Theoretical Understanding of Low x Physics

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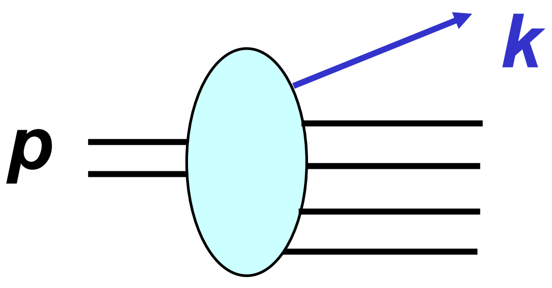
Outline of the Talk

- ❑ The low x physics – why it is interesting
- ❑ Locality of the probes
- ❑ Coherent QCD multiple scattering
- ❑ Dynamical power corrections
- ❑ Discussions and Conclusions

The low x physics

- x: momentum fraction of a parton of a boosted hadron (e.g., a nucleon)



$$k^\mu = x p^\mu + \frac{k^2 + k_T^2}{2 x p \cdot n} n^\mu + k_T^\mu$$

- PQCD ⇔ hard probes
  – collisions with large invariants of 4-momentum exchange: Q^2 > several GeV^2

$$x p \cdot n \sim Q \gg \Lambda_{\rm QCD} \sim \frac{1}{\rm fm}$$

The low x physics (II)

- **Size of a hard probe is very localized and much smaller than a typical hadron at rest**

$$1/Q \ll 2r_0 \sim \mathrm{fm}$$

- **But, it might be larger than a Lorentz contracted hadron:**

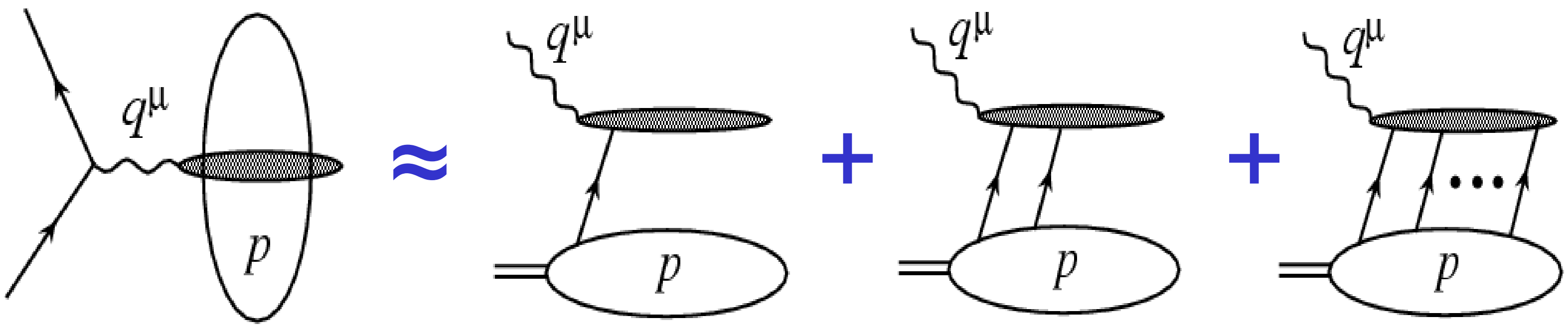
$$1/Q > 2r_0\left(m/p\right)$$

- **low x: uncertainty in locating the parton is much larger than the size of the boosted hadron (a nucleon)**

$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2r_0\frac{m}{p} \quad \Rightarrow \quad x \ll x_c \equiv \frac{1}{2mr_0} \approx 0.1$$

The low x physics (III)

- IF $x<x_c$, a hard probe can interact **coherently** with **more than one** low x partons at a same impact parameters

