Mach cones in heavy ion collisions

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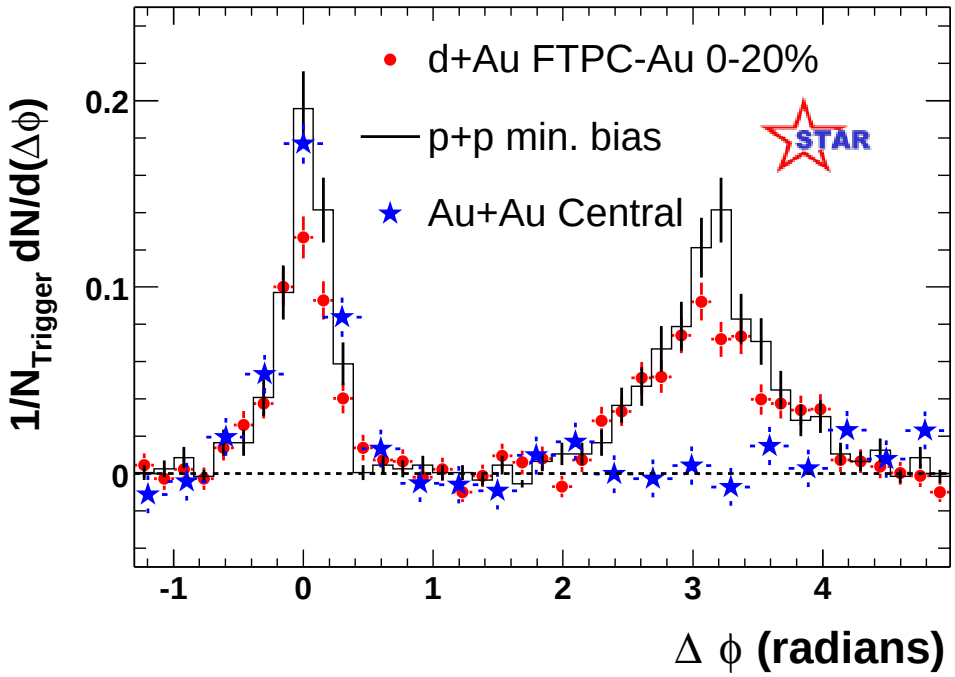
With B.Betz,J.Noronha,M.Gyulassy,I.Mishustin,V.Greco,D.Rischke,H.Stoecker

"jets" of fast particles quickly lose energy by medium-induced radiation. This was conclusively shown to happen, and is usually interpreted as the system at RHIC being very opaque. Where does the missing E/p go?

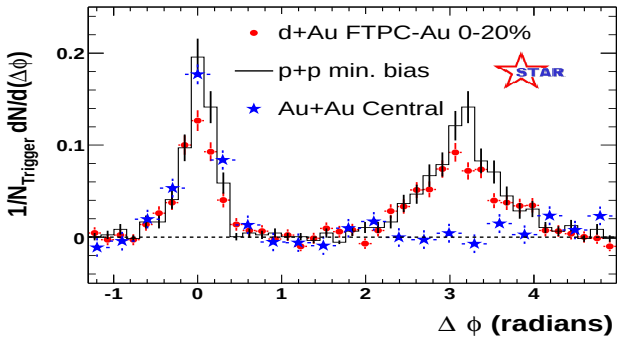


Jet suppression revisited:

$p_T^{near,away} > 2$ GeV

near side

Away side



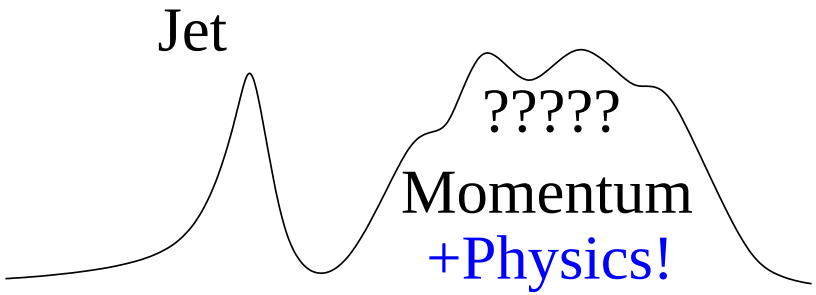
Softening the away−side trigger

$p_T^{near} > 2$ GeV

$p_T^{away} > 0$

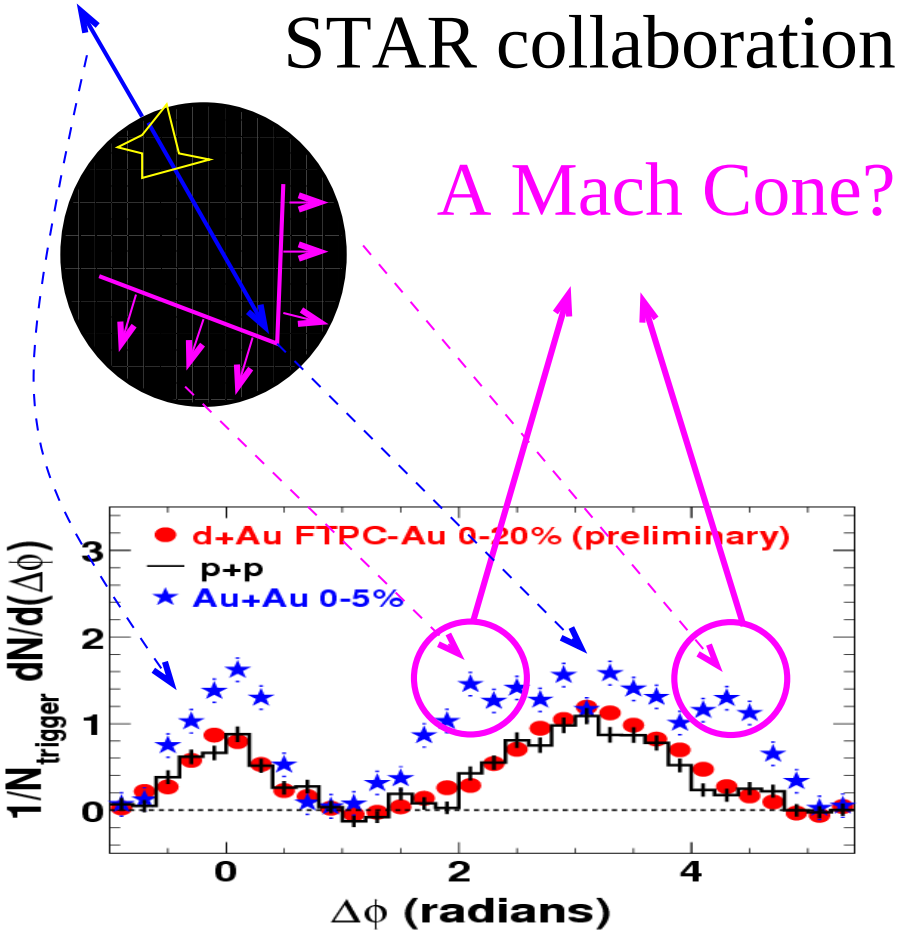
near side

Away side



Correlations between <u>hard</u> particles ($p_T^{near,away} > 2$ GeV) suppressed. By conservation of momentum correlation should <u>reappear</u> when p_T^{away} lowered, hopefully with interesting structures!

Experiment:If we lower trigger, away-side peak reappears and...

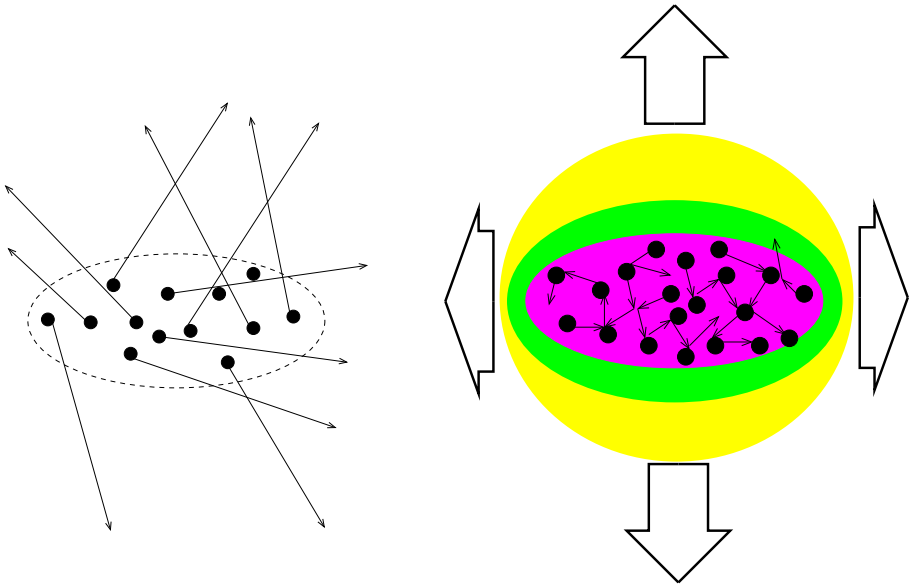


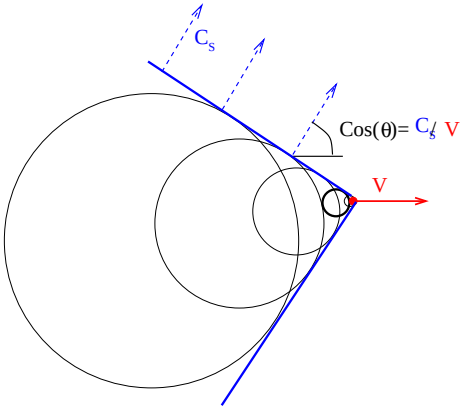
Why wed like it to be a Mach cone:

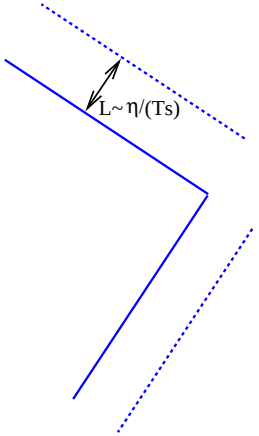
v_2 suggests that the soft degrees of freedom in the RHIC system are thermalized and viscosity is low

A "dust"
Particles ignore each other, their path is independent of initial shape

A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)



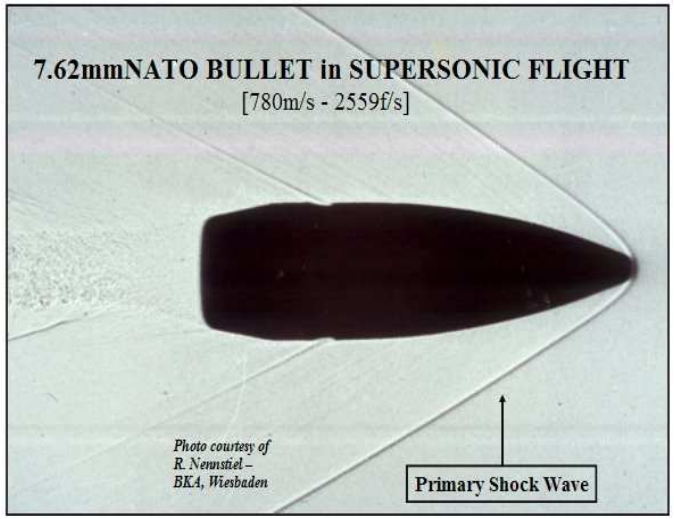




Mach cone angle Sensitive to EoS, $\cos\theta = c_s/v$

Cone killed by high viscosity <u>exponentially</u>, $A(x) \sim A(0)e^{-x/\Gamma}, \Gamma \sim \eta/(Ts)$

<u>IF</u> we see this, we <u>confirm</u> fast thermalization <u>and</u> study fluid's EoS!

