Theory Overview on Longitudinal Spin Physics

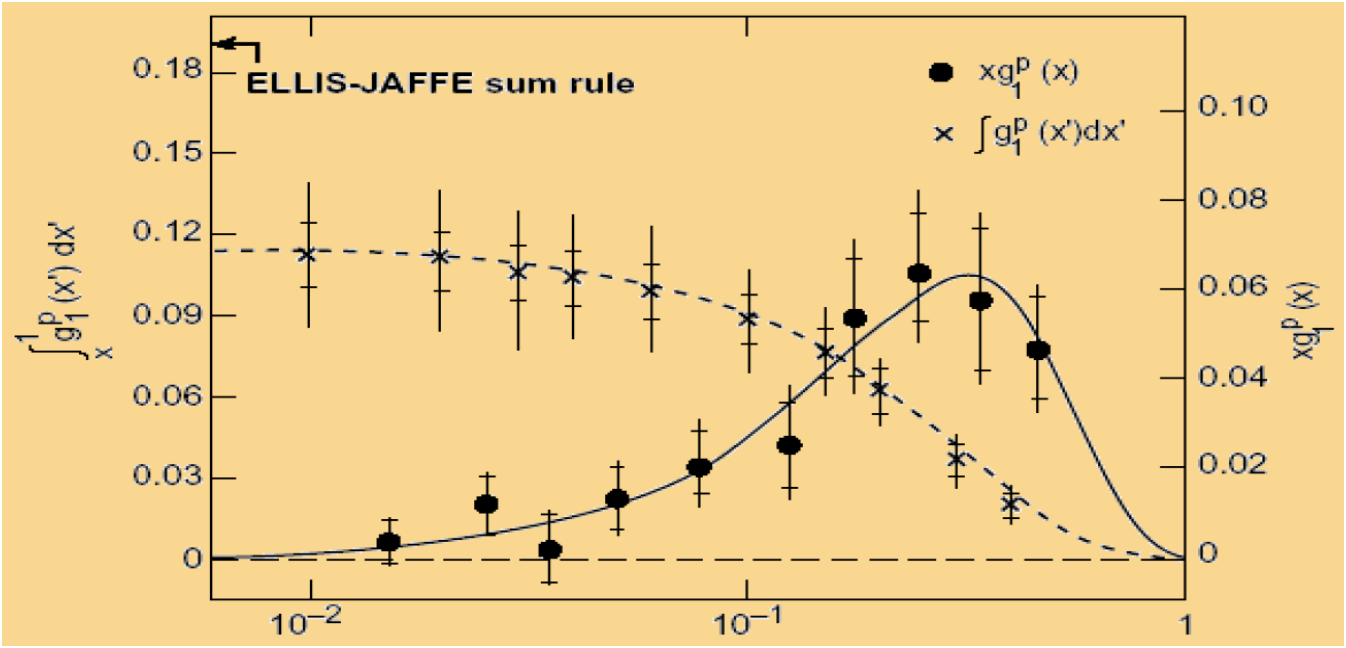
Jianwei Qiu
Brookhaven National Laboratory

Outline

- Twenty years since the "spin crisis"
- Polarized cross sections and spin asymmetries
- QCD factorization and universality of polarized PDFs
- ΔG – the first milestone of RHIC spin program
- W-physics program and quark flavor separation
- Summary and outlook

Over twenty years since the "spin crisis"

- **EMC experiment in 1988/1989 – "the plot":**



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\|}\rangle$$

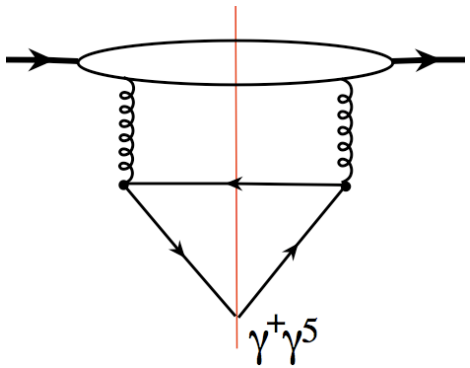
- **"Spin crisis":**

$$\Delta\Sigma = \sum_q \left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$$

Early "solution" to the "crisis"

- **Large ΔG to cancel the "true" Δq:**

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|} | \overline{\psi}_q(0) \gamma^+ \gamma_5 \psi_q(0) | P, s_{\|} \rangle$$



$$\Delta\Sigma \to \Delta\Sigma - \frac{n_f \alpha_s(Q^2)}{2\pi} \Delta G(Q^2)$$

- **What value of ΔG is needed?**

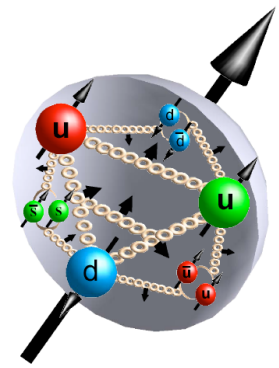
$$\Delta G(Q^2) \sim 2$$ **at Q ~ 1 GeV**

- **Question: How to measure ΔG independently?**

  - ✧ **Precision inclusive DIS**
  - ✧ **Jets in SIDIS**
  - ✧ **Hadronic collisions – RHIC spin**
  - ✧ **…**

Bigger "questions"

- **Beyond ΔG:**
  - ✧ Quark flavor separation?
  - ✧ Antiquark helicity contribution?
  - ✧ Proton's spin structure?
  - ✧ QCD dynamics behind the spin structure?
  - ✧ ...



- **Proton spin = Angular momentum of the proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 | \, \hat{J}_f^z \, | P, S_z = 1/2 \rangle = \frac{1}{2}$$

- **Spin sum rules:**

$$\frac{1}{2} = \left[\frac{1}{2} \Delta\Sigma(\mu^2) + L_q(\mu^2) \right] + J_g(\mu^2)$$

$$\frac{1}{2} = \left[\frac{1}{2} \Delta\Sigma(\mu^2) + L_q(\mu^2) \right] + \left[\Delta G(\mu^2) + L_g(\mu^2) \right]$$

Gluon contribution to proton's spin

❑ Gluon angular momentum:

$$J_g(\mu^2) = \int d^3x \langle P,s|\vec{x}\times(\vec{E}\times\vec{B})|P,s\rangle \equiv \Delta G(\mu^2) + \left(J_g(\mu^2) - \Delta G(\mu^2)\right)$$

Helicity

Transverse motion

❑ Asymptotic limit (Ji):

$$J_q(\mu^2\to\infty) \to \frac{1}{2}\frac{3N_f}{16+3N_f} \qquad J_g(\mu^2\to\infty) \to \frac{1}{2}\frac{16}{16+3N_f}$$

If ΔG is very small, total gluon angular momentum comes from its transverse motion!

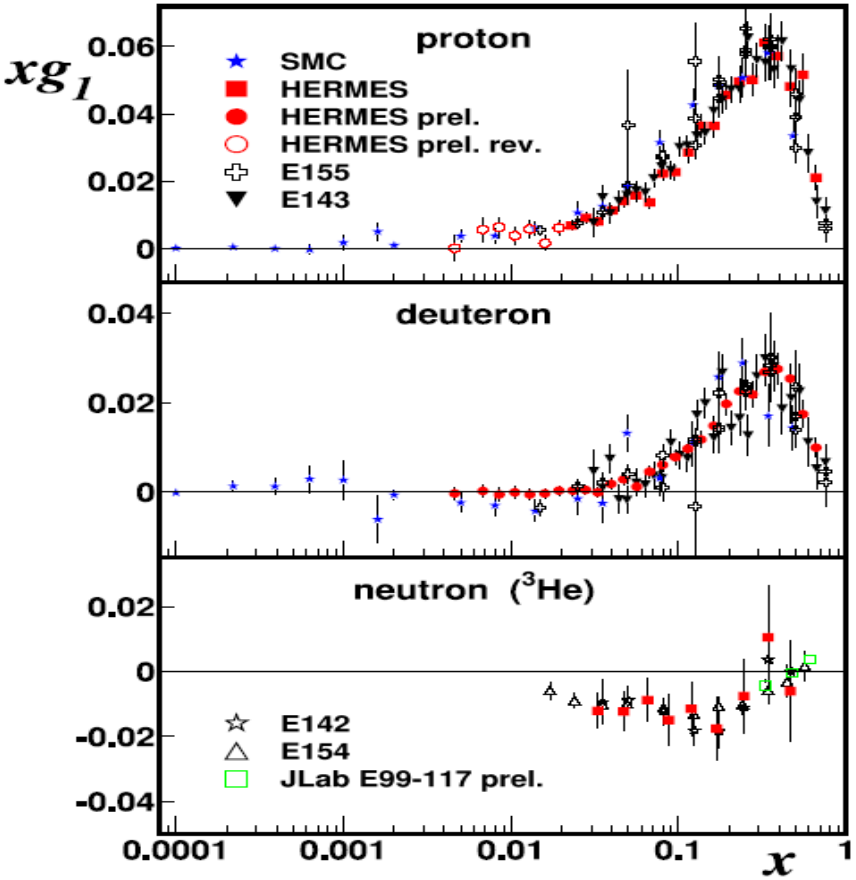
❑ Gluon orbital contribution:

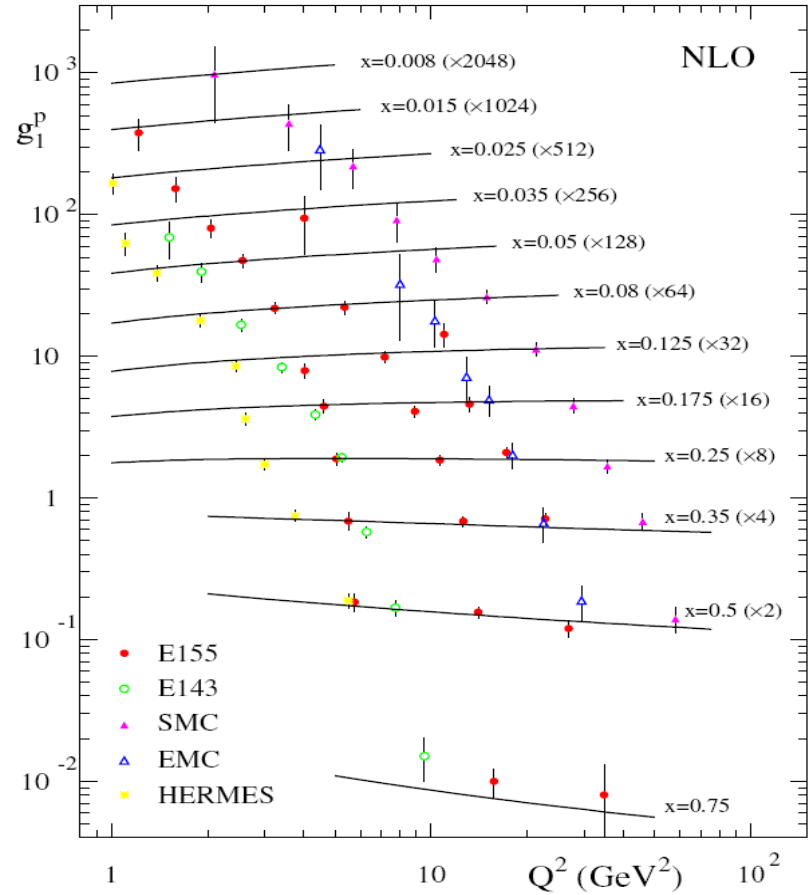
Role of gluon k_T in a longitudinally polarized proton:

➡ **Power corrections to the longitudinal spin asymmetries**

Improvement in over twenty years

- The "Plot" is now much improved:





- Flavor separation – Global QCD analysis:

  SIDIS – HERMES, COMPASS, JLAB

  Hadronic collisions – RHIC

The goals of RHIC spin program

Research plan for the spin physics at RHIC (2005, 2007, 2008)

- Determination of polarized gluon distribution (ΔG) over a large range of momentum fraction x, using multiple probes

- Determination of flavor identified quark and anti-quark polarization using parity violating production of $W^{\pm}$

- Transverse spin phenomena in QCD: The transversity (δq) and the origin of transverse-spin phenomena

Questions

Experiments measure cross sections,

Not ΔG, Δq and $\Delta\bar{q}$!

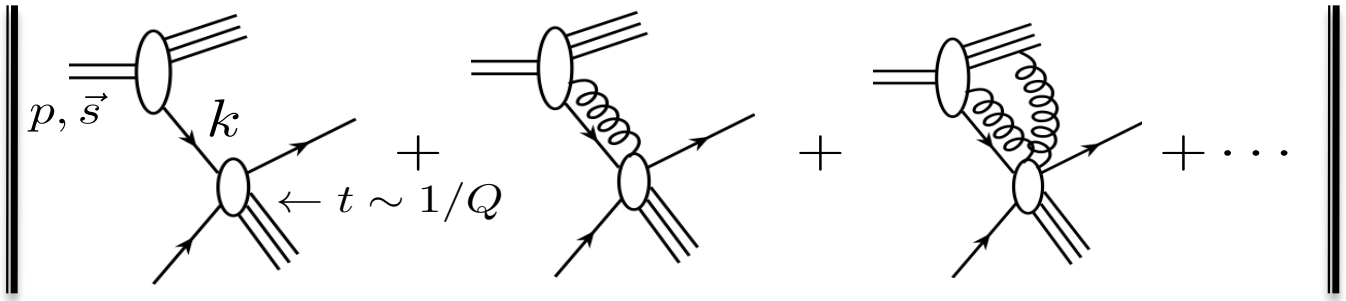
How reliable we can extract these quantities

from

the measured cross sections/asymmetries?

Hadronic cross sections

- **Theorists' view of hadronic cross section:**

$\sigma(Q,\vec{s}) \propto$ 

Every parton could participate in the collision

- **Large momentum transfer simplifies the picture:**

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

Single hard scale ➡ Leading power ➡ Collinear factorization

$$\sigma^{(2)}_{AB}(Q,\vec{s}) = \hat{\sigma}_{ab}(x,x',Q) \otimes f_{a/A}(x,Q,\vec{s}) \otimes \left[f_{b/B}(x',Q) \otimes \cdots\right]$$

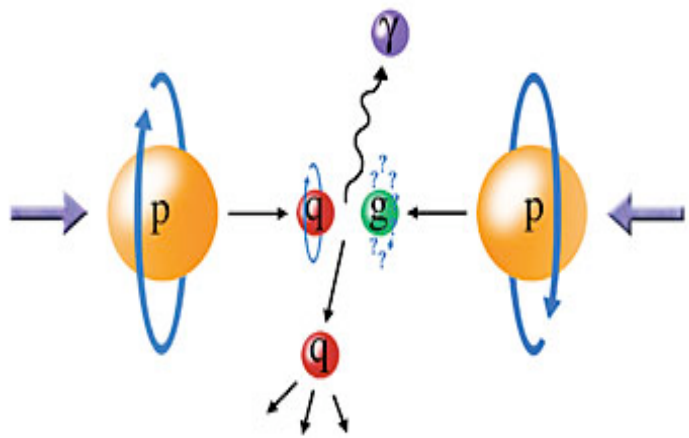
- **Predictive power:**

Short-distance dynamics, PDFs, and FFs

It worked beautifully – great success of QCD!

PQCD collinear factorization

- **Factorization is an approximation:**



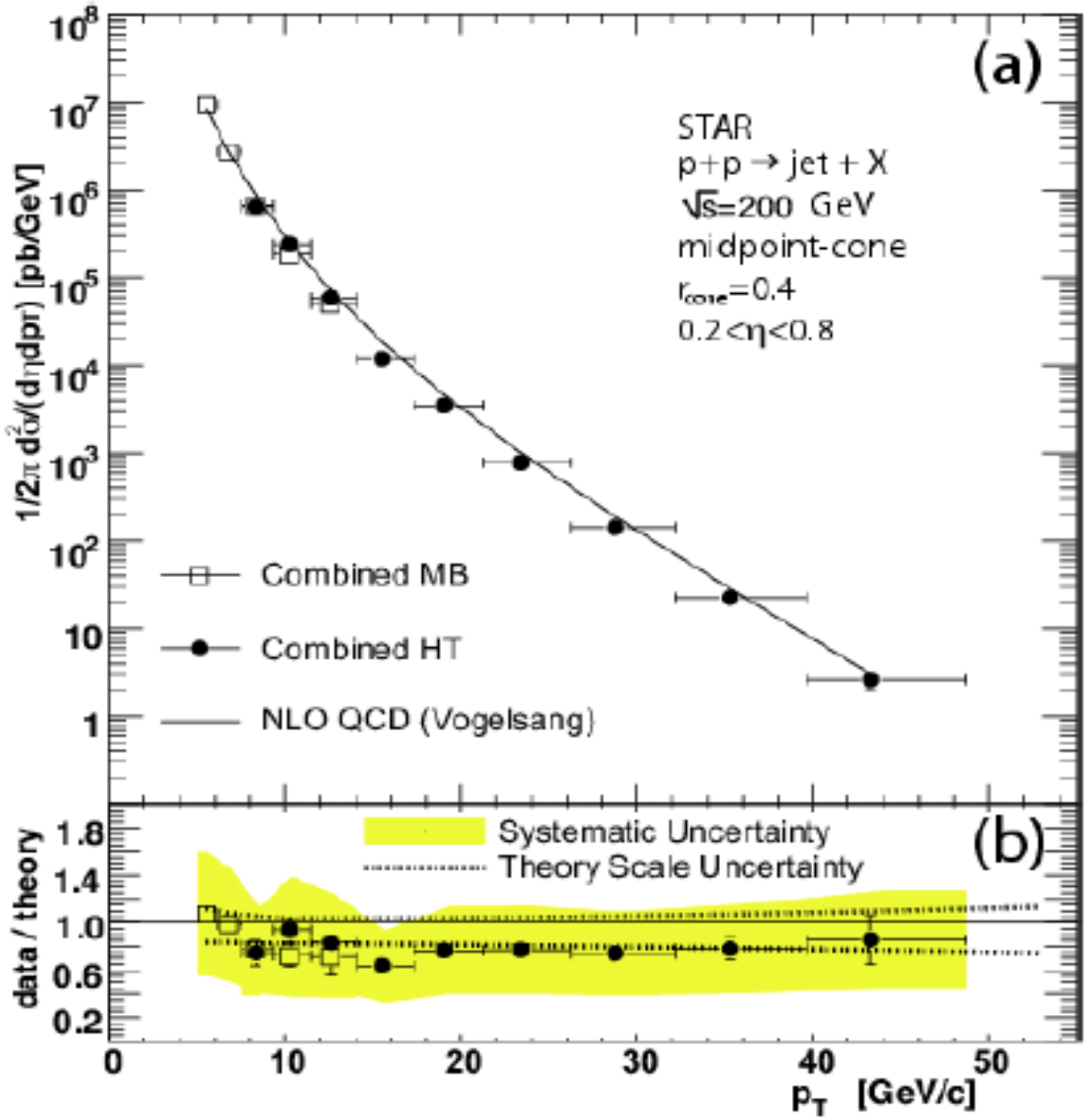
$$\frac{d\sigma}{dydp_T^2} = \int \frac{dx}{x} q(x) \int \frac{dx'}{x'} g(x') \frac{d\hat{\sigma}_{qg\to\gamma q}}{dydp_T^2} + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{Q_s}{P_T}\right)^n$$

Convoluted with a fragmentation function for inclusive single particle production

