Synchrotron Radiation

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What is in this lecture?

This lecture will cover basic concepts of synchrotron radiation:

- properties of SR beams,
- magnetic devices for generating radiation,
- overview of light source facilities and X-ray free electron lasers around the world.

The focus will be in describing and discussing the light source properties

I will tell you how to GENERATE synchrotron light but NOT how to USE it

Outlook

Introduction

I. Radiation sources

II. Trajectory of and Radiation from Moving Charge

III. Particle Accelerators

IV. Synchrotron Radiation

V. An undulator

VI. Free Electron Laser

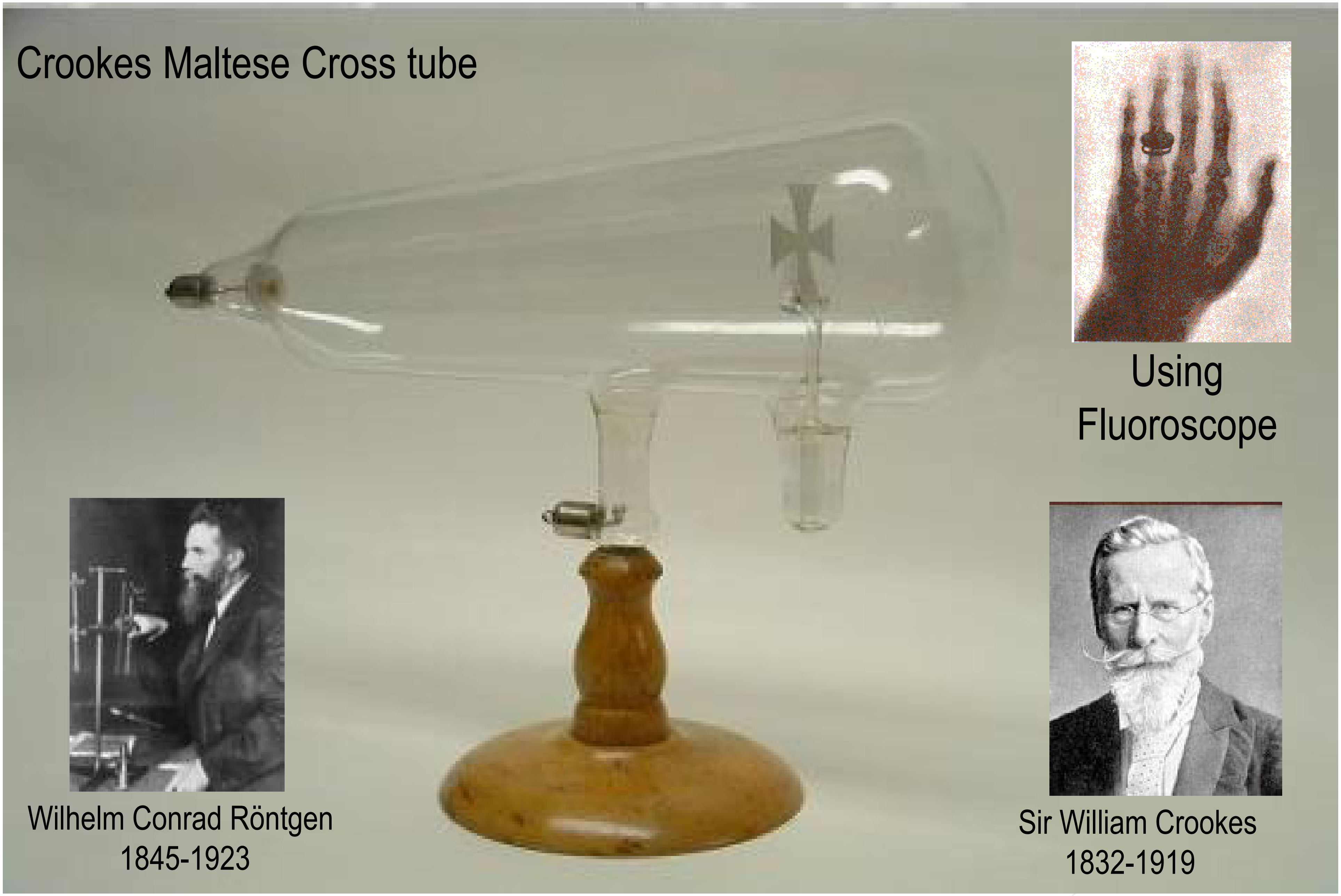
VII. Synchrotron Radiation Facilities

Concluding remarks

Introduction: History of X-rays

100 years later…at Brookhaven Lab

Crookes Maltese Cross tube



Using Fluoroscope

Wilhelm Conrad Röntgen
1845-1923

Sir William Crookes
1832-1919

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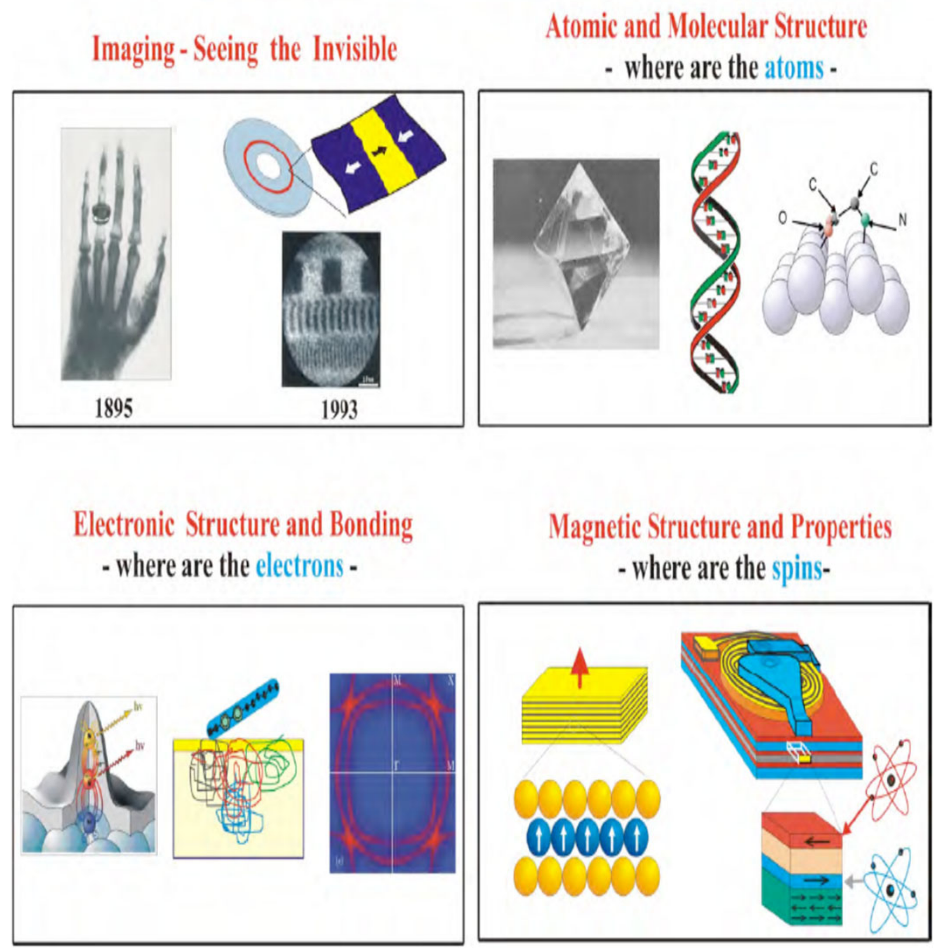
Variables and Parameters

- ***B*** – magnetic field, T
- ***v*** and ***a*** – velocity and acceleration, m/s and m/s^2
- λ - wavelength of light (radiation), m
- ***k*** - wavenumber, 1/m
- $\gamma = E/E_0$, relative particle energy (E_0=0.511 MeV)
- ε_{ph} – photon energy, ε_c – critical photon energy, eV
- ***R, r*** – radius of curvature, m
- ε -- beam emittance, m*rad
- ***I*** – beam current, A
- (α, β, γ) -- Twiss parameters, (n/u, m, 1/m)
- δ -- energy deviation or energy spread
- $\sigma_{x,y}$ – beam size, m
- λ_0, K (K_u) – undulator period length (m), undulator parameter
- *n* – harmonic number
- F_n – photon flux, ph/s
- Φ_n -- angular spectral flux, ph/s/0.1%BW/rad^2

I. Radiation sources

- Modern science needs various radiation sources for various purposes
  - Imaging of materials on micro- and macro-scales
  - Study of chemical processes
  - Study of bonds
  - Study of surfaces

- Sources range by their properties:
  - Wavelength
  - Directivity
  - Flux
  - Brightness
  - Coherence



Following earlier lecture by Q. Shen

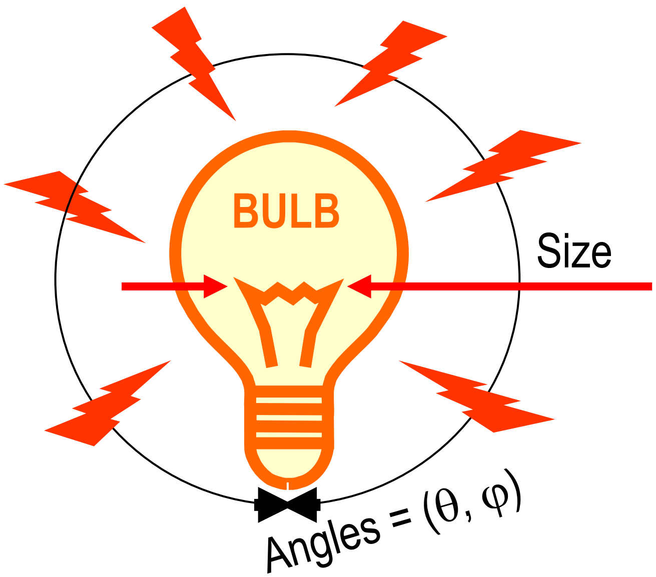
Directivity

- Directivity = maximum directive power *max{D(θ,ϕ)}* among all solid angles of radiation:

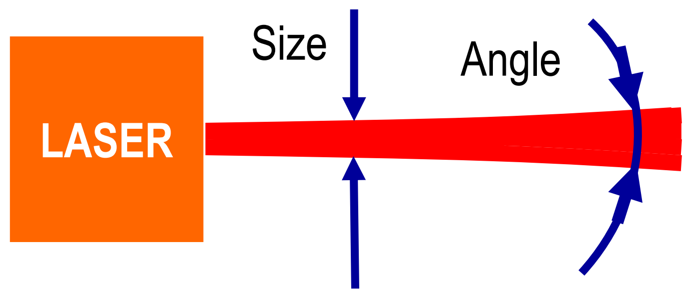
$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{P_{total}}$$

where $U(\theta,\phi)$ is power per solid angle and P_{total} is total radiated power

