$\cos 2\phi$ in SIDIS, Drell Yan, & e^+e^-

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Probing the nucleon's spin structure with transversely polarized beams

- Remarks Transverse Spin effects in TSSAs and AAs in QCD
- ⋆ Reaction Mechanisms: Beyond Co-lineararity ISI/FSI Twist Two vs. Colinear-limit ETQS-Twist Three
- ⋆ Unintegrated "T-odd" TMDs Distribution/ Fragmentation Functions in Spect. FRMWK Correlations btwn intrinsic $k_\perp$, transverse spin S_T
- ⋆ T-odd $\cos 2\phi$ & $\sin 2\phi$ asymmetries
- Conclusions

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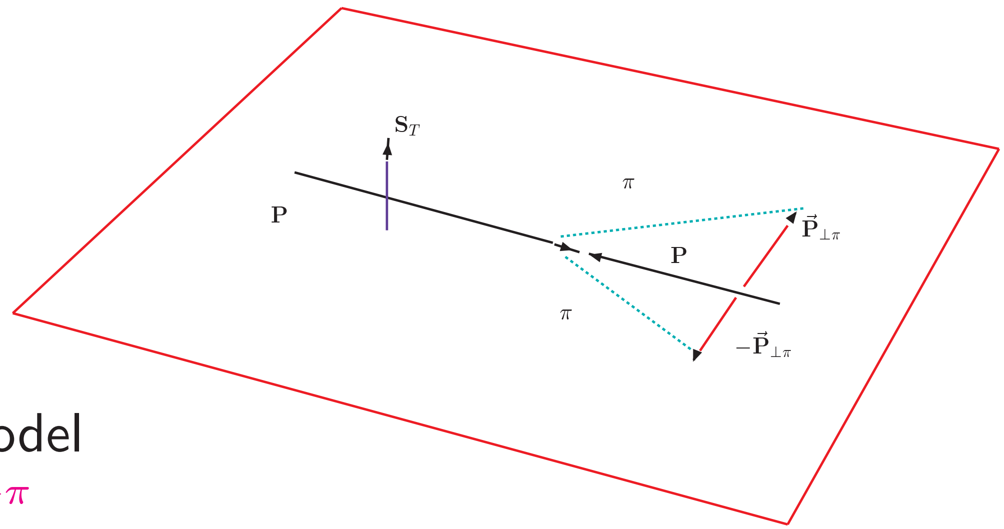
Transverse SPIN Observables SSA (TSSA)

$$\Delta\sigma \sim i\mathbf{S}_T \cdot (\mathbf{P} \times \boldsymbol{P}_{\pi\perp})$$



- ⋆ Co-linear factorized QCD-parton model
  $\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$
  Requires helicity flip in hard part $\Delta\hat{\sigma} \equiv \hat{\sigma}^{+} - \hat{\sigma}^{-}$

- $|\perp/\top\rangle = (|+\rangle \pm i|-\rangle) \Rightarrow A_N = \frac{d\hat{\sigma}^{\perp} - d\hat{\sigma}^{\top}}{d\hat{\sigma}^{\perp} + d\hat{\sigma}^{\top}} \sim \frac{2\, Im\, f^{*+} f^{-}}{|f^{+}|^2 + |f^{-}|^2}$

- ⋆ Requires relative phase btwn helicity amps

- QCD interactions conserve helicity $m_q \to 0$ & Born amplitudes real!

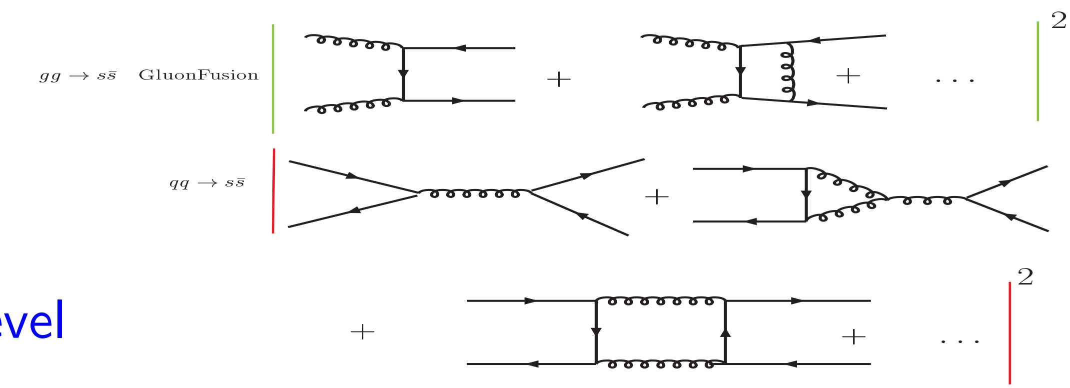
- ⋆ Kane, Repko, PRL:1978 Generally $A_N \sim \frac{m_q \alpha_s}{P_T}$ small

Early test-Λ Production ($pp \to \Lambda^{\uparrow} X$)

- Need strange quark to polarize a Λ

  Dharmartna & Goldstein PRD 1990

$$P_\Lambda = \frac{d\sigma^{pp\to\Lambda^{\uparrow}X} - d\sigma^{pp\to\Lambda^{\downarrow}X}}{d\sigma^{pp\to\Lambda^{\uparrow}X} + d\sigma^{pp\to\Lambda^{\downarrow}X}}$$

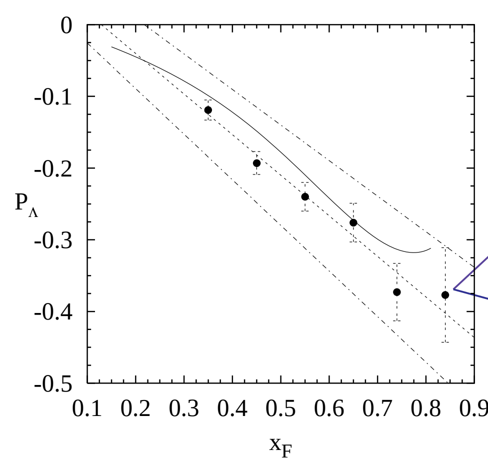


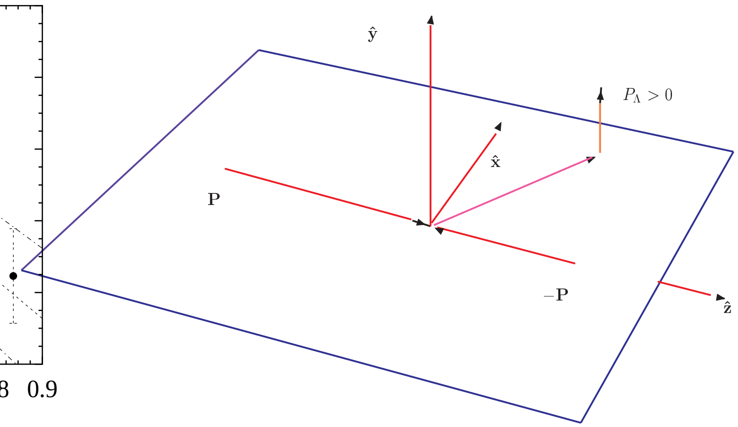
Phases in *hard part* $\hat{\Delta\sigma}$
interference of loops and tree level

- Polarization $P_\Lambda \sim \frac{m_s \alpha_s}{P_T}$–twist 3 & small $\approx 5\%$ as predicted
- *Experiment glaringly at odd with this result*