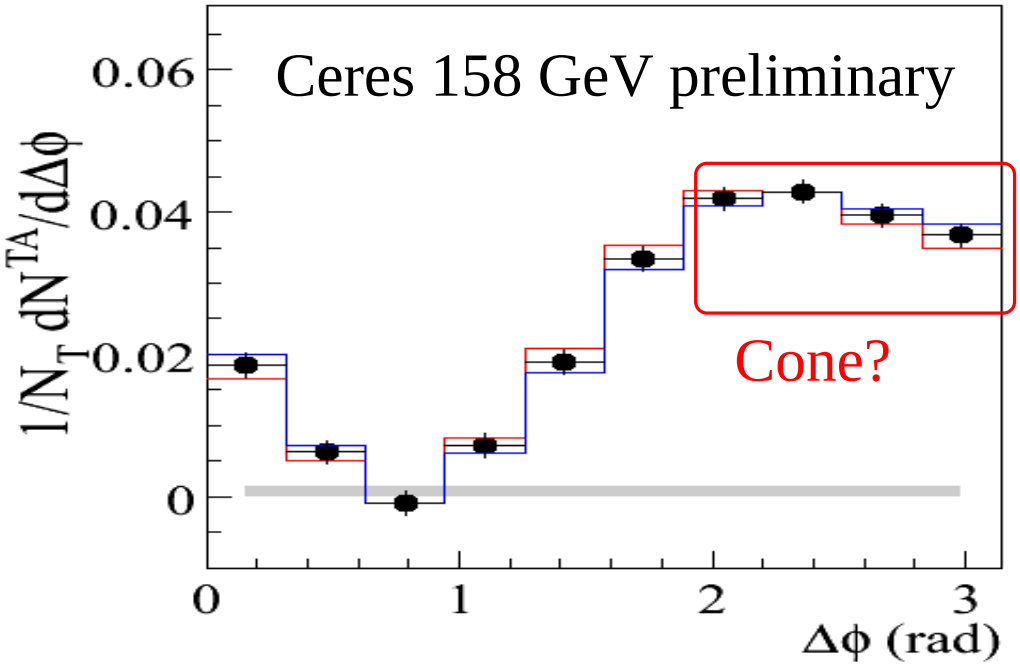


Phenomenon is <u>well known</u> (Mach,19th century, Lifshitz+Landau,...)

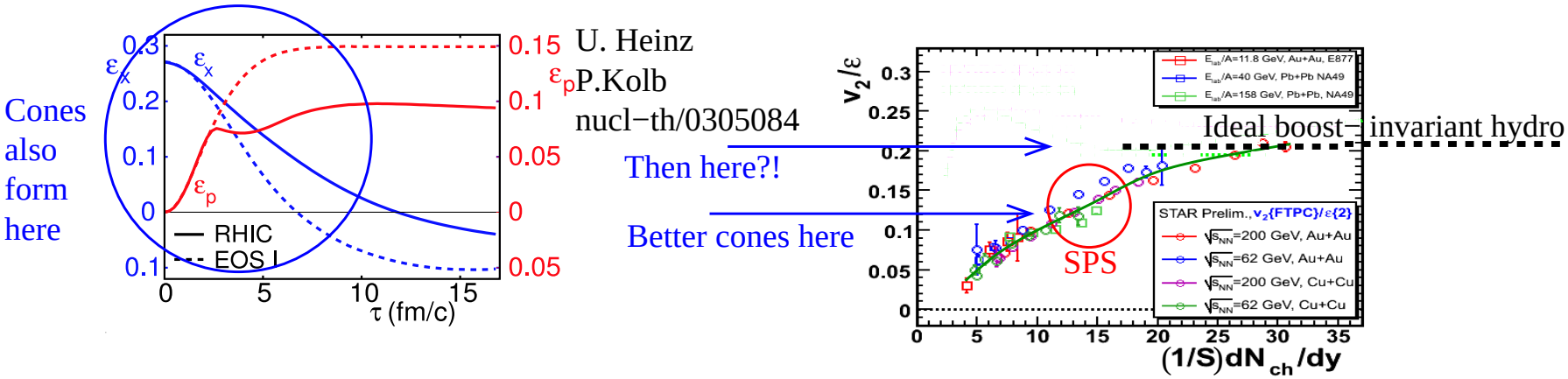
First suggested to be relevant in Heavy ion collisions, H. Stoecker, W. Scheid, W. Greiner,... ,1975

Excplictly linked to Jets in RHIC by J. Casalderrey-Solana, E.V. Shuryak, D. Teaney,also Stoecker, Mishustin, Satarov, ~ 2004

A surprise we should not forget: Mach cones signals also found at SPS!



If you believe in hydrodynamics, this is weird



Time-scales of v_2 and Mach cone formation should be comparable, so the two should be correlated. v_2 at SPS far from ideal hydro limit

Knudsen number For the Mach cones ($\sim$ cone) much larger than v_2 ($\sim$ System), so cone should be more sensitive to viscous effects

What dont we understand? Cone or v_2?

This talk in a sentence: heavy ion collisions $\neq$ "textbook"

- Background non-trivial (flowing, phase transition)
- Non-linear hydrodynamics
- Energy-momentum deposition <u>not</u> trivial, and not well understood
- Freeze-out: We don't see fluid, but particles

Each of these, as we will see, changes the answer radically: Both suppressing a Mach cone when a "perfect fluid" is formed, and by creating a "fake" Mach cone which is actually very different in origin from the one found in textbooks (And whose angle is not trivially correlated with the EoS)

Non-linear/non-equilibrium: Is it a Mach angle, a Shock angle or a "neck" angle?

full hydrodynamics in a nutshell

Conservation equations of energy-momentum coupled to source (jet)

$$\partial_\nu \underbrace{T^{\mu\nu}}_{energy-momentum\ \ tensor} = \underbrace{J^\mu}_{energy-momentum\ \ source}$$

Use local isotropy in frame co-moving with flow (u_μ) to fix $T_{\mu\nu}$

$$T_{\mu\nu} = (e+P)u^\mu u^\nu - pg^{\mu\nu}$$

Use equilibrium EOS to close system of equations,$p = p(e)$ (5 equations to solve for $p, e, u^{x,y,z}$.

Viscosity:

$$T_{\mu\nu} = T_{\mu\nu} + \Pi_{\mu,\nu}(\zeta(e), \eta(e), \partial_\mu u_\nu)$$

This is complicated (Nonlinear, few analytic solutions) To get insights, use linearized hydro (Casadellrey-Solana,Shuryak,Renk,Ruppert).

$$T^{\mu\nu} = \underbrace{T_0^{\mu\nu}}_{diag[e,p,p,p]} + \underbrace{\delta T^{\mu\nu}}_{\ll T_0^{\mu\nu}}$$

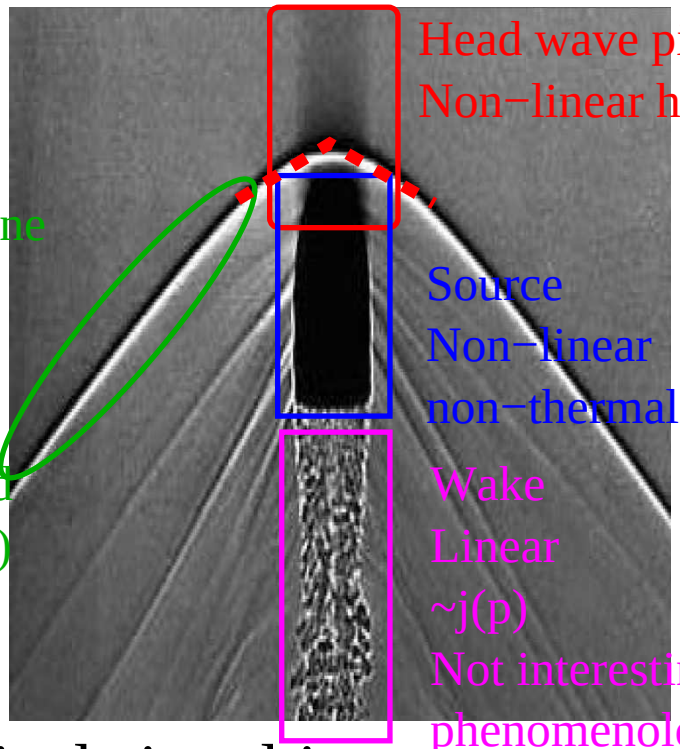
Flow small, only pressure/density (sound waves):In terms of $\epsilon = \delta T^{00}, g_L = T^{0L}, \vec{g}_T = T^{0T}$ where L and T are parallel/perpendicular to jet

$$\partial_t \epsilon + ikg_L = J^0$$

$$\partial_t g_L + ic_s^2 k_L \epsilon + \frac{4}{3}\frac{\eta}{e_0 + p_0} k^2 g_L = J^L$$

$$\partial_t \vec{g}_T + \frac{4}{3}\frac{\eta}{e_0 + p_0} k^2 \vec{g}_T = \vec{J}^T$$

Is linearized hydro good? probably not



Rischke,Stoecker,Greiner
PRD42:2283−2292,1990
Maccoll problem (Angular shock)
NOT same as Mach angle
Angle~amplitude,not Cs and v

A Conical signal is not necessarily a Mach cone.

