Lecture IV

Magnetic Design
Coil Optimization

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Coil Designs for Real Magnets

- All magnets use NbTi Superconductor
- All designs use cosine theta coil geometry

Do they really look like having a cosine theta current distribution?

Or that matter, even an elliptical geometry for conductor having a constant current density?
Coil Cross-section Optimization (1)

The optimization of a coil cross-section for a good magnetic design involves:

- Minimizing field harmonics
- Maximizing field (Transfer Function) for a lower number of turns
- Minimizing Peak Field (Max. field on the conductor for given central field)

At first, it appears to be fairly straightforward process, thanks, in part, to the modern automated codes like ROXIE and PAR2DOPT, etc.,

In fact, one can built a magnet based on the optimized coil structure obtained by a relatively new user.
But the advanced cross-section optimization is a bit more involved:
  • One must avoid designs that create mechanical difficulties
  • One should look for flexibility to allow future adjustments
  • Also look for special requirements in each application.

One approach fits all, may not always be a good strategy.

My Experience:
  • The initial design, quiet often sets, the eventual performance of the magnet apart from degree of difficulties in manufacturing the magnet.

  • As compared to building magnets, design process takes a relatively small resources. Spend time in looking for as many possibilities/options as possible.
Field Harmonic Definitions

The field quality in magnets is expressed in terms of the normal and skew harmonic coefficients, $b_n$ and $a_n$ by the following expansion:

$$B_y + iB_x = 10^{-4} \times B_R \sum_{n=1}^{\infty} (b_n + ia_n)[(x + iy)/R]^{n-1}$$

where $x$ and $y$ are the horizontal and vertical coordinates, $B_R$ is the field strength of the primary harmonics at the “reference radius” $R$. The values of the field harmonic are given in the units of $10^{-4}$.

The definition used above (European convention) differs from that used in many U.S. publications (US convention), where $n-1$ is replaced by $n$ and the summation starts from $n = 0$. 
Consider a “Radial Block” between radii $\rho_1$ & $\rho_2$ and angle $\phi_1$ & $\phi_2$ having a current density of $J$. The total current ($I$) and harmonics ($A_n$, $B_n$) are given by:

$$I = \frac{1}{2} J (\rho_2^2 - \rho_1^2) (\phi_2 - \phi_1)$$

$$A_1 = -\frac{\mu_0 J}{2\pi} (\rho_2 - \rho_1) \left[ \cos (\phi_2) - \cos (\phi_1) \right],$$

$$A_2 = -\frac{\mu_0 J R_0}{2\pi} \ln \left( \frac{\rho_2}{\rho_1} \right) \left[ \cos (2\phi_2) - \cos (2\phi_1) \right],$$

for $n \geq 3$

$$A_n = \frac{\mu_0 J}{2\pi} \frac{R_0^{n-1}}{n(n-2)} \left( \frac{1}{\rho_2^{n-2}} - \frac{1}{\rho_1^{n-2}} \right) \left[ \cos (n\phi_2) - \cos (n\phi_1) \right]$$

$$B_1 = -\frac{\mu_0 J}{2\pi} (\rho_2 - \rho_1) \left[ \sin (\phi_2) - \sin (\phi_1) \right],$$

$$B_2 = -\frac{\mu_0 J R_0}{2\pi} \ln \left( \frac{\rho_2}{\rho_1} \right) \left[ \sin (2\phi_2) - \sin (2\phi_1) \right],$$

for $n \geq 3$

$$B_n = \frac{\mu_0 J}{2\pi} \sum_{n=3}^{\infty} \frac{R_0^{n-1}}{n(n-2)} \left( \frac{1}{\rho_2^{n-2}} - \frac{1}{\rho_1^{n-2}} \right) \left[ \sin (n\phi_2) - \sin (n\phi_1) \right]$$

$n = 1$ refers to dipole. $R_0$ is the reference radius.

See my thesis for these and other derivations.
Assume that a current block is between radii 10 cm and 12 cm and it starts at an angle $\Phi = 0$.

Find the subtended angle (or the angle at which the block must end) for

- The normal sextupole harmonic to be zero. Is it a unique solution?
- Assume that you have a dipole symmetry. How many other blocks must be present to generate this symmetry (give number and angles).
- Compute the values of the first three non-zero allowed harmonics in Tesla ($A_n$ and/or $B_n$) at a reference radius of 6 cm.

- Do the above for decapole harmonic to be zero. Is it a unique solution?

- Can you find a solution for which both sextupole and decapole harmonics are zero? The block does not have to start from an angle $\Phi = 0$.

