

Fritzsche,Hollik '02

Renormalization of $\tan\beta$

Need to define physically consistent and generally applicable scheme for $\tan\beta$ in higher orders.

Desirable properties of renormalization schemes:

- Gauge independence
- Numerical stability
- Process independence

Automatically fulfilled for masses and couplings in **on-shell scheme** (in weakly coupled theories)
$\rightarrow$ relation to physical quantities

Gauge dependence of known schemes

- $A^0 Z$ unmixing — Dabelstein '95, Chankowski et al. '94

  $$\hat{\Sigma}_{A^0 Z}(M_A^2) \stackrel{!}{=} 0 \qquad \Rightarrow \qquad \frac{\delta \tan\beta}{\tan\beta} = \frac{1}{M_Z \sin 2\beta} \Sigma_{A^0 Z}(M_A^2)$$

  Equivalently for $H^+ W^+$ unmixing — Wan, Ma, Zhang, Yi '01

- $\overline{\mathrm{DR}}/\overline{\mathrm{MS}}$ scheme

  $$\delta \tan\beta^{\mathrm{fin}} \stackrel{!}{=} 0$$

In contrast to on-shell scheme no relation to physical observables
$\rightarrow$ Problem of gauge-dependence

Determination of dependence on gauge parameter ξ:

Use extended Slavnov-Taylor identities

$\tilde{S}(\Gamma) = S(\Gamma) + \chi \partial_\xi \Gamma$ $\qquad$ $\chi =$ BRS transformation of ξ

$\boxed{\tilde{S}(\Gamma) = 0 \quad \Rightarrow \text{ value of } \tan\beta \ \xi\text{-independent}}$

For $\overline{\text{DR}}$ scheme: $\quad$ $\tilde{S}(\Gamma) = 0$ predicts $\partial_\xi \delta \tan\beta \neq 0$

in contrast to $\overline{\text{DR}}$ condition $\delta \tan\beta^{\text{fin}} \stackrel{!}{=} 0$

$\Rightarrow$ $\overline{\text{DR}}$ scheme violates extended STI

Similar for $A^0 Z$ unmixing

$\boxed{\text{Definition of } \tan\beta \text{ in both schemes is gauge-dependent!}}$

Special case: $\overline{\text{DR}}$ scheme is indep. of R_ξ gauge at 1-loop

Construct gauge-independent prescriptions

- Non-renormalization of tadpoles $\Rightarrow v_{1,2}$ gauge indep.

$$\delta \tan\beta^{\mathrm{fin}} = \mathrm{const.} \left(\frac{\delta t_1}{v_1} + \frac{\delta t_2}{v_2}\right)^{\mathrm{fin}}$$

- Use gauge independence of Lagrangian parameters

e.g. $V_{\mathrm{Higgs}} \supset m_3^2(H_1^1 H_2^2 - H_1^2 H_2^1) + \mathrm{h.c.}:$ $\quad \delta m_3^{\mathrm{fin}} \stackrel{!}{=} 0$

- Relation to physical observables, e.g. Higgs masses:

$$\cos^2 2\beta \stackrel{!}{=} \frac{M_{\mathrm{h}^0}^2 M_{\mathrm{H}^0}^2}{M_{\mathrm{A}^0}^2 (M_{\mathrm{h}^0}^2 + M_{\mathrm{H}^0}^2 - M_{\mathrm{A}^0}^2)}$$

$\rightarrow$ all respect $\tilde{S}(\Gamma) = 0$

Numerical properties

Consider effect on Higgs mass shifts:

$\overline{\text{DR}}$ scheme known to give stable results

Frank, Heinemeyer, Hollik, Weiglein '02

Including 1-loop corrections in $\overline{\text{DR}}$ scheme

$M_{h^0} = 145$ GeV for $\tan\beta = 50$

m_h^{max} scenario, $\mu = M_{A^0}$

$M_{h^0} = 145$ GeV in other schemes:

	$\tan\beta$
$A^0 Z$ unmixing	53.6
no δt_i	674.7
minimal δm_3	540.5
Higgs mass rel.	no real solution

Definition of $\tan\beta$ via physical process

Coarasa et al. '98

$$\left.\begin{array}{ll} \text{e.g.} & \tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(H^+ \to \tau^+\nu_\tau) \\ \text{or} & \tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(A^0 \to \tau\tau) \end{array}\right\} \text{to all orders}$$

$\Rightarrow$ gauge-independent and numerically well-behaved

Problem: Experimentally $\tan\beta$ determined from combination of several processes

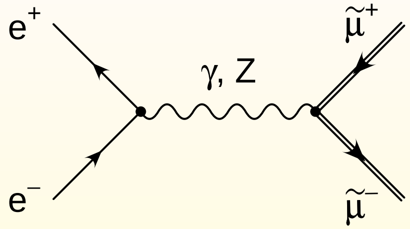
$\to$ no "gold-plated" process for $\tan\beta$

