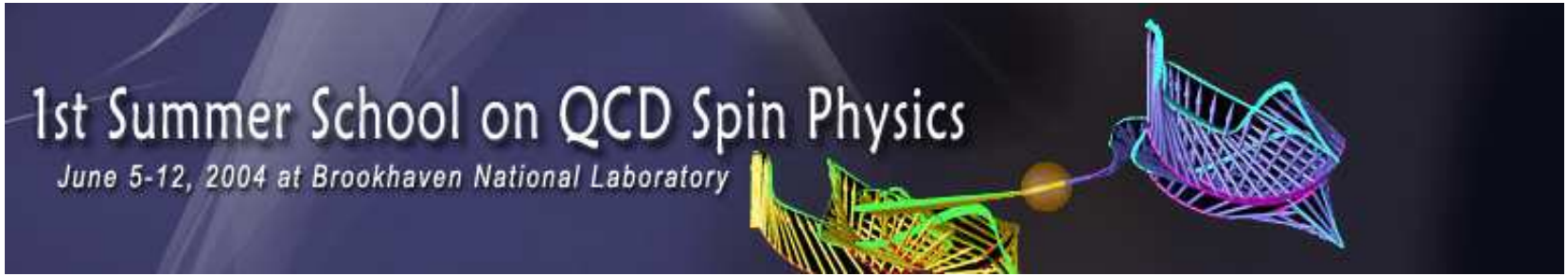


Polarized $\vec{p}\vec{p}$ Collider Physics

Marco Stratmann
University of Regensburg

- **LECTURE I**: *Appetizer, Basics & Foundations*
- **LECTURE II**: *Applications & Examples*

Appetizer: Collider Physics & QCD

Collider experiments are a QCD laboratory . . .

- vast source of information about hadronic structure
- complementary to deep-inelastic scattering experiments

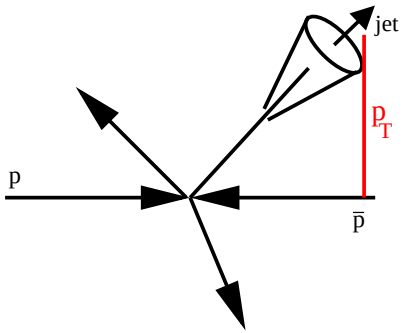
  "gluons are the key players"

- challenge our understanding of strong interactions
- push theorists to perform more and more refined calculations

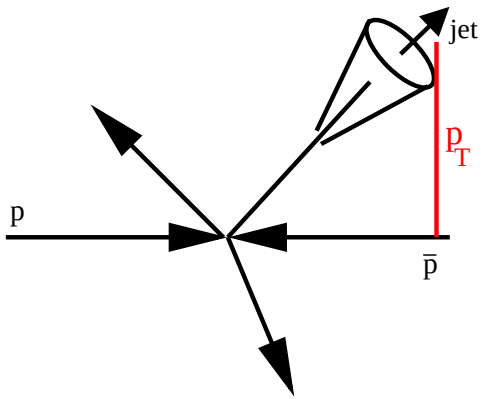
. . . a success story ever since

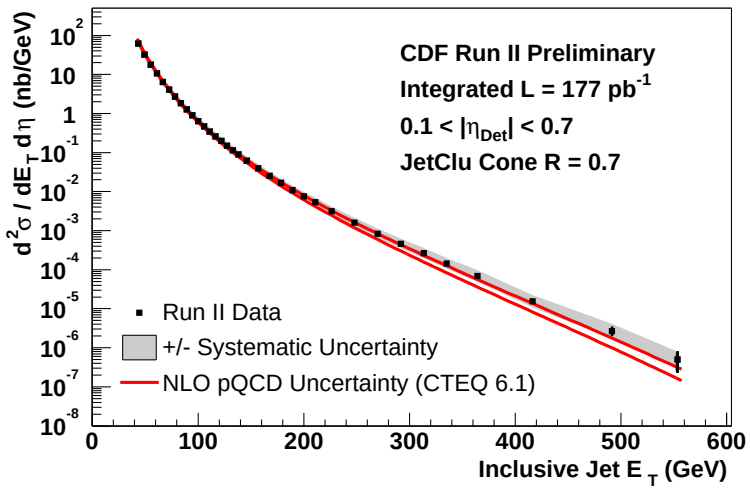
Some examples from Fermilab's TeVatron

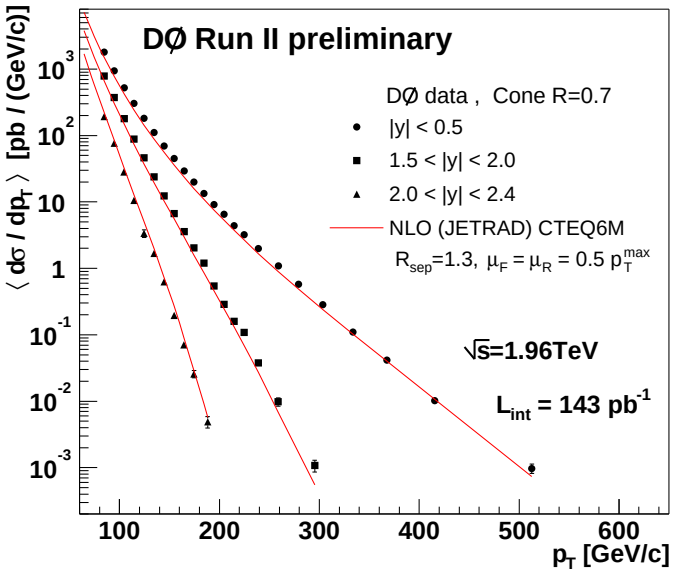
- largest hadron-hadron collider
- *unpolarized* p and $\bar{p}$ beams
- c.m. energy up to $\sqrt{S} = 1.96\,\mathrm{TeV}$
- two major experiments:
  DØ and CDF
- major discovery: top quark
- probes QCD hard interactions in many different high-p_T processes



Fermilab's TeVatron cont.



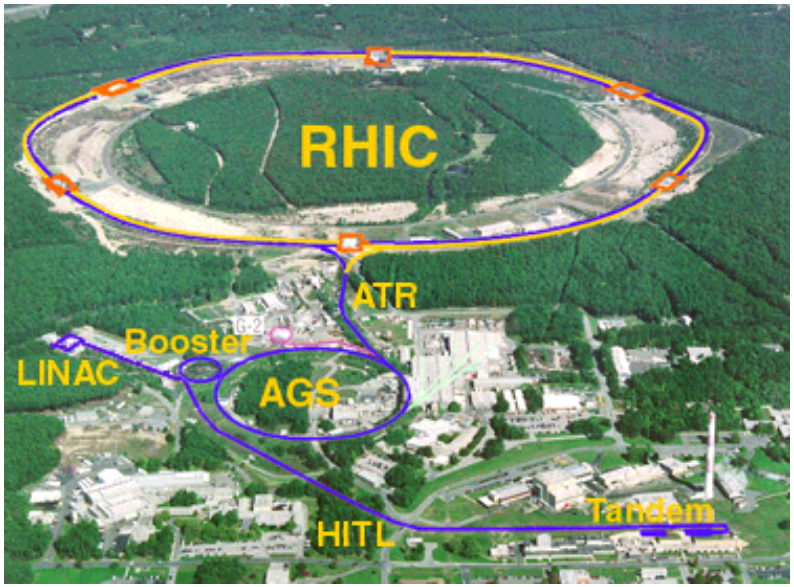




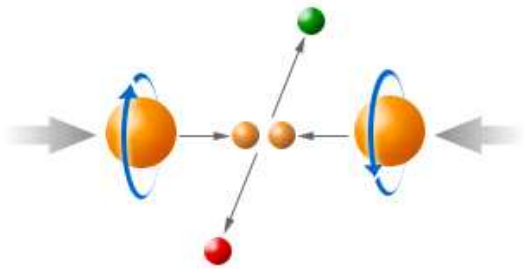
- jet cross section in agreement with expectations from pert. QCD calculations
- similar for other processes: heavy flavors, prompt photons, electroweak bosons, . . .

Local example: BNL's Relativistic Heavy Ion Collider

BROOKHAVEN NATIONAL LABORATORY



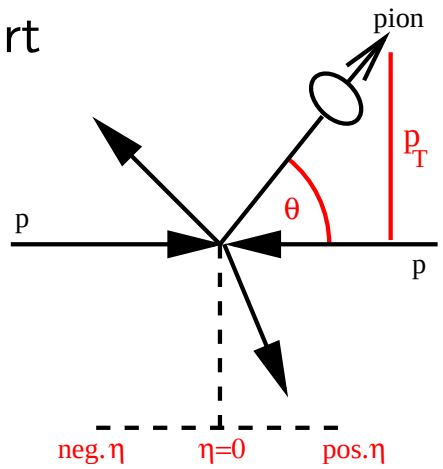
- 1st polarized pp collider
- also heavy-ion beams
- c.m. energy up to $\sqrt{S} = 500\,\mathrm{GeV}$
- two major experiments: PHENIX and STAR
- probes spin dependence of strong interactions in high-p_T processes

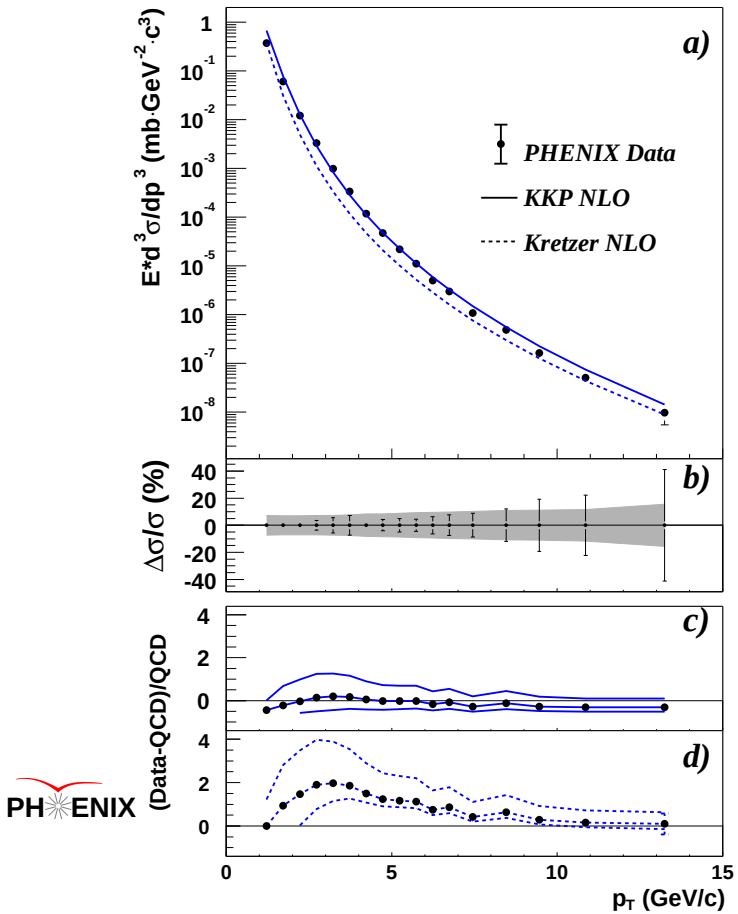


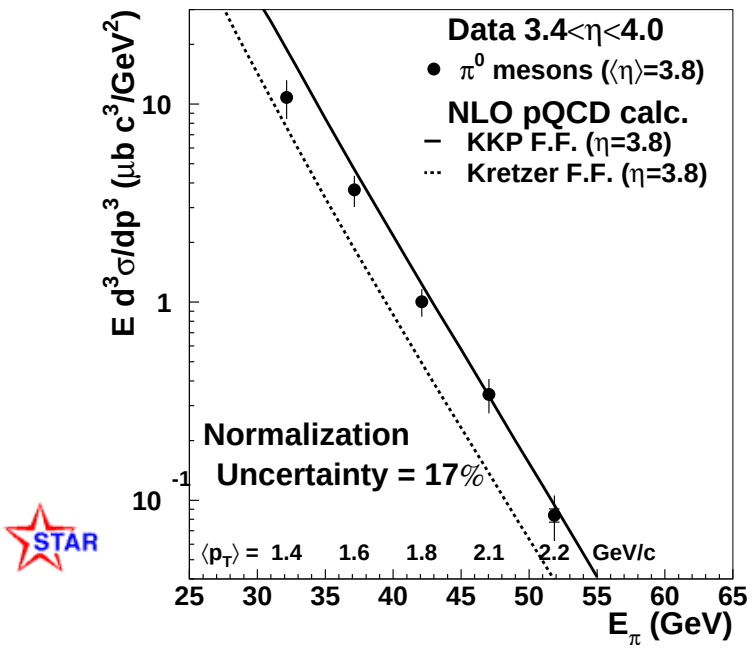
main theme of my lectures

First results from RHIC

- RHIC physics program had a very successful start
- first results on *unpolarized* pion production







Questions to be addressed in the lectures

- How to compute a cross section in perturbative QCD?

  keywords: *parton densities; factorization; polarization; hard cross sections*

- How reliable is a theoretical calculation?

  keywords: *scale dependence*

- How to systematically improve the accuracy of a calculation?

  keywords: *next-to-leading order calculation, resummations*

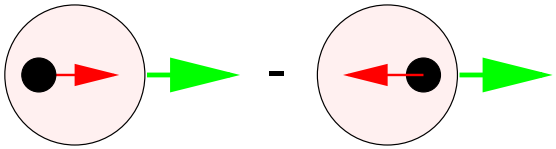
- What are we hoping to learn from RHIC spin experiments?

  keywords: *spin asymmetries; "global analysis"*

Reminder: Parton Densities

Let us focus on longitudinally polarized nucleons:

w.r.t. the nucleon spin $\rightarrow$, parton spins can be either aligned $\rightarrow$ or anti-aligned $\leftarrow$



$$\Delta f(x,\mu) \equiv f_{\rightarrow}^{\rightarrow}(x,\mu) - f_{\leftarrow}^{\rightarrow}(x,\mu)$$

$f_{\rightarrow}^{\rightarrow}(x,\mu)$ [$f_{\leftarrow}^{\rightarrow}(x,\mu)$]: # of partons with *same* [*opposite*] helicity as nucleon carrying a momentum fraction x at a resolution scale μ

- parton f can be a quark, anti-quark, or gluon
- Δf is a non-perturbative object $\rightsquigarrow$ you have to measure it! RHIC !

  [also: ab initio non-perturbative QCD methods like *lattice QCD* $\longrightarrow$ Tom Blum's lectures]

Parton Densities (cont.)

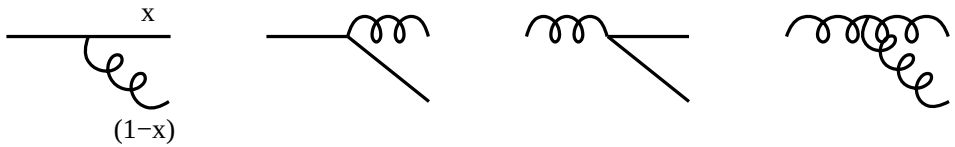
- once measured at scale μ_0, the parton/spin content of the nucleon at a resolution scale $\mu > \mu_0$ is *predicted* by perturbative QCD:

  DGLAP evolution eqs.:

$$\mu \frac{d}{d\mu} \begin{pmatrix} \Delta q(x,\mu) \\ \Delta g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z}, \mu\right)$$

  $\Delta\mathcal{P}_{ij}$: pert. calculable spin-dep. "parton j to parton i splitting functions"

  Ahmed, Ross; Altarelli, Parisi; Mertig, van Neerven; Vogelsang



- parton densities are *universal* → foundation for predictive power of pQCD

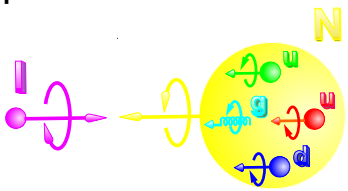
both properties can be verified for the 1st time at RHIC

Parton Densities (cont.)

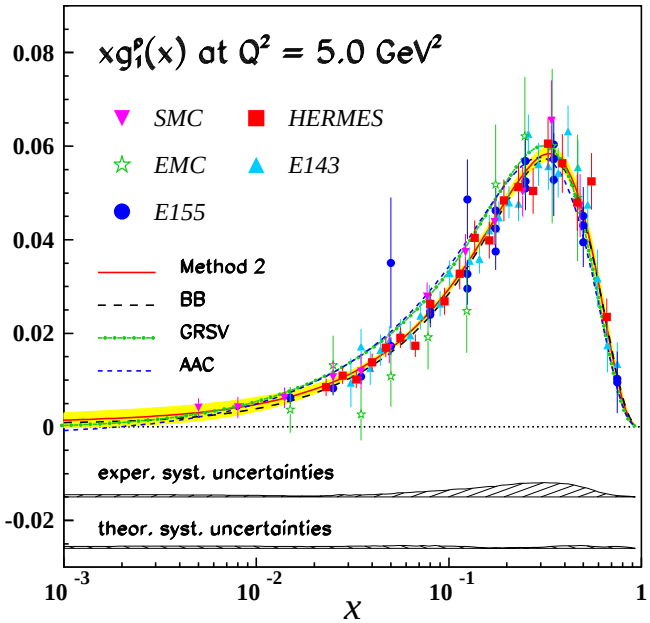
- current knowledge of Δf exclusively from *low energy* lepton-nucleon experiments

$\longrightarrow$ Gerhard Mallot's lectures

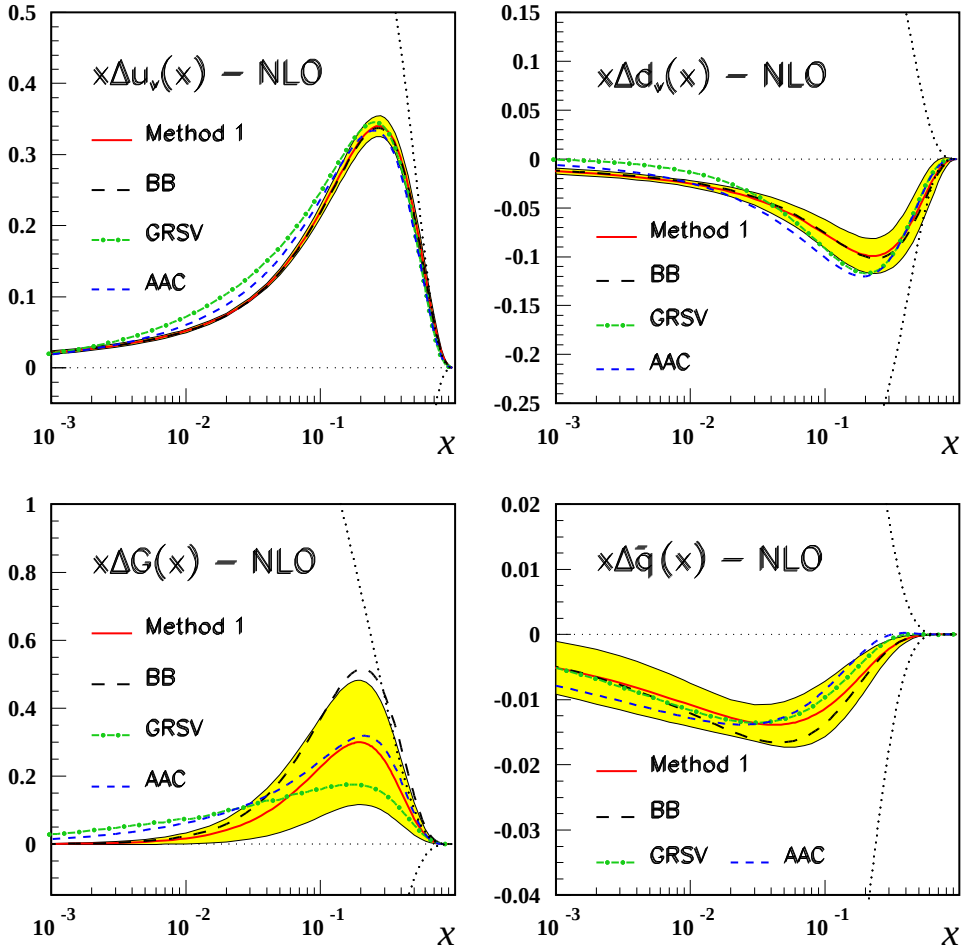
"resolving power" of DIS microscope



$\leftrightarrow$ virtuality Q^2 of γ^*: $R \simeq 1/\sqrt{Q^2}$



extract pdf's $\longrightarrow$ from data



figs. taken from Böttcher, De Nardo

Shopping list for RHIC

we have gained a lot of insight from more than 20 years of fixed target experiments

but we are still not out of the woods -

the jury is still out on some **crucial questions**:

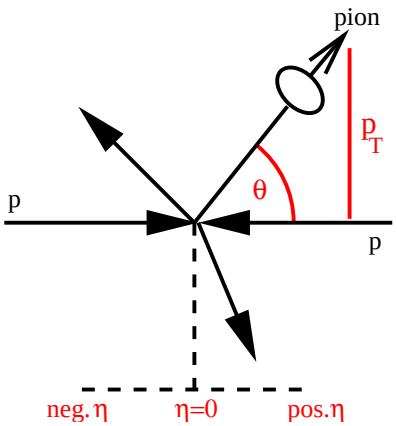
- fixed target experiments can cover only very limited kinematical range
  - haven't yet really seen the $\log \mu^2$ behavior of Δf as predicted by DGLAP
  - extrapolation uncertainties in $\int_0^1 \Delta f(x, \mu^2) dx$ entering nucleon spin "sum rule"
- DIS is an electromagnetic probe
  - no direct coupling to gluons $\rightarrow$ $\Delta g(x, \mu^2)$ completely/largely unknown
  - DIS $\propto$ $(\Delta q + \Delta \bar{q})$ $\rightarrow$ separation of Δu, $\Delta \bar{u}$, Δd, $\Delta \bar{d}$, and Δs?
  - cannot test universality of polarized pdfs from DIS alone

Perturbative QCD approach for hadron-hadron cross sections

starting point:
- exploit universality of pdf's
- invoke *factorization* Libby, Sterman; Ellis et al.; Amati et al.; Collins et al.; . . .

