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Transverse momentum dependent PDFs in high energy processes

Theory Overview and Applications

Piet Mulders

mulders@few.vu.nl

vrije Universiteit amsterdam

NIKHEF

Abstract

Transverse momentum dependent PDFs in high energy processes

P.J. Mulders, NIKHEF & VU University, Amsterdam

The basic idea of PDFs is achieving a factorized description with soft and hard parts, soft parts being portable and hard parts being calculable. At high energies interference disappears and PDFs are interpreted as probabilities. We aim to go beyond the collinear situation. We thus consider besides the dependence on partonic momentum fractions x also the dependence on the transverse momentum p_T of the partons. Experimentally that dependence on transverse momenta shows up in azimuthal asymmetries for produced hadrons or jet-jet asymmetries. In combination with polarization one can explain single spin asymmetries or look at production of polarized Lambdas.

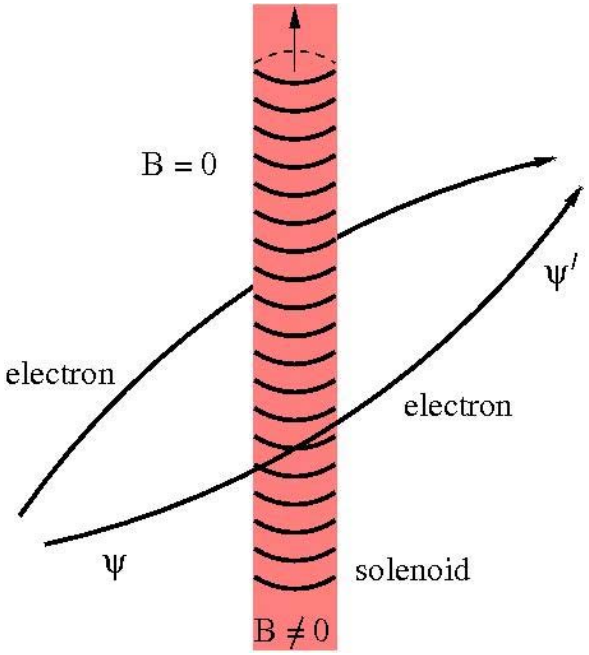
Single spin asymmetries are examples of time-reversal odd (T-odd) observables, which rely on phases in a scattering process. In a gauge theory one naturally expects phases, but these need to be detected through interference or, as it turns out in high-energy scattering processes, by considering transverse momenta ($p_T = iD_T - gA_T$) in a proper color gaug-invariant theoretical description. The relevant transverse momentum dependent (TMD) distribution functions in the partonic correlators can be interpreted as 'single spin correlations' describing transverse spin distributions in an unpolarized hadron or unpolarized quarks in a (transversely) polarized hadron. For gluons similar correlations can be considered. The TMD correlators emerge with specific future or past-pointing gauge links as well as more complex gauge link structures, that depend on the color flow in the hard subprocess, complicating the universality of PDFs. To compare with collinear treatments, one can consider specific p_T-weighted cross sections and recover the Qiu-Sterman description of e.g. single spin asymmetries in terms of gluonic pole matrix elements.

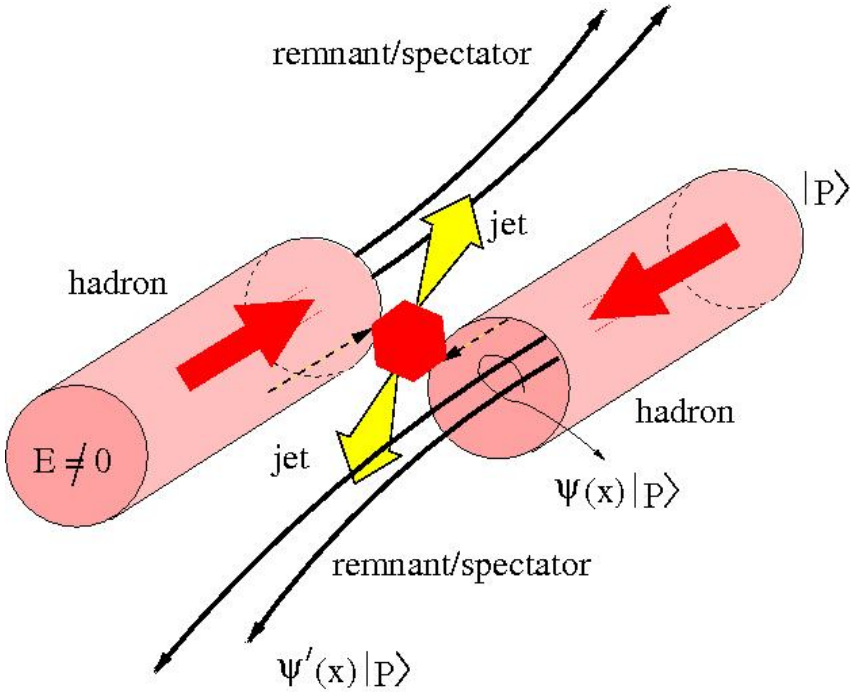
The study of TMD correlators thus breaks the simple universality of quark and gluon distributions multiplying a hard cross section, but the resulting description offers various new possibilities to employ particular spin-momentum correlations of partons in high energy processes, while such correlations of course also offer new insight into hadron structure.

Issues

- Basic goals
  - Try to incorporate small p_T (two scales) extending f(x) to $f(x,p_T^2)$
  - Reduce effects of NLO/NNLO/...
- (Theoretical) complications
  - Operator structure (gauge links) and twist expansion
  - Gluonic interactions essential (Brodsky, Hwang, Schmidt, ...)
  - Effects from soft gluon limit (Qiu, Sterman, Koike, ...)
  - Universality and factorization (Qiu, Yuan, Vogelsang, ...)
- Opportunities/Applications
  - New effects (Sivers, Collins, Boer-M)
  - Combination with polarization is extremely useful, but also jet effects/ broadening/ Lambda production (polarimetry)

Novel: phases in gauge theories



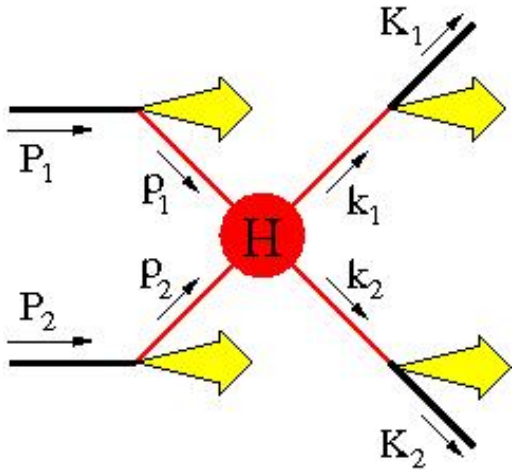


$$\psi' = e^{ie\int ds.A}\psi$$

$$\psi_i(x)\left|P\right\rangle = e^{-ig\int_x^{x'} ds_\mu A^\mu}\psi_i(x')\left|P\right\rangle$$

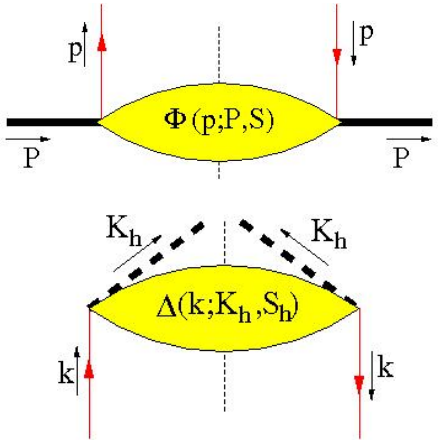
PDFs and PFFs

Basic idea of PDFs is getting a factorized description of high energy scattering processes



$$\hat{\sigma} = |H(p_1, p_2, \ldots)|^2$$

calculable



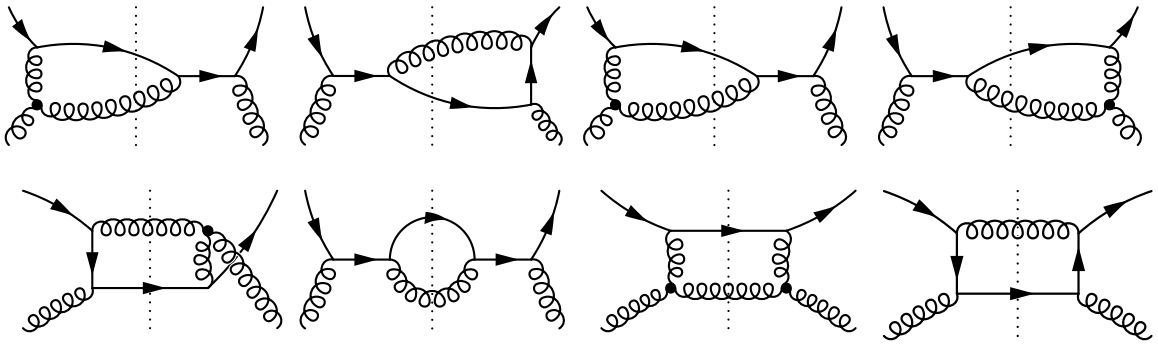
defined (!)
&
portable

$$\sigma(P_1, P_2, \ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$
$$\otimes \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots.$$

