Jet Quenching and Energy Loss: Radiative, Collisional and Dissociative

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Adil, Gyulassy, Horowitz & Wicks : PRC 75, 044906 (2007)
Adil & Vitev : PLB 649, 139-146 (2007)
Wicks & Gyulassy : In Preparation
Jet Quenching at RHIC - I

Light Quarks

Jet Quenching in A-A studied primarily using the nuclear modification factor.

\[ R_{AA} = \frac{dN_{AA}^h}{T_{AA} d\sigma_{pp}^h} \]

Vitev, arXiv:nucl-th/0603010

Note that only radiative energy loss used with fixed path length.
Jet Quenching at RHIC - II
Heavy Quarks

- Electrons from B/D decays
- Radiative Energy Loss using DGLV (both c + b)
- Radiative + Collisional + Geometry (both c + b)
- Radiative + Collisional + Geometry (only c)
- Other approaches use “fragility” and large $\hat{q}$ (AWS)

Wicks et al. arXiv:nucl-th/0512076
Adding Collisional à la WHDG

Pros

- Elastic non negligible for RHIC Physics (Mustafa etc.)
- Elastic “causes” radiative energy loss, include for self consistency
- Different energy loss mechanisms have different dependence on density, energy, path length

Cons

- Neglect finite time corrections (Djordjevic, Peigne et al., Adil et al.)
- Use simplified diffusion to model collisional energy loss fluctuations, not valid for small number of collisions
Finite Time Effects
Calculations for Collisional Energy Loss

QFT 2 -> 2

• Energy Loss as an integral over elastic collision cross-section

\[ \frac{dE}{dx} \propto \int_{t_{min}}^{t_{max}} dt \frac{d\sigma}{dt} (E_f - E_i) \]

• Main Calculations
  – Bjorken, 1982 (First estimate)
  – Braaten & Thoma, 1991 (screening and 2 -> 2 processes at high energy)

Linear Response

• Energy Loss as Joule Work done by ‘induced’ electric field on particle

\[ \frac{dE}{dx} \propto \vec{E}_{ind} \cdot \vec{j} \]

• Main Calculations
  – Thoma & Gyulassy, 1991 (First consistent inclusion of screening)
Energy Loss Calculation

- Use current of one particle traveling at \((E, p)\)

\[
j^{\mu a}_{\infty, (1)}(x, t) = q^a v^\mu \delta(x - v t)
\]

\[
j^{\mu a}_{\infty, (1)}(\omega, k) = 2\pi q^a v^\mu \delta(\omega - k \cdot v).
\]

- Energy Loss is

\[
\frac{dE}{dx} = -i \frac{C_F \alpha_s}{v^2 \pi^2} \int \frac{dk}{k^2} \frac{k \cdot v}{k^2} \left( \frac{1}{\epsilon_\parallel} - \frac{k^2 v^2 - (k \cdot v)^2}{k^2 - (k \cdot v)^2 \epsilon_\perp} \right)
\]

Integral can be done analytically in Leading Logarithm approx.

\[
\frac{dE}{dx} = -\frac{C_F \alpha_s}{2} \frac{m_D^2}{m_D^2} \log \left( \frac{k_{max}}{m_D} \right) f(v)
\]

\[
f(v) = \frac{1}{v^2} \left( v - \frac{1}{2} (1 - v^2) \log \left( \frac{1 + v}{1 - v} \right) \right)
\]

Note that all previous results are presented in leading logarithm approximation.
But Wait…

- The current used in the previous calculation is “infinite time”, i.e. forever live in a plasma of infinite spacetime extent.

- Experimental jets are produced at finite time inside the medium, say at $t = 0$.