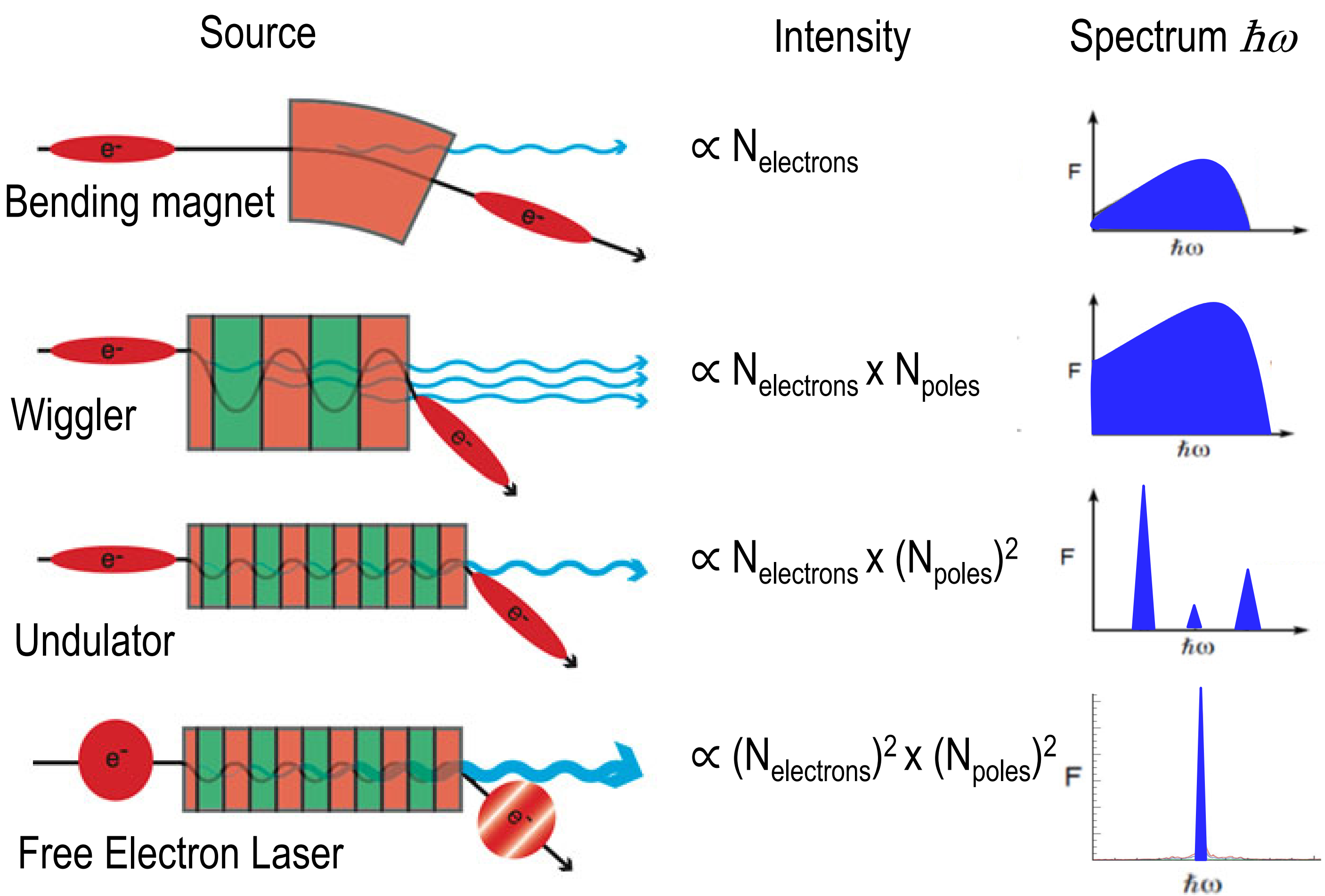
Light bulb: no directivity



Laser: high directivity

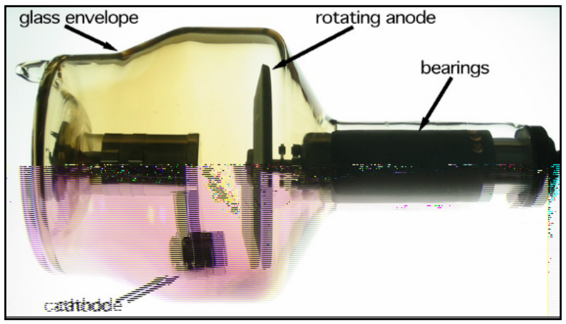


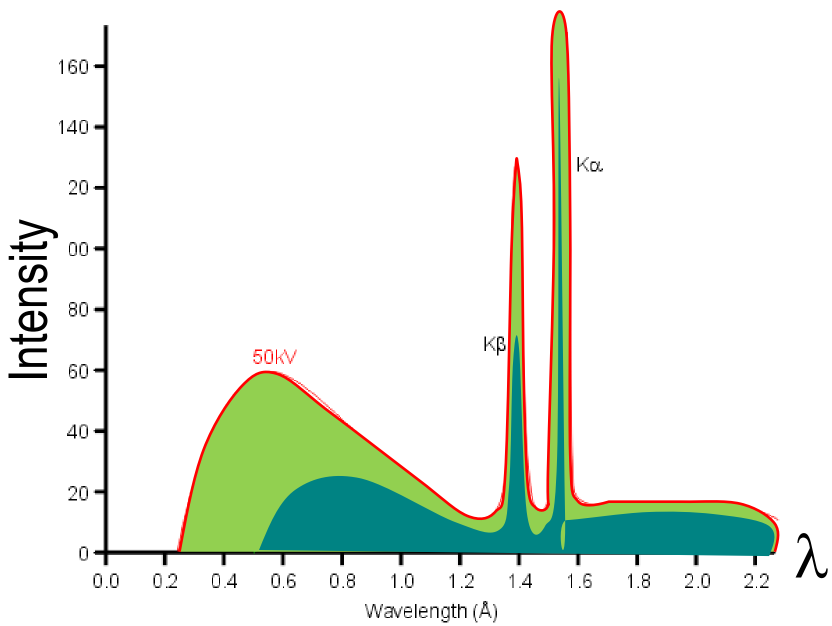
Forms of Synchrotron Radiation

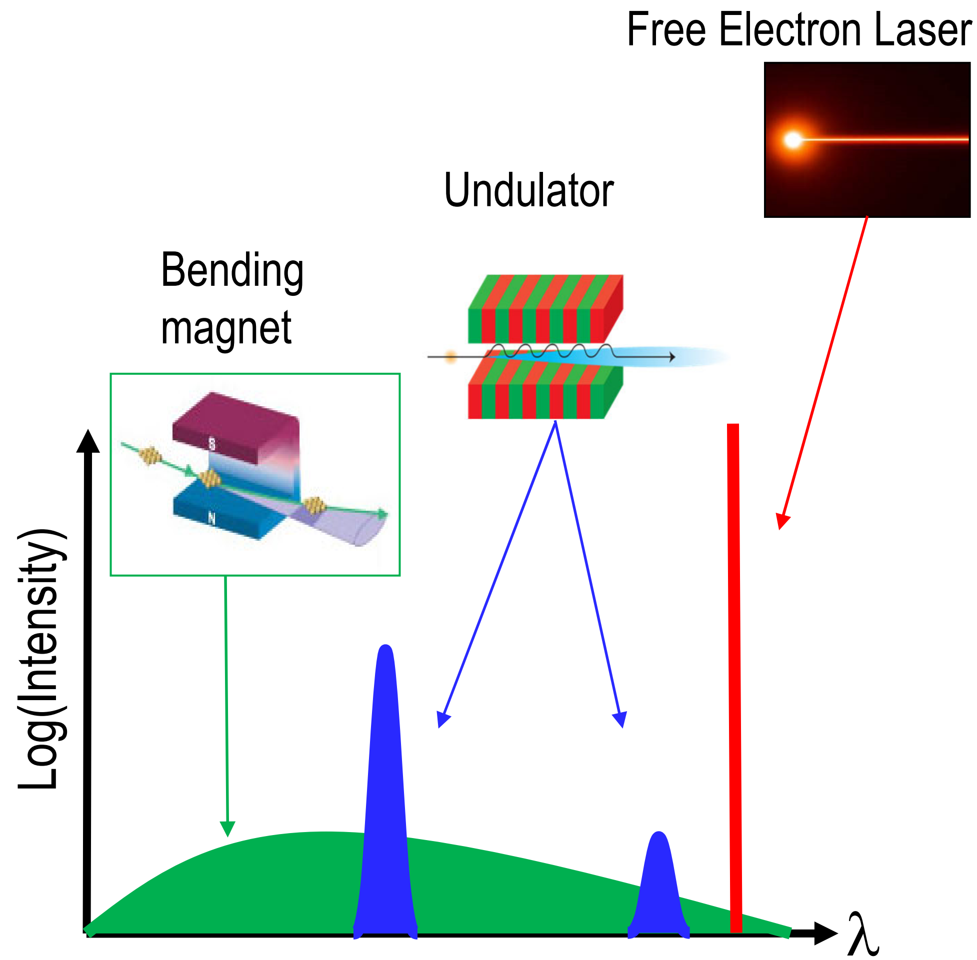


Comparison with an X-ray tube

Rotating Anode Tube







Radiation units / Flux and Brightness

* Photon energy and wavelength

$$h\nu = \hbar\omega = \hbar\, 2\pi c / \lambda$$

electronVolts (eV) and m

* Total Flux = Photons per second

Photons /s

* Spectral Flux = Photons per second per unit of bandwidth

Photons /s /0.1%BW

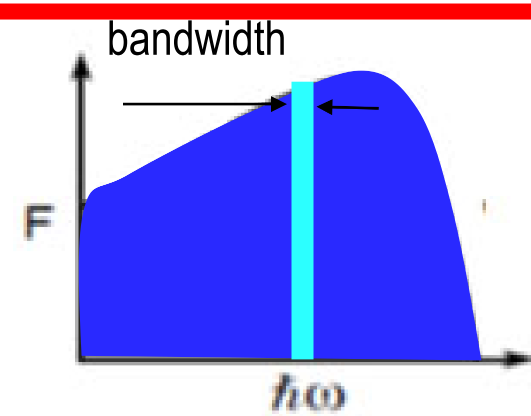
* Brightness =

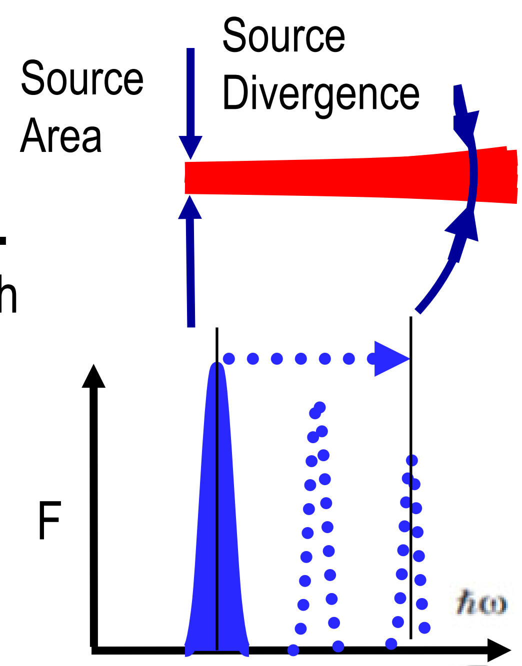
$$\frac{\text{Photons per second}}{\text{Source Area * Source Divergence * bandwidth}}$$

Photons / s / mm^2 / $mrad^2$ / 0.1%BW

* Bandwidth (BW) = $\Delta\lambda/\lambda = \Delta h\nu/h\nu$

* Tunability range $\Delta\lambda$, nm or eV





Synchrotron Light Source versus X-ray tube

- Photons per second
- Ability to tune photon energy

	Photon Flux	Tunability	Brightness
X-ray tube	Low	No	Low (~10^7)
Light Source	High	Yes	High average (~10^{22})
FEL	Medium	Yes	High peak (~10^{33})

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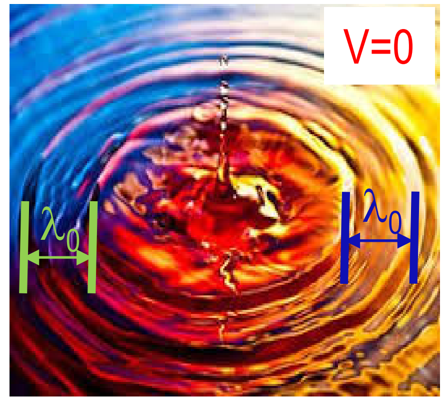
II. Trajectory of and Radiation from Moving Charge

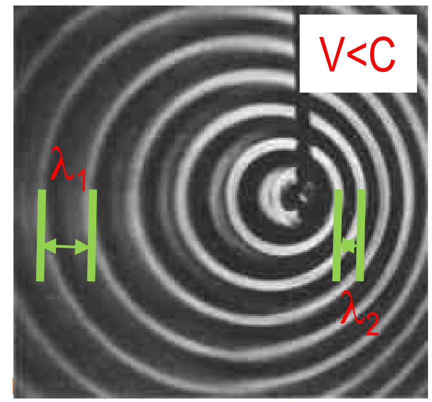
- Compare stationary and moving oscillators →
- Angle-dependent Doppler shift

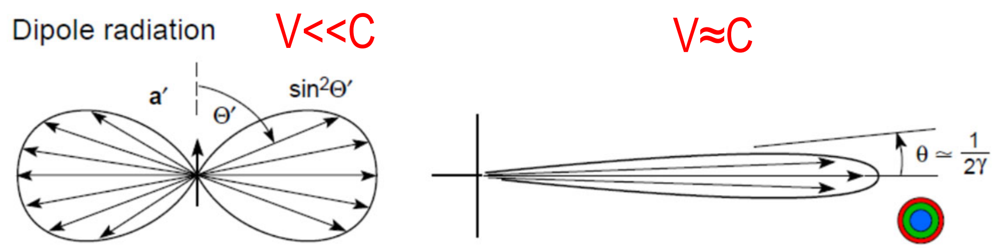
- Compare non-relativistic, stationary oscillator and moving relativistic oscillator

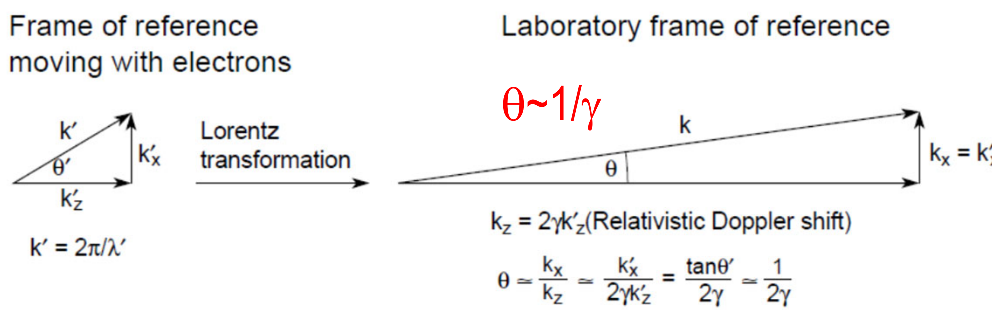
- Directivity: relativistic particle radiates in a narrow forward cone

- $\gamma = E/E_0$, relative particle energy = ratio of full energy to the rest energy (0.511 MeV for electrons)









How do you measure speed of electron current

- In XVIII French clergymen and physicist **Jean-Antoine Nollet** measured speed of electron current.
- In one of his experiments, he gathered about 200 monks into a circle of a mile in circumference
- "… and the whole company, upon the discharge of the (Leyden) phial, gave a sudden spring…"
- Nollet concluded that the speed of electricity propagation was **very high**.

- **In synchrotron light sources we deal with relativistic electrons, i.e.** $\beta \approx 1, \gamma >> 1$

The History And Present State Of Electricity: With Original Experiments
By Joseph Priestley

IN France as well as in Germany experiments were made to try how many perſons might feel the ſhock of the ſame phial. The Abbé Nollet, whoſe name is famous in electricity, gave it to one hundred and eighty of the guards, in the King's preſence; and at the grand convent of the Carthuſians in Paris, the whole community formed a line of nine hundred toiſes, by means of iron wires between every two perſons (which far exceeded the line of one hundred and eighty of the guards) and the whole company, upon the diſcharge of the phial, gave a ſudden ſpring, at the ſame inſtant of time, and all felt the ſhock equally. †

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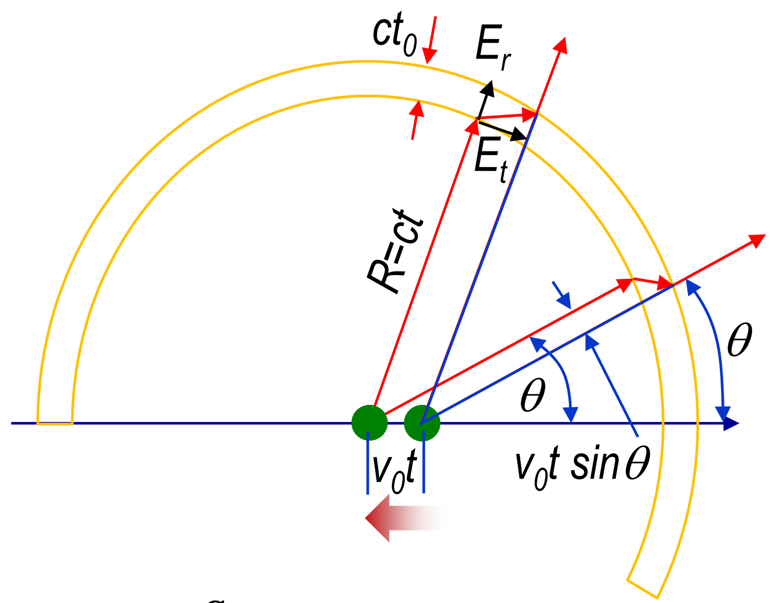
Radiation of decelerating charge

- Charge q decelerates within time t_0 to a full stop
- Pulse of radiation travels to radius $R=ct$
- Geometry of kink in the radial E field



$$\frac{E_t}{E_r} = \frac{v_0 t\ sin\theta}{ct_0} = \frac{a_0 R}{c^2} sin\theta$$

- Now we can find E_t:

$$E_t = \left\{\frac{a_0 R}{c^2} sin\theta\right\}\left\{\frac{q}{4\pi\varepsilon_0 R^2}\right\} = \frac{q a_0 sin\theta}{4\pi\varepsilon_0 c^2}\frac{1}{R} \quad \text{vs} \quad \frac{q}{4\pi\varepsilon_0 R^2}$$

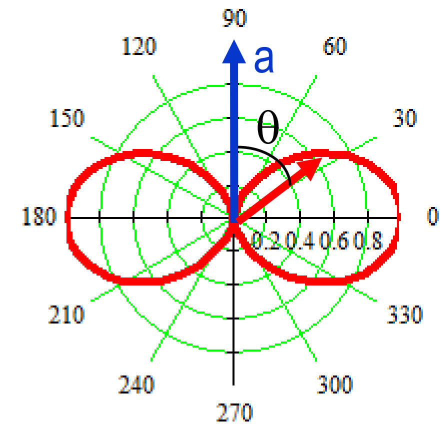
- Power radiated:

$$P = \frac{q^2 |a_0|^2}{6\pi\varepsilon_0 c^3}$$

Power radiated by an accelerated charge

- Size of the radiator is much smaller than radiated wavelength → dipole radiation. a = acceleration.

$$\frac{dP}{d\Omega} = \frac{e^2|a|^2 sin^2\theta}{16\pi^2\varepsilon_0 c^3}$$



Donut-shaped radiation pattern

- Instantaneous power? Integrate over all angles $P = \frac{e^2|a|^2}{6\pi\varepsilon_0 c^3}$

- For relativistic particle the total power transforms into*: $P = \frac{1}{4\pi\varepsilon_0}\frac{2e^2c}{3r^2}\gamma^4$

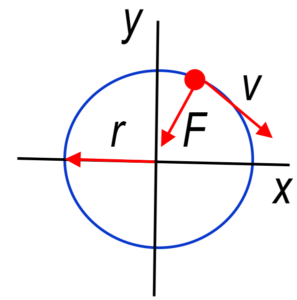
* see example later

Jumping ahead → Characteristic photon energy of radiation scales with electron energy as γ^3:

$\varepsilon_{ph} = \frac{3}{2}\frac{c\hbar}{r}\gamma^3$ Or, in practical units: $\varepsilon_{ph}[keV] = 2218.29\, E^3[GeV]/r[m]$

Problem solving: power radiated by an accelerated charge

Derive Schwinger's formula → Expression 1.11 in J. Schwinger, Phys. Rev. 75, (1949) 1912 (http://journals.aps.org/pr/pdf/10.1103/PhysRev.75.1912)



* Power radiated by an accelerated charge

$$P = \frac{e^2|a|^2}{6\pi\varepsilon_0 c^3} \qquad (*)$$

e = electron charge
c = speed of light
ε_0 = vacuum dielectric constant

*a = acceleration, defined as:

$|a| = \frac{1}{m}\frac{d|p|}{d\tau}$, with relativistic momentum $|p| = \gamma m v$

* Relativistic transformation of time

$$\tau = \frac{t}{\gamma}, \qquad \gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

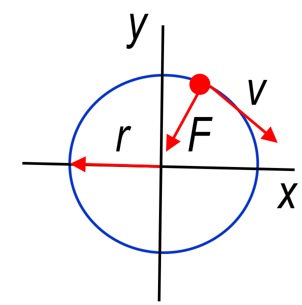
* For a charge moving in a circle, centripetal acceleration

$$|a| = \gamma^2 \frac{d}{dt} v \quad \text{--->} \quad |a| = \gamma^2 \frac{v^2}{r}$$

r = radius of curvature
v = velocity

Problem solving: power radiated by an accelerated charge (Cont'd)

Derive Schwinger's formula → Expression 1.11 in J. Schwinger, Phys. Rev. 75, (1949) 1912 (http://journals.aps.org/pr/pdf/10.1103/PhysRev.75.1912)



substituting into (∗)
$$P = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{2}{3}\cdot\frac{e^2}{c^3}\cdot\gamma^4\cdot\frac{v^4}{r^2}$$

assuming v≈c
$$P = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{2}{3}\cdot\frac{e^2\cdot c}{r^2}\cdot\gamma^4$$

Loss of energy per 1 full turn

$$\Delta E = P\cdot\frac{2\cdot\pi\cdot r}{c}$$

$$\boxed{\Delta E = \frac{e^2}{3\cdot\varepsilon_0}\cdot\frac{\gamma^4}{r}}$$

In practical units

$$\frac{\Delta E}{e\cdot 10^3} = \frac{e^2}{3\cdot\varepsilon_0}\cdot\frac{E^4}{r}\cdot\left(\frac{e\cdot 10^9}{mc^2}\right)^4$$

$$\Delta E[keV] \approx 88.9^* E^4\,[GeV]/r\,[m]$$

For example, at NSLS-II: --> $\Delta E = 288\ keV$

E=3 GeV, r=25m energy loss of a single electron per turn

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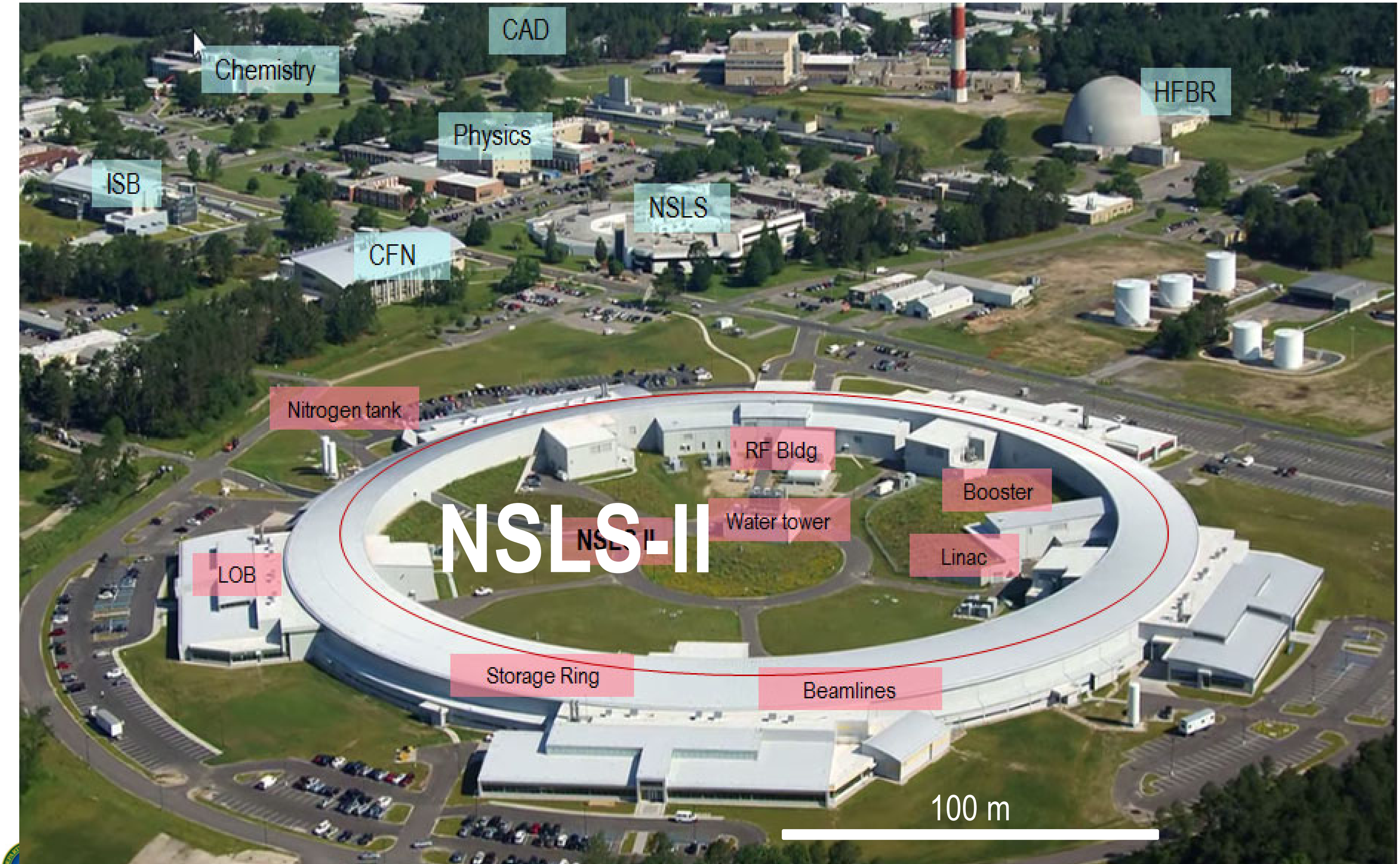
III. Particle Accelerators

