“Local Strong Parity Violation” Workshop,
RHIC/AGS Users Meeting, June 7, 2010

The Chiral Magnetic Effect and Local Strong Parity Violation

D. Kharzeev

BNL

Based on:

DK, hep-ph/0406125 (PLB)

DK, A. Zhitnitsky, arXiv: 0706.1026 (NPA)

DK, L. McLerran, H. Warringa, arXiv:0711.0950 (NPA)

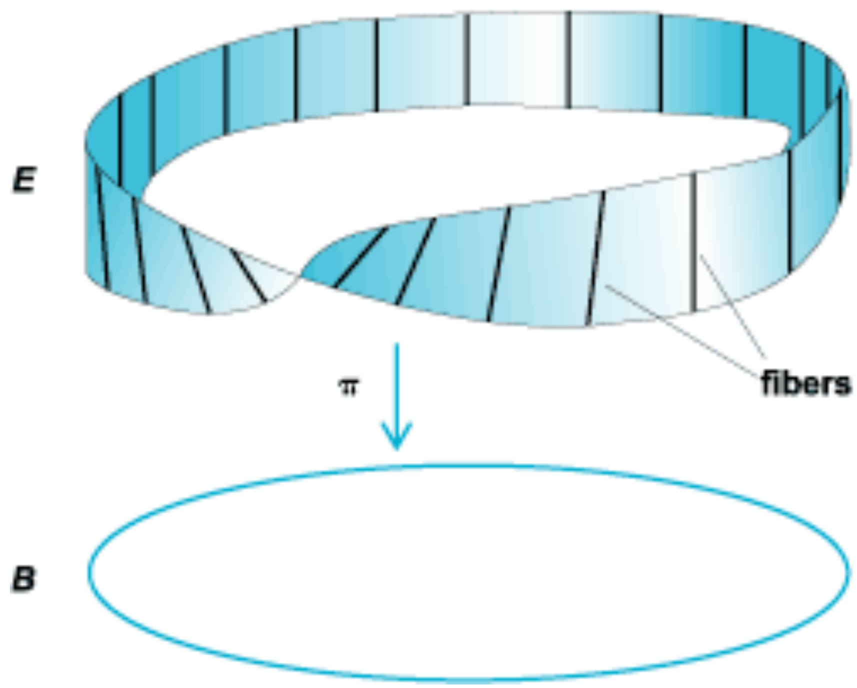
K. Fukushima, DK, H. Warringa, arXiv: 0808.3382 (PRD); 0912.2961 (NPA); 1002.2495 (PRL)

DK, H. Warringa, arXiv: 0907.5007 (PRD)

DK, arXiv: 0911.3715 (Ann. Phys.)

G. Basar, G. Dunne, DK, arXiv: 1003.3464 (PRL)

Gauge fields and topology



Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories "live" in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Topology and Chern-Simons forms

6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω. For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2)\,\mathrm{tr}\,AB$. Calculating from (3.5) shows

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\}\,. \qquad (6.1)$$

What does it mean for a gauge theory?

Chern-Simons theory

CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} \,. \tag{6.1}$$

What does it mean for a gauge theory?

Geometry ⟷ Physics

Riemannian connection ⟷ Gauge field

Curvature tensor ⟷ Field strength tensor

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

Abelian non-Abelian

Chern-Simons theory

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

Remarkable novel properties:

- gauge invariant, up to a boundary term
- topological - does not depend on the metric, knows only about the topology of space-time **M**
- when added to Maxwell action, induces a mass for the gauge boson - different from the Higgs mechanism!
- **breaks Parity invariance**

Chern-Simons theory and the vacuum of Quantum Chromodynamics

Equation:

$$D^{\mu} F^{a}_{\mu\nu} = 0$$

Belavin, Polyakov, Tyupkin, Schwartz;
tunneling events:
't Hooft; Gribov;....

Solution:

$$A^{a}_{\mu}(x) = \frac{2\eta_{a\mu\nu}x_{\nu}}{x^2 + \rho^2}.$$

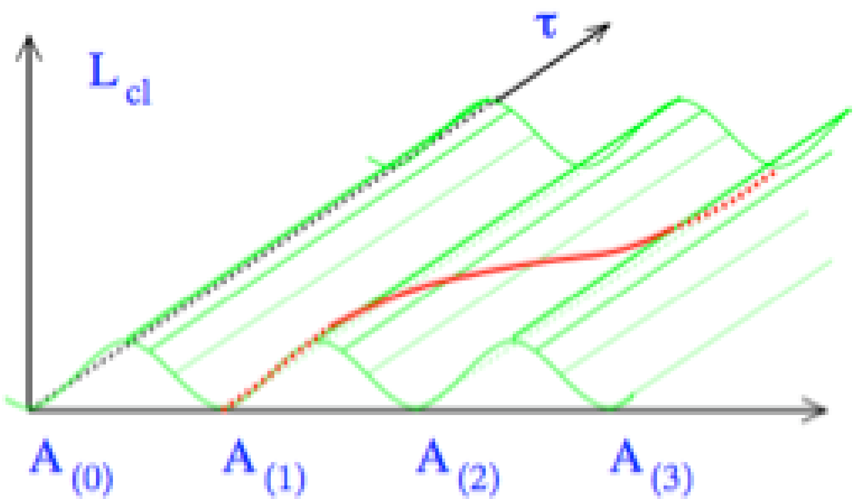
Coupling of space-time and color:

Integer $\quad Q = \int d\sigma_{\mu} K_{\mu}$

$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu, \nu = 1, 2, 3, \\ \delta_{a\mu} & \nu = 4, \\ -\delta_{a\nu} & \mu = 4. \end{cases}$$

$$K_{\mu} \;=\; \frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^{a}_{\alpha}\partial_{\beta}A^{a}_{\gamma} + \frac{1}{3}f^{abc}A^{a}_{\alpha}A^{b}_{\beta}A^{c}_{\gamma}\right)$$ Chern-Simons current

QCD vacuum as a Bloch crystal



"θ - vacuum"

$$|\theta\rangle = \sum_q e^{i\theta q}|q\rangle$$

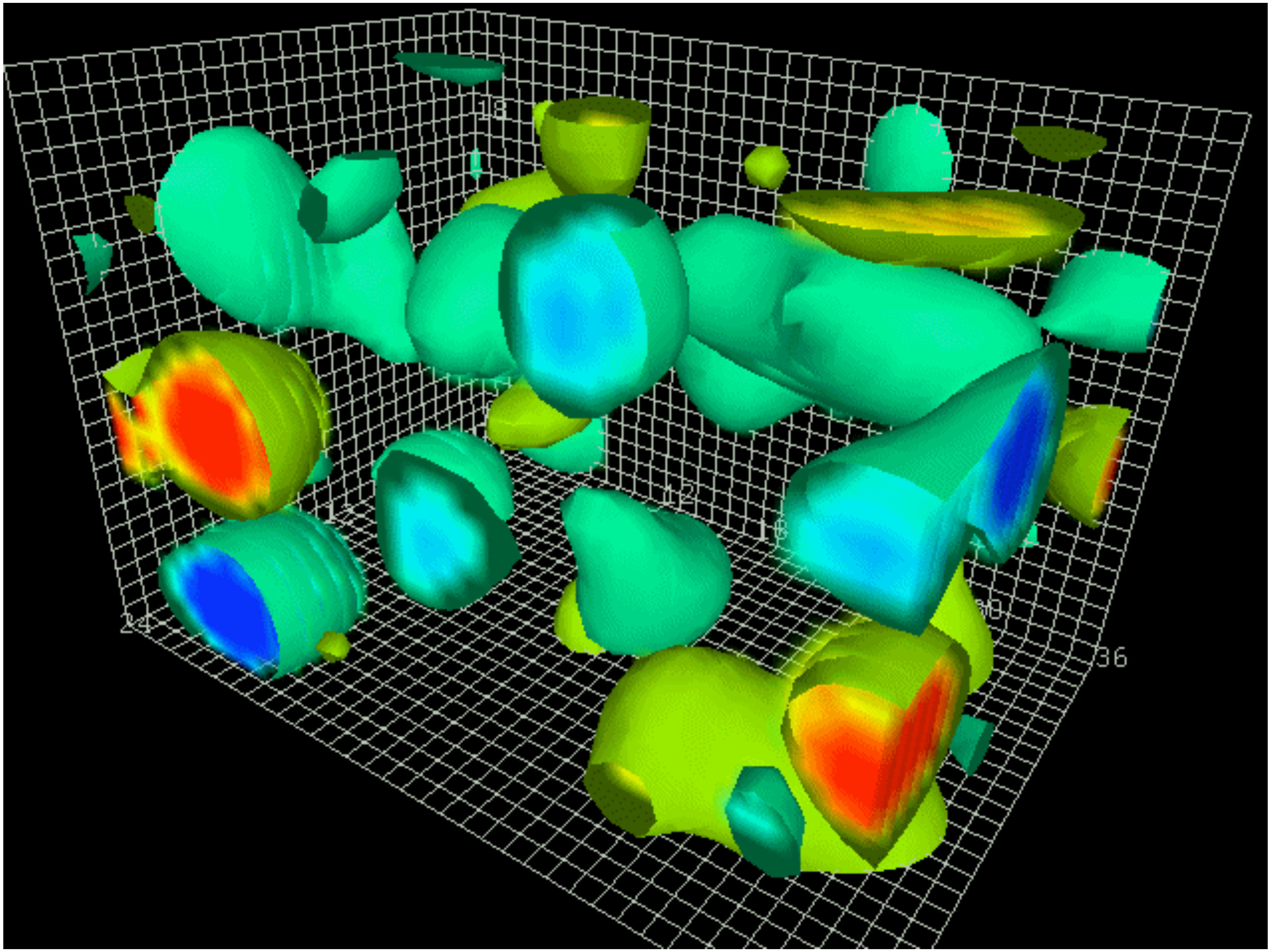
$$\langle \mathcal{O} \rangle = \sum_q f(q) \int_q D[\psi] D[\bar{\psi}] D[A] \exp(i S_{QCD}) \mathcal{O}(\psi, \bar{\psi}, A)$$

$$f(q_1 + q_2) = f(q_1) f(q_2) \longrightarrow f(q) = \exp(i\theta q)$$

85

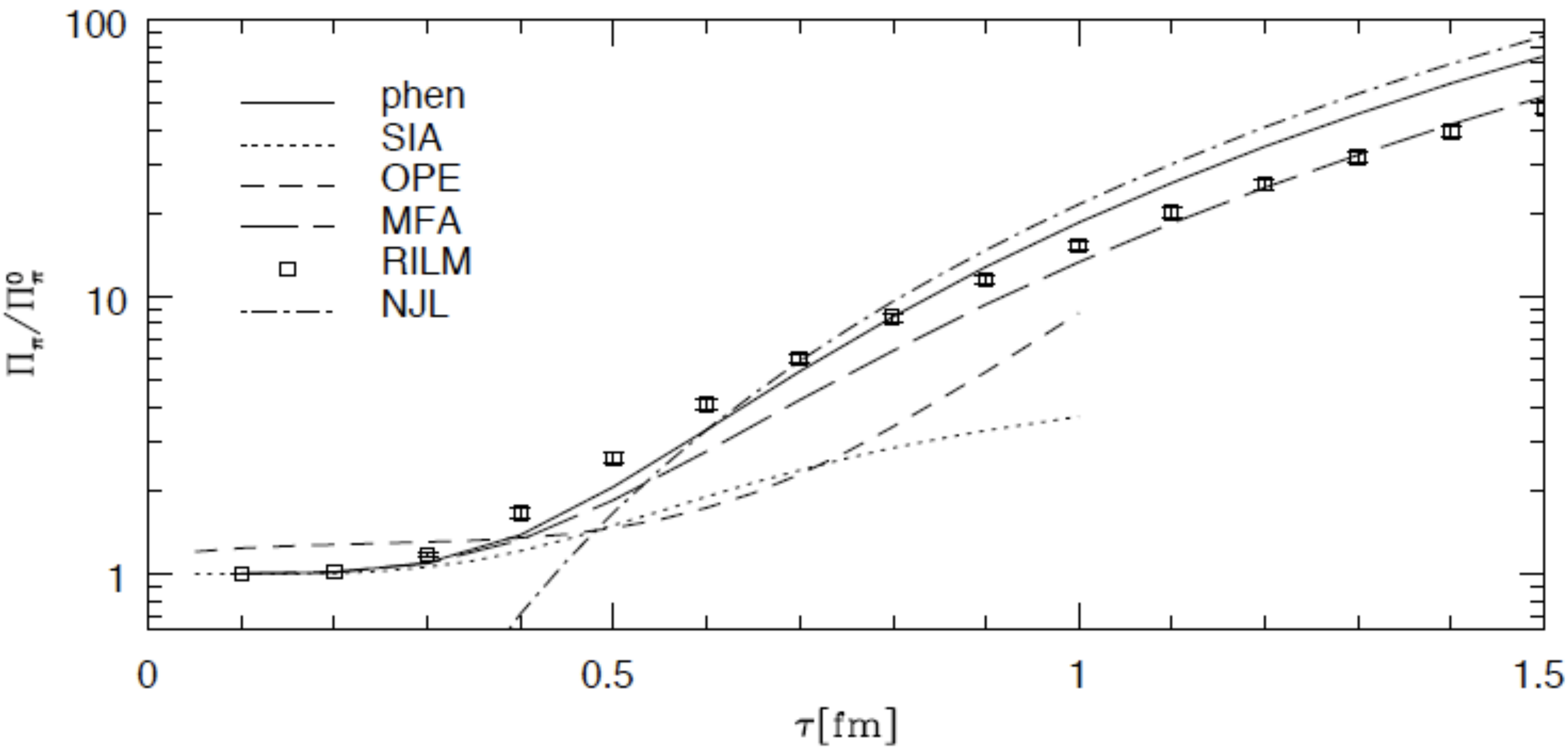
"quasi-momentum" "coordinate"

