Elliptic flow fluctuations in Au+Au collisions at $\sqrt{s_{_{NN}}} = 200$ GeV

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This paper presents the first measurement of event-by-event fluctuations of charged particle elliptic flow in Au+Au collisions at $\sqrt{s_{_{NN}}} = 200$ GeV as a function of collision centrality. The relative non-statistical fluctuations of the elliptic flow coefficient, v_2, are found to be approximately 40%. The magnitude of the observed relative fluctuations in v_2 agrees with predictions based on spatial fluctuations of the participating nucleons in the initial nuclear overlap region. These results are consistent with a scenario where v_2 is directly proportional to the eccentricity of the initial matter distribution on an event-by-event basis. The subsequent evolution of the system does not change this in any measurable way, consistent with hydrodynamic evolution with a very low viscosity. The agreement of the results with event-by-event eccentricity calculations indicates that matter in the initial stage of the collision is produced with a transverse granularity similar to that of the participant nucleons.

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Results from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory suggest that a dense state of matter is formed in ultrarelativistic nucleus-nucleus collisions [1, 2, 3, 4]. Studies of final state charged particle momentum distributions show that the produced matter undergoes a rapid collective expansion transverse to the direction of the colliding nuclei. In particular, for collisions at large impact parameter, the expansion shows a significant anisotropy in the azimuthal angle, strongly correlated with the anisotropic shape of the initial nuclear overlap region. The dominant component of this anisotropic expansion is called "elliptic flow" and is commonly quantified by the second coefficient, v_2, of a Fourier decomposition of the azimuthal distribution of observed particles relative to the event-plane angle.

Elliptic flow has been studied extensively in Au+Au collisions at RHIC as a function of pseudorapidity, centrality, transverse momentum and center-of-mass energy [3, 4, 5, 6, 7]. For Au+Au collisions at RHIC energies, the observed dependence of the elliptic flow signal on centrality and transverse momentum is found to be in good agreement with calculations in hydrodynamic models [7, 8]. This is considered evidence for an early equilibration of the colliding system and a low viscosity of the matter produced in the early stage of the collision process [9]. In such calculations, for given conditions of the produced matter, the elliptic flow magnitude v_2 is found empirically to be proportional to the eccentricity, ϵ, characterizing the transverse shape of the initial nuclear overlap region [10].

Recent measurements of elliptic flow in the smaller Cu+Cu system have shown surprisingly large values of elliptic flow, in particular for the most central collisions where the average eccentricity of the nuclear overlap region is small [11]. The results for Cu+Cu and Au+Au collisions can be reconciled if event-by-event fluctuations in the initial eccentricity are considered. To account for these fluctuations, we have proposed a definition of the eccentricity that does not make reference to the direction of the impact parameter vector, but rather characterizes the eccentricity through the event-by-event distribution of nucleon-nucleon interaction points obtained from a Glauber Monte-Carlo calculation [11, 12, 13]. This "participant eccentricity" is defined as

$$\epsilon_{\mathrm{part}} \equiv \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2}}{\sigma_y^2 + \sigma_x^2}, \quad (1)$$

corresponding to a principal component transformation that maximizes the eccentricity for each event, where

σ_x^2, σ_y^2, and σ_{xy} are the event-by-event (co-)variances of the participant nucleon distributions projected on the transverse axes, x and y. The covariance term, σ_{xy}, which leads to a finite initial eccentricity even for the most central events and has a large effect in the smaller Cu+Cu system, has been found to be crucial for understanding the comparison of Cu+Cu and Au+Au elliptic flow results [11].

Using the probabilistic distribution of interaction points obtained from a Glauber calculation, performed on an event-by-event basis, leads to relative eccentricity fluctuations of $\sigma_{\epsilon_{\rm part}}/\langle\epsilon_{\rm part}\rangle \approx 40\%$ for Au+Au collisions at fixed number of participating nucleons ($N_{\rm part}$) [14]. If v_2 is proportional to ϵ, an event-by-event measurement of elliptic flow should therefore exhibit large fluctuations in v_2, even at fixed $N_{\rm part}$. All previous measurements of elliptic flow were reported for averages over large ensembles of events in bins of centrality. In this letter we present the first measurement of event-by-event v_2 fluctuations as a function of collision centrality in Au+Au collisions at $\sqrt{s_{_{NN}}} = 200$ GeV.