- Short-wavelength radiation is generated by high-energy electrons
- To reach high photon energy (X-rays) we need to accelerate a particle to high (GeV) energy →

→ Particle accelerators

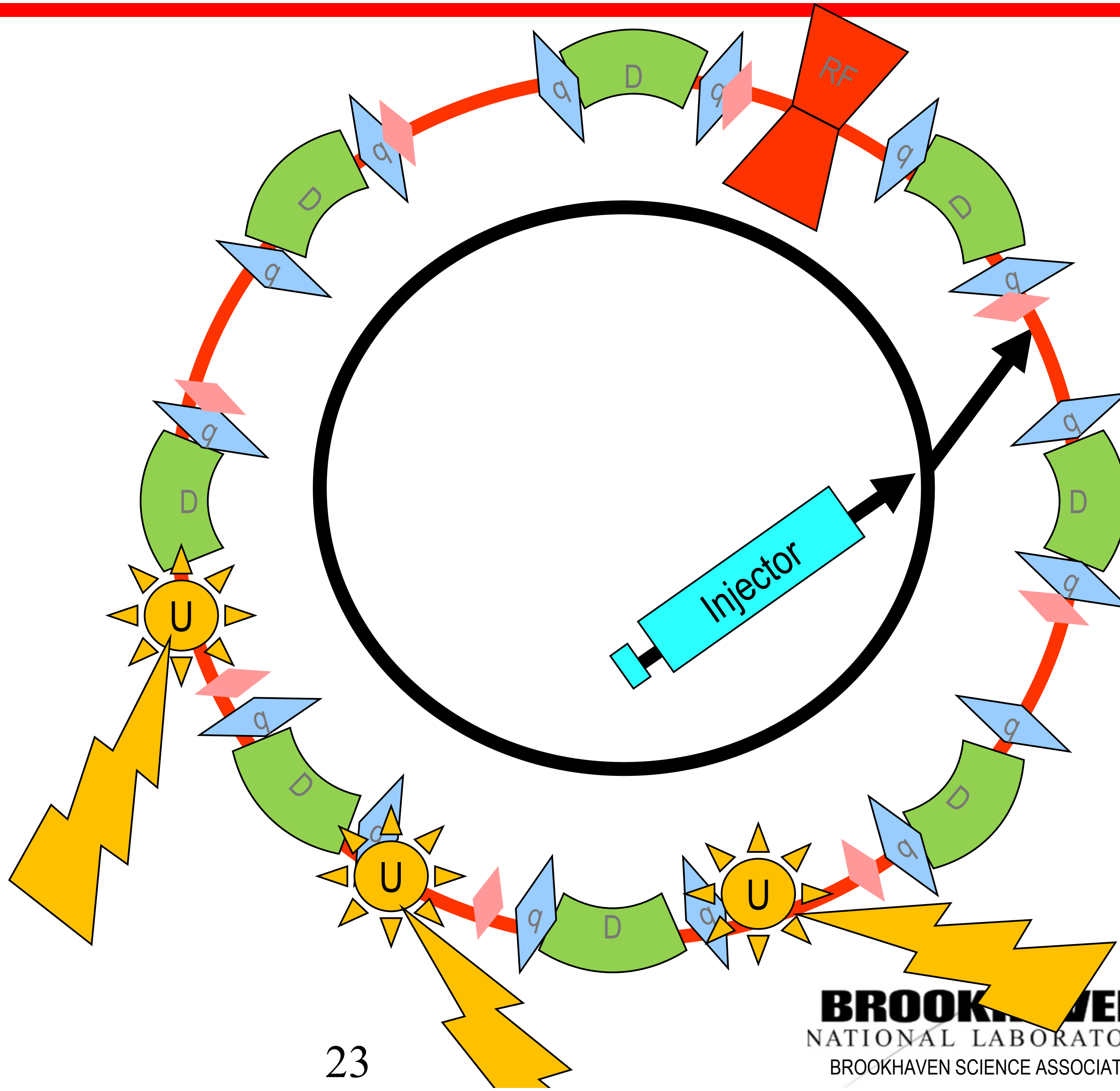
  - Gun (thermionic or RF) emits particles in the energy range of 100 keV -10 MeV
  - Bunching system compresses particles into separate bunches spaced on a scale of RF wavelength
  - Linear accelerators and booster-synchrotrons accelerate bunches to ~GeV energy
  - Bunches are injected into Storage Ring.
  - They circulate in the Storage Ring radiating photons into beam lines
  - Particles trickle out of circulating bunches due to their finite lifetime
  - Losses of particles are replaced with successive injections

Modern Synchrotron Light Source

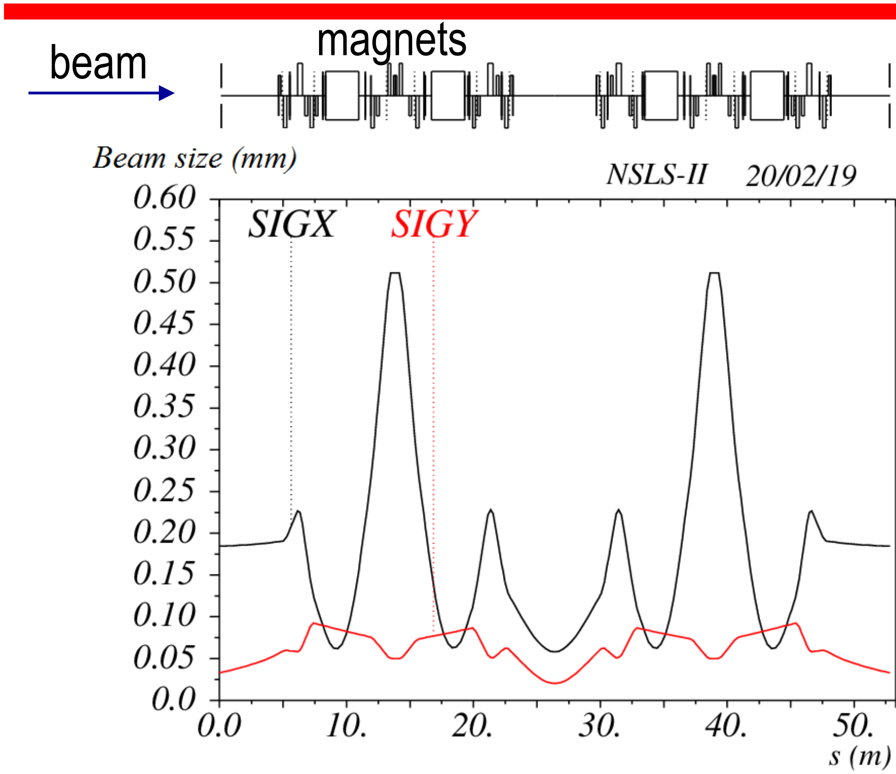


Building an accelerator for Light Source

- Magnet Lattice:
  - Dipoles
  - Quadrupoles
  - Sextupoles
- Undulators
- Vacuum system
- RF system
- Injector

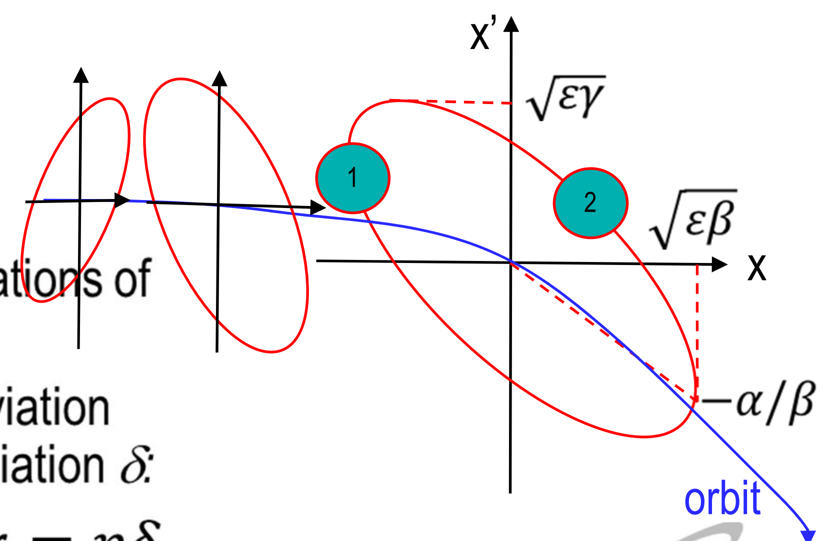


Motion of particles in a particle accelerator



$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

- ε_x *is* electron beam emittance = area of phase space *(x, x')* occupied by beam
- Beam envelopes oscillate while the emittance is preserved



- α, β, γ are Twiss parameters that describe oscillations of beam envelope along accelerator
- Dispersion η describes chromatic effects, i.e. deviation from reference orbit for a particle with energy deviation δ:

$$\Delta x = \eta\delta$$

Source Size and Divergence

- Electron beam envelope is governed by emittance and Twiss parameters
- Root Mean Square beam size σ_x and divergence σ_{xp}
- As an example, we take propagation of a Gaussian beam along a drift

$$x(s) = \sqrt{\varepsilon_x \cdot \beta_x(s)}\cos(\Phi(s) - \phi_0)$$

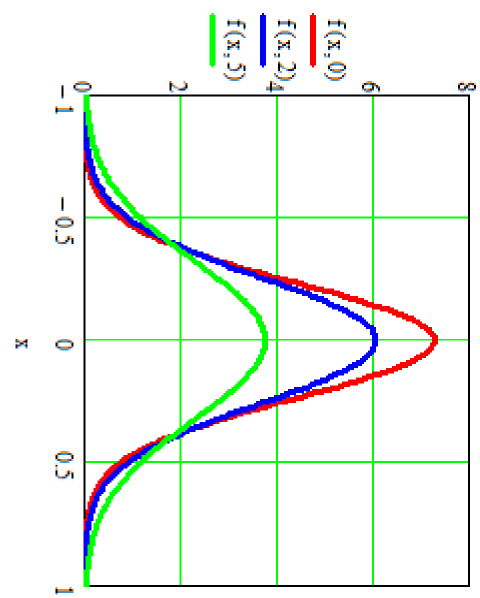
$$\sigma_x(s) = \sqrt{\varepsilon_x \cdot \beta(s)}$$

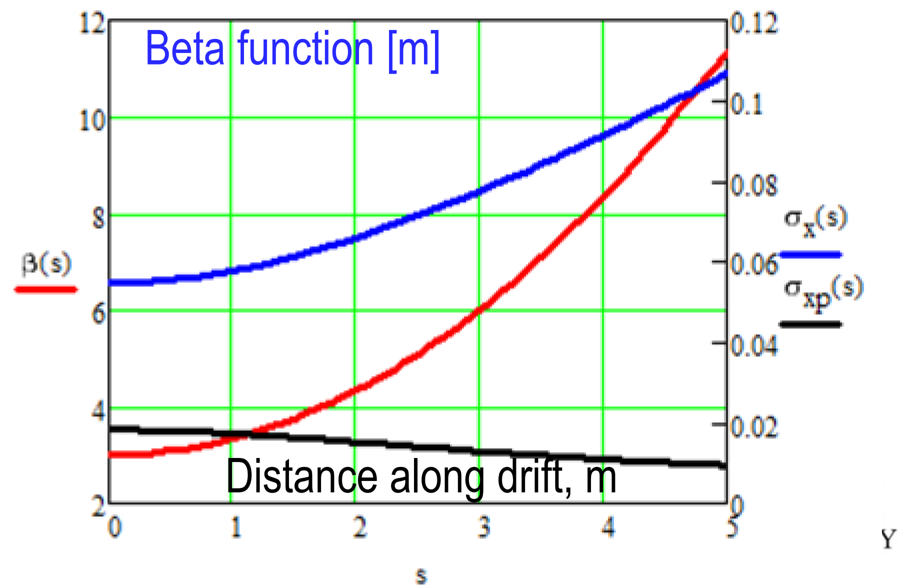
$$\sigma_{xp}(s) = \sqrt{\varepsilon_x} \cdot \sqrt{\frac{1 + \alpha(s)^2}{\beta(s)}}$$

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

Transverse distributions of electrons along the drift:

- at the waist
- 3 m away
- 5 m away





Brightness of Radiation beams

- **Brightness** is defined as photon flux divided by radiation volume
- It depends on electron beam sizes at the source point (bend or undulator)
- It depends on natural source size and divergence of SR beam defined by diffraction effect
- Total source size is a convolution of electron beam and radiation sizes and divergences

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

Σ_x, Σ_z : Photon beam sizes

Σ'_x, Σ'_z: Photon beam divergences

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_R^2} \ ,$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \sigma'^2_R} \ ,$$

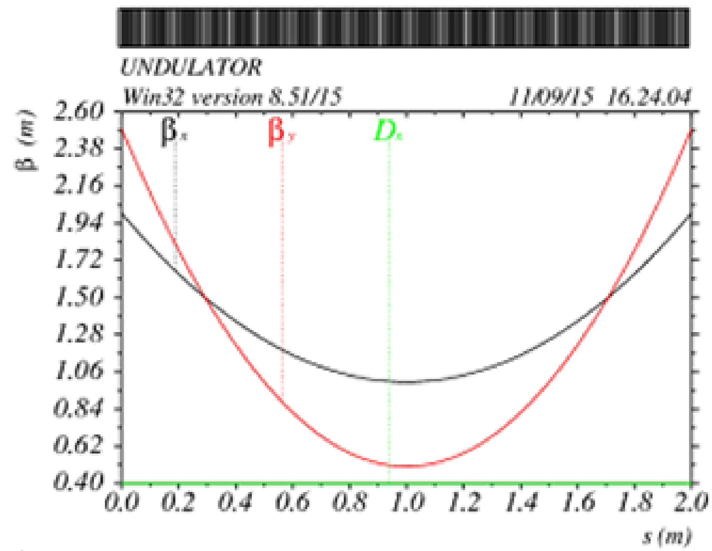
σ_x electron beam size
σ'_x electron beam divergence

σ_R Radiation source size
σ'_R Radiation divergence

Problem solving: radiator source size and divergence

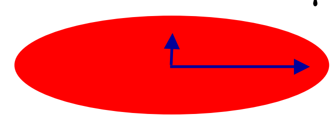
Twiss parameters and emittances

- $\varepsilon_x = 1 \cdot 10^{-9}$ m rad
- $\varepsilon_y = 0.01 \cdot \varepsilon_x = 1\text{x}10^{-11}$ m rad
- $\beta_x = 1.0\ m$ Horizontal beta function
- $\beta_y = 0.5\ m$ Vertical beta function
- $\eta_x = 0.15\ m$ Horizontal dispersion
- $\delta = 0.00055$ fractional energy spread



Source sizes: $\sigma_x = \sqrt{\varepsilon_x \cdot \beta_x + (\eta_x \cdot \delta)^2} = 8.835 \text{ x } 10^{-5}\ meters = 88\mu m$

$\sigma_y = \sqrt{\varepsilon_y \cdot \beta_y} = 2.236\ x\ 10^{-6}\ meters = 2.2\ \mu m$



Source divergences:

$$\sigma_{xp} \approx \sqrt{\varepsilon_x \beta_x^{-1}} = 3.162\ x\ 10^{-5} \quad radians = 32\ \mu rad$$

$$\sigma_{yp} \approx \sqrt{\varepsilon_y \beta_y^{-1}} = 4.472\ x\ 10^{-6} \quad radians = 4.4\ \mu rad$$

