Taking a Spin with the Magnetic Pair Distribution Function



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Outline

- Introduction to magnetic PDF analysis
- Building intuition: Calculated mPDFs from simple systems
- Putting it into practice: mPDF of MnO from two different instruments
- Application to a genuine short-range ordered magnet: NaCaCo₂F₇
- Sharing the love: diffpy.mpdf open-source software

Short-range magnetic order: ubiquitous in modern magnetism



Neutron scattering from magnetic SRO



Neutron scattering from magnetic SRO



Fennel et al, Science 326 415 (2009)

Previous work

- Model magnetic scattering in momentum space
 - Keen & McGreevy, J. Phys.: Cond. Matt. **3** 7383 (1991)
 - Stewart, Andersen, & Cywinski, Phys. Rev. B 78 014428 (2008)
 - Paddison & Goodwin, PRL 108 017204 (2012)
 - Paddison, Stewart, & Goodwin, JPCM **25** 454220 (2013)





Paddison, Stewart, & Goodwin (2013)

Alternative approach in real space

- View and model magnetic correlations directly in real space
 - Blech & Averbach, Physics **1** 31 (1964)
 - Wu, Dmowski, Egami, & Chen, J. Appl. Phys. 61 3219 (1987)
 - Greedan et al, J. Appl. Phys. 67 5967 (1990)
 - Ehlers et al, Phys. Rev. B 81 224405 (2010)





Missing up until now: Analytical expression for the real-space magnetic pair distribution function

Magnetic

<u>Atomic</u>

After CL Farrow & SJL Billinge, Acta A 65 (2009) 232

Magnetic

 $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (\gamma r_0)^2 \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \mathbf{S}_{\perp i} \cdot \mathbf{S}_{\perp j}$

 $\frac{\text{Atomic}}{\mathrm{d}\Omega} = \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$

After CL Farrow & SJL Billinge, Acta A 65 (2009) 232

<u>Magnetic</u>

 $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (\gamma r_0)^2 \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \mathbf{S}_{\perp i} \cdot \mathbf{S}_{\perp j}$

Orientational average:

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{2}{3} NS(S+1)(\gamma r_0)^2 f^2 + \\ (\gamma r_0)^2 f^2 \sum_{i \neq j} \left[A_{ij} \frac{\sin Qr_{ij}}{Qr_{ij}} + B_{ij} \left(\frac{\sin Qr_{ij}}{(Qr_{ij})^3} - \frac{\cos Qr_{ij}}{(Qr_{ij})^2} \right) \right] \\ A_{ij} &= S_i^y S_j^y \quad B_{ij} = 2S_i^x S_j^x - S_i^y S_j^y \end{aligned}$$

$$\frac{\text{Atomic}}{\mathrm{d}\sigma} = \sum_{i,j} f_i(Q) f_j^*(Q) e^{iQ \cdot (\mathbf{r}_i - \mathbf{r}_j)}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = N f^2 + f^2 \sum_{i \neq j} \frac{\sin Q r_{ij}}{Q r_{ij}}$$

After CL Farrow & SJL Billinge, Acta A 65 (2009) 232

<u>Magnetic</u>

 $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (\gamma r_0)^2 \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \mathbf{S}_{\perp i} \cdot \mathbf{S}_{\perp j}$

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Normalize by form factor:

$$S(Q) = 1 + \frac{1}{N} \frac{3}{2S(S+1)} \sum_{i \neq j} \left[A_{ij} \frac{\sin Qr_{ij}}{Qr_{ij}} + B_{ij} \left(\frac{\sin Qr_{ij}}{(Qr_{ij})^3} - \frac{\cos Qr_{ij}}{(Qr_{ij})^2} \right) \right] \quad S(Q) = 1 + \frac{1}{N} \sum_{i \neq j} \frac{\sin Qr_{ij}}{Qr_{ij}}$$

After CL Farrow & SJL Billinge, Acta A 65 (2009) 232

 $\frac{\text{Atomic}}{\mathrm{d}\Omega} = \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$

Magnetic

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (\gamma r_0)^2 \sum_{i,j} f_i(\boldsymbol{Q}) f_j^*(\boldsymbol{Q}) e^{i\boldsymbol{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \mathbf{S}_{\perp i} \cdot \mathbf{S}_{\perp j}$$