Topological number fluctuations in QCD vacuum



D. Leinweber

Extensive role of topological effects in the properties of hadrons

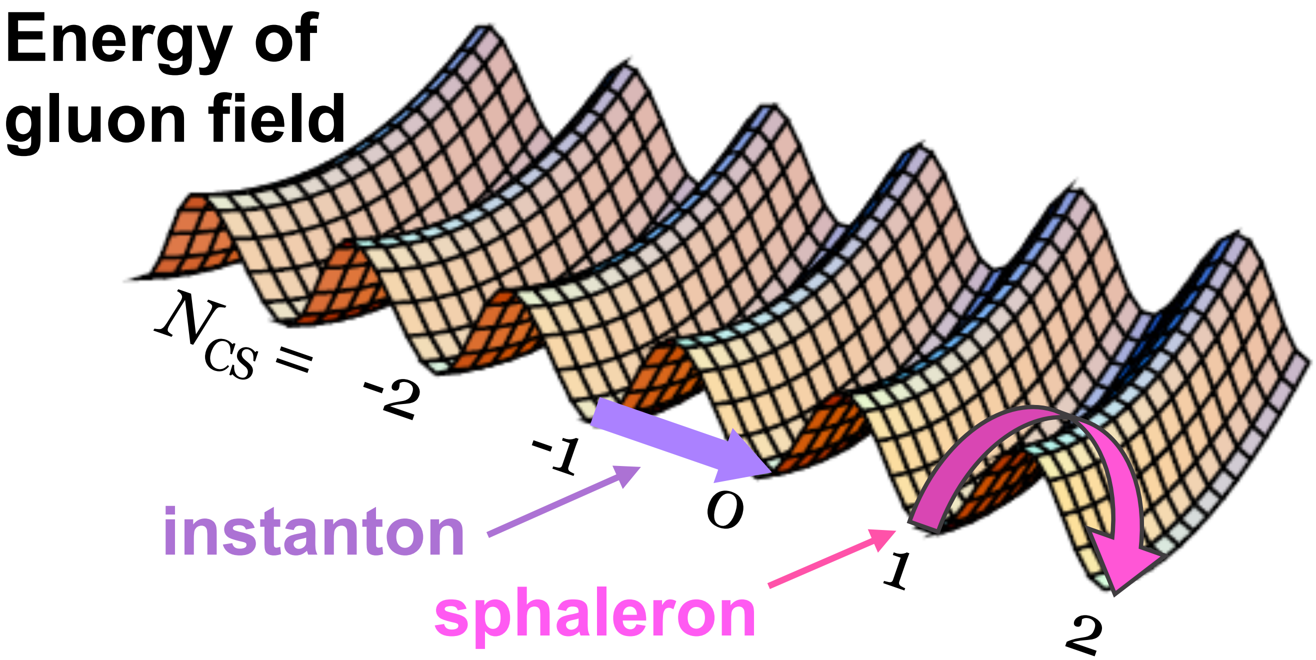


Instantons solve the η' puzzle,
explain why the pion is so light, etc

T. Schafer and E.Shuryak,
Rev. Mod. Phys. 70 (1998) 323

Sphaleron transitions at finite energy or temperature

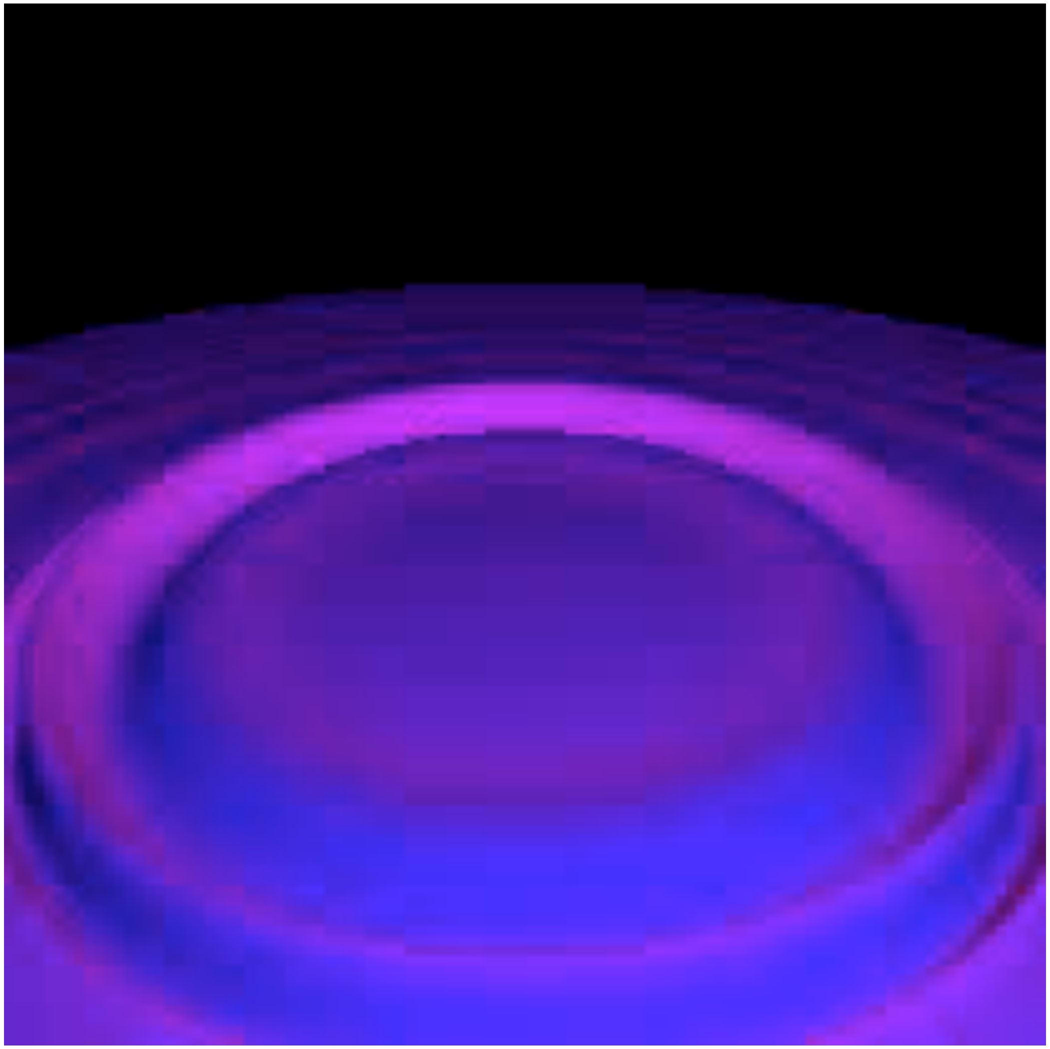
$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int_0^t \langle (q(x)q(0) + q(0)q(x)) \rangle d^4x$$



Sphalerons:
random walk of
topological charge at finite T:

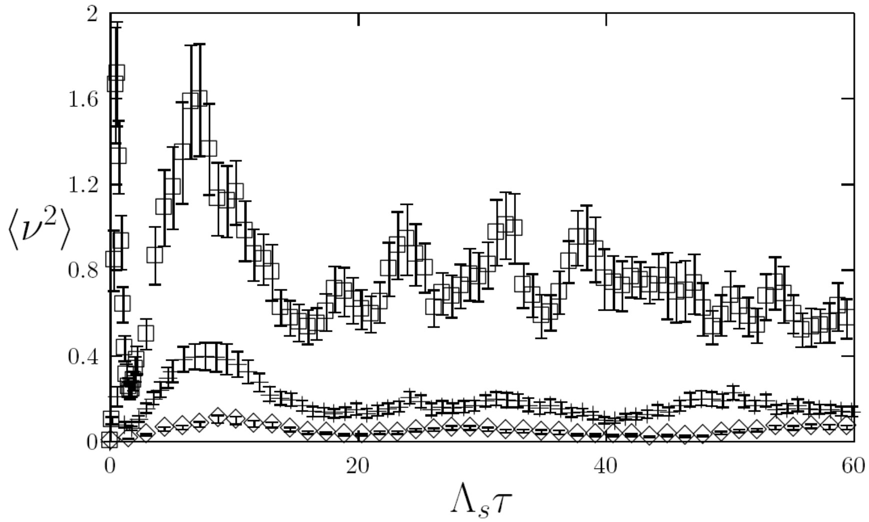
$$\langle Q^2 \rangle = 2\Gamma V t, \quad t \to \infty$$

Sphaleron transitions at finite energy or temperature

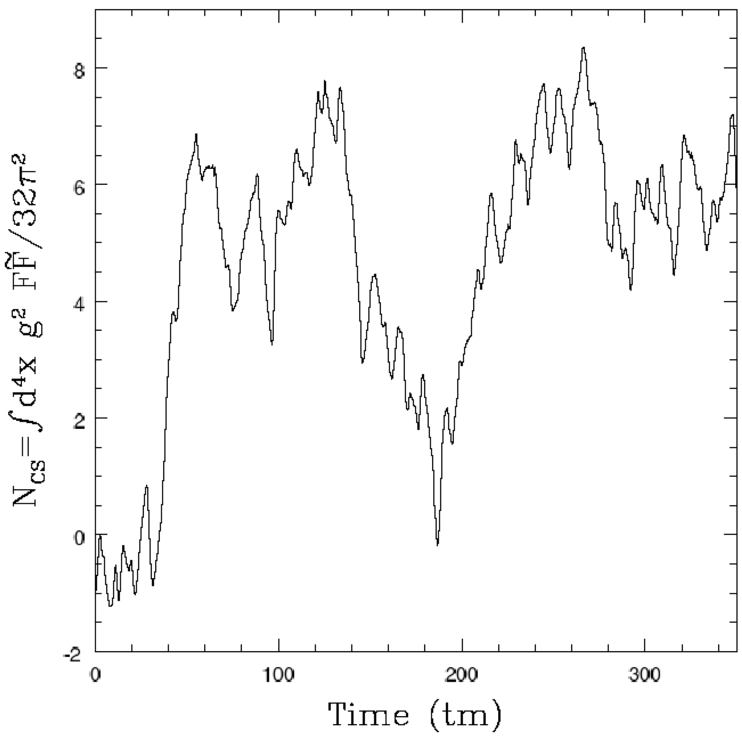


C. Rebbi, http://scv.bu.edu/visualization/gallery

Diffusion of Chern-Simons number in QCD: real time lattice simulations

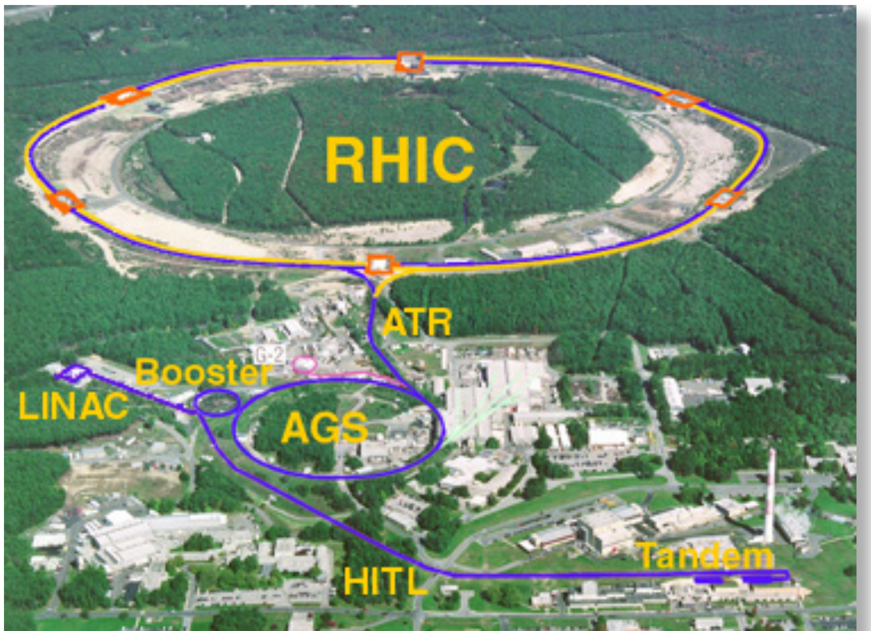


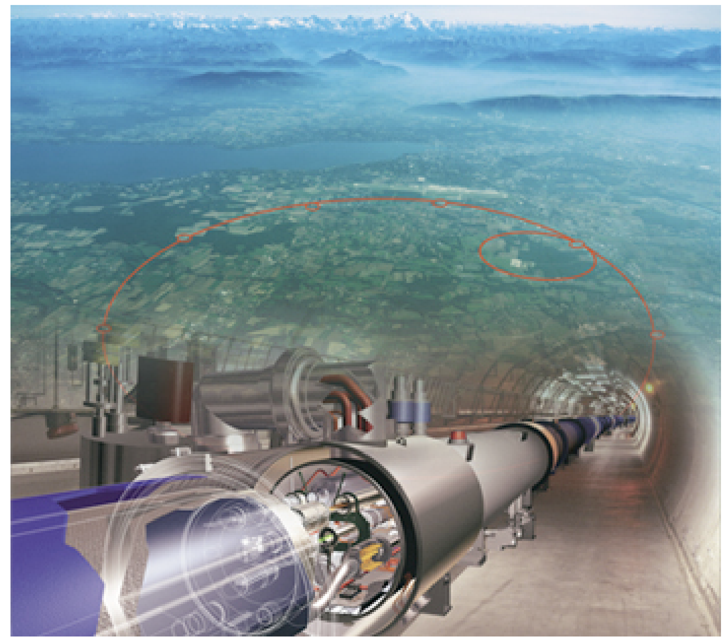
DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002



P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

Experimental test of Chern-Simons dynamics in hot QCD: Heavy ion collisions



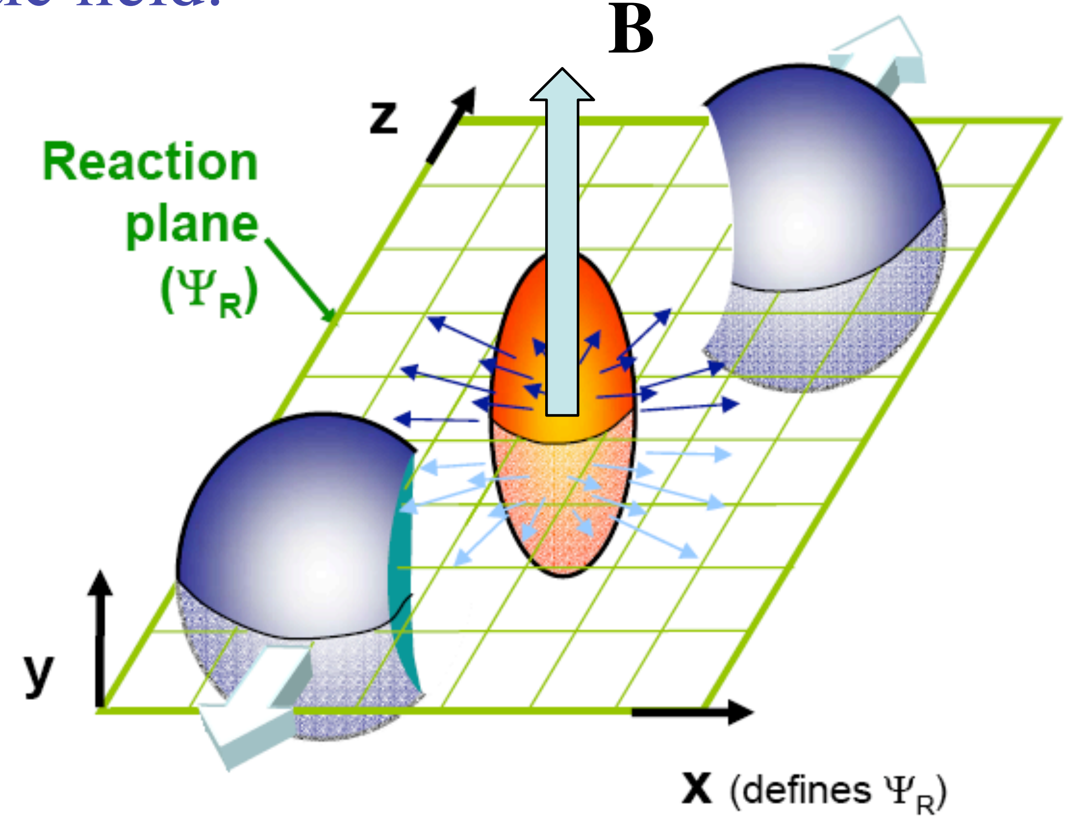


LHC

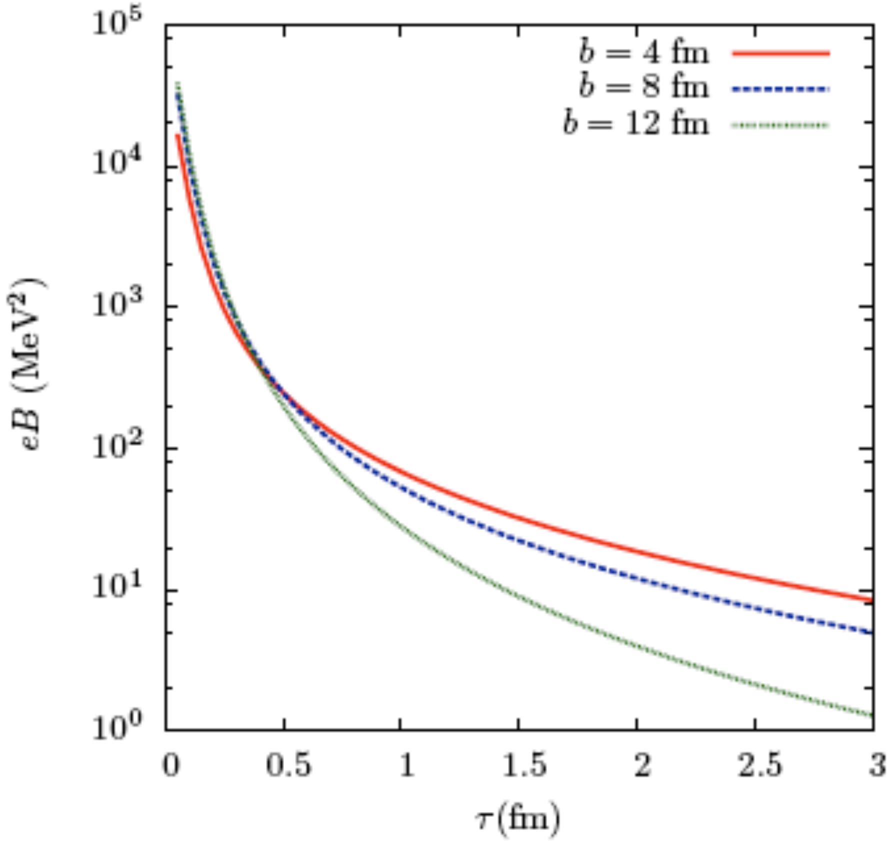
NICA,
JINR

Is there a way to observe topological charge fluctuations in experiment?