What happens if it is in a cylindrical iron cavity having a permeability of
(a) 10, (b) 100 and (c) 1000.  Hint: You can use the method of images.
Homework Assignment
(Two blocks)

Assume that there are two current blocks. First between radii 10 cm and 11 cm and second between 11 cm and 12 cm. Both starts at an angle $\Phi = 0$.

Find the subtended angle (the angle at which block must end) for the normal sextupole harmonic and decapole harmonics to be zero.

Is it a unique solution?

What happens if it is in a cylindrical iron cavity having a permeability of

(a) 10

(b) 100 and

(b) 1,000.

Hint: You can use the method of images.
How to Look for Optimal X-section (1)

1. Look for designs that look similar to cosine theta distribution (experience)

2. Use special techniques/software (artificial intelligence)

3. Cover a large range of combinations and find the best

My recommendation, go for the 3rd option (several thousand cases):

It does not take long to look a large number of possibilities - less than a few seconds per case if the peak field is not computed.

To save time compute peak field only in promising solutions.
Develop a front-end program to automatically create several cases for a series optimization run.

**In this optimization:**
- Vary number of blocks and number of turn in those blocks.
- Vary starting condition of wedges, etc.

Post-process results to select a limited cases with filters for harmonics, etc.
- Compute peak field for these selected cases.

Go back and carefully evaluate and further optimize these few cases for performance, mechanical layout, flexibility, sensitivity, efficiency and any other requirement.
A D07GEN Run

midplane

1 M1 10.00000 0. 1. 30.
enter min,max number of turns for blk [and increment in FOR017]
13 21 2
3 M2 16.00000 0. 1. 30.
enter min,max number of turns for blk [and increment in FOR017]
12 18 3
5 M3 13.00000 0. 1. 4.
enter min,max number of turns for blk [and increment in FOR017]
10 14 1
7 M4 10.0 0.0 0. 4.
enter min,max number of turns for blk [and increment in FOR017]
3 12 1
9 M5 6.00000 0. 1. 4.
enter min,max number of turns for blk [and increment in FOR017]
3 8 1
11 M5 4.0 0.0 0. 4.
enter min,max number of turns for blk [and increment in FOR017]
2 6 1

YOU HAVE SPECIFIED # CASES = 47250
Enter start cycle, end cycle [<cr> for all] 10001 20000
Base design has total turns = 60
ENTER min,max number of total turns 57 59
Do you wish a constraint on a subtotal?
eg total number of turns for inner layer-- [yes/[NO]] Y
Enter First, Last blocks to be included in sum 1 3
ENTER lower,upper limits of sun 20 40
A PARSLCT Run

```
BNLADA>ty LANL-73TX1D.PARSLCT;
LANL quad ***SELECT CASES*** 16-DEC-01 11:18:39 BNLADA$OKA200:[GUPTA.LANL]LANL-73TX1D.D04;1
# WKP BLOCKS RIN ROUT Current Thickness KEYSTONE AzInsul <------ Cable Information
      1     4.0     11. 182165 193045 1. 53.0 3.2 3.45
       Turns  Chisq    T.F.   Pole b2 b4 b6 b8 b10 b12 w1 w2 w3 w4 w5 CASE  ENHin  Bsys
JKDA56 0.720 0.337 43.00 -0.05 0.03 -0.85 0.00 0.00 0.00 0.469 2.185 0.764 9.297 0.303 301 26.598 24.93
JKDC45 0.951 0.341 43.00 -0.06 -0.13 -0.56 0.00 0.00 0.00 0.399 1.368 2.439 8.276 0.533 305 25.430 25.78
JKDC54 0.451 0.342 42.99 -0.13 -0.12 -0.65 0.00 0.00 0.00 0.309 1.542 2.103 8.304 0.755 306 24.999 26.10
JKDC63 0.579 0.342 42.97 -0.07 -0.17 -0.74 0.00 0.00 0.300 1.514 2.084 8.372 0.721 307 24.740 26.28
JOAA46 0.447 0.339 43.00  0.08 -0.03 -0.66 0.00 0.00 0.00 0.493 2.666 0.324 8.964 0.572 346 25.952 25.40
JOAA55 0.570 0.339 42.99  0.00  0.09 -0.75 0.00 0.00 0.499 2.622 0.413 9.046 0.429 347 25.924 25.42
JOAA73 0.641 0.339 43.00  0.05  0.04 -0.80 0.00 0.00 0.476 2.623 0.325 9.126 0.468 349 25.554 25.67
JOAC44 0.690 0.343 42.92  0.01  0.02 -0.63 0.00 0.00 0.307 1.691 2.273 8.287 0.303 350 24.996 26.20

104 cases examined. 8 cases selected for: CHISQ< 5.0000  TP> 0.300  POLE<85.00; |hn|< 1.00 1.00 1.00 1.00 1.00 1.00 1.00
FILENAME = BNLADA$OKA200:[GUPTA.LANL]LANL-73TX1D.D04;1
```
Influence of Coil Parameters in SSC 50 mm Aperture Dipole

Table 4.3.1: The effect of a \(+25 \mu m\) change in a wedge thickness or pole width on the transfer function and the field harmonics in the SSC 50 mm aperture dipole magnet. The field harmonics are calculated with a 10 mm reference radius. The numbering of the wedges is counter-clockwise from the midplane. The pole width is measured from the vertical axis.

