Lattice results on the critical point (phase diagram)

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Y.Aoki, S.Borsanyi, G.Endrodi, C.Holbling, A.Jakovac, S.Katz, S.Krieg, C.Ratti, K.Szabo

Nature 443 ’06 675, JHEP 1009 ’10 73, 1011 ’10 77, 1104 ’11 001

Brookhaven National Laboratory, June 20, 2011

SFB TR55

Outline

User's guide to lattice QCD results

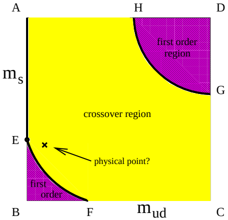
- Full lattice results have three main ingredients

1. (tech.) technically correct: control systematics (users can't prove)
2. (m_q) physical quark masses: m_s/m_{ud} $\approx$28 (and m_c/m_s $\approx$12)
3. (cont.) continuum extrapolated: at least 3 points with $c \cdot a^n$

only a few full results (nature, T_c, spectrum, EoS, m_q, curvature, B_K)

ad 1: obvious condition, otherwise forget it
ad 2: difficult (CPU demanding) to reach the physical u/d mass
BUT even with non-physical quark masses: meaningful questions
e.g. in a world with M_π=M_ρ/2 what would be M_N/M_π
these results are universal, do not depend on the action/technique
ad 3: non-continuum results contain lattice artefacts
(they are good for methodological studies, they just "inform" you)

Phase diagram and its uncertainties



physical quark masses: important for the nature of the transition

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

intermediate quark masses: we have an analytic cross over (no χPT)

F.Karsch et al., Nucl.Phys.Proc. 129 ('04) 614; G.Endrodi et al. PoS Lat'07 182('07);

de Forcrand, S. Kim, O. Philipsen, Lat'07 178('07)

continuum limit is important for the order of the transition:

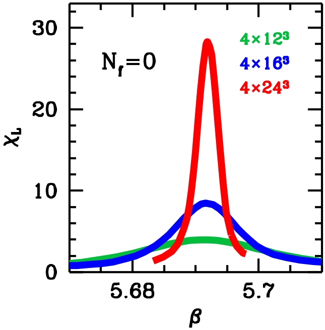
n_f=3 case (standard action, N_t=4): critical $m_{ps}\approx$300 MeV

different discretization error (p4 action, N_t=4): critical $m_{ps}\approx$70 MeV

the physical pseudoscalar mass is just between these two values

Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

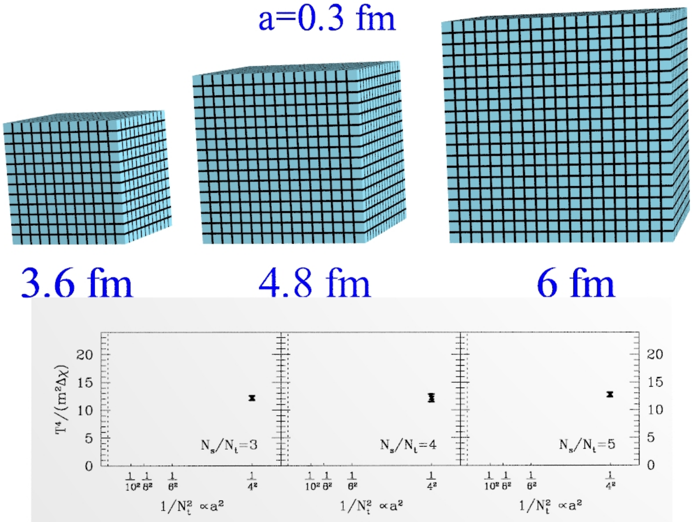
finite size scaling study of the chiral condensate (susceptibility)

$$\chi=(T/V)\partial^2 \log Z/\partial m^2$$

phase transition: finite V analyticity $V\rightarrow\infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated χ_{max}: finite size scaling

Approaching the continuum limuit



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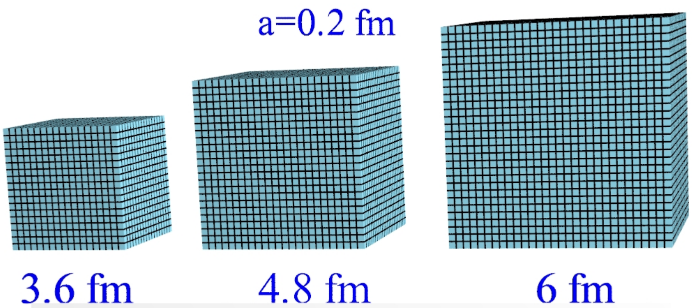

