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An Effective Theory Approach to Unstable Particles

ADRIAN SIGNER

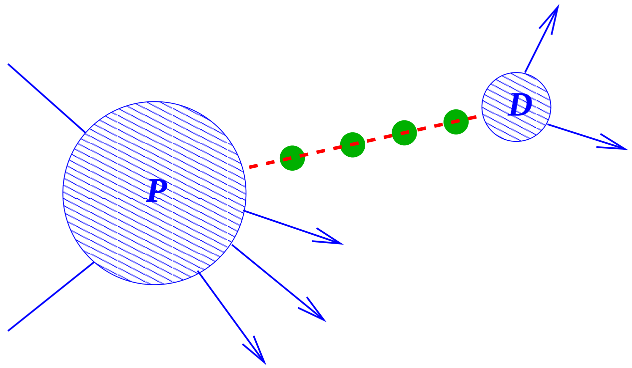
IPPP, UNIVERSITY OF DURHAM

Overview

Problem	• What's special about unstable particles ?
	• Do we have to worry?
Effective Theory	• Scales
	• Strategy of regions $\Longrightarrow$ mode expansion
	• Construction of the effective theory
Example	• Simple model $\Longrightarrow$ $\mathcal{L}$
	• Effective Lagrangian, H"Q"ET + SCET

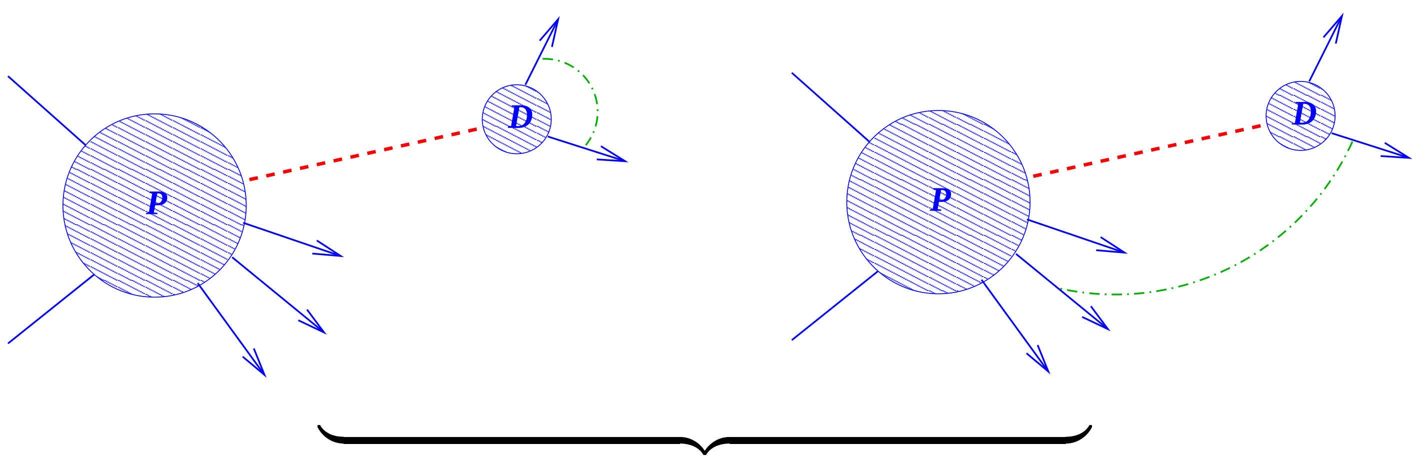
Work done in collaboration with:
M. Beneke (Aachen), A. Chapovsky (Aachen), G. Zanderighi (IPPP)

Problem



$$\mathcal{A}^{(\text{“tree”})} = P(q^2)\,\frac{1}{q^2 - M^2 - \Pi(q^2)}\,D(q^2)$$

Includes higher-order corrections but not a strict order-by-order expansion



(double) pole approximation: include only terms enhanced by resonant propagators, i.e. neglect $\mathcal{O}\left(\frac{\alpha}{\pi}\frac{\Gamma}{M}\right)$ terms. [Stuart; Aeppli et.al.]