  $\longrightarrow$ way to separate long-distance (= non-perturbative) from short-distance (= perturbative) phenomena

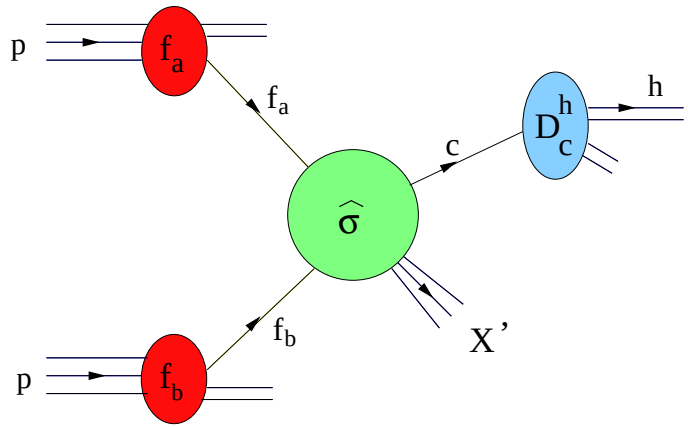
example: *unpolarized* high-p_T single-inclusive hadron production



factorization

$\longrightarrow$

theorem



ingredients: parton densities $f_a(x_a)$, $f_b(x_b)$, fragmentation fcts. $D_c^h(z)$,

hard partonic cross section $\hat{\sigma}$

Factorized cross sections (cont.)

upshot: differential cross section can be (very schematically) written as

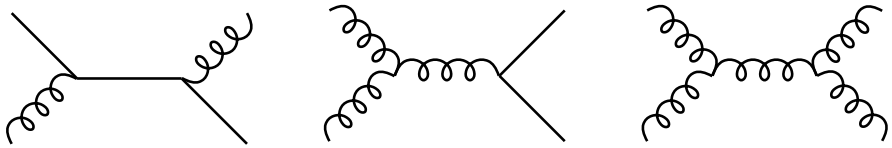
$$d\sigma = f_a \times f_b \times d\hat{\sigma} \times D_c^h$$

However, this a bit too naive $\rightarrow$ several amendments are necessary:

- we observe only the hadron's p_T and η & can control the beam energies
  - we cannot select the partons taking part in the hard scattering
  - we don't know which parton fragments into the hadron

  $\rightarrow$ have to sum over all possible partonic reactions: $a\,b \rightarrow c\,d$

e.g.



$qg \rightarrow gq$ $\quad$ $gg \rightarrow q\bar{q}$ $\quad$ $gg \rightarrow gg$

Factorized cross sections (cont.)

- observed p_T and η not uniquely linked to fixed momentum fractions x_a, x_b, and z_c

  $\longrightarrow$ ordinary products turn into *convolutions*: $\phi(x) = \int_x^1 \frac{dy}{y} \xi(y) \zeta(x/y) \equiv \xi \otimes \zeta$

  $\longrightarrow$ major complication for extraction of pdf's from measured cross sections!!

- separation of long- ($f_{a,b}$, D_c^h) and short-distance ($\hat{\sigma}$) physics not unique

  · controlled by arbitrary *factorization scales* μ_f and μ'_f

  [amount of "parton radiation" included in $f_{a,b}$, D_c^h]

$$d\sigma = \sum_{a,b,c} f_a(x_a, \mu_f) \otimes f_b(x_b, \mu_f) \otimes d\hat{\sigma}_{ab}(x_a, x_b, z_c, \mu_f, \mu'_f) \otimes D_c^h(z_c, \mu'_f)$$

  · another arbitrary scale controlling the running of α_s: *renormalization scale* μ_r

Questions: Is the whole picture completely arbitrary? How to choose these scales?

Factorized cross sections (cont.)

Question: Is the whole picture completely arbitrary? NO!

measured cross section *must not* depend on theoretical conventions

$\rightarrow$ e.g. $\mu_f \frac{d\sigma}{d\mu_f} = 0$ (example of a *"renormalization group eq."*)

$d\hat{\sigma}(x_a, x_b, z_c, \mu_f \rightarrow \mu_f + d\mu_f, \mu'_f)$ dictates scale behavior of $f_{a,b}(x_{a,b}, \mu_f)$

roughly

$$\frac{d\hat{\sigma}(\mu_f)}{d\ln\mu_f} \otimes f_a(x_a, \mu_f) \otimes f_b(x_b, \mu_f) \otimes D_c^h(z_c, \mu'_f) \quad +$$

$$\hat{\sigma}(\mu_f) \otimes \frac{df_a(x_a, \mu_f)}{d\ln\mu_f} \otimes f_b(x_b, \mu_f) \otimes D_c^h(z_c, \mu'_f) \quad +$$

$$\hat{\sigma}(\mu_f) \otimes f_a(x_a, \mu_f) \otimes \frac{df_b(x_b, \mu_f)}{d\ln\mu_f} \otimes D_c^h(z_c, \mu'_f) \quad = \quad 0$$

result: $\frac{df_a(x_a,\mu_f)}{d\ln\mu_f}$ have to obey DGLAP evolution!

Factorized cross sections (cont.)

There is **catch** . . . we work with a perturbative expansion in α_s:

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\, d\hat{\sigma}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\, d\hat{\sigma}^{(2)}}_{\text{NNLO}} + \ldots$$

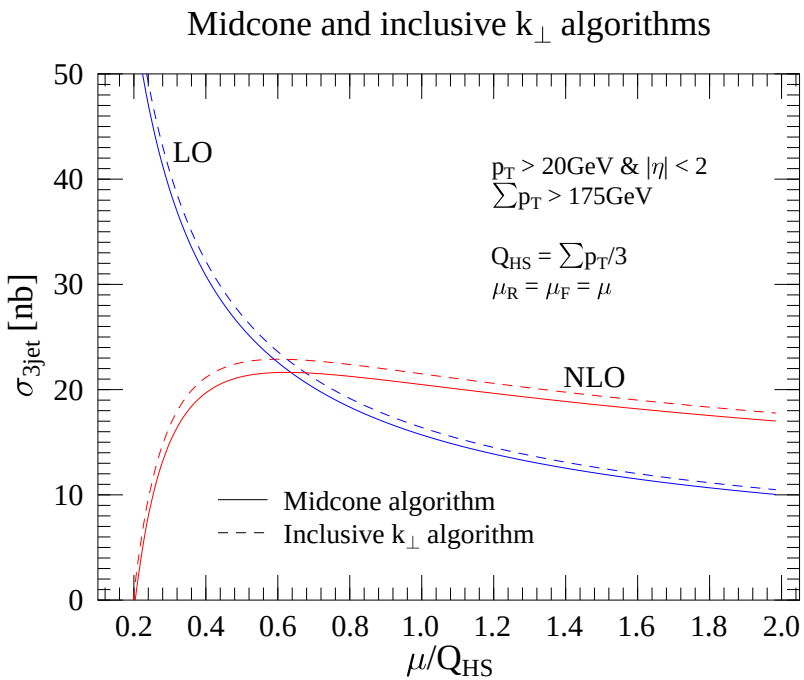
number of partons

complexity of calculation

μ_f cancellation only happens to all orders

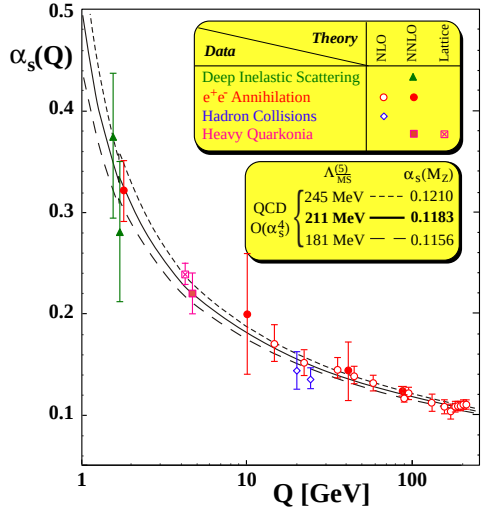
LO	:	no cancellation whatsoever
NLO	:	cancellation starts to work
NNLO	:	better and better
. . .		

$\rightarrow$ motivation for higher order calculations

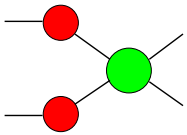


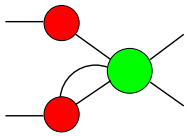
example: 3-jets @ TeVatron (Z. Nagy)

Factorized cross sections (cont.)

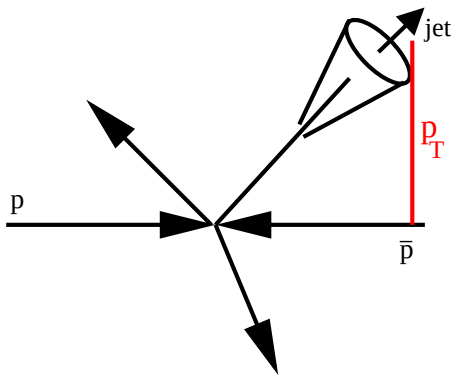


Question: How to choose all these scales μ_f, μ'_f, μ_r ?

for our factorized framework  to work . . .

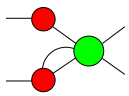
- we have to be in the perturbative domain: $\alpha_s(\mu) \ll 1$
- multi-parton correlations  have to be negligible

$\rightarrow$ **minimum requirement**: at least one hard (= large) scale present, usually p_T



$$\mu_f \simeq \mu'_f \simeq \mu_r = \mathcal{O}(p_T)$$

if $p_T \gtrsim 2\,\mathrm{GeV}$ (no clear borderline!!)

- $\alpha_s(\mu_r) \ll 1 \rightarrow$ pert. expansion should work
-  $\propto \left(\frac{\Lambda}{p_T}\right)^n \rightarrow$ hopefully small

Summary on "factorization" so far

- for the experimentally important class of high-p_T processes we have a well-defined procedure how to compute cross sections in pQCD
- calculations can be systematically improved by going to higher orders in α_s
- desired information on hadronic structure (parton densities) sits inside complicated convolutions, e.g., $d\sigma = \sum_{a,b} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}$
- pQCD results are *not* god given numbers - they have uncertainties as well most prominent ones:
  - residual dependence on choice of scales due to truncation of series

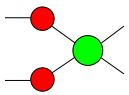
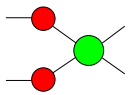
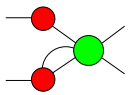
    $\mu_{f,r}$ dependence: important measure of the reliability of pQCD results

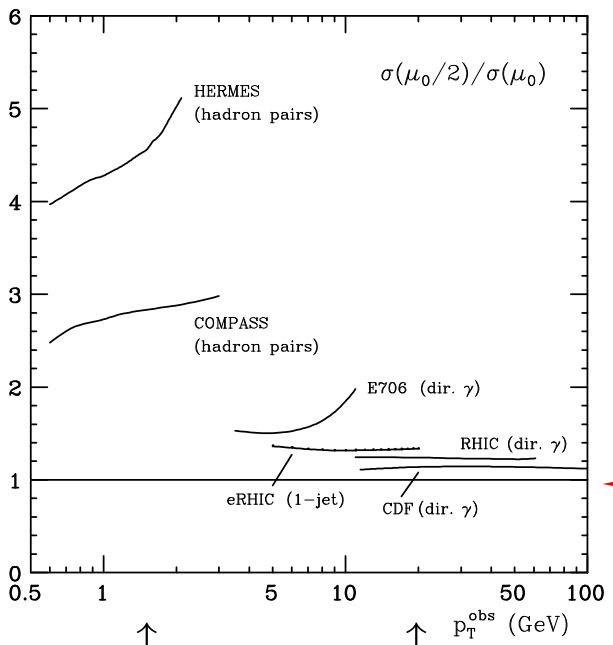
  - unknown size of higher terms in the series

Summary on "factorization" so far (II)

in terms of *reliability* colliders have a big advantage over fixed target experiments:

large c.m.s. energy $\sqrt{S} \rightarrow$ very large p_T-range up to $p_T^{\max} = \sqrt{S}/2$ accessible

- pQCD should be applicable plus  *not*  +  + . . .
- bonus: for same p_T, scale cancellation works better the higher $\sqrt{S}$



typical ratios of cross sections for two choices of μ_f

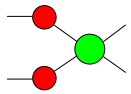
fixed target:
HERMES, COMPASS, E706

collider:
eRHIC, RHIC, CDF

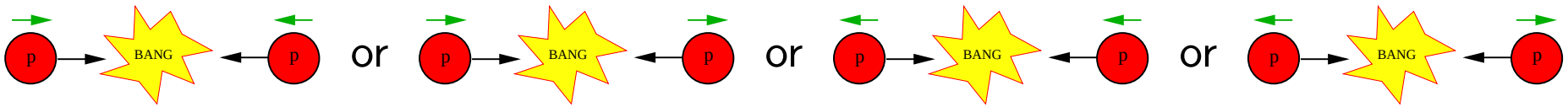
ratio of cross sections should be close to 1

fixed target ✗ collider ✓

Polarized $\vec{p}\vec{p}$ cross sections

next question: How does this "factorized picture"  work out w/ polarization?

with longitudinally polarized protons you can perform *four different experiments*:



label proton spin states by their helicity

$d\sigma(+,+)$ $\quad$ $d\sigma(+,-)$ $\quad$ $d\sigma(-,+)$ $\quad$ $d\sigma(-,-)$

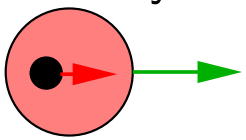
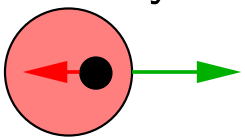
so far we have discussed the unpolarized (*helicity-averaged*) cross section:

$$d\sigma \equiv \frac{1}{2} \cdot \frac{1}{2} \left[d\sigma(+,+) + d\sigma(+,-) + d\sigma(-,+) + d\sigma(-,-) \right]$$

let's have a closer look at what happens to $d\sigma(+,+)$ at the *partonic level* . . .

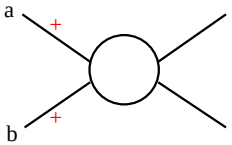
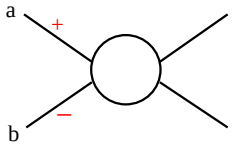
Polarized $\vec{p}\vec{p}$ cross sections (cont.)

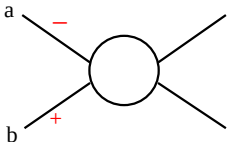
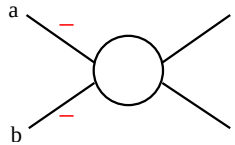
recall: in a proton with helicity $+$ we can find a parton with

helicity $+$  or helicity $-$ 

$$f^{\rightarrow}_{\rightarrow} \equiv f^{+}_{+} \qquad\qquad f^{\rightarrow}_{\leftarrow} \equiv f^{+}_{-}$$

$\rightarrow$ hadronic $d\sigma(+,+)$ decomposes into *four partonic reactions*:

$$= f^{+}_{a,+} \otimes f^{+}_{b,+} \otimes$$  $$+ f^{+}_{a,+} \otimes f^{+}_{b,-} \otimes$$ 

$$+ f^{+}_{a,-} \otimes f^{+}_{b,+} \otimes$$  $$+ f^{+}_{a,-} \otimes f^{+}_{b,-} \otimes$$ 

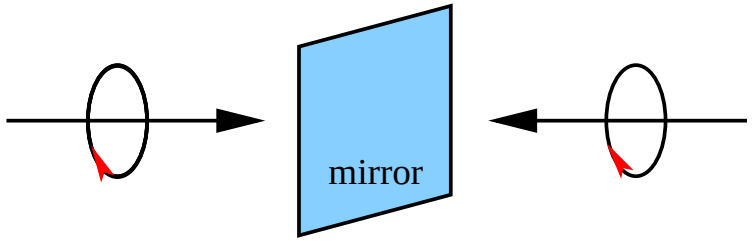
$$= f^{+}_{a,+} \otimes f^{+}_{b,+} \otimes d\hat{\sigma}(+,+) \qquad + f^{+}_{a,+} \otimes f^{+}_{b,-} \otimes d\hat{\sigma}(+,-)$$

$$+ f^{+}_{a,-} \otimes f^{+}_{b,+} \otimes d\hat{\sigma}(-,+) \qquad + f^{+}_{a,-} \otimes f^{+}_{b,-} \otimes d\hat{\sigma}(-,-)$$

and similarly for $d\sigma(+,-)$, $d\sigma(-,+)$, and $d\sigma(-,-)$

Polarized $\vec{p}\vec{p}$ cross sections (cont.)

strong interactions invariant under *parity*:



parity flips helicity $\rightarrow$ $f_+^+ = f_-^-$ and $f_-^+ = f_+^-$

This simplifies everything considerably and one obtains

$$
\begin{aligned}
d\sigma &\equiv \frac{1}{4}\left[d\sigma(+,+) + d\sigma(+,-) + d\sigma(-,+) + d\sigma(-,-)\right] \\
&= \frac{1}{4}\sum_{a,b}(f_{a,+}^+ + f_{a,-}^+) \otimes (f_{b,+}^+ + f_{b,-}^+) \\
&\qquad \otimes \left[d\hat{\sigma}(+,+) + d\hat{\sigma}(+,-) + d\hat{\sigma}(-,+) + d\hat{\sigma}(-,-)\right] \\
&\equiv \sum_{a,b} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}
\end{aligned}
$$

. . . our well-known factorized result for the *unpolarized cross section*

Polarized $\vec{p}\vec{p}$ cross sections (cont.)

But which combination of polarized cross sections contains the desired $\Delta f_{a,b}$?