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Fourier transform:

$$\begin{split} f(r) &= \frac{2}{\pi} \int_0^\infty Q \left(S(Q) - 1 \right) \sin Q r \mathrm{d}Q \\ &= \frac{1}{N} \frac{3}{2S(S+1)} \sum_{i \neq j} \left(\frac{A_{ij}}{r} \delta(r - r_{ij}) + B_{ij} \frac{r}{r_{ij}^3} \Theta(r_{ij} - r) \right) \\ &= \frac{1}{N} \sum_{i \neq j} \frac{1}{r} \delta(r - r_{ij}) \end{split}$$

After CL Farrow & SJL Billinge, Acta A 65 (2009) 232

 $\frac{\text{Atomic}}{\mathrm{d}\sigma} = \sum_{i,j} f_i(Q) f_j^*(Q) e^{iQ \cdot (\mathbf{r}_i - \mathbf{r}_j)}$

 Qr_{ij}

Effect of finite Q_{min} on Fourier transform

Atomic PDF:

$$G_{\text{nuc}}(r) = 4\pi r \left[\rho(r) - \rho_0\right]$$

$$= \frac{2}{\pi} \int_{Q_{\min}}^{\infty} F_{\text{nuc}}(Q) \sin Q r dQ$$

$$\neq \frac{2}{\pi} \int_{0}^{\infty} F_{\text{nuc}}(Q) \sin Q r dQ = 4\pi r \rho(r) = f_{\text{nuc}}(r)$$

(see CL Farrow & SJL Billinge, Acta A 65 (2009) 232)

Magnetic PDF:
$$G_{\rm m}(r) = \frac{2}{\pi} \int_{Q_{\rm min}}^{\infty} F(Q) \sin Q r dQ$$

$$= \frac{2}{\pi} \int_{0}^{\infty} F(Q) \sin Q r dQ - \frac{2}{\pi} \int_{0}^{Q_{\rm min}} F(Q) \sin Q r dQ$$
$$= f(r) - C(m)r$$
Net magnetization

- Zero for antiferromagnets
- Nonzero for ferro/ferrimagnets

A closer look



Example: One-dimensional chain



Example: One-dimensional SDW



Example: Cubic antiferromagnet







2 partners: positive peak, negative baseline4 anti-partners: negative

peak, positive baseline









Types of 3-spin chains: O-P-A, O-A-P: positive peak O-P-P, O-A-A: negative peak

4x2x2 + 4x1x2 + 2x2x2 + 2x1x2 O-A-A O-A-P O-P-A O-P-P

> 36 possibilities: 16 O-P-A or O-A-P (positive peak) 20 O-A-A or O-P-P (negative peak)



3-spin chains:

O-P-A, O-A-P: positive peak O-P-P, O-A-A: negative peak 36 possibilities: 16 O-P-A or O-A-P (positive) 20 O-A-A or O-P-P (negative)

Obtaining the experimental mPDF

Test case: MnO

- T_N = 118 K
- Spins aligned within (111)type sheets, anti-aligned between sheets
- Rhombohedral contraction along [111]
- First material to have its magnetic structure determined! (Schull et al 1951)



The challenge

- Accurately isolate the full magnetic scattering intensity over a sufficient Q-range
 - Polarized neutrons (i.e. xyz/10-pt polarization)
 - Fit to and subtract out nuclear structure
 - More sophisticated approaches: field-dependent measurements, combined x-ray/neutron, etc.
- Sequential/simultaneous nuclear and magnetic PDF analysis

mPDF analysis of MnO





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Conventional neutron scattering from MnO



Obtaining the experimental mPDF

- Fit to and subtract out the nuclear Bragg peaks
- Normalize by the magnetic form factor
- Fourier transform to obtain the real-space mPDF
- Quite noisy due to amplification of small errors in normalization step
- Sufficient to verify mPDF equations and encourage future developments
- Is there another way?



Simultaneous atomic and magnetic PDF

- Perform neutron total scattering experiment just like usual atomic PDF
- Process data according to standard atomic PDF protocols
- Result: additive combination of the usual atomic PDF and the (unnormalized) mPDF
- Model the atomic and magnetic PDFs together



