STAR: rapidity dependence of $p_t$ correlations $\Rightarrow$ shear viscosity

Ask: can realistic hydro with plausible viscosity explain STAR data?

I. Measuring viscosity using correlations
II. Viscosity depends on temperature
III. 2$^{nd}$ order viscous diffusion
IV. Other contributions to correlation measurements

Work in progress!
Transverse Flow Fluctuations

Small variations in transverse flow in each event

Fluid elements flow past one another ⇒ viscous friction

Shear viscosity drives velocity toward the average

\[ T_{zr} = -\eta \frac{\partial v_r}{\partial z} \]

damping of radial flow fluctuations ⇒ viscosity
Evolution of Fluctuations

**momentum current**
small fluctuations

\[ g_t \equiv T_{0r} - \langle T_{0r} \rangle \]

**diffusion equation** for momentum current

\[ \frac{\partial}{\partial t} g_t = \frac{\eta}{sT} \nabla^2 (g_t + \text{noise}) \]

shear viscosity \( \eta \), entropy density \( s \), temperature \( T \)
linearized hydro, shear only \( \Rightarrow \) diffusion; small fluctuations \( \Rightarrow \) Langevin noise

**correlation function**

\[ r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle \]
Hydrodynamic Momentum Correlations

momentum flux density correlation function

\[ r = \langle g_t(x_1)g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle \]

\[ \Delta r = r - r_{eq} \text{ satisfies diffusion equation} \]

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

fluctuations diffuse through volume, driving \( r \rightarrow r_{eq} \)

width in relative spatial rapidity \( y = \sinh^{-1} \frac{z}{\tau} \)
grows from initial value \( \sigma_0 \)

\[ \sigma^2 = \sigma_0^2 + 4 \frac{\eta}{T_s} \left( \frac{1}{\tau_0} - \frac{1}{\tau} \right) \]
Measuring the Correlations

**correlation function**

\[ r = \langle g_t(x_1)g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle \]

**observable:**

\[ C = \frac{1}{\langle N \rangle^2} \left( \sum \text{pairs } p_{ti}p_{tj} \right) - \langle p_t \rangle^2 \]

\[ = \frac{1}{\langle N \rangle^2} \int (r - r_{eq}) \, dp_1 \, dp_2 \]

\[ C \text{ in a rapidity interval } \Rightarrow \eta/Ts \]

– Gavin & Abdel-Aziz
Sharma et al., [STAR], preliminary


\[ \sigma_{central}^2 = \sigma_0^2 + 4 \frac{\eta}{T_s} \left( \frac{1}{\tau_0} - \frac{1}{\tau_f} \right) \]

\[ \sigma_{central} = 1.0 \pm 0.2 \]

most peripheral \( \sim \) non-interacting
\[ \Rightarrow \sigma_{peripheral} \approx \sigma_0 = 0.54 \pm 0.02 \]

formation time \( \tau_0 \sim 1 \) fm
freeze out \( \tau_f \sim 20 \) fm
\( T \approx T_c \approx 170 \) MeV
\[ \Rightarrow \eta/s = 0.17 \pm 0.08 \]
2nd Order Viscous Diffusion

causal transport equation:
\[
\left( \tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\eta}{sT} \left( \nabla_1^2 + \nabla_2^2 \right) \right) \Delta r_g = 0
\]

linearized Israel-Stewart; see e.g. Song and Heinz 0805.1756

relaxation time \( \tau_\pi \sim \frac{\text{mean free path}}{\text{thermal speed}} \)
kinetic theory \( \Rightarrow \tau_\pi = \beta (\eta / sT) \) \( \beta \approx 5 \)

temperature vs time:

entropy production:
\[
\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T \tau}
\]

relaxation equation: causality delays heating; time scale \( \sim \tau_\pi \)
\[
\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi} \left( \Phi - \frac{4\eta}{3\tau} \right) - \left[ \frac{1}{\tau} + \frac{\eta T}{\tau_\pi} \frac{d}{d\tau} \left( \frac{\tau_\pi}{\eta T} \right) \right] \frac{\Phi}{2}
\]
Low Viscosity Only Near $T_c$?

**sQGP + hadronic corona** – Hirano & Gyulassy

supersymmetric Yang-Mills: $\eta/s = 1/4\pi$

pQCD and hadron gas: $\eta/s \sim 1$

**Evolution with varying viscosity**

- $\eta/s$ from H&G
- Bjorken flow + 2nd order hydro
- mixed phase $T \equiv T_c$
  
  $s = fs_Q + (1 - f)s_H$

- freeze out proper time
  
  $\tau_F - \tau_0 \propto (R - R_0)^2$; \hspace{1cm} $\tau_F (b = 0) = 10$ fm

  $\tau_0 = 0.6$ fm, \hspace{1cm} $T_F = 150$ MeV
2nd Order Diffusion with Realistic Viscosity

- Bjorken flow + 2nd order hydro
- mixed phase $T = T_c$

- freeze out proper time and temperature
  \[ \tau_F - \tau_0 \propto (R - R_0)^2; \quad \tau_F(b = 0) = 10 \text{ fm} \]
  \[ T_F = 150 \text{ MeV} \]

Find: higher initial $T_0 \Rightarrow$ smaller $\eta/s$ in central collisions
Broadening of $p_t$ Weighted Ridge

**wanted:** C integrated in $\phi$

**measured:** width of near side peak -- the **soft ridge**

Distribution – further effects
- momentum conservation $v_1$
- elliptic flow $v_2$
- triangularity $v_3$

**Ask:** is rapidity width of ridge modified by production mechanism and effect of **transverse flow** at freeze out?
Initial Broadening: Glasma?