Relativistic ions create a strong magnetic field:



Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory



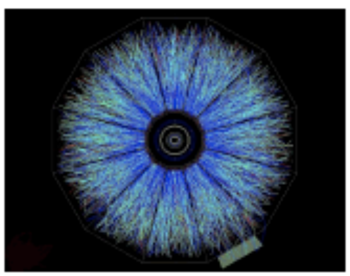
DK, McLerran, Warringa, Nucl Phys A803(2008)227

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

Comparison of magnetic fields

The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau = 0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

From QCD back to electrodynamics: Maxwell-Chern-Simons (axion) theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\, P_\mu J^\mu_{CS}$$

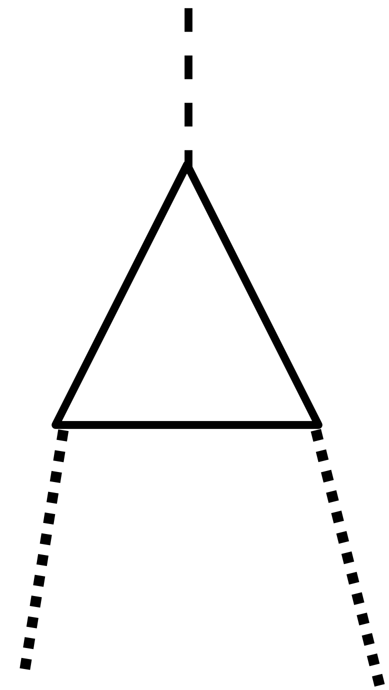
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu\theta = (\dot\theta, \vec{P})$$

$$\vec\nabla \times \vec B - \frac{\partial \vec E}{\partial t} = \vec J + c\left(\dot\theta \vec B - \vec P \times \vec E\right),$$

$$\vec\nabla \cdot \vec E = \rho + c\vec P \cdot \vec B,$$

$$\vec\nabla \times \vec E + \frac{\partial \vec B}{\partial t} = 0,$$

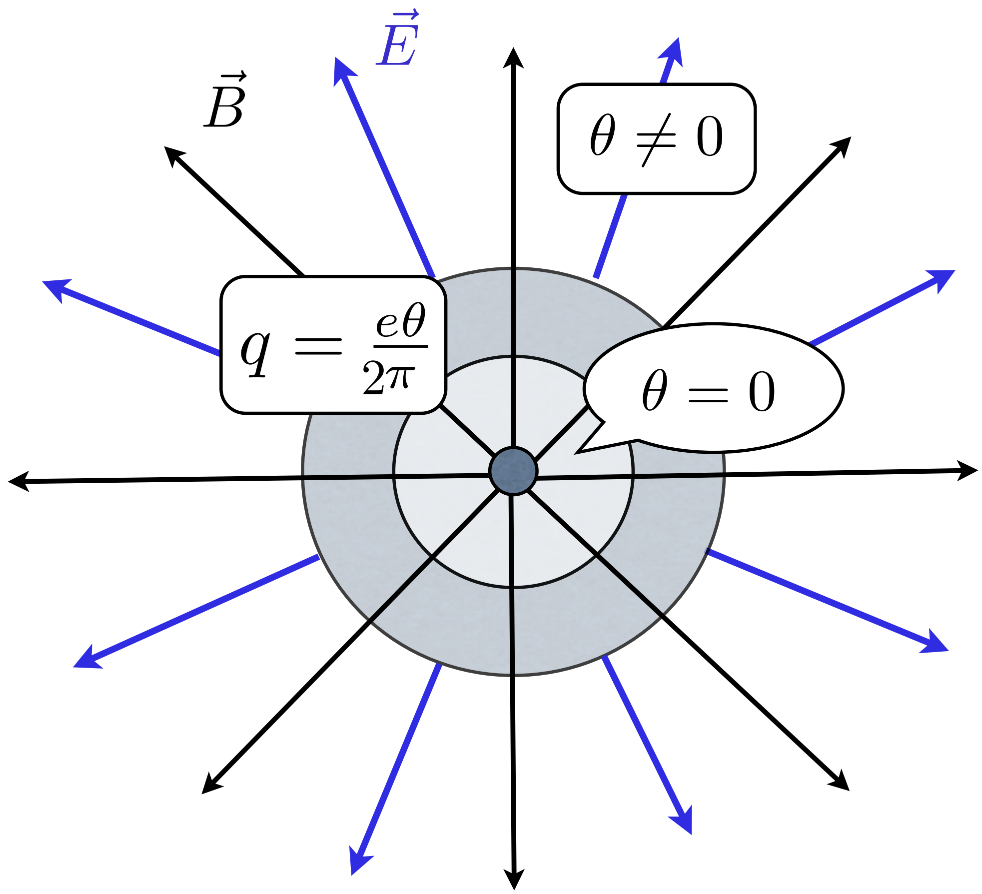
$$\vec\nabla \cdot \vec B = 0,$$



Magnetic monopole at finite θ : the Witten effect

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}$$

$$\vec{P} \equiv \vec{\nabla}\theta$$



E. Witten;

F. Wilczek

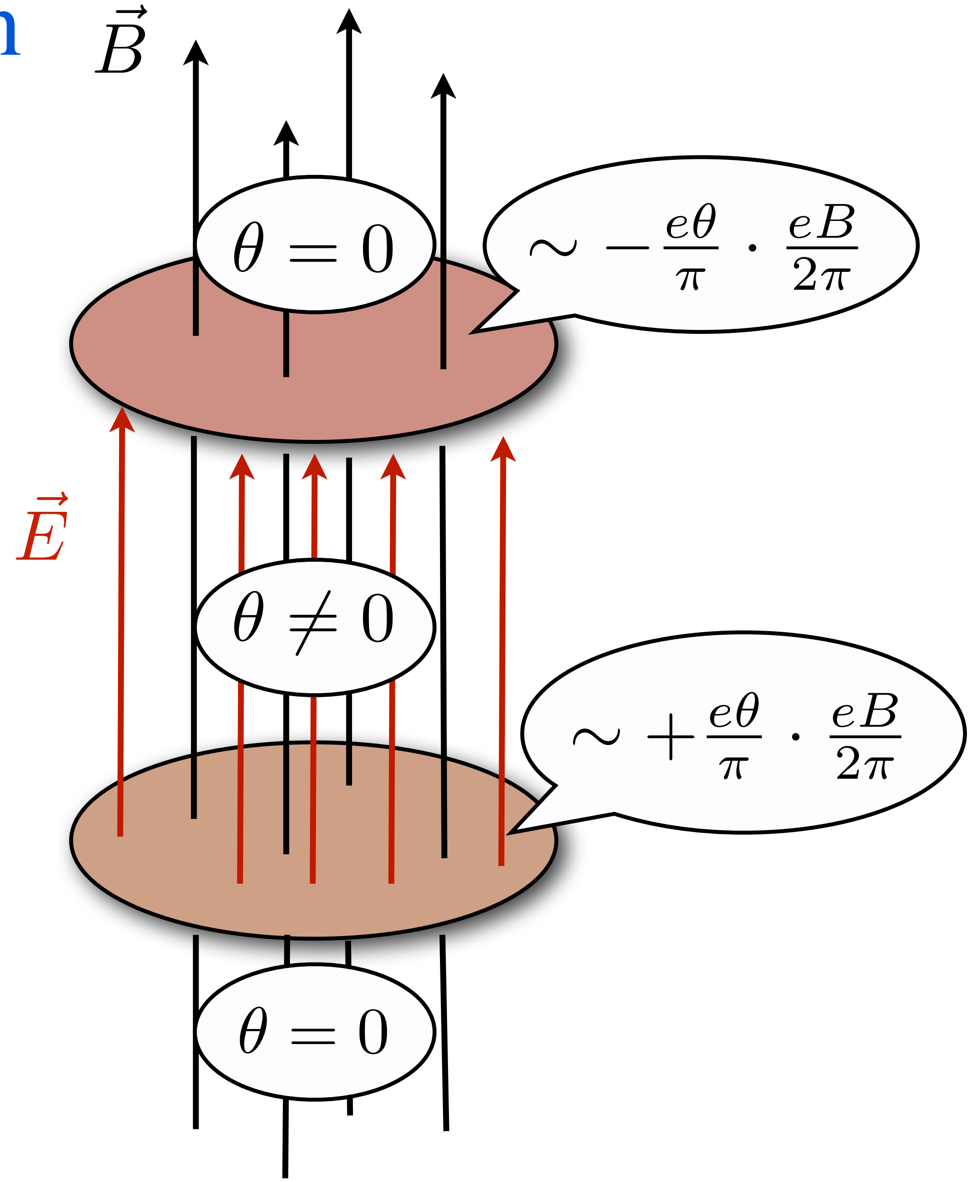
Induced electric charge: $q = c\,\theta\,g = \frac{e^2}{2\pi^2}\,\theta\,g = \frac{e}{2\pi^2}\,\theta\,(eg) = e\,\frac{\theta}{2\pi}$

The Chiral Magnetic Effect I:
Charge separation

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}$$

$$\vec{P} \equiv \vec{\nabla}\theta$$

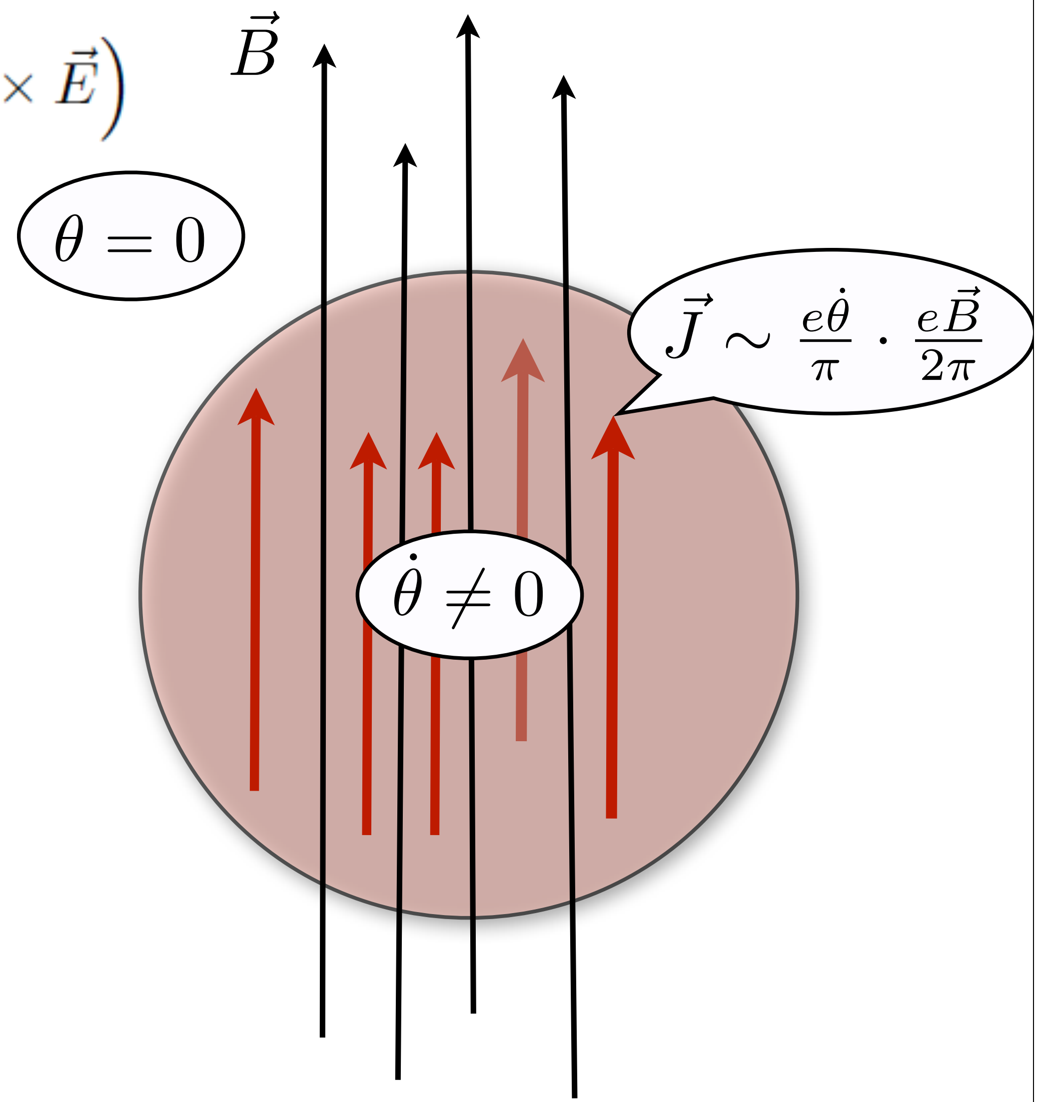
$$d_e = \sum_f q_f^2 \left(e \, \frac{\theta}{\pi} \right) \left(\frac{eB \cdot S}{2\pi} \right) L$$



DK '04;
DK, A. Zhitnitsky '06;
DK arXiv:0911.3715; Annals of Physics (2010)

The chiral magnetic effect II: chiral induction

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)$$



$$\vec{J} = -\frac{e^2}{2\pi^2}\dot{\theta}\vec{B}$$

DK, L. McLerran, H. Warringa '07;
K. Fukushima, DK, H. Warringa '08;
DK, H.Warringa arXiv:0907.5007

Computing the induced current

Fukushima, DK, Warringa, '08

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, vector e.m. current is not conserved:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

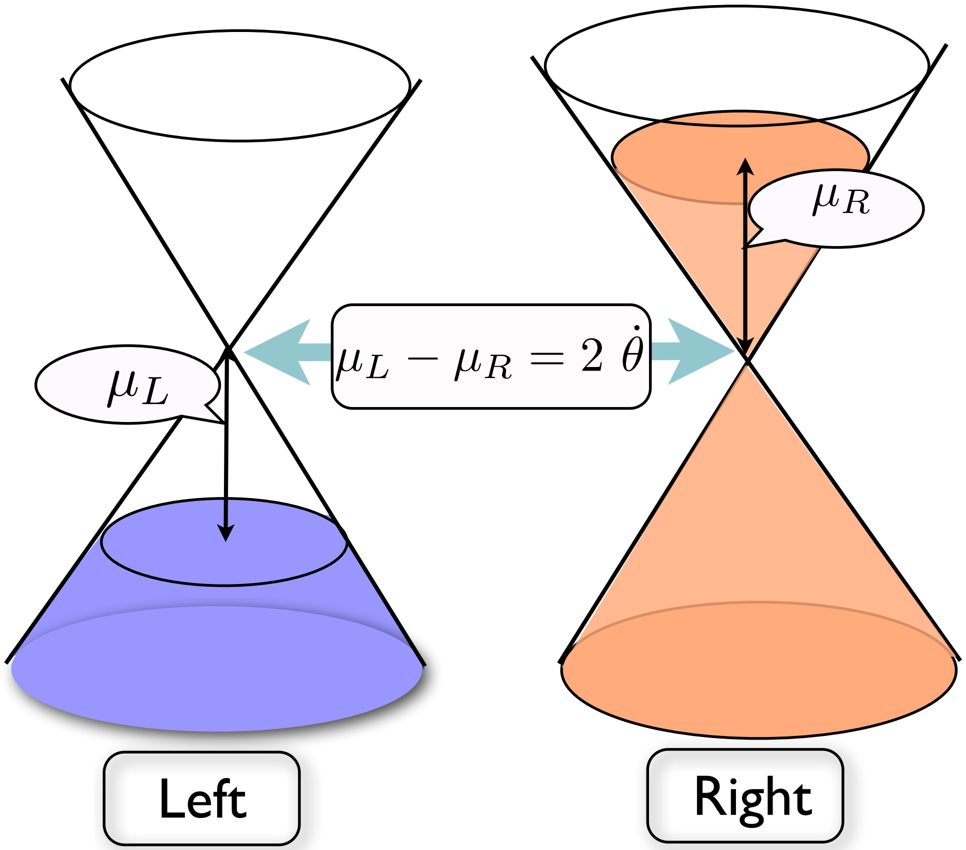
The result:

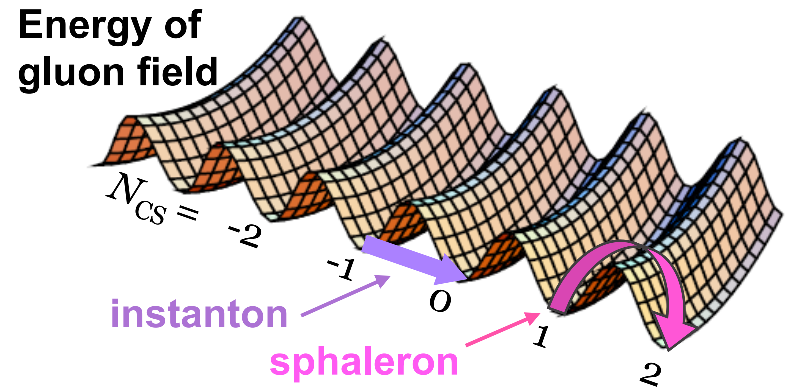
$$\vec{J} = \frac{e^2}{2\pi^2} \, \mu_5 \, \vec{B}$$

also: A.Alexeev, V.Cheianov, J.Froelich, '98; M.Giovannini, M.Shaposhnikov, '97

Coefficient is fixed by the axial anomaly, no corrections

What powers the CME current?



