Viscosity from Transverse Momentum Correlations

Sean Gavin

Wayne State University

STAR: rapidity dependence of p_t correlations $\Rightarrow$ shear viscosity

Ask: can realistic hydro with plausible viscosity explain STAR data?

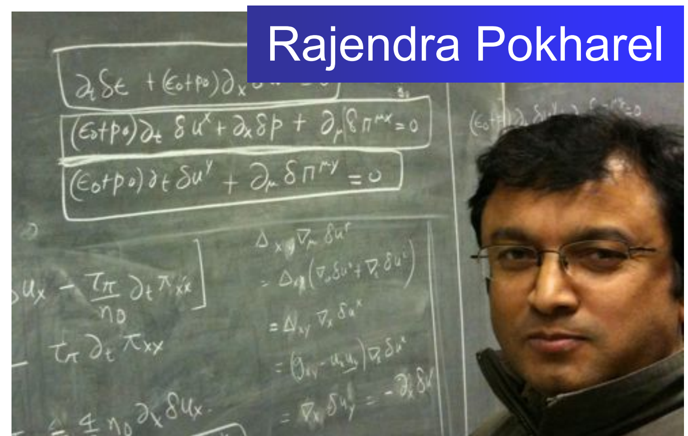
I. Measuring viscosity using correlations

II. Viscosity depends on temperature

III. 2^{nd} order viscous diffusion

IV. Other contributions to correlation measurements

Work in progress!



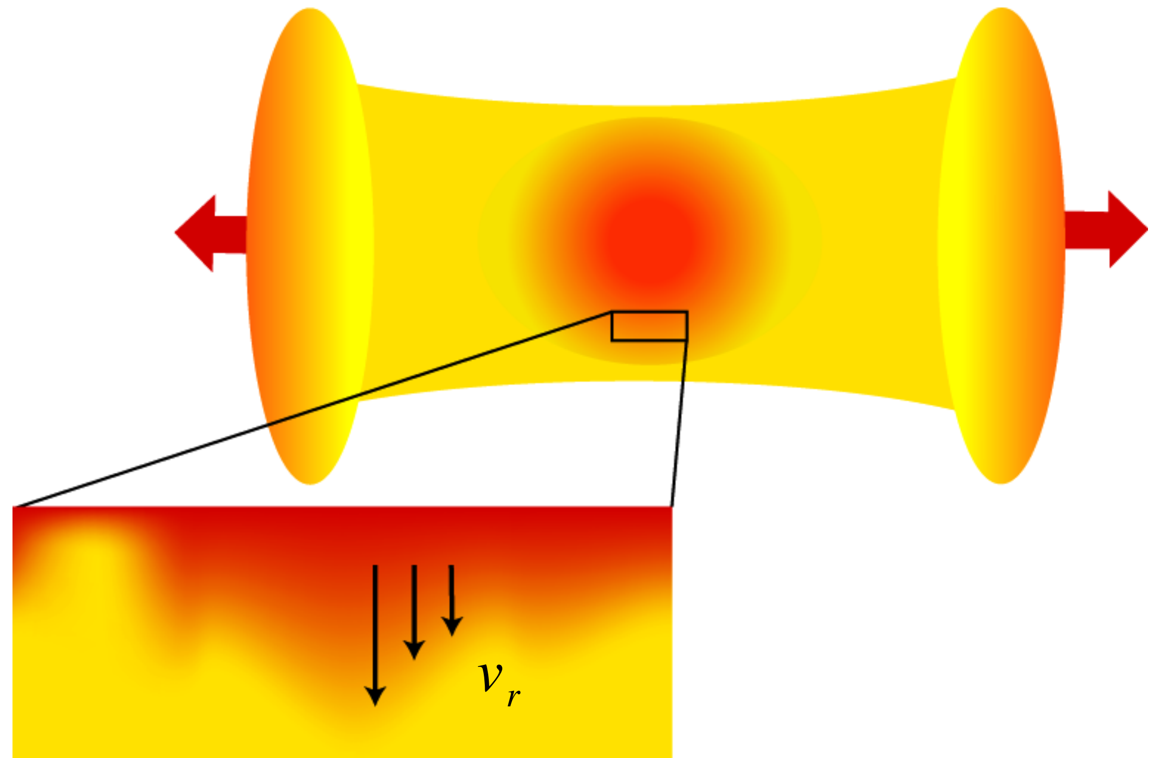
Rajendra Pokharel

George Moschelli

Transverse Flow Fluctuations

small variations in transverse flow in each event

fluid elements flow past one another
$\Rightarrow$ viscous friction

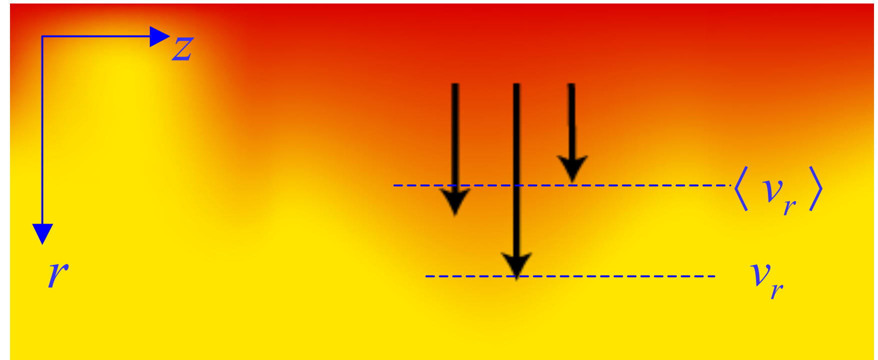


shear viscosity drives velocity toward the average

$$T_{zr} = -\eta\, \partial v_r / \partial z$$

damping of radial flow fluctuations $\Rightarrow$ viscosity

Evolution of Fluctuations



momentum current

small fluctuations

$$g_t \equiv T_{0r} - \langle T_{0r} \rangle$$

diffusion equation for momentum current

$$\frac{\partial}{\partial t} g_t = \frac{\eta}{sT} \nabla^2 \left(g_t + \text{noise} \right)$$