(D)PA

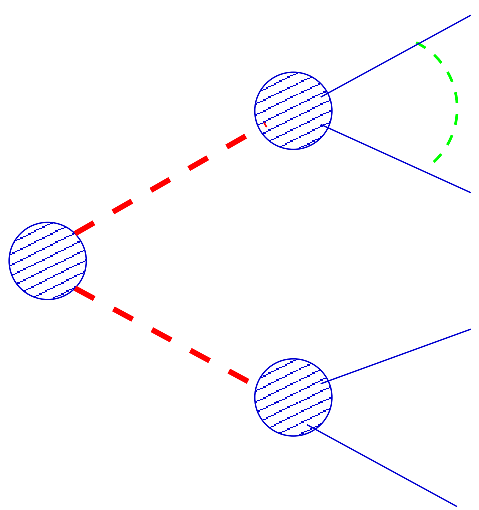
within DPA at one loop [Fadin, Khoze, Martin]

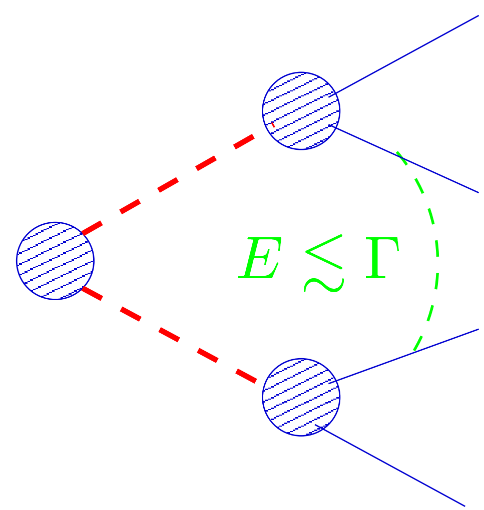
factorizable corrections

correction to production of on-shell unstable particle

non-factorizable corrections

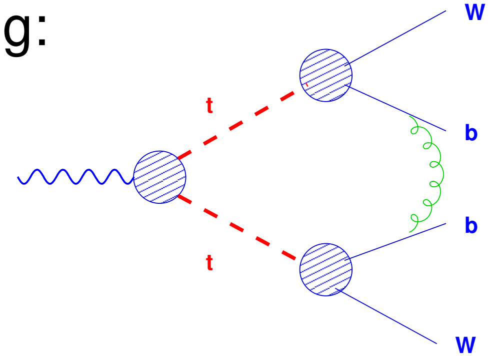
remainder





(D)PA

- Factorizable and non-factorizable corrections are separately gauge invariant
- The separation cannot be done on a diagrammatic basis
- One-loop non-factorizable corrections have been extensively studied. [Fadin, Khoze, Martin; Melnikov, Yakovlev; Beenakker, Berends, Chapovsky; Denner, Dittmaier, Roth, Wackeroth; Jadach, Placzek, Skrzypek, Ward, Was; ...]
- At Linear Collider need to go beyond, e.g: $\Delta m_t \simeq 50(?) \to 25(??)$ MeV
- Problem in QFT !



Beyond DPA

Two ways to go beyond

- higher order in α $\Longrightarrow$ beyond one loop
- higher order in Γ/M $\Longrightarrow$ beyond the pole approximation

Features

- Since $\Gamma \sim \alpha M$ the two corrections are interlinked
- Systematic expansion in two small parameters α and Γ/M
- Different scales involved $\Longrightarrow$ effective theory

Scales

process with unstable particles two scales:		
production/decay time	$t \sim 1/E$	$\Lambda_1 \sim E$
distance travelled	$r \sim 1/\Gamma \cdot E/M$	$\Lambda_2 \sim M\Gamma/E$

two small expansion parameters:

$$\alpha;\ \Lambda_2/\Lambda_1$$

$\Lambda_1 \gg \Lambda_2$ ▶ effective theory

mode	p	$D \equiv p^2 - M^2$
hard	Λ_1	Λ_1^2
res/coll	Λ_1	$\Lambda_1 \Lambda_2$
on shell	Λ_1	0
(u)soft	Λ_2	Λ_2^2

identify relevant modes

▶ “integrate out” unwanted modes

Effective Theory

underlying theory

$\mathcal{L}(\phi_h, \phi_{r/c}, \phi_s)$

dynamical modes:

hard, resonant/collinear, soft

integrate out hard modes

factorizable corrections

non-factorizable corrections

effective theory

$$\mathcal{L} = \sum_n c_n(\mathrm{h})\, O_n(\phi_{r/c}, \phi_s)$$

dynamical modes:

resonant/collinear, soft

Strategy of Regions

- loop integral: $I_{\text{full}} = \int d^dk\, f(k^2, p^2, M^2)$
- separate integral into various contributions coming from separate regions
  $\Longrightarrow$ "strategy of regions" [Smirnov]
- expand integrand: k hard, k soft, k resonant/collinear

$$\left(\int d^dk\, f(k^2, p^2, M^2)\right)_{\text{exp}} = \int d^dk\, f(k_h^2, p^2, M^2)_{\text{exp}} + \int d^dk\, f(k_s^2, p^2, M^2)_{\text{exp}} + \int d^dk\, f(k_r^2, p^2, M^2)_{\text{exp}}$$

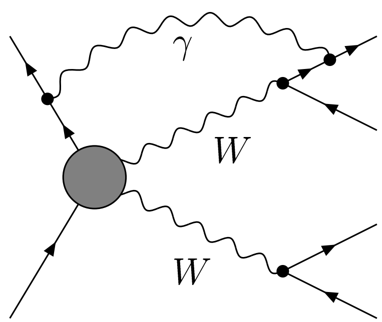
use e.g. $k_h^2 \sim M^2$, $k_s^2 \ll M^2$ in expansion of integrand

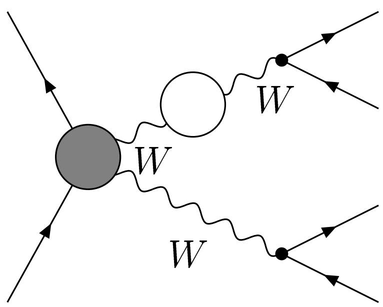
$$I_{\text{full}} \sim \underbrace{\left(\Lambda_1^2\right)^{d/2} I_{\text{hard}}}_{\text{Wilson coefficient}} + \underbrace{\left(\Lambda_2^2\right)^{d/2} I_{\text{soft}} + \left(\Lambda_1\Lambda_2\right)^{d/2} I_{\text{res}}}_{\text{dynamical d.o.f}}$$

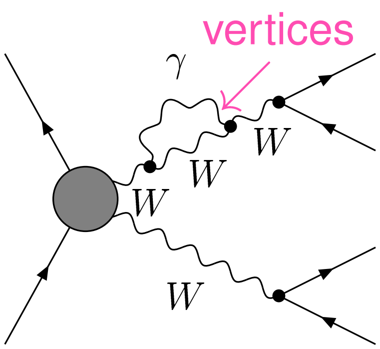
Dimensional regularization is crucial !

Examples

Examples from W-pair production [RacoonWW (Denner et.al.), Beenakker et. al. Jadach et.al.]







LO: $\frac{1}{\Lambda_1\Lambda_2}$	genuinely non-factorizable	factorizable diagram	contributes to both
hard:	$\frac{\alpha\Lambda_1^4}{\Lambda_1^2\Lambda_1^2\Lambda_1\Lambda_1} = \mathrm{LO}\,\frac{\alpha\Lambda_2}{\Lambda_1}$	$\frac{\alpha\Lambda_1^4}{\Lambda_1\Lambda_1(\Lambda_1\Lambda_2)} = \mathrm{LO}\,\frac{\alpha\Lambda_1}{\Lambda_2}$	$\frac{\alpha\Lambda_1^4\Lambda_1^2}{\Lambda_1^2\Lambda_1^2(\Lambda_1\Lambda_2)} = \mathrm{LO}\,\frac{\alpha\Lambda_1}{\Lambda_2}$
soft:	$\frac{\alpha\Lambda_2^4}{\Lambda_2^2(\Lambda_1\Lambda_2)\Lambda_2\Lambda_2} = \mathrm{LO}\,\alpha$	0	$\frac{\alpha\Lambda_2^4\Lambda_1^2}{(\Lambda_1\Lambda_2)\Lambda_2^2(\Lambda_1\Lambda_2)} = \mathrm{LO}\,\alpha$
		$I_{\mathrm{soft}} \sim \int \frac{d^dk}{k^2q^2}$	$I_{\mathrm{soft}} \sim \int \frac{d^dk}{k^2(2kq+D)}$