<table>
<thead>
<tr>
<th>Parameter changed</th>
<th>(\delta TF) (10^{-4}\ T/k_A)</th>
<th>(\delta b_2) (10^{-4})</th>
<th>(\delta b_4) (10^{-4})</th>
<th>(\delta b_6) (10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wedge No. 1</td>
<td>-0.78</td>
<td>-0.24</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>Wedge No. 2</td>
<td>0.42</td>
<td>0.30</td>
<td>0.03</td>
<td>-0.005</td>
</tr>
<tr>
<td>Wedge No. 3</td>
<td>1.16</td>
<td>0.36</td>
<td>-0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Wedge No. 4</td>
<td>-0.29</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Pole Width (inner)</td>
<td>2.0</td>
<td>0.23</td>
<td>-0.03</td>
<td>0.005</td>
</tr>
<tr>
<td>Pole Width (outer)</td>
<td>1.13</td>
<td>0.21</td>
<td>0.00</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Wedge No. 4 is in outer layer.
Influence of Coil Parameters in RHIC 80 mm Aperture Dipole

**Table 4.3.2**: The computed change in the transfer function and field harmonics produced by a $+25 \mu m$ ($0.001''$) change in the wedge thickness, pole width or midplane gap in the RHIC 80 mm aperture arc dipoles. The field harmonics are calculated with a 25 mm reference radius. The numbering of the wedges starts at the vertical and horizontal axis, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta TF$</th>
<th>$\delta b_2$</th>
<th>$\delta b_4$</th>
<th>$\delta b_6$</th>
<th>$\delta b_8$</th>
</tr>
</thead>
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<tr>
<td>changed</td>
<td>$10^{-4} \frac{P}{kA}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
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<tr>
<td>Wedge 1</td>
<td>-0.6</td>
<td>-0.98</td>
<td>-0.122</td>
<td>0.061</td>
<td>0.043</td>
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<tr>
<td>Wedge 2</td>
<td>0.1</td>
<td>0.69</td>
<td>0.423</td>
<td>0.022</td>
<td>-0.050</td>
</tr>
<tr>
<td>Wedge 3</td>
<td>1.1</td>
<td>1.42</td>
<td>-0.090</td>
<td>-0.068</td>
<td>0.041</td>
</tr>
<tr>
<td>Pole Width</td>
<td>1.7</td>
<td>1.11</td>
<td>-0.154</td>
<td>0.039</td>
<td>-0.014</td>
</tr>
<tr>
<td>Midplane Gap</td>
<td>-0.9</td>
<td>-1.68</td>
<td>-0.557</td>
<td>-0.156</td>
<td>-0.050</td>
</tr>
</tbody>
</table>

Notice that the magnitude of change in harmonics by a 25 $\mu$ change, is much large in RHIC dipole than in SSC dipole. **WHY?**
**RHIC Insertion Dipole D0 with single layer coil**

Coil inner radius = 100 mm,
Harmonic reference radius = 65 mm

"D0 MAGNET" : Rough Cross section Optimization Spread sheet

To iterate cross section for b2 & b4 with midplane and pole shims and fixed wedge changes, go to line b65:b72

