TMDs and the $\perp$ Spin Structure of Hadrons

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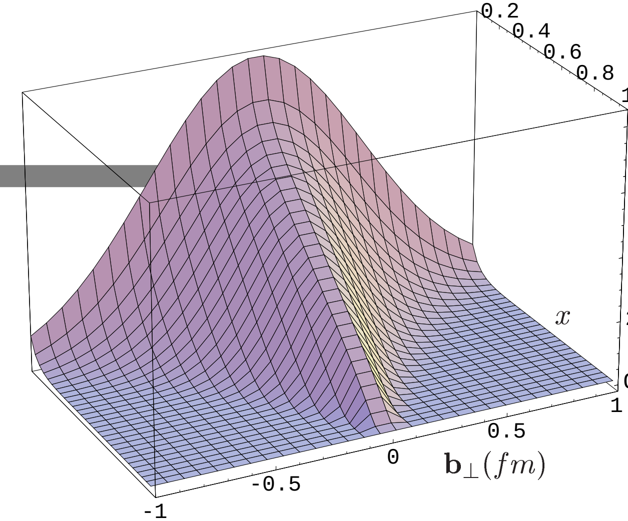
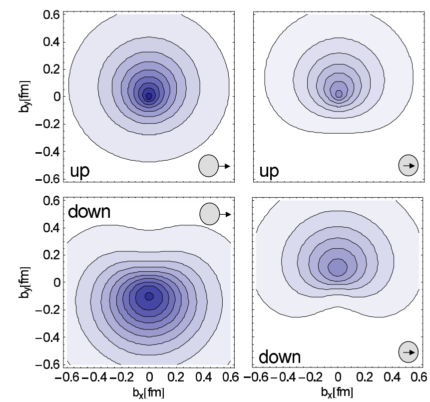
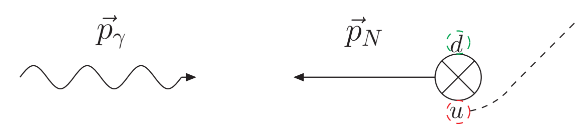
Outline



- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized



- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)



$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow\ \perp \text{SSA in}\ \ \gamma N \longrightarrow \pi + X$$

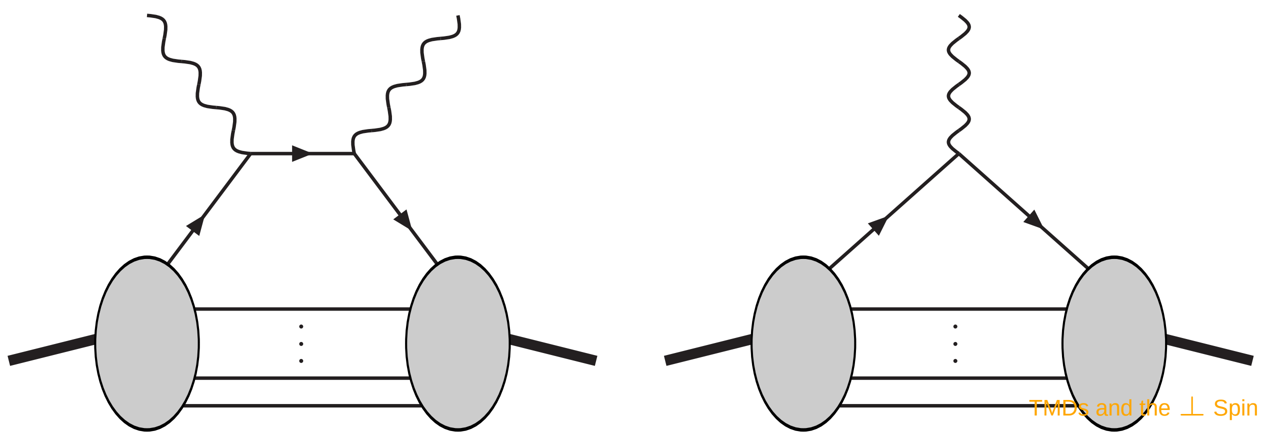
- Quark Gluon Correlations ($g_2(x)$) $\longrightarrow \perp$ force on quarks in DIS
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$