- There must be “retardation” effects, a time delay to set up the induced field.
Accounting for the Retardation

QFT 2 -> 2


\[ H_{int} = -ig\Theta(t - t_0)\Theta(L/v - (t - t_0)) \]
\[ \times \int d^3x \bar{\psi}\gamma^\mu A_\mu \psi \]
- Interaction Hamiltonian adjusted to give interaction only for a finite time

Linear Response

- Work done by S. Peigne et al., 2005.

\[ j^{\mu \alpha}_{(2)}(x, t) = q^\alpha (v_1^\mu \delta(x - v_1 t) - v_2^\mu \delta(x - v_2 t))\Theta(t). \]

\[ j^{\mu \alpha}_{(2)}(\omega, k) = iq^\alpha \left( \frac{v_1^\mu}{\omega - k \cdot v_1 + i\eta} - \frac{v_2^\mu}{\omega - k \cdot v_2 + i\eta} \right) \]
- The current used is adjusted in order to take the into account that the particle is not created at \( t = -\infty \)
- \( V_2 = 0 \) for ease of calculation
PROBLEM!!!!

Significant Difference in the calculations for the retardation length

- Peigne et al. ‘05 include
  - creation radiation
  - 2 particle binding
Removing oranges from apples

\[ \Delta E \sim j_1^*(\omega, k) D_R(\omega, k)(j_1(\omega, k) + j_2(\omega, k)) \]

- \( D_R \) propagates modes with a radiative dispersion relation, need to be removed
- Terms that go like \( j_1^* D_R j_2 \) are obviously due to interactions between particles as opposed to energy loss due to a plasma
The Binding Effect

- Look at finite time current

\[ j_{(2)}^{\mu a}(\omega, k) = i q^a \left( \frac{\nu_1^\mu}{\omega - k \cdot v_1 + i\eta} - \frac{\nu_2^\mu}{\omega - k \cdot v_2 + i\eta} \right) \]

- Schematically write energy loss as

\[ j_1^*(D_{med}^R - D_{vac}^R)(j_1(\omega, k) + j_2(\omega, k)) \]

- The remaining answer is much closer to infinite time
- Peigne previous large suppression is due to this binding with \( v_2 = 0 \)

Note that black line still includes radiative effects
The Result

Collisional Energy Loss after Retardation ~ Infinite Case
Collisional Fluctuations
The Formalism

Similar treatment to:
Arnold, Moore, Yaffe JHEP 0305:051,2003

Massive jet, massless medium

\[
\frac{dN}{d\omega} = \frac{1}{E^2} \frac{1}{\nu (2\pi)^4} \int_{p-\sqrt{(\omega+E)^2-M^2}}^{p+\sqrt{(\omega+E)^2-M^2}} dq \int_{\frac{1}{2} (q+\omega)}^{\infty} dk \int_0^{2\pi} d\phi \times \langle |M|^2 \rangle n(k)(1 \pm n(k'))
\]

\[
\langle |M|^2 \rangle = 4g^4 k_{CF} \left( p^\mu p'^\nu + p'^\mu p^\nu + (M^2 - p.p')g^{\mu\nu} \right) D_{\mu\nu}(q) D_{\mu'\nu'}(q) \left( k^\nu k'^\nu + k'^\nu k^\nu + (m^2 - k.k')g^{\nu\nu'} \right)
\]

Adapted from Wicks, LHC Last Call

\[
\mu_{mag} = \mu_D
\]
The Distributions

Single Collision / Radiation

Poisson Convoluted

Adapted from Wicks, LHC Last Call
The $R_{AA}$

Adapted from Wicks, LHC Last Call
In-Medium Dissociation
Even with the inclusion of collisional energy loss and geometric fluctuations, we still don’t seem to be doing enough.
The Idea of Meson Dissociation

- Assumption has always been free quark energy loss followed by hadronization
  - Good for light quarks but not for heavies
  - Formation time of D meson ~ 1 fm but Pion ~ 10 fm
- Heavy meson tries to form in medium
- Model using coupled rate equations

$$\partial_t f^i(p, t) = -\frac{1}{\langle \tau_{form} \rangle} f^i(p, t) + \int dz \frac{1}{\tau_{diss}(z)} f^j(p, t) \frac{D_{j \rightarrow i}(z)}{z^2}$$

- Dissociation calculated using multiple collision formalism with meson wave functions
**Conceptually Different Approach**

- **Problem:** treated in the same way as light quarks!

