RHIC/AGS Users' Meeting , June 5-9, 2006

Heavy Quarkonium Potentials and Bound States in Quark Gluon Plasma

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- Introduction
- How to extract the Q-$\overline{Q}$ potential from LGT results
- Quarkonium bound states and quark drip lines in quark-gluon plasma
- Conclusions

C.Y.Wong, PRC65,034902('02)
C. Y. Wong, PRC72,034906 ('05)
C.Y.Wong, hep-ph/0509088

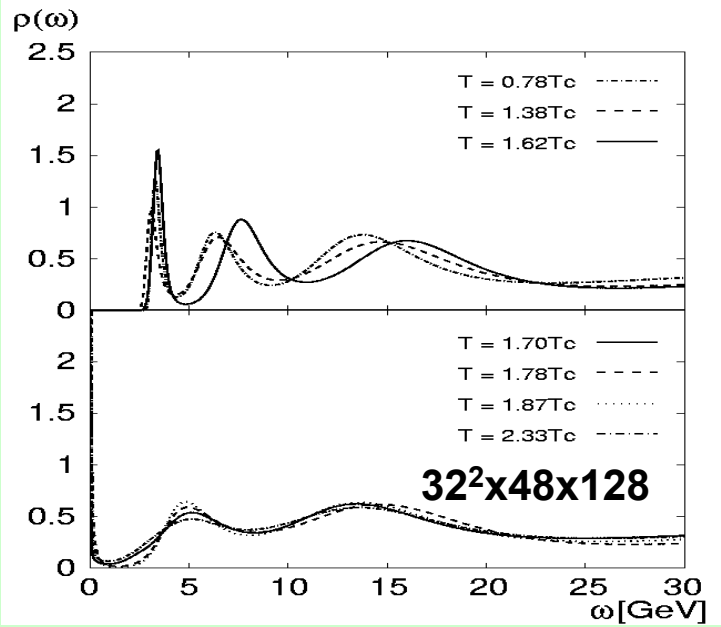
Introduction

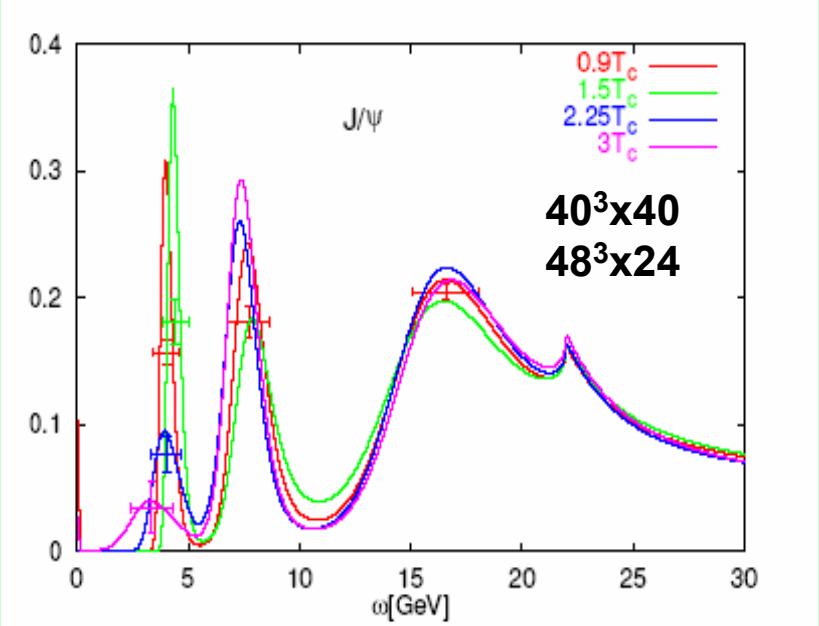
- Two new and surprising results from lattice gauge calculations
  - Lattice spectral function analyses in quenched QCD indicate that J/ψ may be stable up to 1.6Tc
  - Lattice Q-$\overline{Q}$ "potential" appears to be very strong between 1 and 2 Tc

- We would like to use the potential model to check the lattice spectral function analyses

- We would like to inquire whether light quarkonia may be stable in QGP to provide new insights into
  - the recombination of QGP constituents
  - the equation of state of QGP
  - the nature of phase transition
  - the nature of chemical equilibrium and chemical yields

Lattice gauge spectral analyses in the quenched approximation show that the width of J/ψ remains narrow up to $T \leq 1.6\ T_C$





M. Asakawa, T. Hatsuda, and Y. Nakahara, Nucl. Phys. A715, 863 (03)

S. Datta, F. Karsch, P. Petreczky, and I. Wetzorke, Phys. Rev. D69,094507(04)

The drastic change of the spectral function suggests the occurrence of spontaneous dissociation between 1.62 and 1.70Tc.

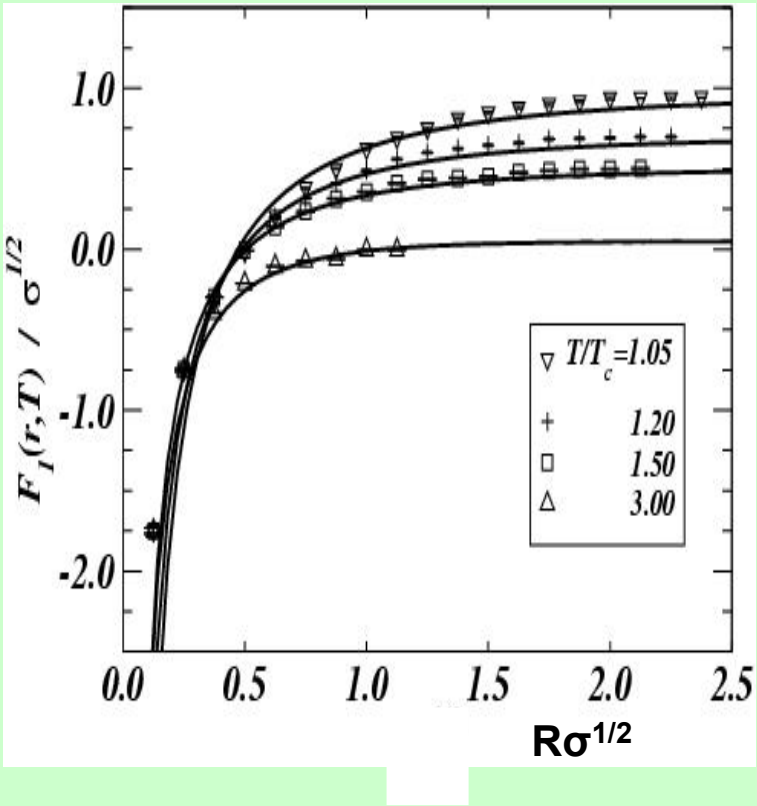
Kaczmarek et al. calculated the color-singlet F_1 and U_1 in the quenched approximation [hep-lat/0309121]

$$\left\langle \mathrm{Tr}L(R)L^{+}(0)\right\rangle = e^{-F_1(R,T)/T}$$

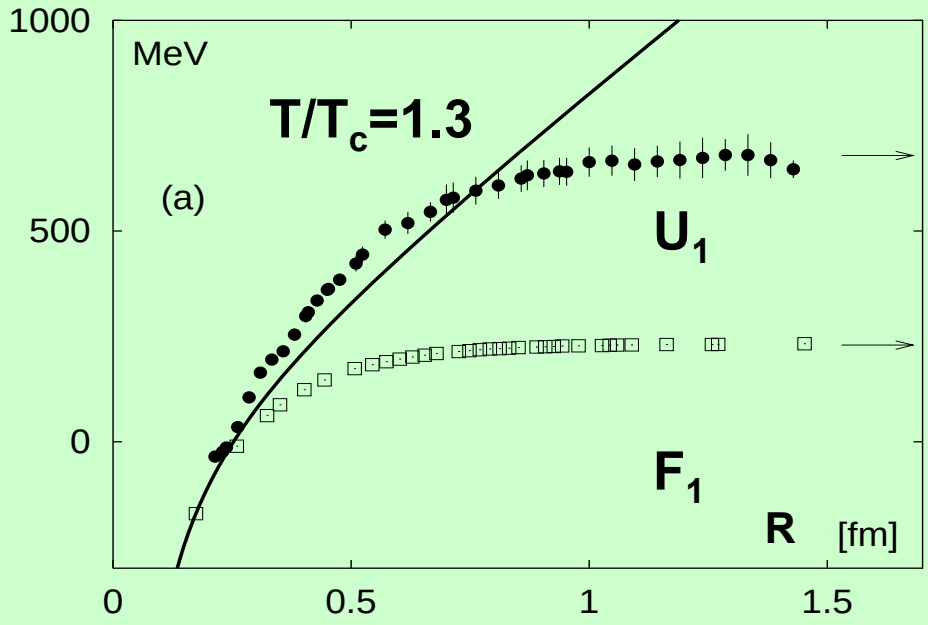
$$U_1(R,T) = F_1(R,T) - T\frac{\partial F_1(R,T)}{\partial T}$$

$$U_1(R,T) = F_1(R,T) + T\,S_1(R,T)$$

$F_1(r,T)$ was calculated in the Coulomb gauge



U_1 is much deeper and broader than F_1 and can hold many more bound states



O. Kaczmarek et al. hep-lat/0506019

Using U_1 as the potential, Shuryak, Zahed, Brown, Lee, and Rho suggested that even light quarkonia may be bound in quark-gluon plasma

What is the Q-$\overline{Q}$ potential?