Hard part: QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate transition amplitudes, e.g. $g^*q \to q$, $q\bar{q} \to g^*$, $g^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- Example:

  $qg \to qg$

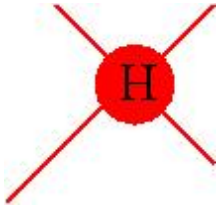


- Calculations work for plane waves

$$\left\langle 0 \left| \psi_i^{(s)}(\xi) \right| p, s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

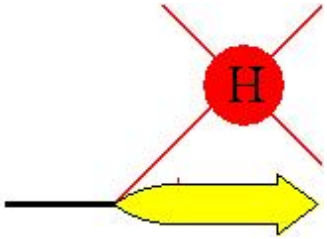
- Use relations such as $u_i(p,s)\,\bar{u}_j(p,s) = (\not{p} + m)_{ij}$

Soft part: hadronic matrix elements



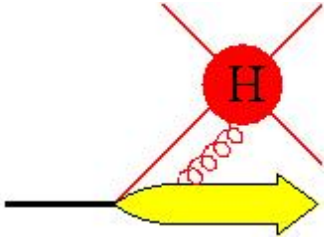
- For hard scattering process involving electrons and photons the link to external particles is the 'one-particle wave function'

$$\left\langle 0 \left| \psi_i(\xi) \right| p, s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$



- Confinement leads to hadrons as 'sources' for quarks

$$\left\langle X \left| \psi_i(\xi) \right| P \right\rangle e^{+ip.\xi}$$



- and 'source' for quarks + gluons

$$\left\langle X \left| \psi_i(\xi) A^{\mu}(\eta) \right| P \right\rangle e^{+i(p-p_1).\xi + ip_1.\eta}$$

- and ....

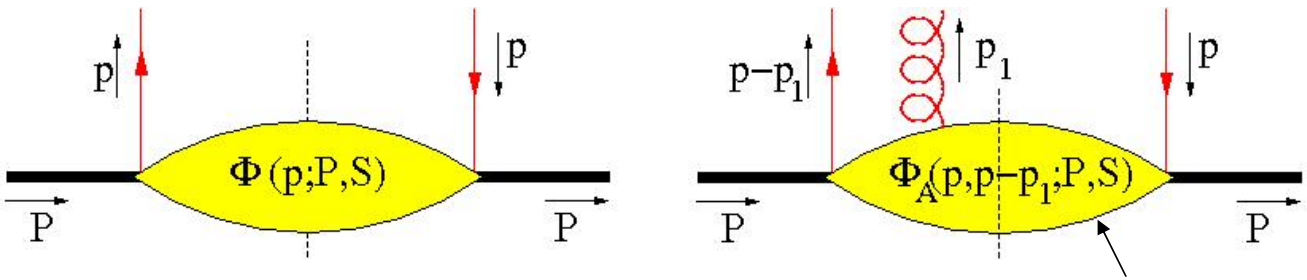
Soft part: hadronic matrix elements

Thus, the theoretical description/calculation for hard processes involves [instead of $u_i(p)\bar{u}_j(p)$] forward matrix elements of the form

$$\Phi_{ij}(p,P) = \int \frac{d^3P_X}{(2\pi)^3 2E_X} < P|\bar{\psi}_j(0)|X >< X|\psi_i(0)|P > \delta(P - P_X - p)$$

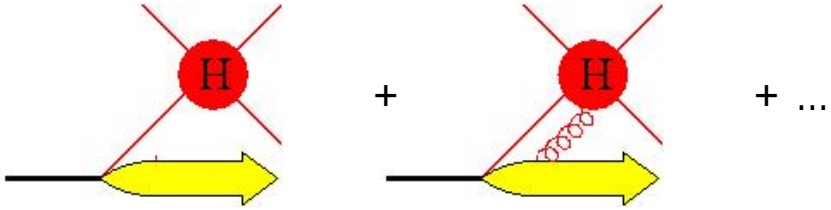
quark momentum

$$= \frac{1}{(2\pi)^4}\int d^4\xi\, e^{i\,p.\xi} < P|\bar{\psi}_j(0)\psi_i(\xi)|P >$$



$$< P|\bar{\psi}_j(0)A^{\alpha}(\eta)\psi_i(\xi)|P >$$

Soft parts: gauge invariant definitions



- Matrix elements containing $A_\mu \sim A^+ p_\mu$ (collinear gluons) produce gauge link

$$U^{[C]}_{[0,\xi]} = \mathscr{P} \exp\left(-ig \int_0^\xi ds^\mu A_\mu \right)$$

- ... essential for color gauge invariant definition

$$\Phi^{[C]}_{ij}(p;P) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip.\xi} \left\langle P \middle| \bar{\psi}_j(0) U^{[C]}_{[0,\xi]} \psi_i(\xi) \middle| P \right\rangle$$

- Not calculable like $u\bar{u}$, but use parametrization (with 'spectral functions')

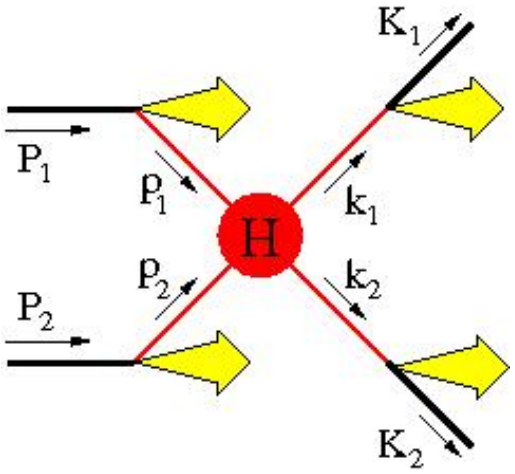
$$\Phi^{[C]}_{ij}(p,P) = (\not{p})_{ij} S^{[C]}_1(p^2, p.P, \ldots) + (\not{P})_{ij} S^{[C]}_2(p^2, p.P, \ldots) + \ldots$$

Ralston and Soper 79, ...

Order: partonic momenta at high energies

- Parton p belongs to hadron P: $p.P \sim M^2$
- For all other momenta K: $p.K \sim P.K \sim s \sim Q^2$
- Introduce a generic lightlike vector n satisfying P.n=1, then n ~ 1/Q
- The vector n gets its meaning in a particular hard process, n = K/K.P

  e.g. in SIDIS: $n = P_h/P_h.P$ (or n determined by P and q; doesn't matter at leading order!)
- Expand quark momenta



$$p = xP^\mu + p_T^\mu + \sigma n^\mu$$

~ Q ~ M ~ M^2/Q

$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

In remainder: retain two scales

Jaffe (1984), Diehl & Gousset, ...

Integrating quark correlators

- ... rather than considering general correlator $\Phi^{[C]}(p,P,...)$, one integrates over $p.P = p^-$ ($\sim M_R^2$, which is of order M^2) and/or p_T

$$\Phi^q_{ij}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \langle P | \bar{\psi}_j(0)\,\psi_i(\xi) | P \rangle_{\xi.n=0}$$

TMD

$$\Phi^q_{ij}(x; n) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \langle P | \bar{\psi}_j(0)\psi_i(\xi) | P \rangle_{\xi.n=\xi_T=0}$$

collinear

- The integration over $p^- = p.P$ makes time-ordering automatic (Jaffe, 1984). This works for $\Phi(x)$ and $\Phi(x,p_T)$
- This allows the interpretation of soft (squared) matrix elements as forward antiquark-target amplitudes, which satisfy particular support properties, etc.
- For collinear correlators $\Phi(x)$, this can be extended to off-forward correlators and generalized PDFs

Boer & Vogelsang

Relevance of transverse momenta?

$$p \approx xP + p_T$$
$$k \approx z^{-1}P + k_T$$

- In a hard process one probes quarks and gluons
- Parton momenta fixed by kinematics (external momenta)

DIS $\quad x = p.n / P.n = Q^2 / 2P.q = x_B$

SIDIS $\quad z = K_h.n / k.n = P.K_h / P.q = z_h$

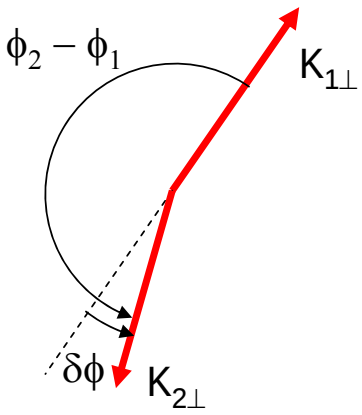
- Also possible for transverse momenta of partons

SIDIS $\quad q_T = q + x_B P - z_h^{-1} K_h = k_T - p_T$

DY $\quad q_T = q - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T}$

2-particle inclusive hadron-hadron scattering

$$q_T = z_1^{-1} K_1 + z_2^{-1} K_2 - x_1 P_1 - x_2 P_2$$
$$= p_{1T} + p_{2T} - k_{1T} - k_{2T}$$



pp-scattering

Second scale!

