EWRC to 2-fermion production at Linear Collider energies : $e^+e^- \to \bar{f}f\ (n\gamma)$

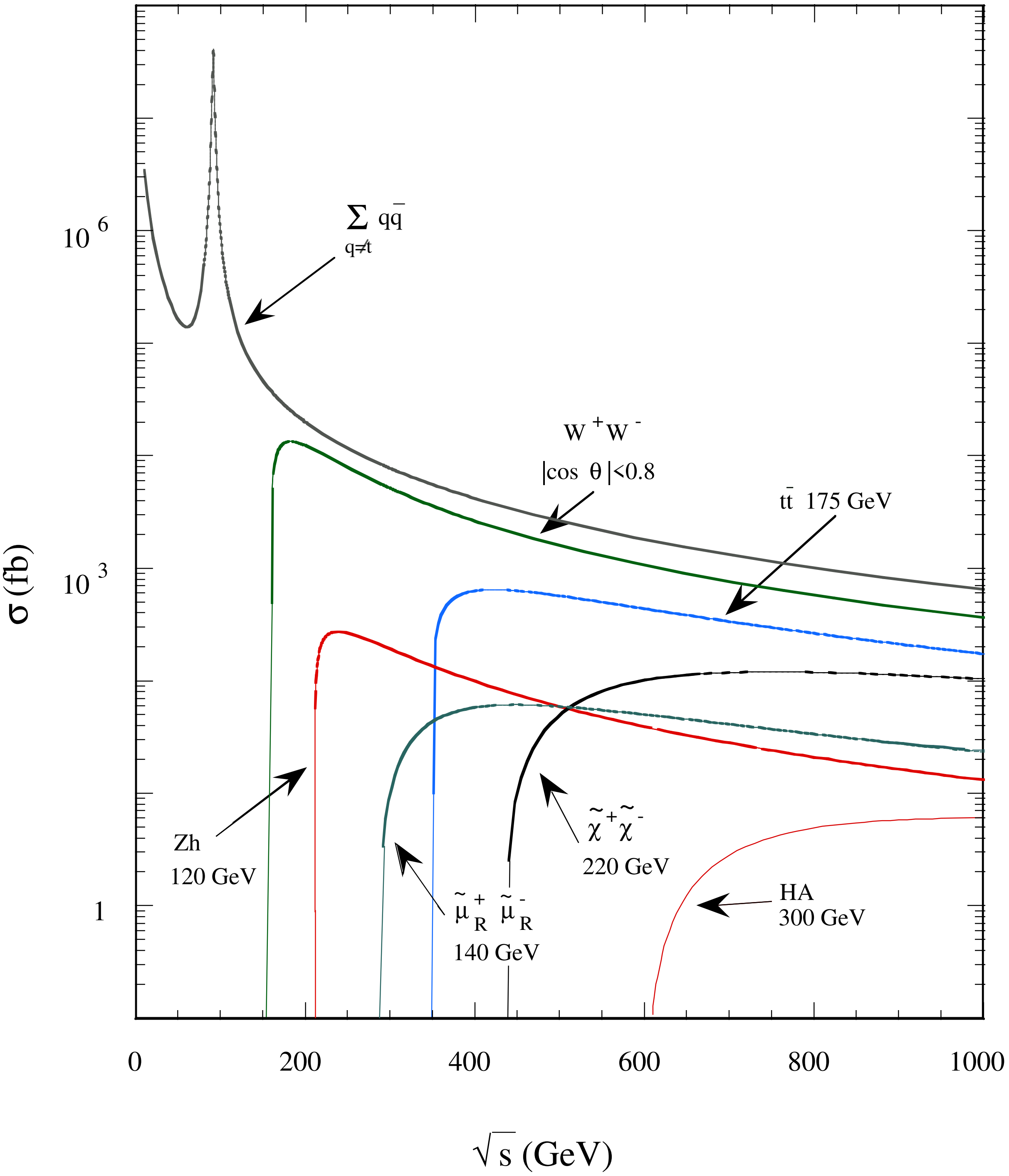
Tord Riemann, DESY Zeuthen

Progress report on work done in collaboration with

Jochem Fleischer, Janusz Gluza, Alejandro Lorca, Oleg Tarasov, Anja Werthenbach

- Introduction
- DIANA
- `topfit` and 2f-production
- Scalar one-loop functions for Bhabha
- Summary

Cross-sections at TESLA - from TDR (2001)



For $\sigma_T \sim$ 0.5 pbarn at $\sqrt{s} = 500$ GeV
and a luminosity of $\mathcal{L} \sim 300$/fbarn,
we may expect about 10^5 top pairs.

Why study $e^+e^- \to \bar{f}f$ and e^+e^- ?

Building Blocks for Experimental Precision Analysis

$\longrightarrow$ Susy and other new physics

$\longrightarrow$ QCD + higher order calculations

$\longrightarrow$ 4-, 5- or 6-fermion production

$\longrightarrow$ Luminosity measurement

Numerical Checking of Available Codes

Nontrivial Testing for Complicated Calculations

DIANA – a DIAgramANAlyser

For DIANA (M. Tentyukov, J. Fleischer) see:

http://www.physik.uni-bielefeld.de/~tentukov/diana.html

What does the program do?

- DIANA reads qgraf output.
  Analyzes the diagrams and produces graphical representation of them.
- Is interpreter of a special text manipulating language (TM).
  Executes the TM-program to insert explicit expressions for the vertices, propagators etc.
  The TM-program produces input text for FORM.

Recent improvements see contribution of Fleischer,Tentyukov,Tarasov at Loops'nLegs, Banz 2002

For Nogueira's qgraph see:

ftp://gtae2.ist.utl.pt/pub/qgraf/

Its output: a list of diagrams, described in a combinatorial fashion.

See also http://www-zeuthen.desy.de/∼alorca/

From here DIANA 3.1 is now downloadable for linux systems, together with a simple example how to use it.

We are creating a new home page

DESY Zeuthen "Loops'n Legs" Home Page

for a more systematic sampling of loop calculational tools.

DIANA: file = create.OneLoop

```
SET _processname          = OneLoop
SET _qgrafname            = "qgraf2"
SET _TOPOLOGYEDITORNAME = tedi
=========================================================
\Begin(model,Standard.model)        \End(model)

\excludevertex(E,e,H;E,e,G0;NU,e,Gp;E,nu,Gm)
=========================================================
\Begin(process)

  ingoing  e(; p1),E(; p4);
  outgoing t(;-p2),T(;-p3);

     loops = 1;                         \End(process)
=========================================================
\Begin(topology,topol.Born)             \End(topology)
=========================================================
\Begin(qgrafoptions)  options=onepi;  \End(qgrafoptions)
=========================================================
\Begin(output) .........

   g Rq =\integrand();                  \End(output)
```

Run this file:

diana -c create.OneLoop
diana -c config.OneLoop

create this way the files:

OneLoop .ps

OneLoopInfo.ps

OneLoop.out contains form expressions for ALL diagrams

Further manipulations with OneLoop.out as INPUT to user-defined form file, one file for each diagram:

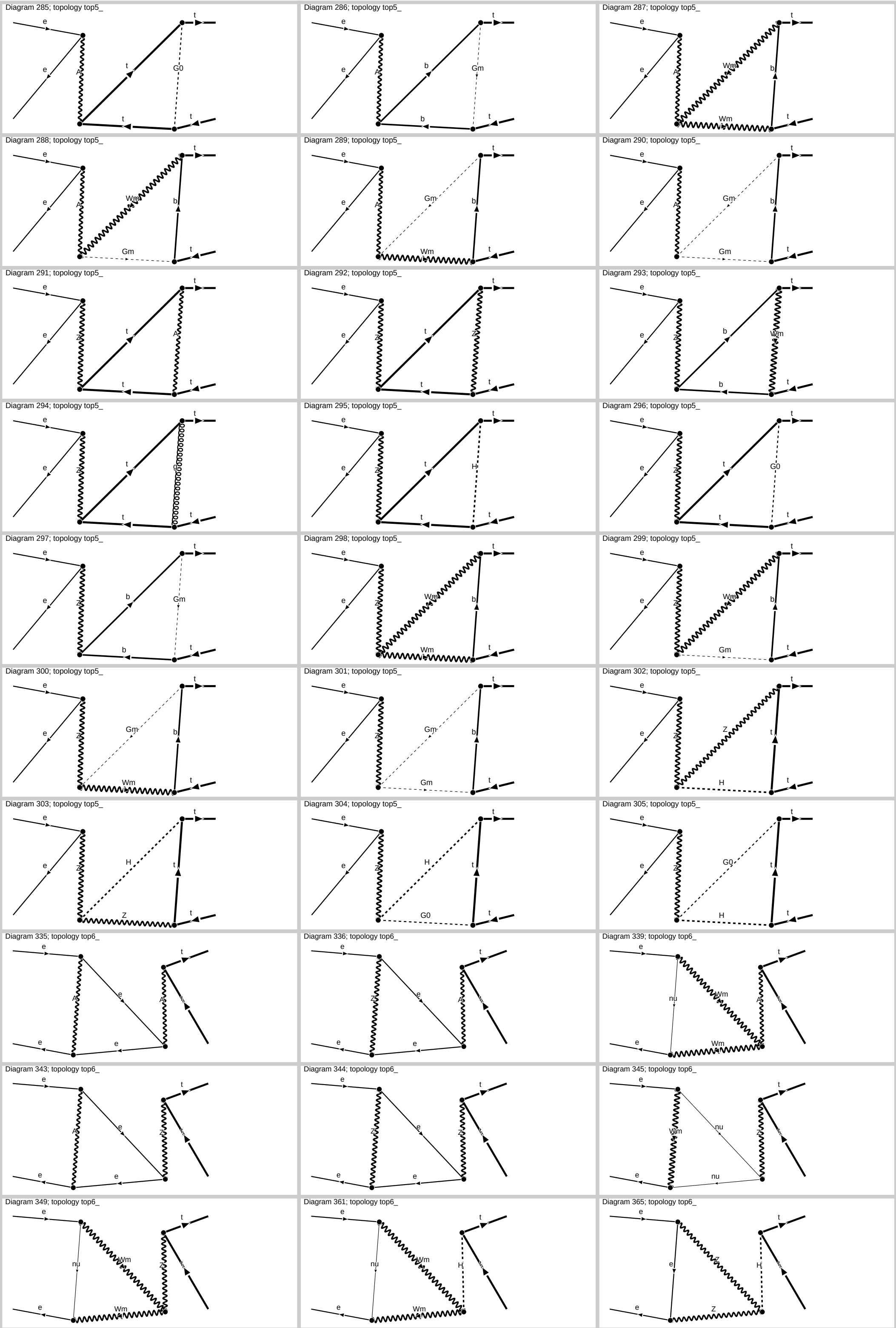
OUT/OneLoop#1.out . . . (binary)
. . .
OUT/OneLoop#last.out (binary)

Further manipulations with these binary files as INPUT to a user-defined form file:

form manipulate_OneLoop.frm

etc.

DIANA OneLoop_page5.ps



5

DIANA OneLoopInfo1037.ps

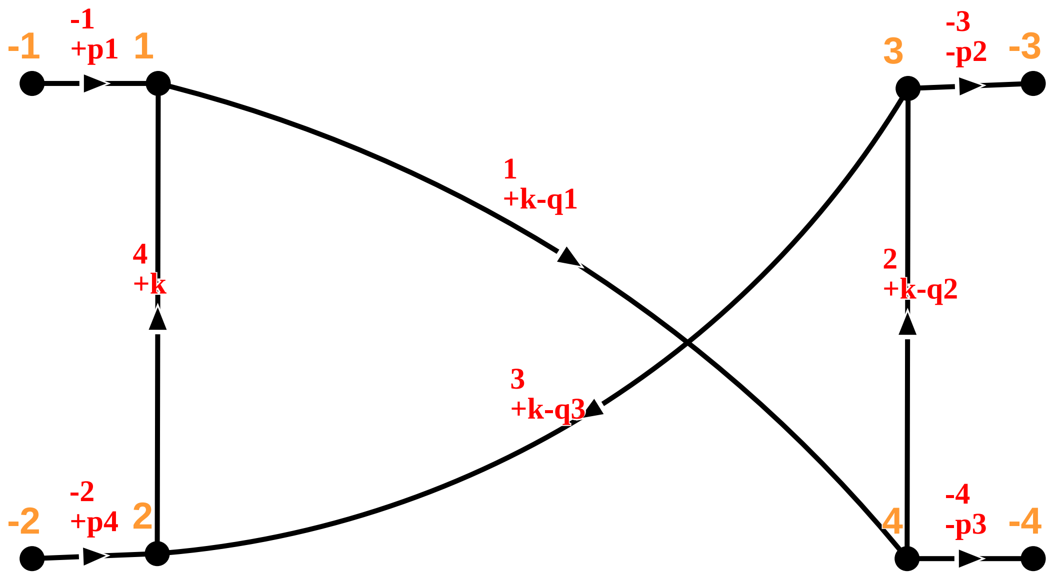
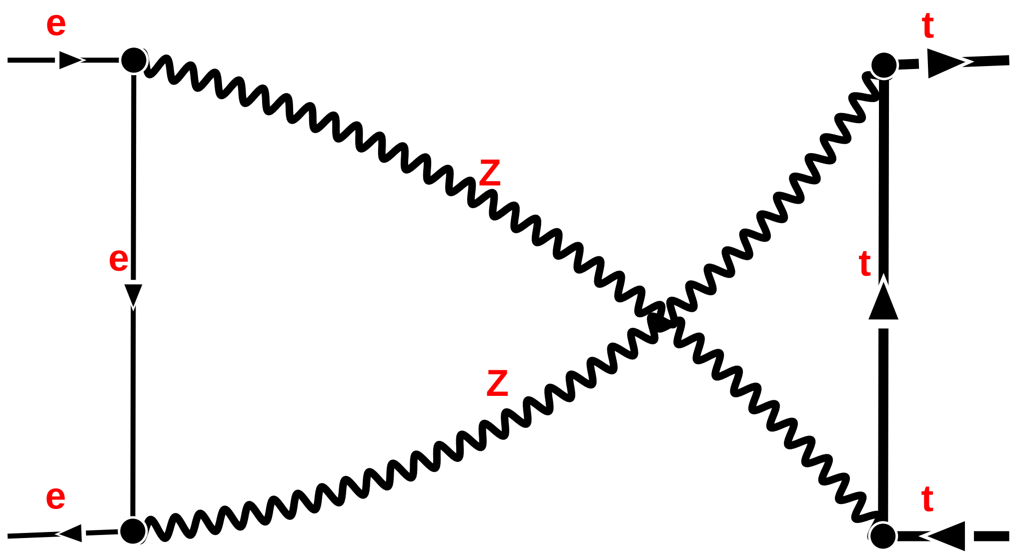