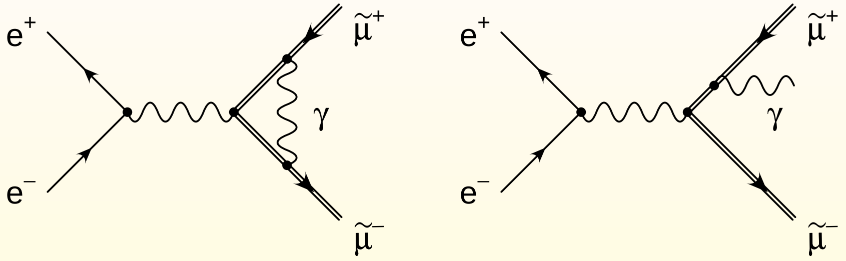
Translation from between process depends on
flavor dependent corrections, QED corrections, experimental cuts, ...

$\to$ technically complicated

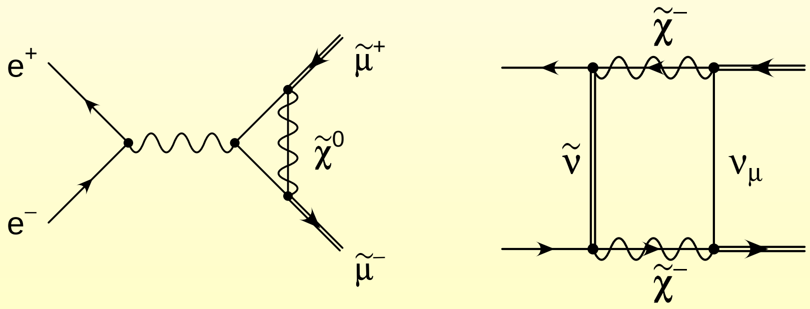
Slepton production: typical diagrams

$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$





universal QED



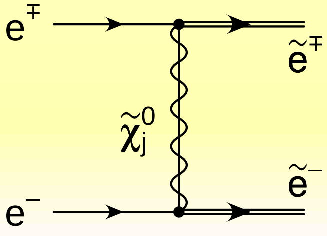
genuine SUSY electroweak

$\tilde{e}$ production: t-channel $\tilde{\chi}^0$ $\quad\to$ additional vertex/box diagrams

$e^+e^- \to \tilde{e}^+\tilde{e}^-$

$e^-e^- \to \tilde{e}^-\tilde{e}^-$

$+$



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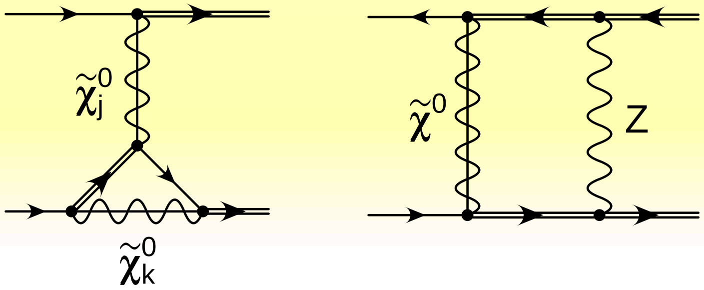


- $\mathcal{O}(500-1000)$ diagrams $\Rightarrow$ computer algebra tools
- Calculation in $\boldsymbol{R_\xi}$ **gauge** as additional check
- Generation of diagrams and amplitudes
  FeynArts Küblbeck, Böhm, Denner '90
  Hahn, Schappacher '01
- Dirac algebra and tensor integral reduction
  FeynCalc Mertig, Böhm, Denner '90
- Numerical evaluation of 1-loop integrals
  LoopTools 't Hooft, Veltman '79