shear viscosity η, entropy density s, temperature T

linearized hydro, shear only $\Rightarrow$ diffusion;   small fluctuations $\Rightarrow$ Langevin noise

correlation function

$$r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

Hydrodynamic Momentum Correlations

momentum flux density correlation function

$$r = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

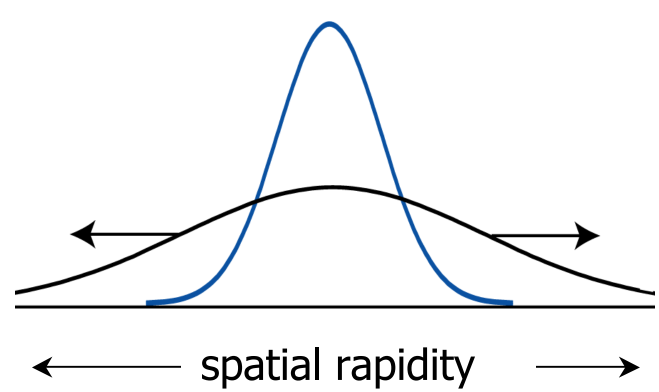
$\Delta r = r - r_{eq}$ **satisfies diffusion equation**

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

fluctuations diffuse through volume, driving $r \rightarrow r_{eq}$

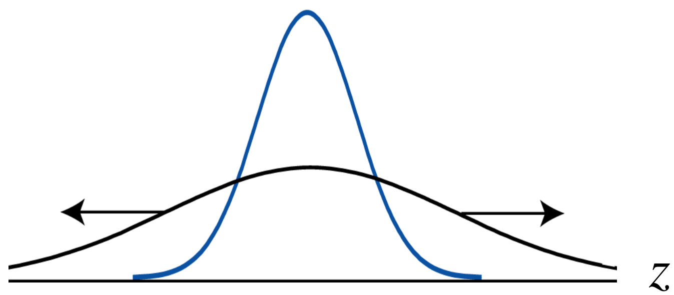
width in relative spatial rapidity $y = \sinh^{-1} z / \tau$
grows from initial value σ_0

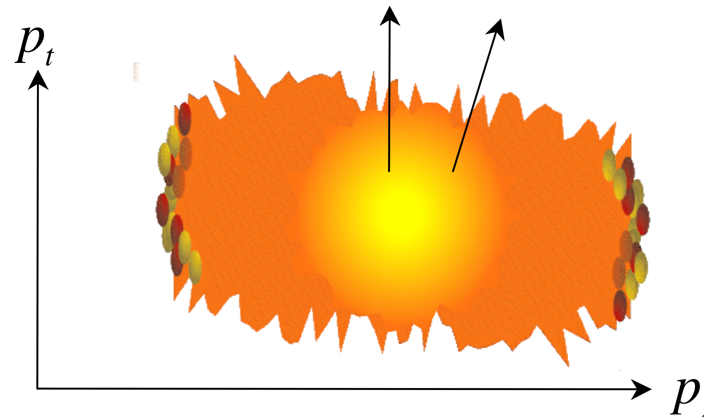
$$\sigma^2 = \sigma_0{}^2 + 4 \frac{\eta}{Ts} \left(\frac{1}{\tau_0} - \frac{1}{\tau} \right)$$



