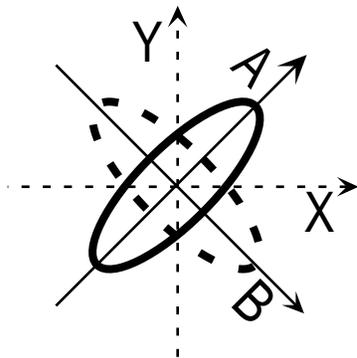


Skew Quadrupole Lattice Design Principles

Skew Quadrupole Lattice \neq Fully Coupled Lattice

The lattices presented here are uncoupled. They are special only in that the betatron eigenplanes (denoted A,B) line up with $\pm 45^\circ$ rather than the horizontal (X) and vertical (Y) axes.

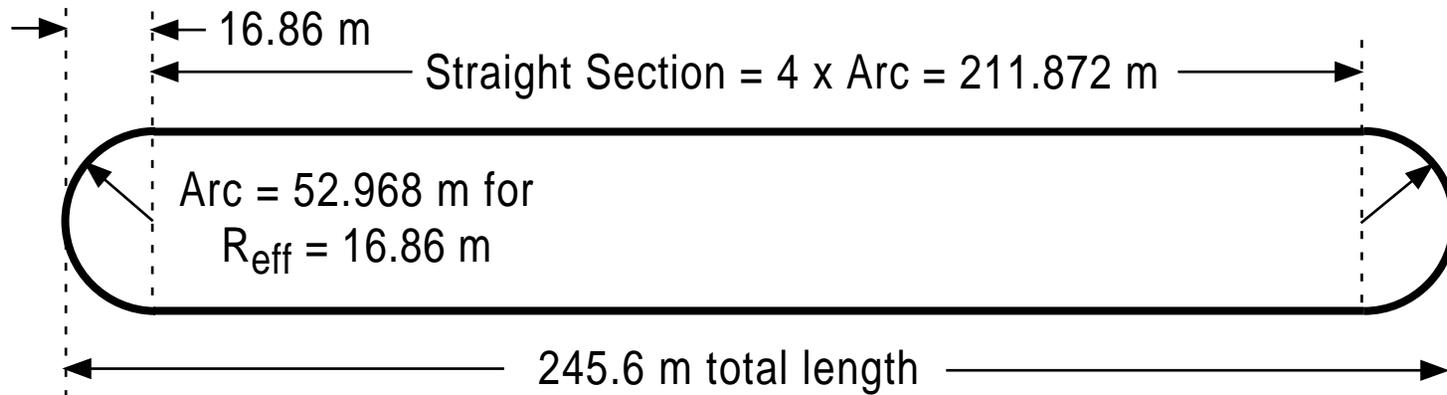


Useful Trick:

Every dipole with bend radius, ρ , has a weak normal focusing component, $k = -1/(2\rho^2)$ added in order to make the focusing cylindrically symmetric.

Ring Layout & Arc Magnet Parameters

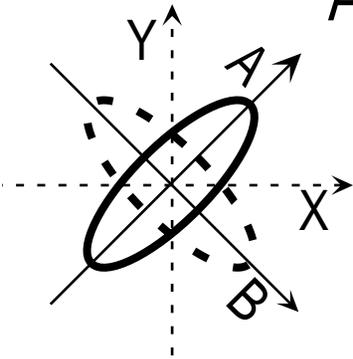
Arc Magnet Parameters: $B_1 = 6.986 \text{ T}$, $G = 0 \text{ T/m}$, $L_1 = 1.10 \text{ m}$
 $B_2 = 3.493 \text{ T}$, $G = 20.19 \text{ T/m}$, $L_2 = 1.55 \text{ m}$



$$\text{Decay Ratio} = \frac{211.872 \text{ m}}{529.680 \text{ m}} = 0.4 \text{ per straight section}$$