As ansatzes:

1. $F_1(r,T)$, the free energy

(Digal et.al `01, Wong `02, Blaschke `05)

2. $U_1(r,T)=F_1(r,T)+TS_1(r,T)$, the internal energy

(Kaczmarek et al.`02, Shuryak et al `04, Alberico `05)

With theoretical justifications:

3. A linear combination of F_1 and U_1

$$U_{Q\bar{Q}}^{(1)}(R,T)=\frac{3}{3+a(T)}F_1(R,T)+\frac{a(T)}{3+a(T)}U_1(R,T)$$

(Wong , PRC 72, 034906 ` 05)

What is the Q-$\bar{Q}$ interaction $U^{(1)}_{Q\bar{Q}}$ in LGT?

$$e^{-\beta F_1(\mathbf{r},T)} = Z(\mathbf{r},T) \Big/ Z_0(T), \tag{1}$$

$$Z(\mathbf{r},T) = \int [dA] W_{Q\bar{Q}}(A,T,\mathbf{r}), \tag{2}$$

$$Z_0(T) = \int [dA] W_0(A,T), \tag{3}$$

$$W_{Q\bar{Q}}(A,T,\mathbf{r}) = tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T), \tag{4}$$

$$W_0(A,T) = \exp\left\{-\frac{1}{4}\int_0^\beta d\tau \int d^3x F_{\mu\nu}F^{\mu\nu}\right\}, \tag{5}$$

We rewrite Eq. (1) as

$$Z_0(T) = \int [dA] e^{\beta F_1(\mathbf{r},T)} tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\}\exp\left\{-\frac{1}{4}\int_0^\beta d\tau \int d^3x F_{\mu\nu}F^{\mu\nu}\right\}. \tag{6}$$

Taking the derivative of this equation with respect to β, we obtain for the derivative of the left-hand side

$$\frac{\partial\{\text{LHS}\}}{\partial\beta} = \frac{\partial Z_0(T)}{\partial\beta} = \int [dA]\left\{-\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_0(A,T), \tag{7}$$

and for the derivative of the right-hand side, we get

$$\begin{aligned}\frac{\partial\{\text{RHS}\}}{\partial\beta} = \int [dA] e^{\beta F_1(\mathbf{r},T)}\Bigg[&\left\{F_1(\mathbf{r},T) + \beta\frac{\partial F_1(\mathbf{r},T)}{\partial\beta} - \frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_{Q\bar{Q}}(A,T,\mathbf{r}) \\ &+ tr\left\{g(T)(A_0(\frac{\mathbf{r}}{2},T) - A_0(-\frac{\mathbf{r}}{2},T))\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T)\Bigg].\end{aligned} \tag{8}$$

We equate the above Eq. (7) to Eq. (8) and get

$$F_1(\mathbf{r},T) + TS_1(\mathbf{r},T) = U_1(\mathbf{r},T), \tag{9}$$

where $S_1(\mathbf{r},T) = -\partial F_1(\mathbf{r},T)/\partial T$, and $U_1(\mathbf{r},T)$ is the total color-singlet internal energy given explicitly by

$$\boxed{U_1(\mathbf{r},T) = U^{(1)}_{Q\bar{Q}}(\mathbf{r},T) + U^{(1)}_g(\mathbf{r},T) - U_{g0}(T),} \tag{10}$$

$$U_{g0}(T) = \int [dA]\left\{\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_0(A,T) \Big/ \int [dA] W_0(A,T), \tag{11}$$

$$U^{(1)}_g(\mathbf{r},T) = \int [dA]\left\{\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_{QQ}(A,T,\mathbf{r}) \Big/ \int [dA] W_{QQ}(A,T,\mathbf{r}), \tag{12}$$

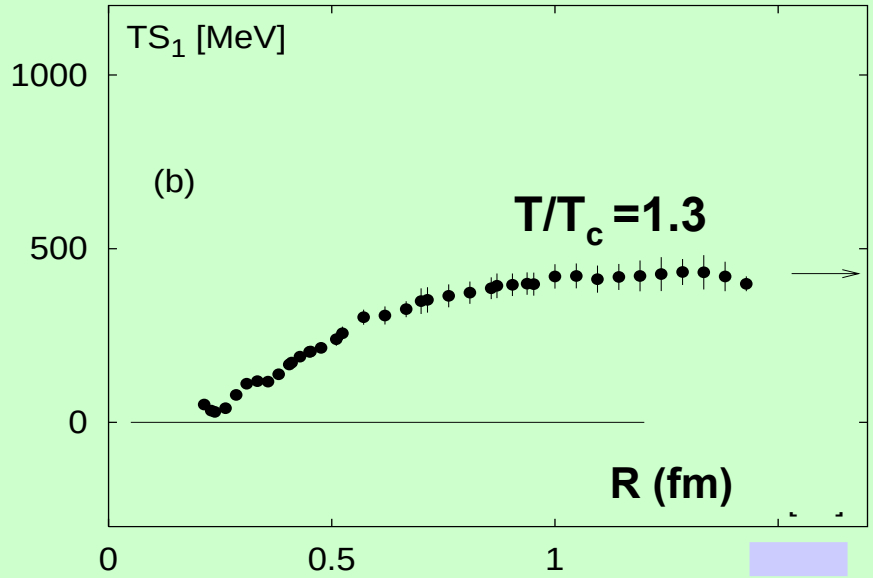
and

$$\begin{aligned}U^{(1)}_{Q\bar{Q}}(\mathbf{r},T) &= \int [dA] tr\left\{g(T)[A_0(\frac{\mathbf{r}}{2},T) - A_0(-\frac{\mathbf{r}}{2},T)]\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T) \\ &\div \int [dA] tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T),\end{aligned} \tag{13}$$

C.Y.Wong,PRC72,034906 ('05)

To extract $Q - \overline{Q}$ potential from lattice gauge F_1 and U_1?

- We need to understand the meaning of U_1.
- We need to understand the meaning of $TS_1 = U_1 - F_1$.



O. Kaczmarek et al. hep-lat/0506019

Why such behavior?

Implications of $S_1(R)$ increases as R increases

We have a system of Q, $\overline{Q}$, and gluons

$$S_1(R) = S_{Q\overline{Q}}(R) + S_{gluon}(R)$$

But, Q and $\overline{Q}$ are held fixed, so $S_{Q\overline{Q}}(R) = 0$.

Therefore, $S_1(R) = S_{gluon}(R)$.

$S_1(R)$ increases as R increases

$\Rightarrow$ $S_{gluon}(R)$ increases as R increases

$\Rightarrow$ Questions :

N_{gluon}, number of gluons, increases as R increases?

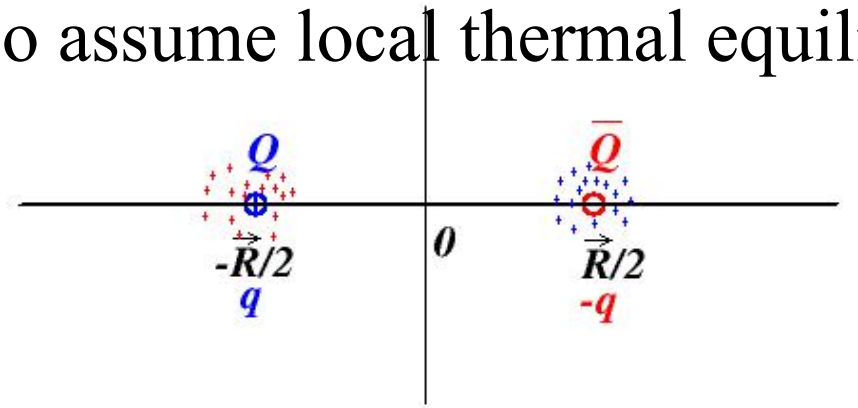
U_{gluon}, gluon internal energy, increases as R increases?

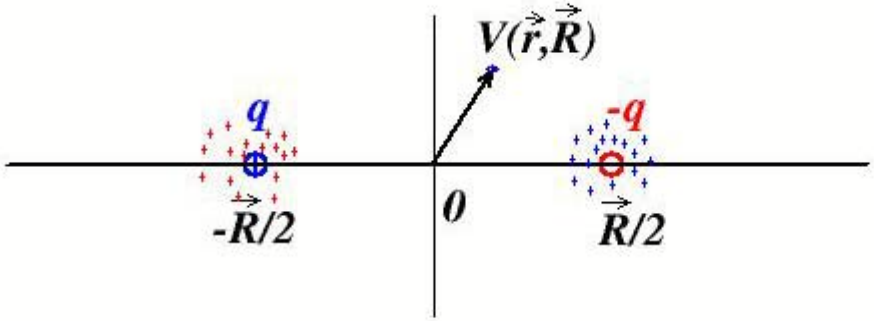
Analogous model of Debye screening

- Q (charge $+q$) at $-\mathbf{R}/2$
- $\overline{Q}$ (charge $-q$) at $\mathbf{R}/2$
- medium: e_+ (charge $+q$), e_- (charge $-q$)
- particles interact with an $e_1 e_2/r$ interaction

We assume e_+ and e_- are massless and are fermions.

