Single Spin Asymmetries

Lecture One

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Outline of Lecture One

- Single spin asymmetry - definition
- Single spin asymmetry before QCD
- Parity and Time-reversal invariance
- Perturbative QCD factorization
- Single spin asymmetry within the collinear factorization
- Outline of Lecture Two

Outline of Lecture Two

- Semi-inclusive deeply inelastic scattering
- K_T- factorization
- Sivers effect and Sivers functions
- Collins effect and Collins functions
- Single spin asymmetry in Drell-Yan
- Open questions and discussion

Single Spin Scattering – (I)

- **Single hadron inclusive process:**

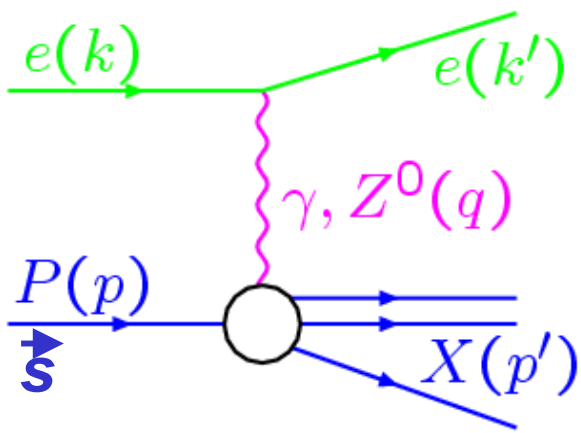
$$A(p, \vec{s}) + B(p') \Longrightarrow C(\ell) + X$$

  - only one initial-state hadron is polarized
  - observed particle $C(\ell)$ is unpolarized, and can be any high transverse momentum particle π, p, γ, or lepton
  - cross section: $\sigma(\ell, \vec{s})$

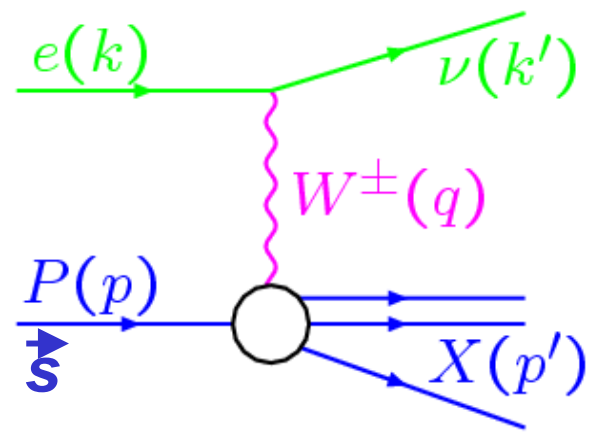
Single Spin Scattering – (II)

❑ Inclusive and Semi-inclusive DIS:

Neutral Current (NC)



Charged Current (CC)



negative four-momentum transfer squared	$Q^2 = -q^2 = -(k-k')^2$
fraction of proton momentum	$x = \frac{Q^2}{2p\cdot q}$
inelasticity	$y = \frac{p\cdot q}{p\cdot k}$
squared cms energy	$s = (k+p)^2 = \frac{Q^2}{xy}$

Single Spin Asymmetries – (I)

- **Spin-avg X-section:** $\sigma(\ell) = \frac{1}{2}[\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})]$
- **Spin-dep X-section:** $\Delta\sigma(\ell, \vec{s}) = \frac{1}{2}[\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})]$
- **Single spin asymmetry:**

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

  - **single longitudinal spin asymmetry:** A_L

    particle spin $\vec{s}$ is parallel to its momentum $\vec{p}$
  - **single transverse spin asymmetry:** A_N

    particle spin $\vec{s}$ is perpendicular to its momentum $\vec{p}$

Single Spin Asymmetries – (II)

- Even though cross section $\sigma(\ell, \vec{s})$ is finite, single spin asymmetries can vanish due to the difference of the polarized cross sections and the fundamental symmetries of the interactions

- A_L vanishes for Parity conserved interactions:

$A_L \neq 0$ for $W^{\pm}$ production
good for parton flavor separation

Single spin asymmetry before QCD

- Almost 40 years ago, A_N in inclusive DIS was proposed by Christ and Lee to test the Time-Reversal invariance:

> In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions
>
> N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)

Parity and Time-reversal invariance

- In quantum field theory, physical observables are given by matrix elements of quantum field operators

- Consider two quantum states: $|\alpha\rangle \quad |\beta\rangle$

- Parity transformation:

$$|\alpha_P\rangle \equiv U_P\,|\alpha\rangle \qquad\qquad |\beta_P\rangle \equiv U_P\,|\beta\rangle$$

$$\langle\alpha_P|\beta_P\rangle = \langle\alpha|\,U_P^\dagger\,U_P\,|\beta\rangle = \langle\alpha|\beta\rangle$$

- Time-reversal transformation:

$$|\alpha_T\rangle \equiv V_T\,|\alpha\rangle \qquad\qquad |\beta_T\rangle \equiv V_T\,|\beta\rangle$$

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\,V_T^\dagger\,V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

A_N for inclusive DIS – (I)

- **DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$

- **Leptionic tensor is symmetric:** $L^{\mu\nu} = L^{\nu\mu}$

- **Hadronic tensor:** $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0)\, j_\nu(y) \, | P, \vec{s}_\perp \rangle$

- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right]$$

- **Vanishing single spin asymmetry:**

$$A_N = 0 \quad \Longleftrightarrow \quad \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0)\, j_\nu(y) \, | P, \vec{s}_\perp \rangle \overset{?}{=} \langle P, -\vec{s}_\perp | \, j^\dagger_\nu(0)\, j_\mu(y) \, | P, -\vec{s}_\perp \rangle$$

A_N for inclusive DIS – (II)

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T\, |P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned}|\beta_T\rangle &= V_T\, \left[j_\mu^\dagger(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle\end{aligned}$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\, |\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned}\Longrightarrow\ & \langle -P, -\vec{s}_\perp|\, \left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y)\, |P, \vec{s}_\perp\rangle\end{aligned}}$$

A_N for inclusive DIS – (III)

❖ **Parity invariance:**

$$1 = U_P^{-1} U_P = U_P^\dagger U_P$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(-y) \, j_\mu(0) \, | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(0) \, j_\mu(y) \, | P, -\vec{s}_\perp \rangle$$
$$= \langle P, \vec{s}_\perp | \, j_\mu^\dagger(0) \, j_\nu(y) \, | P, \vec{s}_\perp \rangle$$

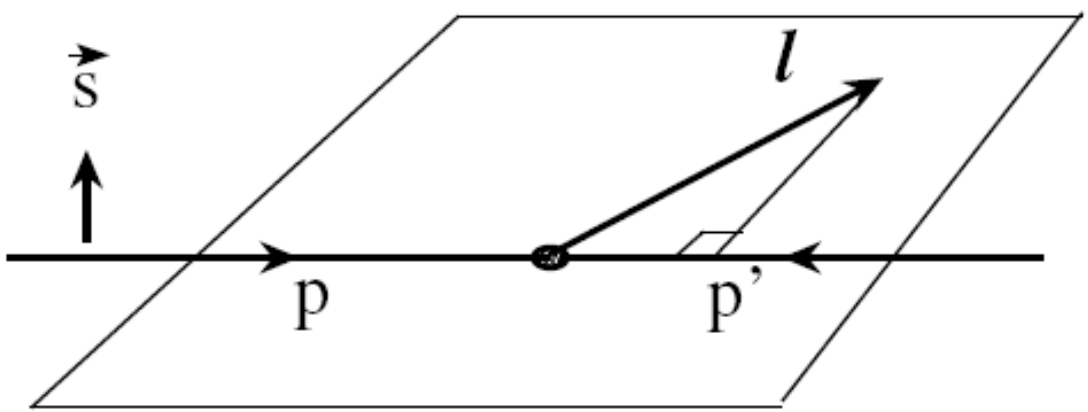
❖ **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} \, [W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)]$$
$$= L^{\mu\nu} \, [W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp)] = 0$$