Skew Combined Function Ring Lattice

Arc Cell Parameters:



$$\beta_{\max}^{(A,B)} = 8.80 \text{ m}$$

$$\beta_{\min}^{(A,B)} = 3.16 \text{ m}$$

$$\Delta\phi = 60^\circ$$

$$\eta_X = (\eta_A + \eta_B) / \sqrt{2}$$

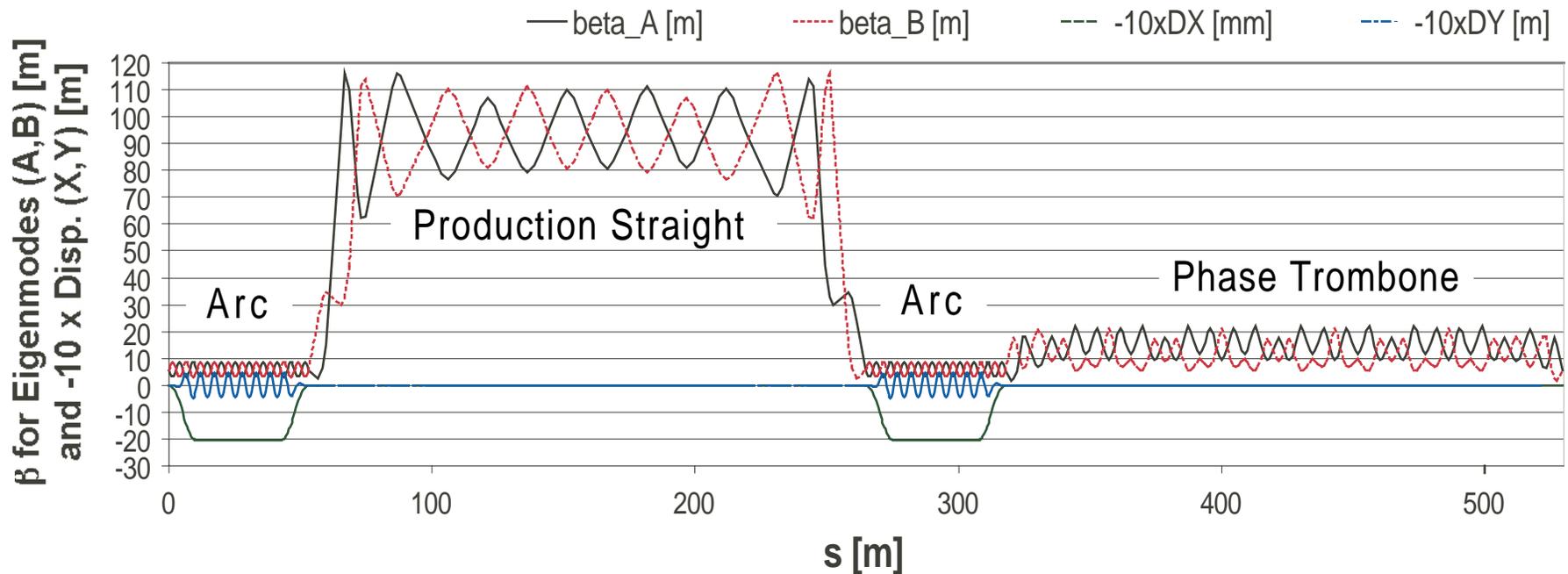
$$\eta_{\text{Long}}^{(A,B)} = 1.782 \text{ m}$$

$$\eta_{\text{Short}}^{(A,B)} = 1.141 \text{ m}$$

$$\eta_Y = (\eta_A - \eta_B) / \sqrt{2}$$

$$\eta_{\max}^X = 2.066 \text{ m}$$

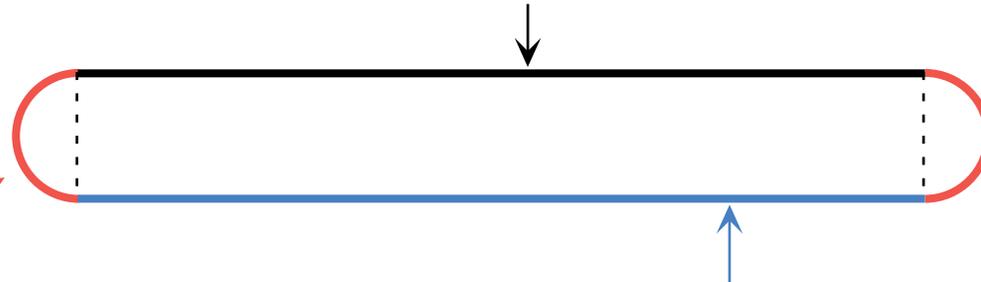
$$\eta_{\max}^Y = 0.484 \text{ m}$$



Storage Ring Lattice Modules and Their Functions

Production straight is 4 times the arc length for 0.4 decay ratio. Also average beta is 91 m to ensure only a 10% average contribution to ν divergence. Will introduce normal quadrupoles for coupling control and additional injection elements.