This one . . .

$$
\begin{aligned}
d\Delta\sigma &\equiv \frac{1}{4}\left[d\sigma(+,+) - d\sigma(+,-) - d\sigma(-,+) + d\sigma(-,-)\right] \\
&= \frac{1}{4}\sum_{a,b}(f^{+}_{a,+} - f^{+}_{a,-}) \otimes (f^{+}_{b,+} - f^{+}_{b,-}) \\
&\qquad \otimes \left[d\hat{\sigma}(+,+) - d\hat{\sigma}(+,-) - d\hat{\sigma}(-,+) + d\hat{\sigma}(-,-)\right] \\
&\equiv \sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab}
\end{aligned}
$$

- defines the spin-dependent $\vec{p}\vec{p}$ cross section measurable at RHIC
- for parity-conserved QCD processes this simplifies further:

$$
\begin{aligned}
d\Delta\sigma &= \frac{1}{2}\left[d\sigma(+,+) - d\sigma(+,-)\right] \quad (\mathrm{hadron-level}) \\
d\Delta\hat{\sigma} &= \frac{1}{2}\left[d\hat{\sigma}(+,+) - d\hat{\sigma}(+,-)\right] \quad (\mathrm{parton-level})
\end{aligned}
$$

Polarized $\vec{p}\vec{p}$ cross sections (cont.)

upshot: to compute spin-dependent cross sections we have a similar factorized "picture" as discussed earlier

one only has to sprinkle some $\Delta' s$ where appropriate

example: *polarized* high-p_T single-inclusive pion production $\vec{p}(P_a)\vec{p}(P_b) \to \pi(P^\pi)X$

long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

$$\frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{dp_T d\eta} = \sum_{abc} \int dx_a\, dx_b\, dz_c\, \Delta f_a(x_a, \mu_f)\, \Delta f_b(x_b, \mu_f)\, D_c^\pi(z_c, \mu'_f)$$
$$\times \frac{d\Delta\hat{\sigma}^{ab\to cX'}}{dp_T d\eta}(x_a P_a, x_b P_b, P^\pi/z_c, \mu_f, \mu'_f, \mu_r) + \mathcal{O}(\frac{\lambda}{p_T})^n$$

short-distance

calculable in pQCD: power series in α_s

power corrections

neglected

Polarized $\vec{p}\vec{p}$ cross sections (cont.)

some final remarks on this section:

- measurable $d\sigma(\pm,\pm)$ all positive (# events for certain polarization pattern)
  - unpolarized cross section $d\sigma = \frac{1}{2}\left[d\sigma(+,+)+d\sigma(+,-)\right]$ always positive
  - sign of polarized cross section $d\Delta\sigma = \frac{1}{2}\left[d\sigma(+,+)-d\sigma(+,-)\right]$ not fixed
  - immediately gives "positivity bound": → Jacques Soffer's lecture

$$|d\Delta\sigma| \leq d\sigma$$

- we will frequently encounter the experimentally relevant spin asymmetry:

$$A_{\rm LL} \equiv \frac{d\Delta\sigma}{d\sigma}$$

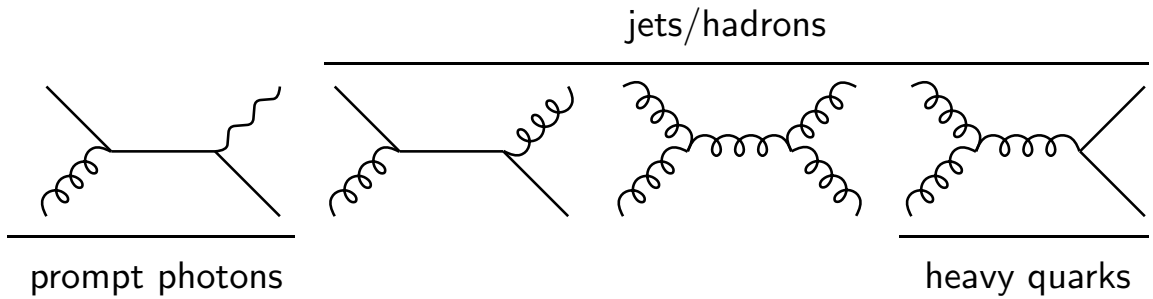
  - note: it's an urban legend that NLO pQCD corrections cancel in $A_{\rm LL}$!!!

Cross sections relevant for RHIC spin

recall major goal of RHIC spin program with longitudinal polarization:

pin down all aspects of helicity pdfs, in particular poorly known Δg

$\longrightarrow$ study processes with a *dominant* gluon contribution in LO:



reaction	LO subprocesses	partons probed	x-range
$pp \longrightarrow$ jets X	$q\bar{q}, qq, qg, gg \longrightarrow \text{jet } X$	Δq, Δg	$x \gtrsim 0.03$
$pp \longrightarrow \pi X$	$q\bar{q}, qq, qg, gg \longrightarrow \pi X$	Δq, Δg	$x \gtrsim 0.03$
$pp \longrightarrow \gamma X$	$qg \longrightarrow q\gamma$, $q\bar{q} \longrightarrow g\gamma$	Δg	$x \gtrsim 0.03$
$pp \longrightarrow Q\bar{Q}X$	$gg \longrightarrow Q\bar{Q}$, $q\bar{q} \longrightarrow Q\bar{Q}$	Δg	$x \gtrsim 0.01$
$pp \longrightarrow W^{\pm}X$	$q\bar{q}' \longrightarrow W^{\pm}$	Δu, $\Delta\bar{u}$, Δd, $\Delta\bar{d}$	$x \gtrsim 0.06$

Cross sections: Born approximation

at $\mathcal{O}(\alpha_s^2)$ (LO = "Born" = "tree-level") one has:

eight relevant QCD $2 \to 2$ parton-parton scattering processes $ab \to cd$

$$
\begin{aligned}
qq' &\to qq' \\
q\bar{q} &\to q'\bar{q}' \\
qq &\to qq \\
q\bar{q} &\to q\bar{q} \\
q\bar{q} &\to gg \\
gg &\to q\bar{q} \\
qg &\to qg \\
gg &\to gg
\end{aligned}
$$

unpol.: boils down to *four* different processes $qq' \to qq'$, $qq \to qq$, $q\bar{q} \to gg$, $gg \to gg$

all other processes related by *crossing symmetry*: $ab \to cd \leftrightarrow a\bar{d} \to c\bar{b}$

polarized: cannot use crossing, e.g., $\vec{q}\vec{\bar{q}} \to gg \neq \vec{q}\vec{g} \to qg$

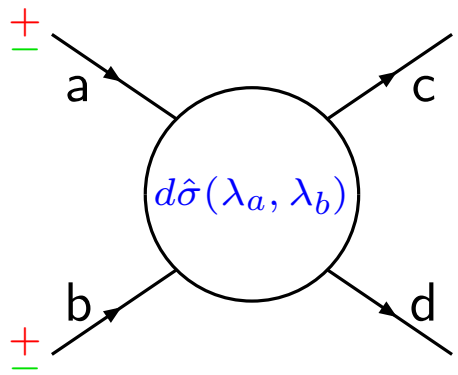
Cross sections (cont.)

computation of LO cross sections is pretty straightforward:

$$a(p_a, \lambda_a) + b(p_b, \lambda_b) \to c(p_c) + d(p_d)$$

$\uparrow$ $\uparrow$

helicities $\lambda_{a,b} = \pm 1$



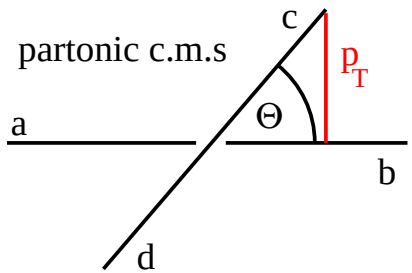
- the helicity-dependent partonic QCD cross sections can be written as

$$d\hat{\sigma}(\lambda_a, \lambda_b) = d\hat{\sigma} + \lambda_a \, \lambda_b \, d\Delta\hat{\sigma}$$

$\to$ no single-spin asymmetries for QCD processes at Born-level

- dependence of $d\hat{\sigma}(\lambda_a, \lambda_b)$ on kinematics conveniently expressed in terms of

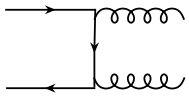
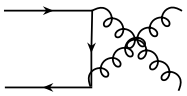
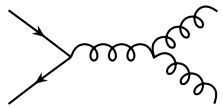
$$\begin{aligned} s &\equiv (p_a + p_b)^2 \\ t &\equiv (p_a - p_c)^2 = -\sqrt{s}\, p_T \, e^{-\eta} \\ u &\equiv (p_a - p_d)^2 = -\sqrt{s}\, p_T \, e^{\eta} \\ \text{where} \quad & \eta = -\ln \tan(\Theta/2) \end{aligned}$$



Cross sections (cont.)

example: $q(p_a, \lambda_a)\, \bar{q}(p_b, \lambda_b) \to g(p_c)\, g(p_d)$

1. step: write down all relevant *Feynman diagrams*

$M_1 =$  $M_2 =$  $M_3 =$ 

2. step: get amplitudes M_i by applying *Feynman rules* for vertices, propagators, and external lines

3. step: compute total amplitude squared $MM^* = |M|^2 = |M_1 + M_2 + M_3|^2$

4. step: attach flux factor $1/2s$ for incoming partons, a *phase space factor* $\frac{d^3p_i}{(2\pi)^3(2E_i)}$ for each produced parton, and four-momentum conservation $(2\pi)^4\delta^{(4)}(p_a + p_b - p_c - p_d)$ to obtain

$$d\hat{\sigma}(s,t,u,\lambda_a,\lambda_b) = \frac{1}{2s}|M|^2(s,t,u,\lambda_a,\lambda_b)\frac{d^3p_c}{(2\pi)^3(2E_c)}\frac{d^3p_d}{(2\pi)^3(2E_d)}(2\pi)^4\delta^{(4)}(p_a+p_b-p_c-p_d)$$

Cross sections (cont.)

after some algebra and "*Dirac*ology" one obtains the full list of LO QCD processes:

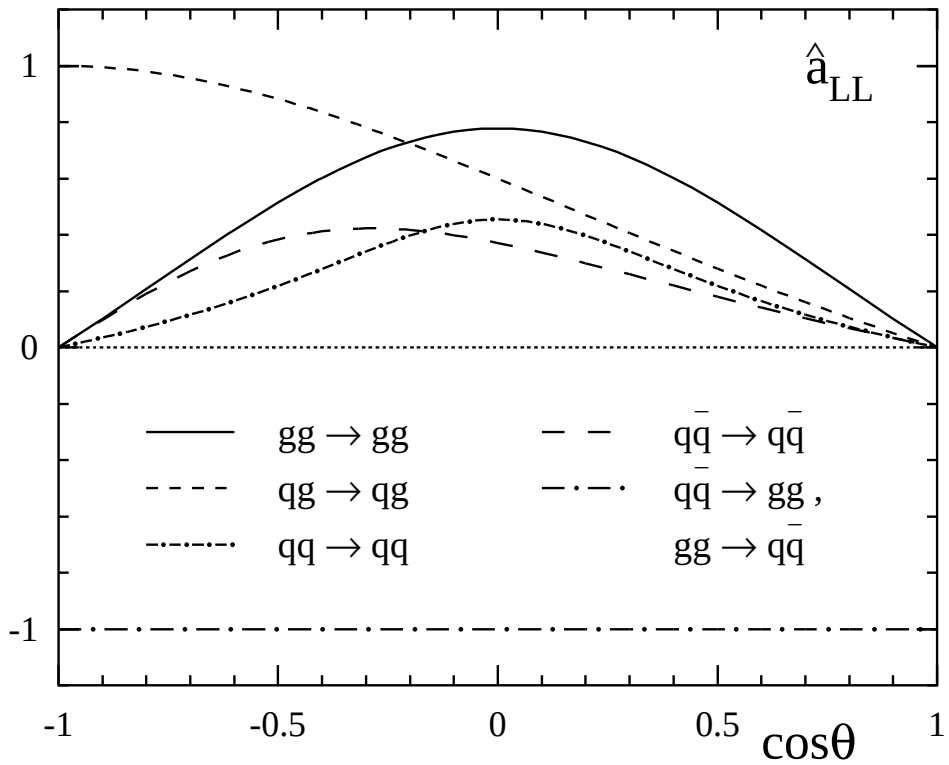
process	$\lvert M\rvert^2(+,+)/g_s^4$	$\lvert M\rvert^2(+,-)/g_s^4$
$qq' \to qq'$	$8s^2/(9t^2)$	$8u^2/(9t^2)$
$q\bar{q} \to q'\bar{q}'$	0	$8(t^2+u^2)/(9s^2)$
$qq \to qq$	$8(s^2/t^2+s^2/u^2-2s^2/(3tu))/9$	$8(u^2/t^2+t^2/u^2)/9$
$q\bar{q} \to q\bar{q}$	$8s^2/(9t^2)$	$8(u^2/t^2+t^2/s^2+u^2/s^2-2u^2/(3st))/9$
$q\bar{q} \to gg$	0	$64(t^2+u^2)/(27ut)-16(t^2+u^2)/(3s^2)$
$gg \to q\bar{q}$	0	$(u^2+t^2)/(3ut)-3(t^2+u^2)/(4s^2)$
$qg \to qg$	$2s^2/t^2-8s^2/(9us)$	$2u^2/t^2-8u^2/(9us)$
$gg \to gg$	$9(2s^2/(ut)-su/t^2-st/u^2)/2$	$9(6-2s^2/(ut)-2u/t^2-st/u^2-2ut/s^2)/2$

and similar for $qg \to q\gamma$ (prompt photons), $q\bar{q} \to l\bar{l}$ (Drell-Yan), $gg, q\bar{q} \to Q\bar{Q}$ (heavy flavors), . . .

Cross sections (cont.)

some observations:

- $gg \to gg$ dominates at $\Theta = 90^\circ$ ($\eta = 0$; $t = u = -s/2$) followed by $qg \to qg$
- most processes have $\hat{a}_{\rm LL} \equiv d\Delta\hat{\sigma}/d\hat{\sigma} > 0$, $q\bar{q}$ annihilation has -1



Cross sections (cont.)

LO pQCD results/predictions

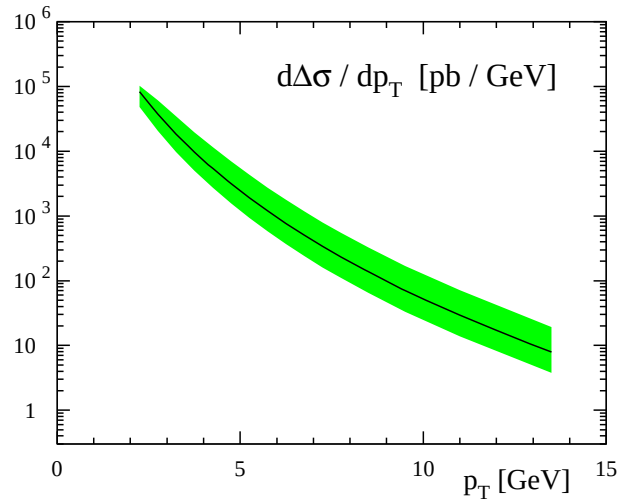
- are useful for quick & dirty estimates of *qualitative* features of a process

- suffer from large theoretical uncertainties

  example:

  $\vec{p}(P_a)\vec{p}(P_b) \to \pi(P^\pi)X$ @ high-p_T

  band: variation $p_T/2 \leq \mu_f \leq 2p_T$



upshot: NLO corrections are in general a must if it comes to numbers

- expect much reduced dependence on unphysical scales $\mu_{f,r}$

- NLO corrections are often sizable

- may effect the sensitivity to Δg and its extraction from data

NLO corrections in a nutshell

Going beyond the LO is in every respect a major enterprise . . .

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\, d\hat{\sigma}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\, d\hat{\sigma}^{(2)}}_{\text{NNLO}} + \ldots$$

number of partons

complexity of calculation

- NLO techniques are well established and most cross sections are available
- NNLO still far from being standard, a lot of progress though

recent example: the NNLO DGLAP evolution kernels P_{ij} Moch, Vermaseren, Vogt

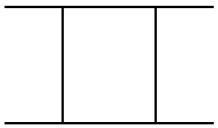
$\rightarrow$ NNLO = pushing computer algebra programs to their limits

	# diagrams	# integrals
LO	18	a few
NLO	350	some more
NNLO	9607	$\sim 10^5$

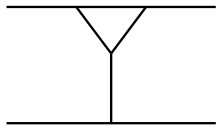
NLO corrections in a nutshell (cont.)

at $\mathcal{O}(\alpha_s^3)$ (NLO) one has to consider:

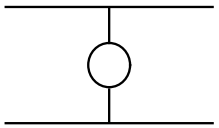
- one-loop (virtual) corrections to all LO $2 \to 2$ processes



"box"



"vertex"



"selfenergy"

- all conceivable $2 \to 3$ parton-parton scattering processes

  this includes additional gluon emission to existing LO processes

$$qq' \to qq'g,\ q\bar{q} \to ggg,\ gg \to ggg, \text{ etc.}$$

  as well as genuine NLO processes not possible at $\mathcal{O}(\alpha_s^2)$

$$qg \to qq'\bar{q}',\ qg \to qq\bar{q}, \text{ etc.}$$

NLO corrections in a nutshell (cont.)

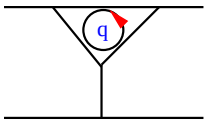
complication: all diagrams have infinities in four dimensions

three types of singularities will appear:

- *ultraviolet (UV) singularities*:

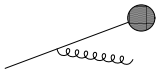
  due to the integration of the unobserved loop-momentum up to infinity

$$\int_0^\infty d^4q$$



- *infrared (UV) singularities*:

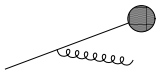
  emission of soft gluons ($E_g \to 0$)



$$\frac{1}{p_q \cdot p_g} = [E_q E_g (1 - \cos\Theta_{qg})]^{-1}$$

- *collinear or mass singularities*:

  collinear emission ($\Theta_{qg} \to 0$)



$$\frac{1}{p_q \cdot p_g} = [E_q E_g (1 - \cos\Theta_{qg})]^{-1}$$

  (not possible for massive quarks)

NLO corrections in a nutshell (cont.)

all singularities have to be tamed before the calculation can make any sense

standard procedure: *dimensional regularization* 't Hooft, Veltman

- continue space-time to $n = 4 - 2\varepsilon$ dimensions
- all infinities manifest themselves as $1/\varepsilon^k$
- free of artifical cut-offs; works systematically to all orders
- $1/\varepsilon^k$ have to be removed before taking the physical limit $\varepsilon \to 0$ in the end

price to pay:

- kinematics/phase space/integrals in n dimensions much more complex
- polarization requires special attention: spin projectors genuine four-dimensional

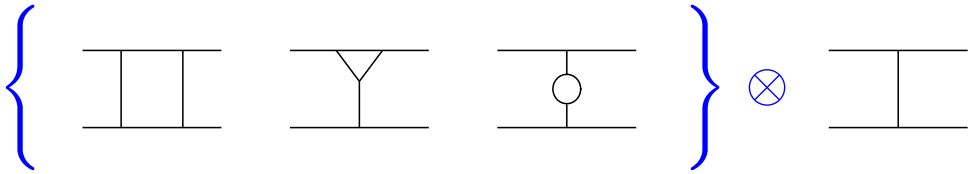
  consistent procedure known $\to$ kinematics even more complex

Breitenlohner, Maison, 't Hooft, Veltman

NLO corrections in a nutshell (cont.)

a closer look at virtual corrections:

at $\mathcal{O}(\alpha_s^3 = g_s^6)$ only the interference of 1-loop and Born amplitudes contributes:



- kinematics described by s, t, u like for a $2 \to 2$ Born process
- IR+UV divergencies show up as $1/\varepsilon$ and $1/\varepsilon^2$
- main technical challenge: computation of loop-integrals

e.g.: $\sim \int d^n q \, \frac{q^\mu [q^\nu [q^\rho]]}{q^2 (q+p_a)^2 (q+p_a-p_c)^2 (q+p_a-p_c-p_d)^2}$

can be reduced to large set of n-dim. scalar integrals Passarino, Veltman

renormalization: UV poles "used" to define physical coupling $g_s \to$ UV finite

Interlude: "philosophy of renormalization"

idea of renormalization:

the quantities we start off from are not physical, e.g., the *bare strong coupling* $g_s^{\rm bare}$

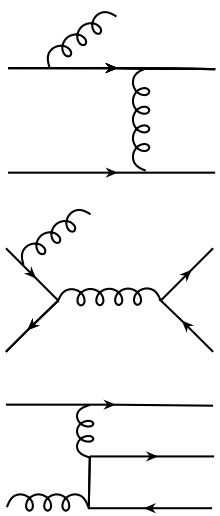
"QCD is renormalizable" = UV poles can be subtracted (at a scale μ_r!) order by order

this procedure defines, e.g., the *physical coupling*: $g_s^{\rm phys} = Z_{\rm ren}\, g_s^{\rm bare}$

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

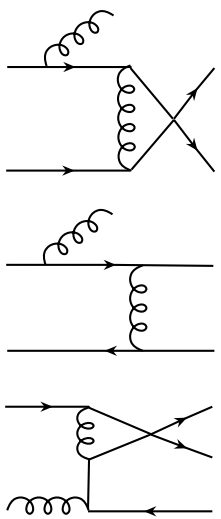
NLO corrections in a nutshell (cont.)

a closer look at $2 \to 3$ corrections: here is a list of some typical diagrams



different flavors

$q \neq q'$



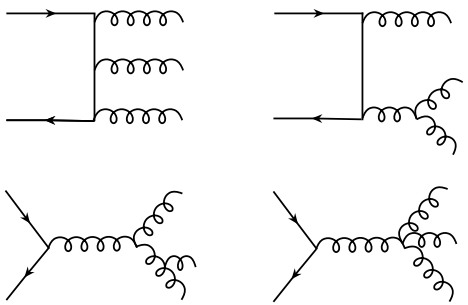
identical flavors

$q = q'$

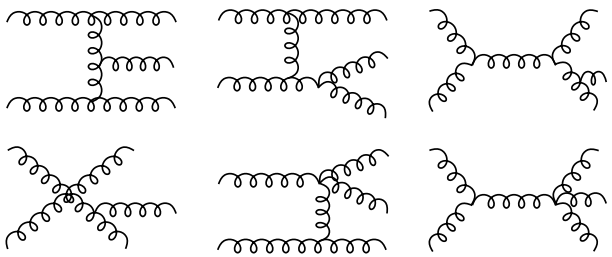
$qq' \to qq'g$

$q\bar{q} \to q'\bar{q}'g$

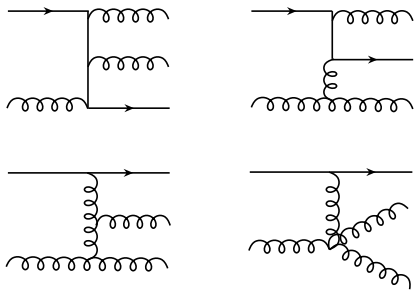
$qg \to qq'\bar{q}'$



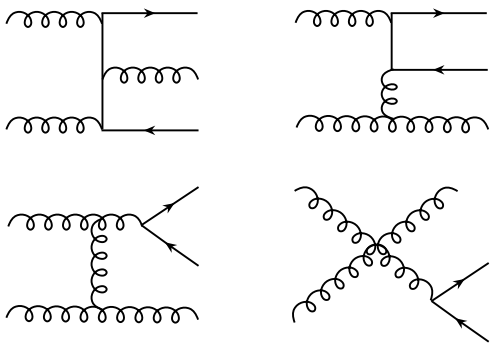
$q\bar{q} \to ggg$



$gg \to ggg$



$qg \to qgg$



$gg \to q\bar{q}g$

NLO corrections in a nutshell (cont.)

main technical challenge here:

- integration over phase space $\frac{d^{(n-1)}p_i}{(2\pi)^{(n-1)}(2E_i)}$ of all unobserved partons

  example: calculation of *single-inclusive* pion cross section:

  typical $2 \to 3$ process: $gg \;\to\; q \;\; (\bar{q}g)$

  fragments: $q \to \pi X$ integrated out

"Monte-Carlo approach":

idea: separate off divergent regions of p.s.

