# Numerical Methods for Predicting Coastal Flooding With Uncertainty

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Columbia Engineering
The Fu Foundation School of Engineering and Applied Science

# Funding

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# Collaborators

*Source: Jocelyn Augustino / FEMA - http://www.fema.gov/photdata/original/38891.jpg*

# Storm Surge

*Reuters - Marc C. Olsen - U.S. Air Force*

# Hurricane Sandy

# Hurricane Irma

# Hurricane Maria

*Mexico Beach, FL - NOAA*

# Hurricane Michael


# Hurricane Harvey


# Storm Surge Vulnerability


*Hoboken Path Station, NJ - Port Authority*

# Transportation Vulnerability

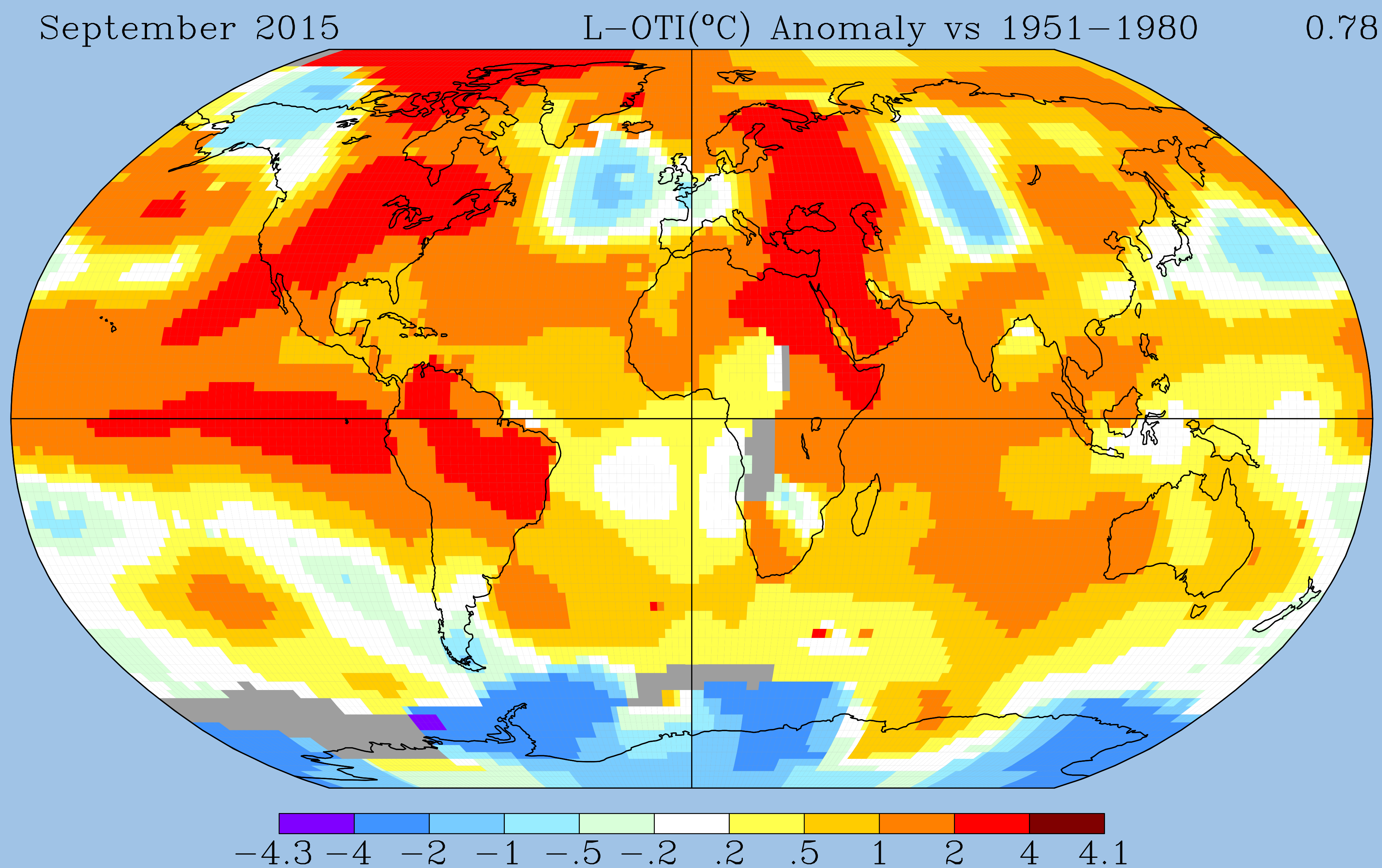
*Iwan Baan - Getty Images*

# Utility Vulnerability

*Tuckerton, NJ - boston.com*

# Residential Vulnerability

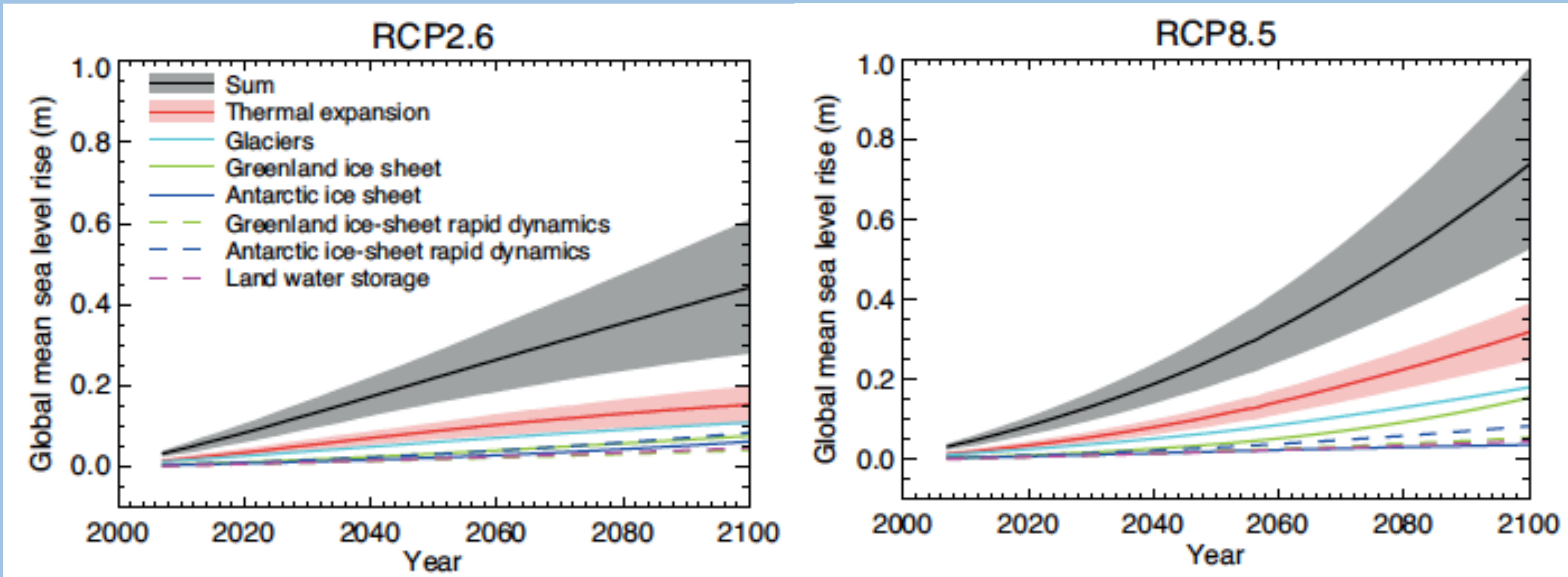
boston.com




Hansen, J., R. Ruedy, M. Sato, and K. Lo, 2010: Global surface temperature change, Rev. Geophys., **48**, RG4004, doi:10.1029/2010RG000345.

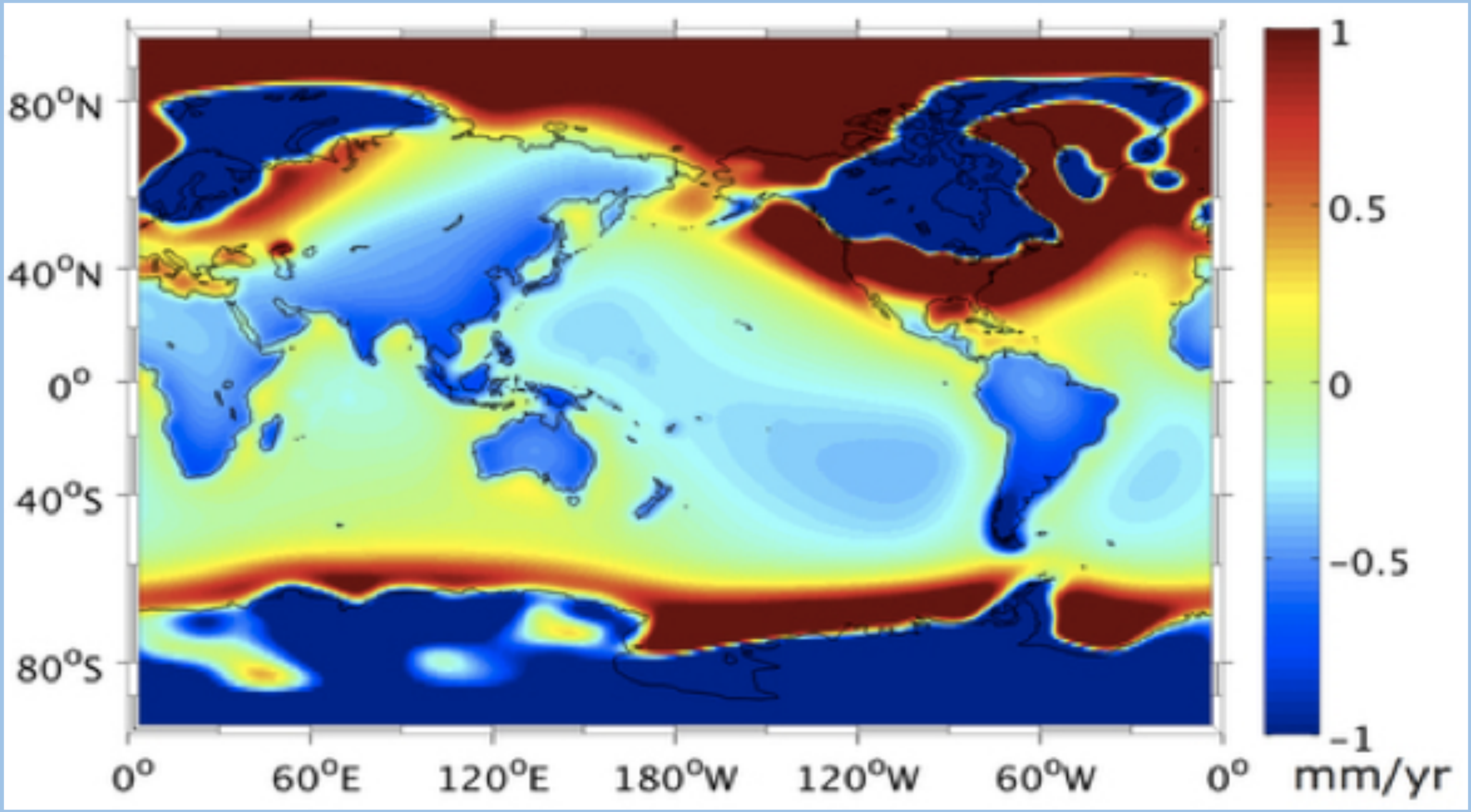
GISTEMP Team, 2015: GISS Surface Temperature Analysis (GISTEMP). NASA Goddard Institute for Space Studies. Dataset accessed 2015-10-13 at http://data.giss.nasa.gov/gistemp/.