Arcs have 10 cells
Beta peak = 8.8 m
Phase/cell = 60°
Cells w/o bending near ends
for dispersion suppression
Skew-sextupoles used either
in central 6 or all 10 cells
for chromaticity correction



Second straight has natural beta about double the arcs but cells are adjusted to make a phase trombone to get 1 unit phase difference between eigenplanes. All adjustments to ring tunes will be done here.

Combined Function Skew Quadrupole Cell Principle

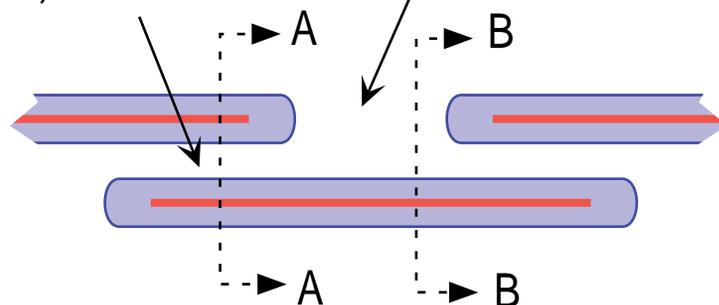
Superconducting coils are in independent flat cryostats which are longitudinally staggered.

Plan View



Full coil overlap gives the full field, $B = 7 \text{ T}$, but $G=0$

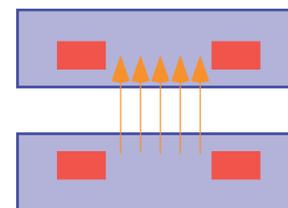
Single coil for half field, $B = 3.5 \text{ T}$, but $G=20 \text{ T/m}$



Elevation View

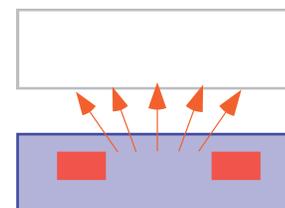
Net result is to use space which is normally wasted between coil ends.