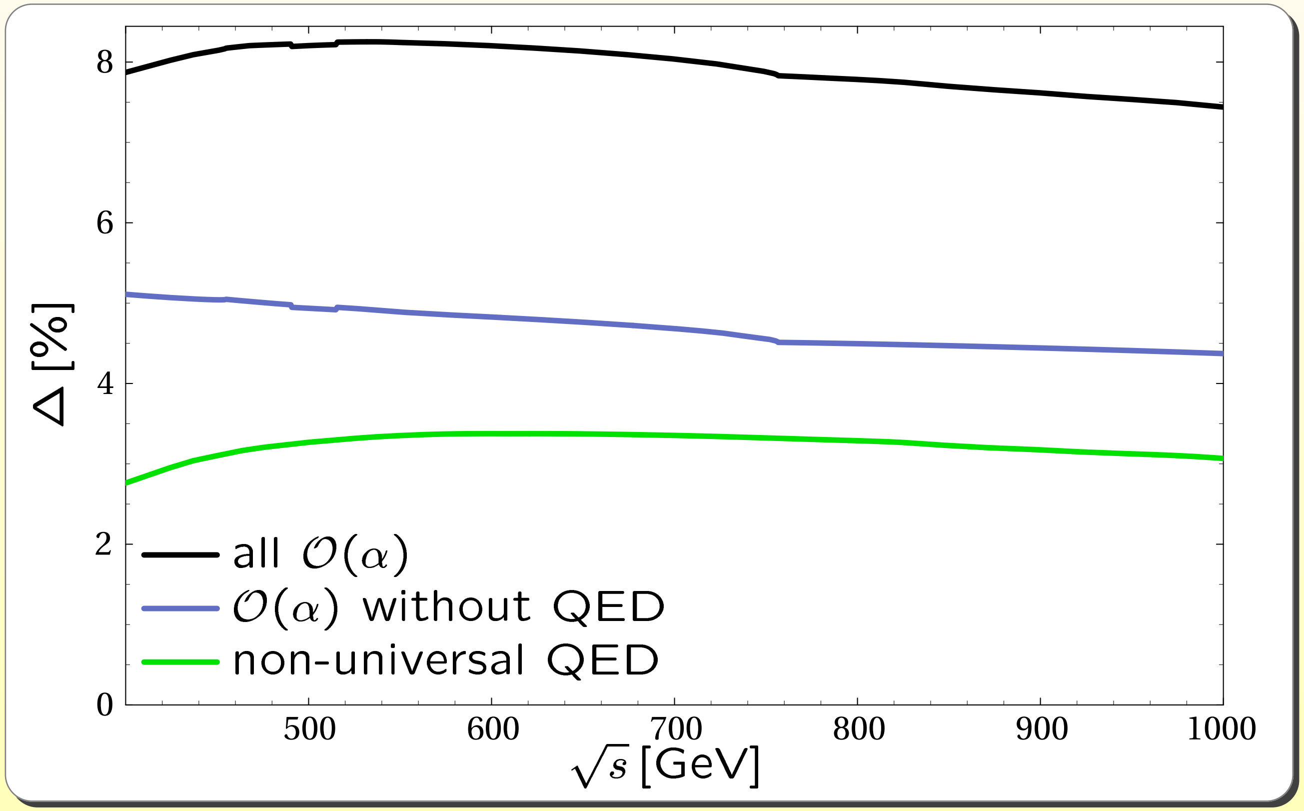
Numerical results

relative corrections

$$\Delta = \frac{\sigma_\alpha - \sigma_{\mathrm{Born}}}{\sigma_{\mathrm{Born}}}$$

$\boldsymbol{\sigma}_{\mathbf{Born}}$ includes universal QED corrections ($\Delta\alpha$, LL ISR)

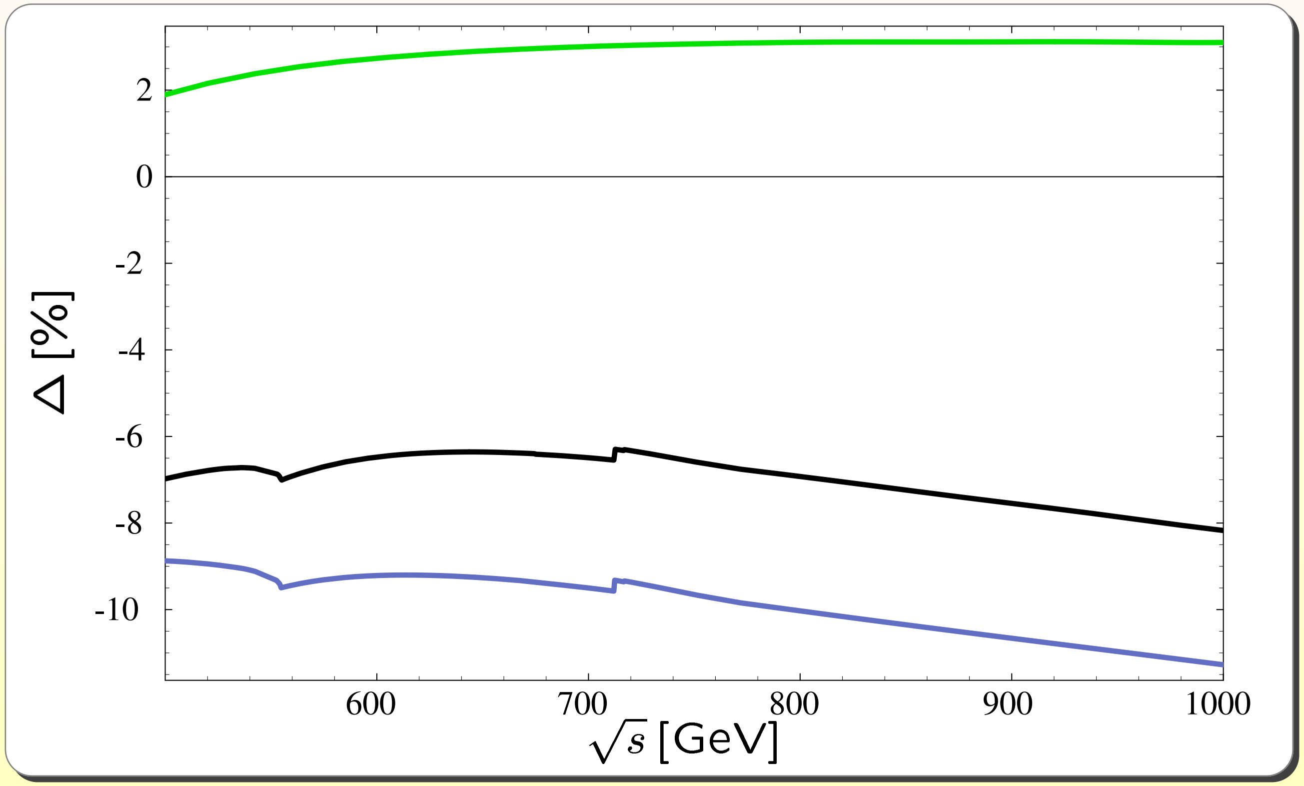
$e^+ e^- \to \tilde{\mu}_{\mathrm{R}}^+ \tilde{\mu}_{\mathrm{R}}^-$