We also assume local thermal equilibrium





The potential at $\vec{r}$ is $V(\vec{r},\vec{R})$, due to Q, $\overline{Q}$, and the charge medium.

Under the potential $V(\vec{r},\vec{R})$ at $\vec{r}$ and local thermal equlibrium

$$f_{\pm}(\vec{r},\vec{p},\vec{R}) = 1 / \left\{ \exp[\,(p + e_{\pm}V(\vec{r},\vec{R}))/T\,] + 1 \right\}$$

$$n_{\pm}(\vec{r},\vec{R}) = (g_{\sigma}/2\pi^2)\int p^2 dp\; f_{\pm}(\vec{r},\vec{p},\vec{R})$$

$$= n_{\pm}^{0}\left\{1 - a_1 e_{\pm}V(\vec{r},\vec{R})/T + a_2[e_{\pm}V(\vec{r},\vec{R})/T]^2\right\}$$

$$N_{+}(R) = \int d\vec{r}\left(n_{\pm}(\vec{r},\vec{R}) - n_{\pm}^{0}\right) = n_{\pm}^{0} a_2 \int d\vec{r}\,[e_{\pm}V(\vec{r},\vec{R})/T]^2$$

$$\sigma_{\pm}(\vec{r},\vec{R}) = \frac{g_{\sigma}}{2\pi^2}\int p^2 dp\left\{-f_{\pm}\ln f_{\pm} - (1 - f_{\pm})\ln(1 - f_{\pm})\right\}$$

$$= \sigma_{\pm}^{0}\left\{1 - b_1 e_{\pm}V(\vec{r},\vec{R})/T + b_2[e_{\pm}V(\vec{r},\vec{R})/T]^2\right\}$$

$$TS_{+}(R) = T\int d\vec{r}\left(\sigma_{\pm}(\vec{r},\vec{R}) - \sigma_{\pm}^{0}\right) = T\sigma_{\pm}^{0} b_2 \int d\vec{r}\,[e_{\pm}V(\vec{r},\vec{R})/T]^2$$

Need $V(\vec{r},\vec{R})$ to obtain $n_{\pm}(\vec{r},\vec{R})$, $\sigma_{\pm}(\vec{r},\vec{R})$

But, how to determine $V(\vec{r},\vec{R})$?

Charge density :

$$\rho(\vec{r},\vec{R}) = q\delta(\vec{r}+\vec{R}/2) + (-q)\delta(\vec{r}-\vec{R}/2) + qn_{+}(\vec{r},\vec{R}) + (-q)n_{-}(\vec{r},\vec{R})$$

$$qn_{+}(\vec{r},\vec{R}) + (-q)n_{-}(\vec{r},\vec{R}) = \underbrace{0}_{\text{lowest order}} + \underbrace{n_0 a_1 q^2 V(\vec{r},\vec{R})/T}_{\text{1st order}} + \underbrace{0}_{\text{2nd order}}$$

Poisson equation :

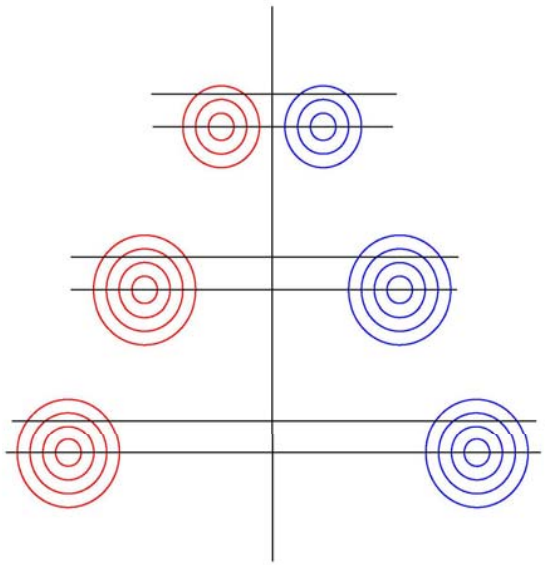
$$\nabla^2 V(\vec{r},\vec{R}) = -4\pi\left(q\delta(\vec{r}-\vec{R}/2) + (-q)q\delta(\vec{r}-\vec{R}/2) + n_0 a_1 q^2 V(\vec{r},\vec{R})/T\right)$$

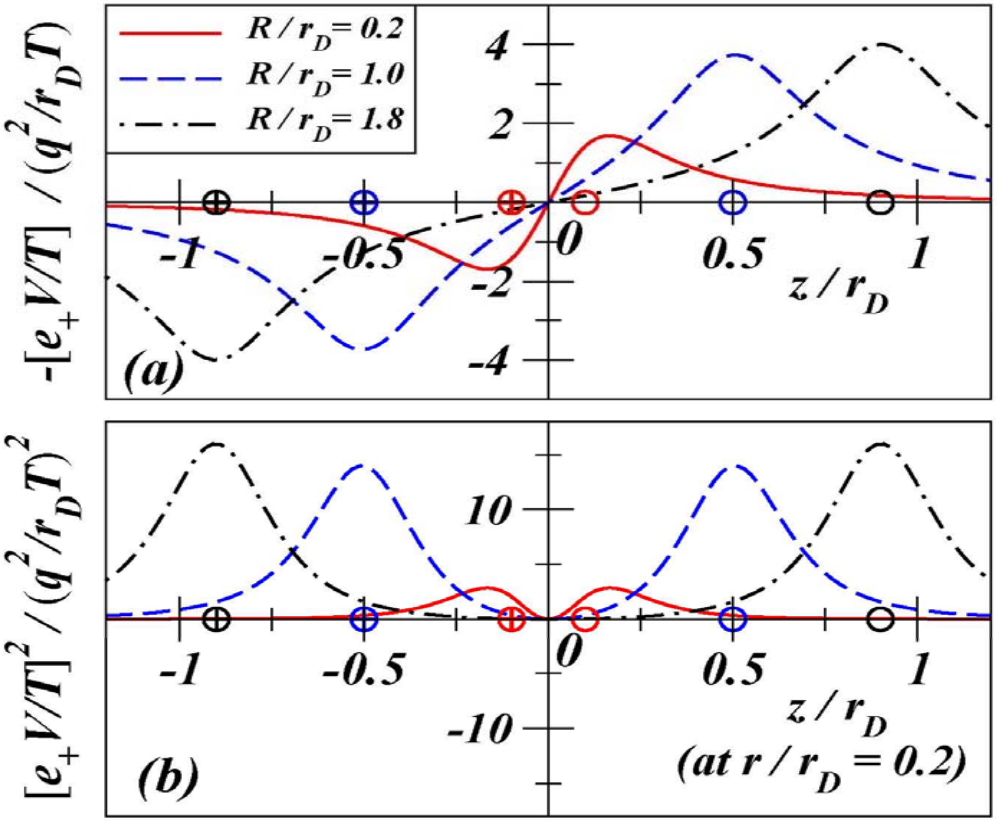
Define $4\pi\, n_0 a_1 q^2 / T = \mu^2 = (\text{Debye mass})^2$