$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- DVCS amplitude

$$\mathcal{A}(\xi,t) \sim \int_{-1}^{1} \frac{dx}{x-\xi+i\varepsilon} GPD(x,\xi,t)$$

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi}\bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \boxed{\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\boldsymbol{\Delta}_\perp^2),\\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\boldsymbol{\Delta}_\perp^2),\end{aligned}}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)

- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

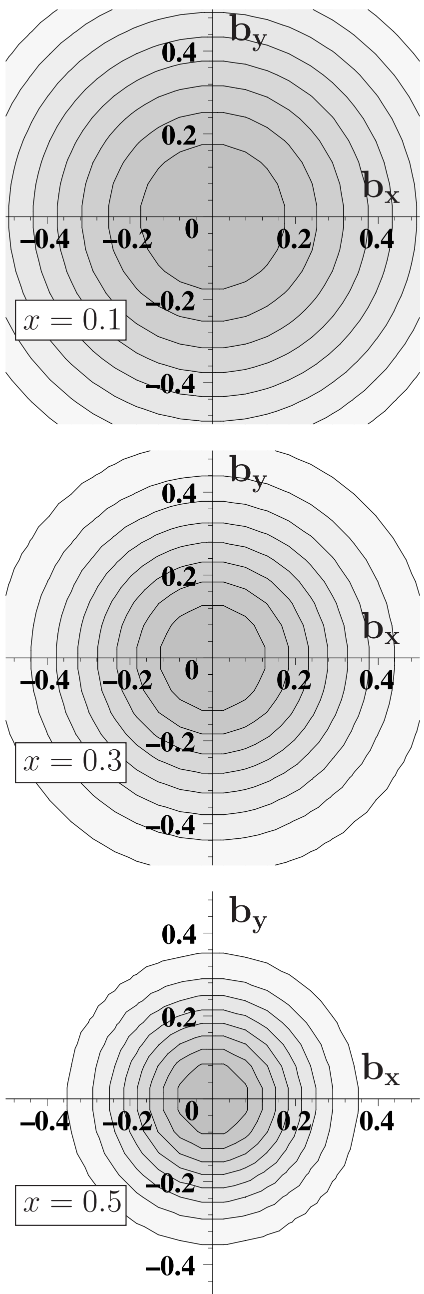
$\hookrightarrow$ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

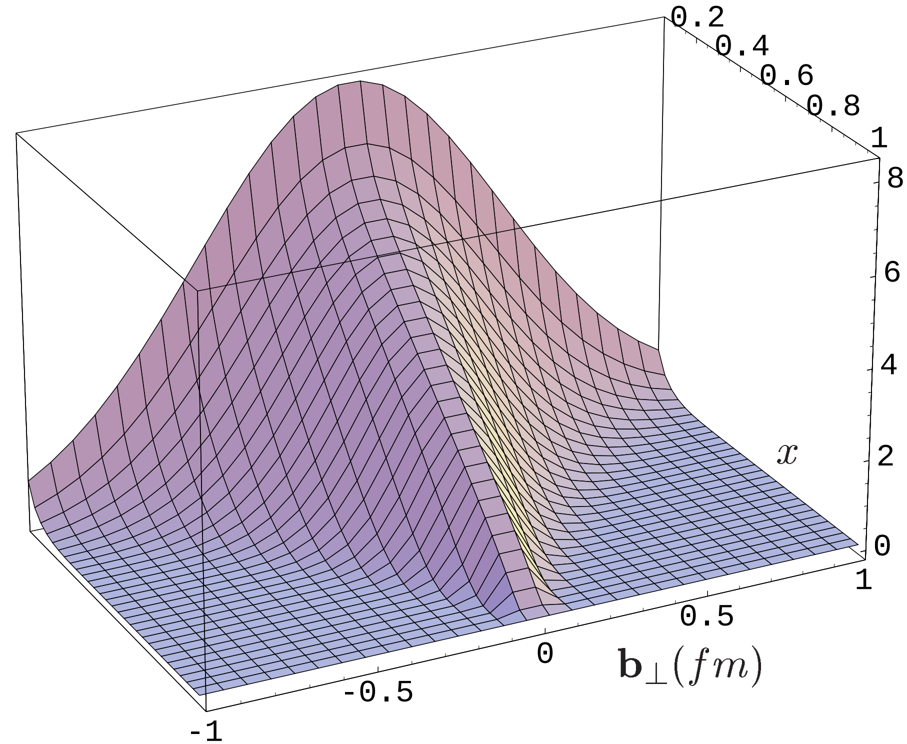
$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

