International Journal of Modern Physics E

A METHOD FOR MEASURING ELLIPTIC FLOW FLUCTUATIONS WITH THE PHOBOS DETECTOR

BURAK ALVER for the PHOBOS collaboration

Laboratory for Nuclear Science, Massachusetts Institute of Technology, 77 Massachusetts Ave.
Cambridge, MA 02139, USA
alver@mit.edu

B.Alver[4], B.B.Back[1], M.D.Baker[2], M.Ballintijn[4], D.S.Barton[2], R.R.Betts[6], A.A.Bickley[7], R.Bindel[7], W.Busza[4], A.Carroll[2], Z.Chai[2], M.P.Decowski[4], E.García[6], T.Gburek[3], N.George[2], K.Gulbrandsen[4], C.Halliwell[6], J.Hamblen[8], M.Hauer[2], C.Henderson[4], D.J.Hofman[6], R.S.Hollis[6], R.Hołyński[3], B.Holzman[2], A.Iordanova[6], E.Johnson[8], J.L.Kane[4], N.Khan[8], P.Kulinich[4], C.M.Kuo[5], W.Li[4], W.T.Lin[5], C.Loizides[4], S.Manly[8], A.C.Mignerey[7], R.Nouicer[2,6], A.Olszewski[3], R.Pak[2], C.Reed[4], C.Roland[4], G.Roland[4], J.Sagerer[6], H.Seals[2], I.Sedykh[2], C.E.Smith[6], M.A.Stankiewicz[2], P.Steinberg[2], G.S.F.Stephans[4], A.Sukhanov[2], M.B.Tonjes[7], A.Trzupek[3], C.Vale[4], G.J.van Nieuwenhuizen[4], S.S.Vaurynovich[4], R.Verdier[4], G.I.Veres[4], P.Walters[8], E.Wenger[4], F.L.H.Wolfs[8], B.Wosiek[3], K.Woźniak[3], B.Wysłouch[4]

[1] *Argonne National Laboratory, Argonne, IL 60439, USA*
[2] *Brookhaven National Laboratory, Upton, NY 11973, USA*
[3] *Institute of Nuclear Physics PAN, Krakow, Poland*
[4] *Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[5] *National Central University, Chung-Li, Taiwan*
[6] *University of Illinois at Chicago, Chicago, IL 60607, USA*
[7] *University of Maryland, College Park, MD 20742, USA*
[8] *University of Rochester,Rochester, NY 14627, USA*

Received (received date)
Revised (revised date)

We have performed the first measurement of elliptic flow (v_2) fluctuations in nucleus-nucleus collisions. In this paper, we describe the analysis method we have developed for this measurement. In this method, v_2 is determined event-by-event by a maximum likelihood fit. The non-statistical fluctuations are determined by unfolding the contribution of statistical fluctuations and detector effects using Monte Carlo simulations. Application of this method to measure dynamical fluctuations in events from a different Monte Carlo event generator is presented.

1. Introduction

Elliptic flow (v_2) is one of the key observables in the understanding of the dynamics heavy ion collisions. The recent measurement of elliptic flow fluctuations,[1] a preliminary version of which was presented at the Quark Matter 2006 Conference,[2] has provided qualitatively new information on the initial conditions of heavy ion collisions and the subsequent collective expansion of the system. In this paper, we discuss the analysis method developed to perform this measurement with the PHOBOS detector. The method was first introduced in a workshop on correlations and fluctuations.[3] In this paper, we will discuss some improvements in the method and the estimation of the systematic uncertainty and present a test of the whole analysis procedure on simulated AMPT[4] events.

2. Method

In the analysis presented, an estimate of v_2 is made event-by-event via a maximum-likelihood fit to the hit distribution on the PHOBOS multiplicity array. The response function of the event-by-event measurement, containing the contribution of statistical fluctuations and detector effects is calculated using simulations of MC events from the HIJING[5] event generator. Non-statistical fluctuations in data are extracted by unfolding the response function from the distribution of the event-by-event measurement.

2.1. *Event-By-Event Measurement*

The PHOBOS detector is composed primarily of silicon pad detectors for tracking, vertex reconstruction, and multiplicity measurements. Details of the setup and the layout of the silicon sensors can be found elsewhere.[6] Key elements of the detector used in this analysis include the silicon vertex detector (VTX), the silicon octagon multiplicity detector (OCT), three annular silicon multiplicity detectors to either side of the collision point (RING), and two sets of scintillating paddle counters for characterizing collision centrality. The PHOBOS multiplicity array (VTX, OCT and RING) covers a large fraction of the full solid angle. At midrapidity, the vertex detector and the octagonal multiplicity detector have different pad sizes and the acceptance in azimuth is not complete. The event-by-event measurement method has been developed to use all the available information from the multiplicity array to measure a single value of the event elliptic flow, while allowing an efficient correction for the non-uniformities in the acceptance.