  P_Λ in $p-p$ scattering-Fermi Lab

  Heller,...,Bunce PRL:1983

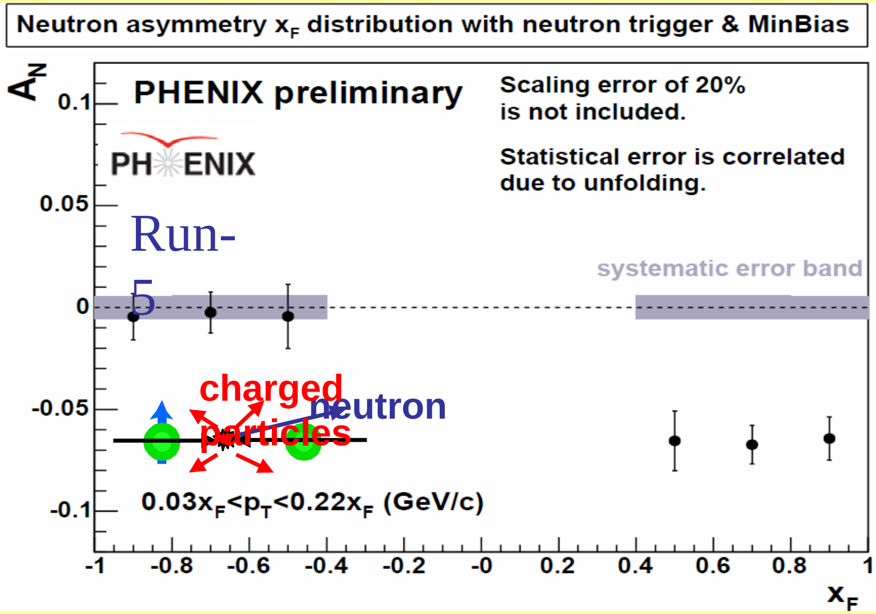


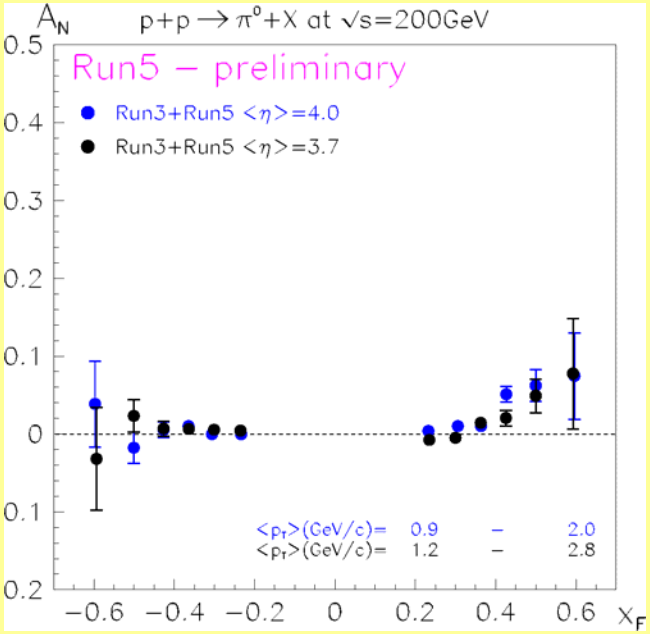


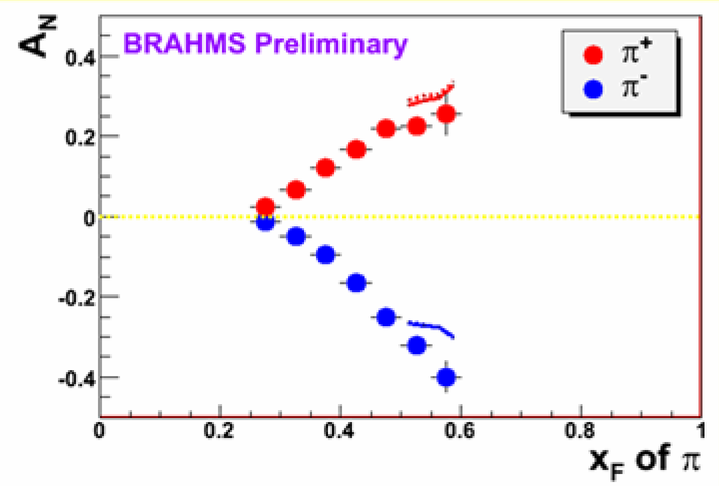
E704 and BNL

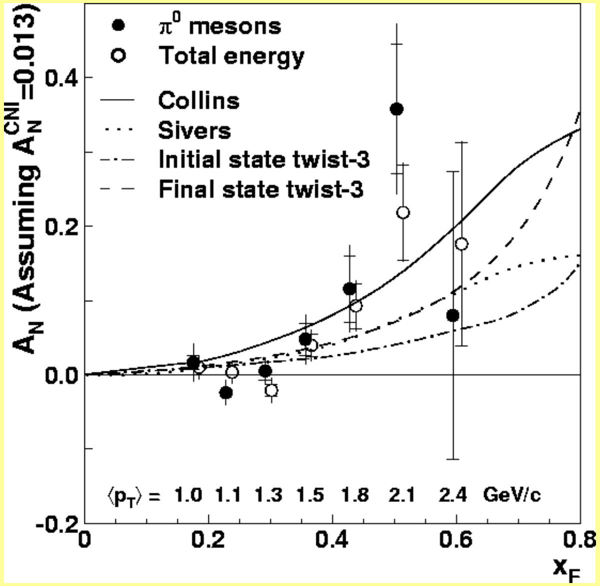
$$p^{\uparrow} p \Rightarrow \pi X$$

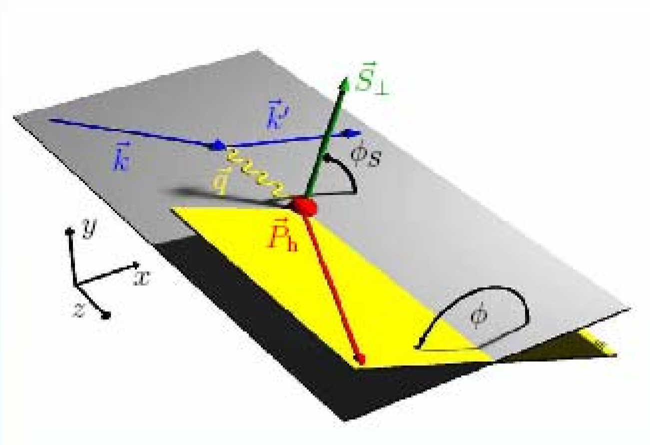
STAR



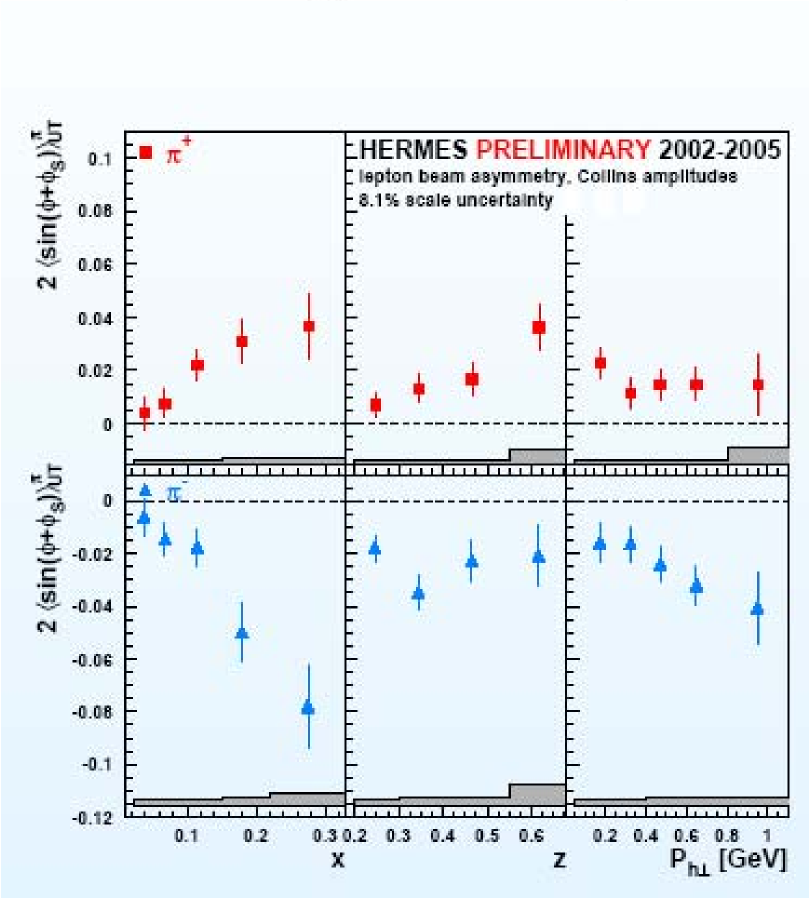


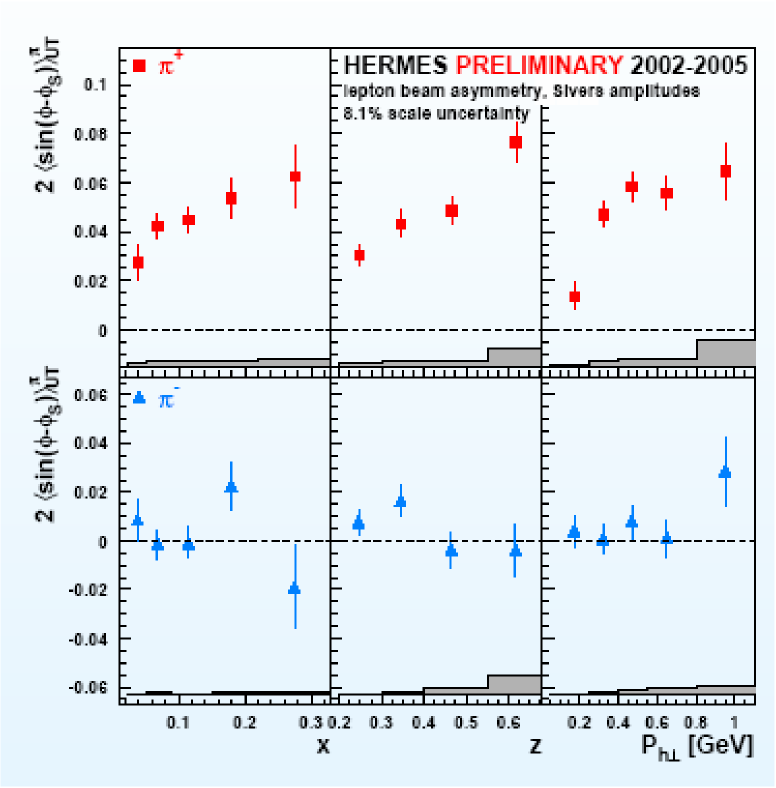




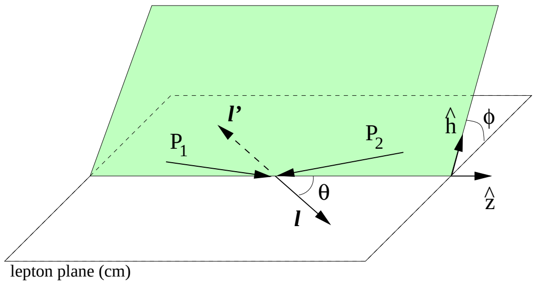


HERMES $ep^{\uparrow} \Rightarrow \pi\ X$





Azimuthal Asymmetry–Unpolarized DRELL YAN



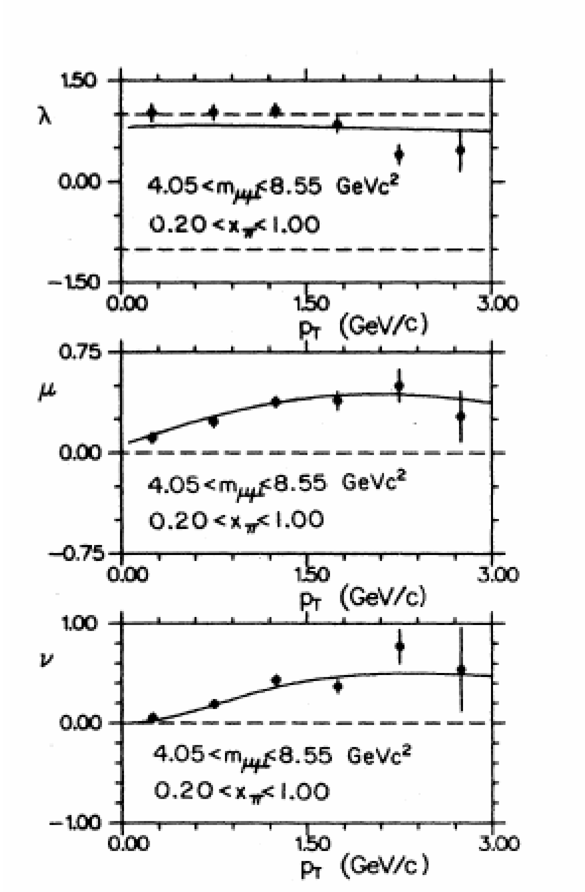
$\pi^- + p \longrightarrow \mu^+ + \mu^- + X$ E615,Conway et al. 1986, NA10, ZPC(1986)

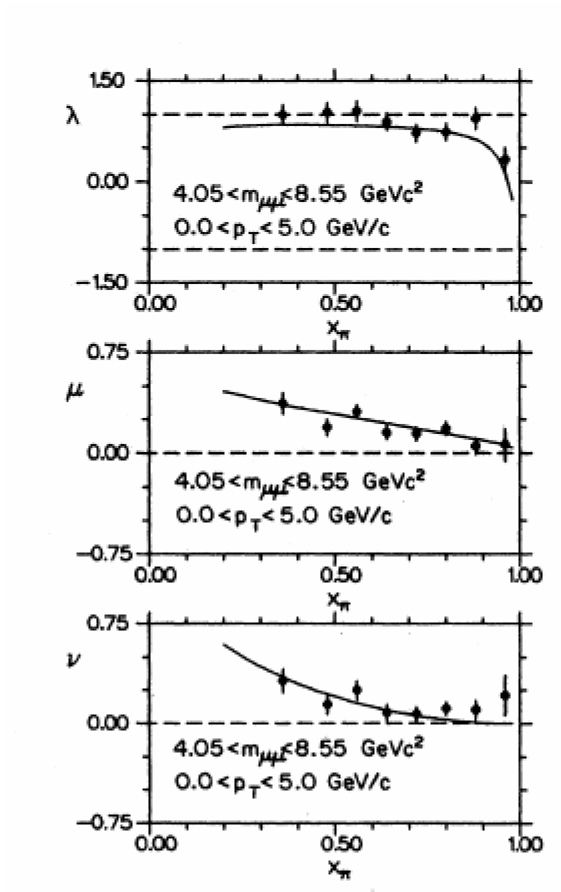
QCD-Parton Model doesn't account for large "AA"

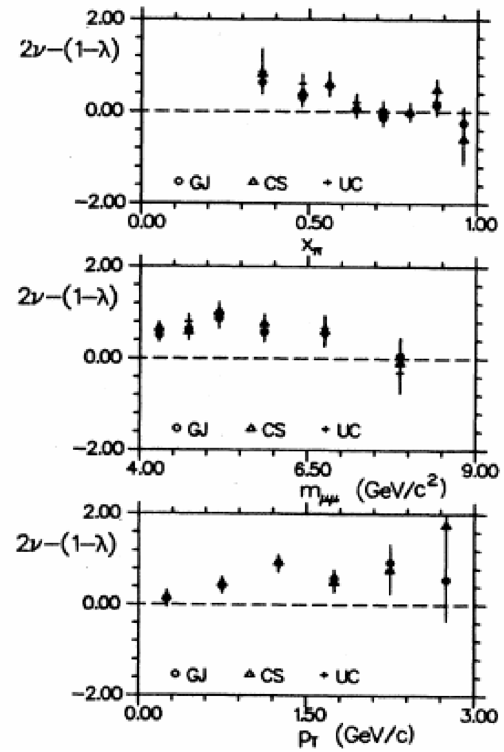
$\lambda,\ \mu,\ \nu$ **depend on** $s, x, m^2_{\mu\mu}, p_T$

$$\frac{dN}{d\Omega} = (\frac{d\sigma}{d^4q})^{-1}\frac{d\sigma}{d^4q d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

- Violation of Lam-Tung relation- (NNLO QCD) $1-\lambda-2\nu \neq 0$ (Mirkes Ohnemus, PRD 1995)
- *Unexpectedly large* $\cos 2\phi$ – $\nu \sim 10-30\%$ AA
- *Suggests a Non-Pertb. origin!*



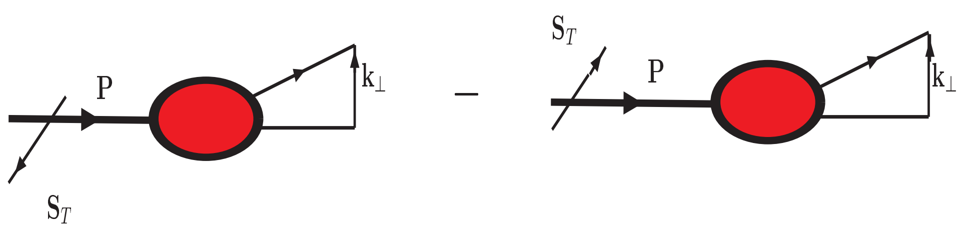




Lam-Tung Relationship Violated

$\boldsymbol{p}_T \sim k_\perp << Q^2$ TSSAs thru "T-Odd" **TMD**

- Sivers PRD: 1990,Collins NPB: 1993, TSSA associated with "T-odd" correlation *transverse* spin and momenta



$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born} \quad \Rightarrow i\boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}_\perp) \to f_{1T}^{\perp}(x, \boldsymbol{k}_\perp)$$

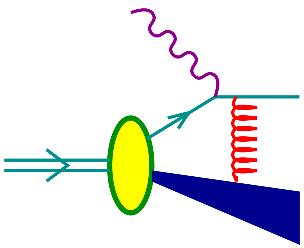
$$\Delta\sigma^{ep^{\uparrow}\to e\pi X} \sim \Delta D^{\perp} \otimes f \otimes \hat{\sigma}_{Born} \quad \Rightarrow i\boldsymbol{s}_T \cdot (\boldsymbol{P} \times \boldsymbol{p}_\perp) \to H_1^{\perp}(x, \boldsymbol{p}_\perp)$$

Mechanism FSI produce phase in TSSAs-*Leading Twist*

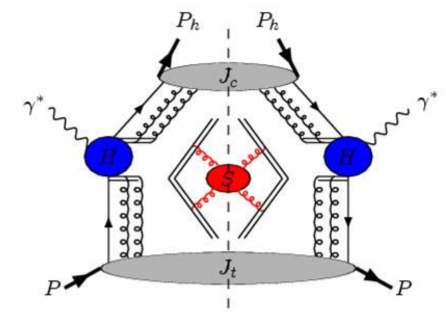
Brodsky, Hwang, Schmidt PLB: 2002 SIDIS w/ transverse polarized nucleon target