- **The proof was originally given for spin-averaged case:**
  - ✧ **The same proof could be carried through for the leading power (LP) contribution to spin-dependent cross section (with a fixed spin state)**
  - ✧ **But, the LP proof does not say anything about the spin dependence of the power corrections – transverse motion – orbital?**

Inclusive jet at 200 GeV

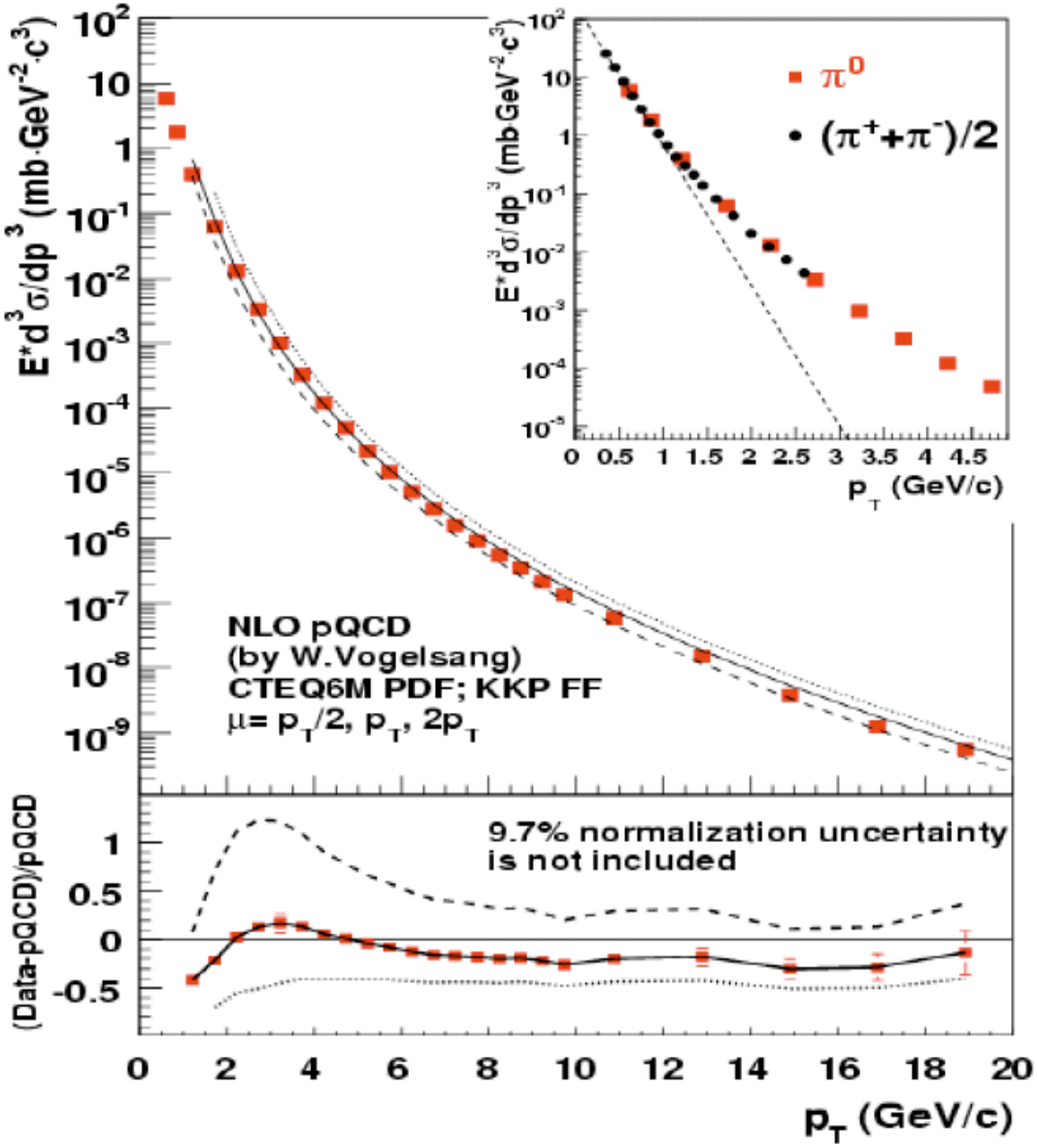
- **STAR:**



PRL97, 252001 (2006)

Inclusive single hadron at 200 GeV

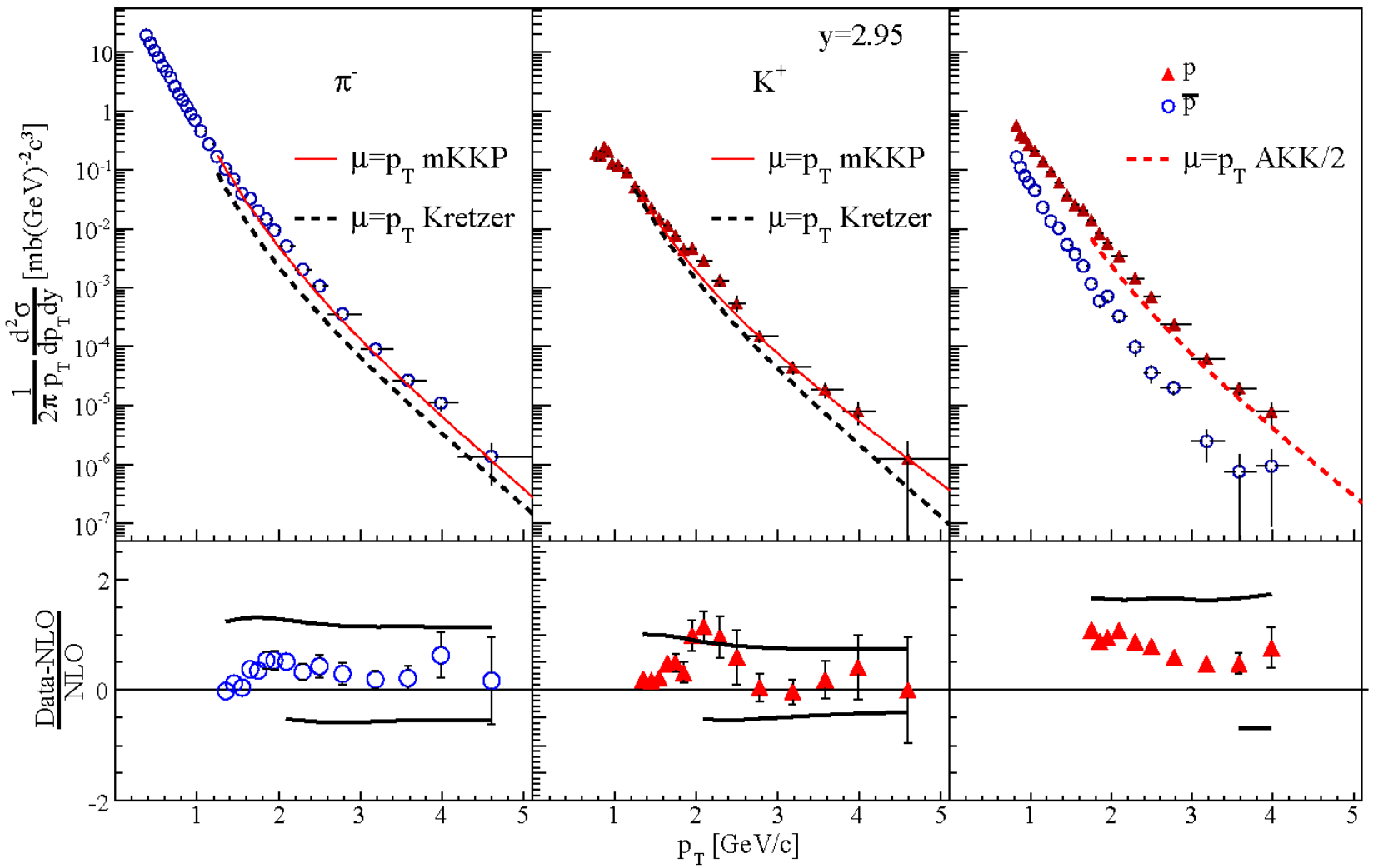
- **PHENIX:**



PRD76, 051106 (2007)

Extending x coverage and particle types

- **BRAHMS:** Large rapidity π,K,p cross sections for p+p, √s=200 GeV



PRL98, 252001 (2007)

Longitudinal spin asymmetries

- **Single spin asymmetries:**

$$A_L \equiv \frac{\sigma(s_\parallel) - \sigma(-s_\parallel)}{\sigma(s_\parallel) + \sigma(-s_\parallel)}$$

Vanishes if parity is invariant

- **Double spin asymmetries:**

$$A_{LL} \equiv \frac{\left[\sigma(s_\parallel, s_\parallel) - \sigma(s_\parallel, -s_\parallel)\right] - \left[\sigma(-s_\parallel, s_\parallel) - \sigma(-s_\parallel, -s_\parallel)\right]}{\left[\sigma(s_\parallel, s_\parallel) + \sigma(s_\parallel, -s_\parallel)\right] + \left[\sigma(-s_\parallel, s_\parallel) + \sigma(-s_\parallel, -s_\parallel)\right]}$$

- **Asymmetries =\= cross sections**
  proportional to the difference of two cross sections

  Leading power pQCD works for spin-averaged cross section does NOT guarantee that the leading power pQCD works for asymmetries

- **Key:** **Power correction is small for spin-averaged cross section**
  =\= power correction is small for the difference of two cross sections

Parton helicity distributions

❑ Quark helicity distribution:

$$\Delta q(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{1}{2} \left[\langle p, s_\| | \overline{\psi}_q(0) \gamma^+ \frac{1+\gamma^5}{2} \psi_q(y^-) | p, s_\| \rangle \right.$$
$$\left. - \langle p, -s_\| | \overline{\psi}_q(0) \gamma^+ \frac{1-\gamma^5}{2} \psi_q(y^-) | p, -s_\| \rangle \right]$$

$$\xrightarrow{\text{P + T}} \quad \Delta q(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p, s_\| | \left[\overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \psi_q(y^-) \right] | p, s_\| \rangle$$