# Questions



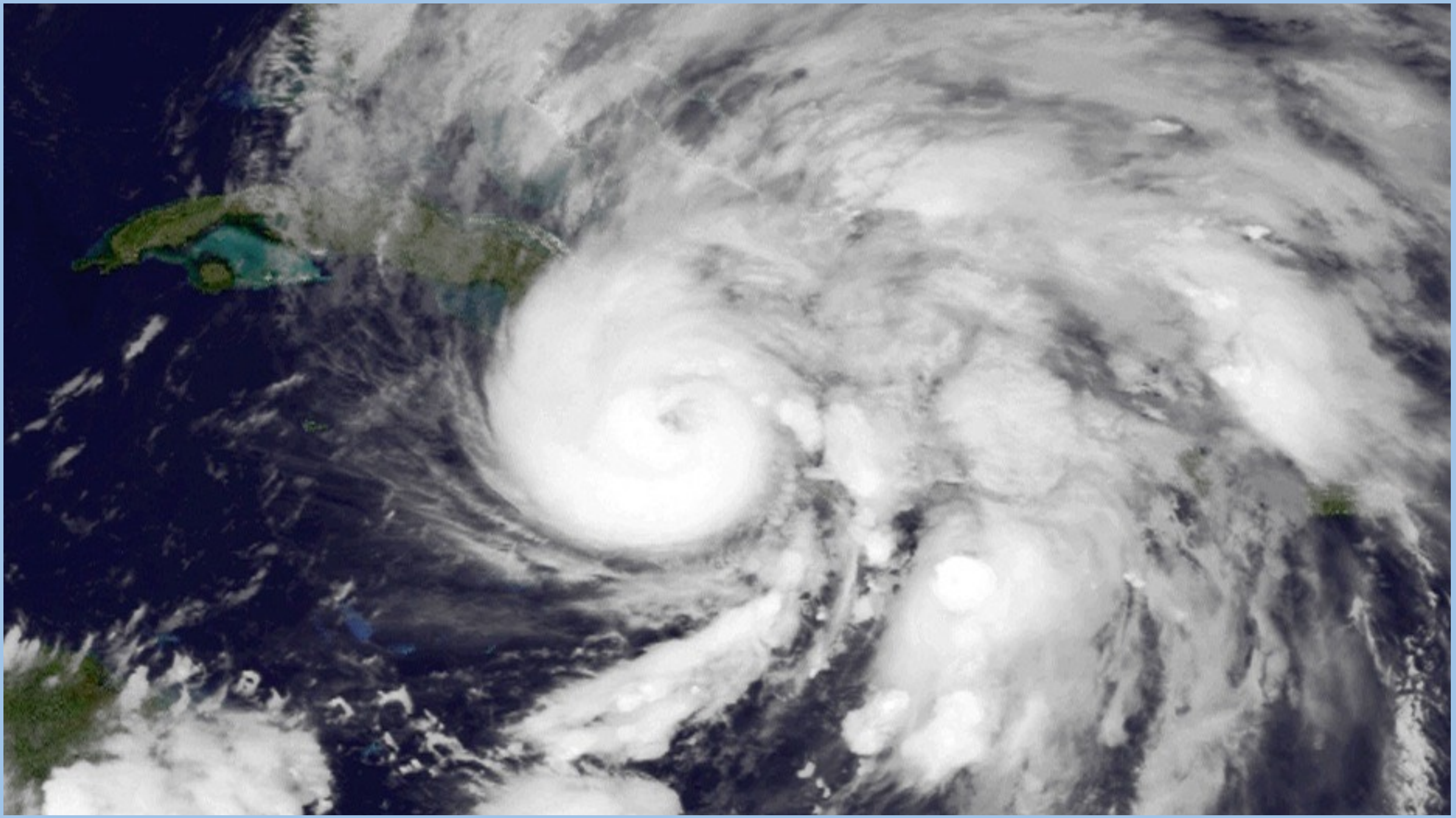

*IPCC, 5th assessment report*

# How does sea-level rise effect surge?



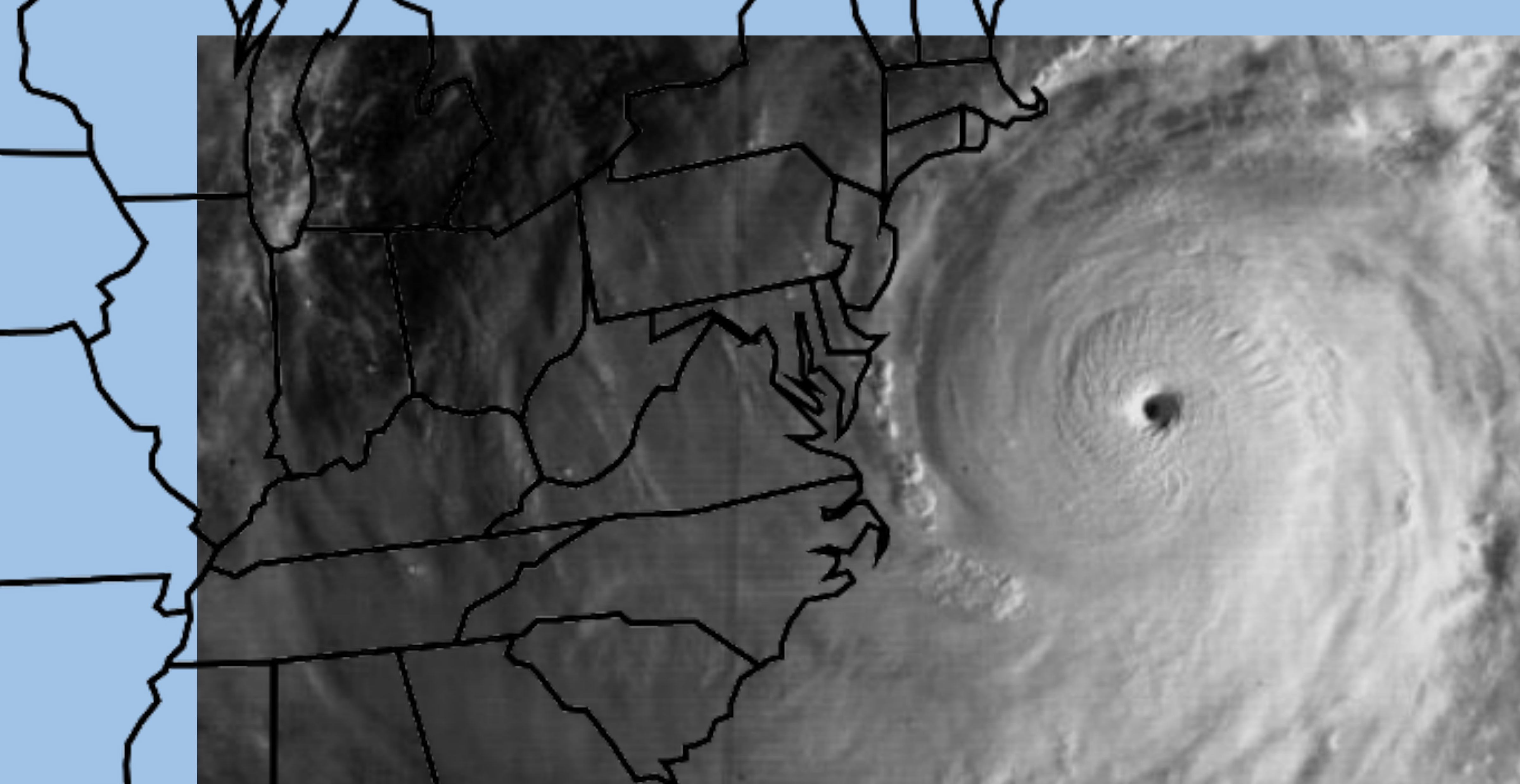

*Hay et al., 2015*

# How does sea-level rise effect surge?



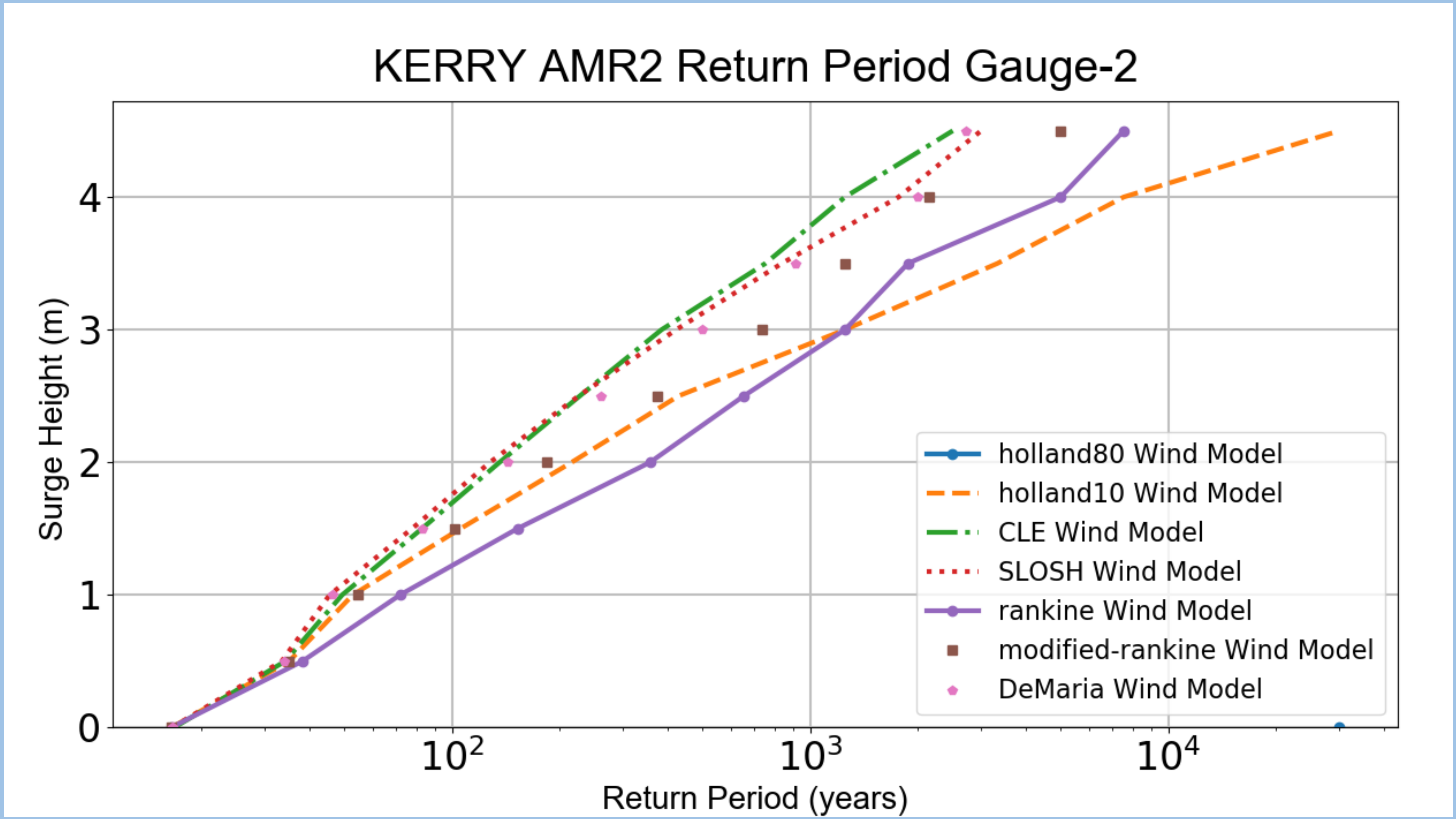
*NASA and NHC*

# Will dangerous storms become more frequent?

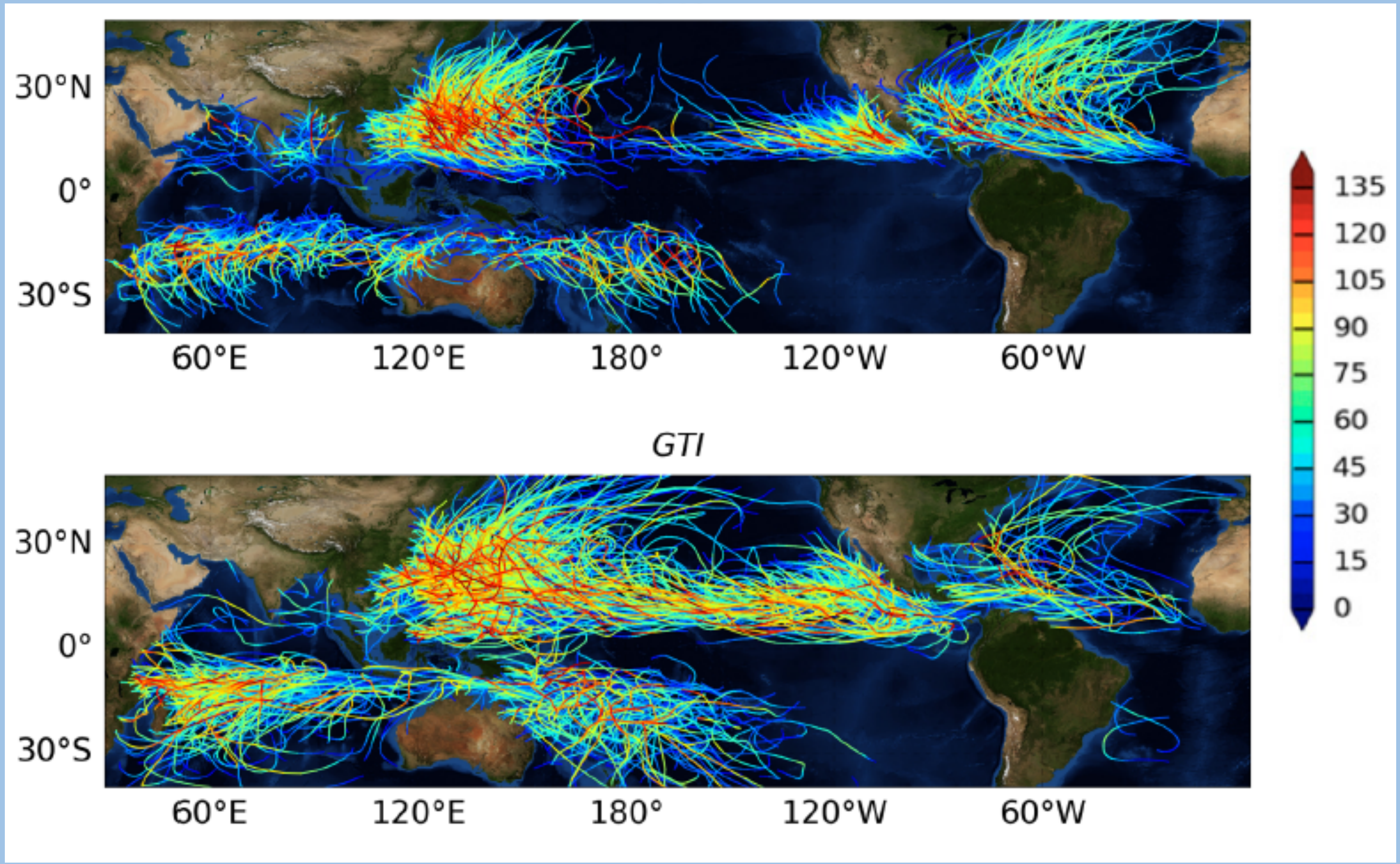


*NOAA*

# Will dangerous storms become more powerful?

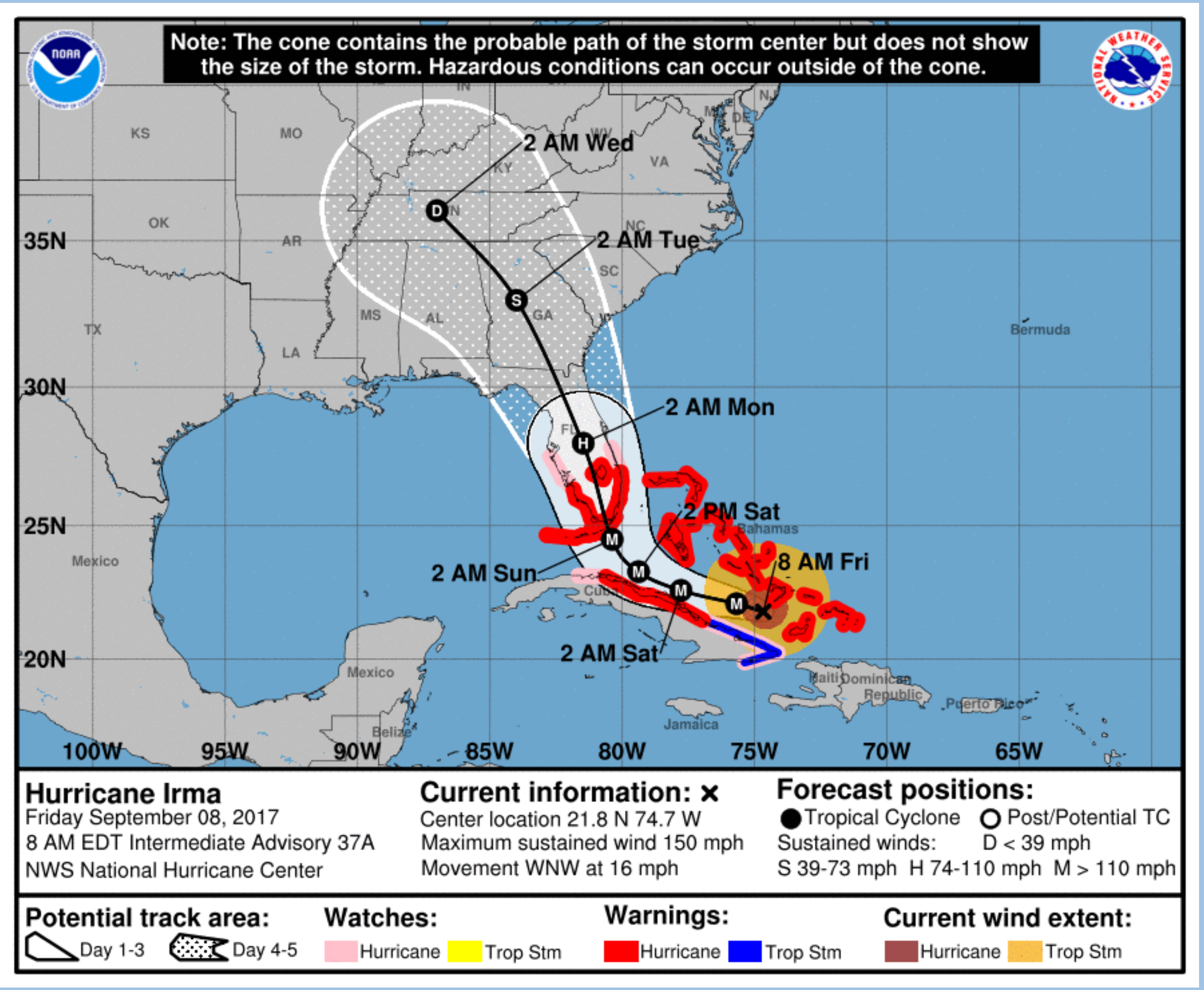



# Can we predict surge probabilities?



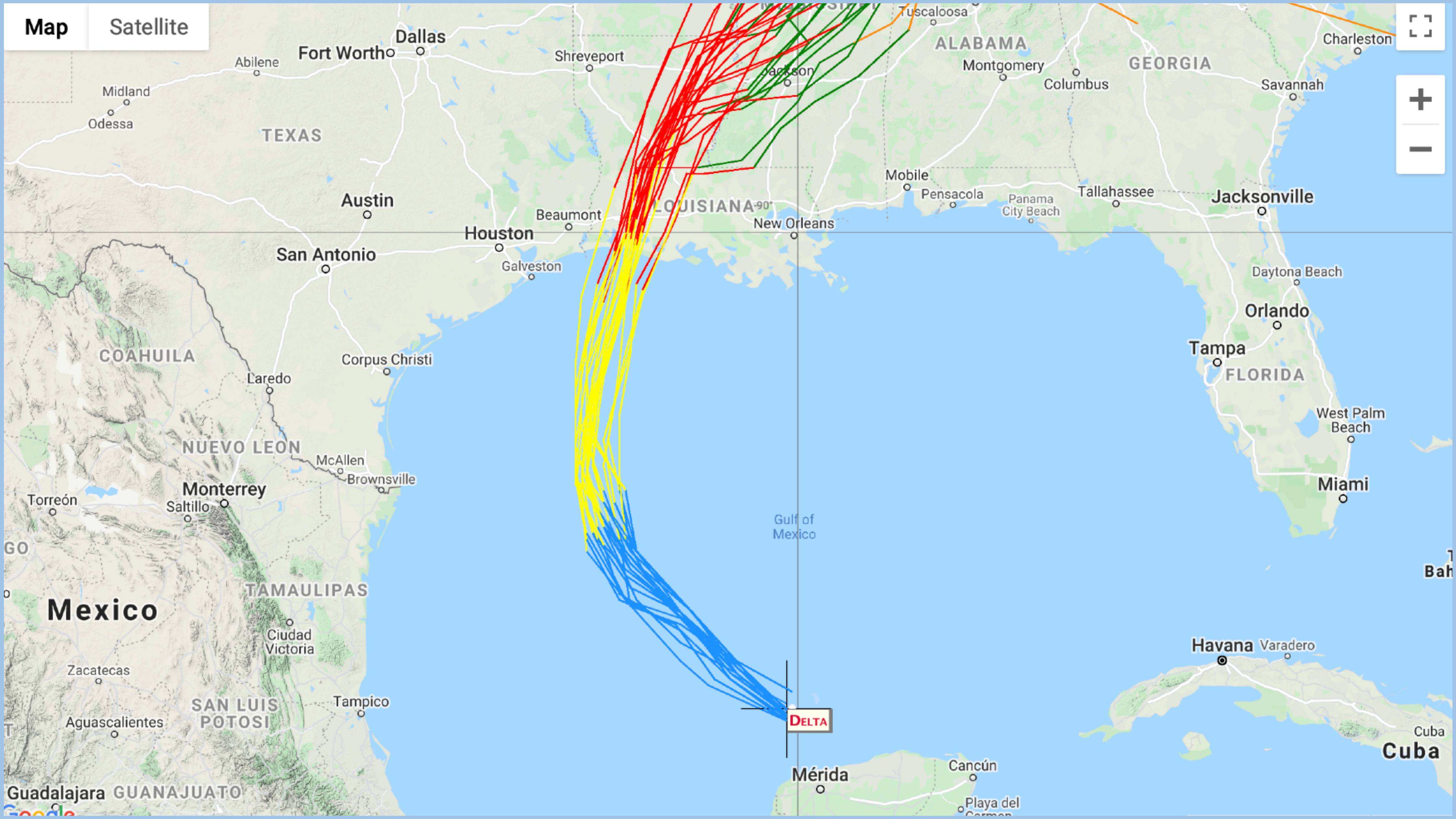

*C. Lee, M. Tippett, S. Camargo, A. Sobel (LDEO - Columbia)*

# Can we predict surge probabilities?




*NOAA - NHC*

# Can we forecast events?




# Can we quantify uncertainty?


# How do we protect ourselves?

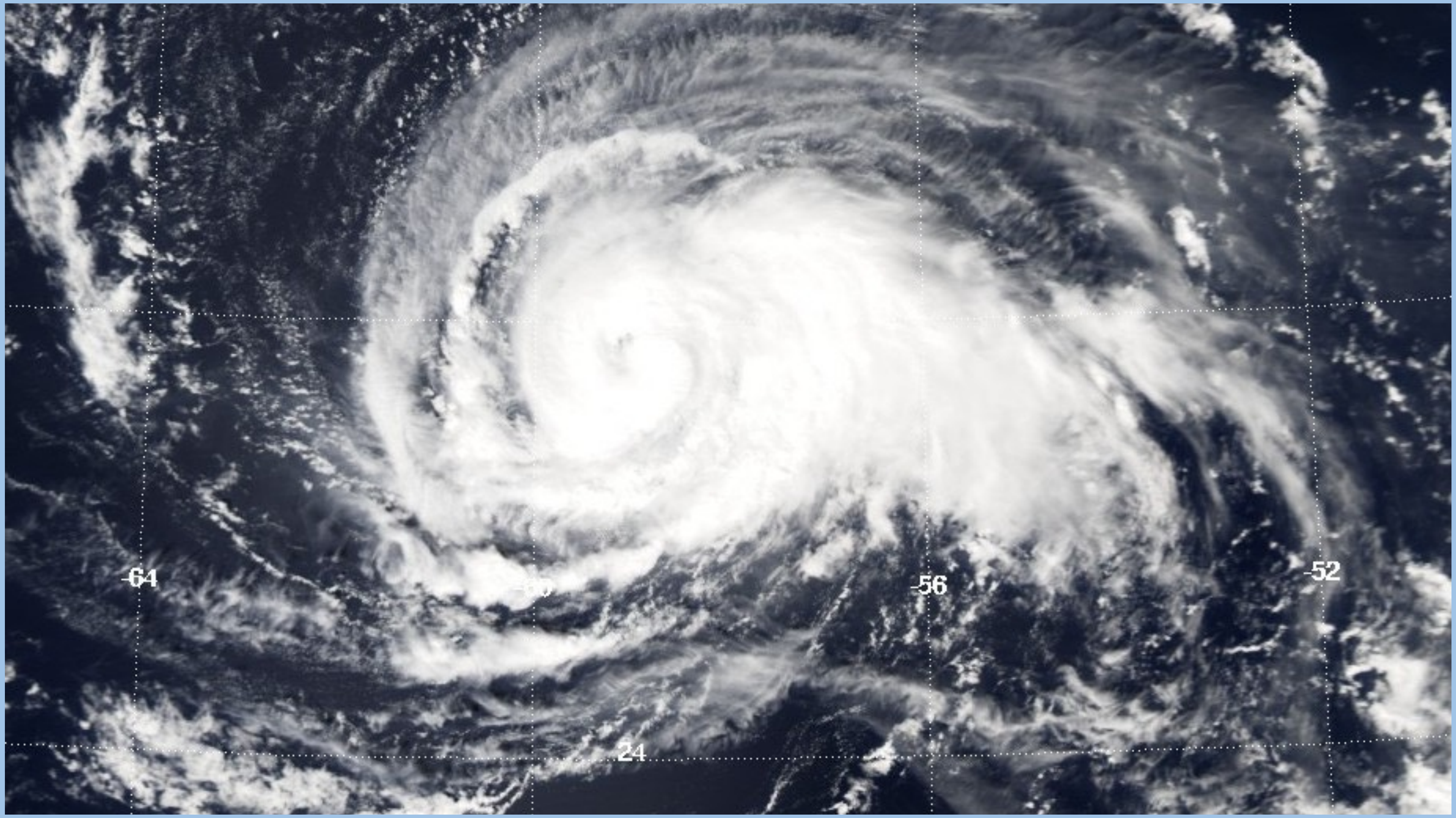
# Can we protect ourselves?


FIGURE 17 - Potential Category 2 hurricane surge at South Ferry (Battery) Subway Station