We need more than one hadron and knowledge of hard process(es)!

Qiu & Sterman, 1997

Relevance of transverse momenta?

- TMD-correlators are not T-invariant
- QCD is T-invariant
- T-odd observables ←→ T-odd TMDs
- Example of T-odd observable: single spin asymmetry

  Left-right asymmetry in $p(P_1)\,p_\uparrow(P_2) \to \pi(K)X$
- Collinear hard T-odd contribution zero ($\sim\alpha_s^2$),

  p_T-contributions remain

$$\varepsilon^{P_1 P_2 S_{2T} K_\pi} \approx \frac{z_\pi}{x_1 x_2}\Big(\varepsilon^{\cancel{p_1 p_2} S_{2T} k} - \varepsilon^{p_{1T} p_2 S_{2T} k} - \varepsilon^{p_1 p_{2T} S_{2T} k} + \varepsilon^{p_1 p_2 S_{2T} k_T}\Big)$$

$\sim s^{3/2}$

$\sim s$

$$p \approx xP + p_T$$
$$k \approx z^{-1}P + k_T$$

... + 'normal' twist three stuff

Gauge-invariant definition of TMDs: which gauge links?

$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \left\langle P \middle| \bar{\psi}_j(0) U_{[0,\xi]}^{[C]} \psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

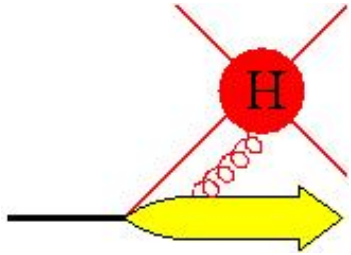
TMD

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \middle| \bar{\psi}_j(0) U_{[0,\xi]}^{[n]} \psi_i(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

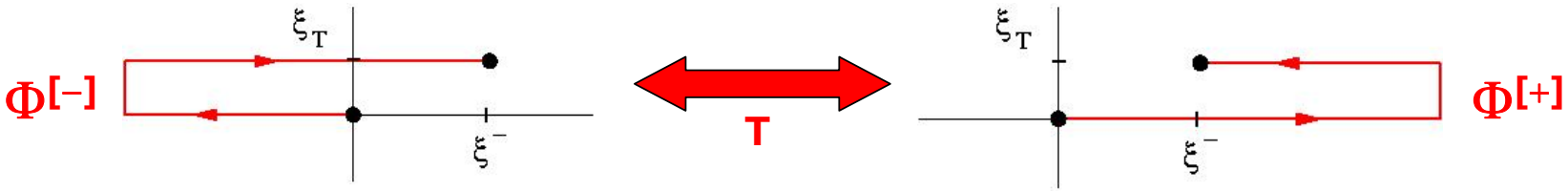
collinear

- Gauge links follow from inclusion of quark-gluon matrix elements

$$U_{[0,\xi]}^{[C]} = \mathscr{P} \exp\left(-ig \int_0^{\xi} ds^{\mu} A_{\mu} \right)$$

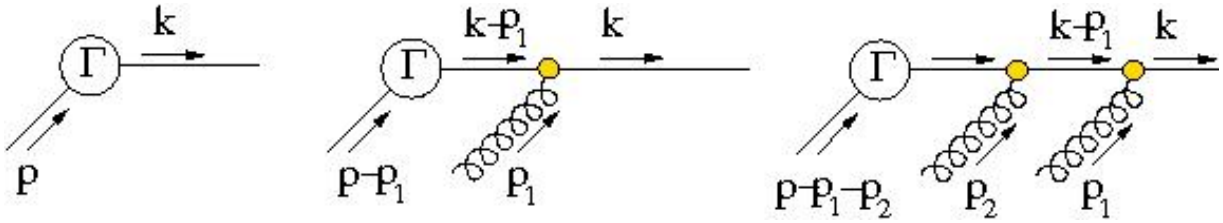


- Basic (simplest) gauge links for TMD correlators:



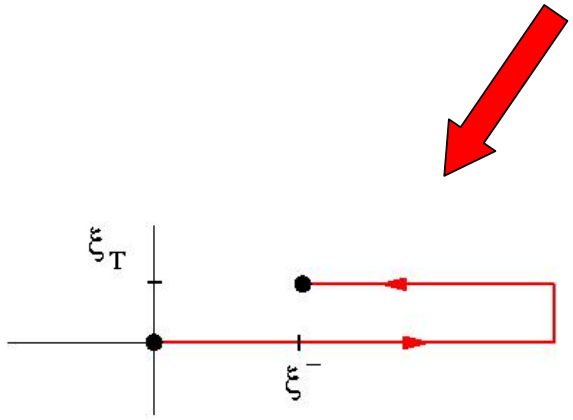
These become a unique 'simple' Wilson line upon p_T-integration

Gauge link calculation for SIDIS



Expand gluon fields and reshuffle a bit:

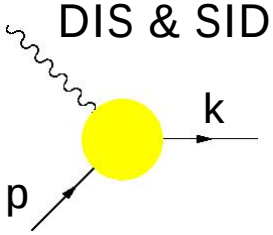
$$A^{\mu}(p_1) = n.A(p_1)\frac{P^{\mu}}{n.P} + iA_T^{\mu}(p_1) + \ldots \;=\; \frac{1}{p_1.n}\left[n.A(p_1)\,p_1^{\mu} + iG_T^{n\mu}(p_1) + \ldots\right]$$



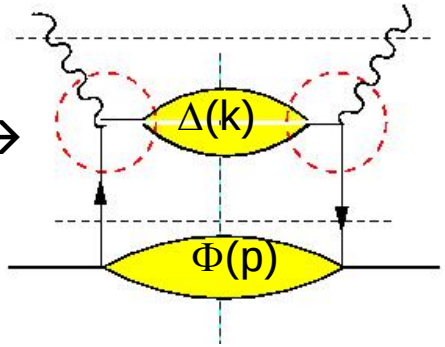
Coupled to a final state parton, the collinear gluons add up to a $U_{[+]}$ gauge link, (with endpoint from transverse reshuffling)

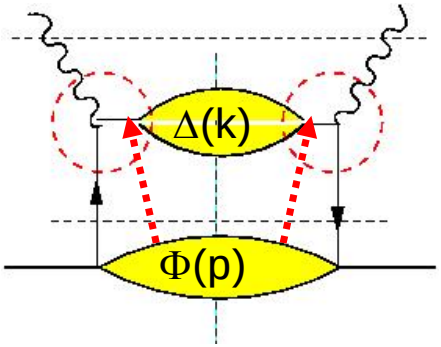
Gauge link calculation for SIDIS

DIS & SIDIS

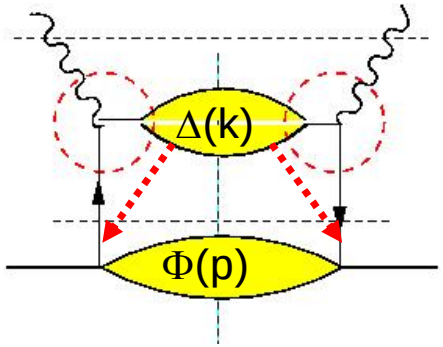


$|M|^2$ →



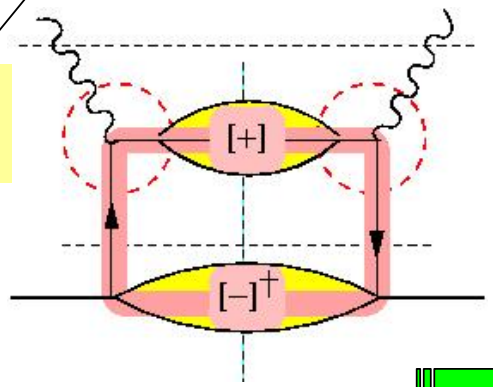


$U^{[k]}_{+\infty}(p)$



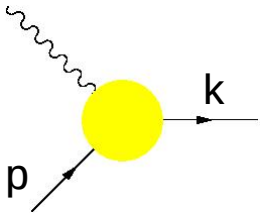
$U^{[p]\dagger}_{-\infty}(k)$

For simple color flow, absorb in Φ(p) and Δ(k)

