STRESS PROBLEMS ASSOCIATED WITH SUPERCONDUCTING AND CRYOGENIC MAGNETS

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INTRODUCTION

The following will be a general discussion of a broad variety of stress problems encountered in the design of superconducting and cryogenic magnets of various types. Details of specific problems will not be discussed. This should be considered merely a design problem check-off list with some comments on approach.

The three fundamental forces are magnetic, thermal, and mechanical. A system design must properly consider their relative interaction.

MAGNETIC FORCES

Magnetic forces arise from four basic interactions. These are:

1. Torque on a magnetic dipole in a uniform (or gradient) field.

2. Forces on a dipole in a gradient field.* To calculate this force one must first know the magnetic properties of the material (i.e., permeability or susceptibility), the geometry of the body, and also, of course, the field gradient. The induced dipole moment will be a function of magnetic properties, geometry (demagnetization factor), and applied field. The force will be proportional to the dipole moment and also to the field gradient.

3. Lorentz interaction. This is, of course, the fundamental load on windings and any current-carrying element, i.e., leads, buswork, etc. It is simply the cross product of current and field.

4. Induction. A time changing field will induce currents in any conducting medium within the field. The resulting Lorentz interaction (and possible heating) is often overlooked.

THERMAL FORCES

Any system in which the operating temperature distribution is different from the initial temperature distribution or is composed of materials having different thermal coefficients (and undergoes a change in temperature) will be subject to thermal stresses. It is important to understand how the system gets from its initial to its operating state since the worst case stresses may occur during this transient. For low temperature systems it is obvious that the system orientation can have an extremely important effect on the transient temperature distribution.

*I hope it is unnecessary to note that there is no net force on a dipole in a uniform field!

MECHANICAL FORCES

In addition to the magnetic and thermal loads there are usually various other loads imposed by vacuum, pressure, dynamic experimental equipment, emergency, failure, start-up conditions, and other requirements which may be part of a total performance specification.

In any system analysis it is important to determine what combination of operating, test, or emergency conditions will give the worse case loads.

STRESSES

In order to calculate stresses one must first know the field and force distribution in the magnetic structure. There exist a number of very powerful computer codes to solve these problems. It is fatuous not to use them for systems of any consequence. There also exist a number of rather elegant general solutions for stress problems of this type. I cannot, however, overemphasize the danger of using general analyses or rules of thumb which have been developed by other people for other types of devices. One must be intimately aware of the assumptions used in order to evaluate realistically whether or not they can be applied to the problem under consideration.

AIR CORE SOLENOID

Here the force distribution gives rise to two principal stresses, the hoop stress or tangential stress in the conductor, and an axial compressive stress on the winding structure. To analyze and design any system one should have an intimate understanding of the elemental forces in the structure and how they are distributed. With this information cumulative loads can be calculated. Models assuming a body force distribution in a homogeneous material having predictable mechanical properties will <u>not</u> give realistic results! In general it is possible to mechanically (and electrically) regionalize any system so that the cumulative loads are within safe limits for all materials in the composite structure. These would include conductors, conductor supports, insulations, spacers, and over-all system support structures.

The dominant stress problem in a solenoid system is the hoop stress which can be easily calculated for an elemental conductor by multiplying the conductor radius in meters by the current in amperes by the axial field in teslas. This will give the hoop force on that elemental conductor in newtons.* This can easily be converted to stress over any arbitrary cross section of conductor and/or conductor support. It is interesting to note that in a solenoid system having an "alpha" (o.d. over i.d. ratio) greater than 2, the peak hoop stress does not occur at the winding i.d. This further tells us that if such a winding structure is a solid monolithic form then peak stresses in the body of the windings will couple to the i.d. and increase that stress level. This emphasizes the practicality of regionalization techniques. It is generally desirable to break up the winding into a number of self-supporting mechanical regions and further to operate these at different current densities.

A simple approach to a stress-limited solenoid design is to start with an inside region of small radial dimension operating in the peak field strength it is desired to produce and having some average radius. With this and the mechanical characteristics of the conductor and support used, one can calculate the current this inner region can carry within safe stress limits. Having thus designed the inner winding region and

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*I very cleverly call this the RIB force.

having calculated the central field contribution of this region, one then moves to a next outer radial region and repeats the process with the new radius of curvature and new (lower) field in which the conductors must operate, etc. Concentric windings are not self-centering and must be rigidly supported against relative radial motion.

In cases where stresses are a problem, one is generally dealing with composite conductors and supports and must, of course, take great care to properly consider all of the relative mechanical characteristics of the materials in use.

THERMAL PROBLEMS WITH COMPOSITE CONDUCTOR/SUPPORTS

This is perhaps most emphatically demonstrated in the design of cryogenic magnets where one is generally trying to support a very high purity, very weak, and very low modulus aluminum conductor in a high field. In order to realize the full potential of their high resistivity ratio, one must operate these conductors in an essentially strain-free condition. It is, however, impossible to find a high modulus support material which does not have a thermal coefficient substantially less than that of aluminum. Thus during cooldown the aluminum conductor either shrinks away from the support material (and must later yield into it), or if it is bonded to its support has a large thermal strain induced.*

SPLITS

When a solenoid is split into a coil pair or multicoil system the problems are complicated somewhat. The major new considerations are the large force between windings and the fact that people do not make splits unless they want lots of free access in a region that ought to be mainly support structure.