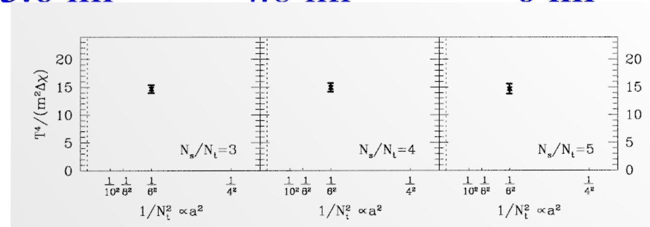

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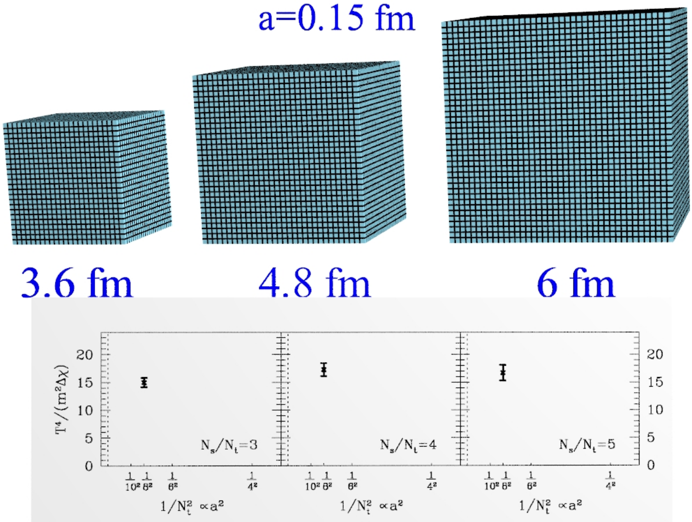

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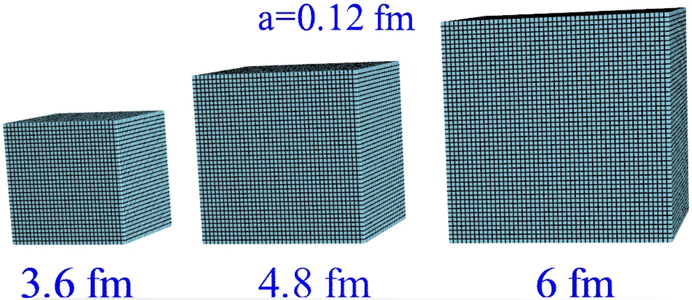

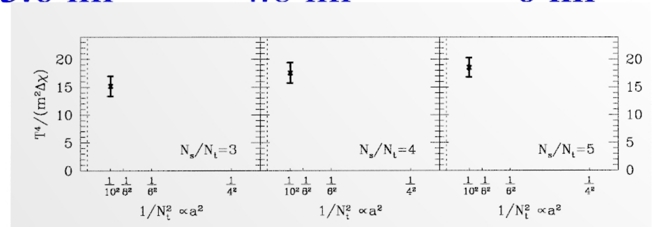

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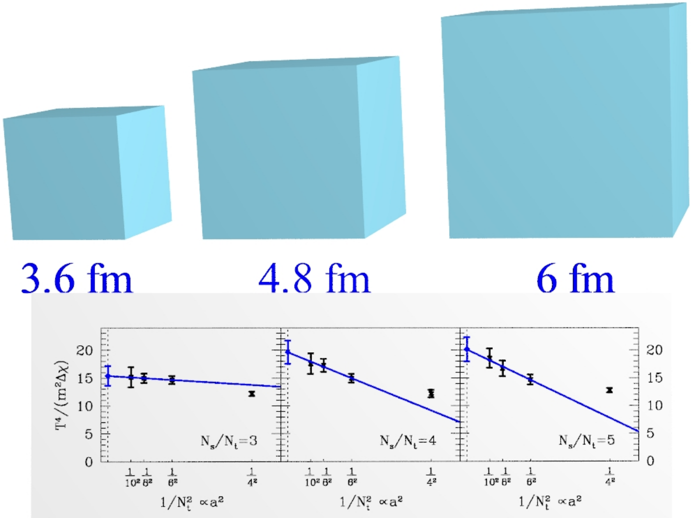
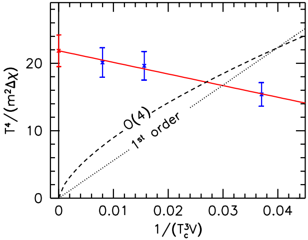