$e\ p^{\uparrow} \to e\pi X$

Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links



$$\Delta\sigma \sim D \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born}$$



Ji, Ma, Yuan: PLB, PRD 2004, 2005 extend factorization of CS-NPB: 81

Collins, Metz: PRL 2005 *Universality & Factorization "Maximally" Correlated*

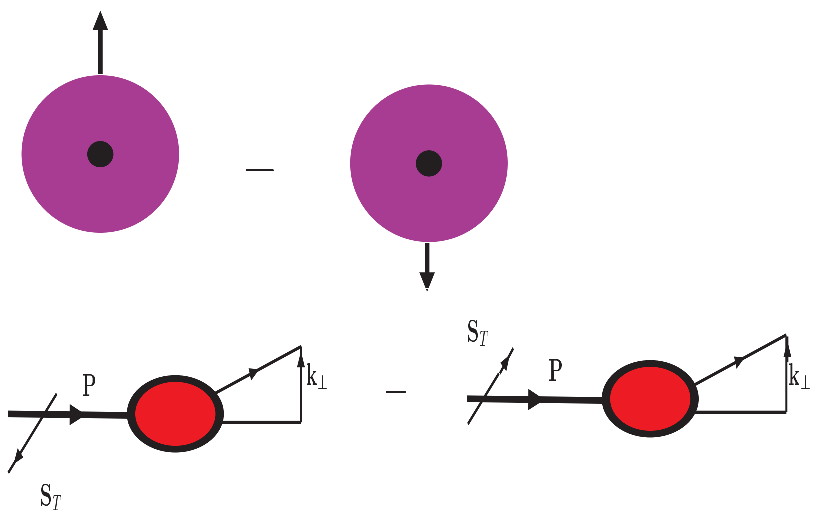
Collins, Qui: arXiv:0705.2141 *Factorization in jepordy for* $H\ H \to\ \ dijets$ *at high* P_T

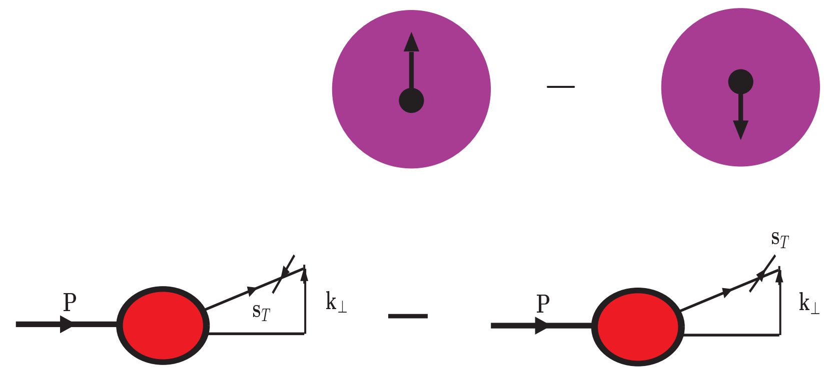
Transversity w/o Target Polarization

Transversely polarized quark in unpolarized Target Boer,Mulders PRD: 1998

Correlation of transversely polarized *quark spin* with intrinsic $\boldsymbol{k}_\perp$

$$i\mathsf{s}_T \cdot (\boldsymbol{k}_\perp \times \boldsymbol{P}) \rightarrow h_1^\perp(x, \boldsymbol{k}_\perp)$$





$h_1^\perp(x, k_\perp)$ number density transversely polarized quarks in unpolarized nucleons

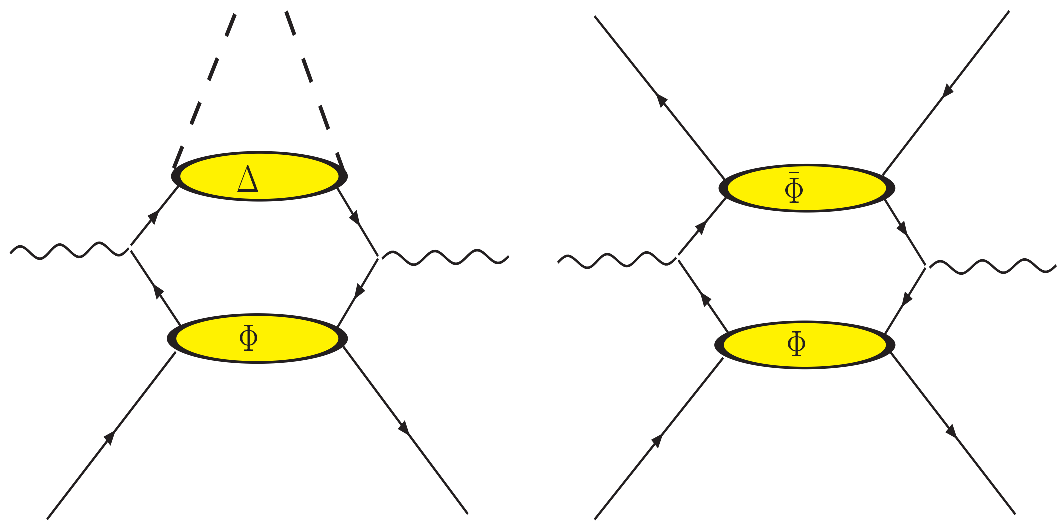
★ Boer, Mulders PRD: 1998 $\cos 2\phi$-AA in unpolarized lepto-production $e\,P \rightarrow e'\,\pi\,X$

★ Boer PRD: 1999 $\cos 2\phi$-AA in Drell Yan $\pi^- + p \rightarrow \mu^+ + \mu^- + X$ or $\bar{p} + p \rightarrow \mu^- \mu^+ + X$
(No Fragmentation!!)

Beyond Co-linear QCD: T-Odd Correlations

Mulders, Levelt, Tangerman, Boer, updates, Bacchetta, Diehl,Goeke,Metz, Schegel(1994, 1996,. . . 2006) incorporated $\boldsymbol{k}_\perp$ T-odd PDFs and FFs relevant to hard scattering QCD at leading subleading twist

$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$



SIDIS Hadronic Tensor

$$2M\mathcal{W}^{\mu\nu}(q, P, P_h) = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\,\mathrm{Tr}[\Phi(x_B, \boldsymbol{p}_T)\gamma^\mu \Delta(z_h, \boldsymbol{k}_T)\gamma^\nu] + (q \leftrightarrow -q\ ,\ \mu \leftrightarrow \nu)$$

Source of T-Odd Contributions to TSSA and AA

- "T-odd" distribution-fragmentation functions enter transverse momentum dependent correlators at *leading twist* Boer, Mulders: PRD 1998

$$\Delta(z,\boldsymbol{k}_\perp)=\frac{1}{4}\{D_1(z,\boldsymbol{k}_\perp)\,\not{n}_- + H_1^\perp(z,\boldsymbol{k}_\perp)\frac{\sigma^{\alpha\beta}k_{\perp\alpha}n_{-\beta}}{M_h} + D_{1T}^\perp(z,\boldsymbol{k}_\perp)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_-^\nu k_\perp^\rho S_{hT}^\sigma}{M_h}+\cdots\}$$

$$\Phi(x,\boldsymbol{p}_\perp)=\frac{1}{2}\{f_1(x,\boldsymbol{p}_\perp)\,\not{n}_+ + h_1^\perp(x,\boldsymbol{p}_\perp)\frac{\sigma^{\alpha\beta}p_{T\alpha}n_{+\beta}}{M} + f_{1T}^\perp(x,\boldsymbol{p}_\perp)\frac{\epsilon^{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_\perp^\rho S_T^\sigma}{M}\cdots\}$$

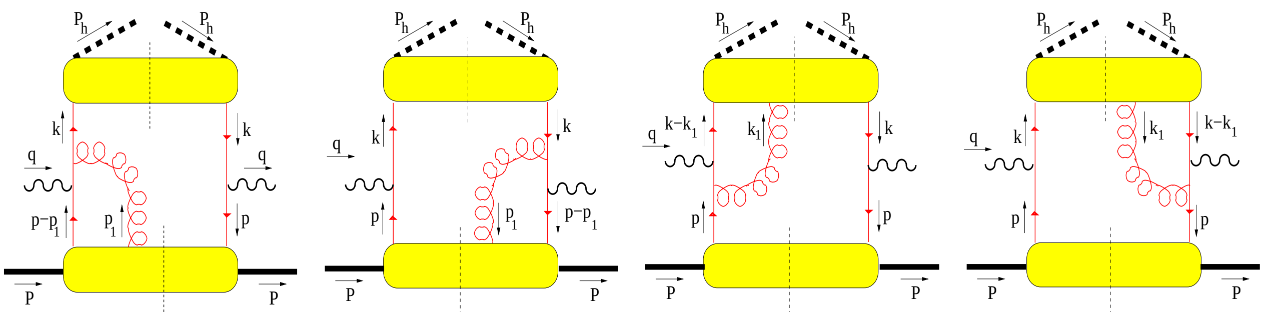
<u>SIDIS cross section</u>

$$\begin{aligned} d\sigma^{\ell N\to\ell\pi X}_{\{\lambda,\Lambda\}} \quad \propto \quad & f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+\frac{k_\perp}{Q}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\cos\phi \\ + \quad & \left[\frac{k_\perp^2}{Q^2}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+h_1^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\right]\cdot\cos 2\phi \\ + \quad & |S_T|\cdot h_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(\phi+\phi_S) \quad \text{Collins} \\ + \quad & |S_T|\cdot f_{1T}^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\sin(\phi-\phi_S) \quad \text{Sivers} \\ + \quad & |S_L|\cdot h_{1L}^\perp\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(2\phi) \quad \text{Kotzinian-MuldersPLB} \end{aligned}$$

T-Odd Effects Incorp. thru Color Gauge Invariant Factorized QCD via Wilson Line

- Leading twist Gauge Invariant Distribution and Fragmentation Functions

  Boer, Mulders: NPB 2000, Ji et al PLB: 2002, NPB 2003, Boer et al NPB 2003



etc . . .

Sub-class of loops in eikonal limit sum up to yield color gauge invariant hadronic tensor *factorized* into distribution Φ and fragmentation Δ correlators

$$\Phi(p,P)=\int\frac{d^3\xi}{2(2\pi)^3}e^{ip\cdot\xi}\langle P|\overline{\psi}(\xi^-,\xi_\perp)\mathcal{G}^\dagger_{[\xi^-,\infty]}|X\rangle\langle X|\mathcal{G}_{[0,\infty]}\psi(0)|P\rangle|_{\xi^+=0}$$

$$\Delta(k,P_h)=\int\frac{d^3\xi}{4z(2\pi)^3}\,e^{ik\cdot\xi}\,\langle 0\,|\mathcal{G}_{[\xi^+,-\infty]}\psi(\xi)|X;P_h\rangle\langle X;P_h|\overline{\psi}(0)\mathcal{G}^\dagger_{[0,-\infty]}|0\rangle|_{\xi^-=0}$$

$$\mathcal{G}_{[\xi,\infty]}=\mathcal{G}_{[\xi_T,\infty]}\mathcal{G}_{[\xi^-,\infty]},\quad \text{where}\quad \mathcal{G}_{[\xi^-,\infty]}=\mathcal{P}exp(-ig\int_{\xi^-}^{\infty}d\xi^-A^+)$$

Study FSI as mechan. for AA & TSSAs from TMDs $e.g.$ BM $h_1^{\perp(1/2)}$, Sivers $f_{1T}^{\perp(1/2)}$ &

- *Practially speaking, Cannot calculate the Quark-Quark Correlator in "Continuum Field Theory"*

$$\Phi_{ij}(x, \vec{p}_T) = \sum_X \int \frac{dz^- d^2 z_T}{(2\pi)^3} \, \mathrm{e}^{ixP^+ z^- - i\vec{p}_T \cdot \vec{z}_T} \langle P, S | \, \bar{\psi}_j(0) \, \mathcal{W}[0, \infty^-] \, |X\rangle \, \langle X | \, \mathcal{W}[\infty^-, z] \, \psi_i(z) \, |P, S\rangle$$

- Use Spectator Framework
- Diquark-model: $|X\rangle \longrightarrow |dq;\ q,\ \lambda\rangle$ one particle-state!
  - Study Wilson line contribution to TMD and FFs
    - ⋆ BHS -2002
    - ⋆ Ji, Yuan 2002 - Sivers Function
    - ⋆ Metz 2002 - Collins Function
    - ⋆ L.G. and Goldstein 2002 - Boer Mulders Function
    - ⋆ Boer, Brodsky Hwang 2003 - Boer Mulders in DY
    - ⋆ Bacchetta Jang Schafer 2004- Sivers Boer Mulders
    - ⋆ Lu Ma Schmidtt 2004/2005 Pion Boer Mulders
  - Spectator Model "Field Theoretic" used study Universality of T-odd Fragmentation Δ_{ij}
    - ⋆ Metz 2002
    - ⋆ Collins Metz 2005
  - Spectator Model Fragmentation T-odd Fragmentation
    - ⋆ L.G., Goldstein 2005, 2007
    - ⋆ Amrath, Bacchetta, Metz 2005
  - PHENO.... many of the above ...