SPS1 scenario

($m_{\tilde{\mu}_{\mathrm{R}}} = 143$ GeV)

$e^+ e^- \to \tilde{\mu}_L^+ \tilde{\mu}_L^-$

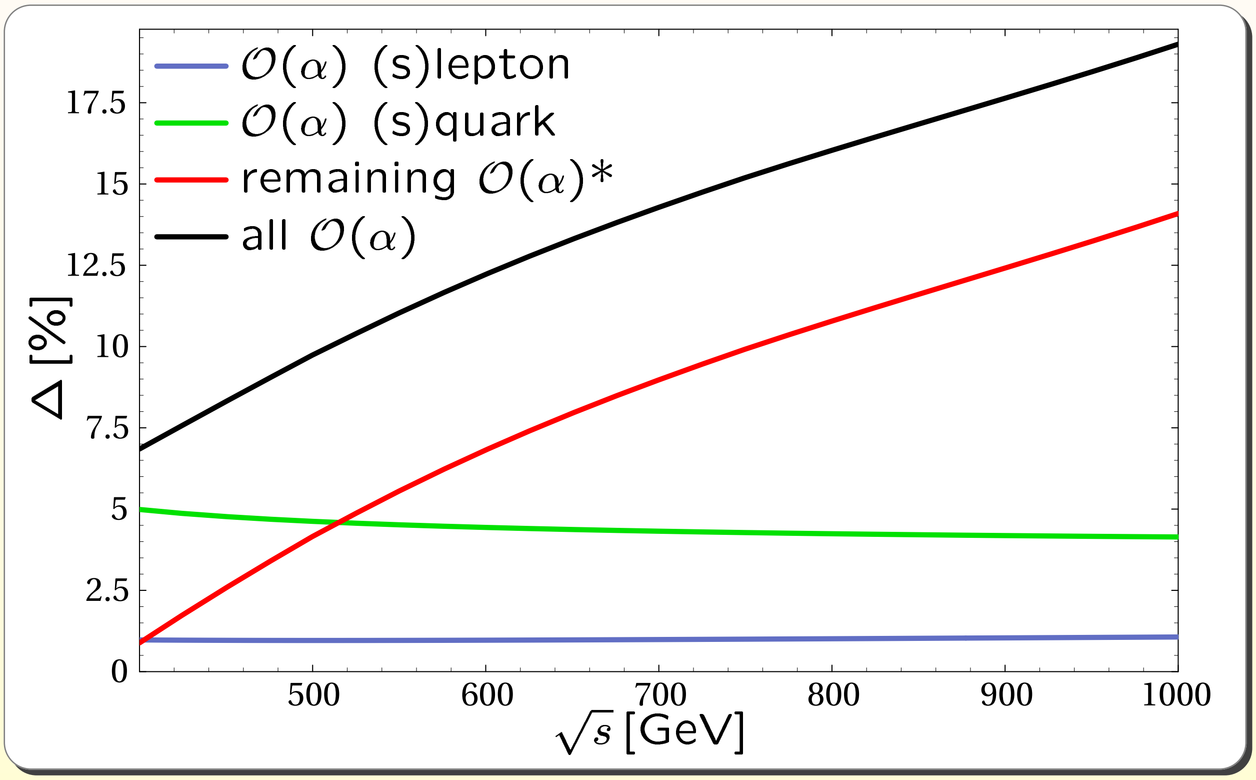


SPS1 scenario

($m_{\tilde{\mu}_L} = 202$ GeV)