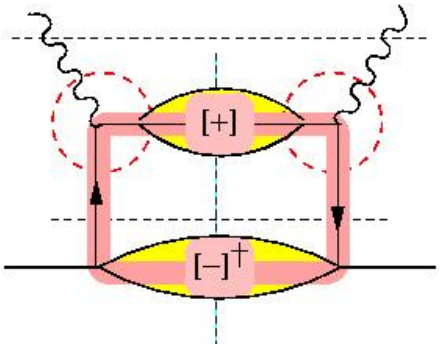


$$\frac{d\sigma}{d^2 q_T} = \Phi_q^{[+]}(p)\hat{\sigma}_{\gamma q \to q}\Delta_q^{[-^\dagger]}(k)$$

Gauge link calculation for SIDIS



$|M|^2 \rightarrow$



$$\frac{d\sigma}{d^2 q_T} = [(\Phi(p) U_{-\infty}^{[p]\dagger}(k))\, \Gamma^* (\Delta(k) U_{+\infty}^{[k]}(p))\, \Gamma\,]_{Tr}$$

kinematical decoupling

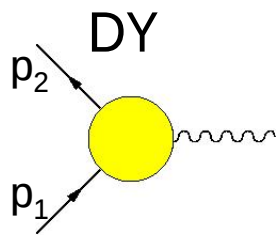
$$= [\Phi(p) U_{-\infty}^{[n_k]\dagger}(k))\, \Gamma^* (\Delta(k) U_{+\infty}^{[n_p]}(p))\, \Gamma\,]_{Tr}$$

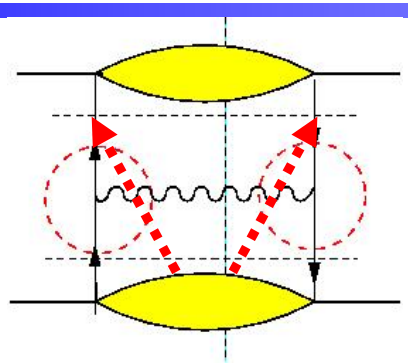
simple color flow!

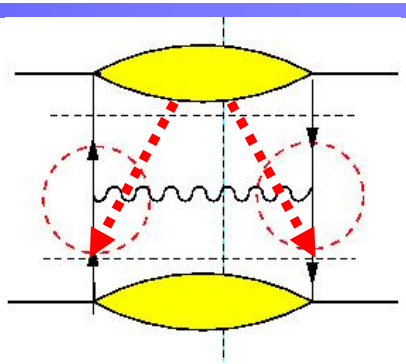
$[U, \Gamma]=0$

$$= [\Phi^{[+]}(p)\, \Gamma^*\, \Delta^{[-^\dagger]}(k)\, \Gamma\,]_{Tr}$$

Gauge link calculation for DY

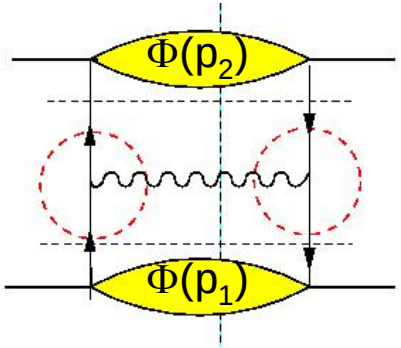




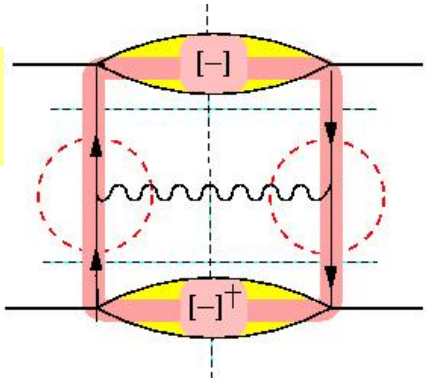


$U_{-\infty}^{[p_2]}(p_1)$ $U_{-\infty}^{[p_1]\dagger}(p_2)$

$|M|^2 \rightarrow$



For simple color flow, absorb in $\Phi(p_1)$ and $\Phi(p_2)$



$$\frac{d\sigma}{d^2 q_T} = \Phi_q^{[-]}(p_1)\Phi_{\bar{q}}^{[-^\dagger]}(p_2)\hat{\sigma}_{q\bar{q}\to\gamma}$$

Gauge link differs from the SIDIS one!

Collinear parametrizations

- Gauge invariant correlators → PDFs
- Collinear quark correlators (leading part, no n-dependence, no link dependence)

Analogy:

$$u(p,s)\,\bar{u}(p,s) = \tfrac{1}{2}(\not{p}+m)(1+\gamma_5 \not{s})$$

$$\Phi^q(x) = \left(f_1^q(x) + S_L g_1^q(x)\gamma_5 + h_1^q(x)\gamma_5 \not{S}_T \right)\frac{\not{P}}{2}$$

$$S \approx S_L \frac{P^\mu}{M} + S_T^\mu$$

- Interpretation: quark momentum distribution $f_1^q(x)$ = q(x), chiral distribution $g_1^q(x)$ = Δq(x) and transverse spin polarization $h_1^q(x)$ = δq(x) in a spin ½ hadron
- Collinear gluon correlators (leading part)

$$\Phi_g^{\mu\nu}(x) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^g(x) + i S_L \varepsilon_T^{\mu\nu} g_1^g(x)\right)$$

- Interpretation: gluon momentum distribution $f_1^g(x)$ = g(x) and polarized distribution $g_1^g(x)$ = Δg(x)

$\Phi(x)$		
χ	even	odd
U	f_1	
L	g_1	
T		h_1

Including flavor index one commonly writes

$$f_1^q(x) = q(x)$$

$$g_1^q(x) = \Delta q(x)$$

$$h_1^q(x) = \delta q(x)$$

$\Phi^g(x)$		
		flip
U	f_1^g	
L	g_1^g	
T		

For gluons one commonly writes:

$$f_1^g(x) = g(x)$$

$$g_1^g(x) = \Delta g(x)$$

$\Delta(x)$		
χ	even	odd
U	D_1	
L	G_1	
T		H_1

TMD parametrizations

- Gauge invariant correlators → distribution functions
- TMD quark correlators (leading part, unpolarized) including T-odd part

$$\Phi^{[\pm]q}(x, p_T) = \left(f_1^{\,q}(x, p_T^2) \pm i h_1^{\perp q}(x, p_T^2) \frac{\not{p}_T}{M} \right) \frac{\not{P}}{2}$$

- Interpretation: quark momentum distribution $f_1^q(x,p_T)$ and its transverse spin polarization $h_1^{\perp q}(x,p_T)$ both in an unpolarized hadron
- The function $h_1^{\perp q}(x,p_T)$ is T-odd (momentum-spin correlations!)
- TMD gluon correlators (leading part, unpolarized)

$$\Phi_g^{\mu\nu}(x, p_T) = \frac{1}{2x} \left(-g_T^{\mu\nu} f_1^{\,g}(x, p_T^2) + \left(\frac{p_T^\mu p_T^\nu + \frac{1}{2} g_T^{\mu\nu}}{M^2} \right) h_1^{\perp g}(x, p_T^2) \right)$$

- Interpretation: gluon momentum distribution $f_1^g(x,p_T)$ and its linear polarization $h_1^{\perp g}(x,p_T)$ in an unpolarized hadron (both are T-even)

$\Phi^{[even]}(x, p_T)$			$\Phi^{[odd]}(x, p_T)$	
χ	even	odd	even	odd
U	f_1			$h_1^{\perp}$
L	g_{1L}	$h_{1L}^{\perp}$		
T	g_{1T}	h_{1T} $h_{1T}^{\perp}$	$f_{1T}^{\perp}$	

$\Phi^{g[even]}(x, p_T)$			$\Phi^{g[odd]}(x, p_T)$	
		flip		flip
U	f_1^g	$h_1^{\perp g}$		
L	g_{1L}^g			$h_{1L}^{\perp g}$
T	g_{1T}^g		$f_{1T}^{\perp g}$	h_{1T}^g $h_{1T}^{\perp g}$