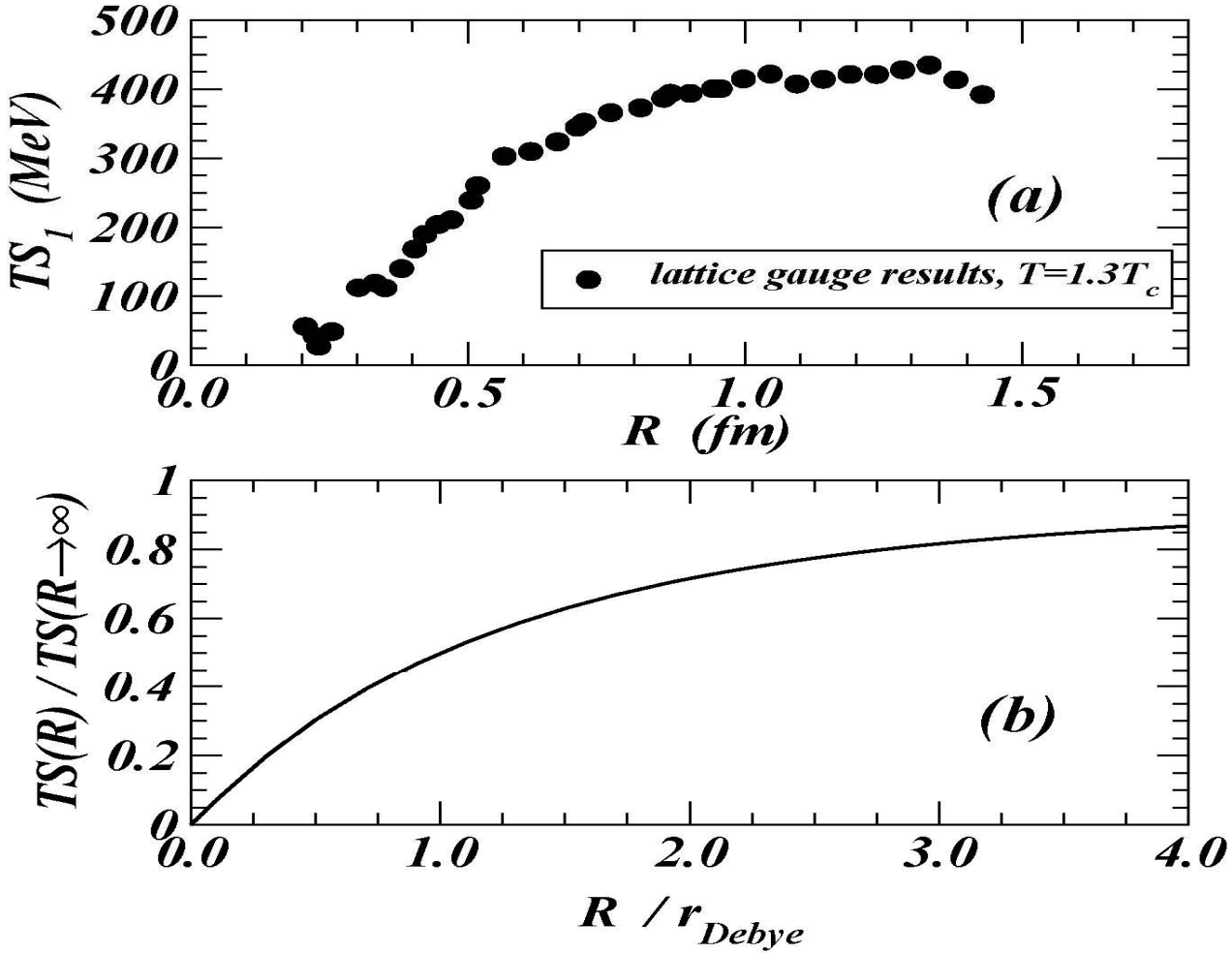
Solution is

$$\boxed{V(\vec{r},\vec{R}) = \frac{q\exp(-\mu r_{+})}{r_{+}} - \frac{q\exp(-\mu r_{-})}{r_{-}} \qquad r_{\pm} = |\vec{r} \pm \vec{R}/2|}$$

$$-e_+V(\vec{r},\vec{R}) = -\frac{q^2\exp(-\mu r_+)}{r_+} + \frac{q^2\exp(-\mu r_-)}{r_-}; \quad r_\pm = |\vec{r} \pm \vec{R}/2|$$







Total medium entropy, particle number, and internal energy increase (and saturate) as R increases.

Schrödinger equation for $Q\overline{Q}$ bound states

Hamiltonian for Q and $\overline{Q}$

$$H = \frac{p_Q^2}{2m_Q} + \frac{p_{\overline{Q}}^2}{2m_{\overline{Q}}} + (\text{all interaction energy, acting on Q and } \overline{\text{Q}})$$

$$= \frac{P_{CM}^2}{2(m_Q + m_{\overline{Q}})} + \frac{p_R^2}{2\mu_{Q\overline{Q}}} + U_{Q\overline{Q}}(R)$$

All interactions acting on Q and $\overline{Q}$ are

$$U_{Q\overline{Q}}(R) = \frac{1}{2}\int \mathrm{d}\vec{r}\left[e_+\delta(\vec{r} + \frac{\vec{R}}{2}) + e_-\delta(\vec{r} - \frac{\vec{R}}{2})\right] V(\vec{r}, \vec{R})$$

$$= \frac{1}{2}\left[e_+ V(\vec{r}, \vec{R})\big|_{\vec{r} = -\frac{\vec{R}}{2}} + e_- V(\vec{r}, \vec{R})\big|_{\vec{r} = \frac{\vec{R}}{2}}\right] \quad \textit{except infinite self-interaction}$$

$$V(\vec{r},\vec{R}) = \frac{q\exp(-\mu r_+)}{r_+} - \frac{q\exp(-\mu r_-)}{r_-}, \quad r_\pm = |\vec{r} \pm \vec{R}/2|$$

Therefore, $$U_{Q\overline{Q}}(R) = -\frac{q^2 e^{-\mu R}}{R} - q^2\mu$$

$$U_{total}(R) = \int d\vec{r}\left[\frac{g}{2\pi^2}\int p^2 dp\,\{f_+(\vec{r},\vec{p},\vec{R}) + f_-(\vec{r},\vec{p},\vec{R})\}\;\; p\right]$$

$$+\frac{1}{2}\int d\vec{r}\left[e_+ n_+(\vec{r},\vec{R}) + e_- n_-(\vec{r},\vec{R})\right] V(\vec{r},\vec{R})$$

$$+\frac{1}{2}\int d\vec{r}\left[e_+\delta(\vec{r}+\frac{\vec{R}}{2}) + e_-\delta(\vec{r}-\frac{\vec{R}}{2})\right] V(\vec{r},\vec{R})$$

$$-\int d\vec{r}\left[u_+^0 + u_-^0\right]$$

$$U_{total}(R) = \int d\vec{r}\left[u_+(\vec{r},\vec{R}) + u_-(\vec{r},\vec{R})\right] + U_{Q\bar{Q}}(R)$$

$$-\int d\vec{r}\left[\;u_+^0 \qquad\qquad + u_-^0 \qquad\qquad\right]$$

$$u_\pm(\vec{r},\vec{R}) = \frac{g}{2\pi^2}\int p^2 dp\;\; f_\pm(\vec{r},\vec{p},\vec{R})\left\{p + \frac{e_\pm V(\vec{r},\vec{R})}{2}\right\}$$

$$= u_\pm^0\left\{1 - c_1 \mathrm{e}_\pm V(\vec{r},\vec{R})/T + c_2[\mathrm{e}_\pm V(\vec{r},\vec{R})/T]^2\right\}$$

$U_{total}(R) = U_{Q\bar{Q}}(R) + U_{medium}(R) - U^0_{medium}$ in Debye screening

cf previous results in lattice QCD, C.Y.Wong, PRC72,0349 06 (05)

$$U_1(R) \;\; = U^{(1)}_{Q\bar{Q}}(R) + U_g(R) - U_{g0}$$

What is the Q-$\bar{Q}$ interaction $U^{(1)}_{Q\bar{Q}}$ in LGT?

$$e^{-\beta F_1(\mathbf{r},T)} = Z(\mathbf{r},T) \Big/ Z_0(T), \tag{1}$$

$$Z(\mathbf{r},T) = \int [dA] W_{Q\bar{Q}}(A,T,\mathbf{r}), \tag{2}$$

$$Z_0(T) = \int [dA] W_0(A,T), \tag{3}$$

$$W_{Q\bar{Q}}(A,T,\mathbf{r}) = tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T), \tag{4}$$

$$W_0(A,T) = \exp\left\{-\frac{1}{4}\int_0^\beta d\tau \int d^3x F_{\mu\nu}F^{\mu\nu}\right\}, \tag{5}$$

We rewrite Eq. (1) as

$$Z_0(T) = \int [dA] e^{\beta F_1(\mathbf{r},T)} tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\}\exp\left\{-\frac{1}{4}\int_0^\beta d\tau \int d^3x F_{\mu\nu}F^{\mu\nu}\right\}. \tag{6}$$

Taking the derivative of this equation with respect to β, we obtain for the derivative of the left-hand side

$$\frac{\partial\{\text{LHS}\}}{\partial\beta} = \frac{\partial Z_0(T)}{\partial\beta} = \int [dA]\left\{-\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_0(A,T), \tag{7}$$

and for the derivative of the right-hand side, we get

$$\begin{aligned}\frac{\partial\{\text{RHS}\}}{\partial\beta} = \int [dA] e^{\beta F_1(\mathbf{r},T)}\Bigg[&\left\{F_1(\mathbf{r},T) + \beta\frac{\partial F_1(\mathbf{r},T)}{\partial\beta} - \frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_{Q\bar{Q}}(A,T,\mathbf{r}) \\ &+ tr\left\{g(T)(A_0(\frac{\mathbf{r}}{2},T) - A_0(-\frac{\mathbf{r}}{2},T))\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T)\Bigg].\end{aligned} \tag{8}$$

We equate the above Eq. (7) to Eq. (8) and get

$$F_1(\mathbf{r},T) + TS_1(\mathbf{r},T) = U_1(\mathbf{r},T), \tag{9}$$

where $S_1(\mathbf{r},T) = -\partial F_1(\mathbf{r},T)/\partial T$, and $U_1(\mathbf{r},T)$ is the total color-singlet internal energy given explicitly by

$$\boxed{U_1(\mathbf{r},T) = U^{(1)}_{Q\bar{Q}}(\mathbf{r},T) + U^{(1)}_g(\mathbf{r},T) - U_{g0}(T),} \tag{10}$$

$$U_{g0}(T) = \int [dA]\left\{\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_0(A,T) \Big/ \int [dA] W_0(A,T), \tag{11}$$

$$U^{(1)}_g(\mathbf{r},T) = \int [dA]\left\{\frac{1}{4}\int d^3x F_{\mu\nu}F^{\mu\nu}\right\} W_{QQ}(A,T,\mathbf{r}) \Big/ \int [dA] W_{QQ}(A,T,\mathbf{r}), \tag{12}$$

and

$$\begin{aligned}U^{(1)}_{Q\bar{Q}}(\mathbf{r},T) &= \int [dA] tr\left\{g(T)[A_0(\frac{\mathbf{r}}{2},T) - A_0(-\frac{\mathbf{r}}{2},T)]\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T) \\ &\div \int [dA] tr\left\{\hat{P}\exp\{\int_0^\beta g(\tau)d\tau A_0(\frac{\mathbf{r}}{2},\tau)\}\exp\{\int_\beta^0 g(\tau)d\tau A_0(-\frac{\mathbf{r}}{2},\tau)\}\right\} W_0(A,T),\end{aligned} \tag{13}$$

C.Y.Wong,PRC72,034906 ('05)

How to subtract $U_g(R)$- U_{g0} from U_1?

