The soft-collinear effective theory

and hadronic B decays

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Outline

- Introduction and motivation
- The soft-collinear effective theory
- Applications
  - Exclusive hadronic form factors: $B \to \gamma \ell \nu$
  - Factorization in color-suppressed nonleptonic $\bar{B}^0 \to D^0 \pi^0$ decays
    Mantry, D.P., Stewart, to appear
- Conclusions and Outlook

Motivation

1. Heavy quark physics

Typical hadron amplitudes in B decays contain uncalculable strong-interaction effects - nonperturbative physics

- Use symmetries of the strong interactions: SU(3), isospin, heavy quark symmetries
- Factorization theorems

$$\mathcal{A} = H \otimes J \otimes S + \text{power corrections}$$

Their proof requires good control over the interplay of soft and collinear physics and power corrections

2. Light quark physics

Factorization in exclusive light hadron processes is well established at leading twist ... Brodsky, Lepage & Efremov, Radyushkin

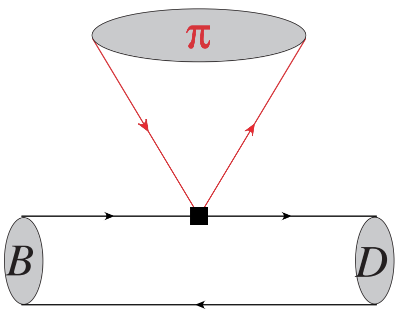
... but power corrections not as well understood

The soft-collinear effective theory (SCET) gives a systematic approach to the study of processes involving energetic particles

Basic setup

Energetic light particles moving close to the light-cone with typical momenta $\rightarrow$ collinear modes

e.g.



$$p_c = (p_+, p_-, p_\perp) \sim (\frac{\Lambda^2}{Q}, Q, \Lambda) = Q(\lambda^2, 1, \lambda)$$

$$n^2 = \bar{n}^2 = 0$$
$$p^+ = p^0 + p^3 = n \cdot p\,,$$
$$p^- = p^0 - p^3 = \bar{n} \cdot p$$

In addition, include soft light and heavy quarks with momenta

$$p_s = (p_+, p_-, p_\perp) \sim (\Lambda, \Lambda, \Lambda) = Q(\lambda^2, \lambda^2, \lambda^2)$$

Construct the effective theory as an expansion in λ.

The soft-collinear effective theory

Bauer, Fleming, Luke (1999)
Bauer, Fleming, D.P., Stewart (2000)
Bauer, Stewart (2001), Bauer, D.P. Stewart (2001,2002)

- Introduce fields for each relevant degrees of freedom

modes	field	$p_\mu = (+, -, \perp)$	p^2
collinear	$A_{n,q}, \xi_{n,p}$	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft	A_s, q_s	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$
usoft	A_μ, q	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

- The fields have well-defined momentum scaling and virtuality
- usoft modes are nonperturbative, but the collinear modes can be included in perturbation theory
- Assign a λ scaling to each effective theory field, chosen such that the kinetic terms in the action are $O(\lambda^0)$

Collinear Lagrangian

- Collinear fields have a fast variation over scales $\bar{n}\cdot x \sim Q^{-1}$ and $x_\perp \sim (Q\lambda)^{-1}$. Introduce momentum labels $\xi_n(x) \to e^{-ip\cdot x}\xi_{n,p}(x)$ such that $\xi_{n,p}(x)$ changes only slowly over $x \sim (Q\lambda^2)^{-1}$
- Split the quark field into two-component fields $q = \xi_{n,p} + \xi_{\bar{n},p}$ ('good' and 'bad' components)
- use the equation of motion to eliminate the small component $\xi_{\bar{n},p} \to$ Lagrangian for the collinear quark field

