# Dynamic Effects in Superconducting Magnets


Animesh Jain

*Brookhaven National Laboratory*

Upton, New York 11973-5000, USA


US Particle Accelerator School on *Superconducting Accelerator Magnets*

Santa Barbara, California, June 23-27, 2003

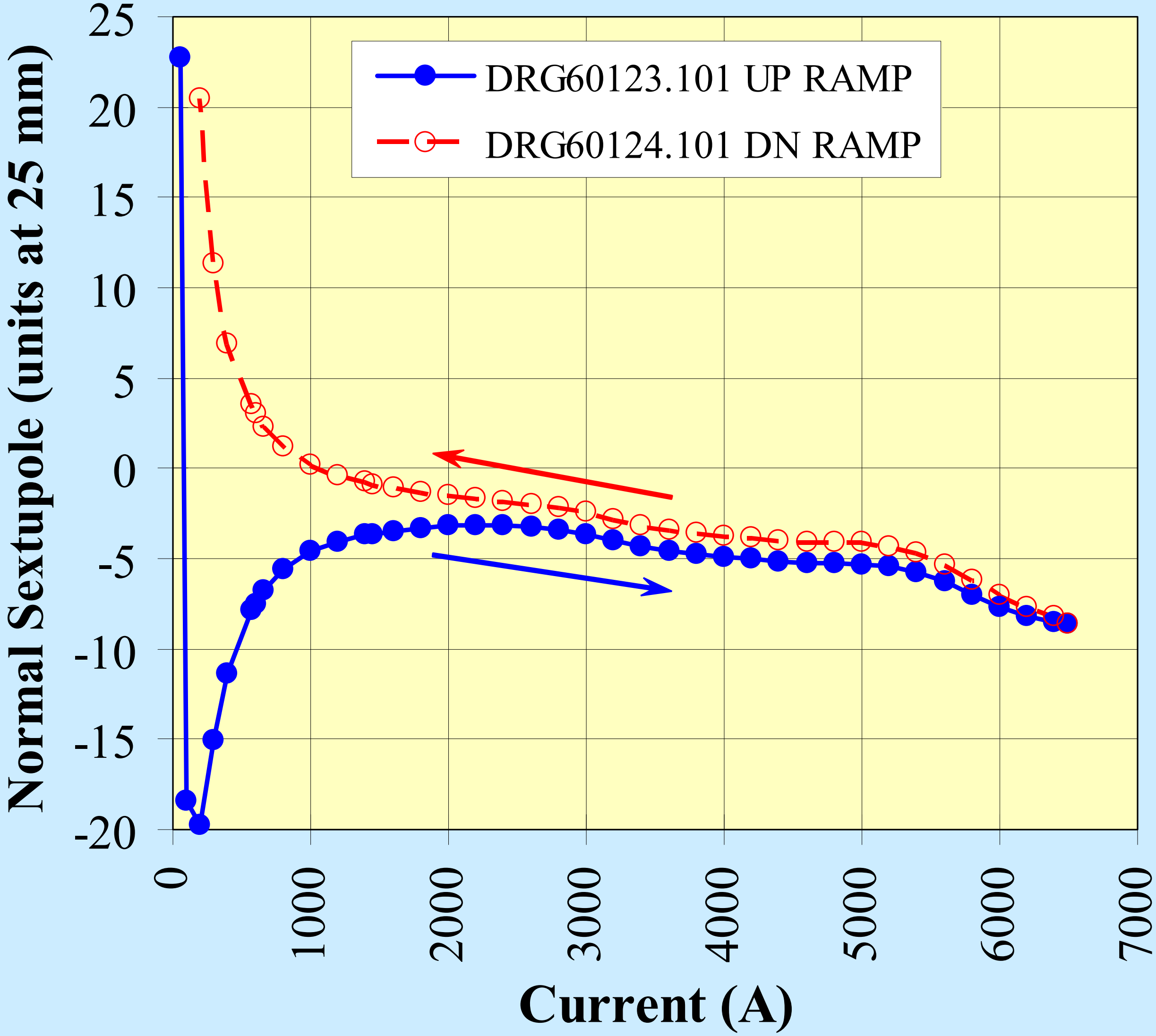
# Introduction

- Any change in the field near a superconductor induces eddy currents. These eddy currents are of a persistent nature due to zero resistance in the superconductor.
- Such eddy currents produce field distortions (harmonics) depending on several factors, such as superconductor properties, ramp direction, ramp rate, etc.
- An understanding of these effects is important in the measurements of superconducting magnets.

# Hysteresis in Harmonics

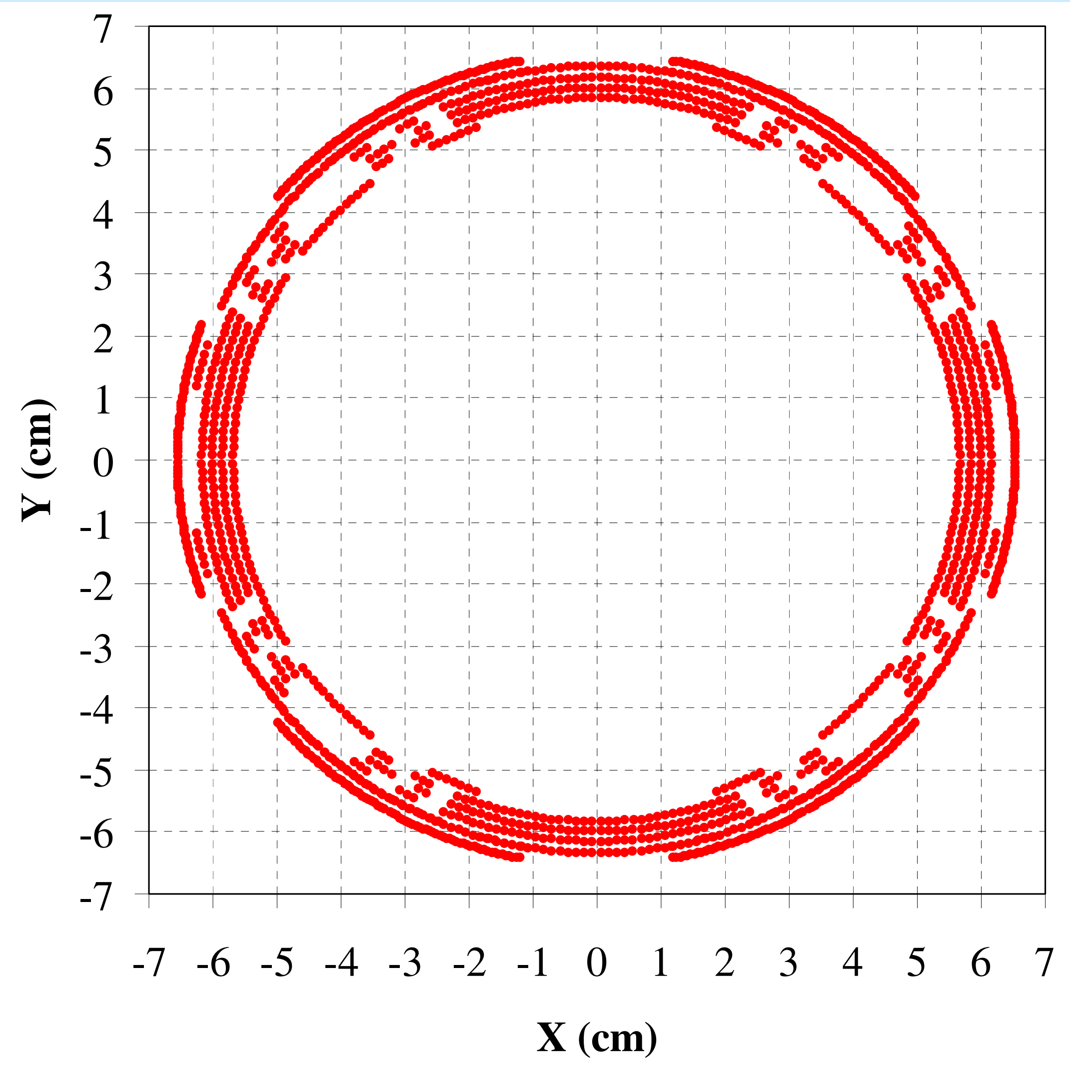
- The dependence of eddy currents on the ramp direction produces a ***hysteresis*** in the harmonics.
- Generally, the eddy currents have symmetries similar to the main field harmonic. Thus, only the allowed harmonics are commonly affected.
- For multilayer magnets with several multipoles, (e.g. corrector packages) the external field may have a symmetry different from the main field. In this case, hysteresis may be seen in some unallowed harmonics also.

# Hysteresis in an Allowed Term




In a dipole magnet, a large negative sextupole is produced at low fields on the UP ramp. This changes to a large positive sextupole on the DOWN ramp.