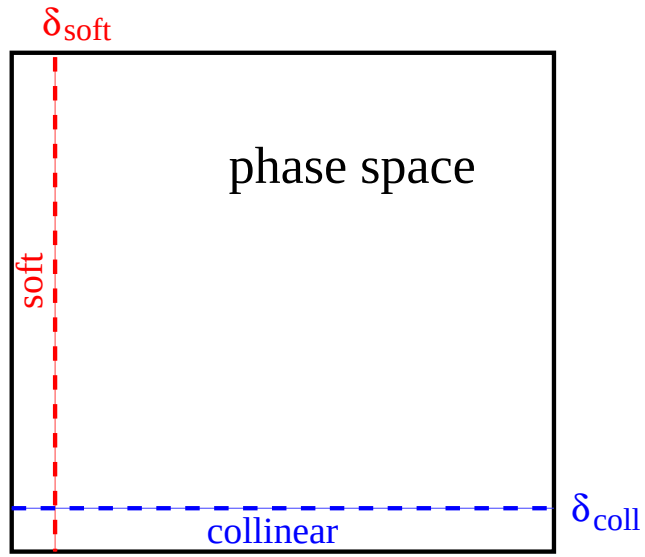
integrate "rest" in 4 dimensions

✓ suitable for complicated observables

✓ exp. cuts easy to build in

× delicate numerical cancellations

× slow → not very suitable for fitting pdfs



NLO corrections in a nutshell (cont.)

"largely analytical approach":

idea: do entire phase space integration analytically in n dimensions

[p.s. best parameterized by two angles $\theta_{1,2}$ in rest frame of the two unobserved partons]

$$d\Delta\hat{\sigma}_{2\to 3} \sim \ldots \int d\theta_1 d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2 \, \Delta \, |M_{2\to 3}|^2$$

extensive partial fractioning boils everything down to

$$I^{(k,l)} = \int \frac{d\theta_1 \ \sin^{1-2\varepsilon}\theta_1 \ d\theta_2 \ \sin^{-2\varepsilon}\theta_2}{(1+\cos\theta_1)^k (1 + A\cos\theta_1 + B\sin\theta_1\cos\theta_2)^l}$$

which can be done *analytically* $\rightarrow$ IR+collinear poles

strength/weakness of method:

- ✓ straightforward, fast numerics
- ✓ well suited for pdf analyses
- × usually limited to single-incl. x-secs
- × exp. cuts not easy to build in

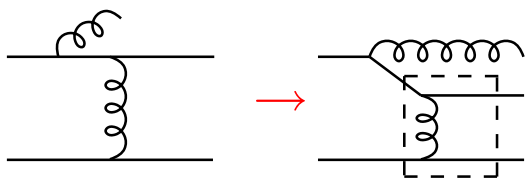
NLO corrections in a nutshell (cont.)

final step: cancellation of remaining singularities *then* $\varepsilon \to 0$

- UV singularities have already been taken care of by renormalizing α_s
- IR singularities cancel in sum of one-loop and $2 \to 3$ contributions

  [essence of Kinoshita, Lee, Nauenberg & Bloch, Nordsieck theorems for "IR-safe" observables]
- remaining collinear singularities have to be removed by factorization

  [factorization = renormalization of bare parton densities and fragmentation fcts.]

e.g.:  $\sim \frac{1}{\varepsilon} \int dx \Delta P_{qq}(x) \Delta\hat{\sigma}_{qq \to qq}$

introduce two arbitrary factorization scales μ_f and μ'_f

initial-/final-state singularities

$\swarrow$ $\searrow$

$\Delta f(x, \mu_f)$ $\qquad$ $D_f^\pi(z, \mu'_f)$

NLO corrections in a nutshell (cont.)

good news: impressive progress recently – list of results relevant for RHIC spin:

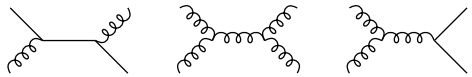
evol. kernels	ΔP_{ij}	NLO		Mertig, van Neerven; Vogelsang
hadrons	$\vec{p}\vec{p} \to H + X$	NLO	•	De Florian; Jäger, Schäfer, MS, Vogelsang
	$p\vec{p} \to \vec{H} + X$	NLO		Jäger et al. (soon)
jets	$\vec{p}\vec{p} \to \mathrm{jet(s)} + X$	NLO	•	De Florian et al.; Jäger,MS, Vogelsang
prompt γ	$\vec{p}\vec{p} \to \gamma + X$	NLO	•	Gordon, Vogelsang; Contogouris et al.
	$\vec{p}\vec{p} \to \gamma\gamma + X$	NLO		Coriano, Gordon
	$p\vec{p} \to \vec{\gamma} + X$	NLO		Vogelsang
γ **+ jet**	$\vec{p}\vec{p} \to \gamma + \mathrm{jet} + X$	NLO		Gordon
γ **+ charm**	$\vec{p}\vec{p} \to \gamma + c + X$	NLO		Berger et al. ($m_c = 0$)
heavy quarks	$\vec{p}\vec{p} \to Q\bar{Q}X$	NLO	•	Bojak, MS
Drell-Yan	$\vec{p}\vec{p} \to (\gamma^*)X$	NLO		Weber; Gehrmann;
		NNLO		Smith et al.
vector bosons	$\vec{p}\vec{p} \to (Z^0, W^\pm)X$	NLO		Weber; Gehrmann
	$p\vec{p} \to (Z^0, W^\pm)X$	NLO	•	Weber; Gehrmann

$\updownarrow$

details/results coming up

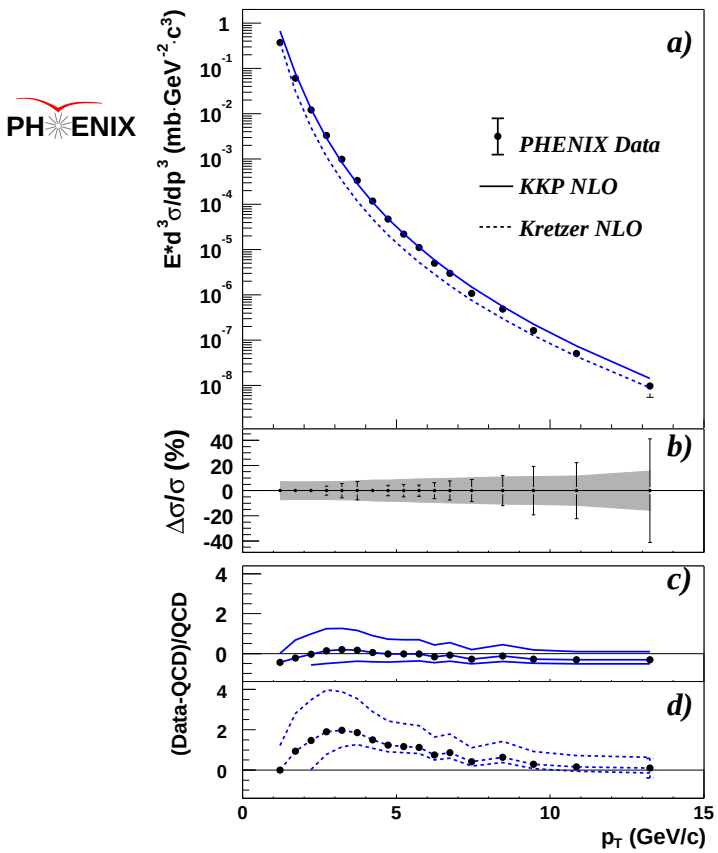
Processes one-by-one

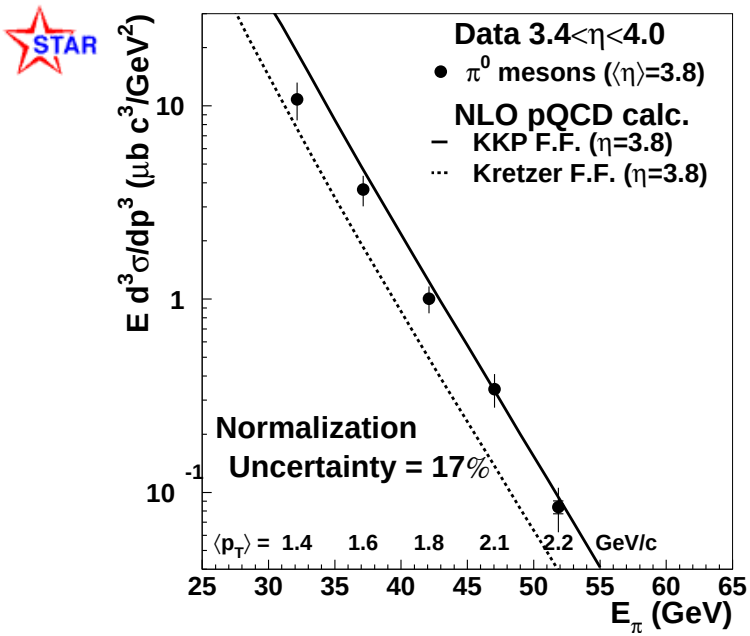
- **high-p_T hadrons:**



NLO: Jäger, MS, Schäfer, Vogelsang; de Florian

recall: $1^{\rm st}$ *unpolarized* measurements at RHIC agree well with pQCD



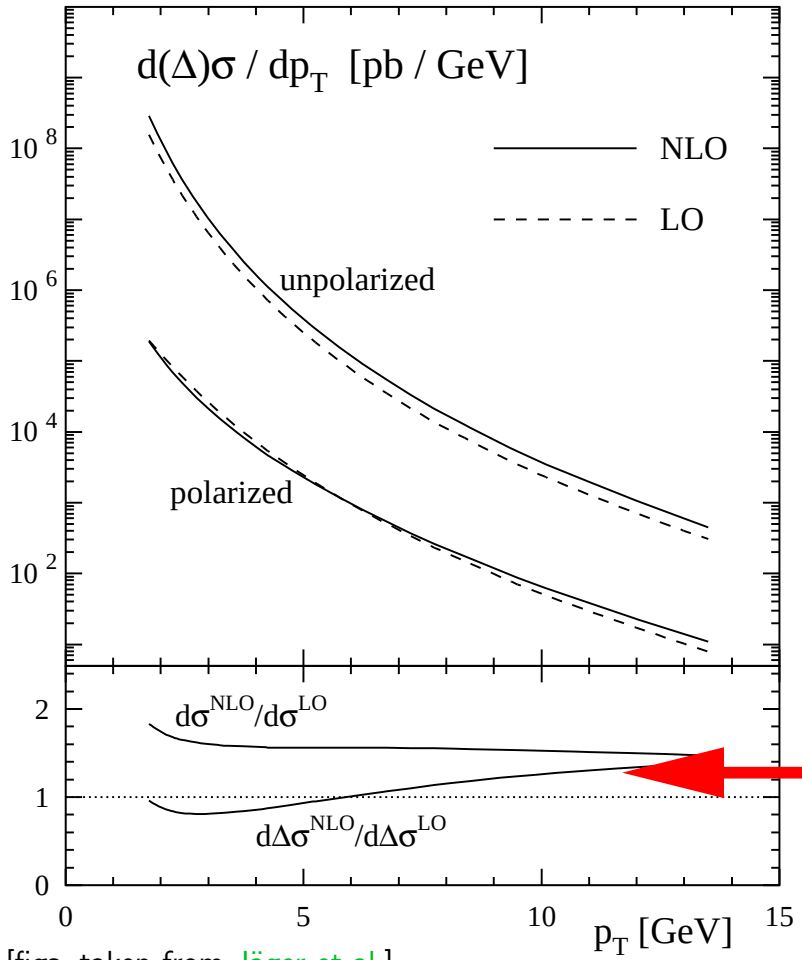


- $D^\pi(z)$ set of KKP favored by data

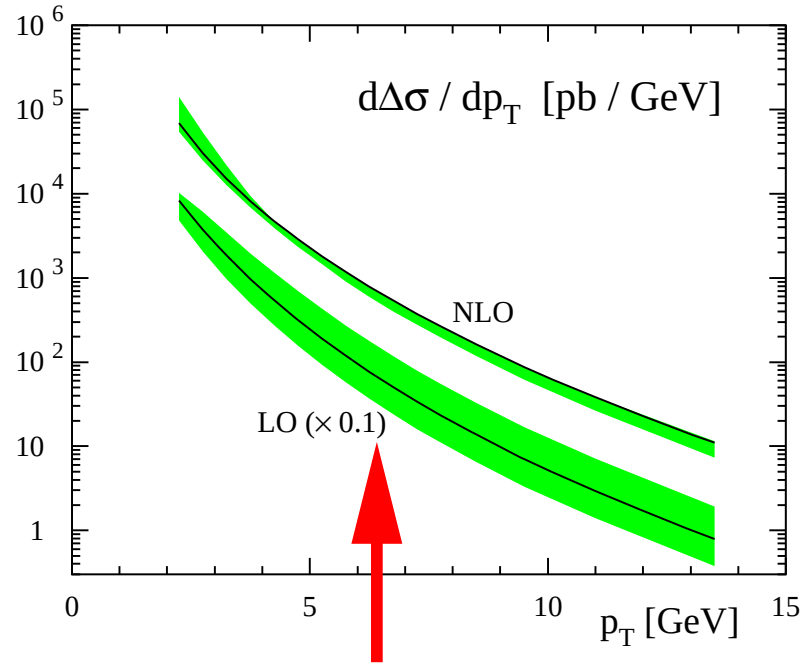
$\longrightarrow$ exciting prospects for similar measurement with polarization

Processes one-by-one (cont.)

a closer look at pQCD results for π^0-production at $\sqrt{S} = 200\,\mathrm{GeV}$, $|\eta| \leq 0.38$:



[figs. taken from Jäger et al.]

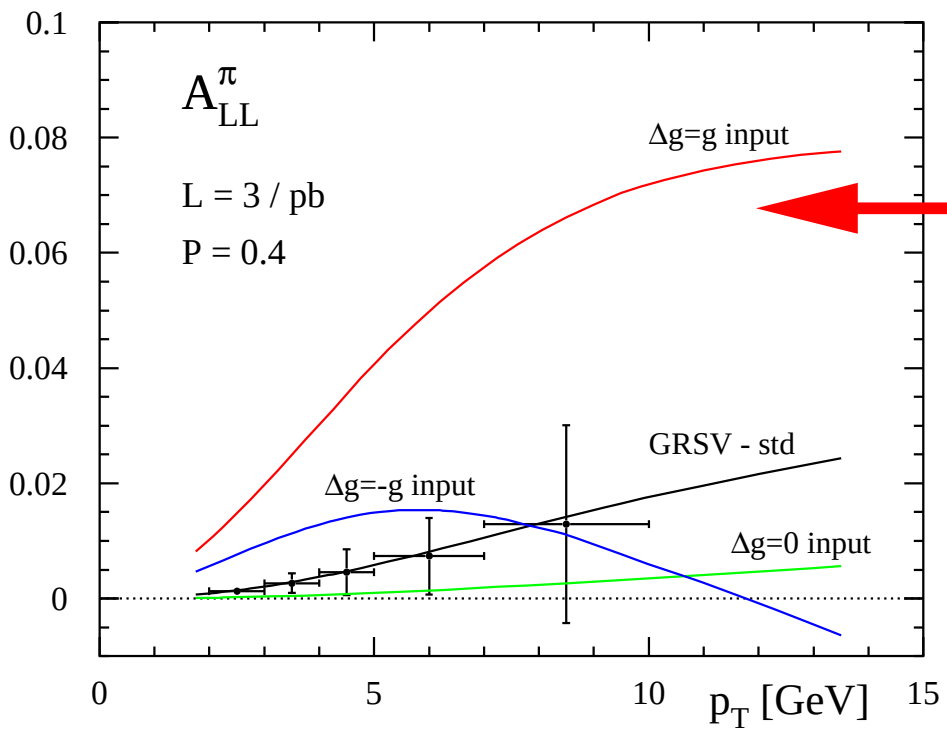


μ_f dependence much reduced in NLO

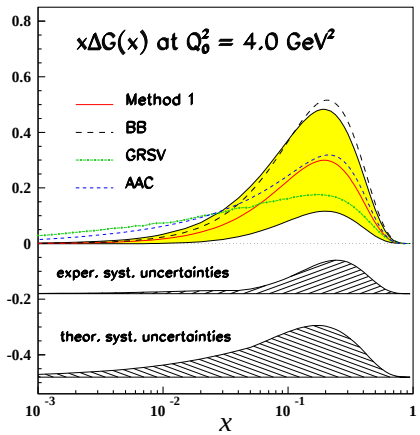
NLO corrections different for $d\Delta\sigma$ and $d\sigma$
$\rightarrow$ do not cancel in $A_{\mathrm{LL}} = d\Delta\sigma/d\sigma$

Processes one-by-one (cont.)