✧ **Fourier Transform of light-cone matrix element:** $\langle p, s_\| | \mathcal{O}_q(y^-) | p, s_\| \rangle$

$$\mathcal{O}_q(y^-) = \overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \psi(y^-)$$

The γ^5 flips the quark helicity at the cut-vertex

✧ **Necessary condition for nonvanish asymmetries – P + T:**

$$\langle p, s_\| | \mathcal{O}_q(y^-) | p, s_\| \rangle \Longleftrightarrow -\langle p, -s_\| | \mathcal{O}_q(y^-) | p, -s_\| \rangle$$

❑ Gluon helicity distribution:

$$\mathcal{O}_g(y^-) = \frac{1}{xp^+} F^{+\alpha}(0) \left[-i\, \varepsilon_{\alpha\beta} \right] F^{+\beta}(y^-)$$

The $i\varepsilon_{\alpha\beta}$ flips gluon helicity at the cut-vertex

QCD Global analysis - I

- **DSSV – 2008:** $\Delta f_j^{1,[x_{\min}-1]}$ at $Q^2 = 10\,\mathrm{GeV}^2$

	$x_{\min} = 0$	$x_{\min} = 0.001$	
	best fit	$\Delta\chi^2 = 1$	$\Delta\chi^2/\chi^2 = 2\%$
$\Delta u + \Delta\bar{u}$	0.813	$0.793\ ^{+0.011}_{-0.012}$	$0.793\ ^{+0.028}_{-0.034}$
$\Delta d + \Delta\bar{d}$	-0.458	$-0.416\ ^{+0.011}_{-0.009}$	$-0.416\ ^{+0.035}_{-0.025}$
$\Delta\bar{u}$	0.036	$0.028\ ^{+0.021}_{-0.020}$	$0.028\ ^{+0.059}_{-0.059}$
$\Delta\bar{d}$	-0.115	$-0.089\ ^{+0.029}_{-0.029}$	$-0.089\ ^{+0.090}_{-0.080}$
$\Delta\bar{s}$	-0.057	$-0.006\ ^{+0.010}_{-0.012}$	$-0.006\ ^{+0.028}_{-0.031}$
Δg	-0.084	$0.013\ ^{+0.106}_{-0.120}$	$0.013\ ^{+0.702}_{-0.314}$
$\Delta\Sigma$	0.242	$0.366\ ^{+0.015}_{-0.018}$	$0.366\ ^{+0.042}_{-0.062}$

- **Hirai-Kumano – 2008:**

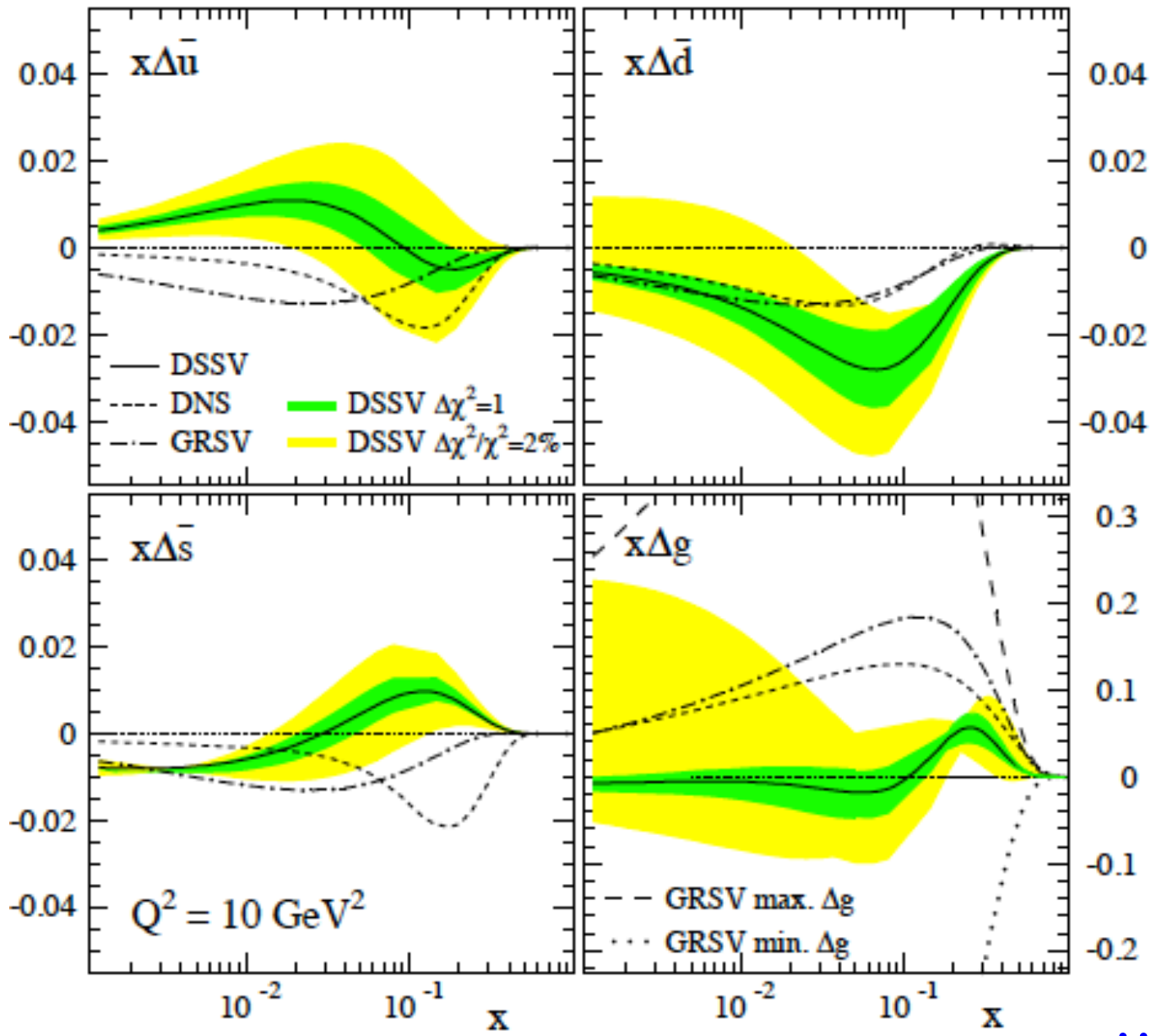
at $Q^2 = 1\ \mathrm{GeV}^2$

Analysis set	DIS	RHIC π^0	E07-011
A	○	–	–
B	○	○	–
C	○	–	○

	Set A		Set B		Set C	
	Positive	Node	Positive	Node	Positive	Node
$\Delta\Sigma$	0.24 ± 0.07	0.22 ± 0.08	0.26 ± 0.06	0.25 ± 0.07	0.24 ± 0.05	0.22 ± 0.05
ΔG	0.63 ± 0.81	0.94 ± 1.66	0.40 ± 0.28	-0.12 ± 1.78	0.63 ± 0.45	0.94 ± 1.09

QCD Global analysis - II

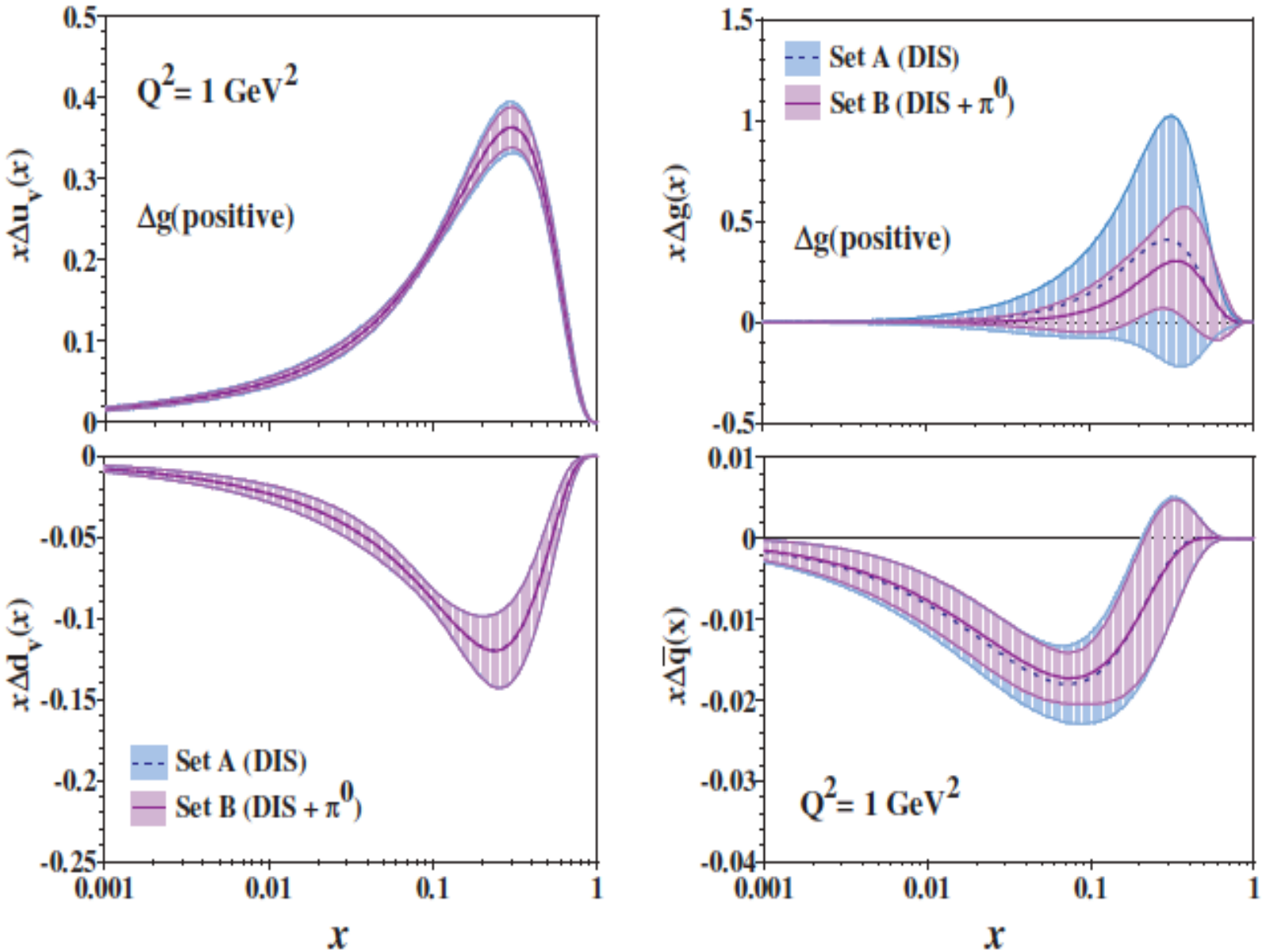
- **DSSV:**



Has a node

QCD Global analysis - III

- **HK:**



Without a node!