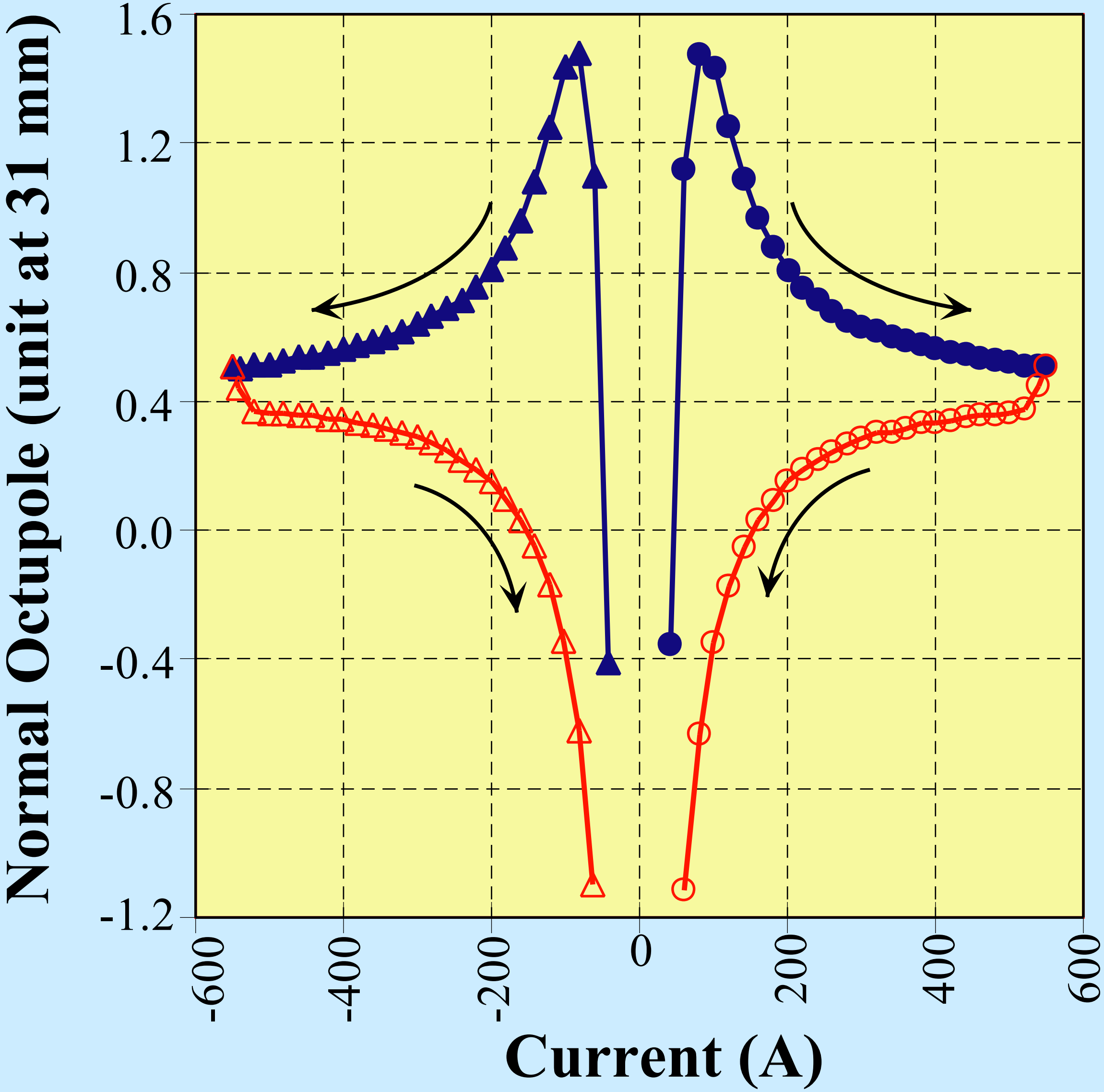
# Hysteresis in an Unallowed Term




Cross section of a quadrupole magnet built at BNL for HERA, DESY. The magnet has concentric layers of normal and skew dipole, normal and skew quadrupole and normal sextupole,

Field seen by conductors in a given layer does not necessarily have the symmetry of that layer

# Hysteresis in an Unallowed Term

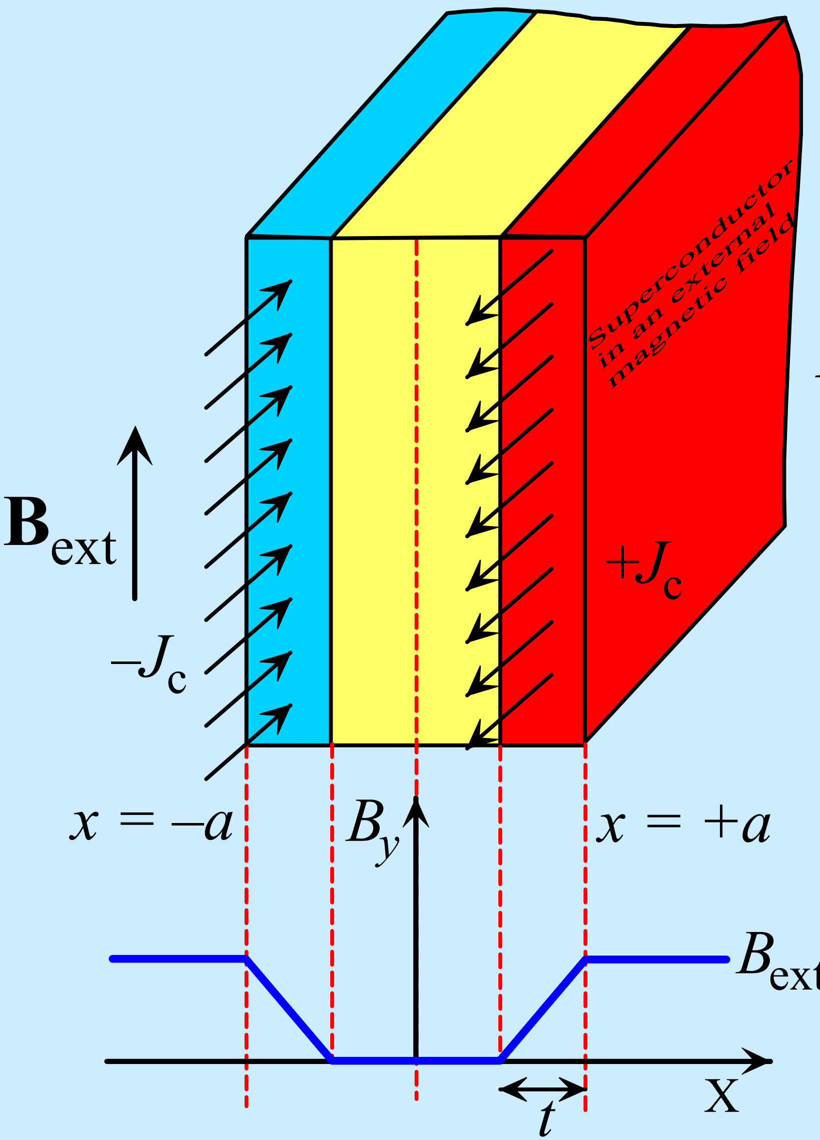



Hysteresis in the octupole term in the quadrupole layer of a multi-layer magnet consisting of several concentric layers of normal and skew multipole magnets.

# Critical State Model

- The hysteresis in certain harmonics resulting from the ramp direction dependence of eddy currents can be understood qualitatively by a simple model known as the ***Critical State Model***.
- In this model, it is assumed that eddy currents, with a density equal to the critical current density, $J_c$, are induced in the superconductor to counteract any change in the external field.

# Superconductor in an External Field



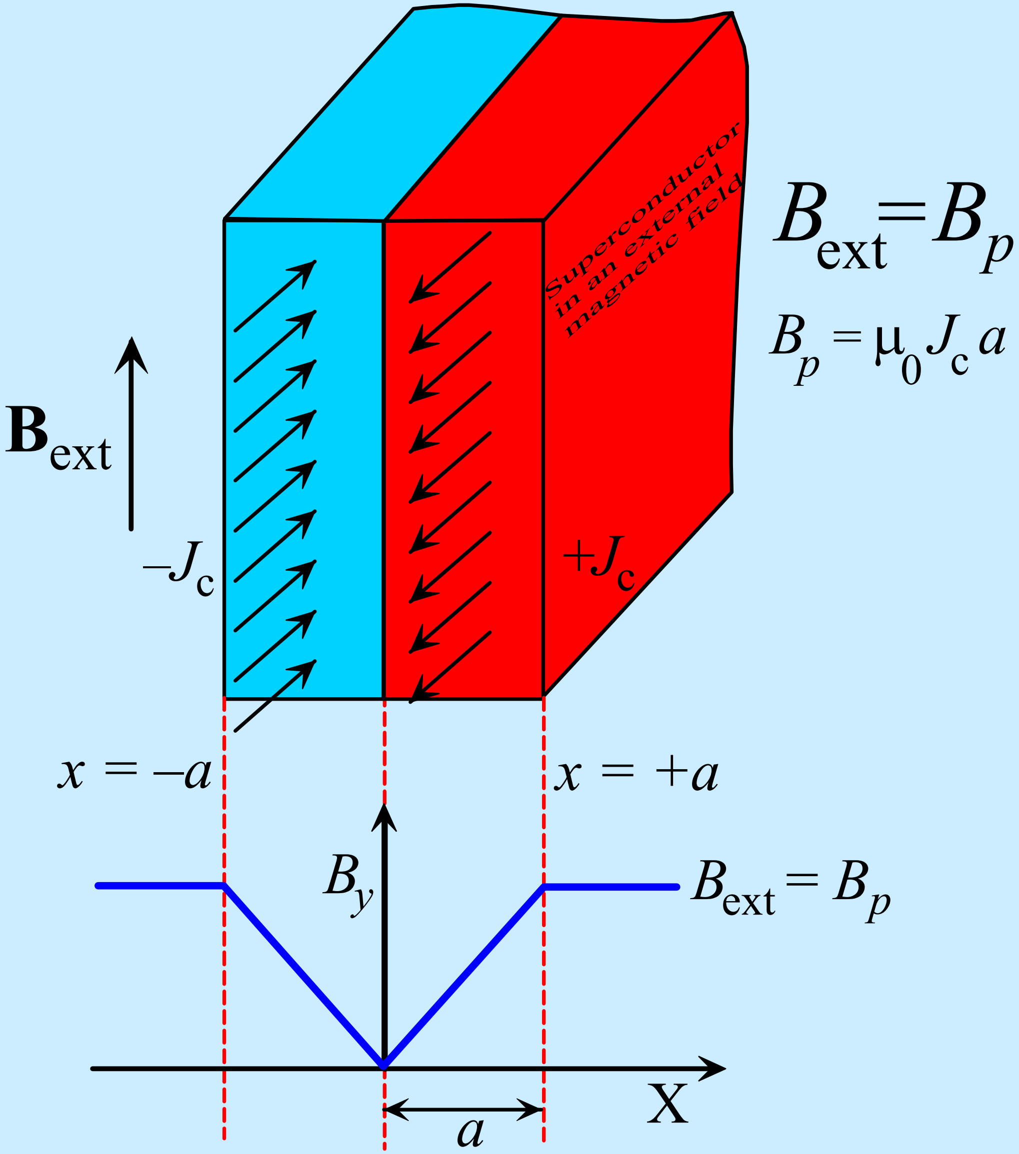

$$B_{\text{ext}} < B_p$$

$$B_p = \mu_0 J_c a$$

$$t = \left(\frac{B_{\text{ext}}}{\mu_0 J_c}\right)$$

For **low external fields** (less than the Penetration Field), persistent currents are localized within a thickness, $t$, sufficient to shield the interior from the external field.