Measuring the Correlations

correlation function $r = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$



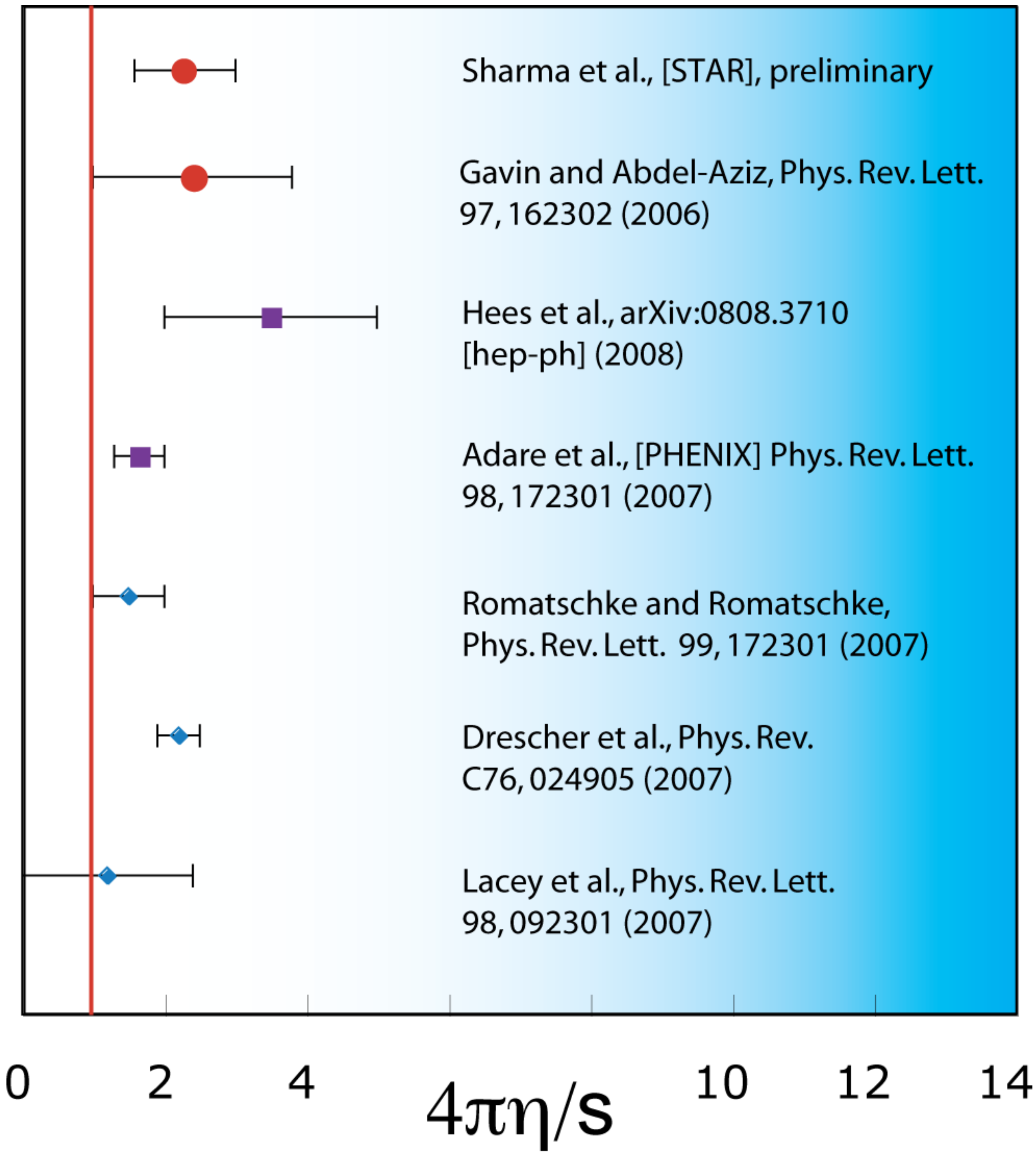


observable: $$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs}} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2$$

$$= \frac{1}{\langle N \rangle^2} \int (r - r_{eq})\, dp_1\, dp_2$$

C* in a rapidity interval ⇒ *η/Ts

– Gavin & Abdel-Aziz


Sharma et al., [STAR], preliminary
Gavin and Abdel-Aziz, Phys. Rev. Lett. 97, 162302 (2006)
Hees et al., arXiv:0808.3710 [hep-ph] (2008)
Adare et al., [PHENIX] Phys. Rev. Lett. 98, 172301 (2007)
Romatschke and Romatschke, Phys. Rev. Lett. 99, 172301 (2007)
Drescher et al., Phys. Rev. C76, 024905 (2007)
Lacey et al., Phys. Rev. Lett. 98, 092301 (2007)
0
2
4
10
12
14
4πη/s

p_t Covariance Measured

STAR Analysis PRELIMINARY

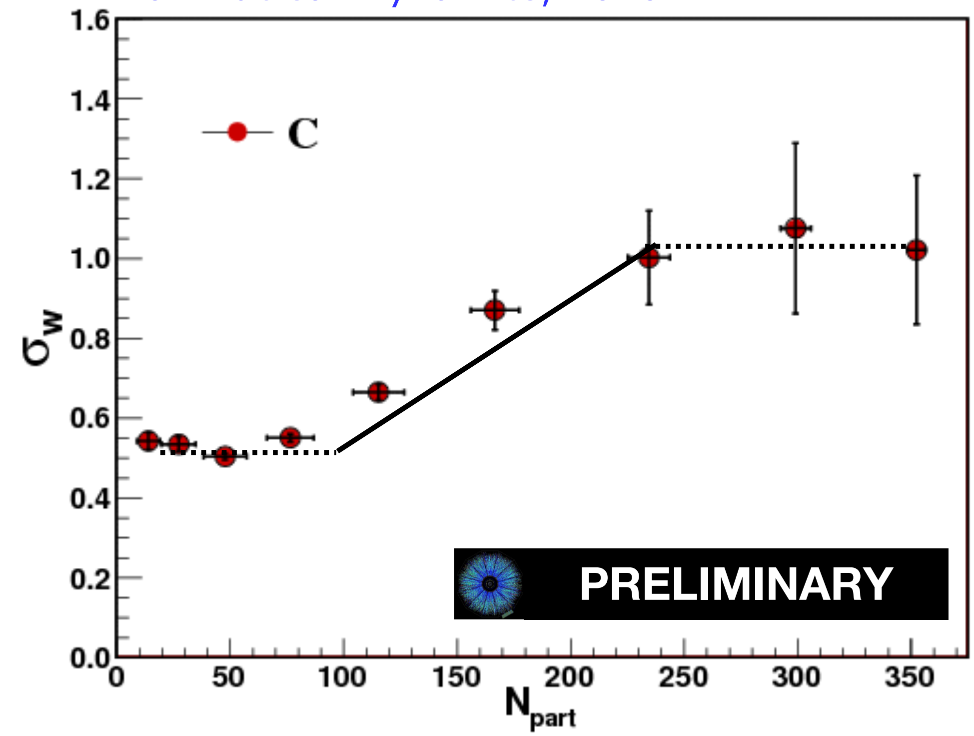
$$\sigma^2_{central} = \sigma_0^2 + 4\frac{\eta}{Ts}\left(\frac{1}{\tau_0} - \frac{1}{\tau_f}\right)$$

$$\sigma_{central} = 1.0 \pm 0.2$$

most peripheral ~ non-interacting

$$\Rightarrow \quad \sigma_{peripheral} \approx \sigma_0 = 0.54 \pm 0.02$$

Sharma, Pruneau et al., Winter Workshop on Nuclear Dynamics, 2010



formation time $\tau_0 \sim 1$ fm
freeze out $\tau_f \sim 20$ fm
$T \approx T_c \approx 170$ MeV

