Heavy quark diffusion and hadronization in dense matter

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Outline

1. Introduction:
   • Heavy quark probe for hot & dense matter

2. HQ probe: a strongly coupled framework
   • Transport coefficient
   • HQ diffusion: Langevin + hydro simulation
   • Hadronization: coalescence vs fragmentation
   •   $R_{AA}$ & $v_2$: thermalization & quenching

3. Charm diffusion: hadronic phase
   • Transport coefficient: quark-hadron duality?!

4. Summary
Introduction: HQ probe

- primordial hard production + number conserved
- thermalization delayed

\[ \tau_Q \approx \frac{m_Q}{T} \quad \tau_q \approx 6 \times \tau_q \geq \tau_{QGP} \]

Heavy quarks make a direct probe for the medium

- HQ energy loss:
  - gluon radiation: dead cone \( \theta \leq m_Q / E \)
  - elastic scattering: \(- \frac{dE}{dx} = \gamma p\)

thermalization rate; diffusion coefficient

\[ \gamma \sim \int |T_{Qq}|^2 (1-\cos \theta) f^q \]
Introduction: HQ probe (continued)

◆ **pQCD elastic collisions** B. Combridge, 1978

![Diagram of pQCD elastic collisions]

various weakly coupled (pQCD) scenarios for HQ transport:
Moore & Teaney, 2005; Gossiaux & Aichelin, 2008; Alberico et al., 2011; J.Uploff et al., 2010; ...
running coupling, Debye screening, K-factor?

◆ **Resonant scattering: non-perturbative** van Hees, Greco & Rapp, 2005, 2006

![Diagram of resonant scattering]

**strongly coupled scenario**
A strongly coupled framework: HQ

Relaxation rate
T-matrix: resonance

Medium/hydro
Langevin simulation
Initial distribution

Hadronization: fragmentation vs Resonance Recombination

D/B: Semi-leptonic decay
Relaxation rate: $T$-matrix

\[ T = V + VT \]

- static approximation: $q_0 \simeq \bar{q}^2 / 2 m_q \ll |\bar{q}|$

- lattice potential: Kaczmarek, 2008

- open/hidden heavy flavor vacuum spectroscopy reproduced: Riek & Rapp, 2010

- heavy quarkonia (bound states)
- heavy quark transport (scattering states)

\[ U = F - T \frac{\partial F}{\partial T} \]

\[ F(t, T) \text{ [MeV]} \]

Common basis & mutual constraints
T-matrix resummation ➔ color singlet and anti-triplet broad Feshbach resonances up to ~1.5 $T_c$

this resonance correlation will be reiterated in our hadronization-coalescence model

T-matrix relaxation rate: a factor ~4-5 larger than LO pQCD at $T=1.2 \, T_c$

T-dependence: screening potential vs light parton density; $p$-dependence: less contribution from threshold Feshbach resonance as $p$ increases
QGP medium: AZHYDRO vs fireball

AZHYDRO: initialization time $\tau_0 = 0.6 \, \text{fm}/c$, boost invariant Cooper-Frye freezeout: the end of mixed phase $e_{\text{dec}} = 0.445 \, \text{GeV}/\text{fm}^3$, $T_{\text{dec}} = 165 \, \text{MeV}$

Elliptic fireball: tune to fit to multi-strange particles’ spectra and $v_2$ at $T_c$ c.f. M.He, R.Fries & R.Rapp, 2010, harder than AZHYDRO, larger flow

$<v_2> = 5.04\% \text{ vs } 4.99\%$

mixed phase $v_s = 0$
**HQ initial distribution**

- Initial $p_T$ spectrum: PYTHIA parametrization
- $B \rightarrow e$ starts to dominate over $D \rightarrow e$ at $p_T \sim 5$ GeV

- Initial spatial distribution: Glauber binary collision density $n_{BC}(x,y)$

\[
\frac{d^2 N_e}{dp_T^2} = C \frac{(p_T + A)^2}{(1 + p_T / B)^\alpha}
\]
A strongly coupled framework: HQ

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D/B: Semi-leptonic decay
Langevin simulation of HQ diffusion

- Langevin + AZHYDRO simulation
  fluid rest frame updates $\rightarrow$ boost to lab frame

$$dx = \frac{P}{E} dt,$$
$$dp = -\Gamma(p) p dt + \sqrt{2D(p + dp)} dt \rho$$

- quenching: early stage when medium particles’ density is high
- $v_2$: develops at later stage when the medium particles’ $v_2$ is large
A strongly coupled framework: HQ