Is the spin asymmetry $A_{\rm LL}$ sensitive to unknown gluon polarization Δg ?



predictions for very different Δg



all compatible with current DIS data

note:

(1) for $p_T \leq 10\,\mathrm{GeV}$: $A_{\rm LL} > 0$

(2) only $3\,\mathrm{pb}^{-1}$ assumed
[design: $320\,\mathrm{pb}^{-1}$]

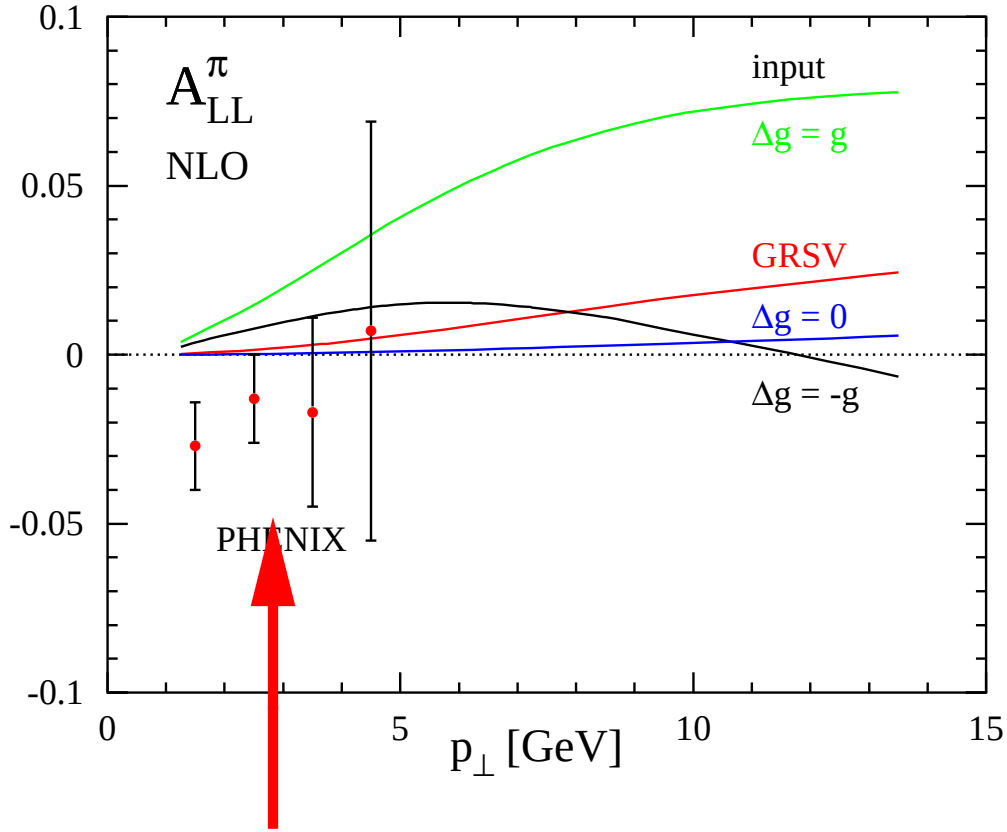
estimate of statistical precision:

$$\delta A_{\rm LL} \simeq \frac{1}{\mathcal{P}^2} \frac{1}{(\mathcal{L} d\sigma_{\rm bin})^{1/2}}$$

$\mathcal{P}$: beam polarization; $\mathcal{L}$: integrated luminosity

Processes one-by-one (cont.)

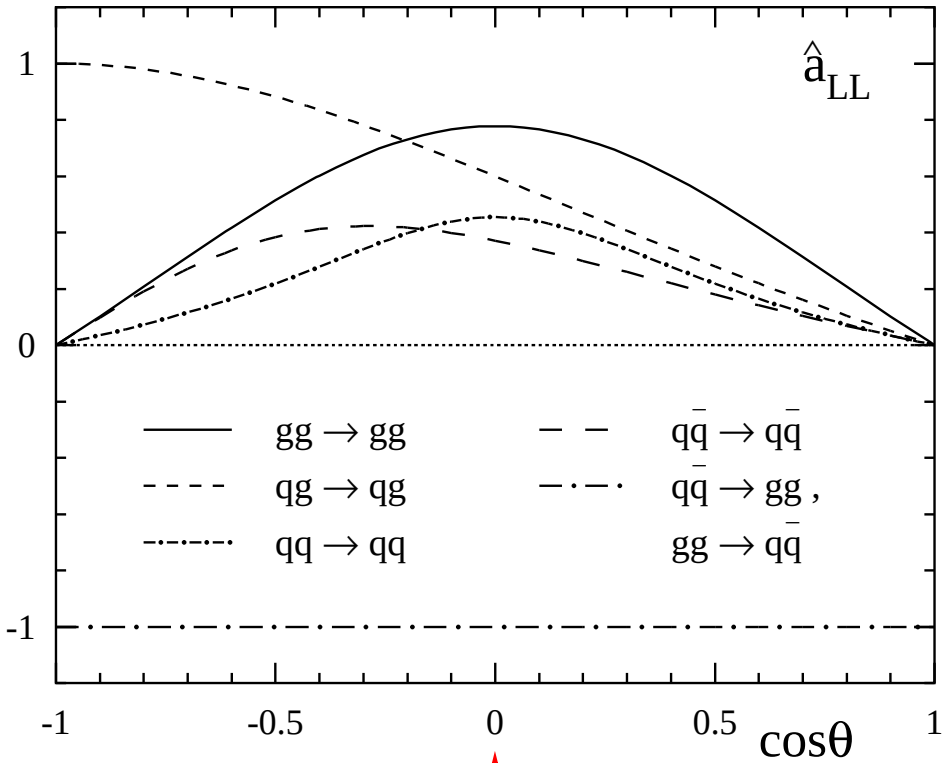
recent exciting development: first results on $A_{\rm LL}$ by PHENIX



trend for $A_{\rm LL} < 0$ at small p_T contrary to expectations

Processes one-by-one (cont.)

How can *that* be?



PHENIX measures at central rapidities

Naive analysis:

need process with $\hat{a}_{\rm LL} < 0$

recall partonic asymmetries

$$gg \to gg \quad \hat{a}_{\rm LL} > 0$$
$$gg \to q\bar{q} \quad \hat{a}_{\rm LL} = -1$$
$$gq \to gq \quad \hat{a}_{\rm LL} > 0$$

conclude:

$gg \to q\bar{q}$ resp. for neg. $A^{\pi}_{\rm LL}$

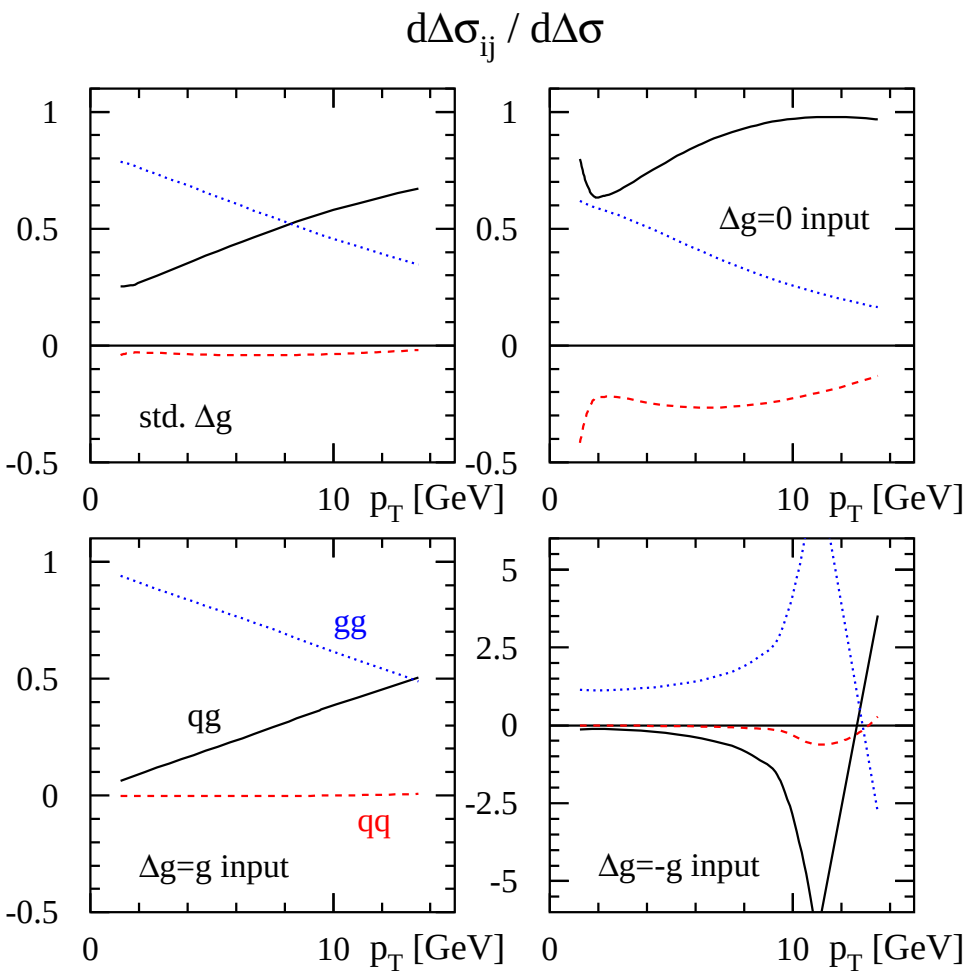
NO!

$$\Delta\hat{\sigma}_{gg\to gg} \simeq 160\ \Delta\hat{\sigma}_{gg\to q\bar{q}}$$
$(\eta \simeq 0)$

Processes one-by-one (cont.)

So - can $\mathbf{A}_{\mathrm{LL}}^{\pi}$ be negative?

Jäger, Kretzer, MS, Vogelsang



subprocess contributions:

fairly independently on what we assume about Δg we find:

gg processes:
dominate for $p_T \lesssim 10\,\mathrm{GeV}$

qg processes:
take over for $p_T \gtrsim 10\,\mathrm{GeV}$

qq processes:
always small unless p_T very large

Processes one-by-one (cont.)

not yet taken into account:

both partons are not probed at the same momentum fraction x

$\rightarrow$ even for $\hat{a}_{\rm LL} > 0$ we can have $A_{\rm LL} < 0$ *if* $\Delta f_a(x_a)\Delta f_b(x_b) < 0$

we can even *analytically* derive a lower bound on $A_{\rm LL}$:

$$\frac{d\Delta\sigma}{dp_T} = \sum_{a,b,c} f_a(x_a) \otimes f_b(x_b) \otimes \frac{d\Delta\hat{\sigma}_{ab}(x_a, x_b, z_c)}{dp_T} \otimes D_c^h(z_c)$$

take $x_T^2 \equiv 4p_T^2/S$ moments $\int dx_T^2 (x_T^2)^{N-1} \ldots \rightarrow$ convolutions turn into products

$$\Delta\sigma^\pi(N) = \left(\Delta g^{N+1}\right)^2 \underset{gg}{\underset{\updownarrow}{\mathcal{A}^N}} + 2\Delta g^{N+1} \underset{qg}{\underset{\updownarrow}{\mathcal{B}^N}} + \underset{qq}{\underset{\updownarrow}{\mathcal{C}^N}}$$

this is a parabola in $\Delta g^N \rightarrow$ minimize!

Processes one-by-one (cont.)

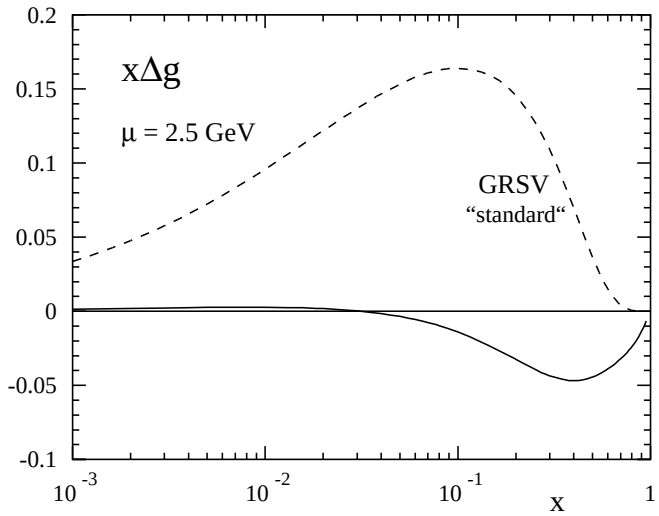
minimization yields: $\Delta\sigma^\pi(N)\Big|_{\rm min} = -\frac{\left(\mathcal{B}^N\right)^2}{\mathcal{A}^N} + \mathcal{C}^N$

$\rightarrow$ negative, but tiny *lower bound* $A^\pi_{\rm LL}\Big|_{\rm min} \simeq \mathcal{O}(-10^{-3}) \ggg$ indications from data

as expected:

the resulting Δg has a node

i.e., $\Delta g(x_a)\Delta g(x_b) < 0$



but still way too early to cry!

- "problem" only in lowest p_T bin where uncertainties are large
- p_T perhaps too small to apply pQCD [it works for $d\sigma/dp_T$ though!]

we need much more data to call this a new "spin surprise"

Processes one-by-one (cont.)

another lesson: around mid-rapidity and for $p_T \lesssim 10\,\mathrm{GeV}$ it is difficult to even pin down the sign of Δg

reason: gg dominance and $\eta \simeq 0 \leftrightarrow x_a \simeq x_b$

What about an A_{LL}^{π} measurement at large rapidities by STAR ?

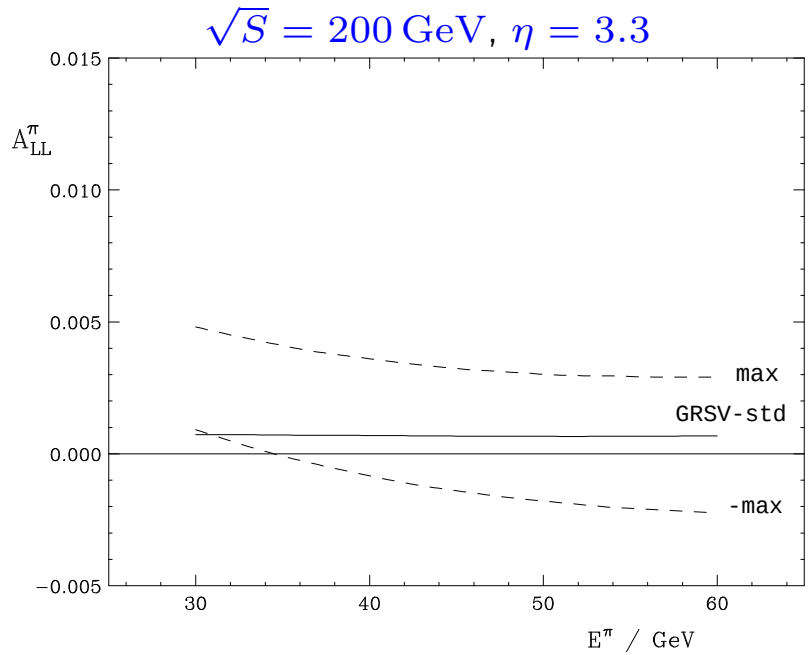
idea: $|\eta| \gg 0$: partonic system boosted

$\longrightarrow$ probes highly asymmetric x_a, x_b

expect: dominance of qg sets in

$\longrightarrow$ sign/size of A_{LL}^{π} tied to sign/size of Δg

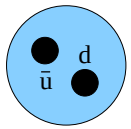
problem: A_{LL}^{π} tiny



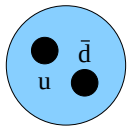
$\langle p_T \rangle =$ 2.3 2.8 3.0 3.4 GeV

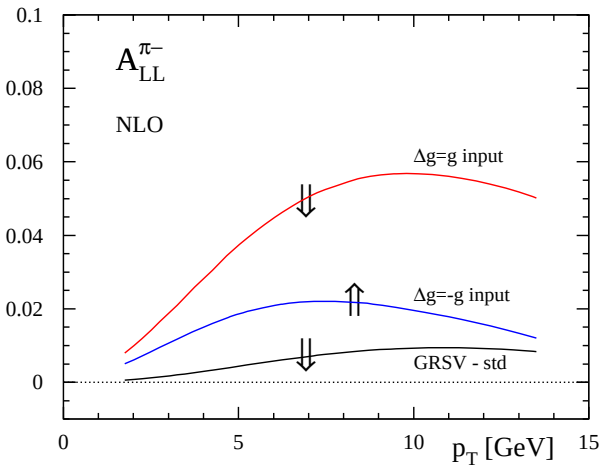
Processes one-by-one (cont.)

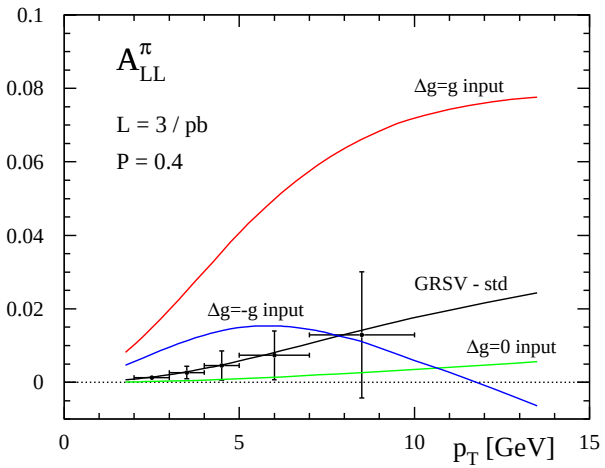
with more luminosity PHENIX can go to higher p_T at $\langle\eta\rangle \simeq 0$ *plus* $A_{\rm LL}^{\pi^\pm}$ vs. $A_{\rm LL}^{\pi^0}$

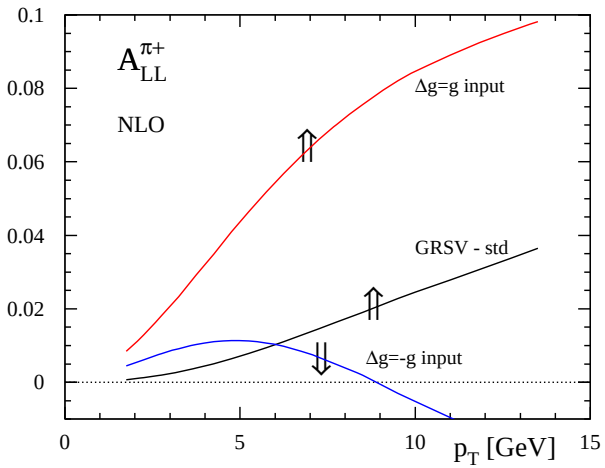
π^- 

π^0

π^+ 





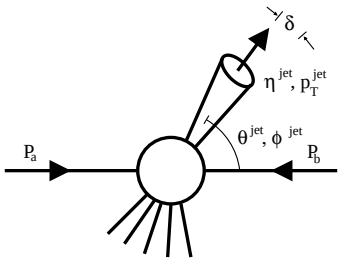


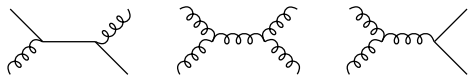
idea: qg starts to dominate for $p_T \gtrsim 5\,\mathrm{GeV}$ and $D_u^{\pi^+} > D_u^{\pi^0} > D_u^{\pi^-}$, $D_g^{\pi^+} = D_g^{\pi^-}$

expect: sensitivity to sign of Δg, e.g., positive Δg: $A_{\rm LL}^{\pi^+} > A_{\rm LL}^{\pi^0} > A_{\rm LL}^{\pi^-}$

Processes one-by-one (cont.)