$$\mathcal{L}_{\rm coll}[\xi_{n,p}, A_{n,q}, A_{\rm us}] =$$

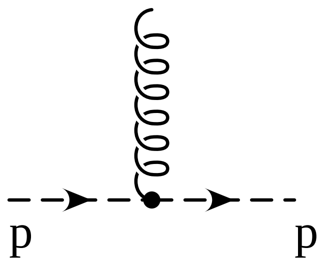
$$\bar{\xi}_{n,p}\left(n\cdot iD + g n\cdot A_{n,q} + i\not{D}_{\perp,c}\frac{1}{\bar{n}\cdot iD_c} i\not{D}_{\perp,c}\right)\frac{\bar{\not{n}}}{2}\xi_{n,p}$$

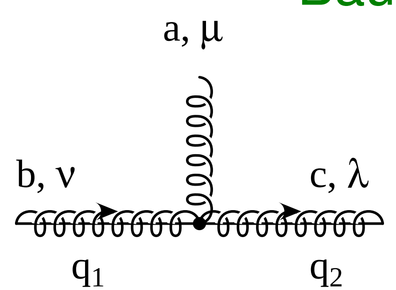
Collinear covariant derivative $iD_c^\mu = \mathcal{P}^\mu + gA_n^\mu$

- The SCET action is nonlocal and describes the couplings of collinear particles to each other and to usoft gluons at leading order in λ

Factorization of usoft modes

- Ultrasoft gluons have eikonal couplings to collinear quarks and gluons $\rightarrow$ their couplings can be resummed to all orders into a Wilson line
Bauer, DP, Stewart

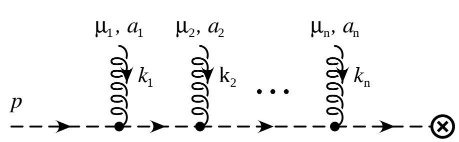




$-ign_\mu T^a$ $\qquad$ $gf^{abc}n_\mu$

Collinear field redefinitions

$$\xi_{n,p} = Y[n\cdot A_{\rm us}]\xi^{(0)}_{n,p}\,, \qquad A_{n,q} = Y[n\cdot A_{\rm us}]A^{(0)}_{n,q}Y^\dagger[n\cdot A_{\rm us}]$$

with $Y[n\cdot A_{\rm us}] =$  $= P\exp\left(ig\int_{-\infty}^{x} d\lambda n\cdot A(n\lambda)\right)$

$\rightarrow$ moves all ultrasoft gluon effects into operators and simplifies cancellations to all orders

$$\mathcal{L}[\xi_{n,p}, A_{n,q}, A_{\rm us}] = \mathcal{L}[\xi^{(0)}_{n,p}, A^{(0)}_{n,q}, 0]$$

Other formal developments

- The SCET can be derived from an underlying symmetry principle: invariance under separate collinear and usoft gauge invariance

$$
\begin{aligned}
&A_c \to U_c A_c U_c^\dagger + U_c[i\mathcal{D}^\mu, U_c^\dagger]\,, \qquad && A_c \to U_{us} A_c U_{us}^\dagger \\
&A_{us} \to A_{us} && A_{us} \to U_{us} A_c U_{us}^\dagger + U_{us}[i\partial^\mu U_{us}^\dagger]
\end{aligned}
$$

  This requires ξ_n to appear in the gauge invariant combination $W^\dagger \xi_n$ with $W[\bar{n}\cdot A_n] = P\exp(ig\int d\lambda \bar{n}\cdot A_n(\lambda\bar{n}))$

- Power corrections to the SCET Lagrangian and currents

  Chay, Kim

  Beneke, Chapovsky, Diehl, Feldmann

  DP, Stewart

- Extend the SCET Lagrangian to include interactions with usoft quarks Beneke, Chapovsky, Diehl, Feldmann

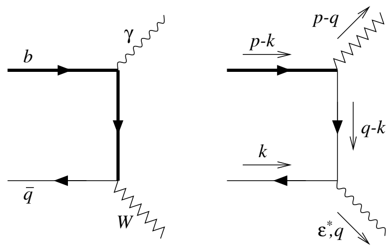
- Reparameterization invariance Chay, Kim