- Note that this does not necessarily imply that 'hadron size' goes to zero as $x \to 1$, as separation $\mathbf{r}_\perp$ between active quark and COM of spectators is related to impact parameter $\mathbf{b}_\perp$ via $\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$.

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Transversely Deformed Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

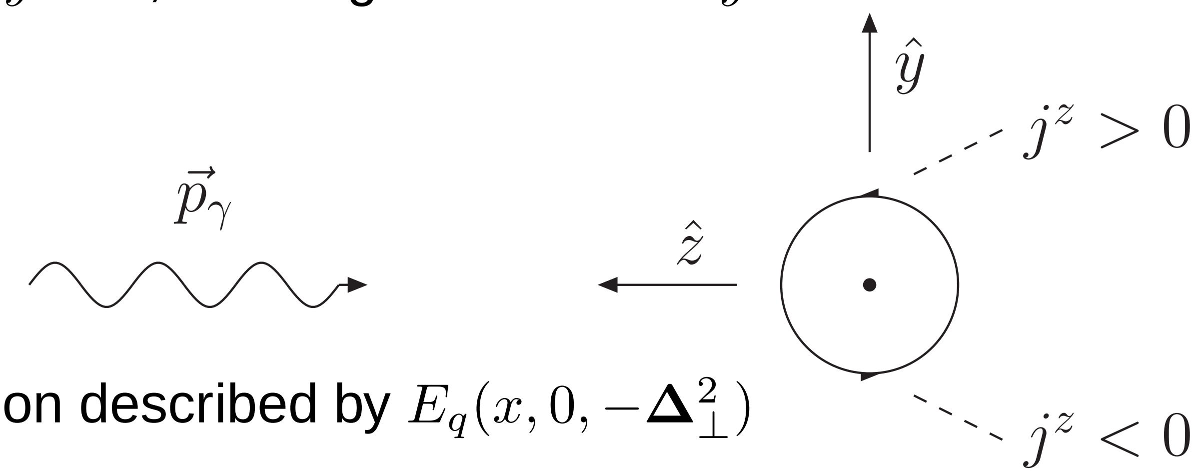
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\hookrightarrow$ j^+ larger than j^0 when quark current towards the γ^*; suppressed when away from γ^*

$\hookrightarrow$ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x .$$

Transversely Deformed PDFs and $E(x,0,-\mathbf{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