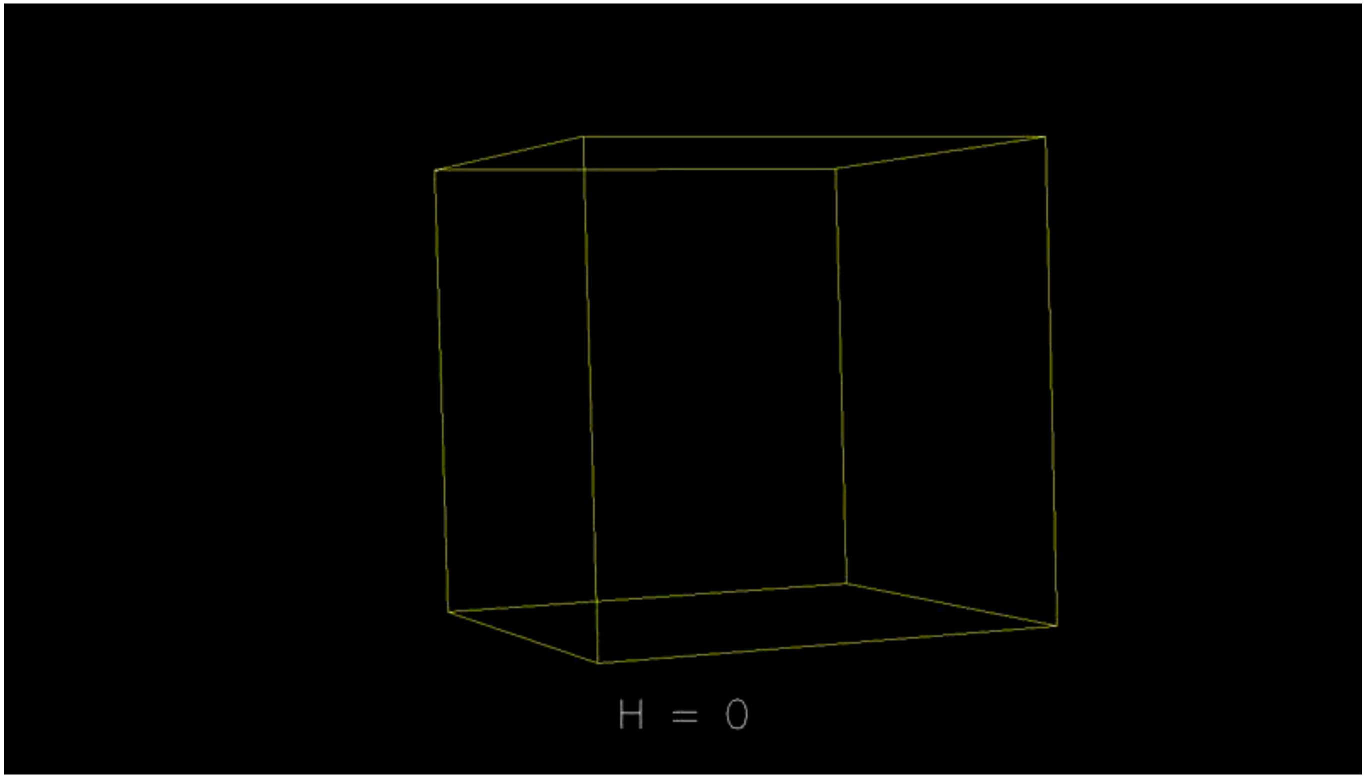


$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$P = \int d^3x\ \vec{J} \cdot \vec{E} = -\dot{\theta}\ \frac{e^2}{2\pi^2} \int d^3x\ \vec{E} \cdot \vec{B} = -\dot{\theta}\ \dot{Q}_5$$

“Numerical evidence for chiral magnetic effect in lattice gauge theory”,

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov, ArXiv 0907.0494; PRD’09



Red - positive charge
Blue - negative charge

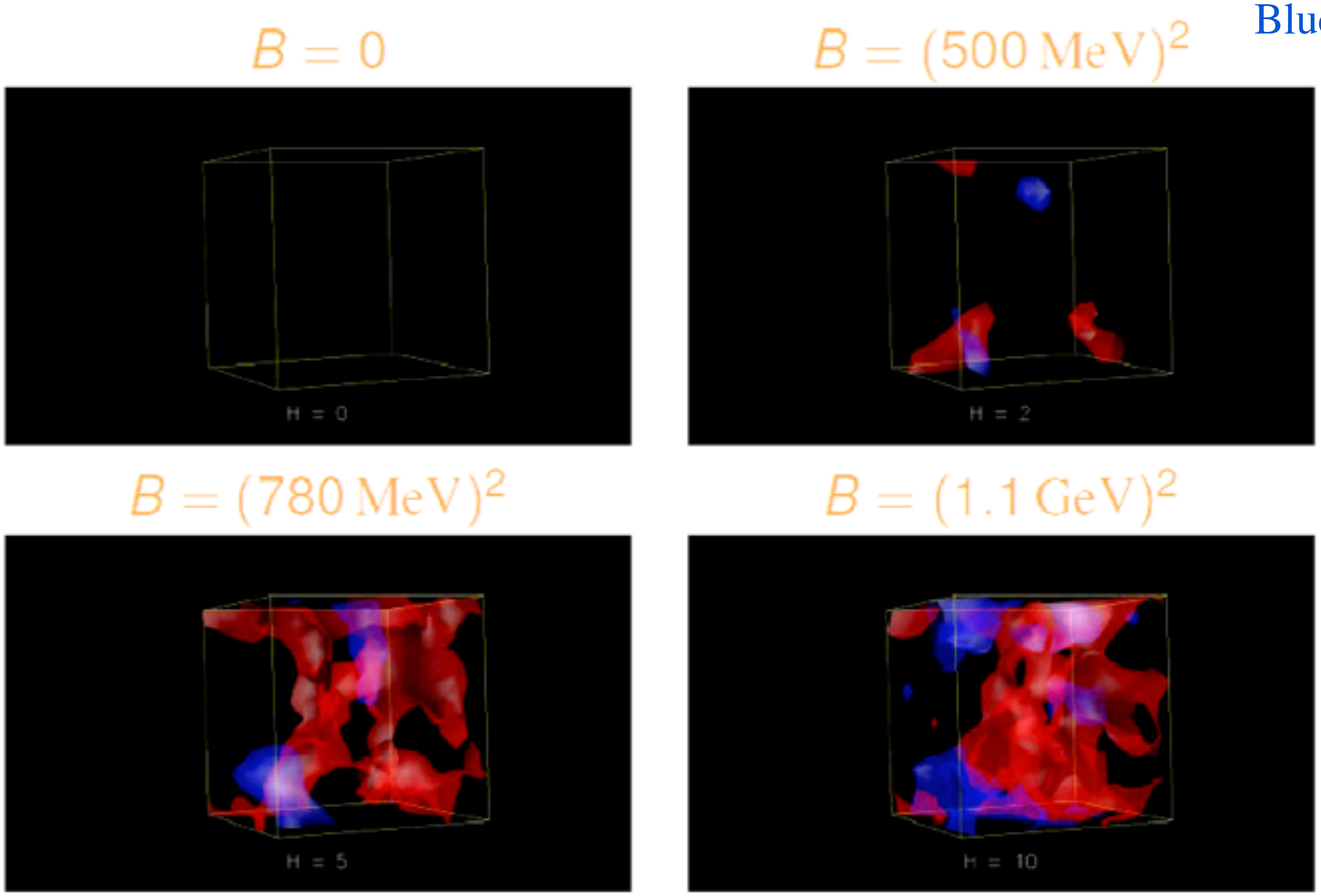
SU(2) quenched, Q = 3; Electric charge density (H) - Electric charge density (H=0)

“Numerical evidence for chiral magnetic effect in lattice gauge theory”,

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov, ArXiv 0907.0494; PRD’09

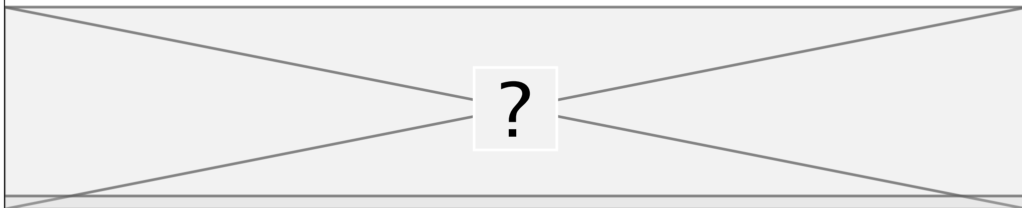
Density of the electric charge vs. magnetic field, 3D time slices

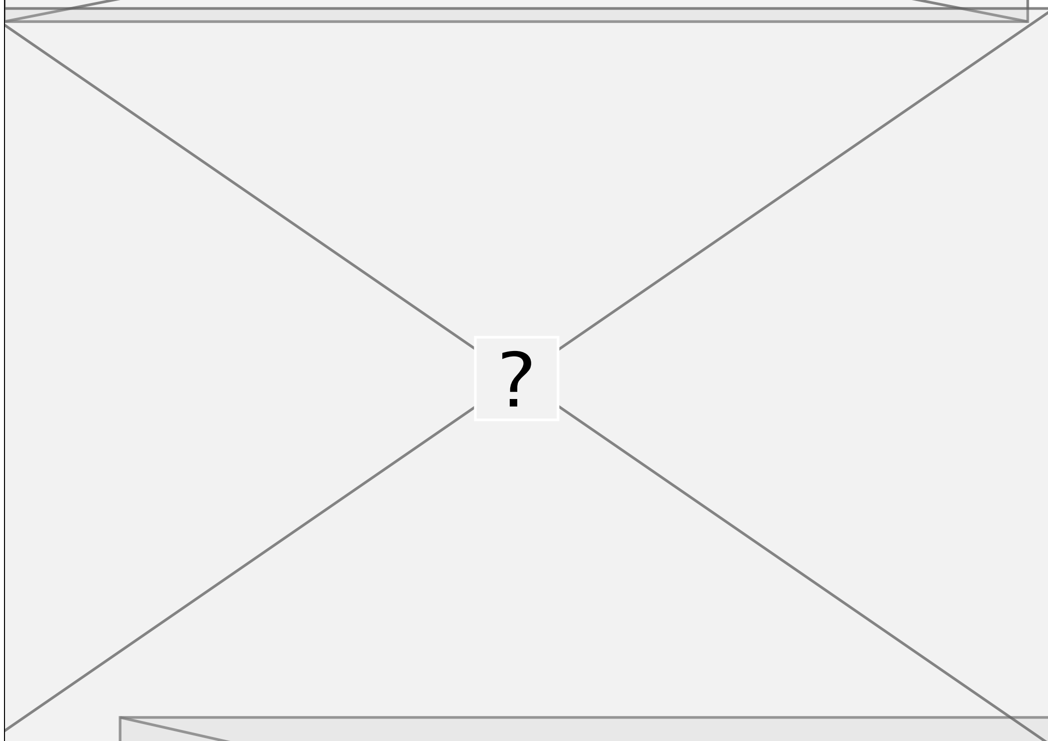
Red - positive charge
Blue - negative charge



note:
B has to be measured in units of the pion mass^2 !

Electric current susceptibility

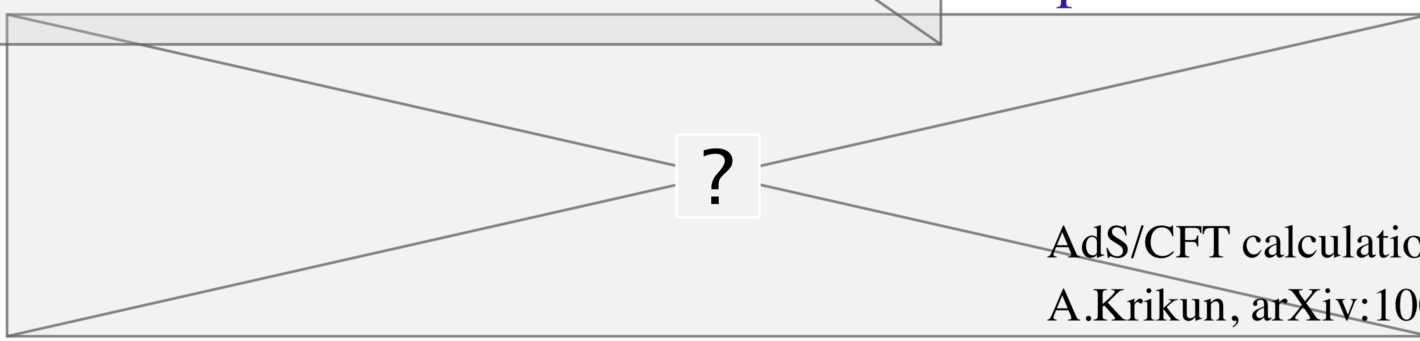




K.Fukushima, DK,
H. Warringa, arXiv:0912.2961

The fluctuations of electric current in magnetic background are anisotropic, the difference of susceptibilities is UV finite.

Lattice data are well reproduced theoretically.

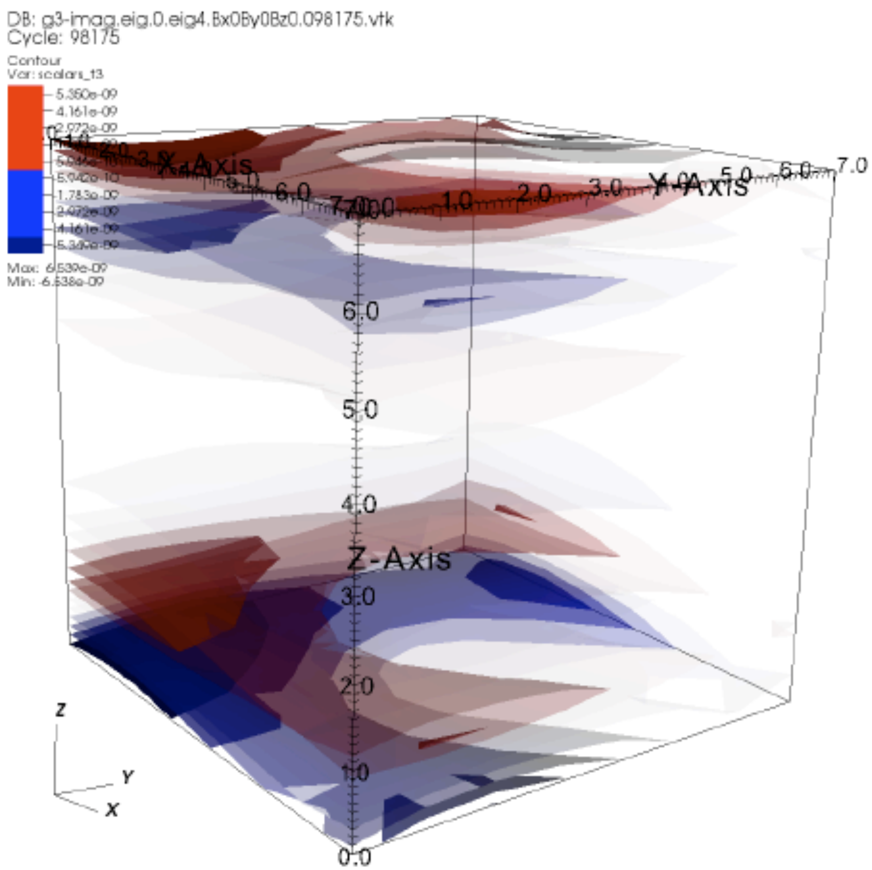


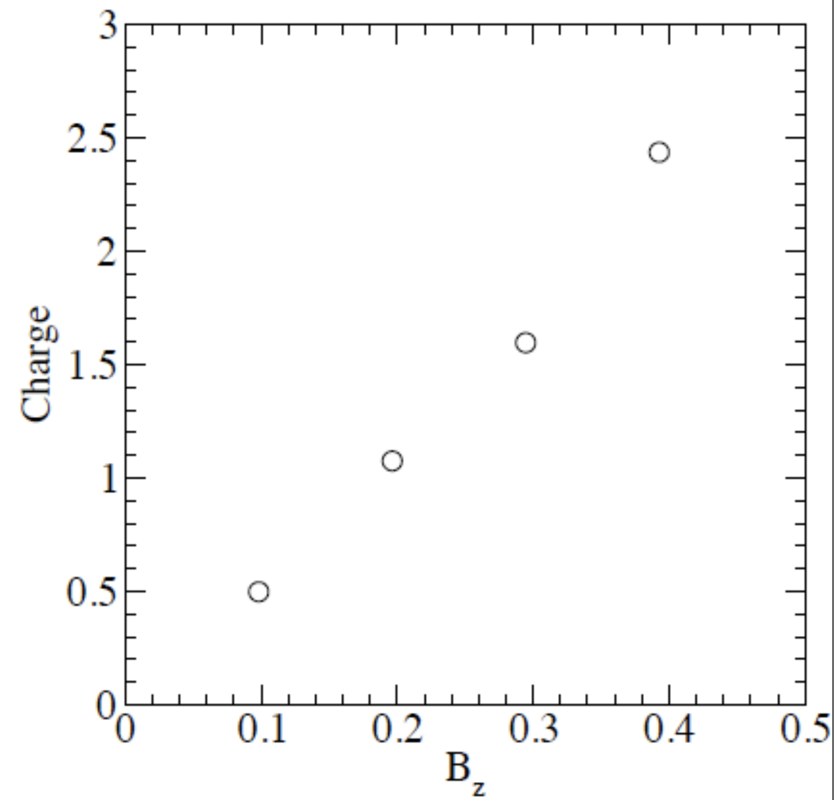
AdS/CFT calculation of susceptibility:
A.Krikun, arXiv:1003.1041

