QCD at small x and parton saturation

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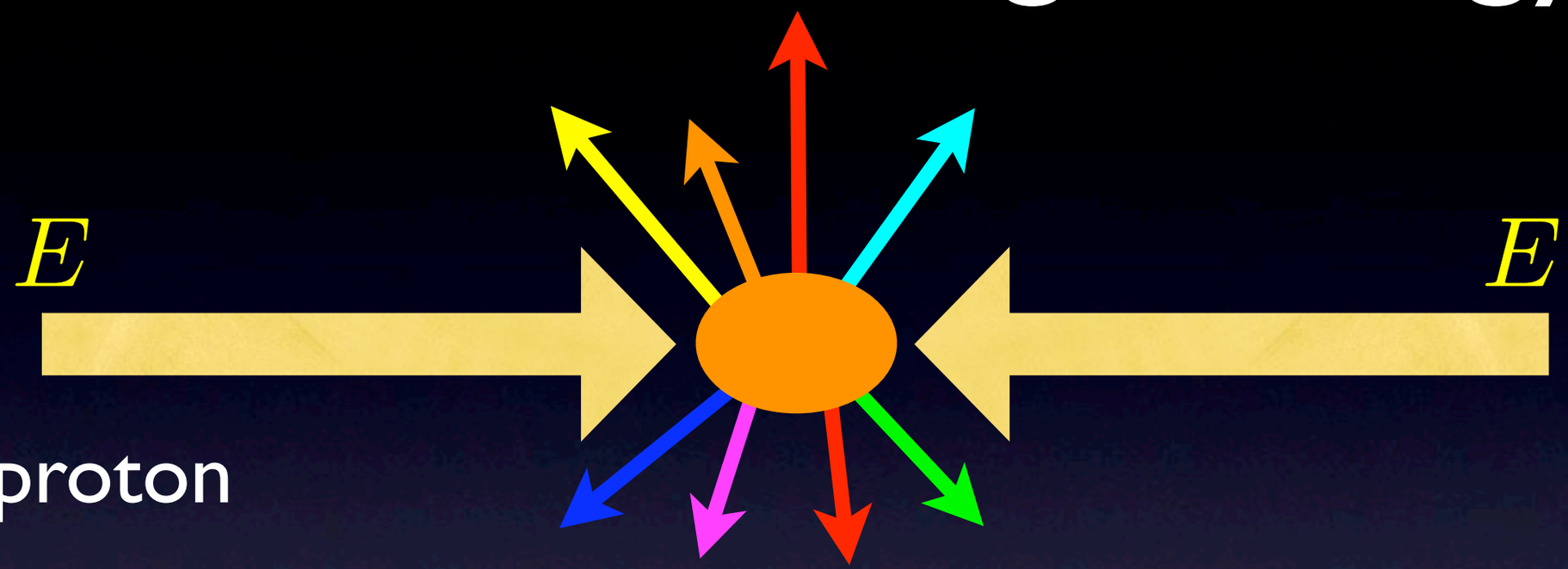
Penn State University & RIKEN Center BNL

RHIC and AGS Users Meeting, BNL, June 2nd, 2009

Outline

- Introduction and motivation:
  - Proton structure at high energy (small x).
  - Gluon radiation.
- Resummation in the evolution at small x.
- Parton saturation.

Collisions at high energy



ex. proton

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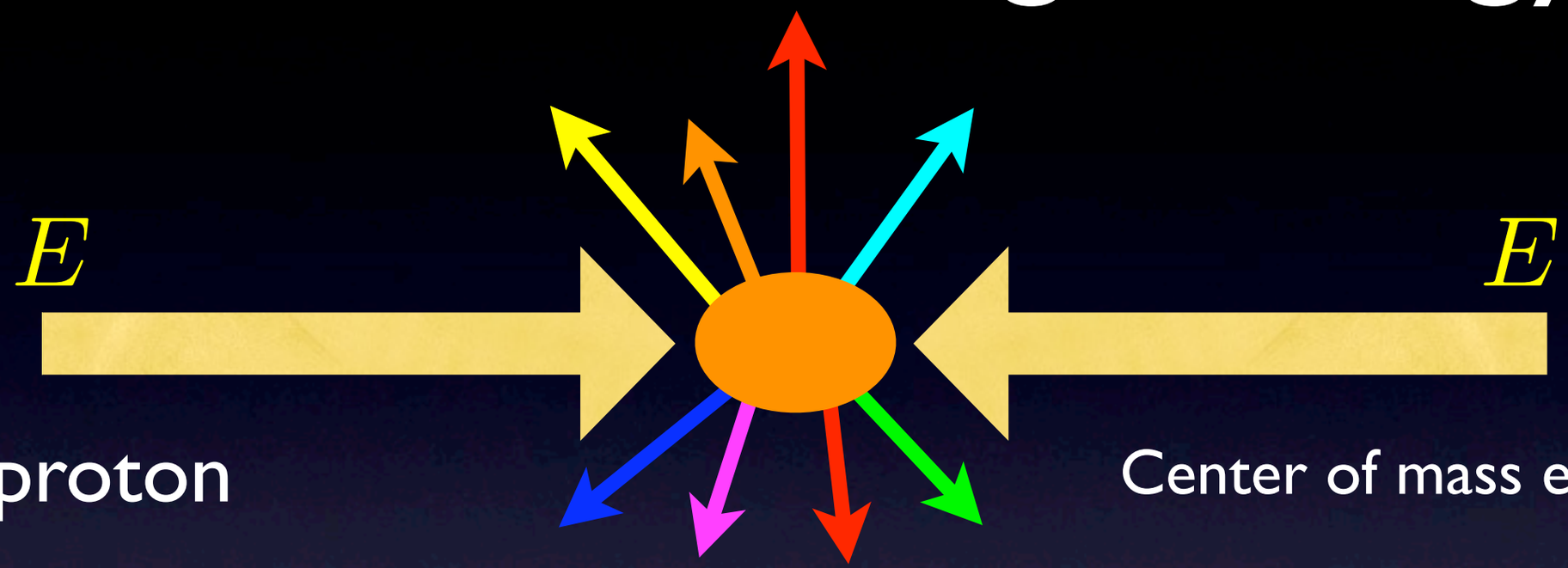

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Center of mass energy

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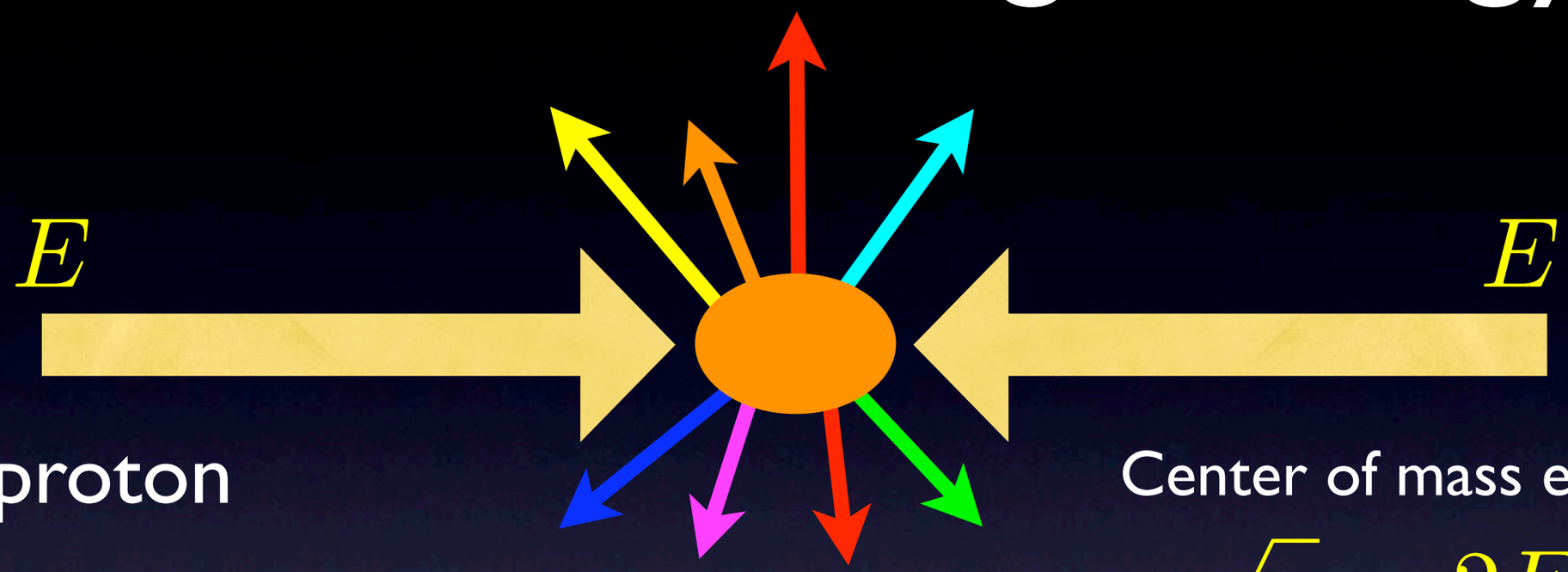

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Center of mass energy

$$\sqrt{s} = 2E \gg m_h$$

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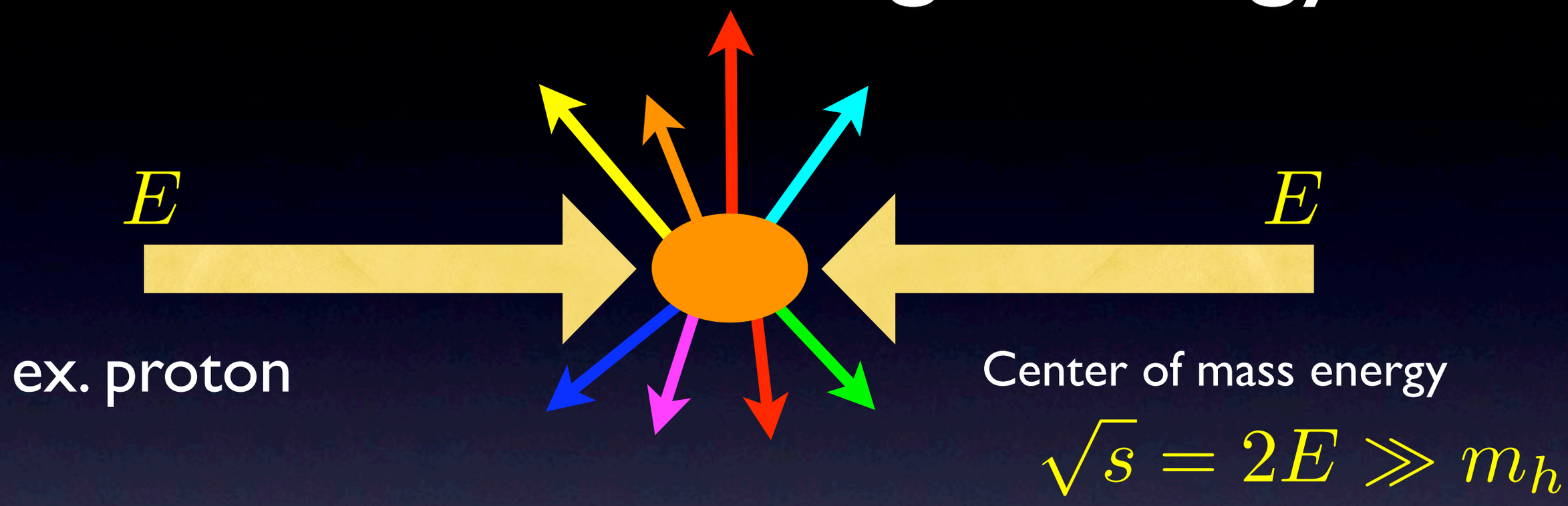

center-of-mass energy:

- RHIC (pp,AA): 200 GeV
- HERA (ep): 300 GeV
- Tevatron (pp): 1.96 TeV
- LHC (pp,AA): 14 TeV, 5.5 TeV
- Cosmic Rays (pA,AA): 100 TeV

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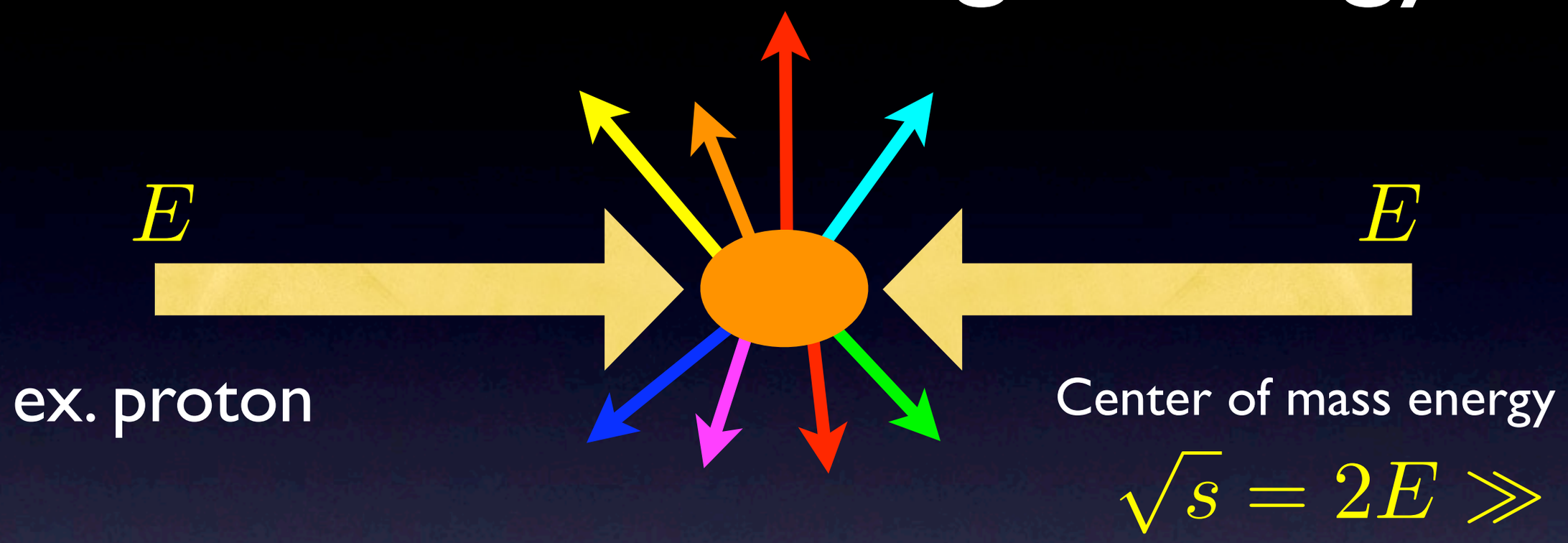

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How well do we know theory of strong interactions - QCD - at these high energies?

Quantum Chromodynamics

QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_{\text{flavours}} \bar{q}_a (i D^\mu \gamma_\mu - m_f)_{ab} q_b$$

Field strength:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\mu A^a_\nu - g f^{abc} A^b_\mu A^c_\nu$$

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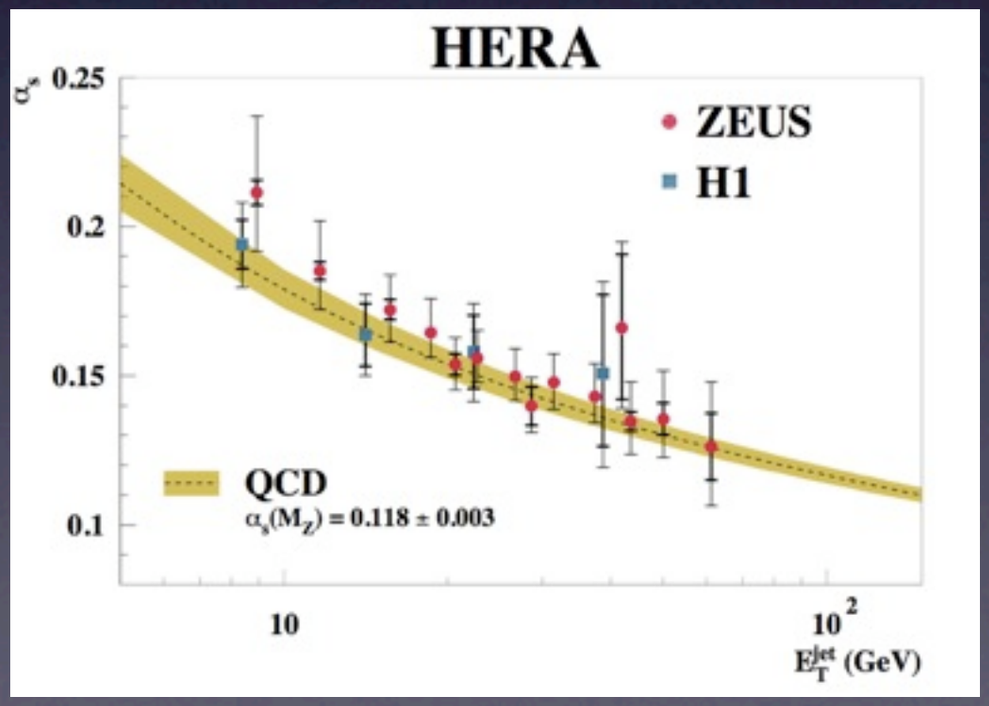

Small scales

Large scales

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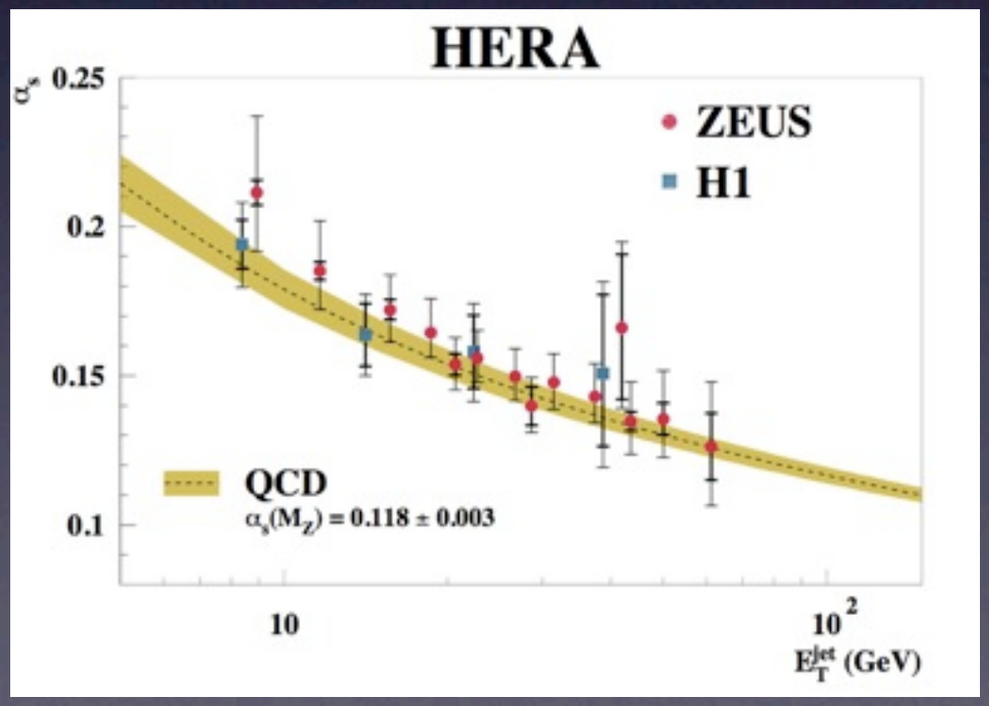

Small scales

Large scales

- Rich and very complicated structure due to non-linear interactions of gluons.
- Emergent phenomena: confinement, Regge trajectories, hadron spectrum.
- Complex dynamics at high energies or at small Bjorken x.

How do we know that gluons play such an important role at high energies?

Deep Inelastic Scattering

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Scattering of electron off a hadron(proton):

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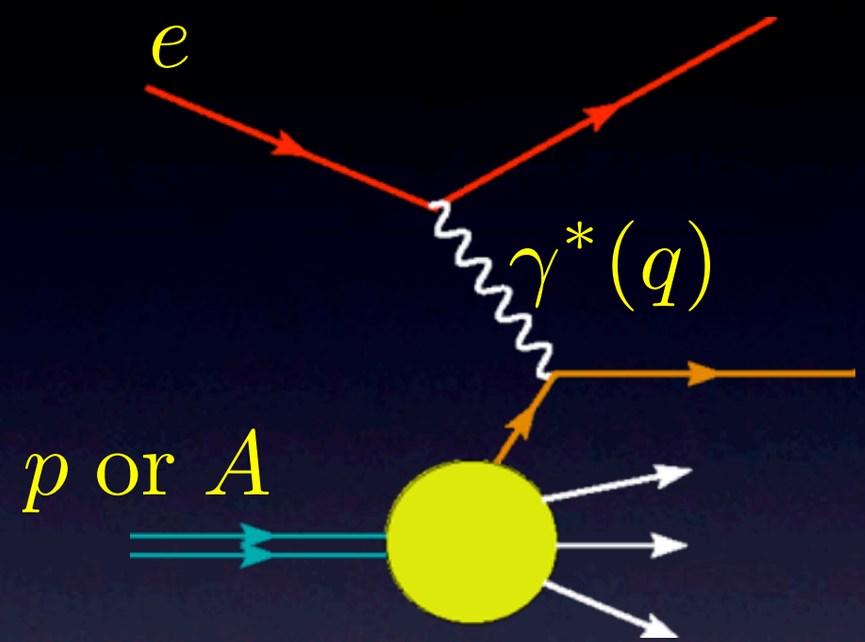

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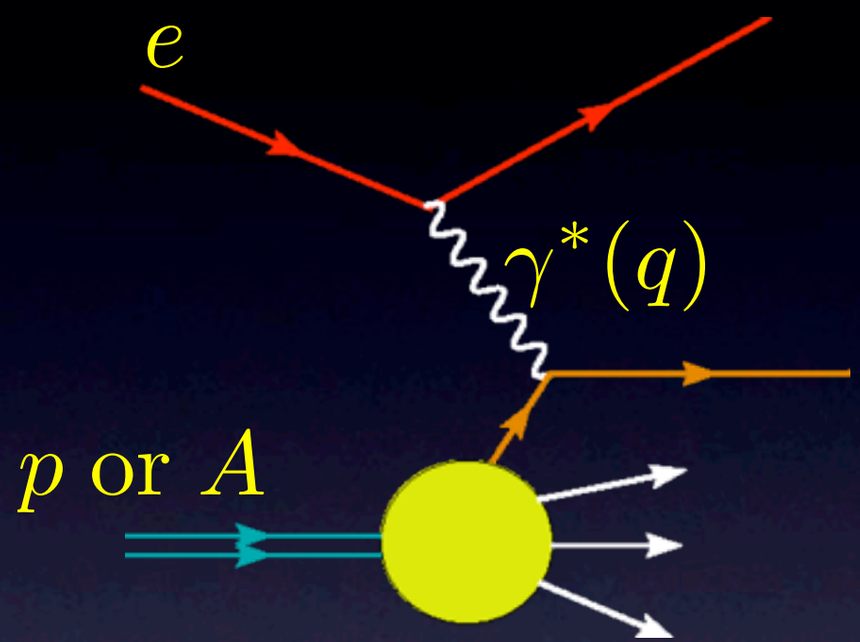

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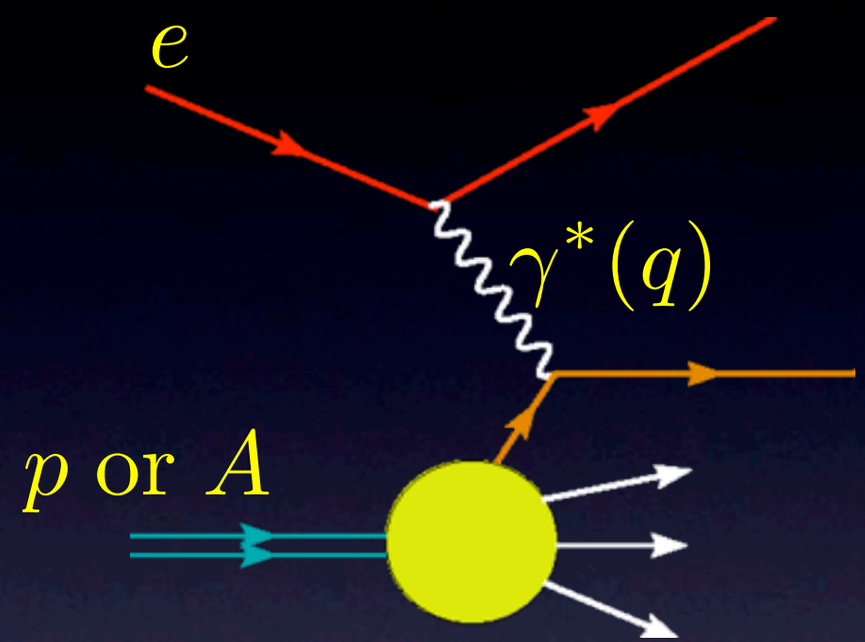

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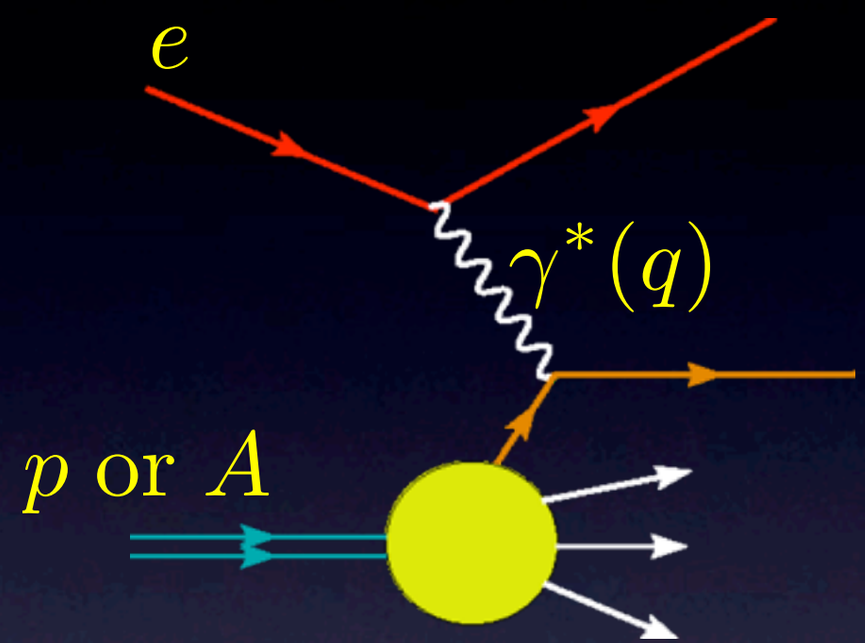

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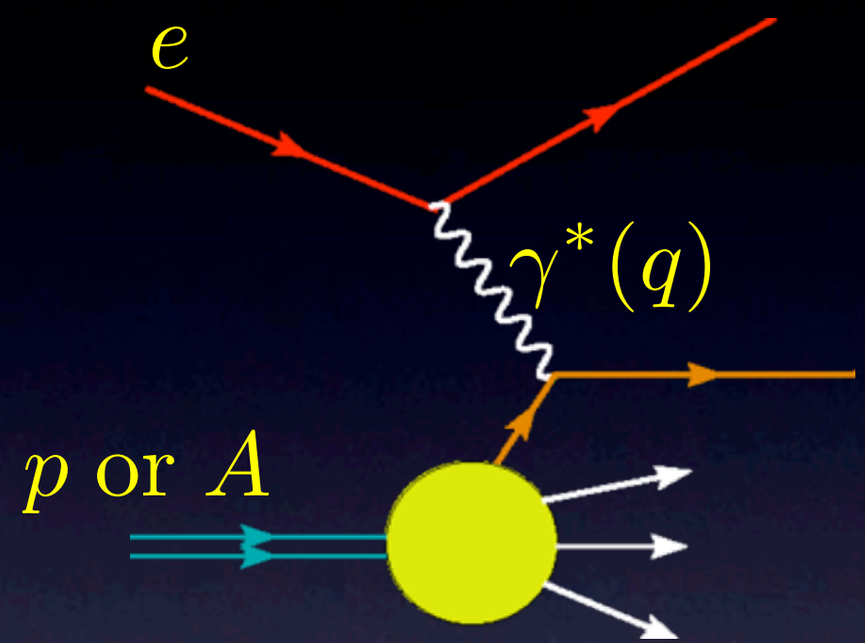

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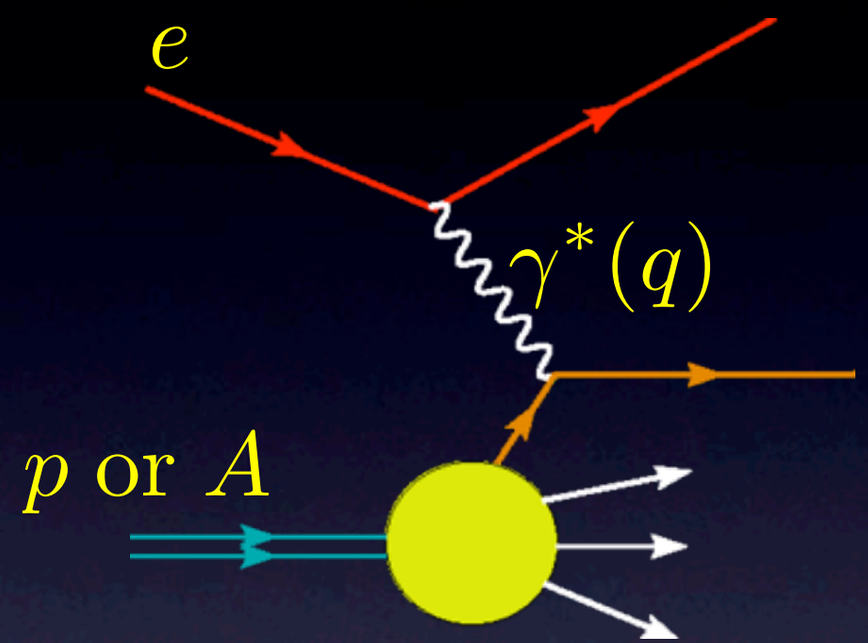

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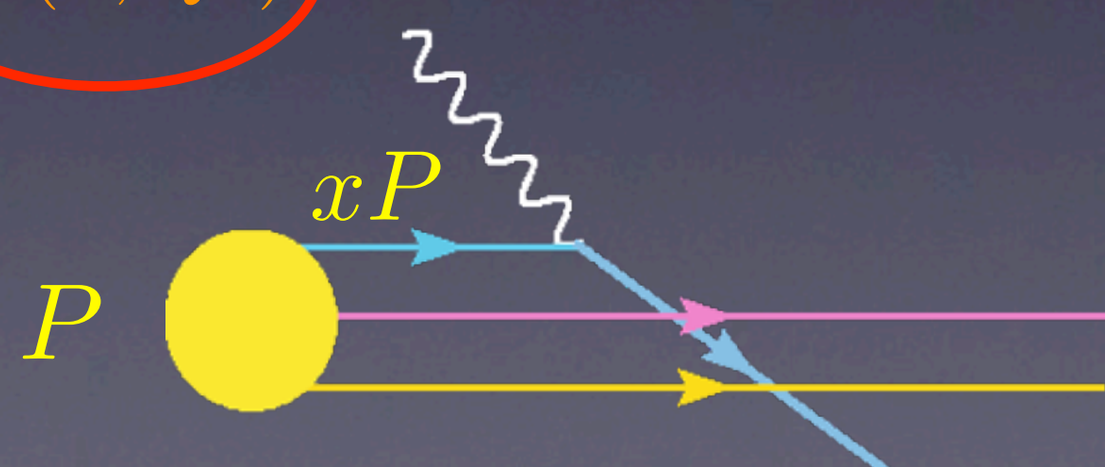
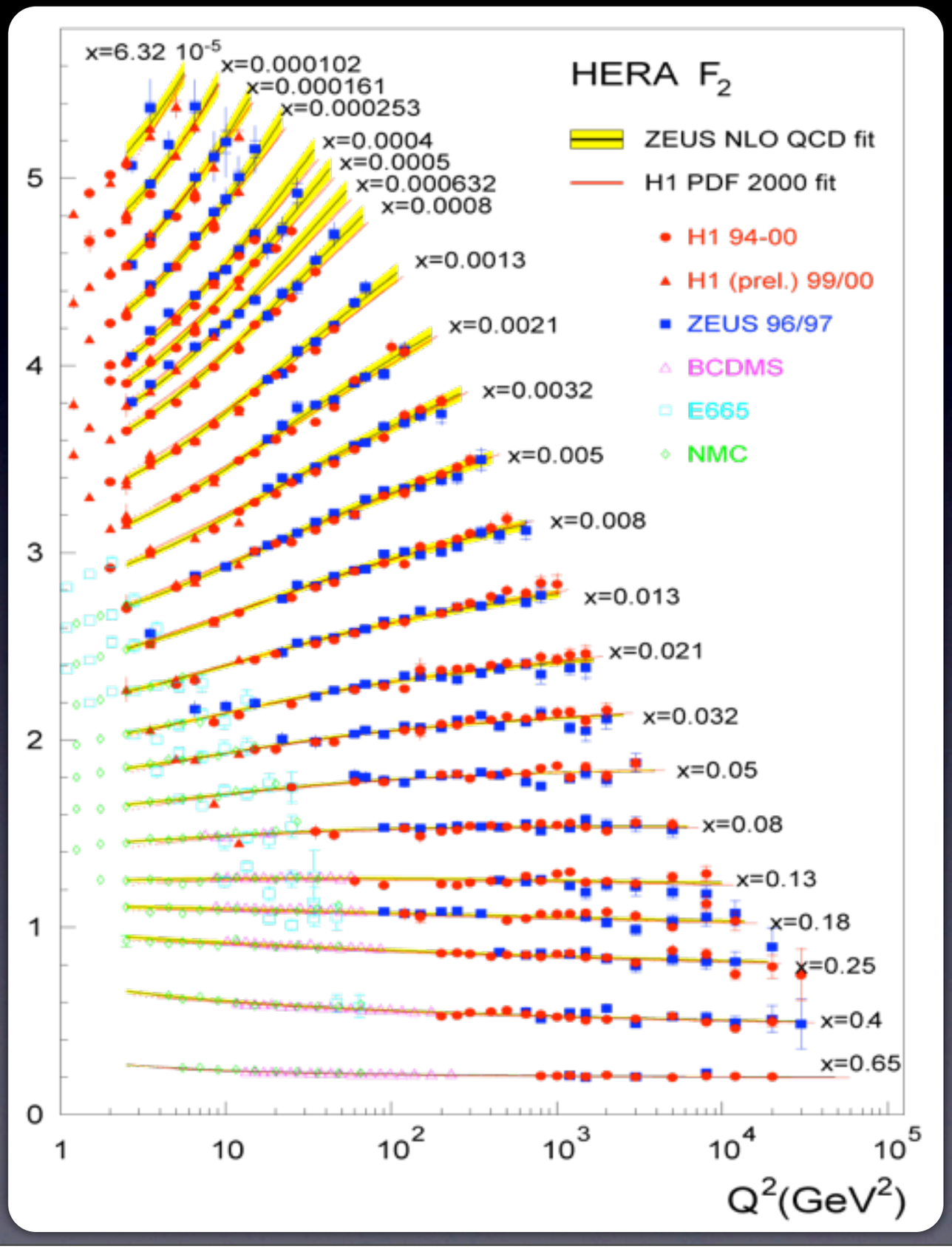


x fraction of the longitudinal momentum of the proton carried by the quark

DIS

$$\frac{d^2\sigma^{ep\to eX}}{dx dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$



Observation of large scaling violations.

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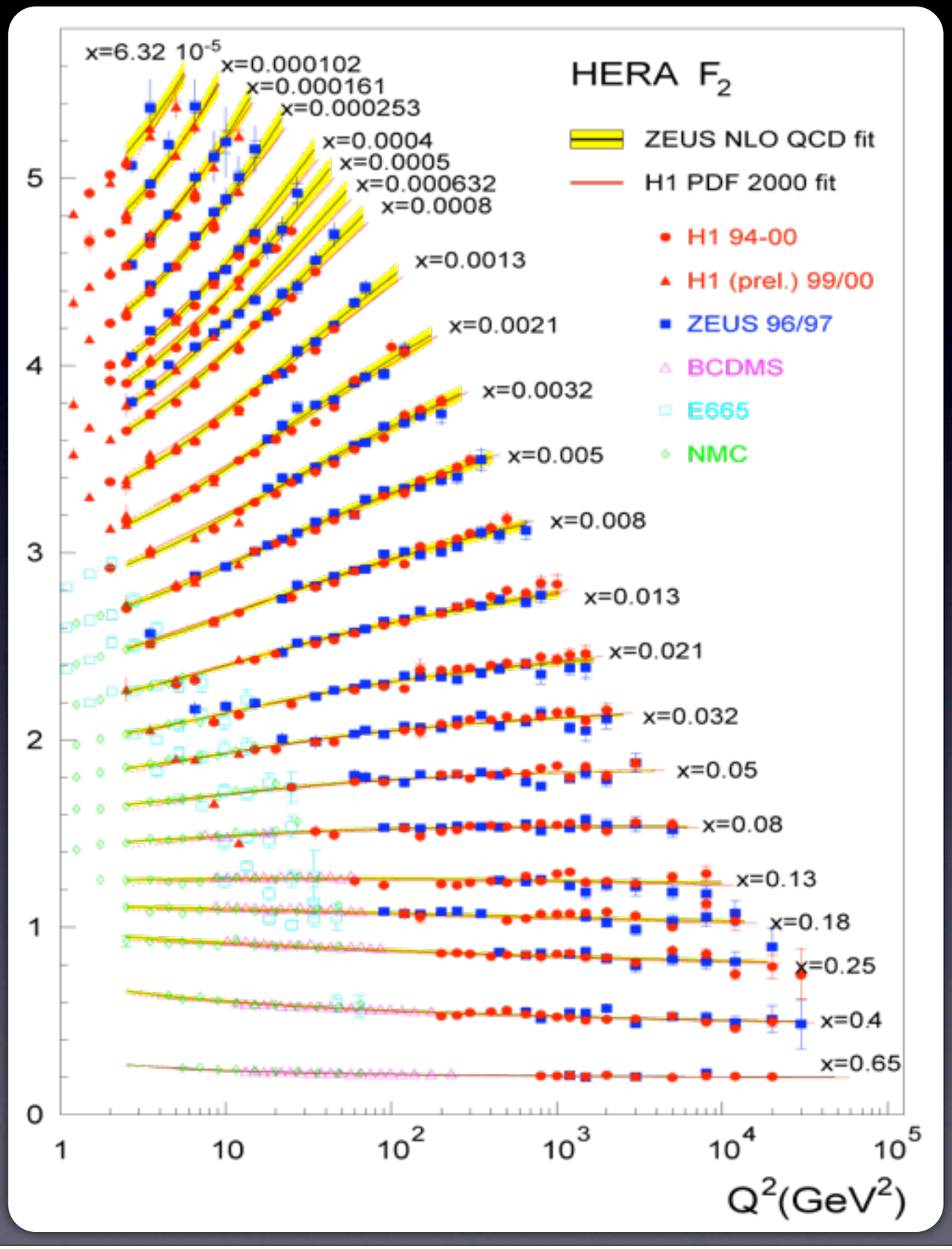

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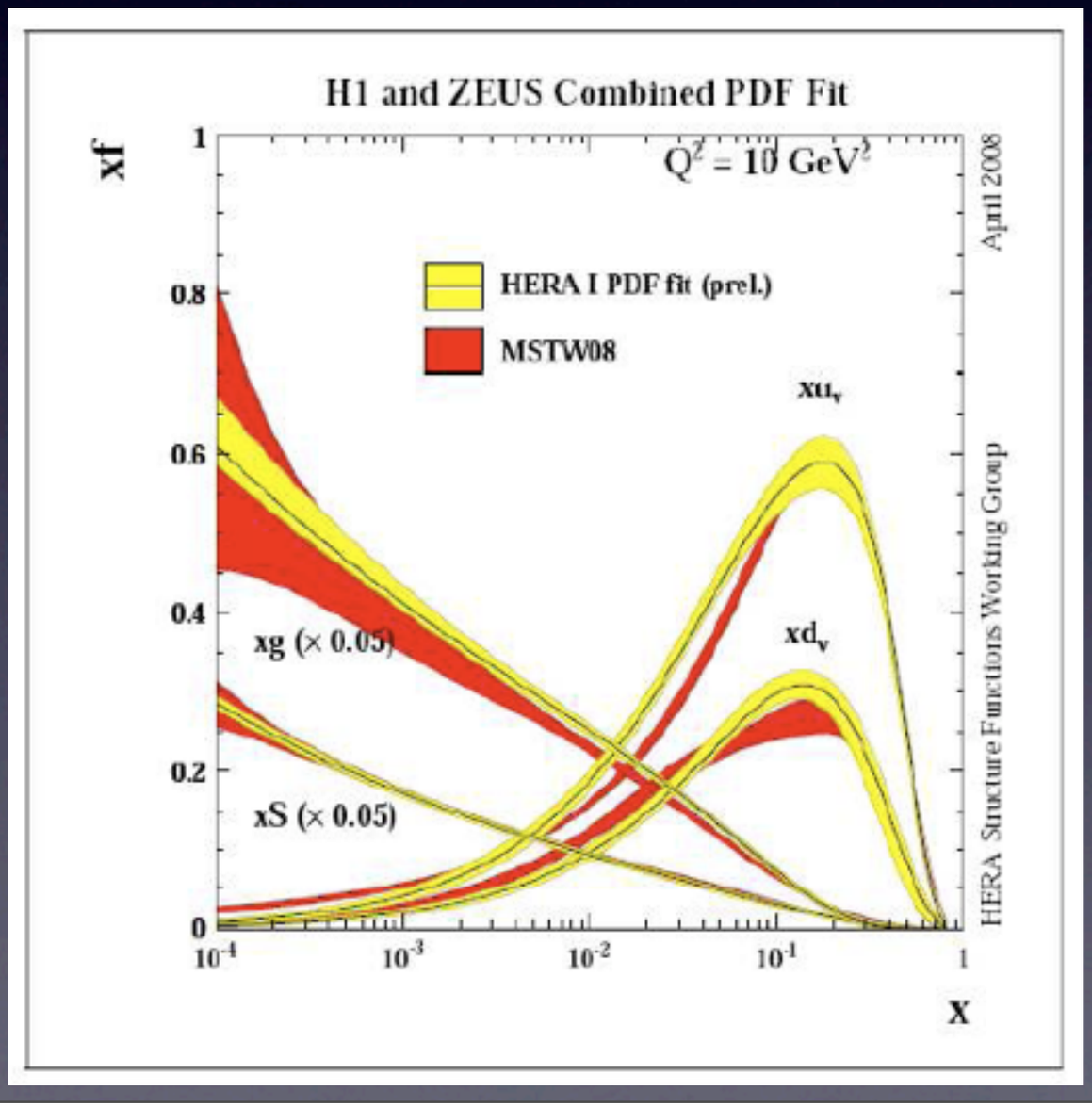