The data shown here were taken with the PHOBOS detector at RHIC during the year 2004. The PHOBOS detector is composed primarily of silicon pad detectors for tracking, vertex reconstruction, and multiplicity measurements. Details of the setup and the layout of the silicon sensors can be found elsewhere [15]. Key elements of the detector used in this analysis include the silicon vertex detector (VTX), the silicon octagon multiplicity detector (OCT), three annular silicon multiplicity detectors to either side of the collision point (RING), and two sets of scintillating paddle counters for characterizing collision centrality.

The Monte Carlo (MC) simulations of the detector performance are based on the HIJING event generator [16] and the GEANT 3.21 [17] simulation package, folding in the signal response for scintillator counters and silicon sensors.

In the analysis presented in this letter, an estimate of v_2 is made event-by-event via a maximum-likelihood fit to the hit distribution on the PHOBOS multiplicity array (VTX, OCT and RING). The response function of the event-by-event measurement, containing the contribution of statistical fluctuations and detector effects is calculated using MC simulations. Non-statistical fluctuations in data are extracted by unfolding the response function from the distribution of the event-by-event measurement.

The PHOBOS multiplicity array covers almost the full solid angle. We parametrize the pseudorapidity dependence, $v_2(\eta)$, with a single parameter, $\nu_2 \equiv v_2(0)$, and a triangular or trapezoidal shape, given by $v_2^{\rm tri}(\eta) = \nu_2\,(1 - \frac{|\eta|}{6})$, or $v_2^{\rm trap}(\eta) = \left\{ \begin{smallmatrix} \nu_2 \text{ if } |\eta|<2 \\ \frac{3}{2}\, v_2^{\rm tri}(\eta) \text{ if } |\eta|\geq 2 \end{smallmatrix} \right.$, respectively. The event-by-event measurement method has been developed to use all the available information from the multiplicity array to measure the elliptic flow at zero rapidity, ν_2, while allowing an efficient correction for the non-uniformities in the acceptance. Taking into account correlations only due to elliptic flow, the probability of a particle with given pseudorapidity, η, to be emitted in the azimuthal angle, ϕ, in an event with elliptic flow magnitude, ν_2, and event-plane angle ϕ_0 is given by

$$p(\phi|\nu_2, \phi_0; \eta) = \frac{1}{2\pi}\left\{1 + 2v_2(\eta)\cos\left(2\left[\phi - \phi_0\right]\right)\right\}. \quad (2)$$

At the points where charged tracks passed through an active silicon detector, energy is deposited in the form of ionization. A pad where energy is deposited is said to be a “hit” [5]. We define the probability density function (PDF) for a hit position (η, ϕ) for an event with ν_2 and event-plane angle ϕ_0 as

$$P(\phi|\nu_2, \phi_0; \eta) = \frac{1}{s(\nu_2, \phi_0; \eta)} p(\phi|\nu_2, \phi_0; \eta), \quad (3)$$

where the normalization parameter $s(\nu_2, \phi_0; \eta)$ is calculated in small bins of η such that the PDF folded with the acceptance is properly normalized for different values of ν_2 and ϕ_0.

For a single event, the likelihood function of ν_2 and ϕ_0 is defined as $L(\nu_2, \phi_0) \equiv \prod_{i=1}^{n} P(\phi_i|\nu_2, \phi_0; \eta_i)$, where the product is over all n hits in the detector. The likelihood function describes the probability of observing the hits in the event for the given values of the parameters ν_2 and ϕ_0. The parameters ν_2 and ϕ_0 are varied to maximize the likelihood function and estimate the observed values, $\nu_2^{\rm obs}$ and $\phi_0^{\rm obs}$, for each event.

The response of the event-by-event measurement is non-linear and depends on the observed multiplicity n. Therefore, a detailed study of the response function is required to extract the true ν_2 distribution from the measured $\nu_2^{\rm obs}$ distribution. Let $f(\nu_2)$ be the true ν_2 distribution for a set of events in a given centrality bin, and $g(\nu_2^{\rm obs})$ the corresponding observed distribution. The true and observed distributions are related by

$$g(\nu_2^{\rm obs}) = \int K(\nu_2^{\rm obs}, \nu_2, n)\, f(\nu_2)\, {\rm d}\nu_2\, N(n)\, {\rm d}n, \quad (4)$$

where $N(n)$ is the multiplicity distribution of the given set of events and $K(\nu_2^{\rm obs}, \nu_2, n)$ is the expected distribution of $\nu_2^{\rm obs}$ for events with fixed input flow ν_2, and constant observed multiplicity n.