“Chiral magnetic effect in 2+1 flavor QCD+QED”,

M. Abramczyk, T. Blum, G. Petropoulos, R. Zhou, ArXiv 0911.1348;
Columbia-Bielefeld-RIKEN-BNL

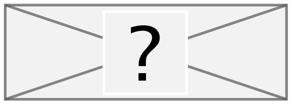
Red - positive charge
Blue - negative charge

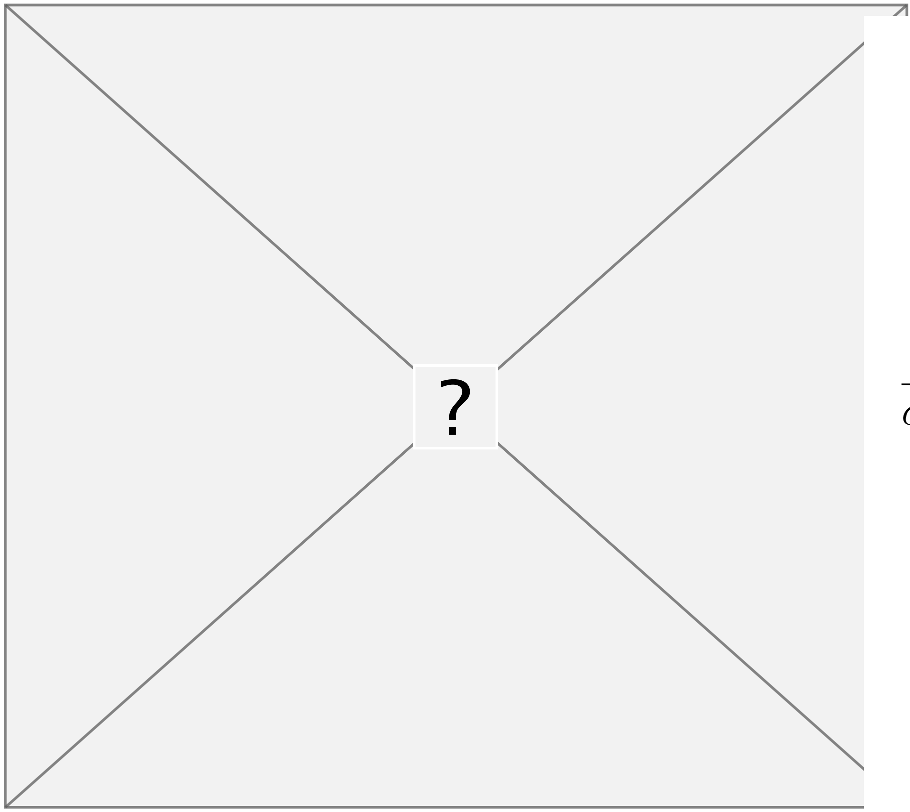




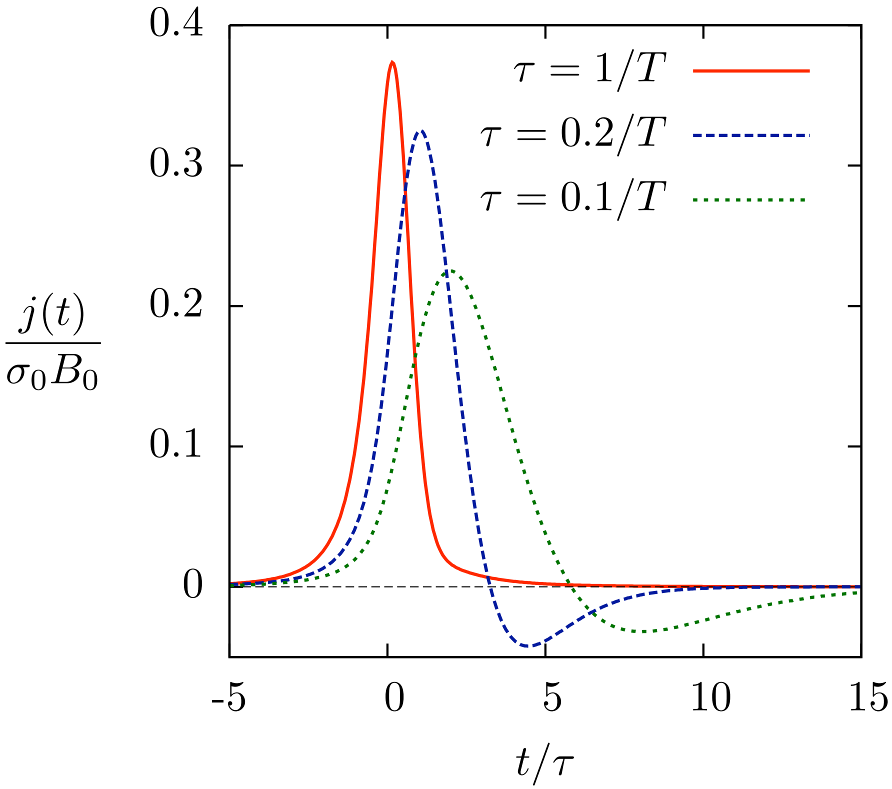
2+1 flavor Domain Wall Fermions, fixed topological sectors, 16^3 x 8 lattice

Chiral magnetic conductivity





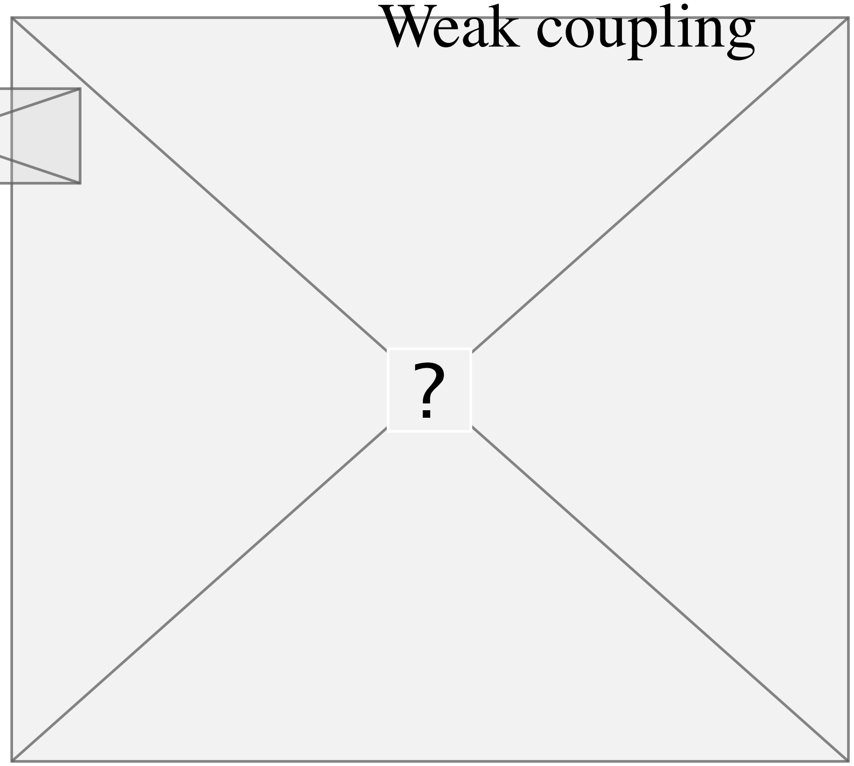
$$\sigma_\chi(\omega = 0, \boldsymbol{p} = 0) \equiv \sigma_0 = \frac{e^2}{2\pi^2}\mu_5$$



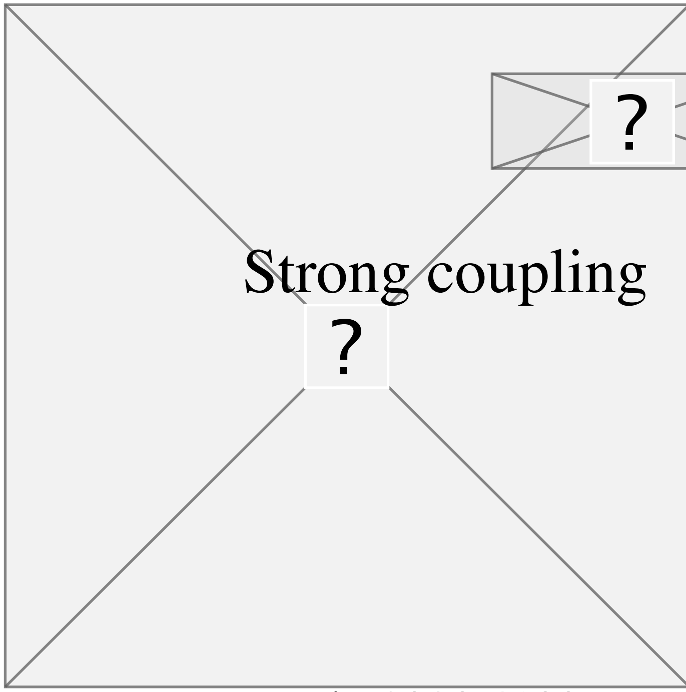
D.K., H. Warringa, Phys Rev D80 (2009) 034028

Holographic chiral magnetic effect: the strong coupling regime (AdS/CFT)

Weak coupling



Strong coupling



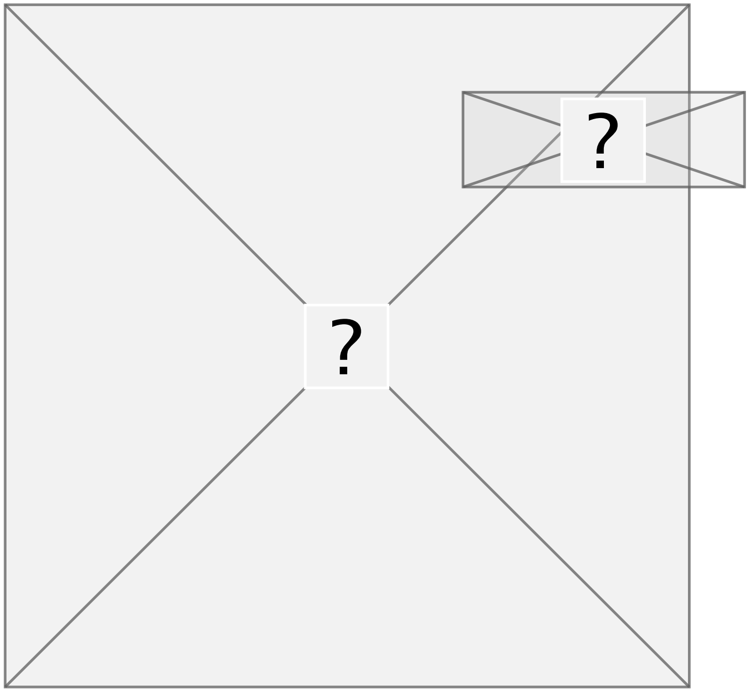
H.-U. Yee, arXiv:0908.4189, JHEP 0911:085, 2009

D.K., H. Warringa
Phys Rev D80 (2009) 034028

A. Rebhan et al, JHEP 0905, 084 (2009), G.Lifshytz, M.Lippert, arXiv:0904.4772;
E. D' Hoker and P. Krauss, arXiv:0911.4518; ...

also: Chiral separation, D. Son and P. Surowka, '09

Holographic CME:
is the current renormalized at strong coupling?



H.-U. Yee, arXiv:0908.4189,
JHEP 0911:085, 2009

H.-U.Yee: No

A. Rebhan et al: Yes (to zero)

Resolved very recently:
V.Rubakov, arXiv:1005.1888;
A. Gynther, K. Landsteiner, F. Pena-Benitez, A. Rebhan,
arXiv:1005.2587

CME current is the same at strong and weak coupling

What carries the current
at strong coupling?

CME in the chirally broken phase

G. Basar, G. Dunne, DK, arXiv: 1003.3464;
Phys.Rev.Lett., in press

"Chiral spiral" in (1+1) theories:

V. Schoen, M. Thies, hep-th/0008175

Gross-Neveu:

$$\mathcal{L} = \bar{q}\, \mathrm{i}\gamma^\mu \partial_\mu q + \frac{1}{2} g^2 \left[(\bar{q}q)^2 - \lambda \left(\bar{q}\gamma^5 q \right)^2 \right] - m_0 \bar{q} q$$

't Hooft:

$$\mathcal{L} = \bar{q}\, \mathrm{i} \not{D} q - \frac{1}{2} \mathrm{tr} F_{\mu\nu} F^{\mu\nu} \,, \qquad \not{D} = \gamma^\mu (\partial_\mu + \mathrm{i} g A_\mu)$$

because of constraints on Dirac matrices in 1+1, explicit form e.g.

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

there is an intricate connection between the vector (baryon) and chiral currents

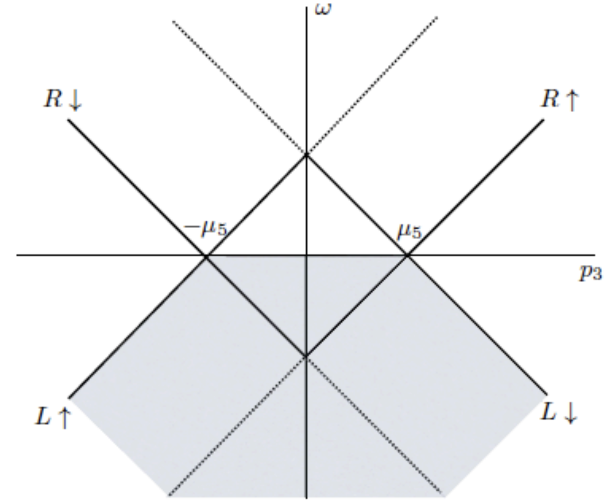
$$j_V^0 = j_A^1 \,, \quad j_V^1 = j_A^0$$

Baryon density - chiral current;
chiral density - vector current

Chiral magnetic spiral

G. Basar, G. Dunne, DK, arXiv: 1003.3464

Plane waves describing the pairing fermions acquire a phase difference due to the chemical potential - the spiral nature of condensates.



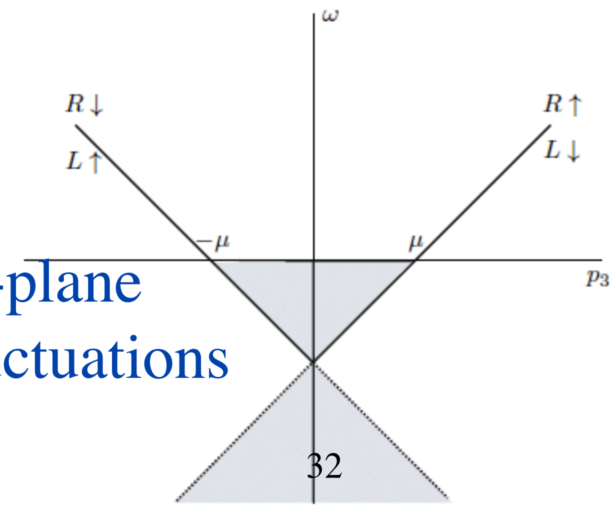
Gapless collective spiral excitation that carries a vector current (at finite chirality) or a chiral current (at finite baryon density).

