Physics-Informed Deep Learning for Traffic State Estimation and Fundamental Diagram Discovery

Sharon Di, Ph.D.
Civil Engineering & Engineering Mechanics
Data Science Institute

Data & innovative-technology driven Transportation Lab

Zhaobin Mo / Dr. Rongye Shi (Civil), Kuang Huang / Prof. Qiang Du (Math)
Traffic State Estimation: Highlights

*Train neural network (NN) w. sparse sensors?*

- Big data
- Small data

**Pattern discovery**
- Traffic Flow Model

**Physical constraint**
- NN regularized by traffic models
  - Outperform w. small data

- LWR data
- NGSIM data

- Train neural network (NN) w. sparse sensors?
- NN regularized by traffic models
  - Outperform w. small data

- LWR data
- NGSIM data
IoT: COSMOS Testbed

Cloud-Enhanced Open Software Defined Mobile Wireless Testbed for City-Scale Deployment

How to leverage data collected from IoT powered by CPS for real-time traffic state estimation?
Physics-Based vs. Data-Driven

Physics-Informed Machine Learning (PIML)

Use of Scientific Theory

Use of Data

Fluid Dynamics

Neuroscience
### Data-Driven Solutions of PDE

\[
\rho_t + \mathcal{N}_x[\rho] = 0, \quad t \in [0, T], \quad x \in \Omega
\]

- \(\rho(t, x)\): the solution of physical value (e.g., density/velocity field)
- \(\mathcal{N}_x[\cdot]\): a nonlinear differential operator
- \(\Omega\): a subset of \(\mathbb{R}^D\), denoting the high-dimensional physical space.

**Goal**  
approximate \(\rho(t, x)\) by a neural network

\[
\text{residual} \quad r \equiv \rho_t - \nabla_x[\rho]
\]

[Raissi-Maziar (2017, 2019)]
Training PIML

- Observation
  - $\hat{\rho}$
  - $\rho$
  - $\text{Observed}$

- Collocation
  - $\text{NN (}N{\theta}\text{)}$

- LWR ($\lambda$)
  - $\rho_t + (\rho U_\lambda(\rho))_x = 0$

Update $\theta$

Loss

$	ext{min Loss}$

End

Update $\lambda$

**Data-Driven Solution of LWR Models**

\[
\rho_t + (\rho u)_x = 0, \quad x \in (0, 1), \ t \in (0, 3)
\]

\[
u = u_{\text{max}} \left(1 - \frac{\rho}{\rho_{\text{max}}} \right)
\]

\[\rho(0, x) = \hat{\rho}_0(x) \text{ (initial condition)}\]

\[\rho(t, 0) = \rho(t, 1) \text{ (boundary condition)}\]

**Observation (labeled)** \(\{(t^i_o, x^i_o), \hat{\rho}^i\}, i = 1, \ldots, N_o\)

**Collocation (unlabeled)** \((t^i_c, x^i_c), i = 1, \ldots, N_c\)

**Loss** = \(\alpha \ \text{MSE}_o + \beta \ r_c\)

Obs. discrepancy

- Initial
- Boundary
- within domain

Traffic density: ground truth

Traffic density: reconstructed from sparse data

*ring road*
Fundamental Diagram (FD) Learner

Observation

Collocation

\[ N \]

Update \( \theta \)

Update \( \lambda \)

End

Observed

\[ \hat{\rho} \]

\[ \rho \]

\[ MSE_0 \]

Loss

Min Loss

\[ \text{FD learner} \ (N_\lambda) \]

ML surrogate

NGSIM: LWR as Physics

\[ \text{exact} - \rho(t, x) \]

Loss = \( \alpha \text{MSE}_o + \beta \ r_c \)

Obs. weight

Physics weight
Fundamental Diagram Learner (FDL)

3 loops

Number of Loop Detectors

Traffic density (normalized)

Estimation Error

- LWR-PIDL+FDL
- LWR-PIDL (using LWR with 3-parameter FD)
- ARZ-PIDL+FDL
- ARZ-PIDL (using ARZ with Greenshields FD)
THANK YOU!

Questions?