Section A-A



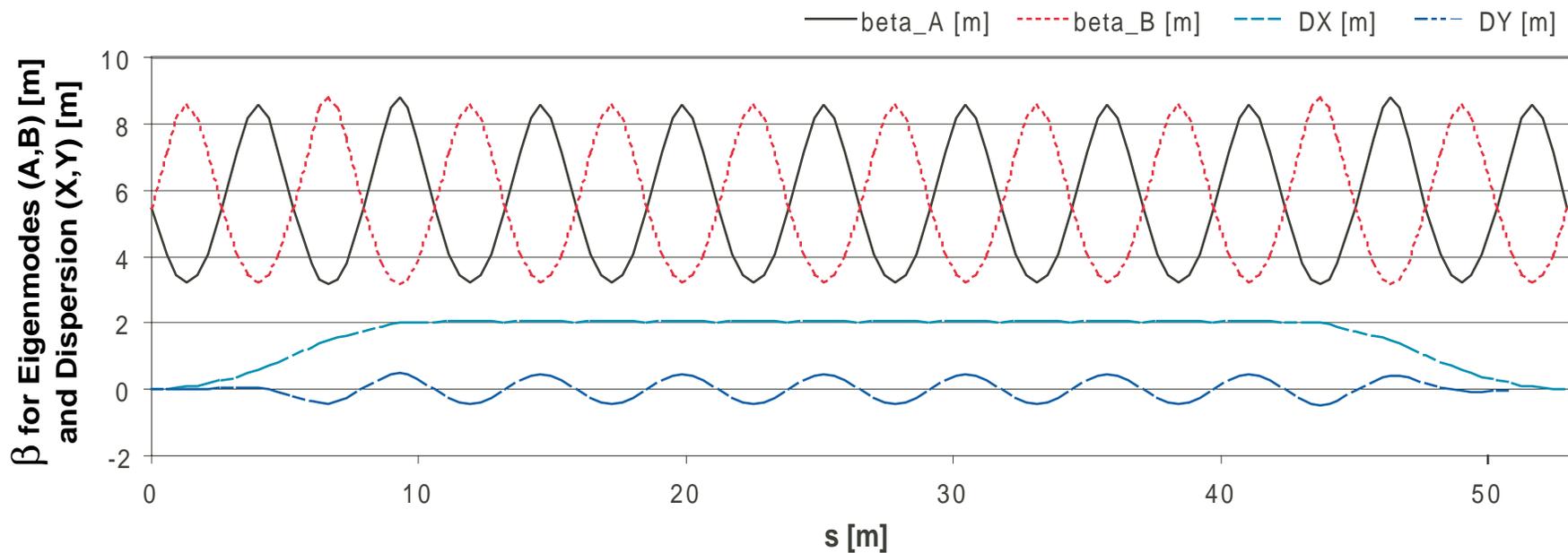
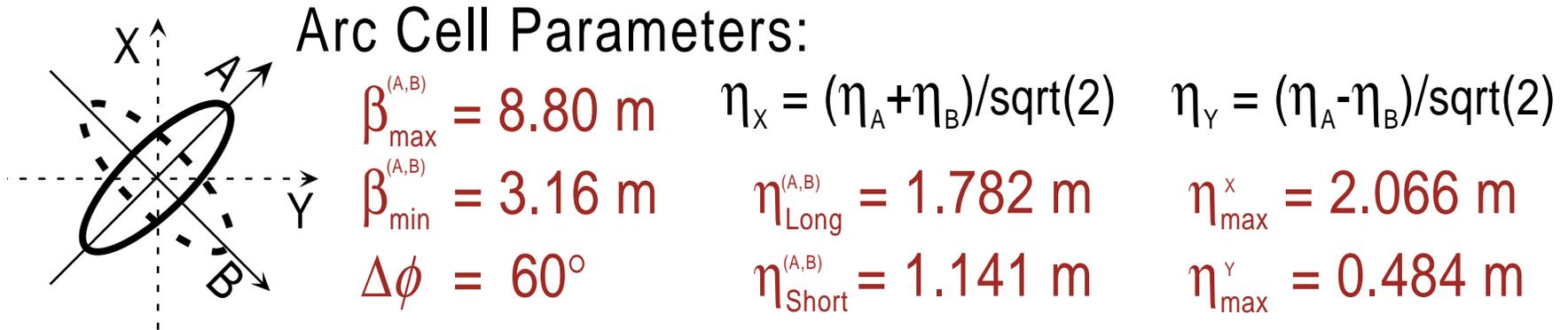
$B = 7 \text{ T}$
 $G = 0 \text{ T/m}$
No skew