- **high-p_T jets:**

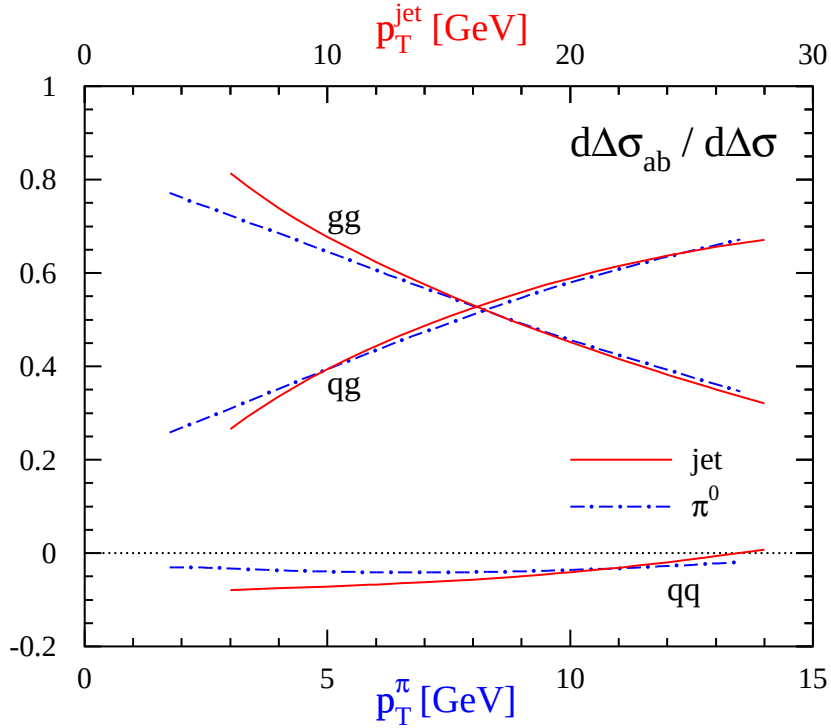




NLO: Jäger, MS, Vogelsang; de Florian, Frixione, Signer, Vogelsang

jet = bunch of particles in a small pencil-like cone; all final-state sing. cancel

jet production proceeds through the same partonic subprocesses as π-production:



π's have roughly $\langle z \rangle \simeq 0.5$:

$\rightarrow$ π with $p_T \simeq$ jet with $2p_T$

comparison to hadrons:

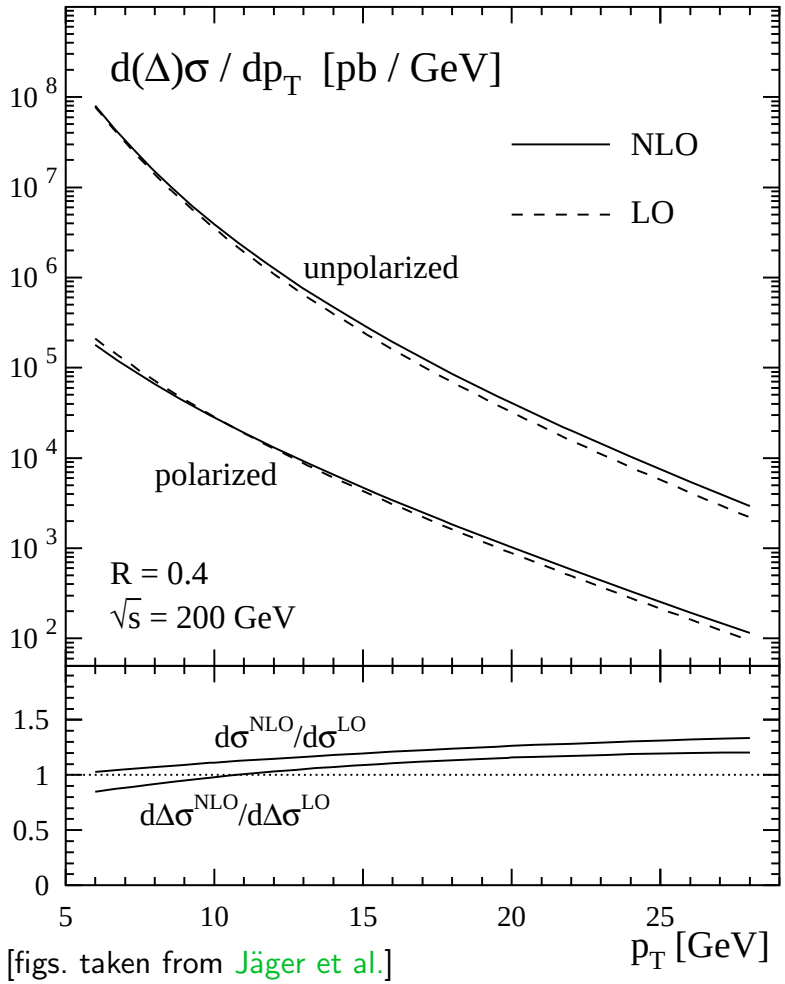
✓ much higher rates

✓ no uncertainties from $D(z)$

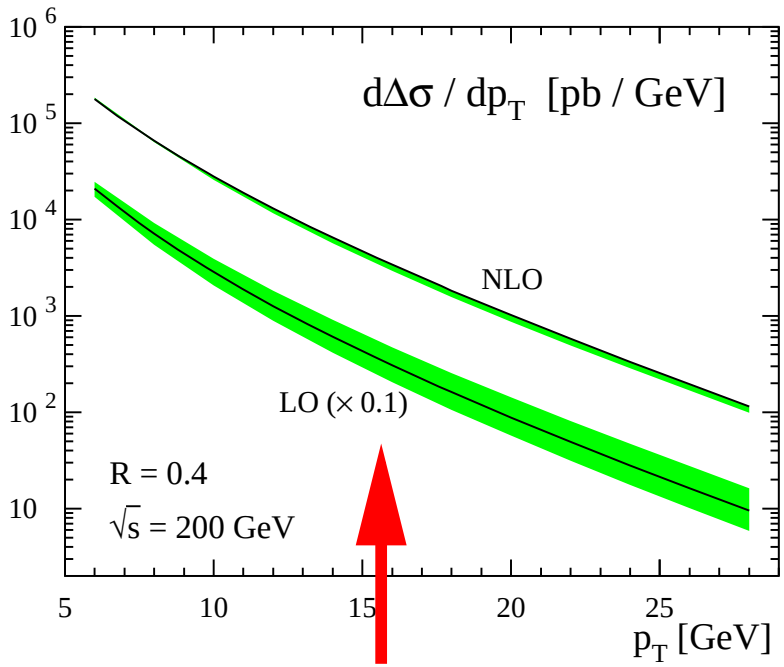
× dependence on precise definition of jet

Processes one-by-one (cont.)

pQCD results for jet-production at $\sqrt{S} = 200\,\mathrm{GeV}$, $R_{\mathrm{cone}} = 0.4$ (SCA), $|\eta| \leq 1$:



[figs. taken from Jäger et al.]



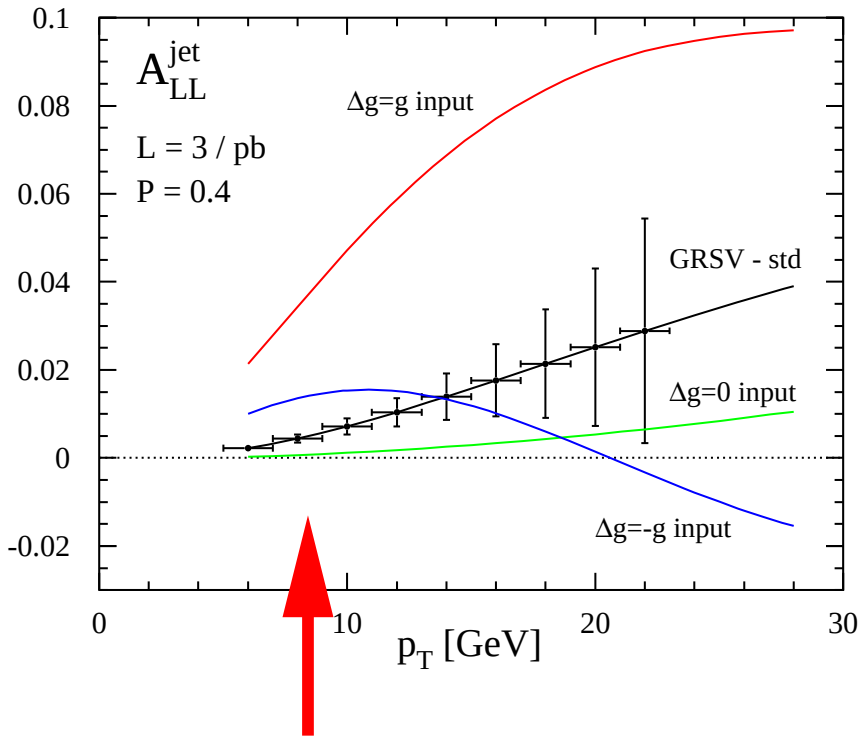
μ_f dependence much reduced in NLO

theoretical uncertainties even smaller than for hadrons

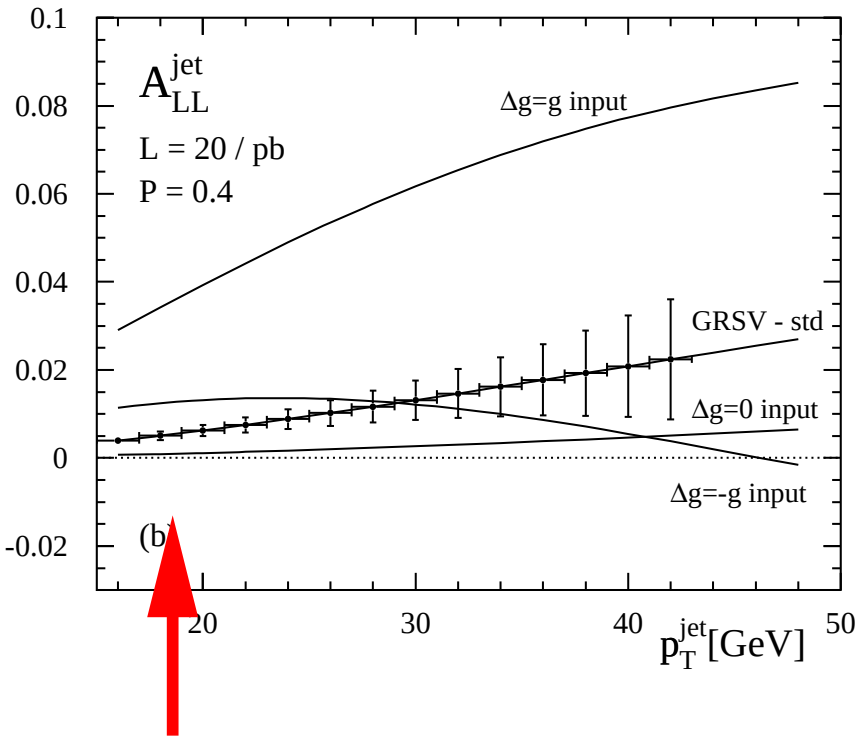
Processes one-by-one (cont.)

not surprisingly, $A_{\rm LL}^{\rm jet}$ is sensitive to gluon polarization Δg: [fig. taken from Jäger, MS, Vogelsang]

$\sqrt{S} = 200\,\mathrm{GeV}$, $R_{\rm cone} = 0.4$ (SCA), $0 \le \eta \le 1$



$\sqrt{S} = 500\,\mathrm{GeV}$, $R_{\rm cone} = 0.7$ (SCA), $|\eta| \le 1$



again: at small p_T no sensitivity to sign of Δg

Processes one-by-one (cont.)

summary on inclusive jets and hadrons:

- offer excellent short-term prospects for a first determination of Δg

  caveat: for sign of Δg one needs measurements up to $p_T \simeq 10 \div 15\,\mathrm{GeV}$

- very important: measurement of *absolute* cross sections not "only" of A_{LL}

  [even unpolarized pp cross sections never before measured at $\sqrt{S} = 200 \div 500\,\mathrm{GeV}$]

- unpolarized "benchmark x-secs" agree very well with pQCD so far

  [even down to unexpectedly small p_T]

- first exciting results from PHENIX on A_{LL}^{π} at small p_T

  if large & negative A_{LL} persists: what is the explanation?

important to collect more data soon

Processes one-by-one (cont.)

long-term goals:

need much higher luminosities, beam polarizations, and/or $\sqrt{S} = 500\,\mathrm{GeV}$

- production of prompt photons $pp \to \gamma X$:

  a supposedly clean signature for Δg

- production of heavy flavors $pp \to c\bar{c}X$, $pp \to b\bar{b}X$:

  extend Δg measurements to smaller x

- study of $W^{\pm}$ boson production:

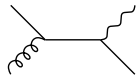
  a very clean tool to gather further information on Δq and $\Delta\bar{q}$ separately

- look for weird things beyond the Standard Model:

  this is always an issue at colliders . . .

Processes one-by-one (cont.)

• high-$\mathbf{p_T}$ prompt photons (plus jets):



NLO: Gordon, Vogelsang; Contogouris et al.; Frixione, Vogelsang

idea: a "classical", supposedly clean signature for Δg

· only two rather than eight LO QCD processes: $qg \to q\gamma$ (dominant) and $q\bar{q} \to g\gamma$

· Δg does not enter squared $\to$ sign of Δg not an issue here

on the downside:

· photons are much less abundant $\to$ need a lot of luminosity

· prompt photon data abandoned in all unpolarized pdf analyses CTEQ, MRST

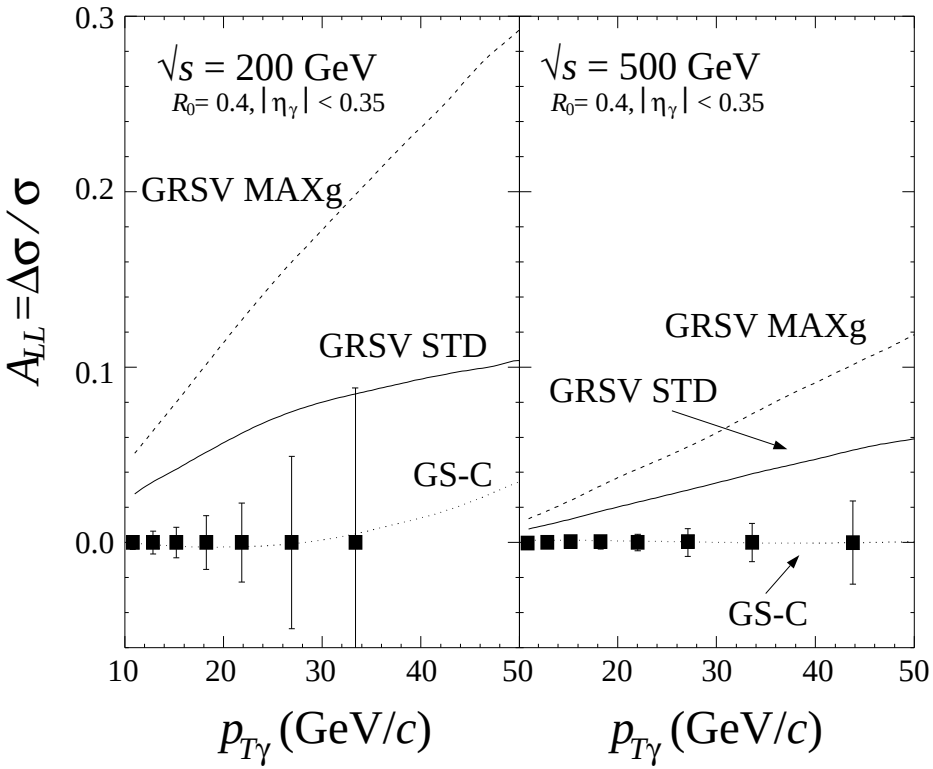
reason: some trouble with fixed target data; high p_T jets give better constraints

positive aspect: problems have pushed theoretical efforts to go beyond NLO

[soft gluon/threshold (and k_T) resummations] Laenen et al.; Catani et al.; Li; Kidonakis Owens; . . .

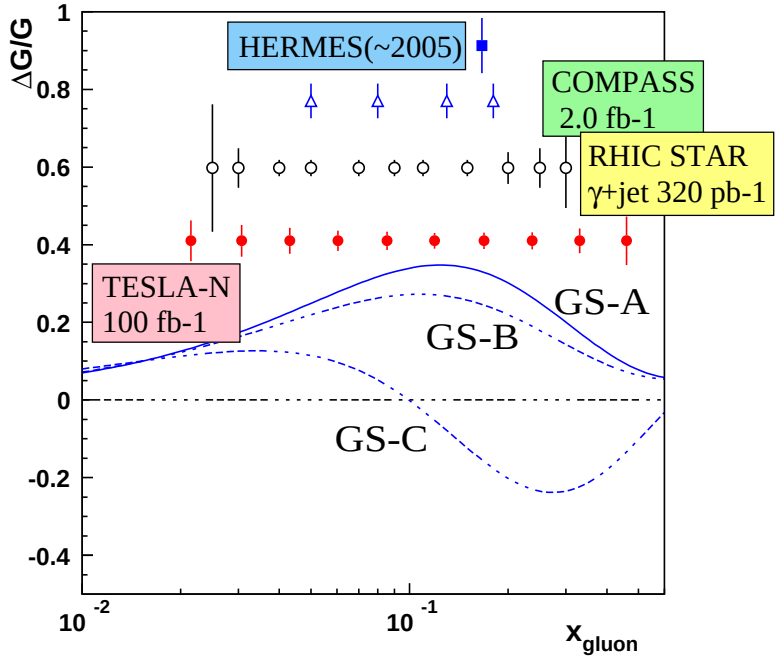
Processes one-by-one (cont.)

$A_{\rm LL}^{\gamma}(p_T)$ shows the expected sensitivity to Δg:



[fig. taken from Frixione, Vogelsang]

not shown: reduced scale dependence in NLO ✓



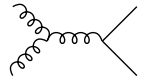
projected accuracy for $\Delta g/g$ from

$\gamma + \mathrm{jet}$ measurement at STAR

(only including $\sqrt{S} = 200\,\mathrm{GeV}$)

Processes one-by-one (cont.)

• heavy flavor production:



NLO: Bojak, MS

idea: sensitivity to Δg through dominance of $gg \to Q\bar{Q}$ (small)

· only one other LO process: $q\bar{q} \to Q\bar{Q}$

· m_Q allows pert. approach even without high p_T; slightly smaller x values accessible

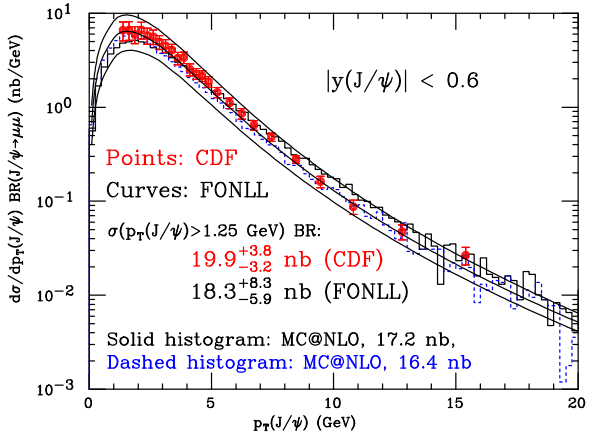
main complication: matching of theory and experiment

· c, b only seen indirectly at RHIC via their decay leptons; exp. c/b separation

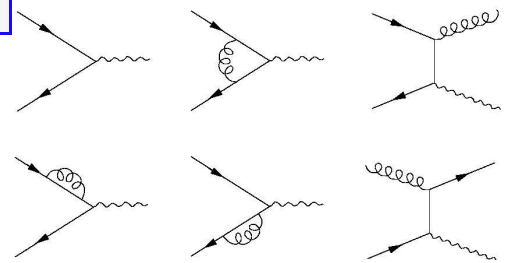
recent development:

· long-standing problem with unpolarized b-cross section at TeVatron gradually disappears

Cacciari et al.



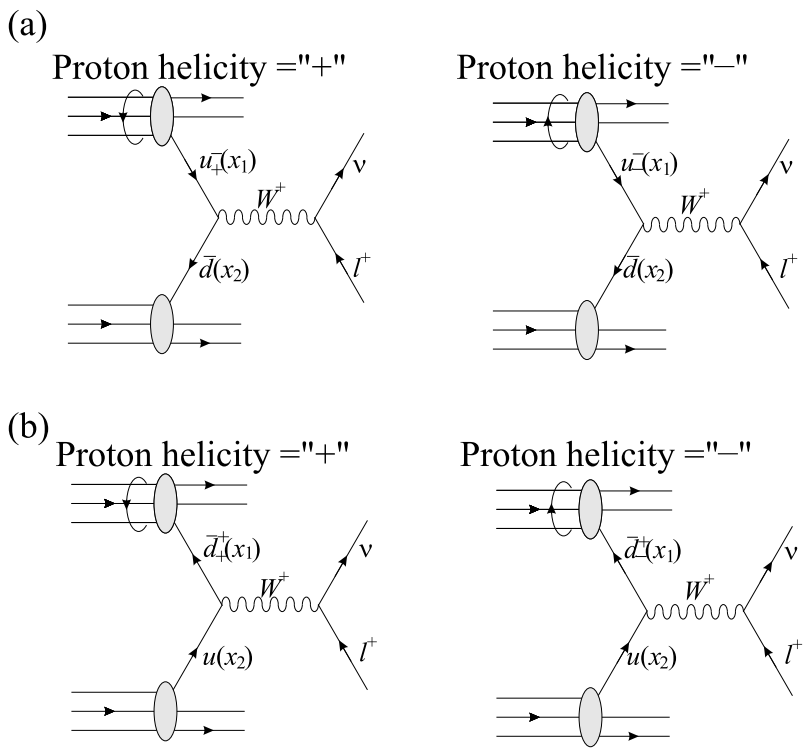
Processes one-by-one (cont.)