*US Army Corps 1995*

# Can we protect ourselves?

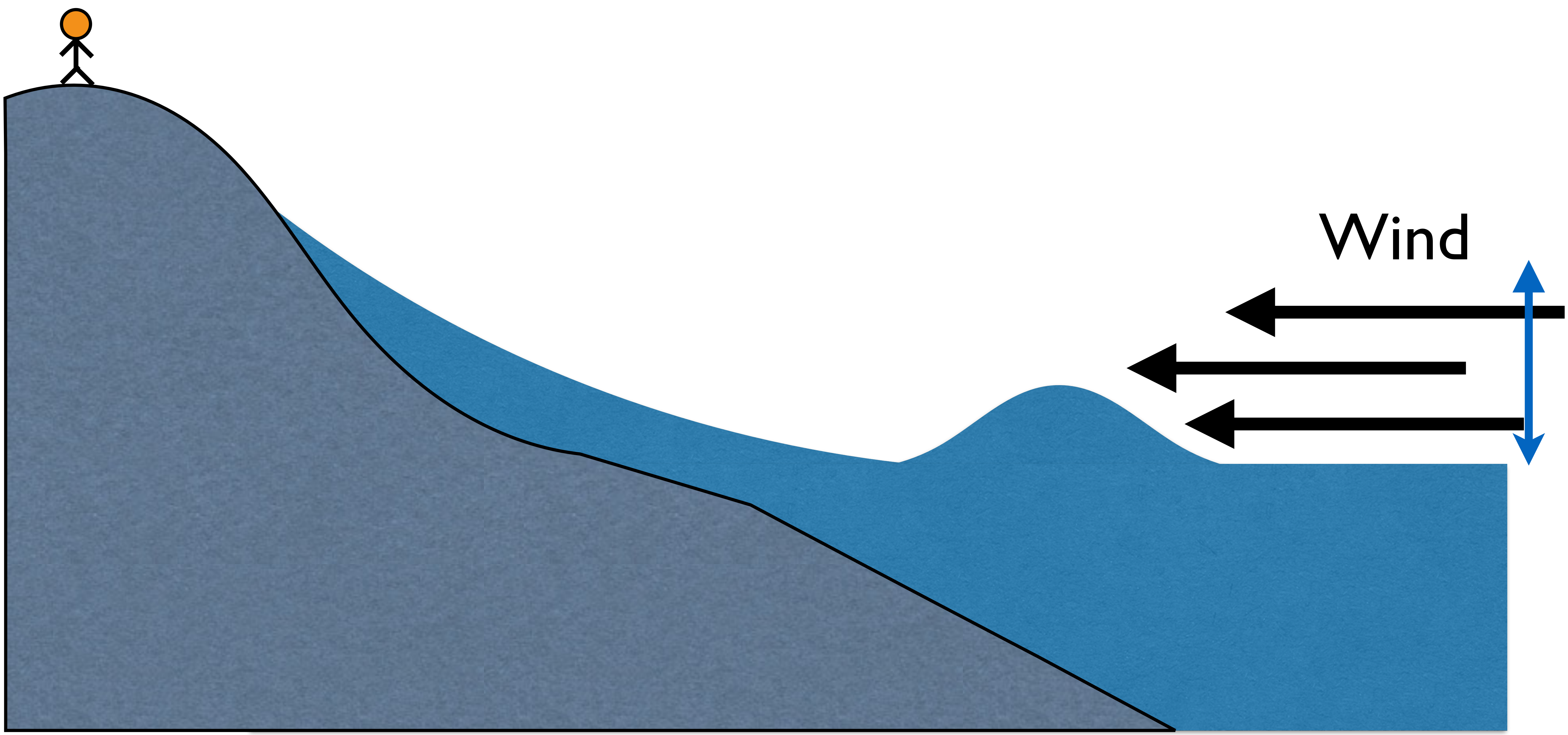

# Overland Precipitation flooding



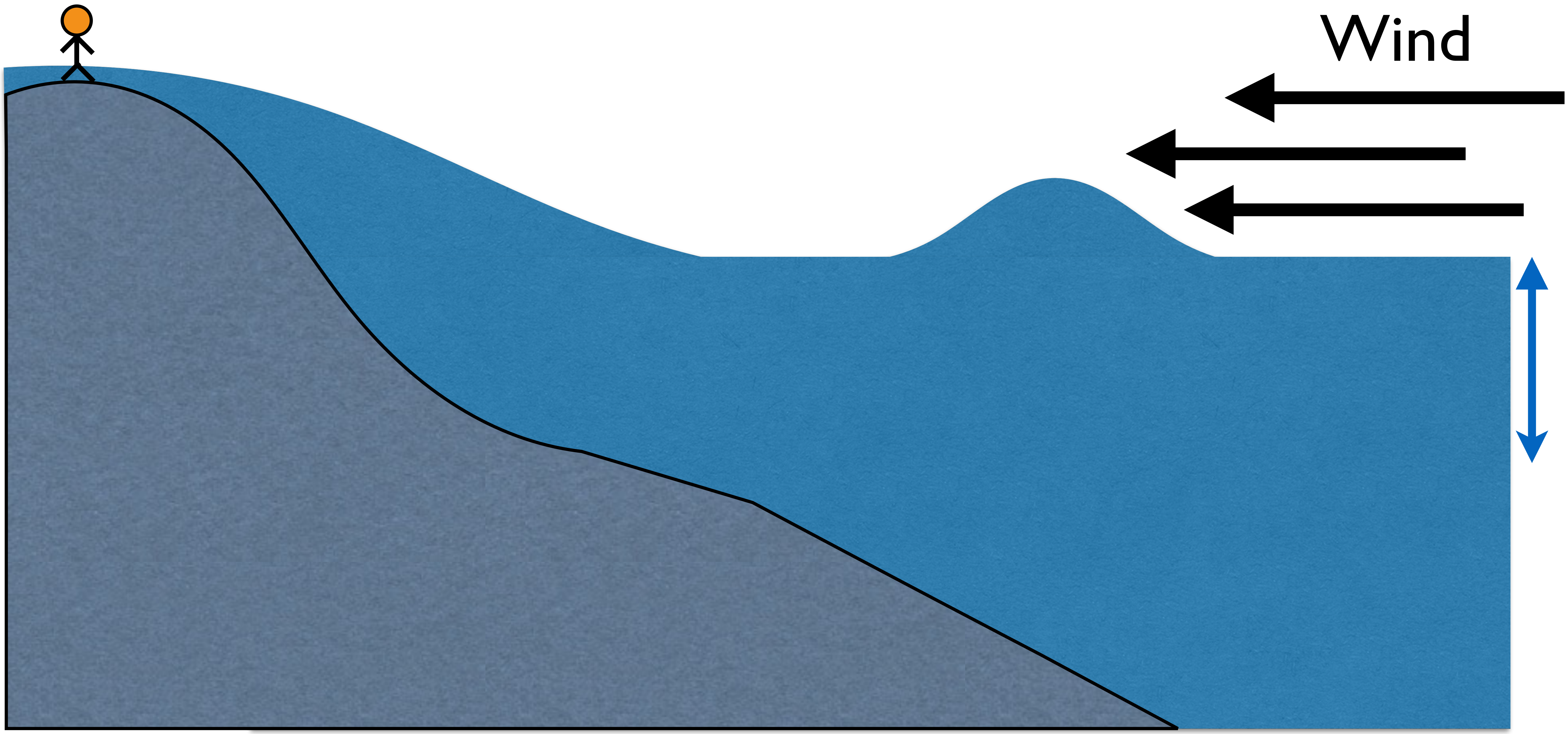

*NASA Modis Satellite*

# Storm Surge Modeling

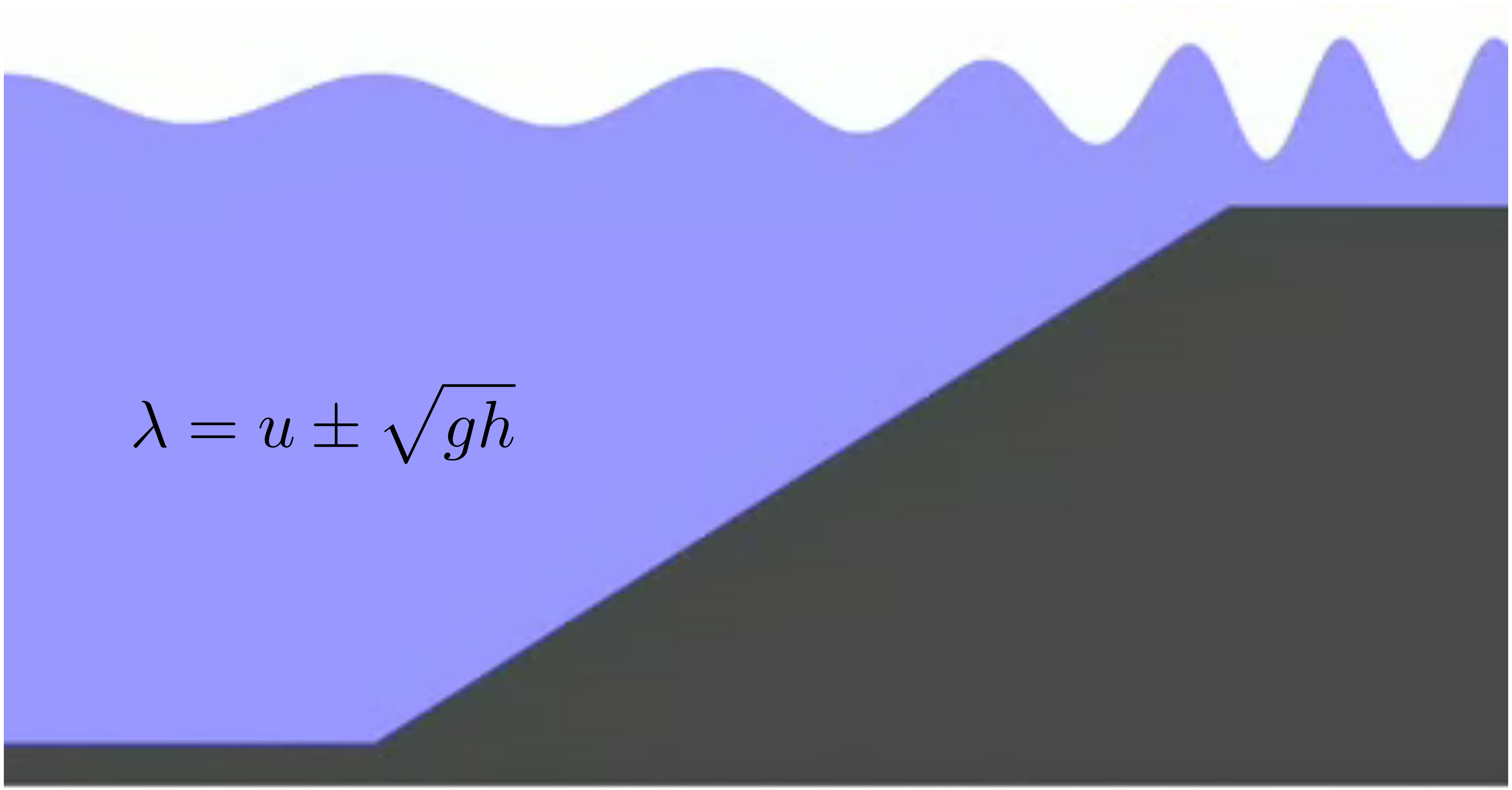
# Storm Surge



# Storm Surge + Sea-Level



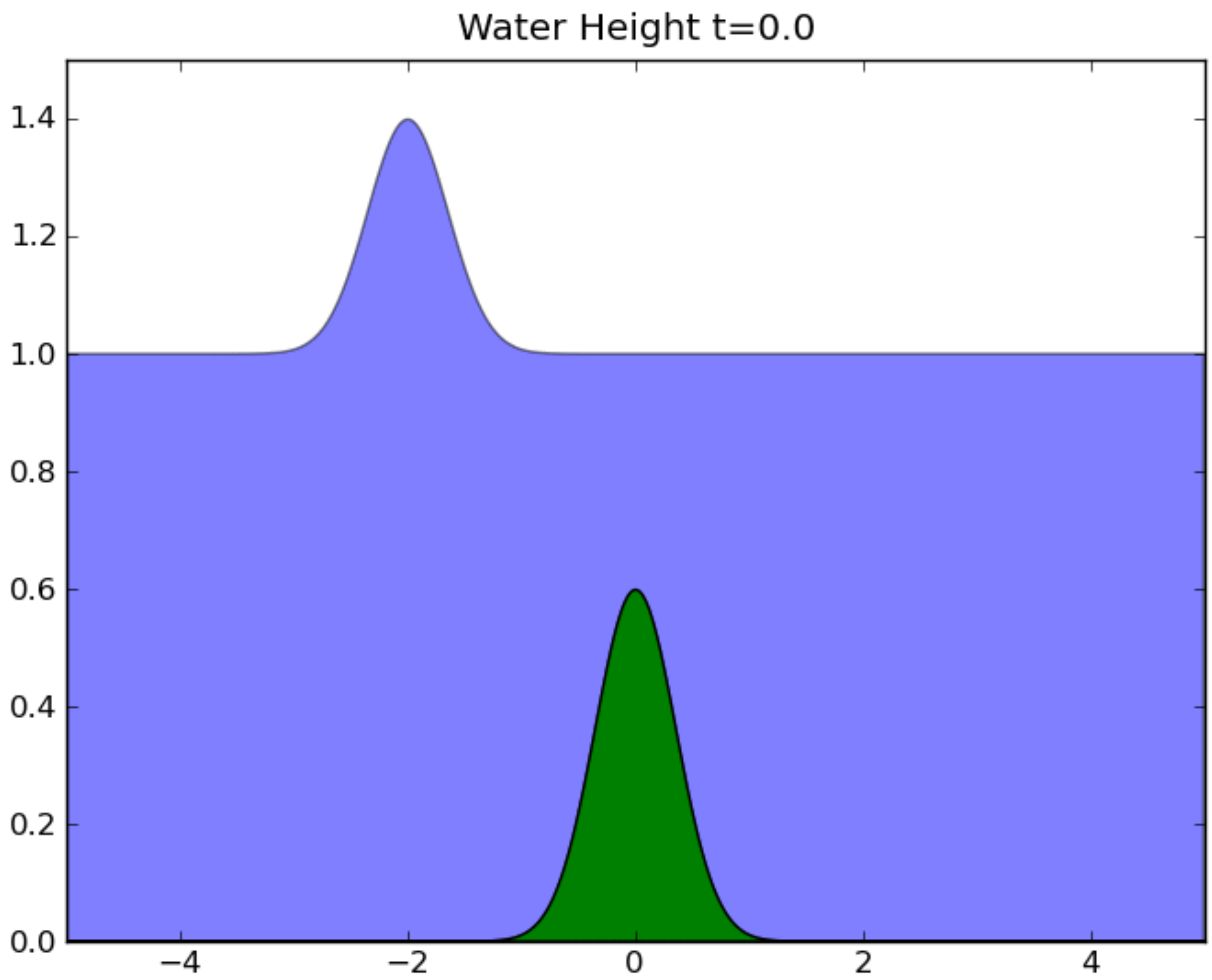
# Shallow Flow



$$\lambda = u \pm \sqrt{gh}$$

*Régis Lachaume*

# Shallow Water - Topography



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# Storm Surge Model

$$h_t + (hu)_x + (hv)_y = 0$$

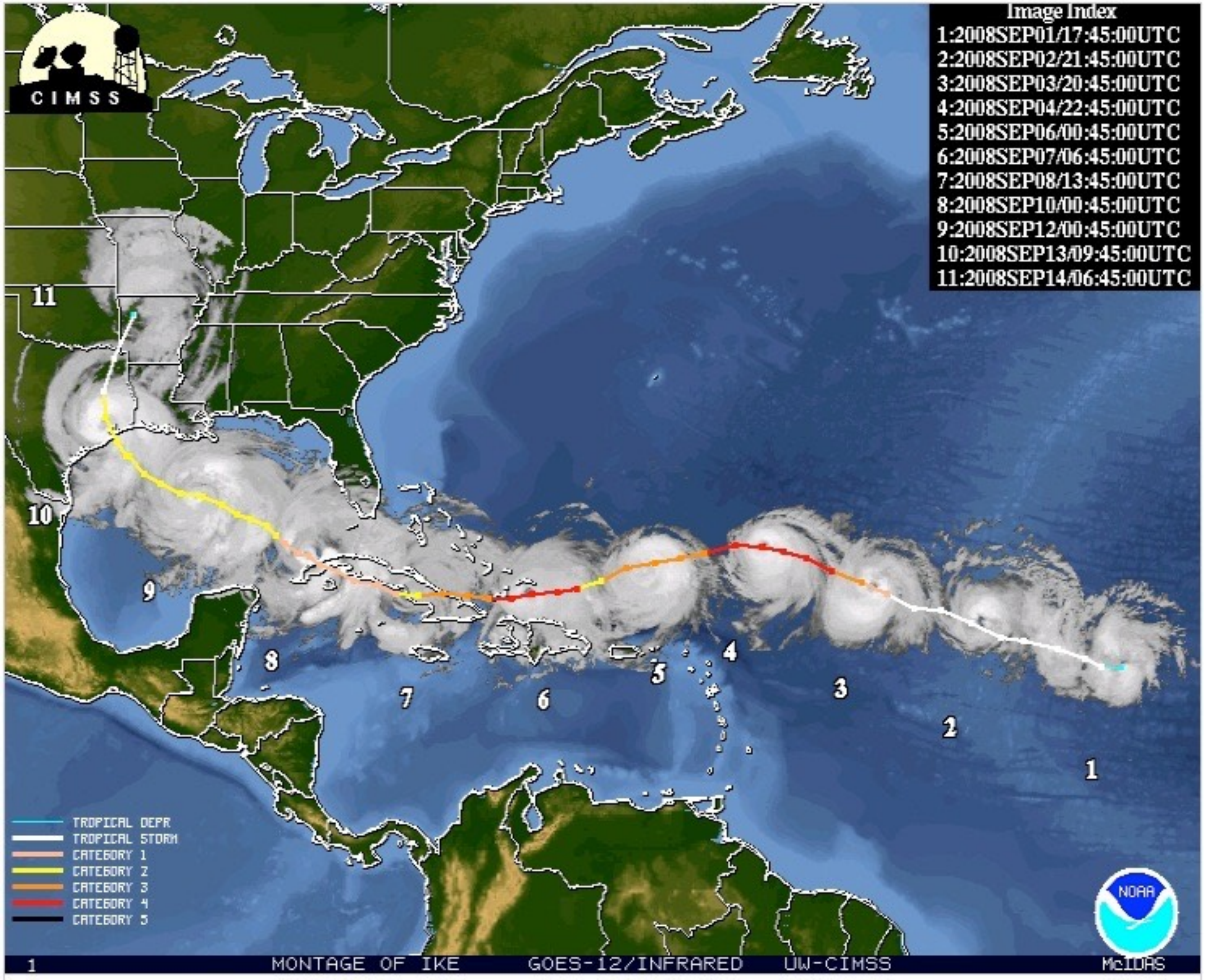
$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y =$$

$$-ghb_x \boxed{+ fhv - \frac{h}{\rho}(P_A)_x + \frac{1}{\rho}(\tau_{sx} - \tau_{bx})}$$

$$(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y =$$

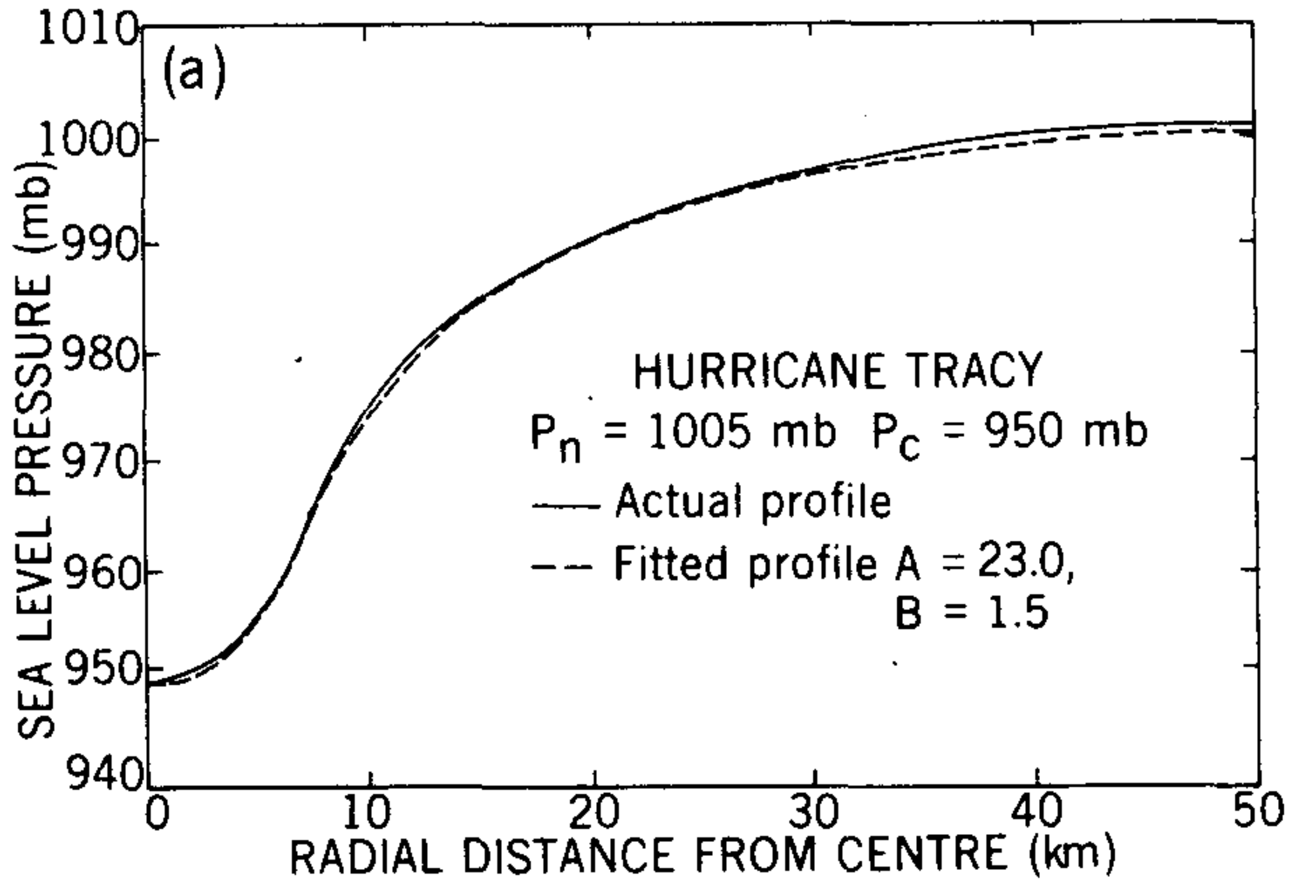
$$-ghb_y \boxed{- fhu - \frac{h}{\rho}(P_A)_y + \frac{1}{\rho}(\tau_{sy} - \tau_{by})}$$

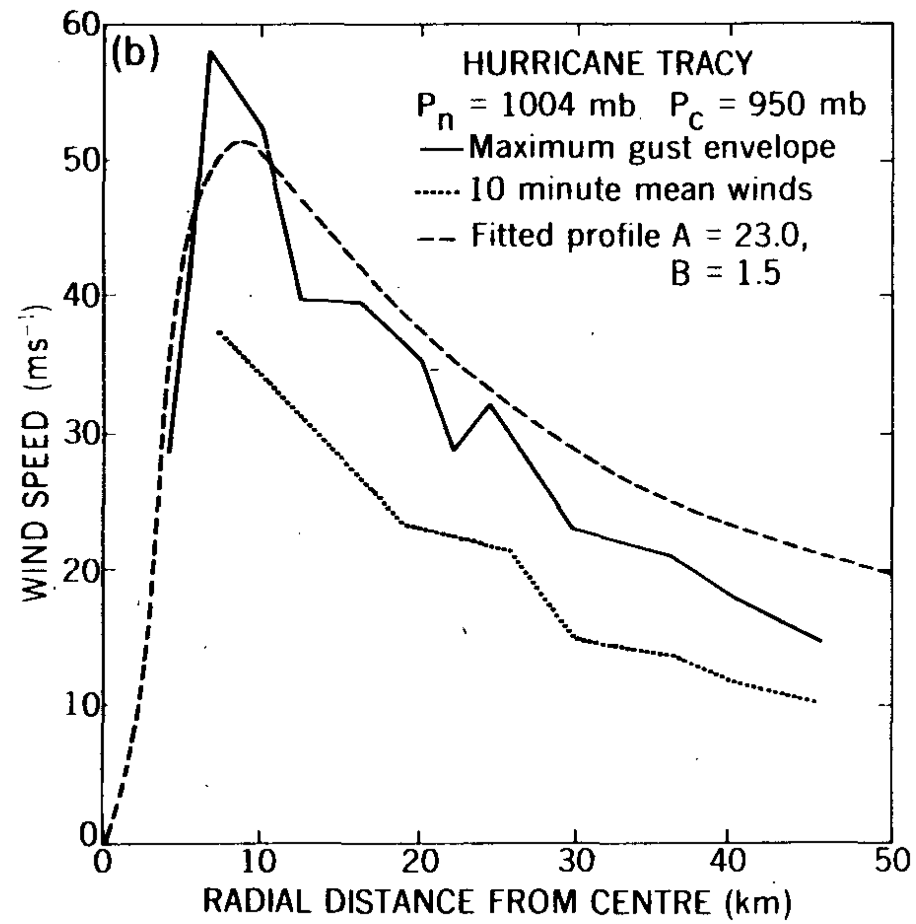
# Storm Representation




*CIMMS: http://cimss.ssec.wisc.edu/tropic2*

# Holland Hurricane Model



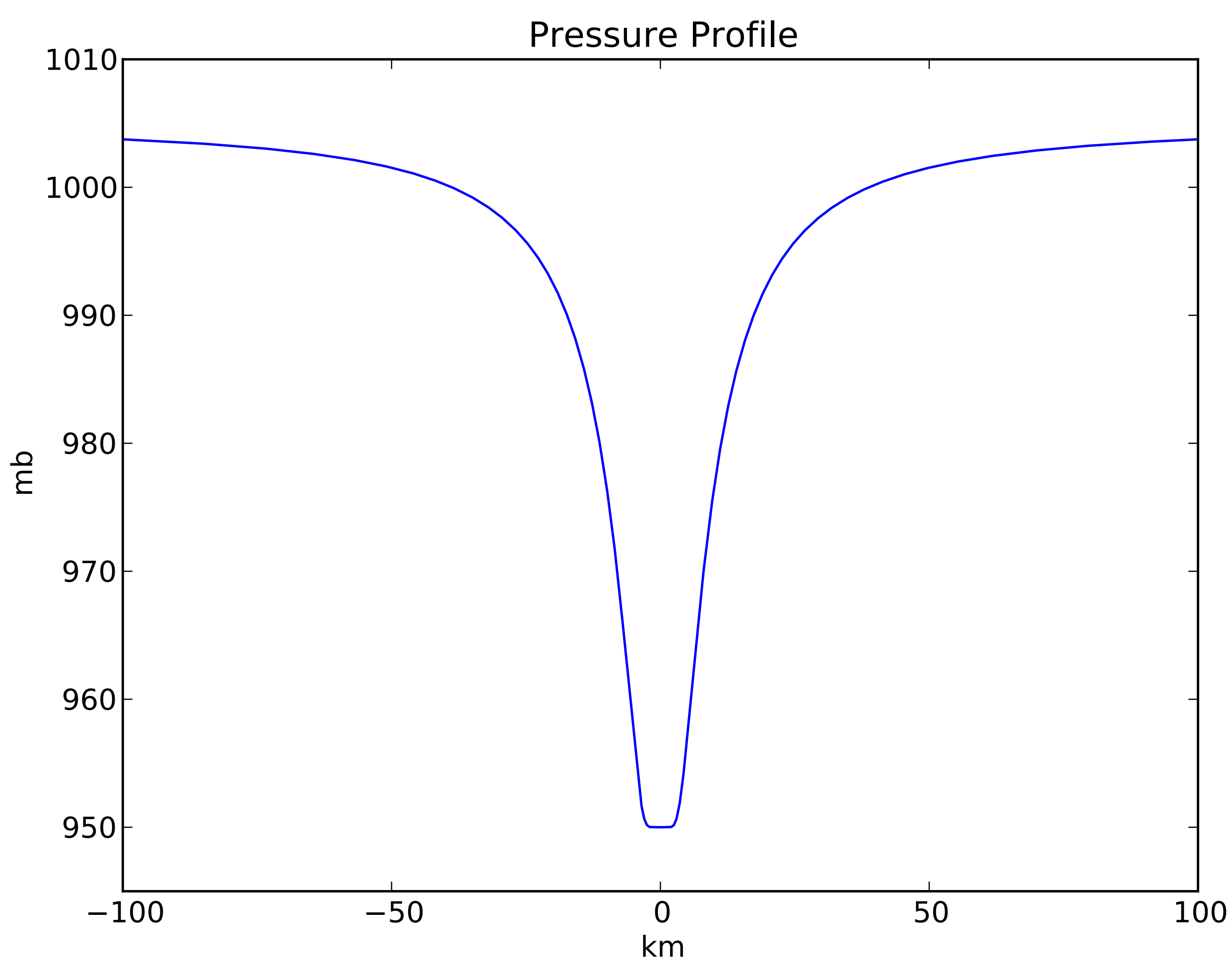



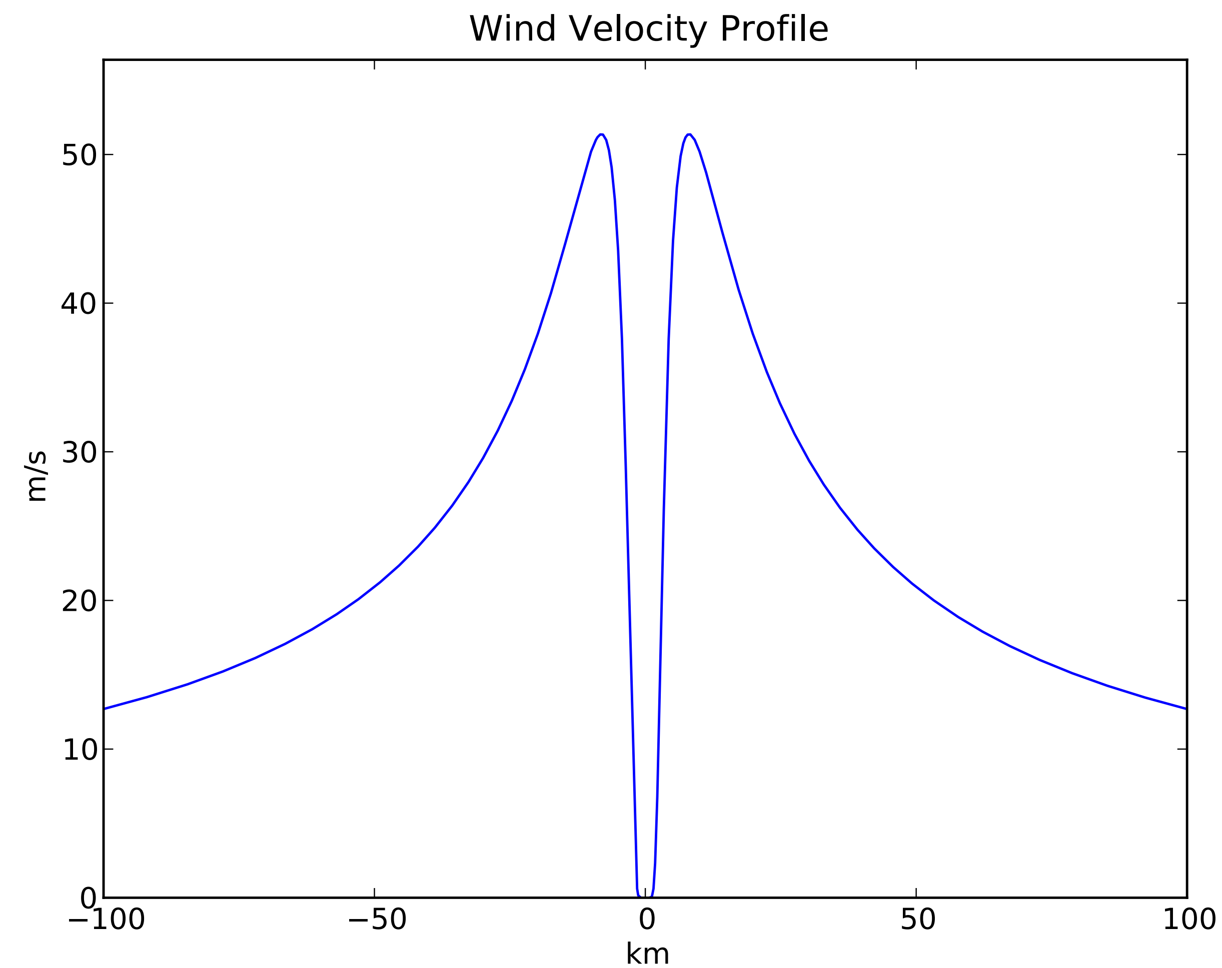

*Holland, G. J. An Analytic Model of the Wind and Pressure Profiles in Hurricanes. Monthly Weather Review 108, 1212-1218 (1980)*

