Risk in power grids

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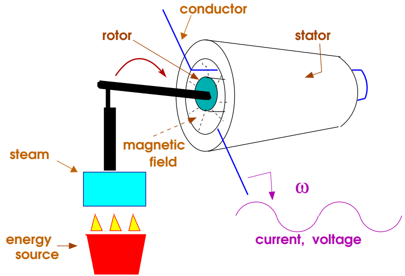
The challenge: power grids under change

- Power grids are getting *old*, and replacing all assets is too expensive, even as demands grow
- Renewables are mandated/desirable, can reduce (!) costs but also introduce *variability*
- Other disruptive technologies: "smart loads", "demand response"
- Power grids are likely to become more *local* as individuals/neighborhoods/towns invest in their own assets (batteries, renewables)
- Large, real-time *variability* recognized as an issue: the grid is not built to handle it
- Many opportunities for mathematics and computing

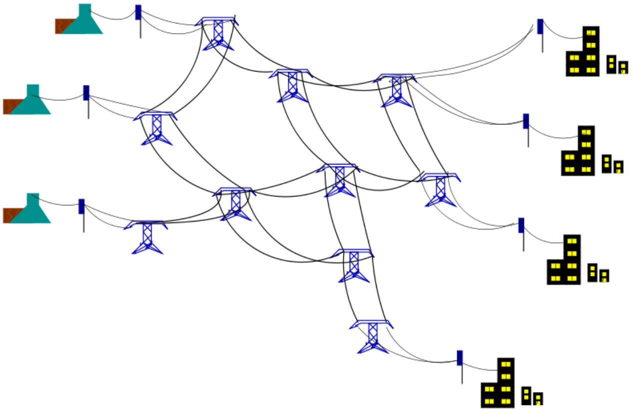
Power engineering for non-power engineers



A generator produces **current** at a given **voltage**.
$\omega \approx 60 \boldsymbol{Hz}$

Ohm's law: power = current x voltage

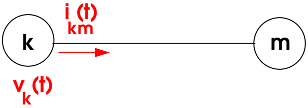
A traditional power system



... many missing details ...

AC Power Flows

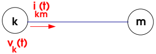
Real-time:



- Voltage at bus k: $v_k(t) \; = \; V_k^{max} \cos(\omega t + \theta_k^V) \; = \; V_k^{max} \, \mathcal{R}\mathrm{e} \, e^{j(\omega t + \theta_k^V)}$
- Current injected at k into km: $i_{km}(t) \; = \; I_{km}^{max} \cos(\omega t + \theta_{km}^I)$.
- Power injected at k into km: $p_{km}(t) \; = \; v_k(t) i_{km}(t)$.

Averaged over period T:

- $p_{km} \; \doteq \; \frac{1}{T} \int_0^T p(t) \; = \; \frac{1}{2} V_k^{max} I_{km}^{max} \cos(\theta_k^V - \theta_{km}^I)$.



- $p_{km} \doteq \frac{1}{T}\int_0^T p(t) = \frac{1}{2} V_k^{max} I_{km}^{max} \cos(\theta_k^V - \theta_{km}^I)$
- $v_k(t) = V_k^{max}\, \mathcal{R}\text{e}\, e^{j(\omega t + \theta_k^V)}, \quad i_{km}(t) = I_{km}^{max}\, \mathcal{R}\text{e}\, e^{j(\omega t + \theta_{km}^I)}$

$$V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$$

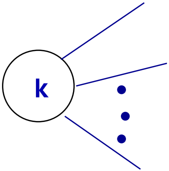
- $p_{km} = |V_k||I_{km}| \cos(\theta_k^V - \theta_{km}^I) = \mathcal{R}\text{e}(V_k I_{km}^*)$
- $q_{km} \doteq Im(V_{km} I_{km}^*)$ and $S_{km} \doteq p_{km} + j q_{km}$

- $V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$ (voltage, current)

$$p_{km} \ = \ \mathcal{R}\mathrm{e}(V_k I_{km}^*), \quad q_{km} \ = \ Im(V_{km} I_{km}^*) \tag{1}$$

$$I_{km} \ = \ \boldsymbol{y_{\{k,m\}}}(V_k - V_m), \quad \boldsymbol{y_{\{k,m\}}} = admittance \text{ of } km. \tag{2}$$