TMD correlators: gluons

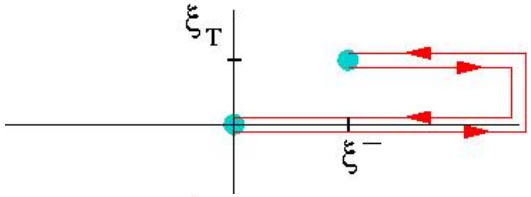
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{ip.\xi}\left\langle P\left|U_{[\xi,0]}^{[C]}F^{n\alpha}(0)U_{[0,\xi]}^{[C']}F^{n\beta}(\xi)\right|P\right\rangle_{\xi.n=0}$$

- The most general TMD gluon correlator contains two links, which in general can have different paths.
- Note that standard field displacement involves C = C'

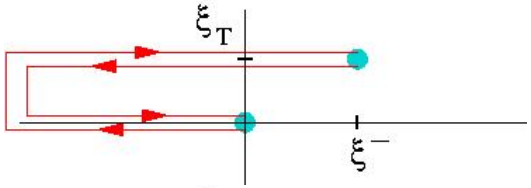
$$F^{\alpha\beta}(\xi)\to U_{[\eta,\xi]}^{[C]}F^{\alpha\beta}(\xi)U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

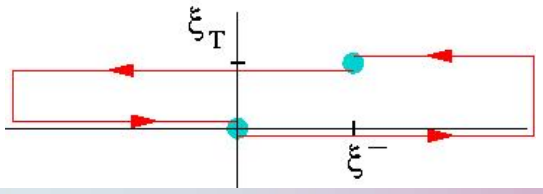
$\Phi_g^{[+,+]}$



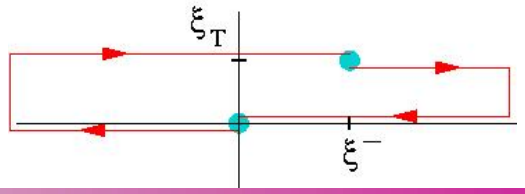
$\Phi_g^{[-,-]}$



$\Phi_g^{[+,-]}$



$\Phi_g^{[-,+]}$



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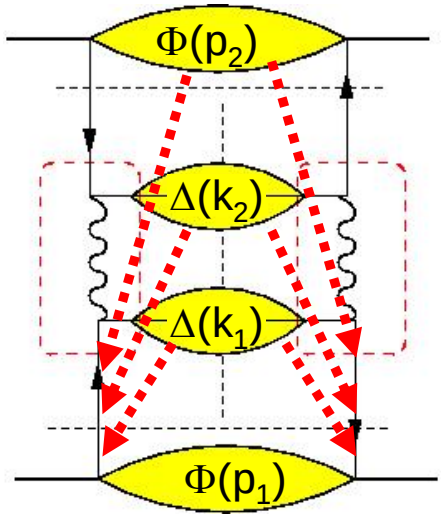
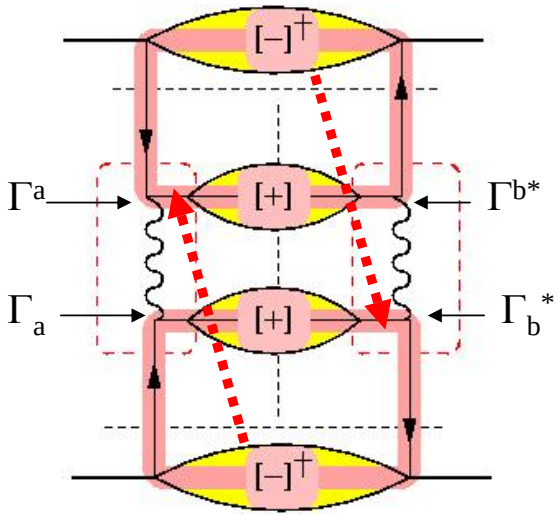
Color entanglement

- Illustrated in quark-quark scattering (even without color exchange)



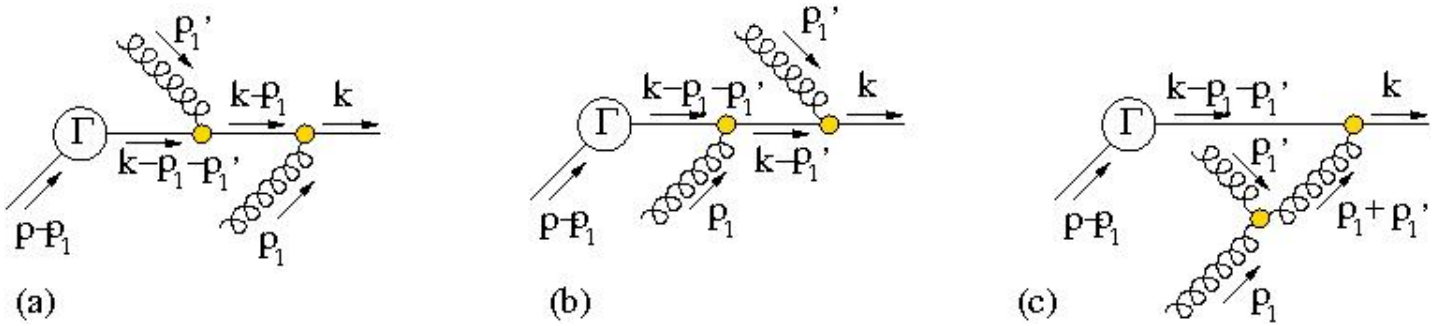
several eikonal factors



$$\sigma \sim [\Phi(p_1)U_-^{[p_1]\dagger}(p_2,k_1,k_2)\Gamma_b^*\Delta(k_1)U_+^{[k_1]}(p_1,p_2,k_2)\Gamma_a]$$
$$\times[\Phi(p_2)U_-^{[p_2]\dagger}(p_1,k_1,k_2)\Gamma^{b*}\Delta(k_2)U_+^{[k_2]}(p_1,p_2,k_1)\Gamma^a]$$

kinematically decoupled, but color entangled!

Color disentanglement?



$$U_{-\infty}^{[k](11)}(p,p')\ldots\Gamma\ldots\psi(p)\ldots\psi(p')=\frac{1}{2}\left\{U_{-\infty}^{[k](1)}(p),U_{-\infty}^{[k](1)}(p')\right\}\ldots\Gamma\ldots\psi(p)\ldots\psi(p')$$

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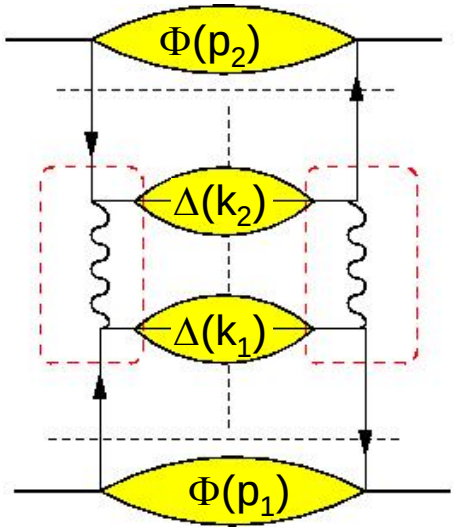
Color disentanglement?



$$...U_{+}^{[n_1]}(p_1)...$$

$$\sigma \sim [\Phi(p_1)U_{-}^{[p_1]\dagger}(p_2,k_1,k_2)\Gamma_b^{*}\Delta(k_1)U_{+}^{[k_1]}(p_1,p_2,k_2)\Gamma_a]$$

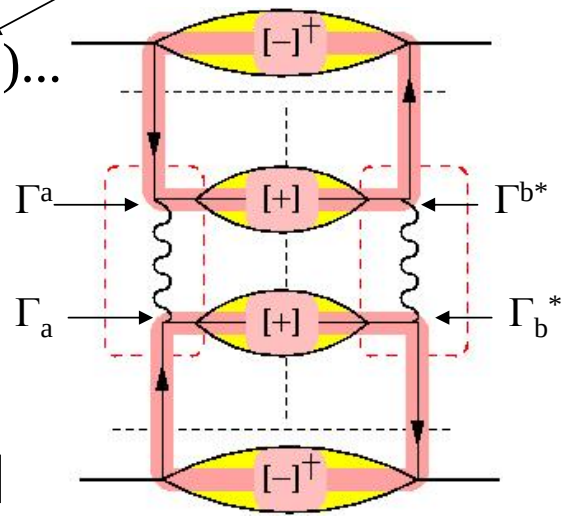
$$\times[\Phi(p_2)U_{-}^{[p_2]\dagger}(p_1,k_1,k_2)\Gamma^{b*}\Delta(k_2)U_{+}^{[k_2]}(p_1,p_2,k_1)\Gamma^{a}]$$

$$...U_{-}^{[n_1]\dagger}(p_1)...\ ?\ ...U_{+}^{[n_1]}(p_1)...$$

$$U_{-}^{\dagger}U_{+} \equiv W(p_1)$$

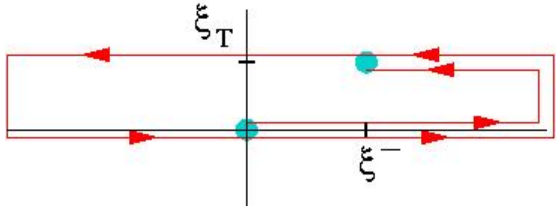
$$\sigma \sim [\Phi^{[(W)+]}(p_1)\Gamma_b^{*}\Delta^{[(W)-^{\dagger}]}(k_1)\Gamma_a]$$

$$\times[\Phi^{[(W)+]}(p_2)\Gamma^{b*}\Delta^{[(W)-^{\dagger}]}(k_2)\Gamma^{a}]$$