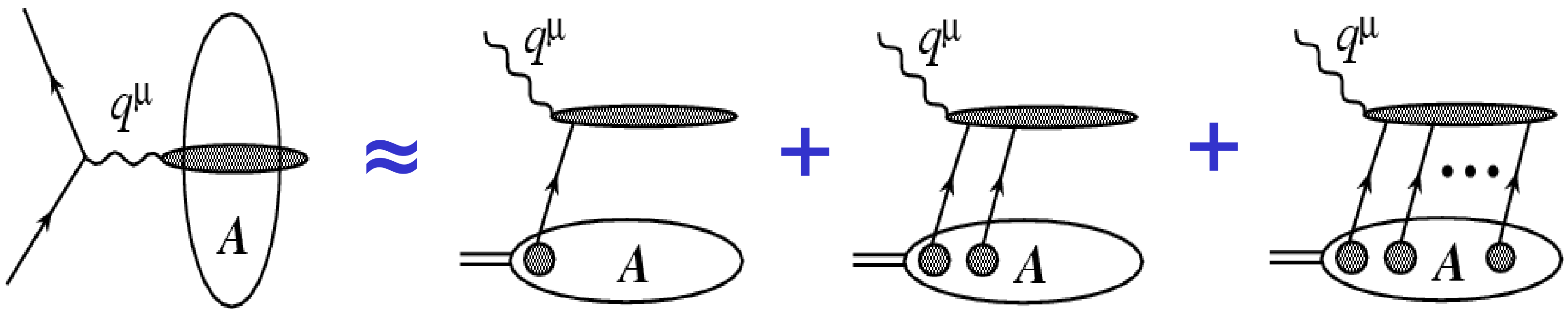


- Each additional **coherent** scattering is suppressed by a factor: $C_p\left(\frac{1/Q^2}{r_0^2}\right)$

  - Coefficient: $C_p \sim \alpha_s(Q)$

The low x physics (IV)

- For a nucleus, if $x \ll x_c / A^{1/3} \approx 0.1 / A^{1/3}$, the probe cannot tell which nucleon the parton is from

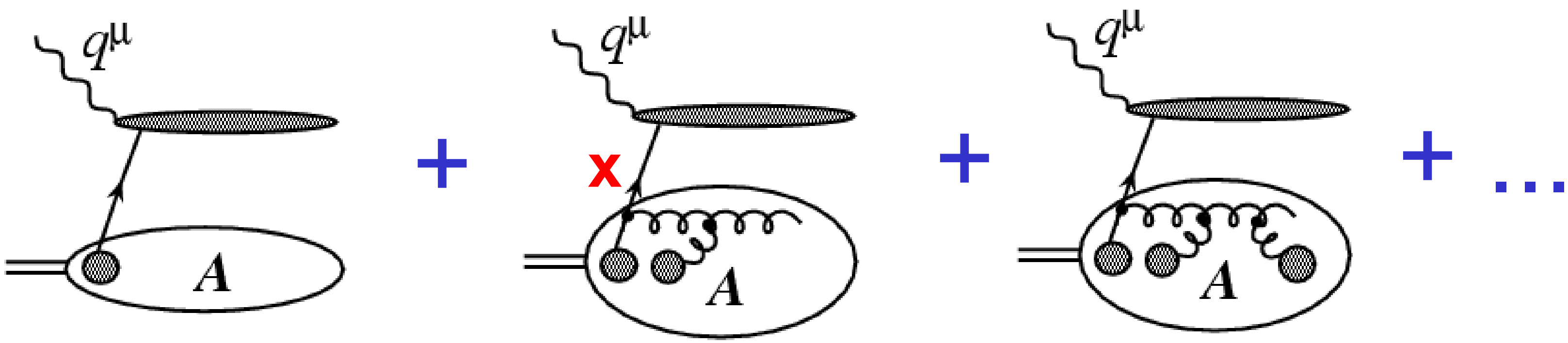


- Each additional coherent scattering is suppressed by a factor: $C_A \left(\frac{1/Q^2}{r_0^2} \right)$

- Coefficient: $C_A \sim \alpha_s(Q) A^{1/3}$

The low x physics (V)

- Low x partons can also interact coherently among themselves if there are more than one at a given impact parameter



- Interaction among low x partons produces a collective feature of whole nucleus

- How many partons are there in a hadron?

- How different coherent interactions contribute to a physical cross section?

Number of partons in a hadron

- depends on definition of parton distributions
- **Collinear factorization** $\Leftrightarrow \quad Q \sim xp \cdot n \gg k_T, \sqrt{k^2}$

Provides systematic ways to quantify high order corrections

Leading Twist

$$\sigma^h_{phys} = \hat{\sigma}^i_2 \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_2^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_4^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_6^{i/h}(x)$$
$$+ \ldots$$

perturbative

Power corrections

Factorization fails at $1/Q^4$ in hadronic collisions

- **K_T–factorization**

$$\Leftrightarrow \quad Q \sim xp \cdot n \approx k_T \gg \sqrt{k^2}$$

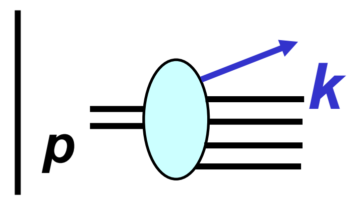
$$\sigma^h_{phys} = \hat{\sigma}^i_2(x, k_T) \otimes T_2^{i/h}(x, k_T)$$

No all-order proof for k_T - factorization

Parton Distribution Functions (PDFs)

- PDF, $\varphi(x,\mu^2)$, is a number density to find a parton of flavor φ = quark, antiquark, gluon with a momentum fraction x at a factorization scale μ^2.