$$\Rightarrow \eta/s = 0.17 \pm 0.08$$

2nd Order Viscous Diffusion

causal transport equation:

$$\left(\tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\eta}{sT}\left(\nabla_1^2 + \nabla_2^2\right)\right)\Delta r_g = 0$$

linearized Israel-Stewart; see e.g. Song and Heinz 0805.1756

relaxation time $\tau_\pi \sim$ (mean free path)/(thermal speed)

kinetic theory $\Rightarrow$ $\tau_\pi = \beta\left(\eta / sT\right)$ $\beta \approx 5$

temperature vs time:

entropy production:
TdS/dt = viscous heating

$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

relaxation equation: causality delays heating; time scale $\sim\tau_\pi$

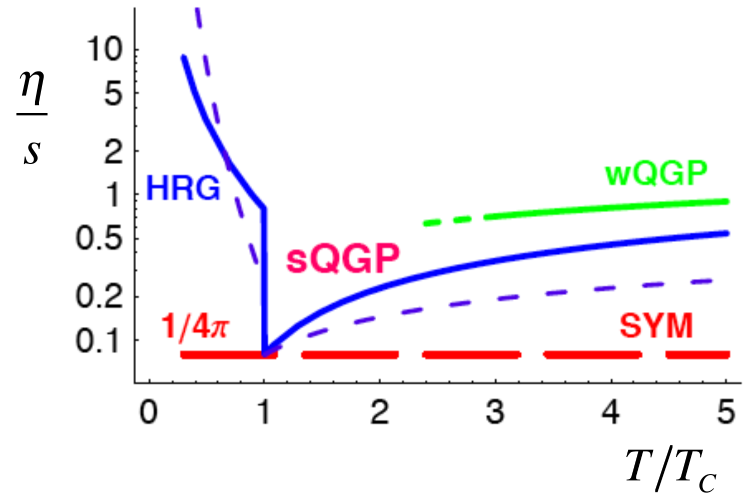
$$\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi}\left(\Phi - \frac{4\eta}{3\tau}\right) - \left[\frac{1}{\tau} + \frac{\eta T}{\tau_\pi}\frac{d}{d\tau}\left(\frac{\tau_\pi}{\eta T}\right)\right]\frac{\Phi}{2}$$

Low Viscosity Only Near T_c?

sQGP + hadronic corona – Hirano & Gyulassy

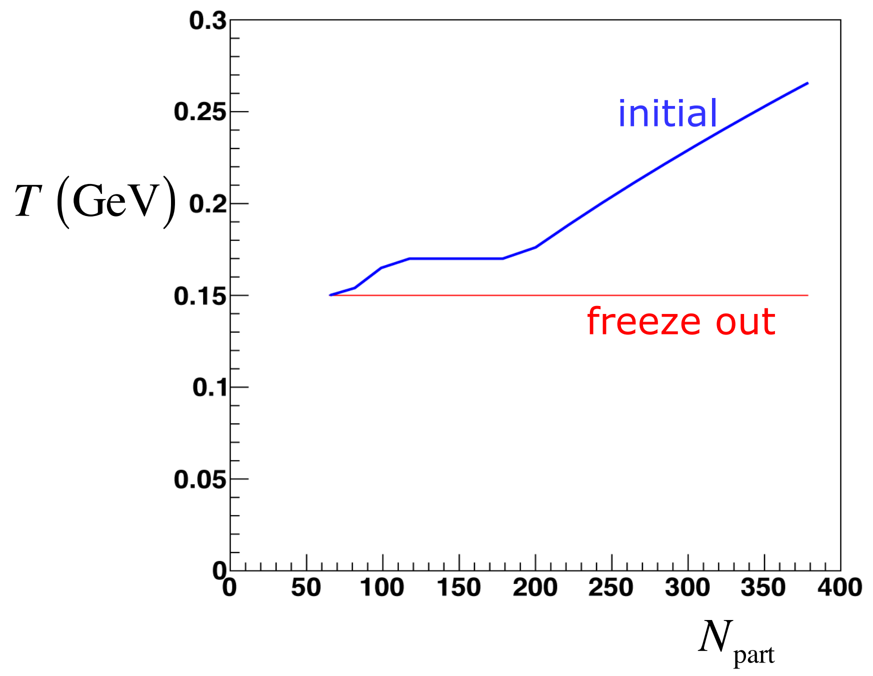
supersymmetric Yang-Mills: $\eta/s = 1/4\pi$

pQCD and hadron gas: $\eta/s \sim 1$



Evolution with varying viscosity

- η/s from H&G
- Bjorken flow + 2nd order hydro
- mixed phase $T \equiv T_c$
  $s = fs_Q + (1-f)s_H$
- freeze out proper time
  $\tau_F - \tau_0 \propto (R - R_0)^2; \quad \tau_F(b=0) = 10\ \text{fm}$
  $\tau_0 = 0.6\ \text{fm}, \quad T_F = 150\ \text{MeV}$