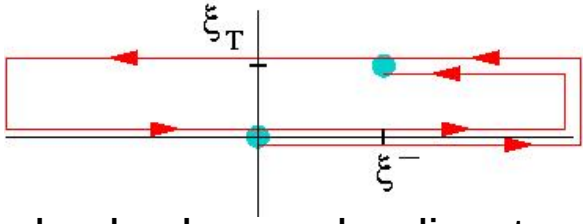


Color disentanglement

- Correlators with more complex gaugelinks, such as



$\Phi^{[(W)+]}$



$\Phi^{[W+]}$

- If only a TMD for *one* hadron is involved color can be disentangled in this way (conjecture)

$$\int d^2 p_T \; ... W^{[n]}(p) ... = \int d^2 p_T \; ... U^{[n]\dagger}_{-\infty}(p) U^{[n]}_{+\infty}(p) ... \; = \; 1$$

- This can be used to simplify the integrated cross sections and to study single weighted azimuthal asymmetries

$$\int d^2 q_T \; q_T^\alpha ... \int d^2 k_{1T} \int d^2 k_{2T} \; ... \delta^2(q_T - k_{1T} - k_{2T})$$

$$= \int d^2 k_{1T} \; k_{1T}^\alpha \int d^2 k_{2T} ... \; + \; \int d^2 k_{1T} \int d^2 k_{2T} \; k_{2T}^\alpha \; ...$$

TMD treatment (tree level!)

✓ In situation that just the p_T-dependence of a single hadron is relevant, we find a TMD correlator with specific process-dependent gauge links:

$$\frac{d\sigma}{d^2 p_{1T}} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots$$

$$\frac{d\sigma}{d^2 k_{1T}} \sim \sum_{D,abc} \Phi_a(x_1)\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

X ... but there is no full TMD factorization

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\Phi_b^{[C_2(D)]}(x_2, p_{2T})\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

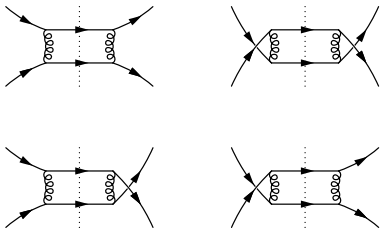
$$+\ldots\ldots\ldots$$

'single-hadron' TMD factorization + ...

TMD treatment (tree level!)

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\, \Phi_b^{[C_2(D)]}(x_2, p_{2T})\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

Note that even for 'single-hadron' TMD factorization, the summation over D is a summation over diagrams *and* color-flow,



e.g. for qq→ qq subprocess:

$$\frac{d\sigma}{d^2 q_T} \sim \Phi_q^{[(W)+]}(1)\, \Phi_q^{[(W)+]}(2) \underbrace{\frac{N_c^2+1}{N_c^2-1}\, \hat{\sigma}^{[D_1]}_{qq\to qq}}_{\hat{\sigma}^{[D_{11}]}} \Delta_q^{[(W)-^\dagger]}(1')\, \Delta_q^{[(W)-^\dagger]}(2')$$

$$+\ \Phi_q^{[W+]}(1)\, \Phi_q^{[W+]}(2) \underbrace{\frac{-2}{N_c^2-1}\, \hat{\sigma}^{[D_1]}_{qq\to qq}}_{\hat{\sigma}^{[D_{12}]}} \Delta_q^{[W-^\dagger]}(1')\, \Delta_q^{[W-^\dagger]}(2') + \ldots.$$

BUT REMEMBER:
THIS EXPRESSION
CANNOT BE COMPLETE!!!

Result for integrated cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a(1) U_{-\infty}^{[1]\dagger}(2,1',\ldots)\Phi_b(2) U_{-\infty}^{[2]\dagger}(1,1',\ldots)\, \hat{\sigma}_{ab\to c\ldots}^{[D]}\, \Delta_c(1') U_{+\infty}^{[1']}(1,2,\ldots)\ldots$$

Integrate into collinear cross section

$$\Phi^{[C]}(x) = \int d^2 p_T \;\; \Phi^{[C]}(x, p_T)$$

Result for integrated cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\, \Phi_b^{[C_2(D)]}(x_2, p_{2T})\, \hat{\sigma}_{ab\to c\ldots}^{[D]}\, \Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

Integrate into collinear cross section

$$\Phi^{[\cancel{C}]}(x) = \int d^2 p_T \;\; \Phi^{[C]}(x, p_T)$$

Gauge link structure becomes irrelevant!

$$\sigma \sim \sum_{abc} \Phi_a(x_1)\Phi_b(x_2)\, \hat{\sigma}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots$$

$$\hat{\sigma}_{ab\to c\ldots} = \sum_D \hat{\sigma}_{ab\to c\ldots}^{[D]}$$

(partonic cross section)

Result for single weighted cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\, \Phi_b^{[C_2(D)]}(x_2, p_{2T})\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

Construct weighted cross section (azimuthal asymmetry)

$$\Phi_\partial^{\alpha[C]}(x) = \int d^2 p_T \;\; p_T^\alpha\, \Phi^{[C]}(x, p_T)$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\, \Phi_b(x_2)\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots + \;\ldots..$$

In each term, dependence on gauge link remains for only *one* of the correlators

- ✓ New info on hadrons (cf models/lattice)
- ✓ Can be handled theoretically
- ✓ Allows T-odd structure (explain SSA)
- ✓ Leading cos 2ϕ effects in DY
- ✓ Jet broadening (weighting of cos 2ϕ)

Result for single weighted cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\, \Phi_b^{[C_2(D)]}(x_2, p_{2T})\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

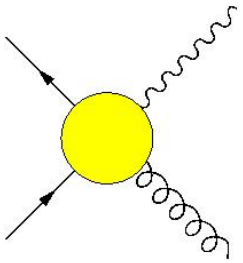
$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots + \ldots\ldots$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots + \ldots\ldots$$

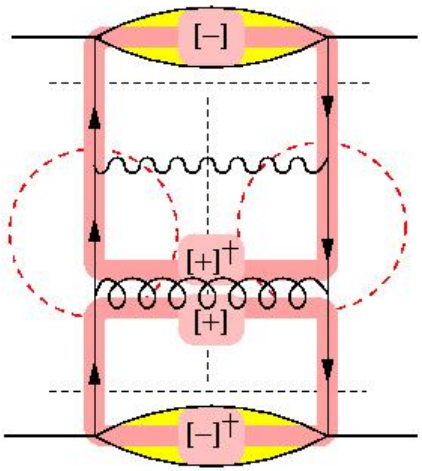
$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots + \ldots\ldots$$

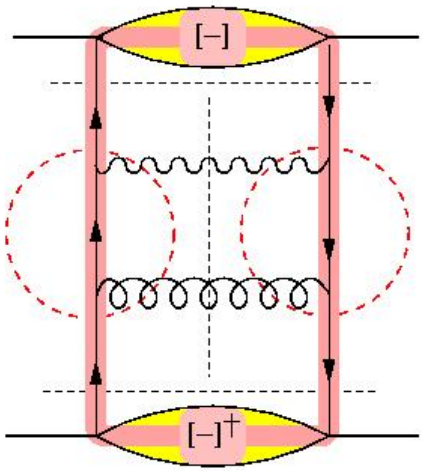
$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\, \hat{\sigma}^{[D]}_{ab\to c\ldots}\, \Delta_c(z_1)\ldots + \ldots\ldots$$

quark + antiquark → gluon + photon



$|M|^2$ →





Four diagrams, each with two similar color flow possibilities

$$\frac{N^2-1}{N}\left[\frac{N^2}{N^2-1}\quad\cdots\qquad -\frac{1}{N^2-1}\quad\cdots\quad\right]$$

$$\frac{d\sigma}{d^2q_T}=\frac{N^2-1}{N}\left[\frac{N^2}{N^2-1}\Phi_q^{[+(W)]}\Phi_{\bar{q}}^{[+^\dagger(W^\dagger)]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}-\frac{1}{N^2-1}\Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}\right]$$