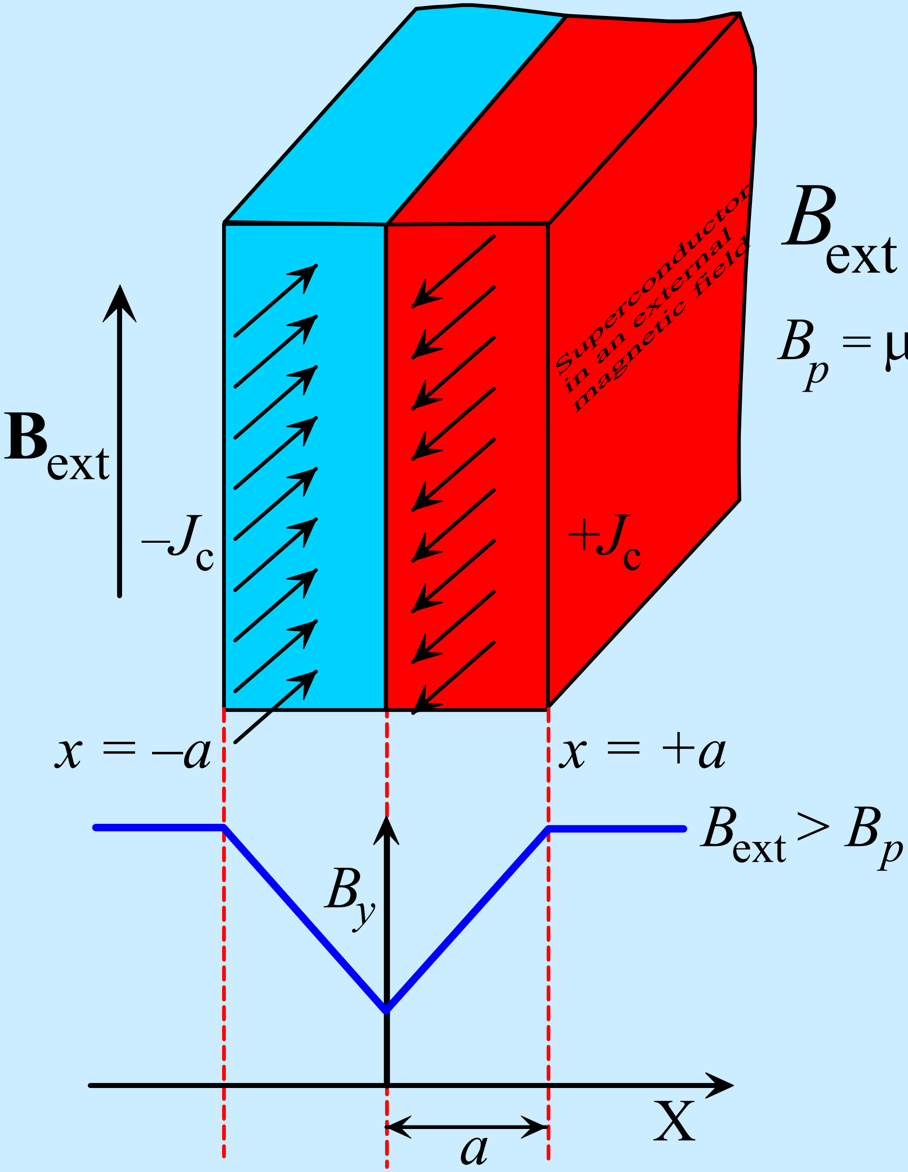
# Superconductor in an External Field




At a certain field (the Penetration Field), persistent currents span the entire volume of the superconductor.

The field at the center is still zero.

# Superconductor in an External Field

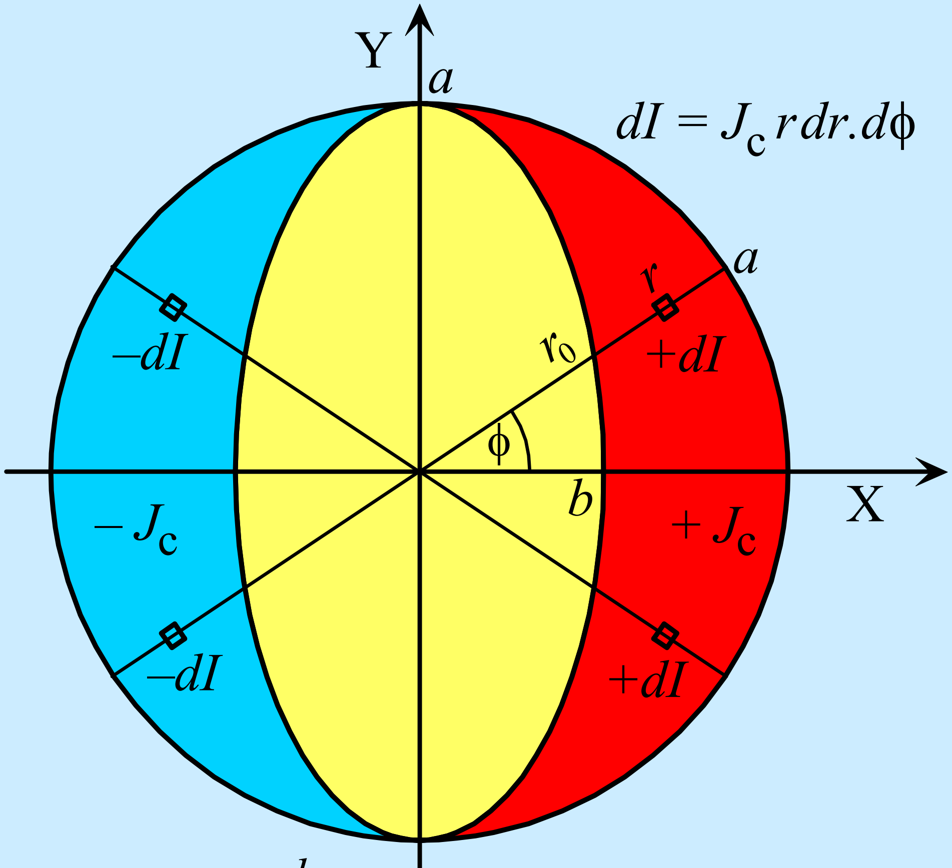



$$B_{ext} > B_p$$

$$B_p = \mu_0 J_c a$$

At external fields **higher than the Penetration Field**, persistent currents span the entire volume of the superconductor, but the interior is not completely shielded.

# Filament Magnetization: 1st Up Ramp



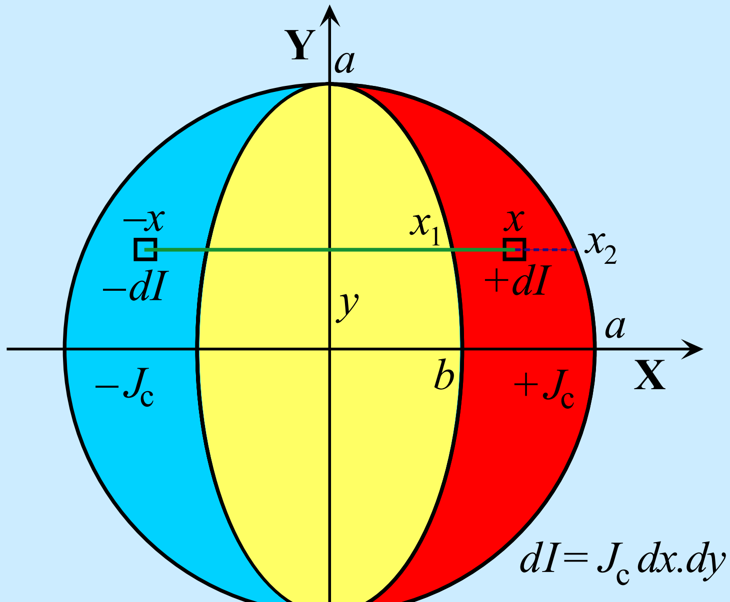

$$r_0 = \frac{b}{\sqrt{\cos^2\phi + e^2\sin^2\phi}}; \; e = b/a = \cos\alpha$$

When the external field is increased from zero to a small value, $B_a$, shielding currents are set up such that the field inside is zero. The boundary of shielding currents may be approximated by an ellipse.

$$\left(\frac{B_a}{B_p}\right) = \left[1 - \frac{\cos\alpha}{(\sin\alpha)/\alpha}\right]$$

$$B_p = \left(\frac{2\mu_0 J_c a}{\pi}\right); \quad B_a \le B_p; \quad 0 \le \alpha \le \pi/2$$

# Magnetic Moment: 1st Up Ramp




$x_1 = b\sqrt{1-(y^2/a^2)}$ $\quad x_2 = \sqrt{a^2-y^2}$

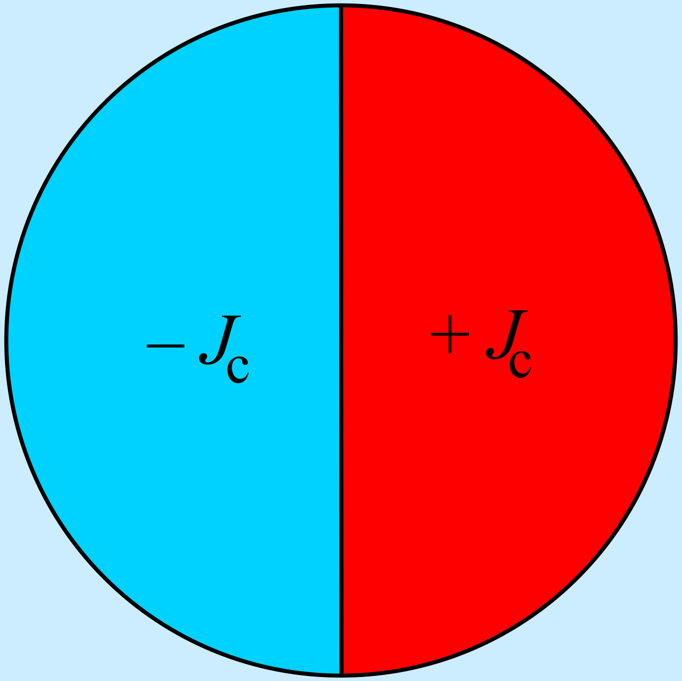
The persistent currents produce a magnetic moment. The Magnetization, or the magnetic moment per unit volume, can be calculated by integrating over elemental loops.

$$M = \frac{-2\mu_0 J_c}{\pi a^2}\int_{-a}^{a} dy \int_{x_1}^{x_2} x\,dx = -\left(\frac{4}{3\pi}\right)\mu_0 J_c\, a\left[1-\frac{b^2}{a^2}\right]$$

$$\left|M_{\text{peak}}\right| = \left(\frac{4}{3\pi}\right)\mu_0 J_c\, a$$

Magnetization is proportional to the critical current density, and the filament diameter.