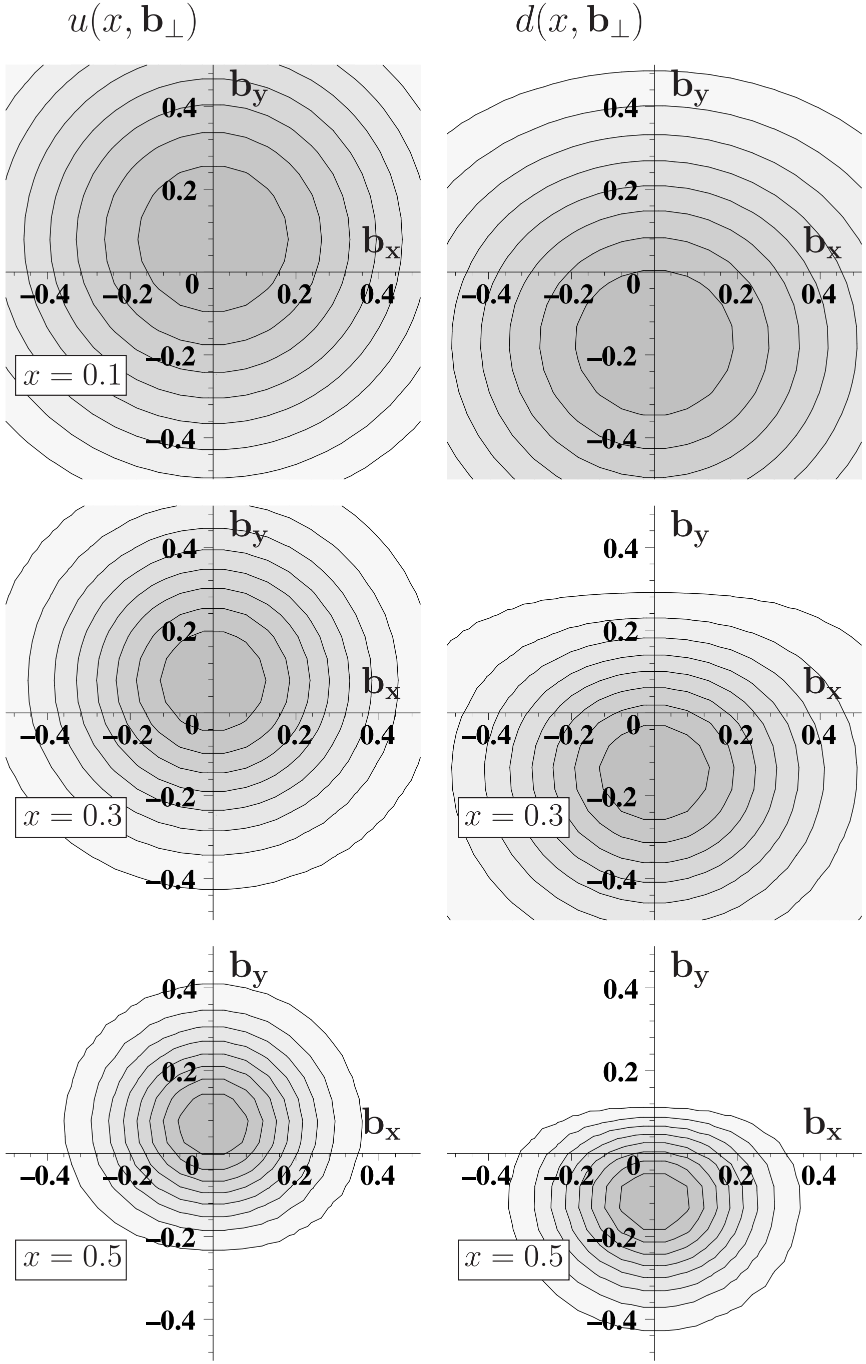
- simple model: for simplicity, make ansatz where $E_q \propto H_q$

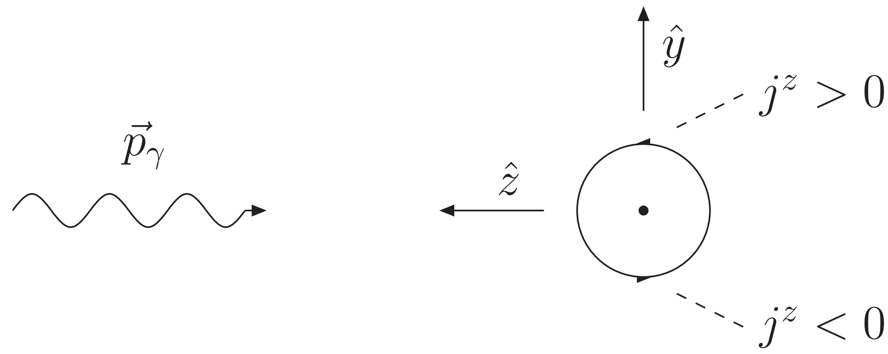
$$\begin{aligned} E_u(x,0,-\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2) \\ E_d(x,0,-\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2) \end{aligned}$$