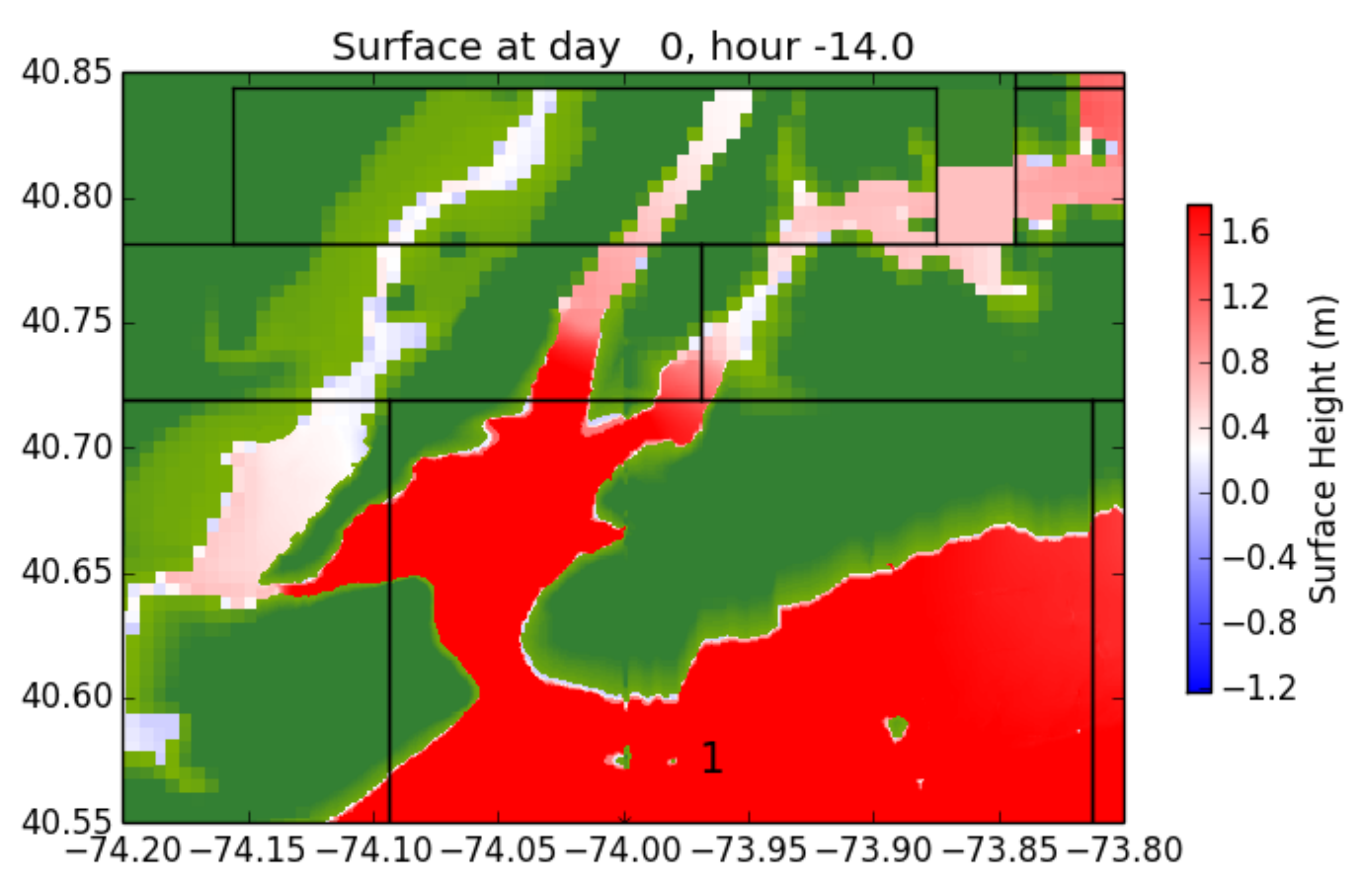
# Holland Hurricane Model

**Wind** $$|W| = \sqrt{\frac{AB(P_n - P_c)e^{-A/r^B}}{\rho_{\text{air}} r^B} + \frac{r^2 f^2}{4}} - \frac{rf}{2}$$

**Pressure** $$P_A = P_c + (P_n - P_c)e^{-A/r^B}$$

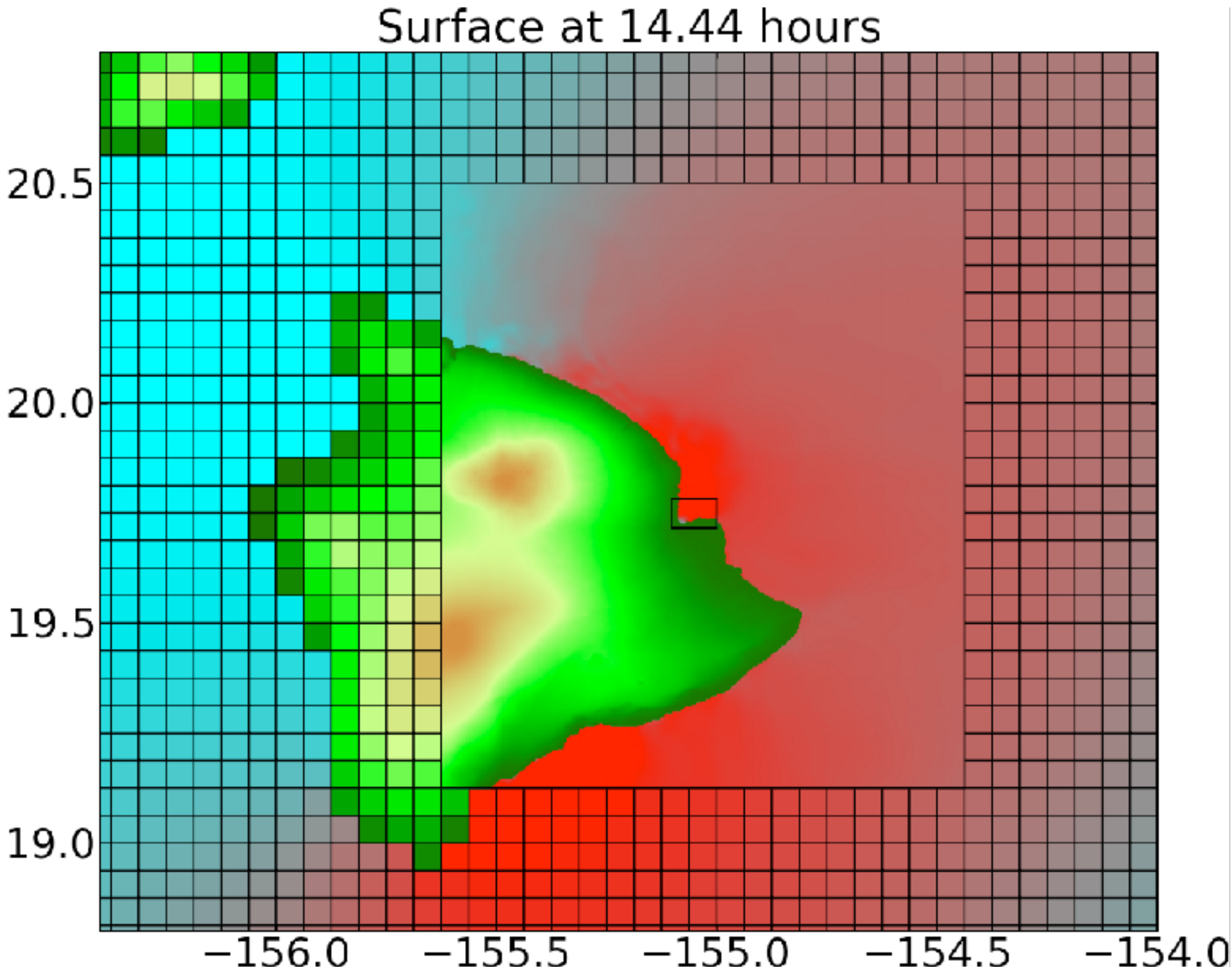


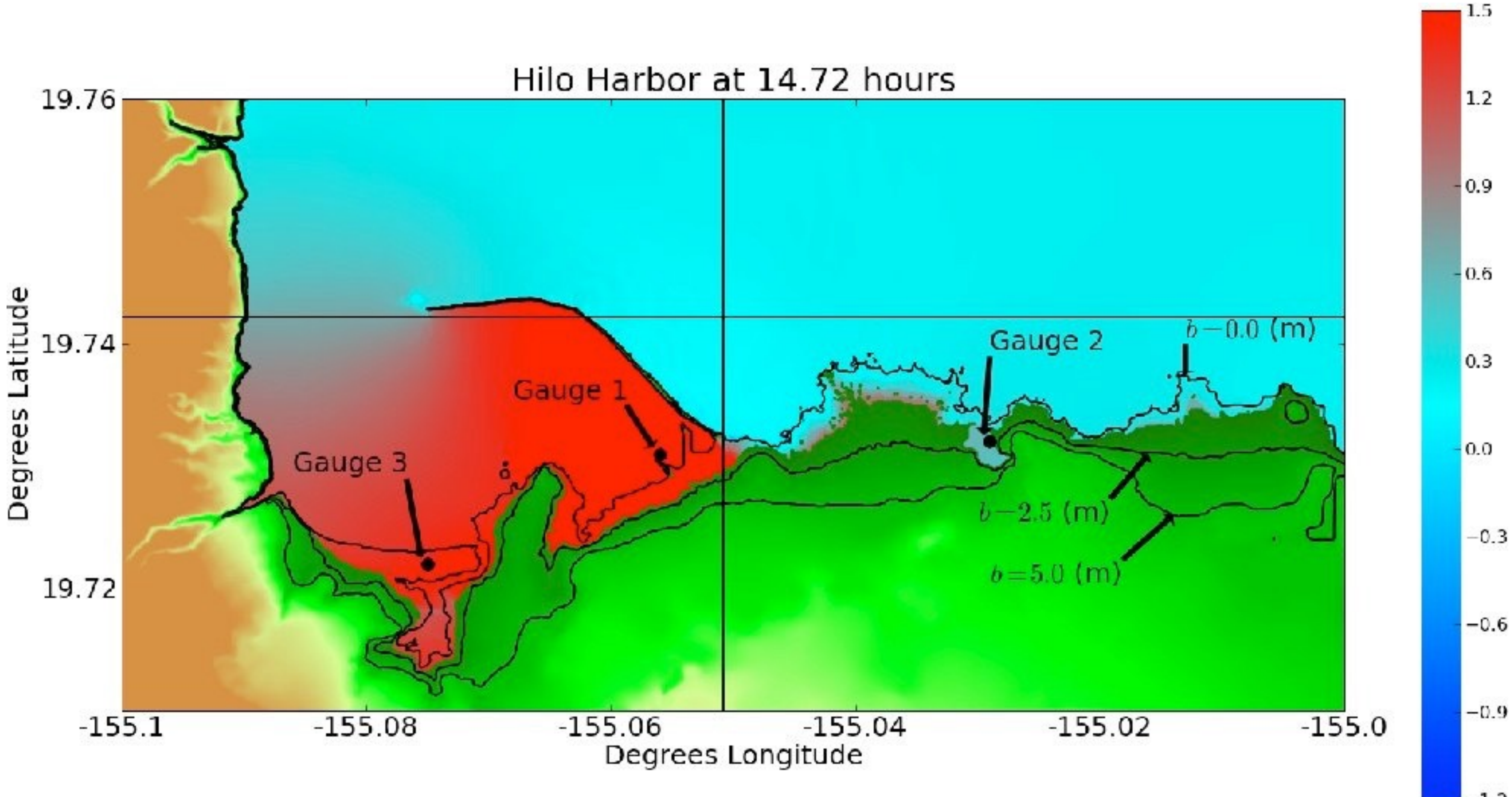



# Storm Surge Computing

# Adaptive Mesh Refinement



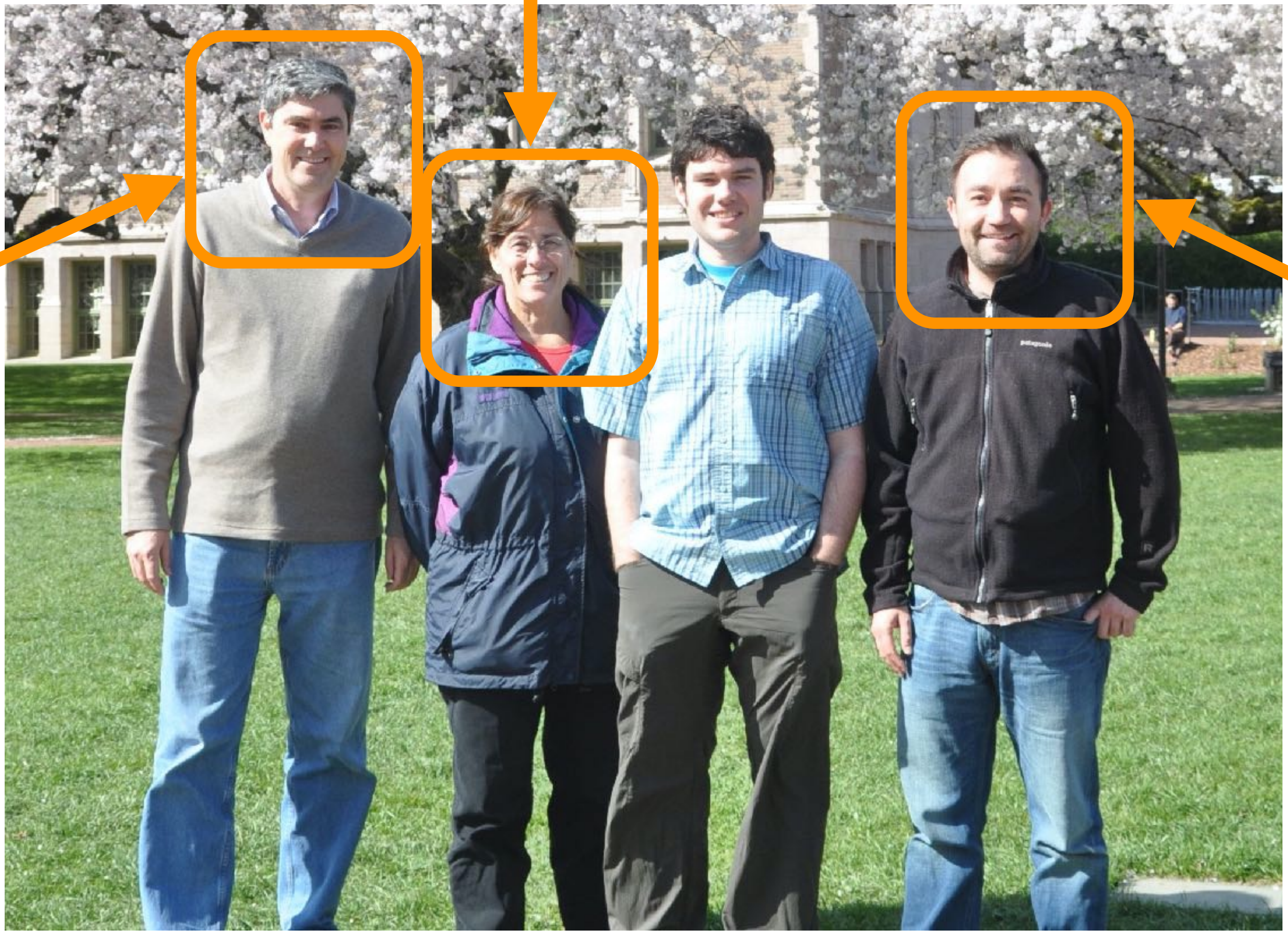
# Adaptive Mesh Refinement



# GeoClaw

Marsha Berger (NYU)

Randy LeVeque (U. Washington)

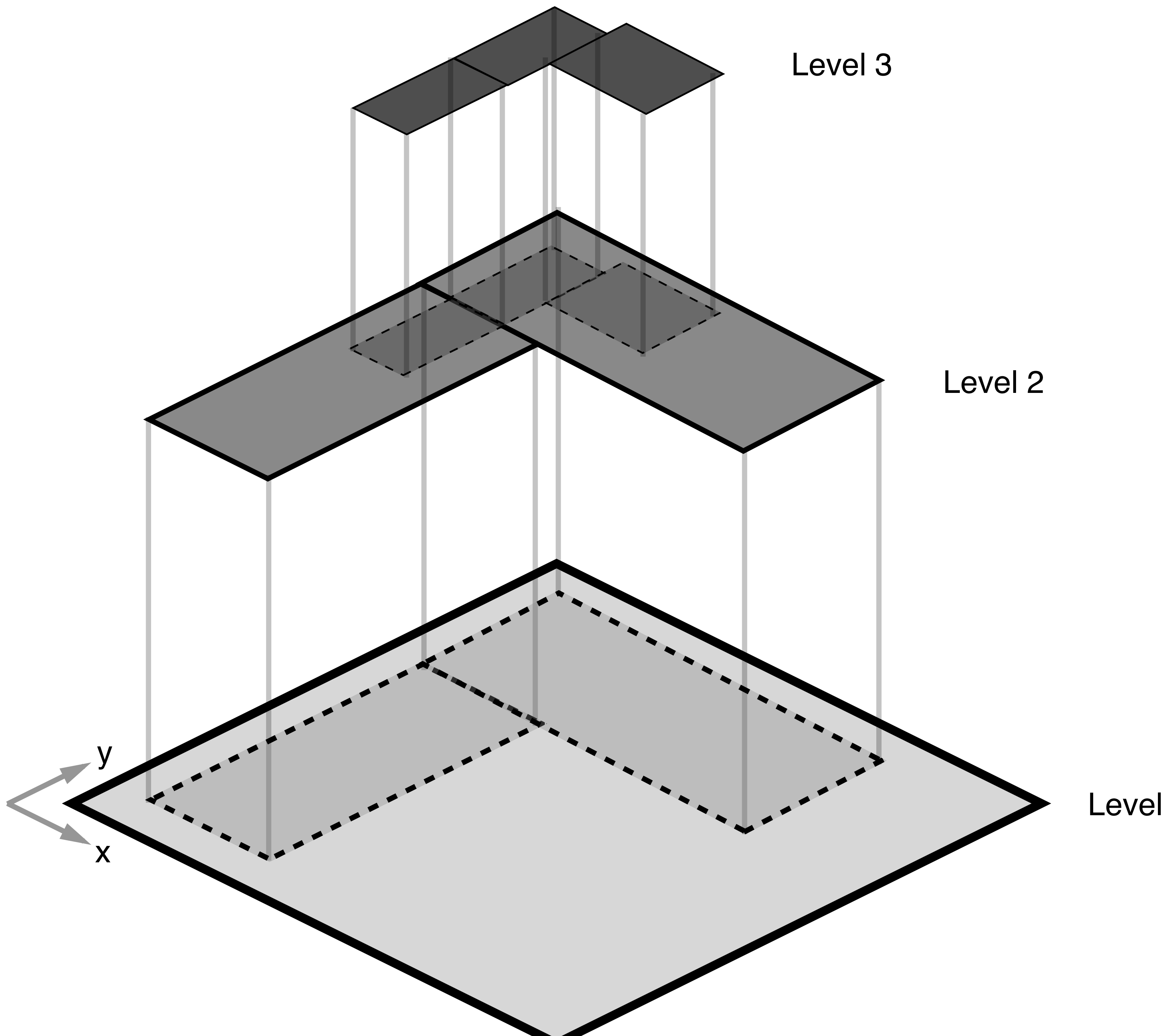


Dave George (USGS)

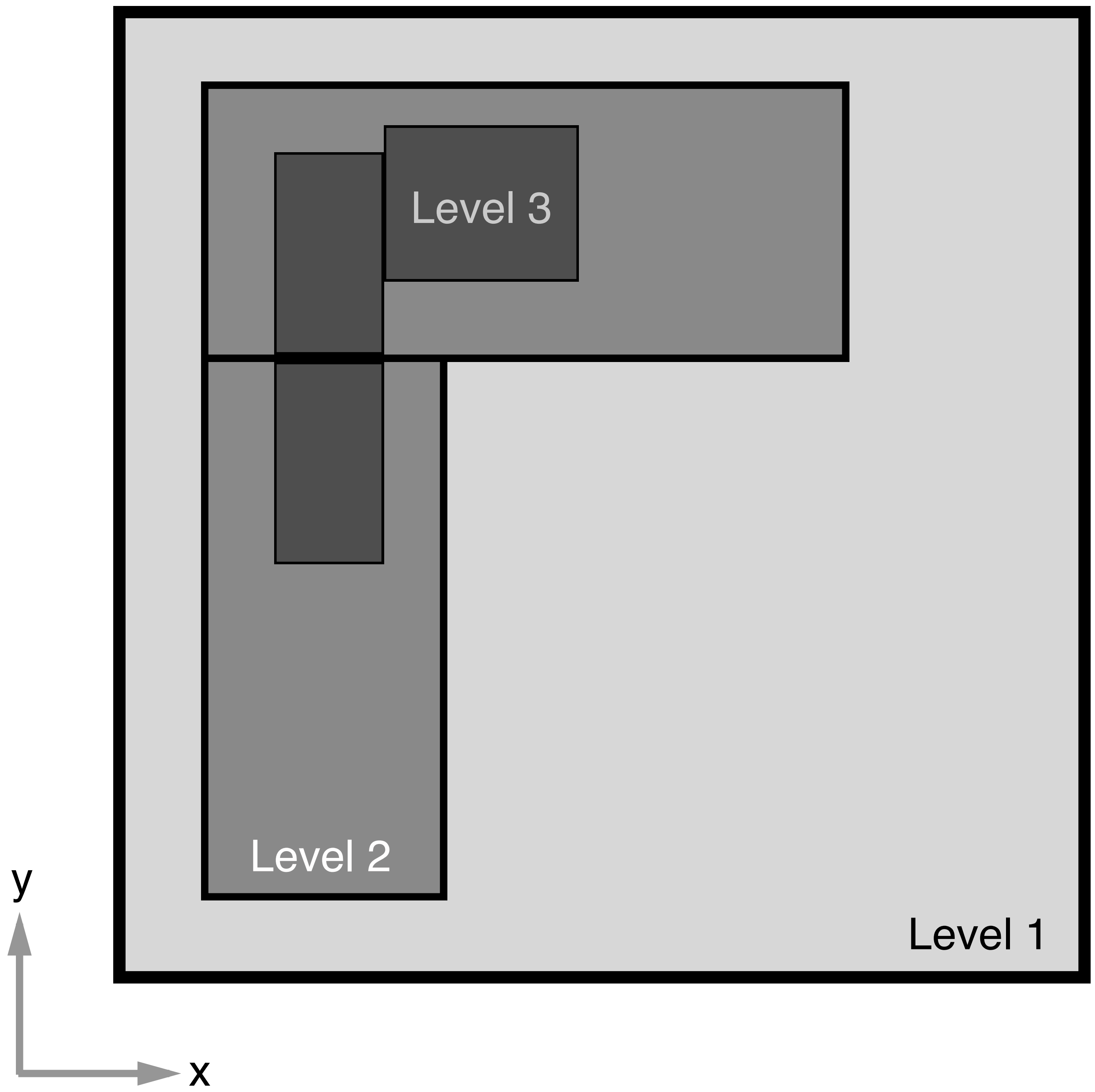
www.clawpack.org

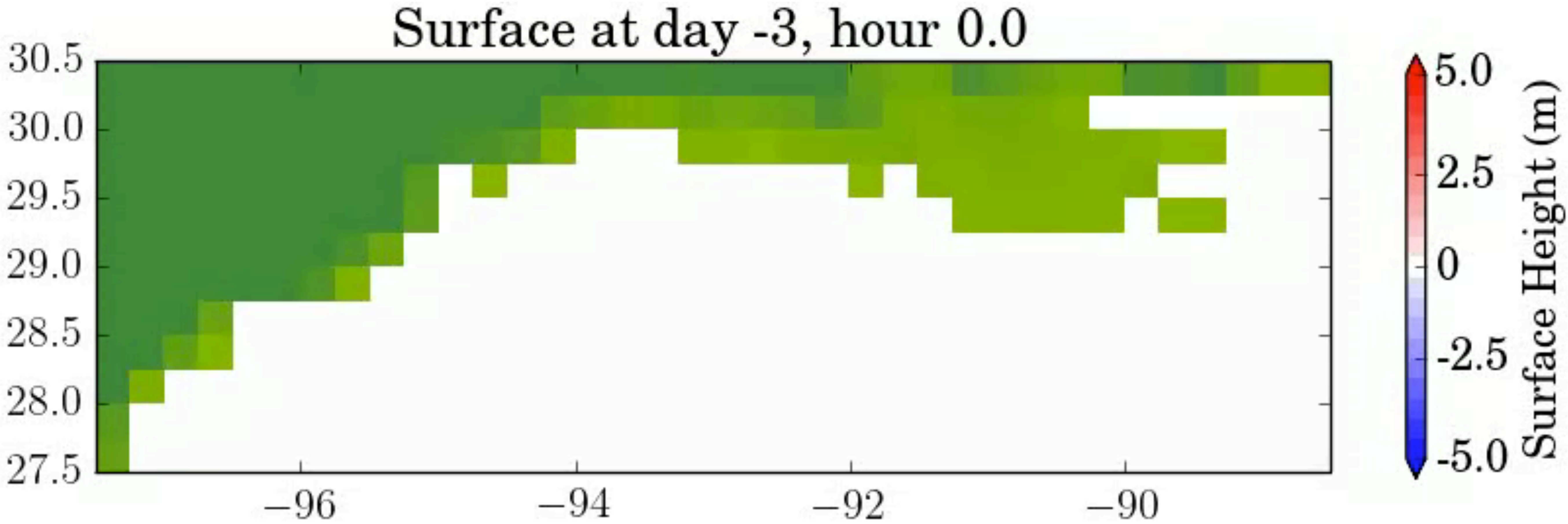
*Berger, M. J., George, D. L., LeVeque, R. J. & Mandli, K. T. The GeoClaw software for depth-averaged flows with adaptive refinement. Advances in Water Resources 34, 1195–1206 (2011).*