# Full Magnetization: Up Ramp




$$B_{\text{applied}} \geq B_p$$

$$B_{\text{inside}} = B_{\text{applied}} - B_p$$

$$\left(\frac{M}{\left|M_{\text{peak}}\right|}\right) = -1$$

Example:

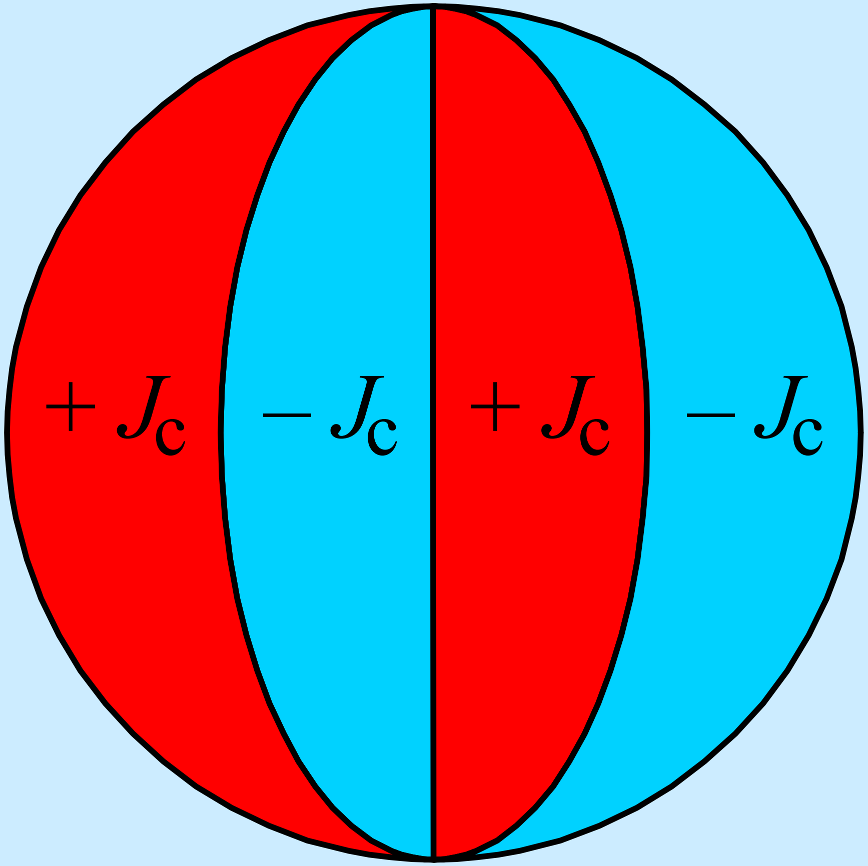
$J_c$ = 2 x 10$^4$ A/mm$^2$
dia. = 2*a* = 6 μm

$B_p$ = 0.048 Tesla

$$B_p = \left(\frac{2\mu_0 J_c\, a}{\pi}\right); \ \left|M_{\text{peak}}\right| = \left(\frac{4}{3\pi}\right)\mu_0\, J_c\, a$$

At fields above the penetration field, the filament is fully magnetized. In this simple model, the magnetization does not change as the field is increased further.

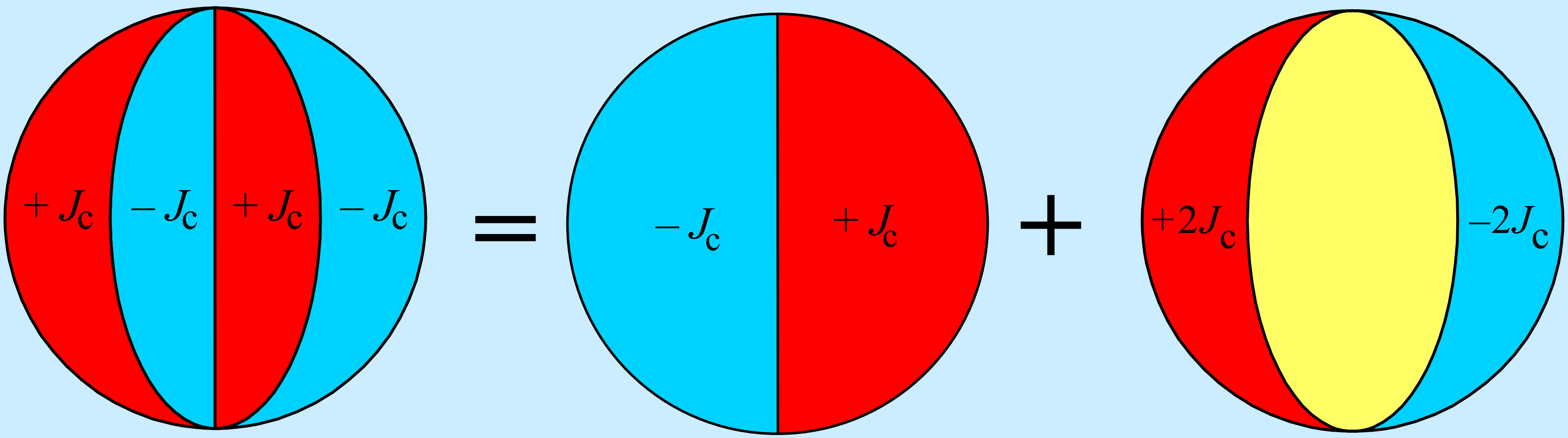
# Magnetization on the Down Ramp




When the field is reduced from the maximum field by an amount $B_0$, new shielding currents are induced, which are opposite in sign to the earlier currents.

The geometry of the new shielding currents is such that a field of $+B_0$ is produced inside the filament.

# Magnetization on the Down Ramp




New distribution at $B_a$ = Distribution at $B_{max}$ + New shielding currents **(Density = 2x$J_c$)**

$$B_a = B_{max} - B_0$$

$$M/|M_{peak}| = -1$$

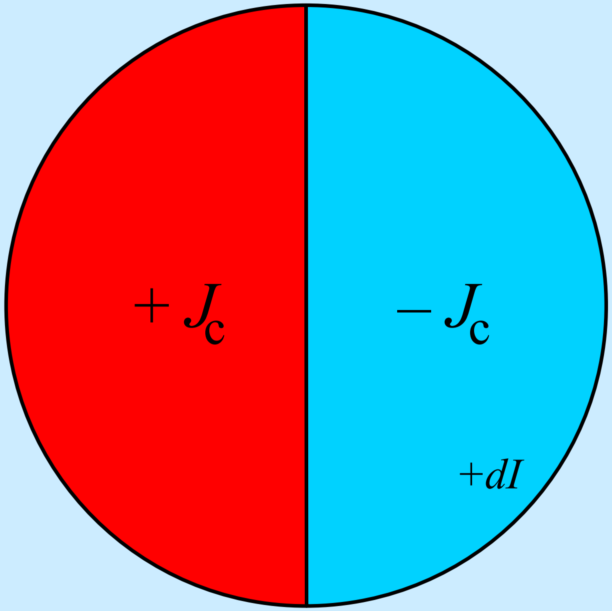
$$|M_{peak}| = \left(\frac{4}{3\pi}\right)\mu_0 J_c a$$

$$B_0 = \left(\frac{4\mu_0 J_c a}{\pi}\right)\left[1 - \frac{\cos\alpha}{(\sin\alpha)/\alpha}\right]$$

$$M = \left(\frac{8}{3\pi}\right)\mu_0 J_c a \sin^2\alpha$$

$$\cos\alpha = b/a$$

# Full Magnetization: Down Ramp



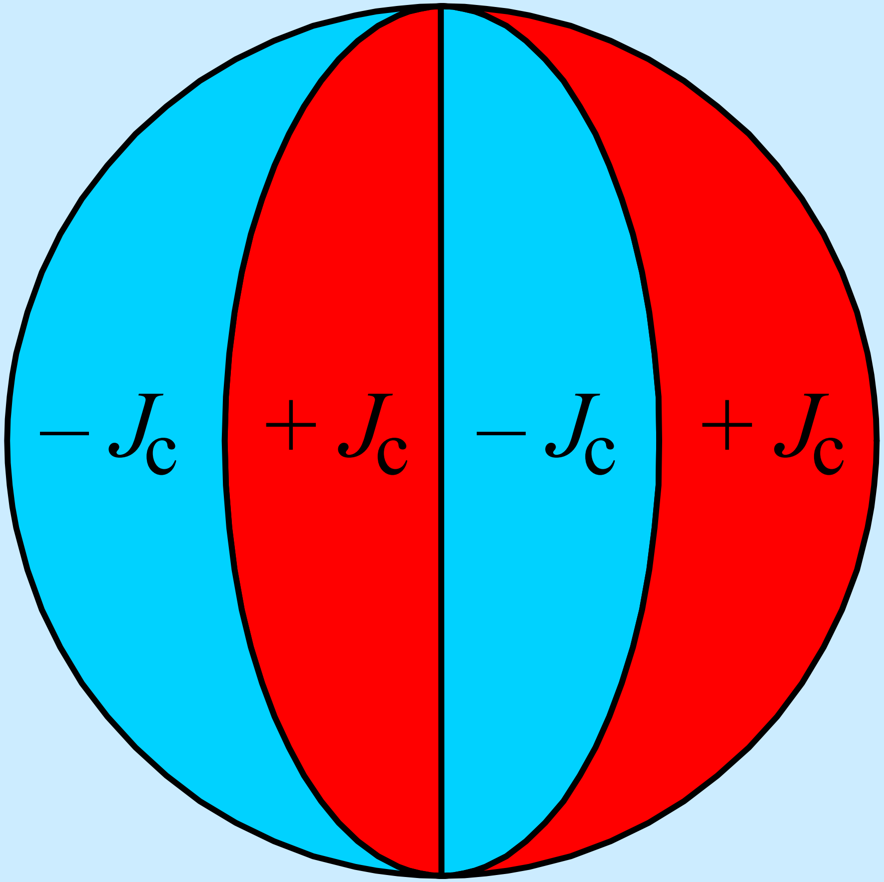

$$B_a \le B_{max} - 2B_p$$

$$\left(\frac{M}{|M_{peak}|}\right) = +1$$

Starting from an unmagnetized state, the filament ends up with a magnetization of $+M_{peak}$ after a cycle to $B_{max}$ and back.

As the field is reduced from a certain maximum value, $B_{max}$, the superconducting filaments are fully magnetized again in the opposite direction. This happens at a field of $B_{max}$ - $2B_p$. This continues until the minimum field, $B_{min}$, as long as the direction of field change is not reversed.

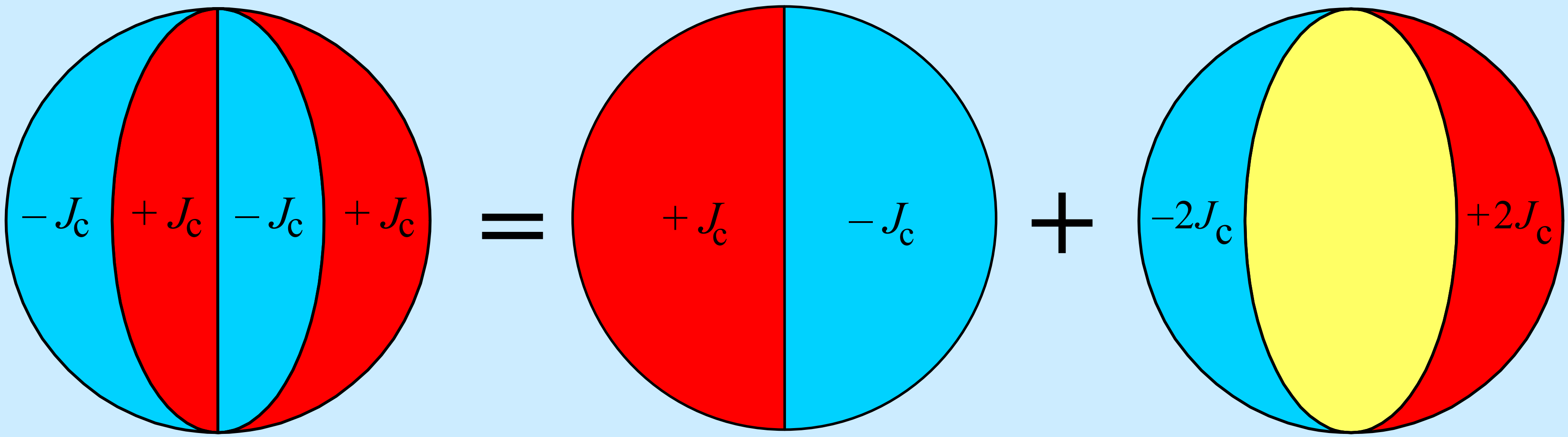
# Magnetization on the 2nd Up Ramp




If the field is increased again after reaching $B_{min}$, new shielding currents are induced, which are opposite in sign to the earlier currents.