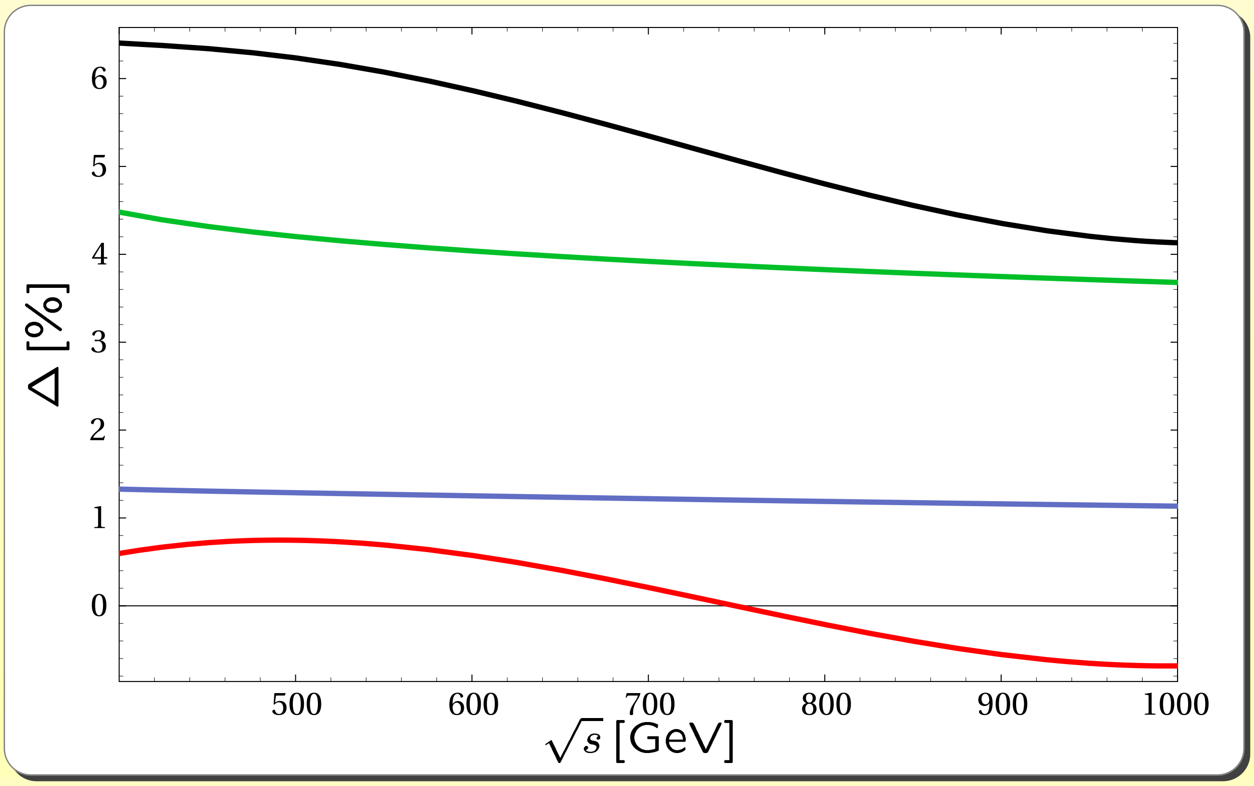
- Radiative corrections depend on all parameters on the MSSM
- Corrections of order 5–10%

$e^+ e^- \to \tilde{e}^+_R \, \tilde{e}^-_R$ SPS1 scenario

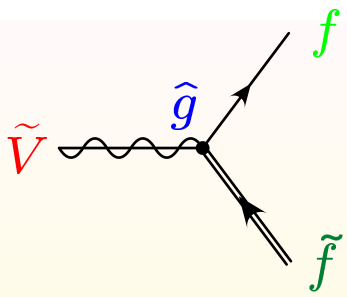


(* incl. QED)

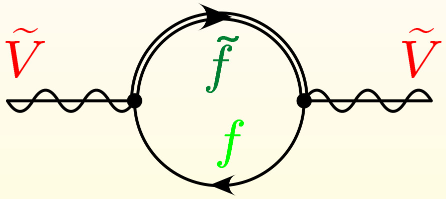
$e^- e^- \to \tilde{e}^-_R \, \tilde{e}^-_R$