Single spin asymmetry corresponds to a T-odd triple product

$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$

- $\vec{p}$ is beam particle's three momentum
- $\vec{\ell}$ is momentum of observed particle
- the phase "i" is required by time-reversal invariance
- covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$



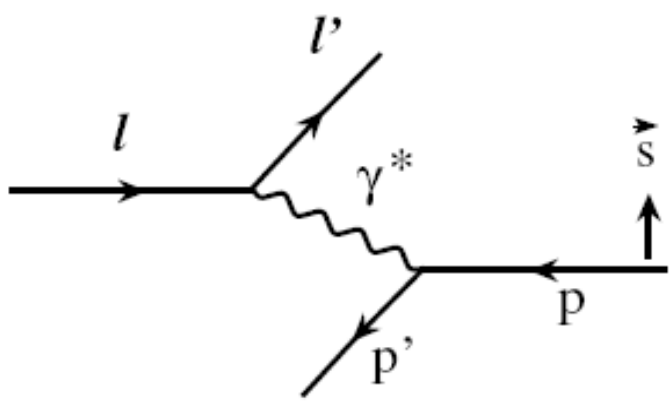
Nonvanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

- Inclusive DIS does not have enough vectors

  Note: q and p can only fix a line

Following examples can generate nonvanishing A_N:

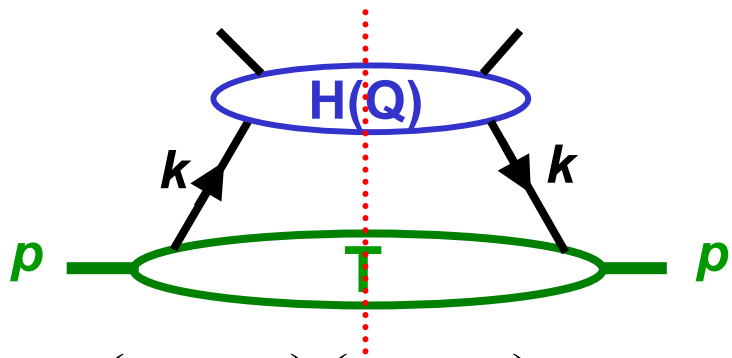
- Single hadron (or photon) at high ℓ_T
- Drell-Yan lepton angular distribution
- Semi-inclusive DIS



- ...

Perturbative QCD factorization

- **Perturbative pinch poles:**



$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})\ \Rightarrow\ \infty\ \text{perturbatively}$$

- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

Nonperturbative matrix element

$$\int\frac{dx}{x}d^2k_T\ \mathrm{H}(Q,k^2=0)\boxed{\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})}$$

Short-distance

Perturbative QCD factorization (II)

- **K_T–factorization** $\Leftrightarrow$ $Q \sim xp\cdot n \approx k_T \sim \sqrt{k^2}$

$$\sigma^h_{phys} = \hat{\sigma}^i_2(x,k_T)\otimes T_2^{i/h}(x,k_T)$$

Next lecture

No all-order proof of k_T – factorization for an arbitrary k_T

- **Collinear factorization:**

$Q \sim xp\cdot n \sim k_T, \sqrt{k^2}$

$$\sigma^h_{phys} = \hat{\sigma}^i_2\otimes[1+\alpha_s+\alpha_s^2+\ldots]\otimes T_2^{i/h}(x)$$
$$+\frac{\hat{\sigma}^i_3}{Q}\otimes[1+\alpha_s+\alpha_s^2+\ldots]\otimes T_3^{i/h}(x)$$
$$+\frac{\hat{\sigma}^i_4}{Q^2}\otimes[1+\alpha_s+\alpha_s^2+\ldots]\otimes T_4^{i/h}(x)$$
$$+\ldots$$