The response function, $K(\nu_2^{\rm obs}, \nu_2, n)$ is determined by performing the event-by-event analysis on modified HIJING events with flow of fixed magnitude ν_2. The flow is introduced by redistributing the generated particles in each event in the ϕ direction according to the probability distribution given by Eq. 2. For the two parameterizations of $v_2(\eta)$, triangular and trapezoidal, used in the event-by-event measurement, the corresponding response functions, $K^{\rm tri}$ and $K^{\rm trap}$, are calculated. Fitting smooth functions through the observed response functions decreases bin-to-bin fluctuations and allows for interpolation in ν_2 and n. The response of a perfect detector can be determined as a function of event multiplicity as described in Ref. [10]. In practice, some empirical modifications to the ideal relation, accounting for the detector effects, significantly improve fits to the response function, leading to

$$K(\nu_2^{\rm obs}, \nu_2, n) = \frac{\nu_2^{\rm obs}}{\sigma^2} \times \exp\left(-\frac{\left(\nu_2^{\rm obs}\right)^2 + \left(\nu_2^{\rm mod}\right)^2}{2\sigma^2}\right) I_0\left(-\frac{\nu_2^{\rm obs}\nu_2^{\rm mod}}{\sigma^2}\right), \quad (5)$$

with $\nu_2^{\rm mod} = (A\,n + B)\nu_2$ and $\sigma = C/\sqrt{n} + D$, and where I_0 is the modified Bessel function. The four parameters (A, B, C, D) are obtained by fits to observed $K(\nu_2^{\rm obs}, \nu_2, n)$ in the modified HIJING samples.

The true event-by-event ν_2 distribution, $f(\nu_2)$, is assumed to be a Gaussian in the range $\nu_2 > 0$, with two parameters, mean $(\bar{\nu}_2)$ and standard deviation (σ_{ν_2}). For given values of the parameters, it is possible to take the integral in Eq. 4 numerically to obtain the expected $\nu_2^{\rm obs}$ distribution. Comparing the expected and observed distributions, the values of $\bar{\nu}_2$ and σ_{ν_2} are found by a maximum-likelihood fit. Midrapidity ($|\eta| < 1$) results from the two parameterizations of $v_2(\eta)$, triangular and trapezoidal, are averaged as $\langle v_2 \rangle = 0.5(\frac{11}{12}\bar{\nu}_2^{\rm tri} + \bar{\nu}_2^{\rm trap})$ and $\sigma_{v_2} = 0.5(\frac{11}{12}\sigma_{\nu_2}^{\rm tri} + \sigma_{\nu_2}^{\rm trap})$, where the factor $\frac{11}{12}$ comes from integration over η.

The induced v_2 fluctuations arising from fluctuations in the number of participating nucleons are calculated by parameterizing the $\langle v_2 \rangle$ versus $N_{\rm part}$ results and folding them with the $N_{\rm part}$ distributions in each centrality bin. The relative contribution of these fluctuations to σ_{v_2} is found to be less than 8%. Results in this letter are presented after subtraction of $N_{\rm part}$ induced fluctuations.