Non-decoupling sfermion corrections

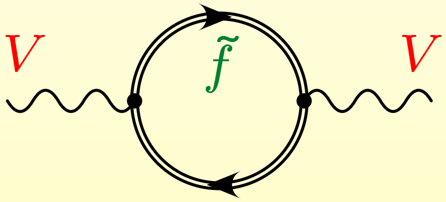


$f/\tilde{f}$ loop corrections to gaugino Yukawa vertex:



$$= C_1 \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

$\tilde{f}$ loop corrections to gauge renormalization:



$$= C_2 \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

$C_1 \neq C_2$

$\rightarrow$ non-decoupling contributions

$$\propto \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

Equivalence of effective gauge and Yukawa couplings

$$g_{\text{gauge}} = \widehat{g}_{\text{Yuk}}$$

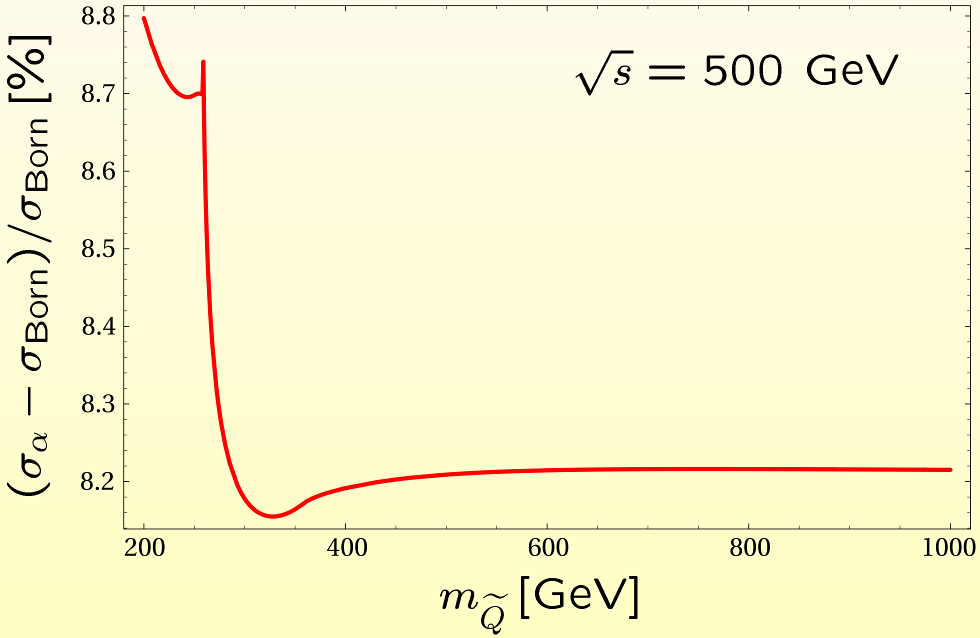
modified at higher orders:

$$g_{\text{gauge,eff}} \neq \widehat{g}_{\text{Yuk,eff}}$$

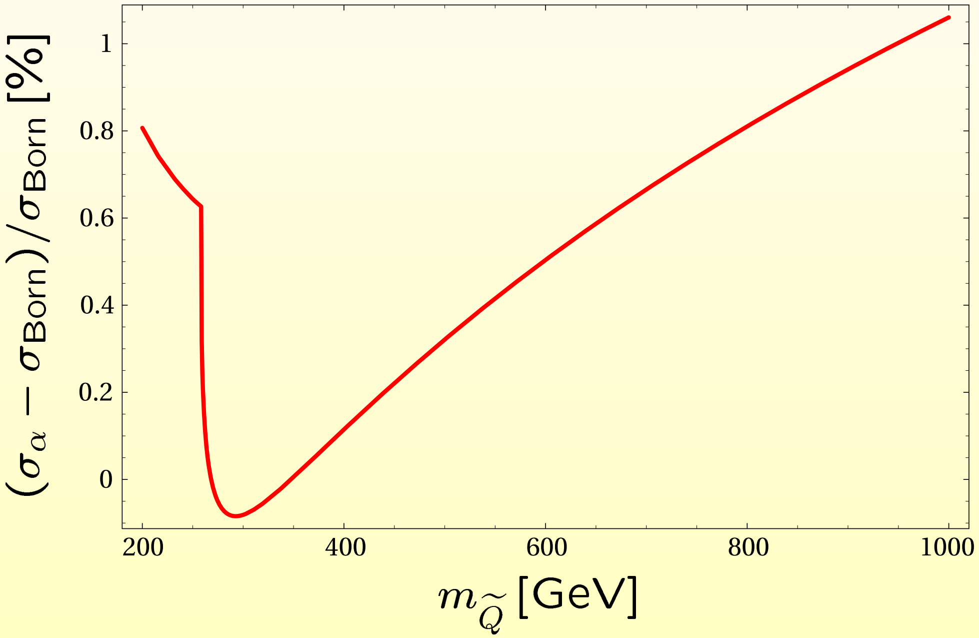
Effect of non-decoupling corrections:

Keep $m_{\tilde{L}}$ fixed, but vary $m_{\widetilde{Q}}$

$\tilde{\mu}_{\mathrm{R}}\tilde{\mu}_{\mathrm{R}}$ production



$\tilde{e}_{\mathrm{R}}\tilde{e}_{\mathrm{R}}$ production

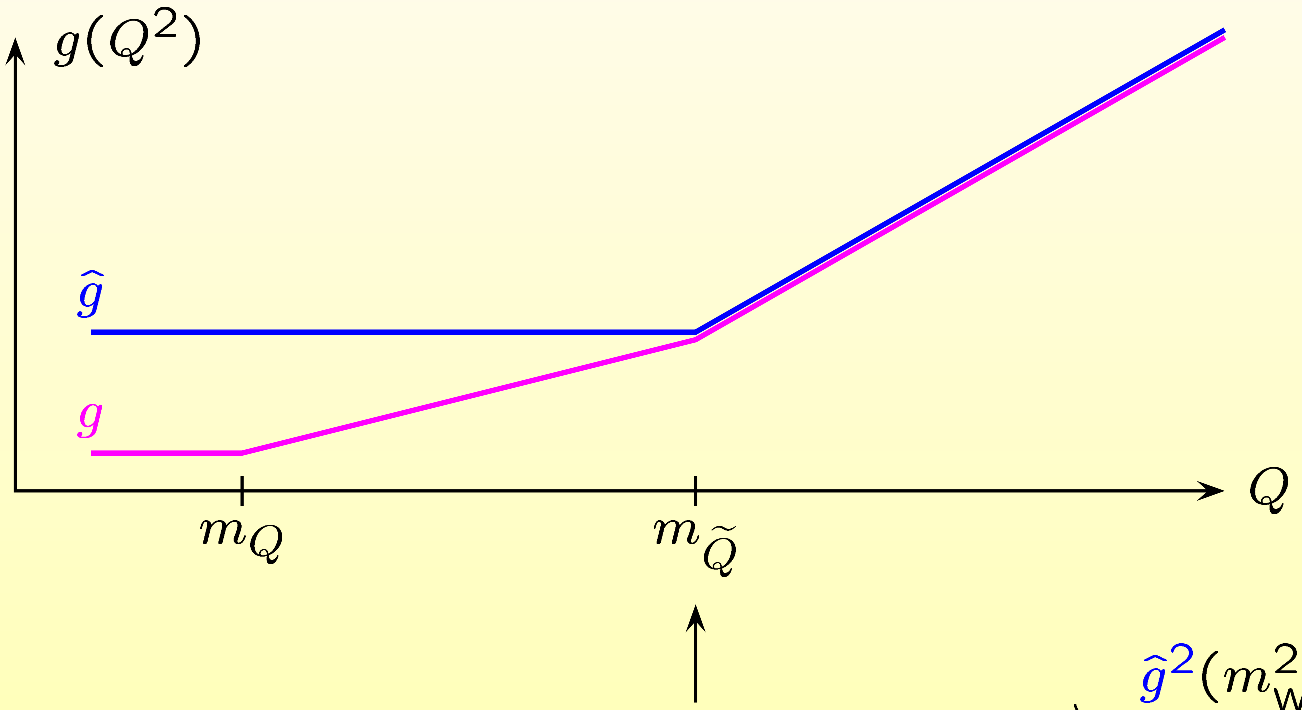


$\rightarrow$ corrections are of $\mathcal{O}(\%)$

Non-decoupling corrections in renormalization group evolution

Nojiri, Fujii, Tsukamoto '96
Cheng, Feng, Polonsky '97

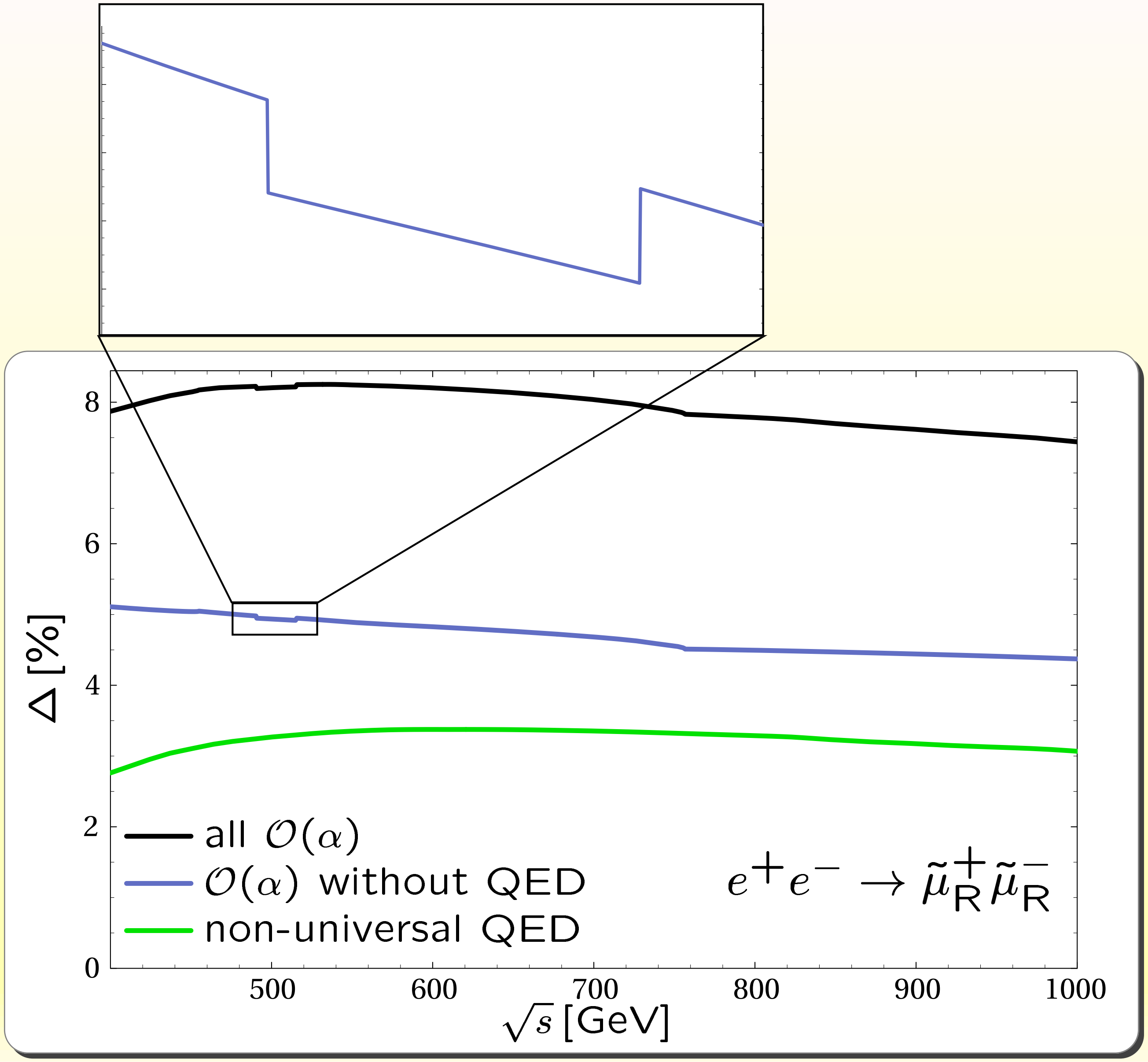
Consider only $q/\tilde{q}$ loops



$\tilde{q}$ decouple from g
q and $\tilde{q}$ decouple from $\widehat{g}$

$$\rightarrow \quad \frac{\widehat{g}^2(m^2_{\text{weak}})}{g^2(m^2_{\text{weak}})} - 1 \propto \log \frac{m_{\widetilde{Q}}}{m_{\text{weak}}}$$

Anomalous thresholds



Discontinuity in $\sigma(s)$

Singularity known as *anomalous threshold*

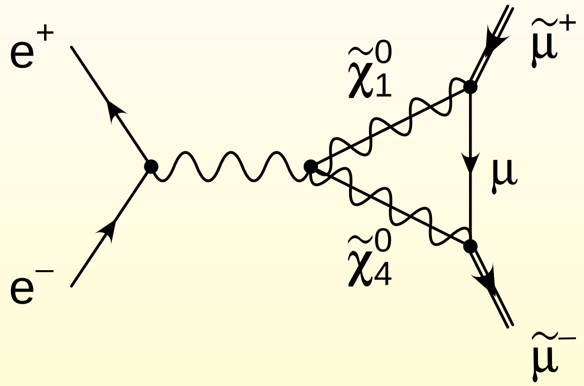
$\to$ Deuteron form factor

Karplus,Sommerfield, Wichmann '58

Landau '59

Anomalous thresholds in vertex graphs

Different in nature to normal two-particle thresholds