Diagram 1037 topology top15_ (unique top15_) momentaset 1 (of 1)



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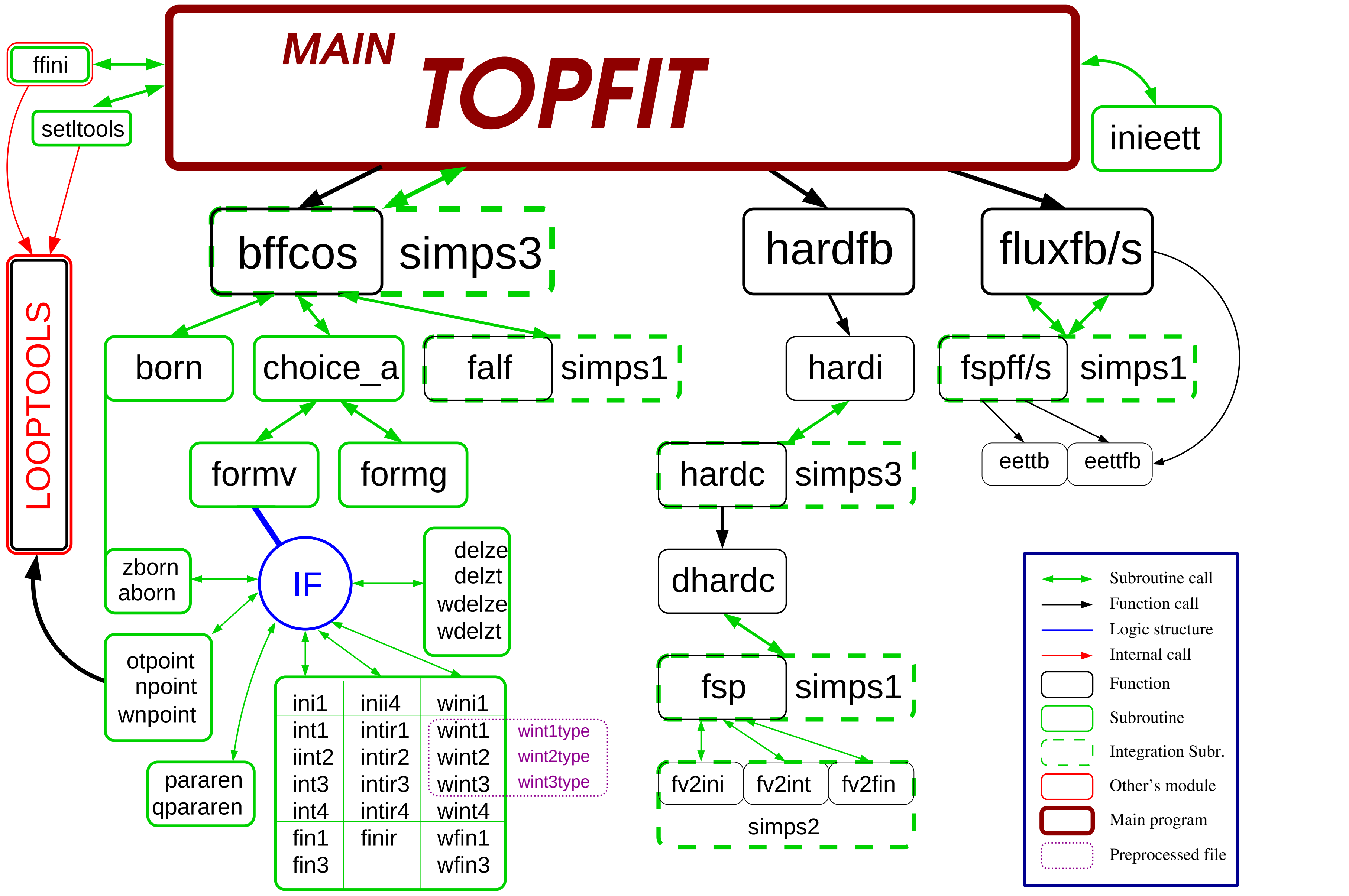
DIANA: file = manipulateBorn.log

```
   FORM version 2.3 Apr 24 1997

Born1 loaded
Born2 loaded
               g bornA=Born1;
               g bornZ=Born2;

   id c^-2*c1^-1 = c^-2/denZ;
   id      c1^-1 =    1/denA;

   .......

   print;
   .end

  bornA = 4 *
         gamma(1,mu1)*gamma(2,mu1)*i_*e^2
       * denA^-1    * ( Qe*Qt );

  bornZ =
         gamma(1,mu1)*gamma(2,mu1)*i_*e^2*s^-2*c^-2
       * denZ^-1
       * ( GVe*GVt - GVe*GAt - GVt*GAe + GAe*GAt )

      +  ..........
```

Flowchart: A. Lorca



The Fortran program topfit

Authors: J.Fleischer A.Leike A.Lorca T.Riemann A.Werthenbach

Features of version March 2003:

– Exactly one-loop

– SM weak loops and/or pure QED

– Input: $\alpha_{qed}(0)$, all masses

– u, c, t, ν: full weak + QED, μ, τ, d, s, b: also – new!

Typical results from a calculation:

– six weak form factors

– $\frac{d\sigma}{d\cos\theta}, \sigma_T, A_{FB}$ with hard bremsstrahlung (some simple cuts)

– $\frac{d\sigma}{d\cos\theta}, \sigma_T, A_{FB}$ without photonic diagrams for use in a Monte Carlo

Special: Two independent calculations
using either package FF or package LoopTools
with dim.reg. for IR-divergency or finite photon mass