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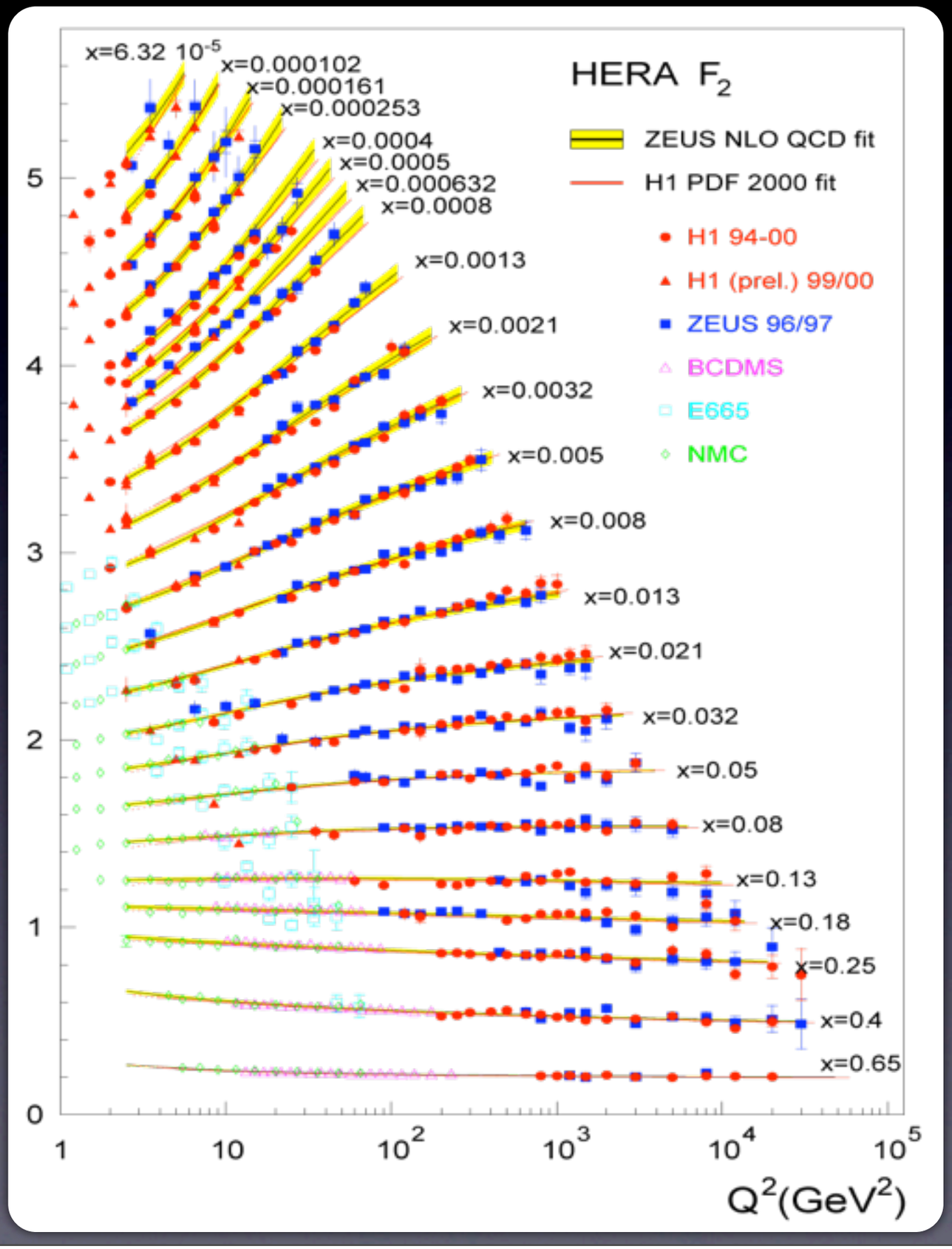
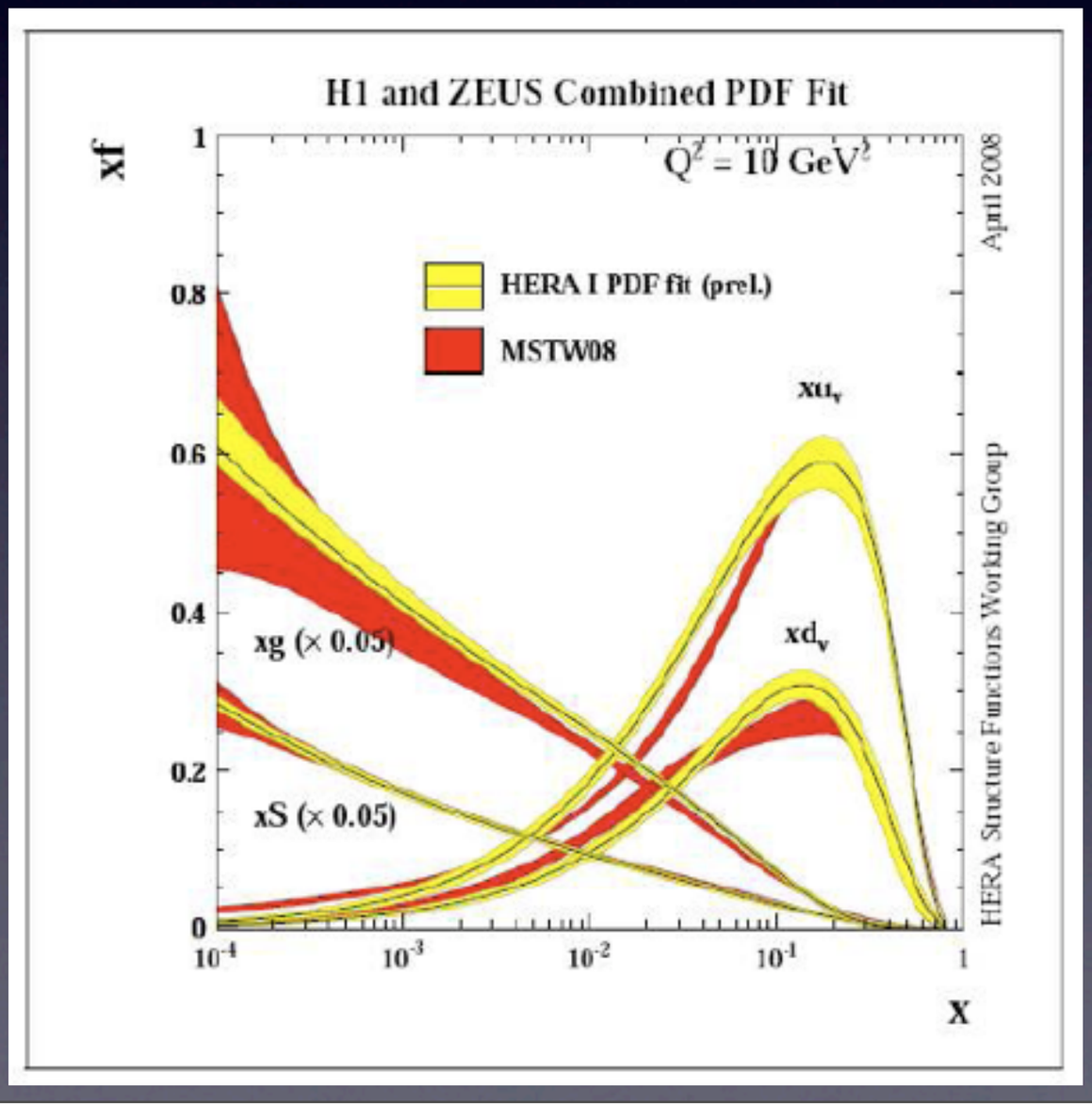
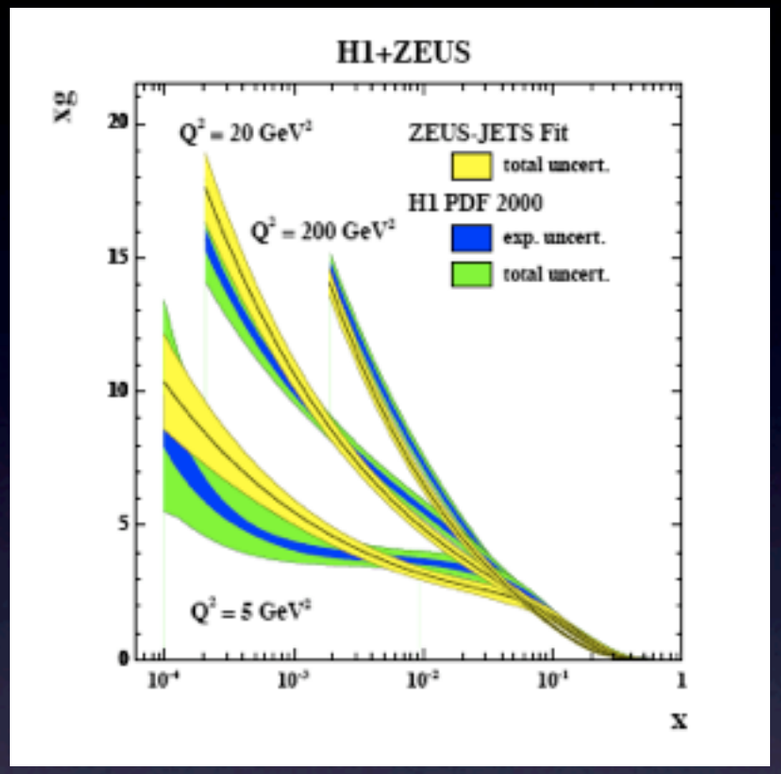


Observation of large scaling violations.

Gluon density dominates at small x!



Estimates of gluon density



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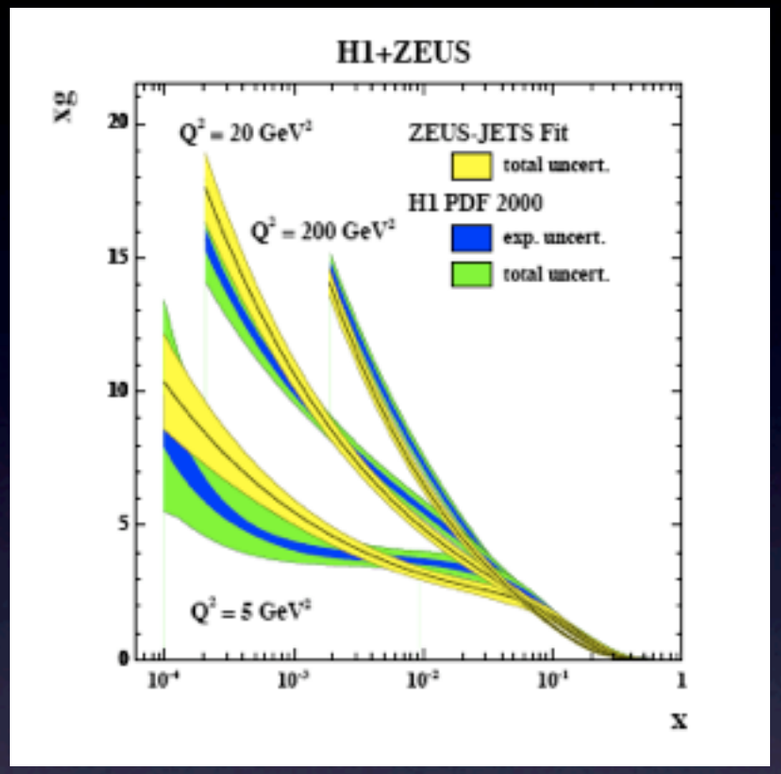

Extrapolation of gluon density to various energies

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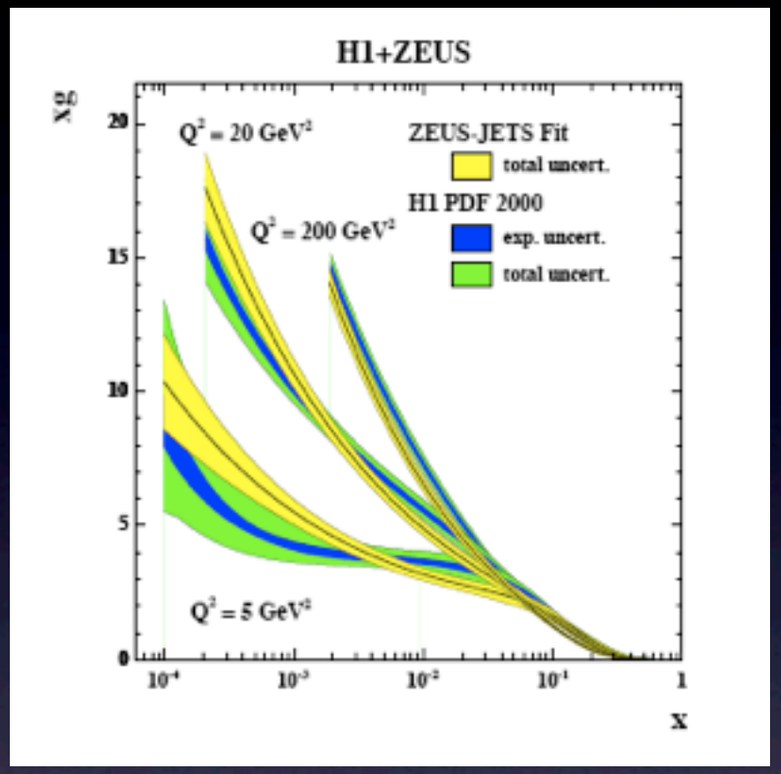

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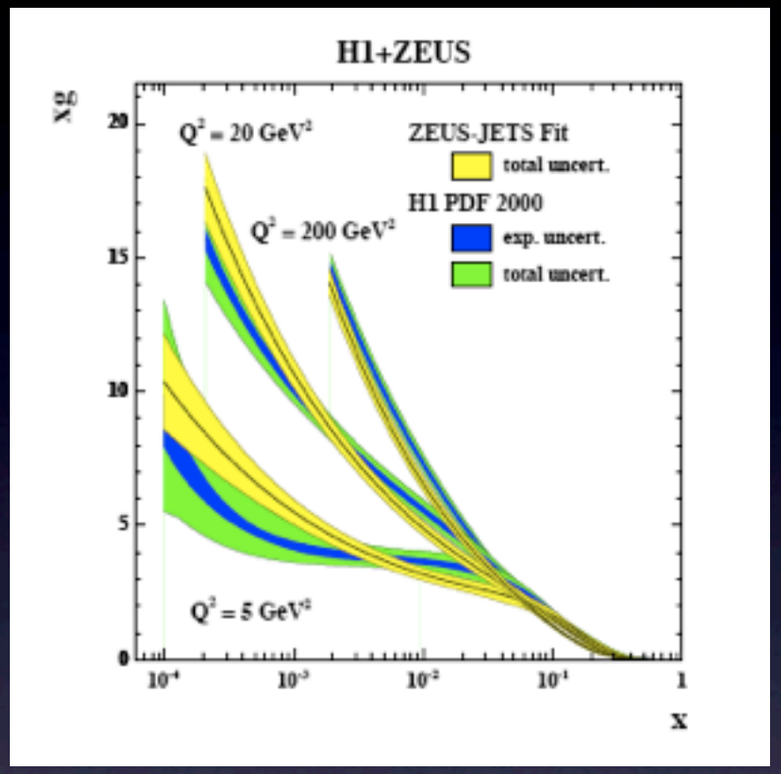

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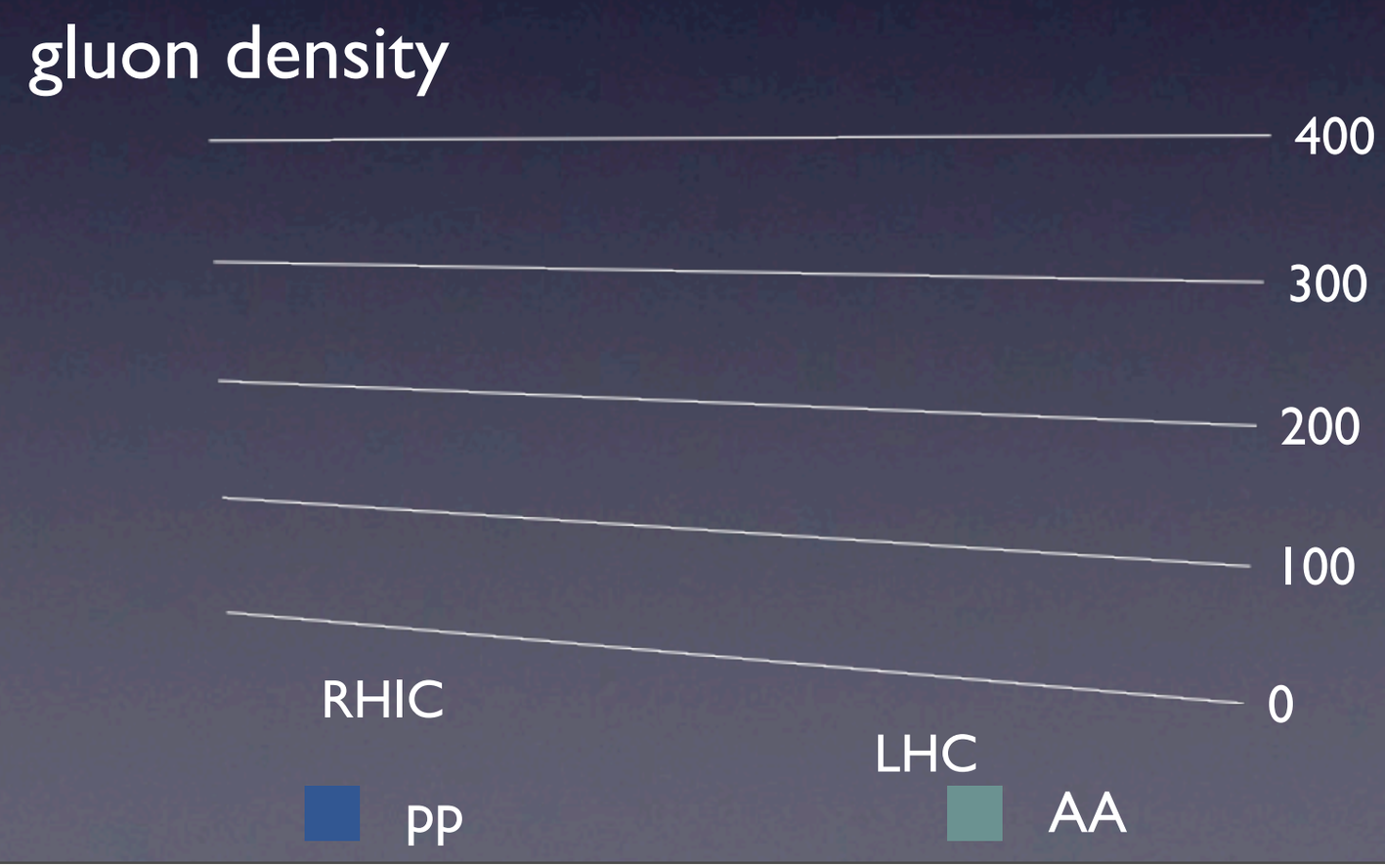

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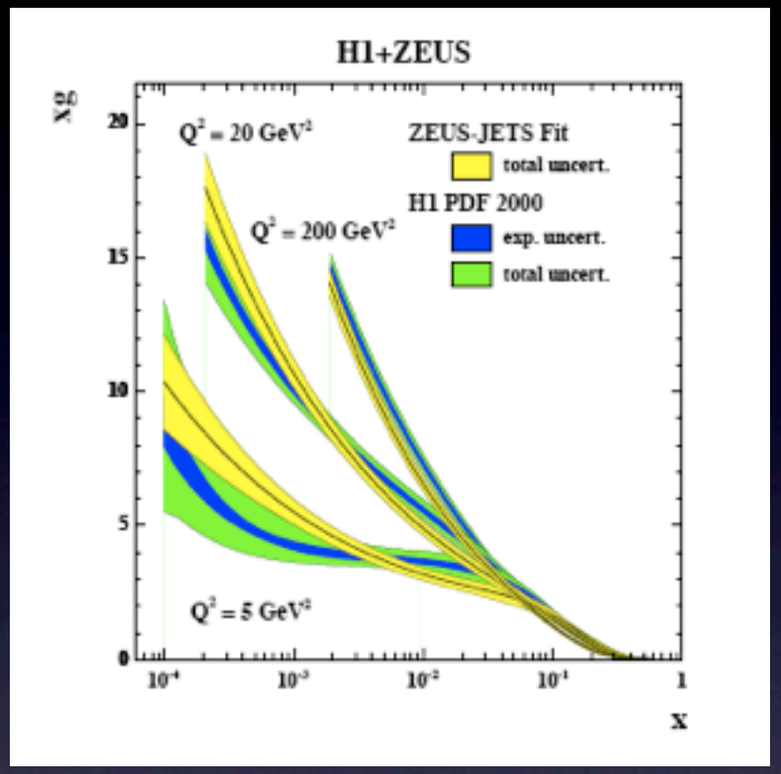

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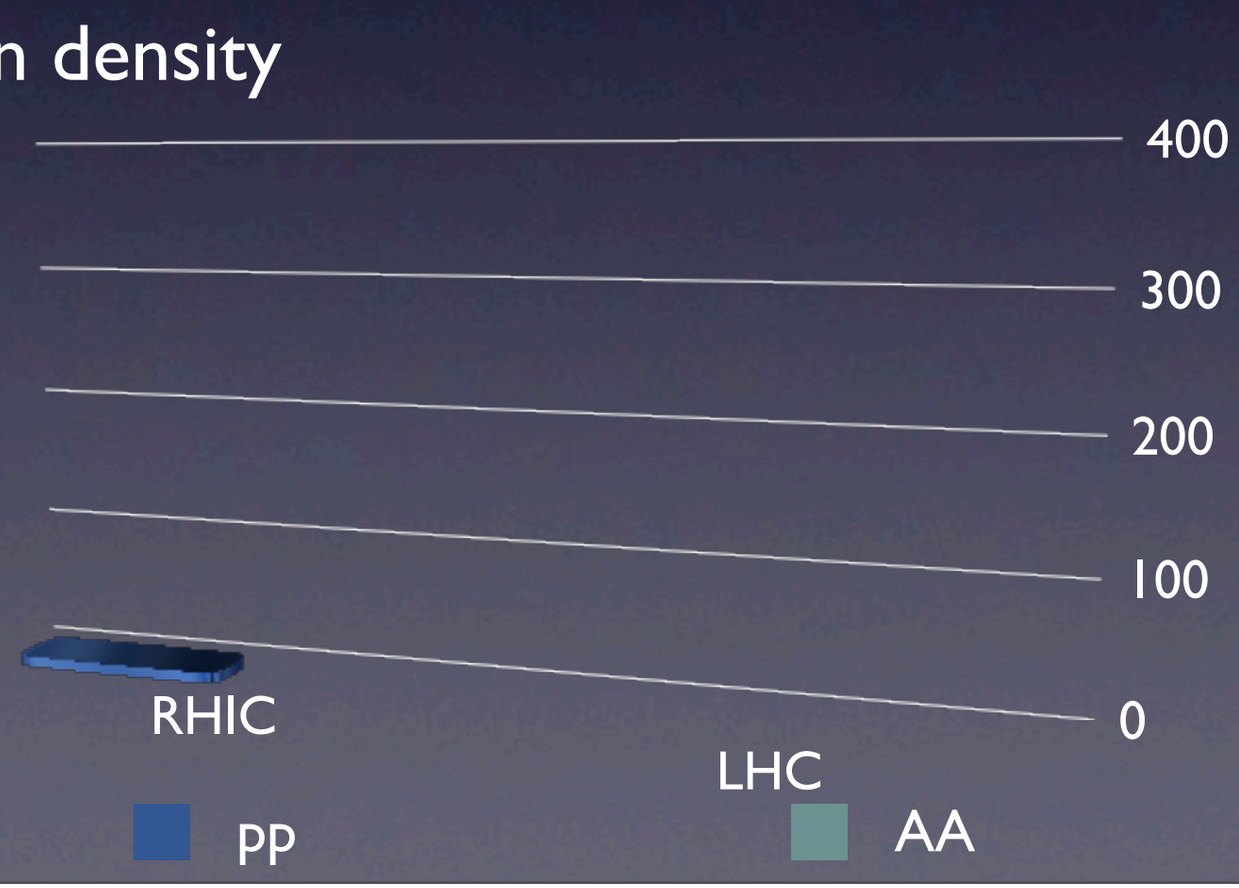

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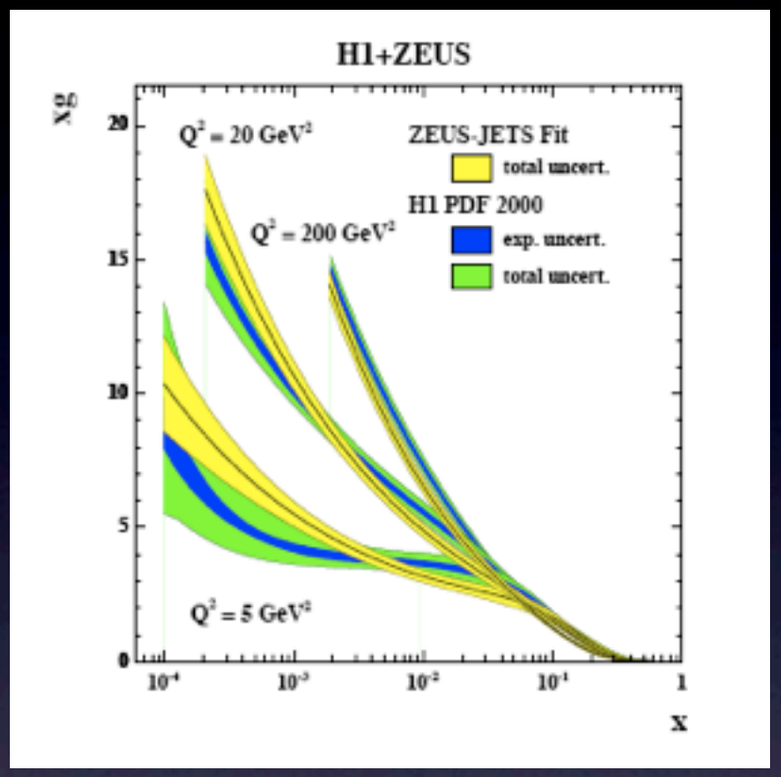

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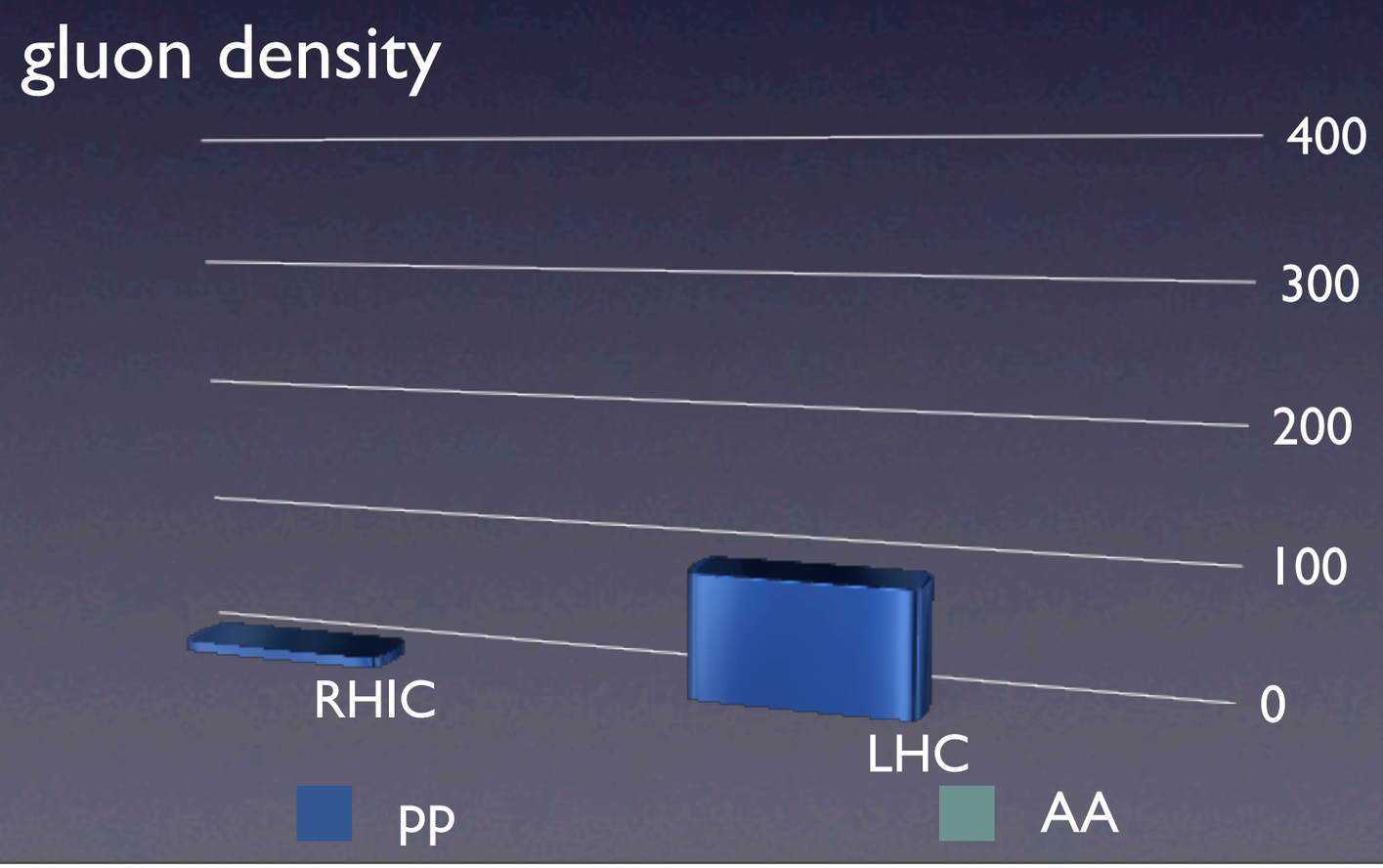

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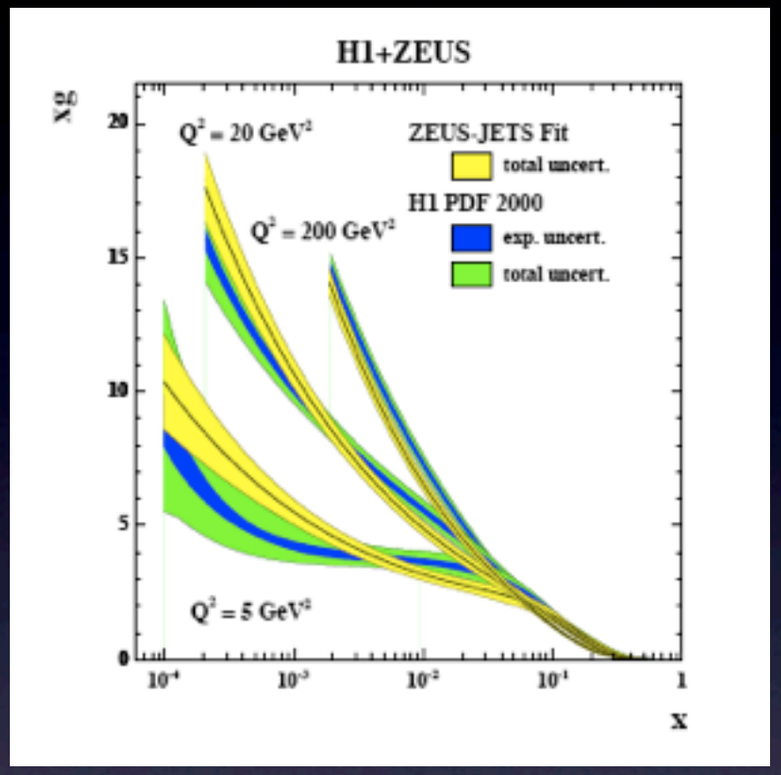

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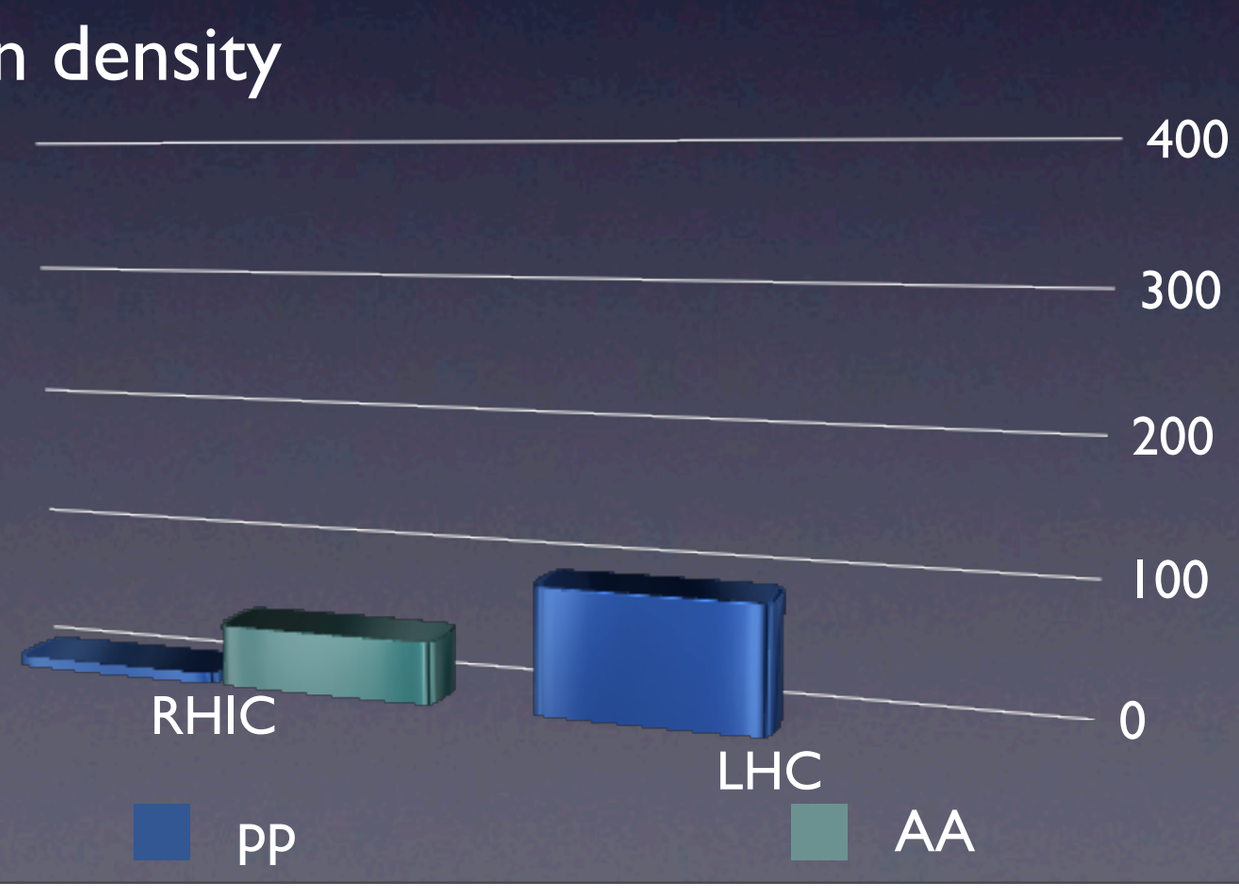

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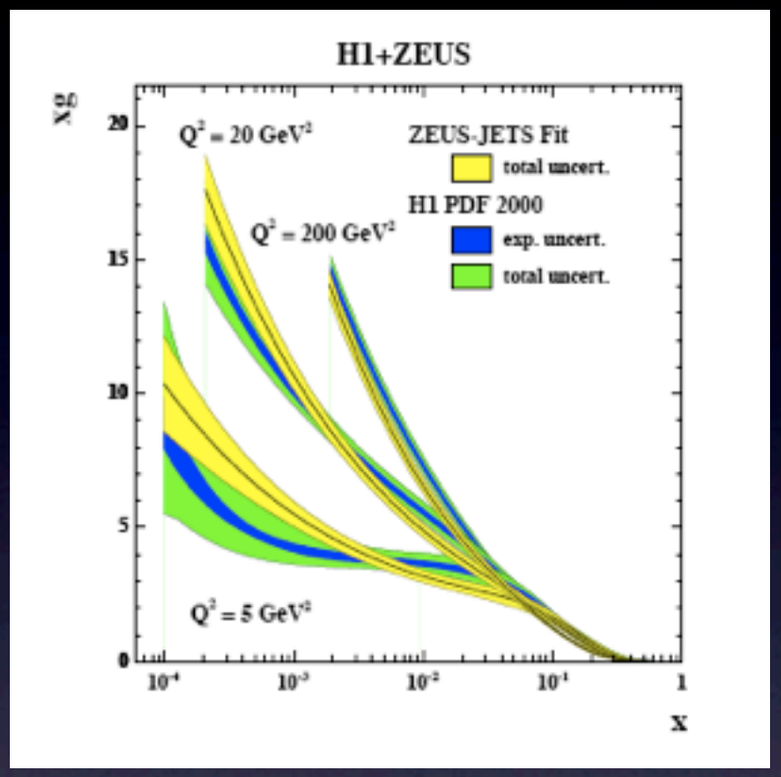

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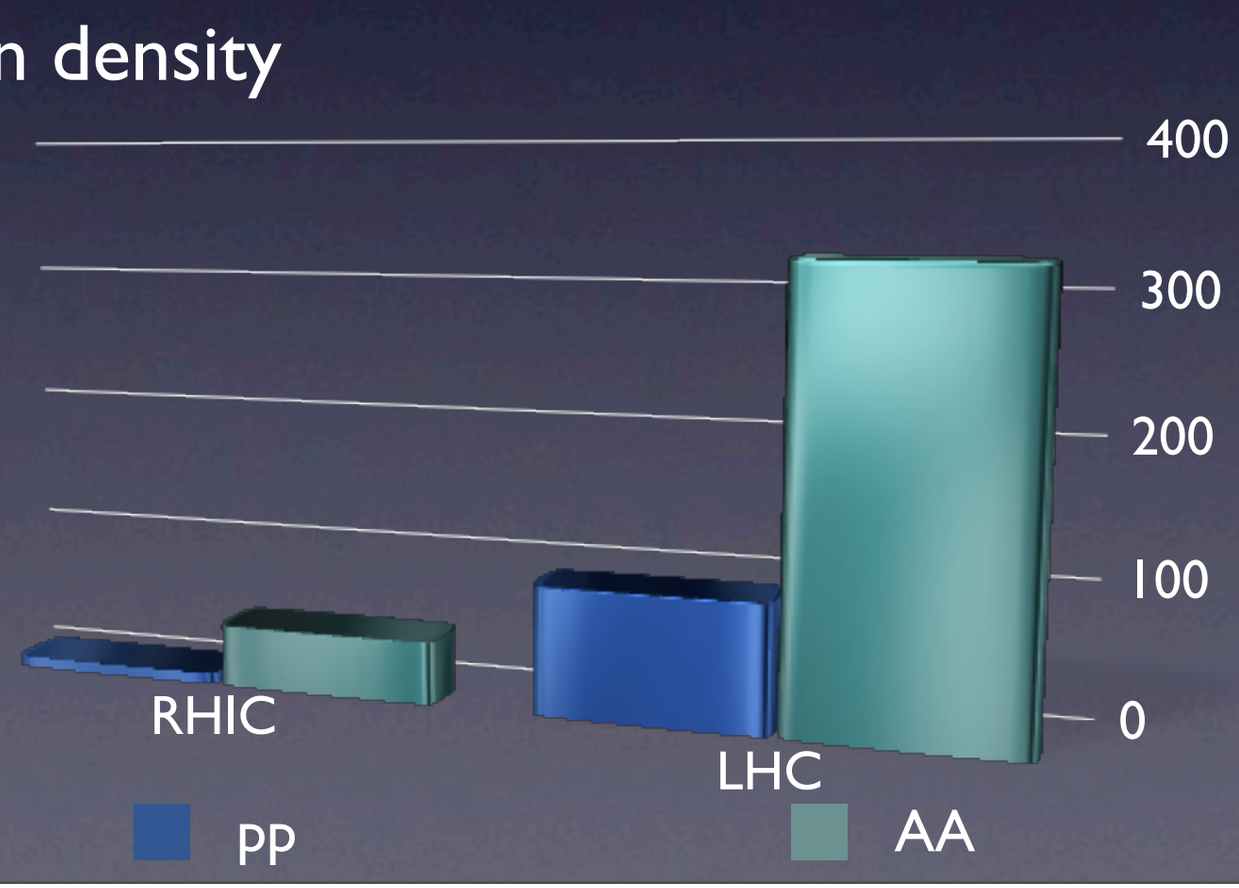

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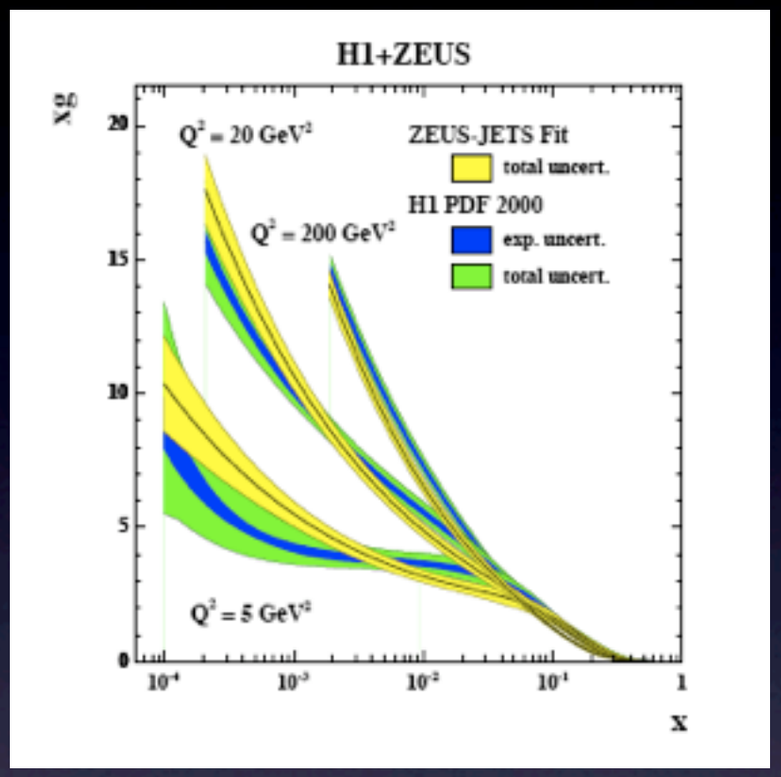

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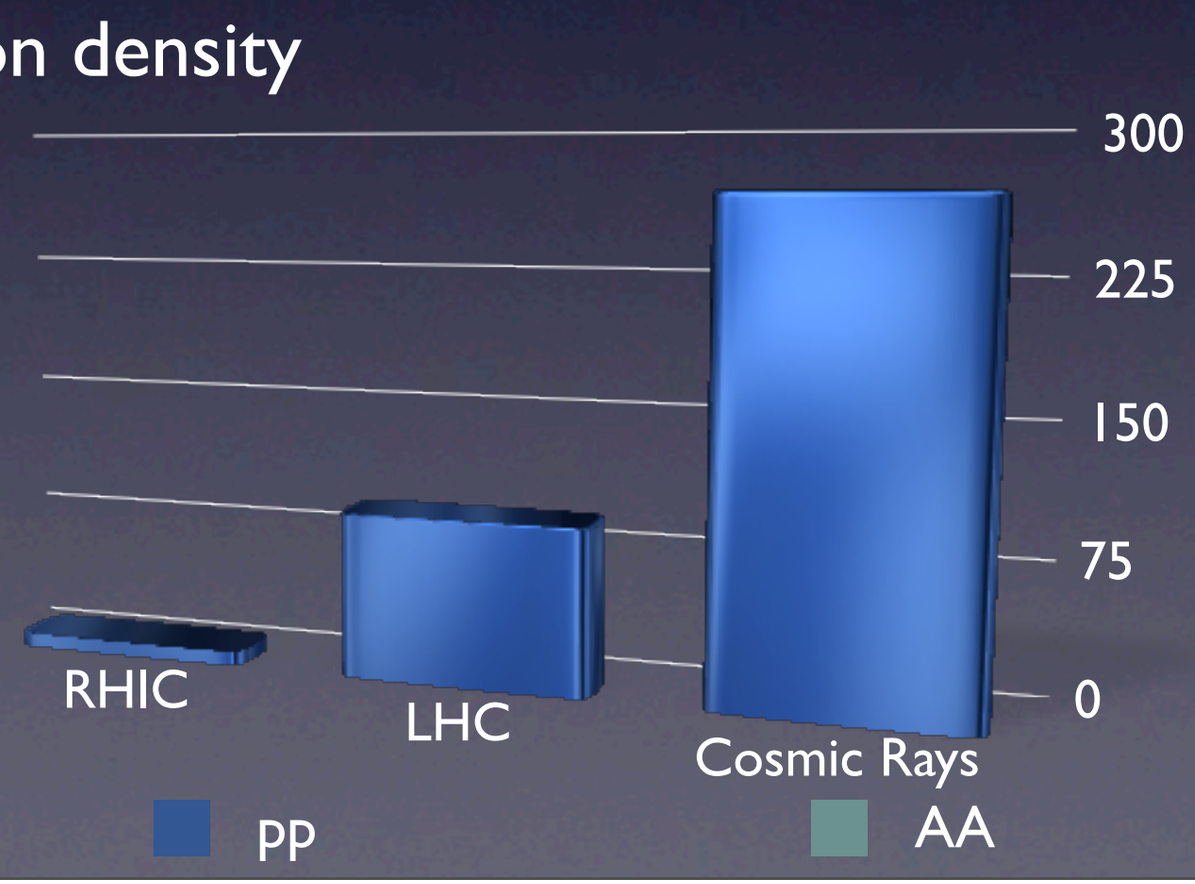

- Increase of the gluon density either via increasing energy or increasing size of the projectile (AA)
- Very small x probed at LHC and Cosmic Ray energies.
- Large gluon densities.