Does initial rapidity width $\sigma_0$ vary with centrality?

Glasma explains ridge height and azimuthal dependence in STAR
Dumitru, Gelis, McLerran & Venugopalan; SG, McLerran & Moschelli

Glasma explains long range correlations in PHOBOS triggered ridge
Dusling, Gelis, Lappi, Venugopalan

but: rapidity dependence doesn't change with centrality in STAR acceptance $-1 < \eta < 1$

⇒ rapidity-independent pedestal

Do other production mechanisms vary with centrality?
strings, ropes, ladders → Pruneau’s talk
**Transverse Flow ⇒ Ridge**

**bulk correlations** – longitudinal string fragmentation

flux tube position $\vec{r}$

**transverse boost**
thermalization and flow

$\vec{v}_t \sim \lambda \vec{r}$

flow ⇒ narrow azimuthal opening angle

$\Delta \phi \sim v_{th} / v_t \sim (\lambda r)^{-1}$

**similar longitudinal narrowing**

$y = \tanh^{-1} \left( \frac{p_z}{E} \right)$

Voloshin; Pruneau, Gavin, Voloshin; Gavin, Moschelli, McLerran; Shuryak; Mocsy & Sorenson
Production and Freeze Out $\Rightarrow$ Broadening

**Freeze Out:** spatial rapidity $\eta_1, \eta_2 \rightarrow$ range of momentum space $y_1, y_2$

Cooper Frye:

$$C \propto \int_{\text{freezeout surface}} f(y_1 - \eta_1) f(y_2 - \eta_2) \Delta r(\eta_1, \eta_2)$$

Boltzmann $f$; blast wave $T$ and $v_r$ from Moschelli & S.G., arXiv:0806.4366

Find: flow effect on **rapidity width** comparable to $T$ dependence of $\eta/s$
Summary: viscosity from $p_t$ correlations

Viscosities from different observables

- viscosity broadens momentum correlations in rapidity
  - integrates over history $\rightarrow$ hadronic contribution
  - depends only on shear viscosity; no bulk contribution
- $v_2$ sensitive to early time
- combined info $\Rightarrow$ more complete picture

Causal transport explains STAR with realistic viscosity

Ridge measurement requires ridge tools

- Production: rapidity width must increase in STAR range
  - long range Glasma correlations don't work $\rightarrow$ Dusling et al.
  - other mechanisms might $\rightarrow$ see Pruneau's talk
- Freeze out: transverse flow effects at freeze out $\sim 10\%$ level
- beam energy scan + identified particles can distinguish contributions
Covariance ⇒ Momentum Flux

\[ C = \frac{1}{\langle N \rangle^2} \left( \sum_{\text{pairs } i \neq j} p_{ti} p_{tj} \right) - \langle p_t \rangle^2 \]

unrestricted sum:
\[ \sum_{\text{all } i,j} p_{ti} p_{tj} = \int p_{t1} p_{t2} dn_1 dn_2 \]

\[ dn = f(x,p) dp dx \]
\[ g_t(x) = \int dp \, p \, \Delta f(x,p) \]

\[ = \int dx_1 dx_2 \left( \int dp_1 \, p_{t1} f_1 \right) \left( \int dp_2 \, p_{t2} f_2 \right) \]
\[ = \langle N \rangle^2 \langle p_t \rangle^2 + \int g(x_1) g(x_2) dx_1 \, dx_2 \]

correlation function:
\[ r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle \]

\[ \int r_g \, dx_1 dx_2 = \left( \sum p_{ti} p_{tj} \right) - \langle N \rangle^2 \langle p_t \rangle^2 = \langle \sum p_{ti}^2 \rangle + \langle N \rangle^2 C \]

\[ C = 0 \text{ in equilibrium } \Rightarrow \quad C = \frac{1}{\langle N \rangle^2} \int (r_g - r_{g,eq}) dx_1 dx_2 \]
Causal Crosses Classic

classic (1st order) > causal (2nd order) for constant coefficients

\[
\left( \tau_\pi \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \frac{\eta}{sT} \left( \nabla_1^2 + \nabla_2^2 \right) \right) \Delta r_g = 0
\]

\[
\tau_\pi \sim (\sigma_{\text{mom}} n)^{-1}
\]

\[
n = N / V \propto t^{-1}
\]

\[
\Rightarrow \tau_\pi \propto \eta/sT \propto \tau \Rightarrow
\]

- wave-like expansion as \( \tau \rightarrow \infty \)
- 2nd order dominates