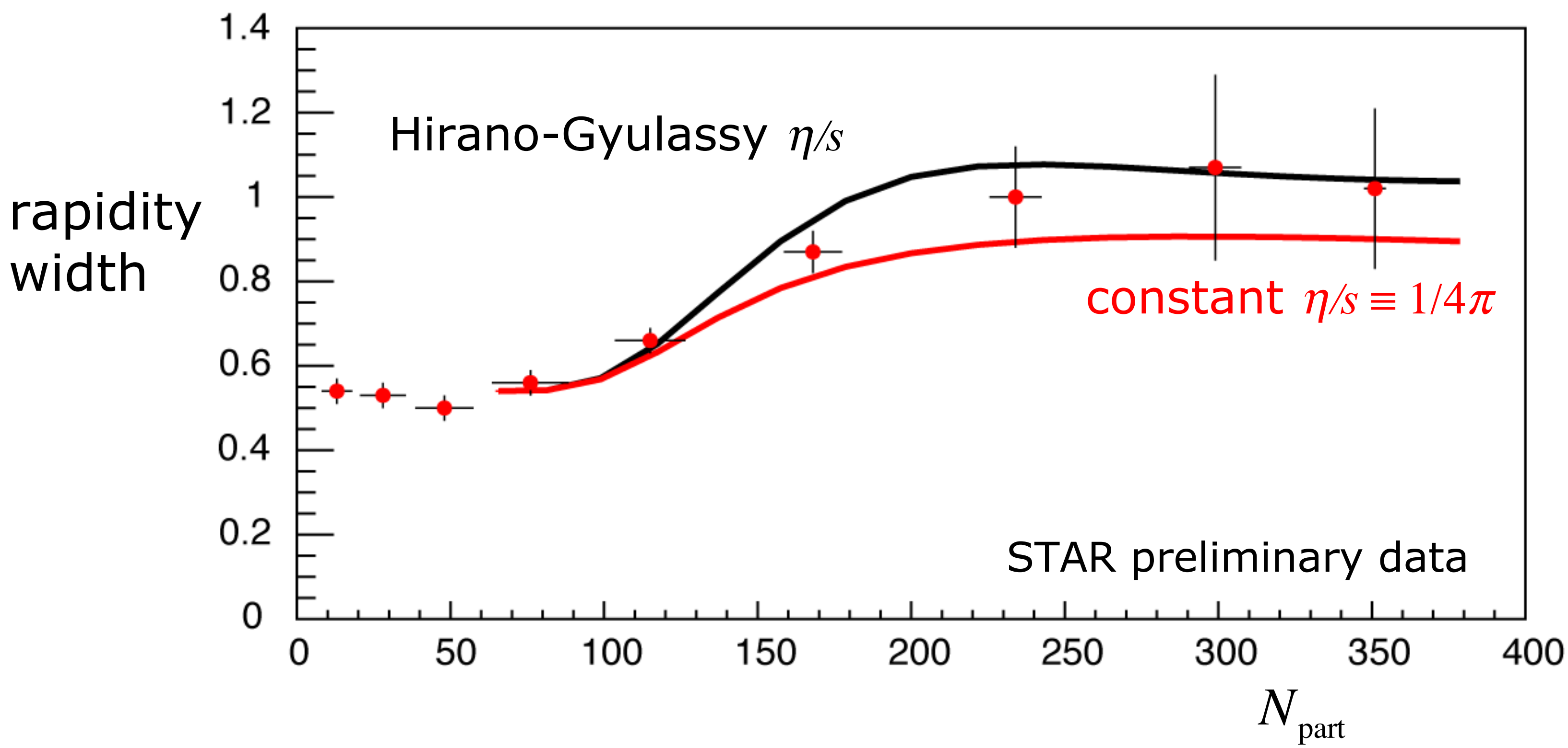
2^{nd} Order Diffusion with Realistic Viscosity

- Bjorken flow + 2nd order hydro
- mixed phase $T \equiv T_c$

- freeze out proper time and temperature

$$\tau_F - \tau_0 \propto (R - R_0)^2; \quad \tau_F(b=0) = 10\ \text{fm}$$
$$T_F = 150\ \text{MeV}$$

Moschelli, Pokharel, S.G



Find: higher initial $T_0 \Rightarrow$ smaller η/s in central collisions

Broadening of p_t Weighted Ridge

STAR Analysis PRELIMINARY

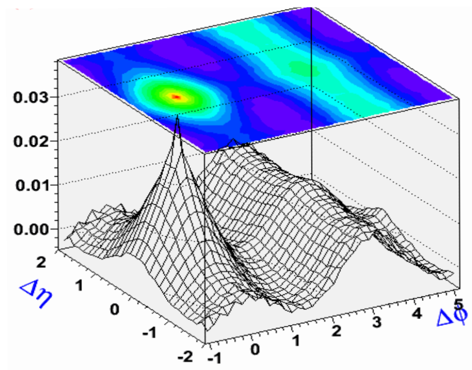
wanted: C integrated in ϕ

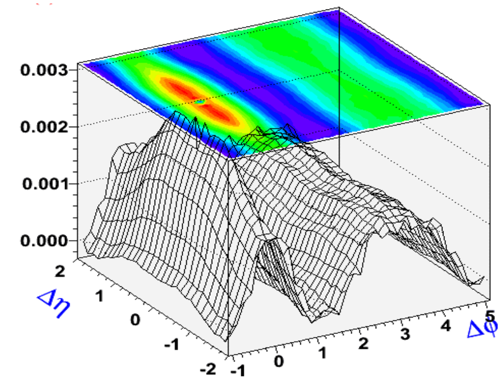
measured: width of near side peak -- the **soft ridge**