Not all signals from thermalized matter are conical

An Insight from string theory

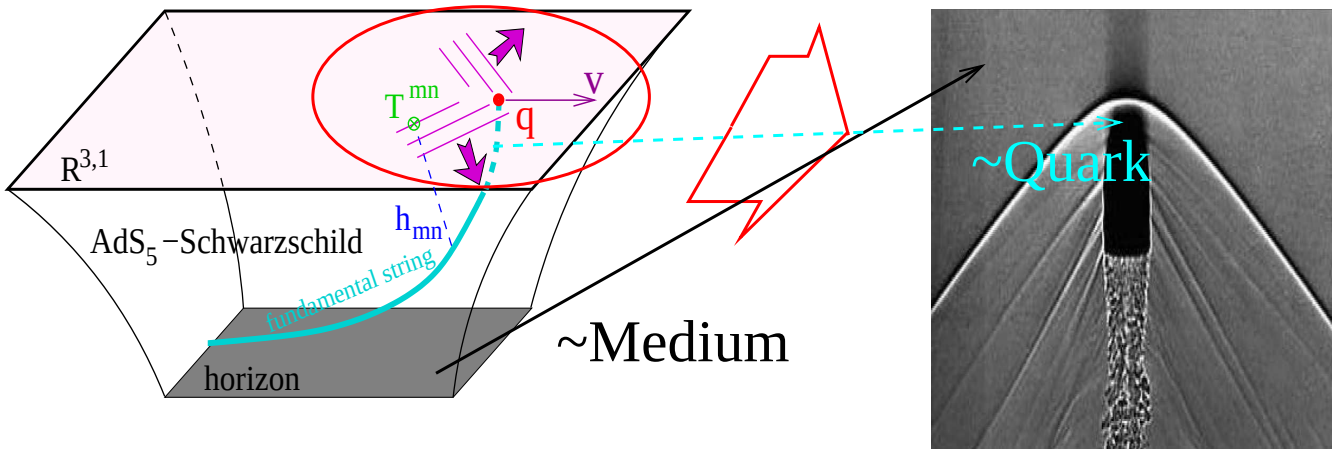
The AdS-CFT correspondence: Every $\left\langle \hat{O}_{CFT} \right\rangle$ a 4D $N_{susy} = 4$ Gauge theory with N_c colors and T'hooft coupling λ, can be calculated by translating to a 10D string theory, with 5 Anti-DeSitter ($\Lambda < 0$) dimensions, 5 dimensions compactified on a sphere, and a string coupling constant of $g_s = \lambda/(4\pi N_c)$

- dictionary between $\hat{O}_{CFT}$ and $\hat{O}_{ADS}$ can be worked out
- Links strongly coupled CFT to weakly coupled perturbative string theory. Infinitely strongly coupled CFT $\Leftrightarrow$ classical supergravity.

If conjecture right, can calculate operator expectation values in infinitely strong CFT!, and check if its $\sim$ Mach cone

E.g. Heavy quark in finite−T medium $\longleftrightarrow$ string in AdS

Joshua J. Friess, Steven S. Gubser, Georgios Michalogiorgakis, Silviu S. Pufu,PRD75:106003,2007 Also A. Yarom



$g_{\mu\nu} \Leftrightarrow T_{\mu\nu}$, Quark $\Leftrightarrow$string

Finite T background $\Leftrightarrow$Black hole in AdS space

$\lambda \to \infty$ $\Leftrightarrow$Classical geometry (Einstein's equations for $g^{\mu\nu}$)

A BIG note of caution: This is NOT QCD (4 SUSYs, no quarks, $N_c, \lambda \to \infty$). This has the potential of introducing qualitative subtle differences.

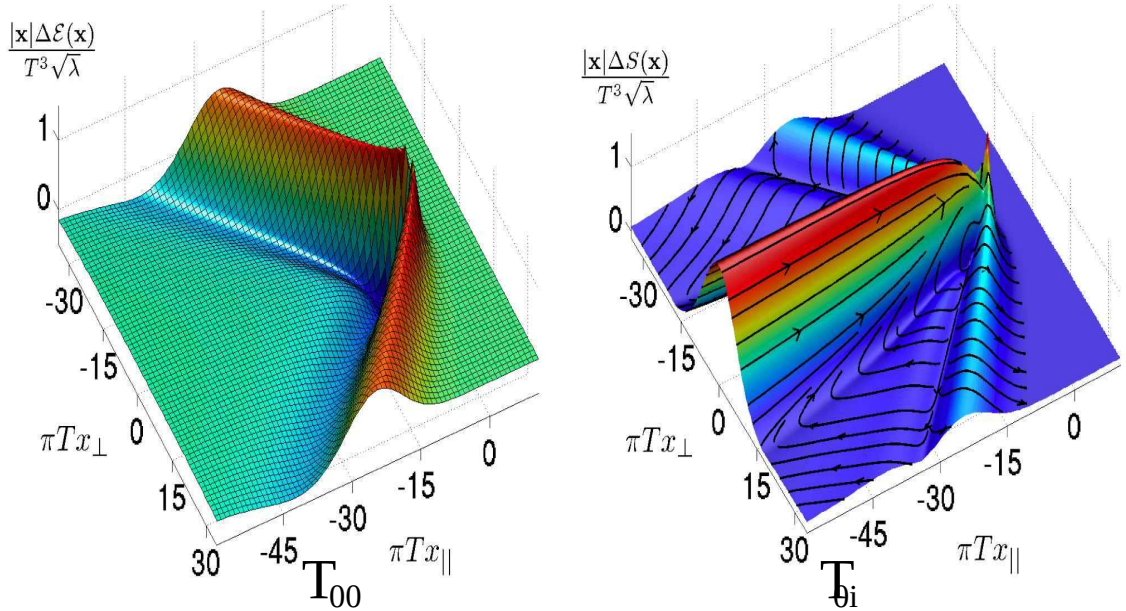
CFT The theory is conformally invariant. No running coupling, no phase transition, no hadrons, no bulk viscosity

QCD Is approximately conformally invariant at weak coupling, big-time non-invariant at strong coupling

For our problem (Understanding the probe-medium correlation in the strong-field limit), this might be OK. But thread with caution: for instance, in CFT jet-medium coupling not so different than medium-medium. probably not so in QCD! Bethe-Heitler/LPM/Factorization limits different?

(Smaller?) warning: heavy quark jet, still not measured

Chesler,Yaffe:arXiv:0712.0050
Also Yarom,Gubser,Noronha,GT,Gyulassy



This is a solution to $T_{\mu\nu}$ from QFT, looks like Mach cone, correct angle, Cone/Wake ratio. GT,J.Noronha,Gyulassy: arXiv:0712.1053: Solution to Navier-Stokes to 1%! after $\sim 1fm$, contribution of viscous tensor also $< 10\%$ It <u>seems</u> the approximations we made are reasonable! But....

Freeze-out: Cooper-Frye vs Bowling ball vs coalescence vs???

Freeze-out: How to get from fluid to particles

Assume $l_{mfp} \sim$ system sizeat some spacetime locus $\Sigma^\mu = (t, \vec{x})$ chosen according to some criterion (critical time, temperature,...). Conservation of energy+local equilibrium... Cooper-Frye formula

$$E\frac{dN}{d^3p} = \int p_\mu d\Sigma^\mu f_{FD/BE}(p_\mu u^\mu, T, \mu_B)$$

Hydrodynamics+freeze-out criterion give flow u_μ, Σ_μ. Currently Σ^μ choice ad-hoc. Resonances decay after freeze-out.
Generally: "Hot spots" give strongly Σ-dependent signal. "Extra Flow" better, as momentum conservation always results in boost
The result: correlation $\frac{d^n N}{d^n(\phi_i - \phi_{jet})}$(NB: Generally n-particle correlation in theory, n+1-particle in experiment!) For 2 particles we get...

The bad news: If hydro linearized (Including AdS/CFT), double peak cone structure smeared away by freeze-out. If $u = (u_{jet}, u_\perp)$,$\Sigma = (1, \vec{0})$,

$$\frac{dN}{p_T dp_T d\phi}|_{y=0} = \int_{\Sigma_T} d\Sigma_\mu P^\mu \left[f(U^\mu, P^\mu, T) - f_{eq}\left(P^0, T_0\right)\right]$$

Background $\vec{U} = 0, T = T_0$, cone $\vec{U} = (U_\perp, U_L), \Sigma^\mu = (1, \vec{0})$ (Isochronous)

$$f(\phi) = 2\pi\, p_T \int_{\Sigma_\perp} dx_1 dx_\perp x_\perp \times \left(\exp\left\{ -\frac{p_T}{T}\left[U_0 - U_1 \cos(\pi - \phi)\right]\right\} I_0(a_\perp) - e^{-p_T/T_0}\right)$$

expanding Bessel function to first order in $p_\perp U_\perp / T$ we get

$$f(\phi) \simeq e^{-p_\perp/T_0} \frac{2\pi\, p_\perp^2}{T_0} \left[\frac{\langle \Delta T \rangle}{T_0} + \langle U_1 \rangle \underbrace{\cos(\pi - \phi)}_{One\ \ Peak\ \ :(} \right]$$

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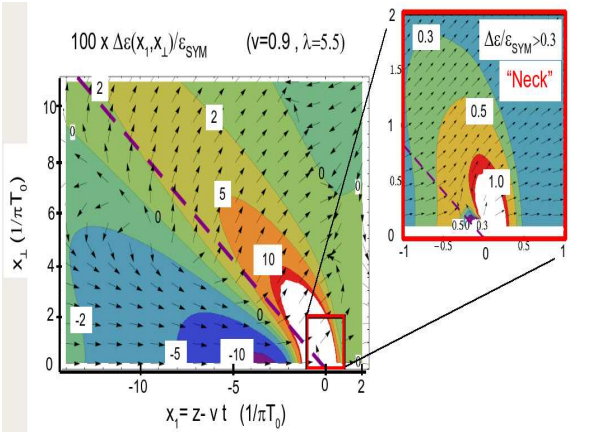
It is important to underline that "one broad peak" is the "Minimum" required by momentum conservation, and can arise out of completely non-locally equilibrated dynamics. Diffusion wake (ie, jet momentum deposition) can also spoil cone signal! (See Barbara's thesis)

Bottom line:Either hydro non-linear corrections big (but then not true "Mach cone", Angle could be different from EoS, initial conditions need to be <u>fine-tuned</u> , (See Barbara's thesis) or $p_T >> T$

Parametrize AdS/CFT solution in terms of $Kn = \Gamma \frac{\nabla . S}{S} \left(\sim \frac{l_{mfp}}{\partial_\mu u} \Big|_{weak \quad coupling} \right)$

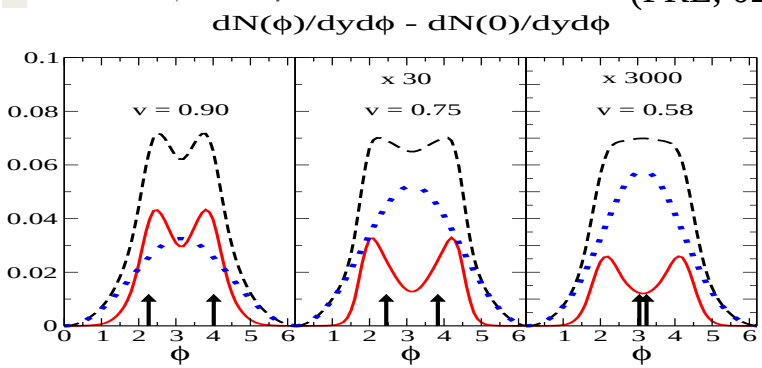


Mach: Kn>3 (~Fluid)

Neck: 3<Kn<1 (~Magneto−fluid?)