In hadron-hadron collisions

LHC

pp 14 TeV

PbPb 5.5 TeV

Cosmic Rays

pp or AA about 100 TeV

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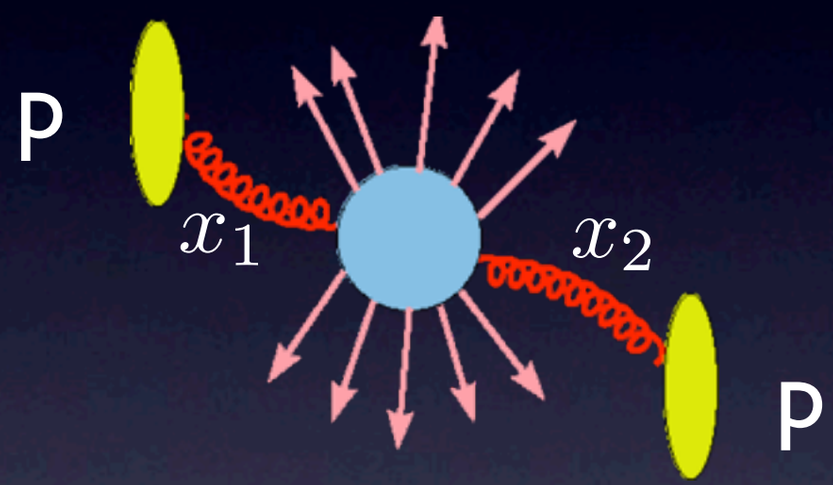

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x_1 x_2

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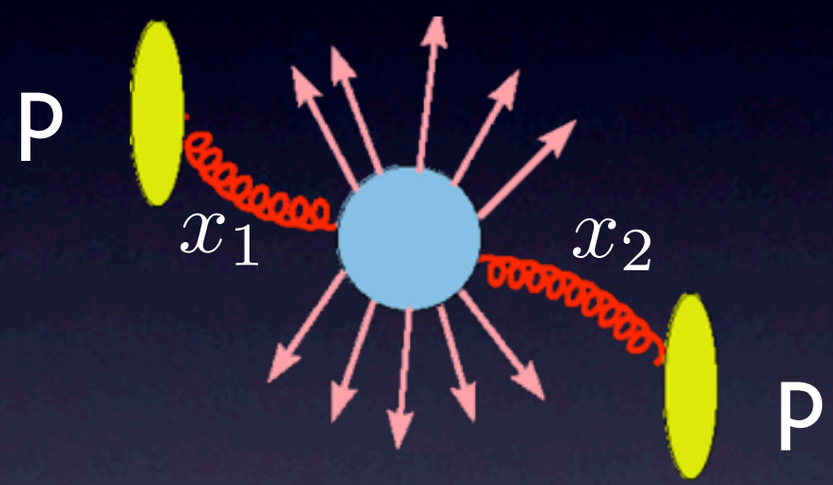

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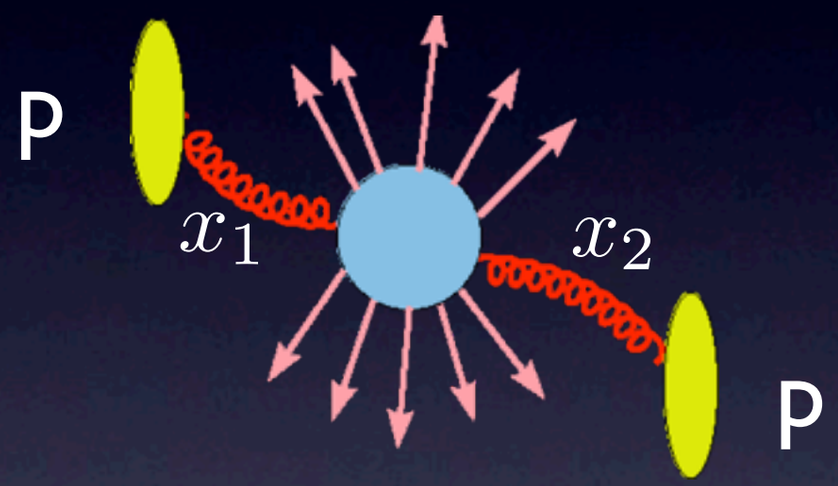

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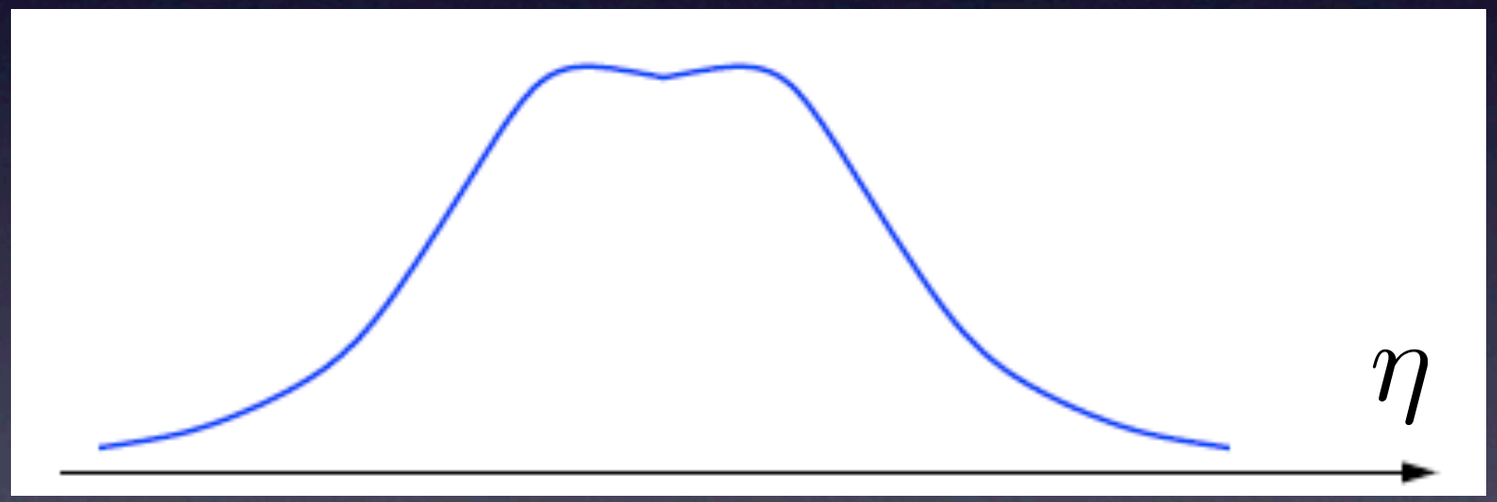
Pseudorapidity distribution of the produced particles



Pseudorapidity $\eta = -\ln(\tan\frac{\theta}{2})$

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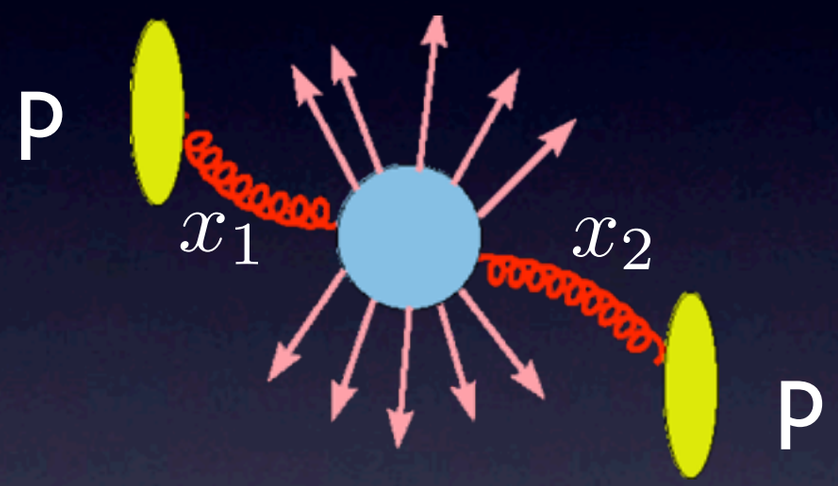

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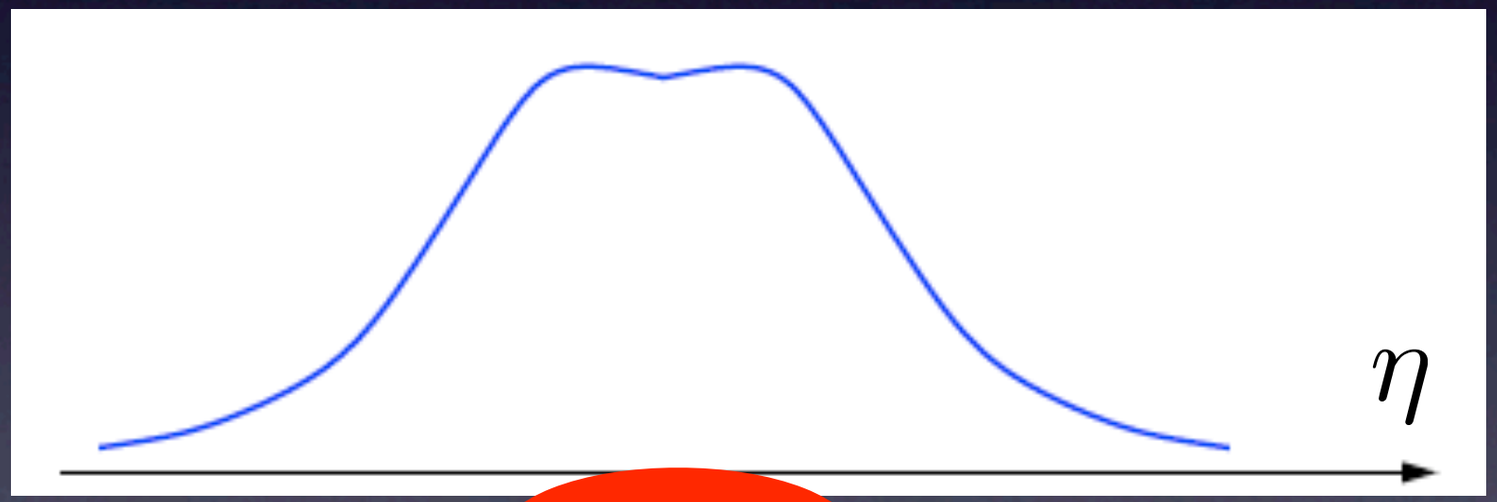

$x_1 \sim x_2$

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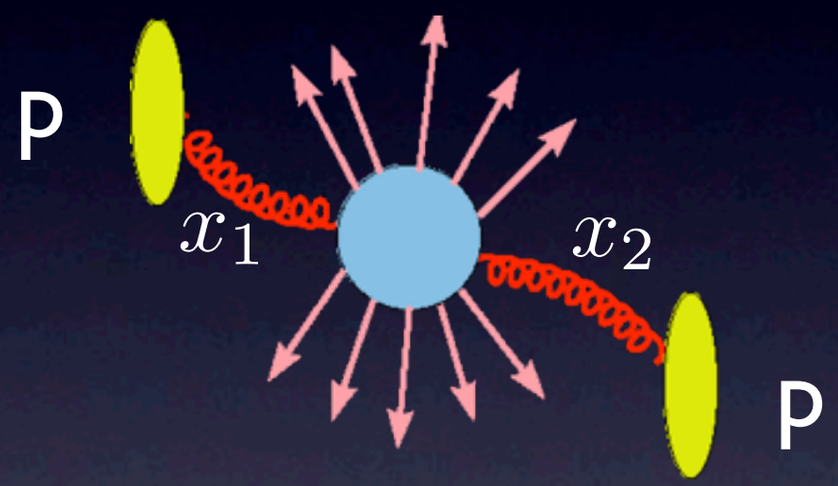

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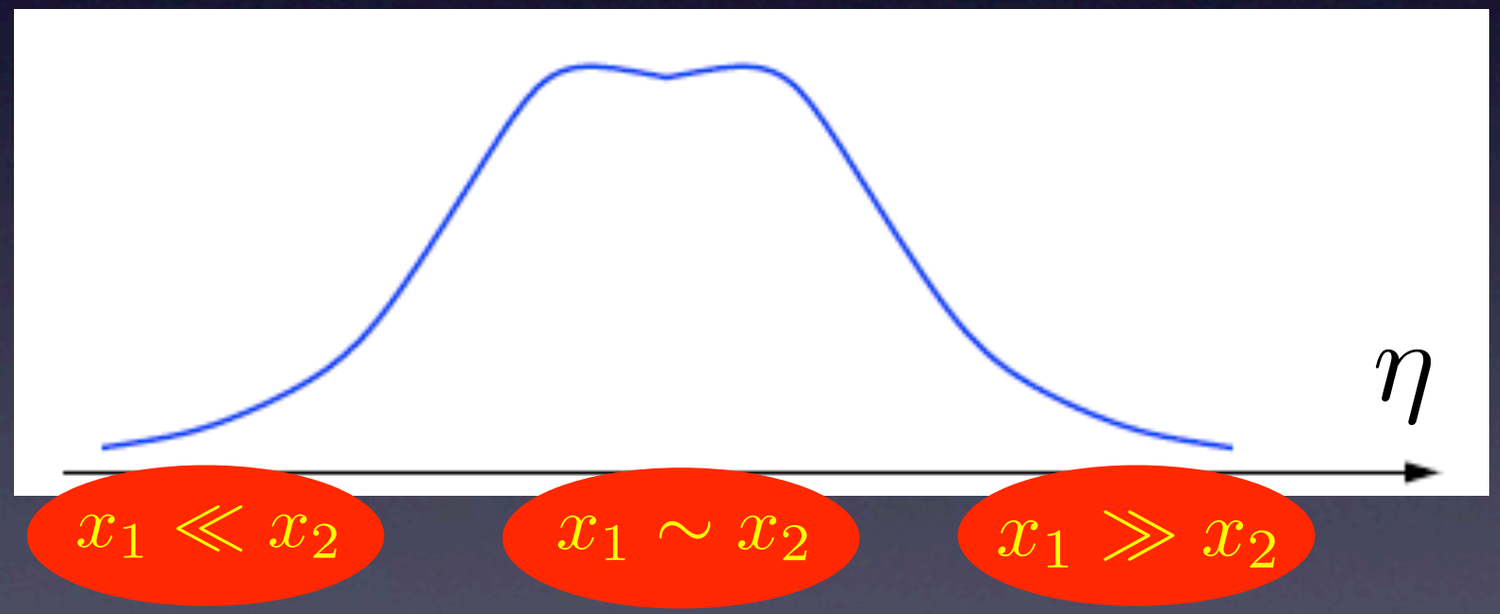

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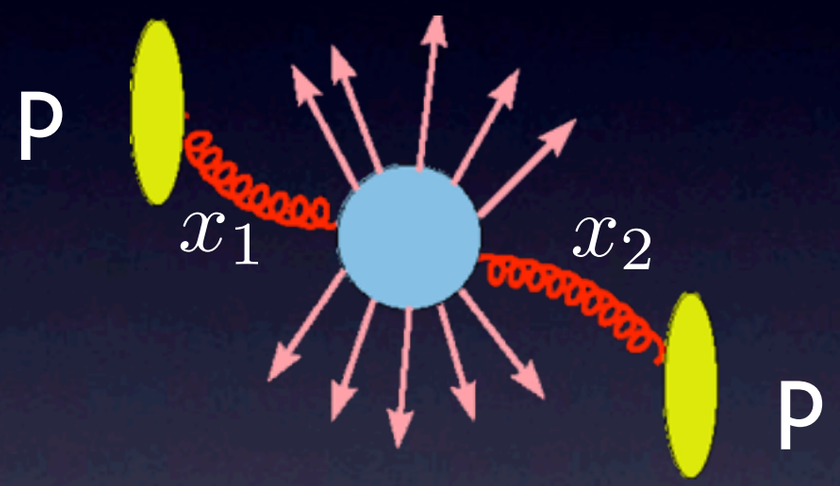


Typical values of x

Scale 2 GeV

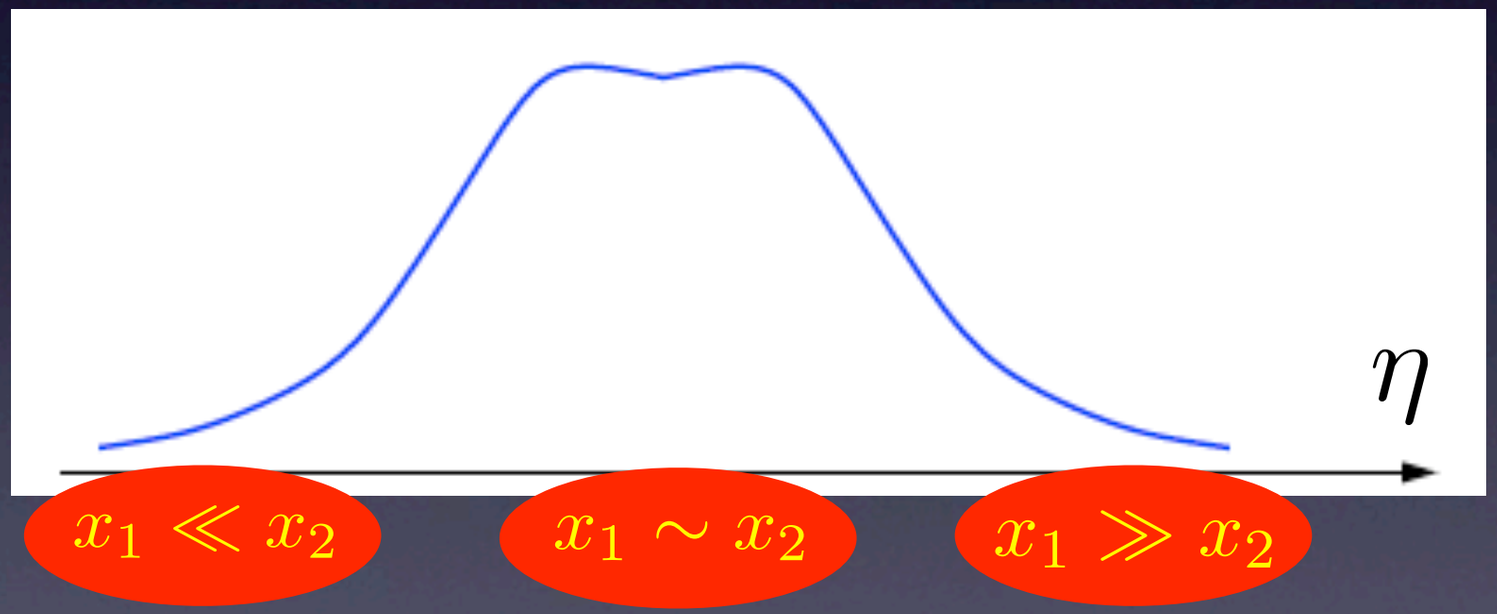
η	0	4
RHIC	0.01	0.0001
LHC	0.0001	0.000001

Cosmic Rays

pp or AA about 100 TeV

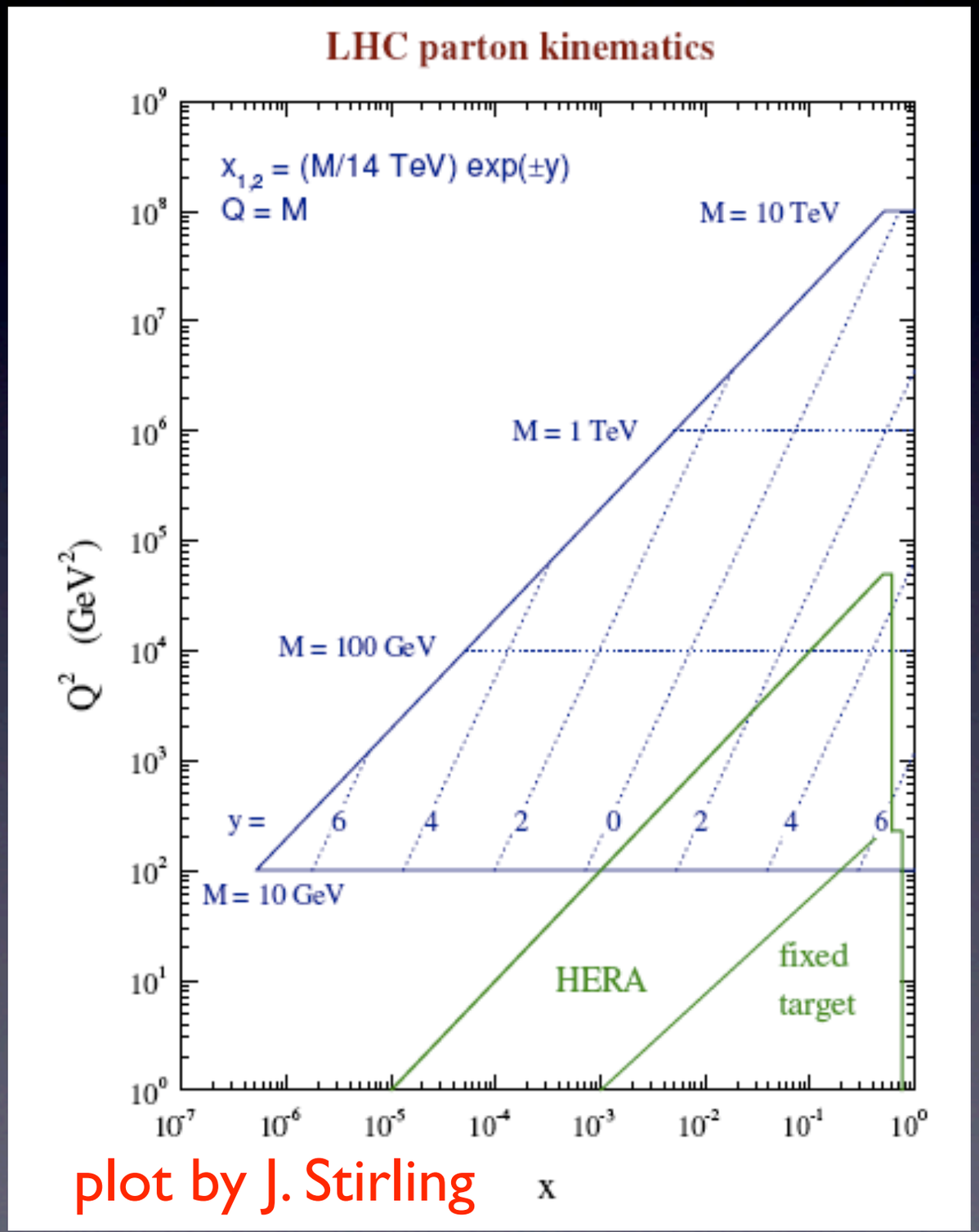
$x_1 \; x_2$ fraction of longitudinal momenta of protons carried by the gluons

Pseudorapidity distribution of the produced particles



Pseudorapidity $\eta = -\ln(\tan\frac{\theta}{2})$

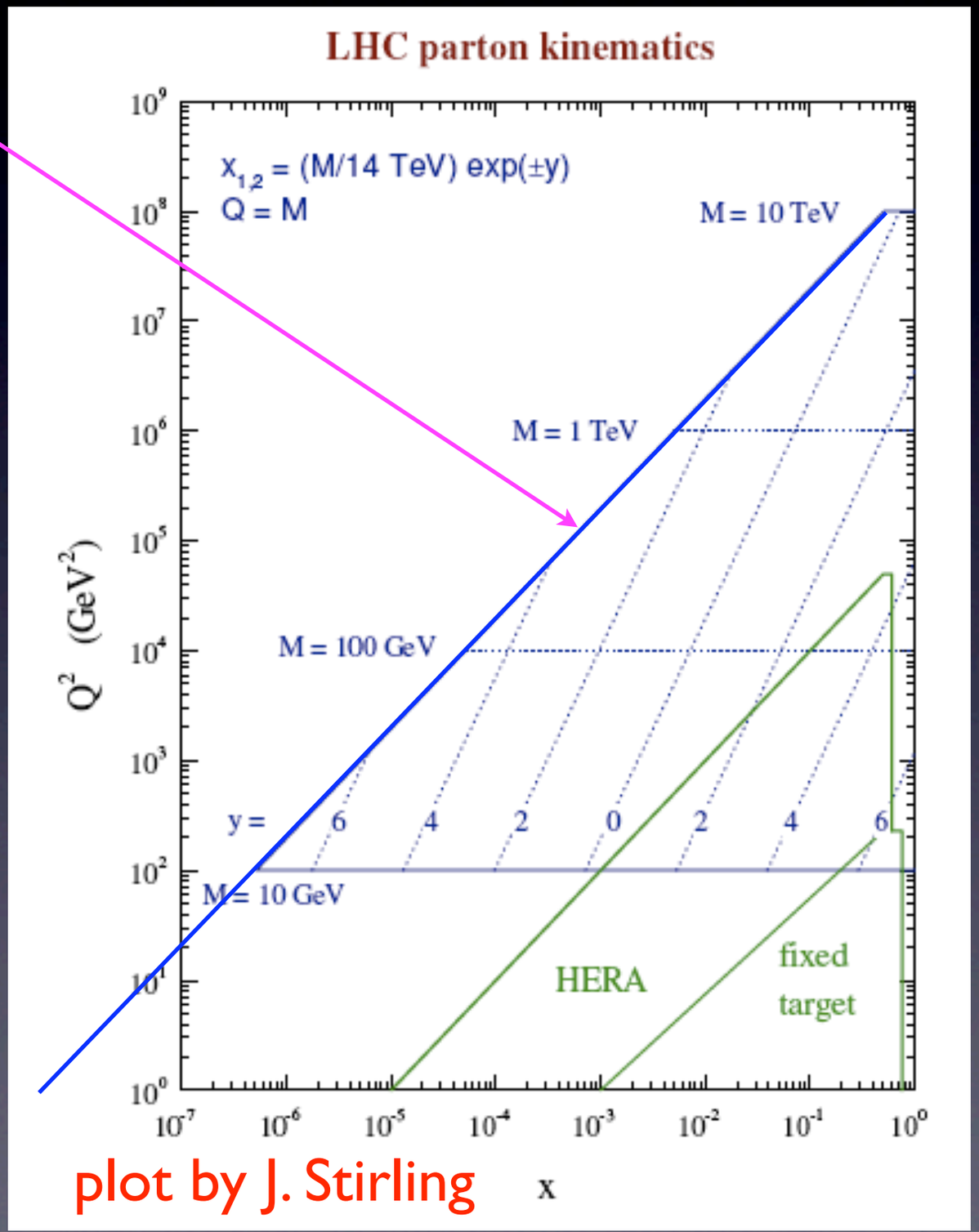
LHC and Auger kinematics



plot by J. Stirling

LHC and Auger kinematics

Kinematic boundary for LHC

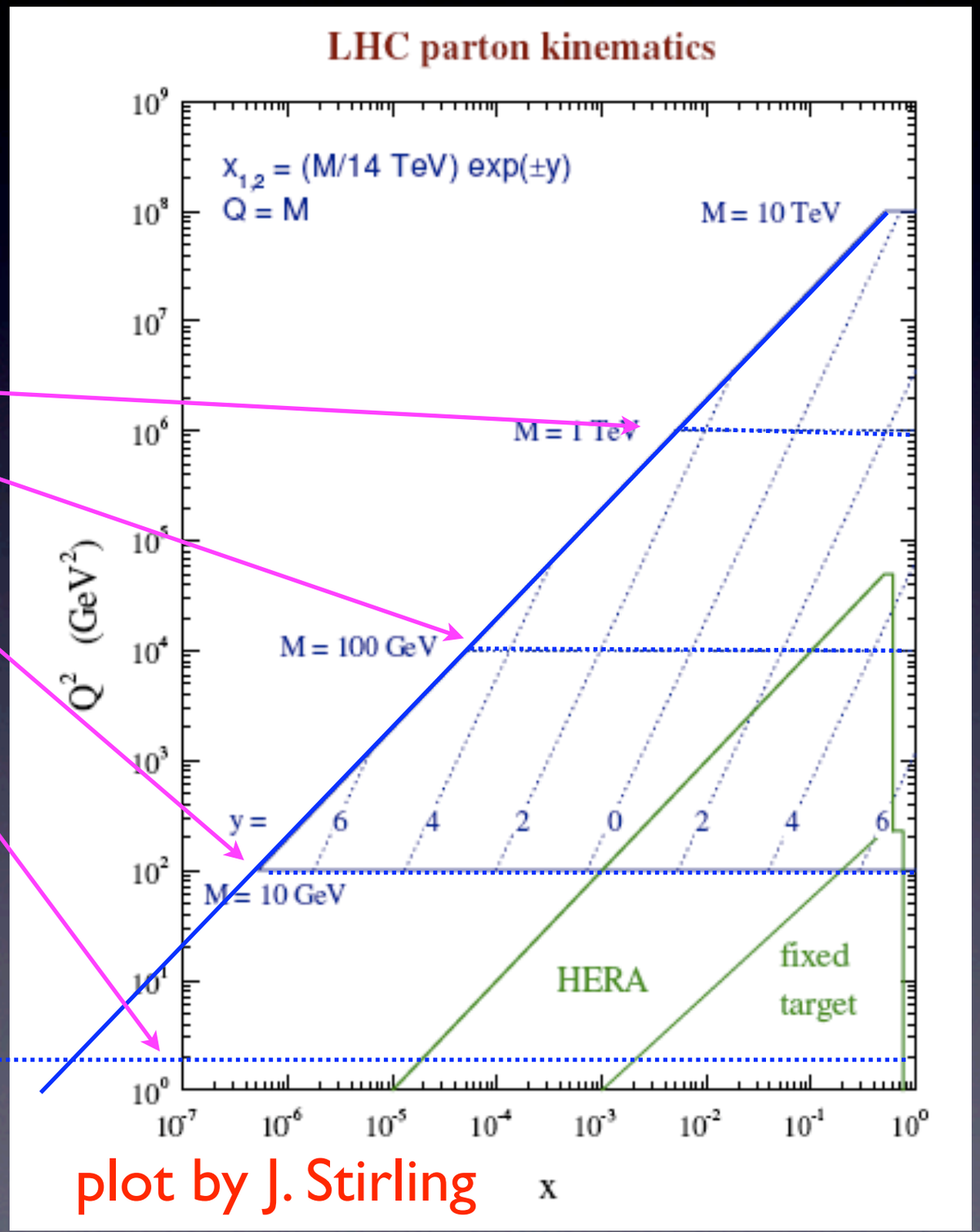


plot by J. Stirling

LHC and Auger kinematics

Kinematic boundary for LHC

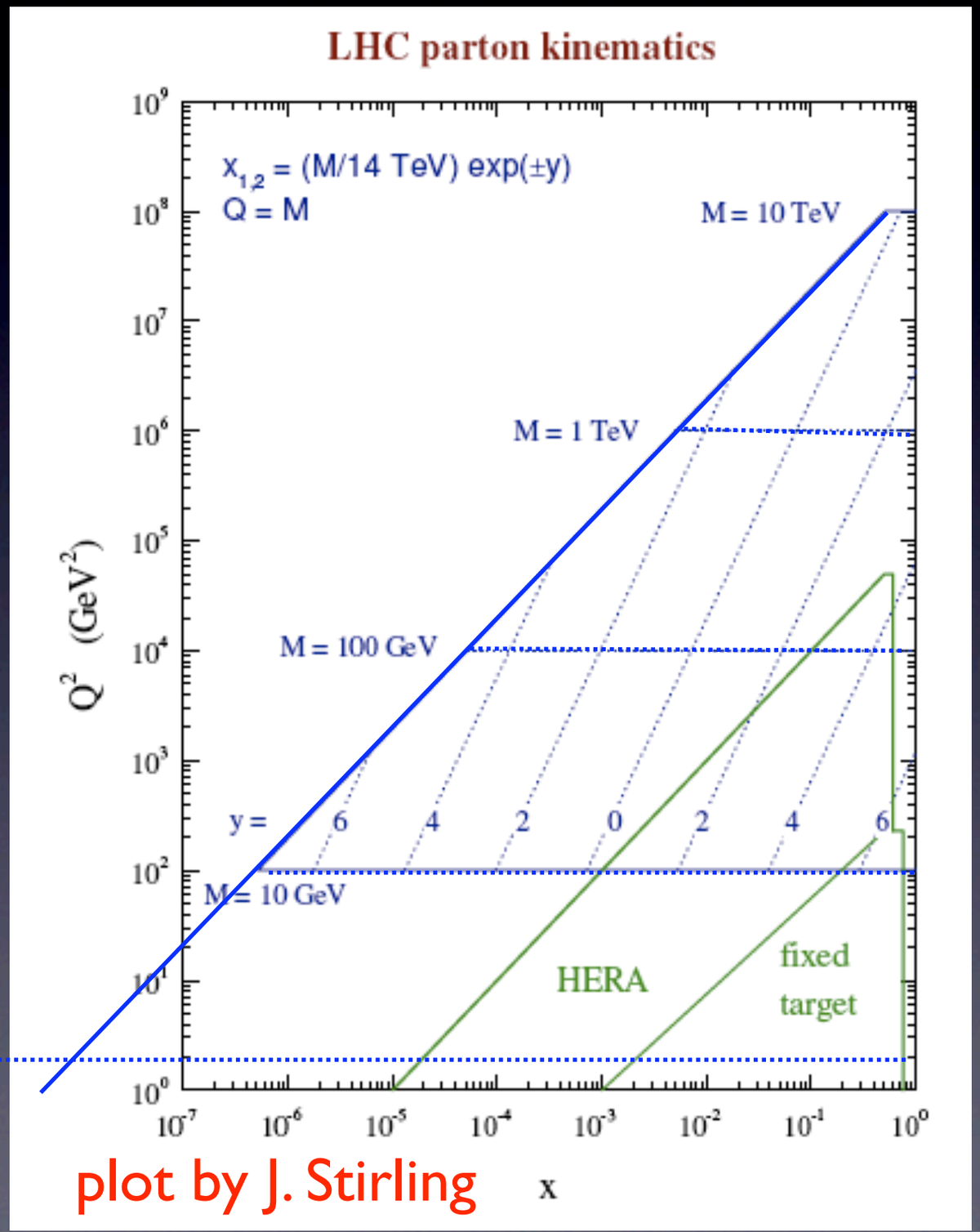
Different scales



plot by J. Stirling

LHC and Auger kinematics

Kinematic boundary for LHC

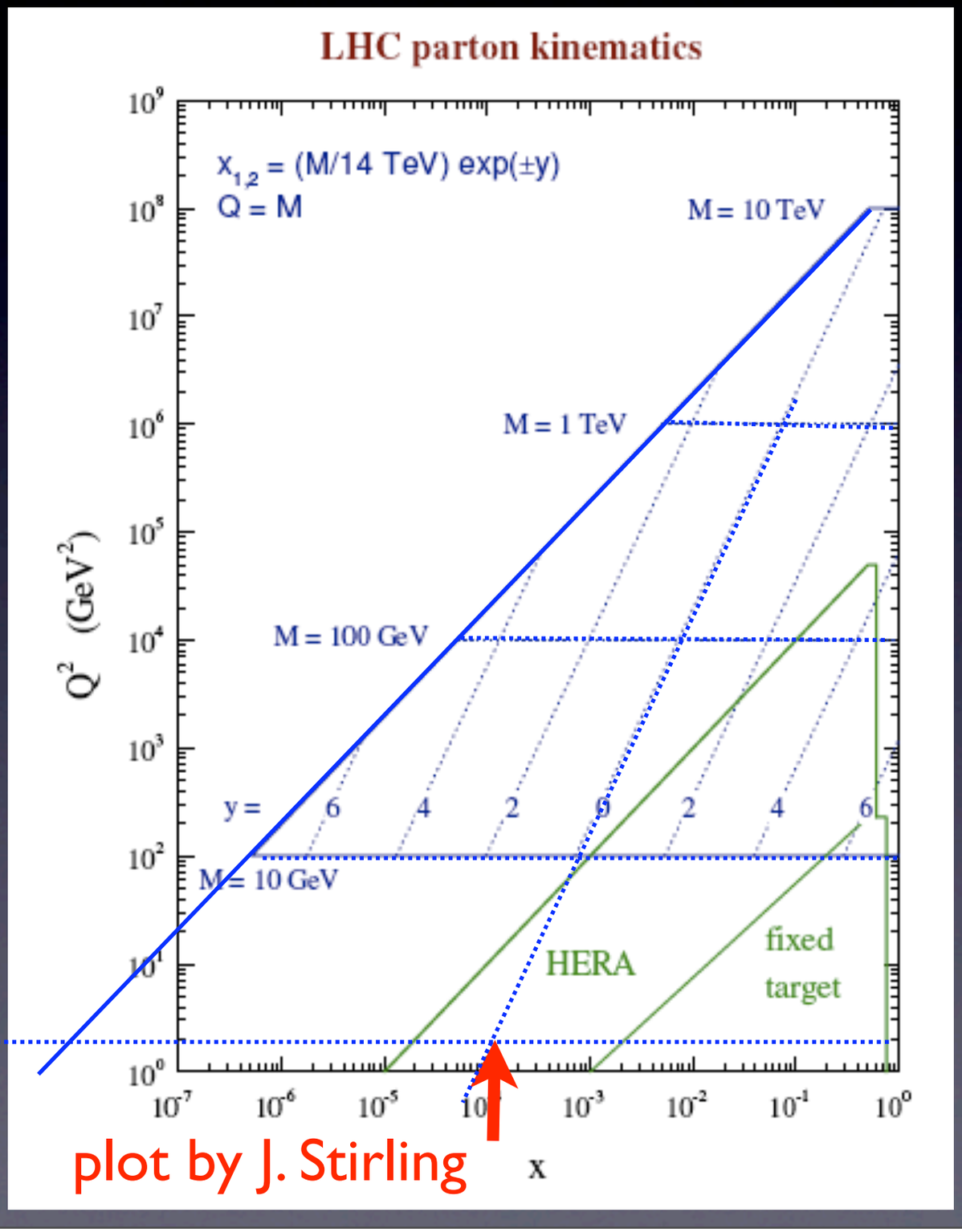


plot by J. Stirling

LHC and Auger kinematics

Kinematic boundary for LHC

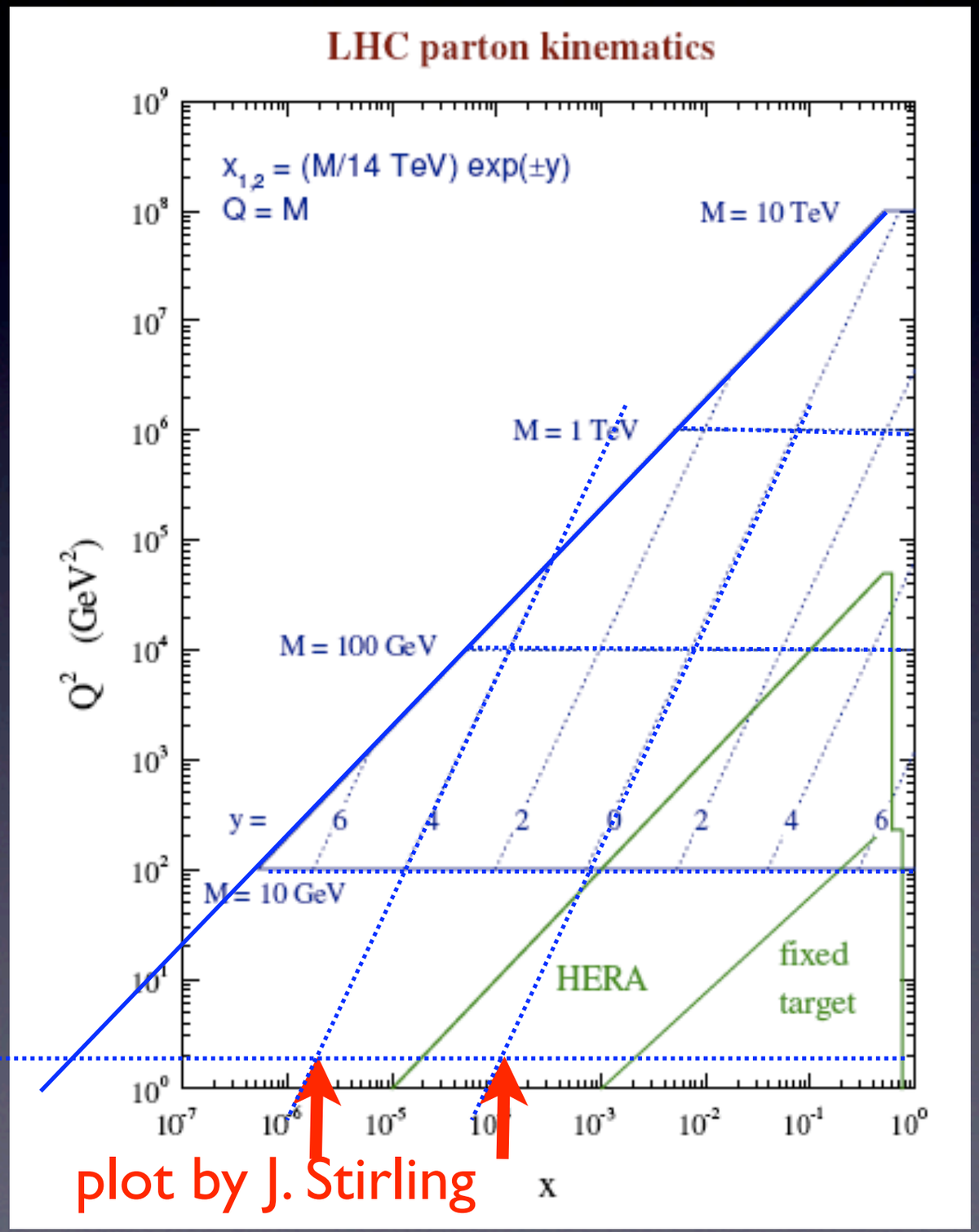
Different rapidities



LHC and Auger kinematics

Kinematic boundary for LHC

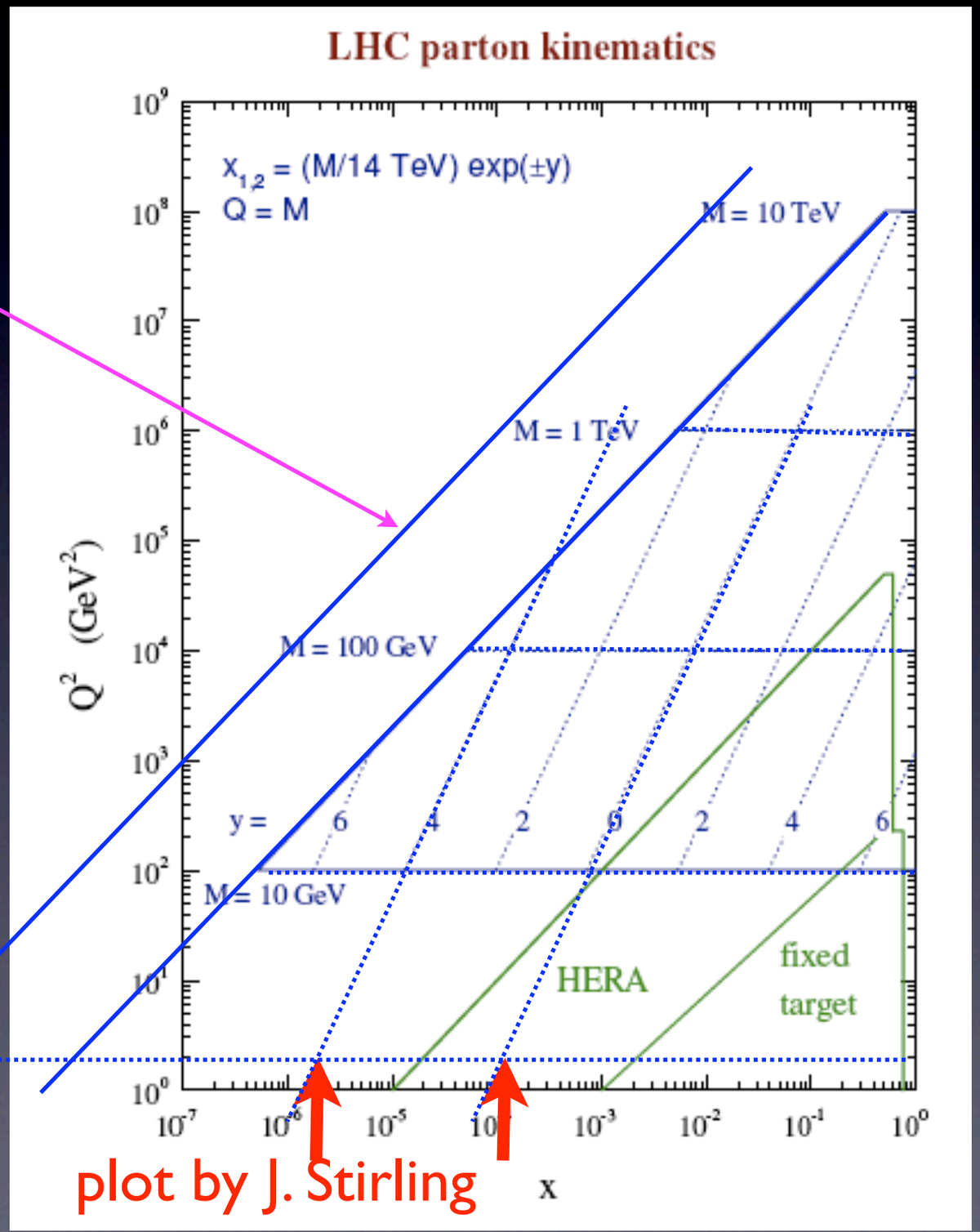
Different rapidities



LHC and Auger kinematics

Kinematic boundary for LHC

Kinematic boundary for Auger: 100 TeV c.m.s.