The Q - $\bar{Q}$ potential is

$$U^{(1)}_{Q\bar{Q}}(R) = U_1(R) - [U_g(R) - U_{g0}] \text{ in quenched QCD}$$

$$U^{(1)}_{Q\bar{Q}}(R) = U_1(R) - [U_{QGP}(R) - U_{QGP0}] \text{ in full QCD}$$

By the first law of thermodynamics for the medium

$$\varepsilon = T\sigma - p\,; \qquad p = \frac{3p}{\varepsilon}\,\frac{\varepsilon}{3} = a(T)\frac{\varepsilon}{3}$$

information on $a(T) = 3p/\varepsilon$ can be taken from the equation of state

$$\varepsilon = \left[\frac{3}{3+a(T)}\right] T\sigma$$

$$\frac{dU_g}{dV} = \left[\frac{3}{3+a(T)}\right] T(\sigma - \sigma_0 + \sigma_0), \qquad TS_1(R)$$

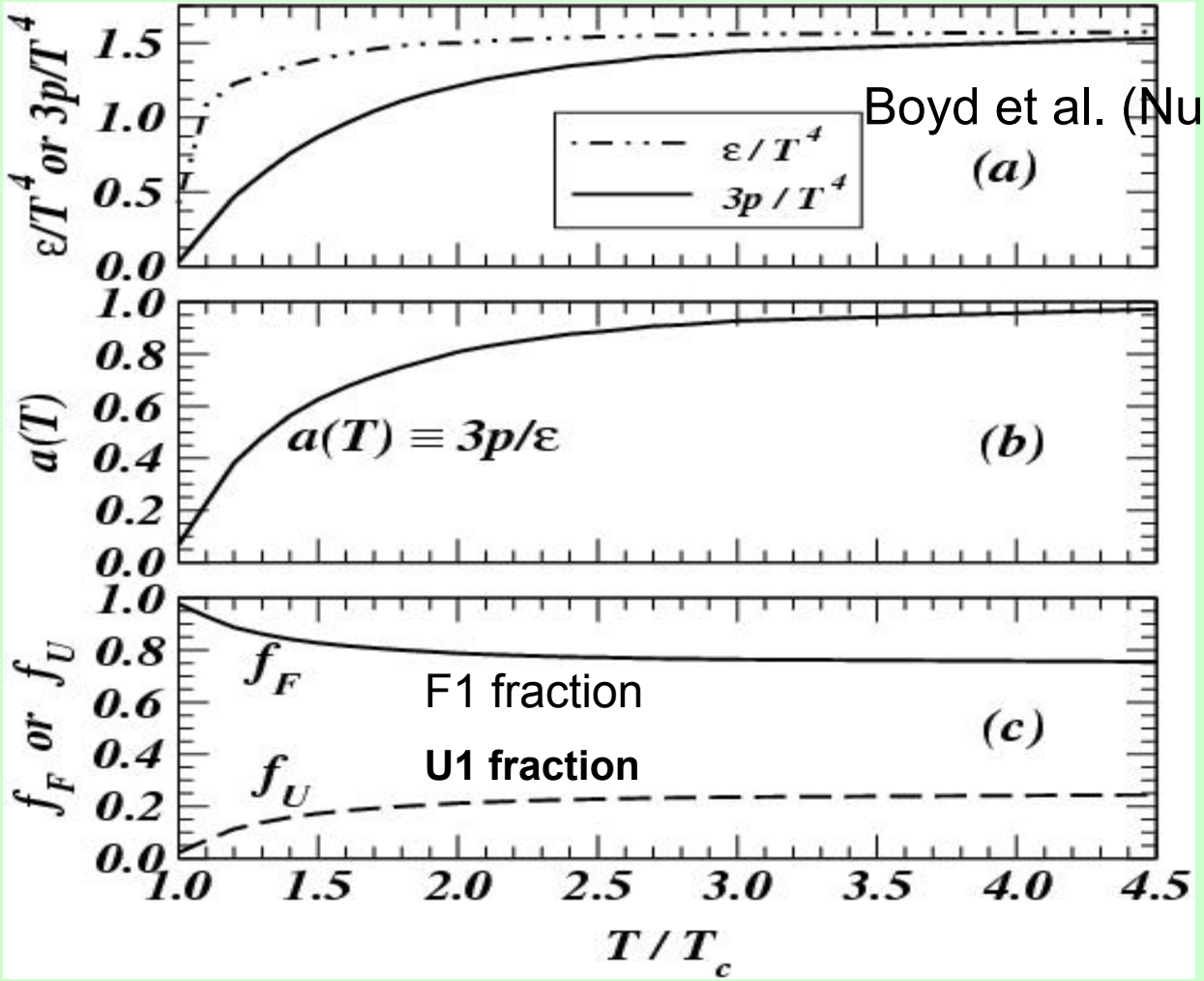
$$U_g(R) = \left[\frac{3}{3+a(T)}\right] \left\{ \boxed{T\int dV(\sigma - \sigma_0)} + T\int dV\sigma_0 \right\}$$

So, $$U_g(R) - U_{g0} = \left[\frac{3}{3+a(T)}\right] TS_g(R) = \left[\frac{3}{3+a(T)}\right] (U_1(R) - F_1(R))$$

Hence, $$U^{(1)}_{Q\bar{Q}}(R) = U_1(R) - [U_g(R) - U_{g0}] = U_1(R) - \left[\frac{3}{3+a(T)}\right](U_1(R) - F_1(R))$$

$$\boxed{U^{(1)}_{Q\bar{Q}}(R) = \frac{3}{3+a(T)} F_1(R) + \frac{a(T)}{3+a(T)} U_1(R)}$$

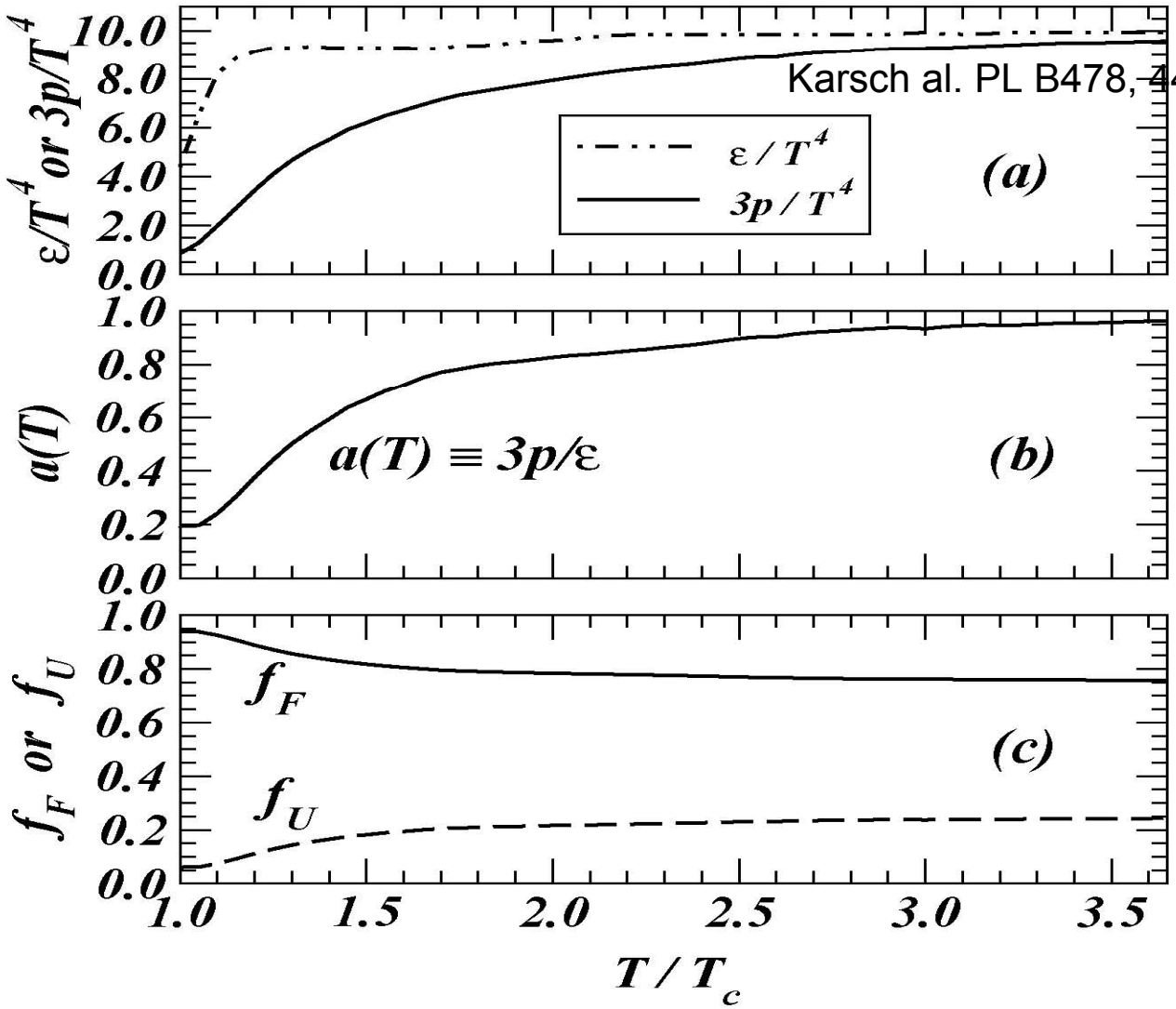
Quenched QCD equation of state



Boyd et al. (Nucl. Phys. B '96)