Normal threshold when e.g. $\tilde{\chi}_1^0$ and $\tilde{\chi}_4^0$ can be produced on-shell

Anomalous threshold:
All three loop particles $\tilde{\chi}_1^0$, $\tilde{\chi}_4^0$ and μ get on-shell

only possible for $m_{\tilde{\chi}_1^0} < m_{\tilde{\mu}} < m_{\tilde{\chi}_4^0}$

at the kinematical point $$s = \frac{m_{\tilde{\mu}}^2 (m_{\tilde{\chi}_1^0}^2 - m_{\tilde{\mu}}^2)^2}{(m_{\tilde{\mu}}^2 - m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_4^0}^2 - m_{\tilde{\mu}}^2)}$$

Conclusions

- Hunt for SUSY involves the test of SUSY coupling relations:

  Fundamental identity
  gauge coupling = Yukawa coupling

- Can be tested to $\lesssim \mathbf{1\%}$ in electroweak sector at future linear collider

  $\rightarrow$ Precise theoretical predictions required

- Complete radiative $\mathcal{O}(\alpha)$ corrections to on-shell $\tilde{\mu}$ and $\tilde{e}$ production

  $\rightarrow$ Corrections to cross-section of 5–10%

  $\rightarrow$ Sizeable effects from sparticles in loops

  $\rightarrow$ Anomalous thresholds