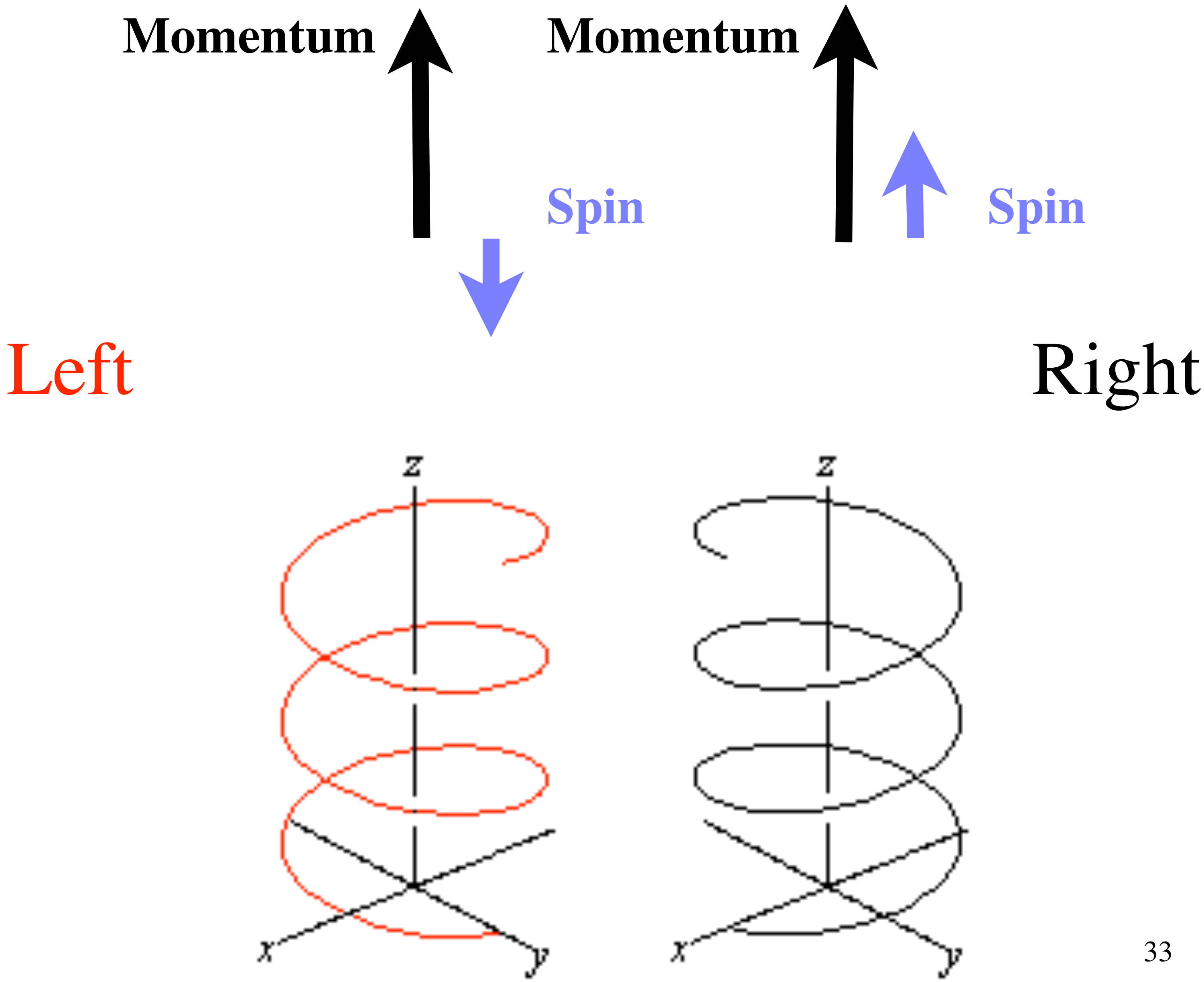
$$\langle J^3 \rangle = \frac{eB}{2\pi}\frac{e\mu_5}{\pi} \qquad \langle J^3_5 \rangle = \frac{eB}{2\pi}\frac{e\mu}{\pi}$$

4 =2x(1+1)

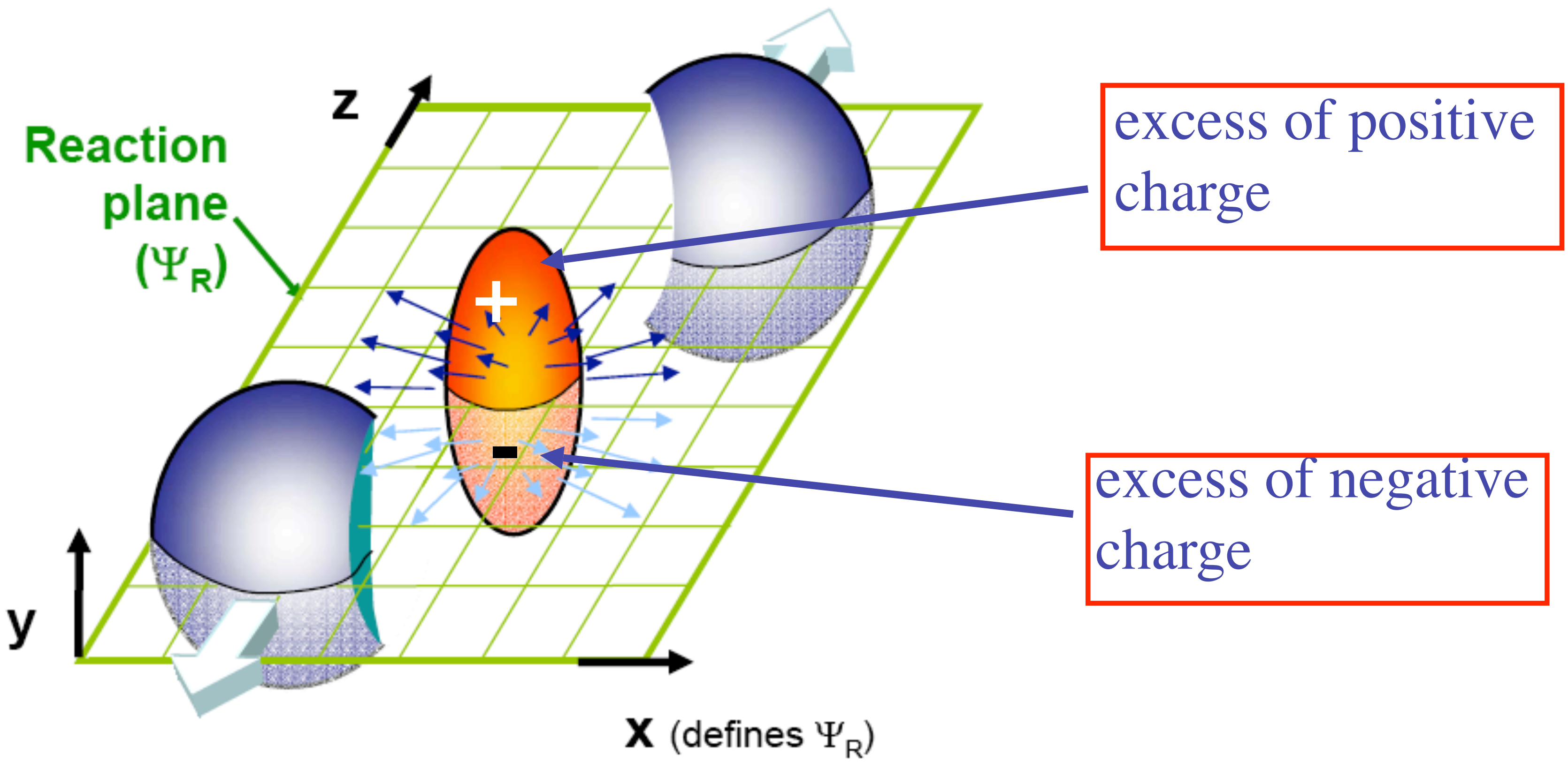
$$\begin{aligned}
\langle J^1 \rangle &= C^2 \cos(2\mu_5\, z - \phi_R) - D^2 \cos(2\mu_5\, z + \phi_L) \\
\langle J^2 \rangle &= -C^2 \sin(2\mu_5\, z - \phi_R) + D^2 \sin(2\mu_5\, z + \phi_L) \\
\langle J^1_5 \rangle &= C^2 \cos(2\mu_5\, z - \phi_R) + D^2 \cos(2\mu_5\, z + \phi_L) \\
\langle J^2_5 \rangle &= -C^2 \sin(2\mu_5\, z - \phi_R) - D^2 \sin(2\mu_5\, z + \phi_L)
\end{aligned}$$

in-plane fluctuations




Momentum
Momentum
Spin
Spin
Left
Right
z
z
x
y
x
y

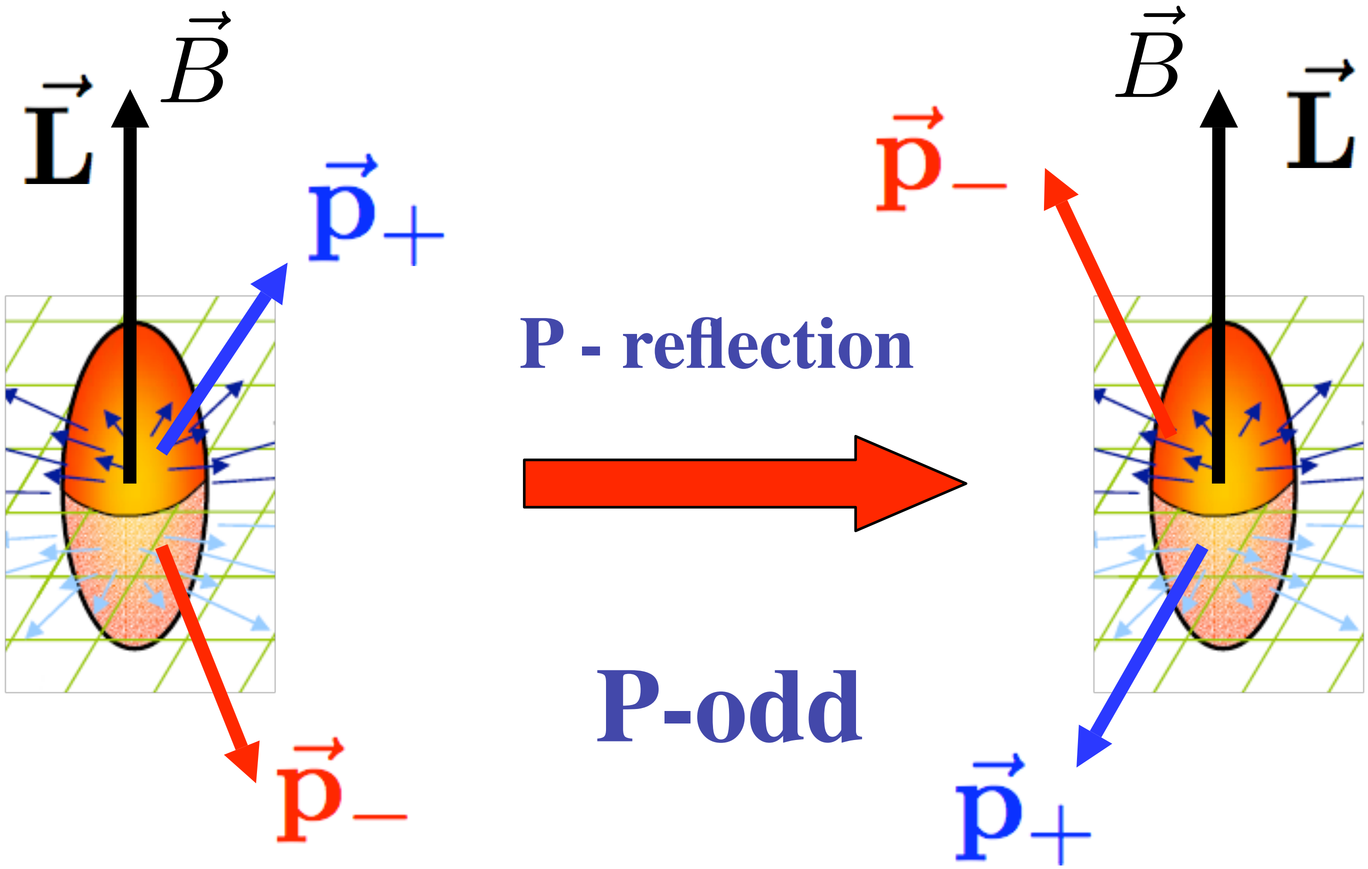
Charge asymmetry w.r.t. reaction plane as a signature of strong P violation



Electric dipole moment of QCD matter!

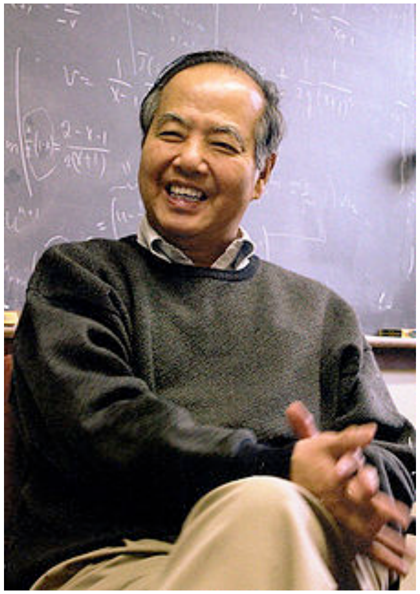
DK, Phys.Lett.B633(2006)260 [hep-ph/0406125]

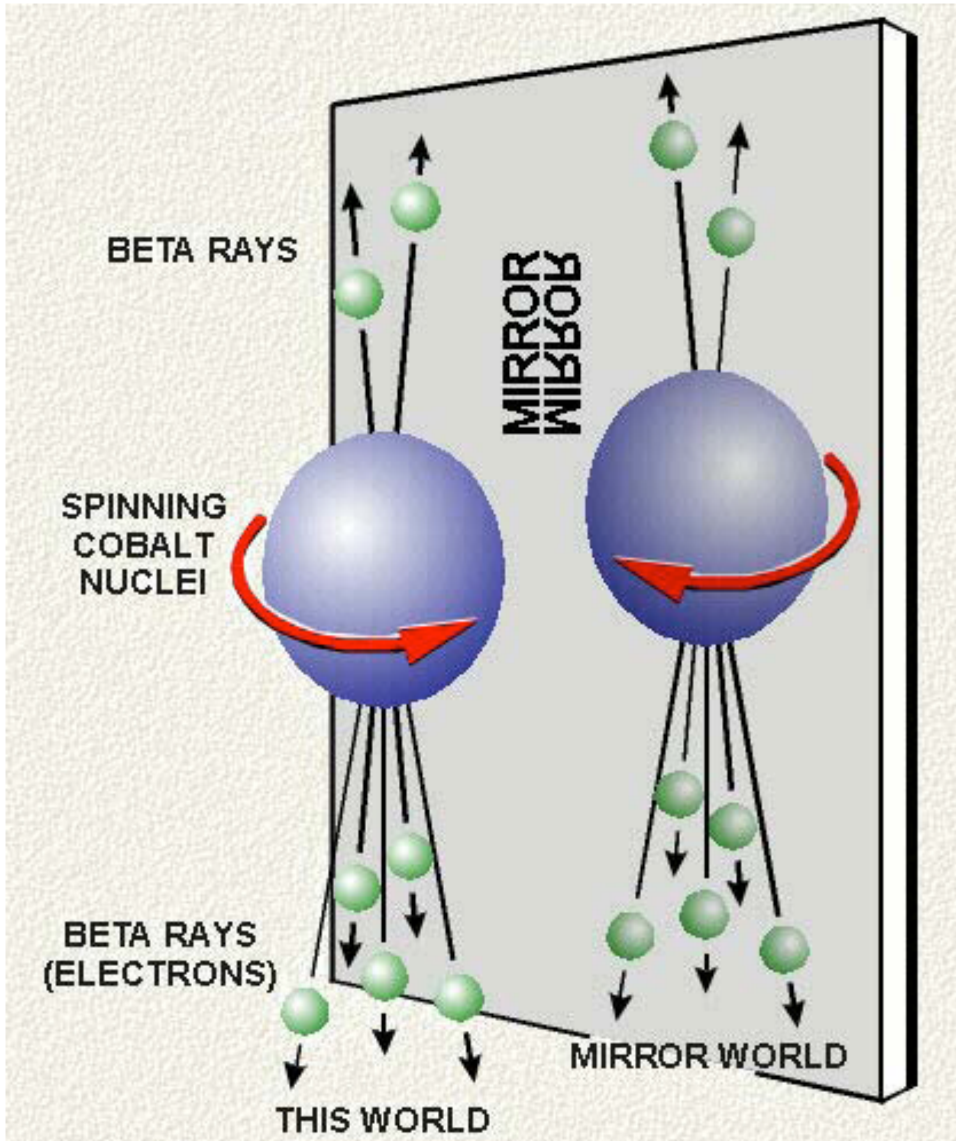
Charge separation = parity violation:



$$\mathcal{P}: \quad \vec{p} \longrightarrow -\vec{p}; \quad \vec{B} \longrightarrow \vec{B}; \quad \vec{L} \longrightarrow \vec{L}$$

Analogy to P violation in weak interactions



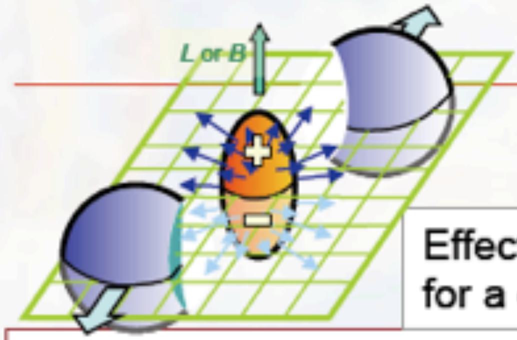


C.S. Wu, 1912-1997

BUT:
the sign of
the asymmetry
fluctuates
event by event

Observable

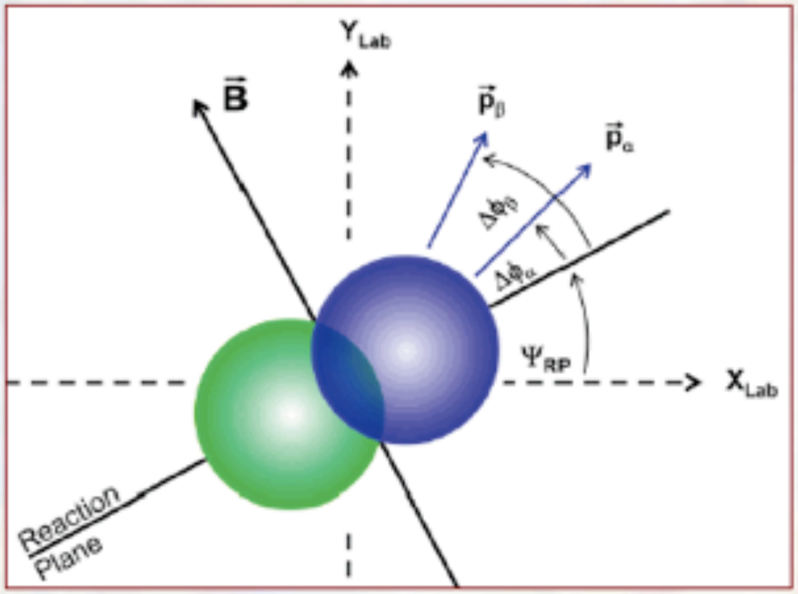
S.A. Voloshin, Phys. Rev. C 70 (2004) 057901