Determination of ΔG

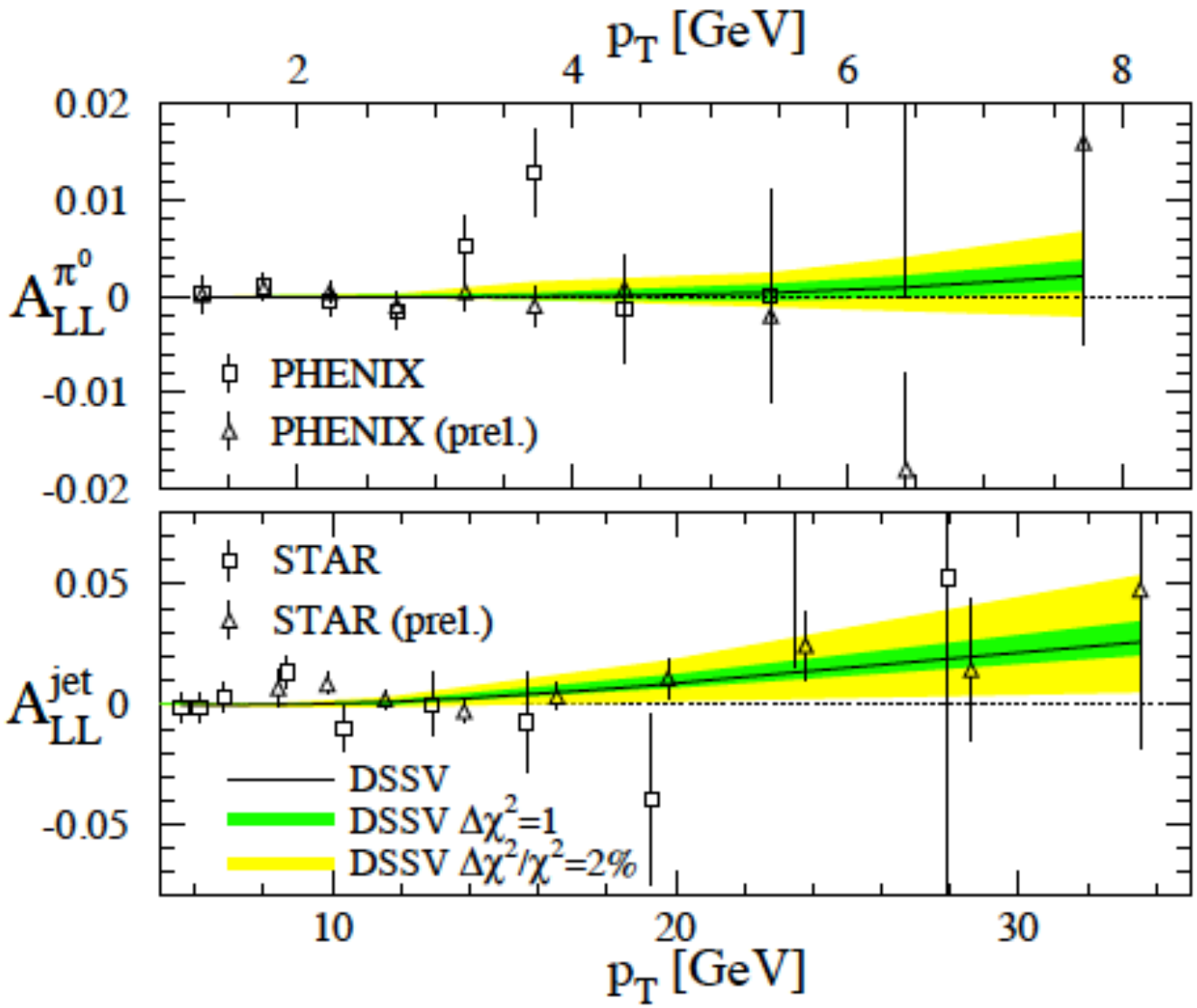
Leading power pQCD factorization theorem
is expected to be valid
for cross section with fixed spin at the same level of
confidence as for spin-averaged observables

Many NLO pQCD calculations
including jet-jet, particle-particle correlations
are available

However, power corrections could be larger
for the asymmetry – difference of two cross sections
if the asymmetry is small