Leading Twist

perturbative

Power corrections

Provides systematic ways to quantify high order corrections

Factorization fails beyond the next-to-leading powers in hadronic collisions

Spin-dependent parton correlations – (I)

- Hadron spin is a long-distance property, and therefore, spin-dependence of a polarized cross section should be in parton correlation functions:

$$\sigma^h_{phys}(Q,\vec{s}_\perp) \approx \hat{\sigma}_i(Q,\mu) \otimes T^h_i(\mu,\vec{s}_\perp)$$

$$\Rightarrow \quad \Delta\sigma^h_{phys}(Q,\vec{s}_\perp) \approx \hat{\sigma}_i(Q,\mu) \otimes \left[T^h_i(\mu,\vec{s}_\perp) - T^h_i(\mu,-\vec{s}_\perp) \right]$$

- Quark correlations:

$$T_i(x;\vec{s}_\perp) = \int \frac{P^+ dy^-}{2\pi} e^{-i\,x\,P^+\,y^-} \langle P,\vec{s}_\perp | \bar{\psi}(0)\,\Gamma_i\,\psi(y^-)\,| P,\vec{s}_\perp \rangle$$

- Real parton correlation functions:

$$T_i^* = T_i \qquad \Leftrightarrow \qquad \gamma^0\,\Gamma_i^\dagger\,\gamma^0 = \Gamma_i$$

$$\Rightarrow \quad \Gamma_i = I, \quad i\gamma_5, \quad \gamma^\mu, \quad \gamma^\mu\gamma_5, \quad \sigma^{\mu\nu} \quad \text{(or } \sigma^{\mu\nu}(i\gamma_5)\text{)}$$

Spin-dependent parton correlations – (II)

❖ **Parton fields under P and T transformation:**

$$U_P\,\psi(y_0,\vec{y})\,U_P^{-1} = \gamma^0\,\psi(y_0,-\vec{y})$$

$$V_T\,\psi(y_0,\vec{y})\,V_T^{-1} = (i\gamma^1\gamma^3)\,\psi(-y_0,\vec{y}) \qquad \boxed{\mathcal{J} = i\,\gamma^1\,\gamma^3}$$

$$\Rightarrow \quad \boxed{\begin{aligned}&\langle P,\vec{s}_\perp|\,\bar{\psi}(0)\,\Gamma_i\,\psi(y^-)\,|P,\vec{s}_\perp\rangle \\ &= \langle P,-\vec{s}_\perp|\,\bar{\psi}(0)\left[\mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger\right]\psi(y^-)\,|P,-\vec{s}_\perp\rangle\end{aligned}}$$

❖ **Quark correlations contribute to polarized X-sections:**

$$T_i(x;\vec{s}_\perp) = -T_i(x;-\vec{s}_\perp) \quad \Rightarrow \quad \mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger = -\Gamma_i$$

$$\boxed{\Gamma_i \;=\; \gamma^\mu\gamma_5,\quad \sigma^{\mu\nu} \quad \text{(or } \sigma^{\mu\nu}\,(i\gamma_5)\text{)}}$$

$\Gamma_i \;=\; I,\quad i\gamma_5,\quad \gamma^\mu$ **contribute to spin-avg X-sections:**

Spin-dependent parton correlations – (III)

- **Twist-2 quark helicity distribution:** $\Delta q(x)$

$$\Gamma_i = \gamma^\mu \gamma_5 \quad \text{with } \mu = +$$

- **Twist-2 quark transversity distribution:** $\delta q(x, \vec{s}_\perp)$

$$\Gamma_i = \sigma^{\mu\nu}\left(i\gamma_5\right) \quad \text{with } \mu = +, \text{ and } \nu = \perp$$