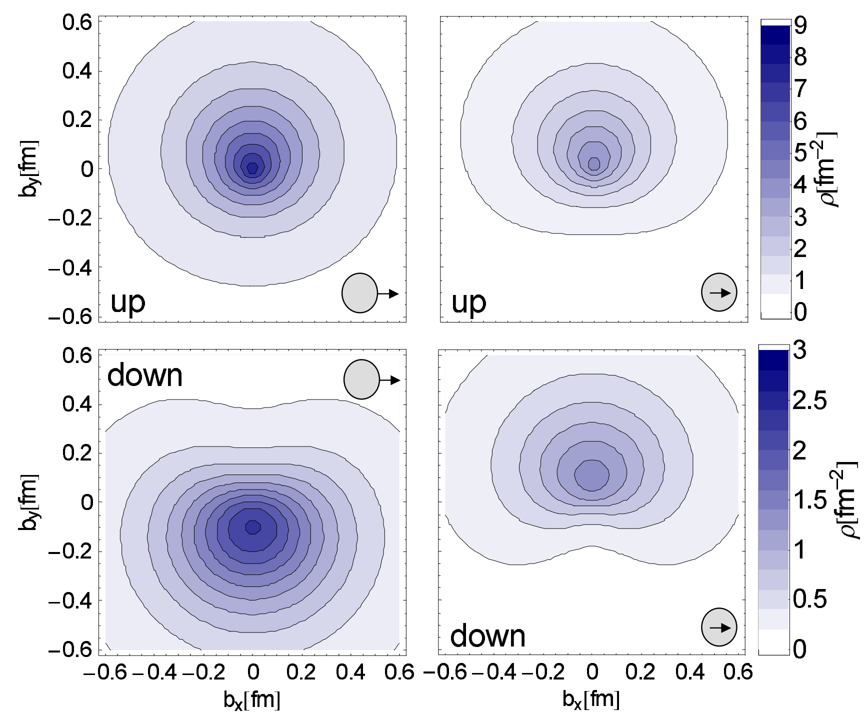
with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!