<table>
<thead>
<tr>
<th>Expected Parameters of the Iterated Design</th>
<th>b2</th>
<th>b4</th>
<th>b6</th>
<th>b8</th>
<th>b10</th>
<th>b12</th>
<th>b14</th>
<th>b16</th>
</tr>
</thead>
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<tr>
<td><strong>Coil Prestress</strong></td>
<td>1</td>
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<td></td>
<td></td>
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<tr>
<td><strong>Coil Size</strong></td>
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<tr>
<td><strong>Target Increase</strong></td>
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<td><strong>in Coil size</strong></td>
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<td><strong>Multiplier</strong></td>
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<tr>
<td><strong>Change(mil)</strong></td>
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<tr>
<td><strong>Target b2n</strong></td>
<td>-8.00</td>
<td>-2.30</td>
<td>-0.16</td>
<td>-0.15</td>
<td>0.016</td>
<td>-0.15</td>
<td>0.039</td>
<td>0.096</td>
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<td><strong>Total Increase</strong></td>
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<td><strong>mil in compression</strong></td>
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<tr>
<td><strong>Midplane(mil)</strong></td>
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<td><strong>PoleShim(mil)</strong></td>
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<td><strong>Decrease Pole</strong></td>
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<tr>
<td><strong>Wedge1(mil)</strong></td>
<td>4</td>
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<tr>
<td><strong>Wedge2(mil)</strong></td>
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<td><strong>Fixed Pole</strong></td>
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<tr>
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<td><strong>Fixed Pole</strong></td>
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<td><strong>Increase Pole</strong></td>
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<tr>
<td><strong>Wedge1+4mil</strong></td>
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<td><strong>Increase Pole</strong></td>
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<tr>
<td><strong>Wedge2+4mil</strong></td>
<td>0</td>
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<td><strong>Increase Pole</strong></td>
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<tr>
<td><strong>Wedge3+4mil</strong></td>
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<td><strong>Increase Pole</strong></td>
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<tr>
<td><strong>Wedge4+4mil</strong></td>
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</tbody>
</table>
**A Flexible Design from the Beginning**

**Design Philosophy:**

- Start out a design from the beginning itself, that allows significant adjustability for field harmonics and mechanical parameters (cable thickness, wedges, etc.).

- A flexible design is generally economical, efficient and produces magnets with better performance. I think it’s a prudent approach.

Geometric: Start with a larger than required shim and midplane cap. Then adjust it, as required without changing the cross-section of the cured coil. One can also adjust the layers of wedge/cable insulation, if needed. These three parameters can adjust, first two allowed harmonics and pre-stress or cable insulation. This approach was used extensively in various RHIC magnets.

Saturation: Start out with holes and fill them with iron rods. Or, punch holes in laminations later.

In KEK, C.F. dipole, one may consider adjusting for quad and sextupole component.
**Change in Midplane Gap to Adjust Harmonics**
(can be easily done by changing the size of the ground-plane insulation cap)

**Table 4.4.1:** The computed and measured change in field harmonics at 25 mm reference radius due to a change in the coil midplane gap. The midplane gap was increased from 0.114 mm to 0.16 mm in the rebuilt 80 mm aperture RHIC model dipole magnet DR5009. In the production magnets, the midplane gap was changed back to 0.114 mm from 0.16 mm to adjust the $b_4$ harmonic.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta b_2$</th>
<th>$\Delta b_4$</th>
<th>$\Delta b_6$</th>
<th>$\Delta b_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computed</td>
<td>-3.0</td>
<td>-1.0</td>
<td>-0.28</td>
<td>-0.09</td>
</tr>
<tr>
<td>Measured</td>
<td>-3.0</td>
<td>-1.0</td>
<td>-0.29</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

**Table 4.5.1:** The measured and computed change in field harmonics caused by an asymmetric increase in the coil-to-midplane gap in the prototype 130 mm aperture RHIC interaction quadrupole QRI002. The gap was increased by 0.1 mm in the horizontal plane only. The harmonics are given at a reference radius of 40 mm.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta b_3$</th>
<th>$\Delta b_5$</th>
<th>$\Delta b_7$</th>
<th>$\Delta b_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computed</td>
<td>-6.8</td>
<td>-1.3</td>
<td>-0.45</td>
<td>-0.16</td>
</tr>
<tr>
<td>Measured</td>
<td>-6.5</td>
<td>-1.2</td>
<td>-0.30</td>
<td>-0.17</td>
</tr>
</tbody>
</table>
RHIC 100 mm Aperture Insertion Dipole: The first magnet gets the body harmonics right

Geometric Field Errors on the X-axis of DRZ101 Body

First magnet and first attempt in RHIC 100 mm aperture insertion dipole
A number of things were done in the test assembly to get pre-stress & harmonics right

Harmonics at 2 kA (mostly geometric).
Measured in 0.23 m long straight section.

<table>
<thead>
<tr>
<th>Reference radius = 31 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
</tr>
<tr>
<td>b2</td>
</tr>
<tr>
<td>b3</td>
</tr>
<tr>
<td>b4</td>
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<td>b5</td>
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<td>b8</td>
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<td>b9</td>
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<tr>
<td>b10</td>
</tr>
<tr>
<td>b11</td>
</tr>
<tr>
<td>b12</td>
</tr>
<tr>
<td>b13</td>
</tr>
<tr>
<td>b14</td>
</tr>
</tbody>
</table>

Later magnets had adjustments for integral field and saturation control. The coil cross-section never changed.