'Single hadron' TMD factorized expression!

cf $qq_{bar} \to \gamma^*$: $$\frac{d\sigma}{d^2q_T}=\Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma}$$

Drell-Yan and photon-jet production

- Drell-Yan

$q_T = p_{1T} + p_{2T}$

$$\frac{d\sigma^{H_1H_2\to\gamma}}{d^2q_T} = \Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma}$$

- Photon-jet production in hadron-hadron scattering

$$\frac{d\sigma^{H_1H_2\to JJ}}{d^2q_T} = \frac{N^2-1}{N}\Big[\frac{N^2}{N^2-1}\Phi_q^{[+(W)]}\Phi_{\bar{q}}^{[+^\dagger(W^\dagger)]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}$$

$$-\frac{1}{N^2-1}\Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}\Big]$$

Drell-Yan and photon-jet production

- Weighted Drell-Yan

$$\left\langle q_T^{\alpha}\,\sigma^{H_1H_2\to\gamma}\right\rangle = \Phi^{\alpha[-]}_{\partial q}\Phi_{\bar{q}}\hat{\sigma}_{q\bar{q}\to\gamma} + \Phi_q\Phi^{\alpha[-^\dagger]}_{\partial\bar{q}}\hat{\sigma}_{q\bar{q}\to\gamma}$$

- Photon-jet production in hadron-hadron scattering

$$\frac{d\sigma^{H_1H_2\to JJ}}{d^2q_T} = \frac{N^2-1}{N}\Big[\frac{N^2}{N^2-1}\Phi_q^{[+(W)]}\Phi_{\bar{q}}^{[+^\dagger(W^\dagger)]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}$$

$$-\frac{1}{N^2-1}\Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}\Big]$$

Drell-Yan and photon-jet production

- Weighted Drell-Yan

Consider only p_{1T} contribution

$$\left\langle q_T^{\alpha}\, \sigma^{H_1 H_2 \to \gamma} \right\rangle = \Phi_{\partial q}^{\alpha[-]} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma} + \ldots$$

- Photon-jet production in hadron-hadron scattering

$$\frac{d\sigma^{H_1 H_2 \to JJ}}{d^2 q_T} = \frac{N^2-1}{N}\Big[\frac{N^2}{N^2-1}\Phi_q^{[+(W)]}\Phi_{\bar{q}}^{[+^\dagger(W^\dagger)]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}$$

$$-\frac{1}{N^2-1}\Phi_q^{[-]}\Phi_{\bar{q}}^{[-^\dagger]}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g^{[-,-^\dagger]}\Big]$$

Drell-Yan and photon-jet production

- Weighted Drell-Yan

$$\left\langle q_T^{\alpha}\, \sigma^{H_1 H_2 \to \gamma} \right\rangle = \Phi_{\partial q}^{\alpha[-]} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma} + \ldots$$

- Weighted photon-jet production in hadron-hadron scattering

$$\left\langle q_T^{\alpha}\, \sigma^{H_1 H_2 \to JJ} \right\rangle = \frac{N^2-1}{N}\Big[\frac{N^2}{N^2-1} \Phi_{\partial q}^{\alpha[+(W)]} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma g} \Delta_g - \frac{1}{N^2-1} \Phi_{\partial q}^{\alpha[-]} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma g} \Delta_g \Big] + \ldots$$

Qiu, Sterman; Koike; Brodsky, Hwang, Schmidt, ...

Result for single weighted cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\Phi_b^{[C_2(D)]}(x_2, p_{2T})\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}^{[D]}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots + \;.....$$

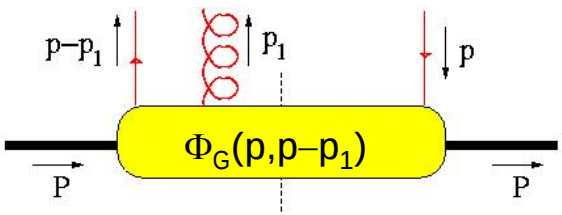
$$\Phi_\partial^{\alpha[C]}(x) = \tilde{\Phi}_\partial^{\alpha[\cancel{U}]}(x) + C_G^{[U(C)]}\pi\,\Phi_G^{\alpha[\cancel{U}]}(x,x)$$

T-even

universal matrix elements

T-odd

(operator structure)



$$\Phi_G^\alpha(x, x - x_1)$$

$\Phi_G(x,x)$ is gluonic pole ($x_1 = 0$) matrix element

Result for single weighted cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\Phi_b^{[C_2(D)]}(x_2, p_{2T})\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots + \ldots\ldots$$

$$\Phi_\partial^{\alpha[C]}(x) = \tilde{\Phi}_\partial^{\alpha[\cancel{C}]}(x) + C_G^{[U(C)]}\pi\,\Phi_G^{\alpha[\cancel{C}]}(x,x)$$

universal matrix elements

Examples are:

$C_G^{[U^+]} = 1$, $C_G^{[U^-]} = -1$, $C_G^{[W\,U^+]} = 3$, $C_G^{[Tr(W)U^+]} = N_c$

Result for single weighted cross section

$$\frac{d\sigma}{d^2 q_T} \sim \sum_{D,abc} \Phi_a^{[C_1(D)]}(x_1, p_{1T})\Phi_b^{[C_2(D)]}(x_2, p_{2T})\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c^{[C_1'(D)]}(z_1, k_{1T})\ldots$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{D,abc} \Phi_{\partial\, a}^{\alpha[C(D)]}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}^{[D]}\,\Delta_c(z_1)\ldots + \ldots..$$

$$\Phi_\partial^{\alpha[C]}(x) = \tilde{\Phi}_\partial^{\alpha[\cancel{C}]}(x) + C_G^{[U(C)]}\pi\,\Phi_G^{\alpha[\cancel{C}]}(x,x)$$

$$\left\langle q_T^\alpha \sigma \right\rangle \sim \sum_{abc} \tilde{\Phi}_{\hat{a}}^{\alpha}(x_1)\Phi_b(x_2)\,\hat{\sigma}_{ab\to c\ldots}\,\Delta_c(z_1)\ldots + \ldots..$$

$$+ \sum_{abc} \pi\Phi_{G\,a}^{\alpha}(x_1, x_1)\Phi_b(x_2)\,\hat{\sigma}_{[a]b\to c\ldots}\,\Delta_c(z_1)\ldots + \ldots..$$

$$\hat{\sigma}_{[a]b\to c\ldots} = \sum_D C_G^{[U(C(D))]}\hat{\sigma}_{ab\to c\ldots}^{[D]}$$

(gluonic pole cross section)

Drell-Yan and photon-jet production

- Weighted Drell-Yan

$$\Phi^{\alpha[-]}_{\partial q} = \tilde{\Phi}^{\alpha}_{\partial q} - \pi \Phi^{\alpha}_{G q}$$

$$\left\langle q_T^{\alpha}\, \sigma^{H_1 H_2 \to \gamma} \right\rangle = \Phi^{\alpha[-]}_{\partial q} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma} + \ldots$$

- Weighted photon-jet production in hadron-hadron scattering

$$\Phi^{\alpha[+(W)]}_{\partial q} = \tilde{\Phi}^{\alpha}_{\partial q} + \pi \Phi^{\alpha}_{G q}$$

$$\left\langle q_T^{\alpha}\, \sigma^{H_1 H_2 \to JJ} \right\rangle = \frac{N^2-1}{N}\Big[\frac{N^2}{N^2-1} \Phi^{\alpha[+(W)]}_{\partial q} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma g} \Delta_g - \frac{1}{N^2-1} \Phi^{\alpha[-]}_{\partial q} \Phi_{\bar{q}} \hat{\sigma}_{q\bar{q}\to\gamma g} \Delta_g \Big] + \ldots$$

$$\Phi^{\alpha[-]}_{\partial q} = \tilde{\Phi}^{\alpha}_{\partial q} - \pi \Phi^{\alpha}_{G q}$$

Drell-Yan and photon-jet production

- Weighted Drell-Yan

$\hat{\sigma}_{[q]\bar{q}\to\gamma}$

$$\left\langle q_T^{\alpha}\,\sigma^{H_1H_2\to\gamma}\right\rangle = \tilde{\Phi}^{\alpha}_{\partial\,q}\Phi_{\bar{q}}\hat{\sigma}_{q\bar{q}\to\gamma} + \pi\Phi^{\alpha}_{G\,q}\Phi_{\bar{q}}\left(-\hat{\sigma}_{q\bar{q}\to\gamma}\right) + \ldots$$