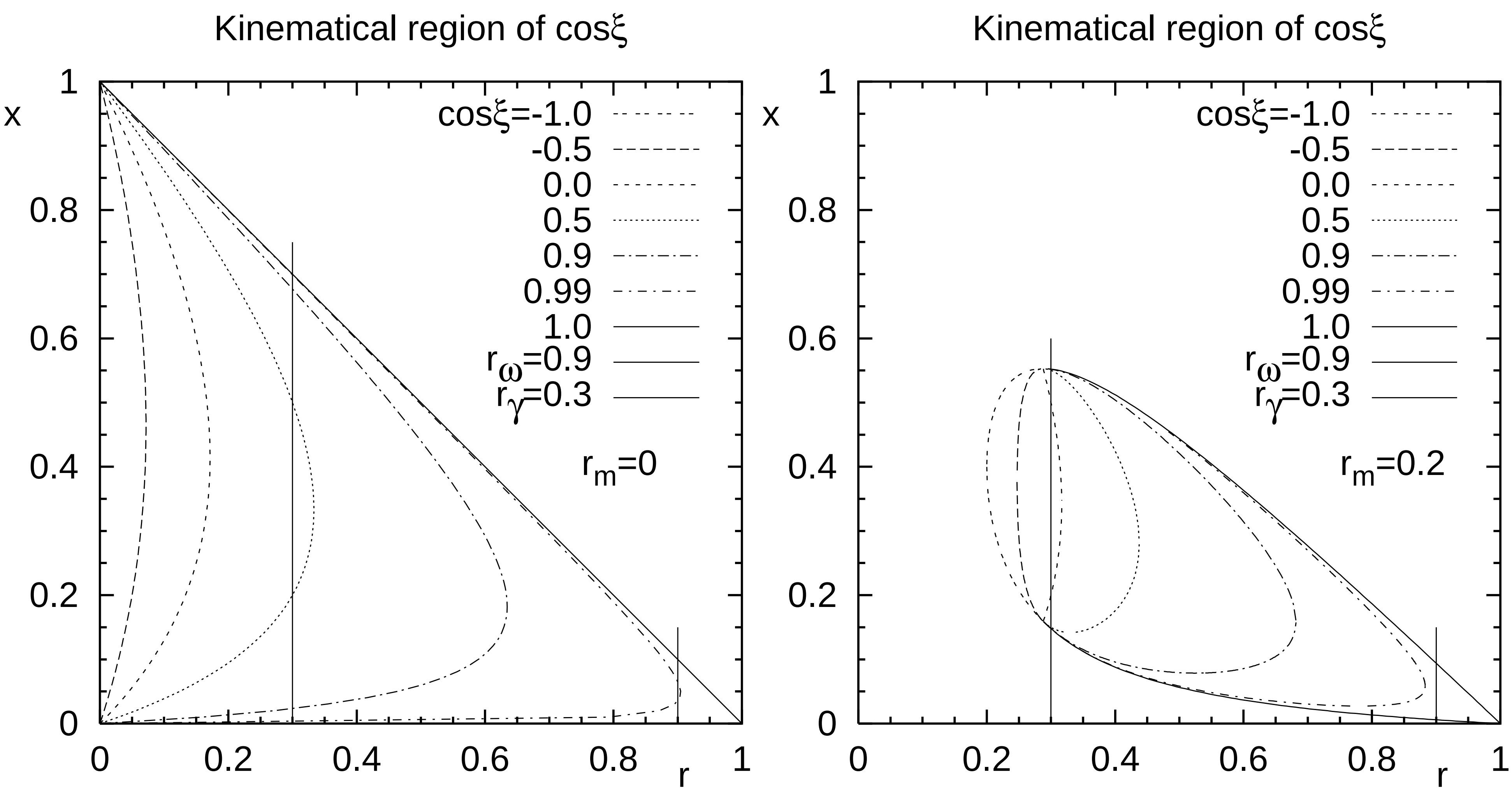
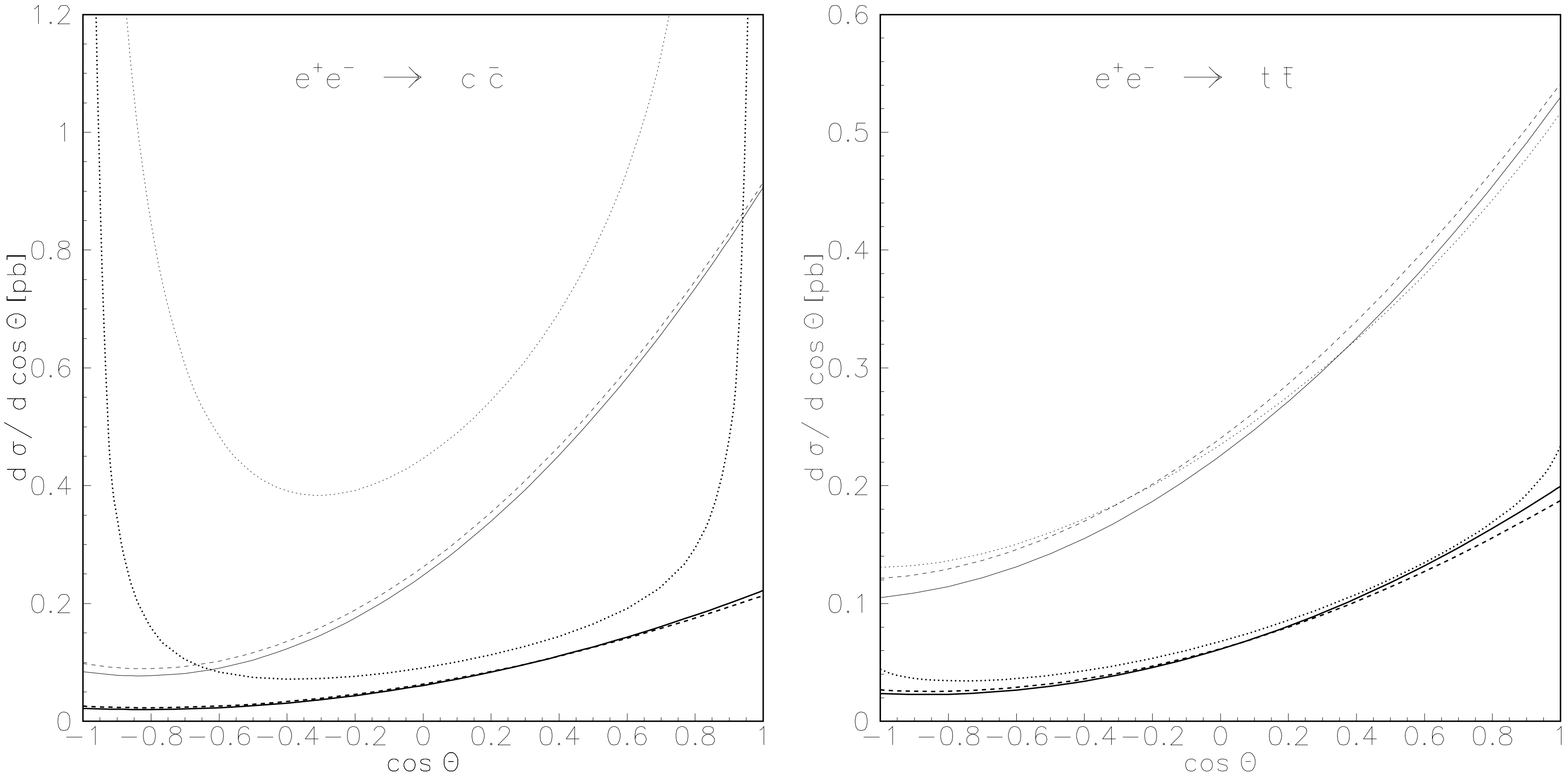