Network Equations



- $V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$ (voltage, current)

$$p_{km} = \mathcal{R}\mathrm{e}(V_k I_{km}^*), \quad q_{km} = Im(V_{km} I_{km}^*) \tag{3}$$

$$I_{km} = \mathbf{y_{\{k,m\}}}(V_k - V_m), \mathbf{y_{\{k,m\}}} = \textit{admittance} \text{ of } km. \tag{4}$$

Network Equations

$$\sum_{km \in \delta(k)} p_{km} = \hat{P}_k, \quad \sum_{km \in \delta(k)} q_{km} = \hat{Q}_k \quad \forall\, k \tag{5}$$

Generator: $\hat{P}_k, |V_k|$ given. Other buses: $\hat{P}_k, \hat{Q}_k$ given.

$$\sum_{km \in \delta(k)} \left[\boldsymbol{g_{km}}(|V_k|^2) - \boldsymbol{g_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{b_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] = \hat{\boldsymbol{P}}_{\boldsymbol{k}}$$

$$\sum_{km \in \delta(k)} \left[-\boldsymbol{b_{km}}(|V_k|^2) + \boldsymbol{b_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{g_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] = \hat{\boldsymbol{Q}}_{\boldsymbol{k}}$$

$$(\boldsymbol{V}_{\boldsymbol{k}}^{\min})^2 \leq |V_k|^2 \leq (\boldsymbol{V}_{\boldsymbol{k}}^{\max})^2,$$

for each bus $\boldsymbol{k} = 1, 2, \ldots$

Also: an **optimization** version

- Minimize cost of operation
- Additional decisions: modify line attributes, requires **binary** variables

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Exploring the Power Flow Solution Space Boundary

Ian A. Hiskens, *Senior Member* and Robert J. Davy

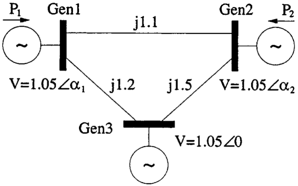


Fig. 6. Three bus system.

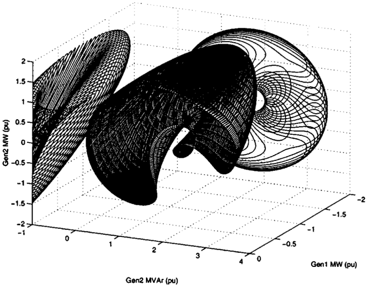


Fig. 13. Solution space, P_1–Q_2–P_2 view.

ACOPF (1960s)

$$
\begin{aligned}
\min \quad & \sum_{k \in G} C_k(G_k) \\
& \sum_{km \in \delta(k)} \left[\boldsymbol{g_{km}}(|V_k|^2) - \boldsymbol{g_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{b_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] \\
& \qquad = \hat{\boldsymbol{P}}_{\boldsymbol{k}} \\
& \sum_{km \in \delta(k)} \left[-\boldsymbol{b_{km}}(|V_k|^2) + \boldsymbol{b_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{g_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] \\
& \qquad = \hat{\boldsymbol{Q}}_{\boldsymbol{k}} \\
& (\boldsymbol{V_k^{\min}})^2 \quad \leq \quad |V_k|^2 \quad \leq \quad (\boldsymbol{V_k^{\max}})^2,
\end{aligned}
$$

for each bus $\boldsymbol{k} = 1, 2, \ldots$

Here:

- $\boldsymbol{G}$ = set of **generators**, $\boldsymbol{G_k}$ = power generated at generator $\boldsymbol{k}$, $\boldsymbol{C_k}$ = cost function at $\boldsymbol{k}$.
- Some simple constraints missing
- Some complex features ommitted

$$\sum_{km \in \delta(k)} \left[\boldsymbol{g_{km}}(|V_k|^2) - \boldsymbol{g_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{b_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] = \boldsymbol{\hat{P}_k}$$

$$\sum_{km \in \delta(k)} \left[-\boldsymbol{b_{km}}(|V_k|^2) + \boldsymbol{b_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{g_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right] = \boldsymbol{\hat{Q}_k}$$

$$(\boldsymbol{V_k^{\min}})^2 \leq |V_k|^2 \leq (\boldsymbol{V_k^{\max}})^2,$$

for each bus $\boldsymbol{k} = 1, 2, \ldots$

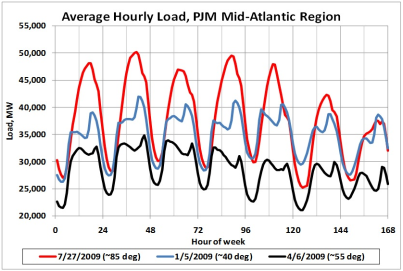
Mathematics, anyone?

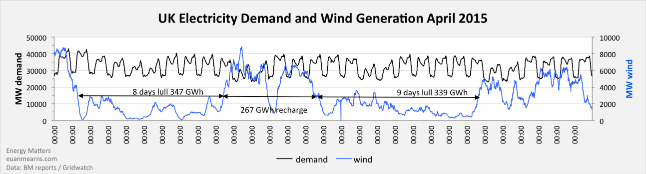
- Log-barrier methods: compute numerical solutions. **But:** no guarantees. **Modern** versions of the problem quite challenging in large-scale cases. 5×10^4 buses $\rightarrow 10^6$ (or more) variables
- Rigorous theory for solution of semi-algebraic problems. Sums-of-squares, SOCPs, RLT, MINLP. Improving, but "not there" yet.

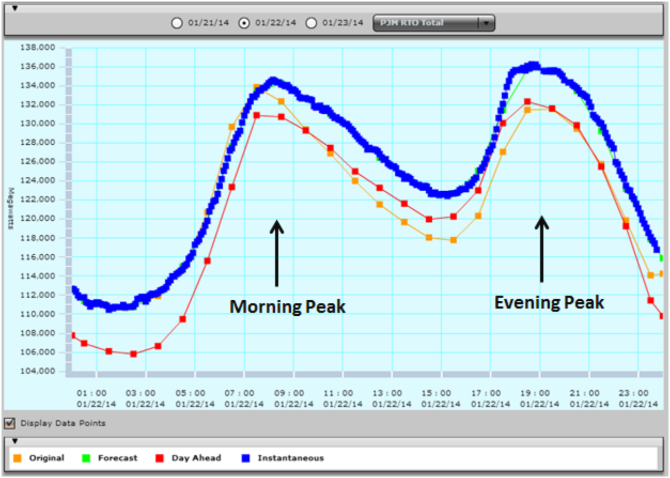
How is this actually used? – a simplified view

1. **Day-ahead computation**
    - Goal is to decide when to operate each generator, and to what extent
    - Uses an (e.g.) hourly profile of demand **estimates**
    - Also outputs economic information ("location marginal prices")
    - **Linearized** optimization model
2. **Shorter time frame computation**
    - This is run e.g. every five minutes
    - Corrects generator output levels
    - Uses more accurate demand estimates
    - Also **linearized** optimization model
3. **But how do we account for demand estimation errors?**