If the applied field is $B_{min}+B_0$, the geometry of the new shielding currents is such that a field of $-B_0$ is produced inside the filament.

# Magnetization on the 2nd Up Ramp




New distribution at $B_a > B_{min}$ = Distribution at $B_{min}$ + New shielding currents **(Density = 2x$J_c$)**

$$B_a = B_{\min} + B_0$$

$$M / |M_{\text{peak}}| = +1$$

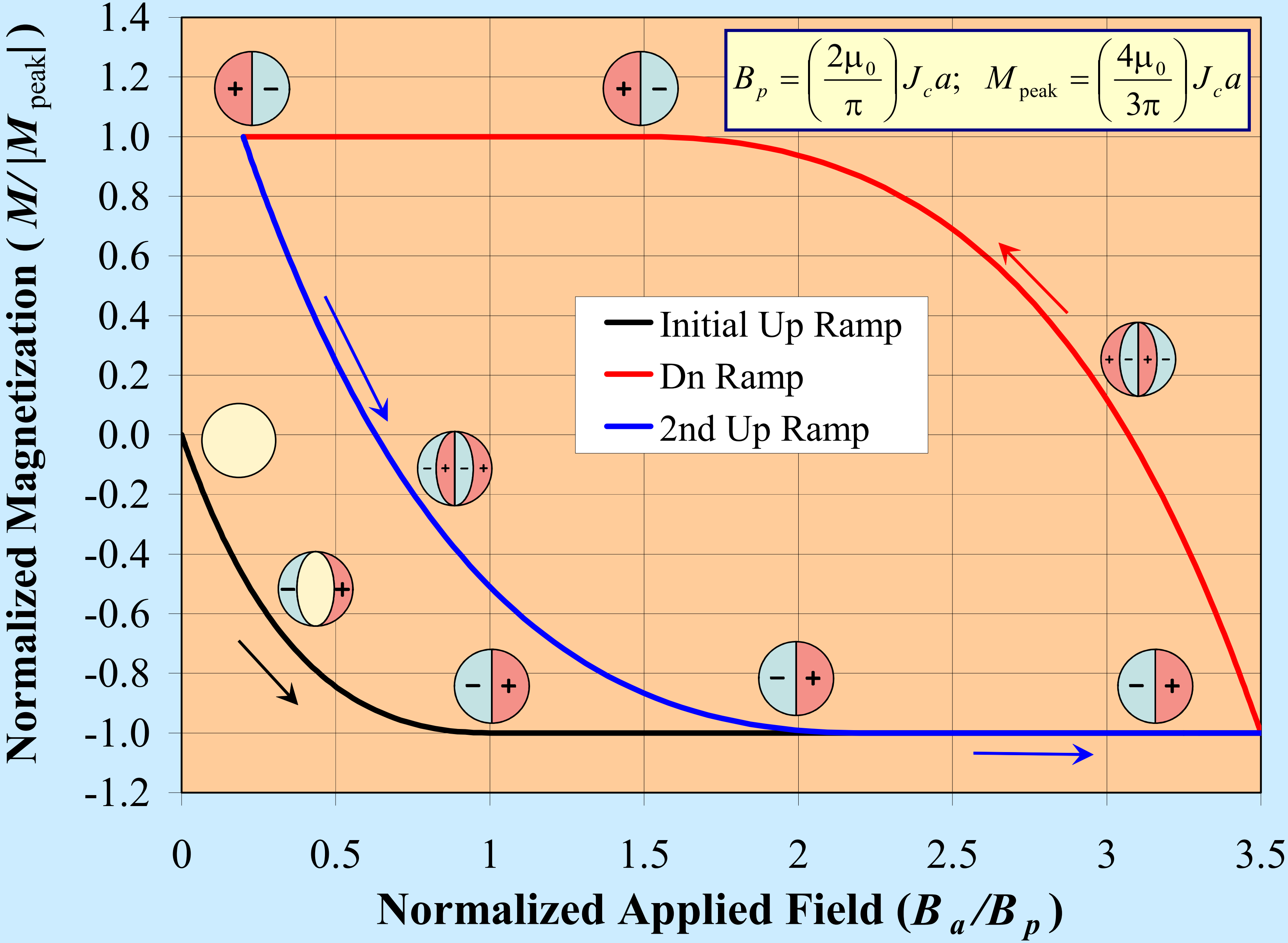
$$|M_{\text{peak}}| = \left(\frac{4}{3\pi}\right) \mu_0 J_c a$$

$$B_0 = \left(\frac{4\mu_0 J_c a}{\pi}\right)\left[1 - \frac{\cos\alpha}{(\sin\alpha)/\alpha}\right]$$

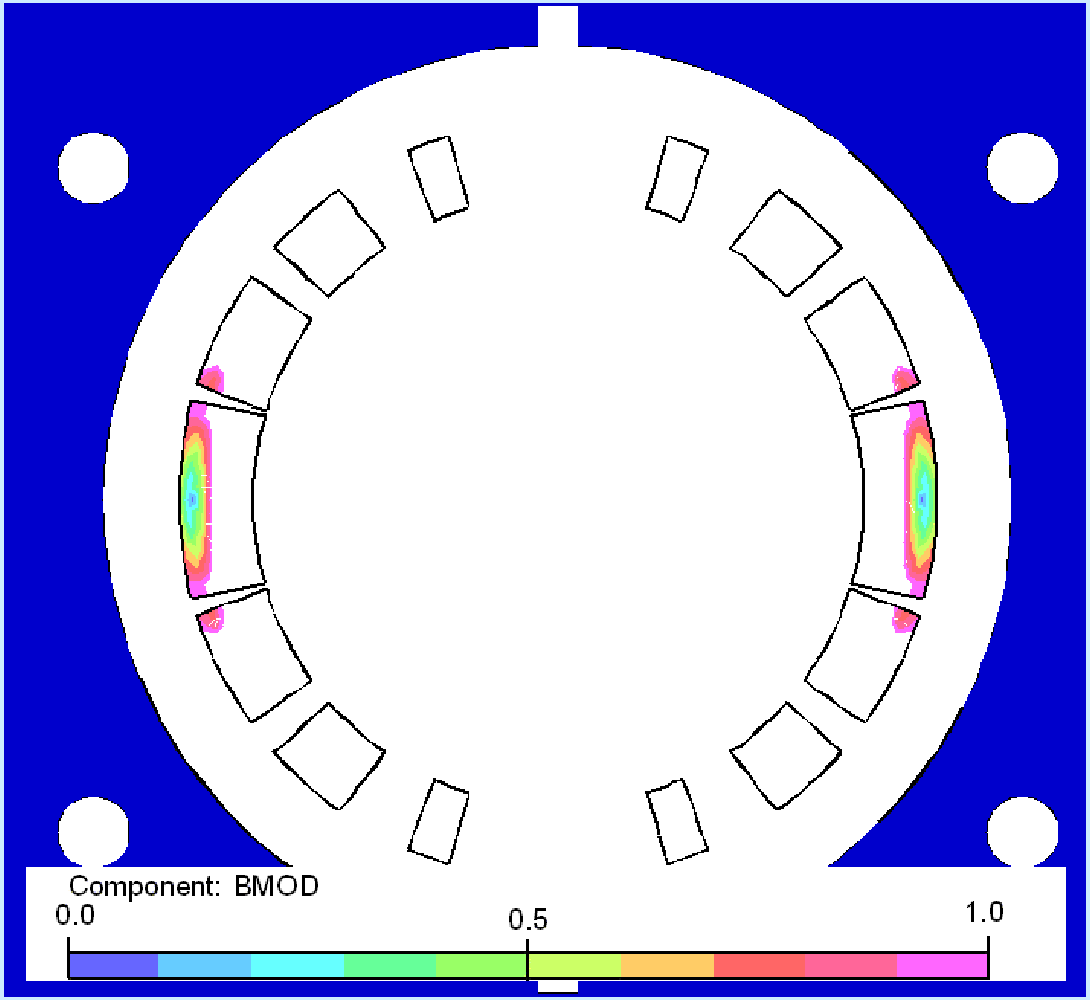
$$M = -\left(\frac{8}{3\pi}\right) \mu_0 J_c a \sin^2\alpha$$

$$\cos\alpha = b / a$$

# Magnetization Vs. Applied Field



# Spatial Variation of Magnetization




Even at the maximum field, some regions of coil may still be well below the penetration field.

Reversing current ramp direction can create a considerably complex shielding current pattern in such regions.

*Regions of magnet coil with field less than 1 Tesla in a RHIC arc dipole at its maximum operating field of 3.45 T. Some regions in the midplane turns can be below the penetration field.*

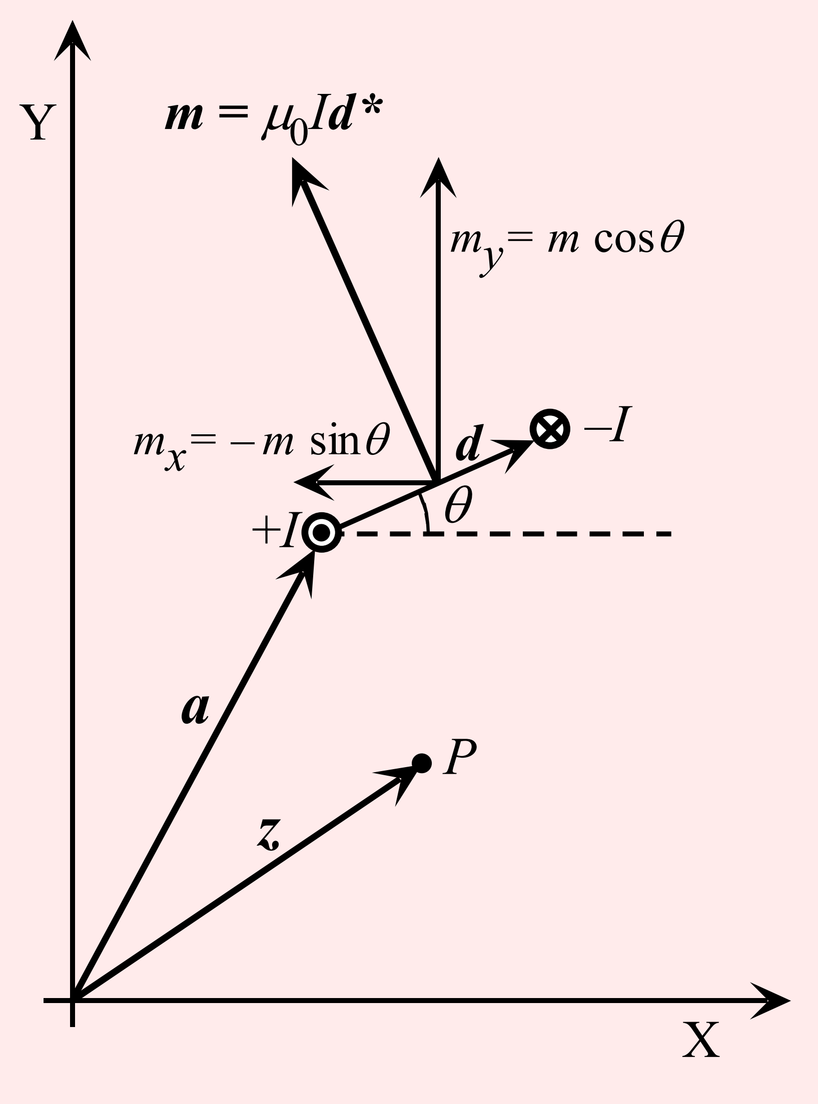
# Calculating Harmonics from Persistent Currents