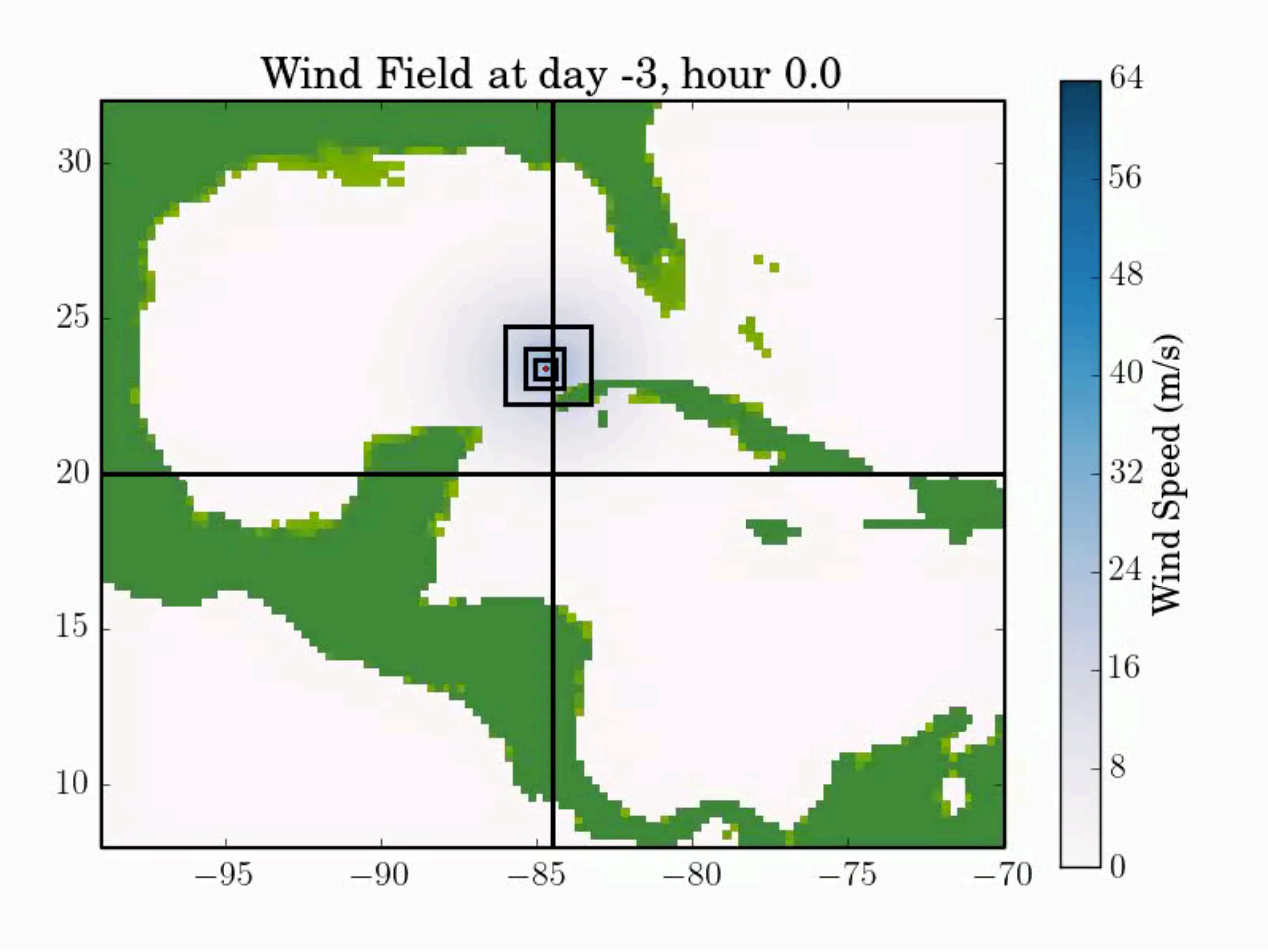
# Adaptive Discretization



# Adaptive Discretization



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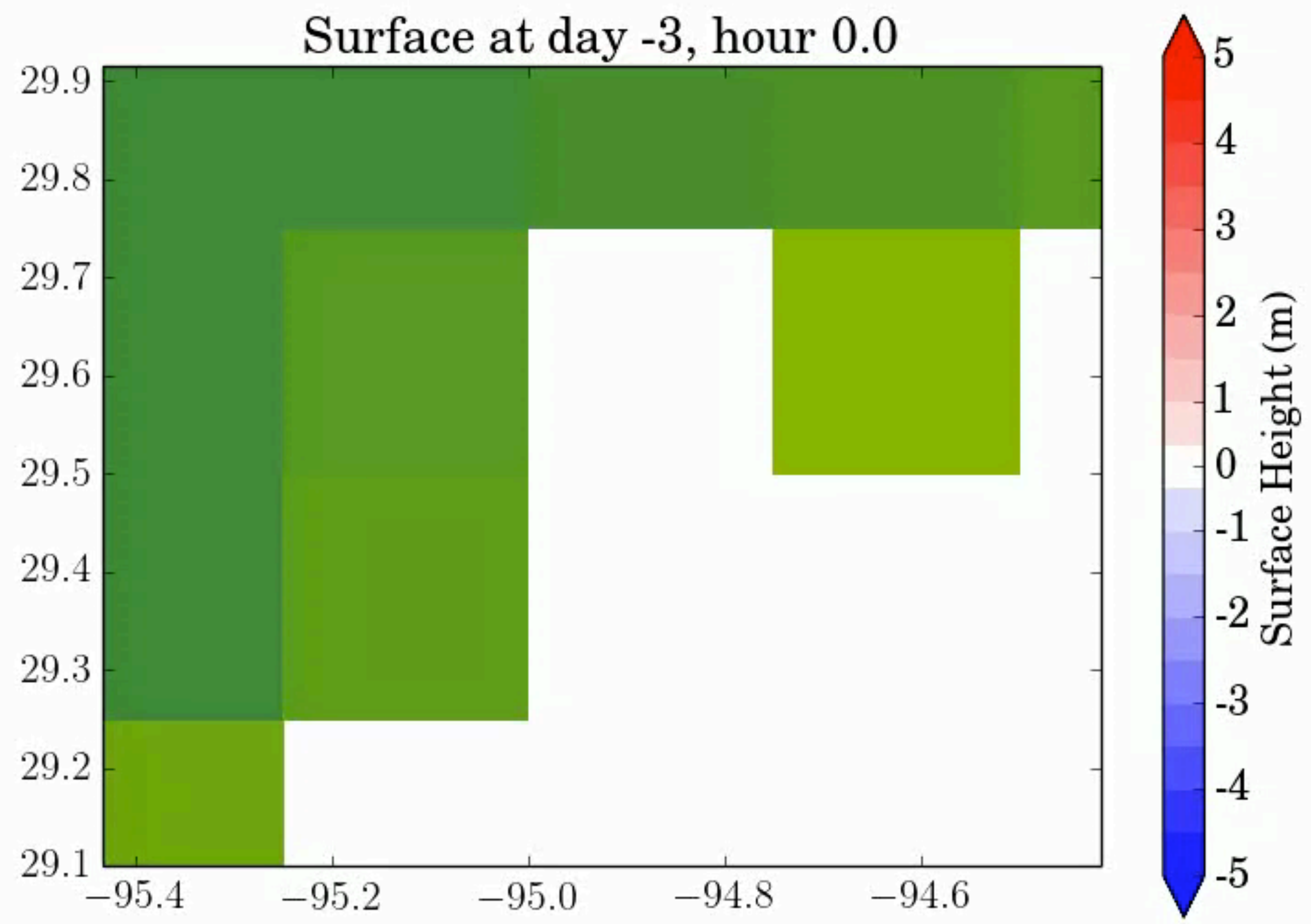
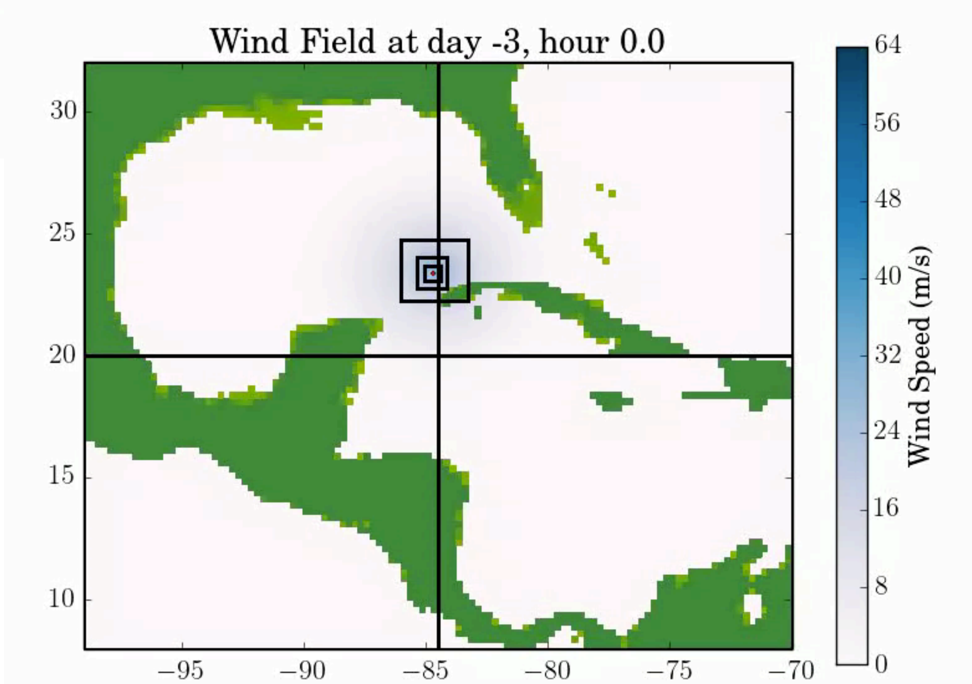
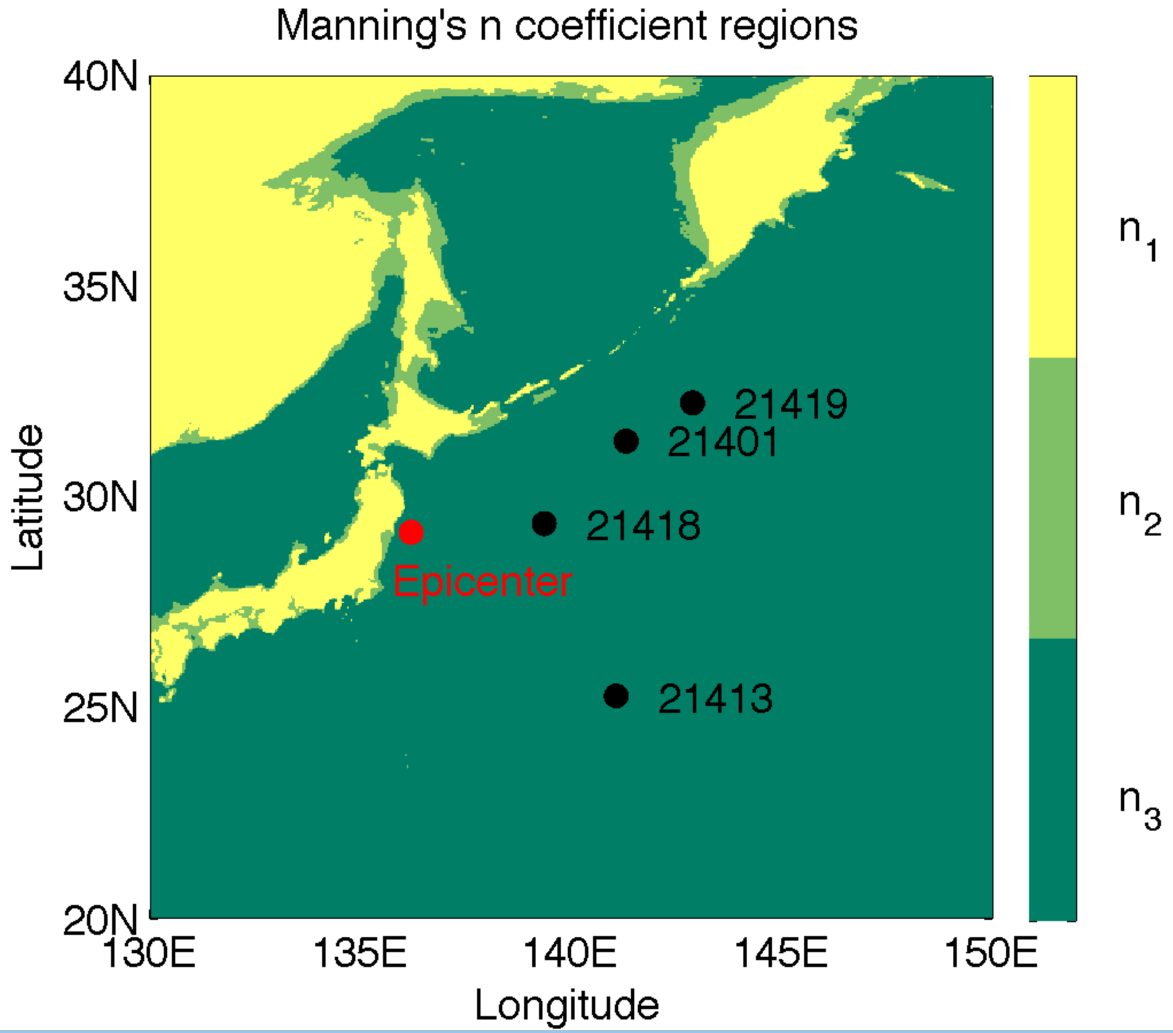
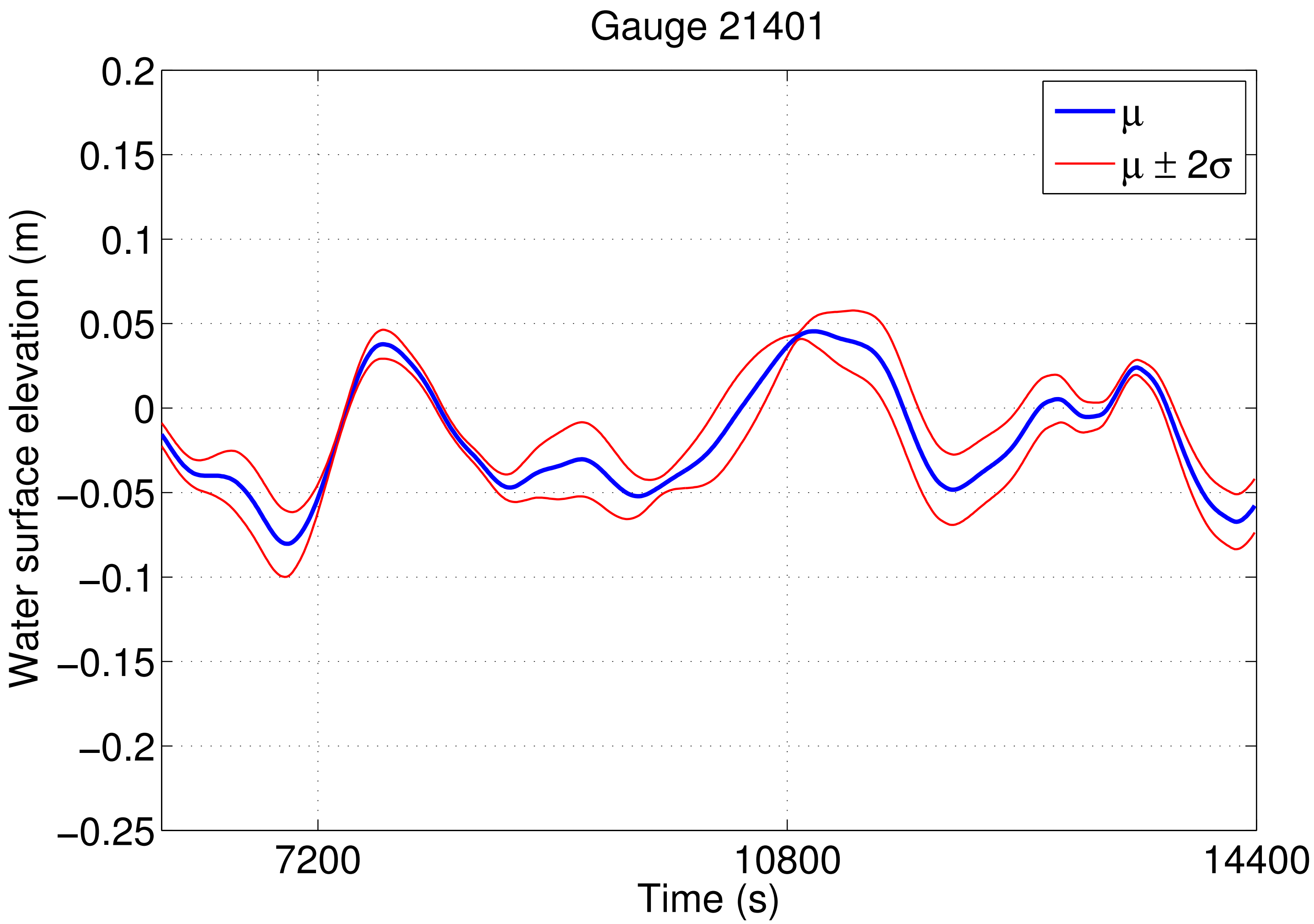


Surface at day -3, hour 0.0
29.9
29.8
29.7
29.6
29.5
29.4
29.3
29.2
29.1
−95.4
−95.2
−95.0
−94.8
−94.6
5
4
3
2
1
0
-1
-2
-3
-4
-5
Surface Height (m)

Wind Field at day -3, hour 0.0
30
25
20
15
10
−95
−90
−85
−80
−75
−70
64
56
48
40
32
24
16
8
0
Wind Speed (m/s)




Sraj, I., Mandli, K.T., Knio, O. M., Dawson, C. N., & Hoteit, I. Uncertainty Quantification and Inference of Manning's Friction Coefficient using DART Buoy Data during the Tohoku Tsunami. Ocean Modelling (2014).

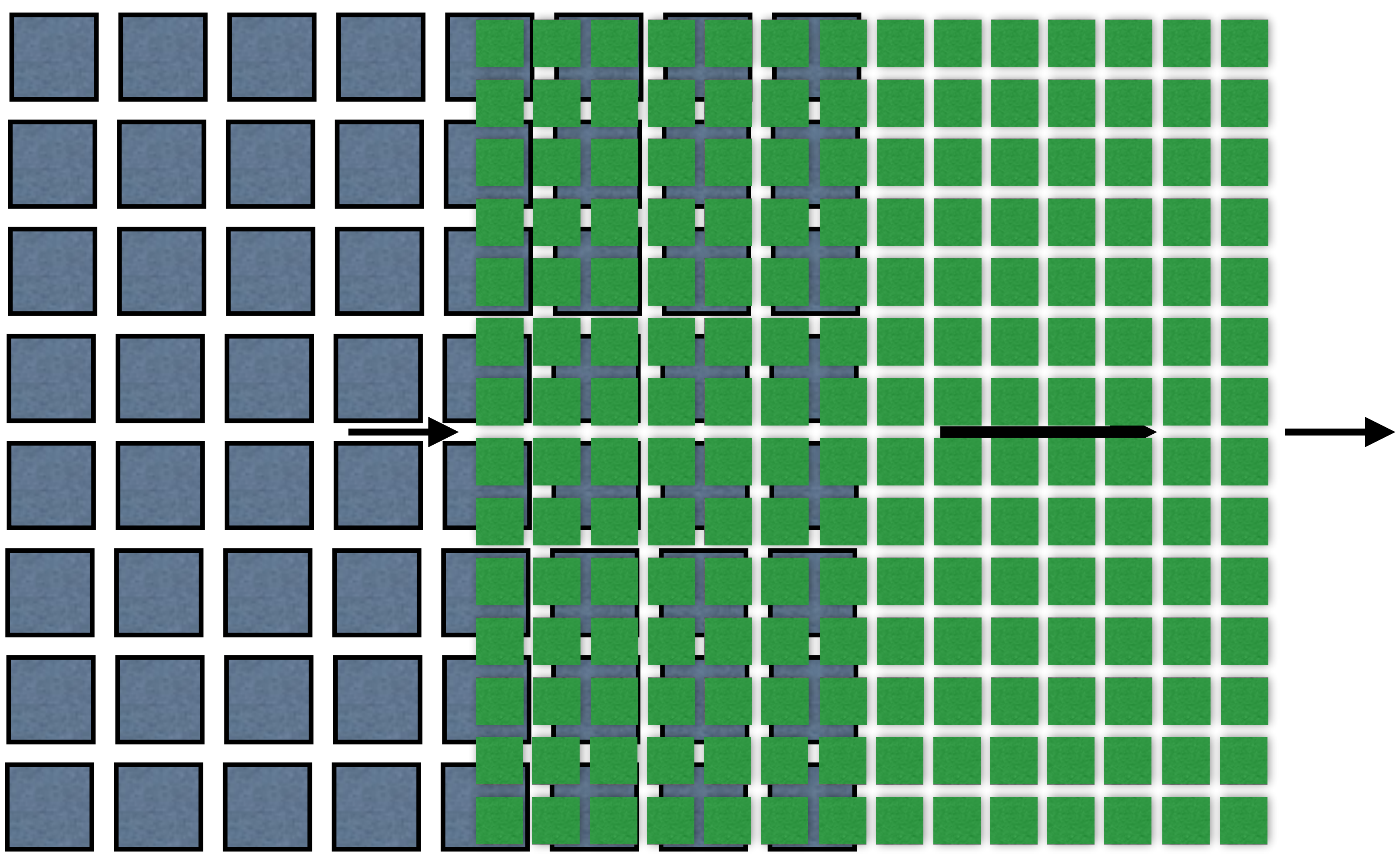
# Reduced Order Models

# Approach

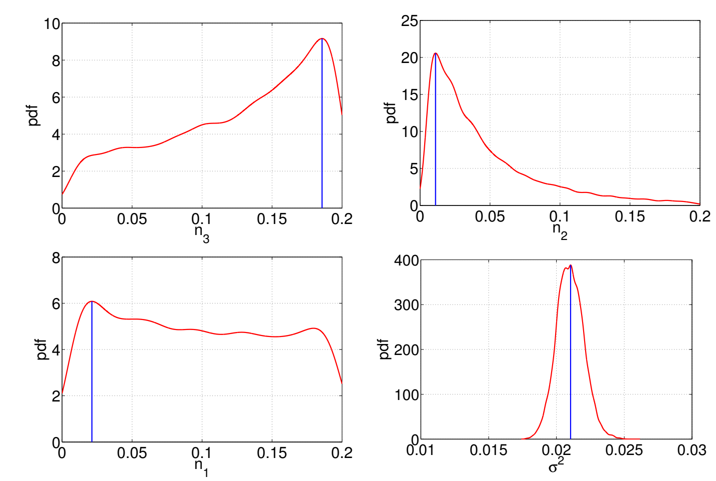
$$G(x, t, \xi) \approx \tilde{g}(x, t, \xi)$$

## Forward Model Runs

## Reduced Order Model



## Parameter Estimation



# Polynomial Chaos Expansions

Quantity of Interest (simulation)

Basis

$$G(x, t, \xi) \approx \sum_{k=0}^{R} g_k(x, t) \psi_k(\xi)$$

Expansion Coefficients

# Spectral Galerkin Projection

$$G(\xi) \approx \sum_{k=0}^{R} g_k \psi_k(\xi)$$

## Orthogonal Polynomials

$$\langle \psi_i, \psi_j \rangle = \int \psi_i(\xi)\, \psi_j(\xi)\, \rho(\xi)\, \mathsf{d}\xi = \delta_{ij} \left\langle \psi_i^2 \right\rangle$$

## Projection

$$g_k = \frac{\langle G, \psi_k \rangle}{\langle \psi_k, \psi_k \rangle} = \frac{1}{\langle \psi_k, \psi_k \rangle} \boxed{\int G \psi_k(\xi) \rho(\xi)\, \mathsf{d}\xi}$$

# POD-Galerkin Method

$$(u(\mu_1), u(\mu_2), \ldots, u(\mu_S)) \qquad \mathcal{O}(N^d N)$$

↓

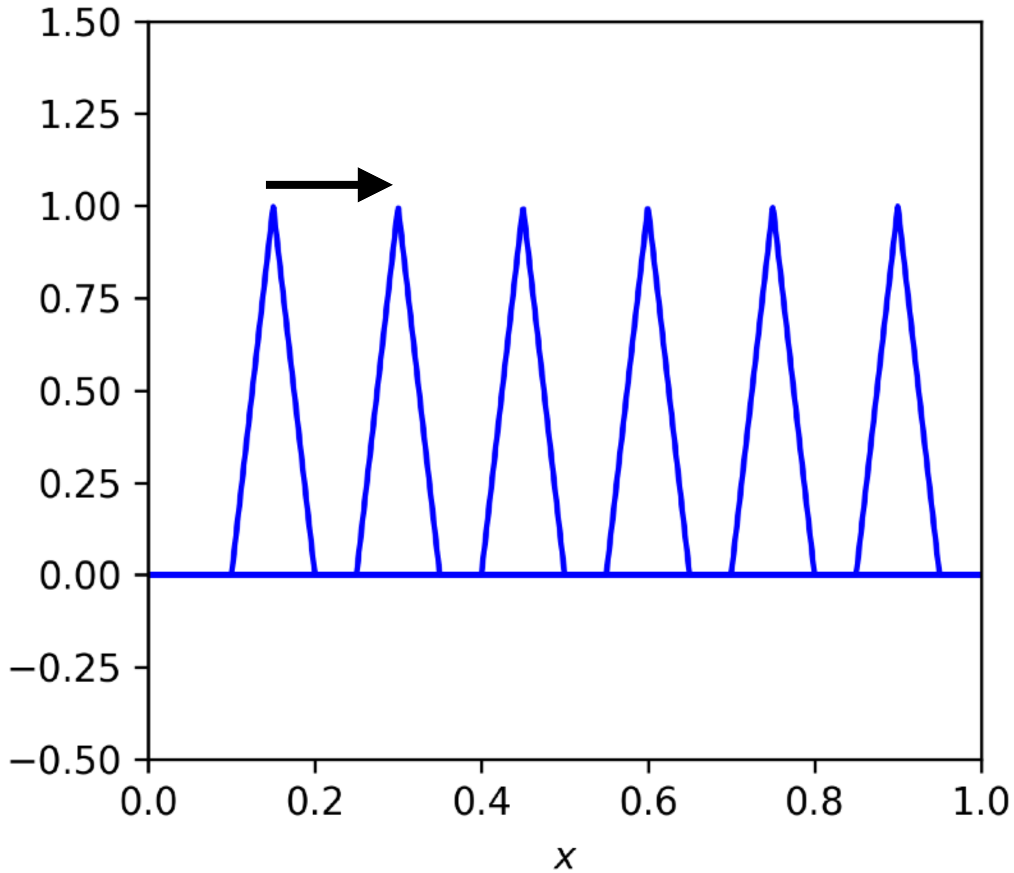
SVD

$$M = \dim(\mathbb{V}_{rb}) \ll \dim(\mathbb{V}) = N$$

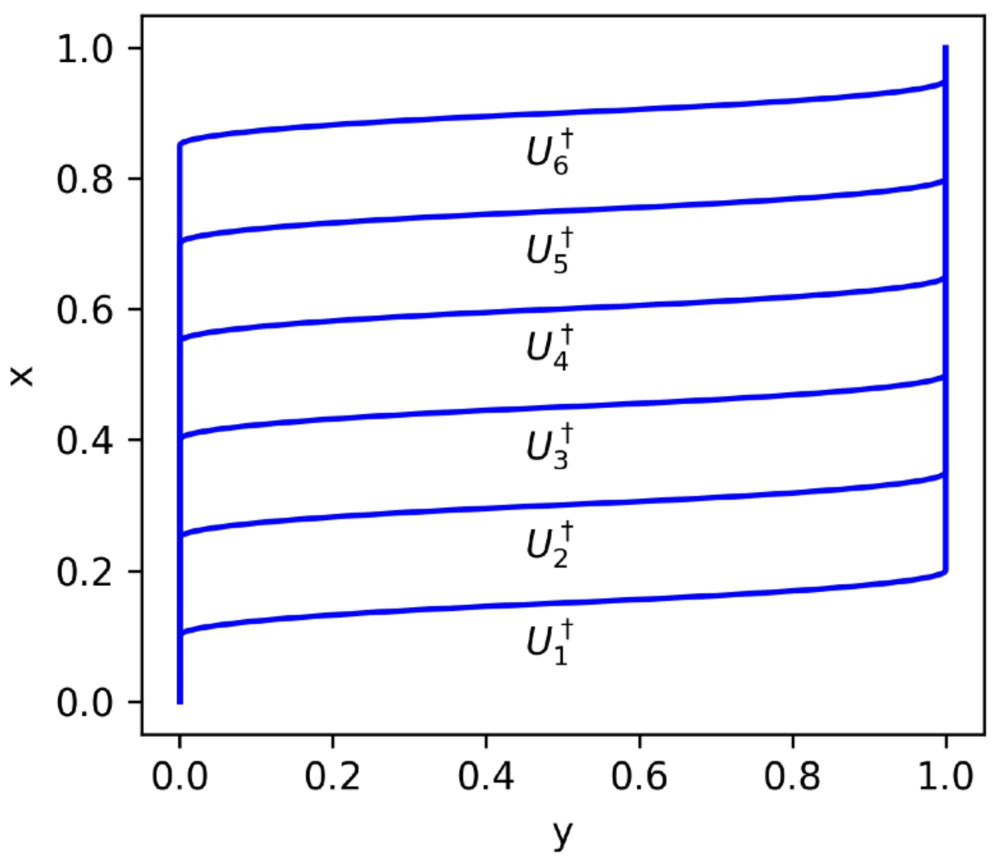
↓

$$u^{n+1}(\mu) = r(u^n(\mu), u^{n+1}(\mu); \xi, \mu) \quad \forall \xi \in \mathbb{V}_{rb} \qquad \mathcal{O}(MN)$$