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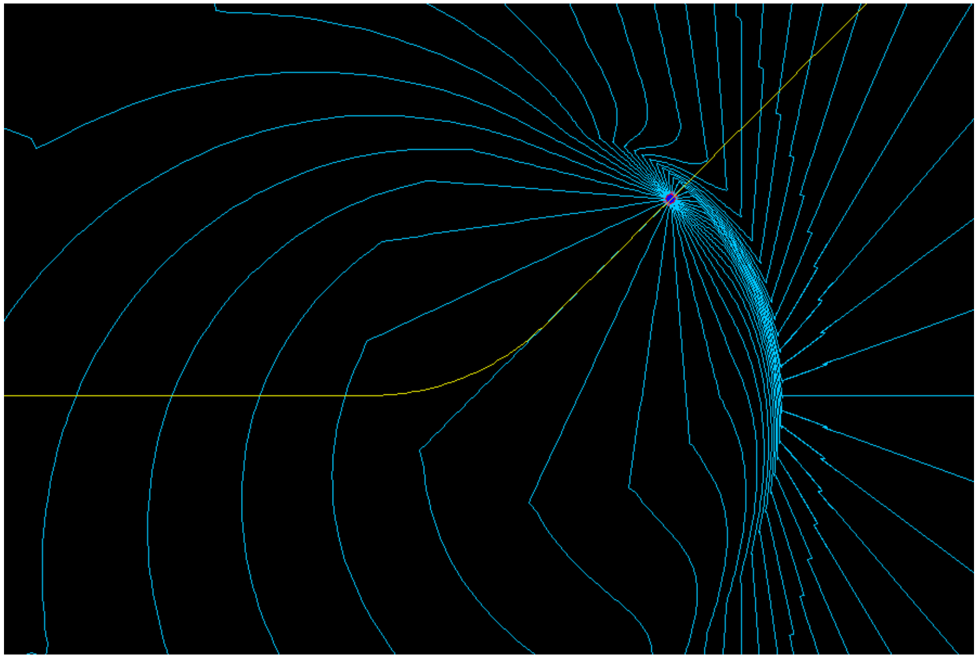


T. Shintake's Radiation 2D:
http://www.shintakelab.com/en/enEducationalSoft.htm

Synchrotron radiation from a bending magnet

* Calculate duration of radiation pulse

$$2\Delta\tau = T_{electron} - T_{light}$$

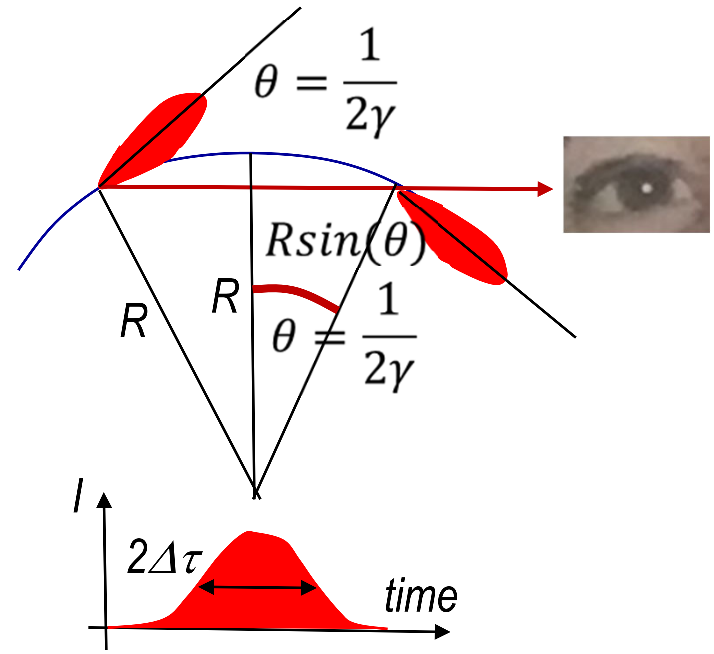
$$2\Delta\tau = \frac{arc\ length}{v} - \frac{light\ path}{c} \approx \frac{R\cdot 2\theta}{v} - \frac{2R\sin(\theta)}{c}$$

$$\theta \approx \frac{1}{2\gamma}, \qquad \sin(\theta) \approx \theta$$

$$2\Delta\tau \approx \frac{R}{\gamma}\left(\frac{1}{v} - \frac{1}{c}\right) \approx \frac{R}{\gamma\beta c}(1-\beta) \approx \frac{m}{2eB\gamma^2}$$

$$with\ R \approx \frac{\gamma mc}{eB}$$

* Frequency ~ $1/\Delta\tau$



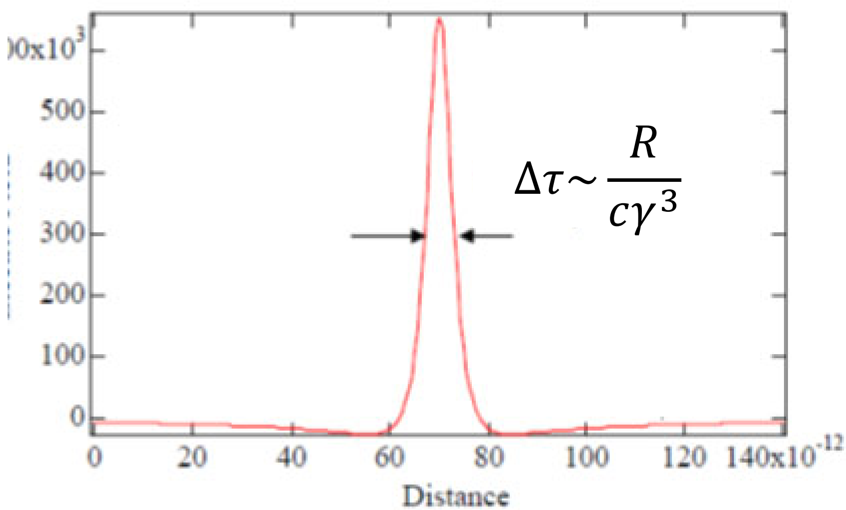
Radiation pulse

* Critical energy of synchrotron radiation from a bending dipole

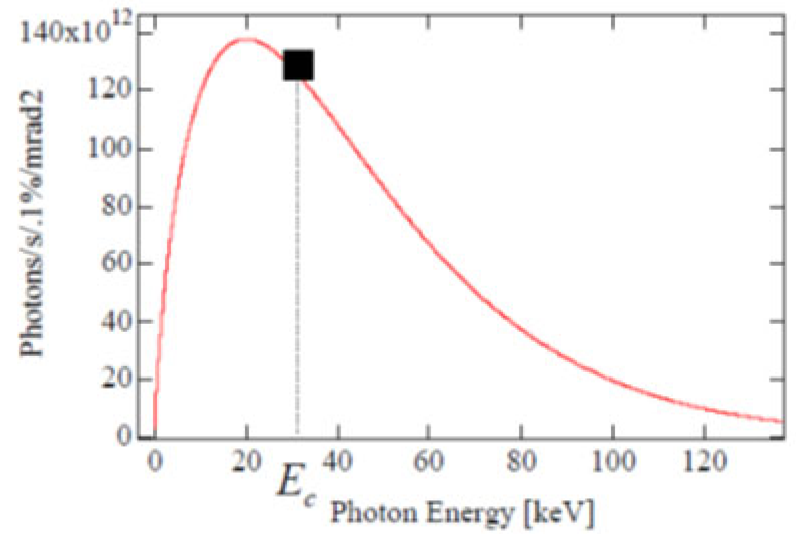
$$\varepsilon_c = h\nu_c = \frac{3}{4\pi}\frac{ehB\gamma^2}{m} = \frac{3}{2}\frac{ch}{r}\gamma^3$$

Critical energy of bending magnet radiation

Electric Field in the Time Domain



Angular Flux in Frequency Domain



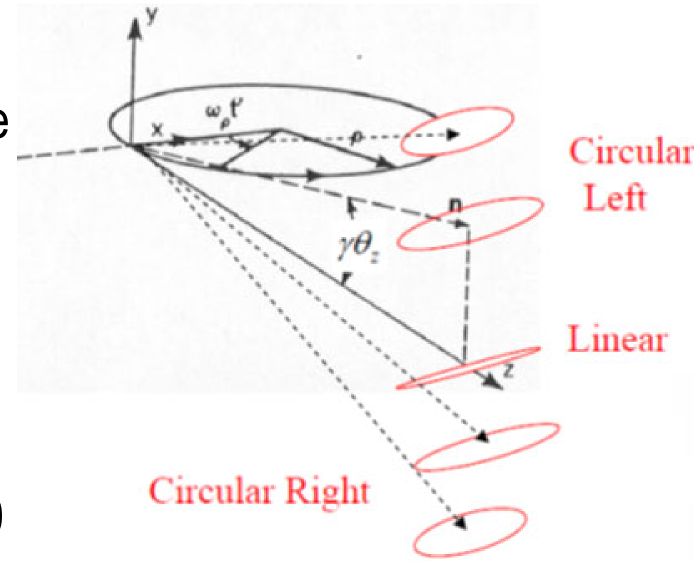
Computed for 6 GeV, I = 200 mA, B =1 tesla

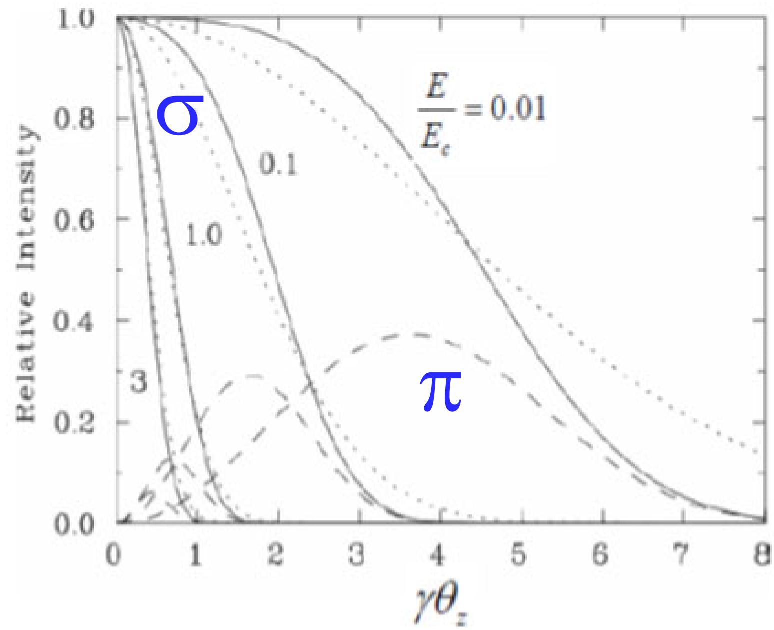
Critical energy of SR from dipole

$$\varepsilon_c = h\nu_c = \frac{3}{4\pi}\frac{ehB\gamma^2}{m} \rightarrow \varepsilon_c[keV] = 0.665 \cdot E^2[GeV] \cdot B[T]$$

Polarization of bending magnet radiation

- Polarization is changing from linear in the midplane to circular left and circular right above and below the midplane
- Intensity of the parallel and perpendicular components of the photon flux as a function of $\gamma\theta$
- The curves are reported for $\varepsilon / \varepsilon_c$=1/3, 1, 10 and 100 and are normalized to the intensity at = 0
- These polarization components are called σ and π



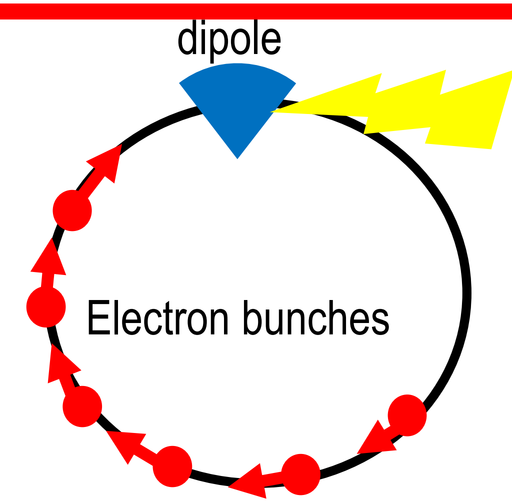


Time structure

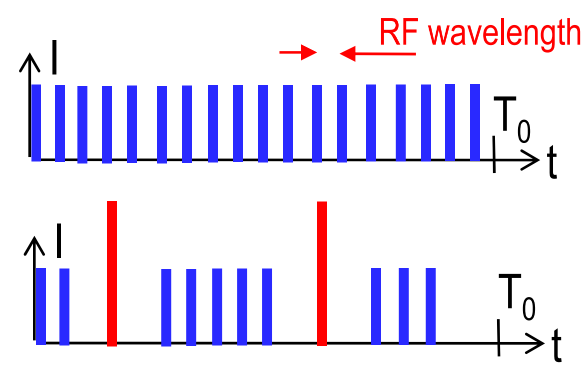
- Storage ring gets filled with several (many) bunches
- Bunches circulate around the ring arriving to the source point once per revolution

- Bunch length can vary between ns and ps depending on the RF frequency
  - There are techniques to select ~100 fs long portion of radiation pulse
  - Free Electron Lasers radiate short pulses down to 6-fs RMS

- Detectors of synchrotron radiation generate signal, which is a function either average intensity or a harmonic of revolution frequency

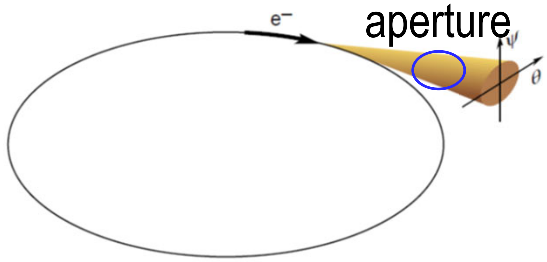


Intensity time patterns



T_0 = time of 1 revolution

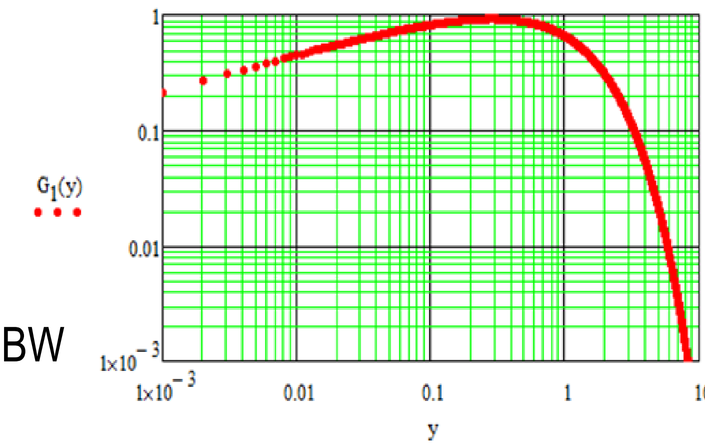
Problem solving: calculation of SR flux through an aperture



$$\frac{d^2F_B}{d\theta d\,\omega/\omega} = 2.46\,10^{13}E_e(GeV)I(A)G_1(E/E_c)\frac{photon\,s/s}{mrad\cdot(0.1\%BW)}$$

y=0.001, 0.002..10

$$G_1(y) = y\cdot\int_y^{\infty} Kn\left(\frac{5}{3},z\right)dz$$



* For NSLS-II: *E=3 GeV, B=0.4 T, I=0.5 A*

$$\varepsilon_c = 0.665\cdot E^2\cdot B = 2.394\ \ keV\ \text{Critical energy}$$

* Calculate flux *at* $\varepsilon_{ph} = 0.1\cdot\varepsilon_c$

$$N_\lambda = 2.457\cdot 10^{13}\cdot E\cdot I\cdot G_1(0.1) =$$
$$= 3.015\ x\ 10^{13}\ \ photons\ /s/0.1\,\%\,BW/mrad$$

* IF a) window is 0.2 mrad, b) there is a filter of 0.02 % BW

Flux per second→
$$Nwf_\lambda = N_\lambda\cdot 0.2\cdot 0.2$$
$$= 1.206\ x\ 10^{12}\ \ photon\,s/s$$

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II. Trajectory of and Radiation from Moving Charge