- Weighted photon-jet production in hadron-hadron scattering

Gluonic pole cross sections

$$\left\langle q_T^{\alpha}\,\sigma^{H_1H_2\to JJ}\right\rangle = [\tilde{\Phi}^{\alpha}_{\partial\,q}\Phi_{\bar{q}}\hat{\sigma}_{q\bar{q}\to\gamma g}\Delta_g$$

$$+\pi\Phi^{\alpha}_{G\,q}\Phi_{\bar{q}}\left(\frac{N^2+1}{N^2-1}\hat{\sigma}_{q\bar{q}\to\gamma g}\right)\Delta_g] + \ldots$$

$\hat{\sigma}_{[q]\bar{q}\to\gamma g}$

Note: color-flow in this case is more SIDIS like than DY

Bomhof, PJM, JHEP 0702 (2007) 029 [hep-ph/0609206]

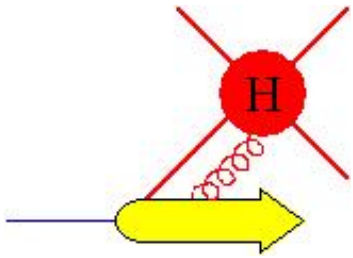
Gluonic pole cross sections

- For quark distributions one needs normal hard cross sections

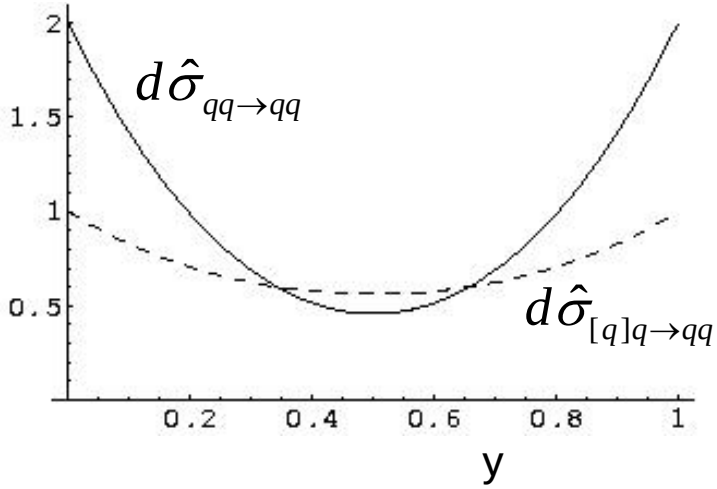
$$\hat{\sigma}_{qq\to qq} = \sum_{[D]} \hat{\sigma}^{[D]}$$

- For T-odd PDF (such as transversely polarized quarks in unpolarized proton) one gets modified hard cross sections

$$\hat{\sigma}_{[q]q\to qq} = \sum_{[D]} C_G^{[U(D)]} \hat{\sigma}^{[D]}$$



(gluonic pole cross section)



- for DIS: $\hat{\sigma}_{\ell[q]\to\ell q} = +\hat{\sigma}_{\ell q\to\ell q}$
- for DY: $\hat{\sigma}_{[q]\bar{q}\to\ell\bar{\ell}} = -\hat{\sigma}_{q\bar{q}\to\ell\bar{\ell}}$

Bomhof, PJM, NPB 795 (2008) 409 [arXiv: 0709.1390]

Universality and factorization for TMDs?

- We can work with basic TMD functions $\Phi^{[\pm]}(x,p_T)$ + 'junk'
- The 'junk' constitutes process-dependent residual TMDs

$$\Phi^{[(W)(W^\dagger)+]}(x,p_T) = \Phi^{[+]}(x,p_T) + \underbrace{\left[\Phi^{[(W)(W^\dagger)+]}(x,p_T) - \Phi^{[+]}(x,p_T)\right]}_{\delta\Phi^{[(W)(W^\dagger)+]}(x,p_T)}$$

$$\Phi^{[W+]}(x,p_T) = 2\Phi^{[+]}(x,p_T) - \Phi^{[-]}(x,p_T) + \delta\Phi^{[W+]}(x,p_T)$$

$$\boxed{\Phi^{[U]}(x,p_T) = \tfrac{1}{2}\Phi^{[even]}(x,p_T) - \tfrac{1}{2}C_G^{[U]}\Phi^{[odd]}(x,p_T) + \delta\Phi^{[U]}(x,p_T)}$$

- Thus: $\Phi^{[[q]g\to qg]} = \frac{1}{2}\Phi^{[even]} - \frac{1}{2}C_G^{[U([q]g\to qg)]}\Phi^{[odd]} + \delta\Phi^{[[q]g\to qg]}$
- The junk gives zero after integrating ($\delta\Phi(x) = 0$) and after weighting ($\delta\Phi_\partial(x) = 0$), i.e. cancelling k_T contributions; moreover it most likely also disappears for large p_T
- Junk pieces might be collected in 'entangled multi-hadron' part.

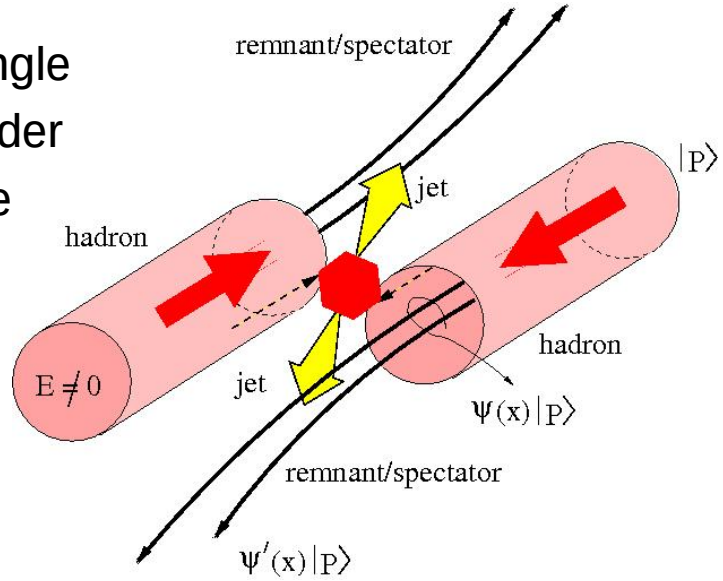
Bomhof, PJM, NPB 795 (2008) 409 [arXiv: 0709.1390]

(Limited) universality for TMD functions

QUARKS	$\Phi^{[even]}(x,p_T)=\frac{1}{2}\Phi^{[+]}+\frac{1}{2}\Phi^{[-]}$	$\Phi(x)$	$\tilde{\Phi}_\partial(x)$
	$\Phi^{[odd]}(x,p_T)=\frac{1}{2}\Phi^{[+]}-\frac{1}{2}\Phi^{[-]}$		$\pi\Phi_G(x,x)$
GLUONS	$\Gamma^{[even]}(x,p_T)=\frac{1}{2}\Gamma^{[+,+]}+\frac{1}{2}\Gamma^{[-,-]}$	$\Gamma(x)$	$\tilde{\Gamma}_\partial(x)$
	$\Gamma_F^{[odd]}(x,p_T)=\frac{1}{2}\Gamma^{[+,+]}-\frac{1}{2}\Gamma^{[-,-]}$		$\Gamma_G^F(x,x)$
	$\Gamma_D^{[odd]}(x,p_T)=\frac{1}{2}\Gamma^{[+,-]}-\frac{1}{2}\Gamma^{[-,+]}$		$\Gamma_G^D(x,x)$
	$\Gamma'^{[even]}(x,p_T)=\frac{1}{2}\Gamma^{[+,-]}+\frac{1}{2}\Gamma^{[-,+]}$	$=\Gamma^{[even]}(x,p_T)+\delta\Gamma(x,p_T)$	

Outlook (work in progress)

- A possible way out to include the color entanglement of different hadrons is to include all (or more) hadrons in *one* soft part
- In p_T-averaged situation it reduces to a product of single-hadron collinear soft parts, for single-weighted case to two single-hadron collinear soft parts, ...
- This may have advantages that a single treatment of factorization (higher order QCD contributions) becomes feasible
- The multi-hadron soft part will most likely involve novel multi-hadron correlations (through color entanglement)



Conclusions

- Transverse momentum dependence, experimentally important for single spin and azimuthal asymmetries, theoretically challenging (consistency, gauges and gauge links, universality, factorization)
- For leading integrated and single-weighted functions factorization is possible, but it requires besides the normal 'partonic cross sections' use of 'gluonic pole cross sections' and it is important to realize that q_T-effects generally come from all partons
- It is still an open issue how to achieve factorization into hard partonic part including its colorflow and a set of (gauge-link dependent, possibly multi-hadron correlating) TMDs (ongoing work with Ted Rogers).

Relevant for past, present and future experiments
(RHIC, COMPASS, JLAB, JPARC, GSI, LHC)