- Divide the entire magnet coil cross section into a suitable number of small segments. For example, each turn can be represented by $N$ strands, uniformly distributed, where $N$ is approximately equal to the actual number of strands in the cable.
- Calculate the local field (magnitude and direction) at each coil segment due to the transport current in the magnet.
- Calculate the critical current density, $J_c$, at each coil segment, based on experimental data and/or empirical parameterization.

# Harmonics from Persistent Currents (Contd.)

- Calculate the magnetic moment of each segment, based on excitation history, copper to superconductor ratio, and filament diameter. The magnetization of each segment is also scaled by a factor $\left(1 - I_{segment} / I_c^{segment}\right)$, where $I_{segment}$ is the transport current carried by the segment and $I_c^{segment}$ is the critical current for that segment. A simple model, such as one based on elliptical boundaries, can be used, although more complex algorithms have also been used.
- Calculate the harmonics produced at the center of the magnet by the magnetic moment of each segment.

# Field due to a Magnetic Moment




Complex magnetic moment, $\boldsymbol{m}$, is:

$$\boldsymbol{m} = m_y + i m_x$$

$$= \mu_0\, I d\,(\cos\theta - i\sin\theta) = \mu_0\, I \boldsymbol{d}^*$$

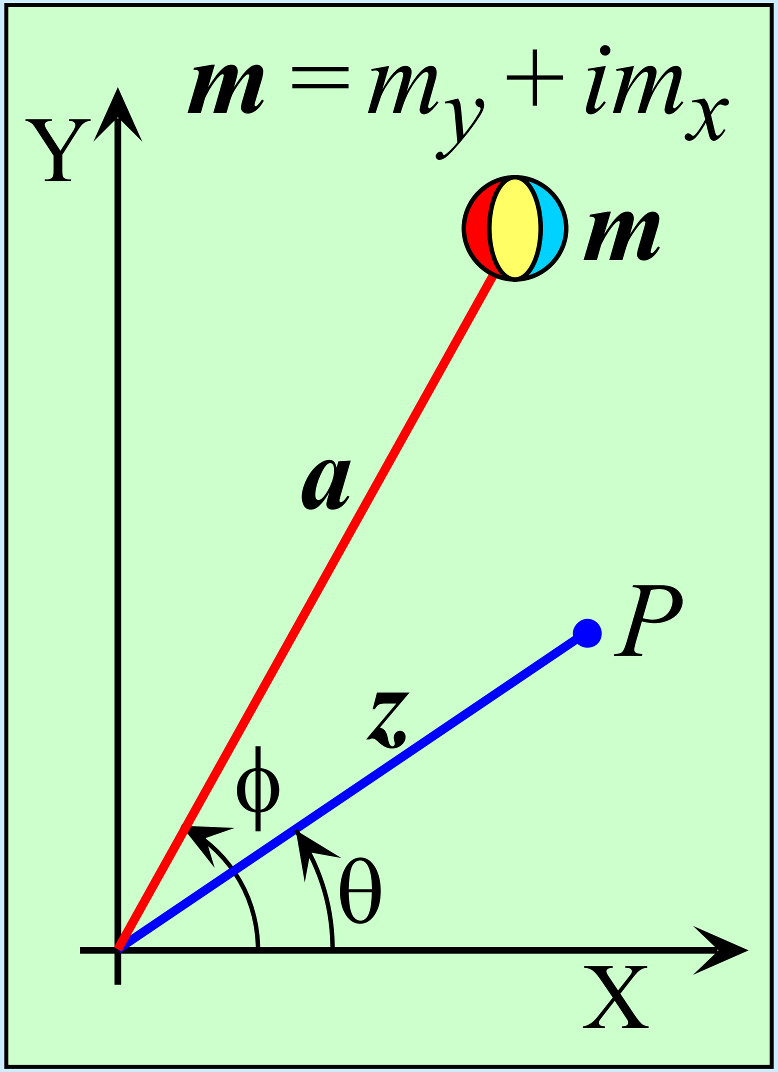
Complex field at point $P$ is:

$$\boldsymbol{B}(\boldsymbol{z}) = \underset{d\to 0}{Lt}\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{1}{\boldsymbol{z}-\boldsymbol{a}} - \frac{1}{\boldsymbol{z}-\boldsymbol{a}-\boldsymbol{d}}\right]$$

$$= \underset{d\to 0}{Lt}\left(\frac{\mu_0 I}{2\pi}\right)\frac{-\boldsymbol{d}}{(\boldsymbol{z}-\boldsymbol{a})(\boldsymbol{z}-\boldsymbol{a}-\boldsymbol{d})}$$

$$\boxed{\boldsymbol{B}(\boldsymbol{z}) = \frac{-\boldsymbol{m}^*}{2\pi(\boldsymbol{z}-\boldsymbol{a})^2}}$$

# Harmonics due to a Magnetic Moment




Field at the point *P* is given by:

$$\boldsymbol{B}(\boldsymbol{z}) = -\left(\frac{\boldsymbol{m}^*}{2\pi \boldsymbol{a}^2}\right)\left[1-\left(\frac{\boldsymbol{z}}{\boldsymbol{a}}\right)\right]^{-2}$$

$$= -\left(\frac{\boldsymbol{m}^*}{2\pi \boldsymbol{a}^2}\right)\sum_{n=1}^{\infty} n\left(\frac{\boldsymbol{z}}{\boldsymbol{a}}\right)^{n-1}$$

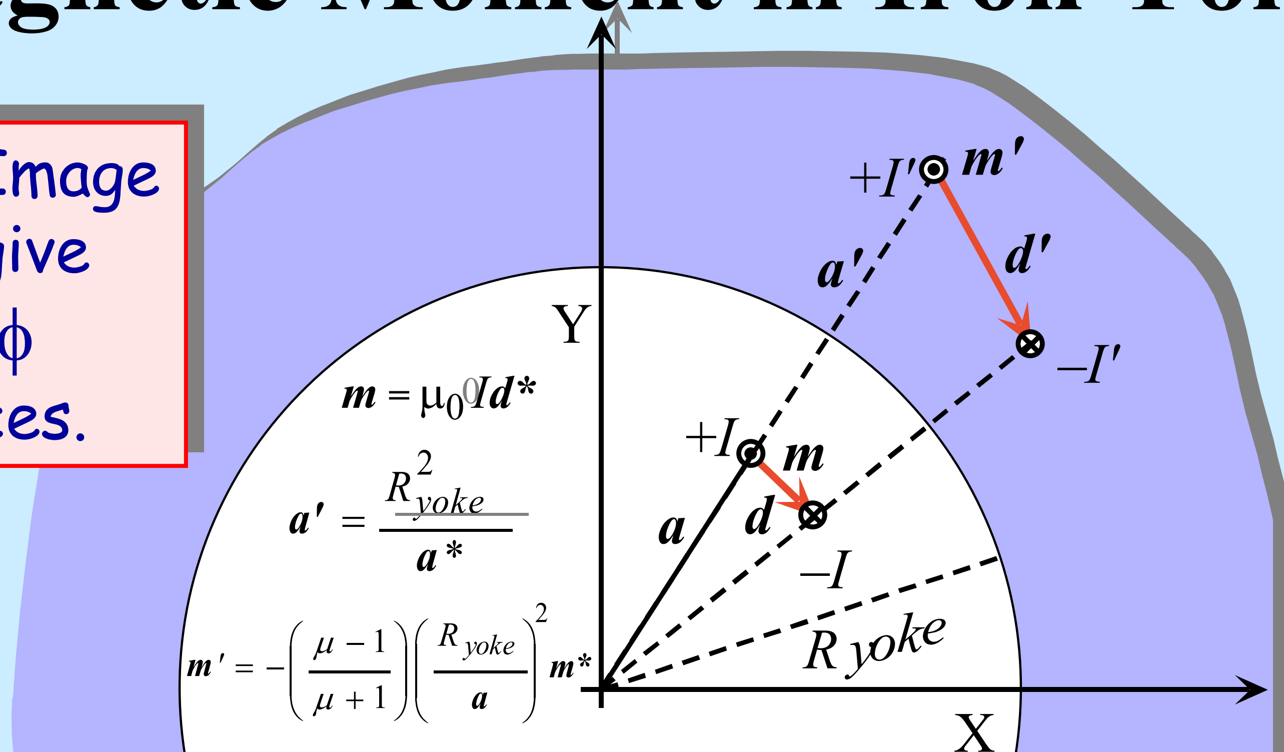
Harmonics are thus given by:

$$B_n + iA_n$$

$$= -\left(\frac{\boldsymbol{m}^* n}{2\pi a^2}\right)\left(\frac{R_{ref}}{a}\right)^{n-1} \exp\left[-i(n+1)\phi\right]$$