Total PDF: nuclear PDF + mPDF

$$G_{tot}(r) = \mathcal{F} \left[Q(S_{tot}(Q) - 1) \right]$$
$$S_{tot}(Q) - 1 = \frac{I_{tot}}{N\langle b \rangle^2} - \frac{\langle b^2 \rangle}{\langle b \rangle^2}$$

$$I_{tot} = I_n + I_m$$

$$G_{tot}(r) = \mathcal{F}\left[Q\left(\frac{I_n}{N\langle b\rangle^2} - \frac{\langle b^2 \rangle}{\langle b\rangle^2}\right)\right] + \mathcal{F}\left[Q\frac{I_m}{N\langle b\rangle^2}\right]$$
$$= G_n(r) + \frac{d(r)}{N\langle b\rangle^2}$$

d(r): "unnormalized" mPDF $d(r) = \mathcal{F}\left[QI_m(Q)\right] \quad \text{cf.} \quad f(r) \sim \mathcal{F}\left[Q\left(\frac{I_m(Q)}{f^2(Q)} - 1\right)\right]$ Properly normalized mPDF $d(r) = f(r) * S(r) - \sqrt{2\pi} \frac{\mathrm{d}S}{\mathrm{d}r}$ $S(r) = s(r) * s(r) = \int \mathrm{d}r' s(r') s(r-r')$ Related to real-space extent of magnetic moment $s(r) = \mathcal{F}[f(Q)] = \int \mathrm{d}Q \exp iQr f(Q)$ **Magnetic form factor**

For details: Frandsen & Billinge, Acta A 71 325 (2015)

"Unnormalized" mPDF: MnO



Atomic PDF fits



Atomic PDF fit: difference curve



Atomic PDF fit: difference curve



Including the mPDF contribution



Ab initio magnetic structure solution

Random initial spin configuration

Ab initio magnetic structure solution



Ab initio magnetic structure solution



mPDF from MnO in paramagnetic phase





Superexchange dominates over direct exchange

Density functional theory (SIC-LSDA/DLM approach), with Julie Staunton at U. Warwick

Summary of methods to obtain the mPDF

- Separate magnetic/nuclear scattering in reciprocal space and do proper normalization
 - Potential for high-resolution mPDF data
 - Data must be treated carefully
 - Should be developed further!
- Perform standard PDF experiment and get nuclear and (unnormalized) magnetic PDFs together
 - Easy and practical; no special effort required!
 - Lower real-space resolution
 - Easiest entry-point for PDF practitioners looking to do mPDF analysis

Handling sub-optimal mPDF data

- What if the mPDF is much smaller than the nuclear PDF?
- What if your model for the atomic structure is of limited quality?



Relative scales of magnetic/atomic PDFs

Ratio of mPDF scale factor to nuclear PDF scale factor:

$$\frac{N_s}{N_a} \frac{\frac{2}{3} (\frac{\gamma r_0}{2})^2 (gJ)^2}{2\pi \langle b \rangle^2}$$

MnO

- $N_s/N_a = 1/2$
- is quite small due to negative scattering length of Mn
- Very large mPDF signal!

NaCaCo₂F₇:

- $N_s/N_a = 2/11$
- is much larger
- Greatly reduced mPDF signal

(Side note: This gives us a way to determine the ordered moment from mPDF measurements!)

mPDF Analysis of NaCaCo₂F₇



Kate Ross



Bob Cava



Jason Krizan

Andrew Wildes





Gøran Nilsen

Gemoetrically Frustrated Magnets

Spins on a triangular lattice





Case 1: Ferromagnetic interactions

Case 2: Antiferromagnetic interactions

Transition Metal Fluoride Pyrochlores



 $NaCaCo_2F_7$



NaSrCo₂F₇



NaCaNi₂F₇

Co compounds: Spins freeze into short-range order below 5 K, remain correlated at higher temperatures





Pyrochlore network: Corner-sharing tetrahedra, HIGHLY FRUSTRATED!

Ross et al, PRB 93, 014433 (2016)

What is the nature of the shortrange magnetic correlations?

NaCaCo₂F₇

- Magnetic Co²⁺ ions on the highly frustrated pyrochlore lattice
- Short-range magnetic correlations freeze below ~3 K
- Promising material for extending studies of frustrated magnetism to strongly correlated 3*d* atoms





Obtaining the mPDF for NaCaCo₂F₇

1. Perform standard PDF measurement on NOMAD, collecting atomic and magnetic $\sqrt[n]{U}_{U}$ PDFs together



2. Refine the atomic structure, subtract high-temperature atomic fit residual from low-temperature fit residual to isolate mPDF



3. Remove high-frequency components that cannot arise from magnetic scattering



Magnetic PDF analysis of NaCaCo₂F₇



Modeling Attempt #1: Start with randomly oriented spins on each site and refine each independently, with scale factor and correlation length



Magnetic PDF analysis of NaCaCo₂F₇



Modeling Attempt #1: Start with randomly oriented spins on each site and refine each independently, with scale factor and correlation length

Frandsen et al, Phys. Rev. Mater. 1, 074412 (2017)

Non-collinear, net antiferromagnetic correlations

> **Collinear antiferromagnetic correlations between neighboring tetrahedra**

$$++++=0$$

Magnetic PDF analysis of NaCaCo₂F₇



Modeling Attempt #2: Same as #1, but with an additional collinear antiferromagnetic component



Significance of the refined spin directions



Refined spins closely match linear combination of ψ_2 and ψ_3 ; fit with either BV nearly as good. Correlation length ≈ 1 nm.