# Hyperbolic PDEs are Low-dimensional

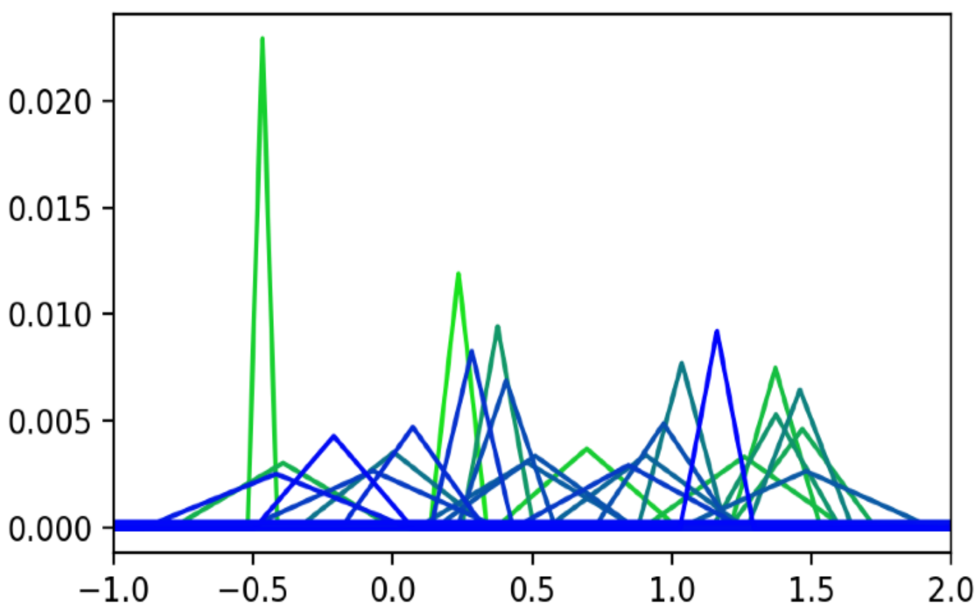


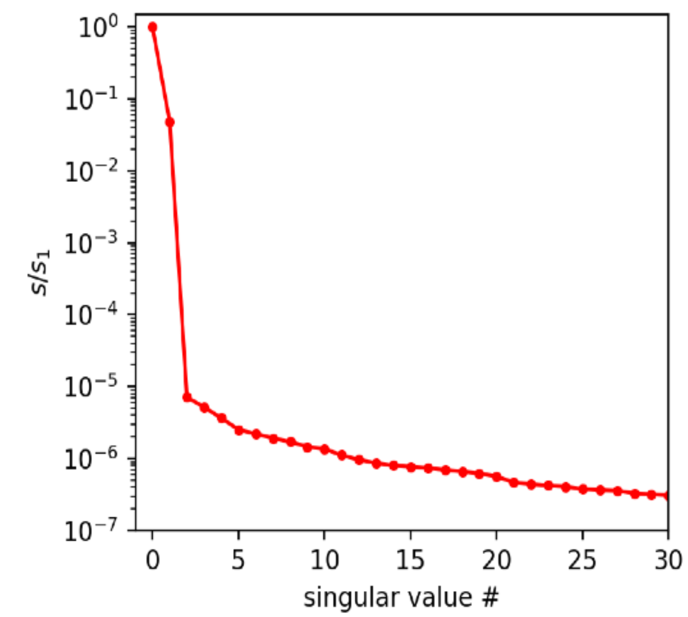

Snapshots are orthogonal

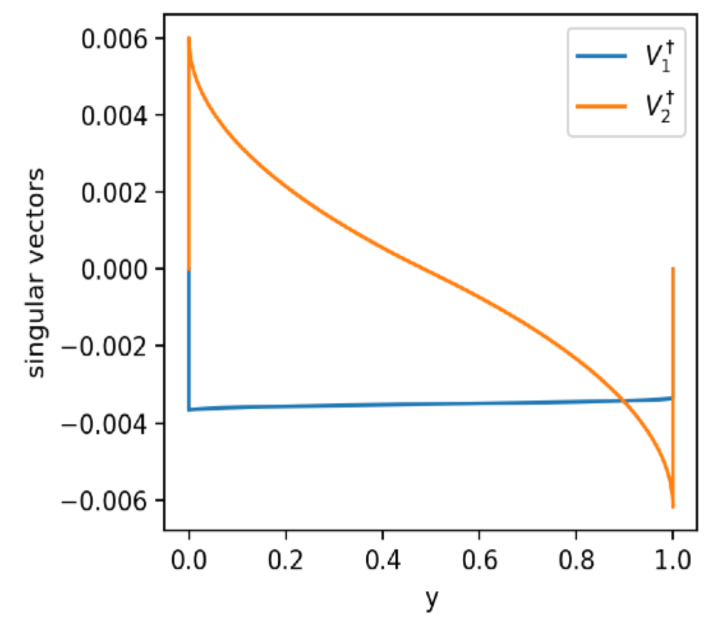



Inverse CDFs are low-rank

# Low-Dimensional Transport Maps



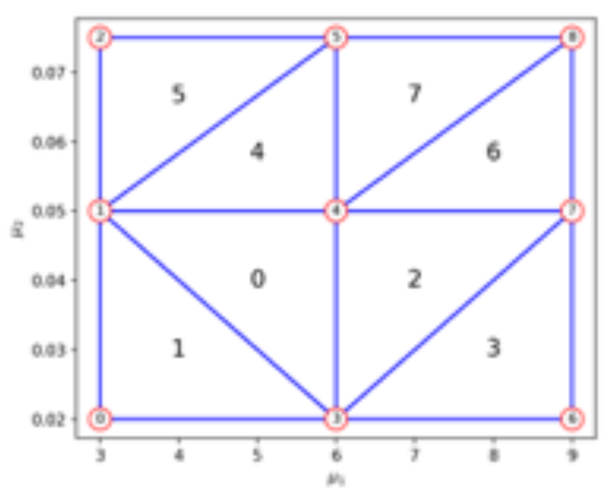



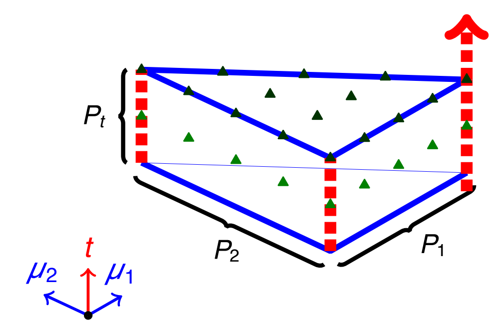



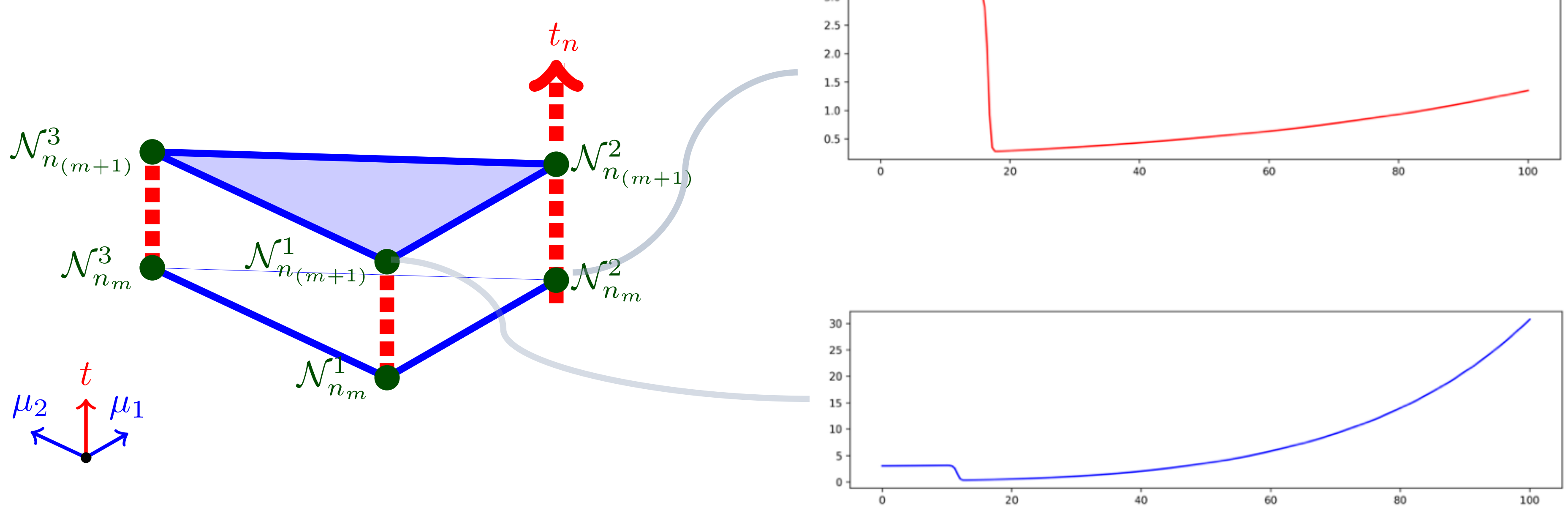

$$u_0(x + \eta_1 w_1(y(x)) + \eta_2 w_2(y(x)))$$

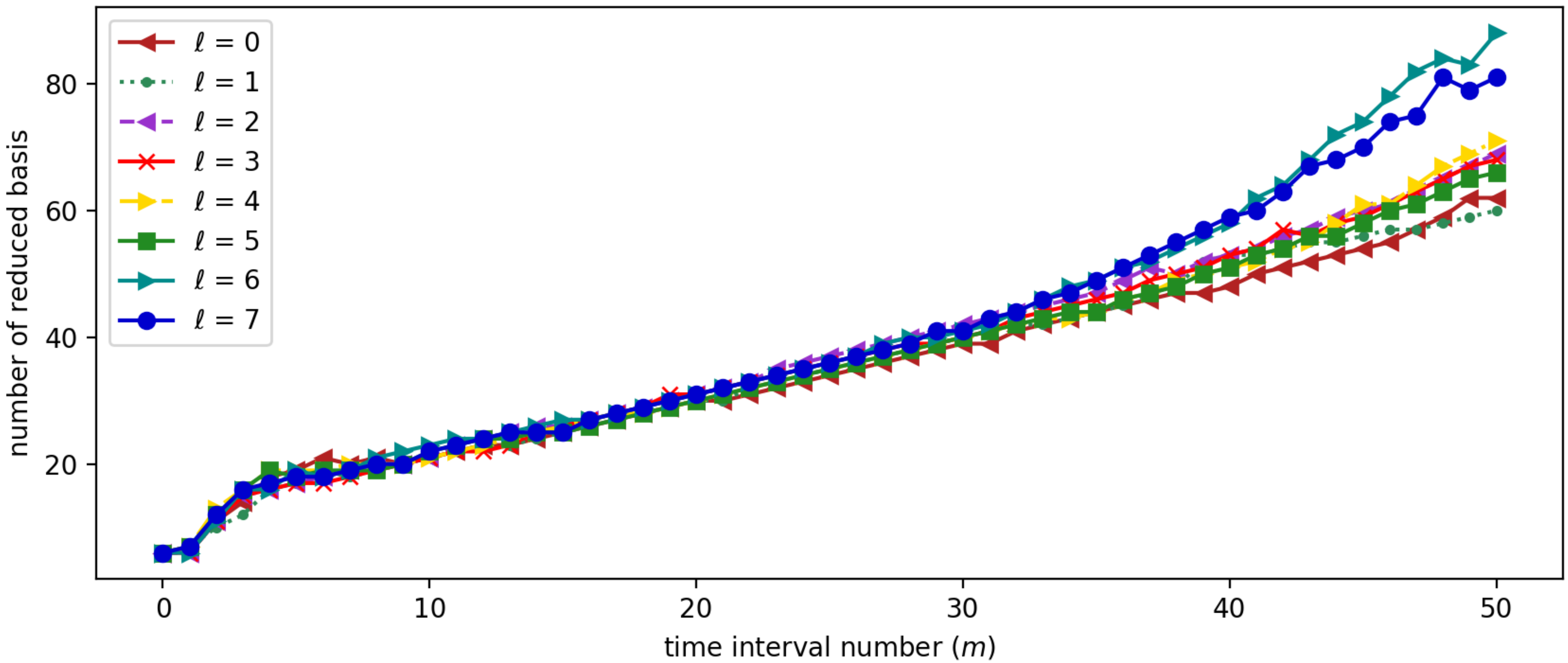
# Example: Burgers' Equation

$$u_t + \left(\frac{1}{2}u^2\right)_x = 0.02e^{\mu_2 x}$$





ℓ = 0
ℓ = 1
ℓ = 2
ℓ = 3
ℓ = 4
ℓ = 5
ℓ = 6
ℓ = 7
number of reduced basis
time interval number (m)
80
60
40
20
0
10
20
30
40
50

# DEIM as a Solution

$$u^{n+1}(\mu) = r(u^n(\mu), u^{n+1}(\mu); \xi, \mu) \quad \forall \xi \in \mathbb{V}_{rb}$$

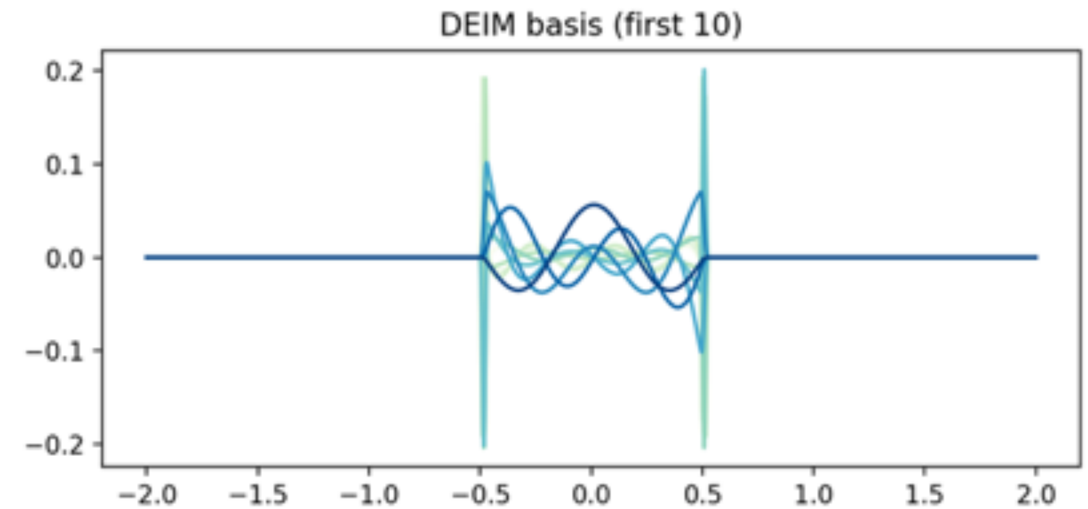
Discrete Empirical Interpolation Method (DEIM)

$$(F(u(x;\mu)), \xi_m) \approx \sum_{p=1}^{P} u(x_p;\mu)\xi_m(x_p)$$

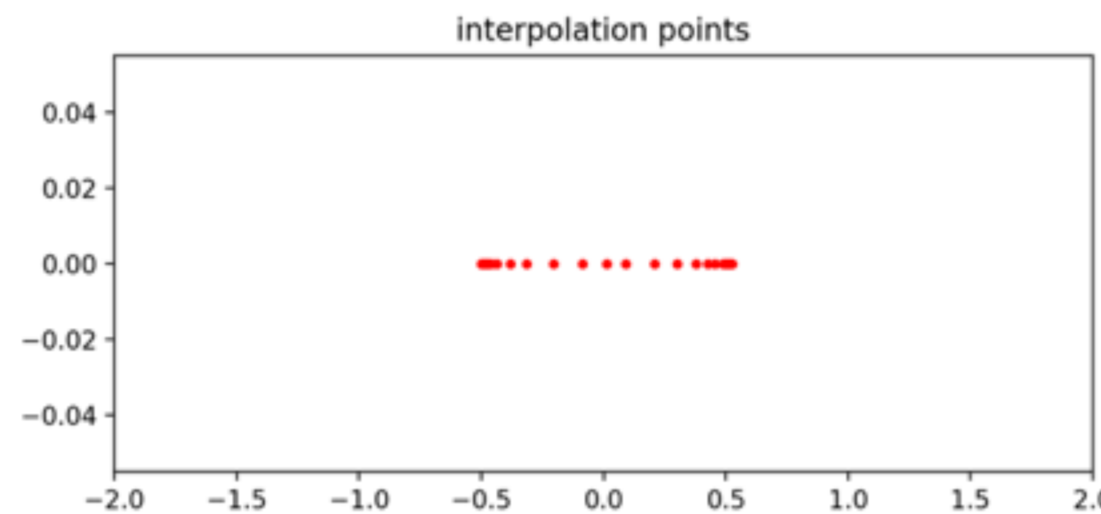
Main Idea = Transport interpolation points