\[
\begin{align*}
[p^+, \frac{M_Q^2}{2p^+}, 0] & \rightarrow [zp^+, \frac{k^2 + m_h^2}{2zp^+}, k] \\
& \quad + [(1 - z)p^+, \frac{k^2}{2(1 - z)p^+}, -k]
\end{align*}
\]

\[
\Delta y^+ \approx \frac{1}{\Delta p^-} = \frac{2z(1 - z)p^+}{k^2 + (1 - z)m_h^2 - z(1 - z)M_Q^2}
\]

\[
\tau_f (p_T = 10 \text{ GeV})
\]

\[
\begin{array}{ccc}
\pi & D & B \\
10 \text{ fm} & 1.0 \text{ fm} & 0.35 \text{ fm}
\end{array}
\]

- Fragmentation and dissociation of hadrons from heavy inside the QGP
Slide from Ivan Vitev, QM 2006

**Lightcone Wave Function**

- **D Meson**
  \[ \psi_\Delta(\frac{\Delta k}{\sqrt{x}}, x, m_1, m_2) \]

- **B Meson**
  \[ \psi_\Delta(\frac{\Delta k}{\sqrt{x}}, x, m_1, m_2) \]

- **The momentum distribution**
  \[ \psi(K, \Delta k; x, m_1, m_2) \]
  \[ = \text{Norm}^2 e^{-\frac{\Delta k^2 + 4m_1^2(1-x) + 4m_2^2 x}{4x(1-x)\Lambda^2}} \times \delta^2(K) \]

- **PDFs**
  \[ \phi_{Q/M}(x) = \int d^2\Delta k d^2K |\psi(K, \Delta k; x, m_1, m_2)|^2 \]
  \[ = \text{Norm}^2 4\pi x(1-x)\Lambda^2 e^{-\frac{m_1^2(1-x) + m_2^2 x}{x(1-x)\Lambda^2}} \]

Collisional Dissociation of Mesons

- Direct and virtual interactions

\[ M_{n-1}(K, k) \left[ e^{-q_n \cdot \vec{v}_K} e^{-q_n \cdot \vec{v}_k} \otimes \left( e^{-q_n \cdot \vec{v}_k} e^{-q_n \cdot \vec{v}_k} + e^{+q_n \cdot \vec{v}_k} e^{+q_n \cdot \vec{v}_k} + e^{-q_n \cdot \vec{v}_k} e^{+q_n \cdot \vec{v}_k} + e^{+q_n \cdot \vec{v}_k} e^{-q_n \cdot \vec{v}_k} \right) \right] M_{n-1}(K, k) \]

- Final state wave function

\[ |\psi_f(k, K)|^2 = \int \frac{d^2k}{(2\pi)^2} \frac{d^2K}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{k}} e^{-\chi \mu^2 \xi b^2} \times e^{-i\vec{B} \cdot \vec{K}} e^{-b^2(\chi \mu^2 \xi + x(1-x)\Lambda^2)} \times \frac{e^{-\frac{K^2}{4\chi \mu^2 \xi}}}{4\pi \chi \mu^2 \xi} \frac{K^2}{4\pi(\chi \mu^2 \xi + x(1-x)\Lambda^2)} \]

- Resummed all interactions above (\(L/L\) enhanced)

Slide from Ivan Vitev, QM 2006

Survival and Dissociations Probabilities

Simple two component system:

\[
\begin{align*}
\psi_0 &\equiv \psi_M \\
\psi_f &\equiv a\psi_M + (b)\psi_{Q\bar{q}}
\end{align*}
\]

Meson
Meson + Free Q-\(q\)bar pair

Survival probability:

\[
P_s(\chi, \xi, \mu) = \left| \int d^2k dx \psi^*_f(k)\psi_0(k) \right|^2
\]

\[
= \left| \int dx \text{Norm}^2 4\pi x(1-x)\Lambda^2 e^{-\frac{m_1^2(1-x)+m_2^2 x}{x(1-x)\Lambda^2}} \right|^2
\]

\[
\times \left[ \frac{2\sqrt{x(1-x)\Lambda^2}\sqrt{\chi\mu^2\xi + x(1-x)\Lambda^2}}{\sqrt{x(1-x)\Lambda^2} + \sqrt{\chi\mu^2\xi + x(1-x)\Lambda^2}} \right]^2
\]

Dissociation probability:

\[
P_d(p_T, m_Q, t) = 1 - P_s(p_T, m_Q, t)
\]