USE OF IRON

In many instances it is sensible to use iron with superconducting and cryogenic systems.

The use of iron in a system changes the field distribution significantly and, therefore, the force and stress distribution. The relative stress problems depend largely on the ultimate field strength it is necessary to produce.† A very large fraction of the total field can be from the magnetized iron contribution. This means that the current in the windings can be reduced, and therefore the forces and winding stresses are also reduced. This is particularly true with systems having a large diameter-to-length ratio (small beta).

The axial loads in low field iron-bound systems are dramatically reduced and, in fact, in most cases can be made arbitrarily low.

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*I am confident that support structures can be designed to accommodate this, and that dynamic resistivity ratios of greater than 1000 can be realized in fields of hundreds of kilogauss, although to my knowledge this has not yet been achieved.

†I would like to note that it is very reasonable to consider the use of iron with these systems up to field strengths of at least 50 kG and perhaps even higher.

With split coils the intercoil forces can be made such that the coils either attract or repel.

The interaction of the windings with the surrounding iron creates some problems but also allows additional design flexibility. In a simple system magnetically centered windings are in a state of unstable equilibrium within the iron structure. Thus if they move away from their magnetic center, they will pull into the iron structure more strongly in that direction.

It is possible to design an over-all geometry in which the windings are stable in at least two directions within the system, and therefore require support in the third direction only. This has obvious implications to the design of low heat leak structures in iron-bound superconducting and cryogenic systems. An interesting possibility is that of a winding which, with a simple feedback system, could be completely magnetically supported within the surrounding iron, and therefore would require no mechanical structure at all between the low temperature bath and the outside world.

TRANSVERSE FIELD SYSTEMS

The next category of systems of general interest might be those that do not have cylindrical symmetry. This would include the various "saddle" and rectangular coil systems for transverse field generation.

Here the basic Lorentz interaction cannot be supported by the conductor and therefore requires some external structure. This relieves the strain on the conductor but is apt to suffer from nonuniform load and stress distribution and high local stresses on conductors and insulations. There is also a general lack of mechanical rigidity likely with this system. Resulting motion could give problems in marginally stable superconducting systems.

Of major consideration with these more complicated support structures are the cryostat and refrigeration costs which may be associated with necessarily higher heat leaks. It is often desirable to operate systems which are traditionally horizontal in a vertical orientation in order to improve thermal design.

FORCE CALCULATION

Here again the most useful method of estimating forces and stresses is simply to look at the elemental and then cumulative Lorentz interaction forces. One must be careful at the ends of the system and must make realistic assumptions of manufacturing tolerances and relative load distributions.

DANGER

The various analogies of magnetic pressure, thick walled cylinders, etc., can be useful but in my estimation only rarely! They suffer from dangerous limitations which tend to get one into a great deal of trouble. If assumed forces do not correlate with a simple $I \times B$ or $J \times B$ calculation something is wrong!

SPECIAL PROBLEMS

There are always certain special problems associated with the design analysis of a magnetic system. One must be careful that he is working to a complete performance requirement specification. Important basic considerations are the following:

1. The effects of time changing fields during field increase or decay should be considered. Under emergency or "quench" conditions the latter might be quite rapid, and additional forces and/or redistribution of forces resulting from conducting loops coupled to the magnetic system, or possible short circuits in the winding structure should be considered.

2. Nonuniform cooldown and localized hot spots during a quench can often create serious thermal stress conditions. Judicious use of materials to take advantage of differential thermal coefficients and prestressing to increase spring constants and/or optimize final operating force distributions should be considered. Thermal walking and loosening must not be allowed.

3. External interactions must be thoroughly evaluated. These might be with magnetic bodies, current leads, buswork, etc.

4. If system operation is dynamic due to constantly changing or pulsed fields or perhaps dynamic external equipment, one must be sure that fatigue characteristics are properly considered. It is important to note that the field, force, and stress solutions associated with fast pulse systems are significantly different from those of steady state or slowly time changing systems.

5. One must consider all possible emergency and failure conditions of the magnet and associated equipment.

6. One must also be sure that he has adequate and accurate information regarding the properties of materials in use and how these properties may change as a function of temperature and field. In general, these are well understood and should present no surprises. One possible exception is the martensitic (nonmagnetic to magnetic) transformation of stainless which can occur if it is cooled in a severely coldworked condition. This could obviously create unexpected stress and, possibly, experimental problems.

Unfortunately there are no good general texts on these problems but a great deal of information can be gleaned from the following. Again one must be extremely careful to select that information which is applicable and correct.

H. Kolm et al., ed., Proc. Intern. Conf. High Magnetic Fields, Cambridge, Mass., 1961, (M.I.T. Press and J. Wiley, 1962).

R. Pauthenet, ed., Actes du Colloque International sur les Champs Magnétiques Intenses, Grenoble, 1966 (No. 166, Editions du Centre National de la Recherche Scientifique, Paris, 1967).

H. Brechna and H.S. Gordon, ed., Proc. Intern. Symposium on Magnet Technology, Stanford, California, 1965 (available from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Va., USA).

H. Hadley, ed., Proc. 2nd Intern. Conf. Magnet Technology, Oxford, 1967 (available from R.C. Pepperell, Rutherford Laboratory, Chilton, Didcot, Berks., England).

One should not overlook the many useful suggestions for stress analysis given by S. Timoshenko, W. Flügge, J.P. Den Hartog, R.J. Roark, and the like!