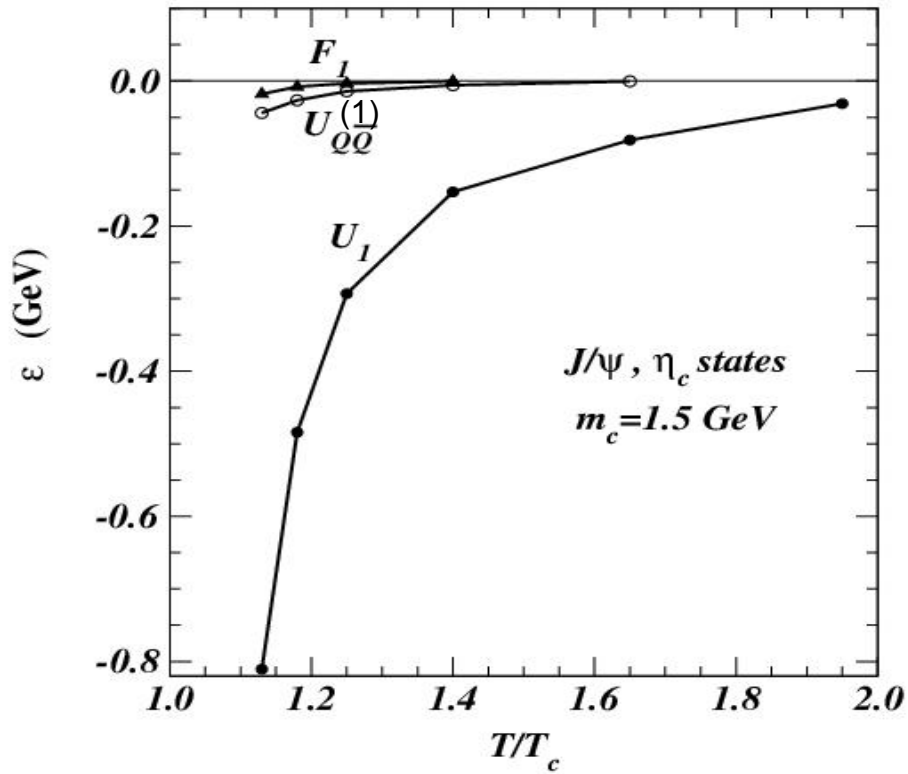
F_1 and U_1 fractions depend on T

Full QCD (2 flavors) equation of state

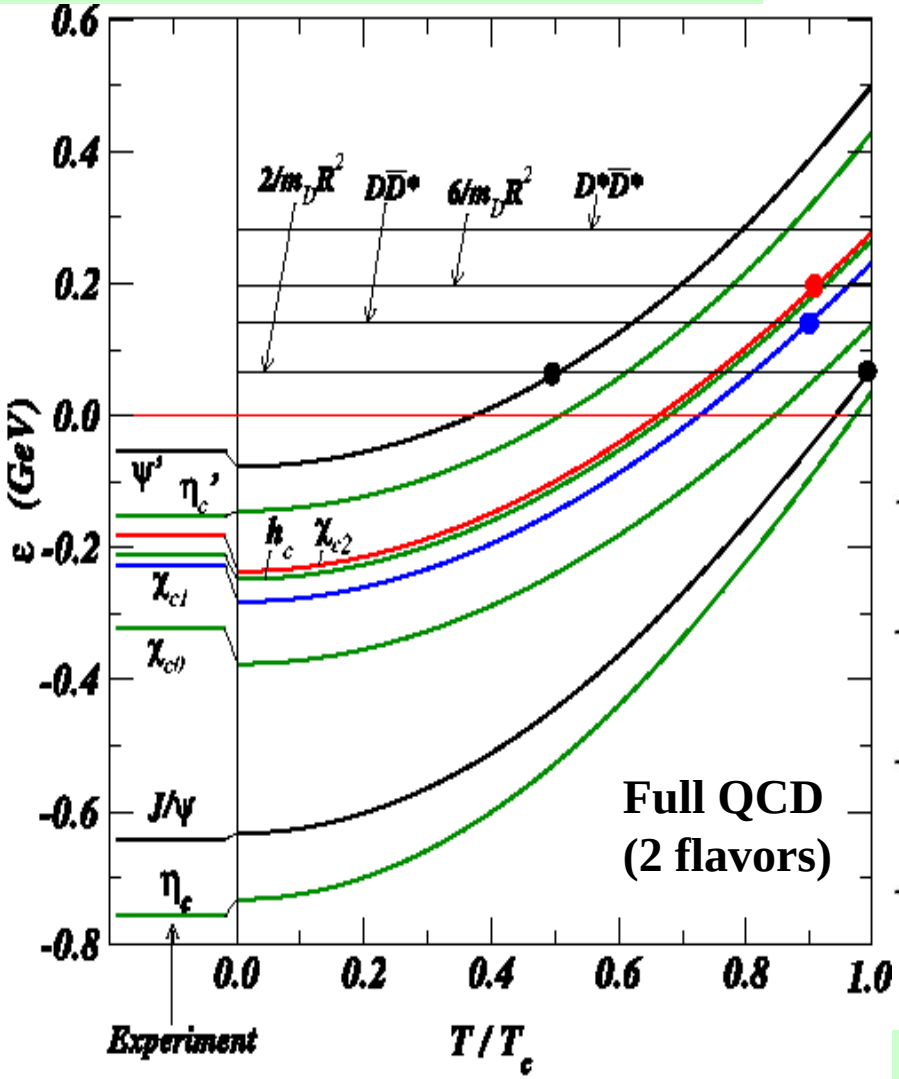


Solve for Q-$\bar{Q}$ bound states

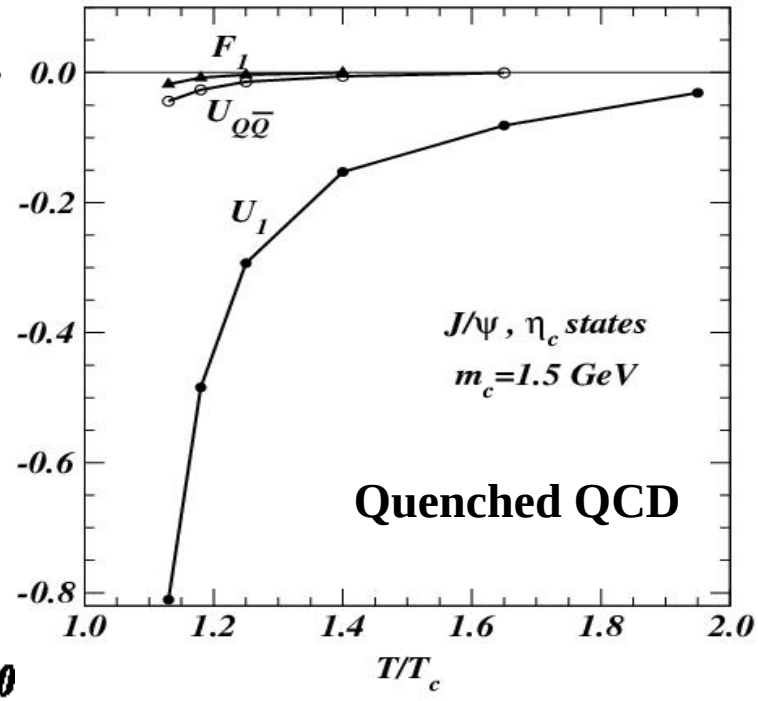
$$\left\{ -\frac{\nabla^2}{2\mu_{red}} + \begin{bmatrix} F_1(r,T) - F_1(r\to\infty,T) \\ U^{(1)}_{Q\bar{Q}}(r,T) - U^{(1)}_{Q\bar{Q}}(r\to\infty,T) \\ U_1(r,T) - U_1(r\to\infty,T) \end{bmatrix} \right\} \psi(r,T) = \varepsilon(T)\psi(r,T)$$



T<Tc



T>Tc



C.Y.Wong, PRC65,034902 (`02)

C.Y.Wong, PRC65,034906 (`05)

Spontaneous dissociation temperatures in quenched QCD

Heavy Quarkonium	Spectral Analysis	$U^{(1)}_{Q\bar{Q}}$ Potential	F_1 Potential	U_1 Potential
J/ψ	$1.62\text{-}1.70T_C$	$1.62T_C$	$1.40T_C$	$2.60T_C$
χ_c , ψ'	0.7 to 1.1 T_C	unbound	unbound	$1.18T_C$
Υ	above $3T_c$	$4.10T_C$	$3.50\ T_C$	$\sim 5.0\ T_C$
χ_b	$1.15\text{-}1.54\ T_C$	$1.19T_C$	$1.10T_C$	$1.73T_C$