RHIC measurement on ΔG

- **DSSV:**

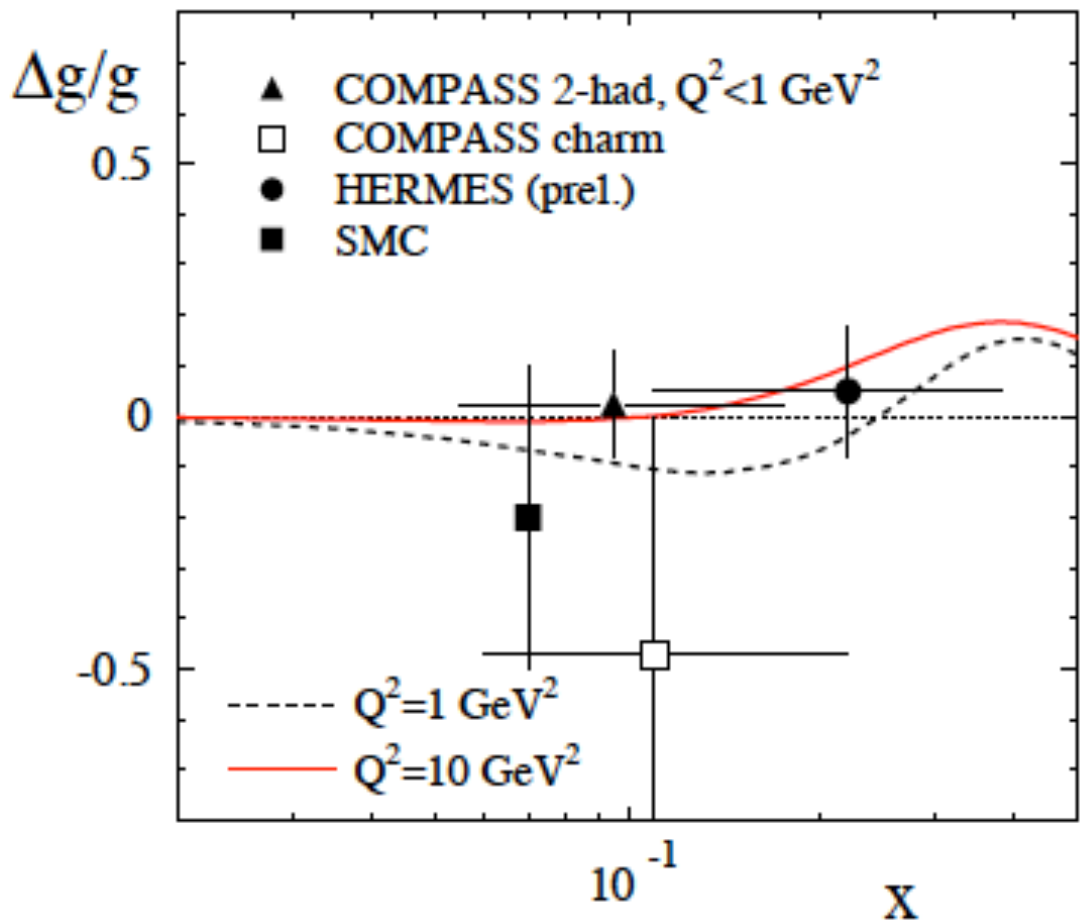


Small asymmetry leads to small gluon "helicity" distribution

Comparison with SIDIS data

- **DSSV:**

Data was not in the analysis



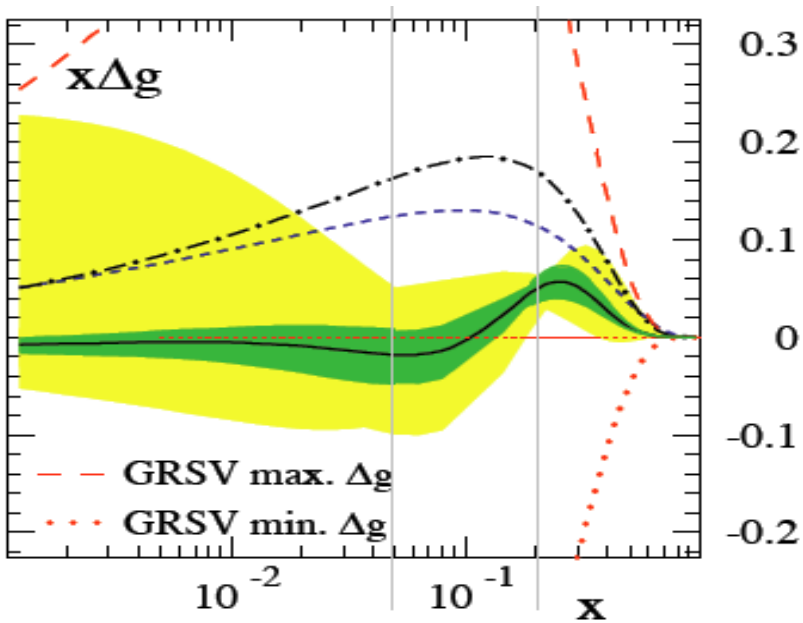
- **Observables:**

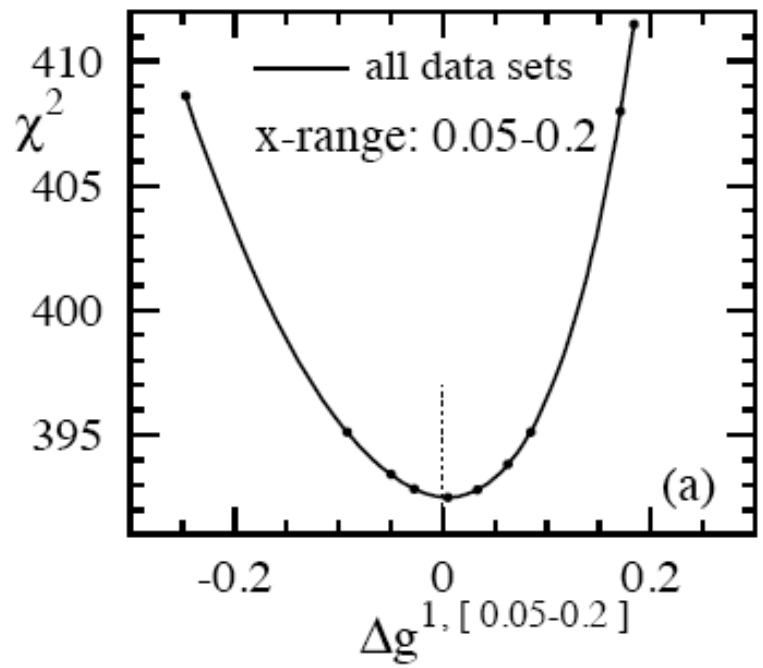
$$\ell p \to h + X \qquad \ell p \to h^+ + h^- + X$$

Current status on ΔG

- **Definition:** $$\Delta G = \int_0^1 dx\, \Delta G(x) = \langle P, s_{\|} | F^{+\mu}(0) F^{+\nu}(0) | P, s_{\|} \rangle (-i\epsilon_{\mu\nu})$$

- **NLO QCD global fit - DSSV:** PRL101,072001(2008)





$$\Delta G \approx \int_{0.001}^1 dx\, \Delta G(x) = -0.084$$

Strong constraint on ΔG from $0.05 \lesssim x \lesssim 0.2$ **If leading twist formulas were used!**

Current "conclusion" on ΔG

- **Anomalous gluonic contribution:**

$$\Sigma(Q^2) = \Sigma(Q^2)_{\rm true} - N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q)$$

Need ΔG ~ 1.5 – 2, which is unlikely

- **Gluonic contribution to proton spin:**

$$J_g(\mu^2 \to \infty) \to \frac{1}{2}\frac{16}{16+3N_f} \sim \frac{1}{4}$$

"without gluon orbital"

Might be ruled out by DSSV based on RHIC data: $\Delta g = -0.084$

But, not by Hirai and Kumano yet:

	Set A		Set B		Set C	
	Positive	Node	Positive	Node	Positive	Node
$\Delta\Sigma$	0.24 ± 0.07	0.22 ± 0.08	0.26 ± 0.06	0.25 ± 0.07	0.24 ± 0.05	0.22 ± 0.05
ΔG	0.63 ± 0.81	0.94 ± 1.66	0.40 ± 0.28	-0.12 ± 1.78	0.63 ± 0.45	0.94 ± 1.09

Could be room for power corrections!

Improvement on ΔG

- NNLO? Probably not yet
- Key: Extrapolation to low x and high x
  - Large x: total contribution might be small due to the steep falling phase space
  - Small x: larger phase space for shower and smaller Q for a fixed collision energy
- Collinear factorization does not work when $Q_s(x) \sim \langle k_T \rangle$

$$G(x) = G^+(x) + G^-(x) \propto \frac{1}{x^{1+\alpha}}$$

$$\Delta G(x) = G^+(x) - G^-(x)$$

at small x

Could be proportional to $\frac{1}{x^\alpha}$

Uncertainty in extrapolation!

- Room for power corrections if the asymmetry is small!

ΔG might not be too small, leading power to asymmetry "cancelled" by subleading power

Power corrections to the asymmetries

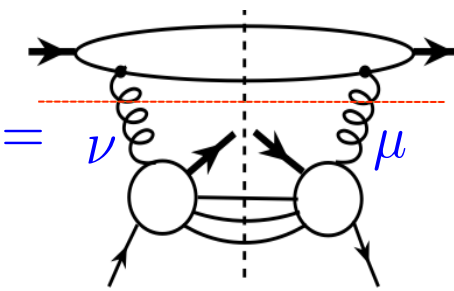
- **Success of leading power calculation for a cross section does NOT guarantee a success for calculating the difference of two cross sections**

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

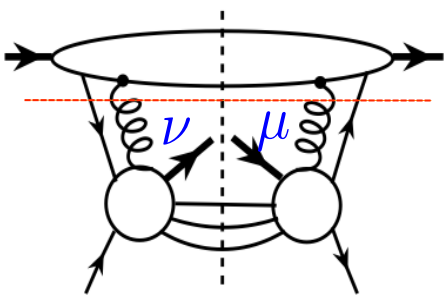
When the leading power contribution is almost canceled, the subleading power contribution can no longer be neglected!

- **Spin flip of high spin states at cut-vertex:**

$d\Delta\sigma(p, s_{\|}) =$



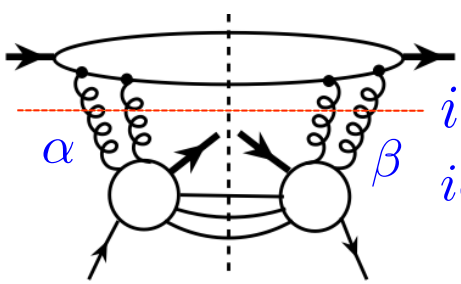
$i\epsilon_{\mu\nu}$ $+$



$i\epsilon_{\mu\nu}\gamma^\alpha$ $+\ldots$

$d_{\mu\nu}\gamma^\alpha\gamma_5$

Power corrections could be at the percent level $+$



$i\epsilon_{\mu\nu}\ d_{\alpha\beta}$ $+\ldots$

$i\epsilon_{\alpha\beta}\ d_{\mu\nu}, \ldots$

Power corrections are process-dependent

- **Existing measurement:** $A_{LL}|_{\rm jet/pion} \propto \Delta G(x) \otimes \Delta G(x')$

  dominated by $(\Delta g(x))^2$-type partonic subprocess

- **Fact: power corrections are process-dependent:**

- **Theory: a nice task for BNL spin summer program (7/14-27)**

- **Experimental tests: direct photon, J/psi, …**

  Observables dominated by different $\Delta g(x)$ combination

Direct photon: $A_{LL} \propto g_1^p(x) \otimes \Delta G(x') + x \leftrightarrow x' + \ldots$

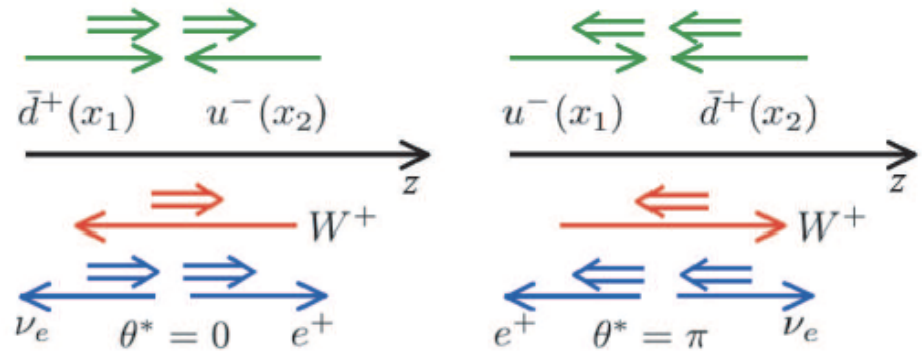
We know something about this one

J/psi:

Dominated by the sub-leading power – producing a color octet relativistic heavy quark pair at short-distance

Quark flavor separation

- **W's are left-handed:**



- **Flavor separation:**

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

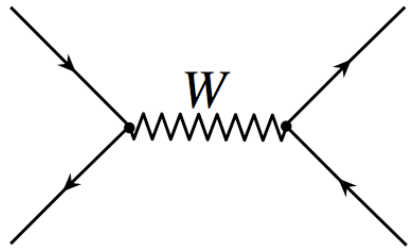
- **Complications:**

High order, W's p_T-distribution at low p_T
RHIC see high p_T lepton from W-decay

High order effect

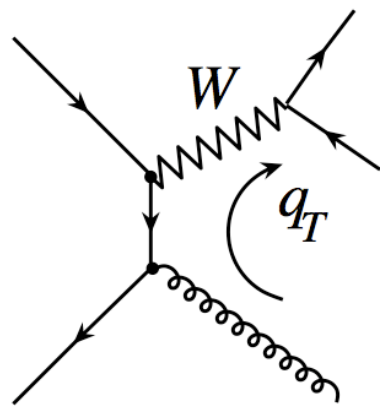
- Fixed order pQCD calculation:

LO:



$$\propto \delta^2(q_T)$$

NLO:



$$\propto \frac{1}{q_T^2} \quad \Rightarrow \infty \quad \text{as} \quad q_T^2 \to 0$$

- All order resummation is needed IF one wants to measure W's p_T-distribution:

CSS formalism – implemented in RHICBOS

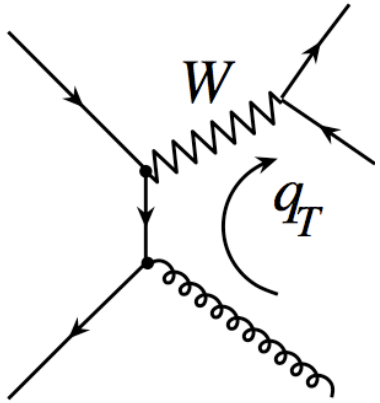
RBRC W-physics workshop (6/24-25)!

