

SAMPLE PARAMETERS OF A TWO-MILE SUPERCONDUCTING ACCELERATOR*†

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The results derived in Ref. 2 have been used to calculate a sample set of parameters¹ for a superconducting version of the two-mile SLAC accelerator. These calculations have been carried out in order to provide a basis for further technical discussions.

BEAM ENERGY

The energy goal of 100 GeV (10 MeV/ft) has been chosen since this energy is sufficiently higher than the 20 GeV capability of the present SLAC accelerator to be of great interest in physics research and yet within anticipated fiscal and technical limitations. It should be remarked here that the gradient of 10 MeV/ft is several times higher than the best gradient achieved to date in a superconducting accelerator. However, superconductivity in its application to accelerators is still in its infancy; therefore, in establishing tentative design parameters for a future accelerator it has been assumed that further advances will make the proposed gradient realistic.

SELECTION OF ACCELERATOR TYPE

As noted in Ref. 2, the ratio of peak-to-average fields in a traveling wave accelerator is significantly less than in a standing wave accelerator (a factor of ≈ 1.6 improvement appears to be achievable). For this reason, the tentative decision has been made to adopt a traveling wave design with rf feedback.

DUTY CYCLE

The possibility of achieving a high duty cycle is one of the attractive features of a superconducting accelerator. However, adopting a 100% duty cycle for the projected 100 GeV two-mile machine would lead to excessive refrigeration costs on the basis of present-day cost experience and estimates. For this reason, the duty cycle of the superconducting machine in this preliminary listing of criteria has been limited to 6%. Even so, this is 100 times the duty cycle of the existing SLAC accelerator and would result in greatly improved experimental statistics.

The filling time of a superconducting accelerator is very long (typically 10 to 100 msec) compared to conventional "room temperature" accelerators (≈ 0.5 to $5 \mu\text{sec}$). The rf pulse length must be many times longer (say 0.3 to 6 sec) in order to allow time for the fields to reach steady state and for particles to be accelerated. Practically speaking, an rf source which is required to produce full power for several

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†This report is based upon an earlier SLAC Technical Note (Ref. 1).

1. W.B. Herrmannsfeldt, R.B. Neal, and P.B. Wilson, Stanford Linear Accelerator Center Technical Note SLAC-TN-68-9 (1968).

seconds must, for cathode emission and heat dissipation purposes, be designed to operate in a cw manner. Moreover, the existing ac distribution system in the SLAC Klystron Gallery has sufficient capacity to supply the required power on a cw basis. Therefore, the design approach which has been adopted is based on initial operation at a 6% duty cycle but with ac and rf capability for a 100% duty cycle. Then, if at a later date the cost of providing refrigeration decreases, the duty cycle could be increased by adding more refrigeration units to the system.

CHOICE OF FREQUENCY

Operation of a superconducting accelerator at the lower microwave frequencies is favored² because of the higher Q's and the resulting lower dissipative losses at these reduced frequencies. The minimum feasible frequency is probably limited by the increasing physical size of the structure, the increasing filling time, and the more stringent phasing and impedance matching tolerances as the frequency is reduced. Moreover, in the case of the projected superconducting accelerator for SLAC, there is a further consideration which merits attention. There is already in existence an expensive high power waveguide system which operates at a particular frequency, viz., 2856 MHz. Considerable cost savings would obviously result if the same frequency were chosen for the superconducting accelerator so that the existing waveguide system could be used. However, it is not yet clear whether or not this choice of a relatively high frequency would result in sufficiently higher refrigeration costs which would outweigh the cost savings just discussed. For this reason, it has been decided to carry out preliminary design studies and cost analyses at two frequencies, 1428 and 2856 MHz, to determine whether either frequency has a definite technical and/or cost advantage over the other. The parameters shown in Table I have therefore been calculated for both frequencies. The basic beam criteria (beam energy, beam current, beam power, and duty cycle) are the same at each frequency. However, the filling times, pulse lengths, and feedback loop characteristics are different at the two frequencies.

r/Q, r, AND Q

The starting points in determining these parameters are: (1) the expected improvement³ in Q at 1300 MHz and 1.85°K by a factor of 10⁶ compared to the Q in the same structure at room temperature; and (2) the room temperature values at 2856 MHz of $Q = 1.3 \times 10^4$ and $r = 56 \text{ m}\Omega/\text{m}$. The following scaling laws are then used to calculate the values given in Table I:

$$\begin{aligned} \text{Improvement factor} &\propto \omega^{-3/2} \\ r &\propto \omega^{-1} \\ Q &\propto \omega^{-2} \end{aligned}$$

The room temperature values at 2856 MHz given above can probably be improved by using a somewhat more complicated cavity geometry (e.g., curving the outer walls, etc.) but such potential improvements have not been taken into account in calculating the values given in Table I.