Model

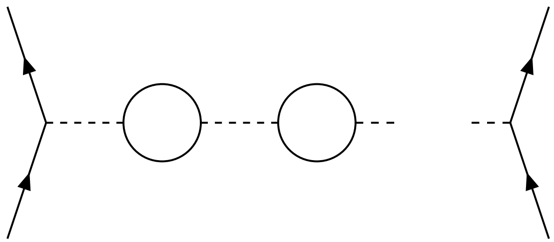
- Scalar, unstable, massive field ϕ (higgs)
- Massless, charged ψ and neutral χ fermion fields (electron and neutrino)
- Abelian gauge field A^μ (photon)

$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - M^2\phi^\dagger\phi + \overline{\psi} i \not{D}\psi + \overline{\chi} i \not{\partial}\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + y\phi\overline{\psi}\chi + y^*\phi^\dagger\overline{\chi}\psi$$

- Process: $\bar{\nu}(p_1)e^-(p_2) \to \bar{\nu}(q_1)e^-(q_2) + X$ with $s \equiv (p_1+p_2)^2 \simeq M^2$
- Expand in α and $\delta \equiv (s^2 - M^2)/M^2 \quad \Longrightarrow \quad \sim \Lambda_2/\Lambda_1$
- fermions: SCET; higgs: H"Q"ET

Double Expansion

$$\mathcal{A} = \sum_{n=0}^{\infty} \mathcal{A}^{n-\mathrm{loop}}; \quad \mathcal{A}^{n-\mathrm{loop}} = \sum_{j=0}^{\infty} \left(\frac{\alpha}{\delta}\right)^{n+1} \delta^j \, c_{n,j}$$



$$\mathcal{A} = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \left(\frac{\alpha}{\delta}\right)^{n+1} \delta^j \, c_{n,j} \equiv \sum_{j=0}^{\infty} \mathcal{A}^{(j)}$$

$\mathcal{A}^{(j)}$ well-defined for $\delta \to 0$

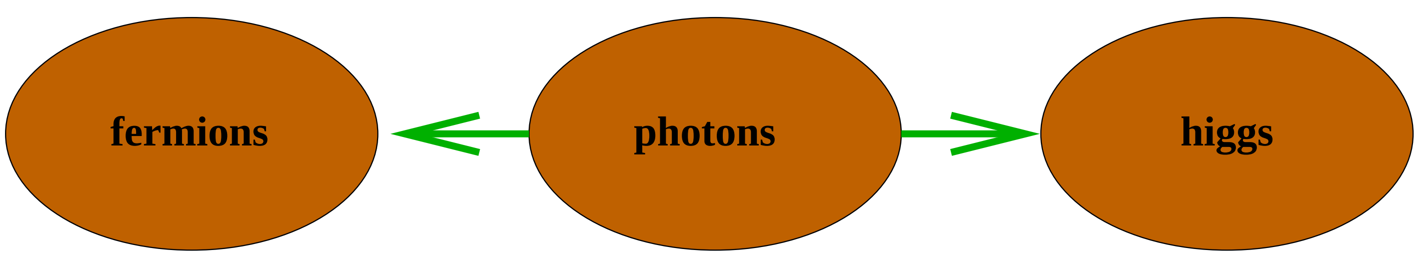
$$\begin{aligned}
\mathcal{A}^{\mathrm{tree}} &= \left(\frac{\alpha}{\delta}\right) c_{0,0} + \left(\frac{\alpha}{\delta}\right) \delta \, c_{0,1} + \left(\frac{\alpha}{\delta}\right) \delta^2 \, c_{0,2} + \ldots \\
\mathcal{A}^{1\mathrm{loop}} &= \left(\frac{\alpha}{\delta}\right)^2 c_{1,0} + \left(\frac{\alpha}{\delta}\right)^2 \delta \, c_{1,1} + \left(\frac{\alpha}{\delta}\right)^2 \delta^2 \, c_{1,2} + \ldots \\
\mathcal{A}^{2\mathrm{loop}} &= \left(\frac{\alpha}{\delta}\right)^3 c_{2,0} + \left(\frac{\alpha}{\delta}\right)^3 \delta \, c_{2,1} + \left(\frac{\alpha}{\delta}\right)^3 \delta^2 \, c_{2,2} + \ldots
\end{aligned}$$

$$\mathcal{A}^{(0)} \quad + \quad \mathcal{A}^{(1)} \quad + \quad \mathcal{A}^{(2)} \quad + \ldots$$