Figure 1: Phase space with acollinearity cut R_ξ; $v_2 := V_2/s \equiv x$.

a. $R_m = 4m_t^2/s = 0$ b. $R_m = 4m_t^2/s = 0.2$

topfit: charm pairs versus top pairs

$\frac{d\sigma}{dcos\theta}$ for $e^+ + e^- \to c + \bar{c} + \gamma$ and $e^+ + e^- \to t + \bar{t} + \gamma$ at TESLA

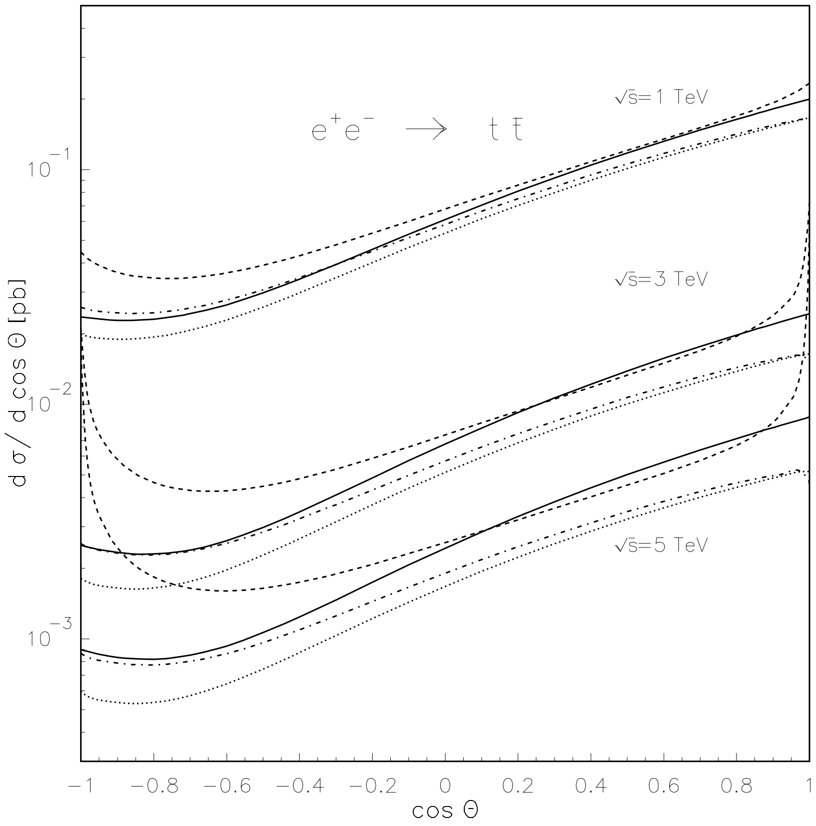


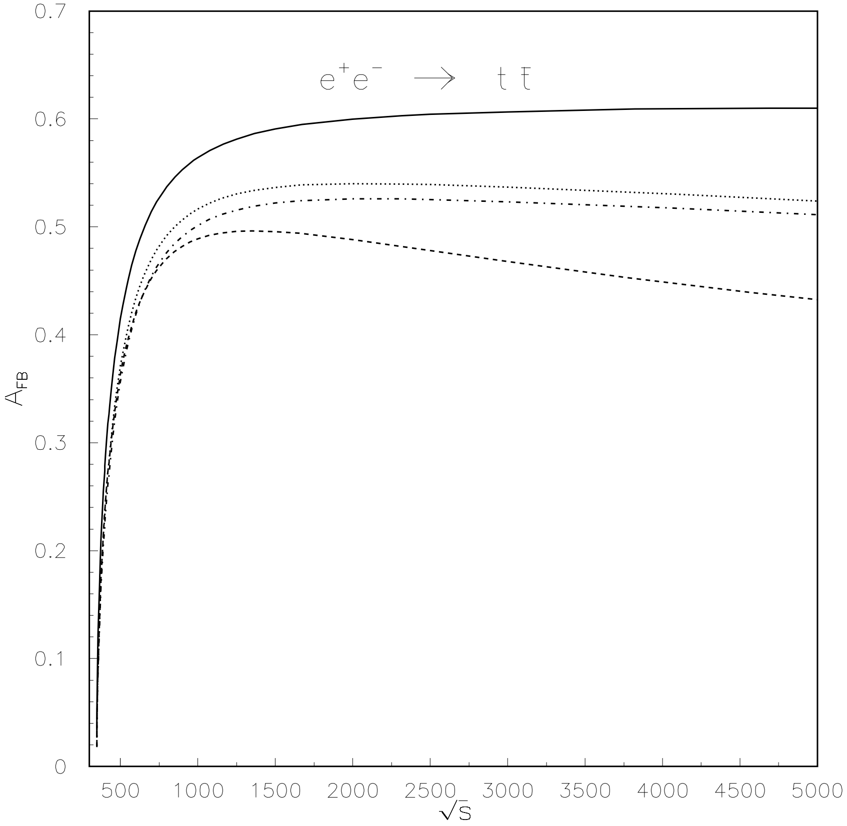
Solid: Born Dashed: weak Dotted: SM = weak + QED (no cut)

Thin lines: $\sqrt{s} = 500$ GeV, thick lines: $\sqrt{s} = 1000$ GeV.

topfit: top pairs at CLIC

$\frac{d\sigma}{dcos\theta}$ for $e^+ + e^- \rightarrow t + \bar{t}(+\gamma)$ at CLIC (A. Werthenbach, TR, March 2003)





Solid: Born Dotted: Weak Dashed: SM = Born+weak+QED (no cut)

Dashed-Dotted: SM = Born+weak+QED (with cuts)

Numerical Results Based on

J. Fleischer, T. Riemann, A. Werthenbach

and

T. Hahn, W. Hollik, C. Schappacher

Complete Electroweak One-loop Corrections to Top-pair Production at TESLA - a Comparison

hep-ph/0202109

(weak corrections, topfit compared to Feynarts)

J. Fleischer, A. Leike, T. Riemann, A. Werthenbach

and

J. Fujimoto, T. Ishikawa, Y. Shimizu

One-loop Corrections to the Process $e^+e^- \rightarrow t\bar{t}$ Including Hard Bremsstrahlung

hep-ph/0203220

(also hard bremsstrahlung, topfit compared to Grace)

Top-quark Pair-production Cross section

Conventions for comparison with Karlsruhe team T. Hahn, Ch. Schappacher and W. Hollik:

$$
\begin{aligned}
\Gamma_Z &= 0 \\
\alpha = \frac{e^2}{4\,\pi} &= 1/137.03599976 \\
E_\gamma^{\max} &= \sqrt{s}/10 \\
M_w &= 80.4514958 \text{ GeV} \\
M_z &= 91.1867 \text{ GeV} \\
M_h &= 120 \text{ GeV} \\
m_e &= 0.00051099907 \text{ GeV} \\
m_t &= 173.8 \text{ GeV} \\
m_b &= 4.7 \text{ GeV} \\
m_\mu &= 0.105658389 \text{ GeV} \\
m_u &= 0.062 \text{ GeV} \\
m_d &= 0.083 \text{ GeV} \\
m_\tau &= 1.77705 \text{ GeV} \\
m_c &= 1.5 \text{ GeV} \\
m_s &= 0.215 \text{ GeV}
\end{aligned}
$$