III. Particle Accelerators

IV. Synchrotron Radiation

V. An undulator

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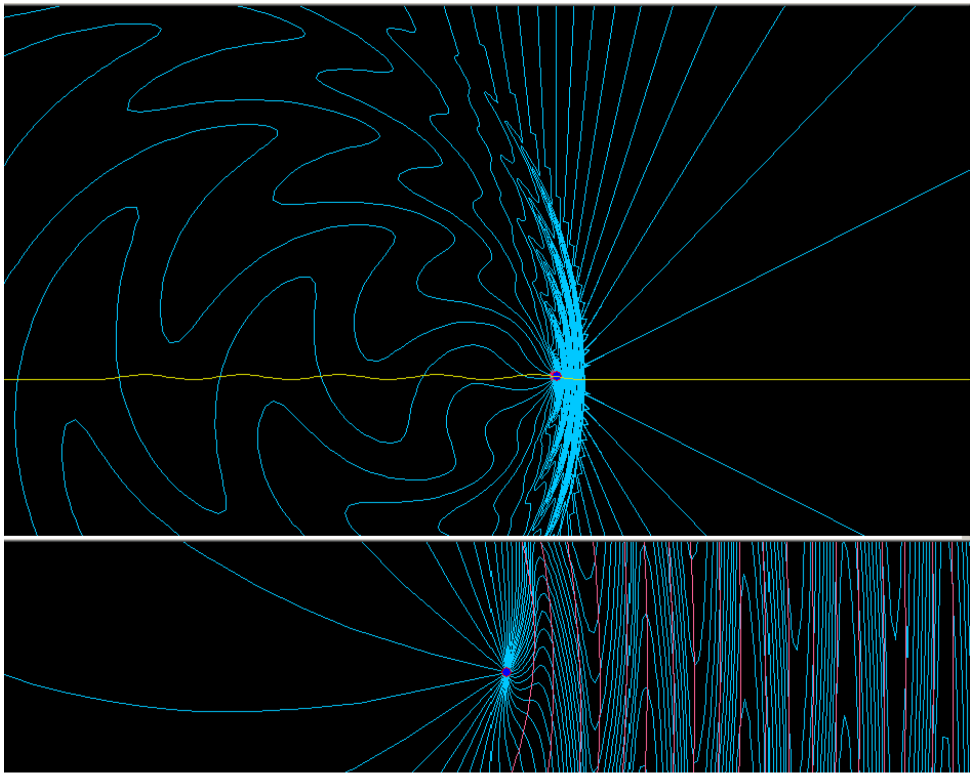
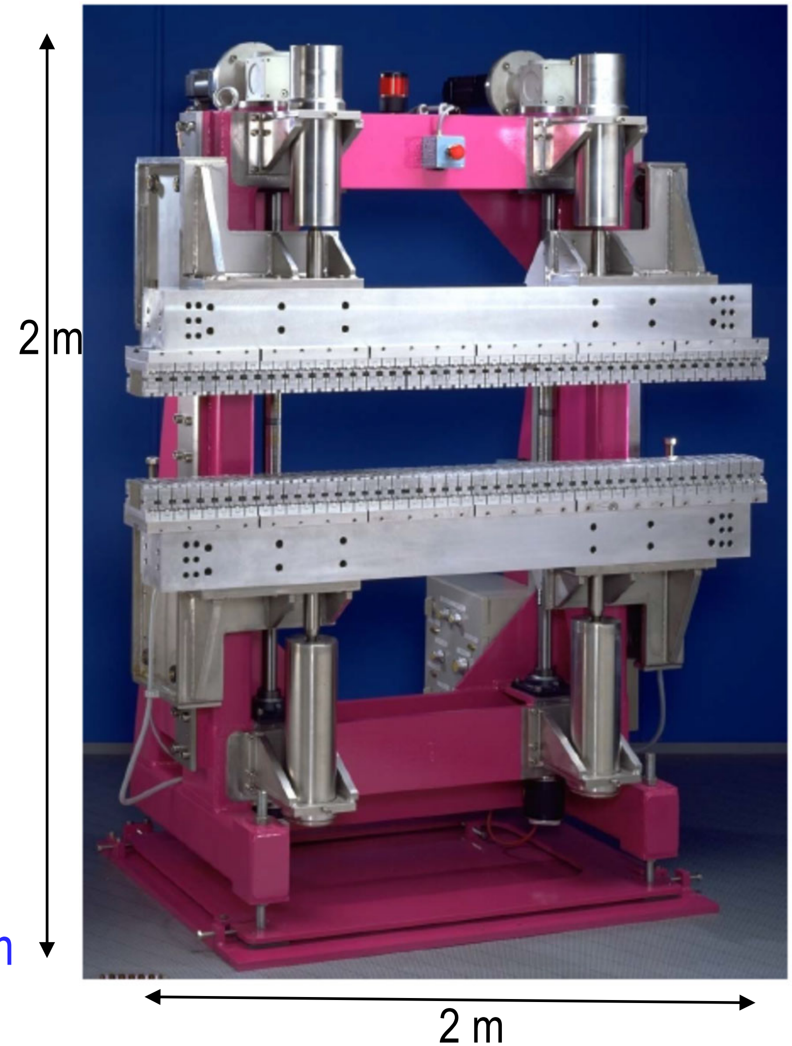
Concluding remarks

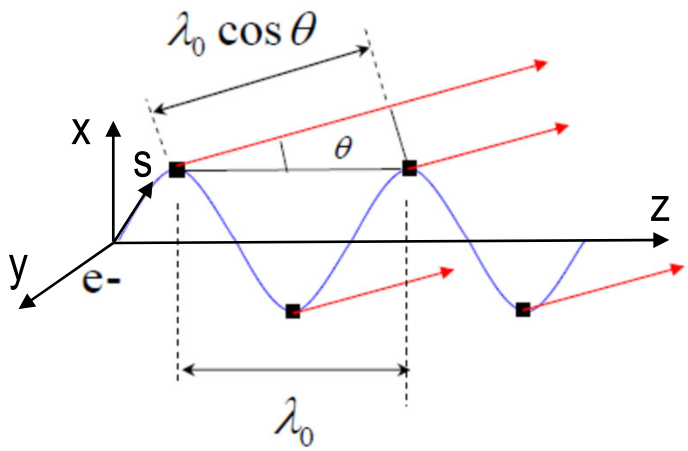
V. An undulator



T. Shintake's Radiation 2D:
http://www.shintakelab.com/en/enEducationalSoft.htm



Electron Trajectory in an undulator



Lorentz force $\gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$

Relativistic factor

$$\gamma = \frac{1}{\left(1 - \frac{{v_s}^2}{c^2}\right)^{1/2}}$$

Assumption

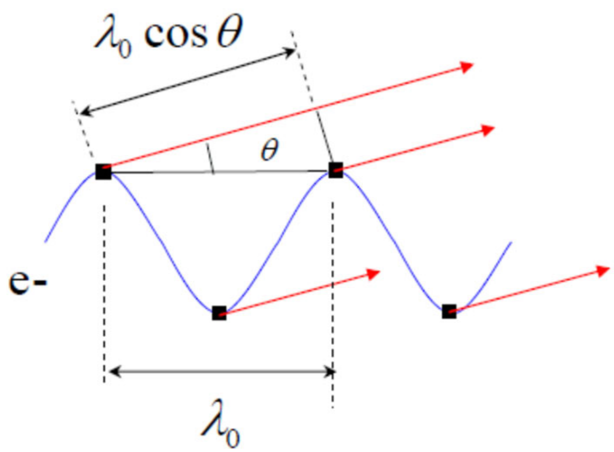
$v_{x,z} \ll v_s \approx c$

$$\gamma m \frac{dv_x}{dt} = e\left(v_y B_z - v_z B_y\right)$$

Angle of trajectory $\frac{v_x(s)}{c} = \frac{e}{\gamma mc}\int_0^s B_y(s')ds'$

Coordinate of trajectory x(s)= $\frac{e}{\gamma mc}\int_0^s \int_0^{s'} B_y(s'')ds''\, ds'$

Electron trajectory in a planar undulator



$$\vec{B} = \left(0, B_0 \cdot \sin\left(2\pi \frac{s}{\lambda_0}\right), 0\right)$$

Only one sinusoidal field component

$$\frac{v_x}{c} = \frac{K}{\gamma} \cos\left(2\pi \frac{s}{\lambda_0}\right)$$

* We will derive these expressions later

$$x = -\frac{\lambda_0}{2\pi} \frac{K}{\gamma} \sin\left(2\pi \frac{s}{\lambda_0}\right), with\ v_z = 0, z = 0$$

$$K = \frac{eB_0\lambda_0}{2\pi mc} \approx 0.0934 \cdot B_0\ [T] \cdot \lambda_0[mm]$$

K is a fundamental dimensionless parameter
Called K-parameter or deflection parameter

Undulators: K~1
Wigglers: K>>1

Example: undulator with period 20 mm and field 1 T → K=1.84

Wavelengths of harmonics of Undulator Radiation

- Wavelength of n^{th} harmonic

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2(\theta_x^2 + \theta_z^2)\right)$$

- Photon energy (ε_{ph}) of n^{th} harmonic

$$\varepsilon_n[keV] = \frac{9.5 \cdot n \cdot E^2[GeV]}{\lambda_0[mm] \cdot \left(1 + \frac{K^2}{2} + \gamma^2(\theta_x^2 + \theta_z^2)\right)}$$

- n = 1, 2, 3, … harmonic number,
- λ_0: undulator period,
- $E=\gamma mc^2$: electron energy,
- K: deflection parameter,
- θ_x, θ_z: observation angles with respect to undulator axis

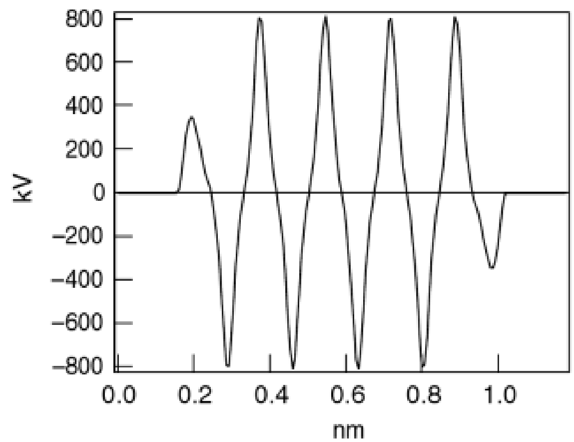
Undulator Radiation: increasing K

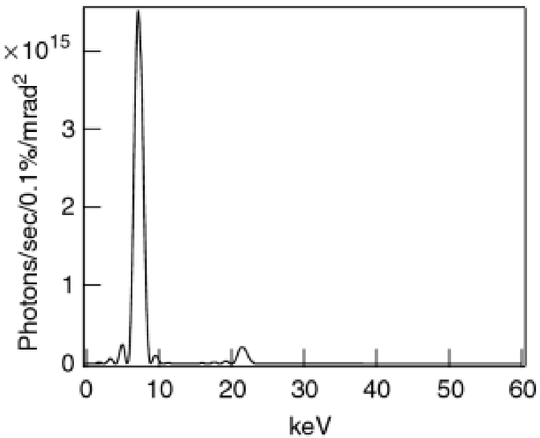
- Horizontal component of Electric field seen by observer on-axis
- For a **small K** the field is sinusoidal with the spectrum dominated by a single peak at the fundamental harmonic frequency
- For **large K**, the field is "spiky", leading to a series of narrow spectral peaks = high harmonics

* Electric Field

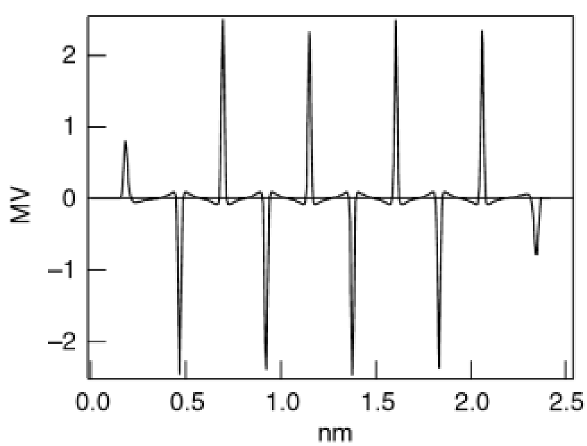
* Radiation Spectrum

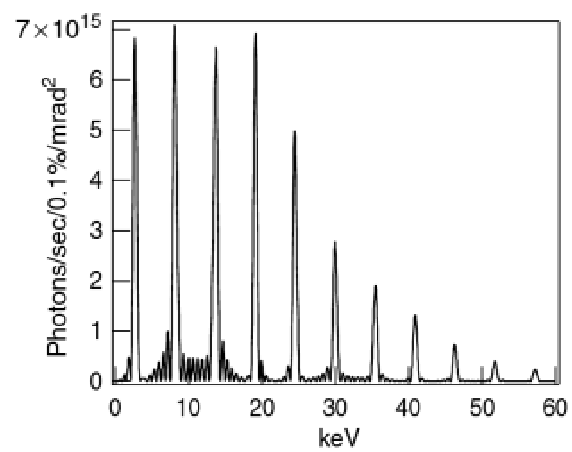
"Small" K=0.5



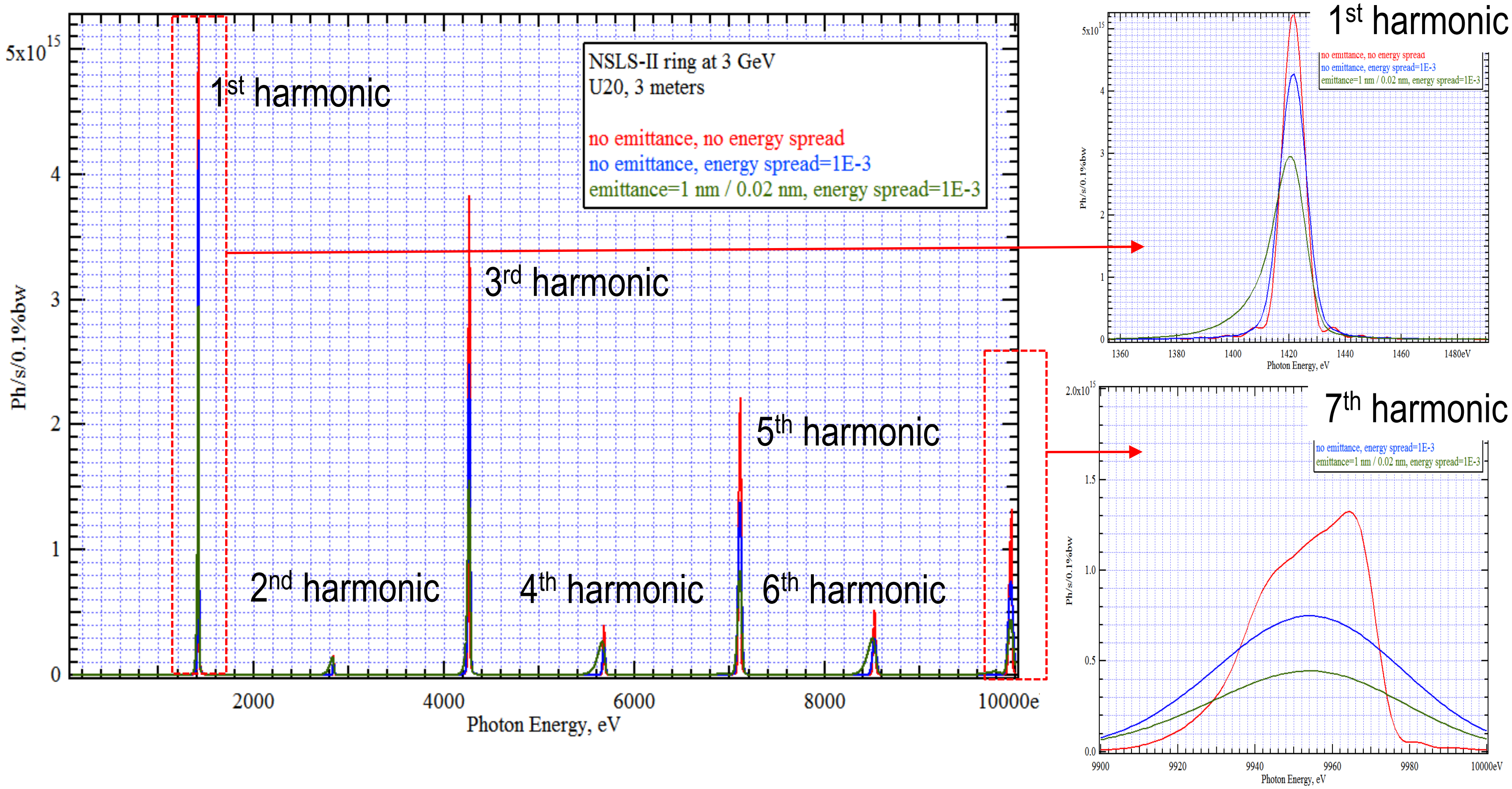


"Large" K=5





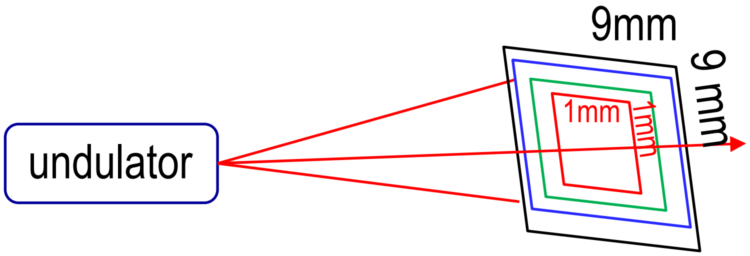
Spectrum: Impact from the finite source size

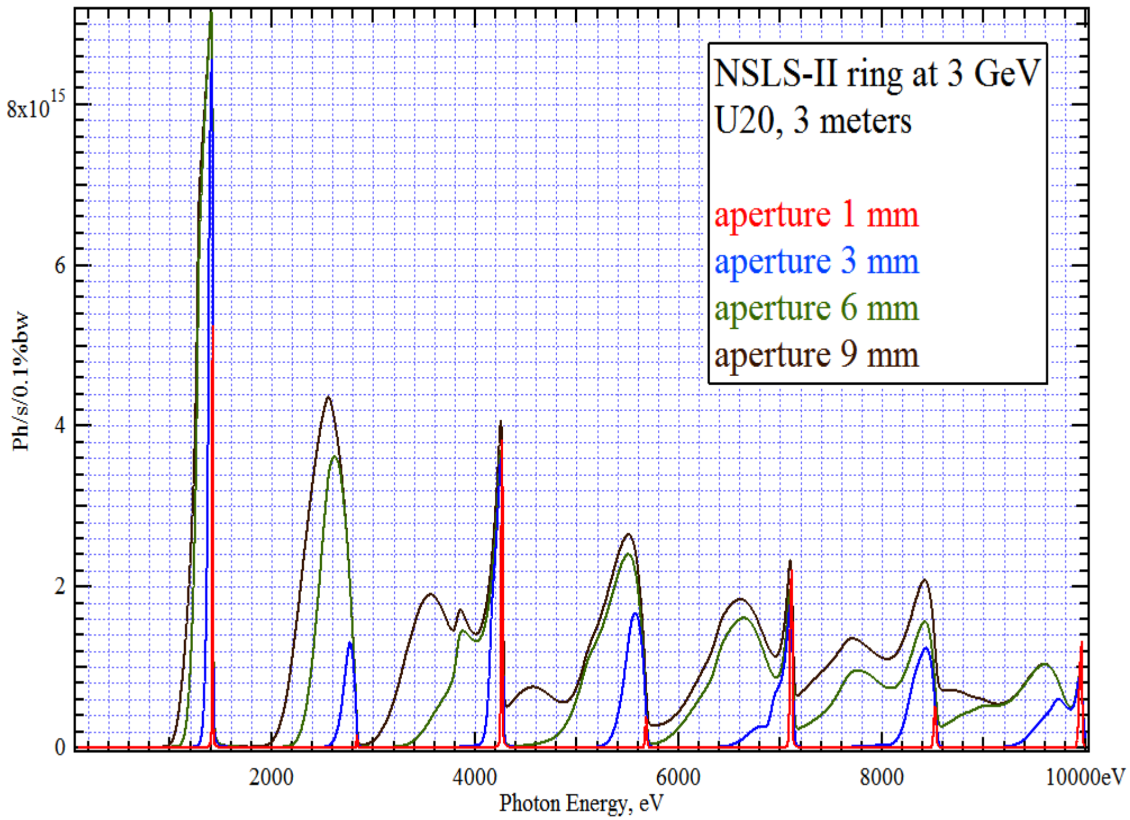


- Two contributions to the source size: emittance and energy spread
- Distortion of lineshape; reduction of peak flux