The maximum likelihood method was applied for this purpose. We parametrize the pseudorapidity dependence, $v_2(\eta)$, with a single parameter, $\nu_2 \equiv v_2(0)$, and a triangular or trapezoidal shape, given by $v_2^{\rm tri}(\eta) = \nu_2\,(1 - \frac{|\eta|}{6})$, or $v_2^{\rm trap}(\eta) = \left\{ \begin{smallmatrix} \nu_2 \text{ if } |\eta|<2 \\ \frac{3}{2}\, v_2^{\rm tri}(\eta) \text{ if } |\eta|\geq 2 \end{smallmatrix} \right.$, respectively. These functions describe the main features of the pseudorapidity dependence of v_2 over a range of centralities.[7] Taking into account correlations only due to elliptic flow, the probability of a particle with given pseudorapidity, η, to be emitted in the azimuthal angle, ϕ, in an event with elliptic flow magnitude, ν_2, and event-plane angle ϕ_0 is given by

$$p(\phi|\nu_2, \phi_0; \eta) = \frac{1}{2\pi}\left\{1 + 2v_2(\eta)\cos\left(2\left[\phi - \phi_0\right]\right)\right\}. \quad (1)$$

At the points where charged tracks passed through an active silicon detector, energy is deposited in the form of ionization. A pad where energy is deposited is said to be a "hit".[8] We define the probability density function (PDF) for a hit position (η, ϕ) for an event with ν_2 and event-plane angle ϕ_0 as

$$P(\phi|\nu_2, \phi_0; \eta) = \frac{1}{s(\nu_2, \phi_0; \eta)} p(\phi|\nu_2, \phi_0; \eta), \tag{2}$$

where the normalization parameter $s(\nu_2, \phi_0; \eta)$ is calculated in small bins of η such that the PDF folded with the acceptance, $A(\eta', \phi)$, is properly normalized for different values of ν_2 and ϕ_0:

$$s(\nu_2, \phi_0; \eta) = \int_{\eta-\Delta\eta}^{\eta+\Delta\eta} A(\eta', \phi) p(\phi|\nu_2, \phi_0; \eta), \mathrm{d}\phi \mathrm{d}\eta'. \tag{3}$$

For a single event, the likelihood function of ν_2 and ϕ_0 is defined as $L(\nu_2, \phi_0) \equiv \prod_{i=1}^{n} P(\phi_i|\nu_2, \phi_0; \eta_i)$, where the product is over all n hits in the detector. The likelihood function describes the probability of observing the hits in the event for the given values of the parameters ν_2 and ϕ_0. Maximizing $L(\nu_2, \phi_0)$ as a function of ν_2 and ϕ_0 allows us to measure ν_2^{obs} event-by-event.

2.2. *The Response Function*

The response of the event-by-event measurement is non-linear and depends on the observed multiplicity n. Therefore, a detailed study of the response function is required to extract the true ν_2 distribution from the measured ν_2^{obs} distribution. Let $f(\nu_2)$ be the true ν_2 distribution for a set of events in a given centrality bin, and $g(\nu_2^{\mathrm{obs}})$ the corresponding observed distribution. The true and observed distributions are related by

$$g(\nu_2^{\mathrm{obs}}) = \int K(\nu_2^{\mathrm{obs}}, \nu_2, n)\, f(\nu_2)\, \mathrm{d}\nu_2\, N(n)\, \mathrm{d}n, \tag{4}$$

where $N(n)$ is the multiplicity distribution of the given set of events and $K(\nu_2^{\mathrm{obs}}, \nu_2, n)$ is the expected distribution of ν_2^{obs} for events with fixed input flow ν_2, and constant observed multiplicity n.

The response function, $K(\nu_2^{\mathrm{obs}}, \nu_2, n)$, is determined by performing the event-by-event analysis on modified HIJING[5] events with flow of fixed magnitude ν_2. The flow is introduced by redistributing the generated particles in each event in the ϕ direction according to the probability distribution given by Eq. 1. These modified events are run through GEANT[9] to simulate the PHOBOS detector response. For the two parameterizations of $v_2(\eta)$, triangular and trapezoidal, used in the event-by-event measurement, the corresponding response functions, K^{tri} and K^{trap}, are calculated. Fitting smooth functions through the observed response functions decreases bin-to-bin fluctuations and allows for interpolation in ν_2 and n. The response of a perfect detector has been determined as a function of event multiplicity by Ollitrault.[10] In practice, some empirical modifications to the ideal relation, accounting for the detector effects, significantly improve fits to the response function, leading to

$$K(\nu_2^{\rm obs},\nu_2,n)=\frac{\nu_2^{\rm obs}}{\sigma^2}\,\exp\left(-\frac{\left(\nu_2^{\rm obs}\right)^2+\left(\nu_2^{\rm mod}\right)^2}{2\sigma^2}\right)I_0\left(-\frac{\nu_2^{\rm obs}\nu_2^{\rm mod}}{\sigma^2}\right),\qquad(5)$$

with $\nu_2^{\rm mod}=(A\,n+B)\nu_2$ and $\sigma=C/\sqrt{n}+D$, and where I_0 is the modified Bessel function. The four parameters (A,B,C,D) are obtained by fits to observed $K(\nu_2^{\rm obs},\nu_2,n)$ in the modified HIJING samples.