Head: Kn<1 (~Coherent field?)

Gyulassy,Noronha,GT:0807.2235
(PRL, 02:102301,2009)



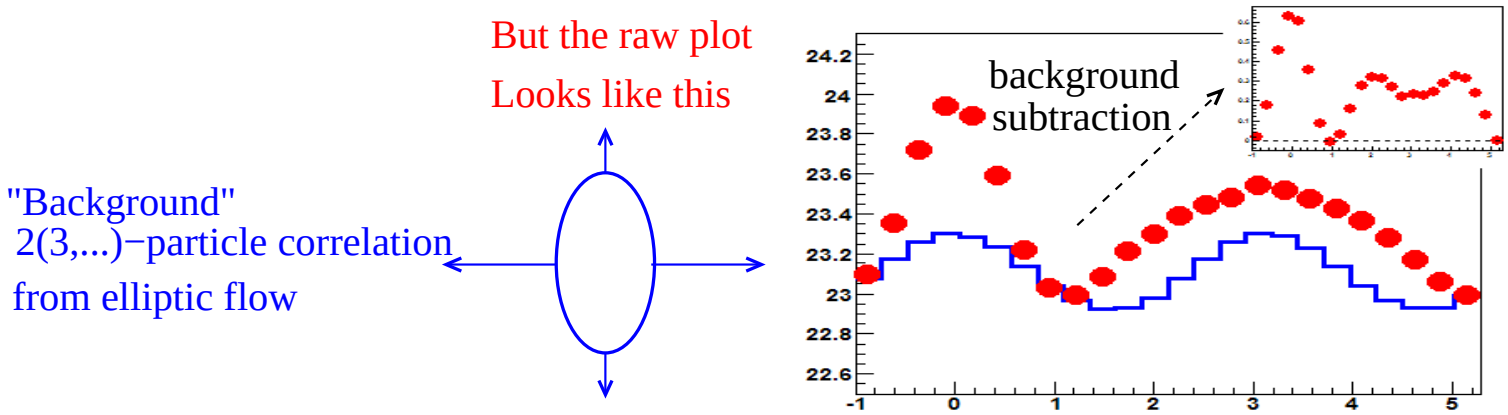
Cooper−Frye freeze−out*:
STRONG "cone−like" signal from
NON−thermalized "neck"
BUT NOT "real" cone
does not obey Mach's law
probably less sensitive to flow

*viscous corrections to linearized signal
twice suppressed, so approx not bad even in neck region

A conical signal
is not necessarily a
Mach cone!

Test by:
−mach cone angle varyingg
v for heavy quarks
−Flow sensitivity in
non−central collisions

The two component model



Subtract "background" correlation from corrlation due to jet. "Guess" normalization (ZYAM,ZYA1,...). Do the same on theory-side, subtract Cooper-Fryed jet and jetless events, convolute with "near-side" signal

Unfortunately, this is WRONG!(but we dont know how to do better)

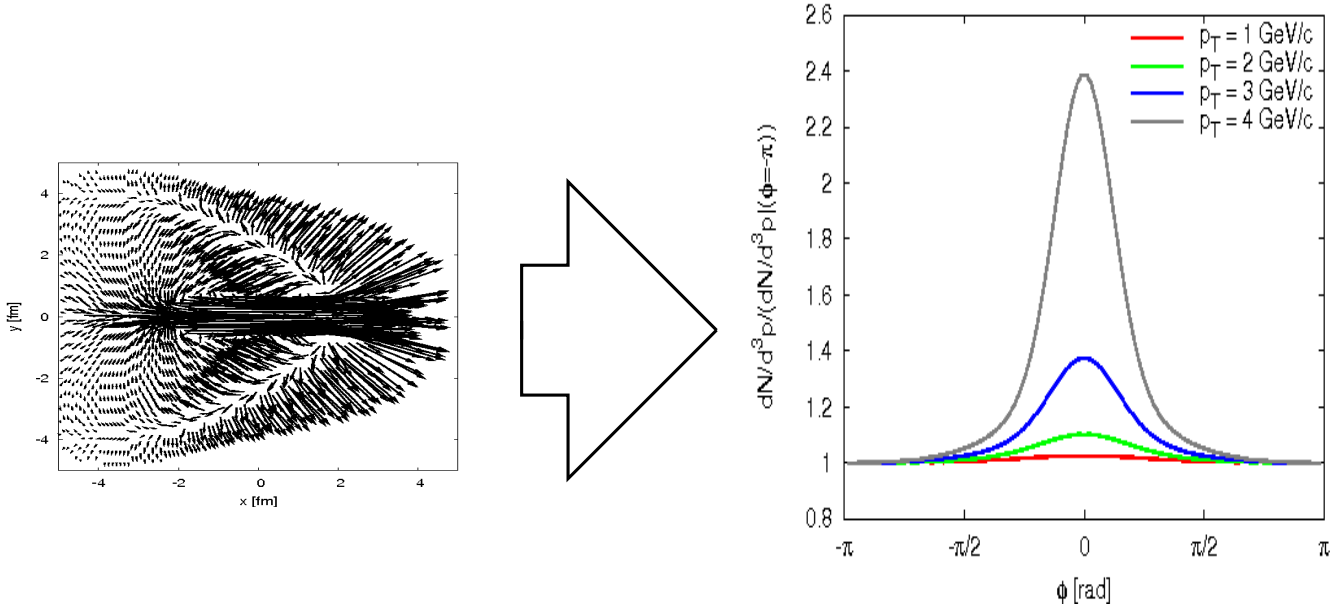
$$\frac{dN}{d\phi} \sim \exp\left[\beta p_T \left(v_T + \sum u_n \cos(2n\phi)|_{background} + \sum u_n \cos(n\phi)\Big|_{mach}\right)\right]$$

$$= \frac{dN}{dp_T}\left(1 + 2v_n \cos(n\phi)\right)$$

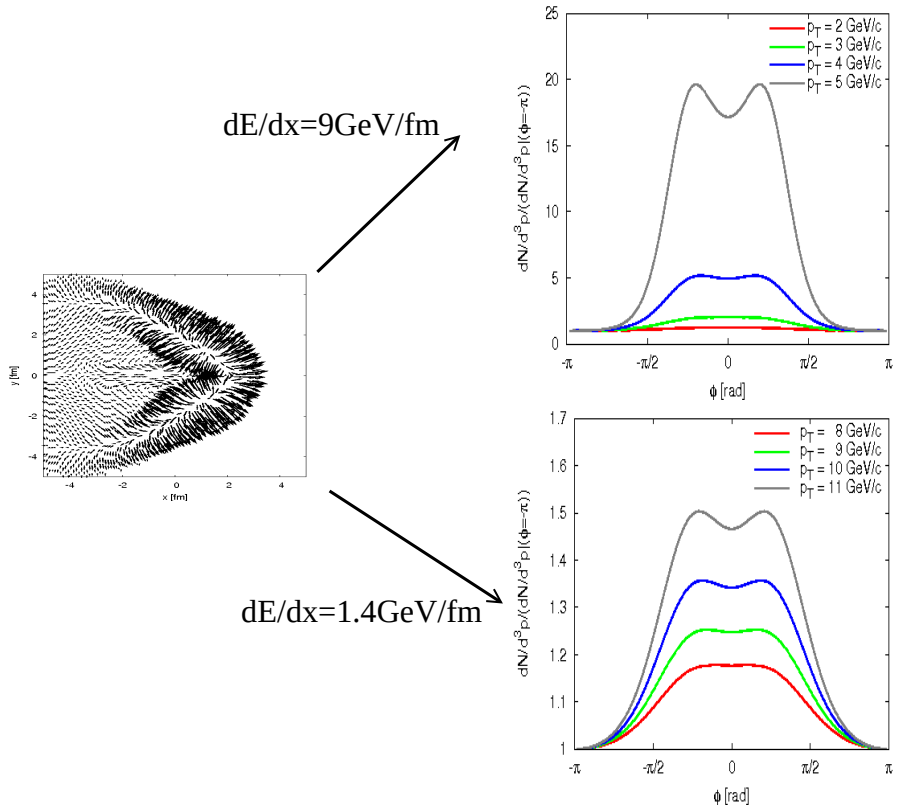
lets remember our Bessel functions!

$$\int_{-\pi}^{\pi} \cos(nx) e^{\alpha \cos mx} \sim I_{n-m}(x) \neq 0$$

Even in the linearized hydrodynamics, Cooper-Frye introduces mixing.
Two component model is <u>wrong</u>, but not clear what is better and as "intuitive" ("Jetty" vs "non-jetty" v_2 measurements?)



Betz, Gyulassy, Stoecker, Torrieri: As expected, diffusion wakes are phenomenologically useless! Yield a generic "peak" indistinguishable from any other jet energy loss mechanism!



Energy deposition works better: Cone structure, correct angle. Signal increases with p_T (Blue-shift), only strong at very high away-side p_T

Other freeze-out models: "The bowling ball"

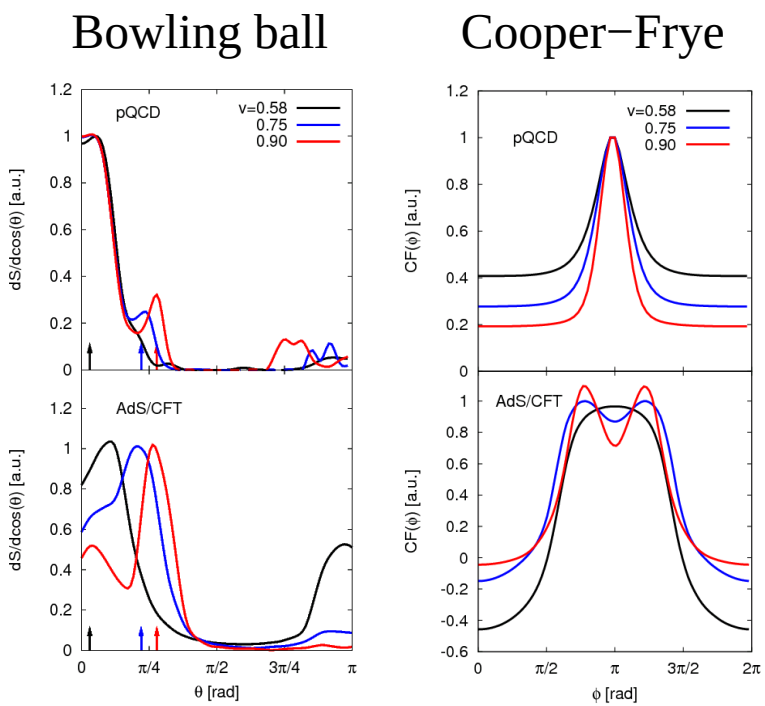
Basic idea: we do not know what happens in the later stages of the collision. High viscosity/mean free path could significantly change the initially conical (or Diffusion-wakey) observed particle distribution... but energy-momentum is <u>conserved</u>. Instead of measuring particle correlations, howe about measuring <u>momentum</u> correlations?

$$\frac{dE}{d\phi} = \int_V r dr dz T_{\mu\nu} d\Sigma^\mu \delta \left(\frac{\vec{T^{0i}}}{|(T_i^i)^2|} - e_{\vec{\phi}} \right)$$

Difference with Cooper-Frye: No thermal broadening.
Physical interpretation: Clusters?