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2. R.B. Neal, Stanford Linear Accelerator Technical Note SLAC-TN-68-1 (1968) (also reported to this Summer Study in slightly modified form, p. 111).
 3. P.B. Wilson, private communication.

AVERAGE BEAM POWER

For initial operation at 6% duty cycle, an average beam power of 0.3 MW has been assumed. This is approximately the same as the maximum average beam power achieved by the present SLAC accelerator. Since a beam conversion efficiency of $\approx 100\%$ should be achieved in the superconducting version, a total rf power of 0.3 MW average and 4.8 MW peak will be required.

KLYSTRON POWER

To take maximum advantage of equipment already available in the SLAC Klystron Gallery the same number of klystrons (≈ 240) as used with the present SLAC accelerator will be employed. Thus, the rf peak power and average power from each klystron will be 20 kW and 1.2 kW, respectively. However, as noted earlier, each klystron will be rated at 20 kW average power which will permit later increase of duty cycle if desired.

Fortunately, the input section of the existing SLAC modulators (ac transformer, silicon rectifiers, filter, control gear, etc.) has the proper rating to supply a 20 kW klystron on a cw basis. The output of the filter section is ≈ 20 kV which means that a tube which has an efficiency of 50% will have a cathode current of 2 A and a perveance of 0.7×10^{-6} amp volt $^{-3/2}$. It is also fortunate that the existing variable voltage substations along the klystron gallery can be used without modification.

PEAK BEAM CURRENT

The peak beam current is obtained by dividing the peak beam power by the beam energy, i.e.,

$$i_p = \frac{4.8 \times 10^6}{100 \times 10^9} = 48 \text{ } \mu\text{A}$$

POWER DISSIPATED IN ACCELERATOR

From Eq. (4) of Ref. 2, the power dissipated can be written

$$P_d = \frac{V^2}{rL} \times \text{duty cycle}$$

where V is the total beam energy, r is the shunt impedance per unit length, and L is the total accelerator length. Thus

$$\text{At 1428 MHz: } P_d = \frac{10^{22} \times 0.06}{3.44 \times 10^{13} \times 3 \times 10^3} = 5800 \text{ W}$$

$$\text{At 2856 MHz: } P_d = \frac{10^{22} \times 0.06}{1.72 \times 10^{13} \times 3 \times 10^3} = 11\,600 \text{ W}$$

COST OF REFRIGERATION

Based on cost estimates³ made at the W.W. Hansen Laboratories of Physics, the cost of a refrigeration unit capable of cooling to 1.85°K can be approximated by the following formula:

$$C \approx \$12\,000 P^{0.6}$$

where P is the dissipative power in watts which must be removed by the refrigeration unit. Let N be the number of separate refrigeration units, so that $NP = P_d =$ the total power dissipated in the accelerator and feedback loops. The total cost C_T can then be written

$$\begin{aligned} C_T &= N \, 12\,000 \left(\frac{P_d}{N} \right)^{0.6} \\ &= \$12\,000 N^{0.4} P_d^{0.6} \end{aligned}$$

Thus, the total cost (neglecting distribution costs) is reduced as the total number of individual refrigeration units is reduced. In this study, it has been assumed that $N = 15$ since this is the present number of separate power substations along the accelerator and has been found to be a convenient number for control purposes. Then, the estimated costs are:

$$\text{At 1428 MHz: } C_T = (12\,000)(15)^{0.4} (5800)^{0.6} = \$6.4 \text{ M}$$

$$\text{At 2856 MHz: } C_T = (12\,000)(15)^{0.4} (11\,600)^{0.6} = \$9.7 \text{ M}$$

These estimates may change significantly after several more years of cryogenic experience and development and hopefully will be reduced.