Collinear AF component: lowenergy (< 10 meV) excitations observed from single-crystal INS; $\xi \approx 4 \text{ Å}$

Temperature Dependent mPDF analysis



Correlations persist to unusually high temperatures!

Comparison to polarized neutron scattering



Comparison to polarized neutron scattering



NaCaCo₂F₇ Summary

- ► Refinement of local magnetic structure against mPDF data confirms single-crystal results: ψ_2/ψ_3 + collinear AF
- New fluoride pyrochlores are promising platform for frustrated magnetism at relatively high temperatures
- MPDF enables real-space inspection and analysis of shortrange magnetic correlations in frustrated materials





I WANT YOU TO START DOING mPDF

- diffpy.mpdf (<u>http://www.diffpy.org/products/mPDF.html</u>)
 - Open-source python package
 - Runs on Linux and MacOS
 - Requires other packages in the DiffPy-CMI suite as dependencies
- Features
 - Calculate the normalized and unnormalized mPDF from an arbitrary magnetic structure
 - Build magnetic structures of arbitrary complexity using propagation vectors or a magnetic unit cell
 - Include multiple magnetic species in one structure
 - Easily refine magnetic models against mPDF data
 - Atomic/magnetic PDF co-refinements within DiffPy-CMI, or sequential refinements with convenient interface to PDFgui
 - Easily customizable for your own tailored calculators/refinements
 - Do simple 3D visualizations of your magnetic structure
 - Calculate magnetic scattering pattern from magnetic structure
 - And many more features!

Installation

Conda installation

conda config --add channels diffpy
conda config --add channels benfrandsen

conda create --name diffpy python=2.7
source activate diffpy

conda install diffpy.mpdf

Installation from source

Download source files from https://github.com/benfrandsen/diffpy.magpdf

python setup.py install

Pip installation

pip install diffpy.mpdf

MPDFcalculator Calculates mPDF from an array of spin vectors and their positions

MagStructure Generate an array of spin vectors and positions

MPDFcalculator Calculates mPDF from an array of spin vectors and their positions



Specify basis vectors, propagation vectors, magnetic form factor, etc



Specify basis vectors, propagation vectors, magnetic form factor, etc

Getting started with diffpy.mpdf

Tutorials and example scripts on the CMI Exchange github page:

https://github.com/diffpy/cmi_exchange/tree/master/cmi_scripts/mpdf

Simple example: Calculating the mPDF from a spin dimer

We will now create a very simple magnetic structure consisting of just two spins and then calculate the corresponding mPDF.

In [1]: ### Import the necessary Libraries
import numpy as np
import matplotlib.pyplot as plt
from diffpy.mpdf import *

Set all plots to be inline %matplotlib notebook

Create a MagStructure object
mstr = MagStructure()

Create two atoms in the structure
mstr.atoms = np.array([[0,0,0],[4,0,0]])

Create two spin vectors corresponding to the atoms. Let's make them antiferromagnetic. S=np.array([0,0,1]) mstr.spins = np.array([S,-S])

Create the MPDFcalculator object and load the magnetic structure into it mc = MPDFcalculator(mstr)

Calculate and plot the mPDF!
r,fr = mc.calc() # Use calc() if you want to extract the numerical results of the calculation
mc.plot() # Use plot() if you just want to plot the mPDF without extracting the numerical arrays.

<IPython.core.display.Javascript object>



Email me and I will help you get started! benfrandsen@byu.edu

Summary

- mPDF method provides a way to inspect and model magnetic correlations in real space
- Data can be obtained from conventional neutron beamlines: both standard diffractometers and PDFoptimized instruments (get the mPDF for free along with your atomic PDF!)
- Protocols for successful treatment of weak and/or noisy mPDF signal
- diffpy.mpdf: free and open-source software for mPDF analysis
- Successful application to a variety of magnetic materials with widely varying correlation lengths

References

- Acta A 70 3 (2014)
- Acta A **71** 325 (2015)
- *PRL* **116** 197204 (2016)
- *PRB* 94 094102 (2016)
- Phys. Rev. Mater. 1, 074412 (2017)
- More to come...

Thank you!