2.3. *Calculation of Dynamical Fluctuations*

The true event-by-event ν_2 distribution, $f(\nu_2)$, is assumed to be a Gaussian in the range $\nu_2>0$, with two parameters, mean $(\bar{\nu}_2)$ and standard deviation (σ_{ν_2}). For given values of the parameters, it is possible to take the integral in Eq. 4 numerically to obtain the expected $\nu_2^{\rm obs}$ distribution. Comparing the expected and observed distributions, the values of $\bar{\nu}_2$ and σ_{ν_2} are found by a maximum-likelihood fit. Midrapidity ($|\eta|<1$) results from the two parameterizations of $v_2(\eta)$, triangular and trapezoidal, are averaged as $\langle v_2\rangle=0.5(\frac{11}{12}\bar{\nu}_2^{\rm tri}+\bar{\nu}_2^{\rm trap})$ and $\sigma_{v_2}=0.5(\frac{11}{12}\sigma_{\nu_2}^{\rm tri}+\sigma_{\nu_2}^{\rm trap})$, where the factor $\frac{11}{12}$ comes from integration over η.

The induced v_2 fluctuations arising from fluctuations in the number of participating nucleons are calculated by parameterizing the $\langle v_2\rangle$ versus $N_{\rm part}$ results and folding them with the $N_{\rm part}$ distributions in each centrality bin. The relative contribution of these fluctuations to σ_{v_2} is found to be less than 8% in real data.

2.4. *Determination of the Systematic Uncertainty*

Systematic errors have been investigated in three main classes: variations to the event-by-event analysis, response of the measurement to known input σ_{v_2}, and intrinsic differences between HIJING events and data.

Various modifications to the event-by-event analysis have been applied. Corrections, used in the hit-based event-plane analysis,[8,11] to account for signal dilution due to detector occupancy and to create an appropriately symmetric acceptance have been applied to both HIJING and data events. Hit definitions have been varied. These changes lead to at most 4% variations in the observed relative fluctuations demonstrating a good understanding of the response function.

The determination of the response function and the final fitting procedure have been studied by performing the analysis on sets of modified HIJING events with varying input σ_{v_2}. Differences between input and reconstructed σ_{v_2} are identified as a contribution to the systematic uncertainty. The sensitivity of the measurement is observed to be limited for very low $\langle v_2\rangle$ values. Therefore the 0-6% most central events, where the reconstructed $\langle v_2\rangle$ is below 3%, have been omitted.

Differences between HIJING and data in terms of $dN/d\eta$, $v_2(\eta)$ and particle correlations other than flow (non-flow correlations) can, in principle, lead to a miscalculation of the response function. A sample of MC events has been generated, in

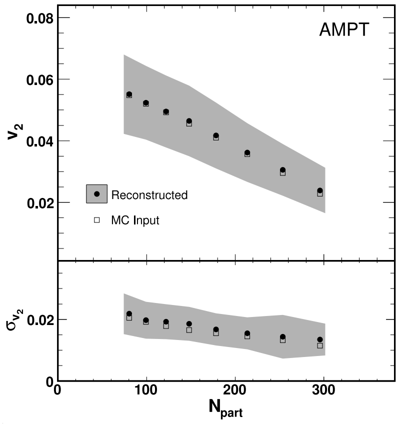

Fig. 1. $\langle v_2 \rangle$ (top) and σ_{v_2} (bottom) versus $N_{\rm part}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated with AMPT. Filled symbols show reconstructed results from detector simulations. The band shows 90% confidence level systematic errors. Open symbols show results reconstructed from the generated primary particles.