Top pair production: "FeynLoop" and topfit

$\sqrt{s} = 500$ GeV: $e^+ + e^- \to \bar{t} + t$ at TESLA

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{weak}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$
-0.9	0.1088 3919 4075 0.1088 3919 4075	0.09866 4253 0.09866 4253	0.1242 59037 1 0.1242 59037 6	0.1140 8410 0.1140 8409 5578
-0.5	0.1422 7506 9392 0.1422 7506 9392	0.1285 0790 0.1285 0790	0.1568 48371 9 0.1568 48371 8	0.1430 8121 0.1430 8120 5165
0.	0.2254 7046 4032 0.2254 7046 4033	0.2023 9167 0.2023 9167	0.2402 6680 4 0.2402 6680 3	0.2171 8801 0.2171 8800 9765
0.5	0.3546 6647 0331 0.3546 6647 0332	0.3151 1724 0.3151 1723	0.3688 8650 7 0.3688 8650 5	0.3293 3727 0.3293 3727 3947
0.9	0.4911 4371 5766 0.4911 4371 5767	0.4307 1437 0.4307 1437	0.5033 3751 2 0.5033 3750 8	0.4429 0817 0.4429 08167343

Agreement confirms accuracy of electroweak calculations of two groups (06 March, 2002):
upper line: T.Hahn et al., lower line: topfit, magenta: March 2003, A.Lorca

Results: Numerical agreement for c, b-quarks and τ lepton (A.Lorca)

- Born: ~ 12 digits

  !! m_e not fully neglected at kinematics by Thomas.

  ✓ Solved! Easy to restore by hand, full 16 digits agreement.

- Born+QED+Soft: ~ 11 digits

  ✓ Achieved desired agreement.

- Born+Weak: ~ 6 digits

  !! Higgs and unphysical gauge fixing fields $(\chi, \phi^{\pm})$ partially discarded in Thomas' comparisons:

```
SetOptions[InsertFields,
  Model -> "SMc", Restrictions -> NoLightFHCoupling]
```

  ? Other possible m_e effects (divergencies cancellations).

Results: Numerical comparison (A.Lorca)

$b-$quarks pair production: $e^+ + e^- \to b + \bar{b}\ (+\gamma)$

at linear collider: $\sqrt{s} = 500$ GeV, $E_{max}(\gamma_{\rm soft}) = \sqrt{s}/10$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm B+weak}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{{\rm All}\ \mathcal{O}(\alpha^{\ni})}$	Group
−0.9	0.035 947 210 200	0.042 347 4	0.037 629 4	topfit
−0.9	0.035 947 210 200	0.042 347 8	0.037 629 8	*FeynLoop*
−0.5	0.052 846 991 429	0.055 564 4	0.049 542 2	topfit
−0.5	0.052 846 991 429	0.055 564 7	0.049 542 4	*FeynLoop*
0.0	0.134 448 437 257	0.135 139 0	0.121 176 2	topfit
0.0	0.134 448 437 256	0.135 139 3	0.121 176 5	*FeynLoop*
0.5	0.283 246 237 852	0.291 227 2	0.264 541 2	topfit
0.5	0.283 246 237 851	0.291 227 8	0.264 541 8	*FeynLoop*
0.9	0.450 665 853 761	0.482 564 5	0.447 083 2	topfit
0.9	0.450 665 853 760	0.482 565 4	0.447 084 1	*FeynLoop*

Nice independent agreement up to 6th digit!

FeynLoop numbers gently supplied by Thomas Hahn, Thanks!

Top pair production: topfit and Grace, angular distribution

$\cos\theta$	$\omega/\sqrt{s}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{tot}}$
-0.9	T : 0.1	0.108839194075	+0.098664253	+0.11408410	0.13144
	T : 0.00001	0.108839194075	−0.017474702	−0.002054858	0.13229
	G : 0.00001	0.108839194076		−0.002054859	0.13206(12)
-0.5	T : 0.1	0.142275069392	+0.12850790	+0.14308121	0.15973
	T : 0.00001	0.142275069392	−0.029702340	−0.015129038	0.16029
	G : 0.00001	0.142275069393		−0.015129039	0.16013(13)
$+0.0$	T : 0.1	0.225470464033	+0.20239167	+0.21718801	0.23638
	T : 0.00001	0.225470464033	−0.058010508	−0.043214169	0.23476
	G : 0.00001	0.225470464033		−0.043214168	0.23513(14)
$+0.5$	T : 0.1	0.354666470332	+0.31511723	+0.32933727	0.35651
	T : 0.00001	0.354666470332	−0.109721291	−0.095501257	0.35062
	G : 0.00001	0.354666470332		−0.095501252	0.35104(17)
$+0.9$	T : 0.1	0.491143715767	+0.43071437	+0.44290816	0.48796
	T : 0.00001	0.491143715767	−0.179672655	−0.16747886	0.47768
	G : 0.00001	0.491143715767		−0.16747886	0.47709(21)

Various differential cross sections. The upper and lower numbers correspond to the `topfit` (T) and GRACE (G) approach, respectively, $\sqrt{s} = 500$ GeV.

Why study 2-loop Bhabha scattering now?

- Prominent luminosity-monitoring reaction for e^+e^- colliders
- 2loop Bhabha: Experimental accuracy points to 0.1%
- New Physics reach: $e^+e^-e^+e^-$ four fermion operators
  small angles: t-channel dominates
- Relatively simple in the low-angle case: QED is by far dominating
  Although at TESLA also Z exchange will play a role there
- In theory, we have now an understanding of 2loop problems
  nobody expected 9 years ago
  1994: 2nd Loops'nLegs (Teupitz) had first bulk of 2loop talks