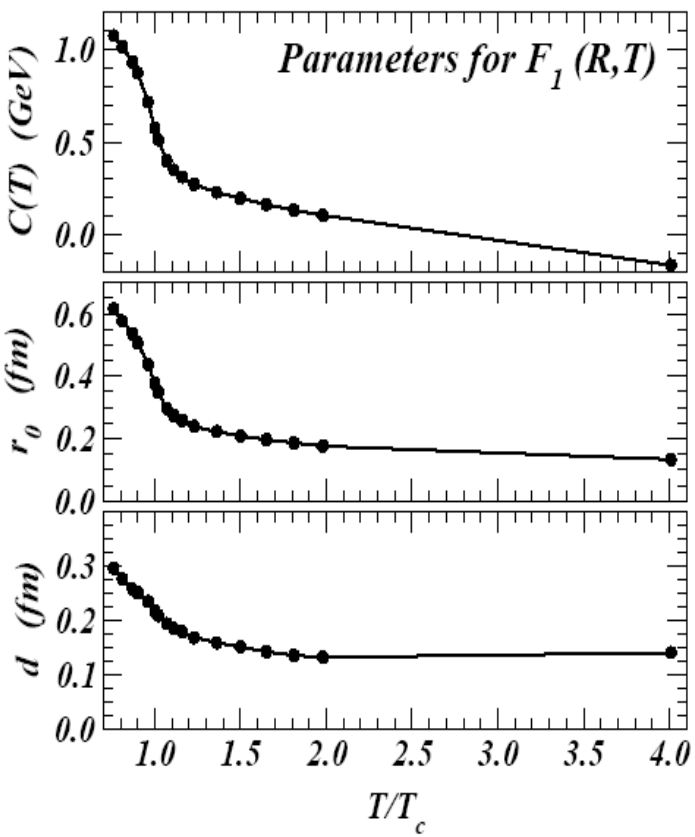
Quenched QCD & Full QCD

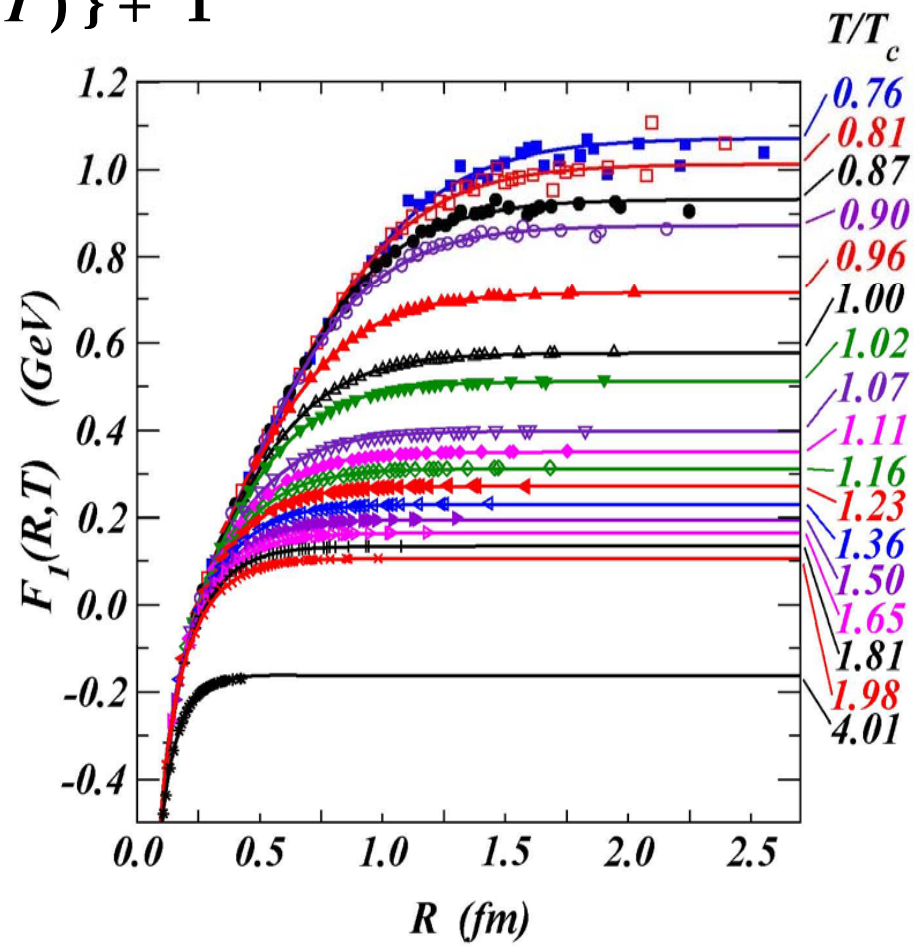
- Quenched QCD is inadequate as it neglects the effects of dynamical quarks
- Dynamical quarks provide additional screening
- F_1 and U_1 for full QCD (with 2 flavors) have been obtained by Kaczmarek et al. ´05
- The equation of state for full QCD (with 2 flavors) has been obtained by Karsch et al. ´02.

Full QCD with two flavors

$$F_1(R,T) = -\frac{4}{3}\frac{\alpha_s}{R} f(R,T) + C(T)[1 - f(R,T)], \qquad \alpha_s = 0.30,$$