Slide from Ivan Vitev, QM 2006
Rate Equations

- Notation and initial conditions

\[
\begin{align*}
    f^Q(p_T, t) &= \frac{d\sigma^Q(t)}{dyd^2p_T}, \quad f^Q(p_T, t = 0) = \frac{d\sigma^Q_{PQCD}}{dyd^2p_T} \\
    f^H(p_T, t) &= \frac{d\sigma^H(t)}{dyd^2p_T}, \quad f^H(p_T, t = 0) = 0,
\end{align*}
\]

\[
\frac{1}{\langle \tau_{\text{diss}}(p_T, t) \rangle} = \frac{\partial}{\partial t} \ln P_d(p_T, m_Q, t)
\]

\[
\langle \tau_{\text{form}}(p_T, t) \rangle = \sum_i \int_0^1 dz D_{H_i/Q}(z) \tau_{\text{form}}(z, p_T, m_Q, t)
\]

- Coupled ordinary differential equations

\[
\begin{align*}
    \partial_t f^Q(p_T, t) &= -\frac{1}{\langle \tau_{\text{form}}(p_T, t) \rangle} f^Q(p_T, t) \\
    &\quad + \frac{1}{\langle \tau_{\text{diss}}(p_T/\bar{x}, t) \rangle} \int_0^1 dx \frac{1}{x^2} \phi_{Q/H}(x) f^H(p_T/x, t)
\end{align*}
\]

\[
\begin{align*}
    \partial_t f^H(p_T, t) &= -\frac{1}{\langle \tau_{\text{diss}}(p_T, t) \rangle} f^H(p_T, t) \\
    &\quad + \frac{1}{\langle \tau_{\text{form}}(p_T/\bar{z}, t) \rangle} \int_0^1 dz \frac{1}{z^2} D_{H/Q}(z) f^Q(p_T/z, t)
\end{align*}
\]

\[
\text{Loss term} \quad \text{Gain term}
\]
Quenching of Non-Photonic Electrons

- **PYTHIA** used to decay all B- and D-mesons / baryons into $(e^+e^-)$

- Suppression $R_{AA}(p_T) \sim 0.25$ is large

- Similar to light $\pi^0$, however, different physics mechanism

- B-mesons are included. They give a major contribution to $(e^+e^-)$

Predictions also made for Cu+Cu (RHIC) and Pb+Pb (LHC)
Conclusions - Finite Time Coll

- Must always be careful and compare apples to apples
  - We did a complete and careful calculation of collisional energy loss
  - After removal of radiative and unrealistic binding effects two different calculations give very similar results making collisional energy loss a viable component

- The binding effect is real (but not simple singlet)
  - Jets are created back to back and their interactions is part of the p-p process and leads to induced energy loss
  - Need to calculate and include binding effect for $v_2 \neq 0$ and a more realistic charge correlation
  - Maybe helpful for $v_2$ at moderate $p_T$?

- Finite time calculation in expanding medium
Conclusion - Collisional Fluctuations

• Fluctuations of all Sorts are Important
  – Geometric fluctuations matter, no fixed lengths
  – Collisional fluctuations are important and non Gaussian.
  – Not a simple drag-diffusion process.

• Significant Remaining Uncertainties
  – Value of $\alpha_s$ / Running (Vitev, Peshier …)
  – Interference b/w Radiative/Collisional, Poisson
Conclusions - Dissociation

• The vacuum fragmentation approximation is not as good for heavy quarks as it is for light quarks
• One can get large energy loss due to repeated dissociation and fragmentation of hadrons
  – Energy loss comes from the peaking of wavefunctions in ‘x’
• Bottom quark quenched at the same level as charm quark
  – This is a unique signature of the process
  – $p_T$ Dependence of Energy Loss also a good discriminator
• Need to implement further details
  – Accompanied partonic energy loss
  – Geometric fluctuations
Acknowledgments

I would like to thank the organizers for inviting me.