Section B-B



$B = 3.5 \text{ T}$
 $G = 20 \text{ T/m}$
Skew Quad

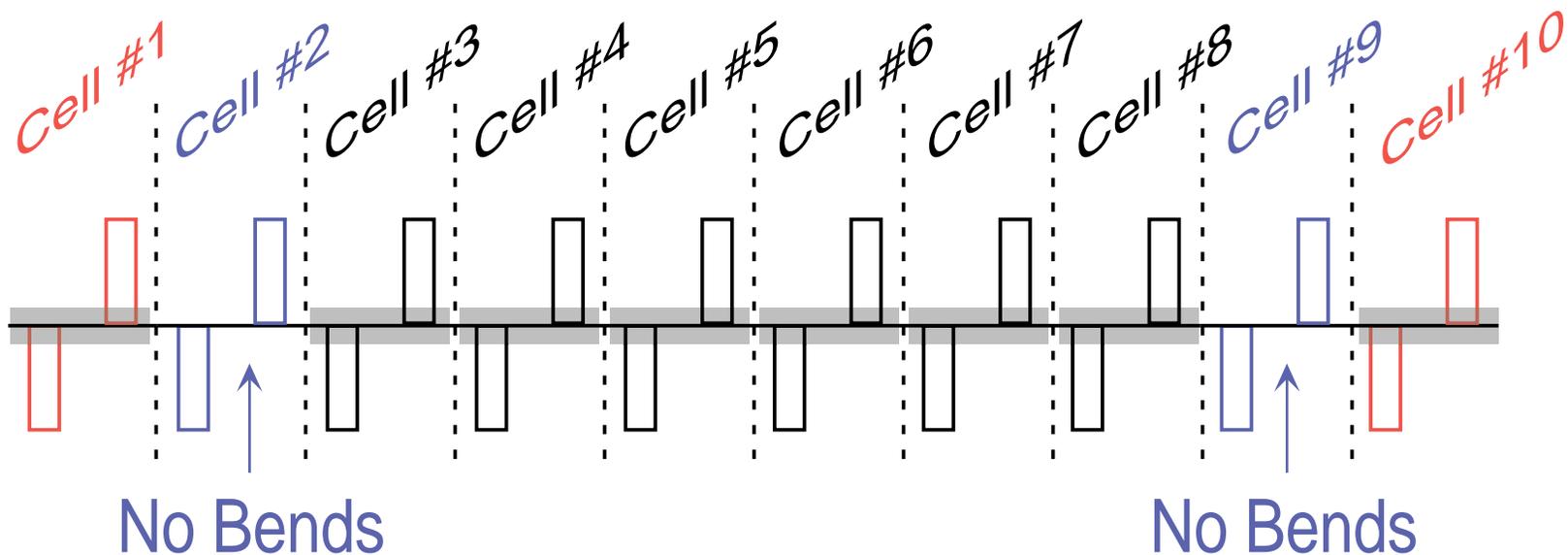
Arc Lattice Functions Detail



Phase advance = $1.\overline{666}$ & chromaticity contribution = -1.635 in both planes.

Arc Lattice Dispersion Suppression Scheme

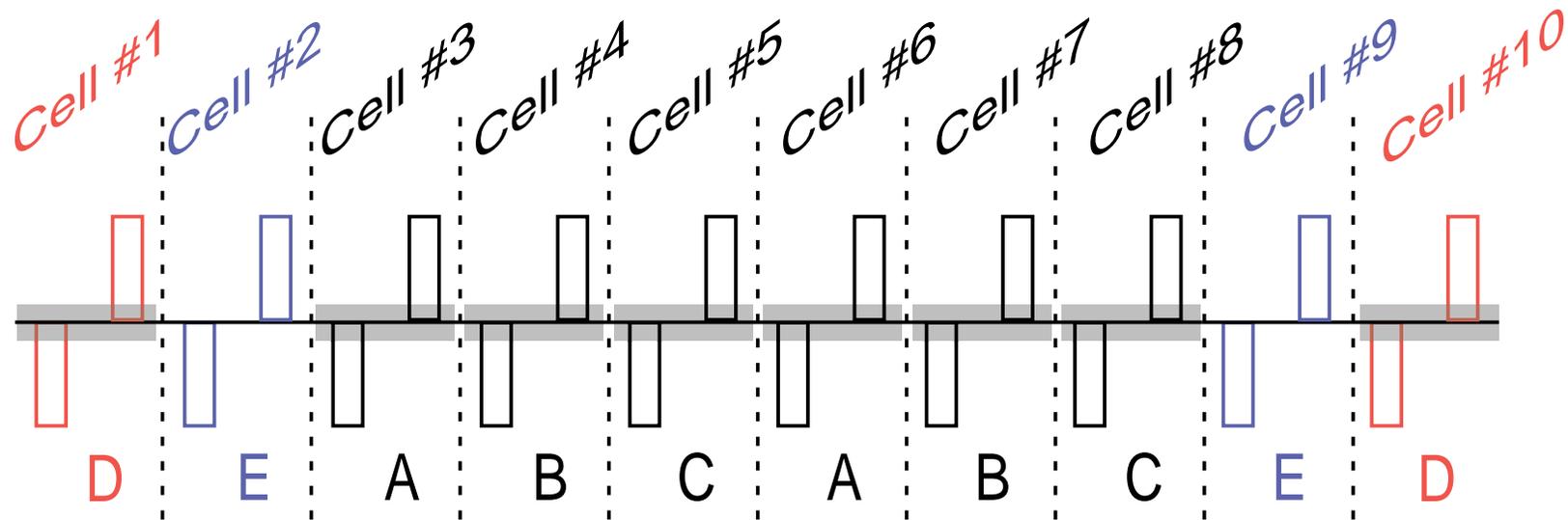
- With 60° phase advance, the central six cells make up an achromat.
- Peak dispersion is reduced via two additional cells at each arc end where one of the cells has zero dipole bending.