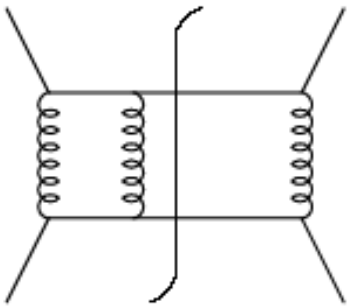
- **Twist-3 quark-gluon correlation functions:**

$$T_F(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1 P^+ y_1^- + i(x_2 - x_1) P^+ y_2^-}$$
$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \left[\epsilon^{s_T \sigma n \bar{n}} F_\sigma^{\ +}(y_2^-)\right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

Three field operator does not have the probability interpretation of normal parton distributions

A_N in hadronic collisions

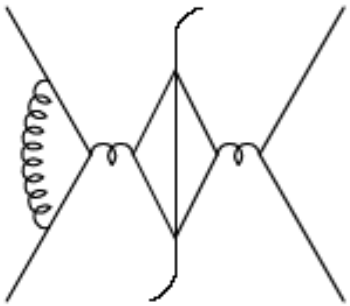
- pQCD was first used to study single transverse-spin asymmetry by Kane, Pumplin, and Repko in 1978



$+$ c.c.

– imaginary part of the loop provides the phase

– quark mass provides the needed spin flip



$+$ c.c.

– $A_N \propto \frac{m_q}{\ell_T} \langle p, \vec{s}_T | \bar{\psi} \Gamma \psi | p, \vec{s}_T \rangle$ where $\Gamma = \gamma^+ \gamma_5 \gamma_T, \ldots$

Even number of γ's

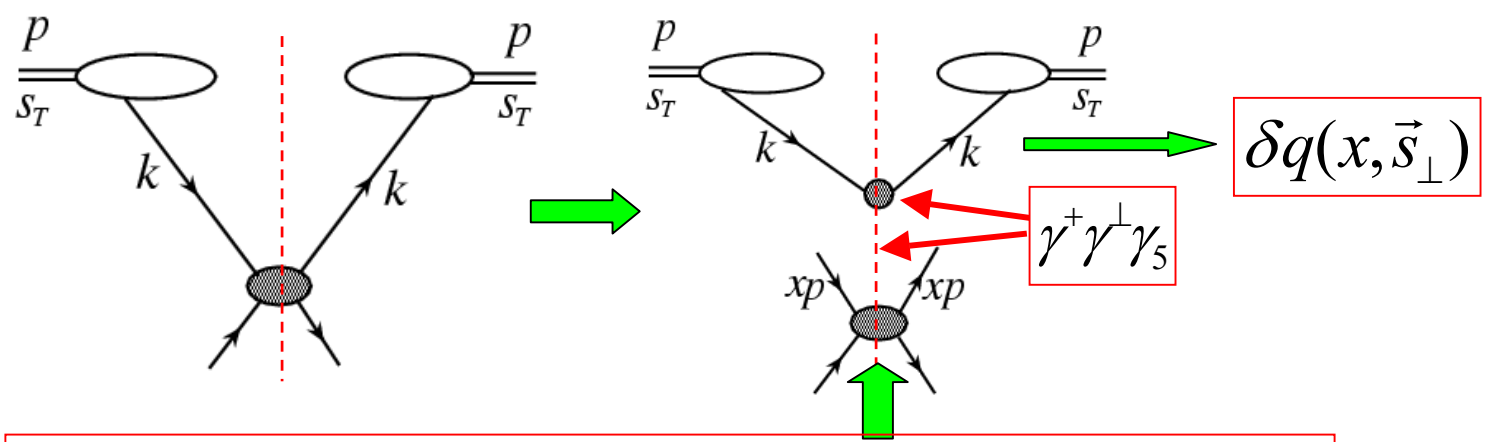
- The fact that $A_N \propto m_q$ indicates that A_N is a twist-3 effect in QCD perturbation theory