Effective particle distribution for a certain Q.

$$\frac{dN_\alpha}{d\phi} \propto 1 + 2v_{1,\alpha}\cos(\Delta\phi) + 2\,v_{2,\alpha}\cos(2\Delta\phi) + ... + 2a_{1,\alpha}\sin(\Delta\phi) + 2\,a_{2,\alpha}\sin(2\Delta\phi) + ...,$$

$$\Delta\phi = (\phi - \Psi_{RP})$$



- The effect is too small to observe in a single event
- The sign of Q varies and $\langle a \rangle = 0$ (we consider only the leading, first harmonic) ➔ one has to measure correlations, $\langle a_\alpha a_\beta \rangle$, $\mathcal{P}$-even quantity (!)
- $\langle a_\alpha a_\beta \rangle$ is expected to be ~ 10^{-4}
- $\langle a_\alpha a_\beta \rangle$ can not be measured as $\langle \sin\varphi_\alpha \sin\varphi_\beta \rangle$ due to large contribution from effects not related to the orientation of the reaction plane

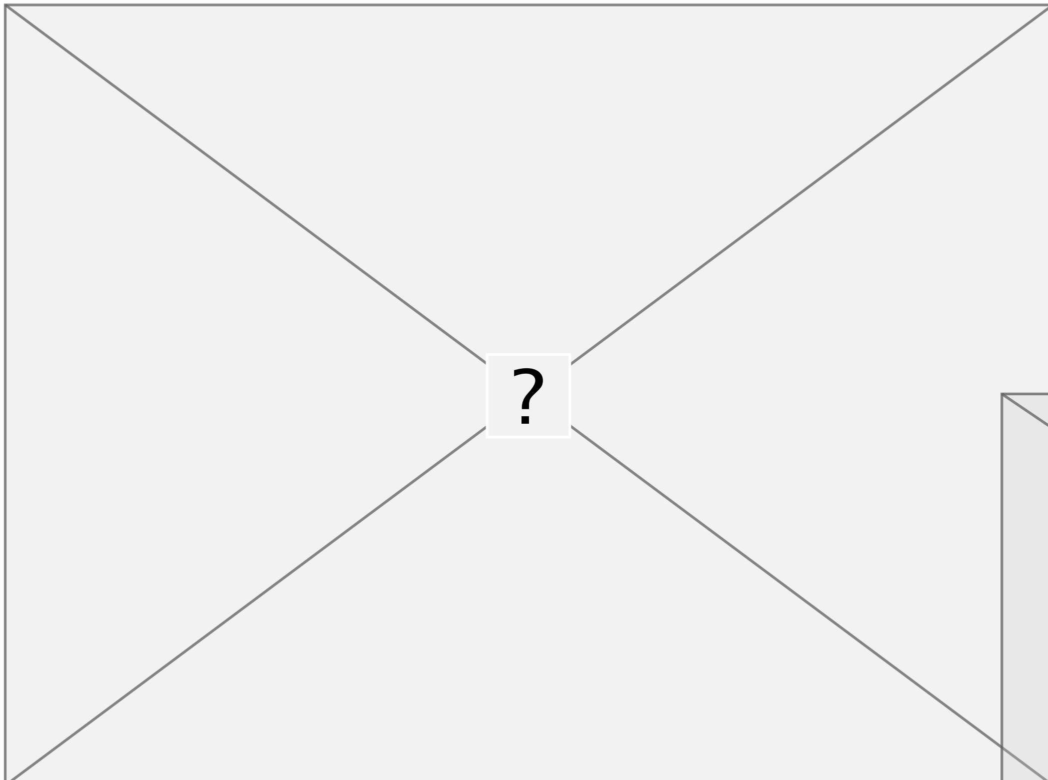
➔ study the difference in corr's in- and out-of-plane

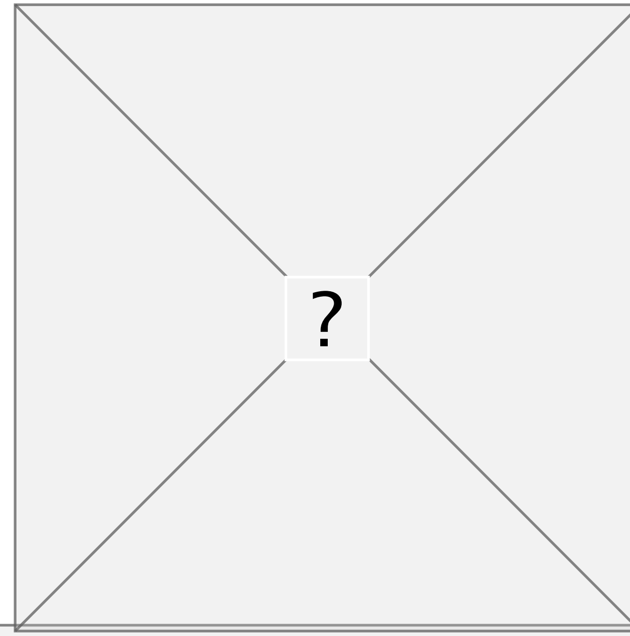
$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP})\rangle =$$
$$= \langle\cos\Delta\phi_\alpha\,\cos\Delta\phi_\beta\rangle - \langle\sin\Delta\phi_\alpha\,\sin\Delta\phi_\beta\rangle$$
$$= [\langle v_{1,\alpha}v_{1,\beta}\rangle + B^{in}] - [\langle a_\alpha a_\beta\rangle + B^{out}].$$

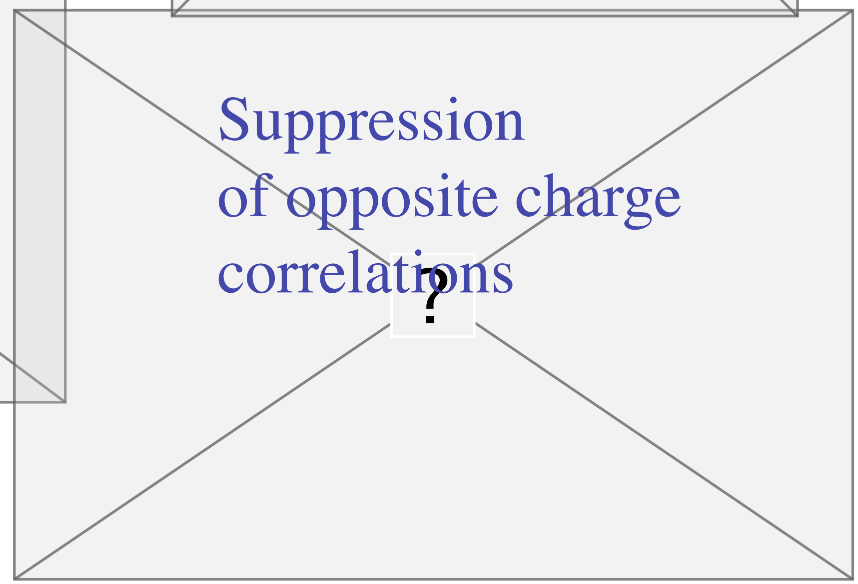
$$B^{in} \approx B^{out}, \quad v_1 = 0$$

A practical approach: three particle correlations: $\langle\cos(\phi_a + \phi_\beta - 2\phi_c)\rangle = \langle\cos(\phi_a + \phi_\beta - 2\Psi_{RP})\rangle\, v_{2,c}$

Theory estimates for Au-Au collisions







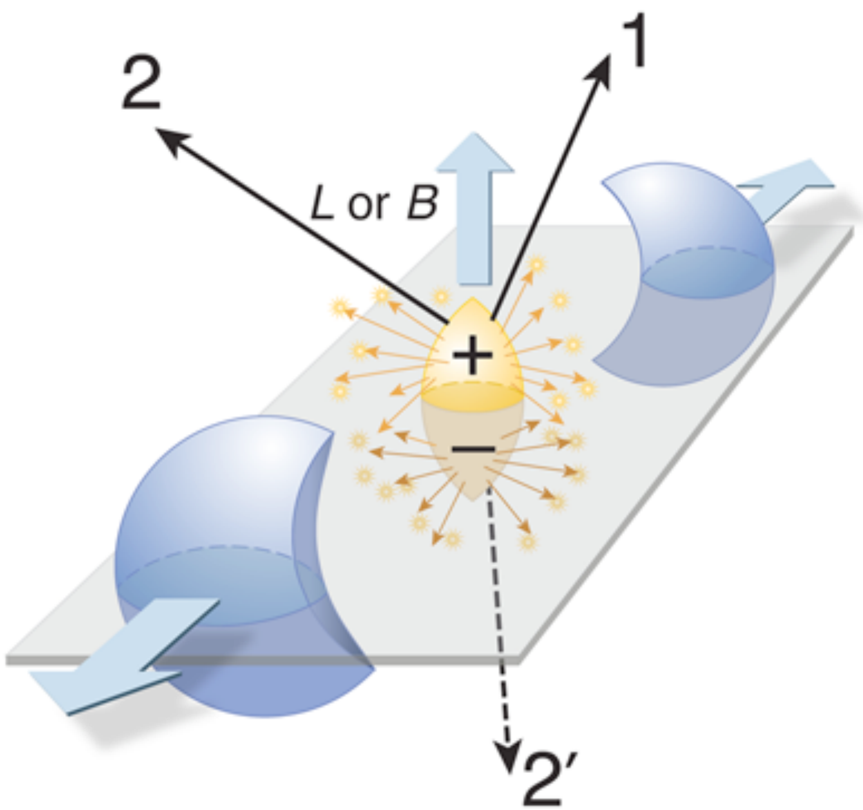
Suppression
of opposite charge
correlations

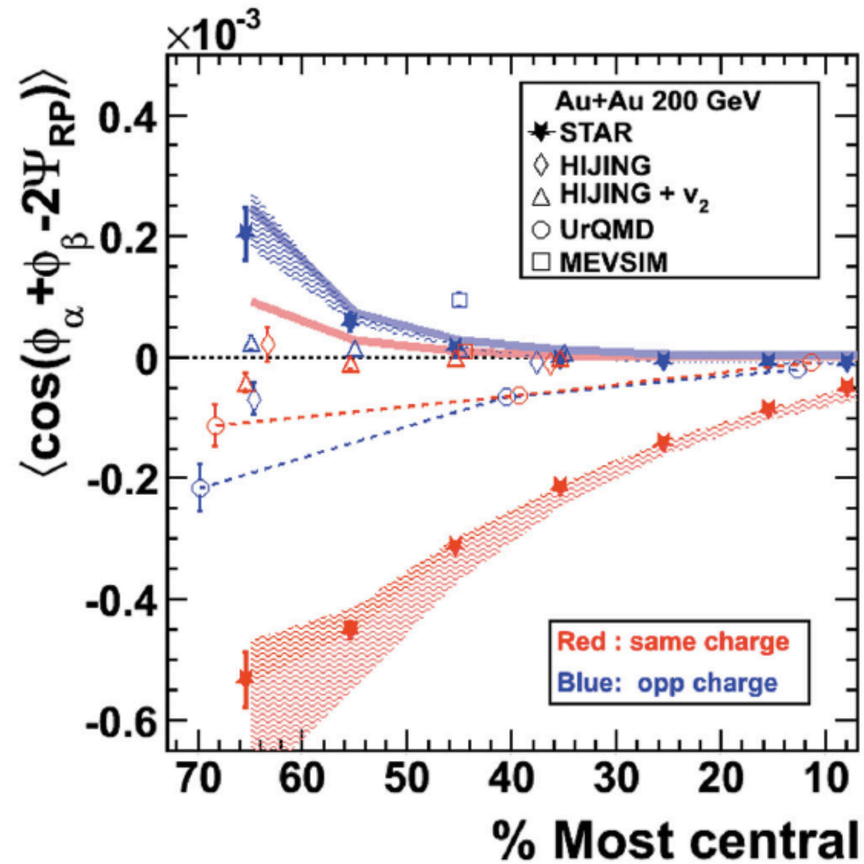
DK, L.McLerran, H.Warringa ‘07

Selected for a Viewpoint in *Physics*
PRL 103, 251601 (2009) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

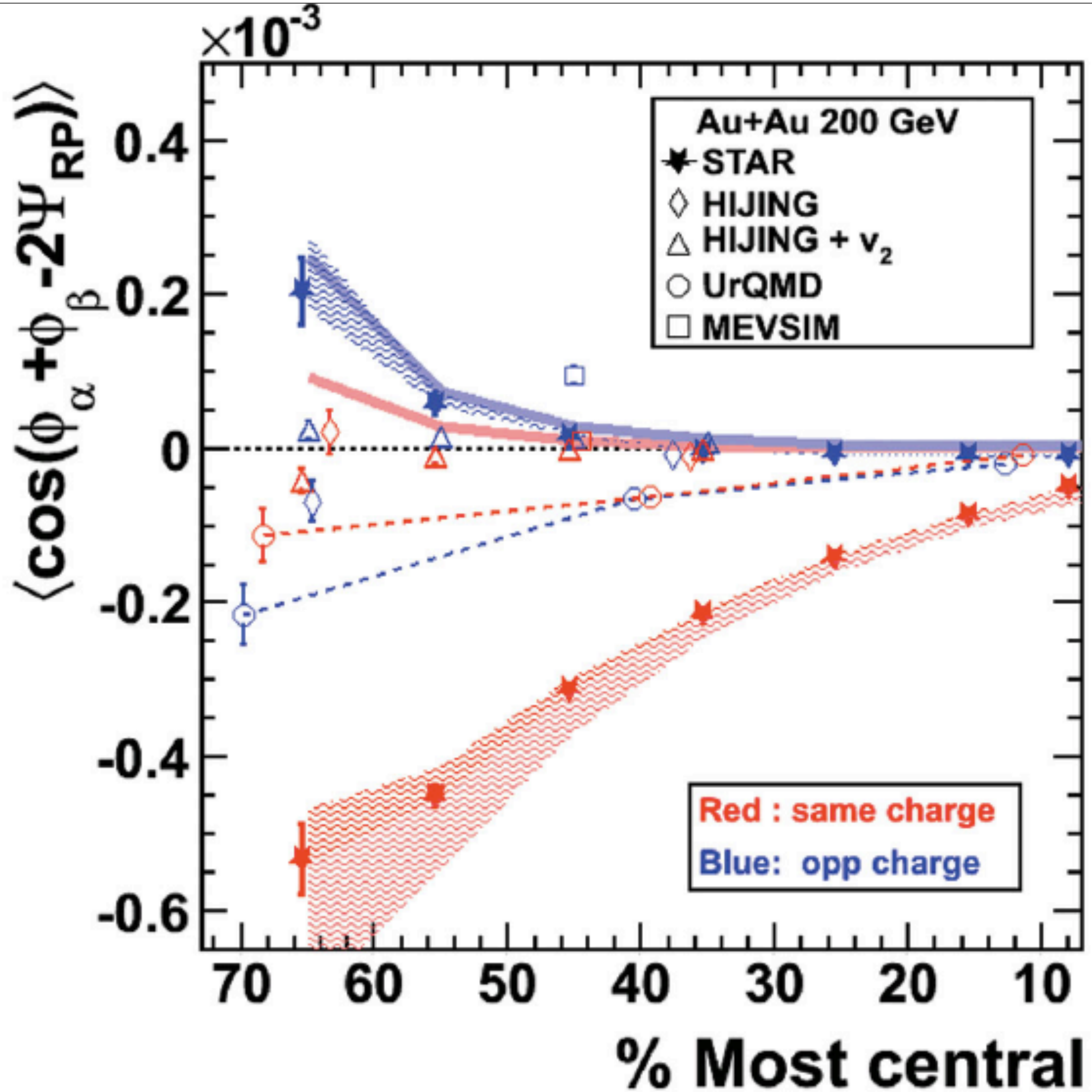
Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)