Result the same for CF vs bowling ball

B.Betz,J.Noronha,GT,M.Gyulassy
PLB in press



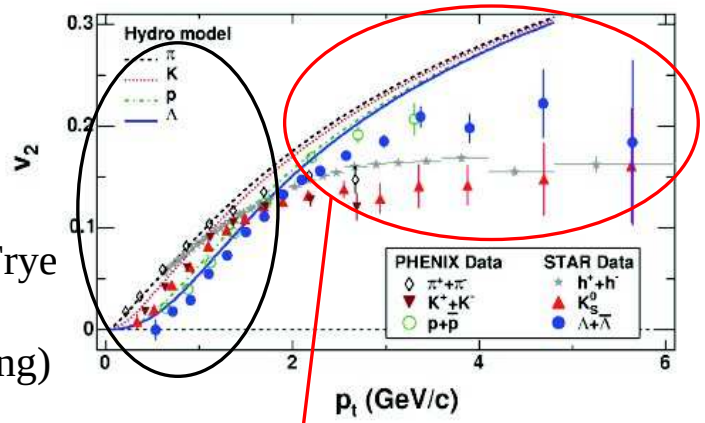
But we only had ideal hydro. Experimentalists might think presenting both energy and particle correlations as the latter are less sensitive to (unknown!) freeze-out dynamics!

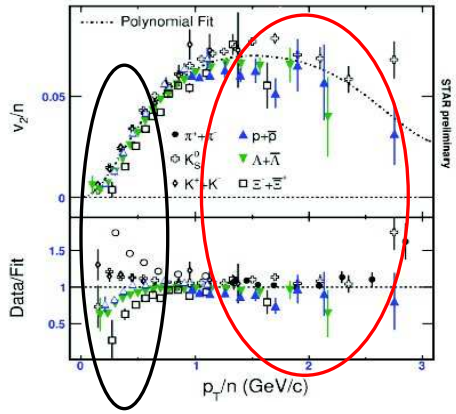
Other freeze-out models: Coalescence

Compilation by STAR collaboration

Coalescence better

Hydro
+Cooper−Frye
applies
(mass scaling)





Coalescence worse

Hydro stops applying (Saturation)

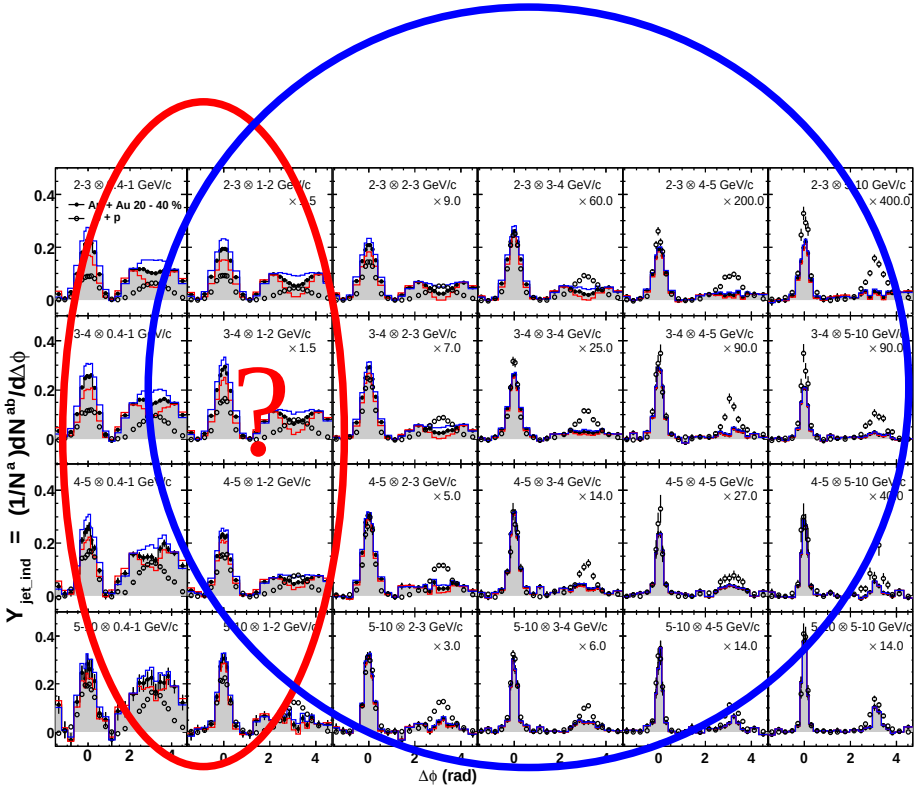
Molnar+Voloshin,PRL.91:092301,2003. nucl-th/0302014

Fries et al: PRC68:044902,2003, : nucl-th/0306027

Cooper-Frye seems to work at lower p_T ($p_T/T_f < 5$ or so), but intermediate p_T a result of <u>coalescence</u> of hydro and/or (non-thermalized) higher p_T partons

PHENIX

Phys.Rev.C78:014901,2008.



Experimental data says conical flow present <u>both</u> in the Hydro regime (Usually frozen out with Cooper-Frye) and in the coalescence regime

A "toy model": Mesons with a δ-Wigner function

$$\frac{dN_M}{d^2P} = g \int d\mathbf{r}_1\, d\mathbf{r}_2 \int d\mathbf{p}_1\, d\mathbf{p}_2\; f_q(\mathbf{p}_1, \mathbf{r}_1) f_q(\mathbf{p}_2, \mathbf{r}_2) f_M(\mathbf{r}_1 - \mathbf{r}_2; \mathbf{p}_1 - \mathbf{p}_2) \delta^2(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2)$$

f_q is the quark distribution described previously

$$f_q(p, \phi_p, \phi_r) = \mathrm{Exp}\left[-\frac{(\mathrm{p}^2 + \mathrm{m}^2)^{1/2}\, \mathrm{U}_0 + \mathrm{p}\, \mathrm{U}_\mathrm{L} \cos\phi_\mathrm{p} + \mathrm{p}\, \mathrm{U}_\mathrm{T} \sin\phi_\mathrm{p} \sin\phi_\mathrm{r}}{\mathrm{T}} \right]$$

Using a simple delta function for the meson Wigner function f_M

$$\frac{dN_M}{d^2P} = \int dr_1\, dr_2\, r_1 r_2 \int d\phi_{1r}\, d\phi_{2r} \int dp_1\, dp_2\, p_1 p_2 \int d\phi_{1p}\, d\phi_{2p}\, f_q(p_1, \phi_{1p}, \phi_{1r}) f_q(p_2, \phi_{2p}, \phi_{2r})$$

$$\frac{1}{r_1} \delta(r_1 - r_2)\, \delta(\phi_{1r} - \phi_{2r}) \frac{1}{p_1} \delta(p_1 - p_2)\, \delta(\phi_{1p} - \phi_{2p}) \delta^2(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2)$$

Therefore we can rewrite it :

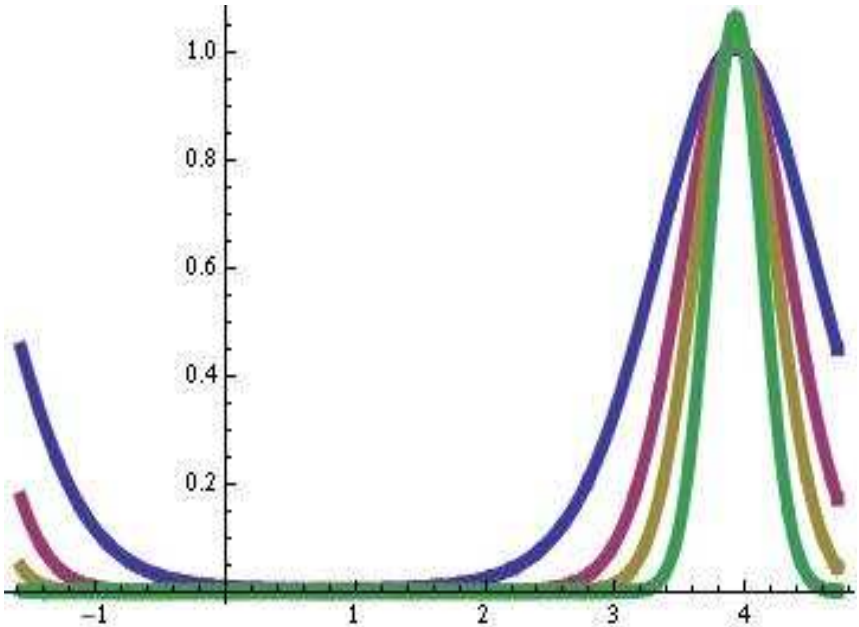
$$\frac{dN_M}{P\,dP\,d\Phi_p} = \int dr_1\, r_1 \int dp_1\, p_1 \int d\phi_{1r} \int d\phi_{1p}\, f_q^2(p_1, \phi_{1p}, \phi_{1r})\, \delta(P - 2p_1)\, \delta(\Phi_p - \phi_p)$$

Now the integral over r is just a constant factor, the delta functions kill the two integral over the momentum space. Substituting these into the convolution integral we have:

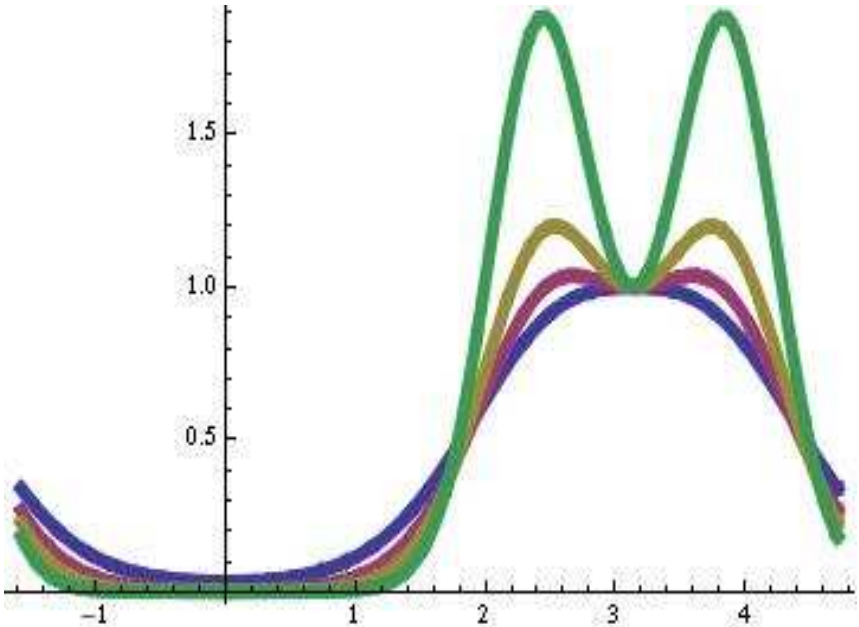
$$\frac{dN}{d^2P} \sim \exp\left[-2\frac{(\mathrm{P}^2/4 + \mathrm{m}^2)^{1/2}\,\mathrm{U}_0}{\mathrm{T}}\right] \exp\left[-\frac{\mathrm{P}\,\mathrm{U_L}\cos\phi_\mathrm{p}}{\mathrm{T}}\right] \int \mathrm{d}\phi_\mathrm{r} \exp\left[\frac{\mathrm{P}\,\mathrm{U_T}\sin\phi_\mathrm{p}}{\mathrm{T}}\sin\phi_\mathrm{r}\right]$$

$$\frac{dN}{d\phi_p} \sim \exp\left[-2\frac{(\mathrm{P}^2/4 + \mathrm{m}^2)^{1/2}\,\mathrm{U}_0}{\mathrm{T}}\right] \exp\left[-\frac{\mathrm{P}\,\mathrm{U_L}\cos\phi_\mathrm{p}}{\mathrm{T}}\right] \mathrm{I}_0\left[\frac{\mathrm{P}\,\mathrm{U_T}}{\mathrm{T}}\sin\phi_\mathrm{p}\right]$$

Quark dN/d ϕ



Meson dN/d ϕ



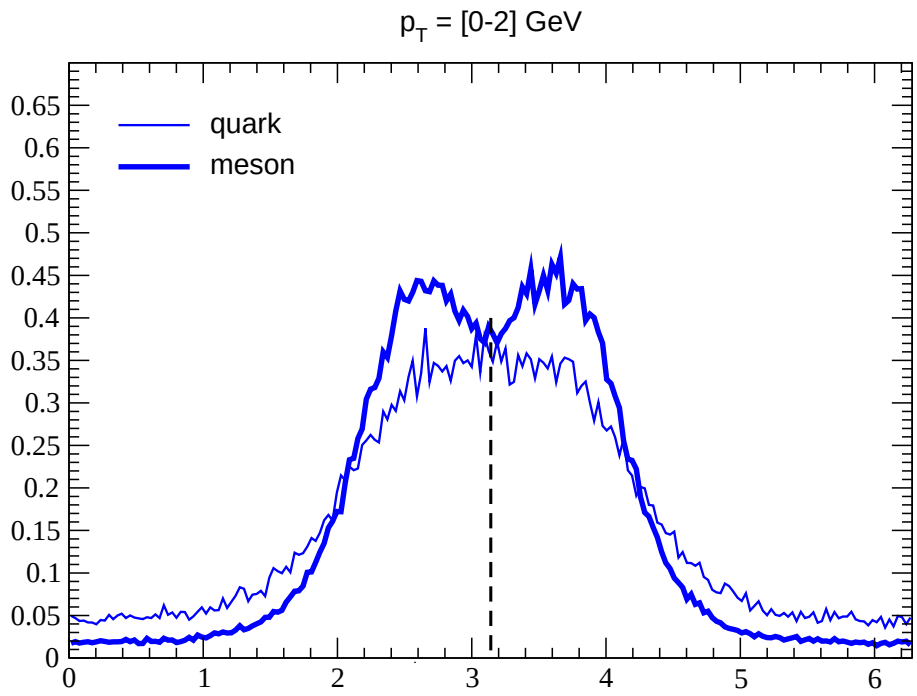
Blue−red−cyan−green: pT=1,1.5,2,3 GeV

Normalization:Peak=1 GT,Greco,Noronha,Gyulassy: QM09

V. Greco, C.M. Ko , P. Levai, PRC68 034904,2003, nucl-th/0305024

- Local in configuration space (Wigner function Gaussian w.r.t. interquark separation)
- Non-collinear and conserving momentum (Constituent quark masses)
- πs constituent ($m_\pi = 700$ MeV)...
- ...But Resonance decay implemented (πs can be $\rho \to \pi\pi$)

Montecarlo calculation confirms the analytical results

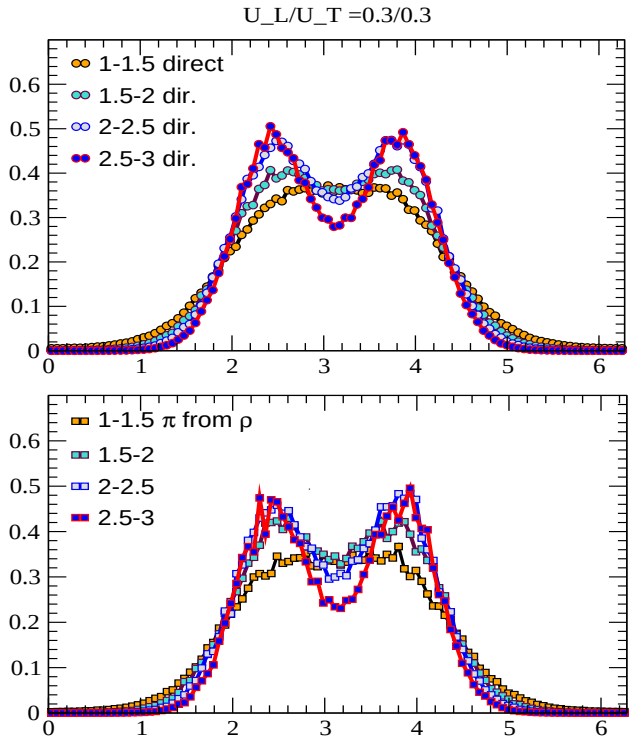


Even angle compatible with experiment
(Gaussian Wigner lowers cone p_T)
Angle not set by medium but by Wigner function (tuned to $p/\pi, v_2$)

Coalescence
vs
Resonance
decay

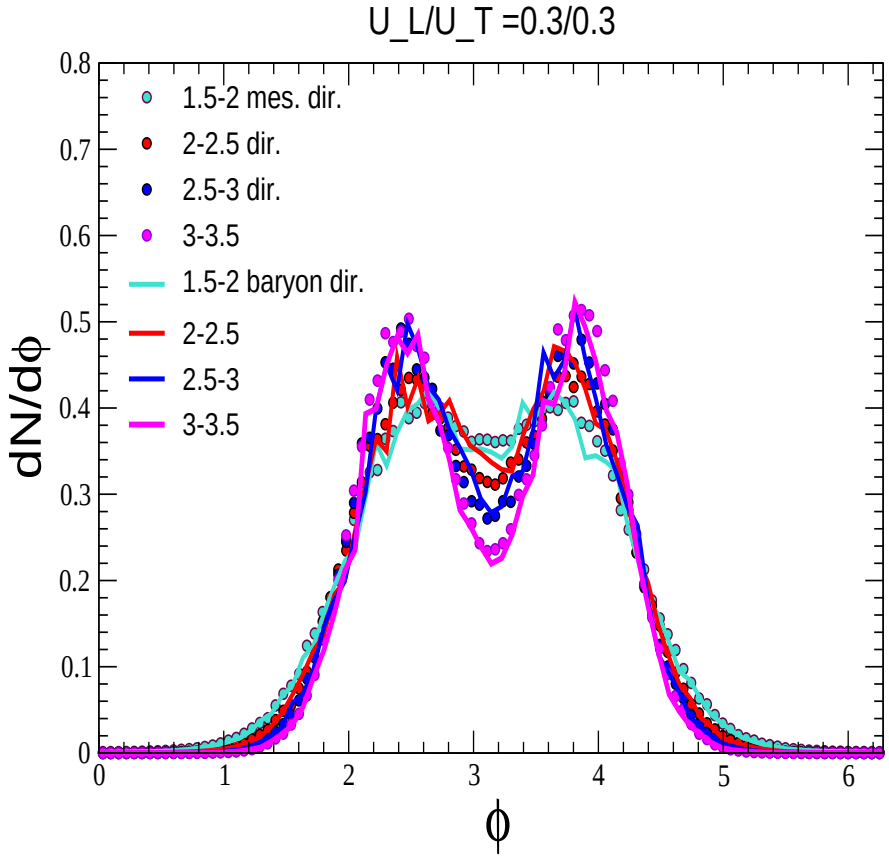
direct pions
vs

ρ ——> π π



Since resonances only correlate in Momentum space (No correlations between particles with different flow), its NOT the inverse of coalescence. Hence, resonance decay alone can not make 2 peaks out of one. Effect of resonance decay on angular Mach-like coalescence also mild

Meson/Baryon



Angle and depth of signal look very similar.

Energy deposition: Diffusion wakes and all that

Source usually (a la Lifshitz-Landau) local

$$J^\mu \sim J_0^\mu \delta(x - vt)$$

For an <u>infinite</u> δ-function, linearization $\delta T^{\mu nu}/T^{\mu nu} \ll 1$ badly broken.
Of course, the δ-function approximation of smeared non-equilibrium distribution

$$\delta(x - vt) \simeq f\left(x - vt, \sigma\right)$$

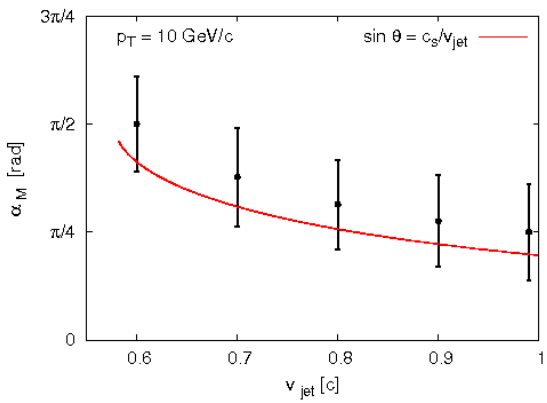
Because full hydrodynamics is non-linear, form of f where $\delta T^{\mu nu}/T^{\mu nu} \sim 1$ can have effects in the linearized ($x \gg \sigma, \delta T^{\mu nu}/T^{\mu nu} \ll 1$) region.