# Combining DEIM with Transport

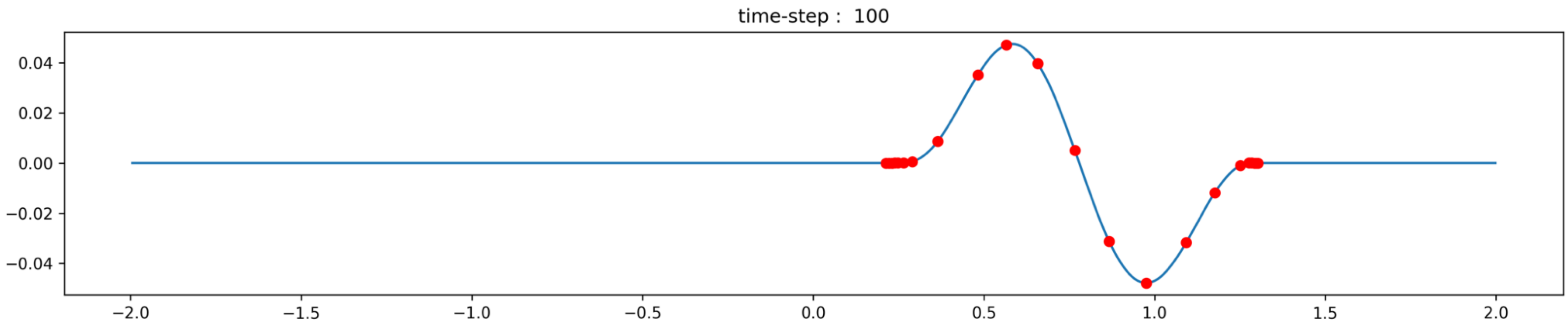
$$\mathbb{V}_{rb} = \{\xi_m\}_{m=1}^M$$



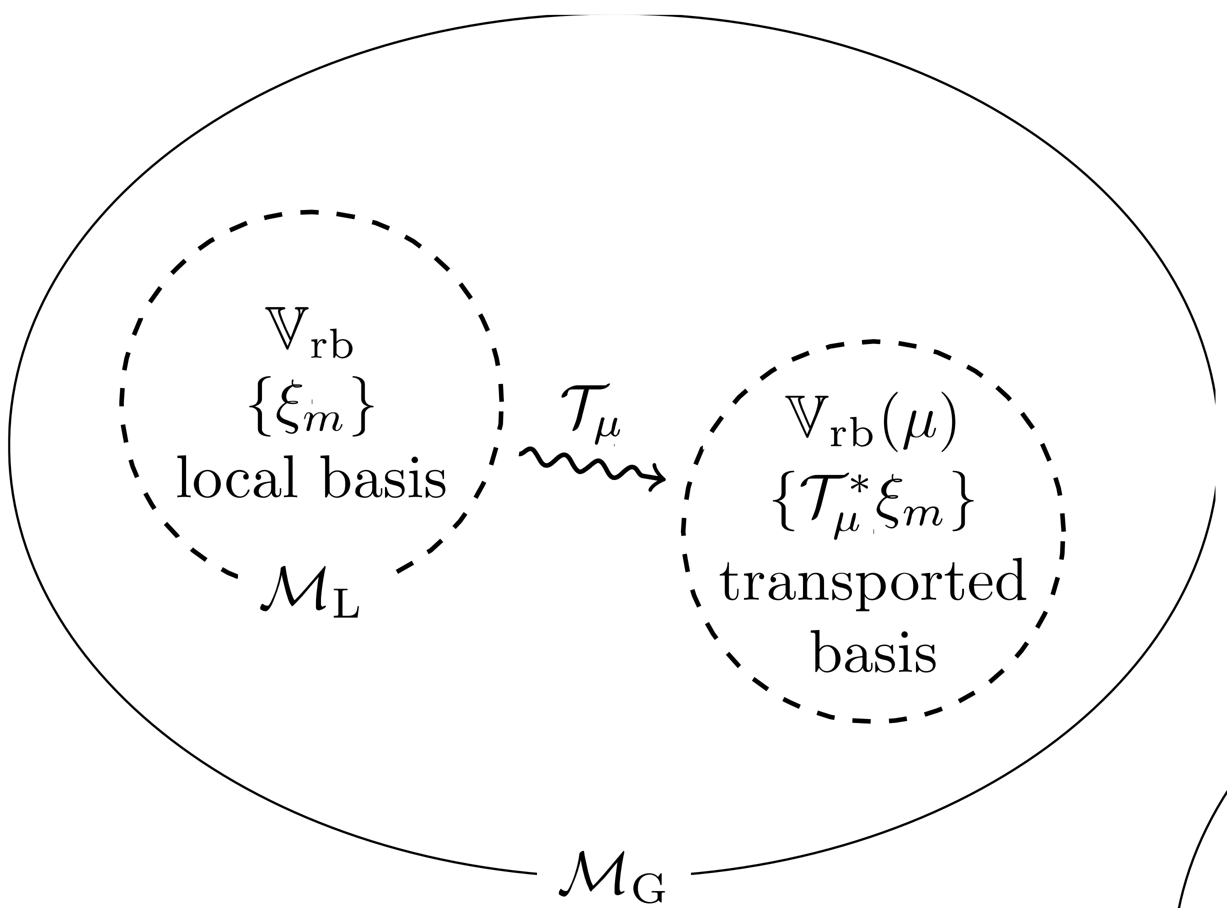

$$\mathcal{I} = \{x_p\}_{p=1}^M$$

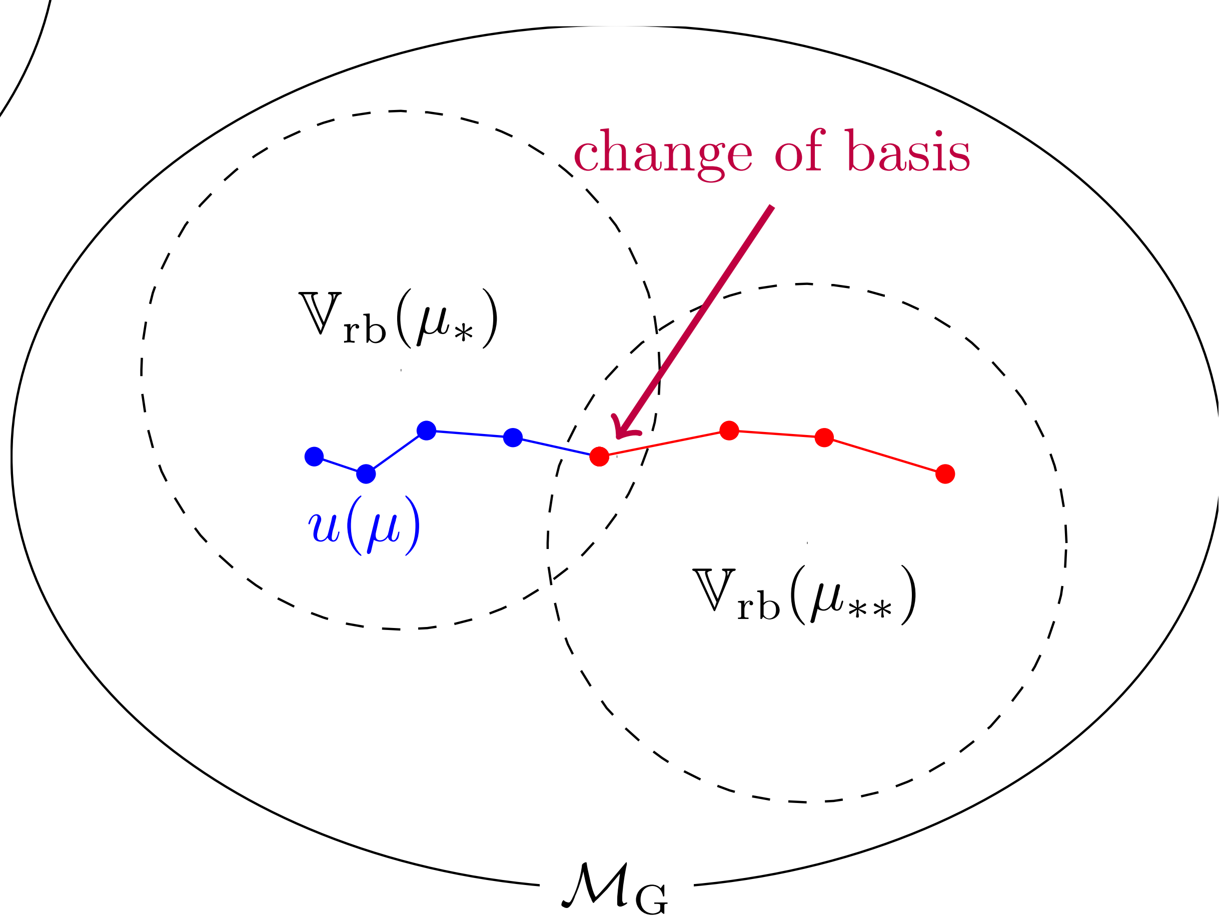


# Advected Basis Points

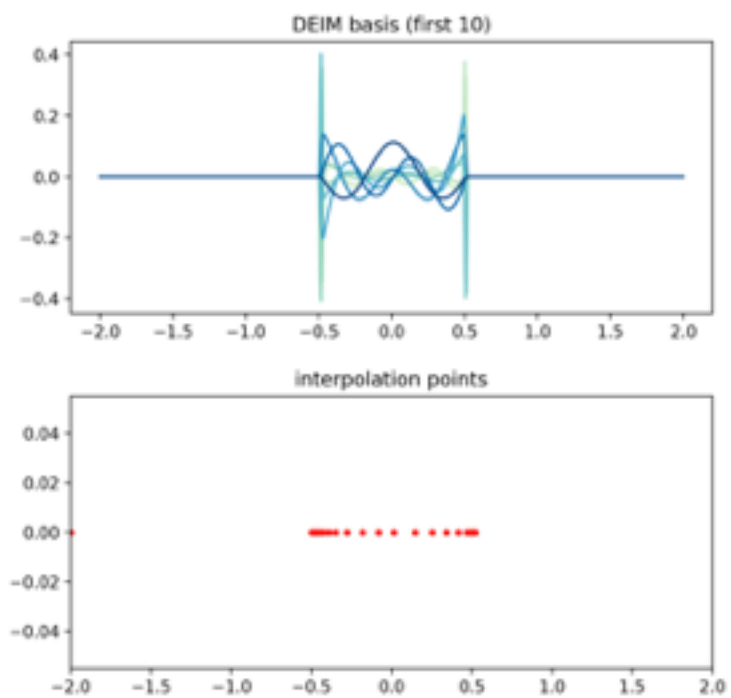


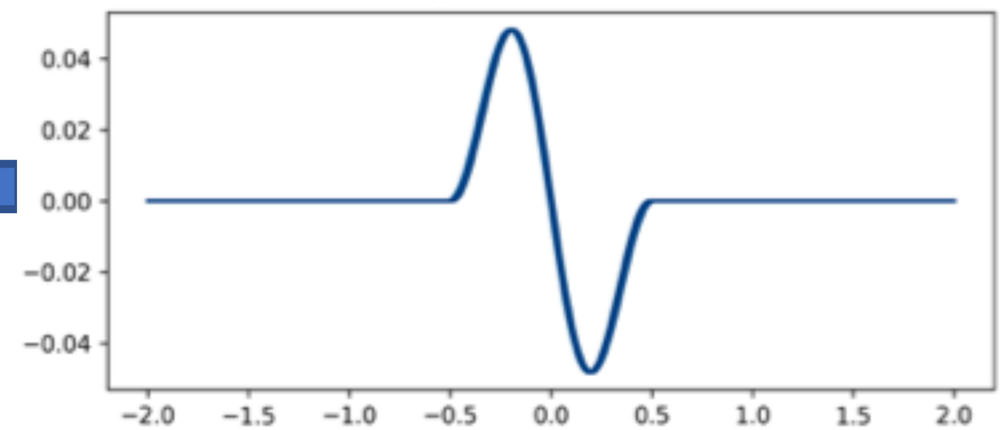
# Moving Basis



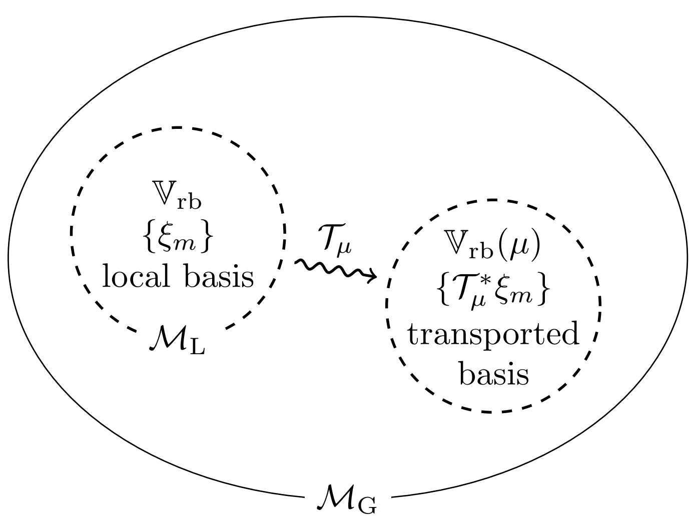



# Example: Translation and Dilation Parameters

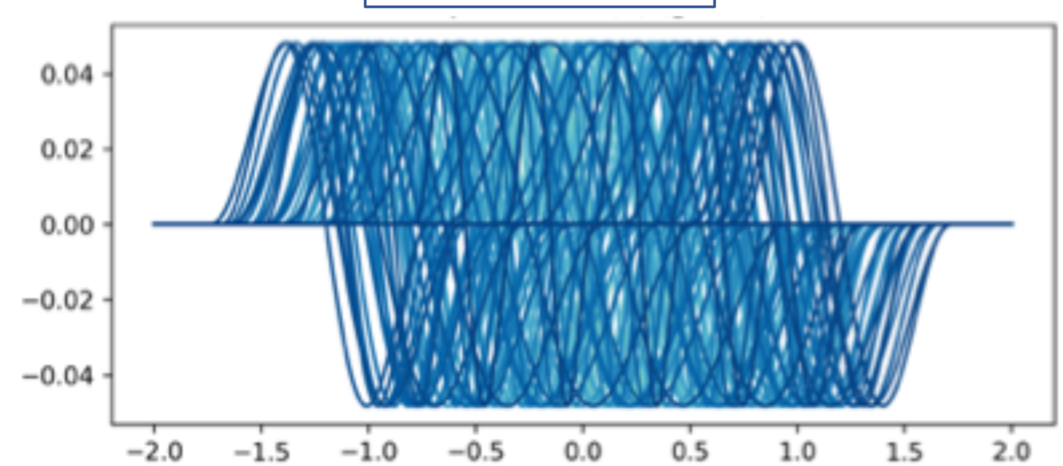


$$u_0\left(\frac{x-\mu_2}{\mu_1}\right)$$




2-dimensional transport map



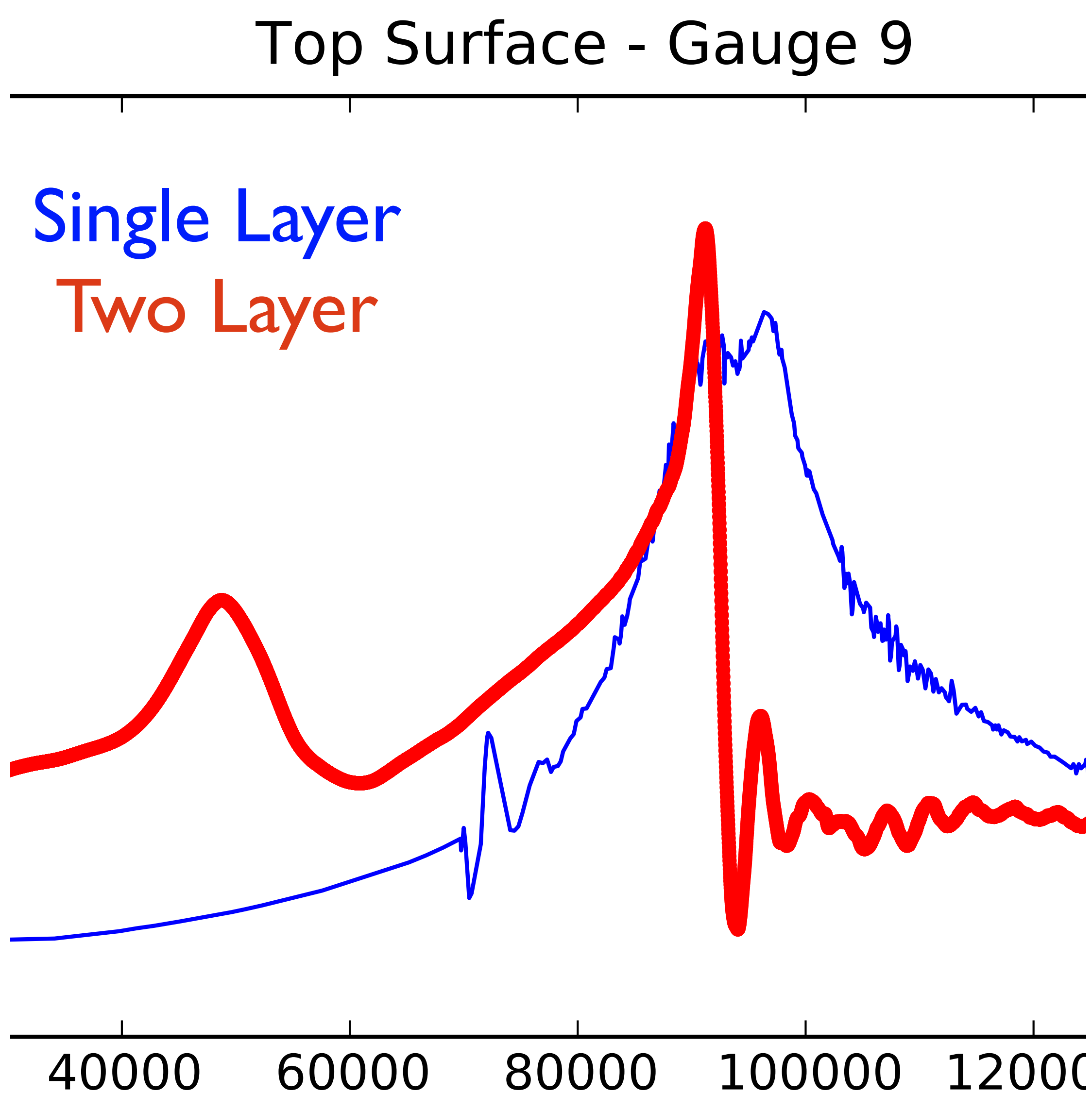
# Outlook

# Storm Surge Computing

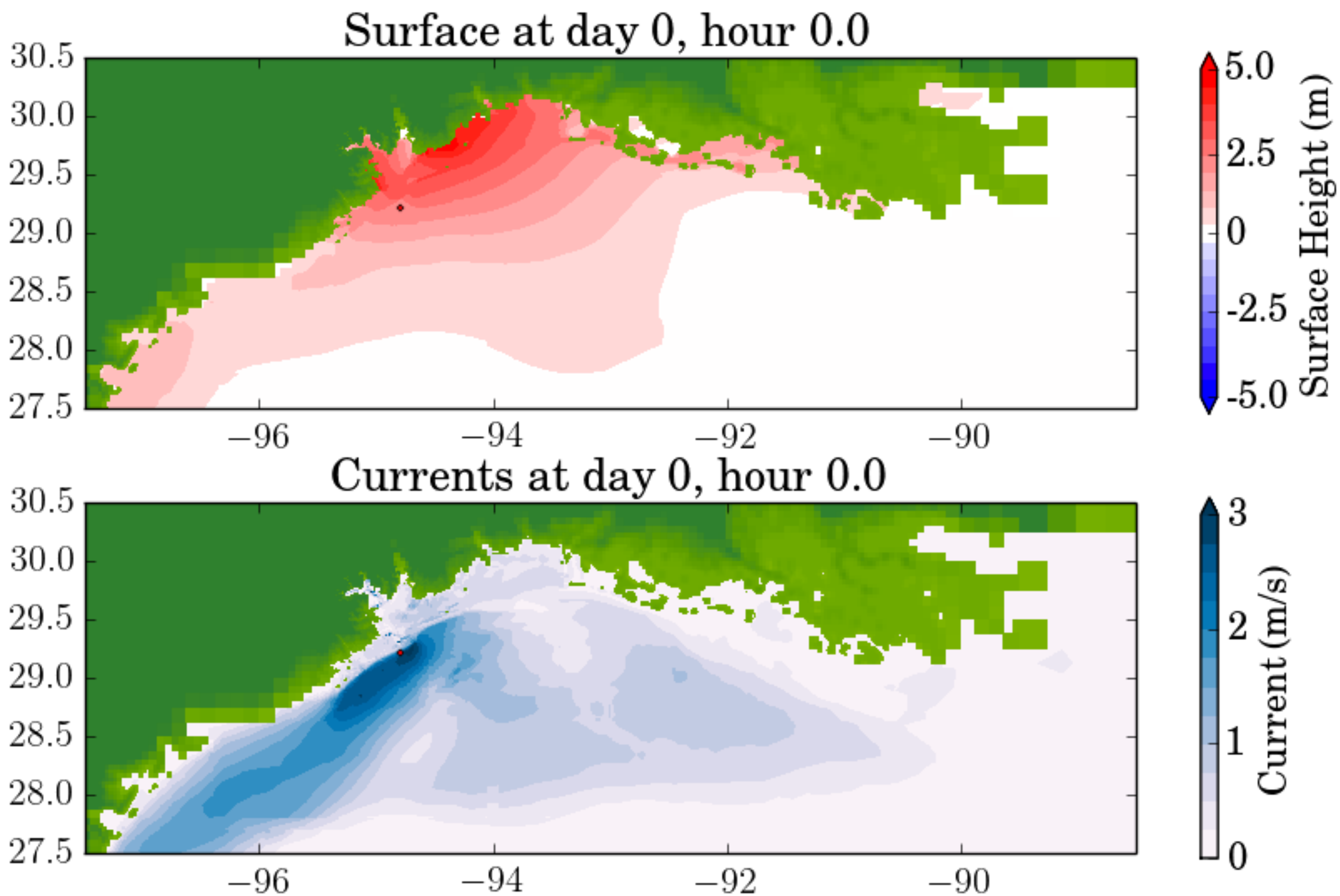
## Adaptive Mesh Refinement

| Package | Cores | Wall Time | Core Time |
|---|---|---|---|
| ADCIRC | 4000 | 35 minutes | 2333 hours |
| GeoClaw | 16 | 2 hours | 32 hours |
| GeoClaw | 4 | 2 hours | 8 hours |

## Multilayer Shallow Water

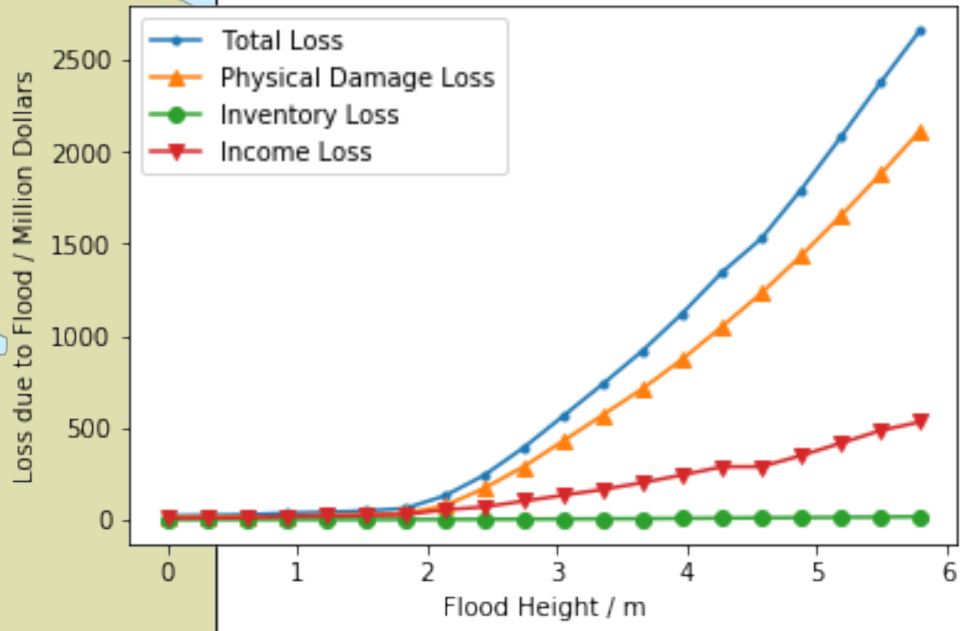


# Storm Surge Forecasting

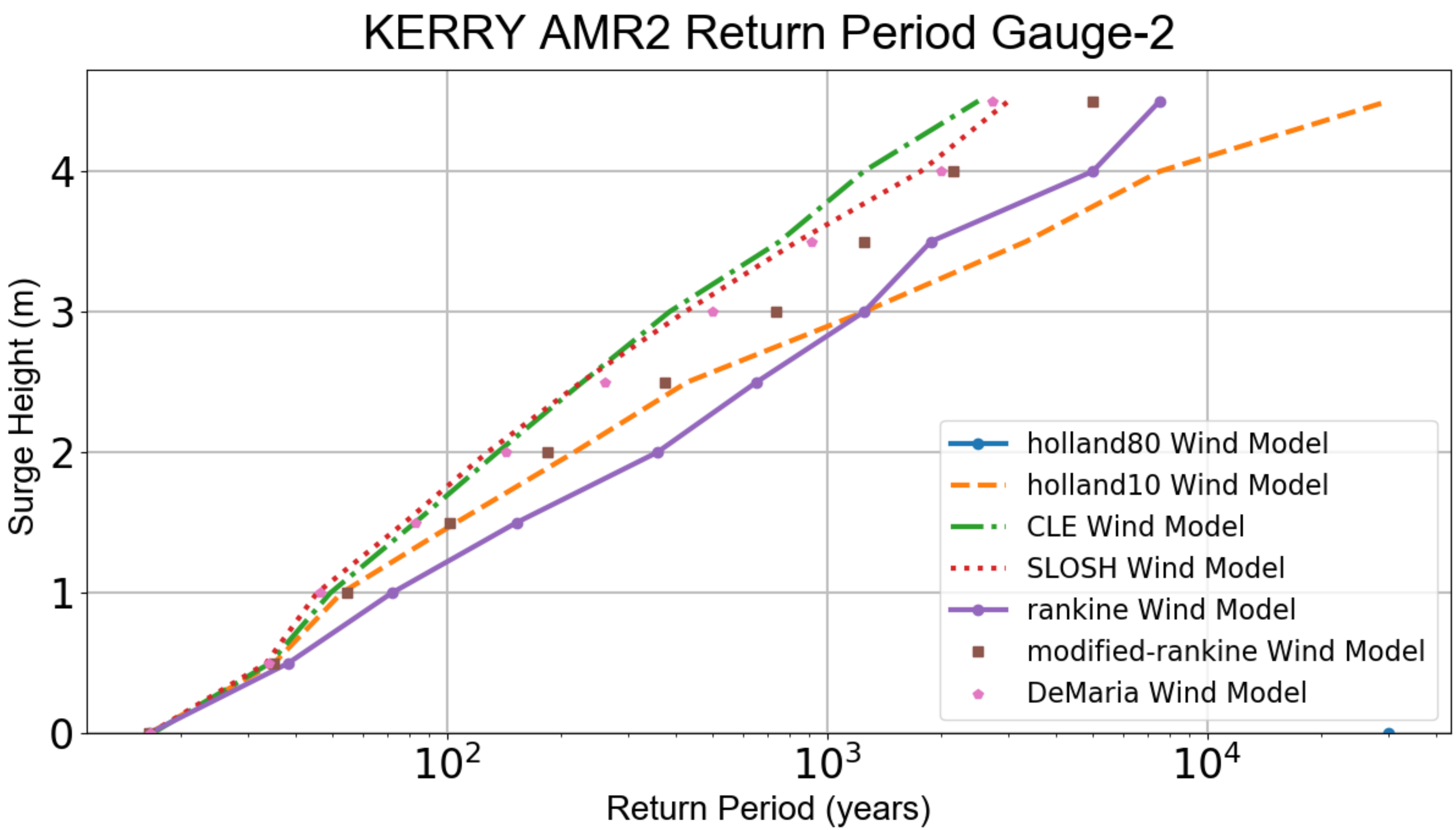



Mandli, K.T. & Dawson, C. N. *Adaptive Mesh Refinement for Storm Surge*. Ocean Modelling 75, 36–50 (2014).

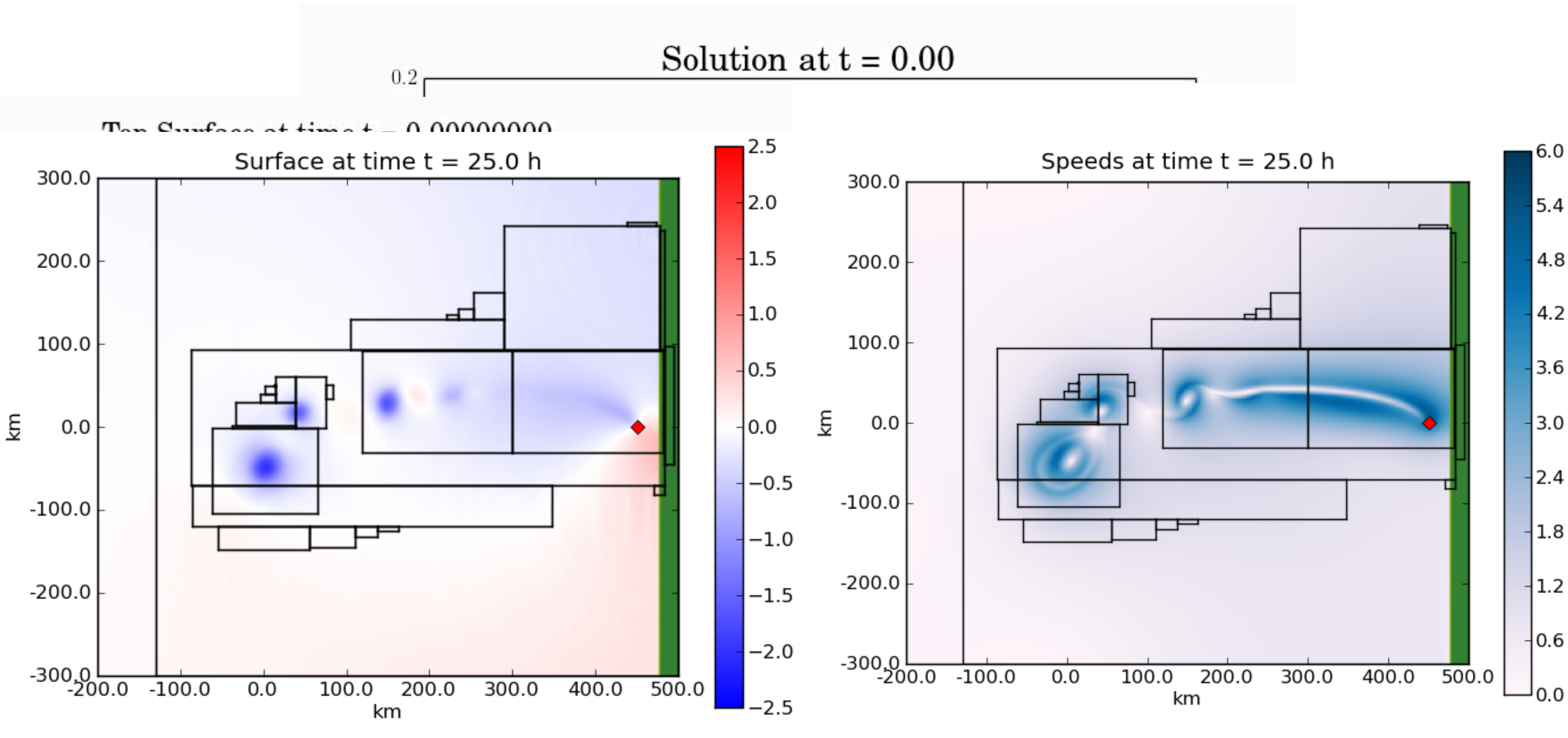
# Multi-Fidelity Models

# Return Curve Sensitivities



# Two-Layer Shallow Water

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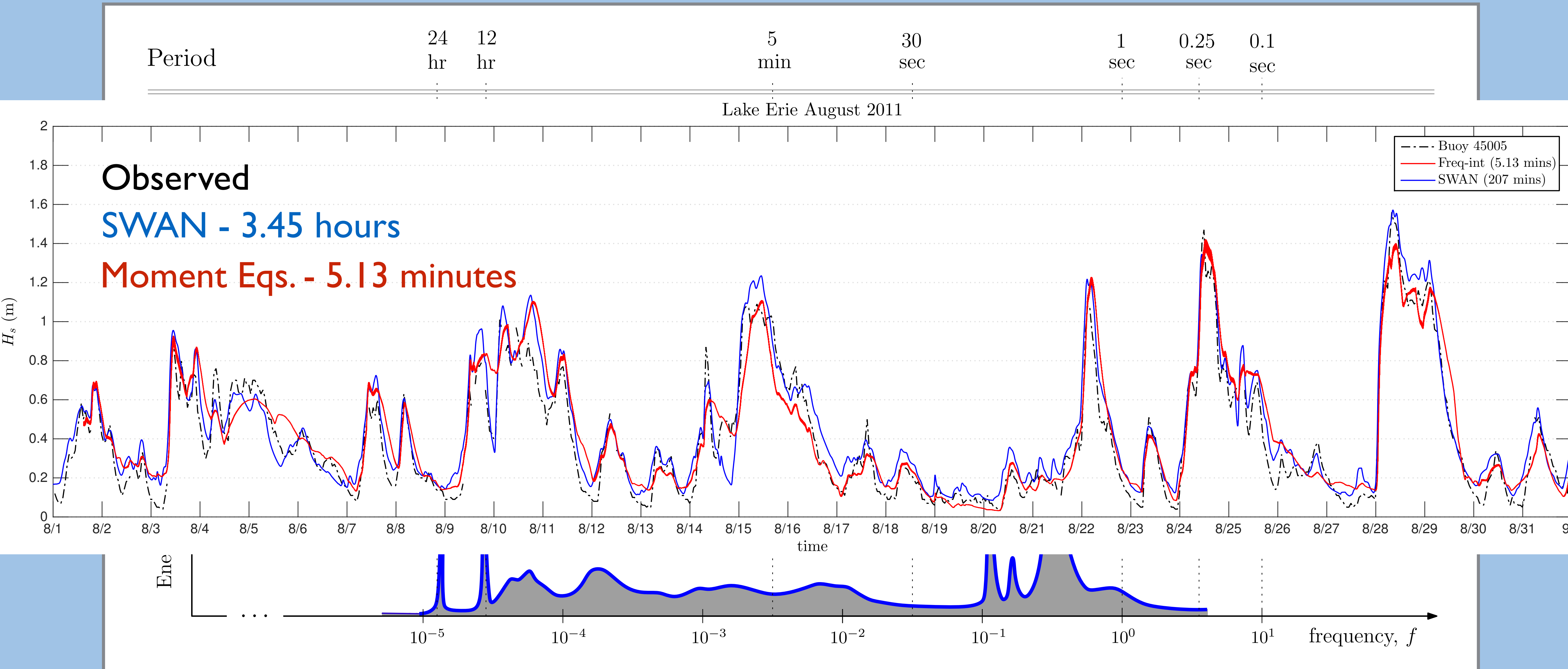