  Manohar, Mehen, DP, Stewart

Application 1: $B \to \gamma e \nu$

- Chirality suppression in leptonic decays $B \to e\bar{\nu}$ can be avoided by adding one photon in the final state $\to$ enhanced branching ratios of $\simeq 10^{-6}$. Burdman, Goldman, Wyler (1995)
- Can give useful information about V_{ub}, f_B

$$A(\bar{B} \to \gamma e \nu) = eQ_u \int d^4x e^{iq\cdot x} \langle 0 | T(\bar{u}\Gamma b)\,, (\bar{u}\not{\epsilon}^* u)(x) | \bar{B}\rangle$$



(a) (b)

Effective theory description (leading order in Λ/E_γ)

Lunghi, D.P., Wyler (2002)

Descotes-Genon, Sachrajda (2002), Bosch et al (2003)

Basic observation: the struck quark moves close to the light cone

$p_\mu = k_\mu - E_\gamma n_\mu \to$ natural description as a collinear particle ξ_n

- Weak current matching

$$\bar{u}\gamma_\mu b = C_1^{(v)}(\omega)\bar{\chi}_{n,\omega}\gamma_\mu h_v + C_2^{(v)}(\omega)\bar{\chi}_{n,\omega}n_\mu h_v + C_3^{(v)}(\omega)\bar{\chi}_{n,\omega}v_\mu h_v + O(\lambda)$$

- The photon can attach to the light quark: direct vs. through one insertion of the usoft-collinear Lagrangian

$$\bar{u}\not{\varepsilon}^* u = \bar{q}\not{\varepsilon}^*\chi_n + i\int d^4x\{(J_\gamma^\mu \cdot \varepsilon_\mu)\,,\mathcal{L}_{q\xi}^{(1)}(x)\}$$

- After the decoupling field redefinition, find factorized form for the decay amplitude

$$A(B^- \to \gamma e\nu) \sim C_1^{(v,a)}(2E_\gamma,\mu)\int dk_+ J(k_+,\mu)\phi_B^+(k_+)$$

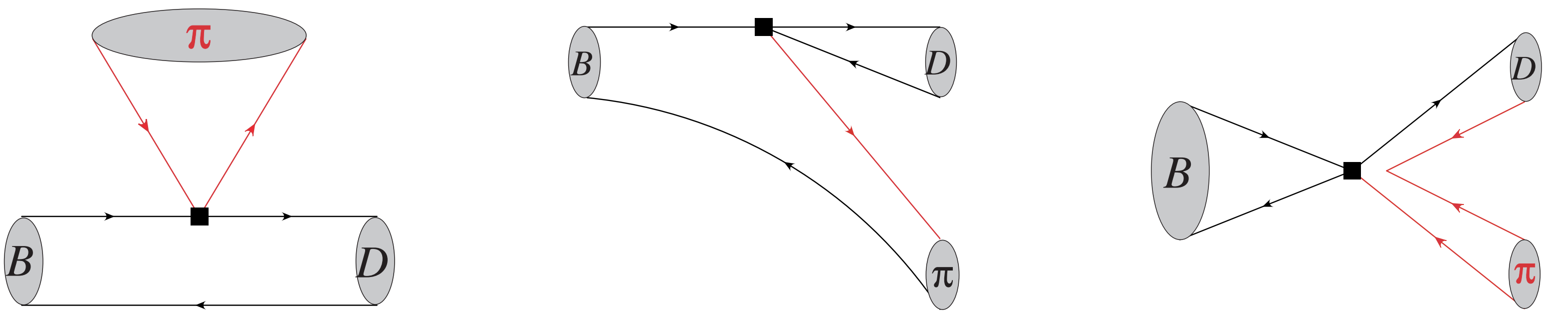
The Wilson coefficients $C_i(\bar{n}\cdot p)$ satisfy a homogeneous RGE $\to$ resum Sudakov logs Descotes-Genon, Sachrajda (2002)