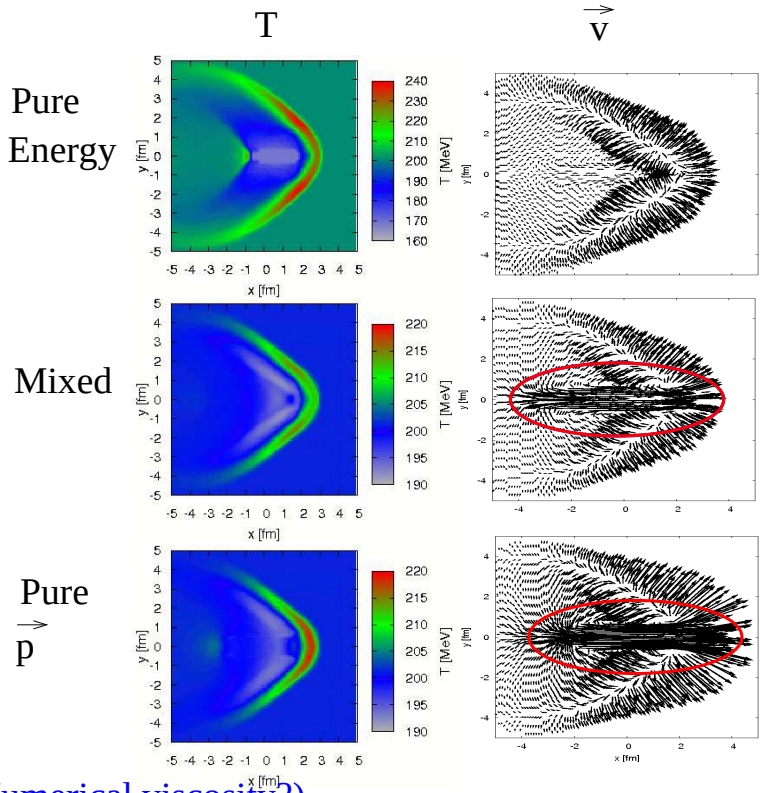
Perhaps when $x \gg \sigma$ these effects go away, but this might be <u>too big</u>.
(In AdS/CFT Far-away dynamics <u>does</u> depend on weather source is a <u>heavy quark</u> or a <u>meson</u>. So near-side dynamics <u>changes</u> far-away result)

Option I:Explore range of J^{μ}s systematically with full hydro;$\sim$ conical, but...

Betz,Gyulassy,Stoecker,Rischke,Torrieri,QM2008 presentation,coming paper
Also J.Casalderrey−Solana, E.V. Shuryak,PRD74 (2006) 085012

(Probably) invisible T pattern independent of source





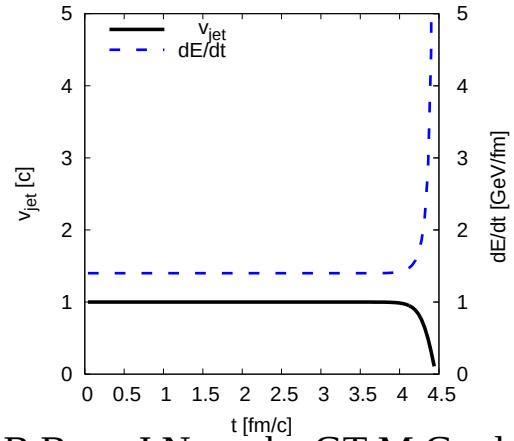
But flow pattern depends on it A LOT!
Momentum deposition creates un−conical "diffusion shock", taking most of the source's energy/momentum

Mach cone angle survives in full hydro (Non linearities no problem. Numerical viscosity?)

"Realistic" GLV/BDMPS calculation forthcoming; LPM effect also likely to spoil Mach signal

This conclusion does not change weather jets are "stopped" or punch-trhough, or weather deposition of momentum is transverse or longitudinal (Vorticity conservation. So a finite η/s could alter this)

Bethe−Block
energy loss assumed

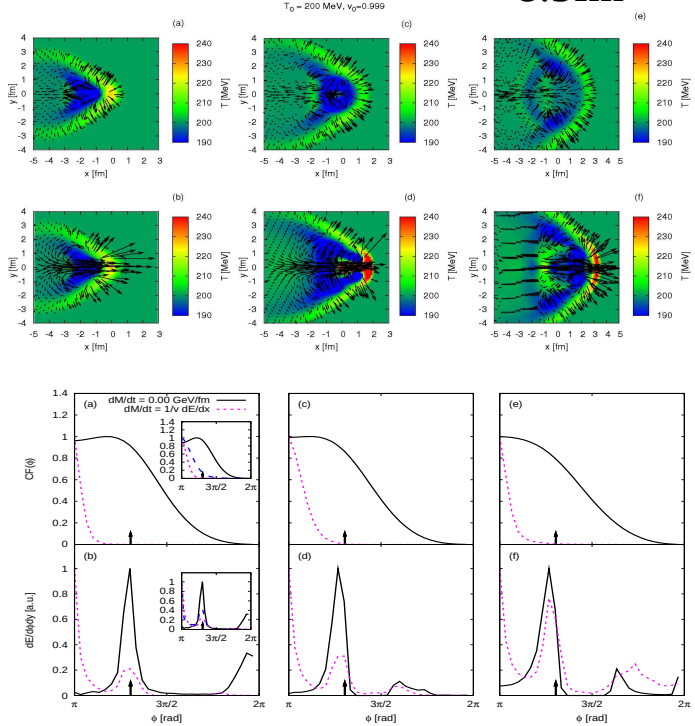


B.Betz, J.Noronha,GT,M.Gyulassy
I.Mishustin,D.Rischke
Phys. Rev. C 79, 034902 (2009)

4.5fm 6.5fm 8.5fm

Pure energy

On−shell



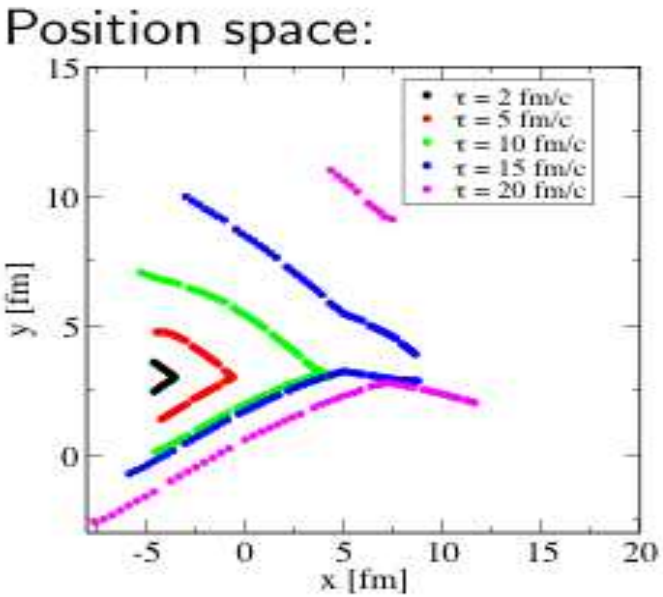
Flow: Distorted cones vs distorted wakes

Effect of flow : Usual relationships with frame co-moving with flow (Satarov, Stoecker,Mishustin,PLB627(2005))

In linearized limit, for ultrarelativistic jet

$\theta = \sin^{-1}\left(c_s/v_{jet}^{comoving\ \ frame}\right) \rightarrow \sin^{-1}\left(c_s\sqrt{\frac{1-v^2}{1-v^2c_s^2}}\right)$

flow $\vec{v}$, given by global hydrodynamics, narrows cone.

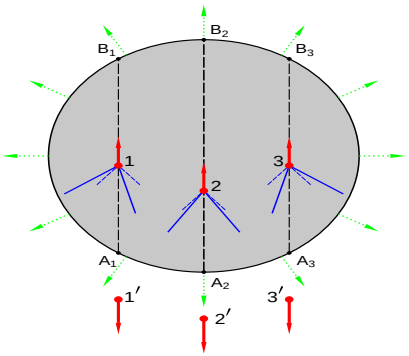


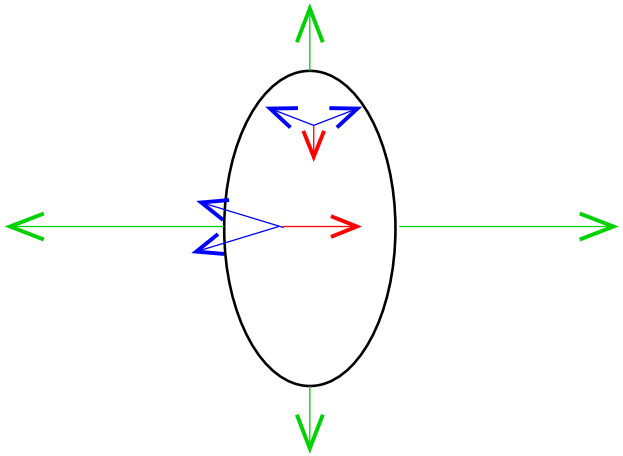
Renk, Ruppert, PRC73:011901,2006

disentangling effect of flow from angle to get EoS might be non−trivial

Transverse flow (Mishustin,Satarov,Stoecker) should "smear" and "narrow" angle

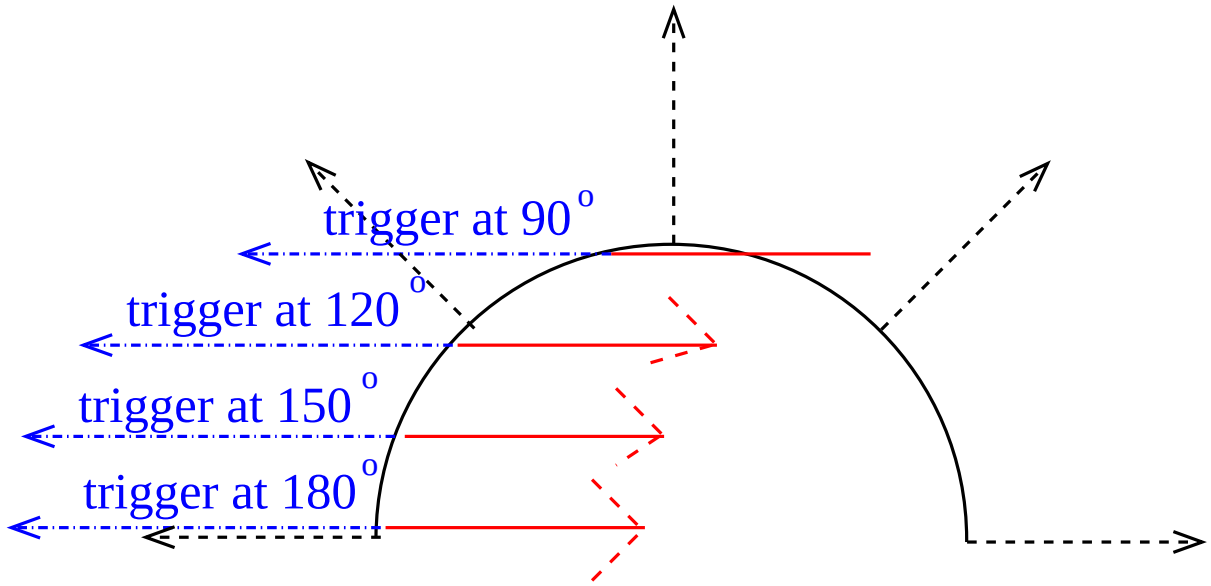
elliptic flow should correlate θ_{mach} to $\phi_{jet} - \phi_{reaction}$ (Unless <u>neck</u> signal?)