$$\varphi(x,\mu^2) \propto \int d^4k\ \delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\left(\left|\ \cdots\ \right|^2 - \mathrm{UVCT}(\mu^2)\right)$$

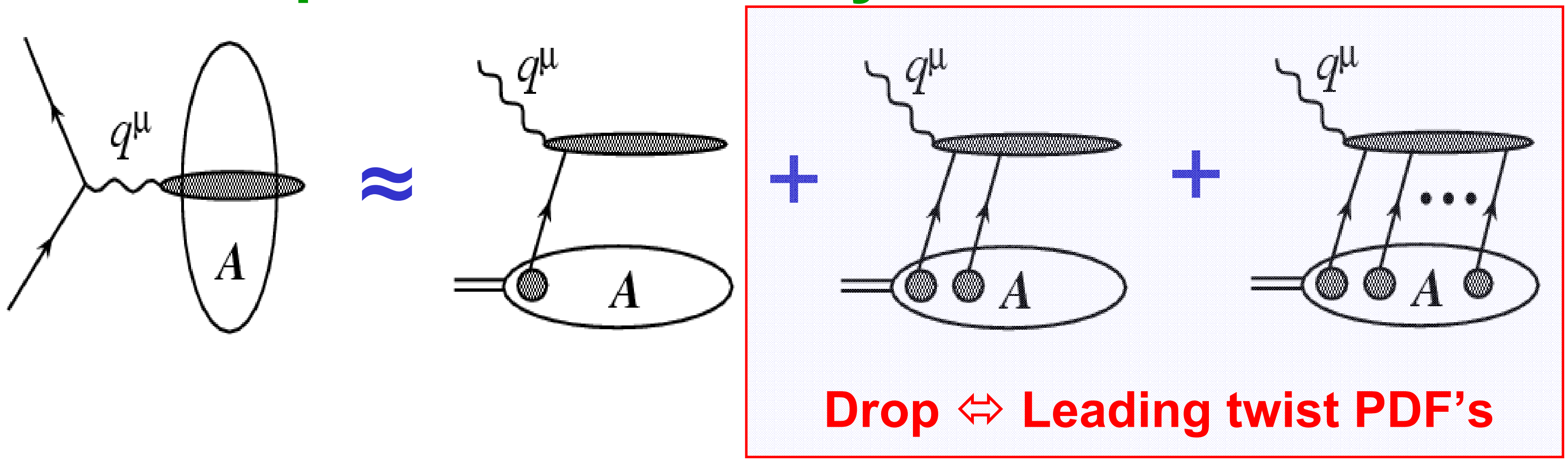


- PDFs depend **choices** of factorization scheme:
  $\overline{\mathrm{MS}}$, MS, DIS, etc.

  CTEQ6M, CTEQ6D, CTEQ6L, …
  MRST, GRV, …

Parton Distribution Functions (II)

❑ **PDFs depend on how they were extracted!**



❖ **the order of perturbative part used to compare with data: LO PDFs, NLO PDFs, …**