Application 2: $\bar{B} \to D\pi$ decays

Mediated by the weak Hamiltonian

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1 (\bar{c}b)(\bar{d}u) + C_2 (\bar{d}b)(\bar{c}u)]$$

Quark level topologies



Tree | Color-suppressed | Annihilation

Naive factorization: $A(\bar{B} \to D\pi) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} a_1 \langle D|\bar{c}b|\bar{B}\rangle \langle \pi|\bar{d}u|0\rangle$

Data:

Decay Mode	CLEO (2002) Br $(\times 10^{-3})$	Factorization $(F_0^{BD}(0) = 0.6)$
$\bar{B}^0 \to D^+\pi^-$	2.68 ± 0.29	3.2
$B^- \to D^0\pi^-$	4.97 ± 0.38	3.2

Arguments for factorization

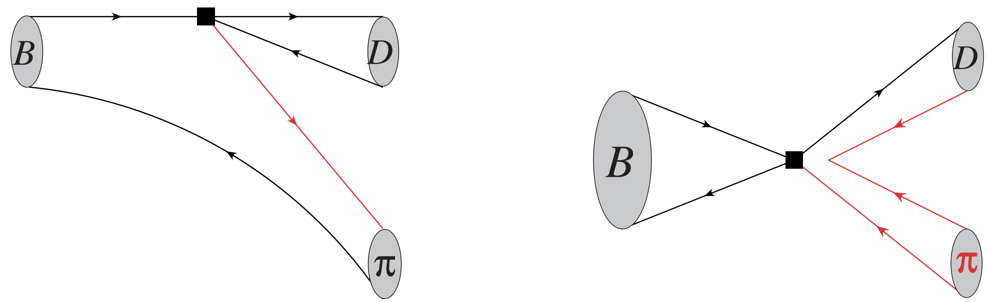
Factorization is observed to work well in class I $B \to D\pi$ decays, but not in class II decays

- Color transparency: the pion is seen as a point particle by the soft gluon fields in the heavy mesons, and decouples at leading order in $1/E_\pi$ Bjorken (1990)
- The Large Energy Effective Theory (LEET) was first proposed as an attempt to explain factorization Dugan, Grinstein (1991)

Problem: does not include collinear gluons!

- One-loop improved factorization Politzer, Wise (1991)
- Extended to 2-loop order + light-light states Beneke, Buchalla, Neubert, Sachrajda (1999)
- All-orders proof in the SCET Bauer, DP, Stewart (2001)

Color-suppressed decays



Consider first large N_c

$$A(\bar{B}^0 \to D^0\pi^0) \sim (C_2 + \frac{C_1}{N_c})\langle D^0\pi^0|(\bar{d}b)(\bar{c}u)|\bar{B}^0\rangle + 2C_1\langle D^0\pi^0|(\bar{d}T^a b)(\bar{c}T^a u)|\bar{B}^0\rangle$$

The $1/N_c$ expansion of the matrix elements gives an enhanced nonfactorizable contribution $(C_1 \sim N_c^0, C_2 \sim N_c^{-1})$