FILLING TIME

From Eq. (43) of Ref. 2, the filling time of the traveling wave accelerator with feedback to 63.2% buildup of the electric field (at maximum beam conversion efficiency) is

$$t_F = \frac{2Q}{\omega} \frac{V}{irL}$$

Thus,

$$\text{At 1428 MHz: } t_F = \frac{2 \times 1.6 \times 10^{10}}{0.895 \times 10^{10}} \frac{10^{11}}{48 \times 10^{-6} \times 3.44 \times 10^{13} \times 3 \times 10^3} = 72 \text{ msec}$$

$$\text{At 2856 MHz: } t_F = \frac{2 \times 4.0 \times 10^9}{1.79 \times 10^{10}} \frac{10^{11}}{48 \times 10^{-6} \times 1.72 \times 10^{13} \times 3 \times 10^3} = 18 \text{ msec}$$

RF PULSE LENGTH

Four of the "filling times" calculated above are required to build up the fields to 98% of the steady-state value. After this buildup, the rf pulse should be on for a period which is long compared to the buildup time in order that the beam duty cycle may be a reasonably large fraction of the rf duty cycle. It will be very desirable to program the beam turn-on during the rf buildup so that the rf and beam loading transients can occur essentially simultaneously. It is not yet clear how successful this technique will be and thus rf pulse lengths in the range of 20 to 80 times the "filling time" are given in Table I.

TIME OFF BETWEEN PULSES

At a 6% duty cycle, the ratio of "time off" to "time on" is just 0.94/0.06. This ratio is used in calculating the "time off" values shown in Table I.

BEAM PULSE LENGTH

For criteria purposes, the beam pulse length has been assumed to be the rf pulse length less one "filling time" for the short rf pulse and less four "filling times" for the long rf pulse.

TEMPERATURE RISE DURING EACH RF PULSE

It is believed that a suitable Dewar capable of enclosing the accelerator and the feedback waveguide at either frequency under consideration would be about 18 in. in diameter and would contain about 20 ℓ /ft of liquid helium. The specific heat of liquid helium is 450 J/ ℓ / $^{\circ}$ K giving a heat capacity of $\approx 10^4$ J/ft/ $^{\circ}$ K. From the calculations of power dissipation made above, the temperature rise per pulse can be calculated as follows:

$$\text{At 1428 MHz: Power dissipation} = 0.58 \text{ W/ft (av)} = \frac{0.58}{0.06} = 9.7 \text{ W/ft (peak)}$$

$$\text{Heat liberated per pulse} = 9.7 \frac{\text{J}}{\text{sec ft}} \times 5.76 \text{ sec} = 56 \text{ J/ft}$$

$$\text{Temperature rise during pulse} = \frac{56}{10^4} = 0.0056^{\circ}\text{K}$$

$$\text{At 2856 MHz: Power dissipation} = 1.16 \text{ W/ft (av)} = 19.3 \text{ W/ft (peak)}$$

$$\text{Heat liberated per pulse} = 19.3 \frac{\text{J}}{\text{sec ft}} \times 1.44 \text{ sec} = 27.8 \text{ J/ft}$$

$$\text{Temperature rise during pulse} = \frac{27.8}{10^4} = 0.0028^{\circ}\text{K}$$

RF FEED INTERVAL

In the present SLAC accelerator, the rf feed interval is 10 ft (the power from each klystron is split into four equal parts). In the superconducting version it has been assumed that the feed interval is 40 ft in order to reduce the total amount of instrumentation and control equipment required. Variation of the principal design parameters with respect to feed interval is shown in Table II. All cases are normalized with respect to the 10 ft feed interval case; all parameters of the 10 ft case are normalized to unity. The same total rf power input and the same beam energy and current are assumed in all cases. Two classes are given in Table II: in class 1 the attenuation parameter τ is held constant as the feed interval is varied; in class 2, τ is proportional to the feed interval. In class 1 the disk aperture must be increased as the feed interval is increased; this causes a decrease in r , the shunt impedance per unit length, as shown. In class 2, the cavity geometry is preserved as the feed interval is varied; therefore r remains constant. The most significant distinctions between the two classes relate to the circulating power P_0 and the bridge ratio g . In class 1, increasing the feed interval requires a decreasing attenuation per unit length in order to keep the total attenuation τ constant; the circulating power must then increase in order to keep the field strength constant. The bridge ratio g in

class 1 varies only sufficiently to compensate for the variation in r . In class 2, however, the circulating power P_O remains constant since the attenuation per unit length remains constant for all feed intervals. The bridge ratio g varies inversely with feed interval.

In the preliminary listing of criteria shown in Table I it has been assumed that class 2 with a 40 ft feed interval will be adopted. This choice results in reduced instrumentation and control requirements and a relatively small value of the bridge ratio g . A reduced value of g eases phasing and matching problems in the feedback loop.