- **$W^{\pm}$ boson production**:

  NLO: Weber; Kamal; Gehrmann; Smith et al.

idea: weak interactions maximally parity violating



$W^{\pm}$ selects parton helicity $\rightarrow$ A_{L} sufficient

in LO one has e.g. Leader, Sridhar; Bourrely, Soffer

$$A_{\mathrm{L}}^{W^+}(y) = \frac{-\Delta u(x_a)\bar{d}(x_b) + \Delta\bar{d}(x_a)u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}$$

where $x_{a,b} = \frac{M_W}{\sqrt{S}} e^{\pm y}$

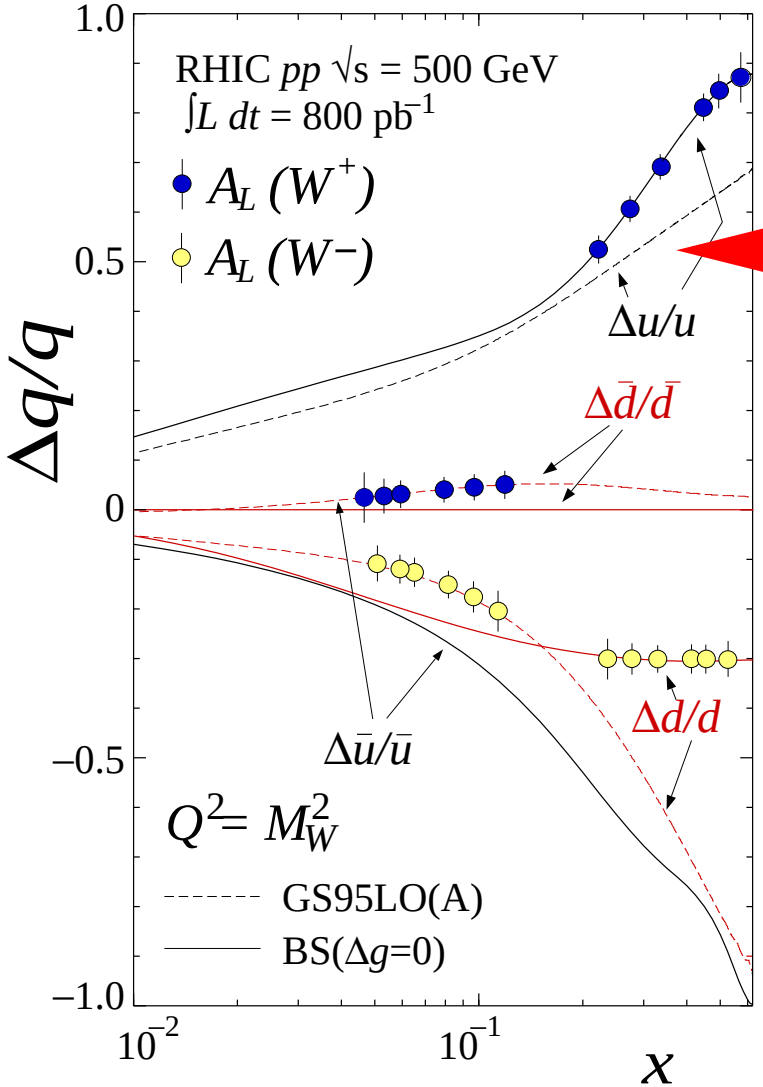
y large & $\left\{ \begin{array}{l} \text{pos. } (x_a > x_b) \\ \text{neg. } (x_a < x_b) \end{array} \right.$: $A_{\mathrm{L}}^{W^+} \simeq \left\{ \begin{array}{l} -\Delta u(x_a)/u(x_a) \\ \Delta\bar{d}(x_a)/\bar{d}(x_a) \end{array} \right.$

$A_{\mathrm{L}}^{W^-}(y)$: $u \leftrightarrow d$ $\rightarrow$ probes $\Delta d/d$ and $\Delta\bar{u}/\bar{u}$

ideal tool for a separation of Δu, Δd, $\Delta\bar{u}$, and $\Delta\bar{d}$

Processes one-by-one (cont.)

expected sensitivity on $\Delta q/q$ from $A_L^{W^\pm}(y)$: [fig. taken from Bunce et al.]



for RHIC kinematics:

Δu, Δd probed at $x \gtrsim 0.2$

$\Delta\bar{u}$, $\Delta\bar{d}$ probed at $0.045 \lesssim x \lesssim 0.12$

complication: detector issues

have to observe $W^\pm$ via decay but

neither PHENIX nor STAR are hermetic

$\rightarrow$ missing E_T cannot be reconstructed

$\rightarrow$ get $y(W)$ only from charged lepton

good news:

full NLO *lepton-level* MC available

Nadolsky, Yuan

QCD analysis of upcoming RHIC data

How to extract the desired information on Δf from data?

executive summary: extraction of pdfs from data is art, science, and voodoo

highly non-trivial exercise:

- recall: pdfs reside inside complicated convolutions

$$d\Delta\sigma = \sum_{a,b} \int \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab}$$

$\longrightarrow$ not directly measurable; no 1:1 relation between data and Δf

- not a priori clear which and where processes can be described by pQCD

- all processes should be analyzed in NLO:

  · recall: less dependent on unphysical scales μ_r and μ_f [= theor. uncertainty]

  · NLO corrections are often sizable

  · NLO closer to experiment [jet definition, cuts, . . .]

QCD analysis (cont.)

experience from corresponding fits to unpolarized data: CTEQ, MRST, GRV, . . .

- experience for about 20yrs. now
- inexhaustible amount of data

technical complications:

- functional form of pdfs not known

  typical ansatz for *each* flavor:

  $$f(x,\mu_0) = N x^{\alpha}(1-x)^{\beta}$$

  $\longrightarrow$ multi-dim. fitting problem

- large number of data required to pin down *all* aspects of pdfs

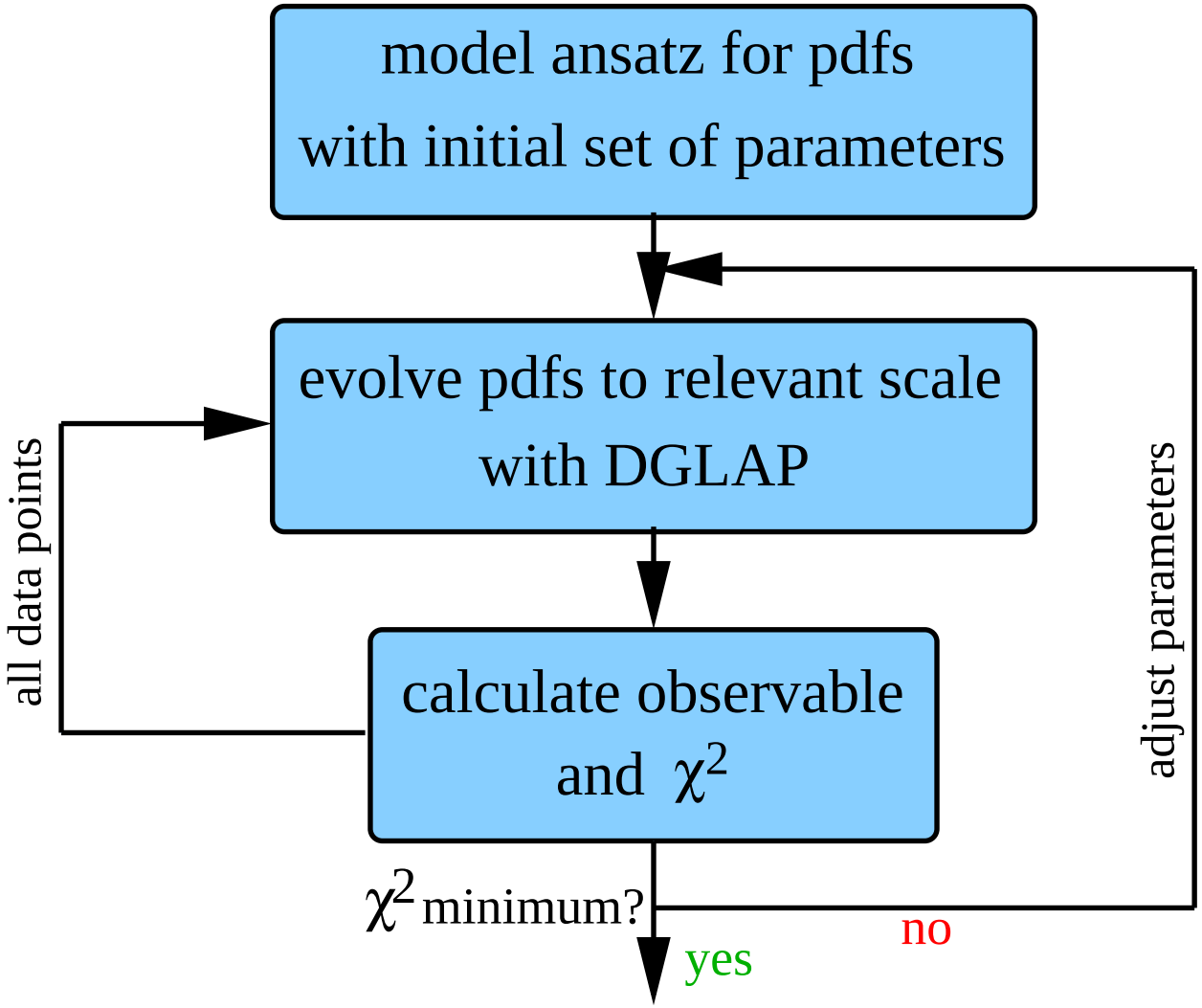
  [e.g. in CTEQ-5: 15 sets from 11 exp. $\longrightarrow$ 1295 pts.]

Process/ Experiment	Leading order subprocess	Parton behaviour probed
DIS $(\mu N \to \mu X)$ $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC*, BCDMS, NMC, E665)	$\gamma^* q \to q$	$u+\bar{u}$; $d+\bar{d}$; $\bar{u}+\bar{d}$; s ($s=\bar{s}$ assumed) plus $\int xg(x,1\text{GeV})dx \simeq 0.35$ $\int(\bar{d}-\bar{u})dx \simeq 0.1$
DIS $(\nu N \to \mu X)$ $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)	$W^* q \to q'$	
DIS (small x**)** F_2^{ep} (H1, ZEUS)	$\gamma^*(Z^*)q \to q$	small x slopes of $\bar{q}$, g: $x\bar{q} \sim x^{-\lambda_{\bar{q}}}$, $xg \sim x^{-\lambda_g}$
DIS (F_L) (NMC, HERA)	$\gamma^* g \to q\bar{q}$	g
$\ell N \to c\bar{c}X$ F_2^c (EMC; H1, ZEUS)*	$\gamma^* c \to c$	c at $x \gtrsim 0.01$; $x \lesssim 0.01$
$\nu N \to \mu^+\mu^- X$ (CCFR)	$W^* s \to c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{4}(\bar{u}+\bar{d})$
$pN \to \gamma X$ (WA70*, UA6, E706)	$qg \to \gamma q$	g at $x \simeq 2p_T^{\gamma}/\sqrt{s}$ $\to x \approx 0.2-0.6$
$pN \to \mu^+\mu^- X$ (E605, E772*)	$q\bar{q} \to \gamma^*$	large x exponent of $\bar{q}$: $\bar{q} \sim (1-x)^{\beta_{\bar{q}}}$
$pp, pn \to \mu^+\mu^- X$ (E866, NA51)	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{u}-\bar{d}$ $0.04 \lesssim x \lesssim 0.3$
$ep, en \to e\pi X$ (HERMES)	$\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$	$\bar{u}-\bar{d}$ $0.04 \lesssim x \lesssim 0.2$
$p\bar{p} \to WX(ZX)$ (UA1, UA2; CDF, DØ) $\to \ell^{\pm}$ **asym** (CDF)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s}$ $\to x \approx 0.13$; 0.05 u/d slope at $x \approx 0.08$
$p\bar{p} \to Q\bar{Q}X$ (CDF, DØ)	$q\bar{q}, gg \to Q\bar{Q}$ with $Q = b, t$	q, g at $x \gtrsim 2m_Q/\sqrt{s}$
$p\bar{p} \to$ **jet** $+X$ (CDF, DØ)*	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s}$ $\to x \approx 0.05-0.5$

* (*) (not) in CTEQ-5 analysis (adapted from MRST)

QCD analysis (cont.)

rough outline of fitting procedure (χ^2 minimization):



QCD analysis (cont.)

main technical challenge of a NLO global analysis:

- NLO expressions for most processes are fairly complex and lengthy
- $d\Delta\sigma^{\rm NLO} = K \cdot d\Delta\sigma^{\rm LO}$ would be nice but is a bad approximation

  $\rightarrow$ computing time required for a multi-dim. fit becomes excessive

possible way out: Kosower; Vogelsang, MS

idea: get rid of convolution integrals in $d\Delta\sigma$ by taking Mellin N moments

$$\int_0^1 dx\, x^{N-1} \left[\int_x^1 \frac{dy}{y} \xi(y) \zeta(x/y) \right] = \int_0^1 dx\, x^{N-1} \left[\int_0^1 dy \int_0^1 dz\, \xi(y) \zeta(z) \delta(x - yz) \right]$$

$$= \int_0^1 dy\, y^{N-1} \xi(y) \int_0^1 dz\, z^{N-1} \zeta(z)$$

$$= \xi^N \zeta^N \quad \text{simple product}$$

QCD analysis (cont.)

- apply to convolutions in cross section formula:

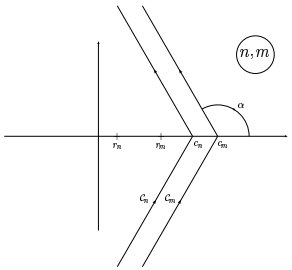
$$d\Delta\sigma \simeq \sum_{a,b} \int dx_a\, dx_b\, \Delta f_a(x_a) \otimes \Delta f_b(x_b) \otimes d\Delta\hat{\sigma}_{ab}(x_a, x_b)$$

Mellin inverse: $\int_{\mathcal{C}_n} dn\, x_a^{-n}\, \Delta f_a^n$ $\quad$ $\int_{\mathcal{C}_m} dm\, x_b^{-m}\, \Delta f_b^m$

$$= \sum_{a,b} \int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm\, \Delta f_a^n\, \Delta f_b^m \left[\underbrace{\int dx_a\, dx_b\, x_a^{-n}\, x_b^{-m}\, \Delta\hat{\sigma}_{ab}(x_a, x_b)}_{\equiv\, \Delta\tilde{\sigma}_{ab}^{n,m}} \right]$$

- pre-calculate time-consuming $\Delta\tilde{\sigma}_{ab}^{n,m}$ on a suitable $n \times m$ grid for all subprocesses and data pts. *once and forever* before the fit

- fast numerical inverse Mellin transformation $\int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm$ on $n \times m$ grids in complex n, m plane



QCD analysis (cont.)

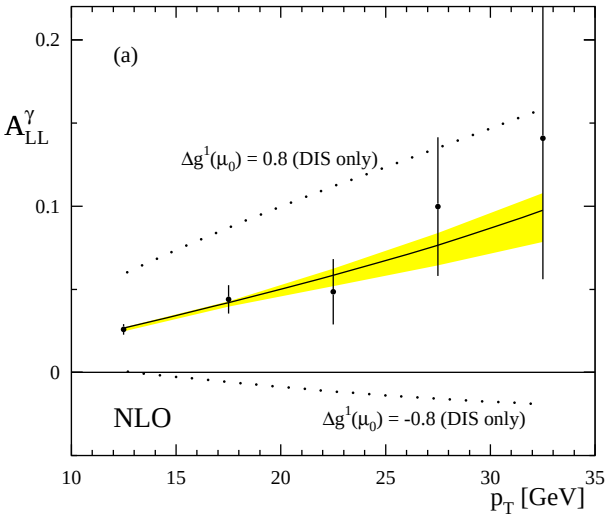
sample analysis of fictitious prompt photon "data": Vogelsang, MS

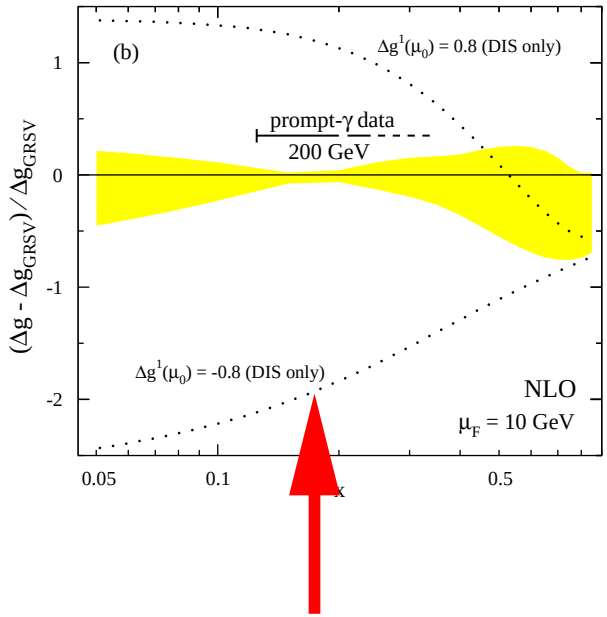
aim: determine x region where Δg predominantly probed by data; gain w.r.t. DIS

NLO "global" QCD fit of DIS + γ data

$\Downarrow$

1σ-bands for A_{LL}^{γ} and Δg





result:
- strong constraint on Δg around $x \simeq 0.15$
- Mellin technique very fast and efficient; applicable to all relevant processes

ideal tool to analyze upcoming RHIC data

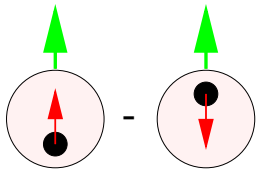
Addendum: transverse polarization

Let's turn spin by 90° – does it matter? Yes!

boosts and rotations do not commute $\longrightarrow$ completely new object

transversity density $\delta\mathbf{q}(\mathbf{x}, \mathbf{Q^2})$: (sometimes also called h_1)

features:

- "as partonic as" q and Δq: h_1 =  Ralston, Soper; Artru, Mekhfi; Jaffe, Ji
- reveals chiral-odd nature if described in helicity-basis

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) \text{ and } |\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$\longrightarrow$ δq is a helicity-flip density (off-diagonal)

$\longrightarrow$ spin-1 $\cancel{\delta g}$ not possible for spin 1/2 targets (protons, . . .) Jaffe, Ji; Artru, Mekhfi; Ji

[would require helicity flip by 2 units]

Addendum: transversity (cont.)

more features of transversity:

- simple DGLAP evolution equations

$$\mu \frac{d\delta q(x,\mu)}{d\mu} = \int_x^1 \frac{dz}{z} \delta P_{qq}(z, \alpha_s(\mu)) \delta q(x/z, \mu)$$

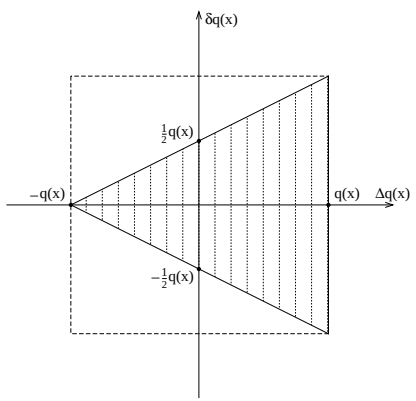
splitting kernels δP_{qq} known up to NLO — Vogelsang; Kumano, Miyama; Koike et al.

peculiar feature of evolution:

all moments $\delta q(\mu) \equiv \int_0^1 x^{n-1} \delta q(x,\mu) dx$ decrease $\rightarrow$ "evolves away"

- so far unmeasured; helpful constraint: Soffer inequality — Soffer; Sivers

$$\underbrace{|\delta q(x,\mu^2)|}_{\text{unknown}} \leq \frac{1}{2} \left[\underbrace{q(x,\mu^2)}_{\text{known (more or less)}} + \underbrace{\Delta q(x,\mu^2)}_{\text{known (more or less)}} \right]$$

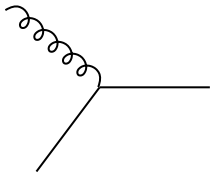


more restrictive than $|\delta q| \leq q$

Addendum: transversity (cont.)

measuring transversity:

difficult! all fundamental interactions preserve chirality

e.g.:  $\overline{\Psi}\gamma^\mu T^a \Psi = \overline{L}\gamma^\mu T^a L + \overline{R}\gamma^\mu T^a R$ where $R, L = \frac{1}{2}[1+\gamma_5]\Psi$

$\longrightarrow$ not accessible, e.g., in deep-inelastic scattering

two ways to explore transversity $\delta\mathbf{q}$ at RHIC:

chirality has to be flipped *twice* to access δq

$\swarrow$ $\searrow$

double-spin single-spin

asymmetries

A_{TT} A_{N}

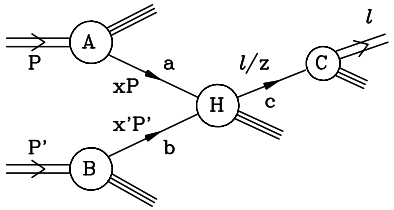
here $\longrightarrow$ Jianwei Qiu's lectures

Addendum: transversity (cont.)