The nature of the QCD transition: analytic

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

the QCD transition is a cross-over

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Collaboration:

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Bielefeld-Brookhaven-Riken-Columbia merged with MILC: 'hotQCD'

Wuppertal-Budapest group: WB

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

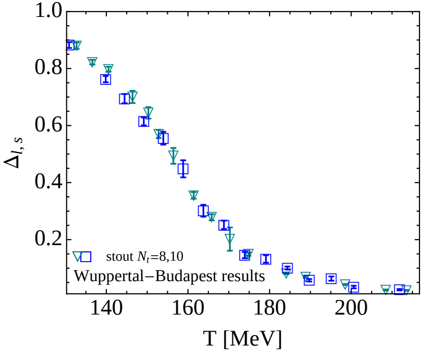
'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

progress in the transition temperature

Wuppertal-Budapest: physical quark masses (m_s/m_{ud} ≈28)
gauge configs: N_t=8,10 in 2006 ⇒ N_t=12 in 2009 ⇒ N_t=16 in 2010

hotQCD 2009: realistic quark masses (m_s/m_{ud} =10)
hotQCD 2010: preliminary: physical quark masses (m_s/m_{ud} =20)



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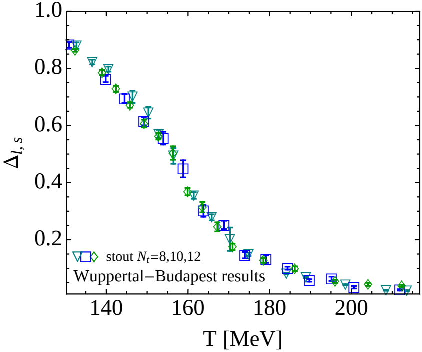

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Wuppertal-Budapest JHEP 1009 '10 73

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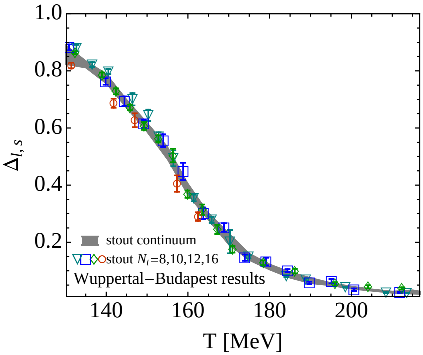

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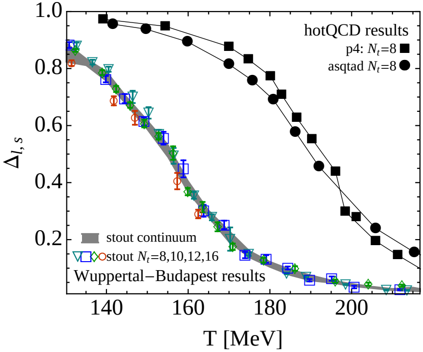

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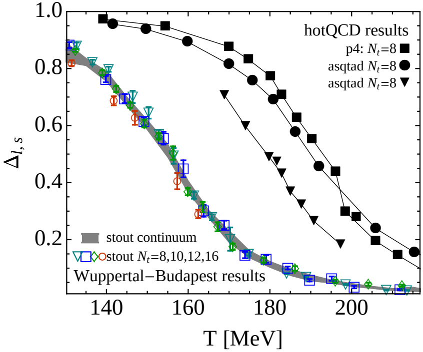

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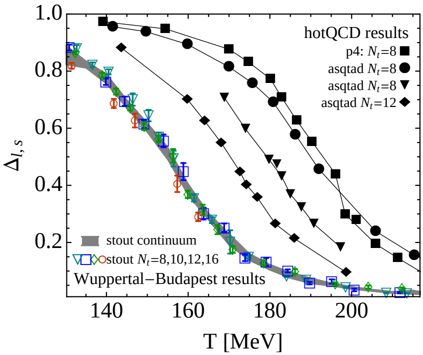