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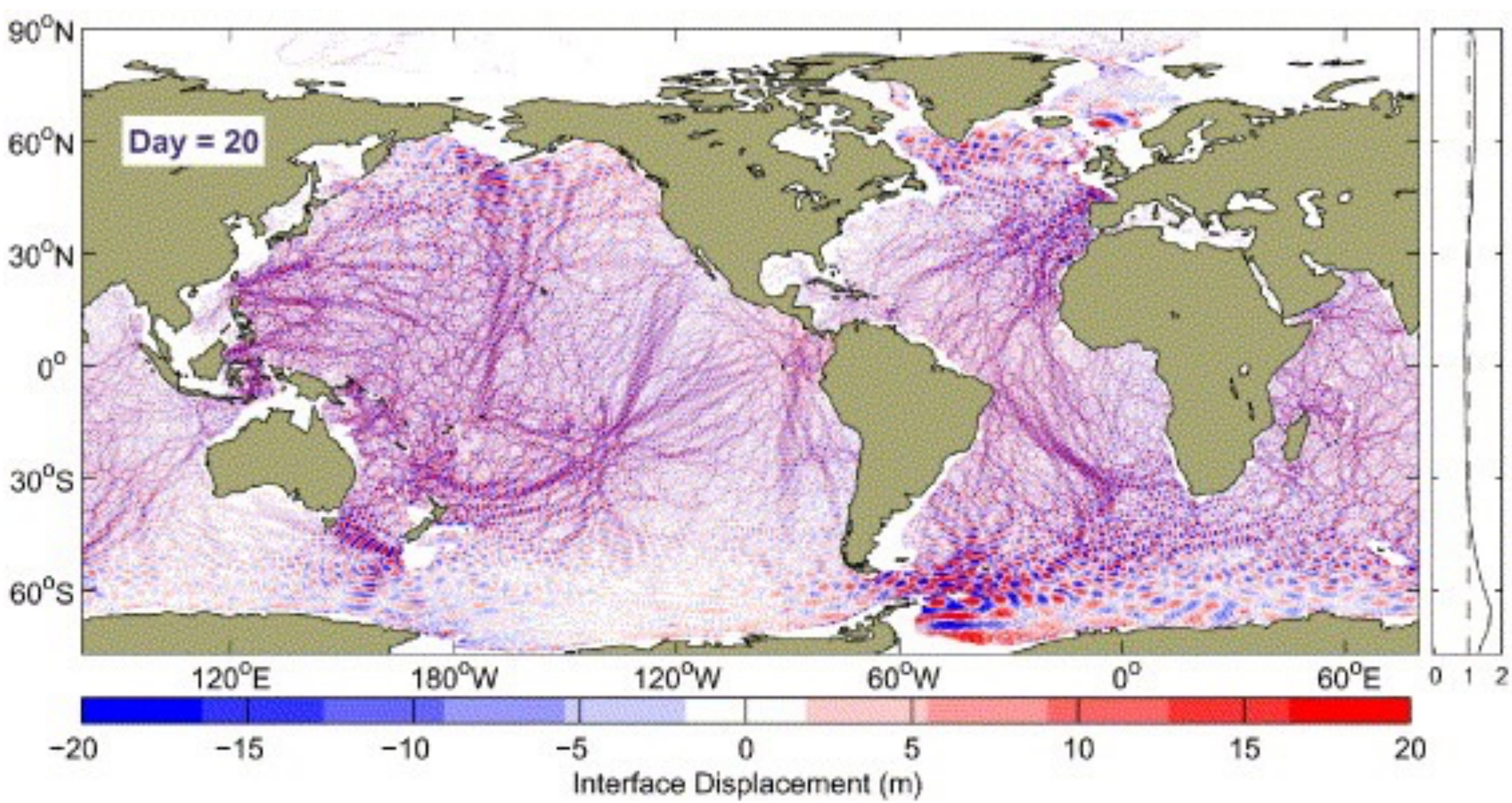
# Air-Sea Waves

# Global Internal Tide Forecasting

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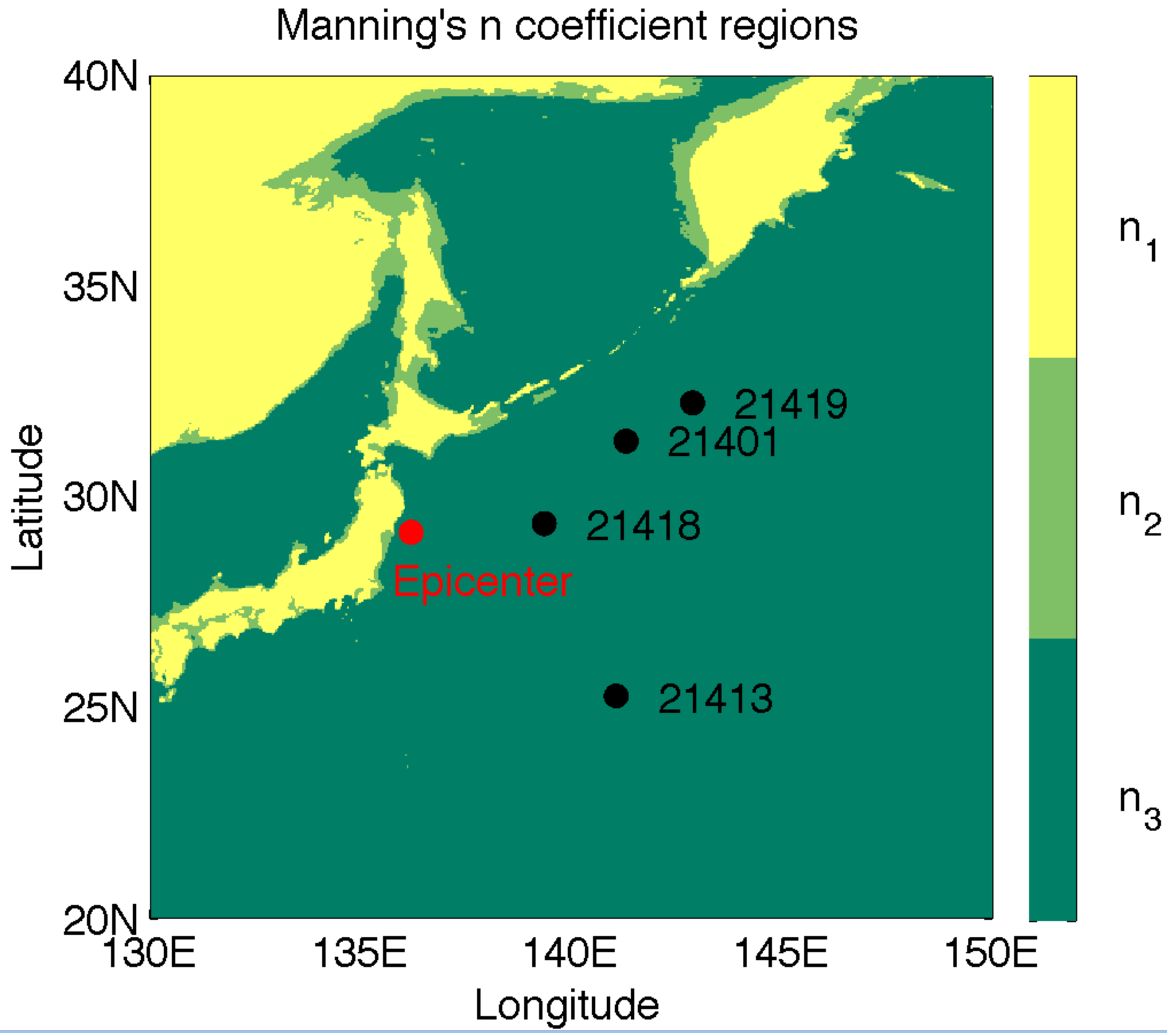


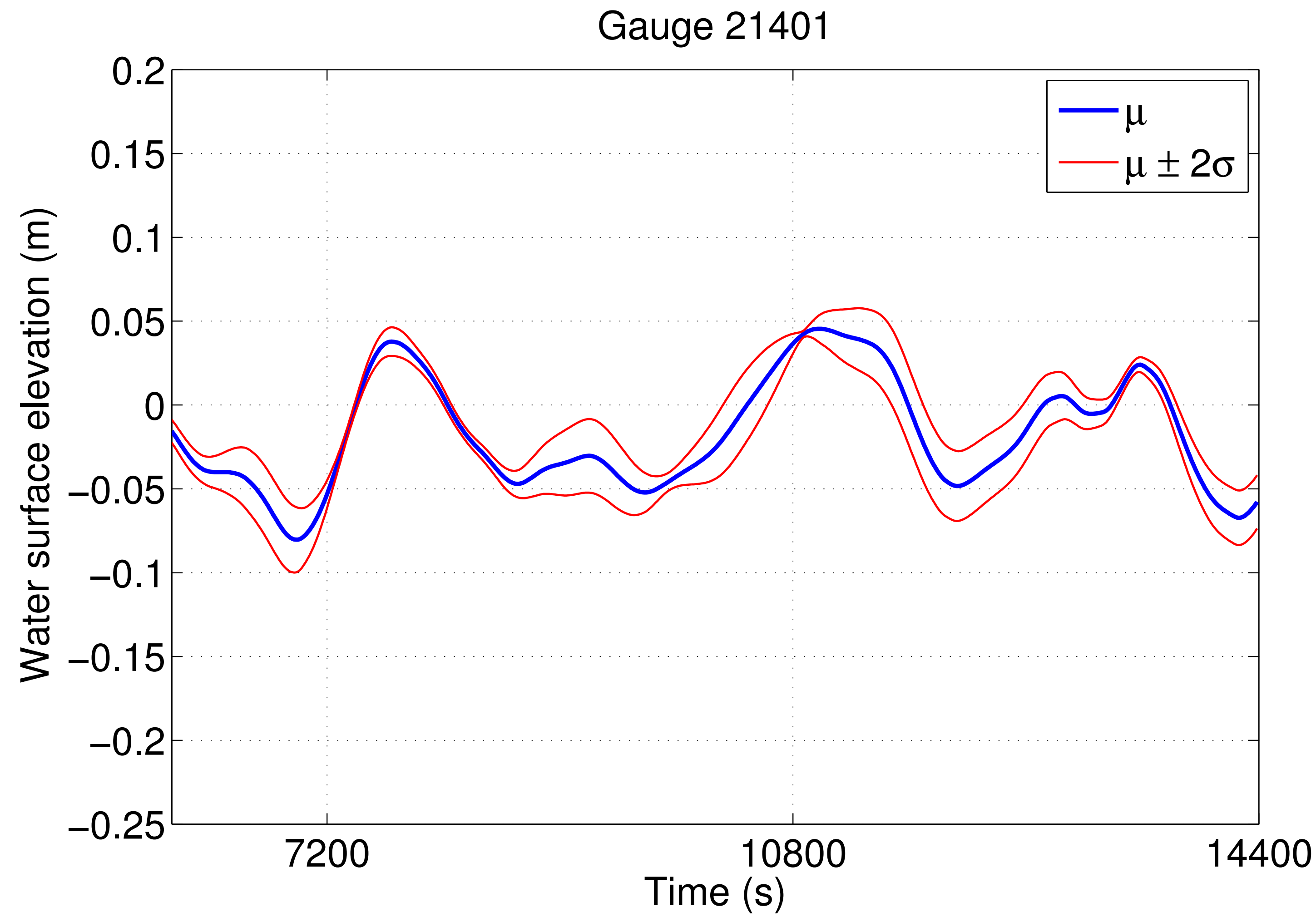


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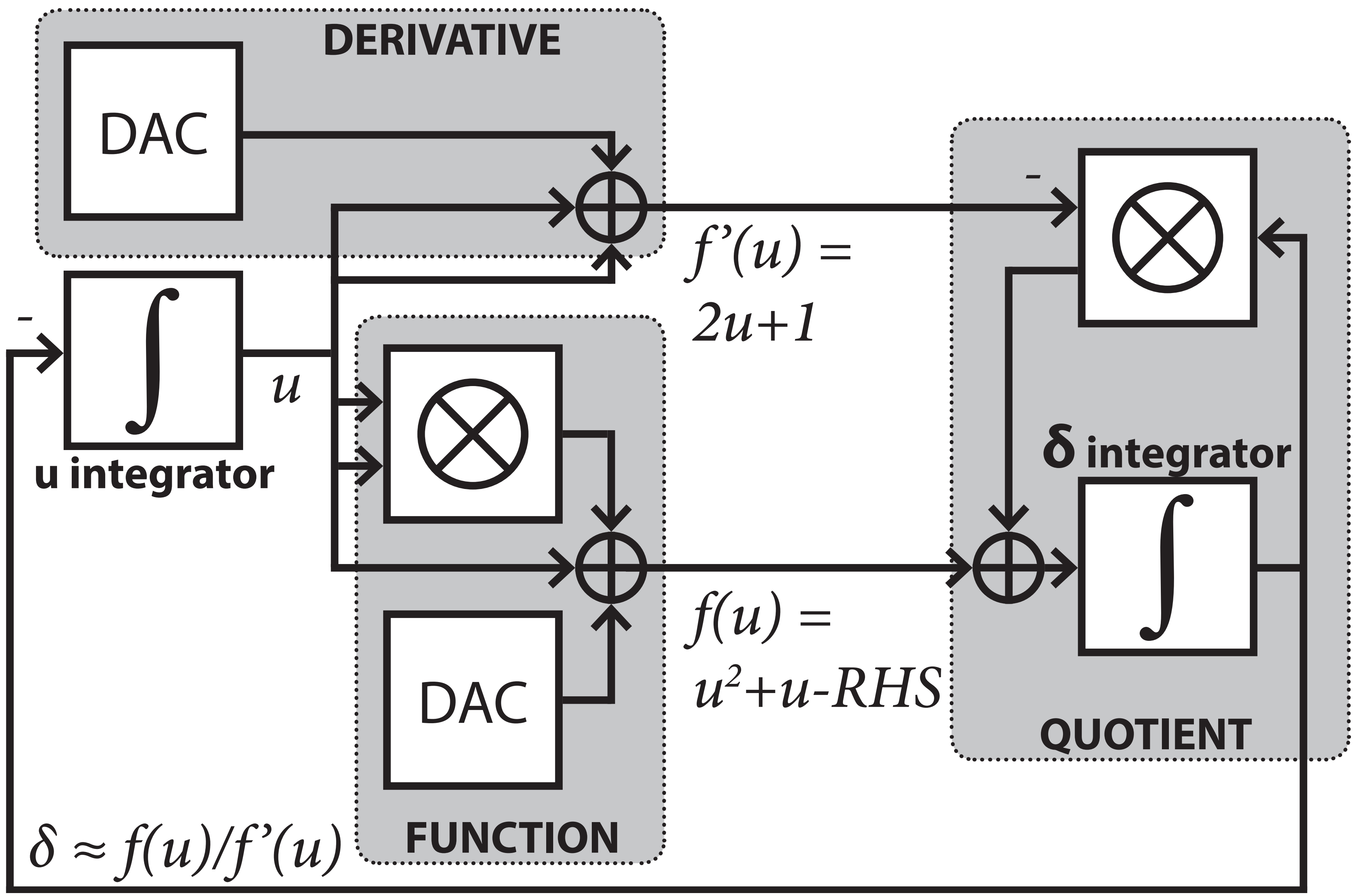
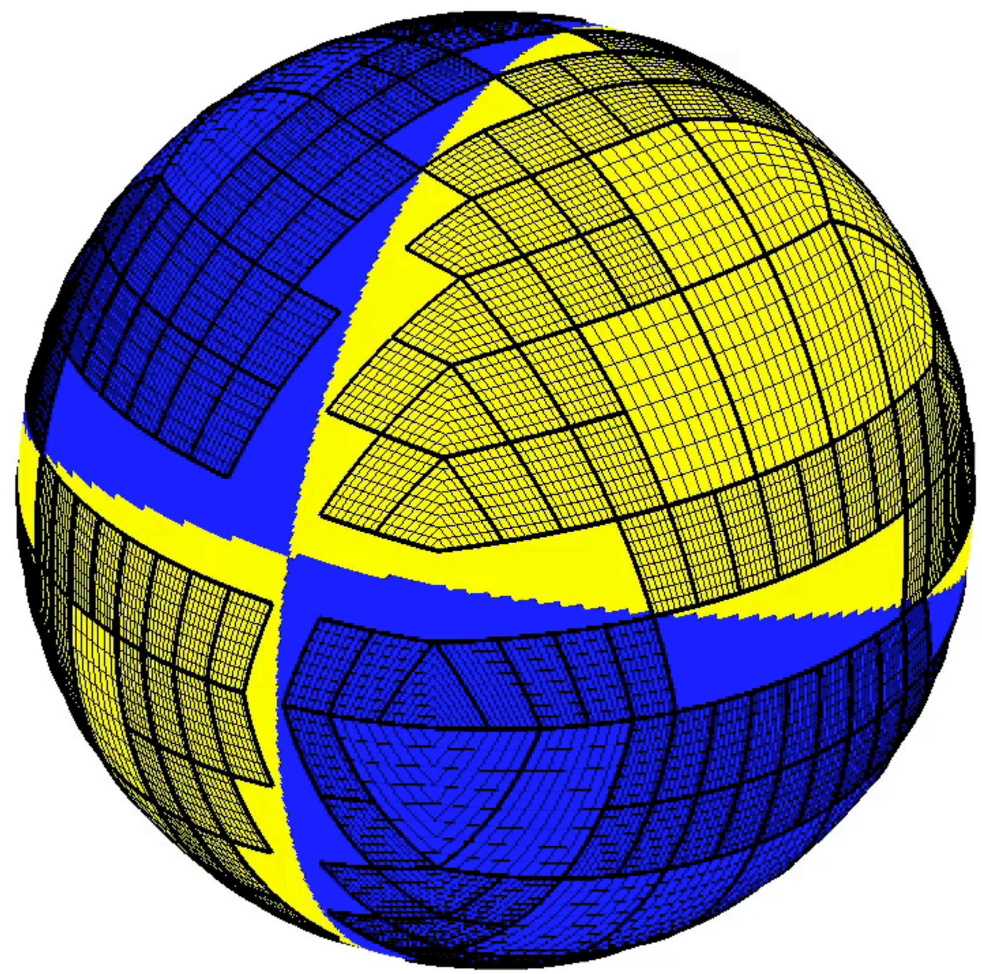
# UQ and Data Assimilation

# "Exotic" Computing



# Ongoing Work



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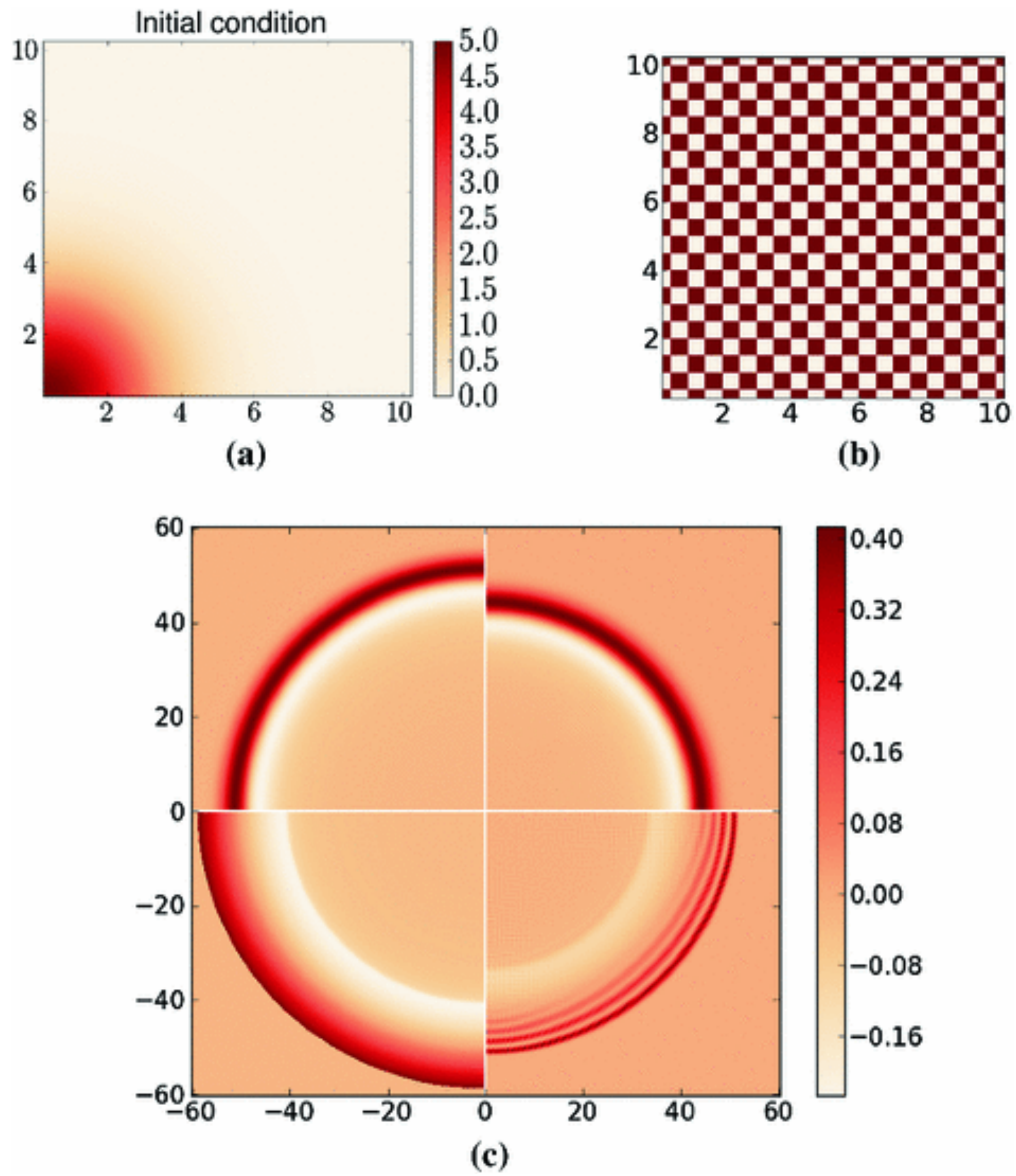
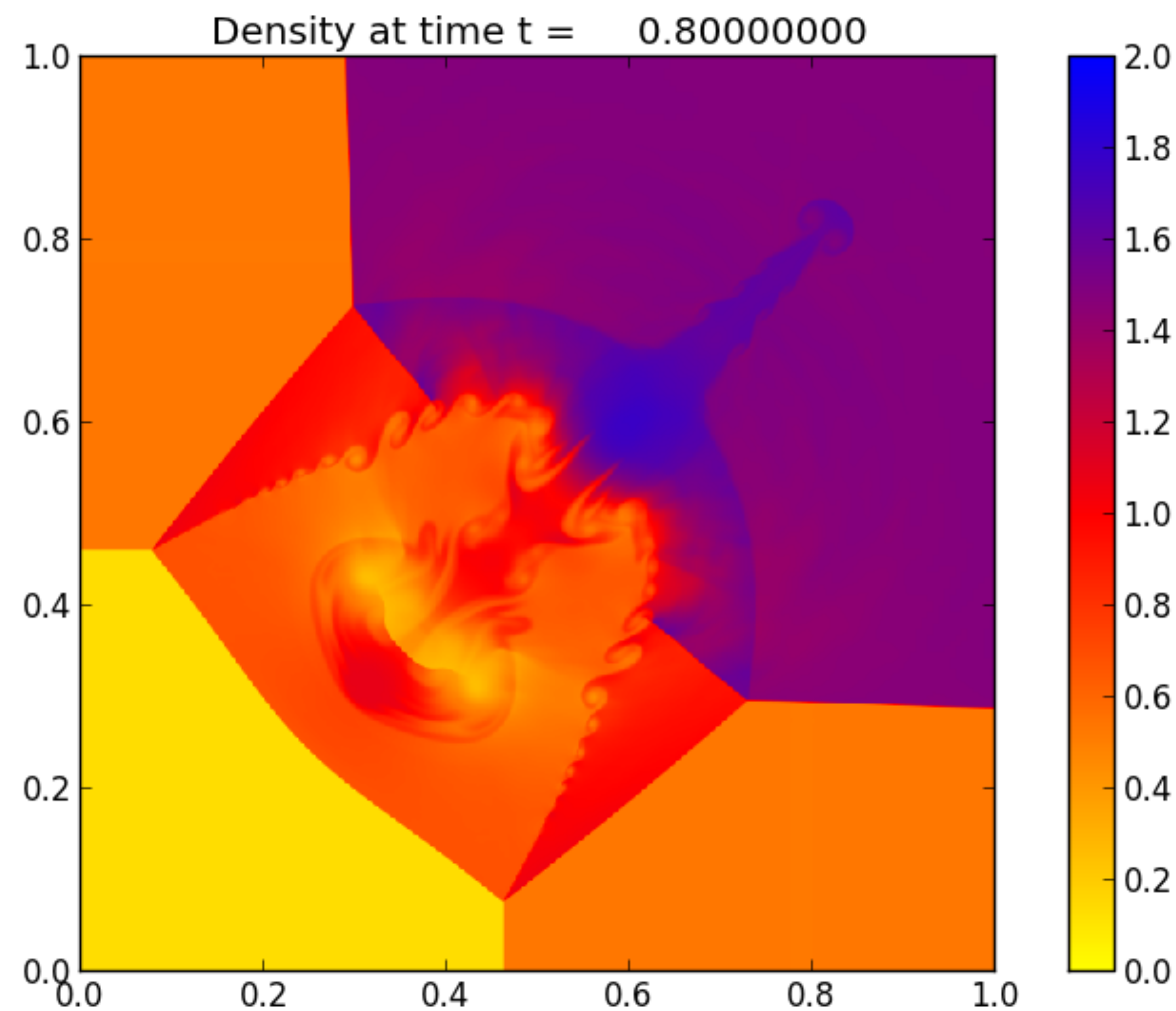
# Ongoing Work

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# Thanks!