All harmonics are within or close to one sigma of RHIC arc dipoles.
Average Field errors $\sim 10^{-4}$ up to 80% of the coil radius

Geometric Field Errors on the X-axis of RHIC DRZ magnets (108-125)

Coil X-section was not changed between 1st prototype and final production magnet
A Flexible & Experimental Design Approach Allowed Right Pre-stress & Right Harmonics

Estimated Integral Mean in Final Set
(Warm-cold correlation used in estimating)

Harmonics at 3kA (mostly geometric)
Reference radius is 31 mm (Coil 50 mm)

<table>
<thead>
<tr>
<th>b1</th>
<th>-0.28</th>
<th>a1</th>
<th>-0.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>b2</td>
<td>-0.26</td>
<td>a2</td>
<td>-3.36</td>
</tr>
<tr>
<td>b3</td>
<td>-0.07</td>
<td>a3</td>
<td>0.03</td>
</tr>
<tr>
<td>b4</td>
<td>0.15</td>
<td>a4</td>
<td>0.48</td>
</tr>
<tr>
<td>b5</td>
<td>0.00</td>
<td>a5</td>
<td>0.04</td>
</tr>
<tr>
<td>b6</td>
<td>0.32</td>
<td>a6</td>
<td>-0.24</td>
</tr>
<tr>
<td>b7</td>
<td>0.00</td>
<td>a7</td>
<td>0.01</td>
</tr>
<tr>
<td>b8</td>
<td>-0.08</td>
<td>a8</td>
<td>0.05</td>
</tr>
<tr>
<td>b9</td>
<td>0.00</td>
<td>a9</td>
<td>0.00</td>
</tr>
<tr>
<td>b10</td>
<td>-0.12</td>
<td>a10</td>
<td>-0.02</td>
</tr>
<tr>
<td>b11</td>
<td>0.03</td>
<td>a11</td>
<td>-0.01</td>
</tr>
<tr>
<td>b12</td>
<td>0.16</td>
<td>a12</td>
<td>0.06</td>
</tr>
<tr>
<td>b13</td>
<td>-0.03</td>
<td>a13</td>
<td>0.03</td>
</tr>
<tr>
<td>b14</td>
<td>-0.10</td>
<td>a14</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*Field errors are $10^{-4}$ to 80% of the aperture at midplane.*
(Extrapolation used in going from 34 mm to 40 mm; reliability decreases)
Computed Peak Fields in SSC Dipole

**Table 6.2.6:** Peak fields in the SSC 50 mm dipole as computed using code MDP.

<table>
<thead>
<tr>
<th>I</th>
<th>$B_o$</th>
<th>$B_{pk}, T$</th>
<th>$\frac{B_{pk}}{B_o}$</th>
<th>Location</th>
<th>$B_{pk}, T$</th>
<th>$\frac{B_{pk}}{B_o}$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.85</td>
<td>6.9058</td>
<td>7.2374</td>
<td>1.048</td>
<td>5%</td>
<td>6.0016</td>
<td>0.869</td>
<td>11%</td>
</tr>
<tr>
<td>7.20</td>
<td>7.2100</td>
<td>7.5595</td>
<td>1.048</td>
<td>5%</td>
<td>6.2660</td>
<td>0.869</td>
<td>11%</td>
</tr>
</tbody>
</table>
Computed Performance of SSC Dipole

Table 6.2.7: Expected quench performance of the SSC 50 mm dipole with 5% cable degradation ($J_c = 2612.5 \text{A/mm}^2$) and at 4.35 K temperature. $S_{\text{quench}}$ is the computed current density in the copper at quench and $S_{6.7T}$ at the design field of 6.7 Tesla.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Cu/Sc Ratio</th>
<th>$B_{ss}$ tesla</th>
<th>$I_c$ A</th>
<th>$B_{\text{margin}}$ %over 6.7T</th>
<th>$T_{\text{margin}}$ kelvin</th>
<th>$S_{\text{quench}}$ A/cm²</th>
<th>$S_{6.7T}$ A/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner</td>
<td>1.7</td>
<td>7.149</td>
<td>7126</td>
<td>6.7</td>
<td>0.519</td>
<td>736</td>
<td>681</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>7.273</td>
<td>7273</td>
<td>8.6</td>
<td>0.625</td>
<td>788</td>
<td>715</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>7.399</td>
<td>7411</td>
<td>10.4</td>
<td>0.730</td>
<td>853</td>
<td>759</td>
</tr>
<tr>
<td>Outer</td>
<td>2.0</td>
<td>7.268</td>
<td>7267</td>
<td>8.7</td>
<td>0.580</td>
<td>919</td>
<td>834</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>7.445</td>
<td>7470</td>
<td>11.1</td>
<td>0.709</td>
<td>980</td>
<td>865</td>
</tr>
</tbody>
</table>