ACCELERATOR ATTENUATION FACTOR (τ)

In Table I, it has been assumed that each 40 ft section of the superconducting accelerator will consist of cavities of the same geometry as an "average" cavity of the present 10 ft SLAC sections. Each existing 10 ft SLAC section has an attenuation parameter $\tau = 0.57$ at room temperature. Four of these sections arranged contiguously with a single feed would therefore result in a τ of 2.28. At 1.85°K, the Q of these sections would be improved by the factor $4.0 \times 10^9 / 1.3 \times 10^4 = 3.08 \times 10^5$. Thus at 2856 MHz and 1.85°K, the attenuation parameter would be $\tau = 2.28 / 3.08 \times 10^5 = 74.0 \times 10^{-7}$ nepers. For the same cavity geometry scaled proportional to wavelength, $\tau \propto \omega^3$, and thus at 1428 MHz and 1.85°K, $\tau = 74 \times 10^{-7} / 2^3 = 9.24 \times 10^{-7}$ nepers.

FEEDBACK ATTENUATION FACTOR (γ)

In Table I it has been assumed that the loss in the feedback loop is 10% of the accelerator loss. Since the feedback loop consists principally of straight waveguide, this estimate is probably higher than the actual loss will turn out to be.

BRIDGE RATIO (g)

The required bridge ratio can be calculated from the following relation [Eq. (20) of Ref. 2]:

$$g = \frac{P_T}{2\tau i^2 rL}$$

Thus,

$$\begin{aligned} \text{At 1428 MHz: } g &= \frac{4.8 \times 10^6}{18.48 \times 10^{-7} \times (48)^2 \times 10^{-12} \times 3.44 \times 10^{13} \times 3 \times 10^3} \\ &= 1.095 \times 10^4 \end{aligned}$$

$$\begin{aligned} \text{At 2856 MHz: } g &= \frac{4.8 \times 10^6}{148 \times 10^{-7} \times (48)^2 \times 10^{-12} \times 1.72 \times 10^{13} \times 3 \times 10^3} \\ &= 0.273 \times 10^4 \end{aligned}$$

CIRCULATING POWER (P_O)

The normalized circulating power P_O/P_S at maximum beam conversion efficiency is just equal to $1 + g$ [Eq. (6) of Ref. 2]. Since the input power per klystron, P_S , is

20 kW, the circulating powers in the two cases become:

$$\text{At 1428 MHz: } P_o = 1.095 \times 10^4 \times 20 \times 10^3 = 219 \text{ MW}$$

$$\text{At 2856 MHz: } P_o = 0.273 \times 10^4 \times 20 \times 10^3 = 54.6 \text{ MW}$$

REQUIRED PHASING ACCURACY IN LOOP

The vector diagram of the voltages in the feedback loop at steady state is shown in Fig. 1. The input from the rf source into the loop is $V_s/(1+g)^{1/2}$. The net feedback voltage vector is $x^{-1} g^{1/2}/(1+g)^{1/2} V_o$, where x is the total attenuation due to loss in the walls and to beam loading. At maximum beam conversion efficiency $x = g^{1/2}/(1+g)^{1/2}$ [Eq. (5) of Ref. 2]. In this case and with $\varphi = 0$ all the vectors in Fig. 1 lie along the same line and the following relation holds:

$$\frac{V_s}{(1+g)^{1/2}} + \frac{g}{1+g} V_o = V_o$$

or

$$\frac{V_o}{V_s} = (1+g)^{1/2}$$

When $\varphi \neq 0$, the following solution of the vector diagram may be obtained using the law of cosines³:

$$V_o^2 + \frac{1}{x^2} \left(\frac{g}{1+g} \right) V_o^2 - \frac{2}{x} \left(\frac{g}{1+g} \right)^{1/2} V_o^2 \cos \varphi = \frac{V_s^2}{1+g}$$

Substituting $x = g^{1/2}/(1+g)^{1/2}$ and $\cos \varphi \approx 1 - \frac{1}{2} \varphi^2$ yields the result

$$\left(\frac{V_o}{V_s} \right)^2 \approx (1+g)(1 - g^2 \varphi^2)$$

or

$$\frac{\delta V_o}{V_o} \approx - \frac{(g\varphi)^2}{2}$$

Thus, in order for the loop gain to be reduced by no more than 1% by the loop phase error, it is necessary that

$$|g\varphi| \leq (0.02)^{1/2} = 0.1414$$

Using the value of g calculated above, the required loop phase accuracies become:

$$\text{At 1428 MHz: } \varphi \leq \frac{0.1414}{1.095 \times 10^4} = 1.29 \times 10^{-5} \text{ rad} = 7.4 \times 10^{-4} \text{ deg}$$

$$\text{At 2856 MHz: } \varphi \leq \frac{0.1414}{0.273 \times 10^4} = 5.16 \times 10^{-5} \text{ rad} = 29.6 \times 10^{-4} \text{ deg}$$

CONCLUSIONS

The tentative parameters calculated in this report and summarized in Table I have been set down to guide further technical and cost studies. It is highly probable that these parameters will be changed as a result of future experience and developments. Many additional questions must be investigated theoretically and experimentally before the basic design framework of a two-mile superconducting accelerator can be considered established.