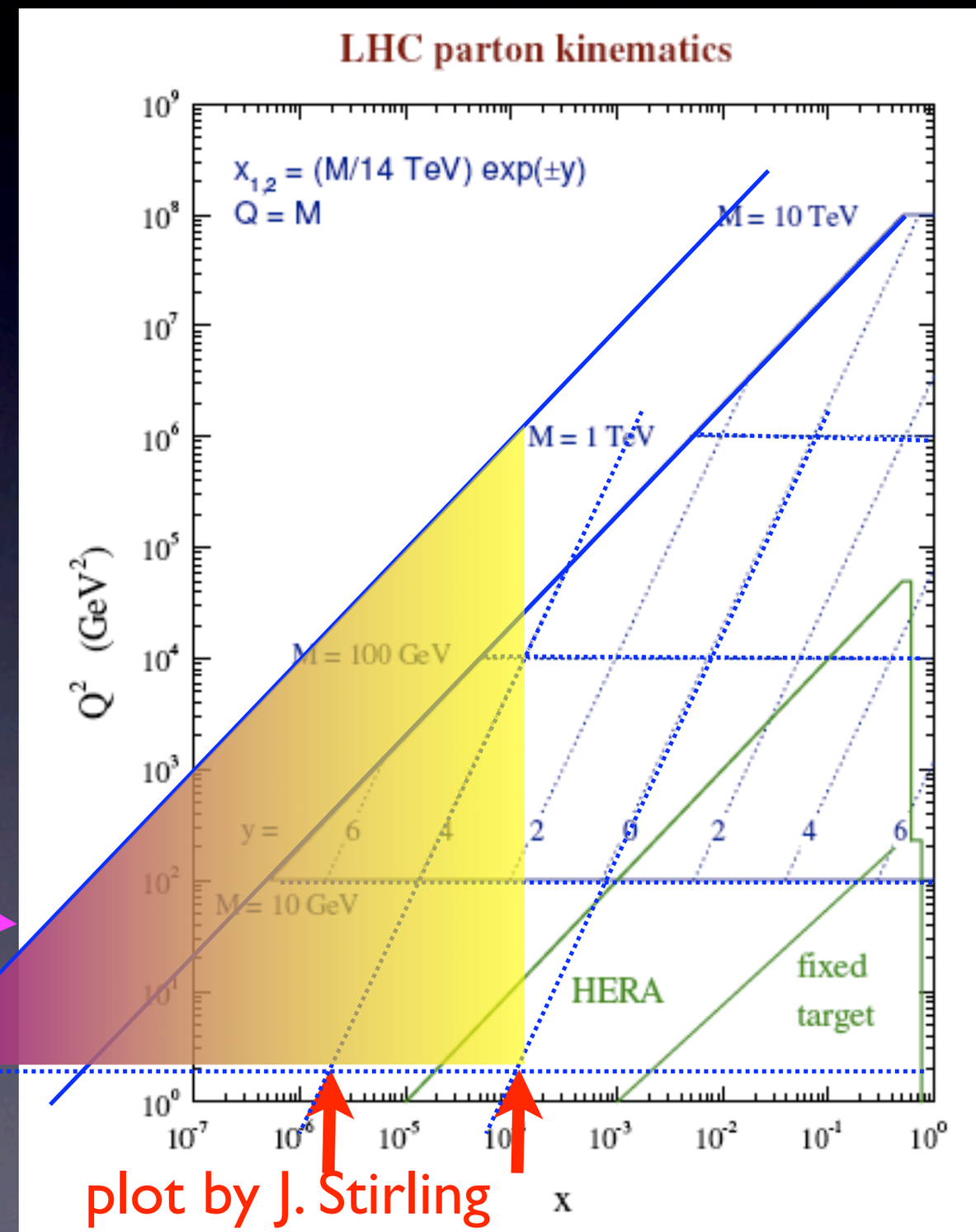


LHC and Auger kinematics

Kinematic boundary for LHC

Kinematic boundary for Auger: 100 TeV c.m.s.

Very large region with very small x!

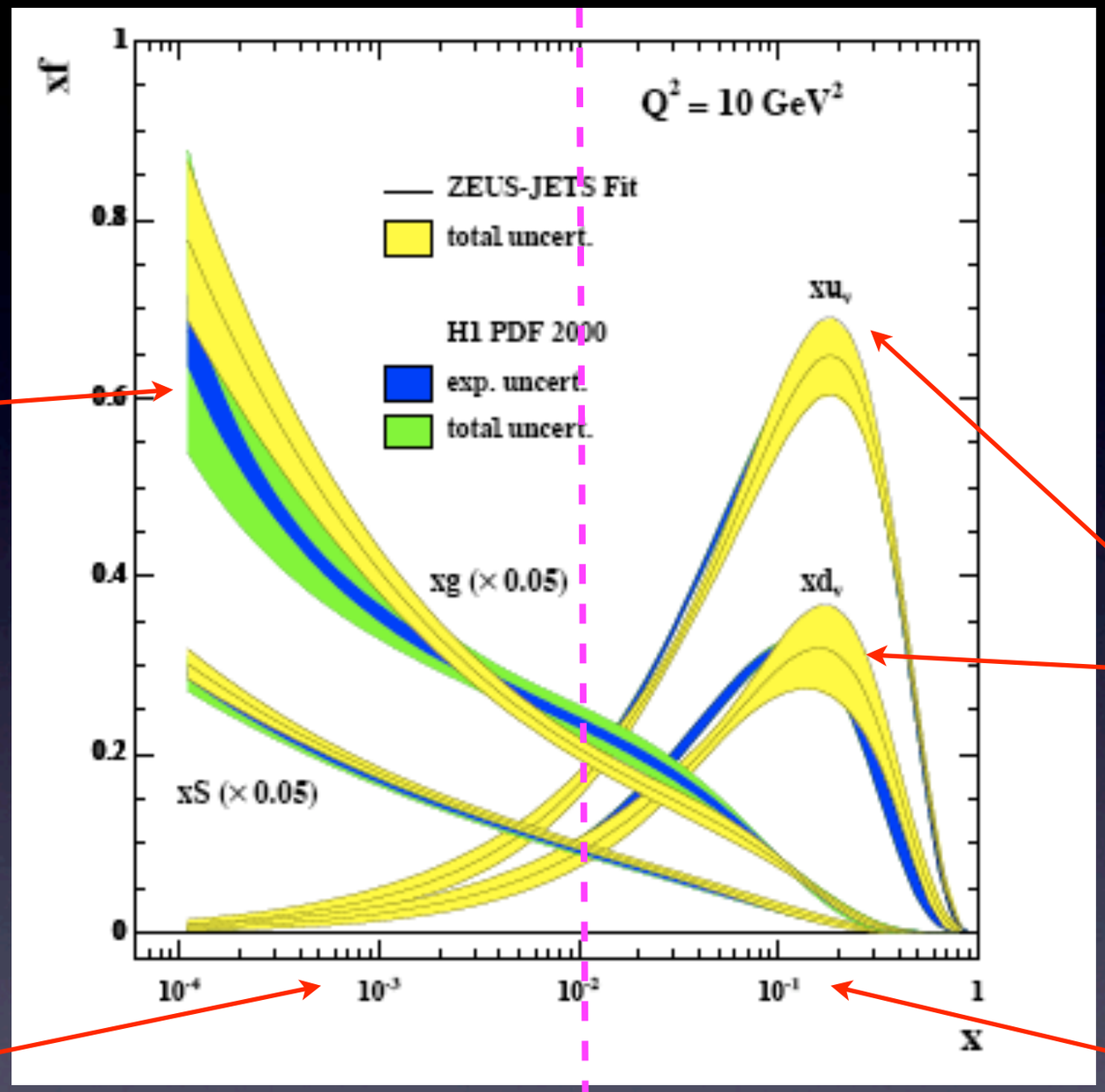


$x \sim 10^{-9}$

plot by J. Stirling

Electron-Hadron scattering at very high energy

Sea and gluons: (soft) slow fields



Valence quarks: fast fields

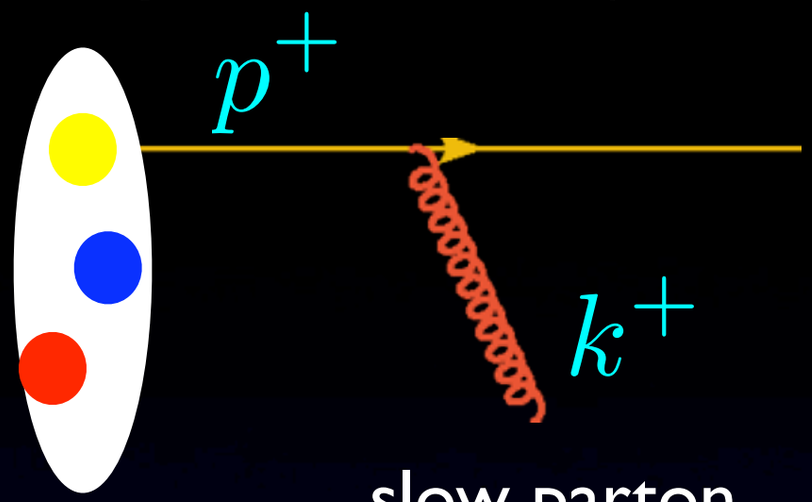
small x

large x

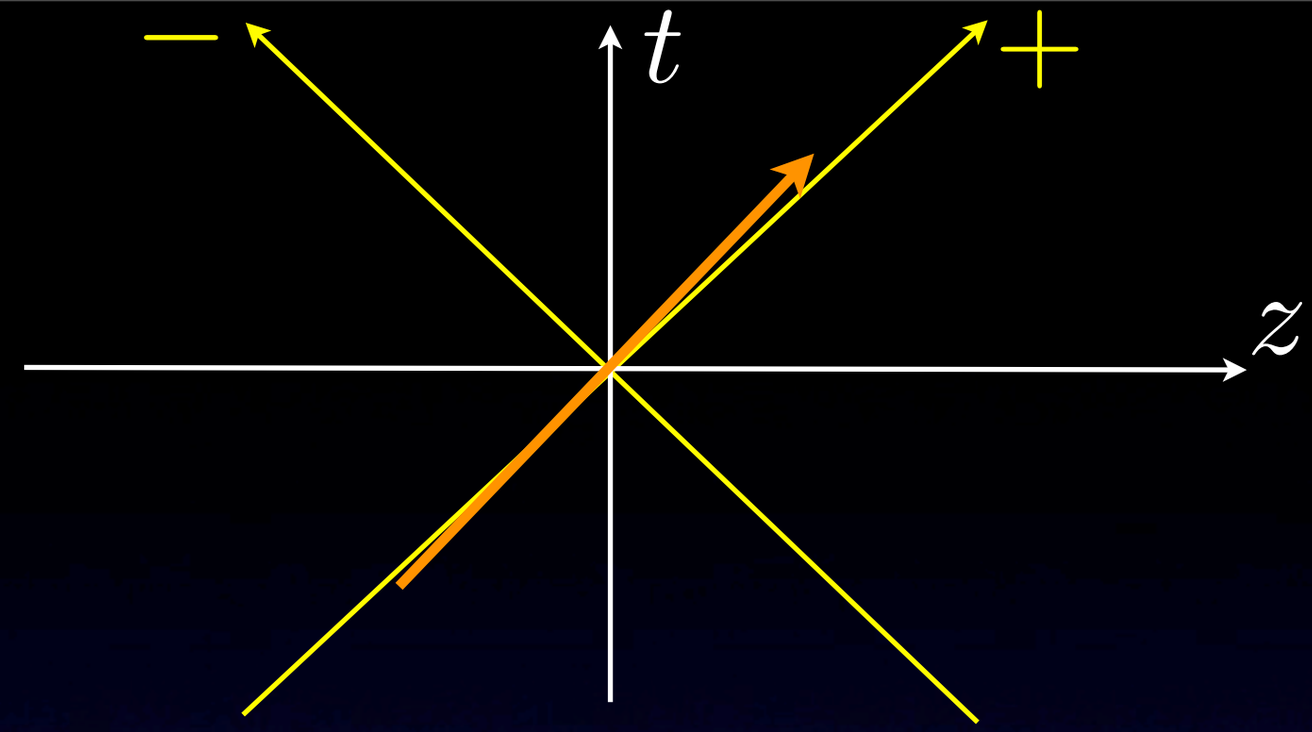
separation scale

Separation between fast and slow fields

Very fast hadron(nucleus)



slow parton



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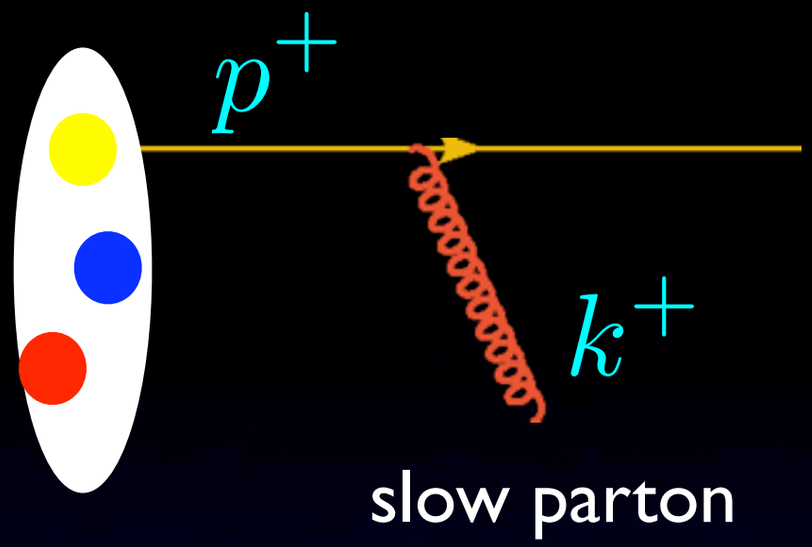
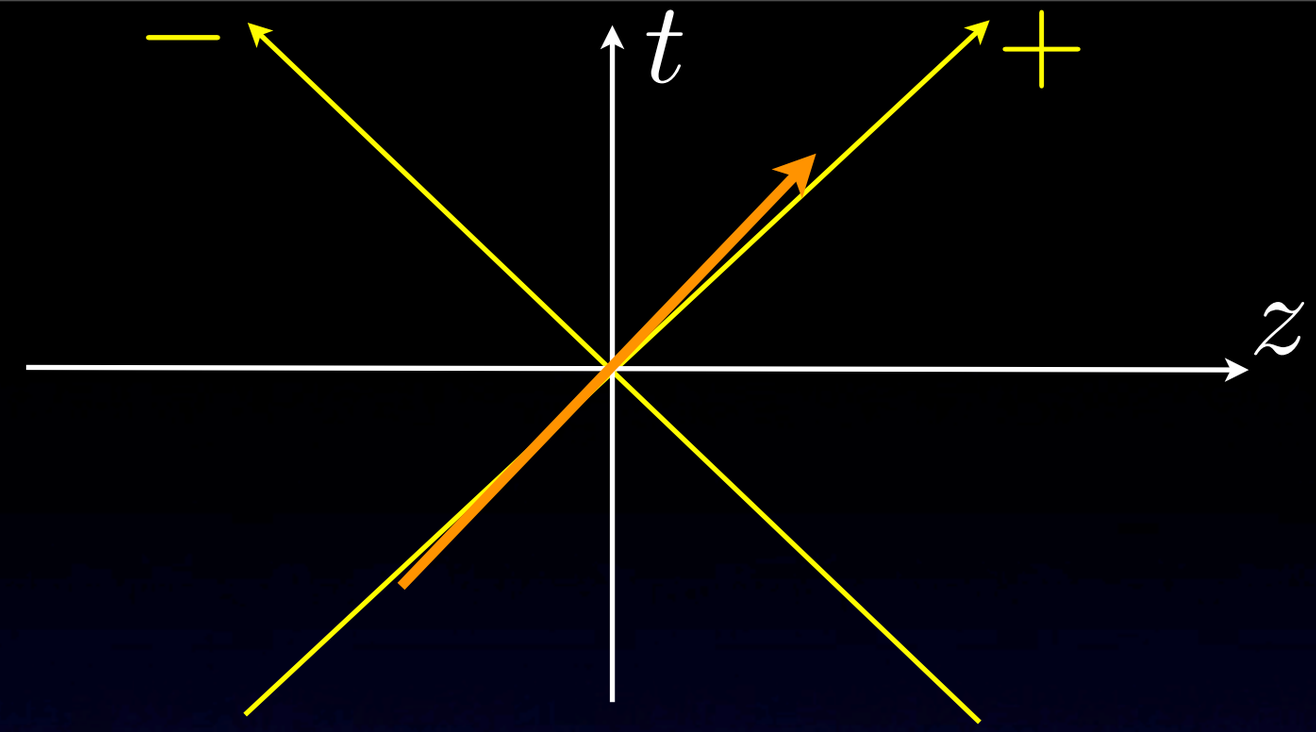
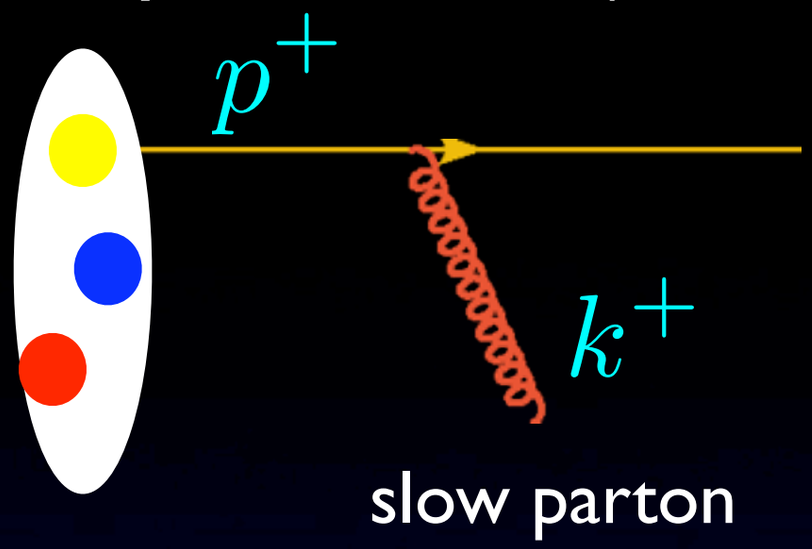
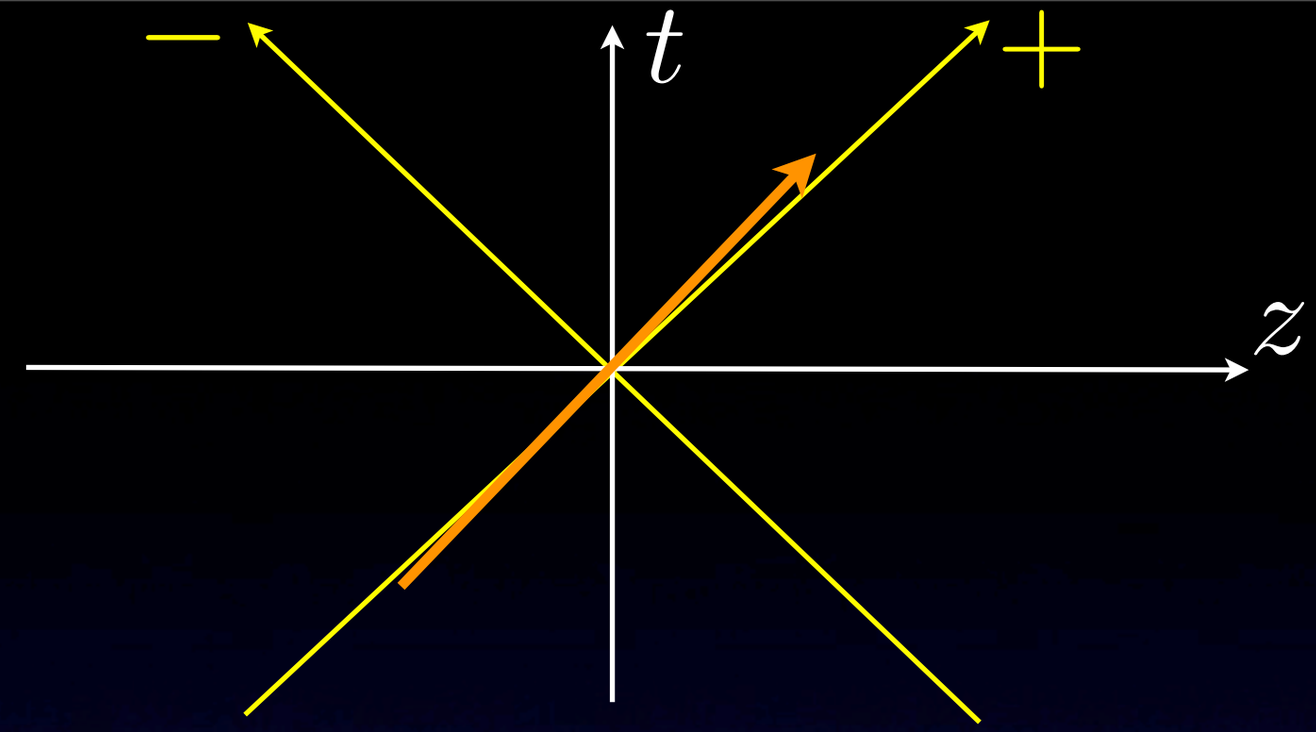




Gribov's spacetime picture: fast parton exists for a long time, slow parton is a short fluctuation.

Very fast hadron(nucleus)

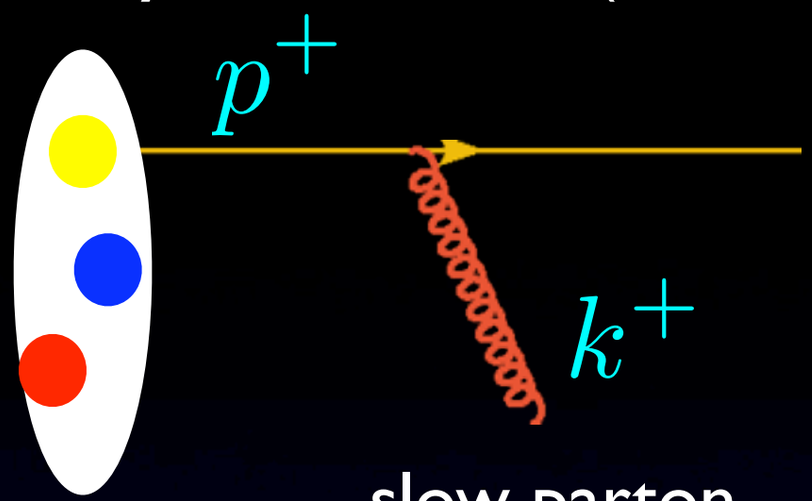




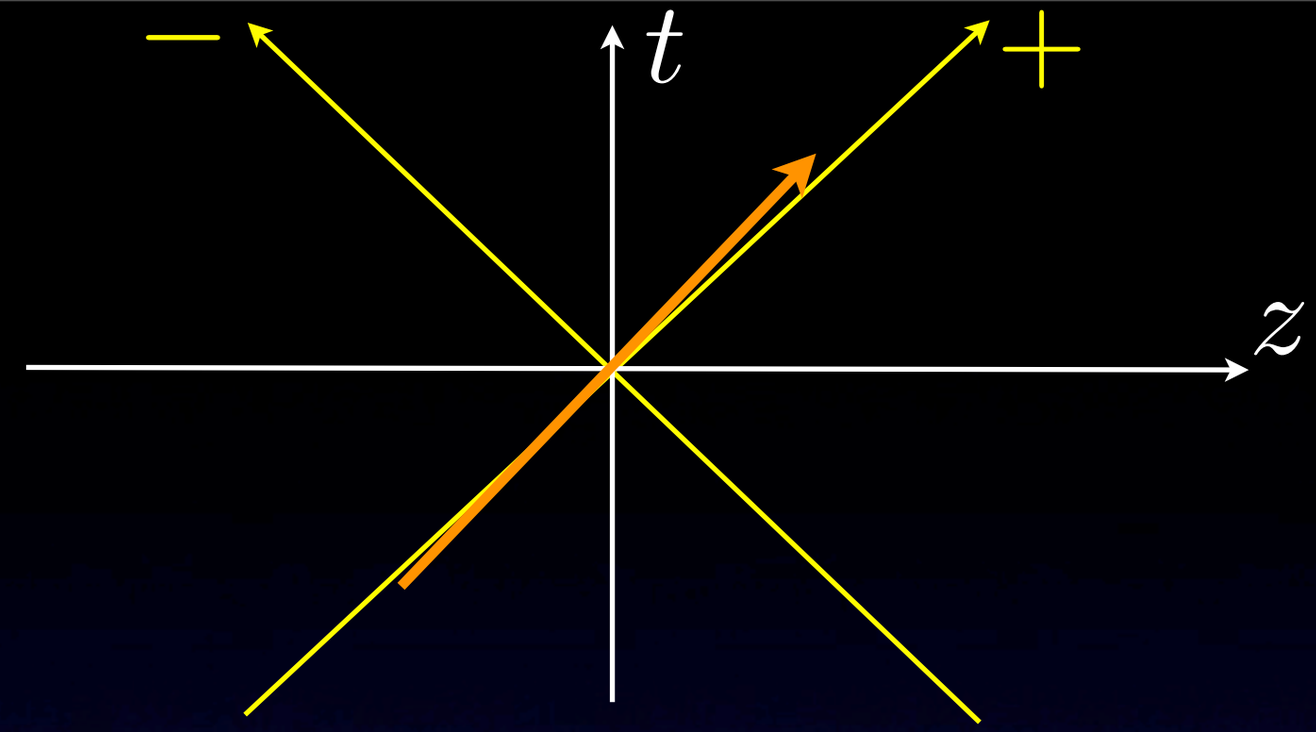
Gribov's spacetime picture: fast parton exists for a long time, slow parton is a short fluctuation.

Lifetime on the lightcone $\Delta x^+(p) \sim \frac{1}{p^-} \sim \frac{p^+}{m_{\rm T}^2}$

Very fast hadron(nucleus)



slow parton



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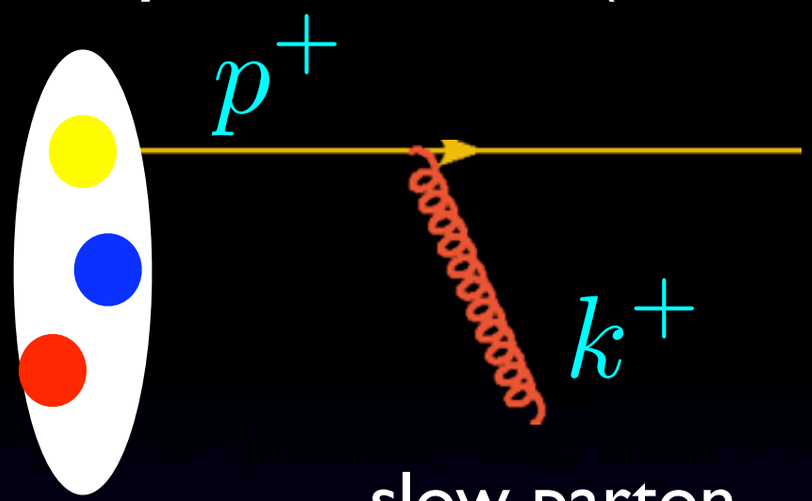
LC momentum:

$$p^+ = \frac{1}{\sqrt{2}}(E + p_z)$$

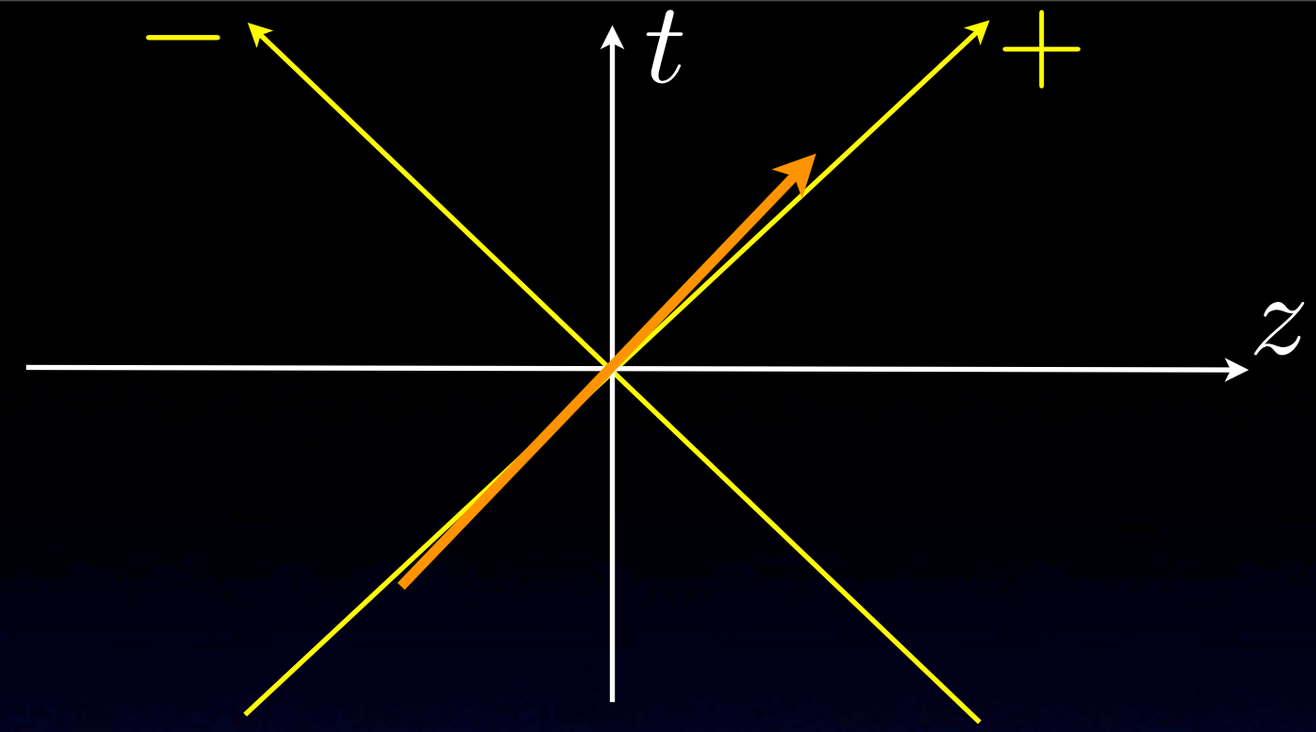
LC energy:

$$p^- = \frac{1}{\sqrt{2}}(E - p_z)$$

Very fast hadron(nucleus)



slow parton



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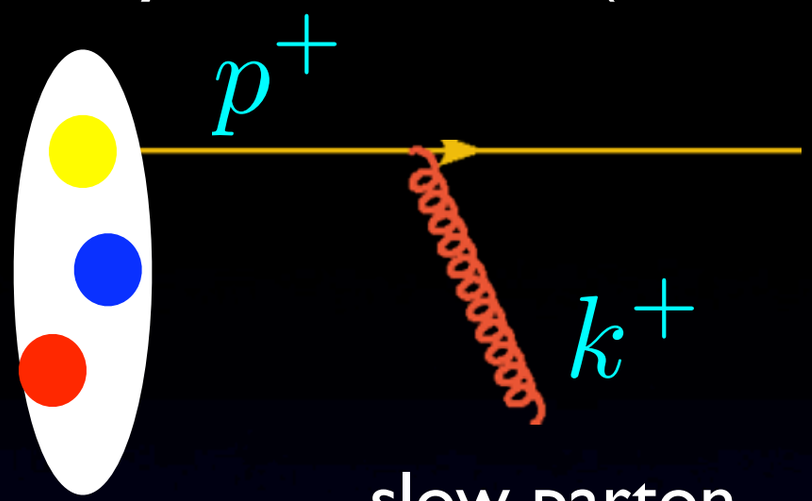
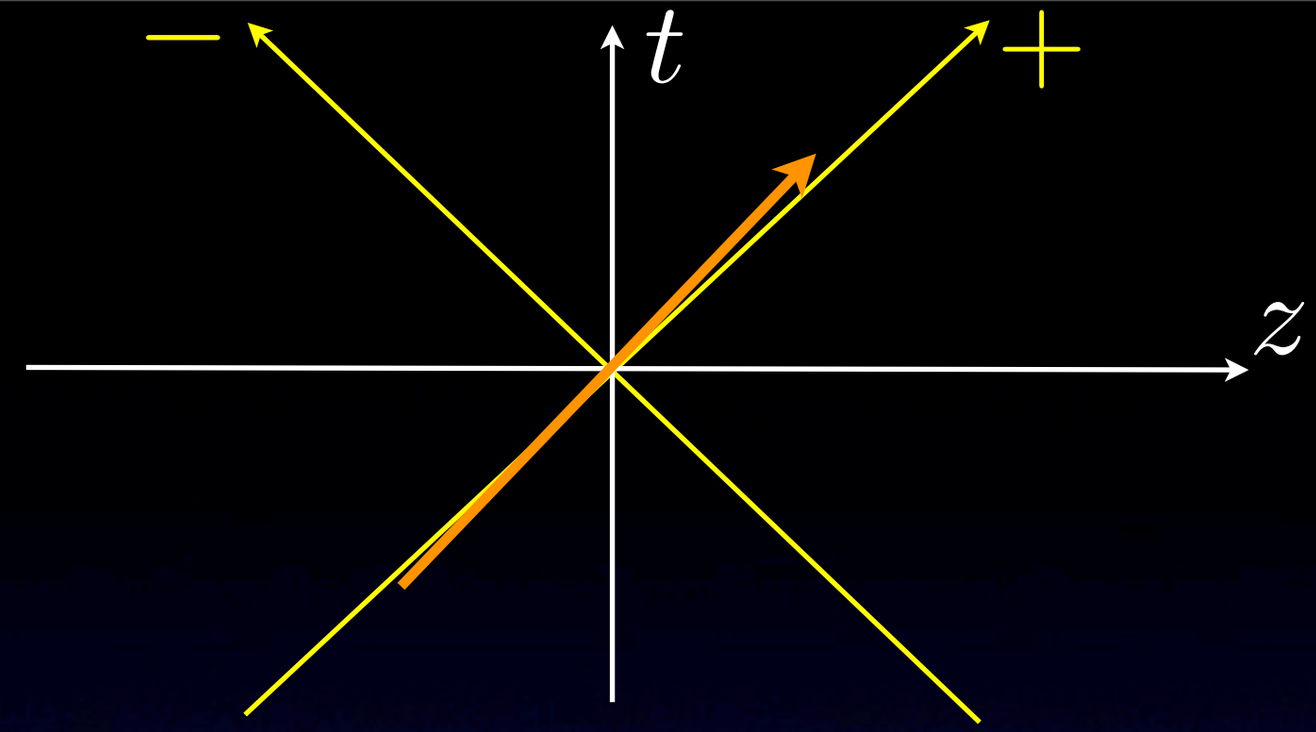


slow parton



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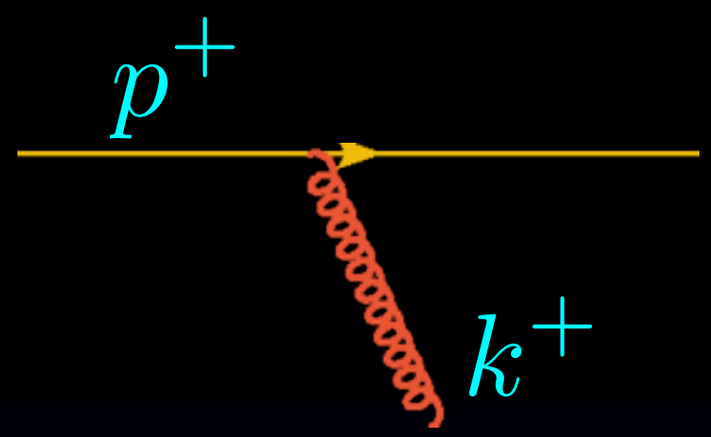
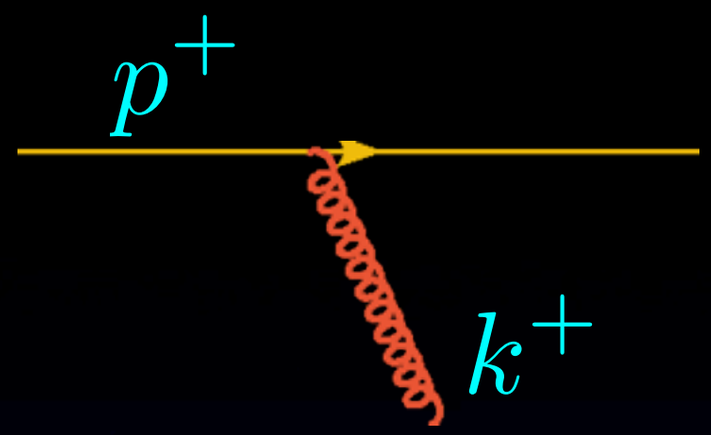
LC energy:

$$p^- = \frac{1}{\sqrt{2}}(E - p_z)$$

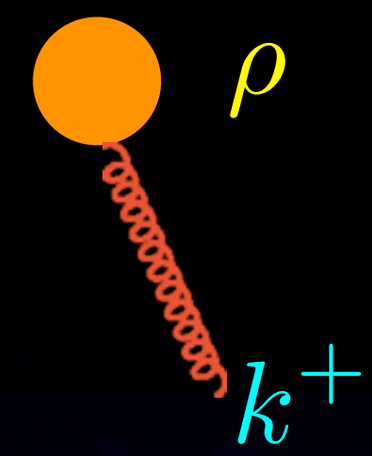
- Slow partons can only see the total charge of the fast partons.