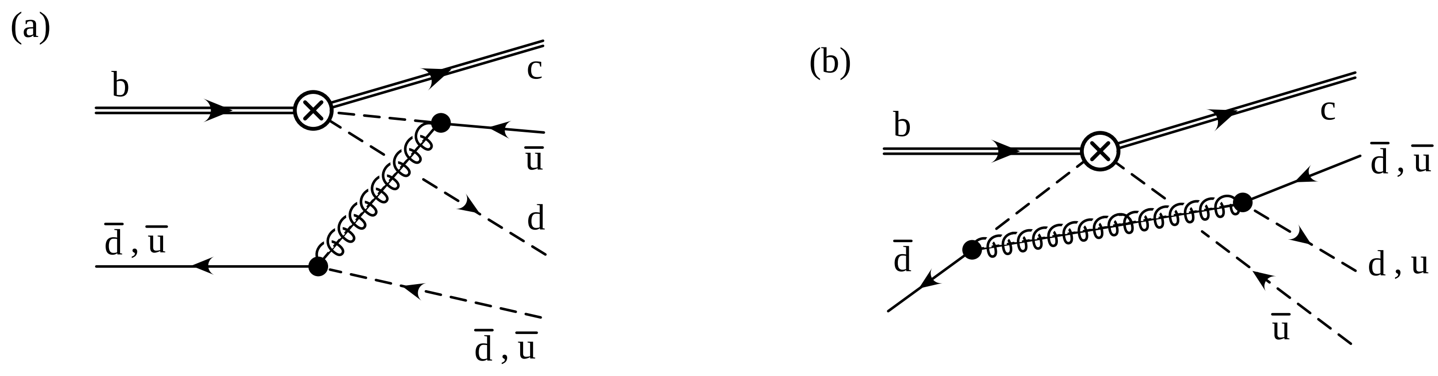
$$\langle D^0\pi^0|(\bar{d}b)(\bar{c}u)|\bar{B}^0\rangle = F_0 + \frac{1}{N_c^2}F_2 + \cdots$$

$$\langle D^0\pi^0|(\bar{d}T^a b)(\bar{c}T^a u)|\bar{B}^0\rangle = \frac{1}{N_c}G_1 + \cdots$$

Factorization is not expected to hold for color-suppressed decays in the large N_c limit!

Take $E_\pi \gg \Lambda$

Mediated by a single class of SCET operators $T\{\mathcal{O}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$



When matched onto SCET this gives a factorization formula:

$$A_0^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}_{L,R}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, S^{(i)}_{L,R}(k_1^+, k_2^+)\, \phi_M(x)$$

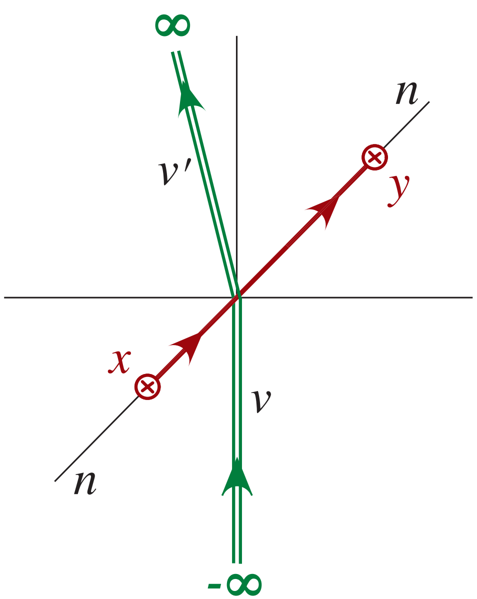
two new non-perturbative soft functions $\qquad (i = 0, 8)$

$$O^{(0,8)}_{L,R}(k_1^+, k_2^+) = \Big[(\bar h^{(c)}_{v'} S)\Gamma^h \{1, T^a\}\, (S^\dagger h^{(b)}_v)\, (\bar d\, S)_{k_1^+} \Gamma_s \{1, T^a\} (S^\dagger u)_{k_2^+} \Big]$$

$$\langle D^{(*)0} | O^{(0,8)}_{L,R}(k_1^+, k_2^+) | \bar B^0 \rangle \to S^{(0,8)}_{L,R}(k_1^+, k_2^+)$$ same for D and D^*