Spectrum: effect of sampling aperture

- Opening aperture broadens spectral bandwidth
- Spectrum exhibits only **odd** harmonics on axis
- **Even** harmonics shows up off-axis
- Energy – angle distribution of undulator radiation
- Monochromators select narrow spectral bandwidth





Angular Spectral Flux: an estimate

Angular spectral flux

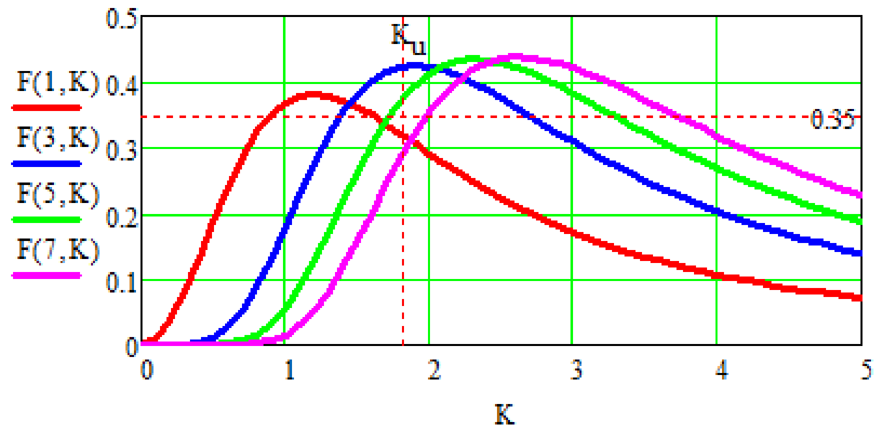
$$\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(\theta_x,\theta_z,\lambda)$$

angular spectral

$$F(n,K) := \frac{n^2 \cdot K^2}{\left(1+\frac{K^2}{2}\right)^2} \cdot \left(Jn\left(\frac{n-1}{2}, \frac{n \cdot K^2}{4+2 \cdot K^2}\right) - Jn\left(\frac{n+1}{2}, \frac{n \cdot K^2}{4+2 \cdot K^2}\right)\right)^2$$

If $\theta_x=\theta_z=0$ and $\lambda=\lambda_n$

$$\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(0,0,\lambda) = \alpha \frac{I}{e} N^2 \gamma^2 F_n(K)$$



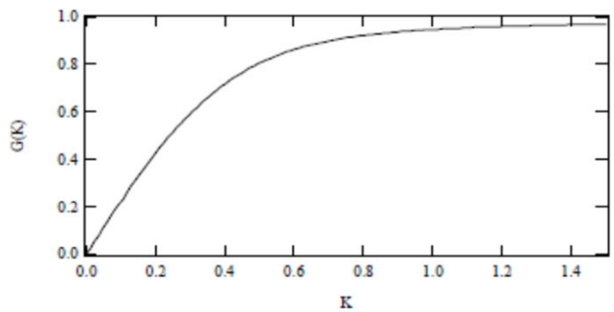
In usefull Units

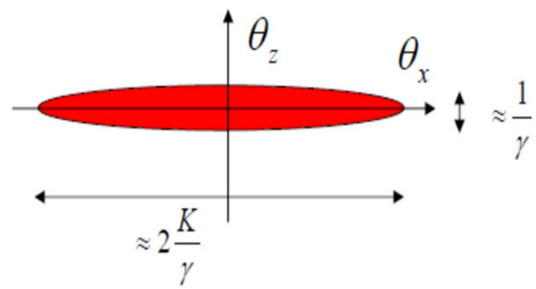
$$\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(0,0,\lambda)[\text{Phot/s/.1\%/mrad}^2] = 1.744\times 10^{14} N^2 E^2[GeV] I[A] F_n(K)$$

Undulator Radiation: Power and Power Density

$$Power[kW] \approx 0.633 \cdot B^2\,[T] \cdot E^2[GeV] \cdot I[A] \cdot L[m]$$

$$\frac{Power}{Solid\ Angle}[W/mrad^2] \approx 10.84 \cdot B\,[T] \cdot E^4[GeV] \cdot I[A] \cdot L[m] \cdot N \cdot G(K)$$

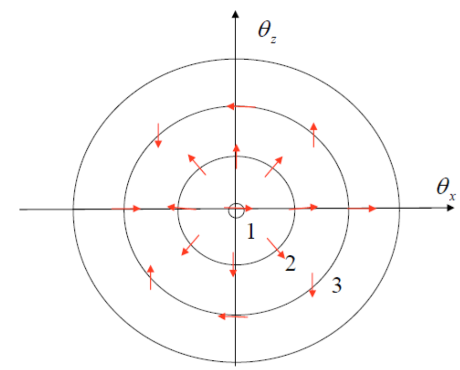




For an undulator : Length = 5 m, Field = 0.75 T, Period =35 mm

Ring	Energy	Current	Power	Power Density
				@ 10 m on-axis
	[GeV]	[A]	[kW]	[kW/mm2]
SuperACO	0.8	0.5	0.57	0.0023
SLS	2.4	0.3	3.08	0.11
ESRF	6	0.2	12.82	2.96

Undulator Radiation: Polarization



- Planar field undulators produce linearly polarized radiation

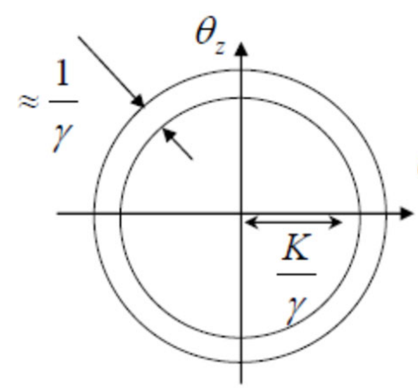
- Ellipsoidal undulator:

- Field: $\vec{B} = \left(B_x \cdot \sin\left(2\pi \frac{s}{\lambda_0} + \varphi\right), B_z \cdot \sin\left(2\pi \frac{s}{\lambda_0}\right), 0\right)$

- Velocity: $\vec{v} = \left(\frac{K_x}{\gamma} \cdot cos\left(2\pi \frac{s}{\lambda_0}\right), \frac{K_z}{\gamma} \cdot cos\left(2\pi \frac{s}{\lambda_0} + \varphi\right), 0\right)$

with $K_{x,z} = \frac{eB_{x,z}\lambda_0}{2\pi mc^2}$

If $K_x = K_y$ (helical undulator): radiation footprint is a ring with harmonic = 1



$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K_x^2}{2} + \frac{K_y^2}{2} + (\gamma\theta_x)^2 + (\gamma\theta_y)^2\right)$$

The polarisation is in general ellipsoidal depending on direction and wavelength

Undulator Radiation: Brightness

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_R^2}$, $\sigma_R = \frac{\sqrt{\lambda_n L}}{2\pi}$ Radiation source size

$\Sigma'_x = \sqrt{\sigma'^2_x + \sigma'^2_R}$, $\sigma'_R = \frac{1}{2}\sqrt{\frac{\lambda_n}{L}}$ Radiation divergence

Σ_x, Σ_z : Photon beam sizes Σ'_x, Σ'_z: Photon beam divergences

Limiting cases of Brightness:

i/ $\sigma_x, \sigma_z << \frac{\sqrt{\lambda_n L}}{2\pi}$ and $\sigma'_x, \sigma'_z << \sqrt{\frac{\lambda_n}{2L}} \Rightarrow B_n = \frac{F_n}{(\lambda_n/2)^2}$ Diffraction Limit

ii/ $\sigma_x, \sigma_z >> \frac{\sqrt{\lambda_n L}}{2\pi}$ and $\sigma'_x, \sigma'_z >> \sqrt{\frac{\lambda_n}{2L}} \Rightarrow B_n = \frac{F_n}{4\pi^2 \varepsilon_x \varepsilon_z}$

iii/ *Intermediate with Optimum Beta Function* $\Rightarrow$

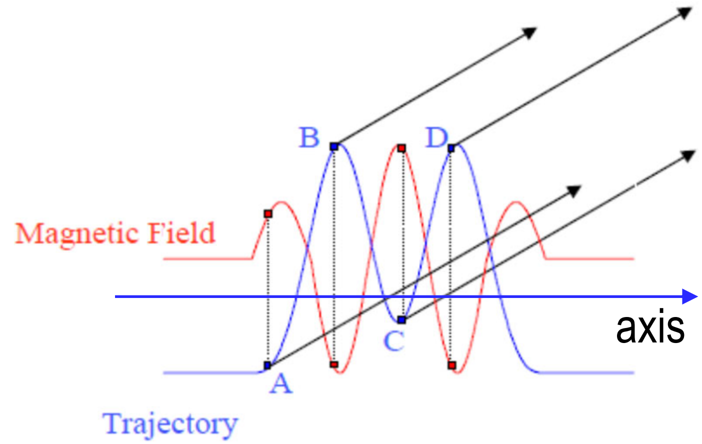
$$B_n = \frac{F_n}{4\pi^2 (\varepsilon_x + \frac{\lambda_n}{4\pi})(\varepsilon_z + \frac{\lambda_n}{4\pi})}$$

ε_x, ε_y are electron beam emittances (WG #24, 25)

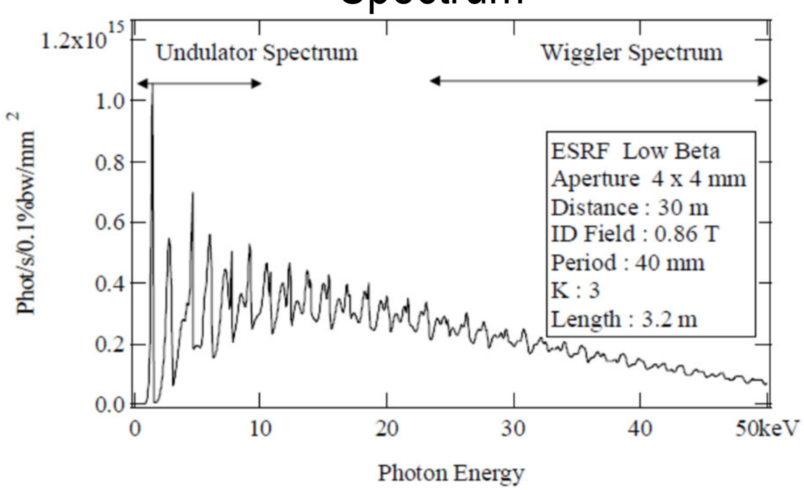
F_n is a photon flux in units of ph/sec

Wigglers

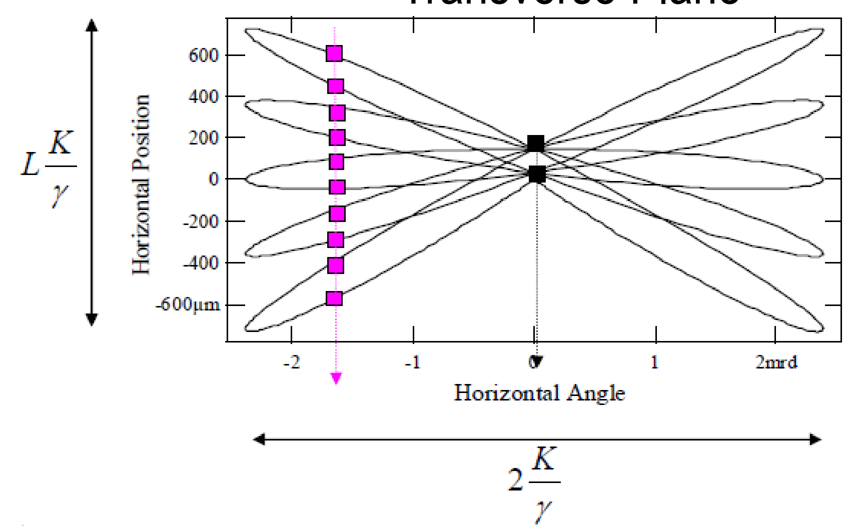
- Devices with K>>1
  - Two source points per period (N periods in total)
  - No constructive interference between poles: ~ 2N dipole sources
  - Continuous spectrum
- Flux and Brightness grow like 2N
- Polarization: linear in Median Plane, depolarized off-axis
- Multiple sources when observed off-axis



Spectrum



Transverse Plane

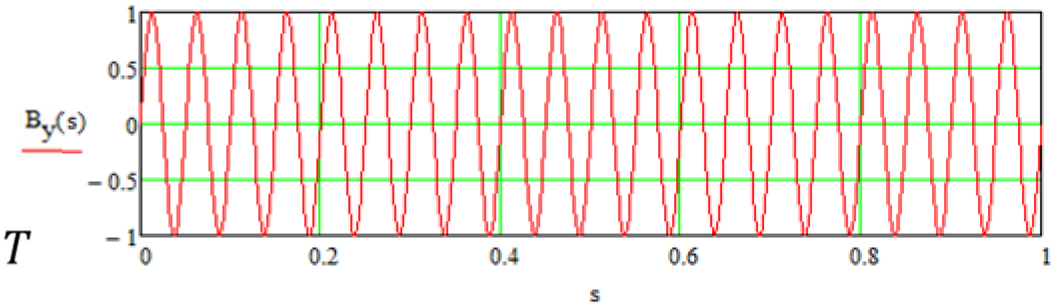


Problem solving: trajectory in a planar undulator

$\lambda_0 = 0.05$ undulator period, m

$k = \frac{2 \cdot \pi}{\lambda_0}$ wavenumber, $1/m$

$B_0 = 1$ Peak field in the undulator, T



$$B_x(s) = B_z(s) = 0 \quad B_y(s) = B_0 \cdot \sin(k \cdot s)$$

Equation of motion if field B:

$$\frac{d}{dt}\vec{p} = e \cdot \vec{v} \times \vec{B} \quad \rightarrow \quad \frac{d}{dt}p_x = e \cdot v_z \cdot B_y \qquad \gamma \cdot m \cdot \frac{d}{dt}v_x = e \cdot v_z \cdot B_y$$

$$\gamma \cdot m \cdot \frac{d}{dt}z \cdot \frac{d}{dz}v_x = e \cdot v_z \cdot B_y \quad \rightarrow \quad \gamma \cdot m \cdot \frac{d}{dz}v_x = e \cdot B_y$$

Trajectory angle:

$$\alpha_x = \frac{v_x}{\beta \cdot c} = \frac{e}{\gamma \cdot m \cdot \beta \cdot c} \cdot \int_0^{\lambda_0} B_y\, ds \text{ with } \alpha_{x.max} = \frac{e}{2 \cdot \pi \cdot \gamma \cdot m \cdot \beta \cdot c} B_0 \cdot \lambda_0$$

Problem solving: trajectory in a planar undulator (Cont'd)

Define undulator **deflection parameter**

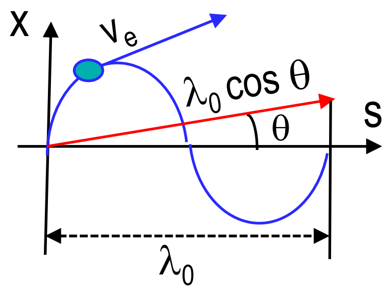
$$K = \frac{e}{2 \cdot \pi \cdot m \cdot c} \cdot B_0 \cdot \lambda_0$$

$K = 0.934 \cdot B_0 \cdot \lambda_0$, where B_o is in Tesla, λ_o is in *cm*

Maximum angle excursion of particle's trajectory: $\alpha_{x.max} = \frac{K}{\gamma}$

$x = \int_0^{\lambda_0} \alpha_x \, ds$ Maximum trajectory amplitude: $x_{max} = \frac{K}{2 \cdot \pi \cdot \gamma} \cdot \lambda_0$

Problem solving: wavelength of UR harmonics



1. Find longitudinal velocity of electron

$$\frac{1}{\gamma^2} = 1 - \beta^2 \quad \rightarrow \quad \frac{1}{\gamma^2} = 1 - \left(\frac{v_x}{c}\right)^2 - \left(\frac{v_s}{c}\right)^2$$

with trajectory angle $\frac{v_x}{c} = \frac{K}{\gamma} \cos(k \cdot s)$

$$\frac{1}{\gamma^2} + \left(\frac{K}{\gamma} \cdot \cos(k \cdot s)\right)^2 = 1 - \left(\frac{v_s}{c}\right)^2 \quad \rightarrow \quad \frac{v_s}{c} = 1 - \Delta \text{ and } \Delta \ll 1$$

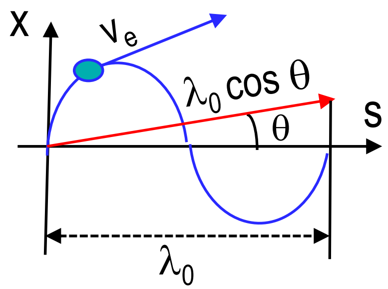
relativistic case

Then $1 - \left(\frac{v_s}{c}\right)^2 \approx 2\left(1 - \frac{v_s}{c}\right)$ and $\frac{v_s}{c} = 1 - \frac{1}{2 \cdot \gamma^2} - \left(1 + \frac{K^2}{2}\right)$