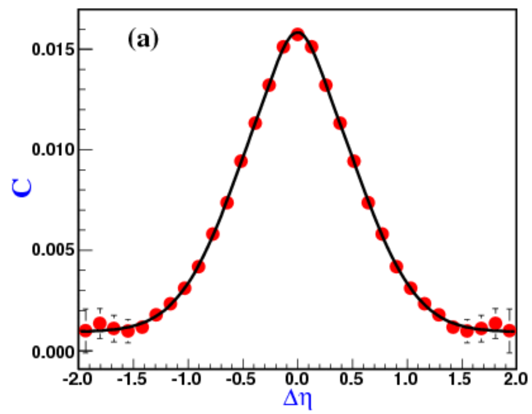
Distribution – further effects

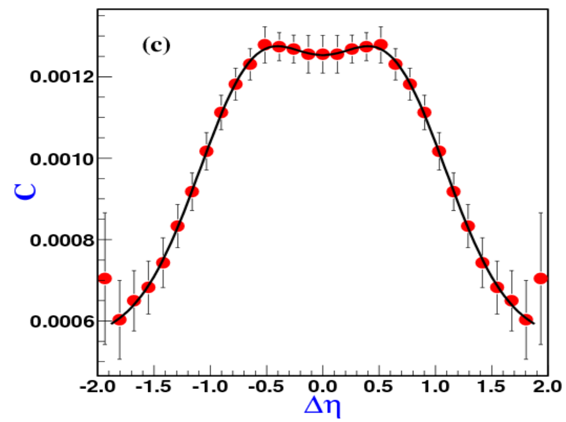
- momentum conservation v_1
- elliptic flow v_2
- triangularity v_3

Peripheral 70-80% | Central 0-5%



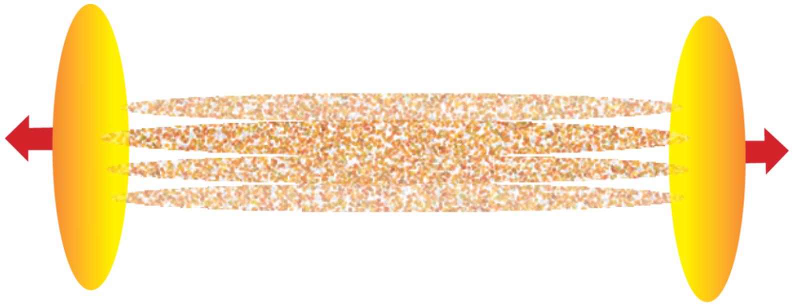






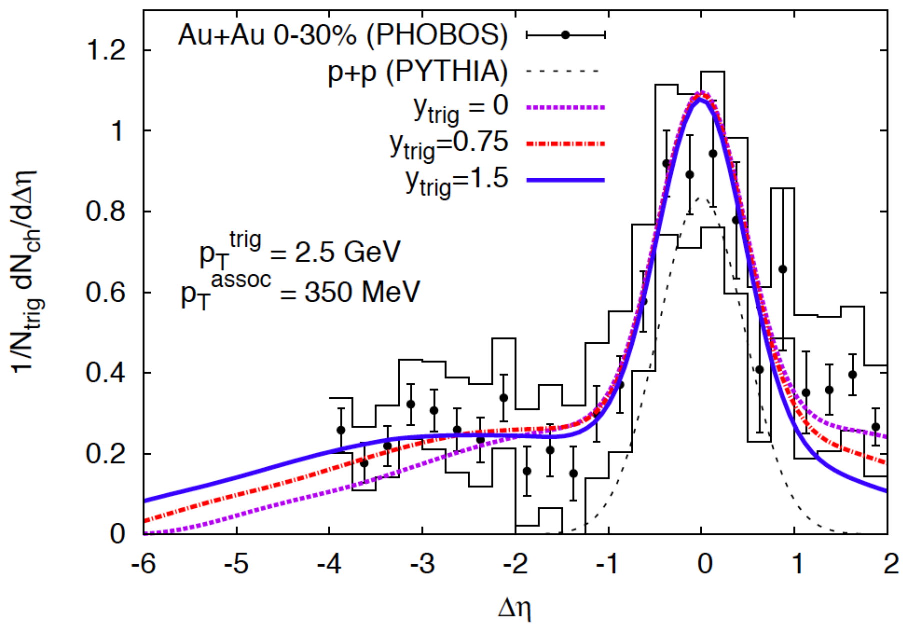
Ask: is **rapidity width of ridge** modified by **production mechanism** and effect of **transverse flow** at freeze out?

Initial Broadening: Glasma?



Does initial rapidity width σ_0 vary with centrality?

Glasma **explains** ridge height and azimuthal dependence in STAR
Dumitru, Gelis, McLerran & Venugopalan;
SG, McLerran & Moschelli



Glasma **explains** long range correlations in PHOBOS triggered ridge
Dusling, Gelis, Lappi, Venugopalan

but: rapidity dependence doesn't change with centrality in STAR acceptance $-1 < \eta < 1$

$\Rightarrow$ **rapidity-independent pedestal**

Do other production mechanisms vary with centrality?
strings, ropes, ladders $\rightarrow$ Pruneau's talk