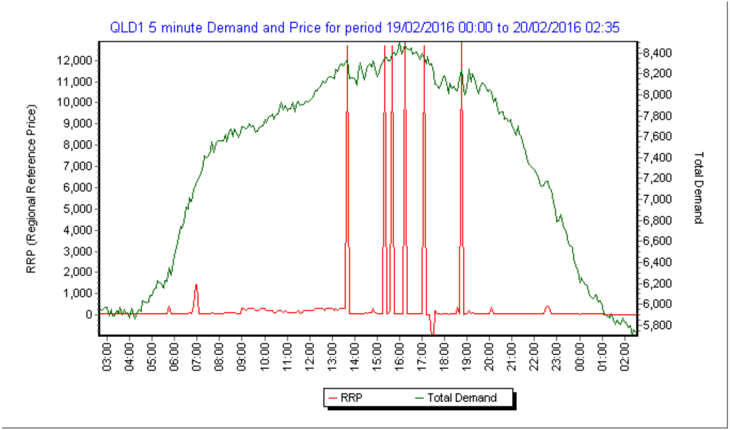
Managing changing demands



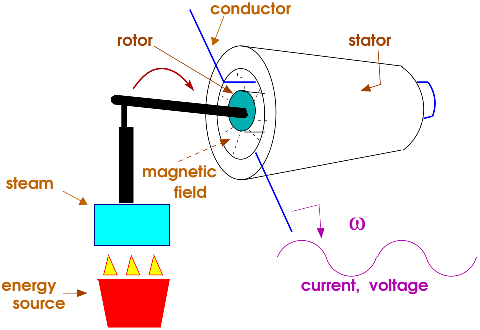




01/21/14
01/22/14
01/23/14
PJM RTO Total
138,000
136,000
134,000
132,000
130,000
128,000
126,000
124,000
122,000
120,000
118,000
116,000
114,000
112,000
110,000
108,000
106,000
104,000
Megawatts
Morning Peak
Evening Peak
01 : 00 01/22/14
03 : 00 01/22/14
05 : 00 01/22/14
07 : 00 01/22/14
09 : 00 01/22/14
11 : 00 01/22/14
13 : 00 01/22/14
15 : 00 01/22/14
17 : 00 01/22/14
19 : 00 01/22/14
21 : 00 01/22/14
23 : 00 01/22/14
Display Data Points
Original
Forecast
Day Ahead
Instantaneous


QLD1 5 minute Demand and Price for period 19/02/2016 00:00 to 20/02/2016 02:35
RRP (Regional Reference Price)
12,000
11,000
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
8,400
8,200
8,000
7,800
7,600
7,400
7,200
7,000
6,800
6,600
6,400
6,200
6,000
5,800
Total Demand
03:00 04:00 05:00 06:00 07:00 08:00 09:00 10:00 11:00 12:00 13:00 14:00 15:00 16:00 17:00 18:00 19:00 20:00 21:00 22:00 23:00 00:00 01:00 02:00
RRP
Total Demand

What happens when there is a generation/load mismatch



Frequency response:

mismatch $\Delta \boldsymbol{P}$ $\Rightarrow$ **frequency change** $\Delta\omega \approx -c\,\Delta P$

AGC, primary and secondary response (simplified!, abridged!)

Suppose generation vs loads balance spontaneously changes (i.e. a net imbalance)?

- AC frequency changes proportionally (to first order) near-instantaneously
- **Primary response.** (very quick) Inertia in generators contributes electrical energy to the system
- **Secondary response.** (seconds) Suppose **estimated** generation **shortfall** $= \Delta P$. Then:

Generator g changes output by $\alpha_g \Delta P$

- $\sum_g \alpha_g = 1$, $\alpha \geq 0$, $\alpha > 0$ for "participating" generators
- **Preset** participation factors
- $\Delta\omega$ sensed by control center, which issues generator commands

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Generator $\boldsymbol{g}$ changes output by $\alpha_{\boldsymbol{g}} \Delta \boldsymbol{P}$

- $\sum_{\boldsymbol{g}} \alpha_{\boldsymbol{g}} = 1$, $\alpha \geq 0$, $\alpha > 0$ for "participating" generators
- **Preset** participation factors
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AGC, primary and secondary response (the future)

Suppose generation vs loads balance spontaneously changes (i.e. a net imbalance)?

- AC frequency changes proportionally (to first order) near-instantaneously
- **Primary response.** (very quick) Inertia in generators contributes electrical energy to the system
- **Secondary response.** (seconds) Suppose **estimated** generation **shortfall** $= \Delta P$. Then:

Generator, **or battery, or photovoltaic** g changes output by $\alpha_g \Delta P$

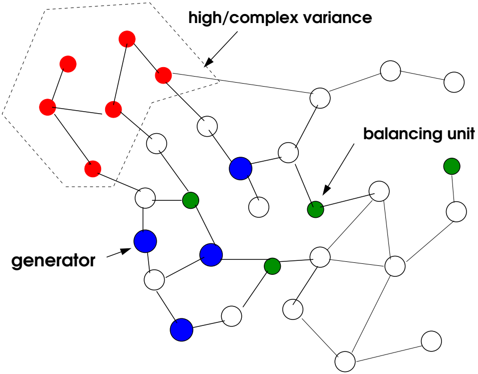
- $\sum_g \alpha_g = 1$, $\alpha \geq 0$, $\alpha > 0$ for "participating" generators
- **Preset** participation factors
- $\Delta\omega$ sensed by control center, which issues generator commands

What is **risk**?