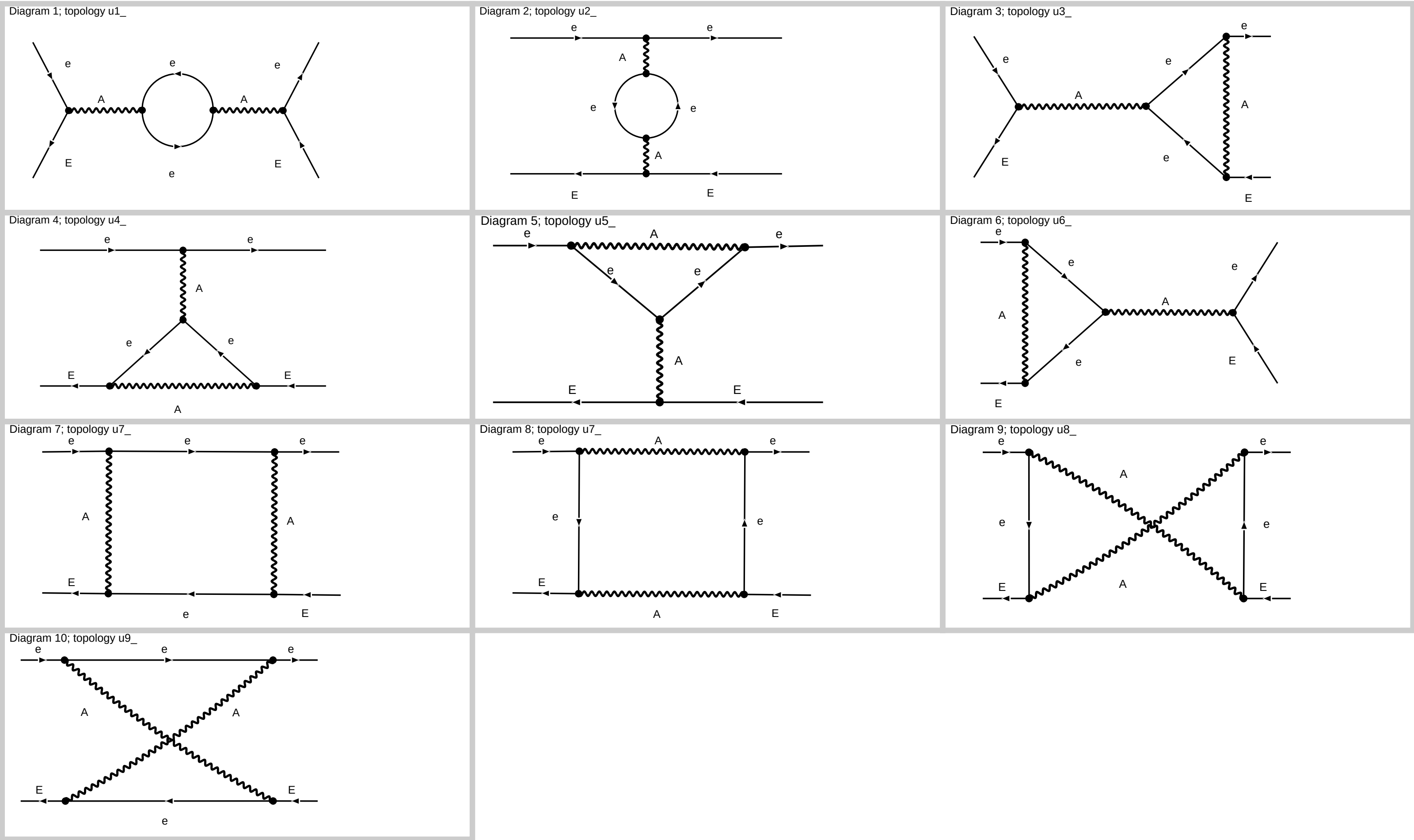


Diagram 1; topology u1_
Diagram 2; topology u2_
Diagram 3; topology u3_
Diagram 4; topology u4_
Diagram 5; topology u5_
Diagram 6; topology u6_
Diagram 7; topology u7_
Diagram 8; topology u7_
Diagram 9; topology u8_
Diagram 10; topology u9_

The one-loop matrix elements

The on-shell diagrams with amplitudes A_j, B_j:

$$\text{Diagram} \quad = \quad \sum_{j=1}^{12} A_j \mathcal{M}_j^{(s)} + \sum_{j=1}^{12} B_j \mathcal{M}_j^{(t)},$$

Define for $\mathcal{M}_j^{(s)}$ (s-channel):

$$[G_f \otimes G_i] \quad = \quad [\bar{U}(-p_2)\; G_f \; V(-p_3)] \times [\bar{V}(p_4)\; G_i \; U(p_1)]$$

and

$$\begin{aligned}
\mathcal{M}_1 &= [\,1 \otimes 1\,]\; F_1 \\
\mathcal{M}_2 &= [\,\not{p}_4 \otimes 1\,]\; F_2 \\
\mathcal{M}_3 &= [\,1 \otimes \not{p}_2\,]\; F_3 \\
\mathcal{M}_4 &= [\,\not{p}_4 \otimes \not{p}_2\,]\; F_4 \\
\mathcal{M}_5 &= [\,\gamma_\mu \otimes \gamma_\mu\,]\; F_5 \qquad \leftarrow \text{Born-type} \\
\mathcal{M}_6 &= [\,\gamma_\mu \not{p}_4 \otimes \gamma_\mu\,]\; F_6 \\
\mathcal{M}_7 &= [\,\gamma_\mu \otimes \gamma_\mu \not{p}_2\,]\; F_7 \\
\mathcal{M}_8 &= [\,\gamma_\mu \gamma_\nu \otimes \gamma_\nu \gamma_\mu\,]\; F_8 \\
\mathcal{M}_9 &= [\,\gamma_\mu \gamma_\nu \not{p}_4 \otimes \gamma_\nu \gamma_\mu\,]\; F_9
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{10} &= [\gamma_\mu \gamma_\nu \otimes \gamma_\nu \gamma_\mu \not{p}_2]\, F_{10} \\
\mathcal{M}_{11} &= [\gamma_\mu \gamma_\nu \not{p}_4 \otimes \gamma_\nu \gamma_\mu \not{p}_2]\, F_{11} \\
\mathcal{M}_{12} &= [\gamma_\mu \gamma_\nu \gamma_\rho \otimes \gamma_\rho \gamma_\nu \gamma_\mu]\, F_{12}
\end{aligned}$$

Relations between amplitudes of *one* diagram:

$$\begin{aligned}
A_3 &= A_2 \\
A_7 &= A_6 \\
A_{10} &= A_9 \\
A_8 &= \frac{1}{4} A_1 - \frac{m_e}{2} A_2 + (\ A_{12}\ ) \\
A_9 &= \frac{1}{4m_e} A_1 - \frac{1}{2} A_2 - \frac{1}{2} A_6 \\
A_{11} &= \frac{1}{4m_e^2} A_1 - \frac{1}{2m_e} A_2
\end{aligned}$$

Thus, there are at most six independent amplitudes.

All four boxes may be expressed by them.

Massive scalar one-loop functions in d dimensions

See also:

Contributions by Fleischer, Tarasov, Tentyukov, TR, Werthenbach at Loops and Legs 2002, http://www-zeuthen.desy.de/theory/radcor02/ and refs. therein

Structure of the 6 s-channel $\gamma\gamma$-box amplitudes

$$
\begin{aligned}
A_i \quad = \quad & a_0(s,t,m,d) \times A_0(m^2) \\
& + b_t(s,t,m) \times B_0(t|m^2,m^2) \\
& + b_0(s,t,m) \times B_0(s|0,0) \\
& + c_t(s,t,m) \times \frac{1}{(d-4)}\left[\frac{A_0(m^2)}{m^2} - B_0(t|m^2,m^2)\right] \qquad \leftarrow \text{IR-div.}C_0 \\
& + c_s(s,t,m,d) \times C_0(s,m^2,m^2|0,0,m^2) \\
& + d_0(s,t,m,d) \times D_0(m^2,m^2,m^2,m^2|t,s|m^2,0,m^2,0) \\
& + e_0(s,t,m)
\end{aligned}
$$

Derived using Tarasov's algorithm.