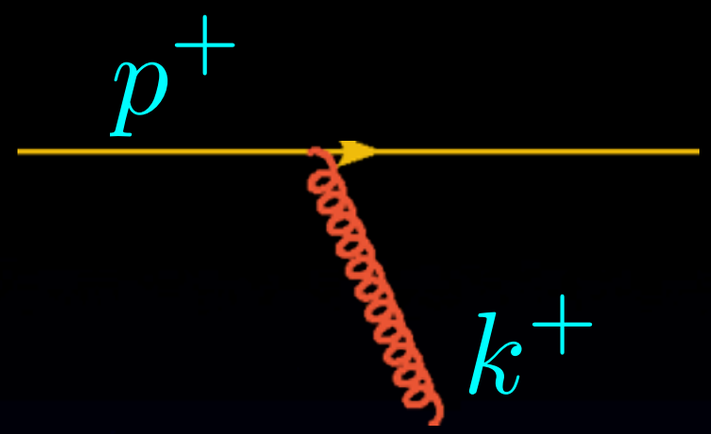
$$p^+ \gg k^+$$



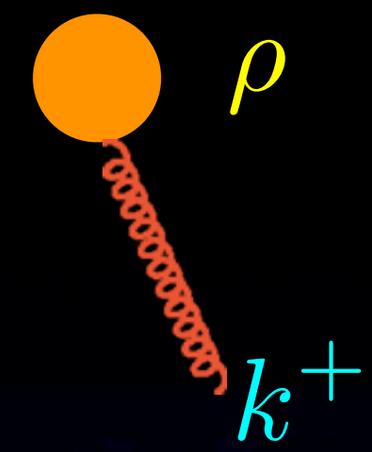
$p^+ \gg k^+$



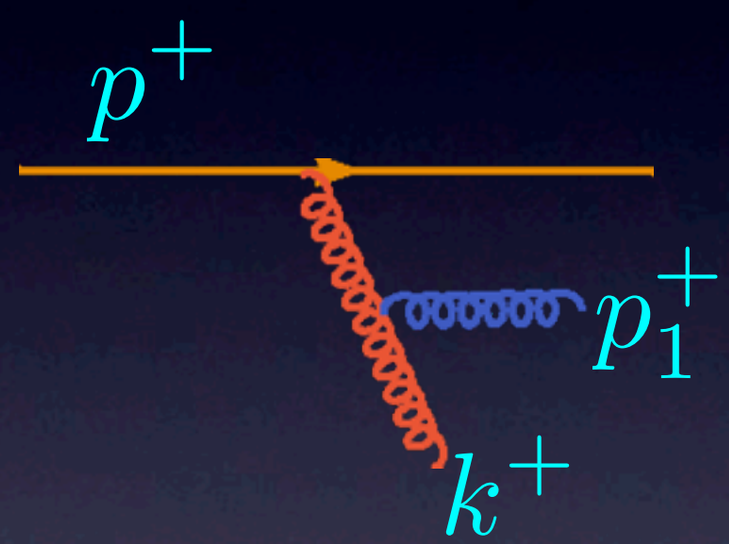
charge

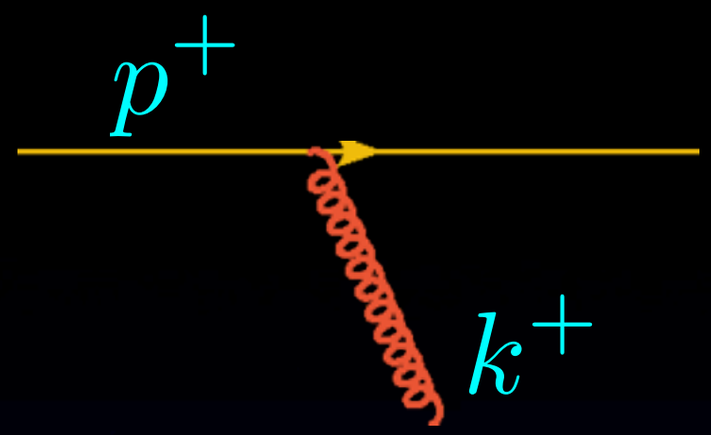


$p^+ \gg k^+$



charge





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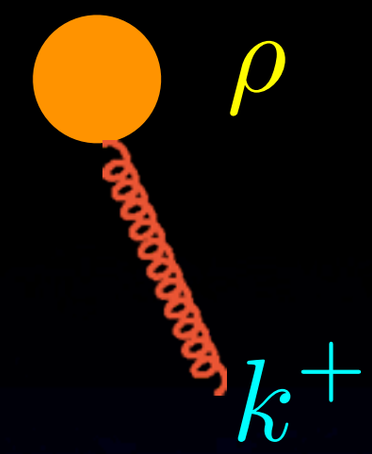
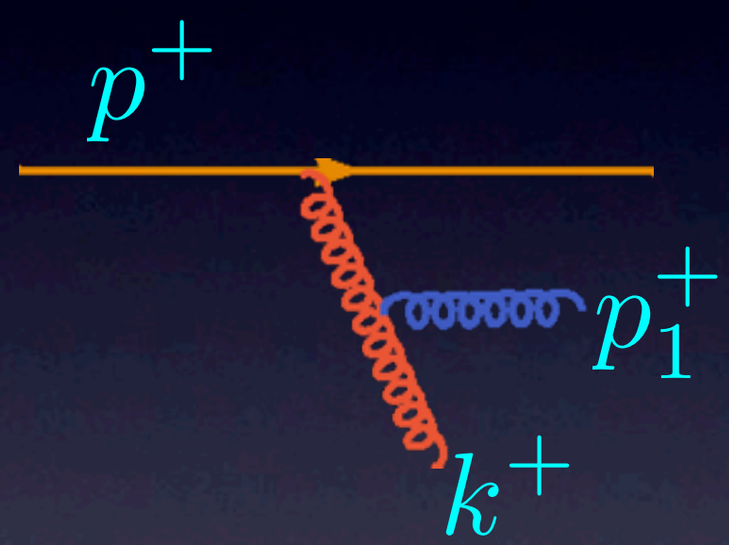


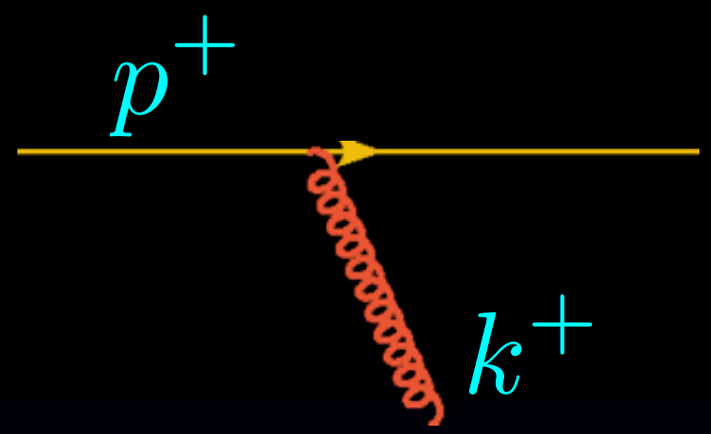
charge



One gluon emission

$p^+ \gg p_1^+ \gg k^+$

Separation of scales



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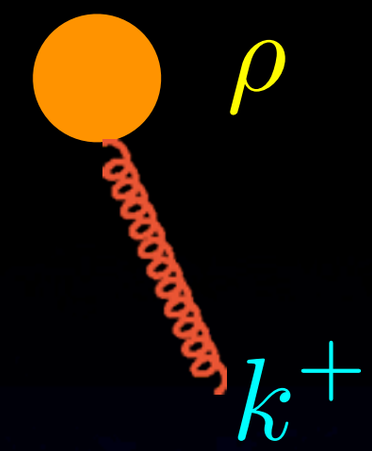

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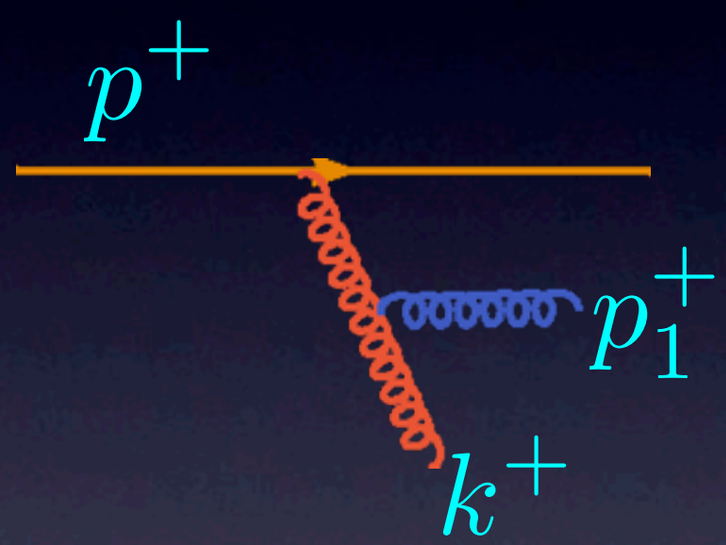

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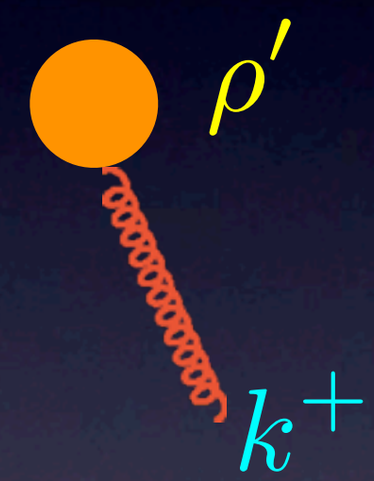
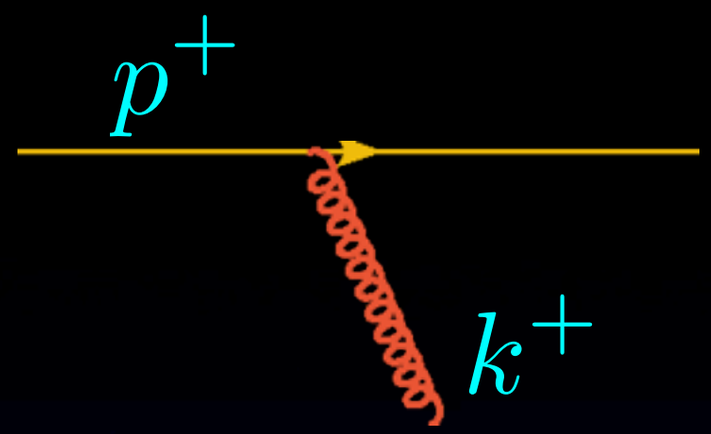


Renormalized charge



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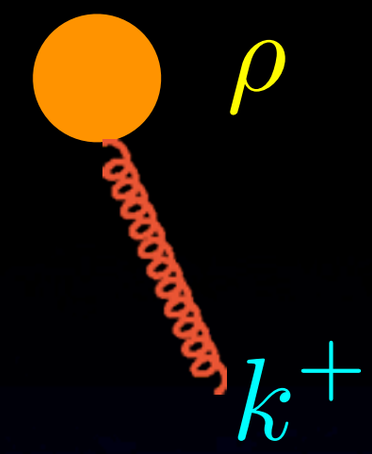
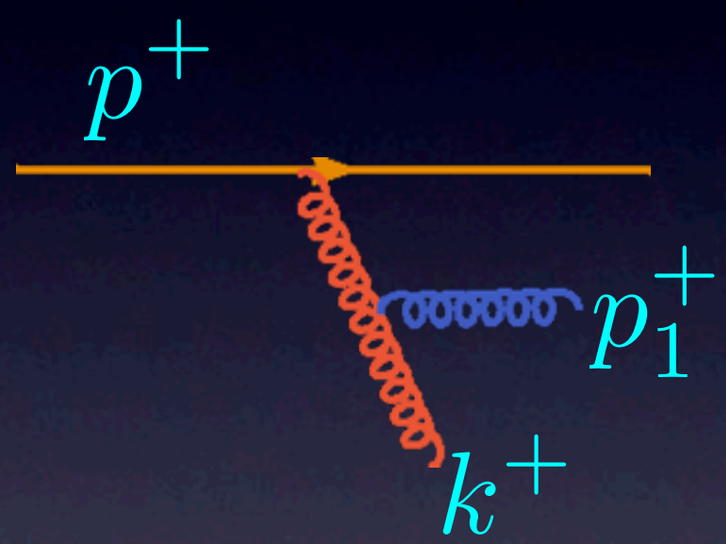


charge



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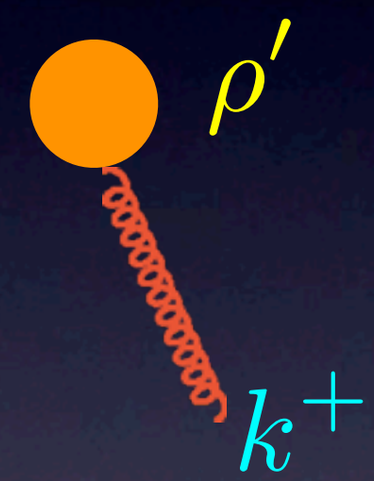
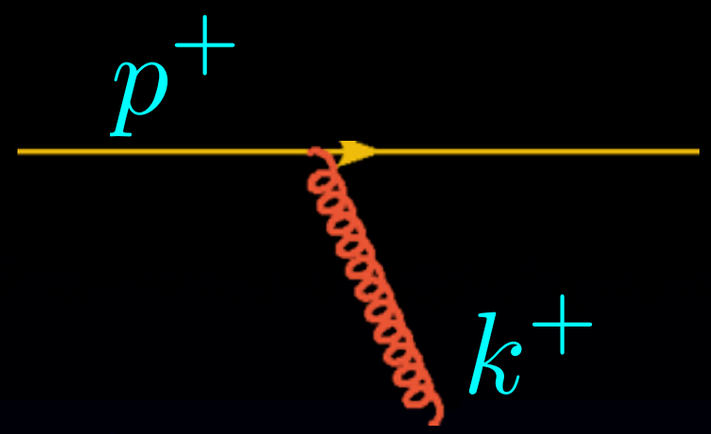
Separation of scales



Radiation of gluons:
Bremsstrahlung

Renormalized charge



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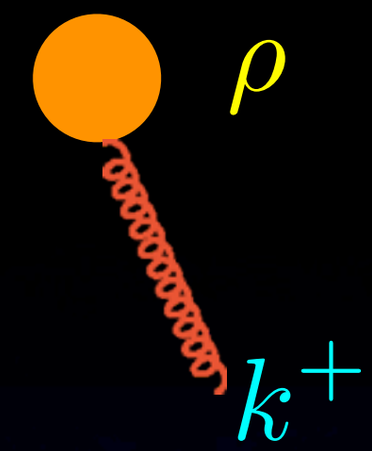
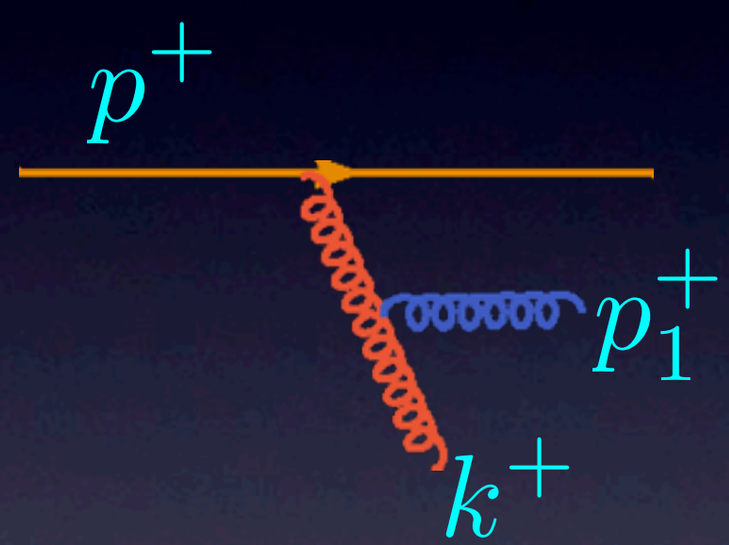


charge



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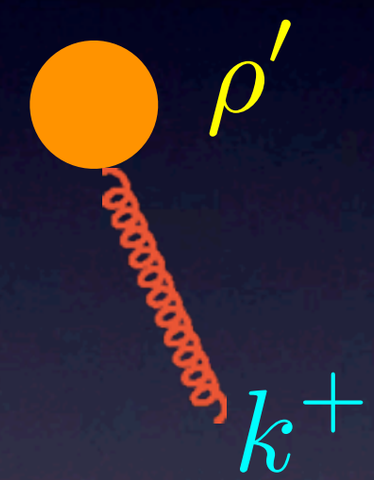


Radiation of gluons:
Bremsstrahlung

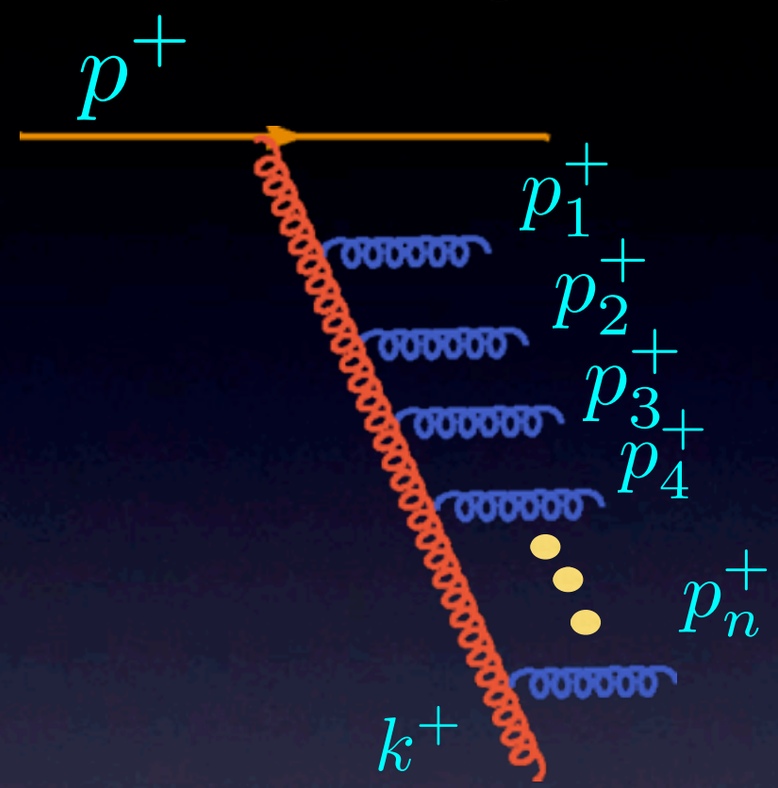
Renormalized charge

The effect of the additional gluon emission is to renormalize the effective color charge.

Generalization to many soft gluon emissions

Generalization to many soft gluon emissions

Cascade of the n soft gluons

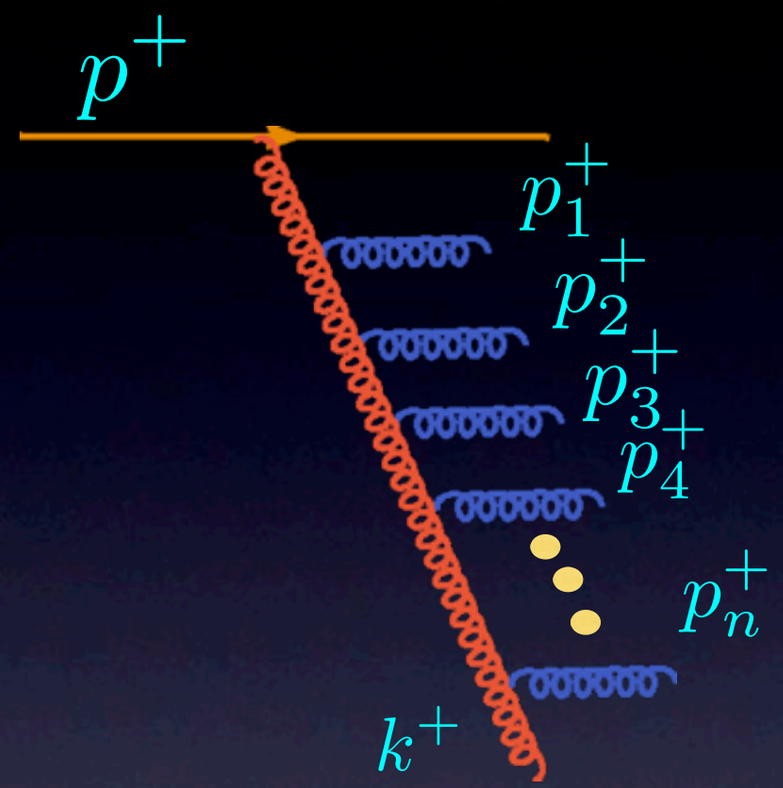


Strong ordering (in longitudinal momenta)

$$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$$

Generalization to many soft gluon emissions

Cascade of the n soft gluons



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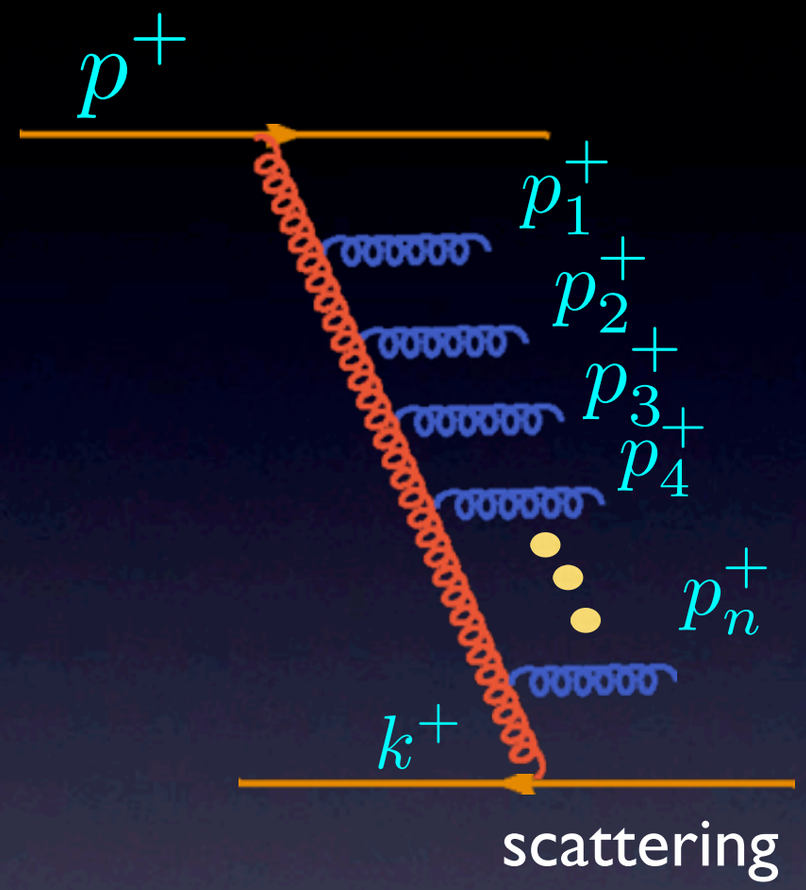
Note: transverse momenta are not ordered

Generalization to many soft gluon emissions

Cascade of the n soft gluons



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Note: transverse momenta are not ordered

$$k^+ = xp^+$$

$$\frac{\alpha_s N_c}{\pi} \int_{k^+}^{p^+} \frac{dp_1^+}{p_1^+} = \frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}$$

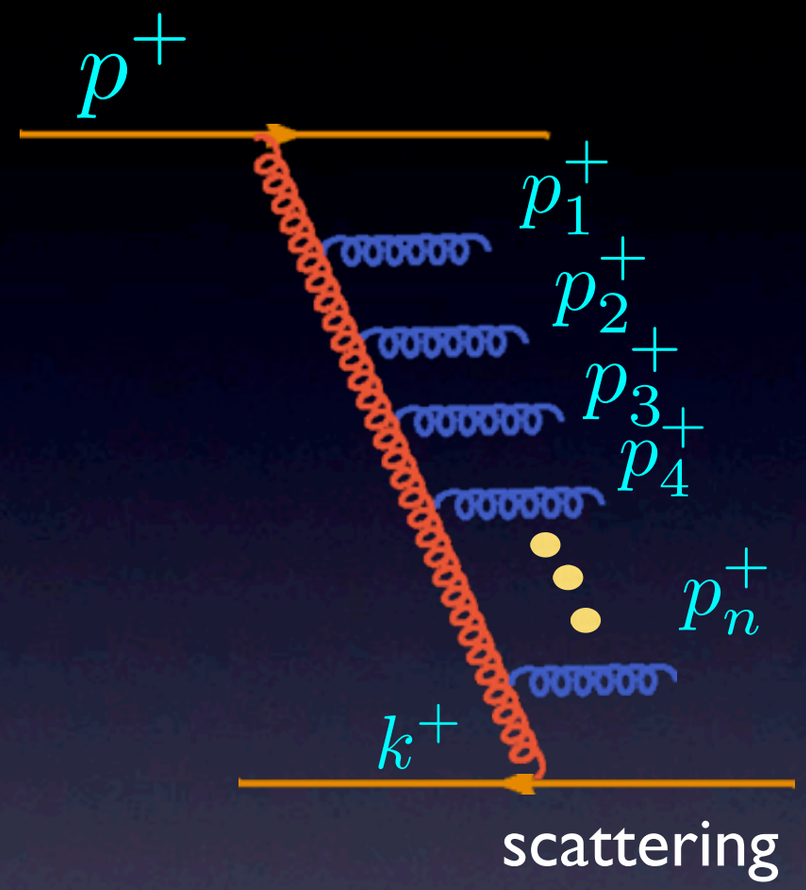
Large logarithm

Nested logarithmic integrals

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} \right)^n$$

Generalization to many soft gluon emissions

Cascade of the n soft gluons



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Large logarithm

Nested logarithmic integrals

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} \right)^n$$

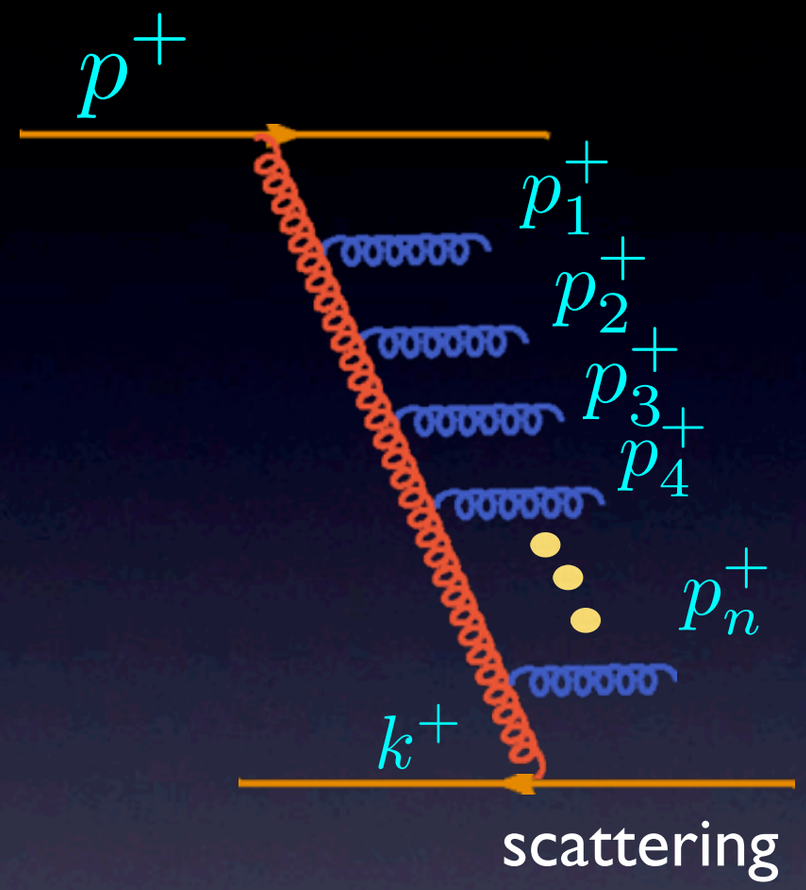
Resummation of the gluon emissions performed by the Bethe-Salpeter type equation

$$\frac{df_g(x, k_T^2)}{d \ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2 k_T' \mathcal{K}(k_T, k_T') \; f_g(x, k_T')$$

I.Balitsky, V.Fadin, E.Kuraev, L.Lipatov

Generalization to many soft gluon emissions

Cascade of the n soft gluons



Strong ordering (in longitudinal momenta)

$$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$$

Note: transverse momenta are not ordered

$$k^+ = xp^+$$

$$\frac{\alpha_s N_c}{\pi} \int_{k^+}^{p^+} \frac{dp_1^+}{p_1^+} = \frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}$$

Large logarithm

Nested logarithmic integrals

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} \right)^n$$

Resummation of the gluon emissions performed by the Bethe-Salpeter type equation

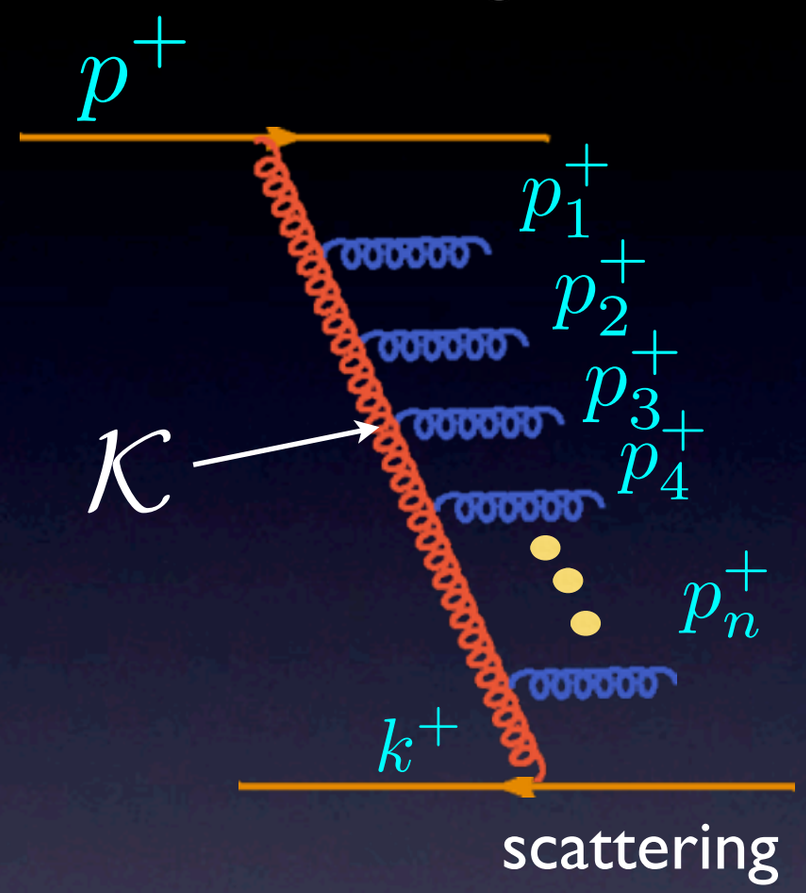
$$\frac{df_g(x, k_T^2)}{d \ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2 k'_T \mathcal{K}(k_T, k'_T) \; f_g(x, k'_T)$$

I.Balitsky, V.Fadin, E.Kuraev, L.Lipatov

integral over transverse momenta

Generalization to many soft gluon emissions

Cascade of the n soft gluons



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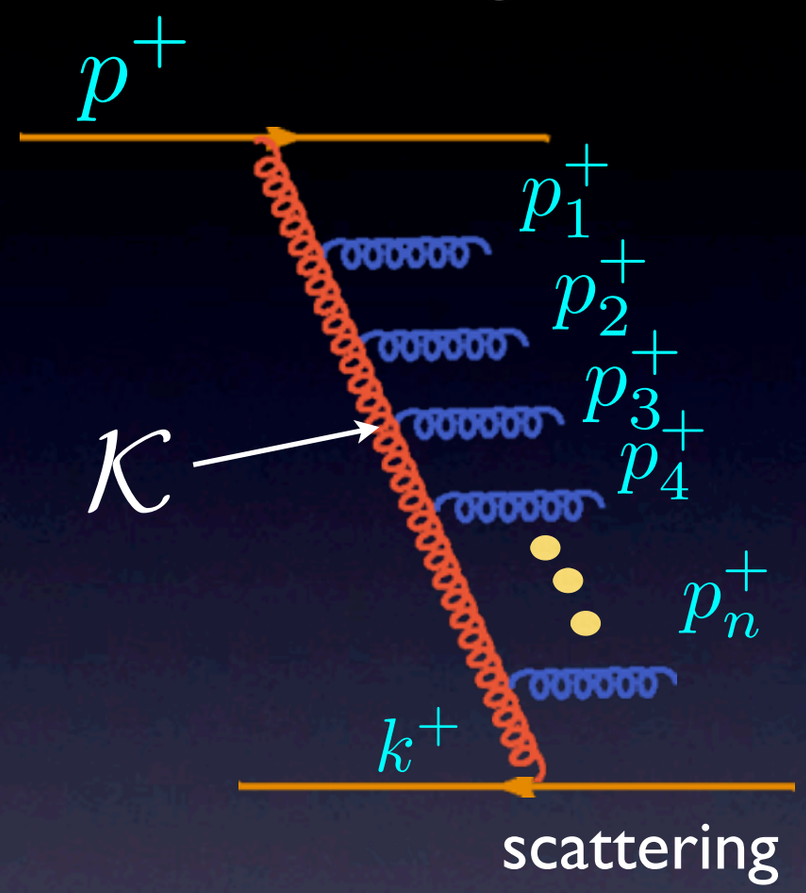

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unintegrated gluon density at small x

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V.Fadin,L.Lipatov, G.Camici,M.Ciafaloni

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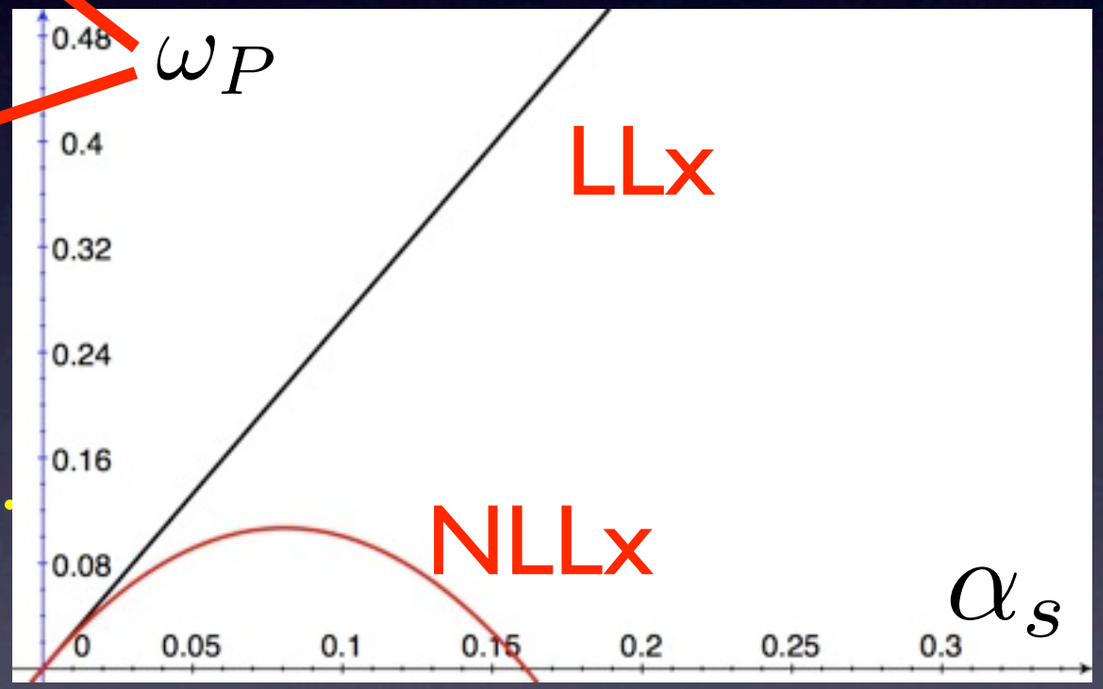

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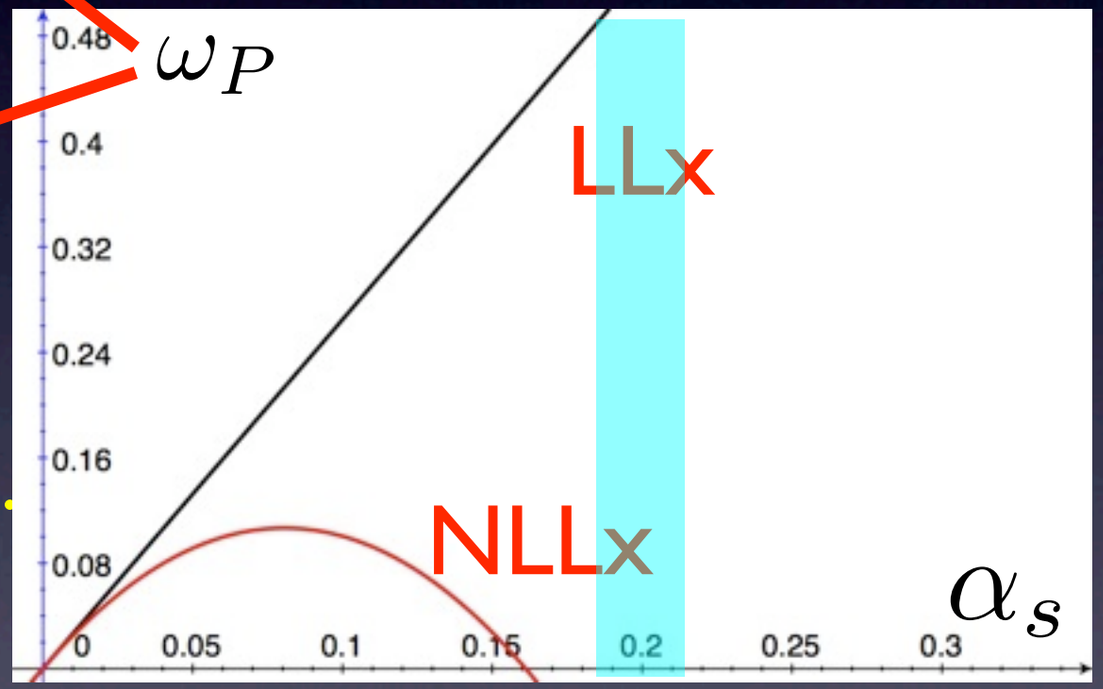

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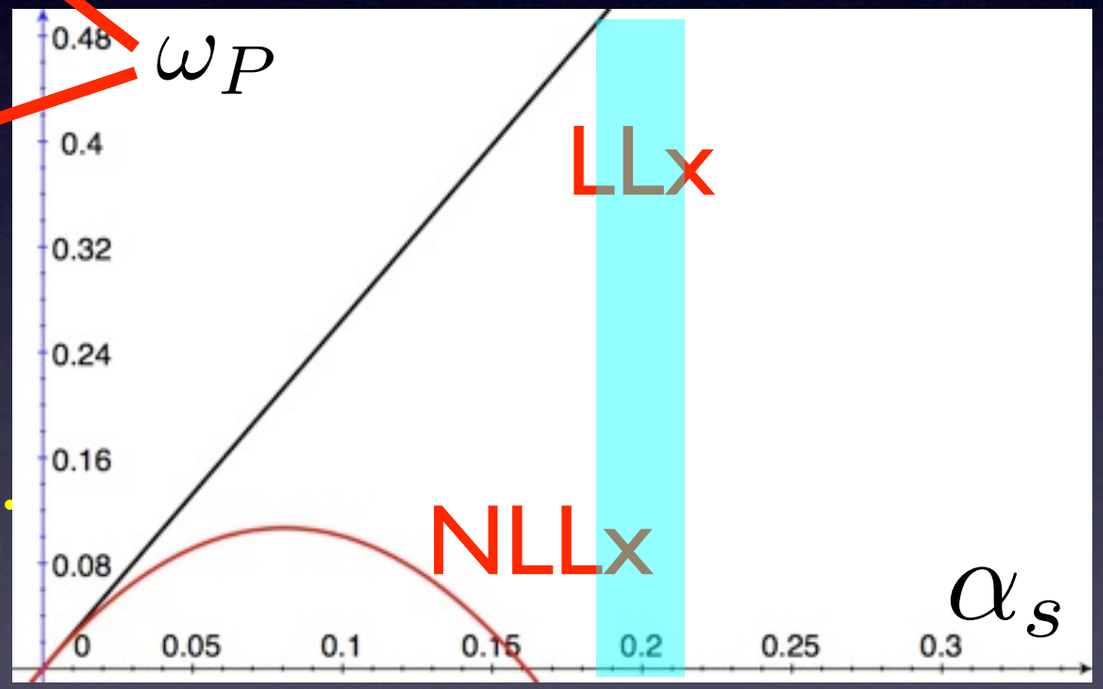
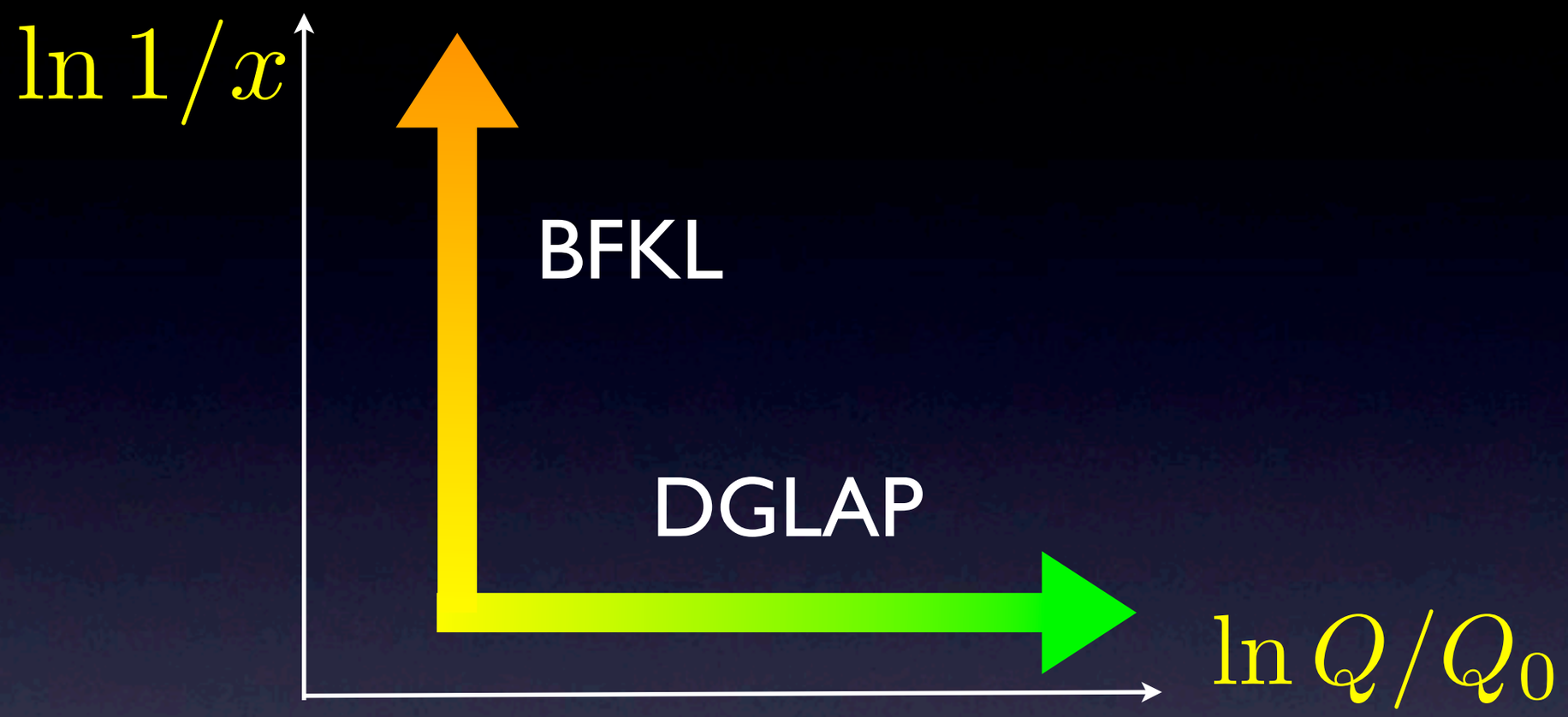


relevant values of α_s

Leading logarithmic approximation, not compatible with the experimental data.
Very large next-to-leading correction!