Averaging the expression above across a single undulator period:

$$\frac{v_s}{c} = 1 - \frac{1}{2 \cdot \gamma^2} \cdot \left(1 + \frac{K^2}{2}\right)$$

Problem solving: wavelength of UR harmonics (Cont'd)



2. Now we use synchronizm between electron and light: every period light wave slips away with respect to electron by a single period of wave

$$\lambda_R = c\left(\frac{\lambda_0}{v_s} - \frac{\lambda_0}{c}\cdot\cos(\theta)\right) \approx \lambda_0\cdot\left(1 - \frac{v_c}{c} + \frac{\theta^2}{2}\right) \quad \text{for small } \theta$$

Then

$$\lambda_R = \frac{\lambda_0}{2\cdot\gamma^2}\cdot\left(1 + \frac{K^2}{2} + \gamma^2\cdot\theta^2\right)$$

And for the n^{th} harmonic: $\lambda_{Rn} = \frac{\lambda_0}{2\cdot n\cdot\gamma^2}\cdot\left(1 + \frac{K^2}{2} + \gamma^2\cdot\theta^2\right)$

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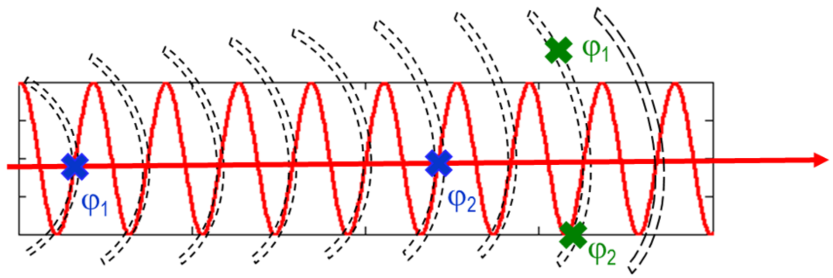
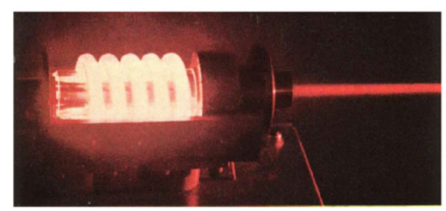
Concluding remarks

VI. Free Electron Laser

- Basics concepts of coherence
  - Optical coherence: phases of propagating radiation wavefront correlate and do not change with time
  - Interference effects come from high degree of coherence (e.g. Thomas Young's two-slit interferometer)
- Two types of coherence:
  - Temporal coherence = concerns correlation between phases (φ_1, φ_2) separated longitudinally
  - Spatial coherence = concerns correlation between phases (φ_1, φ_2) separated transversely
- Temporal coherence corresponds to narrow spectrum
- Transverse coherence corresponds to transverse collimation



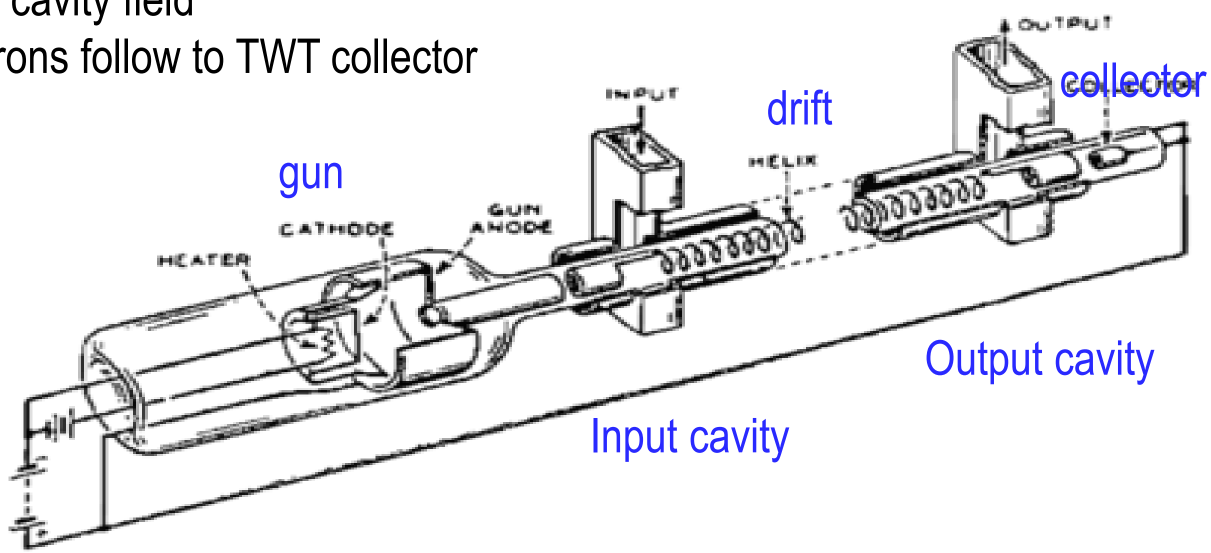
Ruby laser, 1960



* Lasers produce coherent beams of optical radiation
* Can we build a laser using free electrons as the active medium?

Travelling Wave Tube

- Travelling Wave Tube is an RF generator that uses free electrons as active medium
- TWT amplify injected RF signal
- How does it work?
  - Uniform flow of electrons emerge from heated cathode and get accelerated in the gun with hollow anode
  - They interact with the injected RF field in the Input cavity → energy (velocity) modulation
  - Energy (velocity) modulation converts into bunching along the following drift
  - Bunching occurs at RF frequency →longitudinal density of electrons is modulated with RF
  - Modulated electron beam enters Output cavity tuned in resonance with modulation and give energy to the cavity field
  - "Spent" electrons follow to TWT collector



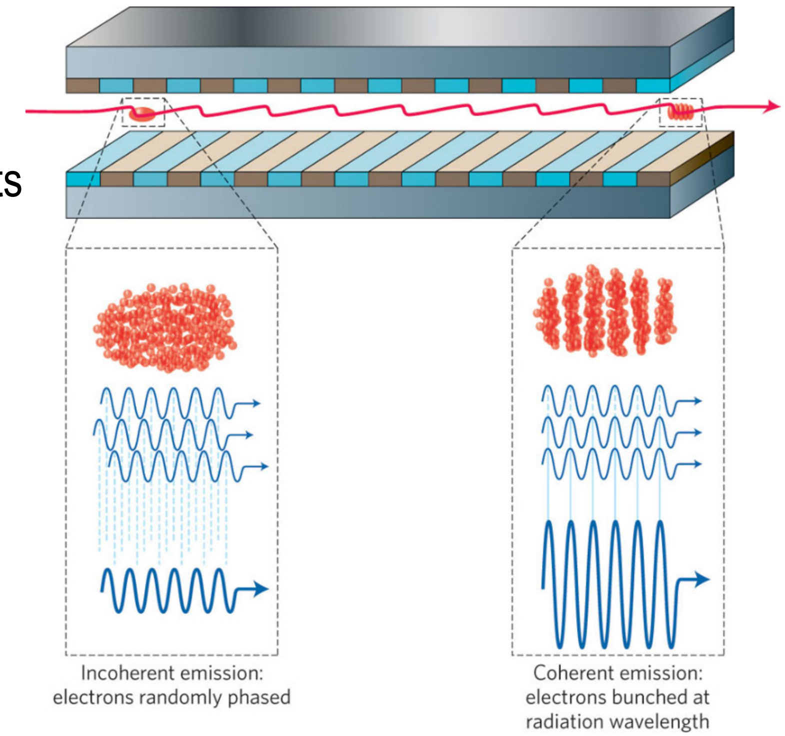
Principle of FEL

- FELs operate using principle that is similar to that of TWT
- AMPLIFIER (HIGH GAIN) FEL
  - Long undulator
  - Spontaneous emission from start of undulator interacts with electron beam.
  - Interaction between light and electrons grows giving microbunching
  - Increasing intensity gives stronger bunching giving, in turn, stronger emission

  → High radiation intensity achieved in single pass

- OSCILLATOR (LOW GAIN) FEL
  - Short undulator
  - Spontaneous emission trapped in an optical cavity
  - Trapped light interacts with successive e- bunches leading to microbunching and coherent emission

  → High radiation intensity achieved over many passes of electron beam



Output power $P \propto N_e^2$, where N_e is number of electrons within coherence length →

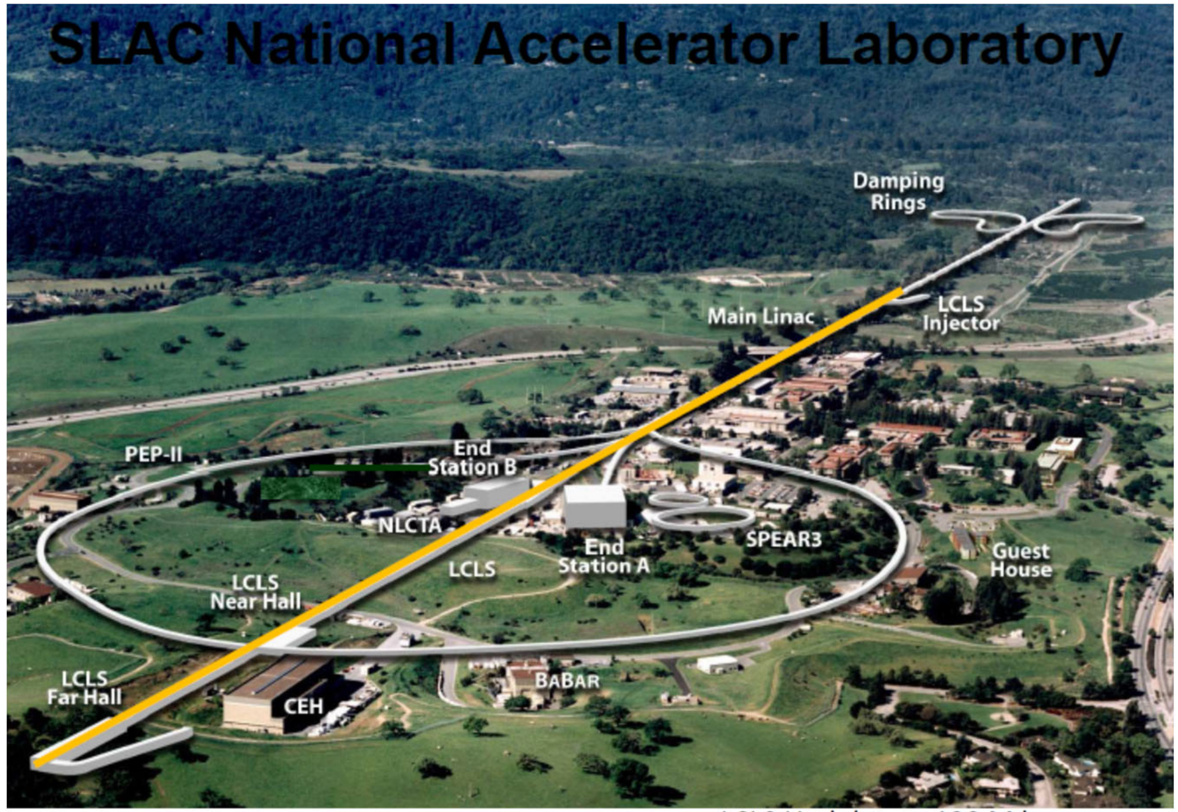
$$P_{FEL} \approx N_e \times P_{undulator}$$

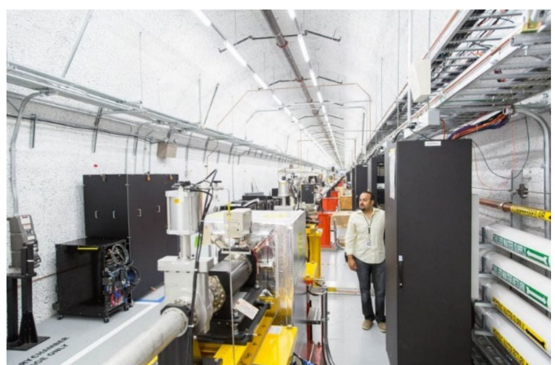
http://www.nature.com/nphoton/journal/v4/n12/fig_tab/nphoton.2010.239_F2.html

Modern X-ray FEL (LCLS)

- Linac Coherent Light Source is powered by 14 GeV beam from SLAC linear accelerator
- 2-mile-long linear accelerator is followed by 130 m long undulator
- Self Amplified Spontaneous Emission FEL lases down to 10 keV photon energy
- Free Electron Laser is commissioned in 2009
- Several instruments are in operations in 0.25-2 keV and 5-9 keV energy ranges
- LCLS-II project is under way!

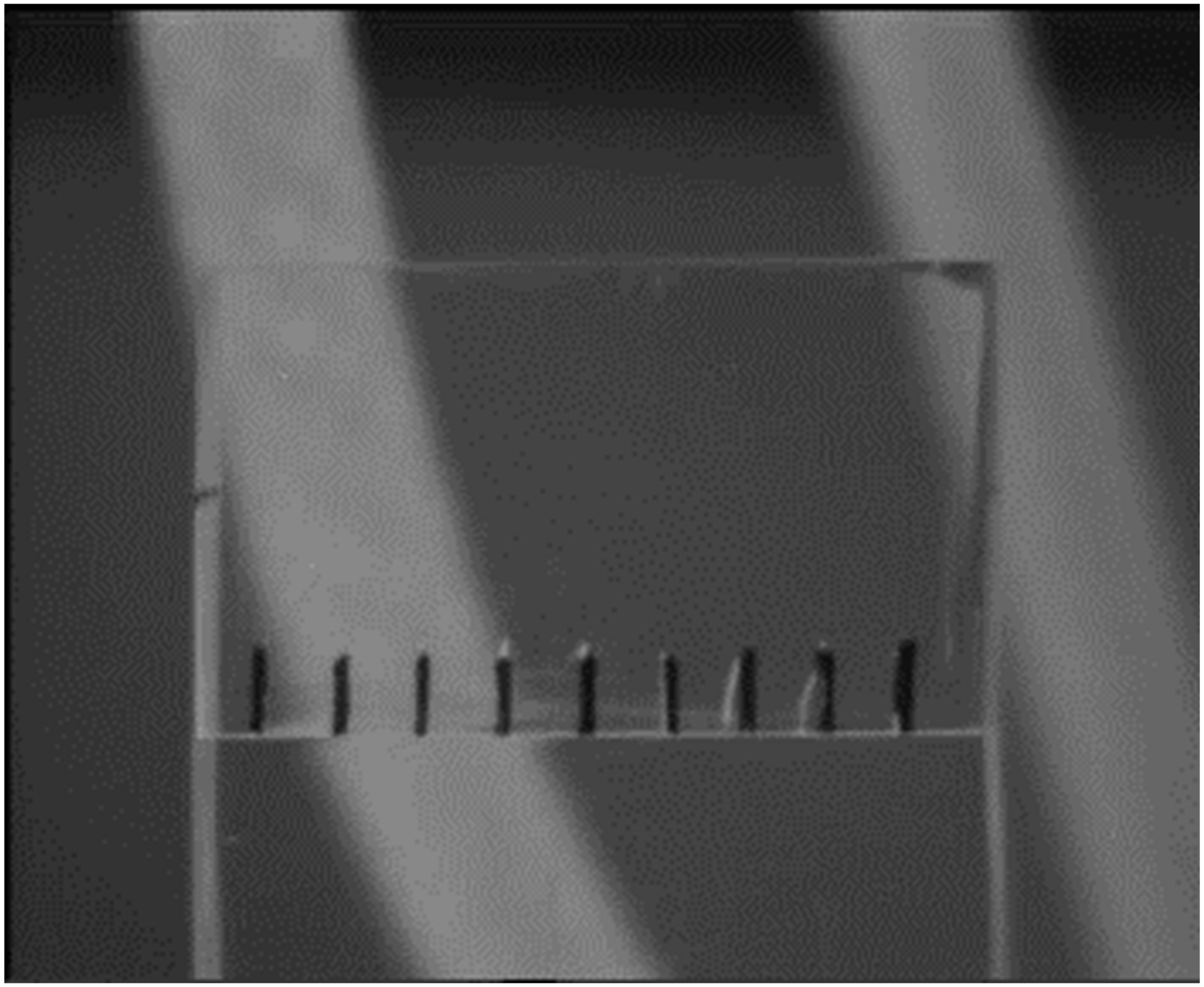
https://portal.slac.stanford.edu/sites/lcls_public/Pages/status.aspx





LCLS Undulator ~130 M long

High average power Free Electron Laser



Melting a piece of plastic with 400 W, 100 um wavelength FEL
Budker Institute of Nuclear Physics

Problem solving: estimating the power of a Star Wars blaster

The Heavy Blaster was a powerful laser pistol from the time of the Galactic Republic through the Galactic Civil War in Star Wars

A problem: Estimating the power of a Star Wars blaster

1. Boiling a glass of water with the blaster in 1 sec.

   $C_{water} = 4.18 \quad J/g/C$ specific heat capacity

   $g=200$ *gram*, weight of glass of water

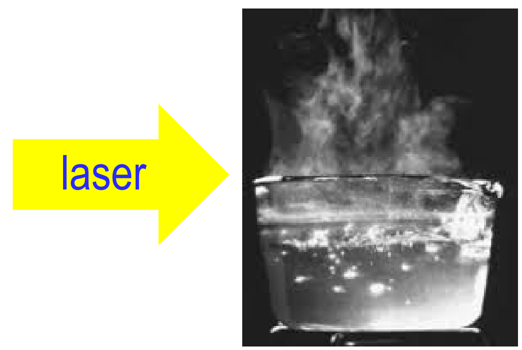
$\Delta T = 80\ C$, Temperature increment from room temperature

$$\Delta Q = c_{water} \cdot g \cdot \Delta T = 6.688\ x\ 10^4$$

67 kJ is required, which is equivalent to **67 kW of average power in 1 sec**!