The scalar functions A_0, B_0, C_0, D_0 have to be calculated in d dimensions. See e.g. $C_0(s,m^2,m^2|0,0,m^2)$.

Have a look at $D_0(m^2,m^2,m^2,m^2|t,s|m^2,0,m^2,0)$.

The others are known, e.g. Berends, Böhm, Buza, Scharf (1994).

Main tool to calculate Feynman integrals - *integration by parts identities*:

$$\prod_{i=1}^{L} \int d^d k_i \frac{\partial}{\partial k_{r\mu}} \left\{ R_{\{\mu\}}\left(\{k\},\{p\}\right) \prod_{j=1}^{N} P^{\nu_j}_{\bar{k}_j, m_j} \right\} \equiv 0,$$

where R is an arbitrary tensor polynomial.

- 'tHooft and Veltman, NPB44 (1972) 189:
  First relation for Feynman integrals derived with *integration by parts identity* in momentum space
- Chetyrkin and Tkachov, NPB192 (1981) 159:
  The integration by parts method: After differentiation substitute
  $k_i p_l = \frac{1}{2}(k_i^2 + p_l^2 - (k_i - p_l)^2)$
- Tarasov, PRD54 (1996) 6479:
  Generalized recurrence relations with *shifted dimensions* d: After differentiation substitute

$$p_{l\mu} \int k_{i\mu} \prod_{r=1}^{L} d^d k_r \prod_{j=1}^{N} P^{\nu_j}_{\bar{k}_j, m_j} = p_{l\mu} T_\mu(\{p\}, \{\partial\}, \mathbf{d}^+) \int \prod_{r=1}^{L} d^d k_r \prod_{j=1}^{N} P^{\nu_j}_{\bar{k}_j, m_j}$$

Combining these methods gives many recurrence relations connecting integrals with different indices ν_j and shifts of d.

Problems:

- **how to use these identities in a systematic way for any integral?**
- **how many master integrals are needed for the reduction of any integral?**

Example: 3-point function for Bhabha

For **two zero masses** and final on-shell momenta we have e.g. (J.Fleischer, O.Tarasov et al.)

$$J_G = -C_0(p^2, m^2, m^2 | 0, 0, m^2) \qquad \leftarrow \text{is NOT IR-divergent !}$$

$$\frac{J_G}{\Gamma(\epsilon)} = \frac{(m^2)^{\frac{d}{2}-3}}{2(d-3)} \, {}_2F_1\left[\begin{array}{c} 1, 1\,; \\ \frac{d-1}{2}\,; \end{array} 1 - \frac{p^2}{4m^2}\right]$$

$$- \frac{\sqrt{\pi}\,\Gamma\left(\frac{d-2}{2}\right)(-p^2)^{\frac{d-4}{2}}}{2^{d-2}\Gamma\left(\frac{d-1}{2}\right) m^2} \, {}_2F_1\left[\begin{array}{c} 1, \frac{d-2}{2}\,; \\ \frac{d-1}{2}\,; \end{array} 1 - \frac{p^2}{4m^2}\right],$$

while for usual 1-loop calculations it is sufficient to work with A_0, B_0, C_0, and D_0.

These Passarino-Veltman functions lead to $\ln(X)$ and $\mathrm{Li}_2(X)$.

New techniques like recurrence relations, difference equations etc.

The scalar function D_0 in d dimensions

New approach see e.g. Tarasov, Loops'nLegs 2000, Bastei, and Fleischer, Tarasov, Tentyukov, Loops'nLegs 2002, Banz

$$
\begin{aligned}
D_0 &\equiv D_0(m^2, m^2, m^2, m^2 | t, s | m^2, 0, m^2, 0) \\
&= \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2} \int \frac{d^d k}{N_1 N_2 N_3 N_4} \\
&= \frac{2}{s} \left[C_0(t, m^2, m^2) + \int dx dy dz \ldots \right] \\
&= D0i(dd0 | m^2, m^2, m^2, m^2 | t, s | m^2, 0, m^2, 0)^{LoopTools} + \mathcal{O}(\epsilon) \\
&= \frac{2}{s} \left[\frac{-1}{\sqrt{\lambda(t, m^2, m^2)}} \ln \frac{t - 2m^2 - \sqrt{\lambda}}{t - 2m^2 + \sqrt{\lambda}} \right] \times \ln \frac{m_\gamma^2}{-s - i\varepsilon} + \mathcal{O}(\epsilon) \\
&= \text{Kampé de Feriét + Appel hypergeometric functions} \qquad (1)
\end{aligned}
$$

with $d = 4 - 2\epsilon$ and $\ln m_\gamma^2 = 1/\epsilon - \gamma_E + \ln(4\pi\mu^2)$.

Simple expression (in $d = 4$) see e.g. Berends,Gaemers,Gastmans (1973).

$$D_0 = \frac{2}{s}\left[-\Gamma(1+\epsilon)\int_0^1 dxdy(y)\frac{1}{M_{xy}^2}\left(\frac{4\pi\mu^2}{M_{xy}^2}\right)^{\epsilon}\right.$$ $\leftarrow$ conventional IR-div.C_0

$$\left. + \Gamma(1+\epsilon)\int_0^1 dxdydz(y^2z)\frac{(1+\epsilon)P^2+(1-2\epsilon)Y^2+2(1-\epsilon/2)C}{M_{xyz}^4}\left(\frac{4\pi\mu^2}{M_{xyz}^2}\right)^{\epsilon}\right]$$

where P, Y, C, M^2 depend on s, t, m^2 and x, y, z.

Alternatively, and evaluated for any d,

$$D_0 \approx \frac{\Gamma(\epsilon)}{(m^2)^{\epsilon}}\frac{4}{(d-3)s(t-4m^2)}KdF\left(-m^2z, 1-\frac{4m^2}{s}\right)$$ $\leftarrow$ Kampé de Fériet

$$- B_0(t|0,0)\frac{2}{m\sqrt{s}(t-4m^2)}F_1\left(\frac{d-3}{2}, 1, \frac{1}{2}, \frac{d-1}{2}, \frac{sz}{4}, 1-\frac{s}{4m^2}\right)$$ $\leftarrow$ Appel hypergeom.

$$-\frac{\Gamma(\epsilon)}{(m^2)^{\epsilon}}\frac{4}{s(t-4m^2)}F_2\left(\frac{d-3}{2}, 1, 1, \frac{3}{2}, \frac{d-2}{2}, \frac{s}{s-4m^2}, -m^2z\right)$$ $\leftarrow$ Appel hypergeom.

with $B_0(t|0,0) \sim \Gamma(\epsilon) \sim 1/(d-4)$ and

$$z = -4\left(\frac{1}{s}+\frac{1}{t-4m^2}\right)$$

Summary

Important steps have been undertaken in recent years for massive Feynman diagrams.
In the long term, it seems to be realistic to approach the study of many interesting massive processes.

As long as we have reason to believe in a future Linear Collider, we should go on with the study of 2-loop corrections in the Standard Model and perhaps with SUSY.