progress in transition temperature

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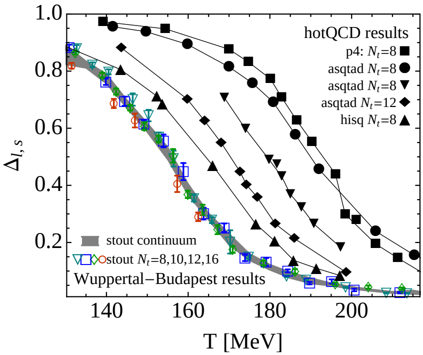
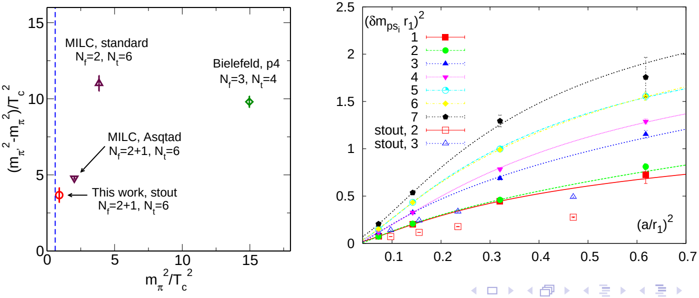


Chiral symmetry/pions

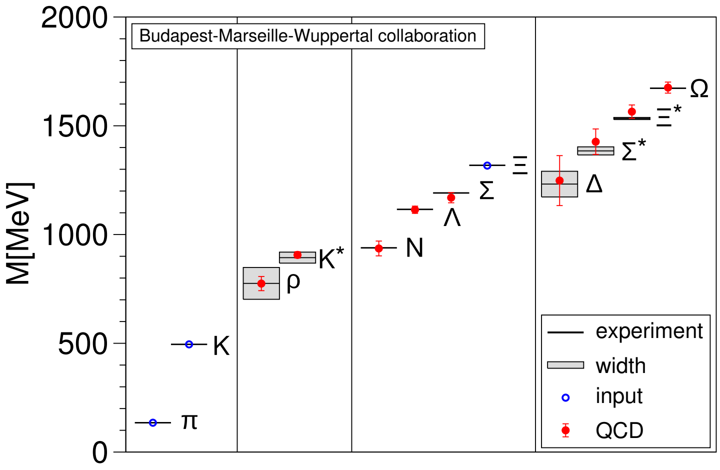
Wuppertal-Budapest: JHEP 0601 (2006) 089. [hep-lat/0510084]

transition temperature for remnant of the chiral transition:
balance between the f's of the chirally broken & symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

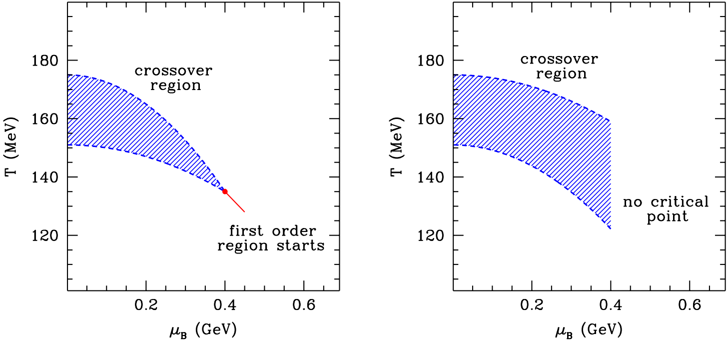
staggered QCD: 1 ($\frac{3}{16}$) pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact $\Rightarrow$ splitting disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact



Final result for the hadron spectrum



Scenarios for $\mu > 0$



Does the crossover region shrink or expand?

The curvature can affect the existence of the critical endpoint

Estimate: if $\mu_{crit} = 360$ MeV $\rightarrow \Delta\kappa \approx 0.02$

Finite chemical potential: the sign problem

at μ=0 the fermion matrix is γ_5 hermitian: $M^\dagger = \gamma_5 M \gamma_5$
easy to check $\Longrightarrow$ eigenvalues: either real or conjugate pairs
det(M) is real, which is not true any more for non-vanishing μ

importance sampling (algorithms) for complex det(M) does not work

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

sign problem $\Rightarrow$ from 2001 new methods to go to μ>0

Fodor-Katz: multiparameter reweighting (hep-lat/0104001, PLB)
Bielefeld-Swansee: det(M) Taylor expanded (hep-lat/0204010, PRD)
de Forcrand-Philipsen: imaginary μ (hep-lat/0205016, Nucl.Phys.B)
D'Elia-Lombardo: imaginary μ (hep-lat/0209146, PRD)

the three methods look different, they are essentially the same

Overlap improving multi-parameter reweighting

one wants to calculate the following path integral

$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha, U)] \det M(U, \alpha)$$