Leading twist contributions to A_N within the collinear factorization

- PQCD calculable partonic parts should not depend on light current quark mass

Need even number of γ's in any quark loop trace

- Leading twist does not contribute to the A_N

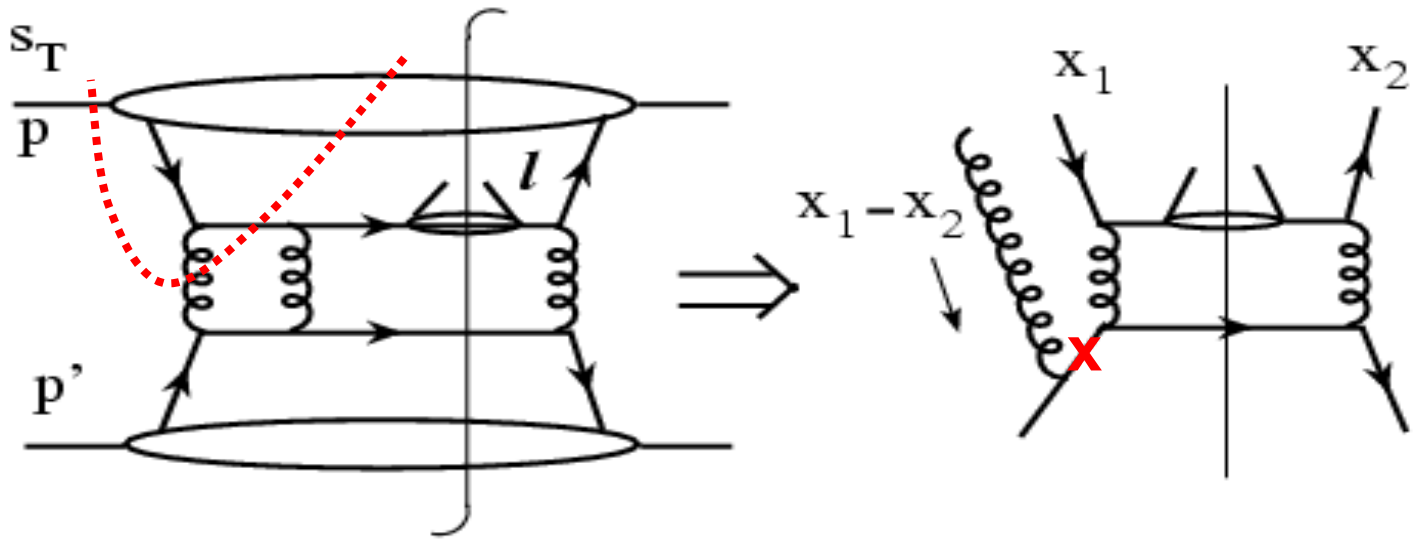


- need an even number γ's:
- need a gluon to contract with the extra transverse index

$$\Gamma_i = I, \quad i\gamma_5$$
$$\sigma^{\mu\nu}\,(i\gamma_5)$$

Twist-3

A_N from polarized twist-3 correlations



- Unpinched pole to give the phase: $i\delta(x_1 - x_2)$
- Spin flip from interference between a quark state and a quark-gluon composite state
- Observed hadron momentum provides the 3rd vector

$$A_N \propto i\, \vec{s}_\perp \cdot (\vec{p} \times \vec{\ell})$$

Single spin asymmetry within the collinear factorization

❖ **Generic twist-3 factorized contributions**

$$\Delta\sigma_{AB\to h}(\vec{s}_T) = \sum_{abc} T^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes f_{b/B}(x') \otimes \hat{\sigma}_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z) \quad (1)$$

Provides hadron spin dependence

$$+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T)$$

transversity

Even γ's !

$$\otimes \left\{ f_{b/B}(x') \otimes \hat{\sigma}'_{ab\to c}(\vec{s}_T) \otimes D^{(3)}_{c\to h}(z_1, z_2) \right\} \quad (2)$$

Even γ's !

$$(3) \left\{ + f^{(3)}_{b/B}(x'_1, x'_2) \otimes \hat{\sigma}''_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z) \right\}$$

(1) Calculated by Qiu and Sterman, Phys. Rev. D, 1999
(2) Not yet be calculated
(3) Calculated by Kanazawa and Koike, Phys. Lett. B, 2000

(3) << (1)