Numerical Methods for Predicting Coastal Flooding With Uncertainty

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Collaborators
Storm Surge

Source: Jocelyn Augustino / FEMA - [http://www.fema.gov/photdata/original/38891.jpg](http://www.fema.gov/photdata/original/38891.jpg)
Hurricane Irma
Hurricane Maria
Mexico Beach, FL - NOAA

Hurricane Michael
Hurricane Harvey
Scientifically, single-event attribution may not be the most rational activity… but humans (and governments, and businesses) are motivated by real events.
Transportation Vulnerability
Residential Vulnerability

Tuckerton, NJ - boston.com

How does sea-level rise effect surge?

IPCC, 5th assessment report
How does sea-level rise effect surge?
Will dangerous storms become more frequent?
Will dangerous storms become more powerful?
Can we predict surge probabilities?
Can we predict surge probabilities?

C. Lee, M. Tippett, S. Camargo, A. Sobel (LDEO - Columbia)
Can we forecast events?
Can we quantify uncertainty?
How do we protect ourselves?
Can we protect ourselves?
FIGURE 17 - Potential Category 2 hurricane surge at South Ferry (Battery) Subway Station

US Army Corps 1995

Can we protect ourselves?
Overland Precipitation flooding
Storm Surge Modeling

NASA Modis Satellite
Storm Surge

Wind

Kyle T. Mandli
Storm Surge + Sea-Level

Wind
Shallow Flow

\[ \lambda = u \pm \sqrt{gh} \]

Régis Lachaume
Shallow Water - Topography

Water Height t=0.0
Storm Surge Model

\[ h_t + (hu)_x + (hv)_y = 0 \]

\[(hu)_t + \left( hu^2 + \frac{1}{2}gh^2 \right)_x + (huv)_y = -ghb_x + fhv - \frac{h}{\rho}(P_A)_x + \frac{1}{\rho}(\tau_{sx} - \tau_{bx}) \]

\[(hv)_t + (huv)_x + \left( hv^2 + \frac{1}{2}gh^2 \right)_y = -ghb_y - fhu - \frac{h}{\rho}(P_A)_y + \frac{1}{\rho}(\tau_{sy} - \tau_{by}) \]
Storm Representation

CIMMS: http://cimss.ssec.wisc.edu/tropic2
Holland Hurricane Model

Wind

\[ |W| = \sqrt{\frac{AB(P_n - P_c)e^{-A/r^B}}{\rho_{\text{air}} r^B}} + \frac{r^2 f^2}{4} - \frac{rf}{2} \]

Pressure

\[ P_A = P_c + (P_n - P_c)e^{-A/r^B} \]
Adaptive Mesh Refinement

Hilo Harbor at 14.72 hours

Gauge 1

Gauge 2

Gauge 3

b = 0.0 (m)

b = 2.5 (m)

b = 5.0 (m)
Adaptive Discretization
Adaptive Discretization

Reduced Order Models
Approach

\[ G(x, t, \xi) \approx \tilde{g}(x, t, \xi) \]
Polynomial Chaos Expansions

Quantity of Interest (simulation)

\[ G(x, t, \xi) \approx \sum_{k=0}^{R} g_k(x, t) \psi_k(\xi) \]

Basis

Expansion Coefficients
Spectral Galerkin Projection

\[ G(\xi) \approx \sum_{k=0}^{R} g_k \psi_k(\xi) \]

Orthogonal Polynomials

\[ \langle \psi_i, \psi_j \rangle = \int \psi_i(\xi) \psi_j(\xi) \rho(\xi) \, d\xi = \delta_{ij} \langle \psi_i^2 \rangle \]

Projection

\[ g_k = \frac{\langle G, \psi_k \rangle}{\langle \psi_k, \psi_k \rangle} = \frac{1}{\langle \psi_k, \psi_k \rangle} \int G(\xi) \psi_k(\xi) \rho(\xi) \, d\xi \]
POD-Galerkin Method

\[ (u(\mu_1), u(\mu_2), \ldots, u(\mu_S)) \leq O(N^d N) \]

\[
\begin{align*}
\text{SVD} & \\
M = \dim(\mathbb{V}_{rb}) \ll \dim(\mathbb{V}) = N \\
& \leq O(MN)
\end{align*}
\]

\[ u^{n+1}(\mu) = r(u^n(\mu), u^{n+1}(\mu); \xi, \mu) \quad \forall \xi \in \mathbb{V}_{rb} \]
Hyperbolic PDEs are Low-dimensional

Snapshots are orthogonal

Inverse CDFs are low-rank
Low-Dimensional Transport Maps

\[ u_0(x + \eta_1 w_1(y(x)) + \eta_2 w_2(y(x))) \]
Example: Burgers’ Equation

\[ u_t + \left( \frac{1}{2} u^2 \right)_x = 0.02 e^{\mu_2 x} \]
DEIM as a Solution

\[ u^{n+1}(\mu) = r(u^n(\mu), u^{n+1}(\mu); \xi, \mu) \quad \forall \xi \in \mathbb{V}_{rb} \]