Results

- Color-suppressed $B \to D\pi$ amplitudes are suppressed by Λ/Q relative to color-allowed
- S is complex, gives a non-perturbative strong phase which is independent of M and choice of D vs. D^*



predict
equal strong phases $\delta^D = \delta^{D^*}$
equal amplitudes $A_0^D = A_0^{D*}$

corrections to this are $\alpha_s(m_b)$, Λ/Q

- At tree level $T^{(i)}(x)$ = constant

$$J^{(i)} \sim \frac{\alpha_s(\mu_0)}{x\, k_1^+\, k_2^+}$$ which gives $$A \sim \int \frac{\phi_\pi(x)}{x} \int \frac{S^{(i)}(k_1^+, k_2^+)}{k_1^+\, k_2^+}$$

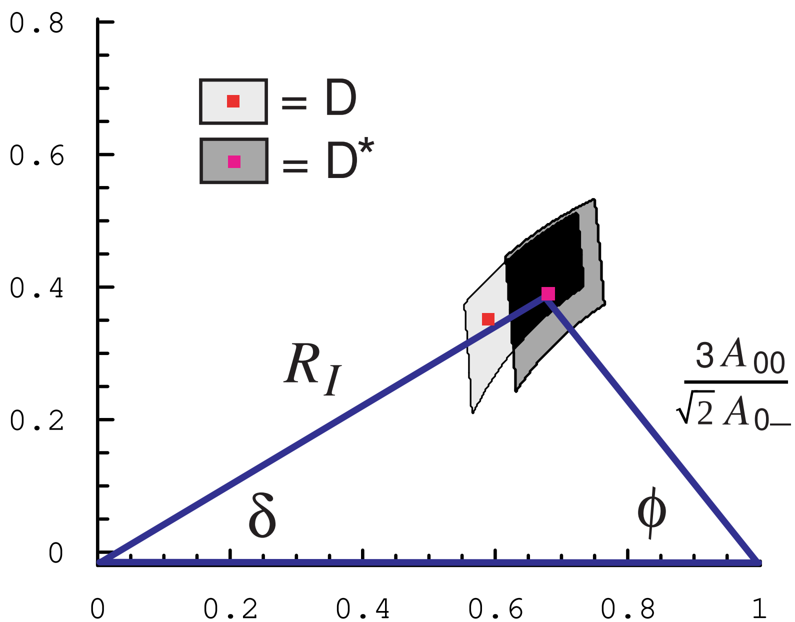
Predictions

#1) The non-perturbative functions drop out in ratios:

$$R_\pi \equiv \frac{|A(\bar{B}^0 \to D^{*0}\pi^0)|}{|A(\bar{B}^0 \to D^0\pi^0)|} = 1\,, \qquad \frac{\delta^{D^*\pi}}{\delta^{D\pi}} = 1$$

Expt (average of Belle and CLEO, 2001):

$$Br(\bar{B}^0 \to D^0\pi^0) = (0.29 \pm 0.05) \times 10^{-3}\ , \qquad \delta^{D\pi} = 30^{\circ}{}^{+8^\circ}_{-14^\circ}$$
$$Br(\bar{B}^0 \to D^{*0}\pi^0) = (0.25 \pm 0.07) \times 10^{-3}\,, \qquad \delta^{D*\pi} = 30^\circ \pm 6^\circ$$



Good agreement with data

Predictions

#2) Predictions for ρ

$$R_\rho \equiv \frac{|A(\bar{B}^0 \to D^{*0}\rho^0)|}{|A(\bar{B}^0 \to D^0\rho^0)|} = 1\,, \qquad \frac{\delta^{D^*\rho}}{\delta^{D\rho}} = 1$$

Experimental limits:

$$Br(\bar{B}^0 \to D^0\rho^0) < 0.39 \times 10^{-3}$$
$$Br(\bar{B}^0 \to D^{*0}\rho^0) < 0.56 \times 10^{-3}$$