Note that because of its lack of bending the focusing is different in the no bend cells and this leads to a small mismatch. This can be fixed by adjusting its length and quadrupole strength to match the other cells, the approach used here, or if the cell length is kept constant, by adjusting the skew quadrupole strengths in both of the outer cells.

Skew Sextupole Chromaticity Correction Schemes

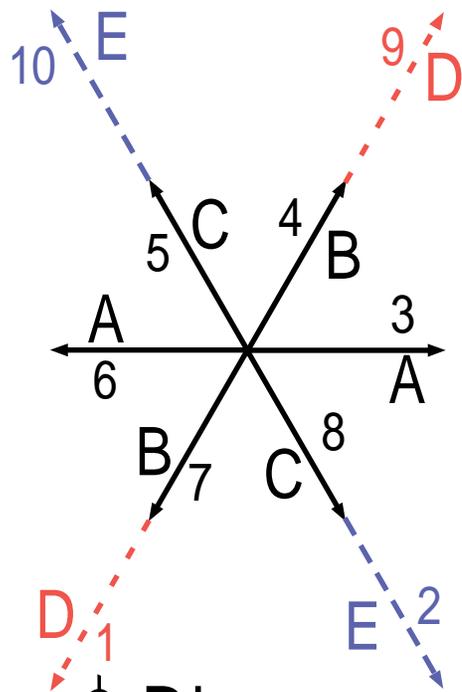
With a phase advance per cell of 60° it is possible to make a chromaticity correction scheme using the central six cells (3-8) very similar to the original HERAe correction scheme which is very flexible and exhibits good dynamic aperture.



Note that for the skew quadrupole lattices considered here, we find it much better to use skew sextupoles than normal sextupoles for chromaticity correction as the needed strengths are at least a factor 4.5 lower using skew sextupoles!

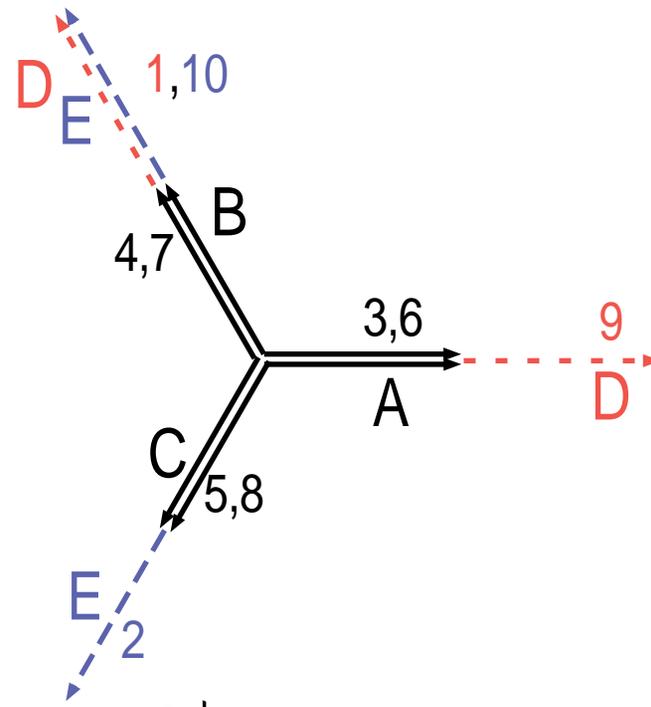
Skew Sextupole Chromaticity Correction Schemes

For central 6 skew sextupoles both first order and higher order moments cancel!



ϕ Diagram

Phase Advance = 60°



2ϕ Diagram

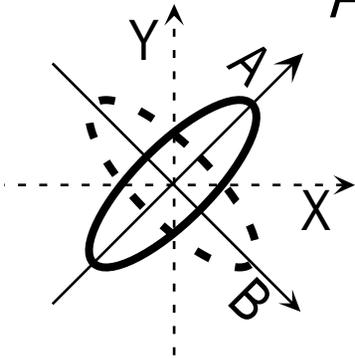
Phase Advance = 120°

Using all 10 skew sextupoles reduces strength needed for linear chromaticity correction but could drive resonances unless strengths are carefully adjusted.