- Relaxation rate
  - T-matrix: resonance

Medium/hydro → Langevin simulation → Hadronization: fragmentation vs Resonance Recombination → D/B: Semi-leptonic decay

Initial distribution
Hadronization: Resonance Recombination

Hadronization = Resonance formation $car{q} \rightarrow D$

consistent with T-matrix findings of resonance correlations towards $T_c$

Realized by Boltzmann equation Ravagli & Rapp, 2007

\[
p^\mu \partial_\mu f_M(t, \vec{x}, \vec{p}) = -m \Gamma f_M(t, \vec{x}, \vec{p}) + p^0 \beta(\vec{x}, \vec{p}).
\]

\[
\beta(\vec{x}, \vec{p}) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \\
\times \sigma(s) v_{rel}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)
\]

\[
\sigma(s) = g^2 \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2) + (\Gamma m)^2}
\]

Equilibrium limit

Energy conservation + detailed balance

**Equilibrium Quark ➔ Equilibrium Meson**

- **Kolb-Heinz AZHYDRO**: space-momentum correlation at freezeout included, excellent equilibrium mapping achieved; boost invariance preserved
  
  c.f. M. He, R. J. Fries & R. Rapp, 2010

- **RRM** (compared to instantaneous coalescence models) quite facilitates the description of the transition from low $p_T$ (equilibrium) to intermediate $p_T$ (kinetic) region in heavy-light quark recombination
Hadronization: coal. vs frag.

- charm quark coal. prob. based on scattering rate:
  \[ P_{\text{coal}}(p_t) = \tau_{\text{res}} \gamma_Q(p_t) \]
- supplemented by fragmentation

\[ D(z) = \delta(z-1) \]

At \( p_T \sim 1.5-4.0 \) GeV, coalescence adds momentum to \( D, R_{AA} \) increases from c to D

At low \( p_T \), D-\( R_{AA} \) gets a dip, not present for charm; flow depletion on D, captured by c-q RRM

Coalescence acts as an extra interaction, driving D-spectrum closer to equilibrium, which also explains D-spectrum slightly below c at high \( p_T \)
Hadronization: coal. vs frag. (cond.)

- coal. vs frag.: relatively normalized with the calculated coal. prob.
- fragmentation: preserves the HQ $v_2$ from c/b $\rightarrow$ D/B
- coalescence: adds $v_2$ from the light quarks to D/B mesons

- RRM properly accounts for space-momentum correlation built up via Langevin + hydro for HQ + q $\Rightarrow$ enables us to analyze medium flow effect
What’s missing: medium flow

◆ multi-strange hadrons \( \phi, \Omega, \Xi \) decouple close to \( T_c \)
  
  STAR, 2005; Mohanty & Xu, 2009; M.He, R.Fries & R.Rapp, 2010
  
  ➔ Important to tune hydro flow to fit to multi-strange hadrons’ spectra at \( T_c \)

◆ AZHYDRO underestimates the flow close to \( T_c \); mixed phase \( v_s=0 \) ➔ lattice EOS

◆ fireball model (Page8): tuned to fit to multistrange hadrons’ spectra and \( v_2 \) at \( T_c \) ➔ light quark spectrum harder ➔ D spectrum via RRM harder !!!
Medium flow effect via RRM (continued)

- RRM admits equilibrium mapping between quark and meson distributions, thus able to capture the remarkable flow effect via heavy-light coalescence.

- D-meson v2 receives a modest increase at $p_T > 2.5$ GeV, also due to larger flow.
Medium flow effect via RRM (continued)

- Dip more prominent
- Bump shifted to higher $p_T$
- $C \rightarrow D$ $R_{AA}$, more increase

- Qualitatively/semi-quantitatively reproduce the data behavior: dip, bump
- Fireball model with larger medium flow gets better agreement with data
- Correct flow at $T_c$ important!
A strongly coupled framework: HQ

Relaxation rate
T-matrix: resonance

Medium/hydro

Langevin simulation

Hadronization: fragmentation vs Resonance Recombination

Initial distribution

D/B: Semi-leptonic decay
Non-photonic decay electrons

- Monte-Carlo simulation, free quark decay $c(b \rightarrow s(e + nu, m_b=5.28, m_c=1.87, m_s=0.5, m_e=0.0005 \text{ GeV};$ inclusive branching ratios: $c:11.5\% ; b: 10.4\%$
- matrix elements $\langle |M|^2 \rangle \propto (p_s \cdot p_{\nu})(p_c \cdot p_e)$ and $\langle |M|^2 \rangle \propto (p_c \cdot p_e)(p_b \cdot p_{\nu})$
- hadronic form factors little effect, checked
- fireball model: better agreement of $R_{AA}$ $\Rightarrow$ correct medium flow important! $v_2$ smaller $\Rightarrow$ hadronic phase diffusion?
D&D₀*, D*&D₁’ : chiral partners, large pion s-wave decay width ~300 MeV, Fuchs, et al., 2006; BELLE Colla., 2004; also verified by Chiral Unitarized Approach