LO NLO NNLO

Modes

Soft-Collinear Effective Theory		+	Heavy "Quark" Effective Theory	
fermions			higgs	
$p^\mu = (n_+p)\frac{n_-}{2} + (n_-p)\frac{n_+}{2} + p_\perp$			$q^\mu = Mv^\mu + k^\mu; \quad q_\perp = q^\mu - v^\mu(qv)$	
$n_\pm^2 = 0, \quad n_+n_- = 2$			$v^\mu \equiv (q_1^\mu + q_2^\mu)/\sqrt{s}, \quad v^2 = 1$	
hard:	$p \sim M$		hard:	$k^\mu \sim M$
(u)soft:	$p \sim M\delta$		resonant:	$k^\mu \sim \delta$
collinear:	$p_\perp \sim M\delta^{1/2};\ n_+p \sim M;\ n_-p \sim M\delta$			



The $\phi^\dagger\phi$ Operator

propagator: $\dfrac{i}{s - M^2 - \Pi(s)}$

hard part of self energy: $\Pi_h(s) = M^2 \sum_{n,j} \alpha^n \delta^j \, \Pi_h^{(n,j)}$

position of pole: $\bar{s} = M^2 - iM\Gamma$

define: $\phi_v(x) \equiv e^{iMvx}\phi(x)$; $\Delta \equiv \dfrac{\bar{s} - M^2}{M^2}$

$$
\begin{aligned}
\mathcal{L} &= D_\mu \phi^\dagger D^\mu \phi - M^2 \phi^\dagger \phi \\
&\rightarrow \phi_v^\dagger \left(2M\,(vk) + (vk)^2 + k_\perp^2 - M\Delta\right) \phi_v \\
&\rightarrow \underbrace{\phi_v^\dagger \, 2M \left((vk) - \frac{\Delta}{2}\right) \phi_v}_{\mathcal{O}(\alpha,\delta)\ LO} + \underbrace{\phi_v^\dagger \left(k_\perp^2 + (vk)\Delta - \frac{\Delta^2}{4}\right) \phi_v}_{\mathcal{O}(\alpha^2,\delta^2)\ NLO} + \ldots
\end{aligned}
$$

$\mathcal{L}$ describes the propagation of resonant ϕ-fields and their interactions with soft gauge bosons. Need $\Pi_h^{(1,0)}$ for LO and $\Pi_h^{(2,0)}$, $\Pi_h^{(1,1)}$ for NLO.

The $\overline{\psi}\psi A^\mu$ Operator

For each direction n_- of an external fermion we have operators
$\overline{\psi}_c \psi_c A_c,\ \overline{\psi}_c \psi_c A_s,\ \overline{\psi}_c \psi_s A_c + \mathrm{h.c.}$
define: $\psi_c = \xi + \eta; \quad \xi \equiv \frac{\not{n}_- \ \not{n}_+}{4}\psi; \quad \eta \equiv \frac{\not{n}_+ \ \not{n}_-}{4}\psi;$
scaling: $\xi \sim \delta^{1/2}; \quad \eta \sim \delta; \quad \psi_s \sim \delta^{3/2}$