It is possible to arrange the skew sextupoles in families to cancel both the off-energy beta-beat and its derivative (the sine-like & cosine-like terms).

Ring Lattice Functions With 0.5% Momentum Offset*

Arc Cell Parameters for $\Delta p/p = 0.005$:

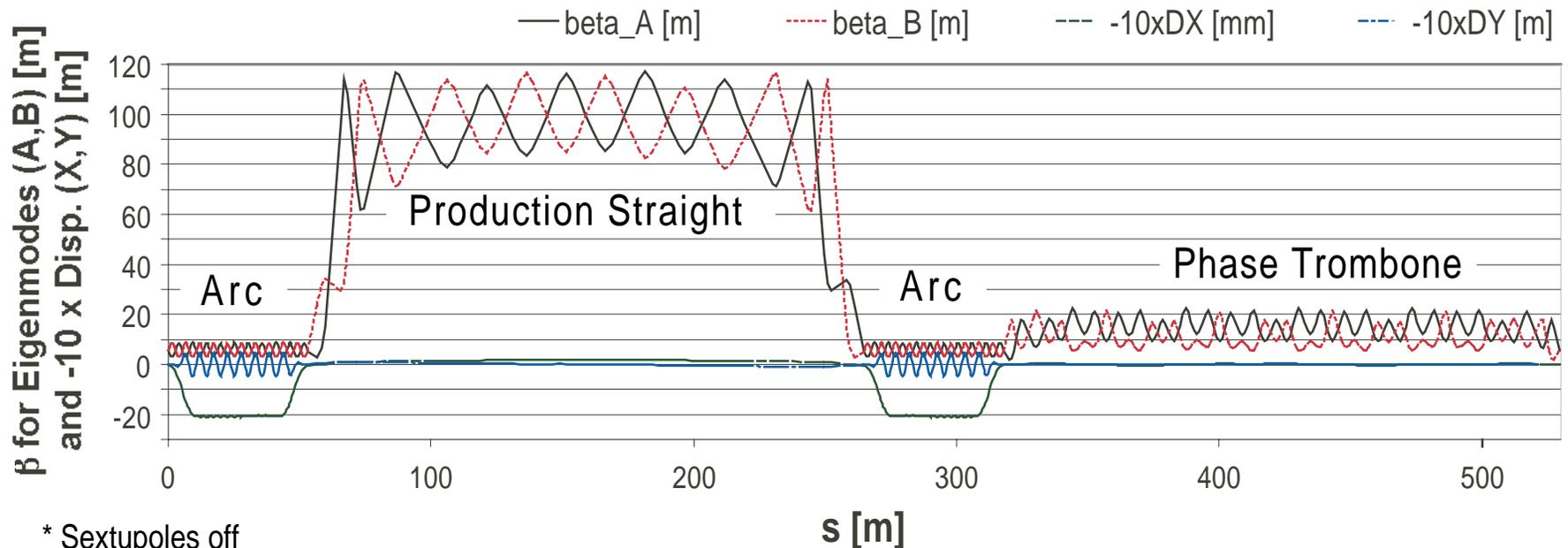


$$\beta_{\max}^{(A,B)} = 8.92 \text{ m} \quad \eta_X = (\eta_A + \eta_B) / \sqrt{2} \quad \eta_Y = (\eta_A - \eta_B) / \sqrt{2}$$

$$\beta_{\min}^{(A,B)} = 3.17 \text{ m} \quad \eta_{\text{Long}}^{(A,B)} = 1.814 \text{ m} \quad \eta_{\max}^X = 2.106 \text{ m}$$

$$\Delta\phi = 59.8^\circ \quad \eta_{\text{Short}}^{(A,B)} = 1.165 \text{ m} \quad \eta_{\max}^Y = 0.459 \text{ m}$$