# Magnetic Moment in Iron Yoke

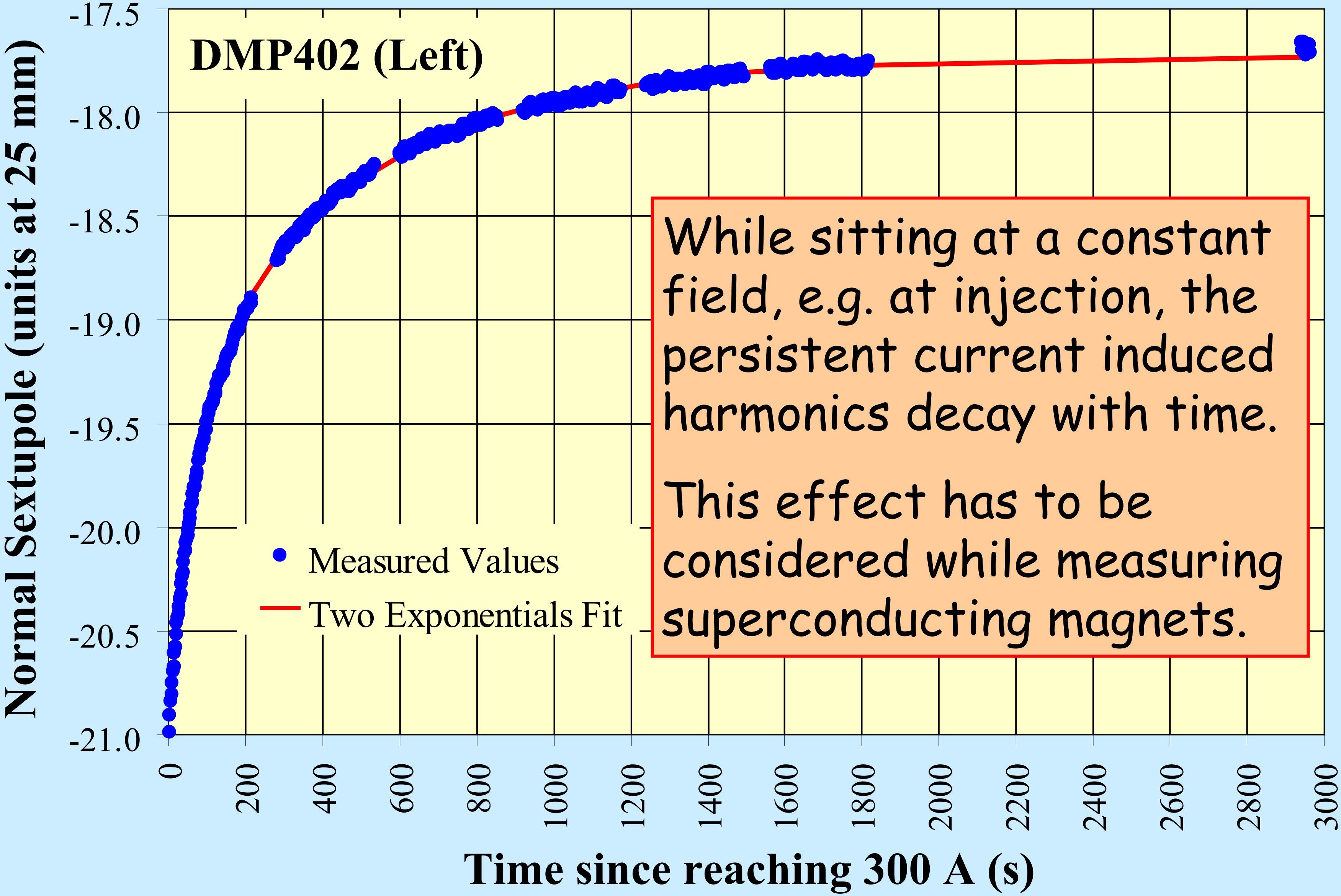
Main and Image moments give different $\phi$ dependences.




$$B_n + iA_n = -\left(\frac{n}{2\pi a^2}\right)\left(\frac{R_{ref}}{a}\right)^{n-1} \times$$

$$\left[ \boldsymbol{m}^* \exp\{-i(n+1)\phi\} - \left(\frac{\mu-1}{\mu+1}\right)\left(\frac{a}{R_{yoke}}\right)^{2n} \boldsymbol{m} \exp\{-i(n-1)\phi\} \right]$$

# Time Decay of Harmonics




While sitting at a constant field, e.g. at injection, the persistent current induced harmonics decay with time.

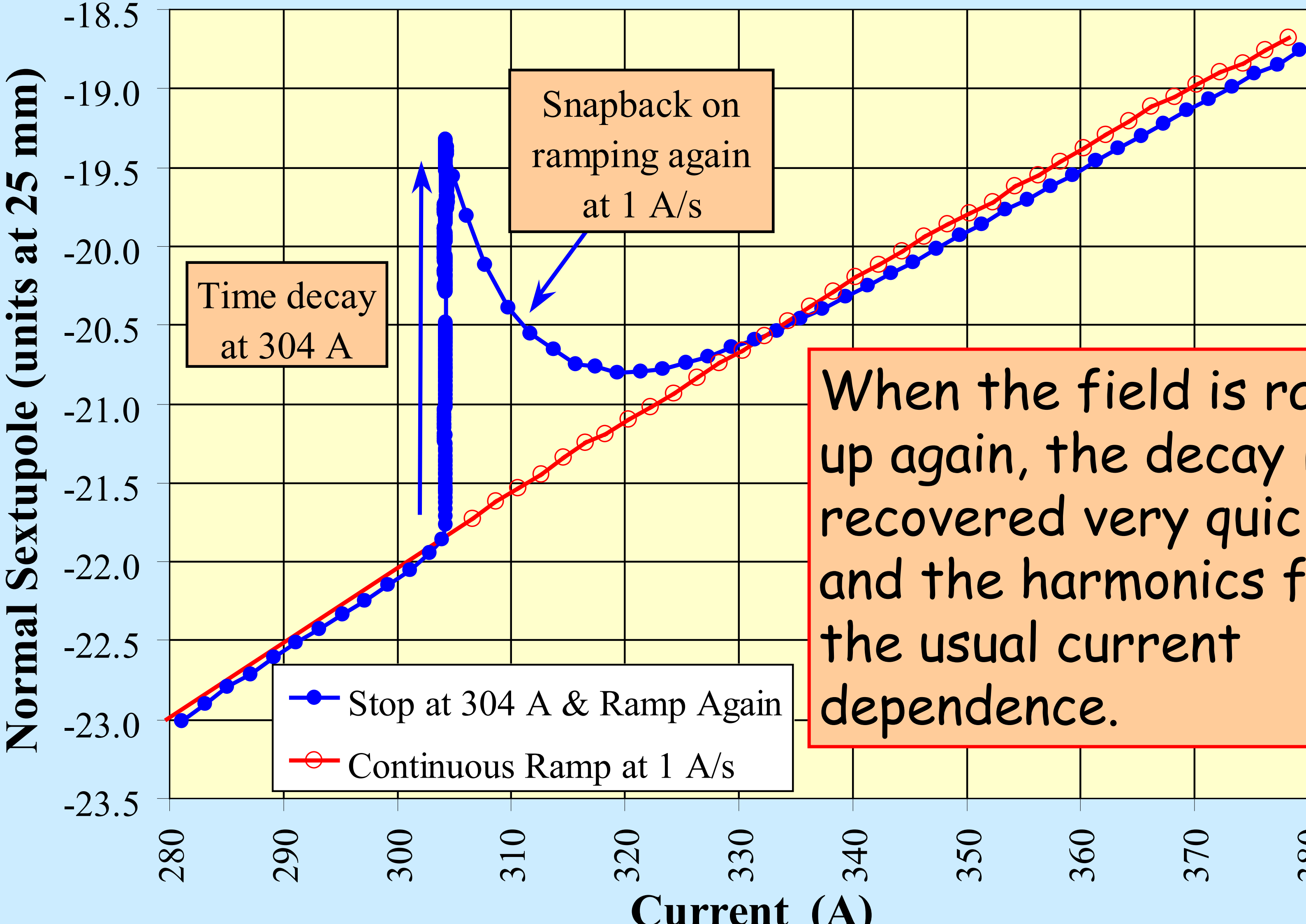
This effect has to be considered while measuring superconducting magnets.

# Time Decay of Harmonics

- **Flux Creep:** is caused by thermal activation and Lorentz forces due to transport current, effectively reducing $J_c$ with time. This is temperature dependent, and produces a logarithmic decay.
- **Boundary Induced Coupling Currents:** Different strands in a cable may carry different currents (e.g. due to spatial variations in time derivative of the field). These coupling currents produce periodic axial variation of the field. As the variations decay, regions which were at a higher field jump to the down ramp branch. Can vary magnet to magnet.

Main effect

# Snapback of Harmonics

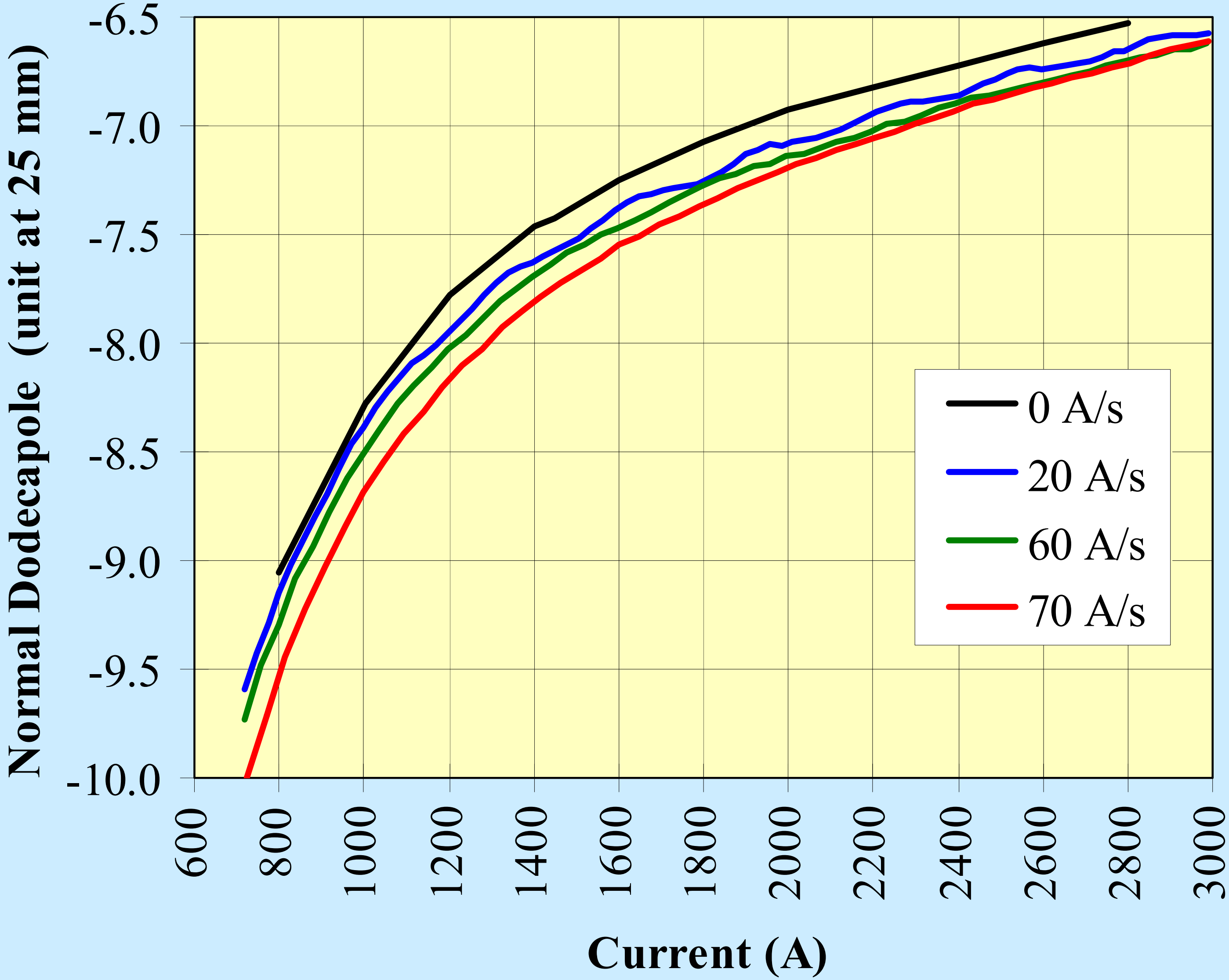



When the field is ramped up again, the decay is recovered very quickly, and the harmonics follow the usual current dependence.