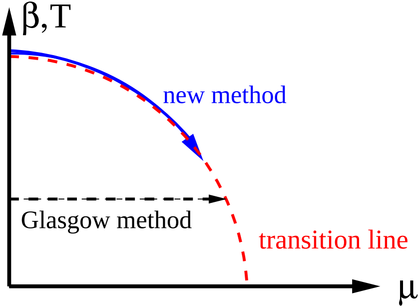
α: parameter set (gauge coupling, mass, chemical potential)
for some parameters α_0 importance sampling can be done

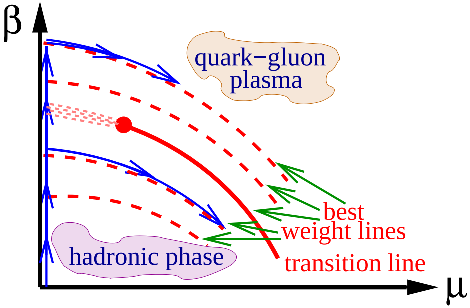
$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha_0, U)] \det M(U, \alpha_0)$$
$$\{\exp[-S_{bos}(\alpha, U) + S_{bos}(\alpha_0, U)] \det M(U, \alpha) / \det M(U, \alpha_0)\}$$

first line: measure; curly bracket: observable (will be measured)
e.g. transition configurations are mapped to transition ones

reweighting factor (ratio of the determinants) can be expressed by the eigenvalues of the (reduced) fermion matrix: closed formula for any μ

Compare with Glasgow (Ferrenberg-Swendsen)





Glasgow method $\Rightarrow$ multiparameter reweighting
single parameter (μ) $\Rightarrow$ two parameters (μ and β)
purely hadronic $\Rightarrow$ transition configurations

map transition configurations to transition ones

Equivalence of the methods (formal/numerical)

(recent lattice review at μ=0 and μ>0: Fodor-Katz 0908.3341)

det(M) can be given by the eigenvalues of M' (transformed) at μ=0

$$\det M(\mu) = e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i)$$

observable at μ>0 or μ_I is given by the observable and λ_i at μ=0

$$Pl(\beta,\mu) = \langle Pl \exp[\Delta\beta Pl] e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i) \rangle$$

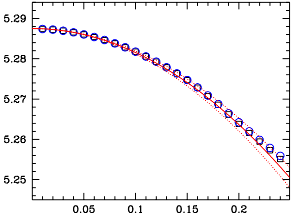
det(M) or Pl(β,μ) can be trivially Taylor expanded (Bielefeld-Swansee)
termination of the series & stochastic determination of the coefficients
$\Longrightarrow$ do not expect this method to work for as large μ as the full one

det(M)$>$0 for imaginary μ: impartance sampling still works
determine the phase line $T_c(\mu_I)$ (e.g. use a quadratic/quartic fit)
plug real μ into the same quadratic/quartic function: $c_2\mu^2$+ $c_4\mu^4$
formally: numerical determination of the (μ^2,μ^4) Taylor coefficients

Equivalence of the methods (formal/numerical)

$\Rightarrow$ for moderate μ Taylor and μ_I agree with reweighting

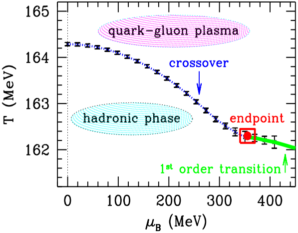
take n_f=2 setting of de Forcrand-Philipsen: $\beta_c(\mu)$ upto 4 digits

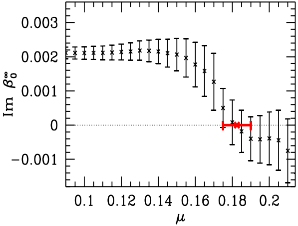


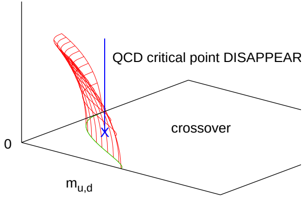
solid/dotted: imaginary μ & error; box: reweighting; circle: Taylor
for larger μ values higher order terms are getting more important

what to choose (depends on the question):
for this particular case imaginary μ has the largest CPU demand;
next one is reweighting; cheapest is Taylor (does not work for large μ)

Critical endpoint discussion (controversy?)







all results are from coarse lattices (a=0.3 fm, read our abstract!)