High p_T lepton asymmetry

- Hard to construct W's at RHIC:

High p_T lepton asymmetry

– mostly from decay of W's



- Two approaches:

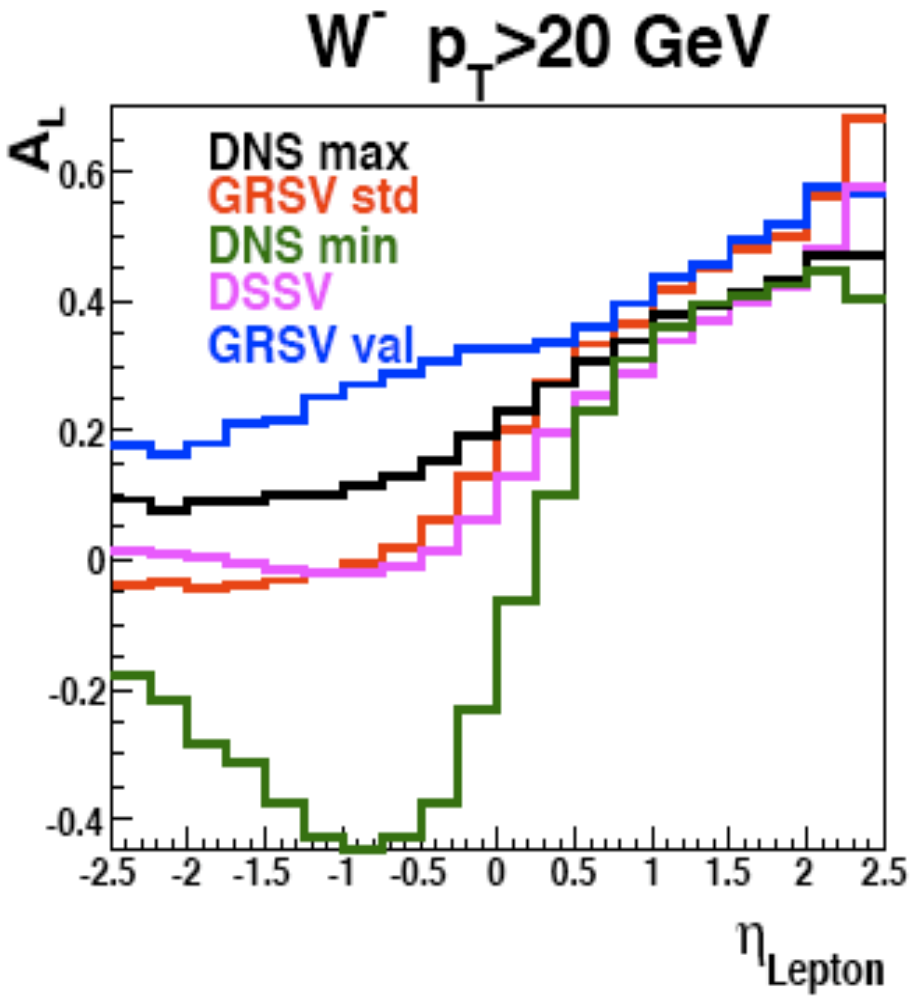
  - RHICBOS: resummed W p_T-distribution, calculate lepton asymmetry from W-decay of the resummed distribution

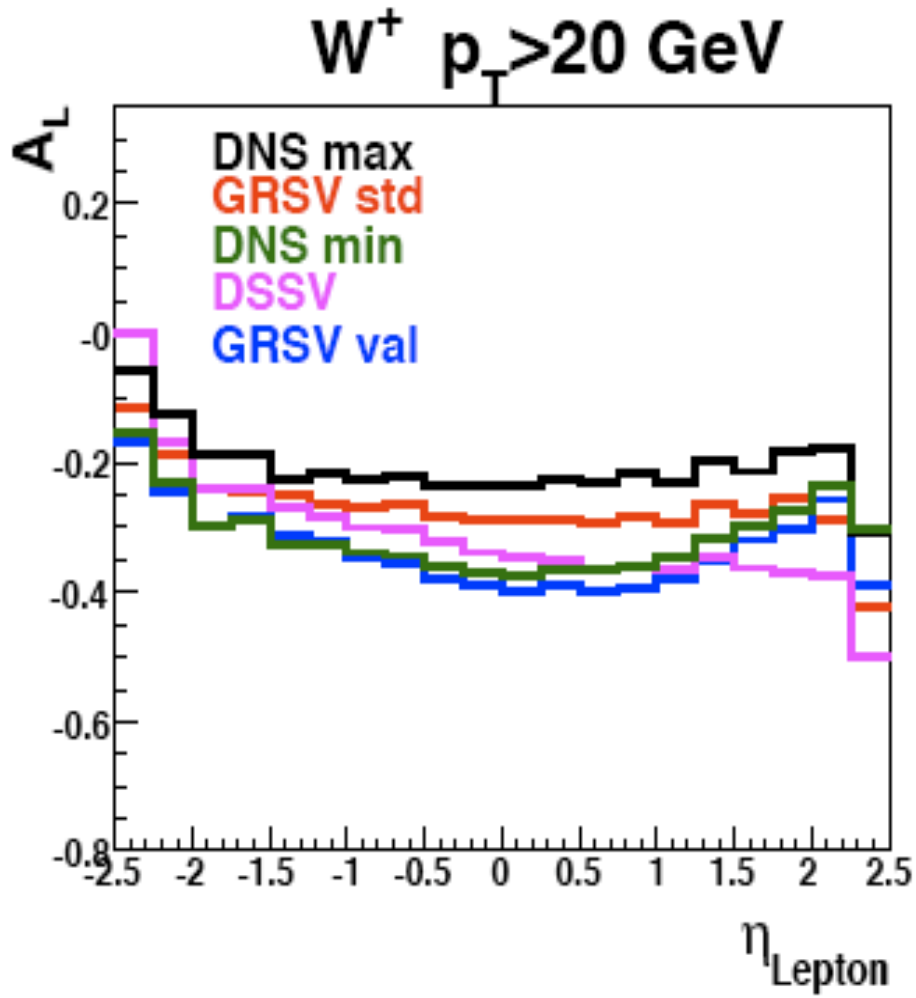
    Uncertainty: non-perturbative effect and off-diagonal contribution to lepton angular distribution

  - de Florian – Vogelsang: fixed order (NLO) lepton distribution – logarithmic divergence is integrable

