Additional Notes for Elliot's Lectures

More complete treatment of spin for massless particles.

Rest system does not exist, so cannot base on rest state $|\vec{p}, m\rangle$.

Take $p^{\mu}_{st} = (p, 0, 0, p)$

$$[\hat{W}_\mu, \hat{W}_\nu]\,|p_{st}; \lambda\rangle = i p\,[\epsilon_{\mu\nu\rho 0} + \epsilon_{\mu\nu\rho 3}]\,\hat{W}^\rho\,|p_{st}; \lambda\rangle$$

Therefore when acting on $|p_{st}; \lambda\rangle$:

$$[\hat{W}^1, \hat{W}^2] = 0 \quad \overset{\leftarrow \text{compare} \rightarrow}{\longleftrightarrow} \quad [\hat{P}^1, \hat{P}^2] = 0$$

$$[\hat{W}^3, \hat{W}^1] = i p \hat{W}^2 \longleftrightarrow [\hat{J}^3, \hat{P}^1] = i \hat{P}^2$$

$$[\hat{W}^3, \hat{W}^2] = -i p \hat{W}^1 \longleftrightarrow [\hat{J}^3, \hat{P}^2] = -i \hat{P}^1$$

$$\therefore \left(\frac{\hat{W}^3}{p}, \hat{W}^1, \hat{W}^2\right) \xleftrightarrow[\text{algebra}]{\text{same}} (\hat{J}^3, \hat{P}^1, \hat{P}^2)$$

$\Rightarrow$ eigenvalues of $\hat{W}^1, \hat{W}^2$ like momentum components : NOT QUANTIZED

Postulate: Physical massless particles correspond to eigenvalue 0 for $\hat{W}^{1,2}$.

$$\hat{W}^{1,2} |p_{st}; \lambda\rangle = 0$$

Hence

$$\hat{W}^{\mu} |p_{st}; \lambda\rangle = (pJ_3, 0, 0, pJ_3) |p_{st}; \lambda\rangle$$

$$\therefore \hat{W}^{\mu} \hat{W}_{\mu} |p_{st}; \lambda\rangle = 0$$

How do we know $|\lambda|$ = integer? or half int?

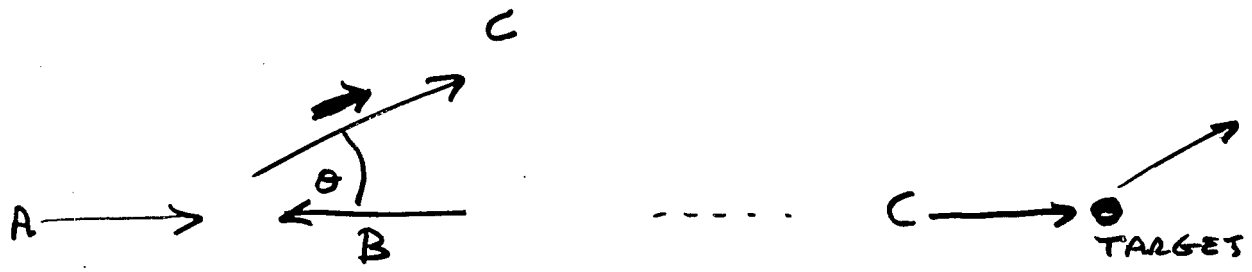
Under rotation by angle α about $\underline{p}$

$$|\underline{p}; \lambda\rangle \rightarrow e^{-i\alpha \frac{\hat{\underline{J}} \cdot \underline{p}}{|\underline{p}|}} |\underline{p}, \lambda\rangle$$

$$= e^{-i\alpha\lambda} |\underline{p}, \lambda\rangle$$

For $\alpha = 2\pi$ require phase $= +1$ for bosons, $= -1$ for fermions.

Importance practically of transformation rules :—



C.M.