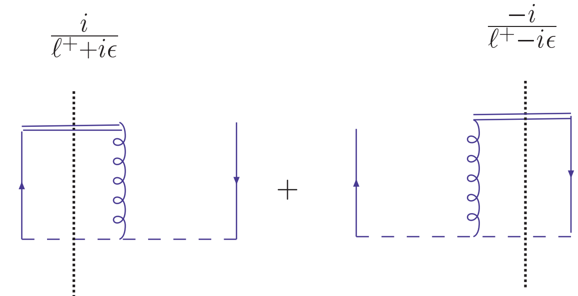
Mechanisms explored thru T-odd Contribution in SIDIS:

Impacts predictions at (HERMES, JLAB 6 & 12 GeV program)

$\cos 2\phi$ Asymmetry in SIDIS- "Boer Mulders Effect"

⋆ In spectator framework point-like nucleon-quark-diquark vertex logarithmically divergent asymmetries, Goldstein, L.G., ICHEP 2002; hep-ph/0209085)

$$h_1^{\perp(s)}(x, k_\perp) = f_{1T}^{\perp(s)}(x, k_\perp)$$



- Asymmetry-weighted function $h_1^{(1)\perp}(x) \equiv \int d^2k_\perp \frac{k_\perp^2}{2M^2} h_1^\perp(x, k_\perp^2)$ *diverges*

- Gaussian Distribution in $k_\perp$ L.G., Goldstein, Oganessyan, PRD 67 (2003)

$$h_1^\perp(x, k_\perp) = \alpha_s \mathcal{N}_s \frac{M(m + xM)(1-x)}{k_\perp^2 \Lambda(k_\perp^2)} \mathcal{R}(k_\perp^2, x)$$

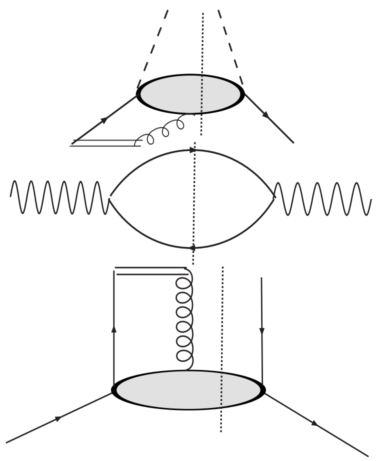
with

$$\mathcal{R}(k_\perp^2, x) = \exp^{-2b(k_\perp^2 - \Lambda(0))} \Big(\Gamma(0, 2b\Lambda(0)) - \Gamma(0, 2b\Lambda(k_\perp^2))\Big)$$

Observable: $\cos 2\phi$ SIDIS Convol. of ISI & FSI thru Gauge link

$$\frac{d\sigma}{dxdydzd^2P_\perp} \propto f_1 \otimes D_1 + \frac{k_T}{Q} f_1 \otimes D_1 \cdot \cos\phi + \left[\frac{k_T^2}{Q^2} f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp\right] \cdot \cos 2\phi$$

Boer-Mulders-Effect: (unpolarized processes)



$$F_{UU}^{\cos(2\phi_h)} = \int d^2p_T d^2k_T\, \delta^{(2)}\left(\vec{p}_T - \vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h}\right) \frac{\vec{k}_T \cdot \vec{p}_T - 2(\vec{h}\cdot\vec{k}_T)(\vec{h}\cdot\vec{p}_T)}{Mm_\pi} h_1^\perp H_1^\perp$$

- The INPUT
  - ⋆ Boer Mulders, Mulders
  - ⋆ Collins Function

- Theoretical Issues
  - ⋆ Sign of Boer Mulders and Mulders function
  - ⋆ Universality of Collins Function

Spectator Framework: BM $h_1^{\perp(1/2)}$, Sivers $f_{1T}^{\perp(1/2)}$, $f_1(x)$ and $h_{1L}^{\perp}$

- *Quark-Quark Correlator*

$$\Phi_{ij}(x,\vec{p}_T) = \sum_X \int \frac{dz^- d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{ixP^+z^- - i\vec{p}_T\cdot\vec{z}_T} \langle P,S|\,\bar{\psi}_j(0)\,\mathcal{W}[0,\infty^-]\,|X\rangle\,\langle X|\,\mathcal{W}[\infty^-,z]\,\psi_i(z)\,|P,S\rangle$$

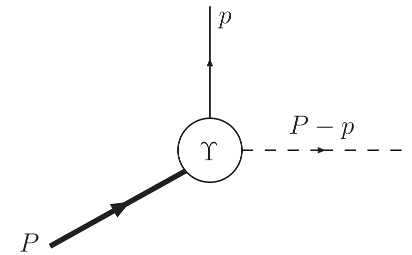
- Diquark-model: $|X\rangle \longrightarrow |dq;\, q,\, \lambda\rangle$ one particle-state!

  Thomas and Melnitchouk 1994, Mulders Rodrigues 1997

- Two kinds of diquarks: Scalar (spin 0) and Axial-vector (spin 1).

  Specification of Nucleon-Diquark-Quark vertex:

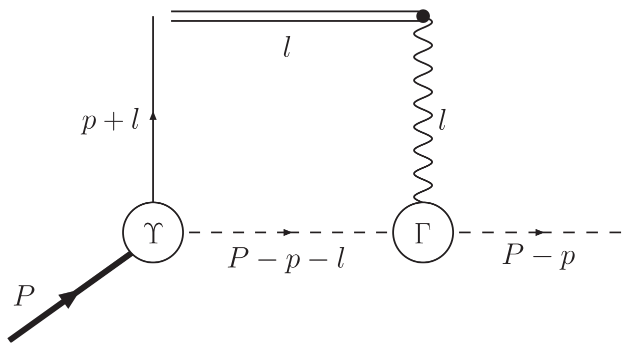
$$\langle dq; P-p, \lambda|\,\psi_i(0)\,|P,S\rangle =$$



- Ingredients, sufficient for T-even PDFs, e.g. $f_1^{(u)}$ and $f_1^{(d)}$

$$\Upsilon^\mu_{ax} = \frac{g(p^2)}{\sqrt{3}}\gamma_5\left(\gamma^\mu - R_g\frac{P^\mu}{M}\right), \Upsilon^\mu_{sc} = g(p^2)$$

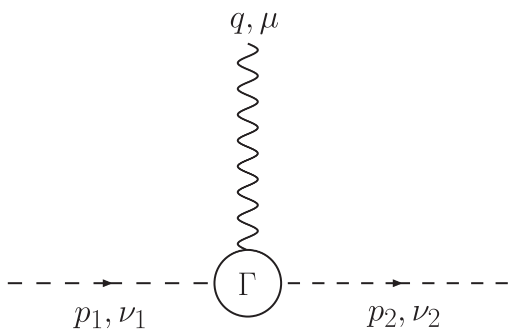
- T-odd PDFs: consequence of Gauge link $\longrightarrow$ 1 Gluon exchange approximation



- gauge boson-axial vector diquark couplingBacchetta et al. 2005

$$\Gamma_{ax}^{\mu\nu_1\nu_2} = -ie_{dq}\left[g^{\nu_1\nu_2}(p_1+p_2)^{\mu} + (1+\kappa)\left(g^{\mu\nu_2}(p_2+q)^{\nu_1} + g^{\mu\nu_1}(p_1-q)^{\nu_2}\right)\right]$$

Account for composite diquark thru anomalous mag moment κ in vertex



- axial-vector diquark and scalar diquark propagator:

$$\mathcal{D}_{ax}^{\mu\nu}(P-p-l) = \frac{-i\left(g^{\mu\nu} - \frac{(P-p-l)^{\mu}(P-p-l)^{\nu}}{m_s^2}\right)}{(P-p-l)^2 - m_s^2 + i0} \quad ; \quad \mathcal{D}_{sc}(P-p-l) = \frac{-i}{(P-p-l)^2 - m_s^2 + i0}$$

$\longrightarrow$ Loop-Integral (axial-vector diquark):

$$\int \frac{d^4l}{(2\pi)^4}\, g((l+p)^2) g(p^2) \mathcal{D}_{\rho\eta}(P-p-l) \left(\sum_\lambda \epsilon^*_\sigma \epsilon_\mu\right) \Gamma^{\lambda\rho\sigma}_{ax} \frac{n_{-\lambda}}{[l^+ + i0]} \times$$
$$\frac{\mathrm{Tr}\left[(\not{P}+M)\left(\gamma^\mu - R_g \frac{P^\mu}{M}\right)(\not{p}-m_q)\gamma^+\gamma^i(\not{l}+\not{p}+m_q)\left(\gamma^\eta + R_g\frac{P^\eta}{M}\right)\gamma_5\right]}{\left[l^2-\lambda^2+i0\right]\left[(l+p)^2-m_q^2+i0\right]}$$

- Simplification the numerator $\rightarrow$ sort by powers of loop-momentum l

$$\boxed{\longrightarrow J^{(i)\alpha_1\ldots\alpha_i} = \int \frac{d^4l}{(2\pi)^4} \frac{g((l+p)^2) g(p^2) l^{\alpha_1}\ldots l^{\alpha_i}}{[(v\cdot l)+i0]\left[l^2+i0\right]\left[(l+p-P)^2-m_s^2+i0\right]\left[(l+p)^2-m_q^2+i0\right]}}$$

- $v = \left[1^-, 0^+, \vec{0}_T\right]$, $l^+ \rightarrow 0$, $l^- \rightarrow \infty$, $\alpha_k = -$ $\Rightarrow$*Light cone divergence*

- Regularization procedure Collins:NPB 1982 , Ji, Ma, Yuan PLB: 2004 :
  1) Clean procedure: (Gamberg, Hwang, Metz, MS, 2006)
  Introduction of Wilson lines *off* the light cone, $v = \left[1^-, \lambda^+, \vec{0}_T\right]$,

$$\longrightarrow h_1^{\perp,ax}(x, \vec{p}_T^{\,2}, v) \propto \ln\left(\frac{v^2}{v\cdot P}\right)$$

2) Phenomenological procedure: Form factor $g(p^2)$

$$\boxed{g(p^2) = N^{2n}\frac{\left[p^2-m_q^2\right]F(p^2)}{\left[p^2-\Lambda^2+i0\right]^n}}$$

$\longrightarrow$ additional pole produces additional factor $[l^+]^n$ in numerator $\longrightarrow$ Regularization.

- Transverse Integral:
$h_1^{\perp,ax} \propto \int d^2l_T \frac{F((l+p)^2)\left[A\vec{l}_T^4+B\vec{l}_T^2(\vec{p}_T\cdot\vec{l}_T)+C\vec{l}_T^2+D(\vec{p}_T\cdot\vec{l}_T)+E\right]}{\left[\vec{l}_T^2\right]\left[(\vec{l}_T+\vec{p}_T)^2+\tilde{m}_\Lambda^2\right]^3}$

- $E=0$: *no IR-divergence*!

- $A \neq 0$: *UV-divergence* $\Longrightarrow$ Further specification of form factor:

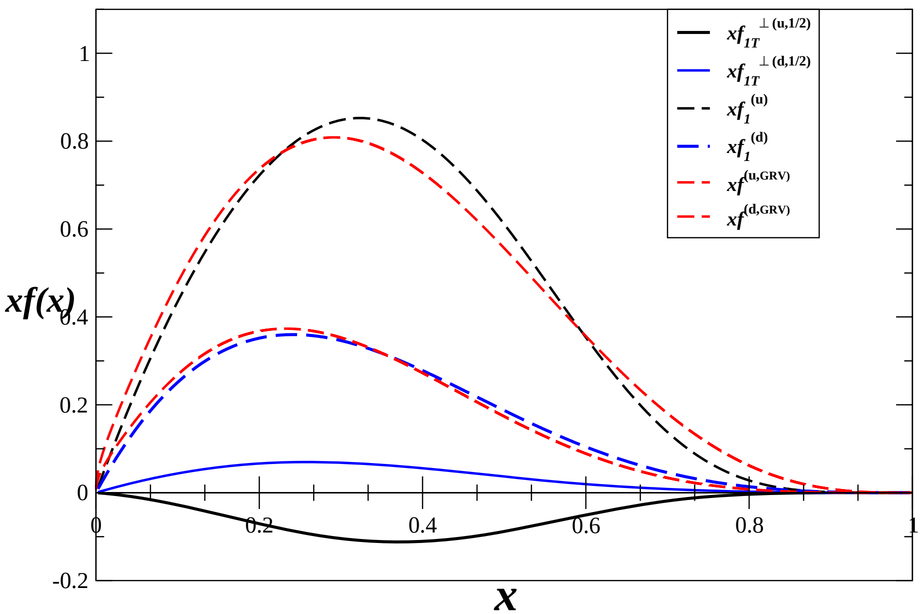
$$g(p^2) = N^2\frac{\left[p^2-m_q^2\right]\,\mathrm{e}^{-b|p^2|}}{\left[p^2-\Lambda^2+i0\right]^3}$$

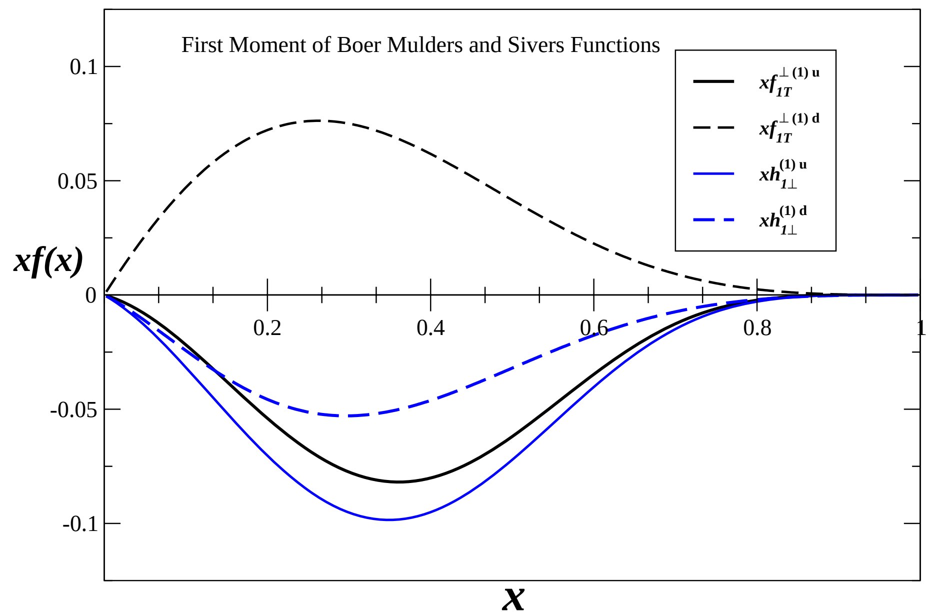
- Integration yields incomplete Gamma-functions $\Gamma(n,x) \equiv \int_x^\infty \mathrm{e}^{-t}t^{n-1}dt\,,\, n>0$.