$$f(R,T) = \frac{1}{\exp\{(R - r_0(T))/d(T)\} + 1}$$

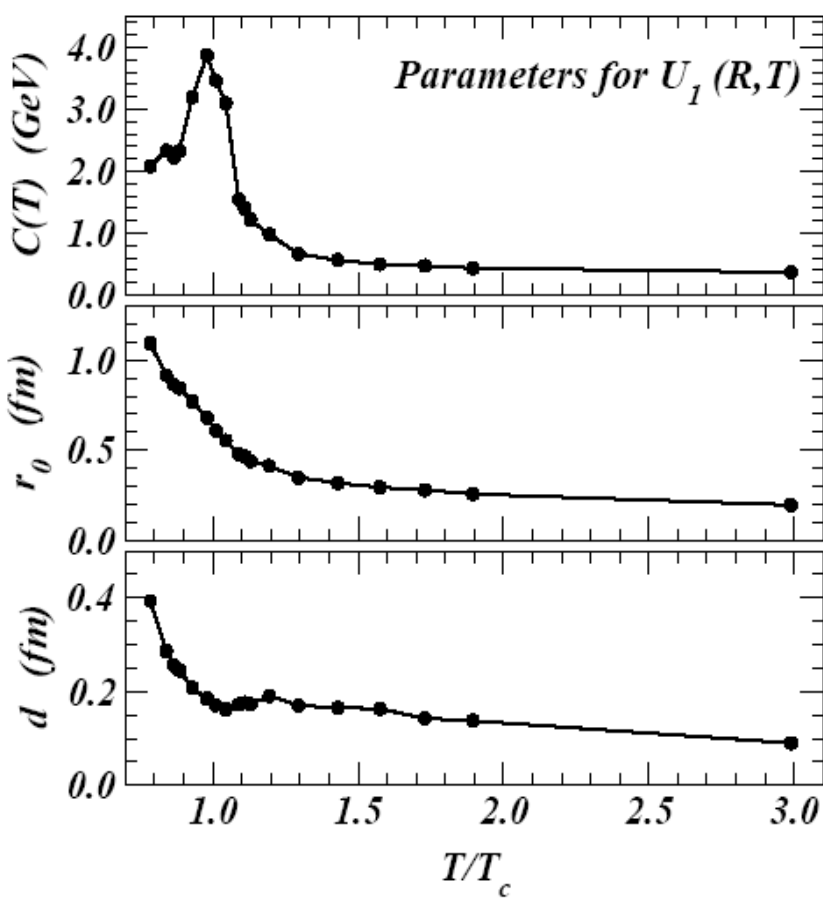


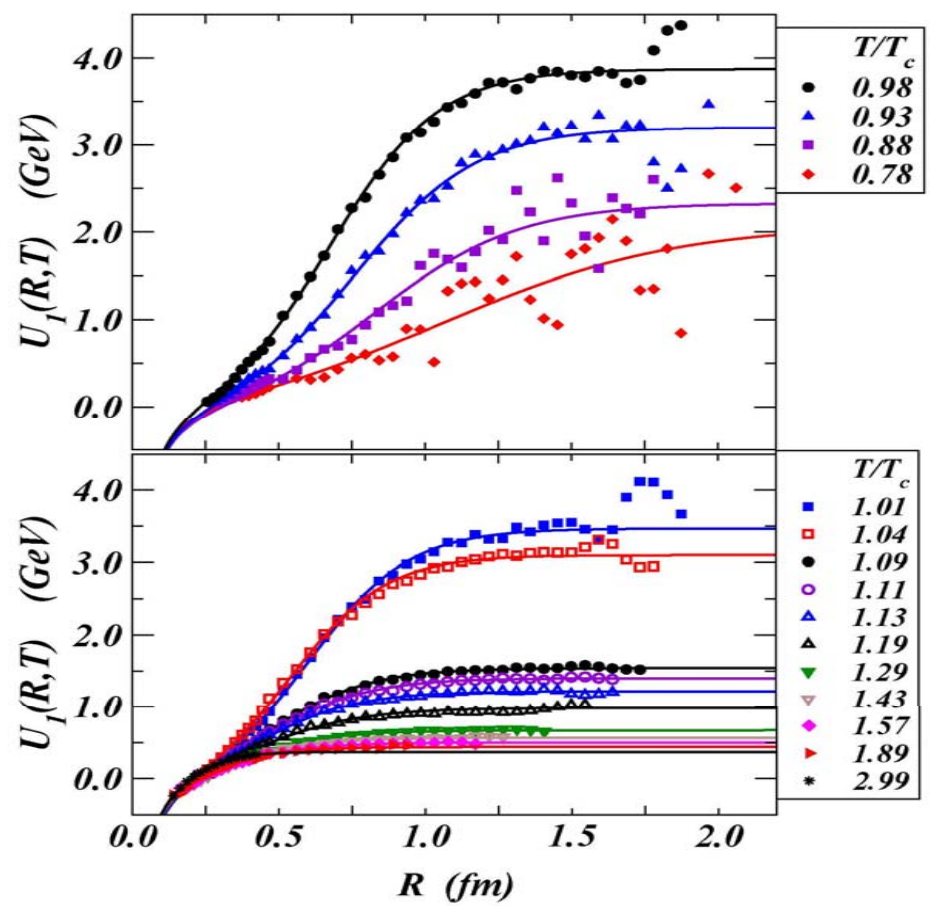


Full QCD with two flavors

$$U_1(R,T) = -\frac{4}{3}\frac{\alpha_s}{R} f(R,T) + C(T)[1 - f(R,T)], \qquad \alpha_s = 0.3,$$

$$f(R,T) = \frac{1}{\exp\{(R - r_0(T))/d(T)\} + 1}$$





Spontaneous dissociation temperatures in quenched QCD & full QCD

Heavy Quarkonium	Quenched QCD $U^{(1)}_{Q\bar{Q}}$	2-flavor QCD $U^{(1)}_{Q\bar{Q}}$
J/ψ	$1.62T_C$	$1.42T_C$
χ_c , ψ'	unbound	unbound
Υ	$4.10T_C$	$3.30\ T_C$
χ_b	$1.18T_C$	$1.22T_C$

Quarkonia in quark-gluon plasma

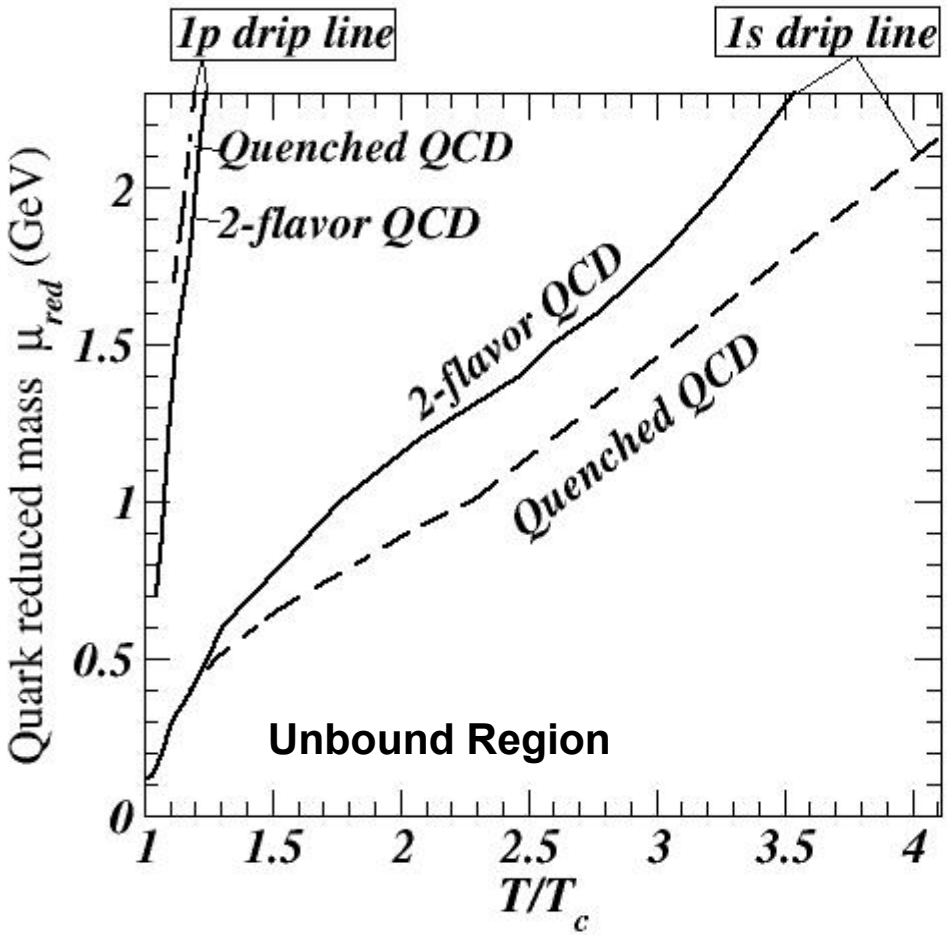
We can treat the quark mass as a variable and obtain the binding energies of a quarkonium as a function of the reduced mass μ_{red} and temperature T.

Stability represented by 'quark drip lines'

- In the (T, quark reduced mass μ_{red}) space, the quark drip line is the line below which a quarkonium is unbound.

- There are 1s drip line, and 1p drip line, etc,…

Quark drip lines in quark-gluon plasma

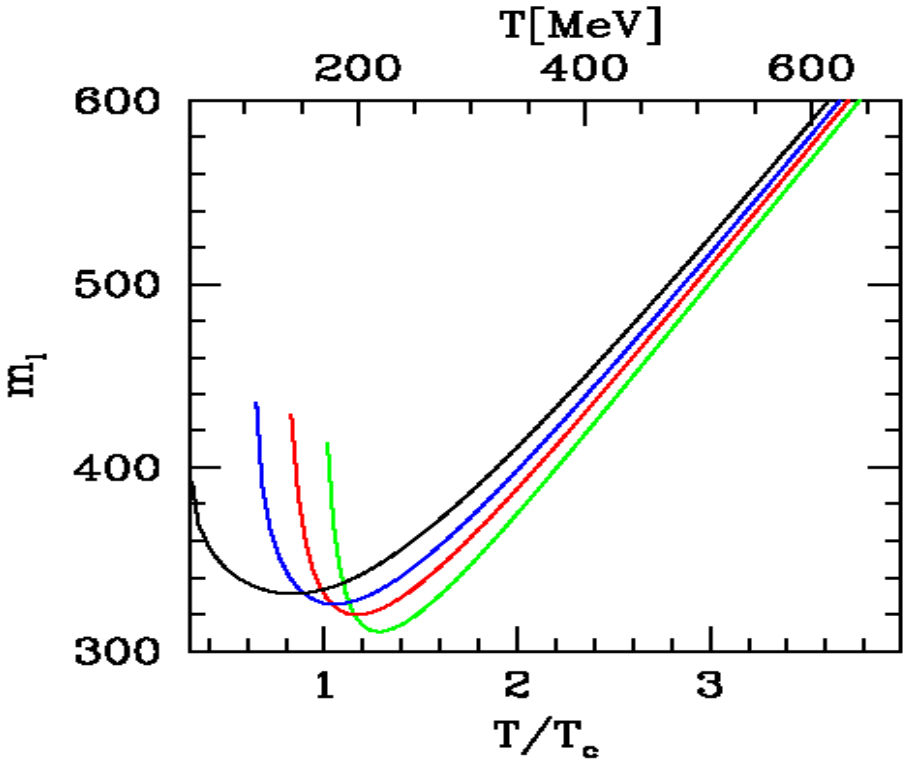


Use quark drip lines to determine the stability of light quarkonia

- Because of the strong interaction, light quarks become quasiparticles and they acquire masses
- These masses can be estimated by studying the equation of state (Levai et al. `98, Szabo et al.`02, Iavanov et al.`05).
- They found that the quasi-particle masses of u, d, and s quarks are

$$m_q \sim 0.3\text{-}0.4 \text{ GeV} \quad \text{for} \quad Tc<T<2Tc.$$

Light quark quarkonia



Szabo et al. JHEP 0306, 008 (`03)
Results from Levai et al `98
and Ivanov et al `05 are similar.

For light quarks with a mass of 300-400 MeV, the quark drip line shows that quarkonia with light quarks can be stable up to 1.06Tc.

Conclusions

- The potential model is consistent with the spectral function analysis, if the Q-$\bar{Q}$ potential is a linear combination of F_1 and U_1,with coefficients that depend on the equation of state.

- The effects of the dynamical quarks modify J/ψ stability only slightly. In quenched QCD, J/ψ dissociates spontaneously at 1.62 T_c. In full QCD with 2 flavors, J/ψ dissociates spontaneously at 1.42 T_c ..

- Light quarkonium states may be stable up to ~1.06 T_c.

The stability of these light quarkonia near T_c provides support to the recombination model at temperatures just above T_c.

Approach to chemical equilibration may start at the end of the QGP phase before the phase transition.