    Uncertainty: non-W source of lepton, from high p_T virtual photon, …

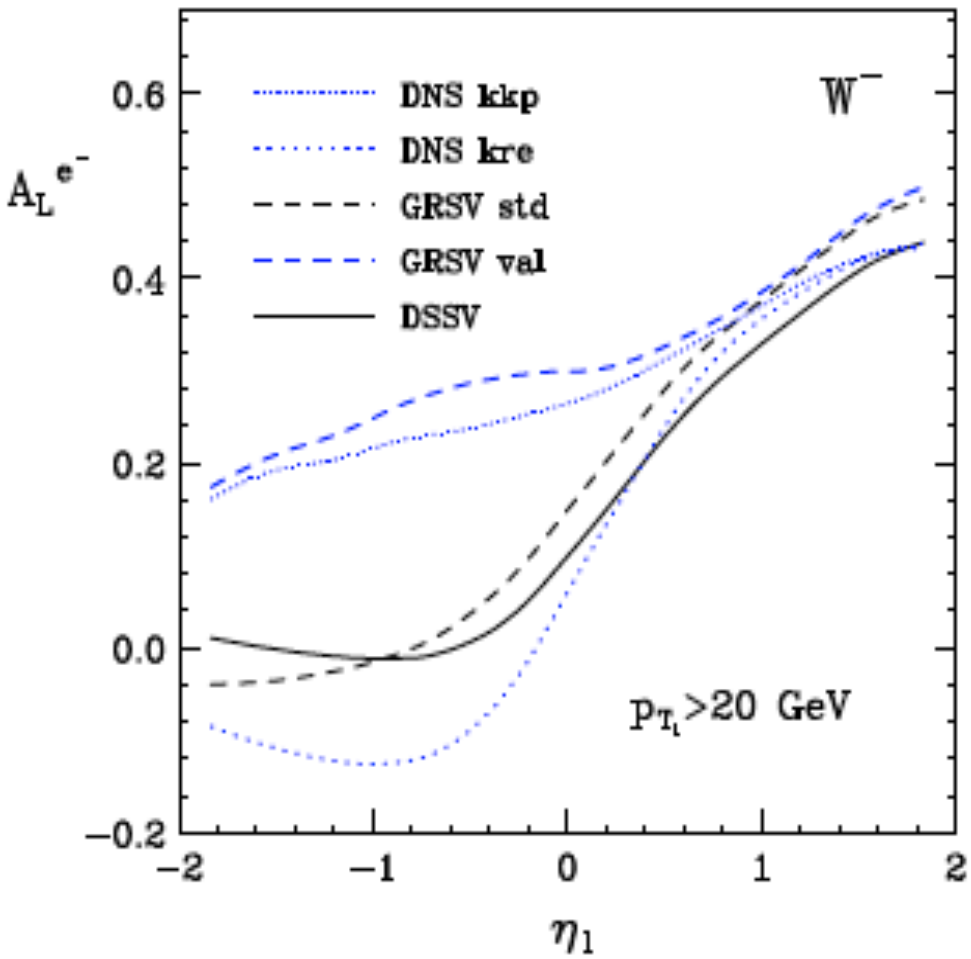
Predicted lepton asymmetry - I

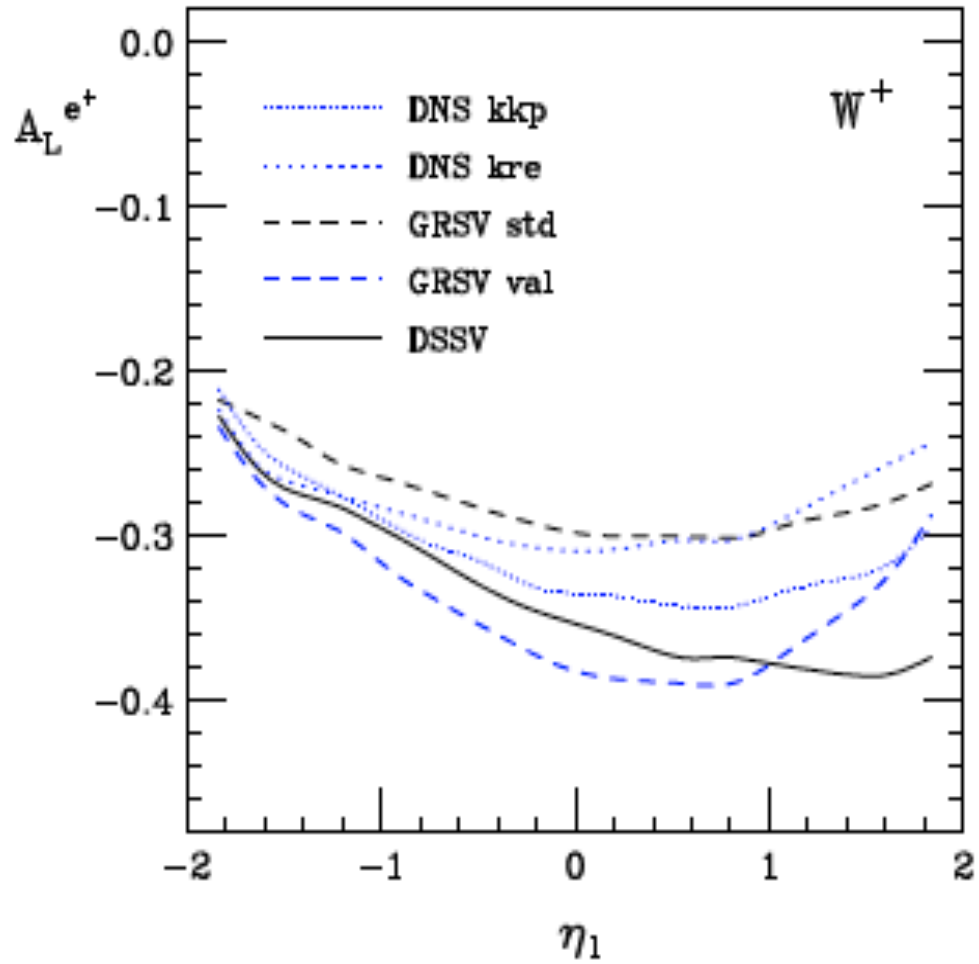




RHICBOS – RS=500 GeV

Predicted lepton asymmetry - II





de Florian and Vogelsang, arXiv:1003.4533

Summary and outlook

- QCD global analysis of existing data indicates that ΔG is unlikely to be larger than 1.5 - 2
- However, existing data are not sufficient to say

  $\Delta G \ll 1/4$ The asymptotic value for gluon angular momentum

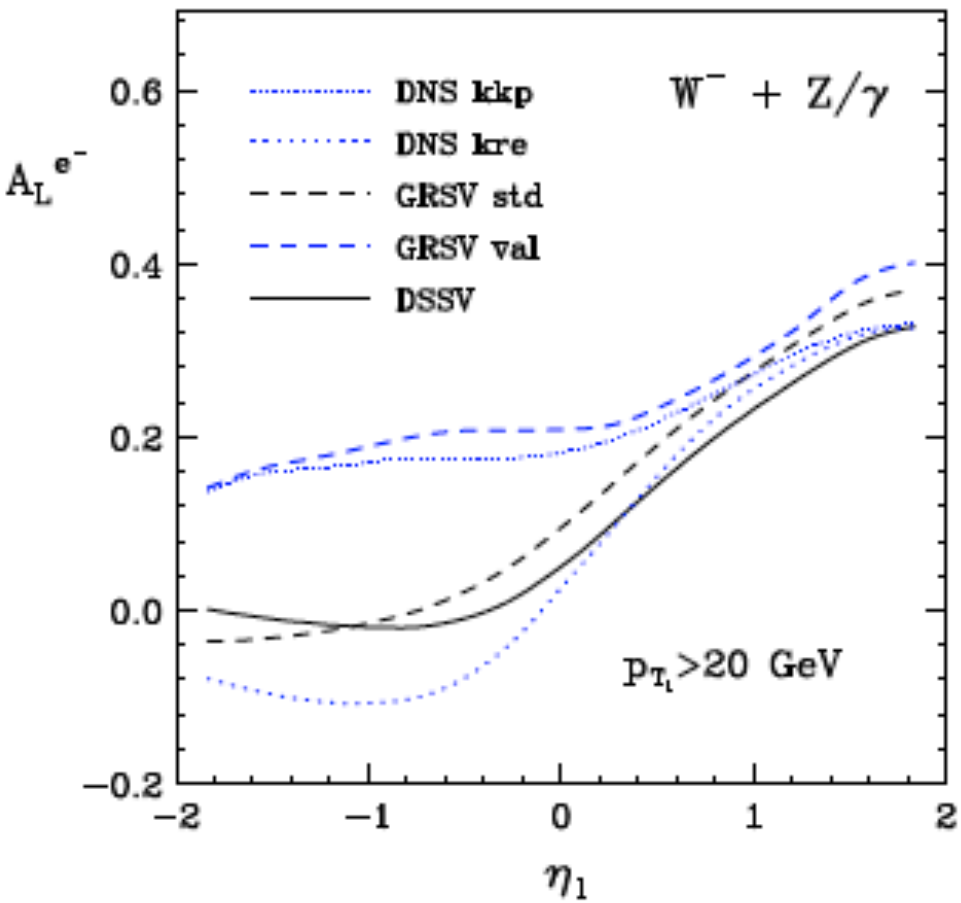
- Knowledge of $\Delta G(x)$ is clearly much more interesting and important than one number ΔG!

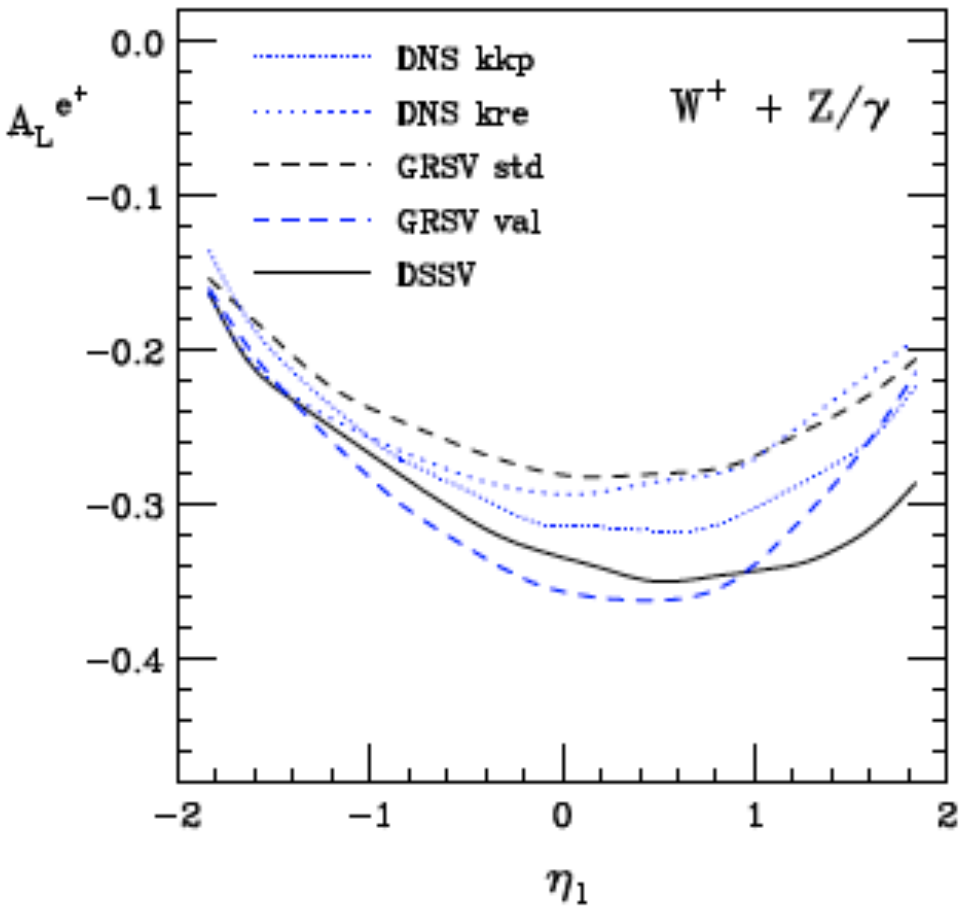
  It was the G(x), not momentum fraction G, that got people excited!

- W-physics program: quark flavor separation at large momentum transfer (much larger than the scale at COMPASS)
- RHIC Spin Program provides unique opportunities to explore QCD dynamics in a regime where no one was able to do before, or in a near future

Thank you!

Predicted lepton asymmetry - III





de Florian and Vogelsang, arXiv:1003.4533