Flavor Dependence: Results & Phenomenolgy

Flavor-dependent PDFs from diquark models: $u = \frac{3}{2}s + \frac{1}{2}a$, $d = a$,

moments: $h_1^{\perp(n/2)}(x) = \int d^2\vec{p}_T \frac{|\vec{p}_T|^n}{2M^n} h_1^{\perp}(x, \vec{p}_T^2)$

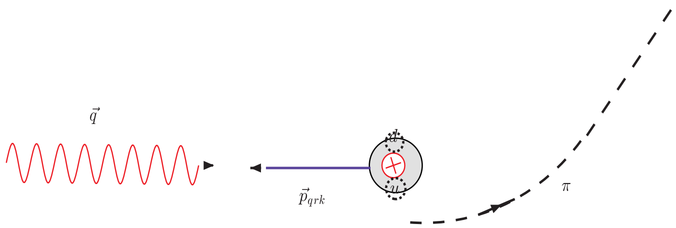




- Comparison to $f_1^{(u,d)}$ (Glück, Reya, Vogt) $\longrightarrow$ parameters of the model, e.g. diquark masses, normalization...
- Comparison to parameterization of Sivers function $f_{1T}^{\perp}$ $\longrightarrow$ size and sign of FSI.
- Boer Mulders up and down are negative is spectator model

Quark Transversity & Boer Mulders Function
GPDs-Impact Parameter PDFs

- Correlations transverse-spin & intrinsic $k_\perp$ serves fix sign Boer Mulders



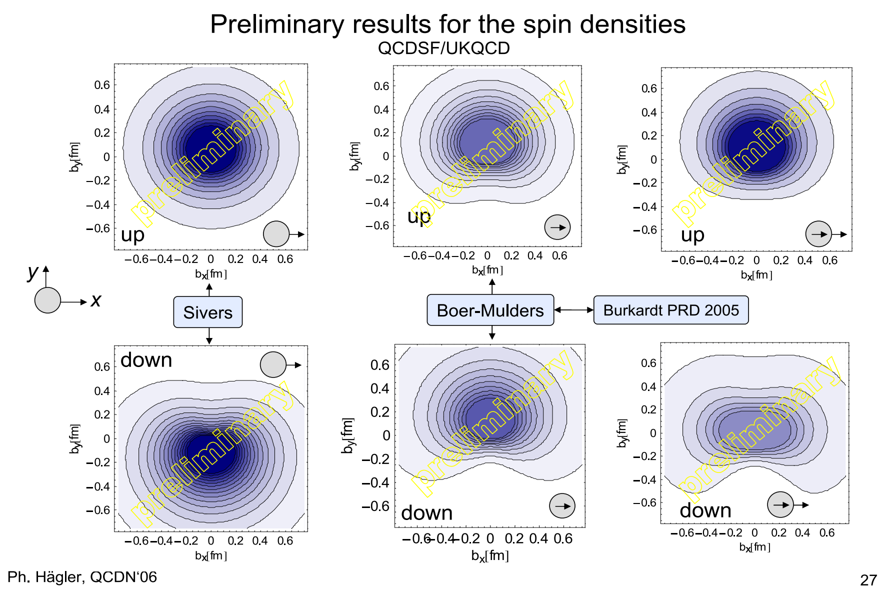
- $\delta q^X(x, \mathbf{b}_\perp) \leftrightarrow h_1^{\perp q}$ WHERE $\delta q^X(x, \mathbf{b}_\perp) = -\frac{1}{2M}\frac{\partial}{\partial\, by}(2\tilde{\mathcal{H}}_T(x, \mathbf{b}_\perp) + \mathcal{E}_T(x, \mathbf{b}_\perp))$
  - ⋆ $d_y^q = \int dx \int d^2\mathbf{b}_\perp \delta q^X(x, \mathbf{b}_\perp) b_y = \kappa_T^q/2M$
- *Transverse distortion* in impact parameter space of transversly polarized quarks in an unpolarized nucleon Burkardt PRD 2005, Diehl, Hägler EPJC 2005

⋆ Implies up and down quark Boer Mulders function-same sign!

- Supports
  - ⋆ Lg N_C arguments Pobylitsa hep-ph/0301236
  - ⋆ Bag Model calculation Yuan PLB 2003
  - ⋆ Implications $\cos 2\phi$ phenomenology in SIDIS & Drell Yan
  - Lattice QCDSF/UKQCD, Hägler et al... calculations of matrix elements on the lattice

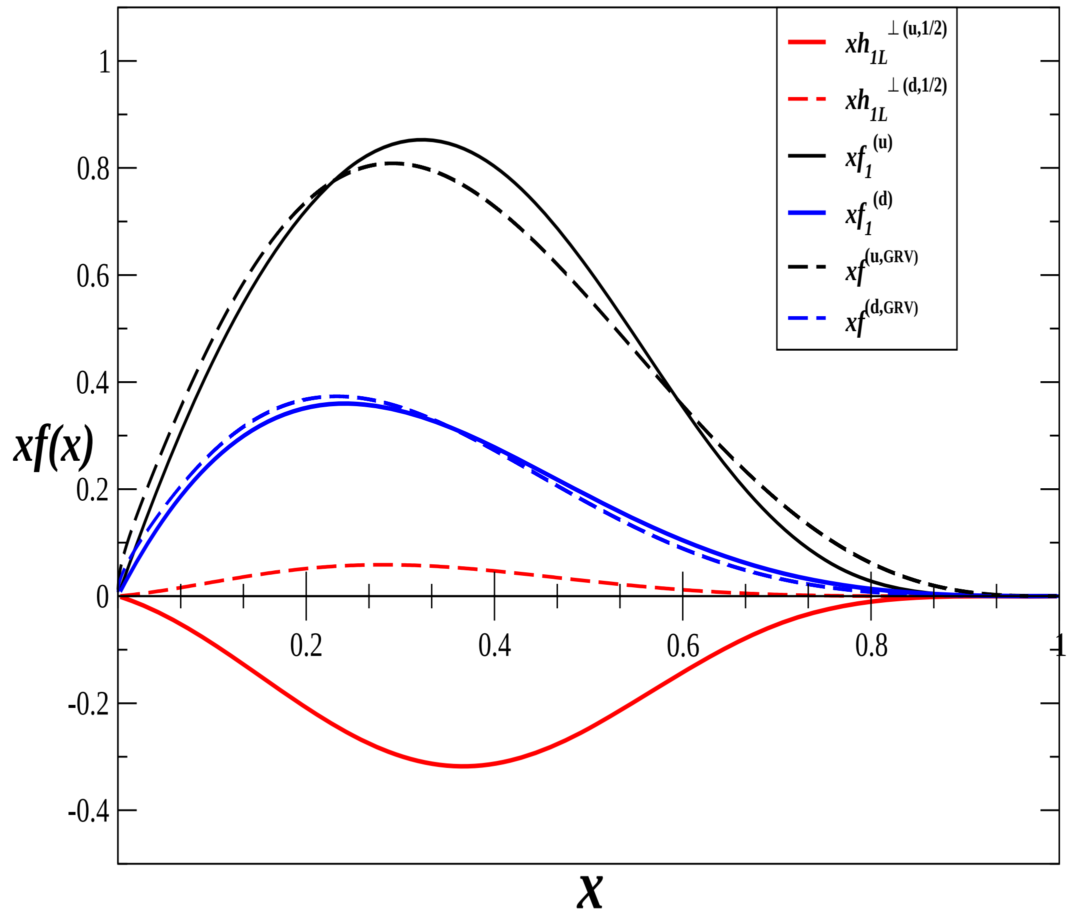
$$\kappa_T = \int dx \bar{E}_T(x, \xi, t = 0) \equiv \bar{B}_{T10(t=0)}$$

$$\kappa_T^{(u)} = \kappa_T^{(d)}$$



see talks of Burkardt and Hägler's Trento 2007

Mulders-Kotzinian-$h_{1L}^{\perp}$



- ⋆ Valence Normalization, $\int_0^1 u(x) = 2$, $\int_0^1 d(x) = 1$
- Black curve- $xu(x)$
- Dashed curve - $xu(x)$ GRV
- Red/Blue curve $xh_{1L}^{\perp(1/2)(u,d)}$

Transverse Momentum Dependent (TMD) Parton Distributions

- Twist-2 TMD parton distributions, parameterization of matrix elements, $f = f(x, \vec{p}_T^2)$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^{+}\mathcal{W}\psi|P,S\rangle\right] = f_1 - \frac{\epsilon_T^{ij} p_T^i S_T^j}{M} \underbrace{f_{1T}^{\perp}}_{\text{Sivers}}$$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^{+}\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \lambda g_{1L} + \frac{\vec{p}_T \cdot \vec{S}_T}{M} g_{1T}$$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}i\sigma^{i+}\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \underbrace{S_T^j \left(\delta^{ij} h_{1T} + \frac{p_T^i p_T^j}{M^2} h_{1T}^{\perp}\right)}_{\rightarrow \text{transversity } h_1(x,\vec{p}_T^2)} + \lambda \frac{p_T^i}{M} h_{1L}^{\perp} + \frac{\epsilon_T^{ij} p_T^j}{M} \underbrace{h_1^{\perp}}_{\text{Boer-Mulders}}$$

- *Time-reversal odd (T-odd)* PDFs $f_{1T}^{\perp}$, $h_1^{\perp}$ $\longrightarrow$ Consequence of the gauge link.
- *Higher twist* parton distributions $\longrightarrow$ matrix elements with $\gamma_\perp^i$, $\gamma_\perp^i\gamma_5$, $\mathbb{1}$, γ_5, $i\sigma^{+-}\gamma_5$, $i\sigma^{ij}\gamma_5$, e.g.

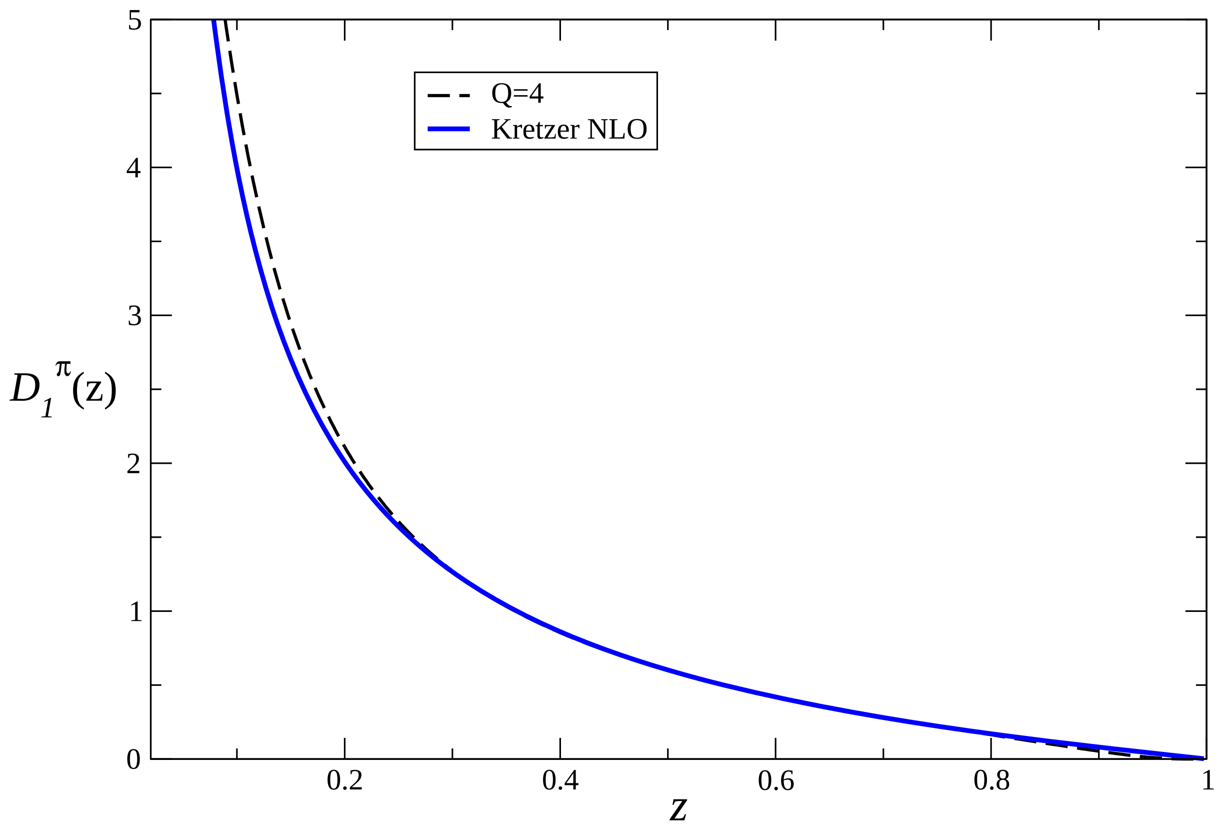
$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma_\perp^i\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \frac{M}{P+}\left(\underbrace{S_T^i \left(\delta^{ij} g_T' + \frac{p_T^i p_T^j}{M^2} g_T^{\perp}\right)}_{\rightarrow \mathrm{g_T(x,\vec{p}_T^2)}} + \lambda \frac{p_T^i}{M} g_L^{\perp} + \frac{\epsilon_T^{ij} p_T^j}{M} g^{\perp}\right)$$

- 8 T-even, 8 T-odd twist-3 parton distributions.