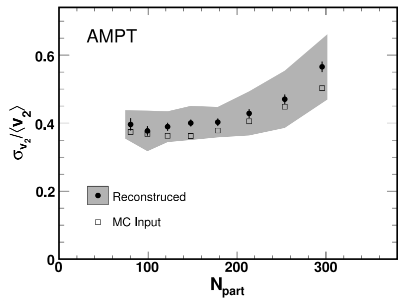

Fig. 2. $\sigma_{v_2}/\langle v_2 \rangle$ versus $N_{\rm part}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated with AMPT. Filled symbols show reconstructed results from detector simulations. The band shows 90% confidence level systematic errors. Open symbols show results reconstructed from the generated primary particles.

which the $dN/d\eta$ distribution of HIJING events is widened by a simple scaling to match the measurements in data within the errors.[12] The difference between results obtained with and without this modification, as well as the difference between results with two different parameterizations of $v_2(\eta)$ are identified as contributions to the systematic uncertainty. The non-flow correlation strength in HIJING was found to be of comparable magnitude to the observed strength in data.[13] A different set of MC events has been generated, in which the flow is introduced by shifting the particle momenta in the azimuthal direction, preserving other correlations. Differences between the results obtained with these MC events, to the results obtained using MC events with only flow correlations, are identified as another contribution to the systematic uncertainty.

Other systematic studies include using a flat, rather than Gaussian, ansatz for the true ν_2 distribution, $f(\nu_2)$, and performing the analysis in different collision vertex and event-plane angle bins. The uncertainty in the contribution of $N_{\rm part}$ induced fluctuations has also been estimated via different parameterizations of the $\langle v_2 \rangle$ versus $N_{\rm part}$ results. Contributions from all error sources described above are added in quadrature to derive the 90% confidence level error.

3. Verification of the Complete Analysis Chain

The whole analysis procedure is tested using simulated AMPT[4] events as "data". The Fig. 1 shows the mean, $\langle v_2 \rangle$, and the standard deviation, σ_{v_2}, of the elliptic flow parameter v_2 at midrapidity as a function of the number of participating nucleons, in AMPT Au+Au events at $\sqrt{s_{_{NN}}} = 200$ GeV for 6–45% most central events. Since the information about the generated v_2 in AMPT events is not readily available, the true $\langle v_2 \rangle$ and σ_{v_2} were extracted from a mixed-event analysis based on the true particle information. The systematic uncertainty in the measurement on the AMPT events was estimated similar to the way it is estimated on real data. The induced v_2 fluctuations due to fluctuations in N_{part} and due to non-flow correlations was not calculated in AMPT since these effects would show in the measurements both on the true particles and the simulations which are compared. The results from the analysis on simulations are observed to be in good agreement with the input.

The uncertainties in $dN/d\eta$ and $v_2(\eta)$, as well as differences between HIJING and the data (in this case AMPT) in these quantities, introduce a large uncertainty in the overall scale in the event-by-event analysis due to the averaging procedure over the wide pseudorapidity range. Most of the scale errors cancel in the ratio, $\sigma_{v_2}/\langle v_2 \rangle$. These "relative flow fluctuations" are shown in Fig. 2 as a function of the number of participating nucleons for AMPT. These results are also observed to be in good agreement with the input MC values also shown in Fig. 2.

In summary, we have discussed the analysis method we have used to measure elliptic flow fluctuations.[1] We have demonstrated that the method gives accurate results within the estimated uncertainty on MC events of a different generator (AMPT) from the one used to calculate the measurement response (HIJING).

This work was partially supported by U.S. DOE grants DE-AC02-98CH10886, DE-FG02-93ER40802, DE-FG02-94ER40818, DE-FG02-94ER40865, DE-FG02-99ER41099, and DE-AC02-06CH11357, by U.S. NSF grants 9603486, 0072204, and 0245011, by Polish KBN grant 1-P03B-062-27(2004-2007), by NSC of Taiwan Contract NSC 89-2112-M-008-024, and by Hungarian OTKA grant (F 049823).

References

1. B. Alver *et al.* [PHOBOS Collaboration], nucl-ex/0702036, submitted to Phys. Rev. Lett.
2. C. Loizides for the PHOBOS Collaboration, nucl-ex/0701049, Proceedings for Quark Matter 2006
3. B. Alver for the PHOBOS Collaboration, Proceedings for workshop on correlations and fluctuations in Florence, PoS(CFRNC2006)023 (2006)
4. Z.W. Lin *et al.* Phys.Rev. **C 72**, 064901 (2005)
5. M. Gyulassy and X. N. Wang, Comput. Phys. Commun. **B 83**, 307 (1994)
6. B.B. Back *et al.* [PHOBOS Collaboration], Nucl. Instrum. Meth. **A 499**, 603 (2003)
7. B.B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. **C72**, 051901(R) (2005)
8. B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **89**, 222301 (2002)
9. R. Brun *et al.*, GEANT3 Users Guide, CERN DD/EE/84-1
10. J. Y. Ollitrault, Phys. Rev. **D 46**, 229 (1992)
11. B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **95**, 122303 (2005)
12. B.B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **91**, 052303 (2003)
13. B.B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. **C 74**, 011901 (2006)