PRODUCTION REACTION

LAB

ANAYZING REACTION

"THEORY" : IN CM C has helicity $\lambda = 1/2$

Test it in LAB via ANAYZING REACTION

i.e. scattering of C depends on its helicity.

In <u>LAB</u> : C DOES NOT have helicity $\lambda = 1/2$, but is some superposition

$$|C : p_c, \lambda = \tfrac{1}{2}\rangle_{LAB} = |C : p_{LAB}, \lambda'\rangle \, \mathcal{D}_{\lambda'\lambda}(\tau_{wick})$$

$$S^\mu(\underline{p}, \underline{s}) = 2\left(\underline{p}\cdot\underline{s},\ M\underline{s} + \frac{\underline{p}\cdot\underline{s}}{E+M}\,\underline{p}\right)$$

Take $\underline{p} = (0, 0, p)$

1) <u>Longitudinal polarization</u>: Take $\underline{s} = (0, 0, \frac{1}{2})$

$\underline{s} \parallel \underline{p}$

$$S^\mu(\underline{p}, \underline{s}_L) = (p, 0, 0, E)$$

General $\underline{p}$: $= (p, E\hat{\underline{p}})$

2) <u>Transverse polarization</u>: $\underline{s}\cdot\underline{p} = 0$

$$S^\mu(\underline{p}, \underline{s}_T) = (0, 2M\underline{s}_T)$$

Example: $\underline{p} = (0, 0, p)$ $\quad \underline{s}_T = (0, \frac{1}{2}, 0)$

$$S^\mu = (0, 0, M, 0)$$

For $M \to 0$, $S^\mu(\underline{p}, \underline{s}_T) \to 0$

$\Rightarrow$ "WRONG" STATEMENT: MASSLESS PARTICLE CANNOT HAVE TRANVERSE POLARIZATION

Consider spin ½ particle: $\underline{p} = (0\ 0\ p)$

$$|\underline{p}, \pm\underline{s}_x\rangle = \frac{1}{\sqrt{2}}\left\{ |\underline{p}, \lambda = \tfrac{1}{2}\rangle \pm |\underline{p}, \lambda = -\tfrac{1}{2}\rangle \right\}$$

$$|p, \pm s_y\rangle = \frac{1}{\sqrt{2}}\left\{ |p, \tfrac{1}{2}\rangle \pm i\, |p, -\tfrac{1}{2}\rangle \right\}$$

(Modulo phases)

FOR A MASSLESS PARTICLE CAN STILL DEFINE THIS SUPERPOSITION OF STATES;

STILL CALL THEM TRANSVERSE spin states.

(of course covariant spin vector = 0)

Some people call these TRANSVERSITY states — BAD, since concept of TRANSVERSITY has different meaning in literature (defined by DALITZ around 1960!)

BUT: for spin 1 transverse spin state is

$$|s=1;\ s_x\rangle = \frac{1}{2}|\lambda=1\rangle + \frac{1}{\sqrt{2}}|\lambda=0\rangle + \frac{1}{2}|\lambda=-1\rangle$$

For MASSLESS particle

$|\lambda=0\rangle$ DOES NOT EXIST.

$\therefore$ CANNOT USE TRANSVERSE SPIN STATE FOR MASSLESS SPIN 1 PARTICLE.

Another way to see this: PHOTON HAS 2 possible helicities $\lambda = \pm 1$.

But state $|s_x\rangle$ has 3 possibilities:

$s_x = -1, 0, 1$

Connection between Dirac spinors and (2s+1) type spinor. (Schematic only)

Under Lorentz transf. $|\mathbf{p}, \lambda\rangle$ has complicated behaviour.

$$\therefore \text{ if } |\mathbf{p}, \lambda\rangle = a^{\dagger}(\mathbf{p}, \lambda)\,|0\rangle$$

transf properties of $a(\mathbf{p}, \lambda)$ will be complicated — involve ℓ <u>and</u> $\mathbf{p}$.