$\mathbf{A}_{\mathrm{N}}$ in brief:

- exciting observable, goes back to the early days of spin:

$$A(p, \vec{s}_T) + B(p') \to C(l) + X \text{ with } C \text{ '=' high-} p_T \ \pi,\ \gamma,\ \ldots$$

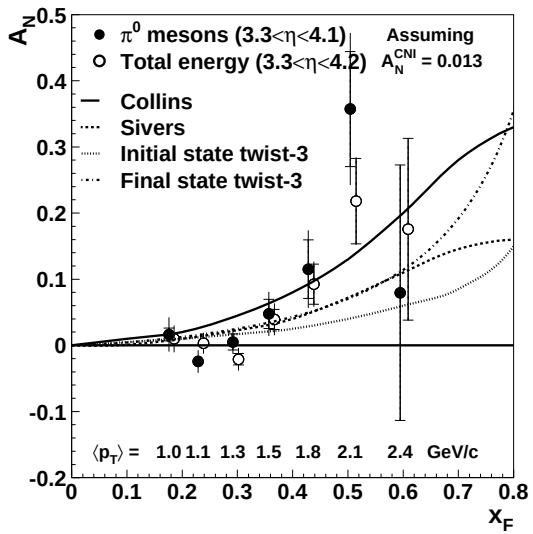


$$\text{measure}: \ A_{\mathrm{N}} = \frac{\Delta_{\mathrm{N}}\sigma}{\sigma} \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

- leading-twist pQCD: $A_{\mathrm{N}} = 0$ but large A_{N} found experimentally ever since

  $\longrightarrow$ explanation requires new non-perturbative objects

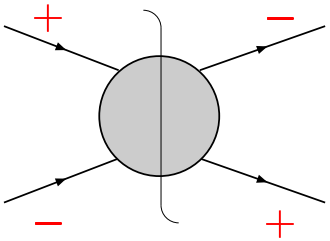
  for the 1st time also seen at $\sqrt{S} = 200\,\mathrm{GeV}$ by STAR



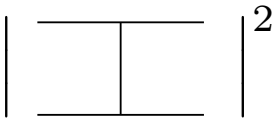
Addendum: transversity (cont.)

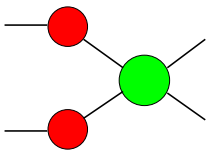
$\mathbf{A}_{\mathrm{TT}}$ in brief:

- $A_{\mathrm{TT}} \equiv \dfrac{d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)}$ is a much more conventional quantity

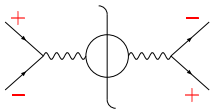
- helicity-flip requirement 

fulfilled for $q\bar{q}$ annihilation

  and for some interference diagrams, not for stuff like 

- standard pQCD framework  applies

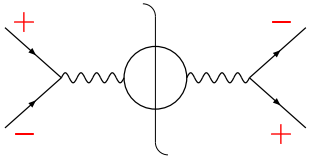
- expect: A_{TT} small due to absence of $g^{\uparrow}g^{\uparrow}$ and $q^{\uparrow}g^{\uparrow}$ processes

  $\longrightarrow$ Drell-Yan process 

of particular interest: no gluon contribution at LO

Addendum: transversity (cont.)

Drell-Yan process:



Ralston, Soper; Cortes, Pire, Ralston; Artru, Mekhfi; Ji; Jaffe, Ji

- only candidate process w/o gluons in LO
- relies on the presence of sea-quark transversity $\delta\bar{q}$

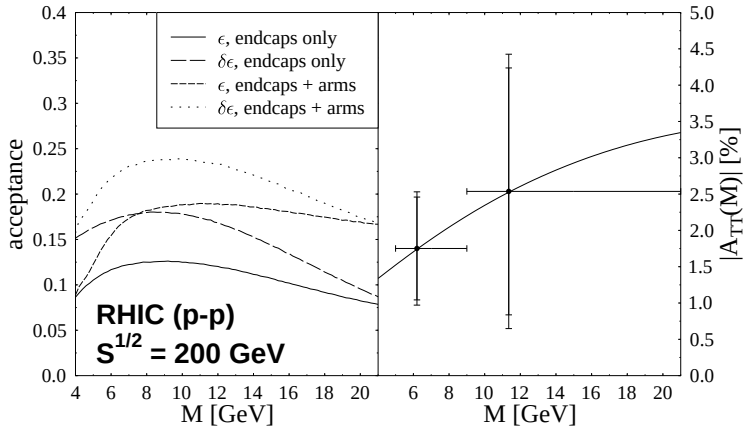
  no gluon density $\delta g \rightarrow$ no $g \rightarrow q\bar{q}$ splitting $\rightarrow$ few sea-quarks?
- experimental "problem": limited acceptance for $\mu^{\pm}$ detection

  PHENIX: $|1.2 \leq y_\mu \leq 2.4|$

estimate of *upper bound* for $A_{\rm TT}$ in NLO QCD:

Martin, Schäfer, MS, Vogelsang

- saturate Soffer bound for δq at $\mu_0 \simeq 0.6\,\mathrm{GeV}$
- includes detector acceptance



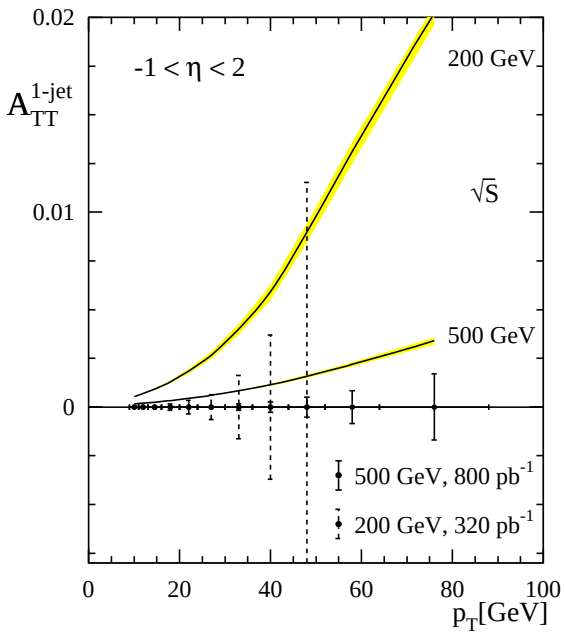
Addendum: transversity (cont.)

single-inclusive high-p_T jet and prompt photon production:

Hidaka et al.; Artru, Mekhfi; Ji; Jaffe, Saito, Soffer, MS, Vogelsang

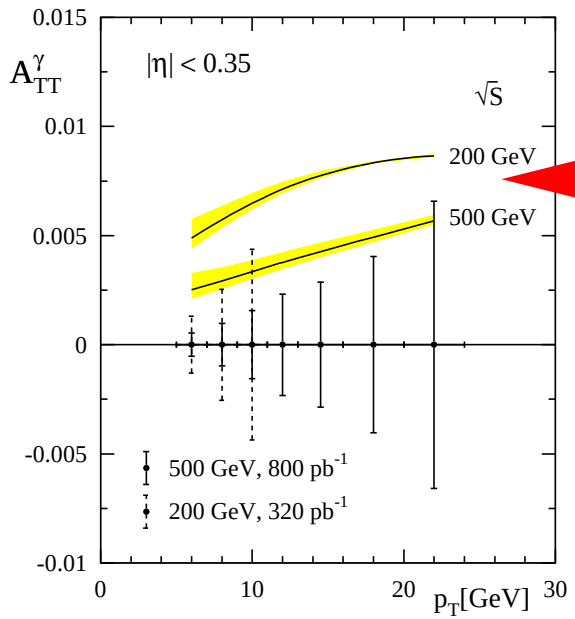
1-jet

STAR



high-p_T photon

PHENIX



some LO estimates of

upper bounds for A_{TT}

Soffer, MS, Vogelsang

$\rightarrow$ A_{TT} very small

Again, NLO QCD corrections are a must: scale dependence, etc.

Addendum: transversity (cont.)

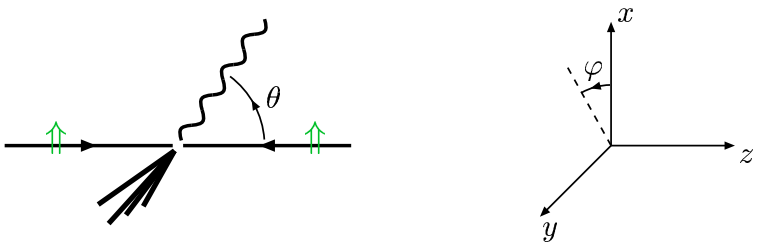
further motivation for NLO:

"technical challenge" & general paucity of NLO calculations with transverse spin

why transverse spin is more complicated to handle:

long. polarization: spin *aligned* with momentum ✓

trans. polarization: spin = *extra spatial direction* ↔ non-trivial azimuthal dep.



$$\text{always of the form} \quad \frac{d^3\delta\sigma}{dp_T d\eta d\phi} \equiv \cos(2\phi) \left\langle \frac{d^2\delta\sigma}{dp_T d\eta} \right\rangle$$

$\longrightarrow$ ϕ integration not appropriate

problem: standard NLO techniques rely on integrations over *full* azimuthal phase space

Addendum: transversity (cont.)

recent progress: simple technique to project out ϕ dependence A. Mukherjee, MS, W. Vogelsang

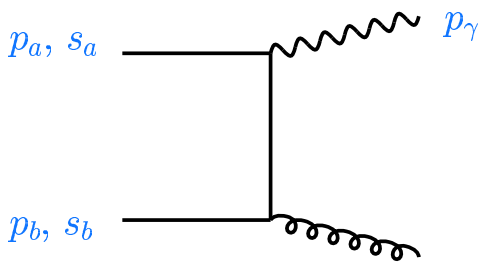
key point: ϕ-dep. always stems from covariant expression

$$\mathcal{F}(p_\gamma, s_a, s_b) = \frac{s}{tu}\left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b) \;+\; \frac{tu}{s}\,(s_a \cdot s_b)\right]$$

$$= \cos(2\phi) \text{ in hadronic c.m.s.}$$

$\longrightarrow$ use $\mathcal{F}$ to project out ϕ covariantly

LO example: $q\bar{q} \longrightarrow \gamma g$



$$p_a \cdot s_a \;=\; p_b \cdot s_b \;=\; 0$$

$$s_a^2 = s_b^2 = -1$$

Addendum: transversity (cont.)

matrix element [use $u(p_a, s_a)\bar{u}(p_a, s_a) = \frac{1}{2}\not{p}_a\,[1 + \gamma_5 \not{s}_a]$,. . .]

$$\delta|M|^2 = (ee_q g)^2 \frac{4C_F}{N_C} \frac{s}{tu} \left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b) \;+\; \frac{tu}{s}\,(s_a \cdot s_b) \right]$$

project with $\mathcal{F}$:

$$\frac{1}{\pi} \int d\Omega_\gamma\, \mathcal{F}(p_\gamma, s_a, s_b)\delta|M|^2 \underset{\updownarrow}{=} (ee_q g)^2 \frac{4C_F}{N_C} = \langle \delta|M|^2 \rangle \;\checkmark$$

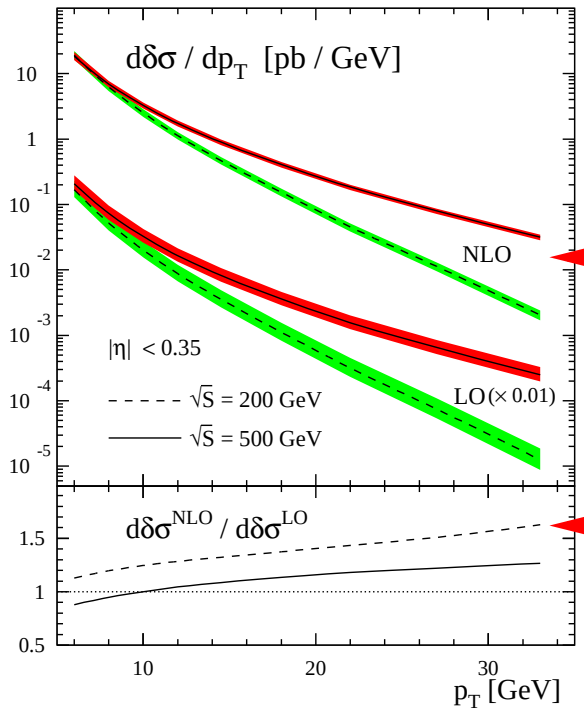
terms involving $p_\gamma \cdot s_a$, $p_\gamma \cdot s_b$ can be integrated "covariantly"

- easily generalized to **NLO calculation** in d dimensions:
  - multiply any $\delta|M|^2$ with $\mathcal{F}(p_\gamma, s_a, s_b)$
  - integrate all resulting scalar products with $s_{a,b}$
  - employ *standard techniques* for phase space integr.
  - restore ϕ dependence afterwards

Addendum: transversity (cont.)

$1^{\rm st}$ NLO result on $\mathbf{p^\uparrow p^\uparrow \to \gamma X}$:

A. Mukherjee, MS, W. Vogelsang



- improved scale dependence
- reasonably small NLO corrections

general conclusions on $\mathbf{A_{TT}}$ at RHIC:

- requires great experimental efforts [in particular, small syst. errors]
- unexpectedly large $A_{\rm TT}$ would constitute a new "spin surprise"

Addendum (II): Further improving pQCD results

resummation: exponentiation of large logarithmic terms in perturbative series

well-known example: DGLAP evolution equation

recall non-singlet evolution: $\mu \frac{dq(x,\mu)}{d\mu} = \int_x^1 \frac{dz}{z} P_{qq}(z, \alpha_s(\mu))\, q(x/z, \mu)$

- convolution turns to simple product in Mellin N moment space
- the solution for moments q^N and scales $\mu > \mu_0$ reads

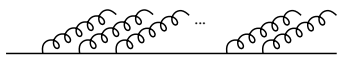
$$q^N(\mu) = q^N(\mu_0) \exp\left[\frac{\alpha_s}{2\pi} P_{qq}^N \log(\mu/\mu_0)\right]$$

$\longrightarrow$ we have resummed gluon emission to all orders in α_s!

to see this expand exponent:

$$\exp[\ldots] = 1 + \frac{\alpha_s}{2\pi} P_{qq}^N \log(\mu/\mu_0) + \frac{1}{2}\left[\frac{\alpha_s}{2\pi} P_{qq}^N \log(\mu/\mu_0)\right]^2 + \ldots$$

$\longrightarrow$ we are getting all terms $\alpha_s^k \log^k(\mu/\mu_0)$



Addendum (II): Further improving pQCD results (cont.)

resummation techniques can be applied in many places . . .

Sterman; Catani, Trentadue; Kidonakis, Oderda, Sterman; Catani, Mangano, Nason, . . .

one more example: single inclusive parton production, e.g., $ab \to cX$

$$\frac{d\hat{\sigma}_{ab}}{dp_T} = \frac{d\hat{\sigma}_{ab}^{\rm LO}}{dp_T} \times \Big[1 + \underbrace{A_1\alpha_s \ln^2(1-\hat{x}_T^2) + B_1\alpha_s \ln(1-\hat{x}_T^2)}_{\rm NLO}\Big]$$

$$+ \ \ldots + A_k\alpha_s^k \ln^{2k} \ln^{2k}(1-\hat{x}_T^2) + \ldots + \text{terms finite at } \hat{x}_T \to 1$$

structure stems from *soft gluon emission* near partonic threshold $\hat{x}_T = 2p_T/\sqrt{s} \to 1$

large logarithms can be resummed (exponentiated) to all orders!

final result (for moments in $\hat{x}_T^2$: $\int_0^1 d\hat{x}_T^2 (\hat{x}_T^2)^{N-1} \ldots$) looks like

$$\hat{\sigma}_{ab}^N \simeq \exp\Big[\ln N \sum_{m=1}^{\infty} \underbrace{a_m(\alpha_s \ln N)^m}_{\rm LL} + \sum_{m=1}^{\infty} \underbrace{b_m(\alpha_s \ln N)^m}_{\rm NLL} + \ldots\Big]$$

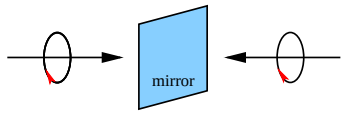
main benefits: large logs included to all orders; scale dependence further reduced

Addendum (III): RHIC as a tool to uncover "New Physics"

at a collider you can always look for weird things, e.g. a

parity-violating single-spin asymmetry for high-$\mathbf{p_T}$ jets:

$$A_L^{PV} \equiv \frac{d\sigma(+)/dp_T - d\sigma(-)/dp_T}{d\sigma(+)/dp_T + d\sigma(-)/dp_T}$$

idea: $A_L^{PV} = 0$ for QCD (recall: QCD invariant under parity 

)

there will be a *small* $A_L^{PV} = 0$ from "QCD $\otimes$ electroweak" interference

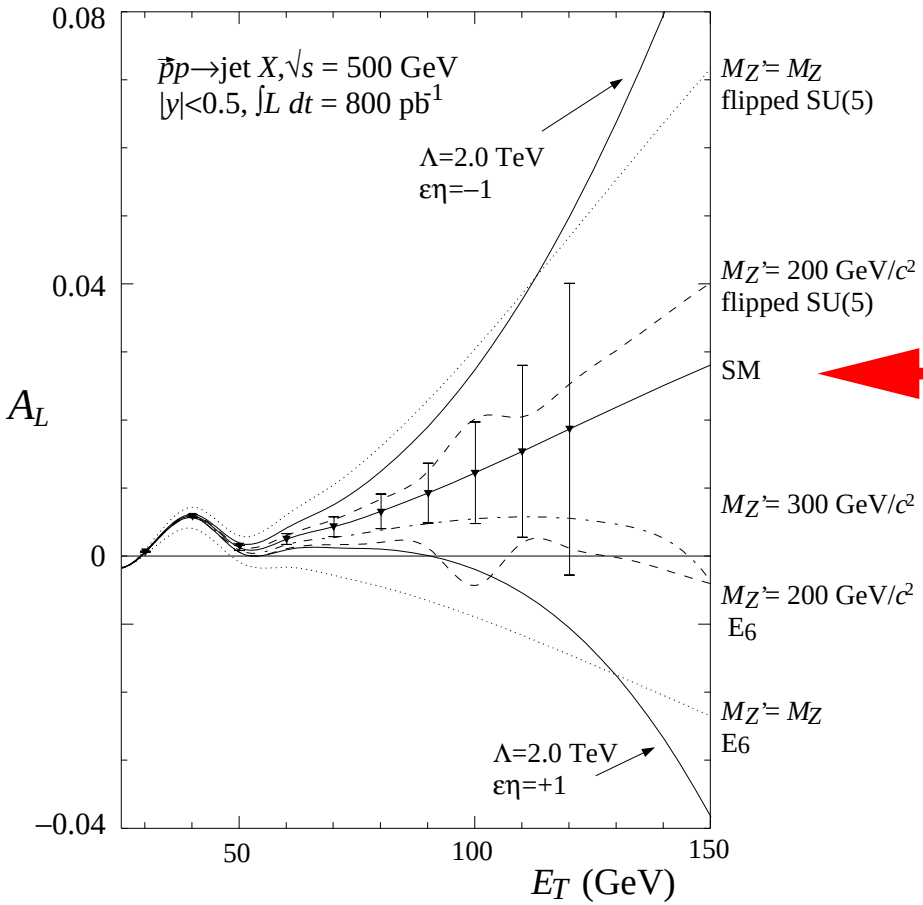
$\longrightarrow$ any new parity-violating interaction can lead to sizable $A_L^{PV} = 0$

Bourrely, Guillet, Soffer; Tannenbaum; Taxil, Virey

- $\sqrt{S}|_{\mathrm{RHIC}} \ll \sqrt{S}|_{\mathrm{TeVatron}}$ but polarization can make RHIC competitive
- could be a unique place to probe chiral structure of possible new interactions

Addendum (III): “New Physics” (cont.)

How would it look like?



[fig. taken from Taxil, Virey]

a typical example:

standard model result

minimum requirements:

- $\sqrt{S} = 500\,\mathrm{GeV}$, high luminosity, and precision
- good knowledge of parton densities and standard model “background”

I haven't mentioned . . .

. . . a lot of interesting things:

- many other observables
- single spin $\rightarrow$ Jianwei Qiu's lectures
- polarization effects in final state
- input from non-perturbative methods like lattice $\rightarrow$ Tom Blum's lectures
- role of Δg in proton "spin sum rule" $\rightarrow$ Xiangdong Ji's lecture
- resummation technique in detail

. . .

Executive Summary

for many years to come

RHIC is *the* place to learn about spin

very exciting times ahead of us