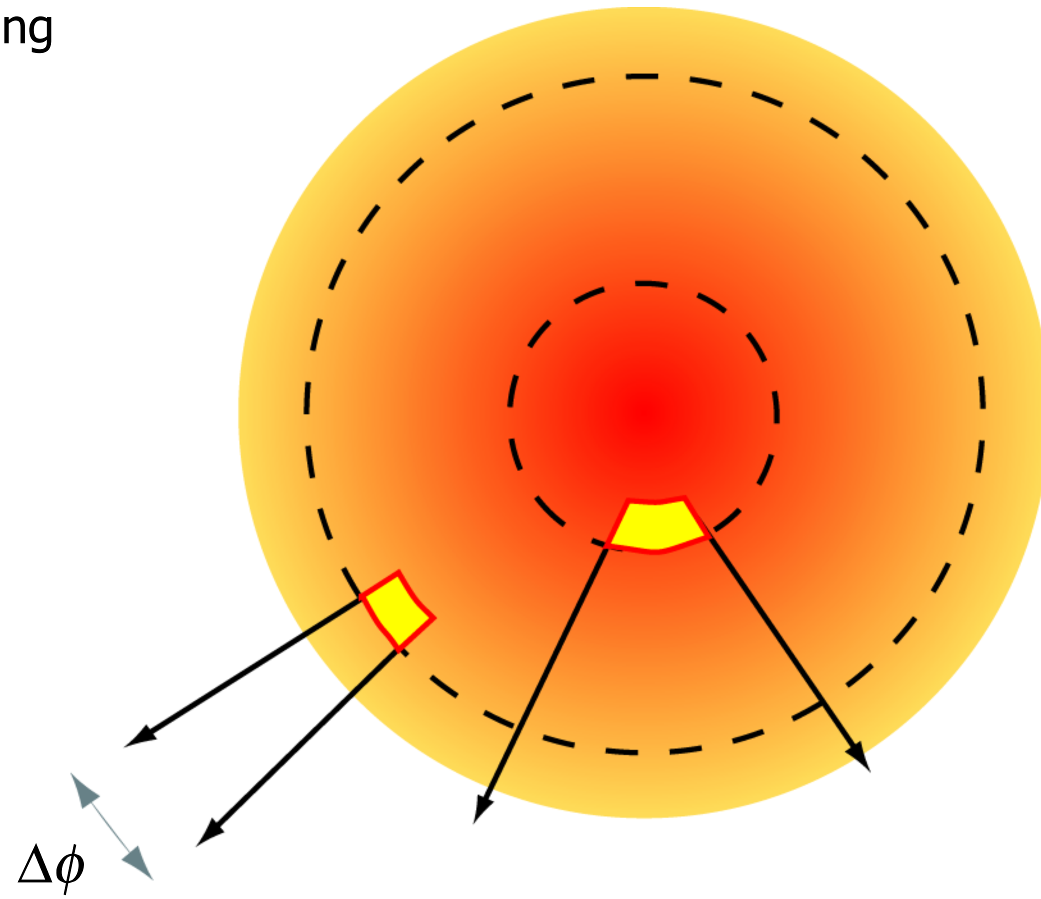
Transverse Flow $\Rightarrow$ Ridge

bulk correlations – longitudinal string fragmentation

flux tube position $\vec{r}$

transverse boost
thermalization and flow
$\vec{v}_t \sim \lambda \vec{r}$

flow $\Rightarrow$ narrow azimuthal opening angle
$\Delta\phi \sim v_{th} / v_t \sim (\lambda r)^{-1}$



similar longitudinal narrowing
$y = \tanh^{-1}\left(p_z / E\right)$

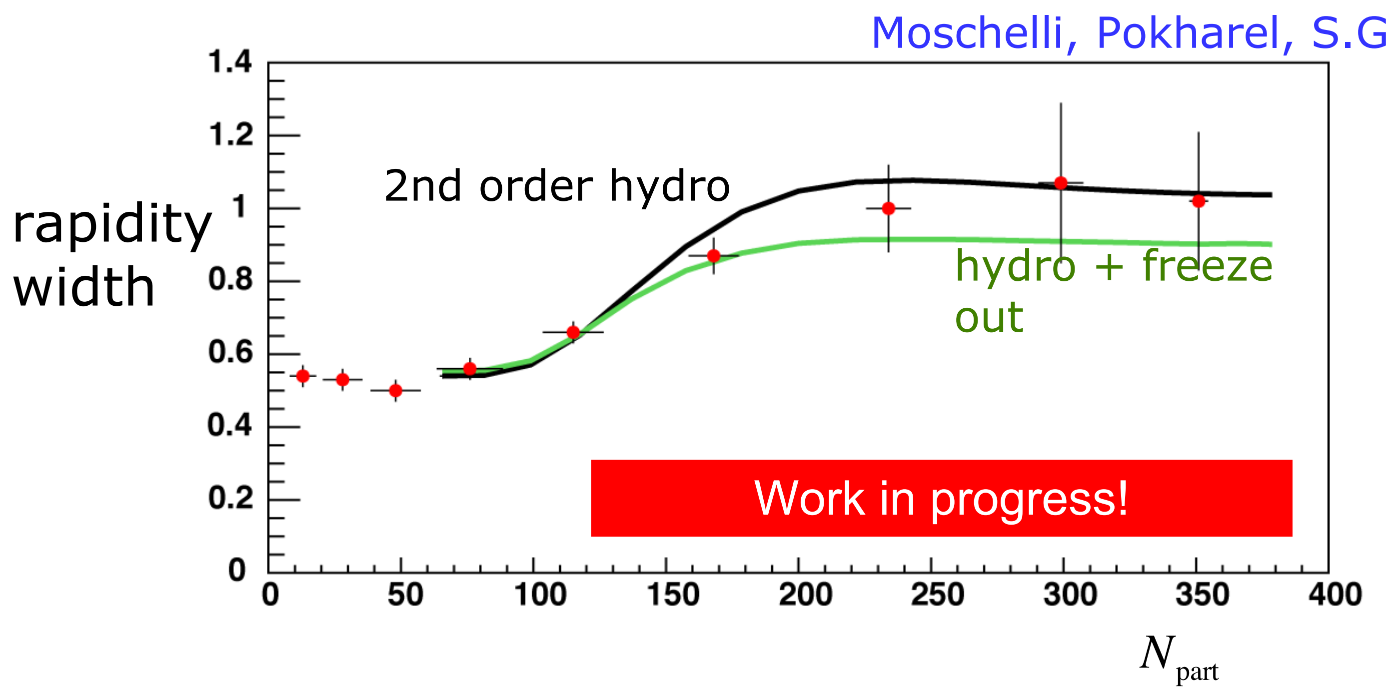
Voloshin; Pruneau, Gavin, Voloshin; Gavin, Moschelli, McLerran; Shuryak; Mocsy & Sorenson