The magnet will quench at the design central field ($B_{\text{design}} = 6.7$ tesla). The field margin is defined as follows:

$$B_{\text{margin}} (\%) = \frac{B_{ss} - B_{\text{design}}}{B_{\text{design}}} \times 100$$
Table 6.2.8: The effect of a 0.05 mm increase in the given parameter on the transfer function and the field harmonics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TF</th>
<th>$b_2$</th>
<th>$b_4$</th>
<th>$b_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>changed</td>
<td>T/kA</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Radius of Block No. 1</td>
<td>0.31</td>
<td>-0.25</td>
<td>-0.10</td>
<td>-0.01</td>
</tr>
<tr>
<td>Radius of Block No. 2</td>
<td>-0.32</td>
<td>0.31</td>
<td>0.12</td>
<td>0.01</td>
</tr>
<tr>
<td>Radius of Block No. 3</td>
<td>-0.12</td>
<td>0.36</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>Radius of Block No. 4</td>
<td>-0.20</td>
<td>0.33</td>
<td>-0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>Radius of Block No. 5</td>
<td>-0.11</td>
<td>-0.04</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Radius of Block No. 6</td>
<td>-0.78</td>
<td>0.22</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>RMS Blocks</td>
<td>0.38</td>
<td>0.27</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>RMS Wedges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of Wedge No. 1</td>
<td>-1.56</td>
<td>-0.48</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Thickness of Wedge No. 2</td>
<td>0.83</td>
<td>0.59</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>Thickness of Wedge No. 3</td>
<td>2.32</td>
<td>0.71</td>
<td>-0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Thickness of Wedge No. 4</td>
<td>-0.57</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Cable thickness inner</td>
<td>2.63</td>
<td>1.08</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>Cable thickness outer</td>
<td>1.99</td>
<td>0.48</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>RMS Cable thickness</td>
<td>2.33</td>
<td>0.83</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Pole angle inner</td>
<td>-4.01</td>
<td>-0.45</td>
<td>0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>Pole angle outer</td>
<td>-2.26</td>
<td>-0.42</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>RMS Pole angles</td>
<td>3.25</td>
<td>0.43</td>
<td>0.04</td>
<td>0.01</td>
</tr>
</tbody>
</table>
The stored energy and the inductance are related through the following formula:

\[ \text{Stored Energy} = \frac{1}{2} \text{Inductance} \times (\text{Current})^2. \]

The inductance decreases at high field as the iron yoke saturates.

The results of POISSON computations for the SSC 50 mm aperture dipole are given at 6.5 kA in Table 6.2.9 for the stored energy per unit length and the inductance per unit length. The total stored energy and the inductance for a 15 m long dipole are also given.

**Table 6.2.9:** Stored Energy and Inductance at 6.5 kA as computed with the code POISSON for the SSC 50 mm aperture dipole.

<table>
<thead>
<tr>
<th>Stored Energy per unit length, kJ/m</th>
<th>105.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stored Energy for 15 m long Dipole, kJ</td>
<td>1575.6</td>
</tr>
<tr>
<td>Inductance per unit length, mH/m</td>
<td>4.972</td>
</tr>
<tr>
<td>Inductance for 15 m long Dipole, mH</td>
<td>74.585</td>
</tr>
</tbody>
</table>
Figure 6.2.5: The magnitude of the components of the Lorentz force on the individual turns in a SSC 50 mm prototype magnet. The radial component of the force \( F_r \) pushes the coil outward and the azimuthal component \( F_a \) compresses the coil towards the midplane (horizontal plane). There are 19 turns in the inner layer and 26 turns in the outer layer of each quadrant.

Computed Lorentz forces at the design field of 6.6 T (6.6 kA).
The table summarizes the forces on the blocks in the example SSC dipole.