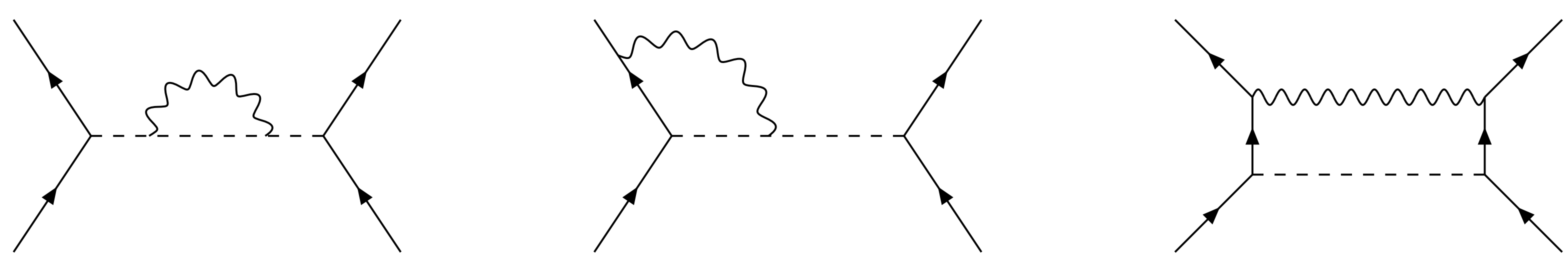
$$\begin{aligned}
\mathcal{L} &= \overline{\psi}_c \, i \not{D} \psi_c \\
&= \bar{\xi}\frac{\not{n}_+}{2} i n_- D \xi + \bar{\eta}\frac{\not{n}_-}{2} i n_+ D \eta + \bar{\xi} i \not{D}_\perp \eta + \bar{\eta} i \not{D}_\perp \xi \\
&\to \bar{\xi}\left(i n_- D + \not{D}_{\perp\, c} \frac{i}{n_+ D_c} \not{D}_{\perp\, c} \right) \xi
\end{aligned}$$

equation of motion $\eta = -\frac{\not{n}_+}{2}\frac{1}{n_+ D}\not{D}_\perp \xi$
(Ultra)soft quarks can be included, but they are further suppressed.
[Bauer, Fleming, Pirjol, Stewart, Beneke, Chapovsky, Diehl, Feldmann]

The Effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm eff} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \phi_v^\dagger\, 2M\left((vD) - \frac{\Delta}{2}\right)\phi_v + \phi_v^\dagger\left(D_\perp^2 + (vD)\Delta - \frac{\Delta^2}{4}\right)\phi_v \\ &+ \overline{\psi}_{\xi,c}\left(in_- D + \not{D}_{\perp\,c}\frac{i}{n_+ D_c}\not{D}_{\perp\,c}\right)\psi_{\xi,c} + \overline{\chi}_{\xi,c}\left(in_-\partial + \not{\partial}_{\perp\,c}\frac{i}{n_+\partial_c}\not{\partial}_{\perp\,c}\right)\chi_{\xi,c} \\ &+ y\, c_{\phi\psi\chi}\phi\bar{\psi}\chi + c_{\psi\psi\chi\chi}\bar{\psi}\chi\bar{\chi}\psi + \ldots\end{aligned}$$

Some examples of $\mathcal{O}(\alpha^2)$ gauge independent / dependent contributions of the full theory

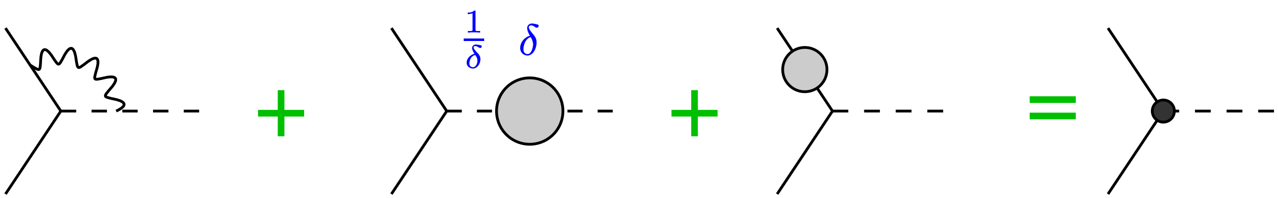


hard	$\alpha^2\delta^{-2} + \alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^0 + \ldots$
soft	$\alpha^2\delta^{-2} + \alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$