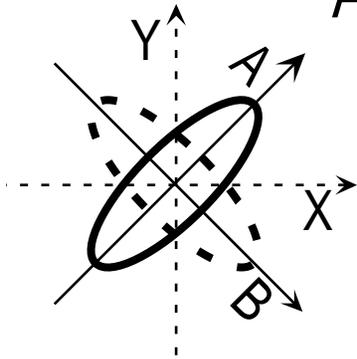
Yields 9 mm max. c.o. error, 5 mm RMS in both A & B eigenplanes but 10.4 mm max. c.o. error horizontal and 2.3 mm max. c.o. error vertical



* Sextupoles off

Ring Lattice Functions With 0.5% Momentum Offset*

Arc Cell Parameters for $\Delta p/p = 0.005$: $\Delta v = 0.0006$

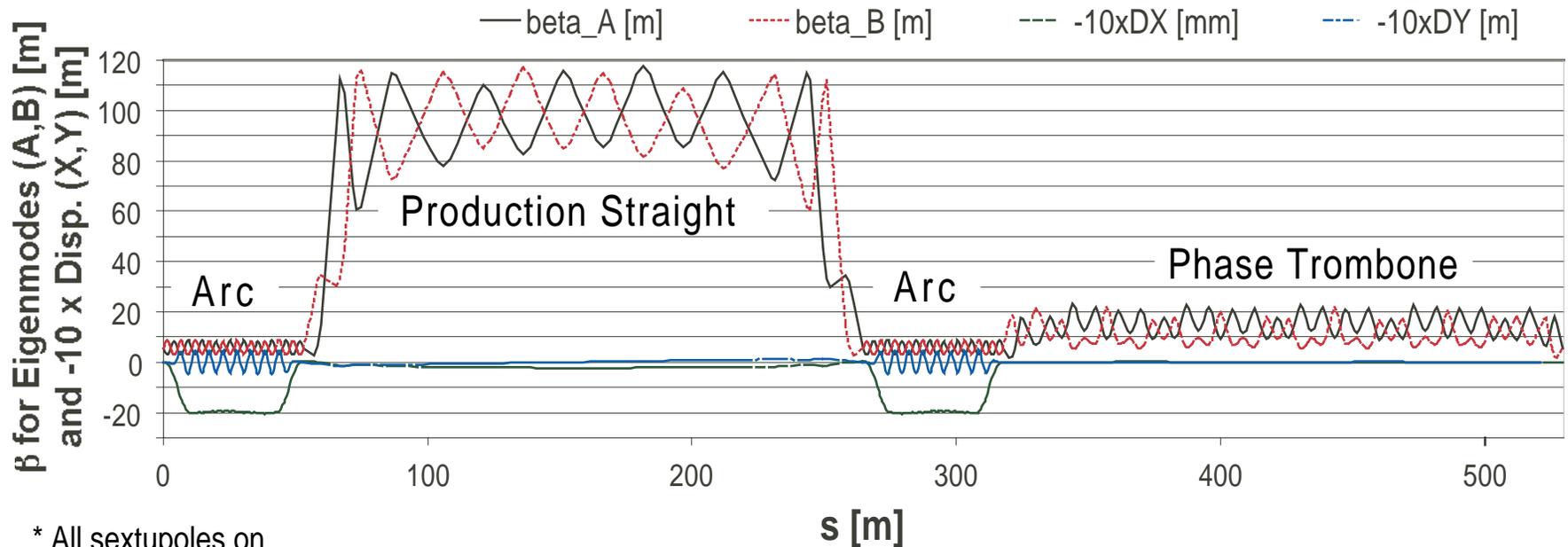


$$\beta_{\max}^{(A,B)} = 8.89 \text{ m} \quad \eta_X = (\eta_A + \eta_B) / \sqrt{2} \quad \eta_Y = (\eta_A - \eta_B) / \sqrt{2}$$

$$\beta_{\min}^{(A,B)} = 3.10 \text{ m} \quad \eta_{\text{Long}}^{(A,B)} = 1.765 \text{ m} \quad \eta_{\max}^X = 2.038 \text{ m}$$

$$\Delta\phi = 60.7^\circ \quad \eta_{\text{Short}}^{(A,B)} = 1.118 \text{ m} \quad \eta_{\max}^Y = 0.457 \text{ m}$$

Yields 9 mm max. c.o. error, 5 mm RMS in both A & B eigenplanes but 10.2 mm max. c.o. error horizontal and 2.3 mm max. c.o. error vertical



* All sextupoles on