Systematic errors have been investigated in three main classes: variations to the event-by-event analysis, response of the measurement to known input σ_{v_2}, and intrinsic differences between HIJING events and data. Various modifications to the event-by-event analysis have been applied. Corrections, used in the hit-based event-plane analysis [5, 6], to account for signal dilution due to detector occupancy and to create an appropriately symmetric acceptance have been applied to both HIJING and data events. Hit definitions have been varied. These changes lead to at most 4% variations in the observed relative fluctuations demonstrating a good understanding of the response function. The determination of the response function and the final fitting procedure have been studied by performing the analysis on sets of modified HIJING events with varying input σ_{v_2}. Differences between input and reconstructed σ_{v_2} are identified as a contribution to the systematic uncertainty. The sensitivity of the measurement is observed to be limited for very low $\langle v_2 \rangle$ values. Therefore the 0-6% most central events, where the reconstructed $\langle v_2 \rangle$ is below 3%, have been omitted. Differences between HIJING and data in terms of $dN/d\eta$, $v_2(\eta)$ and particle correlations other than flow (non-flow correlations) can, in principle, lead to a miscalculation of the response function. A sample of MC events has been generated, in which the $dN/d\eta$ distribution of HIJING events is widened by a simple scaling to match the measurements in data within the errors [18]. The difference between results obtained with and without this modification, as well as the difference between results with two different parameterizations of $v_2(\eta)$ are identified as contributions to the systematic uncertainty. The non-flow correlation strength in HIJING was found to be of comparable magnitude to the observed strength in data [19]. A different set of MC events has been generated, in which the flow is introduced by shifting the particle momenta in the azimuthal direction, preserving other correlations. Differences between the results obtained with these MC events, to the results obtained using MC events with only flow correlations, are identified as another contribution to the systematic uncertainty. Other systematic studies include using a flat, rather than Gaussian, ansatz for the true ν_2 distribution, $f(\nu_2)$, and performing the analysis in different collision vertex and event-plane angle bins. The uncertainty in the contribution of $N_{\rm part}$ induced fluctuations has also been estimated via different parameterizations of the $\langle v_2 \rangle$ versus $N_{\rm part}$ results. Contributions from all error sources described above are added in quadrature to derive the 90% confidence level error.

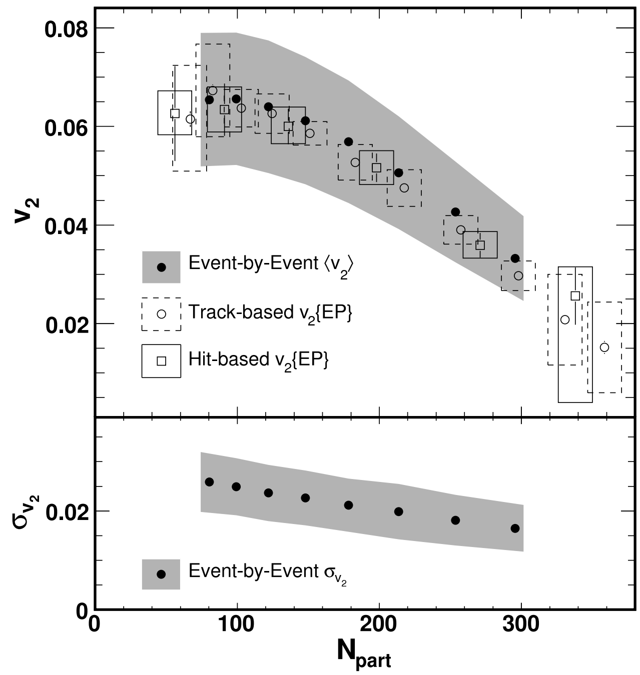

FIG. 1: $\langle v_2 \rangle$ (top) and σ_{v_2} (bottom) versus $N_{\rm part}$ for Au+Au collisions at $\sqrt{s_{_{NN}}} = 200$ GeV. Previously published event-plane v_2 results for the same collision system are shown for comparison [6]. Boxes and gray bands show 90% C.L. systematic errors and the error bars represent 1-σ statistical errors. The results are for $0 < \eta < 1$ for the track-based method and $|\eta| < 1$ for hit-based and event-by-event methods.

Fig. 1 shows the mean, $\langle v_2 \rangle$, and the standard deviation, σ_{v_2}, of the elliptic flow parameter v_2 at midrapidity