Pion Fragmentation Function

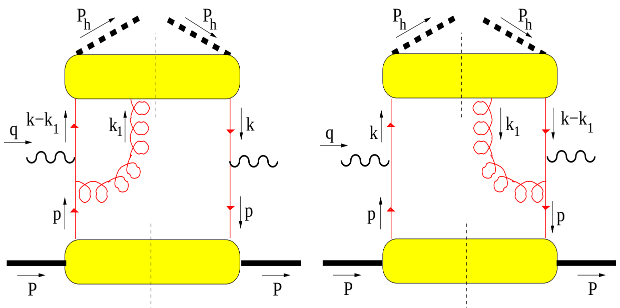
Bacchetta, L.G., Goldstein, Mukherjee in prep

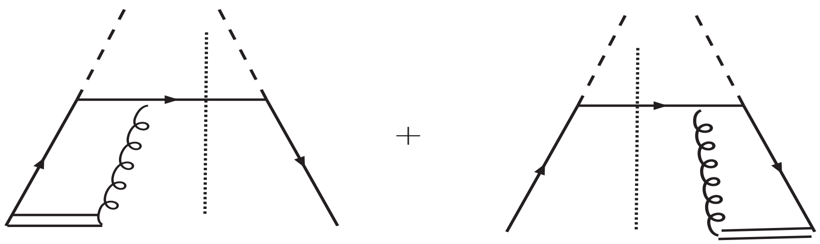
Normalized to Kretzer, PRD: 2000



Gluonic Poles & Gauge Link for T-Odd Collins Function

L.G.,Goldstein,Oganessyan PRD68,2003 $\Delta^{[\sigma^{\perp -}\gamma_5]}(z,k_\perp) = \frac{1}{4z}\int dk^+ Tr(\gamma^-\gamma^\perp\gamma_5\Delta)|_{k^-=P_\pi^-/z}$





$\frac{i}{\ell^- - i\epsilon}$ $\quad Eikonal\, Feynman\, Rules\, Collins \quad '82' \quad$ $\frac{-i}{\ell^- + i\epsilon}$

- *Motivation:color gauge .inv frag. correlator* "pole contribution"
- Gribov-Lipatov Reciprocity 1974, Mulders et al. 1990s for T-even Fragmentation
- Applied to T-odd Fragmentation L.G.,Goldstein,Oganessyan PRD68,2003
- ⋆ Collins Metz PRL 2005 demonstrate to 1-loop, universality. Basis for cut method Amrath et. al.: PRD 2005

Process Dependence: Gauge Link Contribution to Fragmentation Function

L.G., Goldstein, Oganessyan PRD: 2003: Bacchetta, Metz, Jang: PLB: 2003, Amrath et. al.: PRD 2005,
L.G., G. Goldstein & Como Proceedings 2006

- ⋆ Another argument in spectator model use Cauchy's theorem to evaluate the Color Gauge invariant Correlator $\Delta^{[\sigma^{\perp -}\gamma_5]}(z, k_\perp) \sim \mathrm{Tr}(\Delta$
  $sigma^{\perp -}\gamma_5)$

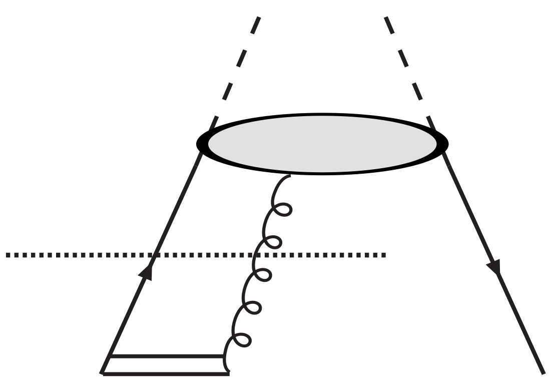
- • Analysis of pole structure in light-cone loop integral ℓ^+indicates a *singular behavior in loop integral-looks like a "lightcone divergence" at first sight*: $\delta(\ell^-)\theta(\ell^-)f(\ell^-)$
- • $f(\ell^-) \propto a_1\ell^- + a_2\ell^{-\,2} + \ldots$ polynomial in ℓ^--vanishes...
- ⋆ Regulate n off light cone, poles outside physical regime

$$\frac{1}{n\cdot\ell \pm i\epsilon} \quad \ldots$$

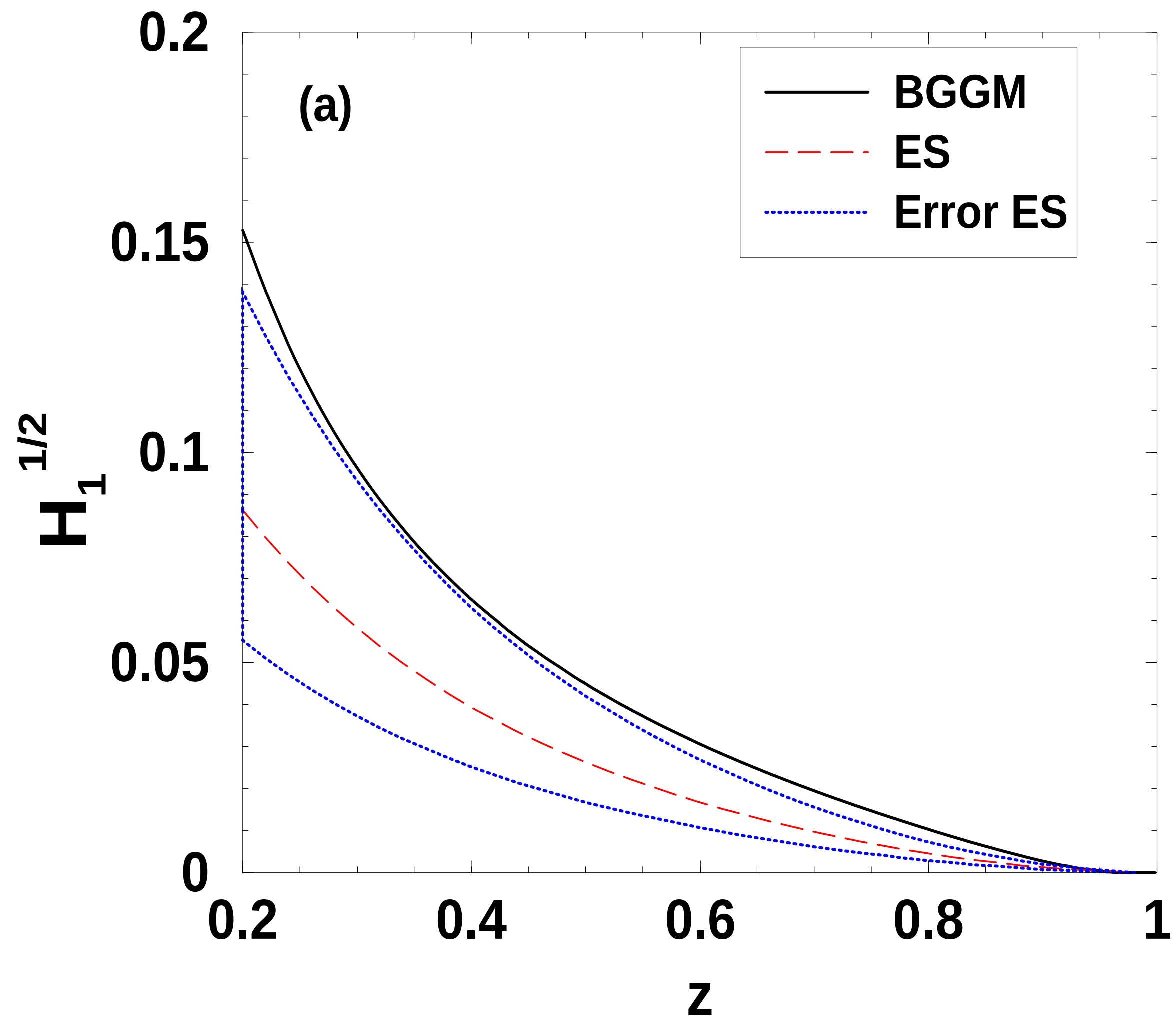
  $n = (n^-, n^+, 0)$ (see CS NPB 1982, LG, Hwang, Metz, Schlegel PBL:2006)

- ⋆ t-channel cut in Eikonal and Spectator vanishes i.e. G.P. contribution zero: Metz 2002, Collins Metz PRL 2005
- ⋆ s-channel cut on Fragmenting quark and gluon contributes
  Reciprocity Fails, "T-odd" Fragemtation Function Universal between e^+e^- and SIDIS to one loop.