- Assuming that all of the power will turn into heat
- In comparison NSLS-II Damping Wiggler radiates **64 kW** of average power at 0.5 A current

Han Solo preferred the laser blaster over a lightsaber



Outlook

Introduction

I. Radiation sources

II. Trajectory of and Radiation from Moving Charge

III. Particle Accelerators

IV. Synchrotron Radiation

V. An undulator

VI. Free Electron Laser

VII. Synchrotron Radiation Facilities

Concluding remarks

VII. Synchrotron Radiation Facilities

- To date there exist more than 50 synchrotron radiation sources in operations in the world serving many areas of science ranging from chemistry, biology, physics, material science, medicine to industrial applications

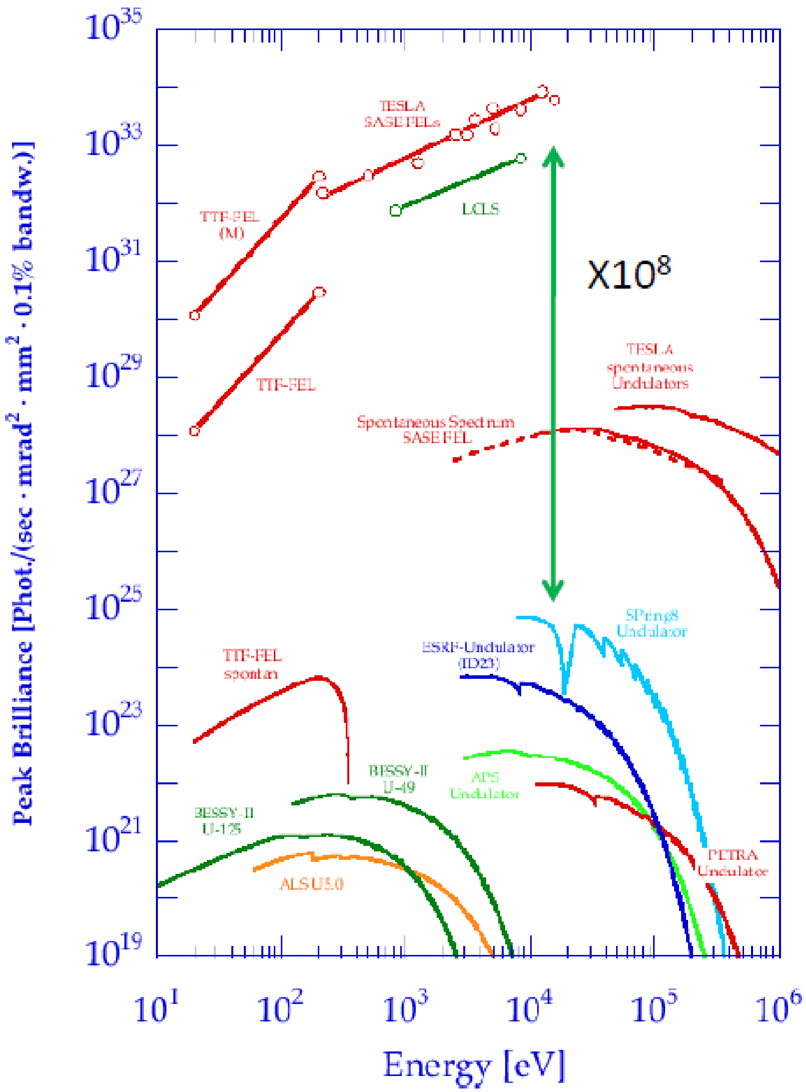
- Early construction of high-energy particle accelerators started essentially after World War II and enabled first generation machines. They were built in order to understand the fundamental laws of matter and particle interactions.
- Second generation: dedicated light source facility with multiple user programs
- Third generation sources started operation in the early 1990s: high brightness, optimized accelerator performance
- Fourth generation machines: diffraction-limited X-ray beams

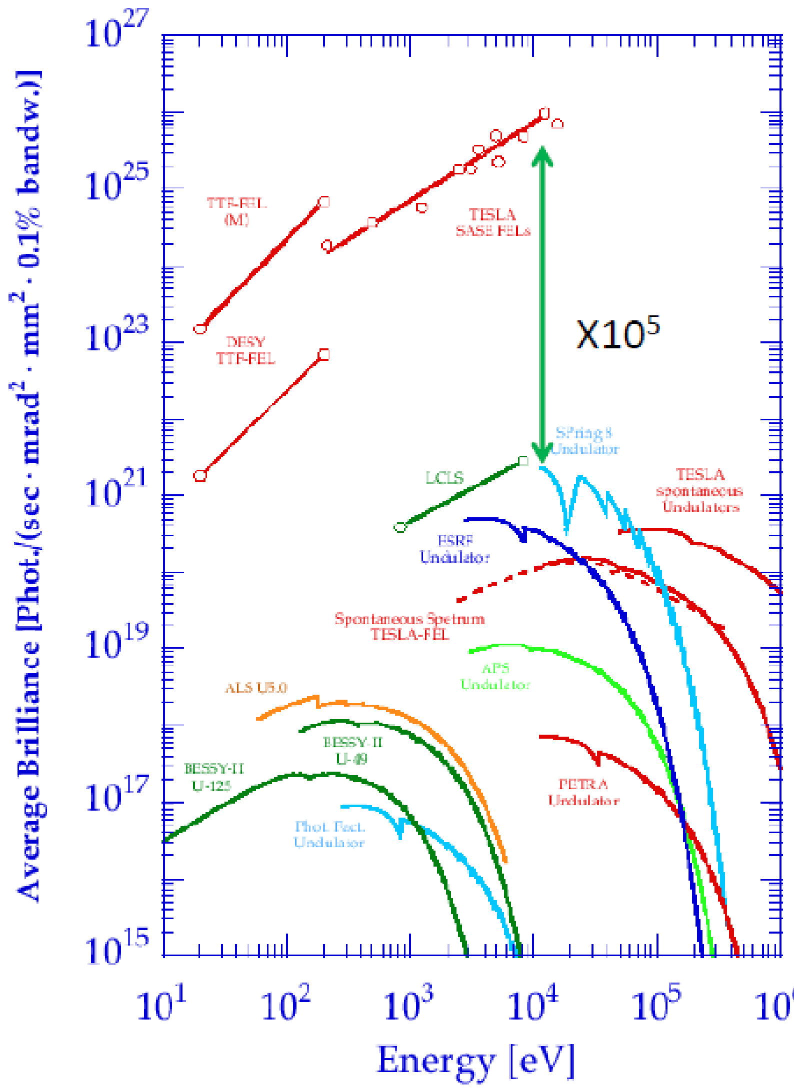
- Figure of merits among light sources are:
  - Photon energy range
  - Photon Flux
  - Photon Brightness
  - Pulse length

http://www.classe.cornell.edu/rsrc/Home/Research/ERL/ErlPubs2005/ERLPub05_22.pdf

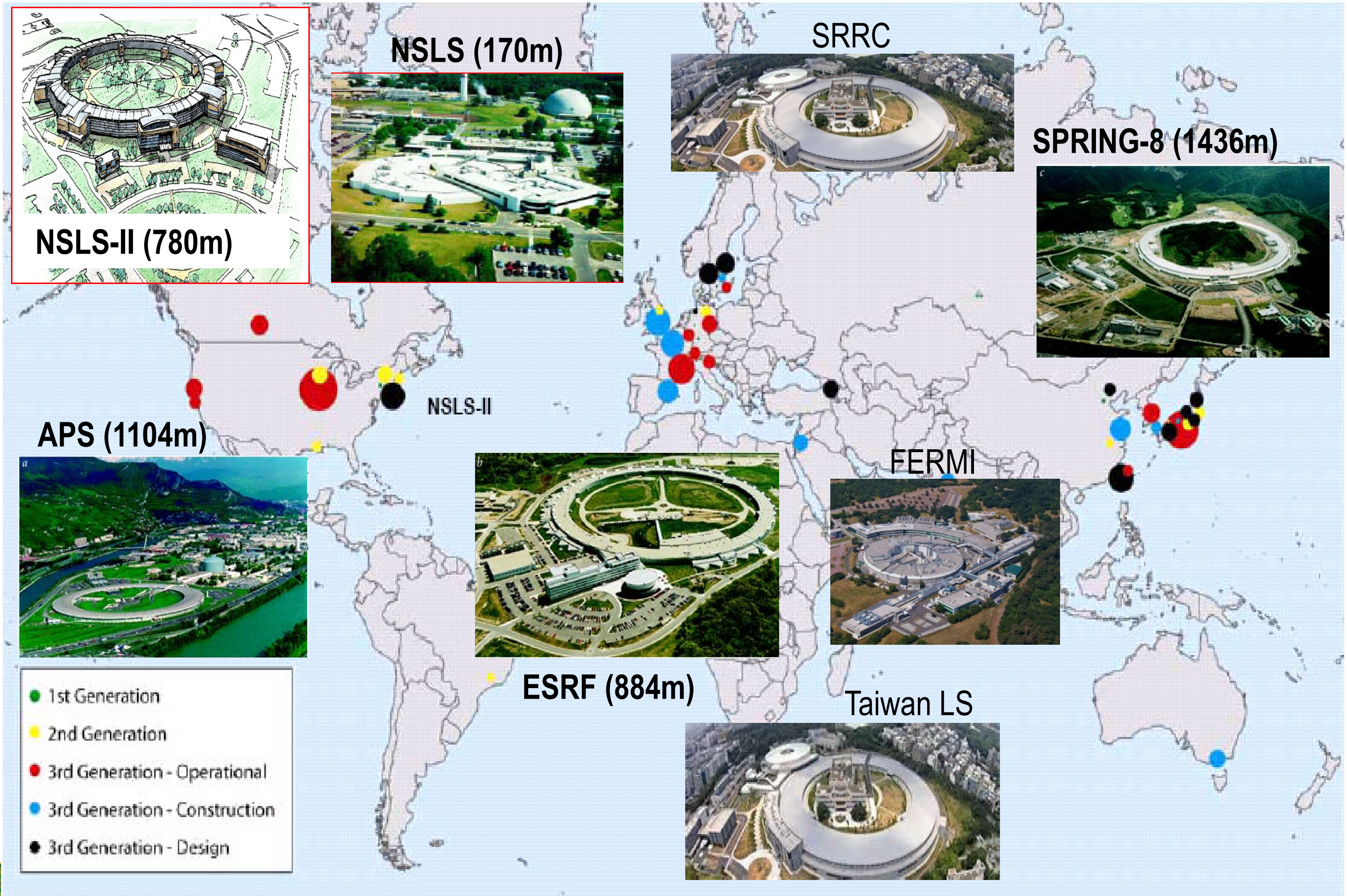
www.lightsources.org/

Figures of Merit: Average and Peak Brightness

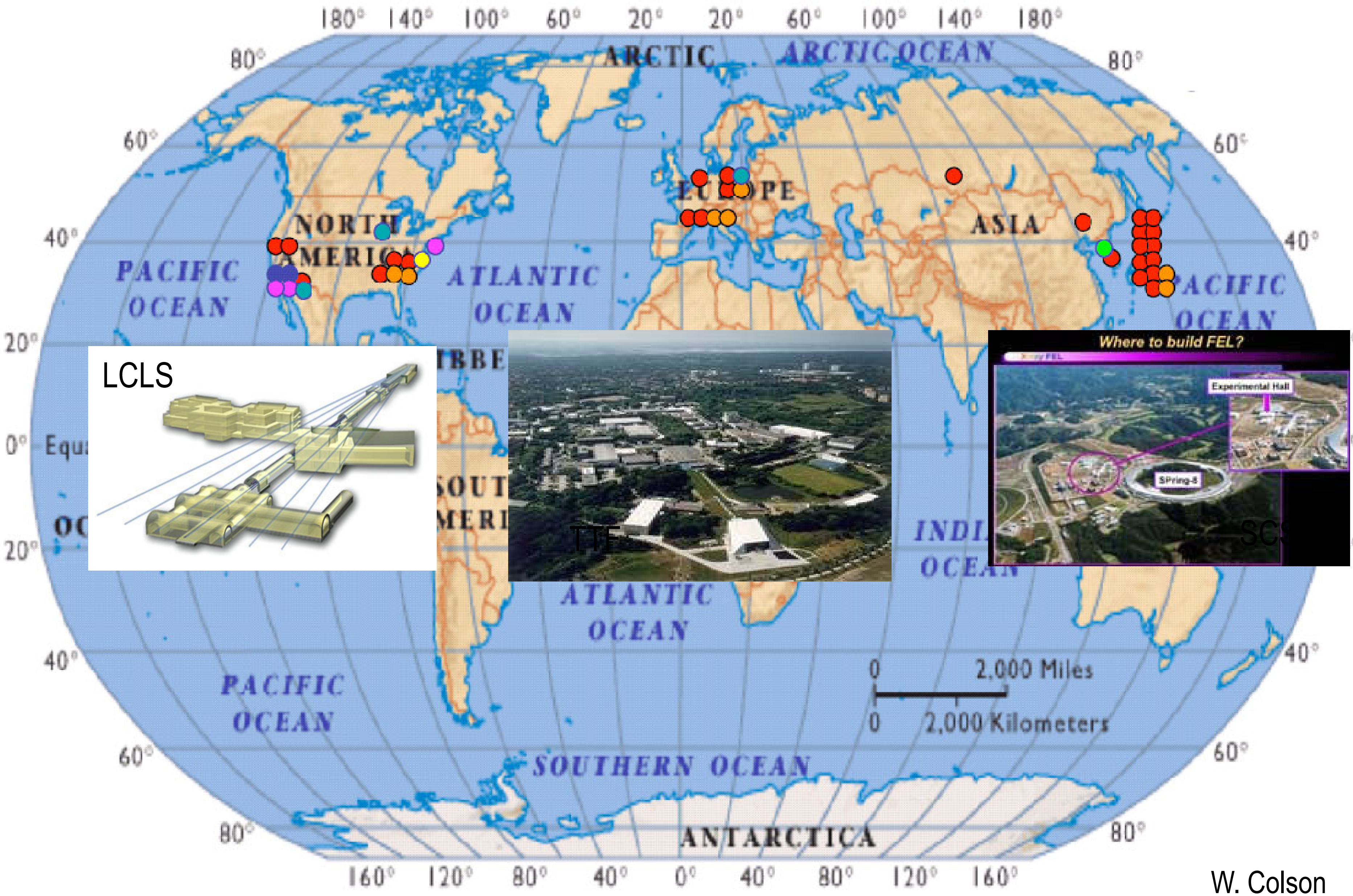




Synchrotron Light Sources Worldwide



Free Electron Lasers worldwide



W. Colson

Concluding remarks

- In this lecture we discussed basic concepts of synchrotron radiation including light source characteristics, particle accelerators, properties of electron beams, undulators and wigglers, their radiation properties, qualitative principles of Free Electron Lasers and concluding by diversity of Synchrotron Radiation facilities
- Going forward you will discover more exciting details of generation, propagation and interaction of light with matter
- Building a modern light source requires broad bandwidth of knowledge: optics, mechanics, electrodynamics, electrical, mechanical, RF, cryo engineering
- What are the light sources of tomorrow?

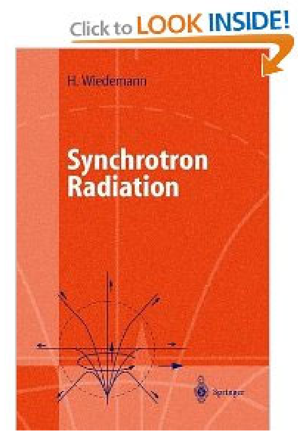
Computer codes for calculating SR

Name	Authors	Platform	Download
B2E & SRW	O. Chubar, P. Elleaume, ESRF	Mac, Windows	http://www.esrf.fr/machine/groups/insertion_devices/Codes/software.html
URGENT	R.P. Walker, Diamond Light Source	Fortran Source	Contact Author
XOP	M. Sánchez del Río , ESRF & R. J. Dejus, APS	Unix, Windows	http://www.esrf.fr/computing/scientific/xop/
SPECTRA	T. Tanaka, SPring8	Unix, Windows Mac	http://www.spring8.or.jp/ENGLISH/facility/bl/insertion/Softs/index.html

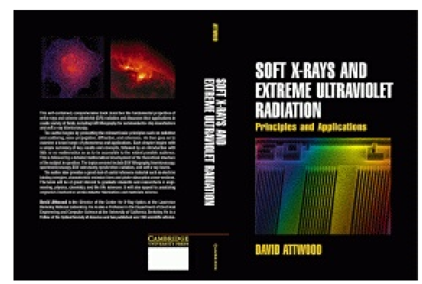
Try also: demo2.sirepo.com/srw

Related reading

- The Physics of Synchrotron Radiation
  by Albert Hofmann, 2007, Cambridge U Press
- Synchrotron Radiation for Materials Science Applications by D. Attwood, Cambridge U Press
- Synchrotron Radiation by H. Weidemann, Springer; 2003 edition



- CERN Accelerator School, CAS, http://cas.web.cern.ch/cas/CAS_Proceedings.html
- US Particle Accelerator School, http://uspas.fnal.gov/materials/index.shtml
- Lectures by Pascal Elleaume http://cas.web.cern.ch/cas/BRUNNEN/Presentations/PDF/I.pdf



- Lectures on the Free Electron Laser Theory and Related Topics by G. Dattoli, A. Renieri, A. Torre, World Scientific,1993 - Science

Acknowledgements

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  - D. Attwood, UCLA
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  - http://www.astec.stfc.ac.uk/ASTeC/Resources/PDF/Thompson_FELs.pdf
  - http://cas.web.cern.ch/cas/Granada-2012/Lectures/GranadaLectures/WolskiFreeElectronLasers.pdf
  - https://www.helmholtz-berlin.de/forschung/oe/fg/mi-synchrotron-radiation/synchrotron/photons/x-ray-pulses/free-electron-lasers/index_en.html