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TABLE I
Parameters of a Two-Mile Superconducting
Accelerator at Two Frequencies

Parameter	f = 1428 MHz	f = 2856 MHz
Length	3000 m	3000 m
\bar{F}/Q	$2.15 \times 10^3 \Omega/\text{m}$	$4.3 \times 10^3 \Omega/\text{m}$
\bar{F}	$3.44 \times 10^{13} \Omega/\text{m}$	$1.72 \times 10^{13} \Omega/\text{m}$
Q	16×10^9	4.0×10^9
Loaded energy (max)	100 GeV	100 GeV
V/L	10 MeV/ft	10 MeV/ft
Duty cycle	0.06	0.06
Peak beam current	48 μA	48 μA
Average beam current	3 μA	3 μA
Peak beam power	4.8 MW	4.8 MW
Average beam power	0.3 MW	0.3 MW
Number of klystrons	240	240
Peak power per klystron	20 kW	20 kW
Average power per klystron	1.2 kW	1.2 kW
Type of rf structure	TW with rf feedback loop	TW with rf feedback loop
Filling time (to 63.2%)	72 msec	18 msec
Power dissipated in accelerator	5800 W (0.58 W/ft)	11 600 W (1.16 W/ft)
Total cost refrigeration	\$6.4 M	\$9.7 M
Pulse length (rf)	1.44 to 5.76 sec	0.36 to 1.44 sec
Pulse length (beam)	1.37 to 5.47 sec	0.34 to 1.37 sec
Time off between rf pulses	22.6 to 90.3 sec	5.65 to 22.6 sec
Normalized peak current (i_n) at η_{max}	7.03	4.97
Accelerator attenuation factor (τ)	9.24×10^{-7}	74.0×10^{-7}
Feedback attenuation factor (γ)	0.924×10^{-7}	7.4×10^{-7}
Bridge ratio (g)	1.095×10^4	0.273×10^4
Circulating power (P_o/P_s) at η_{max}	1.095×10^4	0.273×10^4
Required phasing accuracy in loop for $\delta V_o/V_o \leq 0.01$	7.4×10^{-4} deg	29.6×10^{-4} deg

TABLE II

Variation of Basic Parameters of a Superconducting Accelerator
as a Function of Feed Interval

Two classes are considered: Class 1 with attenuation parameter $\tau = \text{constant}$; class 2 with τ proportional to the distance between fields.

Parameter	Class 1: $\tau = \text{constant}$			Class 2: τ proportional to distance between feeds		
	Distance between feeds			Distance between feeds		
	10 ft	20 ft	40 ft	10 ft	20 ft	40 ft
Attenuation parameter (τ)	1	1	1	1	2	4
Beam energy (V_T)	1	1	1	1	1	1
Peak beam current (i_p)	1	1	1	1	1	1
Shunt impedance (r)	1	0.87	0.77	1	1	1
Total rf input power (P_T)	1	1	1	1	1	1
Input power per feed (P_S)	1	2.0	4.0	1	2.0	4.0
Bridge ratio (g)	1	1.15	1.30	1	0.5	0.25
Normalized circulating power (P_O/P_S)	1	1.15	1.30	1	0.5	0.25
Circulating power (P_O)	1	2.3	5.2	1	1	1
Filling time (t_F)	1	1.15	1.30	1	1	1
Total power dissipated (P_d)	1	1.15	1.30	1	1	1

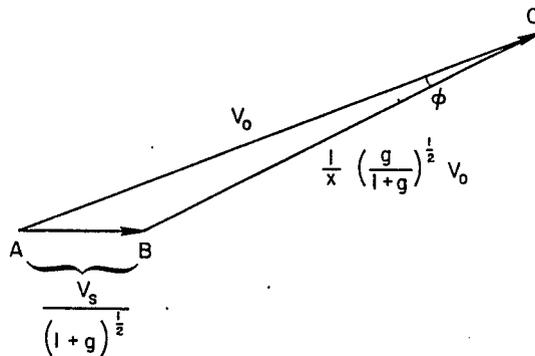


Fig. 1. Steady-state condition in feedback loop with phase error ϕ .