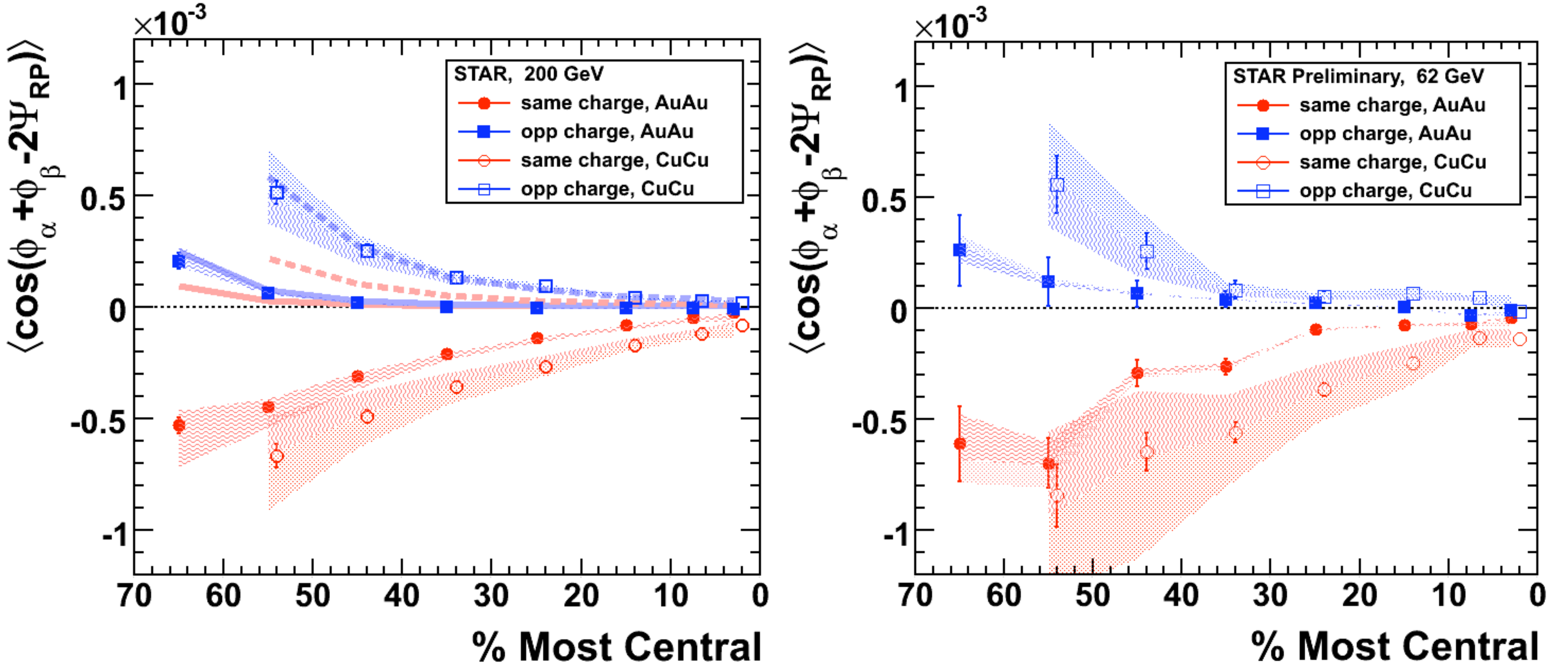


Talk by I. Selyuzhenkov



P-even observable; but: sensitive to P-odd fluctuations

Mass number and energy dependences

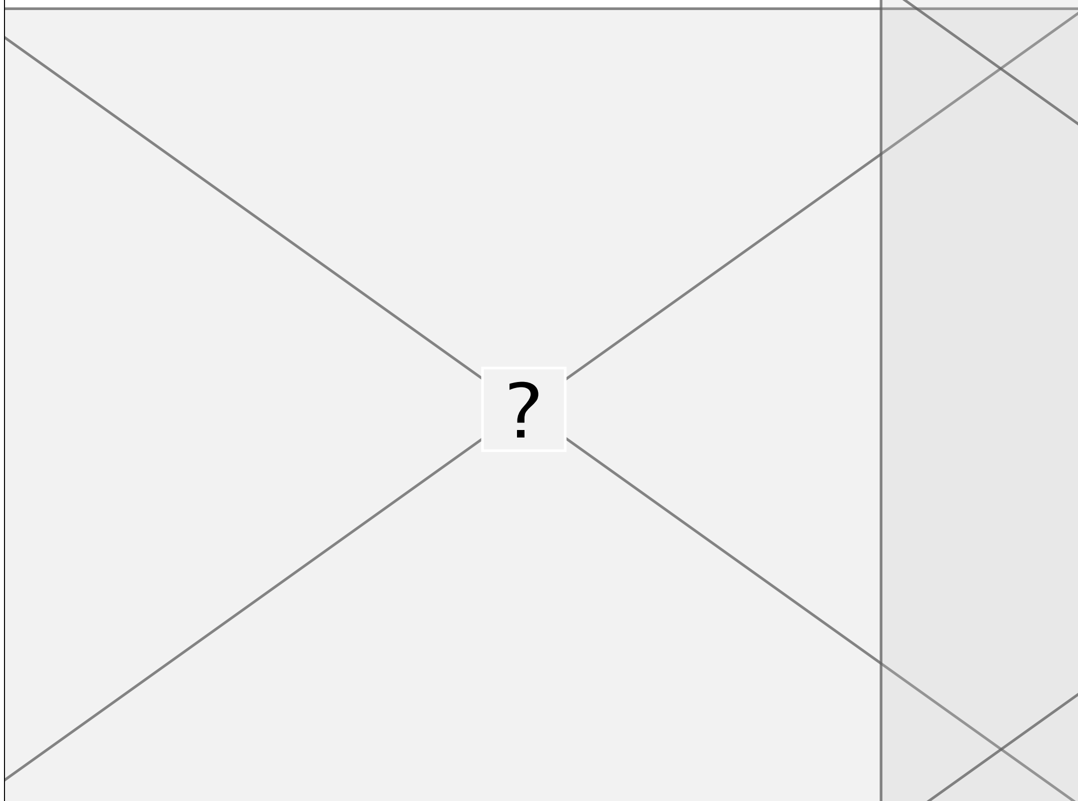


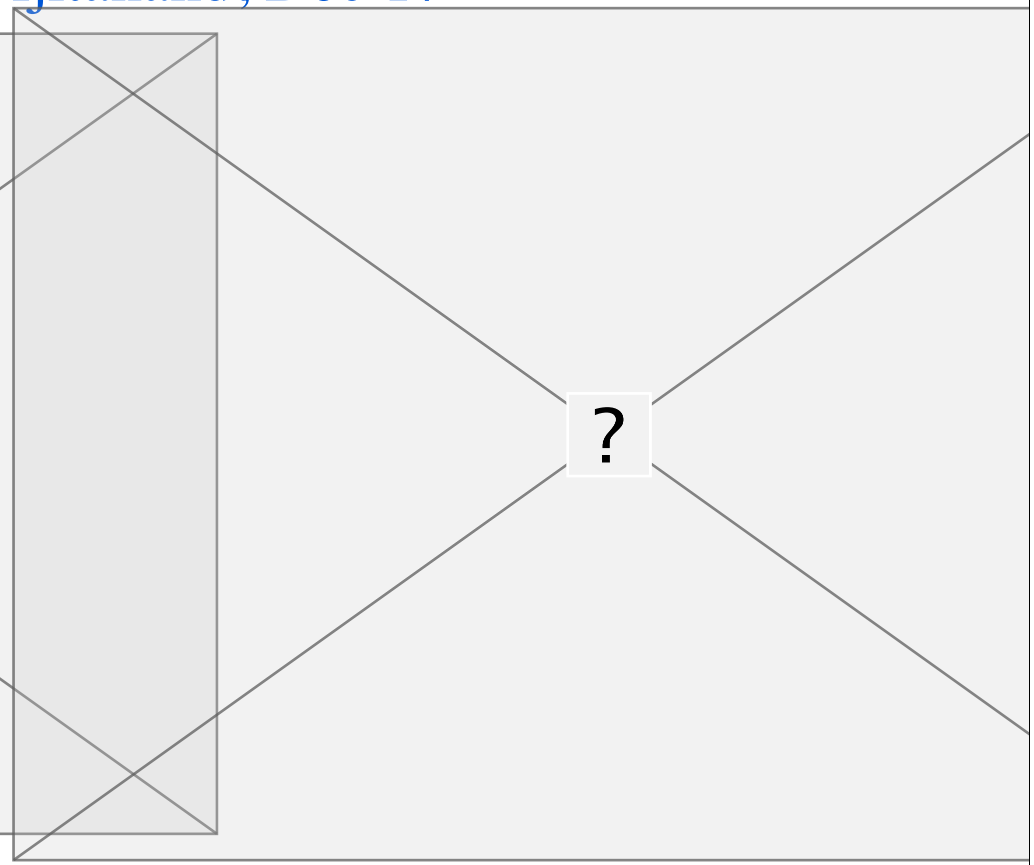
STAR Coll., arXiv:0909.1717
(Phys Rev C)

Expectations for the energy dependence:
slow growth towards low energies
reflecting longer-lived magnetic field,
then gradual disappearance (no QGP):
there has to be a maximum somewhere

PHENIX result I

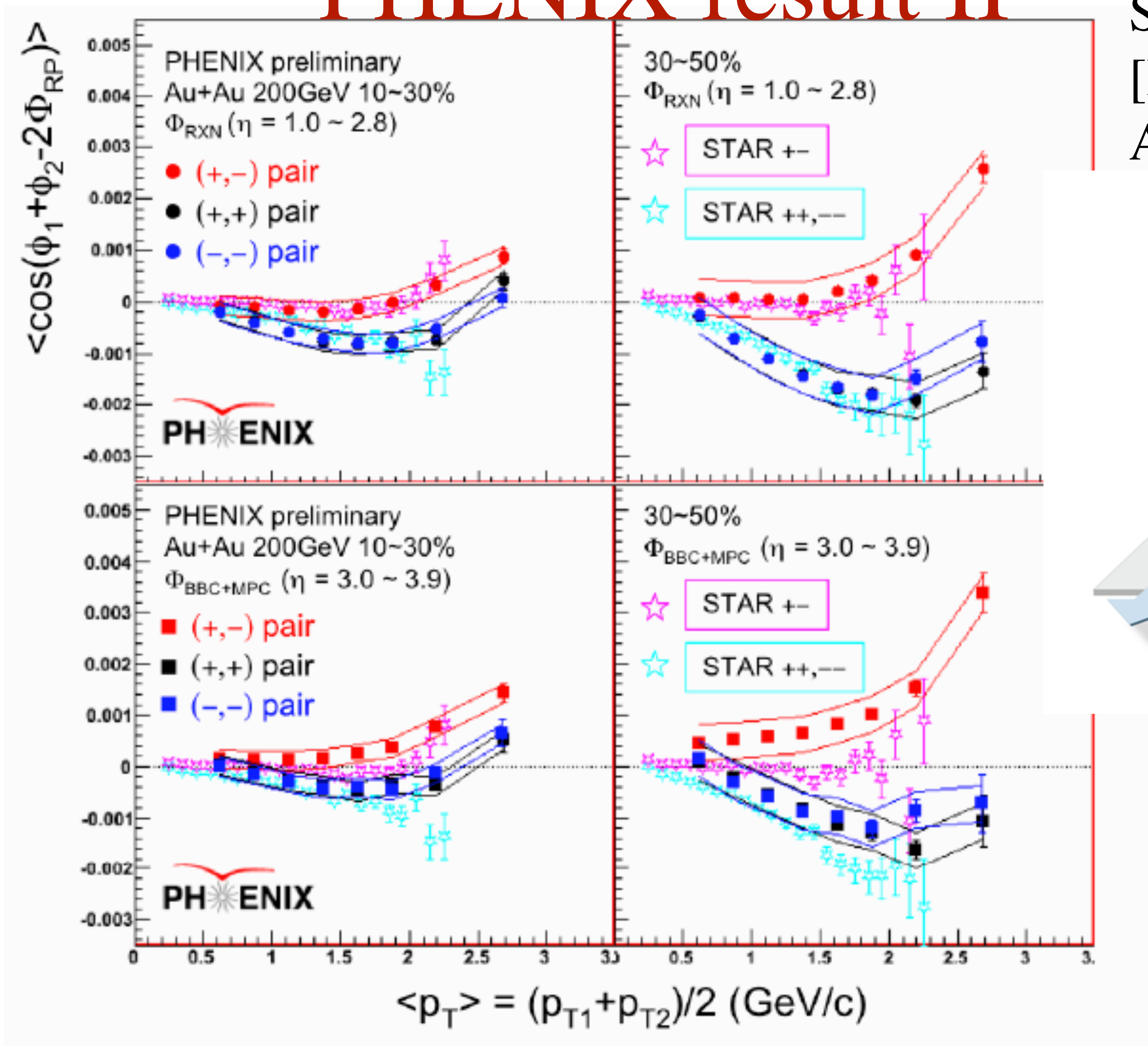
talk by N. Ajitanand, Dec 17

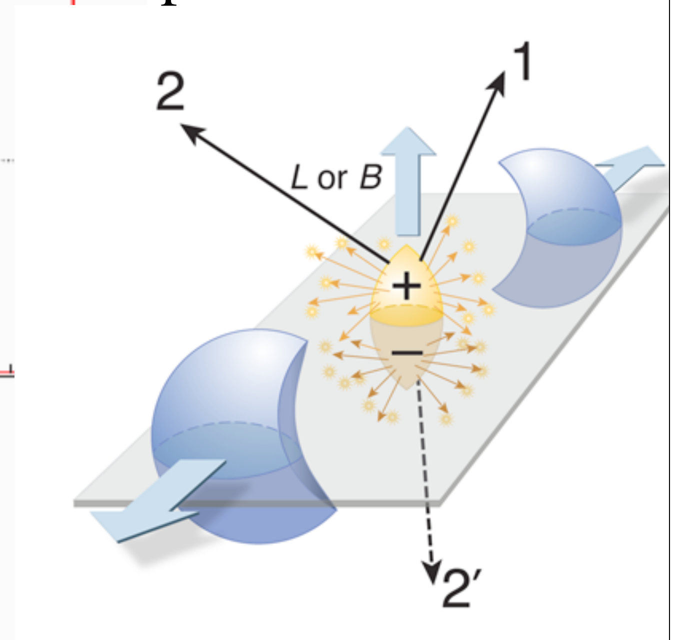




PHENIX result II

S.Esumi et al
[PHENIX Coll]
April 2010





Relatively good agreement between PHENIX & STAR

Are the observed fluctuations of charge asymmetries a convincing evidence for the local parity violation?

A number of open questions that still have to be clarified:

in-plane vs out-of-plane	Talk by A. Bzdak
physics backgrounds	Talks by A. Majumder S. Schlichting F. Wang

Fortunately, a number of analytical and numerical (lattice) tools are available to theorists, and the new data (low energy, PID asymmetries, U-U) will hopefully come - this question can be answered!

Summary

- The existence of topological solutions is an indispensable property of non-Abelian gauge theories that form the Standard Model
- Electric charge separation in the background magnetic field (CME) allows a **direct** observation of a topological effect in QCD
- The existence of the Chiral Magnetic Effect (CME) has been confirmed by several calculations done by different methods, both at weak and strong coupling
- CME has been observed in first-principle lattice QCD calculations
- There is a recent observation of dynamical fluctuations in charge asymmetry at RHIC - an evidence for the CME?

RIKEN-BNL-CATHIE Workshop on

P- and CP-odd Effects in Hot and Dense Matter

April 26-30, 2010

Brookhaven National Laboratory
Long Island, New York, USA

Organizing Committee

- Abhay Deshpande (Stony Brook University/RBRC)
- Kenji Fukushima (Kyoto University)
- Dmitri Kharzeev (BNL)
- Sergei Voloshin (Wayne State University)
- Harmen Warringa (Goethe University)

P- and CP-odd effects in:
nuclear, particle, condensed
matter physics and cosmology

International Advisory Committee

- J. Bjorken (SLAC)
- H. En'yo (RIKEN)
- M. Gyulassy (Columbia University)
- F. Karsch (BNL & Bielefeld)
- T.D. Lee (Columbia University)
- A. Nakamura (Hiroshima University)
- M. Polikarpov (ITEP)
- K. Rajagopal (MIT)
- V. Rubakov (INR)
- J. Sandweiss (Yale University)
- E. Shuryak (Stony Brook University)
- A. Tsvelik (BNL)
- N. Xu (LBNL)
- V. Zakharov (ITEP)

Talks online at
http://quark.phy.bnl.gov/~kharzeev/cpodd/

Additional information and registration at
http://www.bnl.gov/riken/hdm/

Registration deadline: March 1, 2010

Supported by RIKEN BNL Center, Brookhaven National Laboratory and Stony Brook University (Office of Vice-President for BNL Affairs)