Predictions

#3) Predict the ρ to π ratios:

$$R_\rho \equiv \frac{|A(\bar{B}^0 \to D^0\rho^0)|}{|A(\bar{B}^0 \to D^0\pi^0)|} = \# \frac{f_\rho}{f_\pi}\,\frac{\langle x^{-1}\rangle_\rho}{\langle x^{-1}\rangle_\pi} = 1.6 \qquad (\sim 20\% \text{ uncertainty})$$

$$R_\rho^2 = 2.56 \pm 1.0$$

This prediction receives corrections from α_s @ the jet scale $\mu^2 = E_\pi \Lambda$

Experiment:

$$\frac{Br(D^0\rho^0)}{Br(D^0\pi^0)} < 1.34$$

Factorization predicts a larger $B^0 \to D^0\rho^0$ branching ratio

Predictions

#4) The factorization formula relates ratios with charged light mesons

$$R_c = \frac{A(\bar{B}^0 \to D^+\pi^-)}{A(B^- \to D^0\pi^-)} = 1 - \frac{C-E}{T}$$

$$\begin{aligned}
|R_c^{D\pi}| &= \frac{|A(\bar{B}^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|} = 0.76 \pm 0.07 \\
|R_c^{D^*\pi}| &= \frac{|A(\bar{B}^0 \to D^{*+}\pi^-)|}{|A(B^- \to D^{*0}\pi^-)|} = 0.80 \pm 0.06 \\
|R_c^{D\rho}| &= \frac{|A(\bar{B}^0 \to D^+\rho^-)|}{|A(B^- \to D^0\rho^-)|} = 0.80 \pm 0.13 \\
|R_c^{D^*\rho}| &= \frac{|A(\bar{B}^0 \to D^{*+}\rho^-)|}{|A(B^- \to D^{*0}\rho^-)|} = 0.86 \pm 0.16
\end{aligned}$$

- Predict them to be somewhat universal since $f_\rho / f_\pi \sim 1.6$ cancels out

$$\frac{C-E}{T} = \# \frac{f_M \langle x^{-1} \rangle_M}{f_M F_{B\to D}}$$

Predictions

- This mechanism can also be used for other color suppressed decays, like the Cabibbo-suppressed $B \to DK$ modes
- Large logs of $Q^2/(Q\Lambda)$ can be resummed using the RGE in the SCET
- Naive factorization does not hold for color-suppressed $B^0 \to D^0\pi^0$ decays

$$A(\bar{B}^0 \to D^0\pi^0) \neq a_2 \langle D^0|\bar{d}b|\bar{B}^0\rangle\langle\pi^0|\bar{c}u|0\rangle$$

First model-independent treatment of factorization in color-suppressed decays

Other applications

- Factorization in exclusive $B \to \pi e \nu$ form factors - large energy symmetry relations and hard spectator corrections
  Beneke, Feldmann (1999)
  Bauer, DP, Stewart (2002)
- Inclusive $B \to X_s \gamma$ and $B \to X_u e \nu$ decays, resumming Sudakov logs in the endpoint region
  Bauer, Fleming, Luke (2000)
  Bauer, DP, Stewart (2001)
- Simplify traditional proofs of factorization in hard processes
  $e^- p \to e^- X$, $p\bar{p} \to X \ell^+ \ell^-$, $e^- \to \gamma^{(*)} p'$
  Bauer, Fleming, DP, Rothstein, Stewart (2002)
- Applications to charmonium and Υ physics
  Bauer, Chiang, Fleming, Leibovich, Low (2001)
  Fleming, Leibovich (2002)
- Jet physics - smeared distributions are given by two nonperturbative parameters
  Bauer, Manohar, Wise (2003)

Conclusions and outlook

- The soft-collinear effective theory offers a promising approach to the study of processes involving energetic quarks and gluons
- Although motivated by heavy quark problems, it can be also applied to light quark physics

  Rothstein (2003)
- SCET provides a systematic treatment of power corrections. First steps taken, plenty more applications possible
- More SCET talks at LoopFest III...