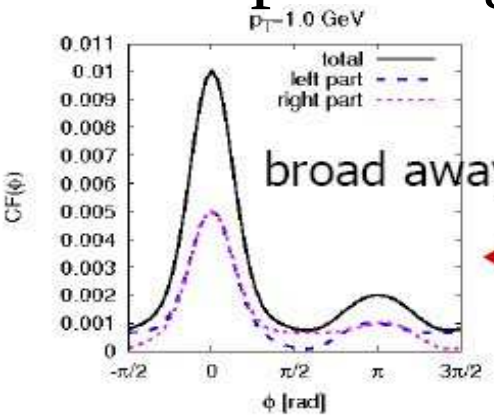
Need hydro studies ,Data binned by centrality,$\phi_{jet} - \phi_{reaction}$

A simulation including Energy-deposition and flowing backoround



Summing over all jet directions, with different jet-flow angle, can give non-trivial correlations after Cooper-Frying

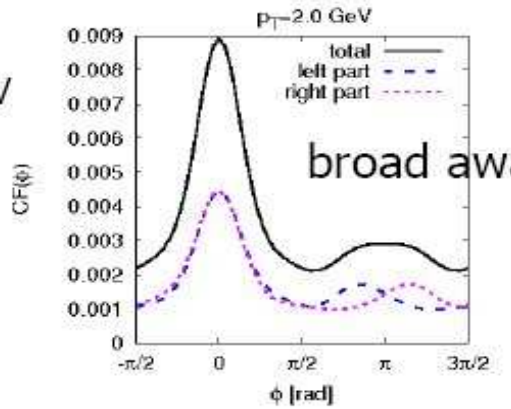
Expanding medium



E_{tot} = 5 GeV

p_T^{trig} = 3.5 GeV

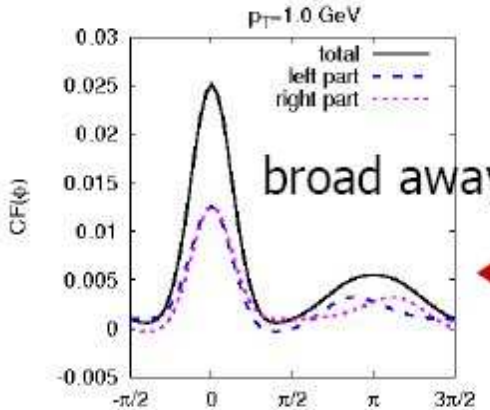
broad away-side peak



broad away-side peak

Pure energy

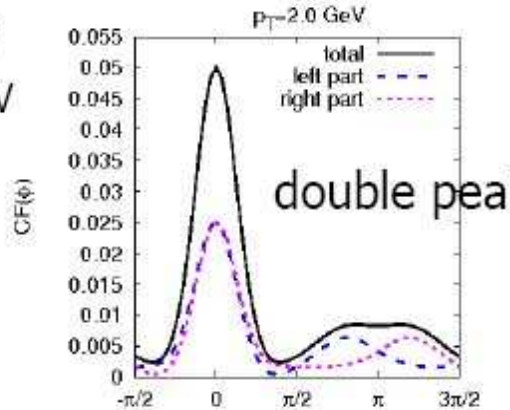
B.Betz QM2009



E_{tot} = 10 GeV

p_T^{trig} = 7.5 GeV

broad away-side peak



double peaked structure

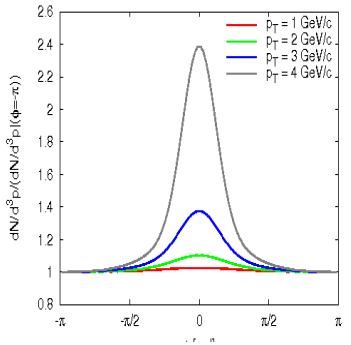
Mixed

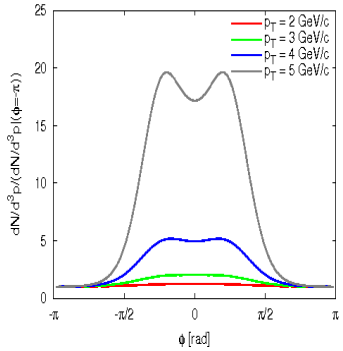
due to

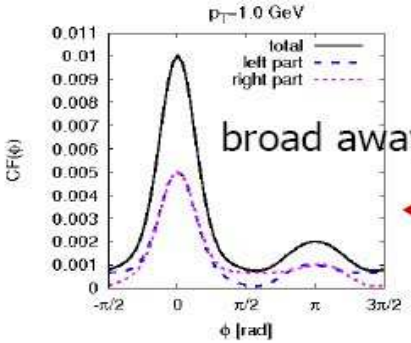
Lets compare.....

Static medium

Expanding medium



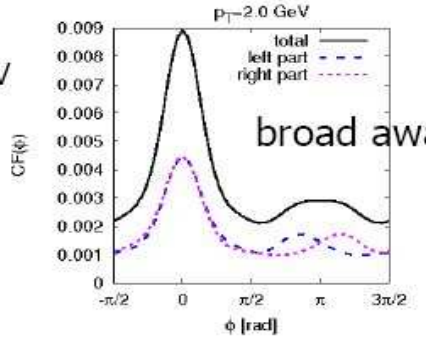




E_{tot} = 5 GeV

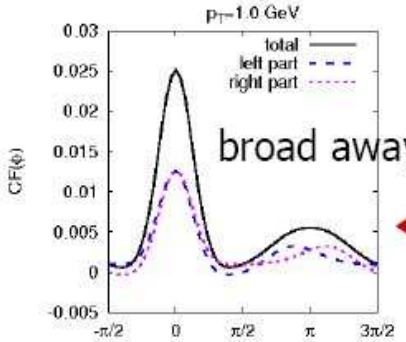
p_T^{trig} = 3.5 GeV

broad away-side peak



broad away-side peak

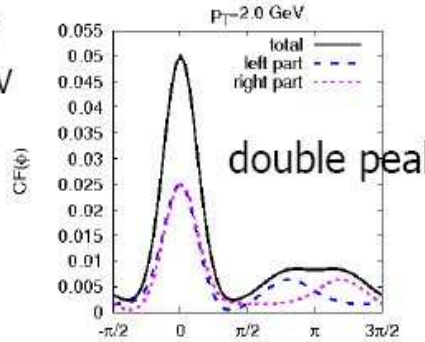
Pure energy



E_{tot} = 10 GeV

p_T^{trig} = 7.5 GeV

broad away-side peak



double peaked structure

Mixed

Results for flowing background opposite to static background.

Pure energy gives you no Mach-cone

Mixed deposition gives a "conical" signal

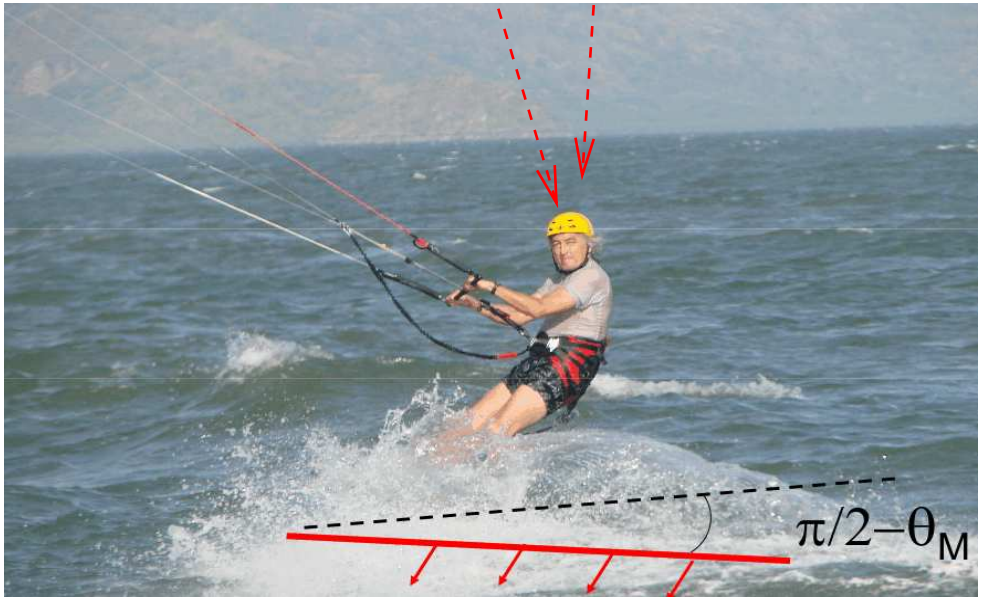
But is it a "true" cone? Or rather a distorted wake? In the 2nd case its a Fake cone, no Mach's law for angle. To distinguish

- Vary v for heavy quark cones, is $\theta_{mach} = \sin^{-1}(c_s/v_{jet})$?
- Bin by $\phi_{trigger} - \phi_{reaction}$ Does cone angle vary?

Instead of a conclusion...

we dont know yet!

And BOTH theoretical AND experimental understanding is needed!



Questions for users

- Heavy quark jets: Does angle depend on v_{jet} in heavy quarks?
- Non-central collisions: Does angle depend on $\psi_{jet} - \psi_{reaction}$?
- 2-component vs harmonics: ZYAM experimental verification? Soft observables in "jetty" samples?
- Energy dependence? WHat to make of Mach cones where ideal hydro fails to describe v_2?
- Mach cones with reconstruced jets (disentangle fragmentation from conical flow)
- More hard-soft correlations with identified particles

Questions for theorists

- So, two-component is bad! What should experimentalists use instead?
- How to distinguish real from fake cones?
- Include <u>all</u> freeze-out effects, survey <u>all</u> freezeout models

Additional slides