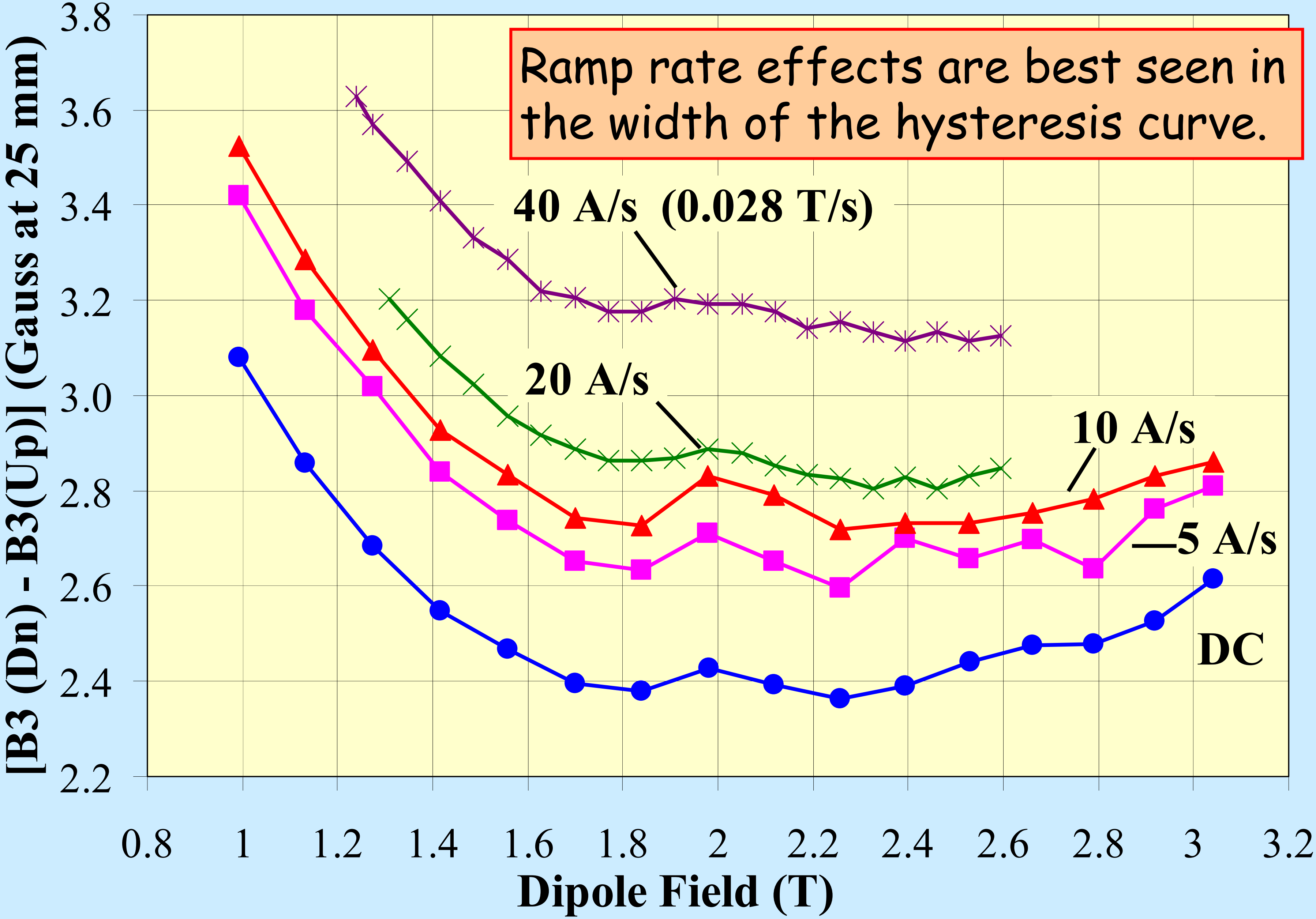
# Ramp Rate Effects

- The field quality is typically measured with rotating coils under DC excitation.
- If the magnet is being ramped at a high ramp rate, eddy currents can cause significant distortion of the field, and thus generate harmonics.
- The extent of distortion depends on the ramp rate and inter-strand resistance in multi-strand cables.

Ramp rate dependence of the allowed 12-pole term measured in a 80 mm aperture quadrupole for RHIC.



# D1L103: Ramp Rate & Magnetization Effects



# Measurements of Dynamic Effects

- The harmonics decay rather rapidly in the beginning.
- The snapback occurs within a few seconds.
- The primary field is also changing with time, particularly when the current is continuously being ramped at a high rate.
- All these factors present unique challenges in the measurement of dynamic effects. The key measurement issue is ***time resolution***.

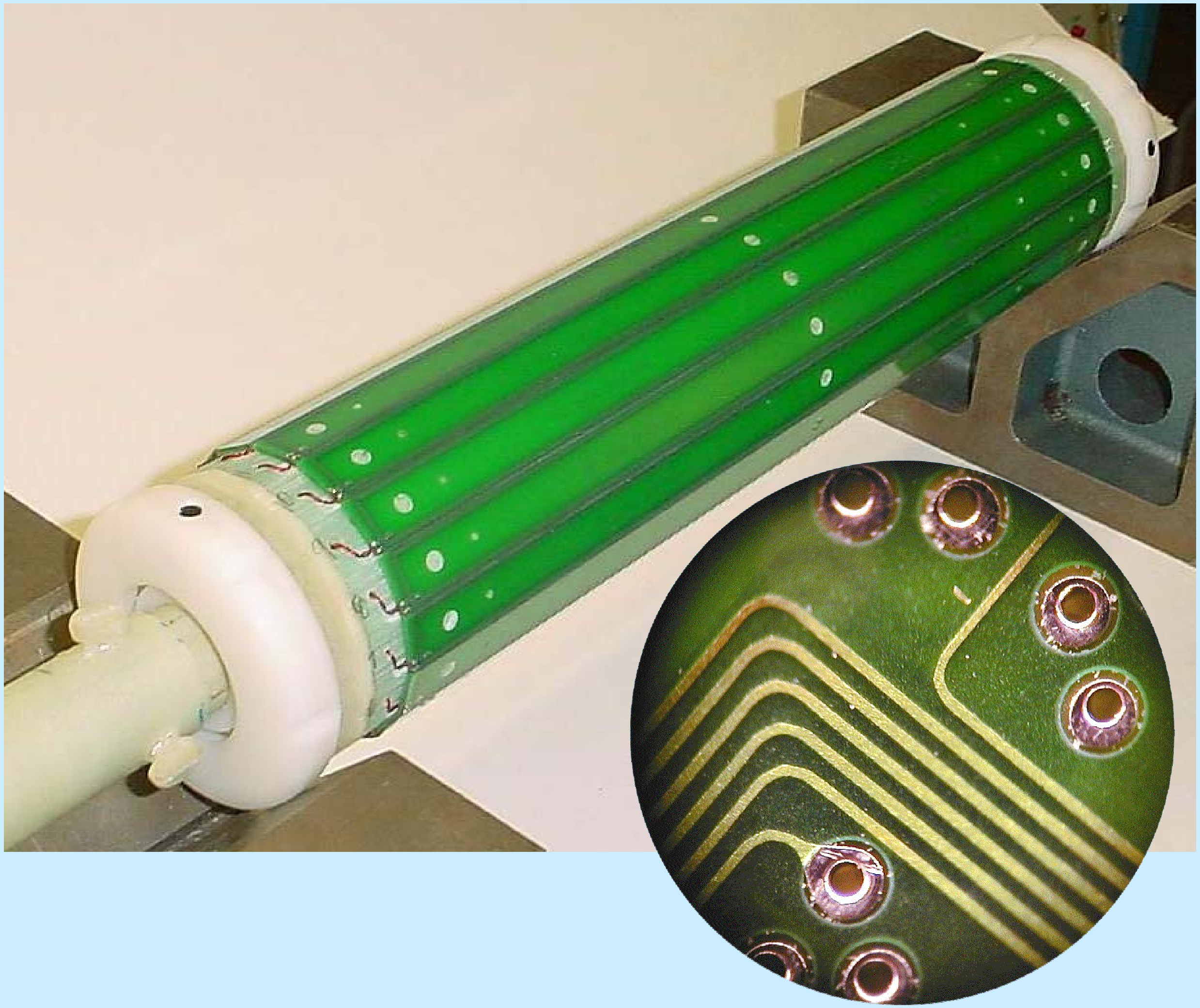
# Techniques for “Fast” Measurements

- Rotate a harmonic coil as fast as practical to improve time resolution.  This allows measurements with ~ 1 s typical resolution. (*OK for time decay and snapback studies*)
- The same technique can be used to measure harmonics during very slow current ramps.
- The technique can be extended to somewhat higher ramp rates by refining the analysis.
- This method is impractical for very high ramp rates, or very short time scales.

# Techniques for "Fast" Measurements

- One could use ***non-rotating probes*** to overcome the time resolution problem.
- Without a rotating probe, one needs a ***multiple probe system*** to get harmonic information.
- A system of 3 ***Hall probes***, for example, can measure the sextupole component. Similarly, ***NMR arrays*** have been built with many probes.
- Intercalibration of individual probes and non-linear behavior are some of the problems that must be addressed in such techniques.

# A Harmonic Coil Array
## (Under development at BNL)



16 Printed Circuit Windings, 10 layers

6 turns/layer

Nominally identical windings due to printed circuits

Non-rotating coil for very fast measurements of harmonics (~50 Hz)

# Summary

- Persistent currents in superconductors produce history dependent harmonics.
- These harmonics decay with time, but also snapback as soon as the ramp is resumed.
- These dynamic effects demand particular care in the measurements of superconducting magnets. Good time resolution is also required.
- Various techniques have been employed for dynamic measurements, but no single technique seems to be the "best". (Promising R&D area.)