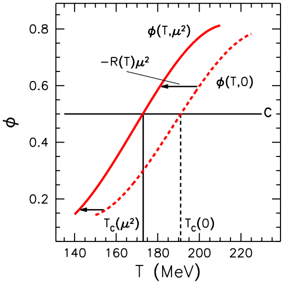
deForcrand-Philipsen: leading order $\Rightarrow$ not stronger, slightly weaker
same from reweighting: $\mu_I/T\approx$1–3 (μ_{crit}: result of the higher orders)

Taylor & radius of convergence (!) only a lower bound: Lee-Yang

full answer (all the way to the continuum) needs much more CPU

The curvature

we change μ and look at the transition curve
it shifts to the left, we look at its value of a fixed C

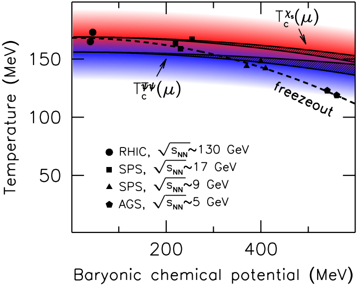


the dimensionless curvature is defined as $\kappa(T) = -T_c(\mu = 0) \cdot R(T)$

$d\kappa/dT$ at T_c tells if the transition is broadening or narrowing
(a point below T_c has a larger or smaller curvature)

Continuum prediction for the curvature: full result

G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, JHEP 1104 2011 001 [1102.1356]



lower solid line: T_c from the chiral condensate
upper solid line: T_c from the strange susceptibility

bands (red and blue) indicate the widths of the transition lines
the widths remain in this order approximately the same
in leading order: no critical point (can be anything)

high statistics in the Taylor method

determining the T dependence needs 10-times more statistics than just one single temperature point
this gives more than just the inflection point
a clear signal for broadening or shrinking can be seen
$a \longrightarrow 0$ could be done with present resources

the Taylor procedure gives only the leading order term(s) in μ
N_t=4 unimproved staggered experience [Fodor-Katz'01, Fodor-Katz'04]
the leading order terms are insensitive to the critical point $\Rightarrow$
evaluation of the whole determinant, we need all the terms in μ

our action (smeared improved): μ-dependent decomposition works
for p4, asqtad or hisq no such eigenvalue structure (det) is known

(it gives certainly more information than just the leading order terms)

memory/CPU requirements for full determinants

N_t=4 & N_s=8,10,12 needed 1 GB memory & 25 CPU years (in '04)
memory requirements grow as N_t^6, CPU requirements as N_t^9

accumulate the same statistics (shown by the first CPU row)
to reach the same μa: exponentially more configs are needed
'05 observation: applicability range $\propto V^{-0.35}$ and $\mu a \propto V^{-0.25}$
$\Rightarrow$ additional increase of the statistics (second CPU$^+$ row)

N_t	4	6	8	10
memory [GB]	1	11	64	244
'04 CPU [kyears]	0.025	1	13	95
'04 CPU$^+$ [kyears]	0.025	1	18	150
machine [year]	cluster	cluster	2 BG/P	15 BG/P

$\Rightarrow$ N_t=6,8,10: our present resources are not enough for that

Summary

- condition for the critical point: crossover at μ=0 with different T_c-s
- new results on $T_c(\mu = 0)$ with several major improvements
  a. at T=0 & T>0 all simulations: with physical quark masses
  b. smaller and smaller lattice spacings: upto N_t=16
  c. finally hotQCD agrees with our (Wuppertal-Budapest) results
- all WB results: in complete agreement with our 2006 findings
- Wilson fermions: theoretically cleaner option
- curvatures at μ=0 and $a \rightarrow$0 was determined
- in leading order no change for the width of the transition
- leading order is not enough to prove/exclude the critical point

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finite size scaling study of the chiral condensate (susceptibility)

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phase transition: finite V analyticity $V \to \infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

two steps (three volumes, four lattice spacings):
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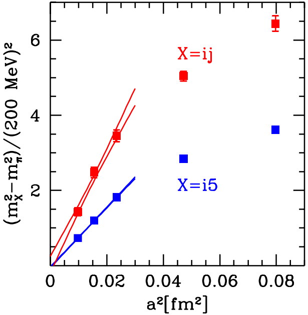
b. using the continuum extrapolated χ_{max}: finite size scaling

Scaling for the pion splitting



scaling regime is reached if a^2 scaling is observed

asymptotic scaling starts only for $N_t \gtrsim 8$ (a$\lesssim$0.15 fm): two messages

a. N_t=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

b. stout-smeared improvement is designed to reduce this artefact

most other actions need even smaller 'a' to reach scaling