p polarized in $+\hat{x}$ direction

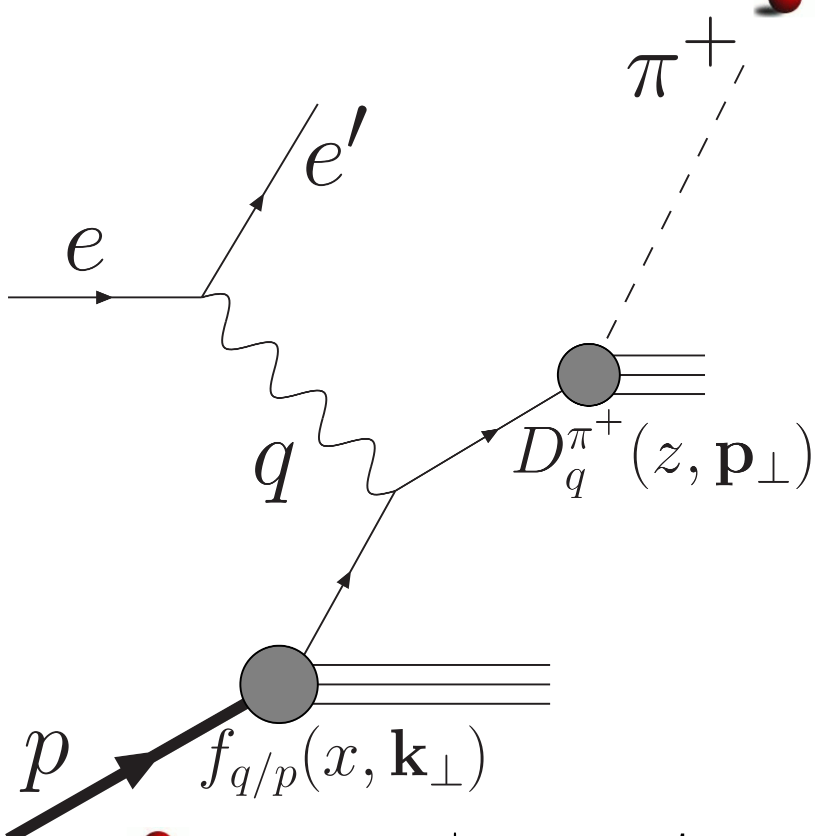






lattice results (Hägler et al.)

SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon
  - $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q
  - $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$
- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

GPD $\longleftrightarrow$ SSA (Sivers)

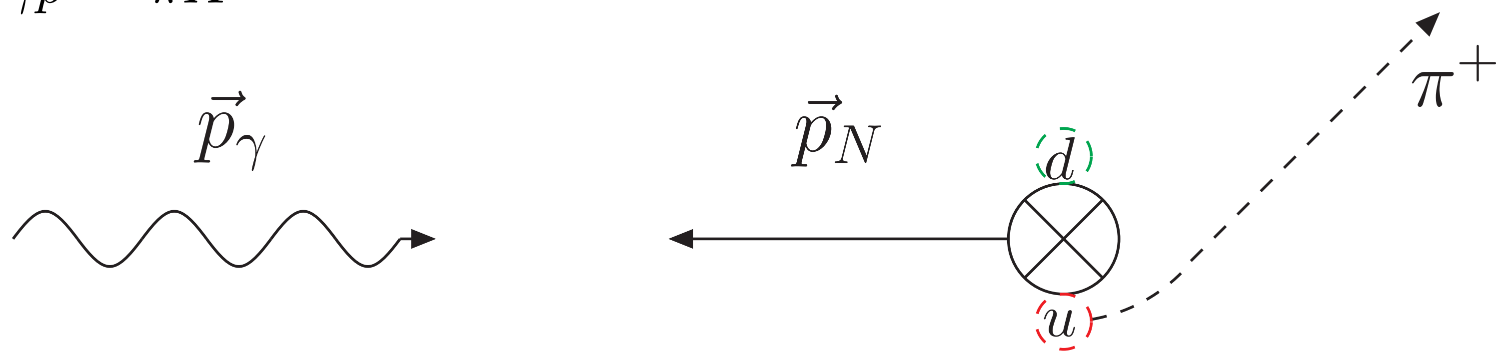
- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $f(x,\mathbf{k}_\perp) = f(x,-\mathbf{k}_\perp) \Rightarrow f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) \neq 0$ (Brodsky, Hwang, Schmidt)
- Why interesting?
  - $\perp$ asymmetry involves nucleon helicity flip
  - quark density chirally even (no quark helicity flip)
  - $\hookrightarrow$ 'helicity mismatch' requires orbital angular momentum (OAM)
  - $\hookrightarrow$ (like κ), Sivers requires matrix elements between **wave function components that differ by one unit of OAM** (Brodsky, Diehl, ..)
  - Sivers requires nontrivial final state interaction phases
  - $\hookrightarrow$ **sensitive to space-time structure of hadrons**

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

SSA evolution

- HERMES p-data shows significant Sivers
- COMPASS (higher Q^2, lower x) p-data no significant Sivers
- higher twist?
- evolution?
  - gluon dressing changes color of active quark
  - evolution destroys long distance color correlation

  $\hookrightarrow$ conceivable that above mechanism for Sivers disappears at high Q^2/low x

  $\hookrightarrow$ suggests rapid evolution of SSAs

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ 'polarized quark distribution' $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve 'higher-twist' distribution functions, such as $g_2(x)$
- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities

Quark-Gluon Correlations (Introduction)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- ‘usually’, contribution from g_2 to polarized DIS X-section kinematically suppressed by $\frac{1}{\nu}$ compared to contribution from g_1

$$\sigma_{TT} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ ‘clean’ separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+ q(0)\right|P,S\right\rangle$$

- $\sqrt{2}G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- matrix elements of $\bar{q}B^x\gamma^+q$ and $\bar{q}E^y\gamma^+q$ are sometimes called color-electric and magnetic polarizabilities
  $2M^2\vec{S}\chi_E = \left\langle P,S\left|\vec{j}_a \times \vec{E}_a\right|P,S\right\rangle$ & $2M^2\vec{S}\chi_B = \left\langle P,S\left|j_a^0\vec{B}_a\right|P,S\right\rangle$
  with $d_2 = \frac{1}{4}(\chi_E + 2\chi_M)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- QED: $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ correlator between quark density $\bar{q}\gamma^+ q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\left(F^{0y} + F^{zy}\right) = -e\sqrt{2}F^{+y}.$$

for charged paricle moving with $\vec{v} = (0, 0, -1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0, 0, -1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- Interpretation of d_2 with the transverse FSI force in DIS also consistent with $\langle k_\perp^y \rangle \equiv \int_0^1 dx \int \mathrm{d}^2 k_\perp\, k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$ in SIDIS (Qiu, Sterman)

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P, S \middle| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^-) \gamma^+ q(0) \middle| P, S \right\rangle$$

  semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

- matrix element defining d_2 same as the integrand (for $x^- = 0$) in the QS-integral:
  - $\langle k_\perp^y \rangle = \int_0^\infty dt F^y(t)$ (use $\mathrm{d}x^- = \sqrt{2}\mathrm{d}t$)

  $\hookrightarrow$ first integration point $\longrightarrow F^y(0)$

  $\hookrightarrow$ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$

  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \left| \bar{q}(0) g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0) \right| P,S \right\rangle \sim f_2$

$\hookrightarrow$ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

$\hookrightarrow$ combine with data for $g_2 \Rightarrow$ disentangle electric and magnetic force

$\hookrightarrow$ combining JLab(E99-117)/SLAC(E155x) data this yields

- proton:

  $\chi_E = -0.082 \pm 0.016 \pm 0.071 \quad \chi_B = 0.056 \pm 0.008 \pm 0.036$
- neutron:

  $\chi_E = 0.031 \pm 0.005 \pm 0.028 \quad \chi_B = 0.036 \pm 0.034 \pm 0.017$

but future higher-Q^2 data for d_2 may still change these results ...

Quark-Gluon Correlations (Estimates)

- What should one expect (magnitude)?
  - if all spectators were to pull in the same direction, force should be on the order of the QCD string tension
    $\sigma \approx (0.45GeV)^2 \approx 0.2GeV^2$
  - however, expect significant cancellation for FSI force, from spectators 'pulling' in different directions
  - $\hookrightarrow$ expect FSI force to be suppressed compared to string tension by about one order of magnitude (more?)
  - $\hookrightarrow$ $|d_2| = \frac{|\langle F^y(0)\rangle|}{M^2} \sim 0.02$
- What should one expect (sign)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$
  - $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton
  - d_2 negative/positive for u/d quarks in neutron
  - large N_C: $d_2^{u/p} = -d_2^{d/p}$
  - consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$

Quark-Gluon Correlations (data/lattice)

- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$ (with large errors)

$\hookrightarrow$ using $M^2 \approx 5\frac{\mathrm{GeV}}{fm}$ this implies

$$\langle F_u^y(0)\rangle \approx -50\frac{\mathrm{MeV}}{fm} \qquad\qquad \langle F_d^y(0)\rangle \approx 28\frac{\mathrm{MeV}}{fm}$$