I gratefully acknowledge many consultations and collaborations with:

M. Gyulassy, I. Vitev, S. Wicks, W. Horowitz, B. Cole, D. Molnar, M. Djordjevic, A. Mueller
Bonus Slides
Linear Response Formulation

\[
\frac{dE}{dx} = \int \frac{d\omega d\mathbf{k}}{(2\pi)^4} e^{-i(\omega - \mathbf{k} \cdot \mathbf{v})} \frac{q^a}{v} \mathbf{v} \cdot \mathbf{E}^a_{\text{ind}}(\omega, \mathbf{k})
\]

\[\mathbf{E}^a = D_R^\perp(\omega, \mathbf{k})(1 - \hat{k} \mathbf{j})^a + \hat{k} D_R^\parallel(\omega, \mathbf{k}) j^0^a\]

\[\mathbf{E}^a_{\text{vac}} = D_R(\omega, \mathbf{k})(\omega \mathbf{j}^a - \mathbf{k} j^0^a)\]

\[D_R(\omega, \mathbf{k}) = \frac{-4\pi i}{(\omega + i\eta)^2 - \mathbf{k}^2}\]

\[D_R^\perp(\omega, \mathbf{k}) = \frac{4\pi i \omega}{k^2 - (\omega + i\eta)^2 \epsilon_\perp(\omega + i\eta, \mathbf{k})}\]

\[D_R^\parallel(\omega, \mathbf{k}) = \frac{-4\pi i}{k_\parallel(\omega + i\eta, \mathbf{k})}\]

\[\epsilon_\perp(\omega, k) = 1 - \frac{\Pi_\perp(\omega, k)}{\omega^2}\]

\[\epsilon_\parallel(\omega, k) = 1 + \frac{\Pi_\parallel(\omega, k)}{k^2}\]

\[\Pi_\perp(\omega, k) = \frac{1}{2} m_D^2 \left(\frac{\omega}{k}\right)^2 \left(1 - \frac{\omega^2 - k^2}{2\omega k} \log \left(\frac{\omega + k}{\omega - k}\right)\right)\]

\[\Pi_\parallel(\omega, k) = m_D^2 \left(1 - \frac{\omega}{2k} \log \left(\frac{\omega + k}{\omega - k}\right)\right)\]

\[m_D^2 = 4\pi \alpha_s T^2 \left(1 + \frac{N_F}{6}\right)\]
Is the Current Correct?

• The actual current should be one modeling a 2 -> 2 process. (Very complicated!!)
  \[ j^{\mu a} = \Theta(t) \left( q_1^a v_1^{\mu} \delta(x - v_1 t) + q_2^a v_2^{\mu} \delta(x - v_2 t - b) \right) \]
  \[ + \Theta(-t) \left( q_1^a v_1'^{\mu} \delta(x - v_1' t) + q_2^a v_2'^{\mu} \delta(x - v_2' t - b) \right) \]

• Averaging over the initial colors of the incoming partons drops interactions to zero

• Enhanced in Peigne current due to
  – \( v_2 = 0 \)
  – Exactly anti correlated charge
The Binding Effect - I

- First look at infinite time with two particles

\[
\begin{align*}
\hat{j}_{\infty, (2)}^{0a}(\omega, k) &= 2\pi q^a (\delta(\omega - k \cdot v) - \delta(\omega)) \\
\hat{j}_{\infty, (2)}^a(\omega, k) &= 2\pi q^a v \delta(\omega - k \cdot v)
\end{align*}
\]

- Note the suppression in the energy loss

\[
\hat{j}^*(D_{med}^R - D_{vac}^R)(\hat{j}_1(\omega, k) + \hat{j}_2(\omega, k))
\]

- Enhanced due to $v_2 = 0$. 

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Radiative Effects: Vacuum Singularity Structure

\[ W = \int dt dx j^a(x, t) \cdot E^a(x, t) \]
\[ = \int \frac{d\omega dk}{(2\pi)^4} j^a(\omega, k) \cdot E^a(\omega, k) \]
\[ W = \int \frac{d\omega dk}{(2\pi)^4} D_R(\omega, k)(\omega(|j_\perp|^2 + |j_\parallel|^2) - k \cdot j^* j^0) \]

\[ D_R(\omega, k) = PV \left( \frac{-4\pi^2 i}{\omega^2 - k^2} \right) + D_{rad}(\omega, k) \]
\[ D_{rad}(\omega, k) = \frac{-4\pi^2}{2k}(\delta(\omega - k) - \delta(\omega + k)) \]

\[ W_{rad} = -\int \frac{dk}{4\pi^2} \{ |j_\perp|^2 + |j_\parallel|^2 - \frac{1}{2} (j^0* j_\parallel + j_\parallel j^0) \} \]
\[ = -\int \frac{dk}{4\pi^2} |j_\perp|^2 \]