Bacchetta, L.G., Goldstein, Mukherjee Re-analysis and Kaons



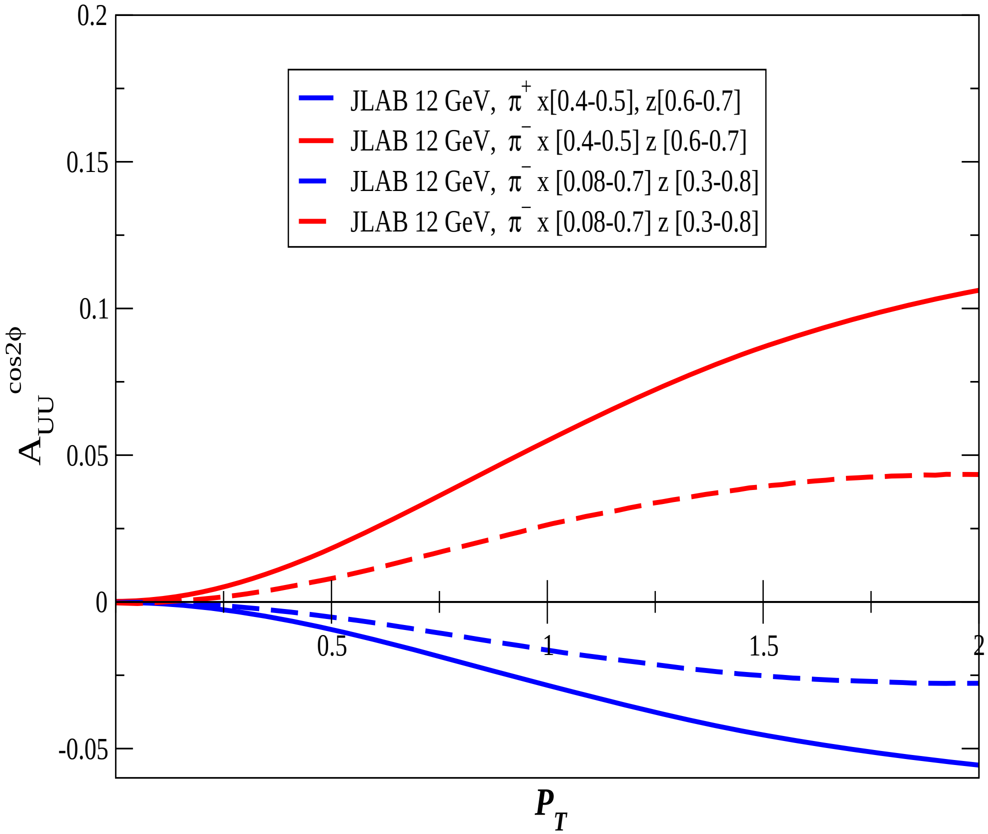
$$\frac{-i}{\ell^{-} - i\epsilon}$$

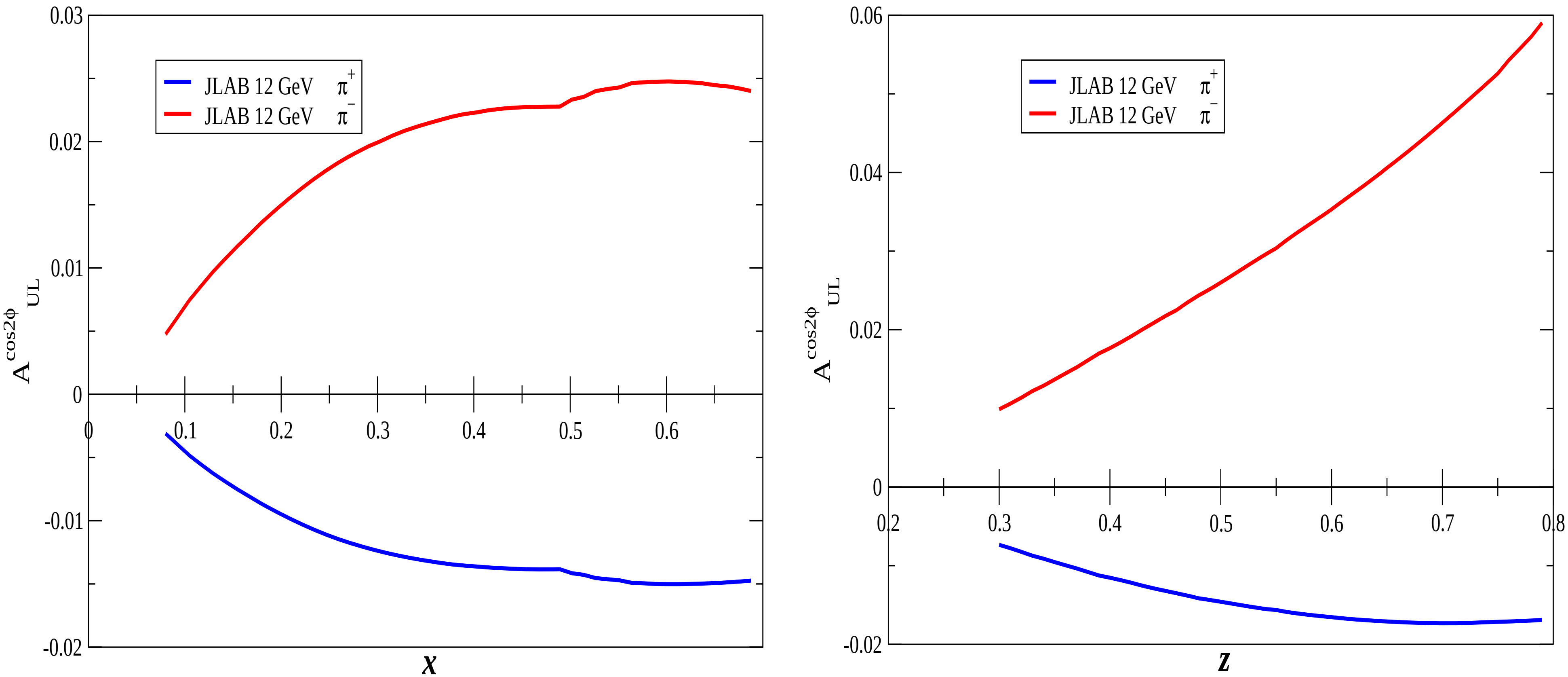


CLAS12 PAC 30-Avakian, Meziani. . . L.G. . .

$$A_{UU}^{\cos(2\phi_h)} \propto \sum_a e_a^2 \int d^2p_T d^2k_T\, \delta^{(2)}\left(\vec{p}_T - \vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h}\right) \frac{\vec{k}_T\cdot\vec{p}_T - 2(\vec{h}\cdot\vec{k}_T)(\vec{h}\cdot\vec{p}_T)}{M m_\pi} h_1^{\perp(a)} H_1^{\perp(a)}$$

Model assumption:
Dis-favored fragmentation
$H_1^{\perp\,(d\to\pi^+)} = -H_1^{\perp\,(u\to\pi^+)}$

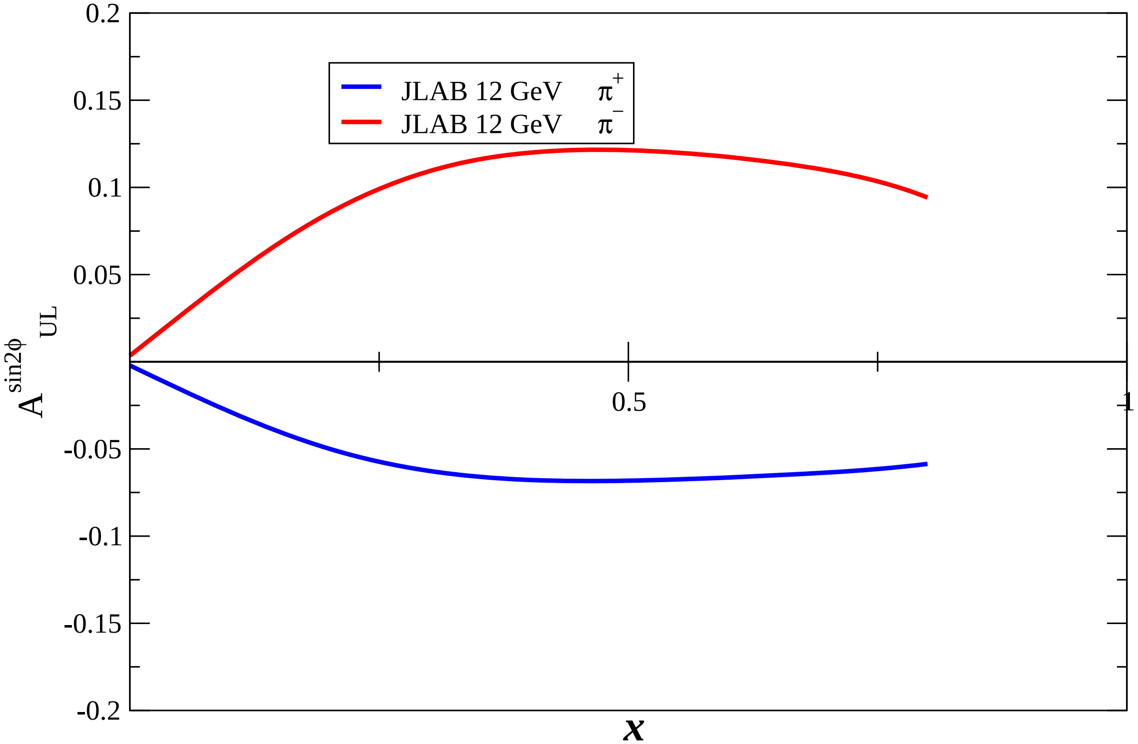



0.03
0.02
0.01
0
-0.01
-0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
$A^{\cos 2\phi}_{UL}$
JLAB 12 GeV π^+
JLAB 12 GeV π^-
x
0.06
0.04
0.02
0
-0.02
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$A^{\cos 2\phi}_{UL}$
JLAB 12 GeV π^+
JLAB 12 GeV π^-
z

CLAS 5.7 PAC 32-Avakian

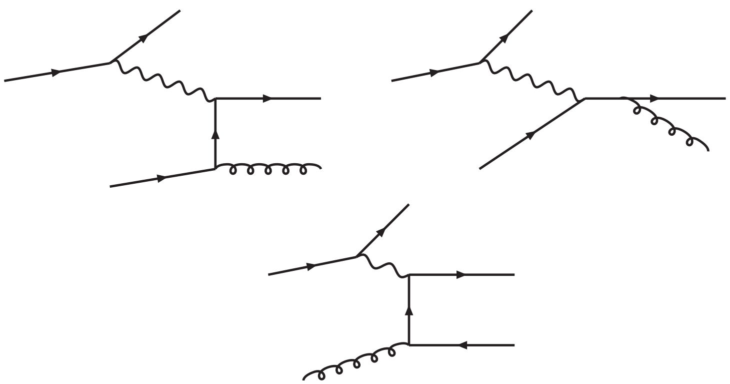
$$A_{UL} = \frac{2(1-y)}{1+(1-y)^2}\frac{h_{1L}^{\perp(1)} H_1^{\perp(1)}}{f_1 D_1}$$

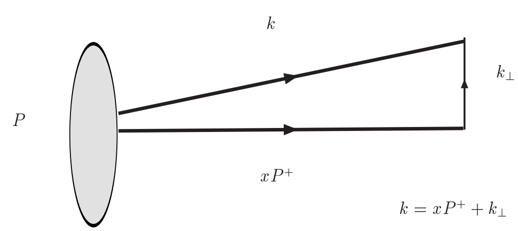
Kotzinian and Mulders PLB 1997



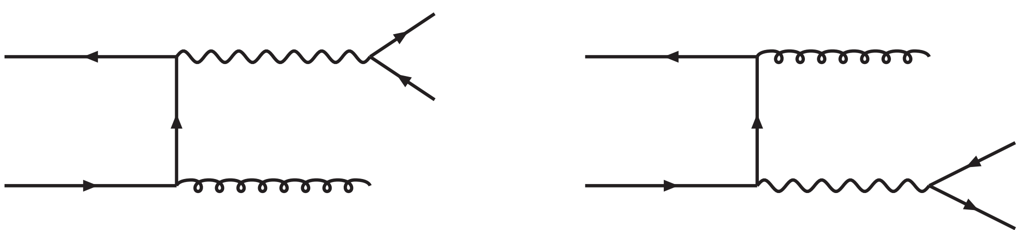
$\cos 2\phi$ JLAB, EIC, GSI, JPARC ...

- Georgi and Mendez 1975, Kroll and König 1982 gluon PQCD ".. gluon bremstrulang competes with convolution of $h_1^\perp \otimes H_1^\perp$
- Cahn Effect: Chay-Ellis PRD 1995,L.G., Goldstein, Oganessyan DIS03-proc 2003,Barone,Ma, PLB: 2006, Anselmino,Boglione,Prokudin, Turk Chay et al PRD: 95
- Qui Sterman Ji Yuan Vogelsang approach 2006

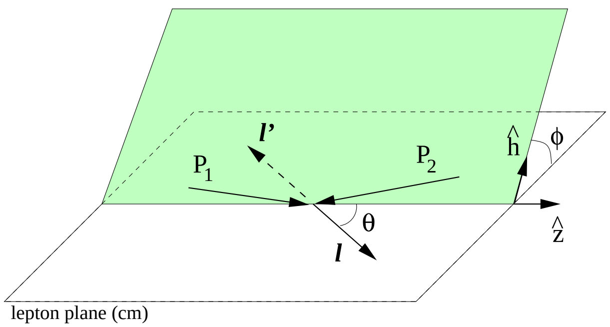




- Gluon bremstrulang Collins PRL: 1979 competes with convolution of $h_1^\perp \otimes \bar{h}_1^\perp$



Unpolarized DRELL YAN $\cos 2\phi$



$$\bar{p} + p \rightarrow \mu^- \mu^+ + X$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin^2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right) \tag{1}$$

Angles refer to the lepton pair orientation in their rest frame relative to the boost direction and the initial hadron's plane. Asymmetry parameters, λ, μ, ν, depend on $s, x, m^2_{\mu\mu}, q_T$

Boer PRD: 1999, Boer, Brodsky, Hwang PRD: 2003 Collins SoperPRD: 1977 subleading twist

- Leading twist $\cos 2\phi$ azimuthal asymmetry depends on T-odd distribution $h_1^\perp$.

$$\nu = \frac{2\sum_a e_a^2 \mathcal{F}\left[(2\boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp - \boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp)\frac{h_1^\perp(x,\boldsymbol{k}_T)\bar{h}_1^\perp(\bar{x},\boldsymbol{p}_T)}{M_1 M_2}\right]}{\sum_{a,\bar{a}} e_a^2 \mathcal{F}[f_1 \bar{f}_1]} \tag{2}$$

Convolution integral

$$\mathcal{F} \equiv \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_\perp \delta^2(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp - \boldsymbol{q}_\perp) f^a(x,\boldsymbol{p}_\perp)\bar{f}^a(\bar{x},\boldsymbol{k}_\perp)$$