Resummation at small x



Use DGLAP information to constrain the BFKL expansion. Very strong constraint!

Take care of both type of logarithms

$\alpha_s \ln 1/x$

$\alpha_s \ln Q/Q_0$

Kinematical constraints and the running coupling.

Resummation

$$f_g(x, k_T) \sim x^{-\omega_P}$$

M. Ciafaloni, D. Colferai, G. Salam, A.Stasto

Before resummation

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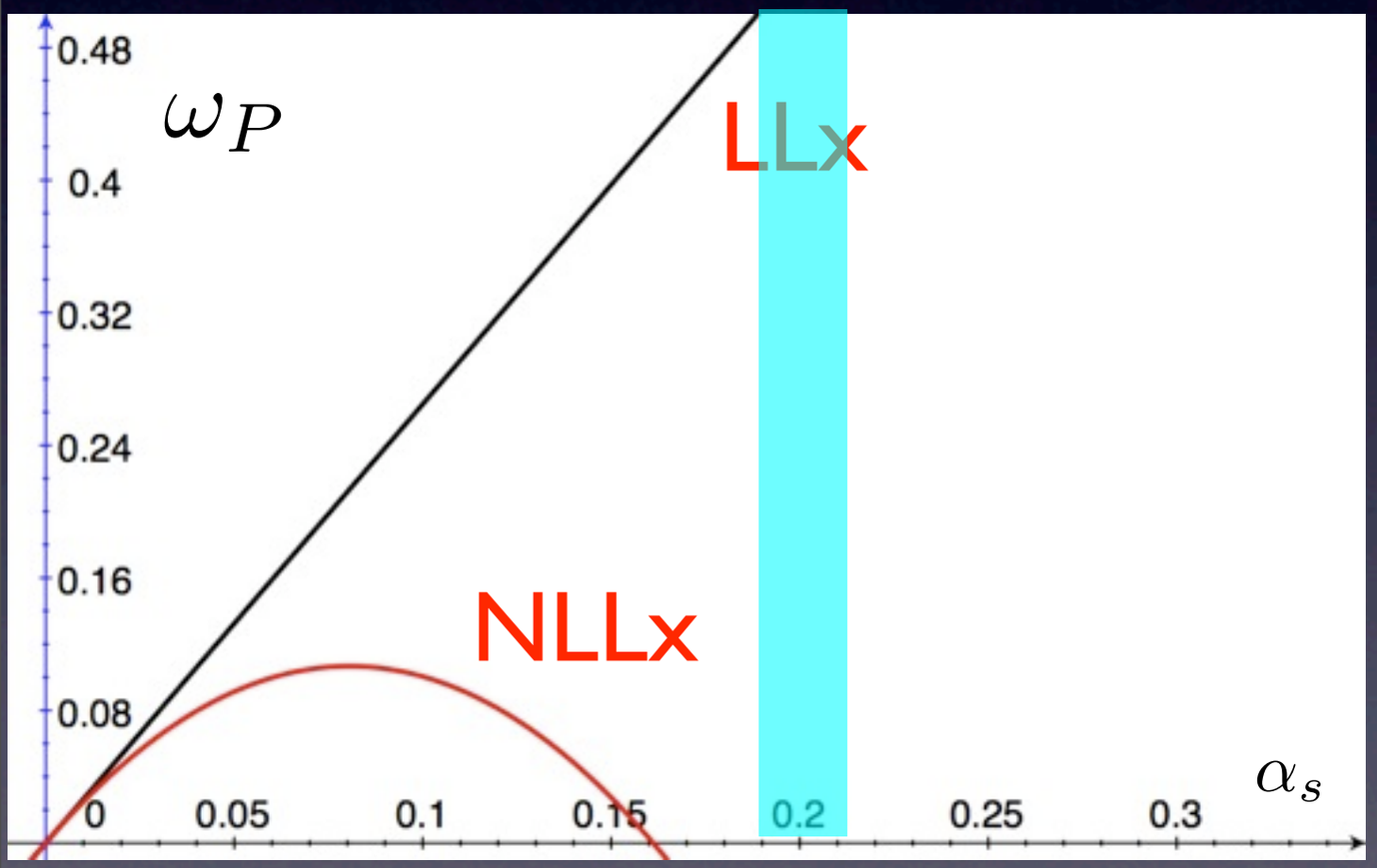

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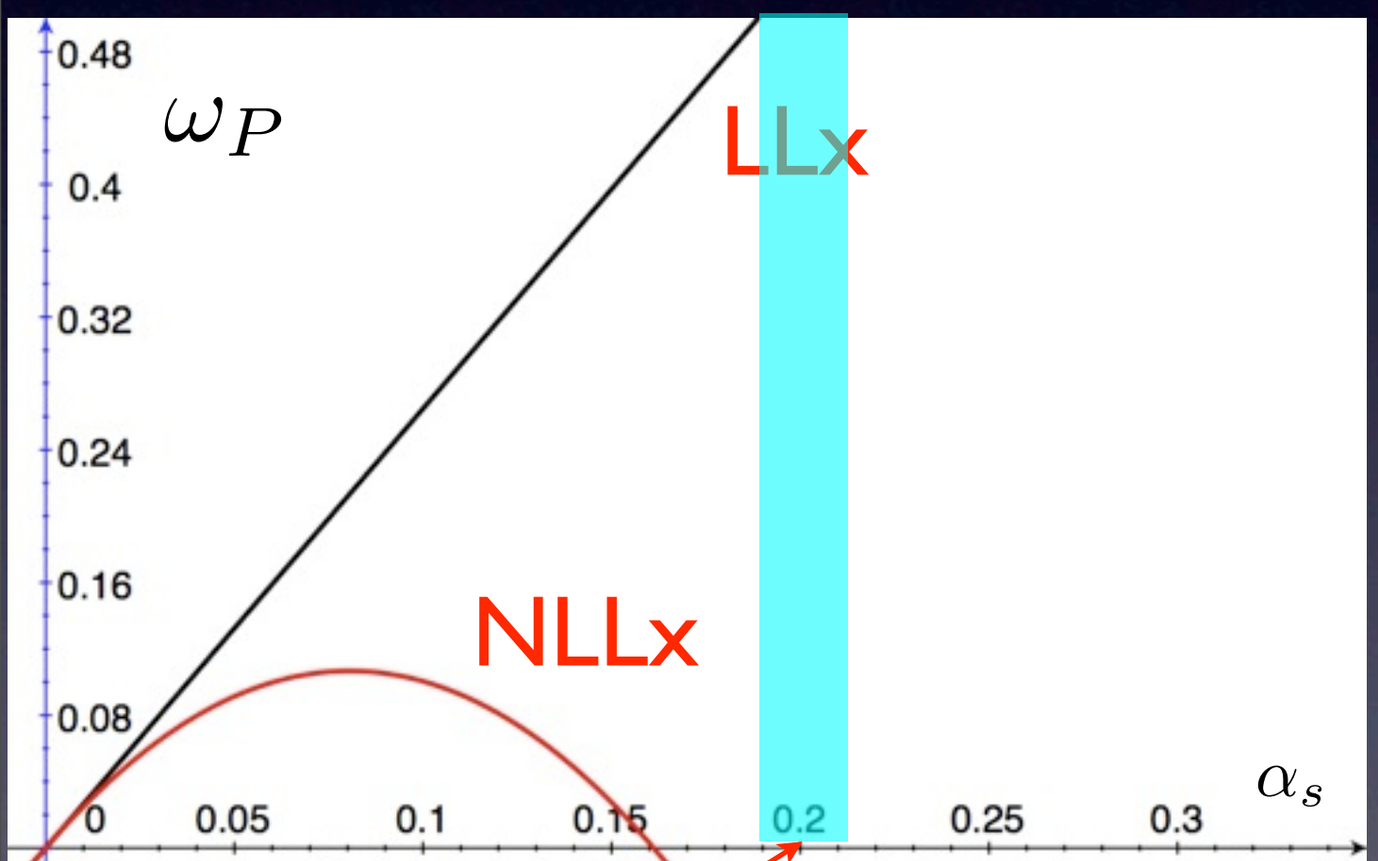

typical values of α_s

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M. Ciafaloni, D. Colferai, G. Salam, A.Stasto

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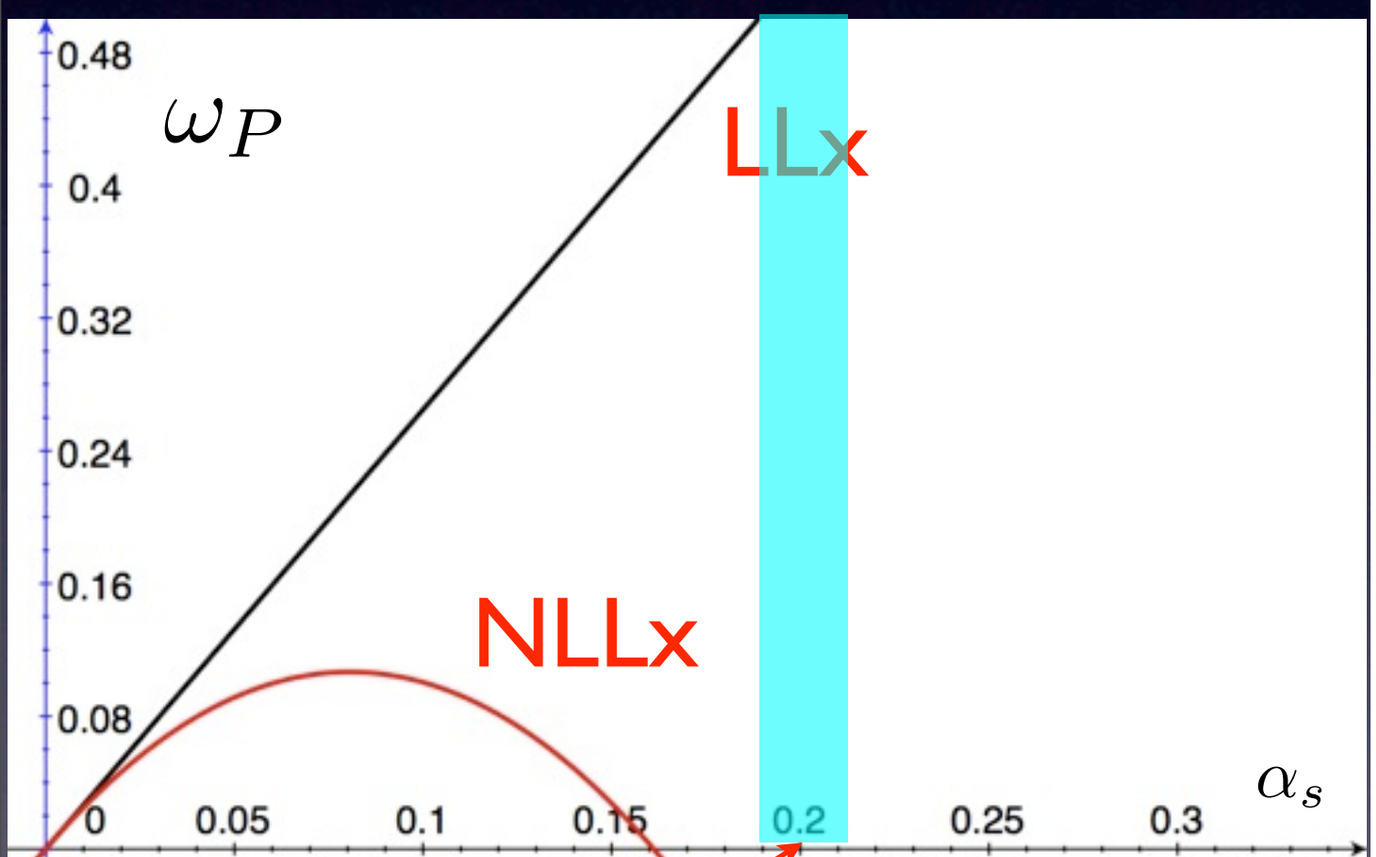
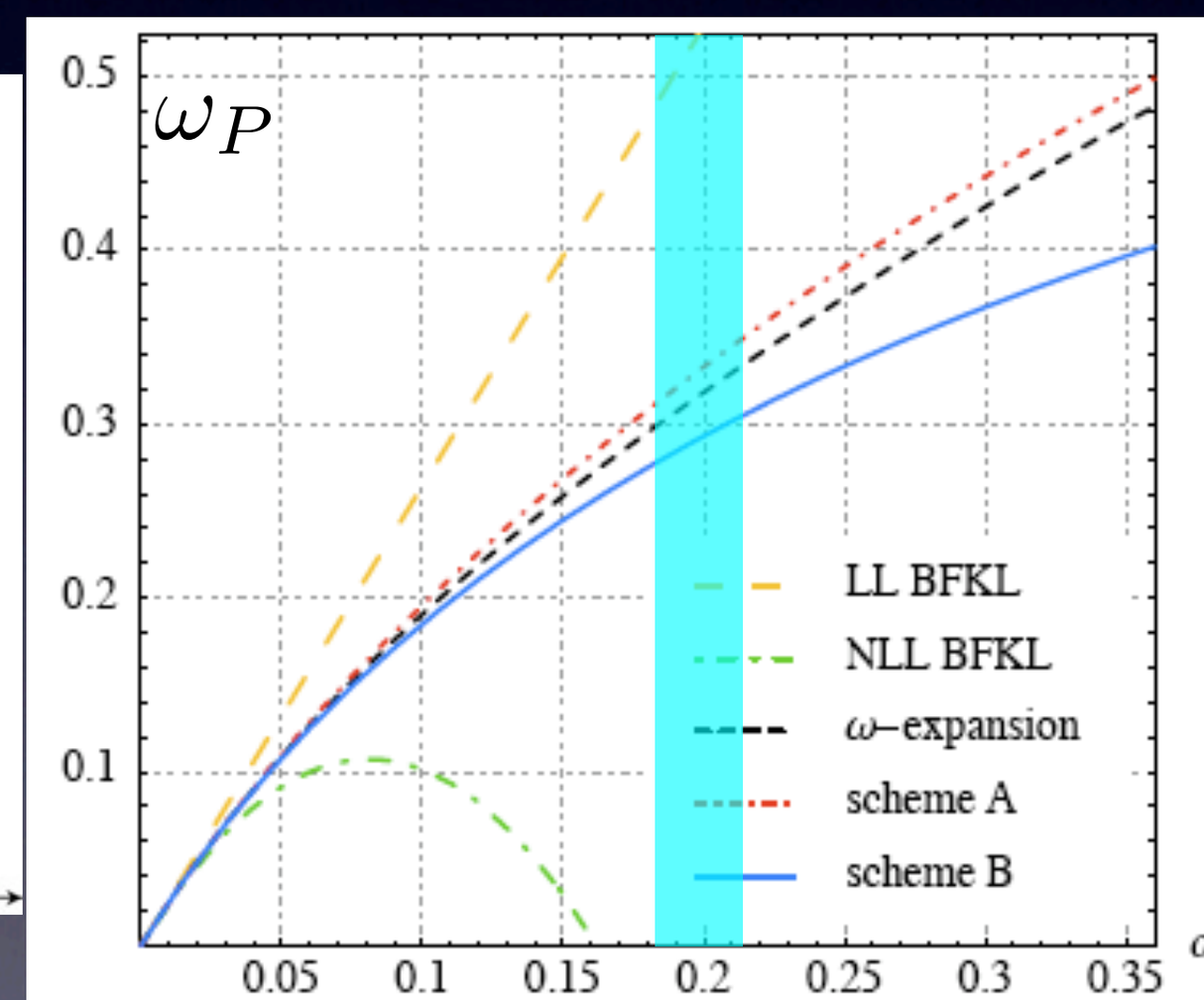


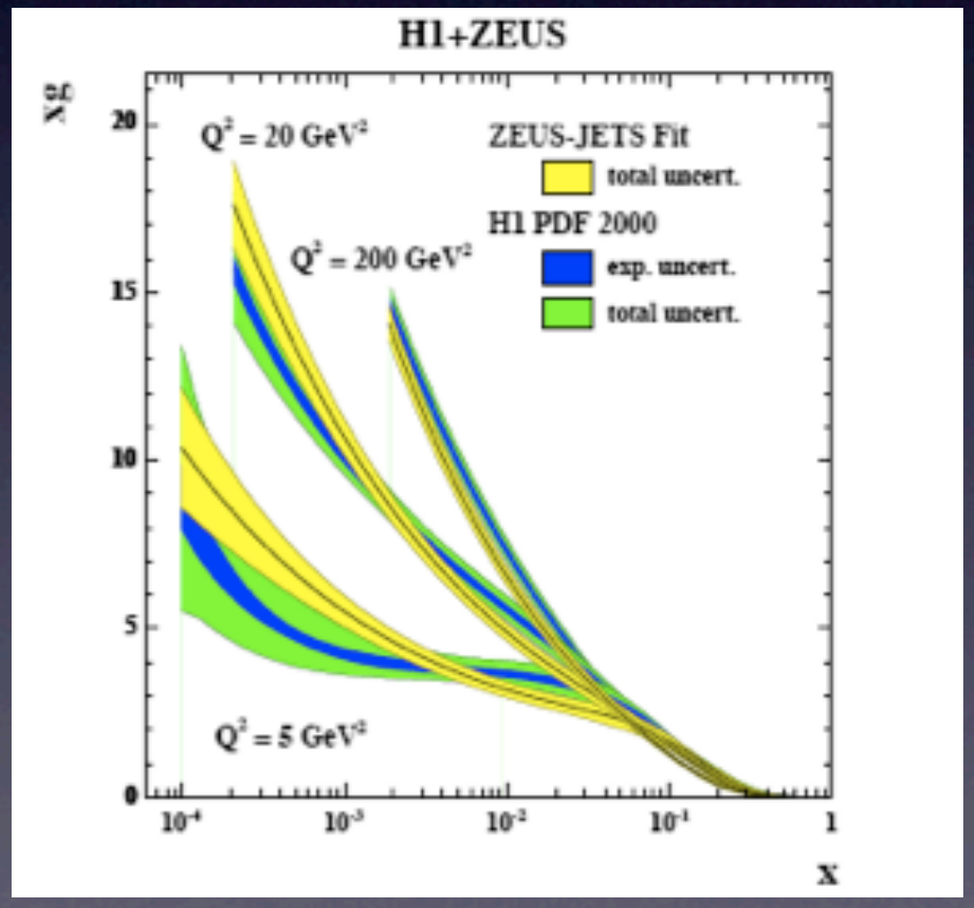
typical values of α_s

After...



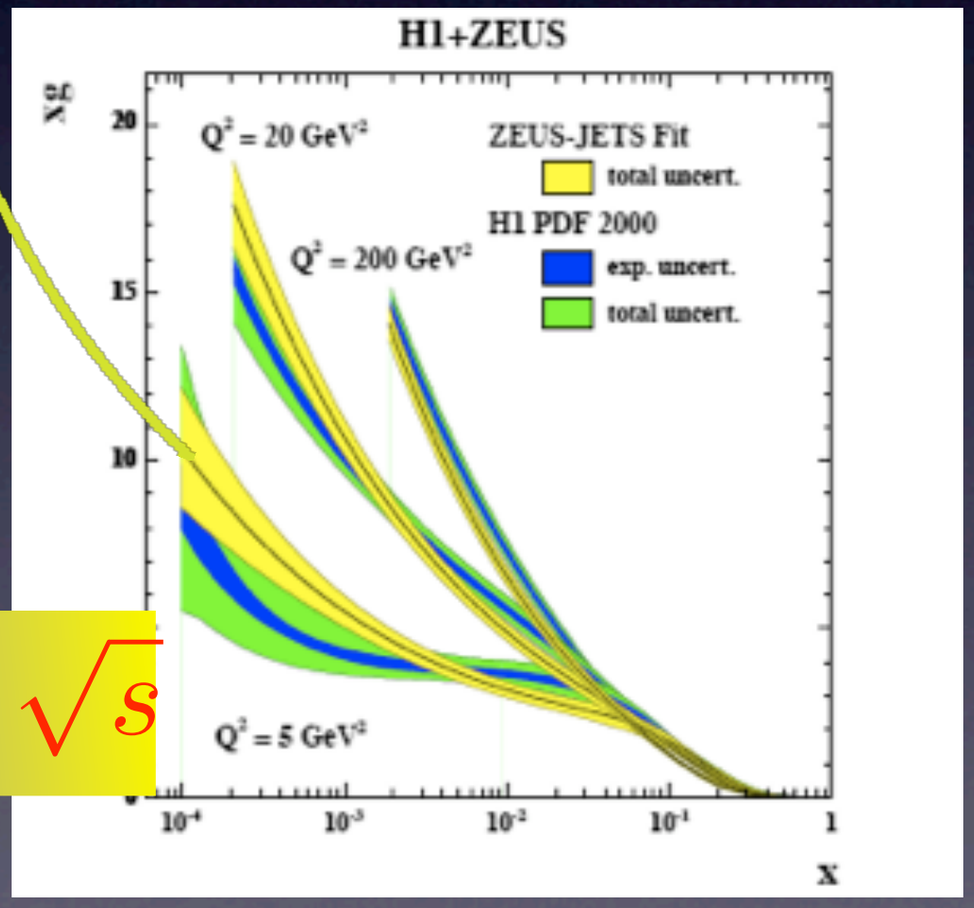
Parton Saturation

Can the gluon density rise forever?
How fast?



Can the gluon density rise forever? How fast?

very large density of gluons



Increase energy $\sqrt{s}$

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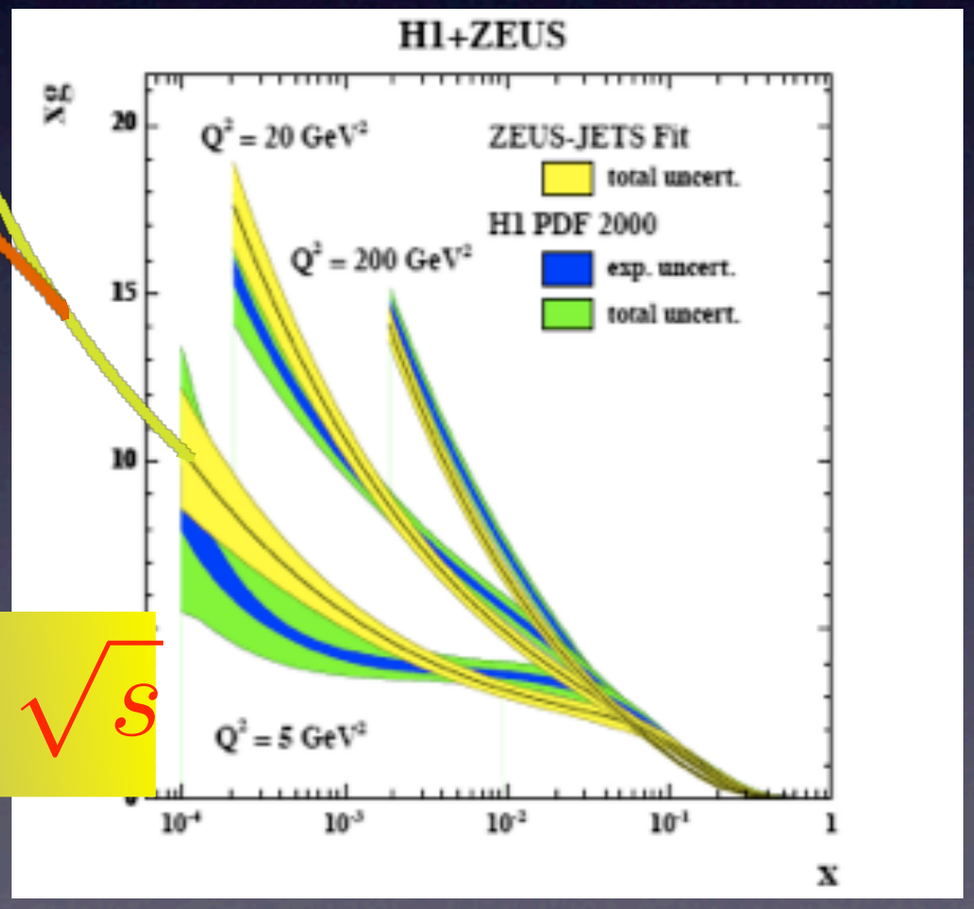

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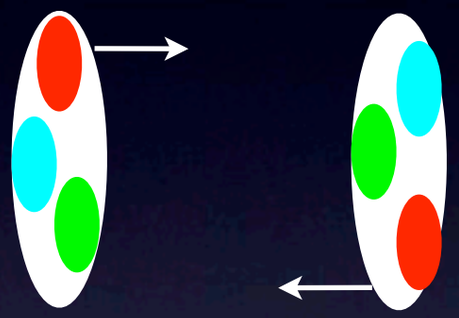

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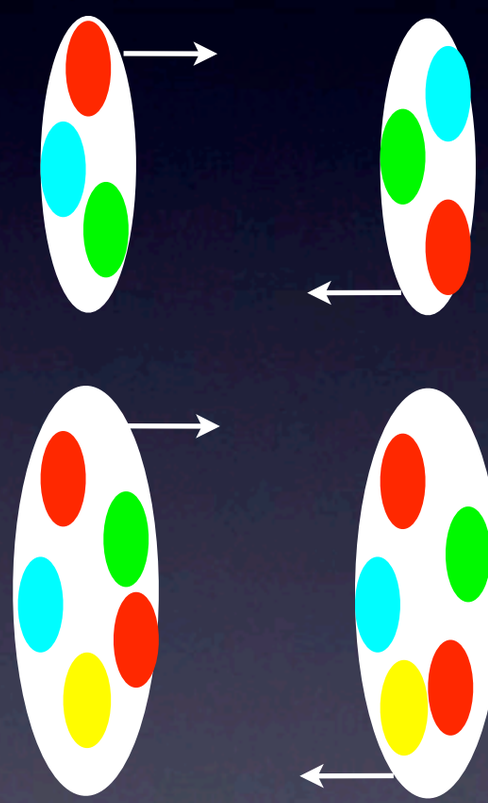

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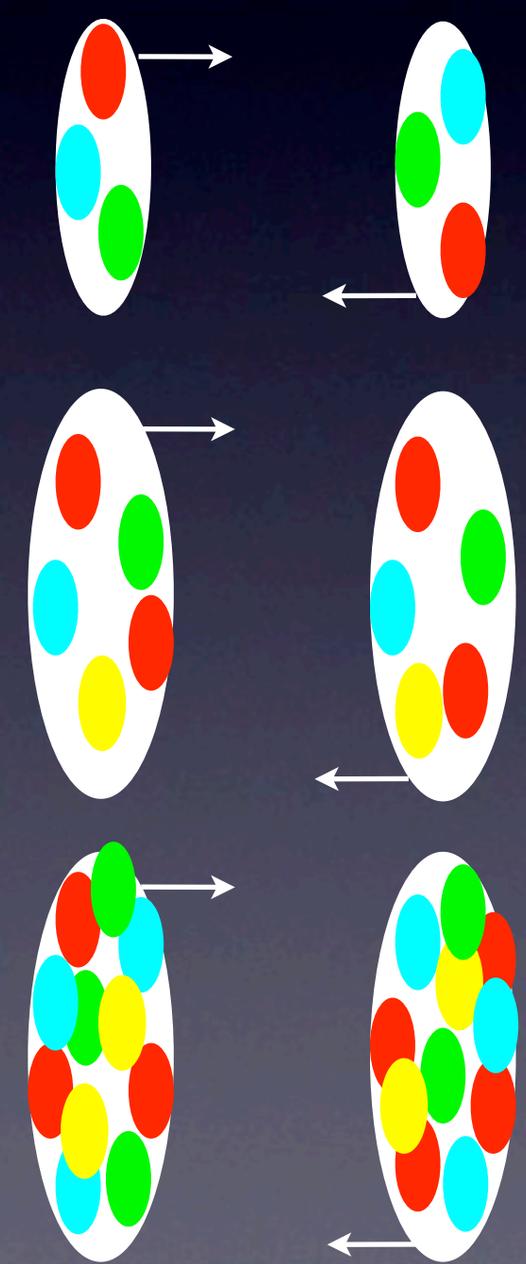

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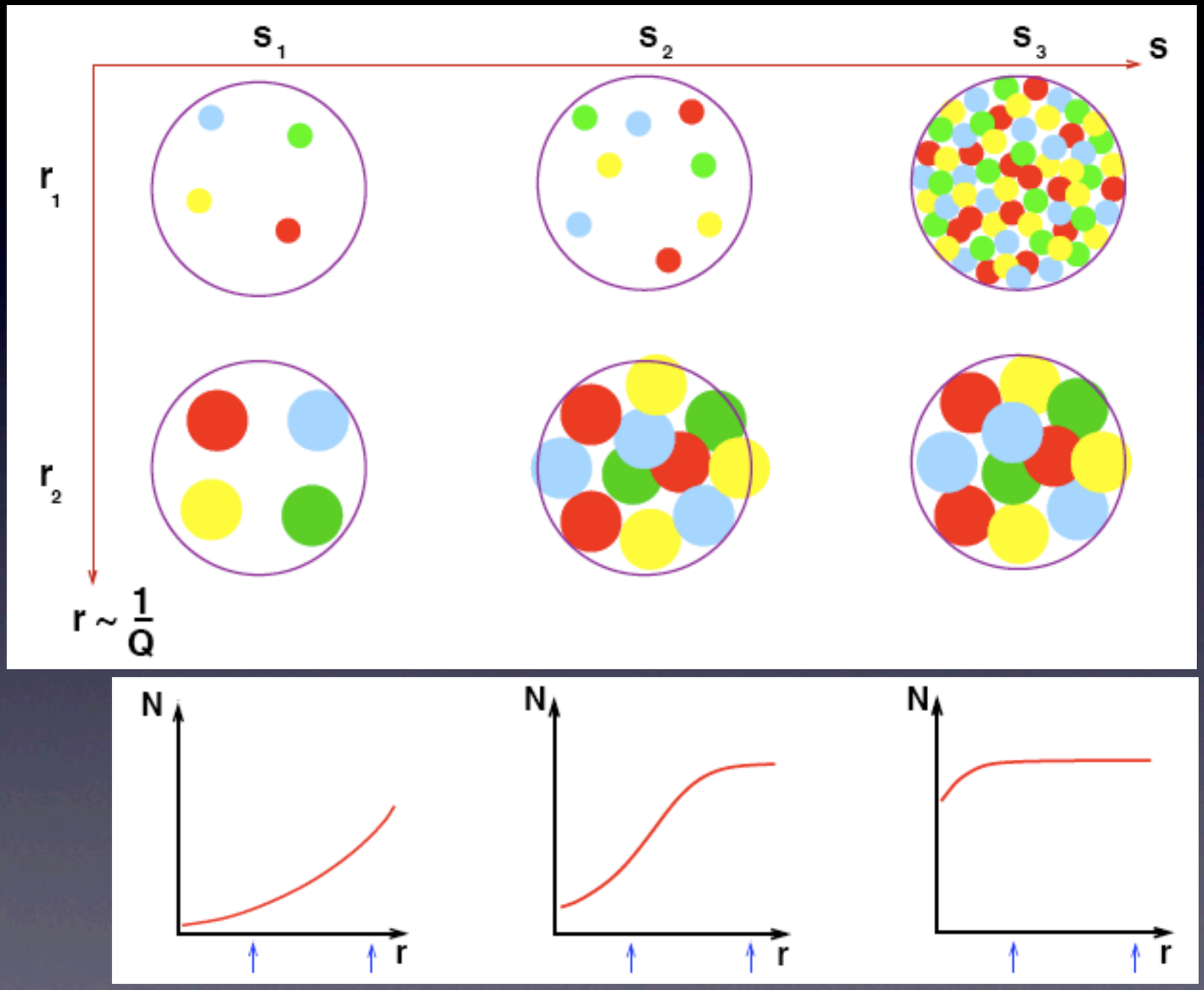
- Increase in the density of the projectile.
- It is limited by the unitarity at fixed impact parameter. Probability of the interaction:

$$N(b) \leq 1$$

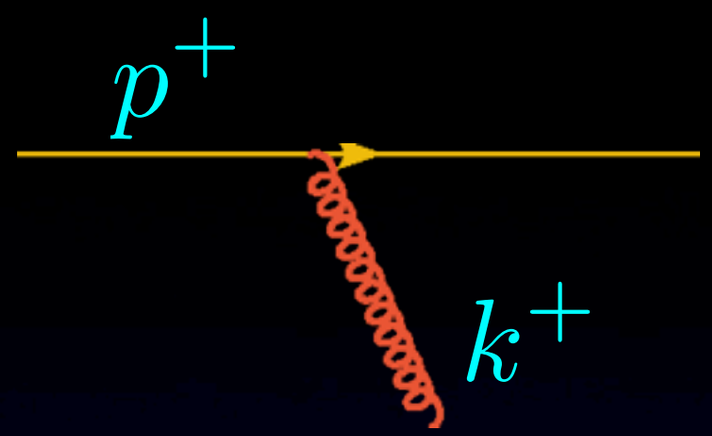
Parton saturation

energy of the interaction

size of the probe



Microscopic mechanism:



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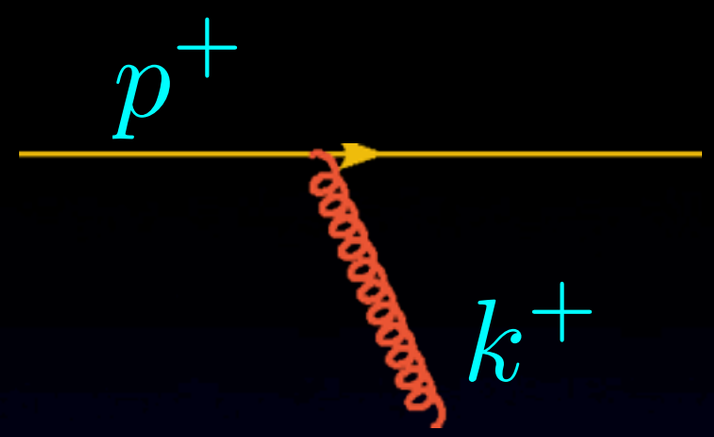
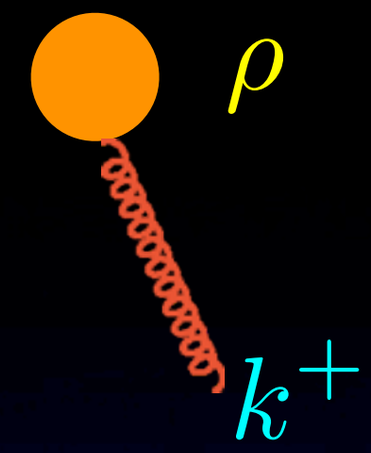


Single charge (source)



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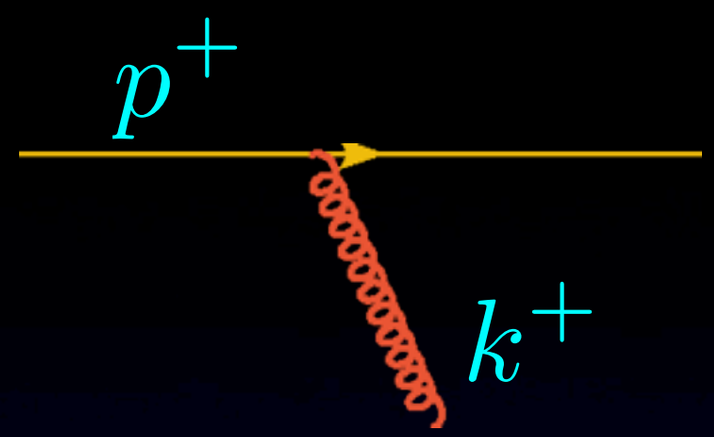

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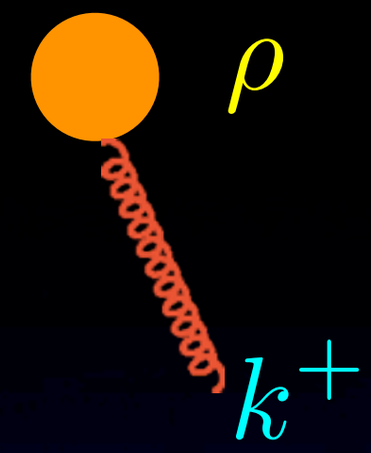

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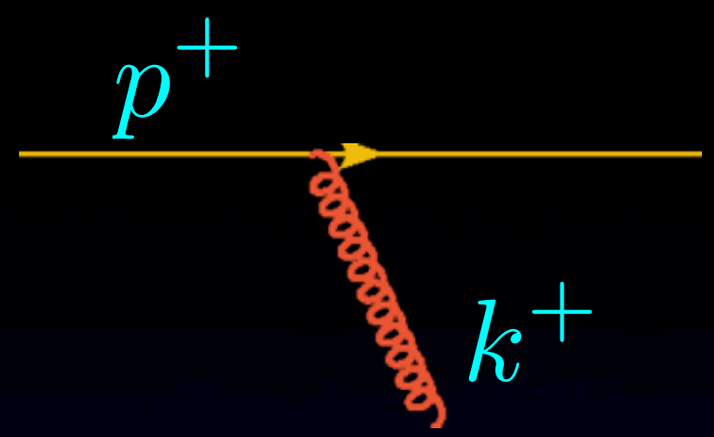

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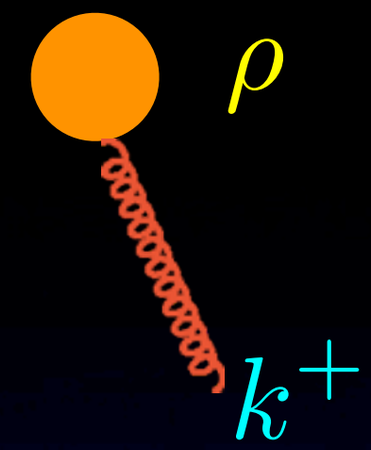

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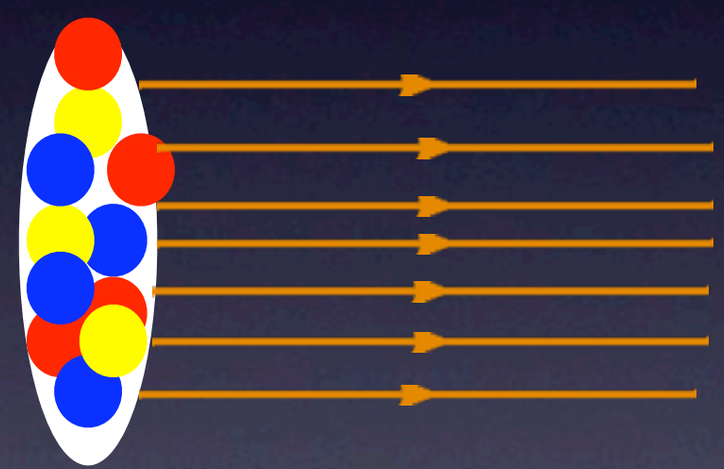

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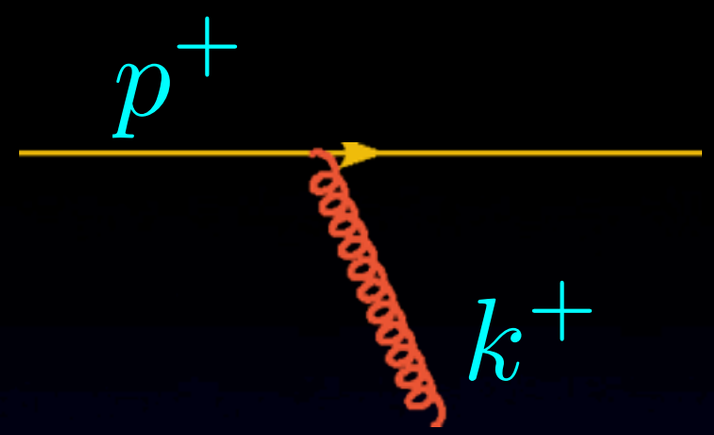

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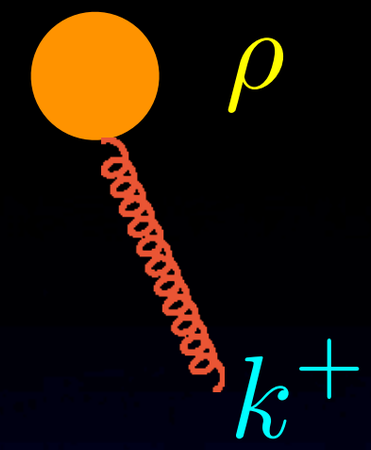

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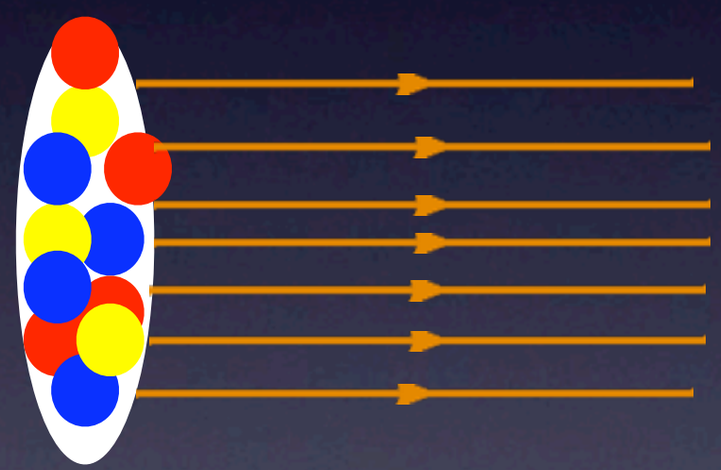
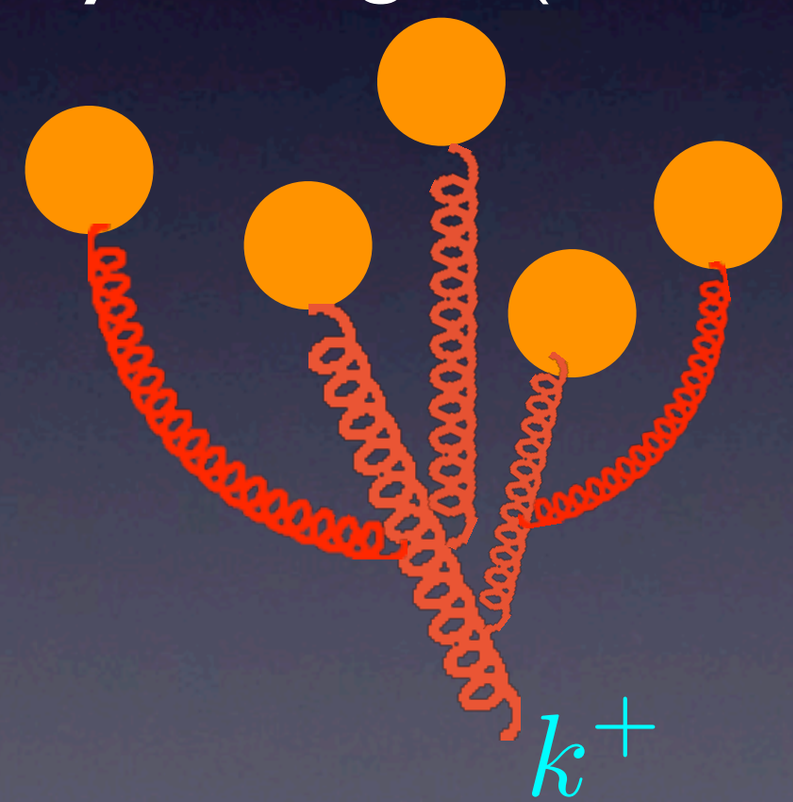