Total Work Done on Current
Work Done by self
Electric Field

Energy Lost to Creation
Radiation Included in our Calculation
Radiative Effects: Medium Singularity Structure

\[ W^\parallel \]
\[ W \]
\[ D^\perp_R(\omega) \]
\[ D^\parallel_R(\omega) \]
\[ W_{\text{rad}}^\parallel \]
\[ W_{\text{rad}} \]

\[ \omega (\text{GeV}) \]
\[ k (\text{GeV}) \]

\[ T = 250 \text{ MeV} \]
\[ N_F = 0 \]
Black \( \omega = k \)
Blue \( \omega = \omega^{\perp}_{\text{pl}}(k) \)
Red \( \omega = \omega^{\parallel}_{\text{pl}}(k) \)

Current

\[ \text{Energy Lost to Creation} \]

Radiation in the medium.
By the way … Ter-Mikaeliyan Effect

- The Radiative Energy Loss enters our calculation as medium - vacuum
- The difference between creation radiation in the medium with respect to the vacuum is the Ter-Mikaeliyan effect
- First studied in QCD by Djordjevic et al. We need to subtract this from our calculations
Subtracting the Radiative Effects

\[ D_R(\omega, k) = D_R(\omega, k) - D_A(\omega, k) + D_A(\omega, k) \]
\[ = D_-(\omega, k) + D_A(\omega, k) \]

\[ D_-(\omega, k) = D_R(\omega, k) - D_A(\omega, k) \]
\[ = -\frac{4\pi^2}{k} (\delta(\omega - k) - \delta(\omega + k)) \]

\[ D^\parallel(\omega, k) = A^\parallel(\omega, k) + 2D^\parallel_{rad}(\omega, k) \]
\[ A^\parallel(\omega, k) = \frac{2\pi^2 m_D^2 \omega^2 (\omega^2 - k^2) \Theta(k + \omega) \Theta(k - \omega)}{k^3(k^2 - \omega^2 + \Pi_\perp(\frac{\omega + im}{k}))(k^2 - \omega^2 + \Pi_\perp(\frac{\omega - im}{k}))} \]
\[ D^\perp(\omega, k) = A^\perp(\omega, k) + 2D^\perp_{rad}(\omega, k) \]
\[ A^\perp(\omega, k) = \frac{-4\pi^2 m_D^2 \omega \Theta(k + \omega) \Theta(k - \omega)}{(k^2 + \Pi_\parallel(\frac{\omega + im}{k}))(k^2 + \Pi_\parallel(\frac{\omega - im}{k}))} \]

Isolates pole structure and one radiative pole can be removed
Advanced Green Function is easily contour integrated over

Predicting LHC

RHIC

LHC

$\pi^0 R_{AA}(p_T)$

$R_{AA}(p_T)$

$dN_g/dy = 1000$

$dN_g/dy = 1750$

PHENIX preliminary

PHENIX

Rad + coll, $\alpha_s = 0.4$

Rad only, $\alpha_s = 0.5$

Rad + coll, $\alpha_s = 0.4$

Rad only, $\alpha_s = 0.5$

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Predicting LHC

RHIC

LHC

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Predicting - LHC

\[ \alpha_s = 0.4 \]

- Charm $dN_g/dy = 1750$
- Bottom $dN_g/dy = 1750$
- Charm $dN_g/dy = 2900$
- Bottom $dN_g/dy = 2900$

\[ R_{AA}(p_T) \]

\[ p_T (\text{GeV}) \]

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D- and B-meson Suppression

- Note the different $p_T$ dependence of B versus D quenching

- **Effective energy loss** (depends on the slope of the spectra)

- **Critical**: finite $\tau_0 = 0.6$ fm formation time to develop density and momentum transfers

- Predictions for Cu+Cu and the LHC

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Slide from Ivan Vitev, QM 2006