D + pion $\rightarrow$ resonance $\rightarrow$ D + pion:

$$A_{1/2} = \sum_{j=0,1,2} \frac{8\pi \sqrt{s}}{k} \frac{(2j + 1)}{(2j_1 + 1)(2j_2 + 1)} s - M_j^2 + i\sqrt{s}\Gamma_j^{tot}$$
Charm diffusion: hadronic resonance gas

- D + K, eta, rho, omega, K*, N, Delta, empirical s-wave cross sections based on ChUA: Lutz et al., 2004, 2006; E. Oset et al., 2007

- $A \sim 0.1 / \text{fm}$ at $T = 180$ MeV, comparable to the non-perturbative T-matrix calculation of charm quark thermal relaxation rate in QGP

- expected modification of D-spectrum at RHIC: $1 - \exp(-A \Delta \tau_{\text{had}}) \approx 20\%$

- spatial diffusion coefficient $D_s = T / (m \Lambda)$, surprisingly close to T-matrix result for charm quark in QGP: quark-hadron duality?!
**Charm diffusion: HRG ➔ \eta/s**

- **Transport coefficient:** \eta/s = (1/5 \sim 1/2)D_s; Danielewicz&Gyulassy, 1985
- \(D_s\) translates into \eta/s = (2-5)/4\pi at \(T=180\) MeV, comparable to J.N-Hostler 2009

![Graph showing D_s vs T and \eta/s vs (T-T_c)/T_c](image)

- Both exhibit a minimum across the quark-hadron transition ➔ duality?!
- The charm diffusion coeffi. provides us with another perspective of looking into the transport properties of sQGP/dense matter
Summary & Conclusion

- A strongly coupled framework for HQ diffusion + hadronization (Hydro + Langevin + RRM) presented
- The role of resonance correlation emphasized:
  (a). resonance contribution (Q-q T-matrix calculation) to heavy quark thermal relaxation
  (b). c-q Resonance Recombination to describe the coalescence hadronization
- Medium flow’s effect (via RRM ) on heavy-meson observables highlighted

- charm hadronic diffusion: quark-hadron duality?!
  Reference: M. He, R. J. Fries & R. Rapp, PLB, in press

Thanks for attention!
Backup 1: Langevin equilibrium

- Elliptic fireball, escape-subtracted
- Uniform initial + modified postpoint Langevin
- Equil: $\Gamma = 40.0 / \sqrt{E}, D = \Gamma ET$
- Analytical, $\exp(-p_u/T)$, boost invariant
- Langevin, 10M

AZHYDRO

AZHYDRO Cooper-Frye freeze-out
- Langevin simulation with $\Gamma = 40 / \sqrt{E}$

Initial charm quark spectrum
Backup 2: RRM equilibrium

elliptic fireball

AZHYDRO

\begin{align*}
m_c &= 1.5, m_q = 0.3, m_D = 1.9, \Gamma_D = 0.1 \\
elliptic fireball, \text{ unif. init. + modified} \\
postpoint Langevin + 1st method RRM \\
Equil: \Gamma = 40.0/\sqrt{E}, D = \Gamma T \\
analytical, \exp(-p_u/T), boost invariance \\
y_D = 0.0, 10M charm \\
y_D = 0.5, 10M charm
\end{align*}
Backup 3: c/b coal.prob. based on scattering rate

◆ Aim: formulate coal.prob. consistent with RRM

◆ Quark scattering rate (vs thermal relaxation rate): $\gamma_Q = n <\sigma_{rel}>$

Breit-Wigner resonant cross section to reproduce color singlet contribution to relaxation rate $\Rightarrow$ scattering rate $\Rightarrow$ boosted to lab frame at the end of Langevin simulation: $\gamma_Q = \gamma_Q(p_t)$

◆ Charm quark scattering time: $\tau_Q = 1/\gamma_Q$. Within this time duration, we can form a D-meson/resonance through c-qbar resonant scattering (RRM). If the resonance formation time allowed by the system evolution (mixed phase duration) $\tau_{res} > \tau_Q$, $\Rightarrow$ $P_{coal}(p_t) = 1$; otherwise, $P_{coal}(p_t) = \tau_{res}\gamma_Q(p_t)$