Many charges (sources)



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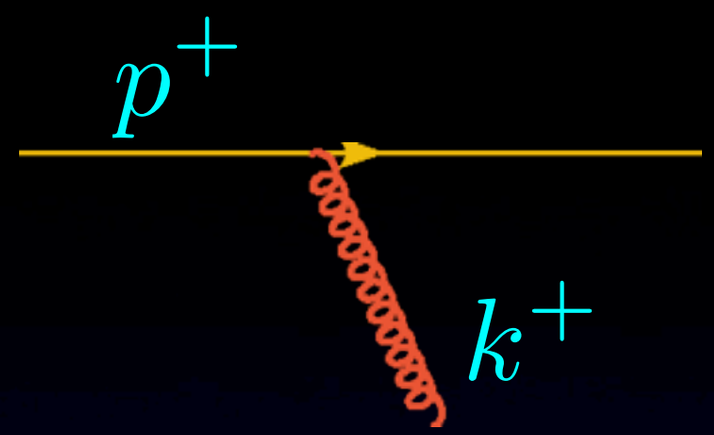

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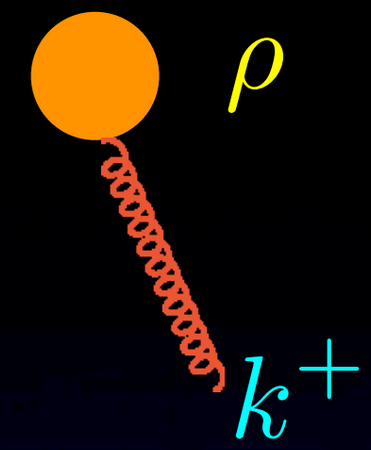

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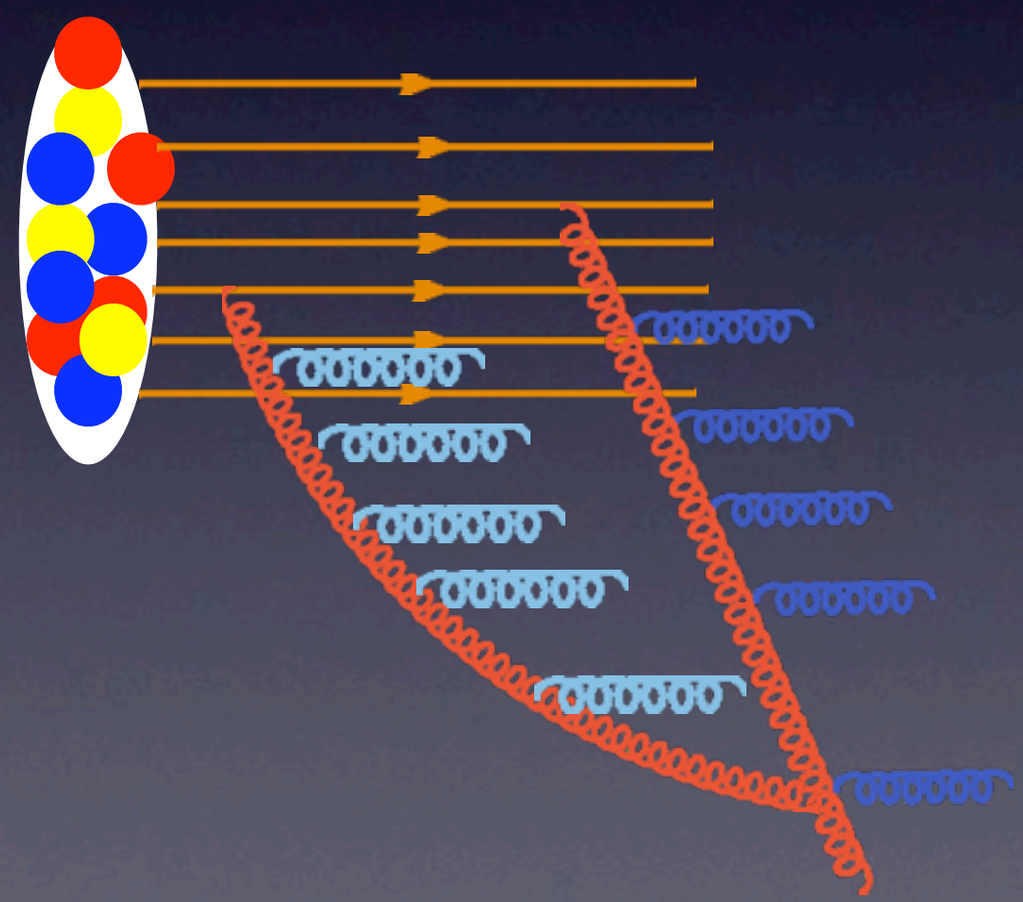

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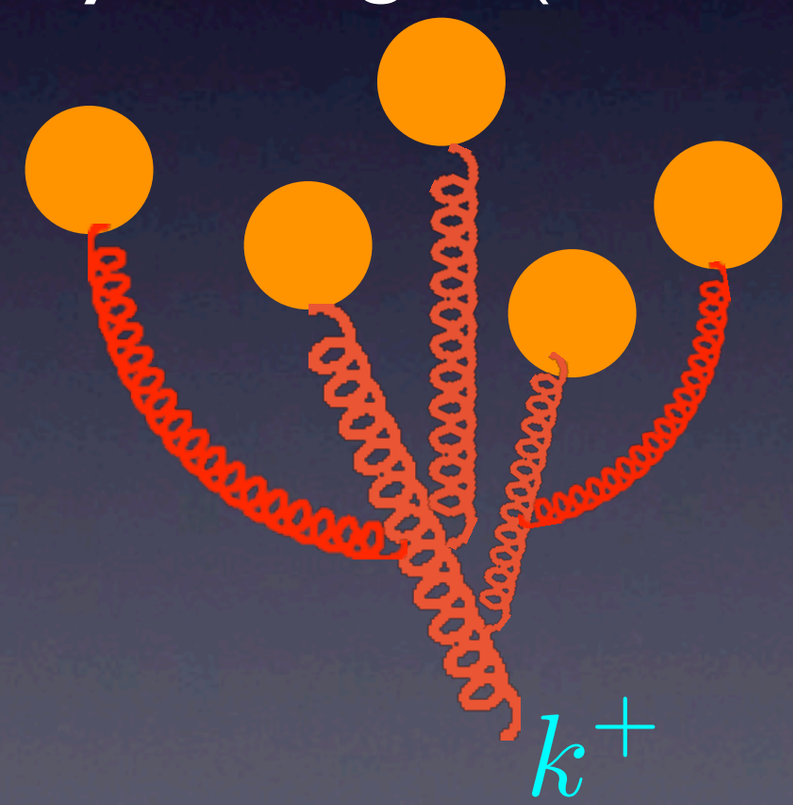

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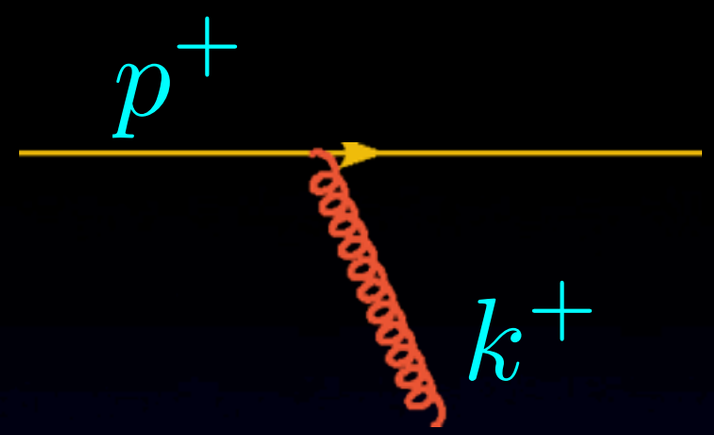

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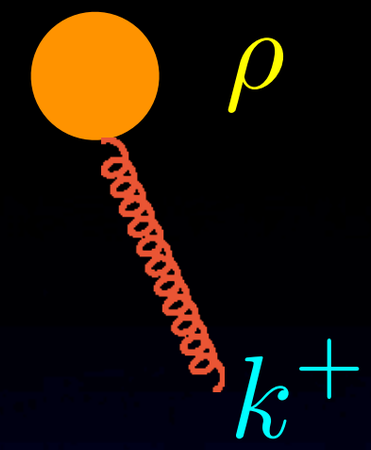

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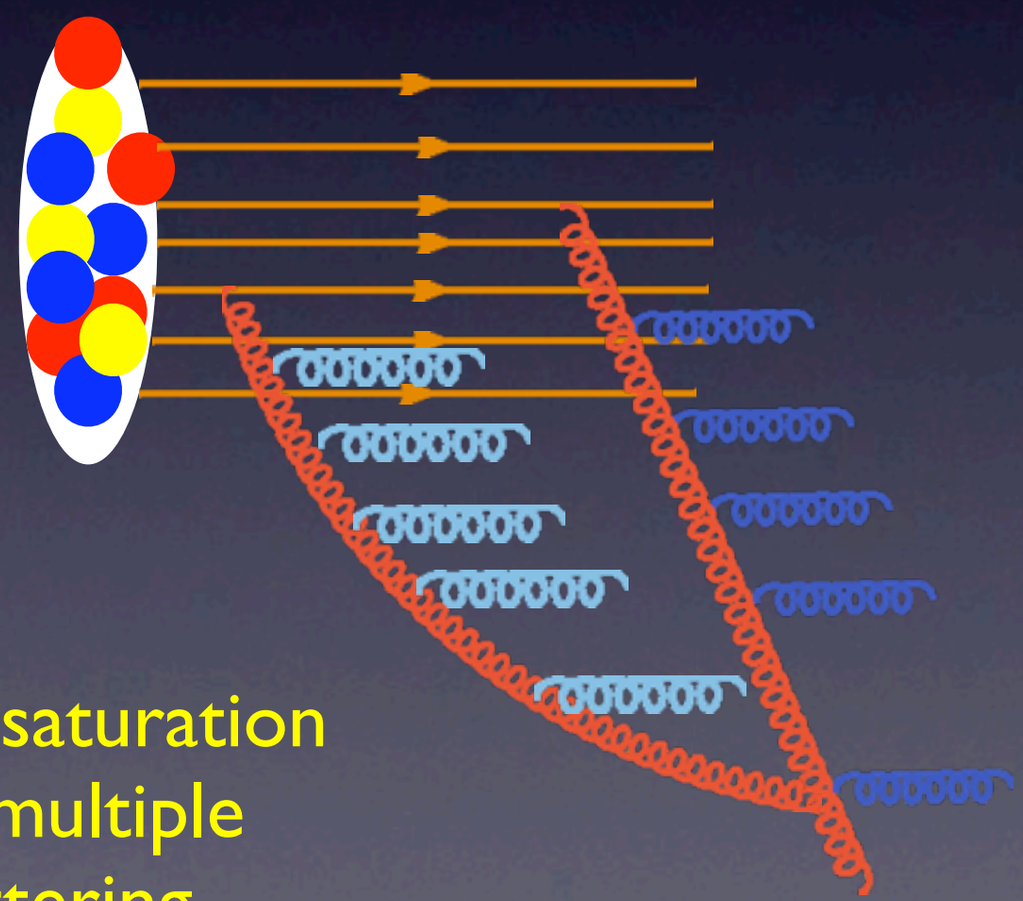
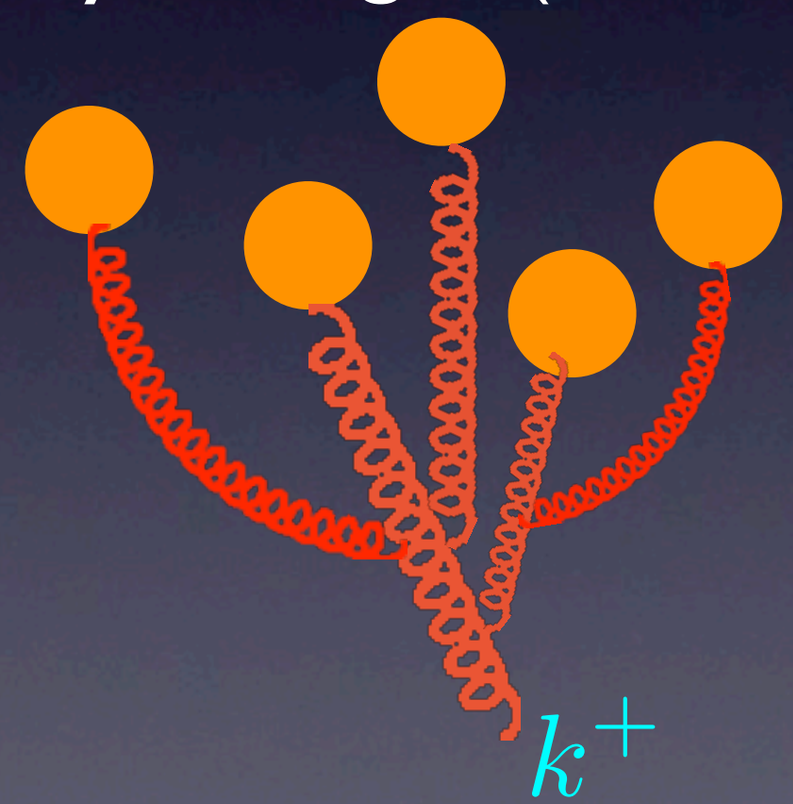


Parton saturation as a multiple scattering.

Many charges (sources)



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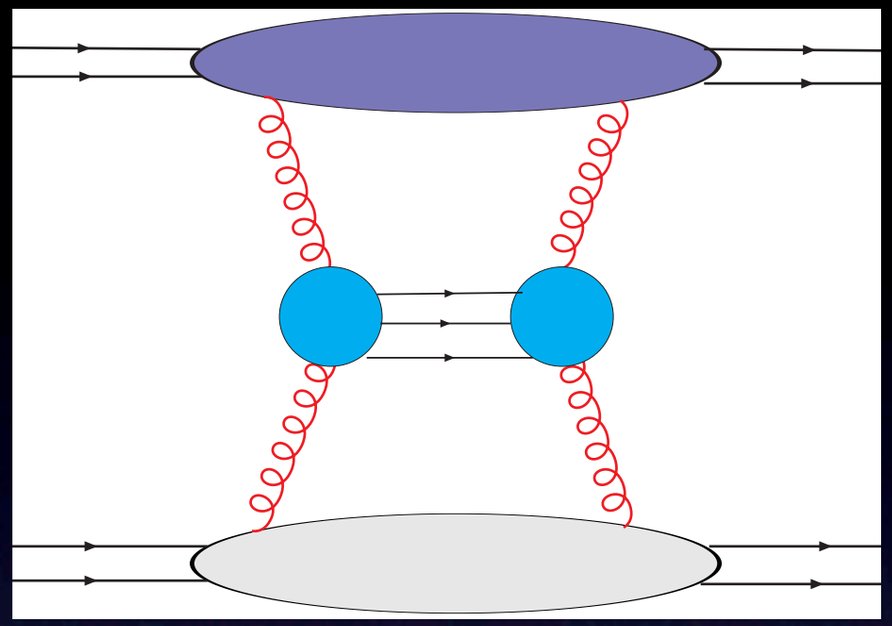

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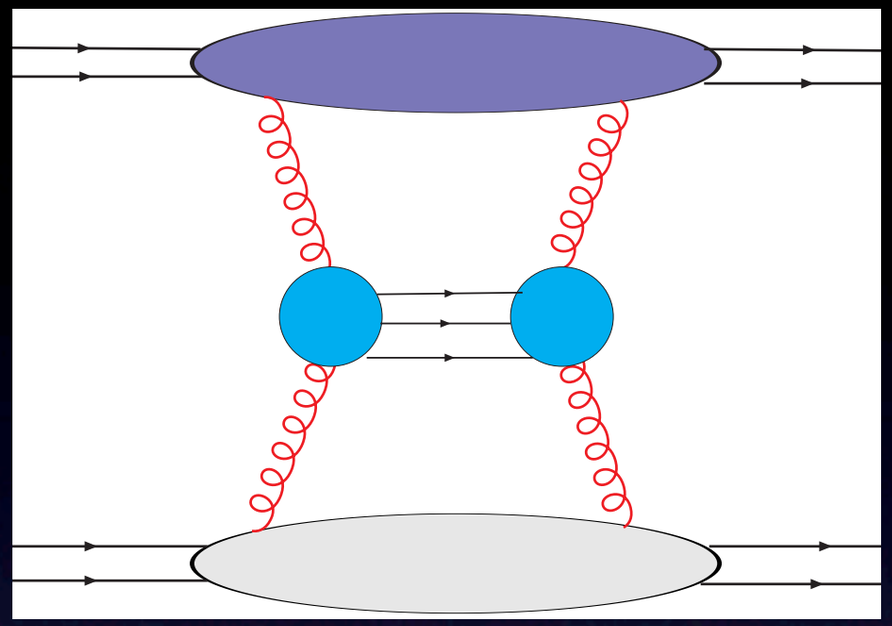

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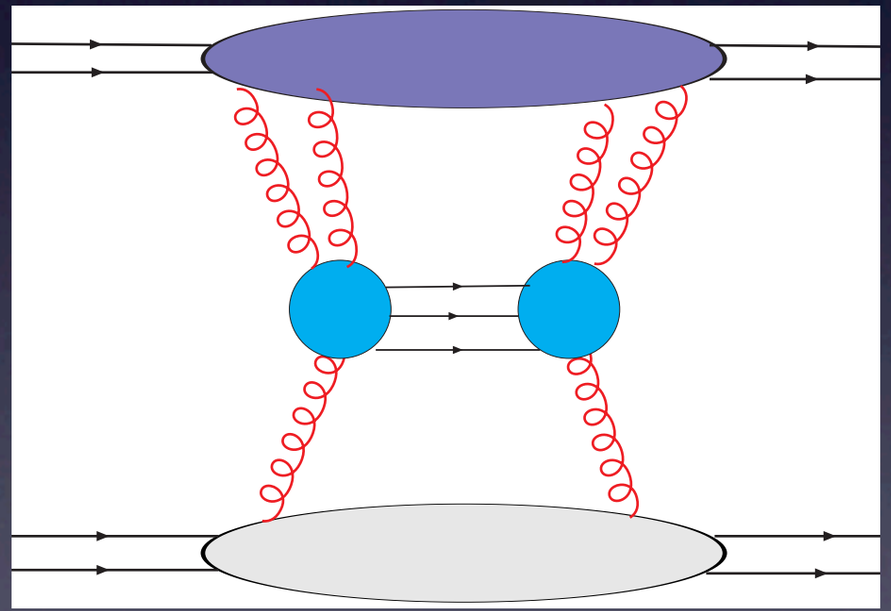

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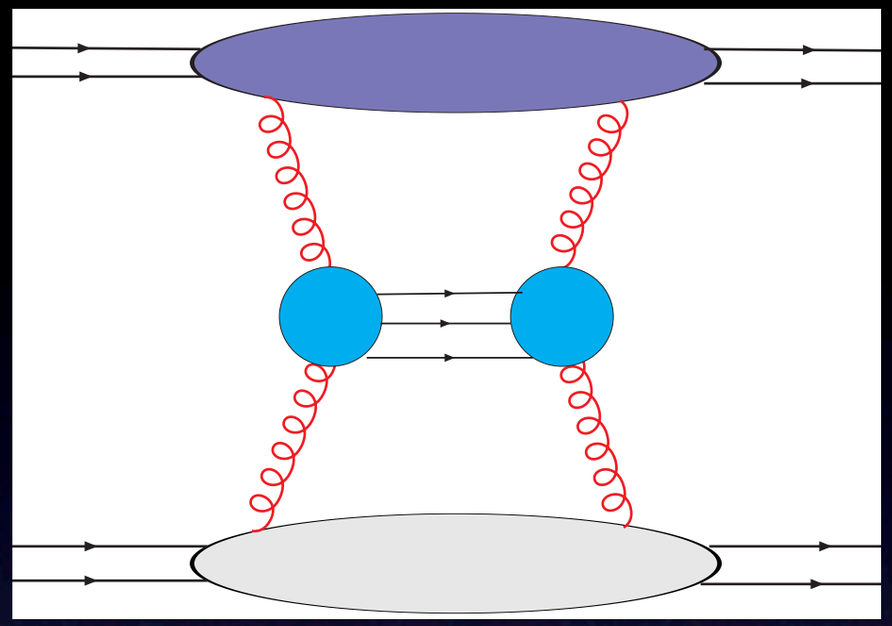

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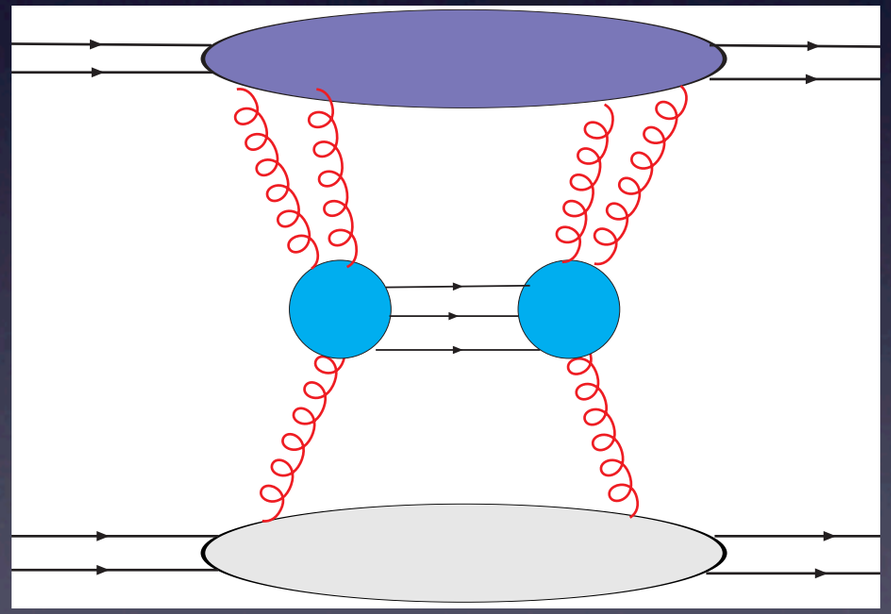

Color Glass Condensate: an effective theory derived from QCD in the limit of high energies and high densities which systematically incorporates higher correlators.

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For small densities in the projectile the evolution equation reduces to the BFKL evolution equation for the gluon density.

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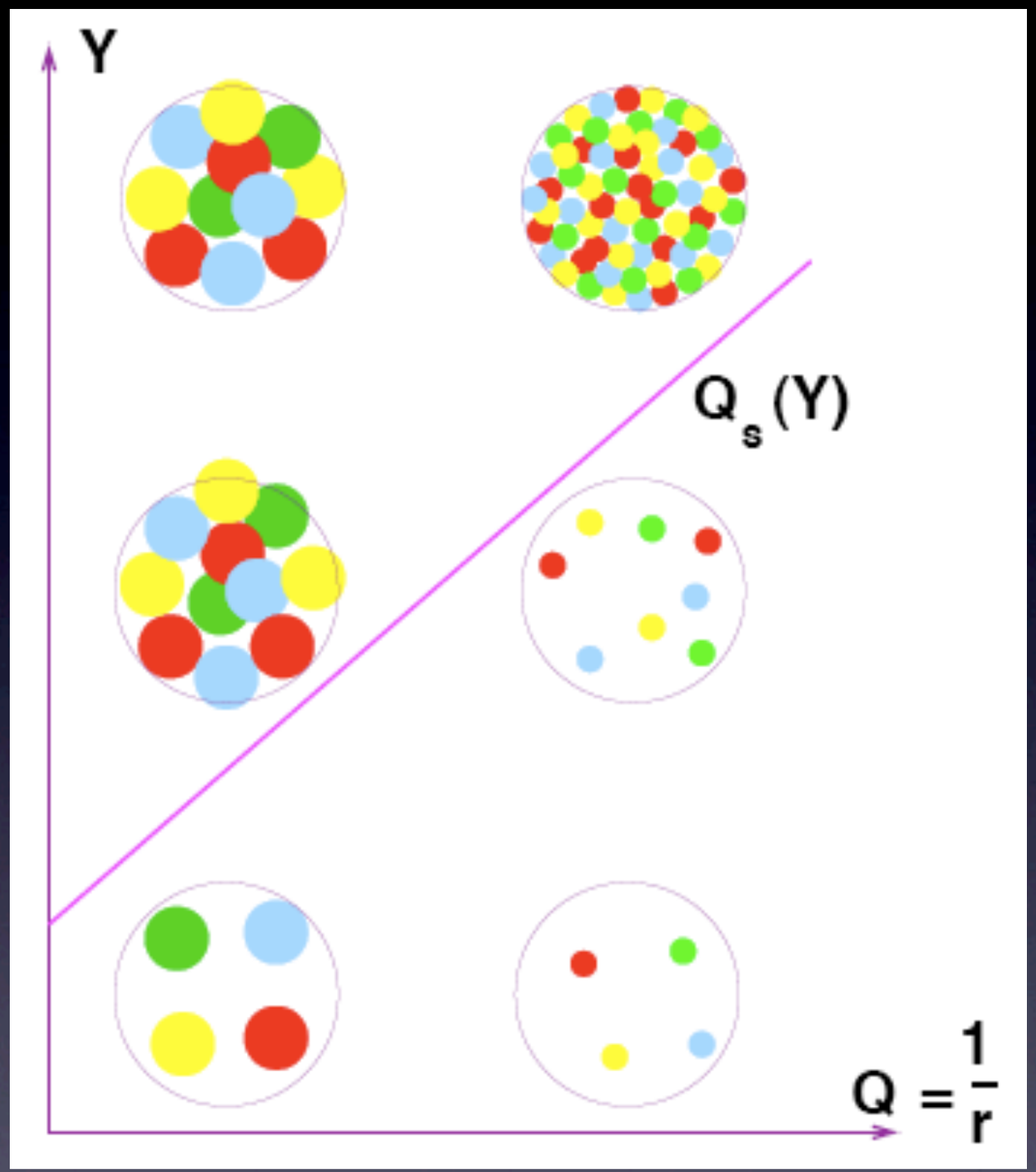
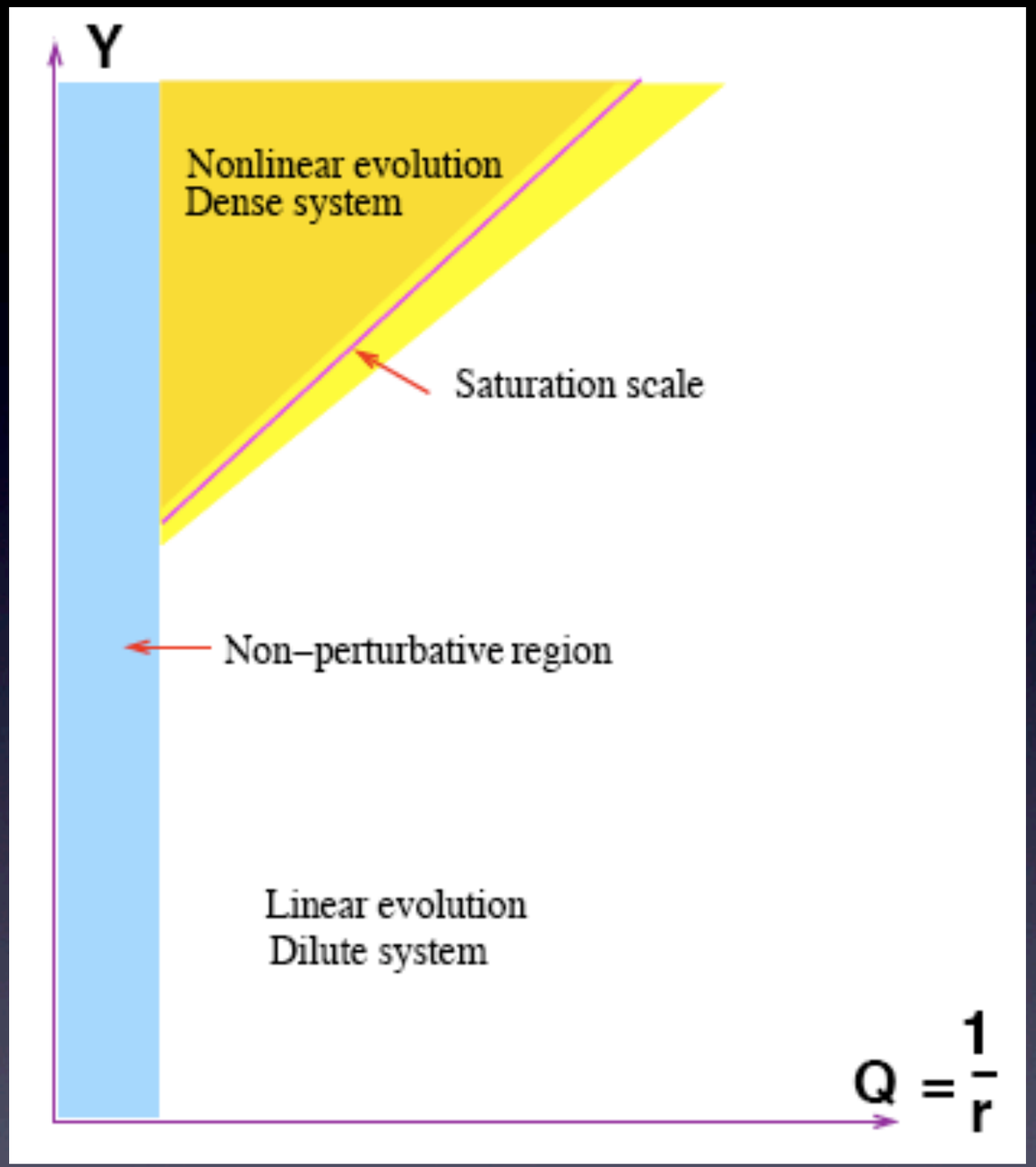
parton saturation

Saturation scale





saturation scale

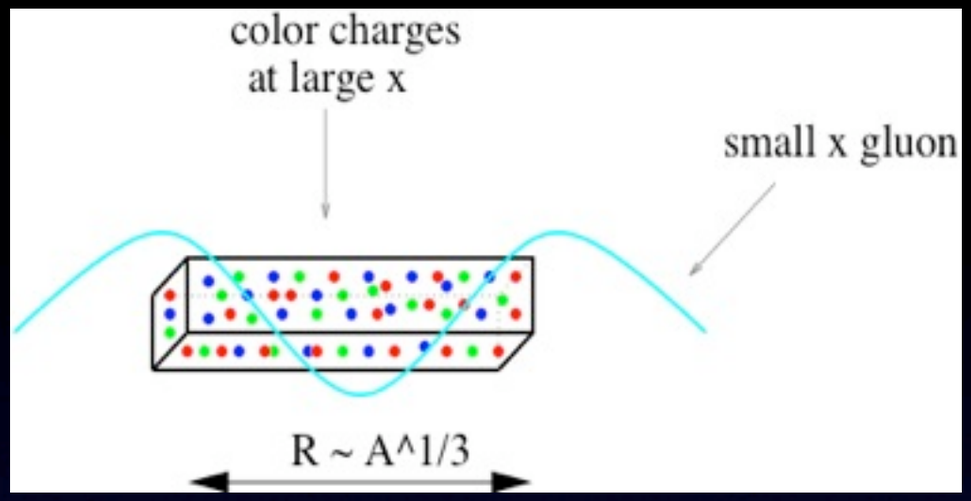
$$Q_s(x) \sim x^{-\lambda}$$

Dynamically generated

Saturation scale grows with A

- Probes interact over distances $L \sim \frac{1}{2m_N x}$

- For $L > 2R_A \sim A^{1/3}$ high-energy probes interact coherently across nuclear size. Very large field strengths.



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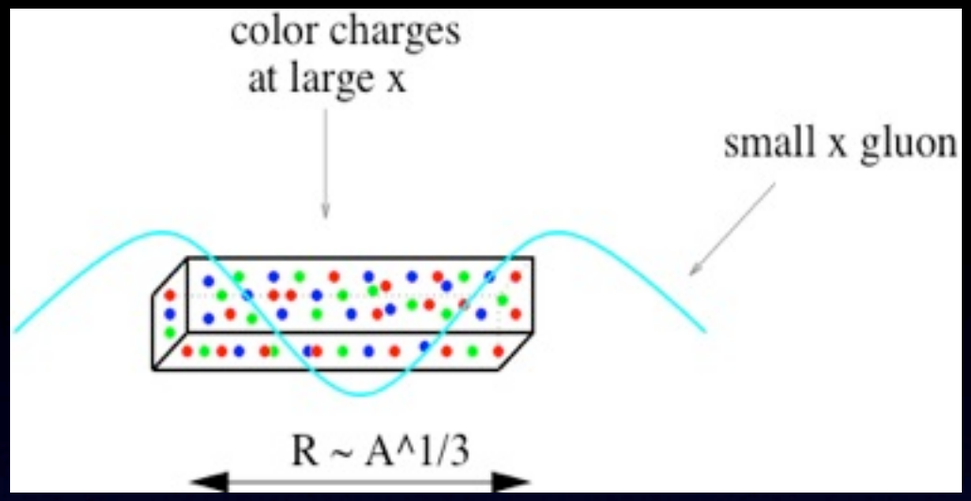

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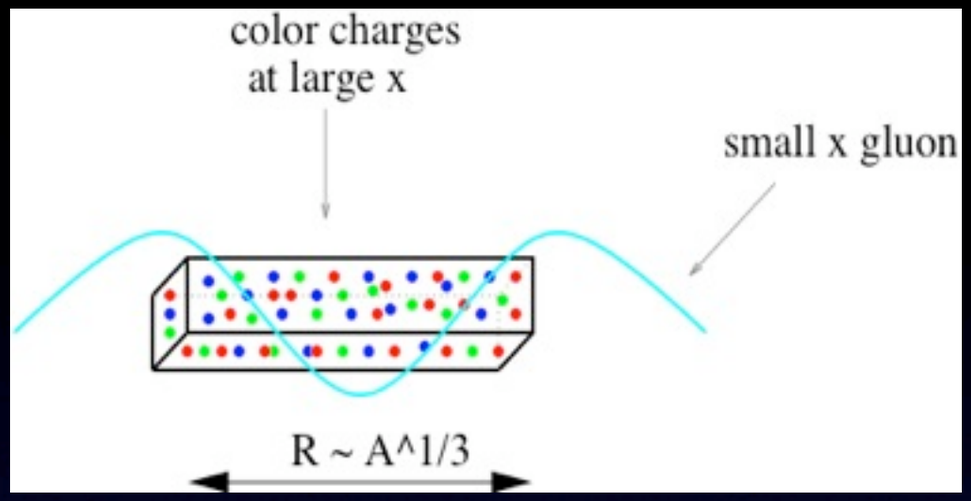

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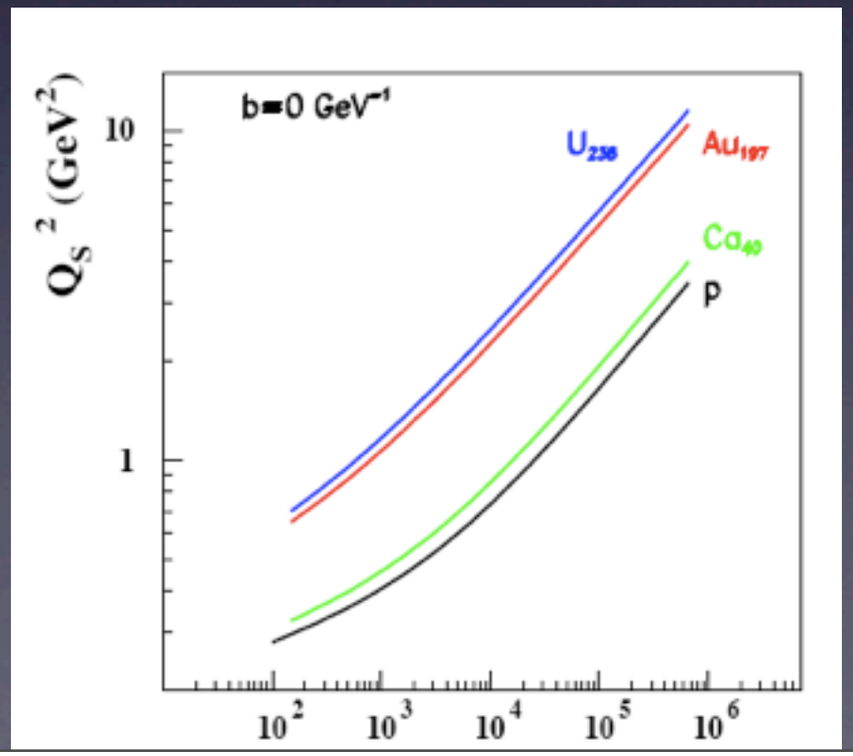
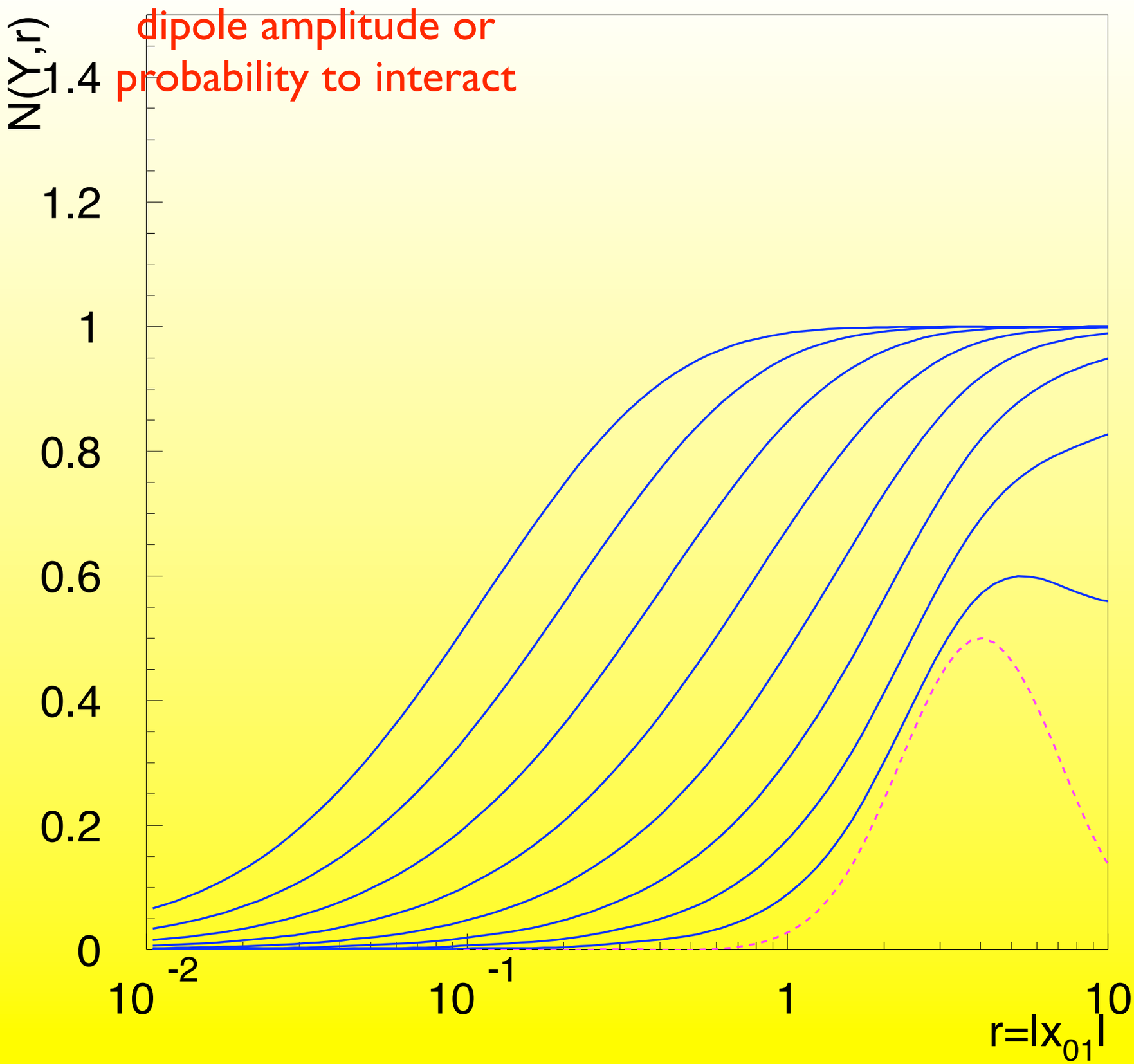
Scattering off nuclei: Saturation is reached for smaller energies due to the enhancement from A.