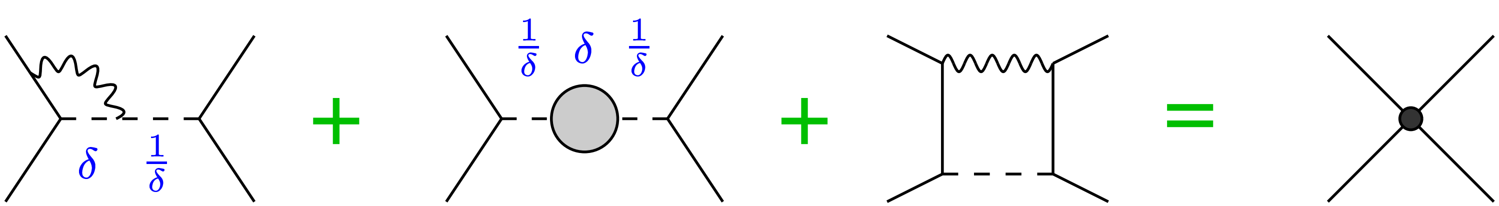
Matching

Matching of $c_{\phi\psi\chi}$

$$= 1 + \frac{\alpha_y}{4\pi}\left[-\frac{5}{4} + \frac{3}{2}\ln\left(\frac{M^2}{\mu^2}\right)\right] - i\,\alpha_y\frac{1}{4} + \frac{\alpha_g}{4\pi}\left[-\frac{1}{\bar{\epsilon}^2}\left(\frac{M^2}{\mu^2}\right)^{-\bar{\epsilon}} - \frac{5}{2\bar{\epsilon}} + \ln\left(\frac{M^2}{\mu^2}\right) - 2 - \frac{\pi^2}{12}\right]$$

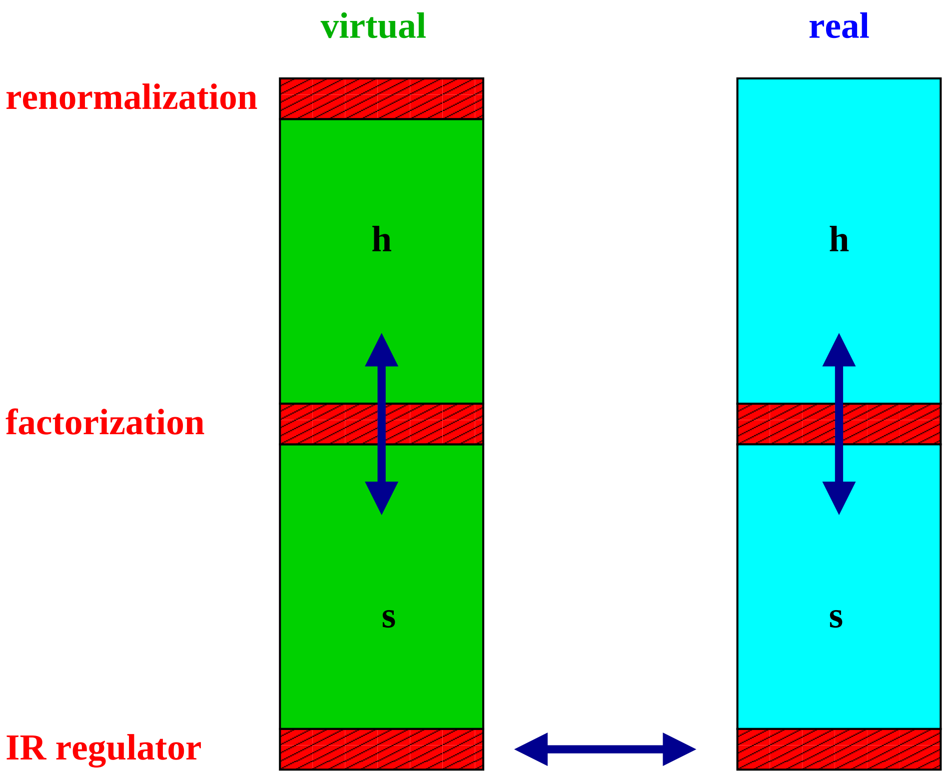


Matching of $c_{\phi\phi\chi\chi}$ (contributes at NNLO)



gauge dependence cancels

Singularities



- requires mode expansion of phase space integrals
- relate PS integrals to loop integrals
- do expansion of PS integrals separately

Conclusions

- Systematic double expansion in α and Γ/M using an effective theory approach
- combination of H"Q"ET and SCET
- strategy of regions $\Longrightarrow$ mode expansion
  - hard contributions $\rightarrow$ Wilson coefficients $\sim$ factorizable corrections
  - soft contributions $\rightarrow$ dynamical d.o.f.$\sim$ non-factorizable corrections
- gauge invariance automatic

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- more complicated/realistic models
- real radiation $\leftrightarrow$ phase-space integration
- compare to "standard" implementations, estimate importance of effects