Discrete Empirical Interpolation Method (DEIM)

\[ (F(u(x; \mu)), \xi_m) \approx \sum_{p=1}^{P} u(x_p; \mu) \xi_m(x_p) \]

Main Idea = Transport interpolation points
\[ \nabla_{rb} = \{ \xi_m \}_{m=1}^M \]

\[ \mathcal{I} = \{ x_p \}_{p=1}^M \]
Advected Basis Points
Moving Basis

\[ \nabla_{rb} \{ \xi_m \} \quad \text{local basis} \quad \mathcal{M}_L \]

\[ \nabla_{rb}(\mu) \quad \{ T_\mu^* \xi_m \} \quad \text{transported basis} \quad \mathcal{M}_G \]

\[ \nu_{rb}(\mu_*) \quad u(\mu) \quad \mathcal{M}_G \]

\[ \nu_{rb}(\mu_{**}) \]

change of basis
Example: Translation and Dilation Parameters

\[ u_0 \left( \frac{x - \mu_2}{\mu_1} \right) \]

2-dimensional transport map

\[ V_{rb}(\mu) \{T_\mu, \xi_m\} \]

Transported basis

\[ V_{rb} \{\xi_m\} \]

Local basis

\[ M_L \]

\[ M_G \]
Outlook
## Adaptive Mesh Refinement

<table>
<thead>
<tr>
<th>Package</th>
<th>Cores</th>
<th>Wall Time</th>
<th>Core Time</th>
</tr>
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<tbody>
<tr>
<td>ADCIRC</td>
<td>4000</td>
<td>35 minutes</td>
<td>2333 hours</td>
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<tr>
<td>GeoClaw</td>
<td>16</td>
<td>2 hours</td>
<td>32 hours</td>
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<tr>
<td>GeoClaw</td>
<td>4</td>
<td>2 hours</td>
<td>8 hours</td>
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</tbody>
</table>

## Multilayer Shallow Water

**Top Surface - Gauge 9**

- **Single Layer**
- **Two Layer**
Multi-Fidelity Models

Lower Manhattan
Numbers of Subway Openings
- 1 - 10
- 11 - 50
- 51 - 125
- Subway Stations

4.27 m (14 ft) Height Flood
- Inundated Area
- Non-inundated Area
- Buildings

Inundated Area
Non-inundated Area
Buildings

Loss due to Flood, Million Dollars
Flood Height / m

Columbia Engineering
The Fu Foundation School of Engineering and Applied Science
Return Curve Sensitivities

KERRY AMR2 Return Period Gauge-2

Surge Height (m)

Return Period (years)

- holland80 Wind Model
- holland10 Wind Model
- CLE Wind Model
- SLOSH Wind Model
- rankine Wind Model
- modified-rankine Wind Model
- DeMaria Wind Model
Observed

SWAN - 3.45 hours
Moment Eqs. - 5.13 minutes


Air-Sea Waves

UQ and Data Assimilation
The square root causes real solutions to exist only when the quadratic equation has the closed form solution:

\[ u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Output using analog-to-digital converters. Integrators are steady, and at that point we can measure the value of the integrators. The quotient would be negated and fed to the input of the derivative. The derivative, a block for doing division using gradient descent, has no special meaning, and a new integrator is added in the middle panel of Figure 3. Viewed edge-on, these pictures show the analog solution compared to the correct solution. The analog accelerator returns only one of the two expected results, depending on the configuration of the system. When we use the analog accelerator, we have considered solving nonlinear equations.

Thus far, we have considered solving nonlinear equations. But nearly all PDEs, there is usually a good initial guess coming from the domain science. Luckily, in the context of solving nonlinear equations, there is normally a good initial condition for the two integrators, one storing the present guess of the present state. We test the continuous Newton's method to be the central choice of zero offers the best guarantee of converging to the actual solution. In the context of solving nonlinear equations, we can modify the Newton's method to be

\[ f(u) = u^2 + u - \text{RHS} \]

\[ f'(u) = 2u + 1 \]

When RHS equals zero, we get two valid solutions, and we need a correct choice of the initial conditions for the two integrators. We show the influence of these choices in the center panel of Figure 3. The analog accelerator returns only one of the two expected results, depending on the configuration of the system. The analog accelerator returns only one of the two expected results, depending on the configuration of the system. When we use the analog accelerator, we have considered solving nonlinear equations. But nearly all PDEs, there is usually a good initial guess coming from the domain science. Luckily, in the context of solving nonlinear equations, there is normally a good initial condition for the two integrators, one storing the present guess of the present state. We test the continuous Newton's method to be the central choice of zero offers the best guarantee of converging to the actual solution. In the context of solving nonlinear equations, we can modify the Newton's method to be
Ongoing Work

Thanks!