Kowalski, Teaney

Saturation and the scattering amplitude



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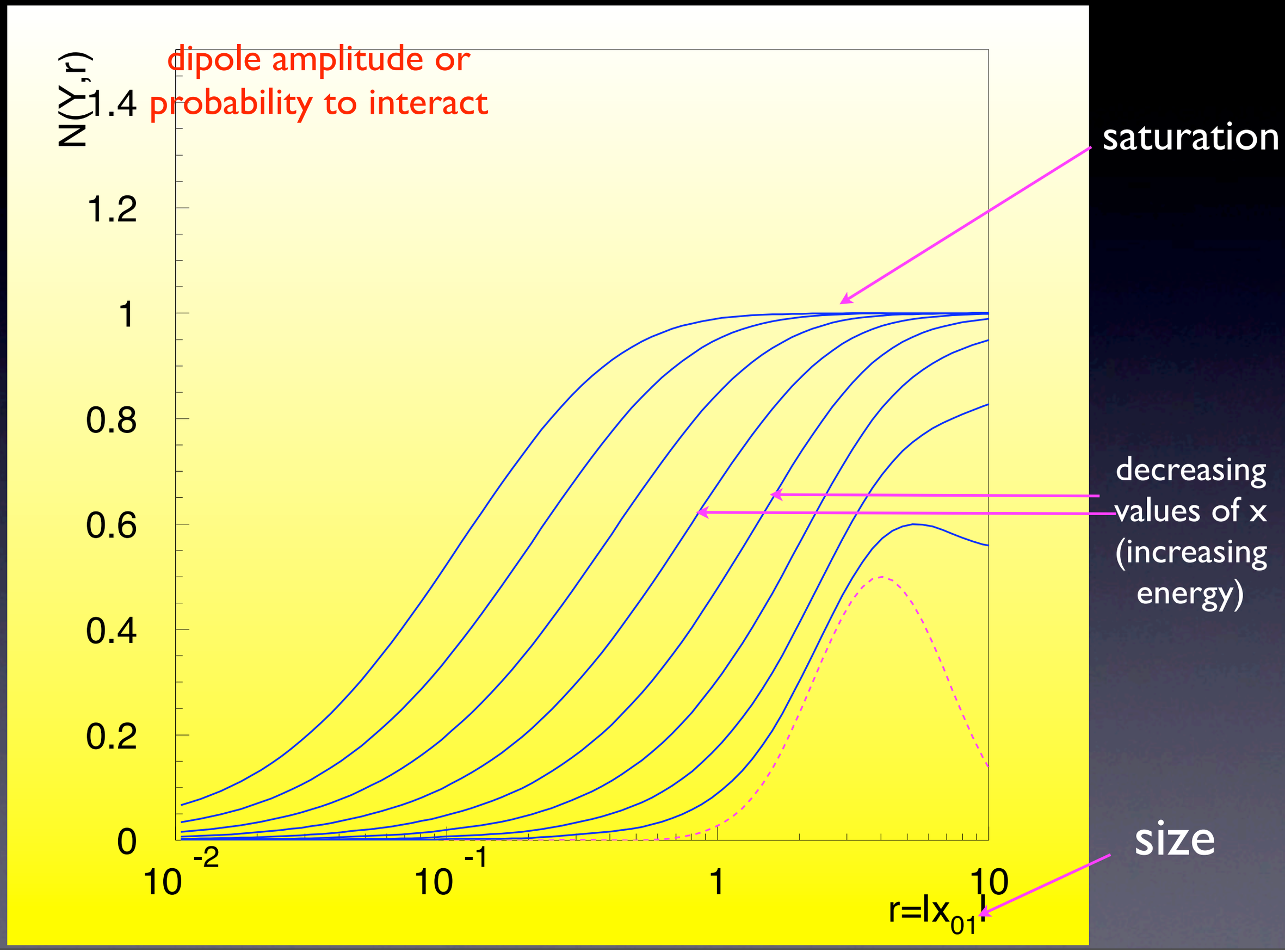

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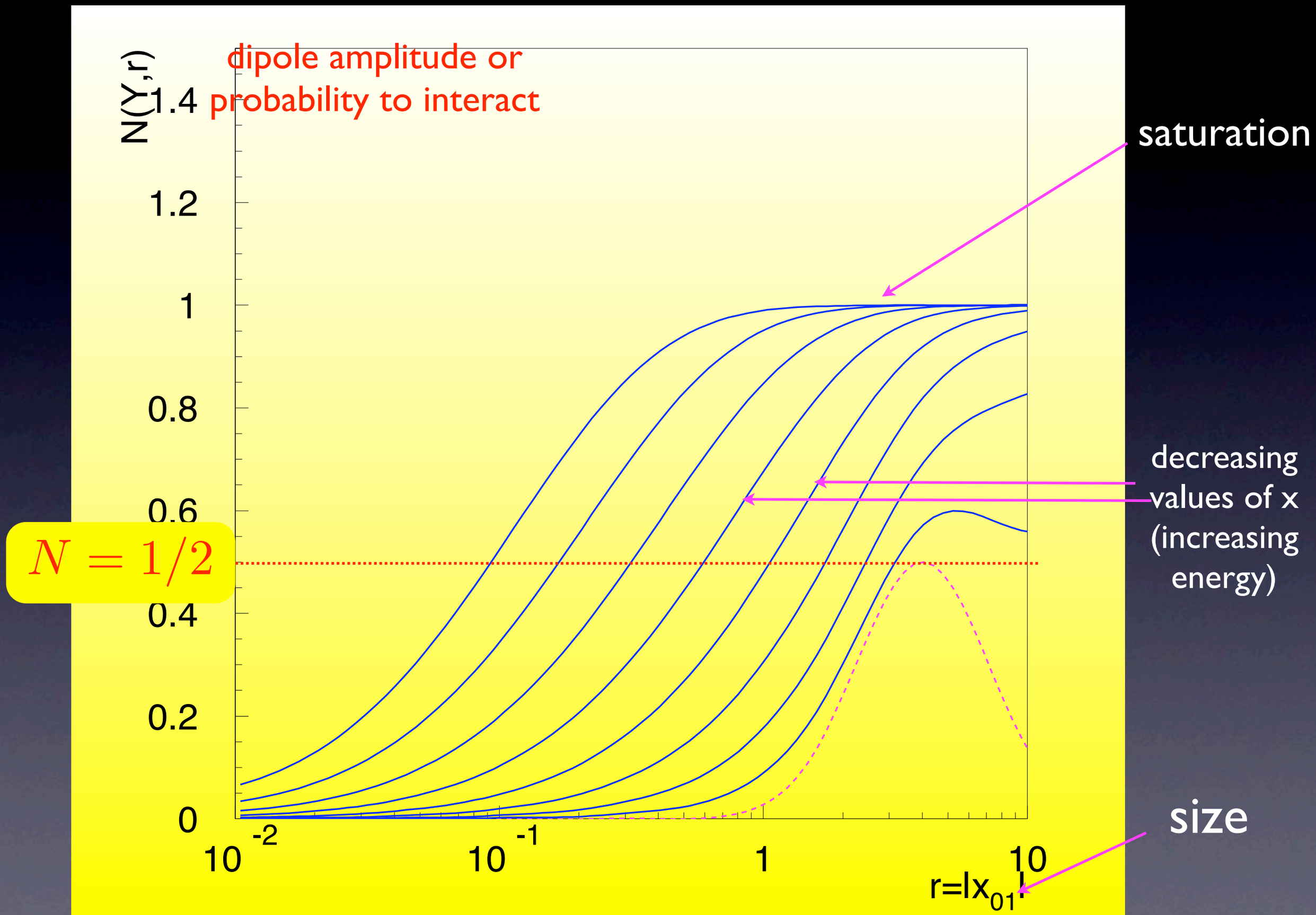

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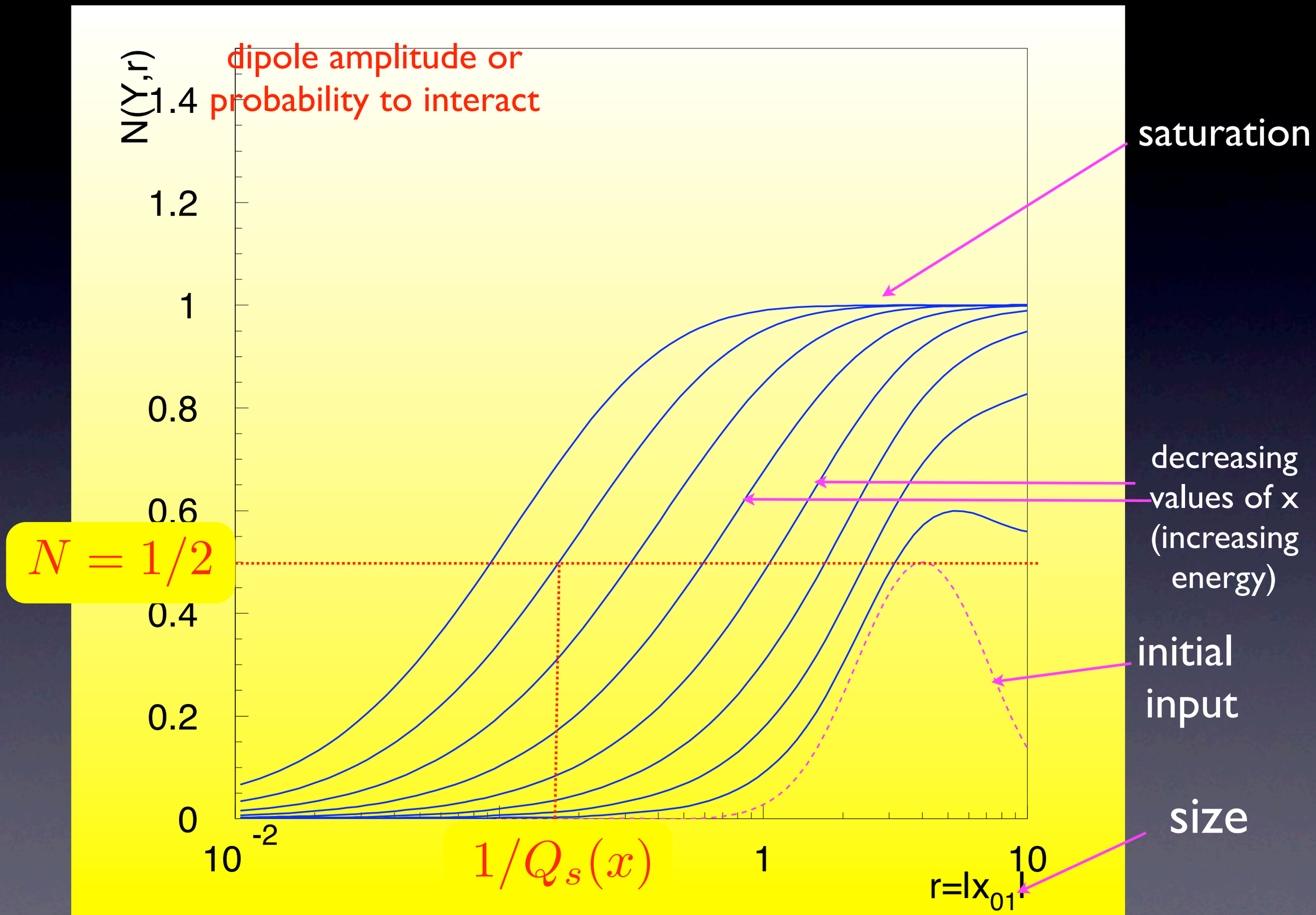

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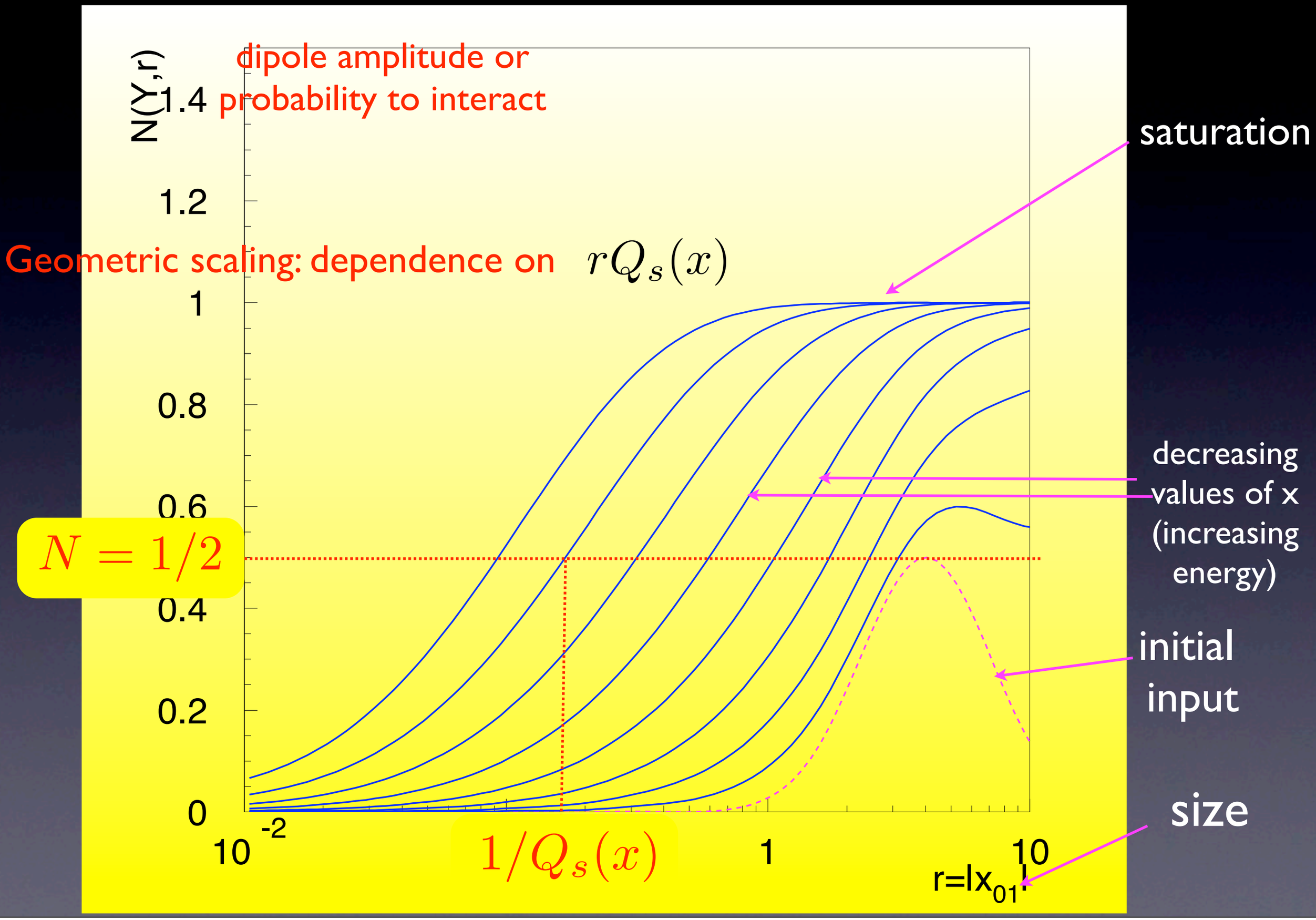

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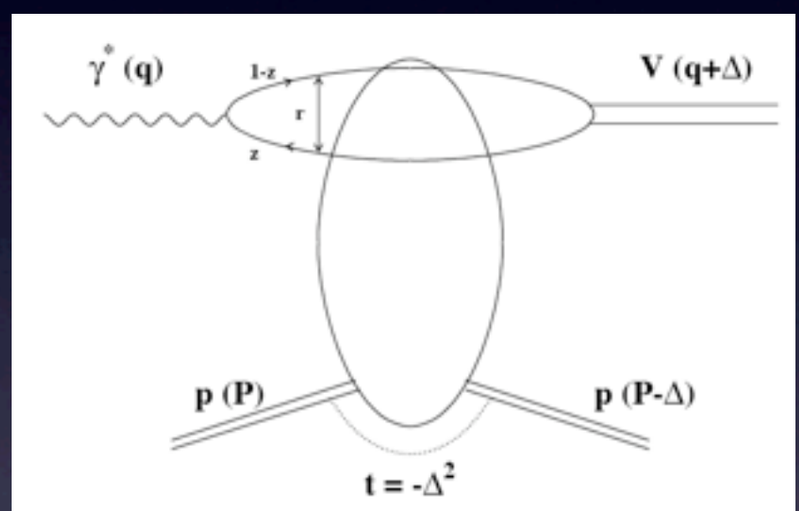

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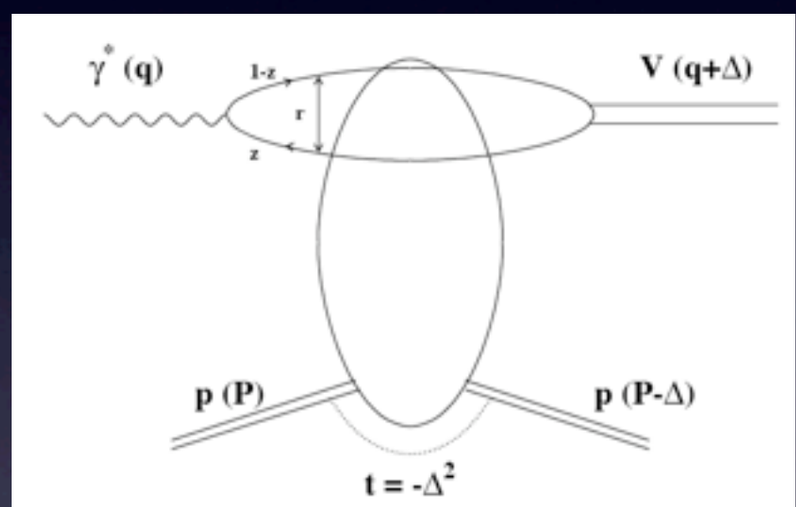

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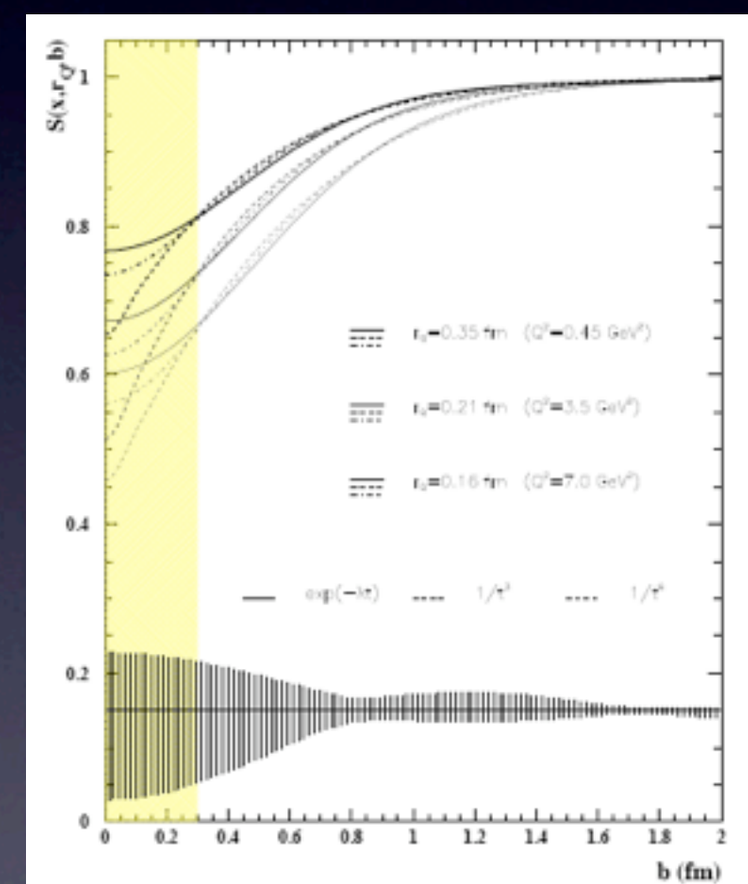

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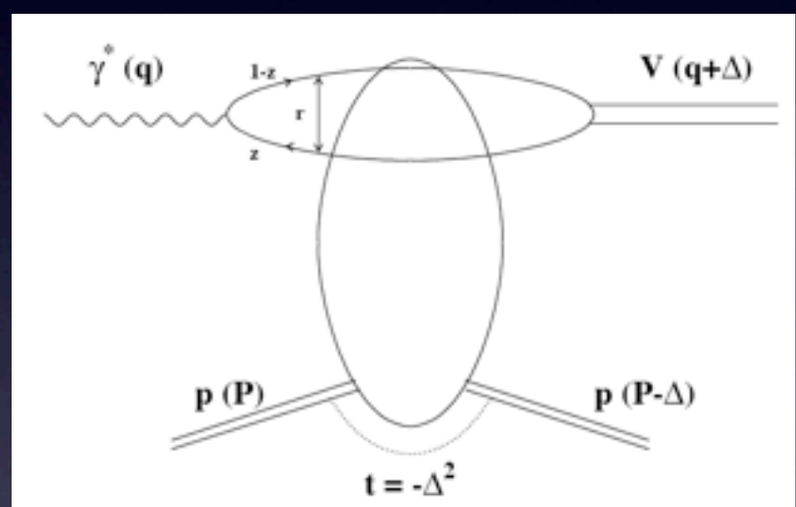

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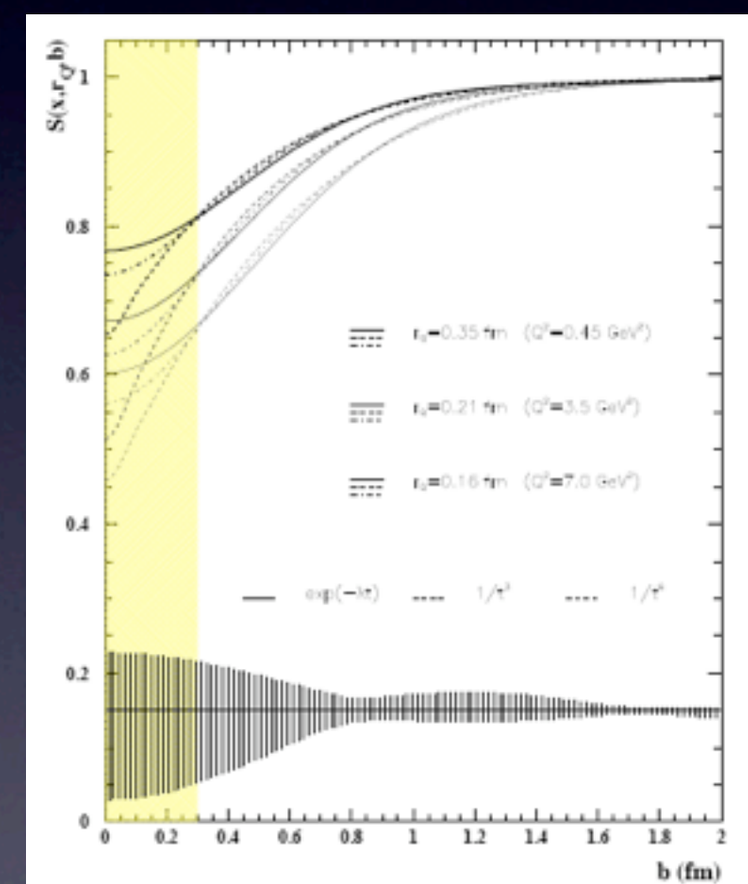

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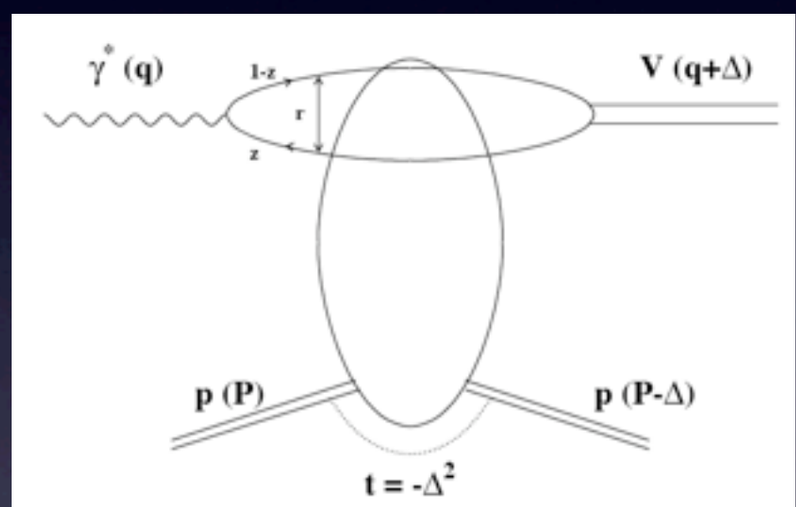

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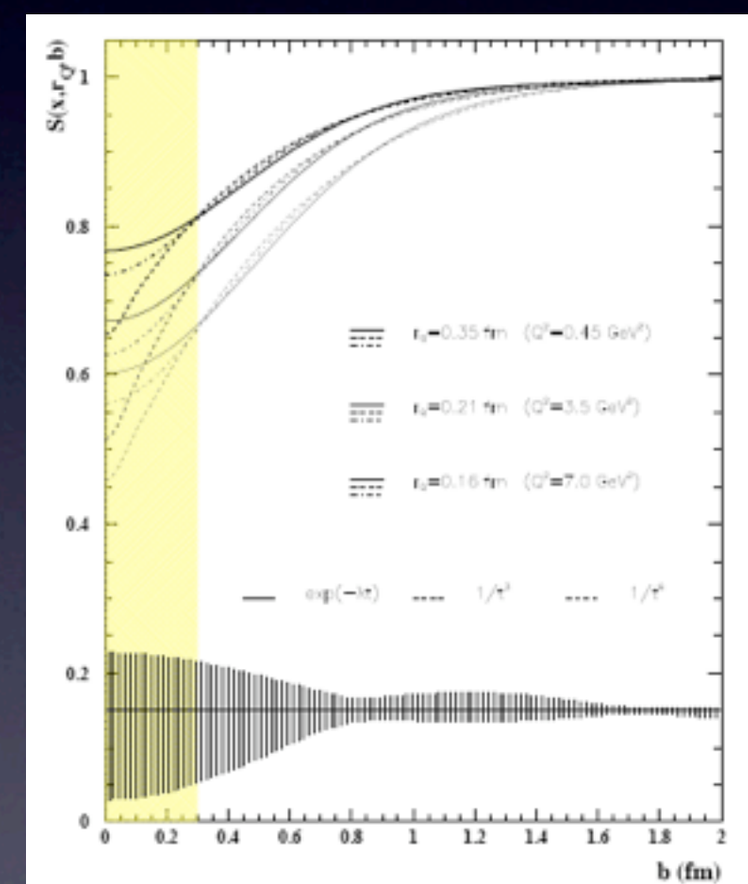

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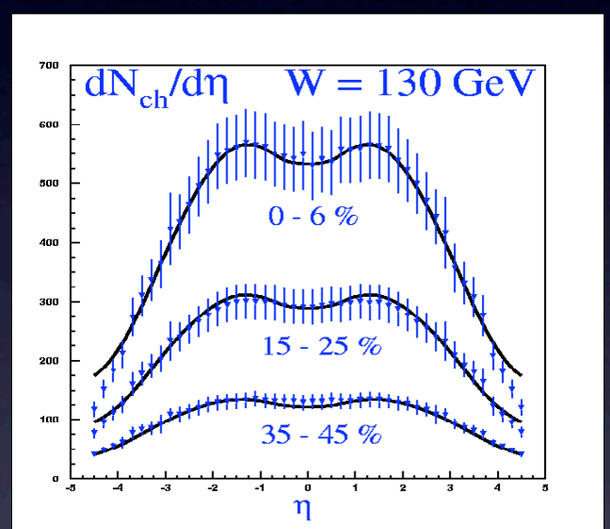
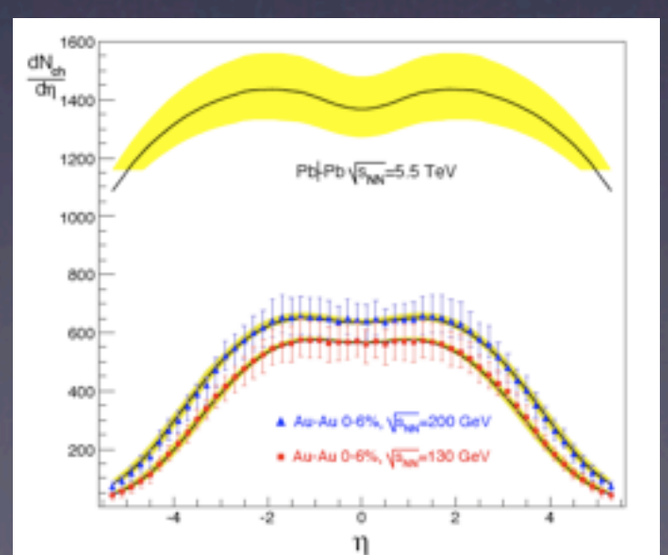


Kharzeev, Levin, Nardi



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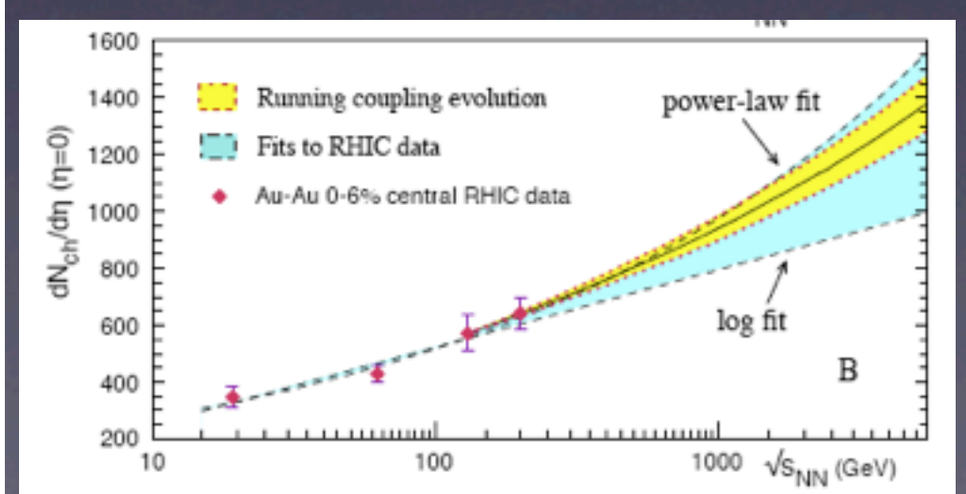

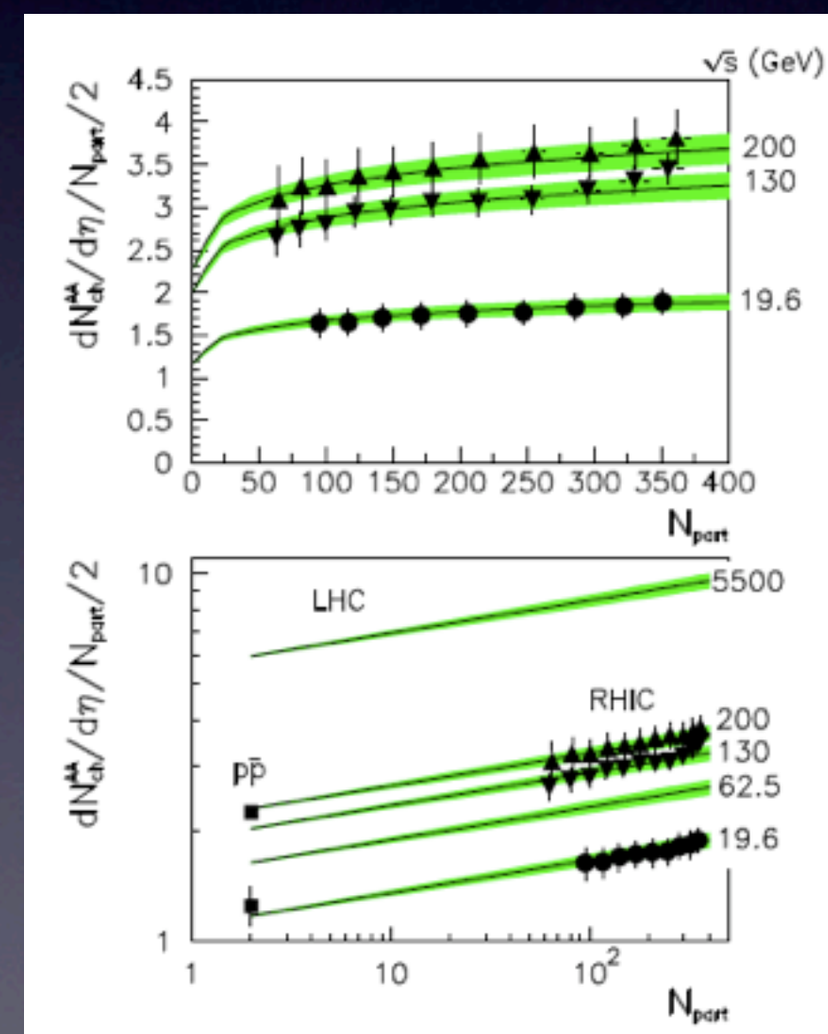

Armesto, Salgado, Wiedemann

Phenomenology: p(d)A

- Forward particle production in p(d)A collisions.
- Idea: fast hadron probing the small x wave function of the dense heavy nucleus.
- p(d)A : suppression of the transverse momentum distribution at forward rapidities.
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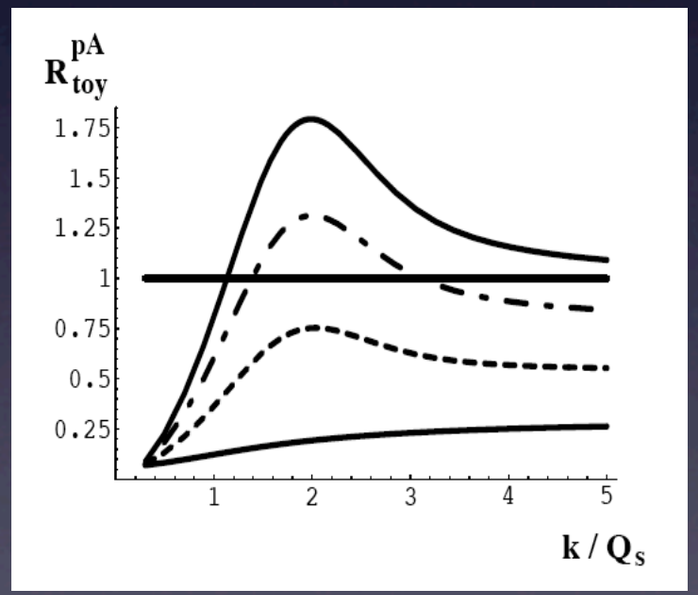

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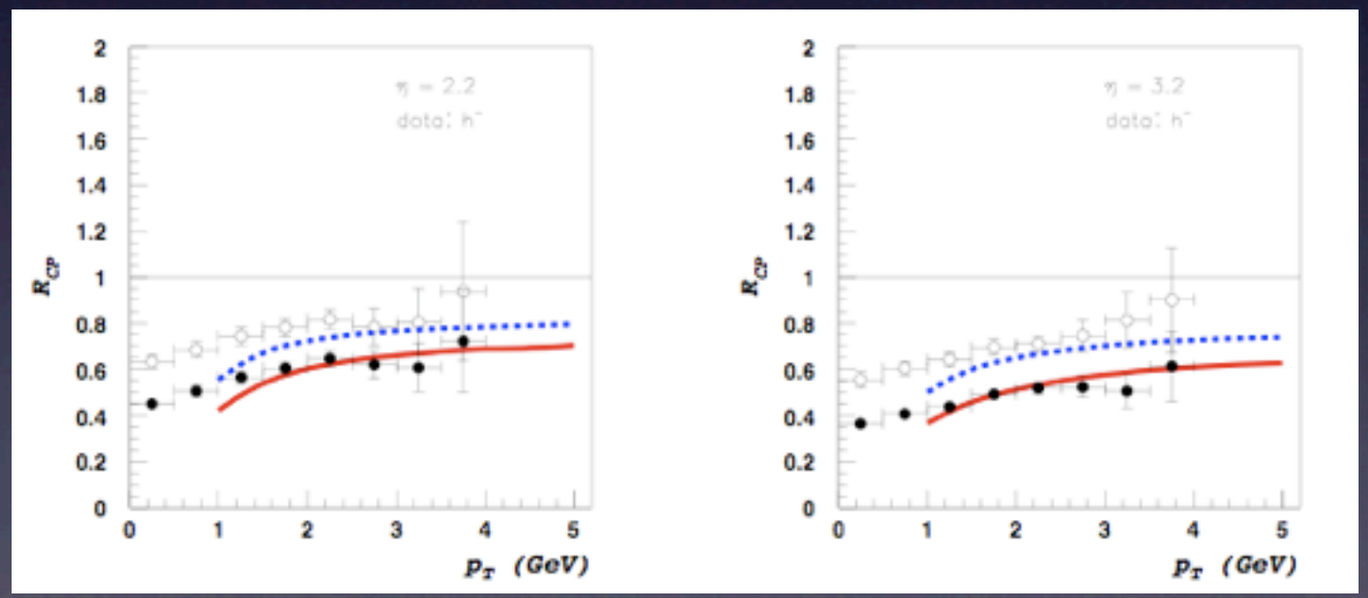

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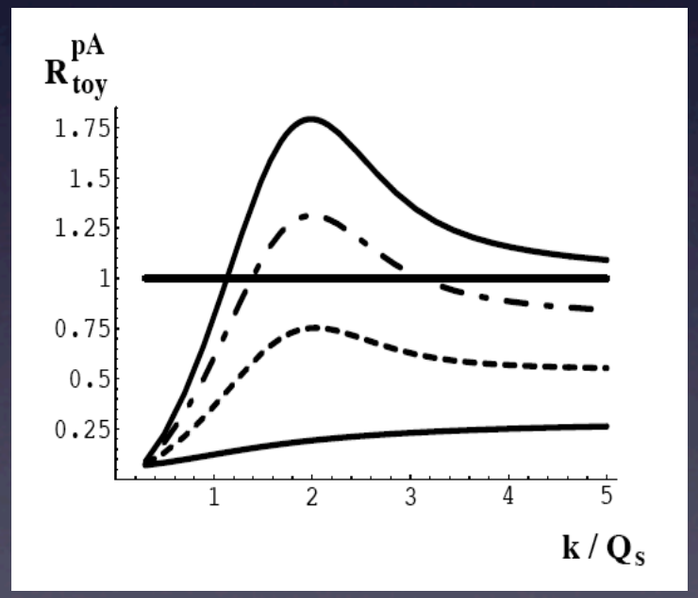

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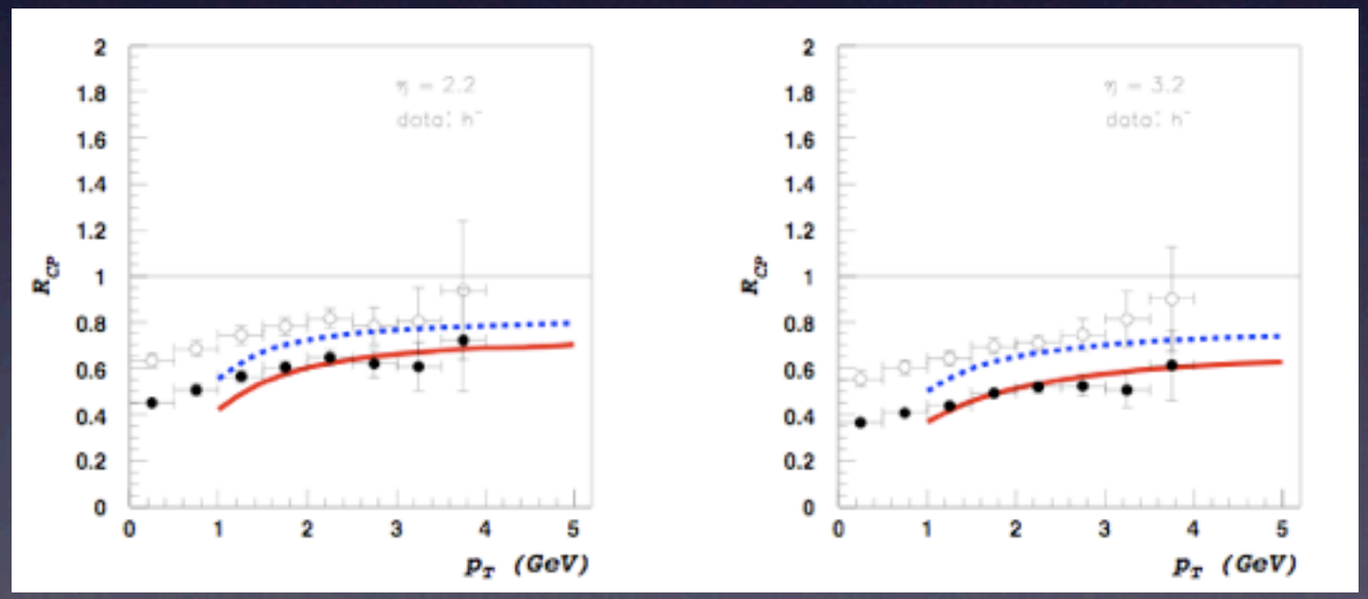

Kharzeev, Kovchegov, Tuchin

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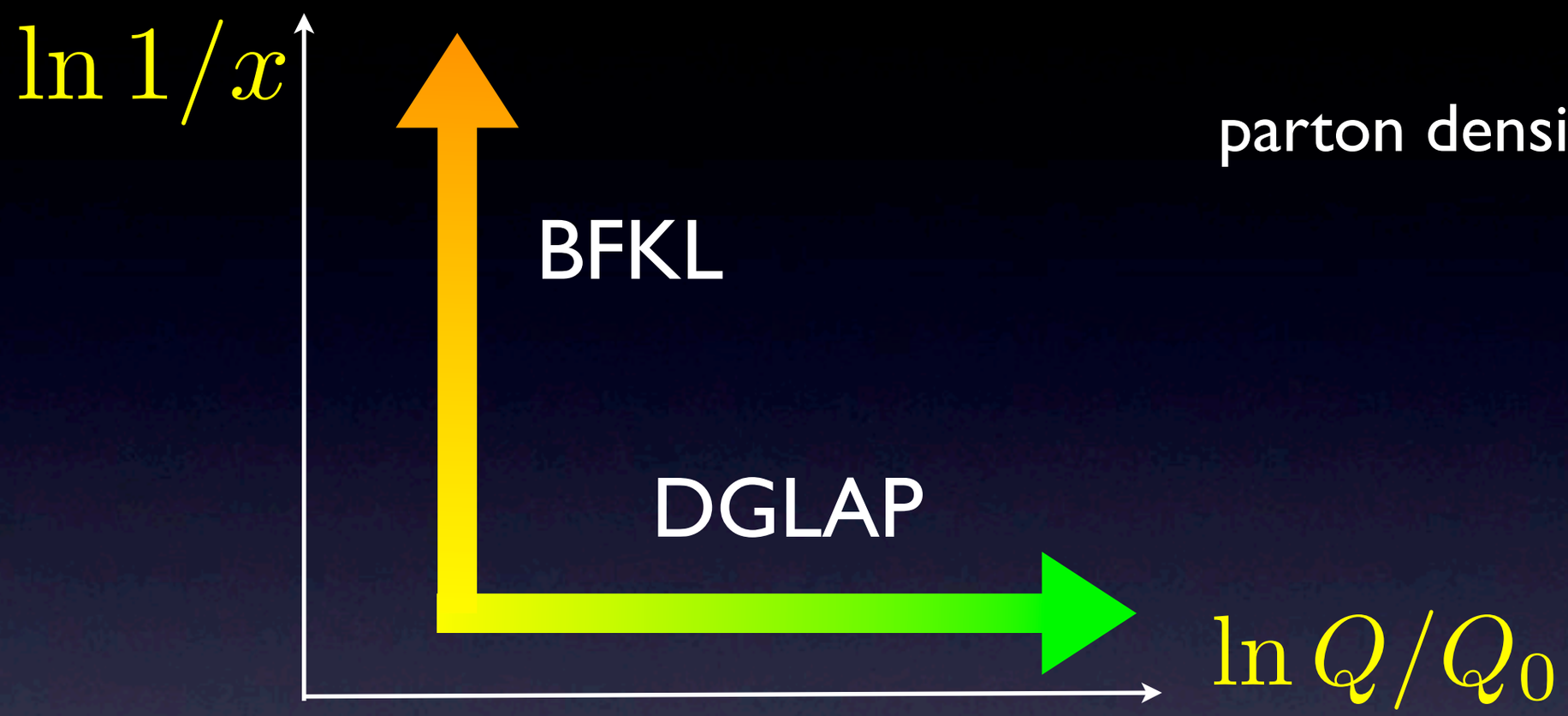
- Successful phenomenology, DIS and pA,AA collisions.

Backup Slides

Parameter space for evolution



parton density $f(x, Q^2)$

DGLAP: evolution of the parton densities with the hard scale (Q in DIS). Operator product expansion + renormalization group. Anomalous dimensions.

Important when $\alpha_s \ln Q/Q_0$ large.

BFKL: evolution with the energy s. Regge limit.
Important when $\alpha_s \ln 1/x$ large

Parton saturation

$f(x, k_T)$ $\longleftrightarrow$ $N(r, x)$ $\quad r \sim 1/k_T$

gluon density

probability to interact

Parton saturation

$f(x, k_T) \longleftrightarrow N(r, x)$ $\quad r \sim 1/k_T$

gluon density

dipole scattering amplitude

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nonlinear equation for dipole amplitude

$$\frac{dN(x,r)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi} \bar{\mathcal{K}}(r,r') \otimes [N(x,r') - N(x,r')N(x,r-r')]$$

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Solution to this nonlinear equation shows scaling:

$$N(r,x) = N(\underbrace{r^2 Q_s(x)^2}_{\text{scaling}})$$

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if $\quad r \geq 1/Q_s(x)$ $\quad$ in the dense regime