- signs consistent with impact parameter picture
- SLAC data ($5GeV^2$): $d_2^p = 0.007 \pm 0.004$, $d_2^n = 0.004 \pm 0.010$
- combined with SIDIS data for $\langle k^y\rangle$, should tell us about 'effective range' of FSI $R_{eff} \equiv \frac{\langle k^y\rangle}{F^y(0)}$
  Anselmino et al.: $\langle k^y\rangle \sim \pm 100\,\mathrm{MeV}$
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ $\kappa^{q/p} \Rightarrow$ sign of deformation

$\hookrightarrow$ attractive FSI $\Rightarrow f_{1T}^{\perp u} < 0$ & $f_{1T}^{\perp d} > 0$

- Interpretation of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as $\perp$ **force on active quark in DIS in the instant after being struck by the virtual photon**

$$\langle F^y(0)\rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

- In combination with measurements of f_2
  - color-electric/magnetic force $\frac{M^2}{4}\chi_E$ and $\frac{M^2}{2}\chi_M$
- $\kappa^{q/p} \Rightarrow \perp$ deformation $\Rightarrow d_2^{u/p} > 0$ & $d_2^{d/p} < 0$ (attractive FSI)
- combine measurement of d_2 with that of $f_{1T}^\perp \Rightarrow$ **range of FSI**
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

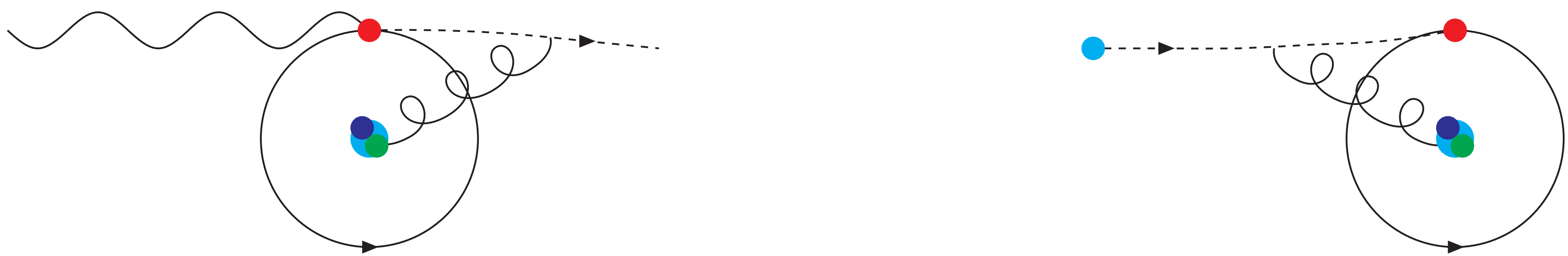
$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:

$$h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longrightarrow$ Boer-Mulders)

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$



a) b)

- time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

$\hookrightarrow$ FSI for knocked out q is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive

What is a Polarizability?

- Polarizability is the relative tendency of a charge distribution, like the electron cloud of an atom or molecule, to be distorted from its normal shape by an external electric field, which may be caused by the presence of a nearby ion or dipole (Wikipedia)
- It may be consistent with this original use of the term to enlarge the definition to encompass all observables that describe the ease with which a system can be distorted in response to an applied field or force
- Suppose one enlarges this definition to encompass ‘how the color electric and magnetic field responds to the spin of the nucleon’

$\hookrightarrow$ many other obeservables also become ‘polarizabilities’, e.g.

  - Δq, as is describes how the quark spin responds to the spin of the nucleon
  - $\vec{\mu}_N$, as it describes how the magnetic field of the nucleon responds to the spin of the nucleon
  - $\vec{L}_q$, as it describes how the quark orbital angular momentum responds to the spin of the nucleon
  - as well as many other ‘static’ properties of the nucleon

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P\exp\left(ig\int_0^\infty d\eta^- A^+(\eta)\right) = 1$$

$\hookrightarrow$ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} f(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \\ &\times \left\langle p, s \left| \bar{q}(y)\gamma^+ U_{[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp]} U_{[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp]} U_{[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]} q(0) \right| p, s \right\rangle \end{aligned}$$

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