<table>
<thead>
<tr>
<th>BLOCK</th>
<th>$F_x$, lb/in</th>
<th>$F_y$, lb/in</th>
<th>$F_x$, N/m</th>
<th>$F_y$, N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1005.46</td>
<td>-108.98</td>
<td>1.76E+05</td>
<td>-1.91E+04</td>
</tr>
<tr>
<td>2</td>
<td>1312.68</td>
<td>-237.56</td>
<td>2.30E+05</td>
<td>-4.16E+04</td>
</tr>
<tr>
<td>3</td>
<td>612.52</td>
<td>-151.97</td>
<td>1.07E+05</td>
<td>-2.66E+04</td>
</tr>
<tr>
<td>4</td>
<td>650.99</td>
<td>-116.62</td>
<td>1.14E+05</td>
<td>-2.04E+04</td>
</tr>
<tr>
<td>5</td>
<td>231.73</td>
<td>-384.60</td>
<td>4.06E+04</td>
<td>-6.74E+04</td>
</tr>
<tr>
<td>6</td>
<td>1208.86</td>
<td>-1371.22</td>
<td>2.12E+05</td>
<td>-2.40E+05</td>
</tr>
<tr>
<td>Total</td>
<td>5022.24</td>
<td>-2370.96</td>
<td>8.80E+05</td>
<td>-4.15E+05</td>
</tr>
</tbody>
</table>
Octupole in Quadrupoles When Quad Assembled Like Dipoles

Measured \( b_3 \) (octupole) \( \neq 0 \)

Also seen in 5c mm Arc Quad (7 unit in both magnets)
A Simple Method For Removing Octupole From Quad

**BASIC Problem**

(a) Trying to assemble quad like dipole

or

(b) Starting yoke i.d. is circular (which becomes non-circular after assembly)

**Fix**

1. Start out of round yoke; so that when assembled, it is round → Too late for that (and too expensive to change)

2. Create deliberate asymmetry in coil to compensate for that → Easy, tested and worked AND/CR: "ICE MAGNITURUZUMEN SHIM" (b2b7)
Goals of End Design

Magnetic Design

• Optimize for low integrated harmonics
• Guide design towards lower peak field without large increase in length
• Compute cross talk and fringe fields

Mechanical Layout

• Minimize strain and tilt of the cable in the end. Minimize large changes
• Cable and entire ends should be well supported (constrained)

In low field magnets, magnetic design drives the end design, whereas, in high field (high force) magnets, the mechanical design must! These guiding principles are common to our all high force magnet designs (including 12 T common coil dipole design).
Ends of Cosine Theta Cable Magnets
Ends of Cosine Theta Cable Magnets
End Harmonic Optimization (conceptual)

Ends without spacer
(large harmonics and peak field)

Ends with spacer
(integrated harmonics & peak field reduced)

- End spacers increase the straight section length of some turns (turns at midplane go further out)
- Now consider the integral field generated by each turn. The harmonic component generated by a turn will depend on the angular location of it. The integral strength will depend on the length.
- A proper choice of end spacer can make integral end-harmonics small. However, note that the local values are large.
- Spacer also reduce the maximum value of field on the conductor (peak field) in the end.
Block Structure

Straight section (6 blocks, 70 turns):
30 20 10 4 3 3 (counting from midplane)
3 3 4 10 20 30 (counting from pole)

End section (8 blocks, 70 turns):
10 5 8 4 13 4 6 20
(counting from pole)

Straight section => pole
3,3,4 => 10
4,10,20 => 5, 8, 4, 13
30 => 4, 6, 20

Equal spacing in “Red Color” blocks is used as harmonic optimization parameters
Small Tilt with monotonic change

Block with Midplane Turns

Block with Pole Turns
AKF indicates the deviation from constant perimeter (hence strain on the cable)

Large Deviation from 1.0 is bad

Small deviation with monotonic change

Block with Pole turns

Block with Midplane turns
Coil End: Design A
A high peak field reduces the magnet quench performance.

- A large effort must be undertaken in 2-d optimization.

Usually about thousand cases are examined to:

- Minimize harmonics
- Find a solution with lower peak field
- Good mechanical turn configuration (wedges, tilt angle, etc).

A series of computer programs have been written to carry out the above optimization in an exhaustive and systematic manner.
Peak Field in the Body of the Magnet

Peak Field Location (pole turn)
Peak Field in the End

An example of an End Design
Peak Field in the End
Peak Field in the End
How does it compare to Body?

In this example, the peak value is larger in the end than in the body of the magnet.

In a typical end design, one removes iron (or increase yoke i.d.) to reduce field in the end.
In cosine theta magnets, the conductors in the Ends are more strained and the mechanical design is generally less robust.