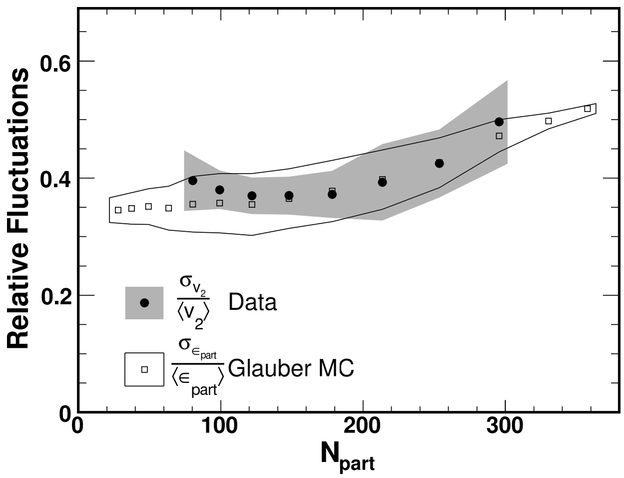

FIG. 2: $\sigma_{v_2}/\langle v_2 \rangle$ versus $N_{\rm part}$ for Au+Au collisions at $\sqrt{s_{_{NN}}}$ = 200 GeV. Open squares show $\sigma_{\epsilon_{\rm part}}/\langle \epsilon_{\rm part} \rangle$ calculated in a Glauber MC. The bands show 90% C.L. systematics errors.

as a function of the number of participating nucleons, in Au+Au collisions at $\sqrt{s_{_{NN}}}$ = 200 GeV for 6–45% most central events. The results for $\langle v_2 \rangle$ are in agreement with the previous PHOBOS v_2 measurements [6], which were obtained with the event-plane method for charged hadrons within $|\eta|<1$. The uncertainties in $dN/d\eta$ and $v_2(\eta)$, as well as differences between HIJING and the data in these quantities, introduce a large uncertainty in the overall scale in the event-by-event analysis due to the averaging procedure over the wide pseudorapidity range. The event-plane method used in the previous PHOBOS measurements has been proposed to be sensitive to the second moment, $\sqrt{\langle v_2^2 \rangle}$, of elliptic flow [20]. The fluctuations presented in this letter would lead to approximately 10% difference between the mean, $\langle v_2 \rangle$, and the RMS, $\sqrt{\langle v_2^2 \rangle}$, of elliptic flow at a fixed value of $N_{\rm part}$. However, a detailed comparison is not possible for our $\langle v_2 \rangle$ measurements due to the scale errors, which dominate the systematic uncertainty on $\langle v_2 \rangle$ and σ_{v_2}.

Most of the scale errors cancel in the ratio, $\sigma_{v_2}/\langle v_2 \rangle$, which defines, "relative flow fluctuations", shown in Fig. 2 as a function of the number of participating nucleons. We observe large relative fluctuations of approximately 40%. MC studies show that the contribution of non-flow correlations to the observed elliptic flow fluctuations is less than 2%. The effect of non-flow correlations has also been found to be small for a preliminary study by the STAR collaboration, measuring elliptic flow fluctuations for charged hadrons near midrapidity [21].

Also shown in Fig. 2 is $\sigma_{\epsilon_{\rm part}}/\langle \epsilon_{\rm part} \rangle$ at fixed values of N_{part} obtained in a MC Glauber simulation. The 90% confidence level systematic errors are estimated by varying Glauber parameters as discussed in Ref. [11]. A striking agreement between the relative fluctuations in the Glauber model participant eccentricity predictions and the observed elliptic flow fluctuations is seen over the full centrality range under study. The observed agreement suggests that the fluctuations of elliptic flow primarily reflect fluctuations in the initial state geometry and are not affected strongly by the latter stages of the collision.

These results are therefore qualitatively consistent with a picture of the collision process in which the shape of the initial stage geometry follows the predictions of the Glauber model and where the initial geometry is translated into the final state azimuthal particle distribution in a hydrodynamic expansion, leading to an event-by-event proportionality between the observed elliptic flow and the initial eccentricity. The results support conclusions from previous studies on the importance of geometric fluctuations of the initial collision region postulated to relate elliptic flow measurements in the Cu+Cu and Au+Au systems [11]. This agreement also provides evidence that matter is created in the initial stage of relativistic heavy ion collisions with a transverse granularity similar to that of the participating nucleons. The results may allow future calculations in hydrodynamic models to constrain the underlying equation of state [22].

In summary, we have presented the first measurement of elliptic flow fluctuations in Au+Au collisions at $\sqrt{s_{_{NN}}}$ = 200 GeV. We show that the magnitude and centrality dependence of these fluctuations agree with predictions for fluctuations of the initial shape of the collision region based on the Glauber model, implying that the latter collision stages do not significantly alter the fluctuation pattern. These results provide qualitatively new information on the initial conditions of heavy ion collisions and the subsequent collective expansion of the system. They suggest that elliptic flow is driven by an event-by-event hydrodynamic expansion originating from the azimuthally anisotropic initial collision region.

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