- **Traditional risk.** A (single) **generator outage** or **line outage** event constitutes risk. Accounted for in day-ahead computation.
- **Novel risk.** Shorter time-frame unusual **demand/generation** patterns imperil **balancing** (AGC).
  - If real-time imbalances are large and *inconveniently located*, balancing capabilities may be insufficient
  - Real-time *volatility* (i.e. "variability") can be especially tricky – rapid, large, back-and-forth changes
  - Adversarial potential? Not **just** bad actors, but also arbitrage potential. Intelligent behavior that works against system safety
  - Power systems are inherently nonlinear – a large infeasible behavior, even if brief, can lead to a large collapse (e.g. generator tripping)
  - Local control and generation can add flexibility and better economics, but also removes protection provided by system inertia

From a current California ISO document:

" … the system may be ill prepared to respond to large amounts of uncertainty when they materialize in real time."



- Endogenous variance + response variance causes system-wide variance
- Variability in physical quantities (voltages, frequency) could become problematic
- From other industries: reducing system variance should be an operational goal unto itself
- Should design balancing structure (the α_g) in a variance-aware fashion

The future: variance-aware power system computations

$$\min \quad \sum_{k \in G} C_k(G_k) \; + \; \underbrace{F(\text{system variance})}_{\text{cost of variance}}$$

$$\sum_{km \,\in\, \delta(k)} \left[\boldsymbol{g_{km}}(|V_k|^2) - \boldsymbol{g_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{b_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right]$$

$$= \hat{\boldsymbol{P}}_{\boldsymbol{k}}$$

$$\sum_{km \,\in\, \delta(k)} \left[-\boldsymbol{b_{km}}(|V_k|^2) + \boldsymbol{b_{km}}|V_k||V_m|\cos(\theta_k - \theta_m) - \boldsymbol{g_{km}}|V_k||V_m|\sin(\theta_k - \theta_m) \right]$$

$$= \hat{\boldsymbol{Q}}_{\boldsymbol{k}}$$

$$(\boldsymbol{V}_{\boldsymbol{k}}^{\min})^2 \quad \leq \quad |V_k|^2 \quad \leq \quad (\boldsymbol{V}_{\boldsymbol{k}}^{\max})^2,$$

for each bus $\boldsymbol{k} = 1, 2, \ldots$

Here:

- $\boldsymbol{G}$ = set of **generators**, $\boldsymbol{G_k}$= power generated at generator $\boldsymbol{k}$, $\boldsymbol{C_k}$ = cost function at $\boldsymbol{k}$.
- Balancing mechanism is part of the optimization (ommitted)
- Model for exogenous variance in loads/generation is assumed
- Needed: equations connecting response to exogenous variance

The future: financial instruments for variance control

- **Example:** the owner of a battery bank, at a price, enters into an agreement with a utility (or ISO) to provide variance reduction (up to a point) during a period of interest
- This is **not** standard balancing: there is a triggering condition and it is up to a point
- Similar to, e.g. *variance swaps*
- The utility, or ISO, could enter into several similar contracts, each covering a different *tranche* of volatility
- Other ideas: derivative contracts, hedging
- Pricing?
- What is variance?

The future: correlation structure recognition, in real-time

→ *Learn* covariance structure as it changes

- PMUs, now and future: very fast sensors (report 100 times per second)
- Will become ubiquitous
- Vast data streams!
- Real-time recognition of *changes* in variance and correlations important
- Approximate PCA, in real time, in a high-dimensional setting
- Some theoretical tools: the noisy power method, and beyond
- Also need to be adversarial, and to anticipate changes

Thanks

ARPA-E funded programs:

- GO competition
- PERFORM program

Kory Hedman, Joe King, Richard O'Neill