Therefore, one would like the peak field in the Ends to be less than that in the body of the magnet, to give conductor a larger margin.
Example (optimized):

Re-adjusted end spacer brings field in the ends down to S.S. level.
Peak Field
Straight Section vs. Ends

Field @ midplane ~2.45 T
Peak Field in S.S. ~3 T @ pole block
Peak Field in Ends in original design ~3.61 T
=> with 2 end spacers between pole blocks ~ 3.36 T
=> Peak field in ends with 3 re-adjusted spacers ~3 T
Block Structure

Straight section (6 blocks, 70 turns):
30 20 10 4 3 3 (counting from midplane)
3 3 4 10 20 30 (counting from pole)

End section (8 blocks, 70 turns):
10 5 8 4 13 4 6 20
(counting from pole)

Straight section => pole
3,3,4 => 10
4,10, 20 => 5, 8, 4, 13
30 => 4, 6, 20

Must avoid large Ultum spacers
(subdivide, if necessary)

Equal spacing in “Red Color” blocks is used as harmonic optimization parameters
End Harmonic Optimization: SMINSQ

Parameters optimized:
- End spacers in block #2 (with 5 turns) and end spacer in block #7 (with 4 turns).
- All spacers within a block have the same size.

Changing the size of two groups of end spacers was adequate to get all harmonics small.

Computed values:
- $B_5 < 1$ unit-meter;
- $B_9$ and $B_{13} < 0.1$ unit-m

Effective Magnetic Length $\sim 15.6$ cm
Mechanical Length $\sim 28$ cm + End Saddle

Block configuration:

(8 blocks, 70 turns):
- 10, 5, 8, 4, 13, 4, 6, 20

(counting from pole)
Field through the Coil Ends

Mechanical length of this end (including end saddle) ~34 cm : ~ 2 coil diameters

Contribution to magnetic length ~16 cm
Field Harmonics through the End

\( b_5 \): dodecapole

End spacers are optimized to produce low integral harmonics
Field Harmonics through the End: $b_9$

End spacers are optimized to produce low integral harmonics.

Harmonic in Gauss

Harmonic in Units

B9(GAUSS)@12cm

b9 at 12 cm in $10^4$ units, normalised to central gradient.
Field Harmonics through the End: $b_{13}$

End spacers are optimized to produce low integral harmonics.

Harmonic in Gauss

Harmonic in Units

$B_{13}(\text{GAUSS})@12\text{cm}$

$b_{13}$ at 12 cm in $10^4$ units, normalised to central gradient

$Z(\text{cm})$

$B_{13}(\text{GAUSS})$

$Z(\text{cm})$
Common Coil Design
(The Basic Concept)

- Simple 2-d geometry with large bend radius (no complex 3-d ends)
- Conductor friendly (suitable for brittle materials - most are - Nb₃Sn, HTS tapes and HTS cables)
- Compact (compared to single aperture LBL’s D20 magnet, half the yoke size for two apertures)
- Block design (for large Lorentz forces at high fields)
- Efficient and methodical R&D due to simple & modular design
- Minimum requirements on big expensive tooling and labor
- Lower cost magnets expected
Field Quality Optimization in the Common Coil Design (Magnet Ends)

Up-down asymmetry gives large skew harmonics, if done nothing. Integrate By.dl 10 mm above and 10 mm below midplane.

An up-down asymmetry in the ends with “no spacer”

By(T) Above midplane
(Integral By.dl = 0.768 Tesla meter)

By(T) Below midplane
(Integral=0.768 Tesla meter)

Up-down asymmetry can be compensated with end spacers. One spacer is used below to match integral By.dl 10 mm above & below midplane.

Proof of principle that it can be removed

By(T) Above midplane
(Integral By.dl = 0.9297 Tesla meter)

By(T) Below midplane
(Integral By.dl = 0.9297 Tesla.meter)

A large Bz.dl in two ends (~1 T.m in 15 T magnet).
- Is it a problem?
- Examine AP issues.
- Zero integral.
- Lead end of one magnet + Return of the next magnet will make it cancel in about ~1 meter (cell length ~200 meters).
- Small v X B.
End harmonics can be made small in a common coil design.

Contribution to integral \((a_n, b_n)\) in a 14 m long dipole \(<10^{-6}\)

<table>
<thead>
<tr>
<th>(n)</th>
<th>(b_n)</th>
<th>(a_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>3</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>-0.005</td>
</tr>
<tr>
<td>5</td>
<td>0.019</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>-0.014</td>
</tr>
<tr>
<td>7</td>
<td>0.025</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>-0.008</td>
</tr>
<tr>
<td>9</td>
<td>-0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>11</td>
<td>-0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Reduction in Peak Field in the Ends of Common Coil

Field and Field lines as computed by OPERA 2-d

Peak field in the ends is minimized by:

- Removing iron over the ends
- Using end spacers between the turns.
A Helical Magnet for AGS at BNL (1)

This magnet uses helical coils to maintain the polarization of the beam as it passes spin resonances in AGS.

Note: Particle Tracking
A Helical Magnet for AGS at BNL (2)
We are now expert in:

2d coil design
   Requires good mechanical, magnetic and flexible design

3d coil design
   Requires good mechanical and magnetic design