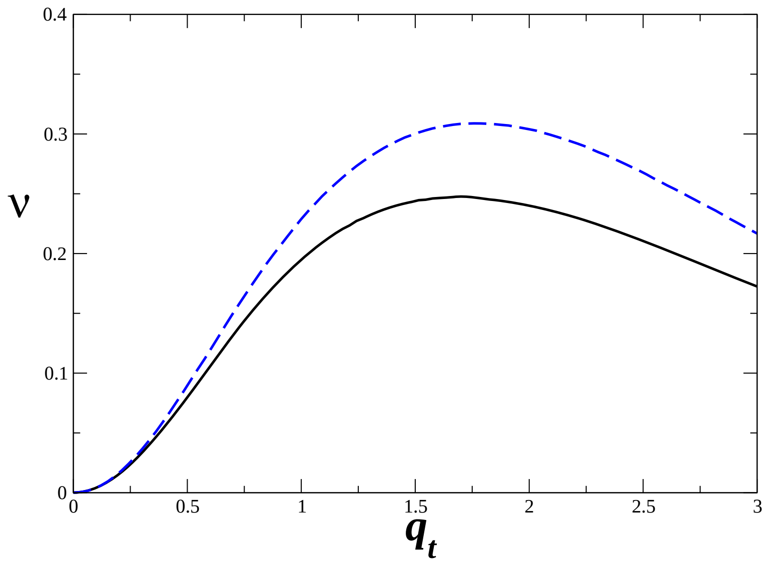
Higher twist comes

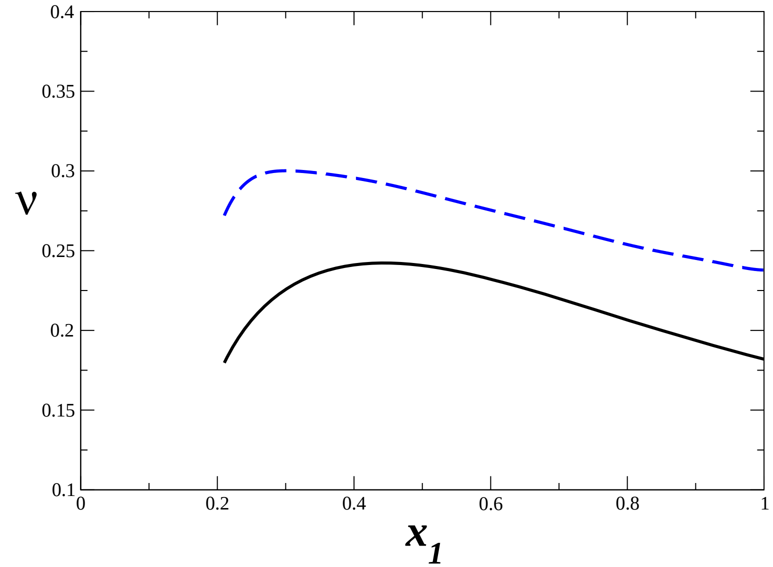
$$\nu = \frac{2\sum_a e_a^2 \mathcal{F}\left[(2\boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp - \boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp)\frac{h_1^\perp(x,\boldsymbol{k}_T^2)\bar{h}_1^\perp(\bar{x},\boldsymbol{p}_T)}{M_1 M_2}\right] + \nu_4[w_4 f_1 \bar{f}_1])}{\sum_{a,\bar{a}} e_a^2 \mathcal{F}[f_1 \bar{f}_1]}$$

where Collins SoperPRD: 1977 subleading twist

$$\nu_4 = \frac{\frac{1}{Q^2}\sum_a e_a^2 \mathcal{F}\left[w_4\, f_1(x,\boldsymbol{k}_\perp)\bar{f}_1(\bar{x},\boldsymbol{p}_\perp)\right]}{\sum_a e_a^2 \mathcal{F}\left(f_1(x,\boldsymbol{k}_\perp)\bar{f}_1(\bar{x},\boldsymbol{p}_\perp)\right)},$$

where the weight $w_4 = 2\left(\hat{\boldsymbol{h}} \cdot (\boldsymbol{k}_\perp - \boldsymbol{p}_\perp)\right)^2 - (\boldsymbol{k}_\perp - \boldsymbol{p}_\perp)^2$





Perform Convolution integral L.G., Goldstein PLB: 2007

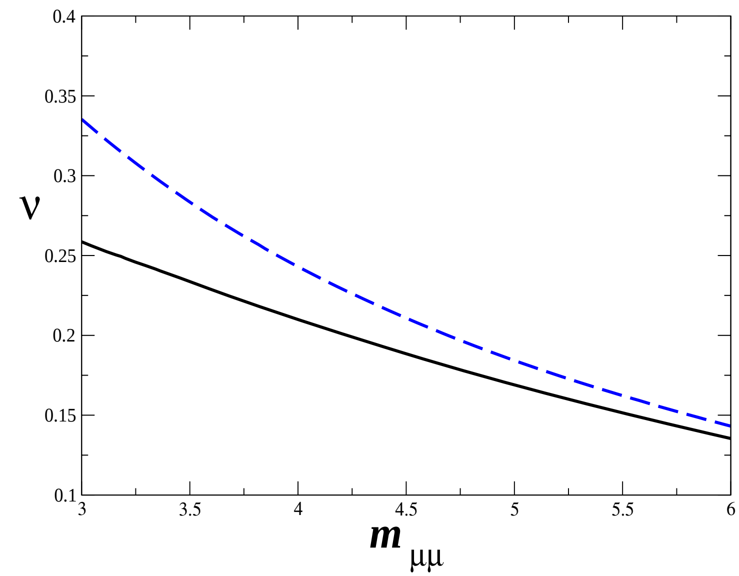
$s = 50\,\mathrm{GeV}^2$, $x = [0.2 - 1.0]$,

$q = [3.0 - 6.0]$ GeV, $\boldsymbol{q}_T = 0 - 2.0\,\mathrm{GeV}$

q_T^2/Q^2 corrections

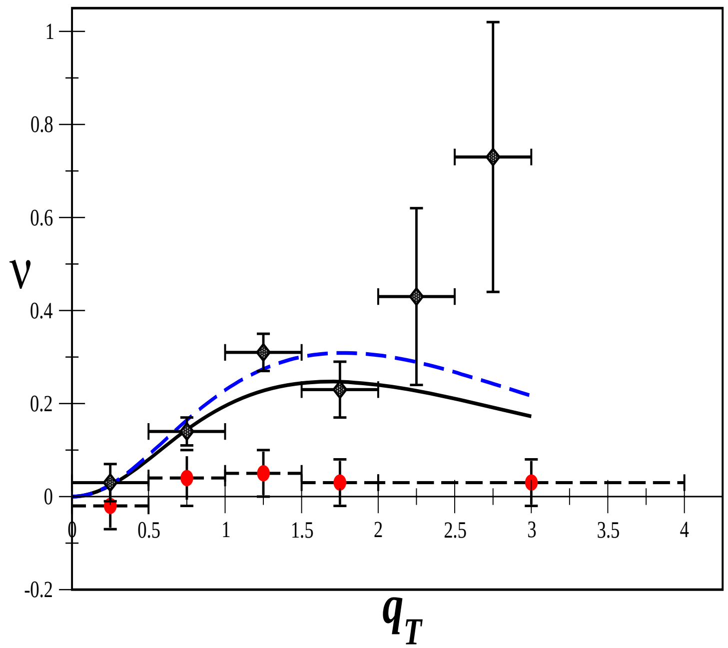
$x_1 x_2 = \frac{Q^2(1+q_T^2/Q^2)}{s}$

q_T/Q can be order 0.5



Sea Quark Boer Mulders

Gamberg & Goldstein Plb 2007



- ν plotted as a function of q_T for $s = 50\ \mathrm{GeV}^2$, $x[0.2 - 1.0]$, and $Q[3-6]\ \mathrm{GeV}/c$. Solid line leading twist contribution ν_2, dashed line leading and sub-leading twist $(\nu_2 + \nu_4)$.
- Data: Diamonds are for E615 $\pi^- + p$ at 252 GeV/c. Circles are for E866 $p + d$ at 800 GeV/c Peng, Zu hep-ex/0609005.
- Horizontal bars refer to bin size, vertical error bars refer to statistical uncertainties.
- One of pair of structure functions in convolution involve sea anti-quarks, the $\bar{h}_1^{\perp(sea)}$ for $N \longrightarrow \bar{u}$ or $\bar{d}$.
- Suppressed by another factor of α_s in our approach (as well as possible kinematic factors) w/ data suggest that sea structure BM function roughly $\frac{1}{3}$ of the magnitude of predicted valence quark structure function.

SUMMARY

- Going beyond the collinear approximation in PQCD recent progress has been achieved characterizing transverse SSA and azimuthal asymmetries through "rescattering" mechanisms which generate T-odd, intrinsic transverse momentum, $k_\perp$, dependent *distribution and fragmentation* functions at leading twist
- Central to this understanding is the role that transversity properties of quarks and hadrons pocess terms of correlations between transverse momentum and transverse spin in QCD hard scattering
- The tranversity programs Belle, HERMES, RHIC, have uncovered large effects and near term Hall-A Transversity will start to check flavor structure of T-odd TMDs
- Future experiments to uncover the Boer Mulders function was approved at JLAB Hall B-CLAS12 proposal on $\cos 2\phi$. Will also be a check on the Collins function

⋆ Azimuthal asymmetries in Drell Yan and SSA can be measured at GSI-PAX, JPARC as well

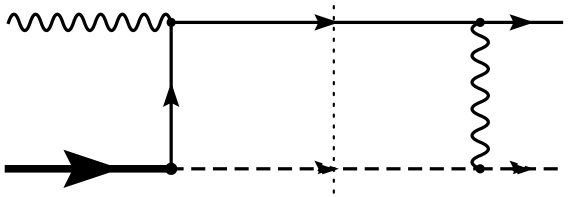
⋆ Transverse spin effects are more than h_1

Longitudinal jet-SSAs in the scalar diquark-spectator-model:

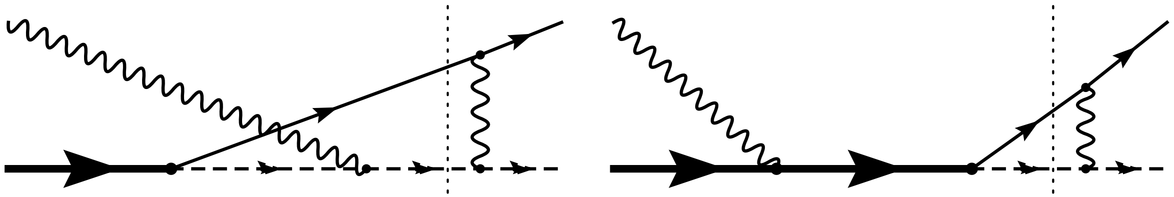
(Afanasev, Carlson, 2003, 2006; Metz, M.S., 2004)

- *Left hand side:* Direct calculation of jet asymmetries $A_{LU} \propto g^{\perp(1)}(x)$, $A_{UL} \propto f_L^{\perp(1)}(x)$

Rescattering effect:

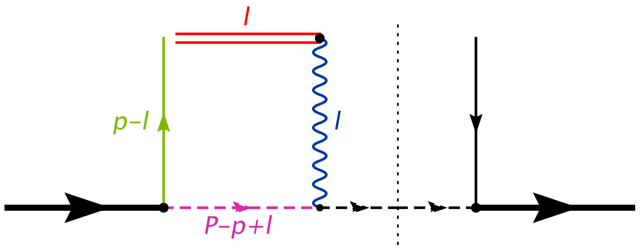


Other diagrams containing final state interactions:



$\Longrightarrow$ Result: Non-vanishing, *finite* asymmetries $\boxed{A_{UL} \neq 0}$, $\boxed{A_{LU} \neq 0}$.

- *Right hand side:* T-odd twist-3 parton distributions in the scalar diquark model
(Gamberg, Hwang, Metz, M.S., 2006)



$$+\ \text{h.c.}\quad \boxed{g^{\perp} \propto \int \frac{d^4l}{(2\pi)^4} \frac{n\cdot(2P-2p+l)\left[\epsilon_T^{ij} p_T^j \left(P^+ l^- - P^- l^+\right) + \epsilon_T^{ij} l_T^j \left(P^- p^+ - P^+ p^-\right)\right]}{[(l\cdot n)+i0]\left[l^2 - i0\right]\left[(P-p+l)^2 - m_s^2 - i0\right]\left[(p-l)^2 - m_q^2 - i0\right]}}$$

For $n = [1^-, 0^+, \vec{0}_T]$ on the light-cone $\longrightarrow$ *Divergence*!

- Regularization: "non lightlike" Wilson lines: $n = [n^-, n^+, \vec{0}_T]$, $\left|\frac{n^+}{n^-}\right| \ll 1$

$$\boxed{g^\perp(x, \vec{p}_T^2, n) \propto \frac{1-x}{x} \ln\left(\frac{n^2}{2(n\cdot P)^2}\right) + \text{finite} + \mathcal{O}\left(\left|\frac{n^+}{n^-}\right|\right)}$$

- Shows LC-divergence explicitly
$\longrightarrow$ same divergence for *all* T-odd twist-3 PDFs, also in quark-target-model.

- Finite Box-graph contributions for twist-2 T-odd PDFs $f_{1T}^\perp$, $h_1^\perp$.

- Regularization procedure: "Tree-level" predictions $(A_{LU} = \text{fin.}) \propto (g^\perp \to \infty)$, $(A_{UL} = \text{fin.}) \propto (f_L^\perp \to \infty)$ $\Rightarrow$ Modification!?

- Factorization theorem for twist-2 observables: (Ji, Ma, Yuan, 2004)

$$\boxed{\frac{d\sigma_{UU}}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} \propto \int d^2p_T\, d^2k_T \left(\int d^2l_T\, S(\vec{l}_T)\delta^{(2)}(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{k}_T + \vec{l}_T)\right) f_1(x_B, \vec{p}_T^2)\, D_1(z_h, \vec{k}_T^2) + \ldots}$$

Soft factor $S(\vec{l}_T)$ due to soft gluon radiation $\longrightarrow$ modifies δ-function.

PDFs/FFs: "non light-like" Wilson lines $\longrightarrow$ "non-light-likeness" parameter $\zeta = \sqrt{\frac{2(P\cdot n)^2}{n^2}}$

- Generalization of "all-order factorization" for twist-3 observables possible?