Production and Freeze Out $\Rightarrow$ Broadening

Freeze Out: spatial rapidity η_1, η_2 $\rightarrow$ range of momentum space y_1, y_2

Cooper Frye: $$C \propto \iint_{\text{freezeout surface}} f\left(y_1-\eta_1\right) f\left(y_2-\eta_2\right) \Delta r(\eta_1,\eta_2)$$

Boltzmann f; blast wave T and v_r from Moschelli & S.G., arXiv:0806.4366



Find: flow effect on **rapidity width** comparable to T dependence of η/s

Summary: viscosity from p_t correlations

Viscosities from different observables

- viscosity broadens momentum correlations in rapidity
  - integrates over history $\rightarrow$ hadronic contribution
  - depends only on shear viscosity; no bulk contribution
- v_2 sensitive to early time
- combined info $\Rightarrow$ more complete picture

Causal transport explains STAR with realistic viscosity

Ridge measurement requires ridge tools

- Production: rapidity width must increase in STAR range
  - long range Glasma correlations don't work $\rightarrow$ Dusling et al.
  - other mechanisms might $\rightarrow$ see Pruneau's talk
- Freeze out: transverse flow effects at freeze out ~10% level
- beam energy scan + identified particles can distinguish contributions

Covariance ⇒Momentum Flux

covariance $$C = \frac{1}{\langle N\rangle^2}\left\langle \sum_{\text{pairs } i\neq j} p_{ti}p_{tj}\right\rangle - \langle p_t\rangle^2$$

unrestricted sum: $$\sum_{\text{all} i,j} p_{ti}p_{tj} = \int p_{t1}p_{t2}dn_1dn_2$$

$$= \int dx_1dx_2\left(\int dp_1\, p_{t1}f_1\right)\left(\int dp_2p_{t2}f_2\right)$$

$$\left.\begin{array}{l} dn = f(x,p)dpdx \\ g_t(x) = \int dp\ p_t\,\Delta f(x,p) \end{array}\right\}$$

$$= \langle N\rangle^2\langle p_t\rangle^2 + \int g(x_1)g(x_2)dx_1\,dx_2$$

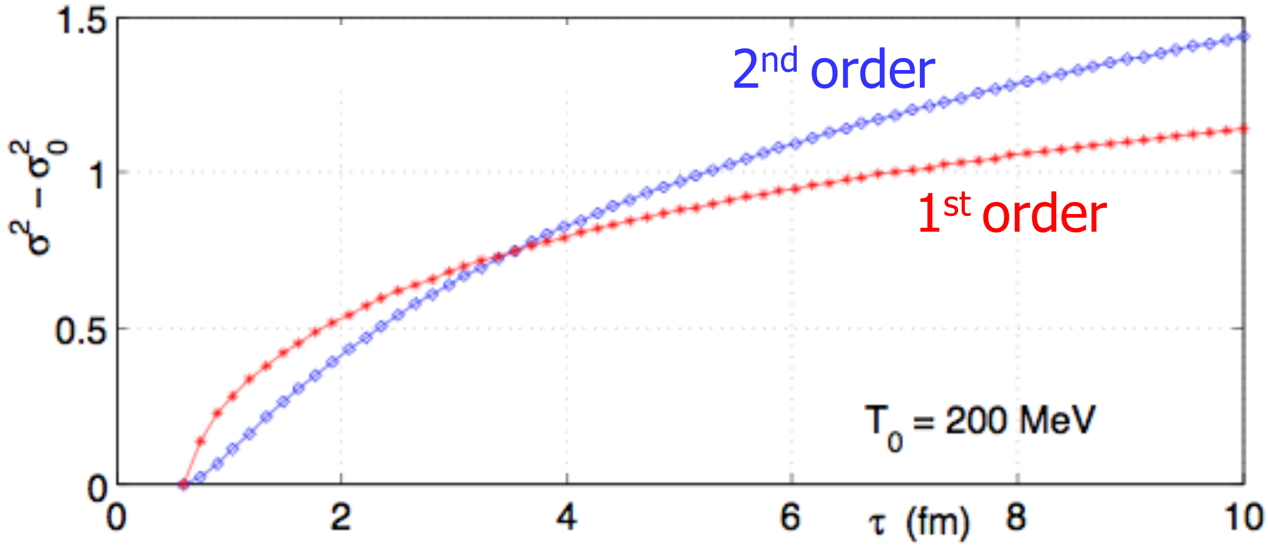
correlation function: $$r_g = \langle g_t(x_1)g_t(x_2)\rangle - \langle g_t(x_1)\rangle\langle g_t(x_2)\rangle$$

$$\int r_g dx_1dx_2 = \left\langle \sum p_{ti}p_{tj}\right\rangle - \langle N\rangle^2\langle p_t\rangle^2 = \left\langle \sum p_{ti}{}^2\right\rangle + \langle N\rangle^2 C$$

$C = 0$ in equilibrium ⇒ $$C = \frac{1}{\langle N\rangle^2}\int (r_g - r_{g,eq})dx_1dx_2$$

Causal Crosses Classic

classic (1st order) > causal (2nd order) for constant coefficients



$$\left(\tau_\pi \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \frac{\eta}{sT}\left(\nabla_1^2 + \nabla_2^2\right)\right)\Delta r_g = 0$$

$$\left.\begin{aligned} \tau_\pi &\sim (\sigma_{mom} n)^{-1} \\ n &= N/V \propto t^{-1} \end{aligned}\right\} \Rightarrow \tau_\pi \propto \eta/sT \propto \tau \;\Rightarrow$$

- wave-like expansion as $\tau \to \infty$
- 2nd order dominates