$\therefore$ (2s+1) component field

$$\psi^{(s)}(x) = \int \frac{d^3\mathbf{p}}{E}\, a(\mathbf{p}, \lambda)\, u(\mathbf{p}, \lambda)\, e^{-ip\cdot x} \cdots$$

will have peculiar L. Transf behaviour.

To build Lagrangians, like to have

$$U(\ell)\,\psi_n(x)\,U^{-1}(\ell) = \sum_m D_{nm}(\ell^{-1})\,\psi_m(\ell x)$$

representation of homogeneous L. Group.

Depends only on ℓ, NOT x^{μ}.

Take spin 1/2 : Wick Hel. Rotn:

$$\mathcal{D}^{(1/2)}_{\lambda'\lambda}(r_W) = \mathcal{D}^{(1/2)}_{\lambda'\lambda}\left[h^{-1}(\ell p)\,\ell\,h(p)\right]$$

ROTATION GROUP

CANNOT SPLIT INTO PRODUCT of MATRICES

Need to use Repres. matrices of Hom. L. Group.

There are two: $\mathcal{D}^{(\frac{1}{2},0)}$, $\mathcal{D}^{(0,1/2)}$

FOR ROTATIONS:

$$\mathcal{D}^{(\frac{1}{2},0)}(r) = \mathcal{D}^{(0,1/2)}(r) = \mathcal{D}^{(1/2)}(r)$$

Two interpretations

$(\frac{1}{2}0)$: $\mathcal{D}^{(\frac{1}{2}0)}(h^{-1}(\ell p))\,\mathcal{D}^{(\frac{1}{2}0)}(\ell)\,\mathcal{D}^{(\frac{1}{2}0)}(h(p))$

Build field $\psi^{(\frac{1}{2},0)}(x)$ using

$$A^{+}(p,\lambda) = \mathcal{D}^{(\frac{1}{2}0)}(h(p))\,a^{+}(p,\lambda)$$

$\Rightarrow \psi_n^{(\frac{1}{2},0)}(x)$: L. Trans : $\mathcal{D}^{(\frac{1}{2},0)}(\ell^{-1})$

IRREDUCIBLE FIELD (2 components)

$(0,\frac{1}{2})$: $\mathcal{D}^{(0,\frac{1}{2})}(h^{-1}(\ell p))\,\mathcal{D}^{(0,1/2)}(\ell)\,\mathcal{D}^{(0,\frac{1}{2})}(h(p))$

Build $\psi^{(0,1/2)}(x)$ using

$$A^{\dagger}(p,\lambda) = \mathcal{D}^{(0,1/2)}(h(p))\,a^{\dagger}(p,\lambda)$$

$\Rightarrow$ $\psi_n^{(0,1/2)}(x)$: L. Transf: $\mathcal{D}^{(0,1/2)}(\ell^{-1})$

Again 2 comps.

PROBLEM : PARITY : Find

$$\mathcal{P}\,\psi^{(\frac{1}{2},0)}(t,\underline{x})\,\mathcal{P}^{-1} = \eta\,\psi^{(0,1/2)}(t,-\underline{x})$$

So to have simple transf under $\mathcal{P}$ need to put together

$$\psi(x) = \begin{pmatrix} \psi^{(\frac{1}{2},0)}(x) \\ \psi^{(0,1/2)}(x) \end{pmatrix}$$ 4 components.

Does <u>not</u> transf IRREDUCIBLY under HOM. L. GROUP

$$\mathcal{D}(\ell) = \begin{pmatrix} \mathcal{D}^{(\frac{1}{2},0)}(\ell) & 0 \\ 0 & \mathcal{D}^{(0,\frac{1}{2})}(\ell) \end{pmatrix}$$

but IRREDUCIBLY under HOM. L. Gp + space INVERSION.

<u>SUMMARY</u>

1) Nice alternative view of Dirac eqn

2) Makes clear why 2 components OK if parity NOT conserved.