❖ **momentum scale of the probe: Q^2**

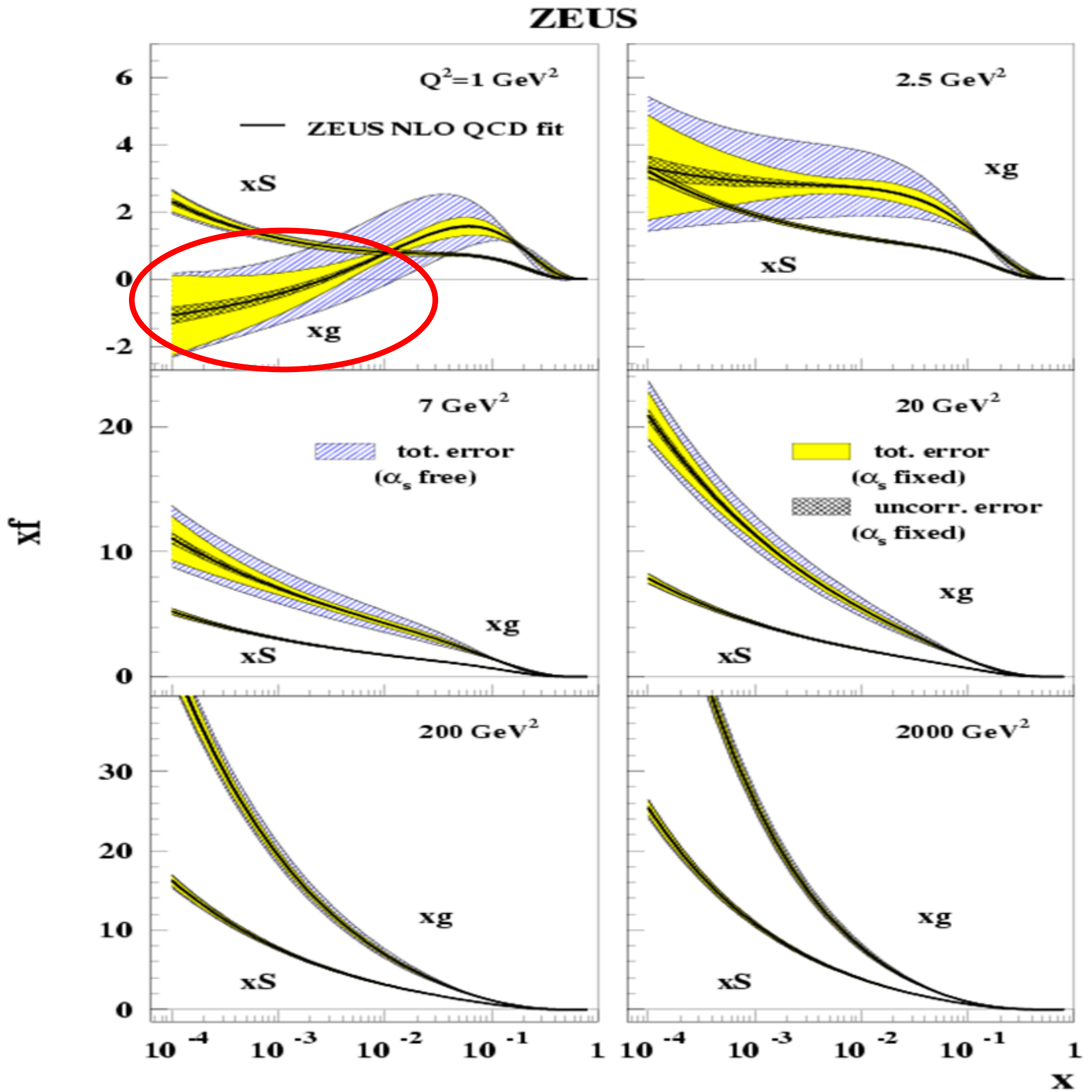
❖ **PDF's Q^2 dependence – evolution equations**

❑ **Beyond LO, PDFs do not have to be positive!**

Negative gluon distribution

- ❑ **NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution**

- ❑ **MRST PDF's have the same features**



Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

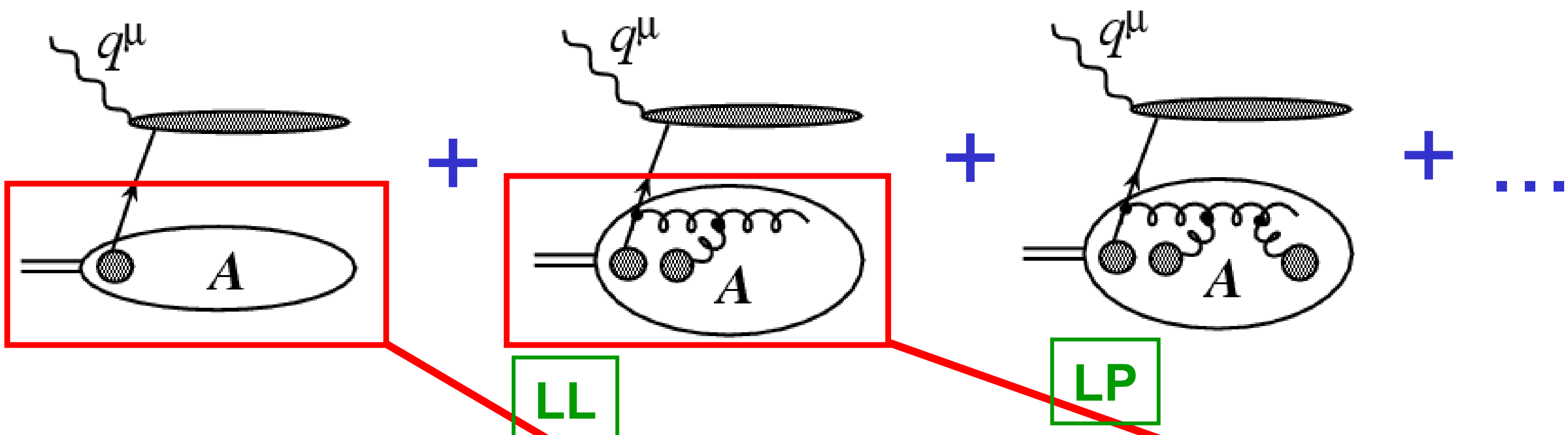
No!

Coherent power corrections to DGLAP

- Negative gluon distribution is not consistent with cross sections where gluon enters at LO order

F_L at low x and low Q^2, Low mass Drell-Yan at moderate Q_T, Low P_T direct photon, etc.

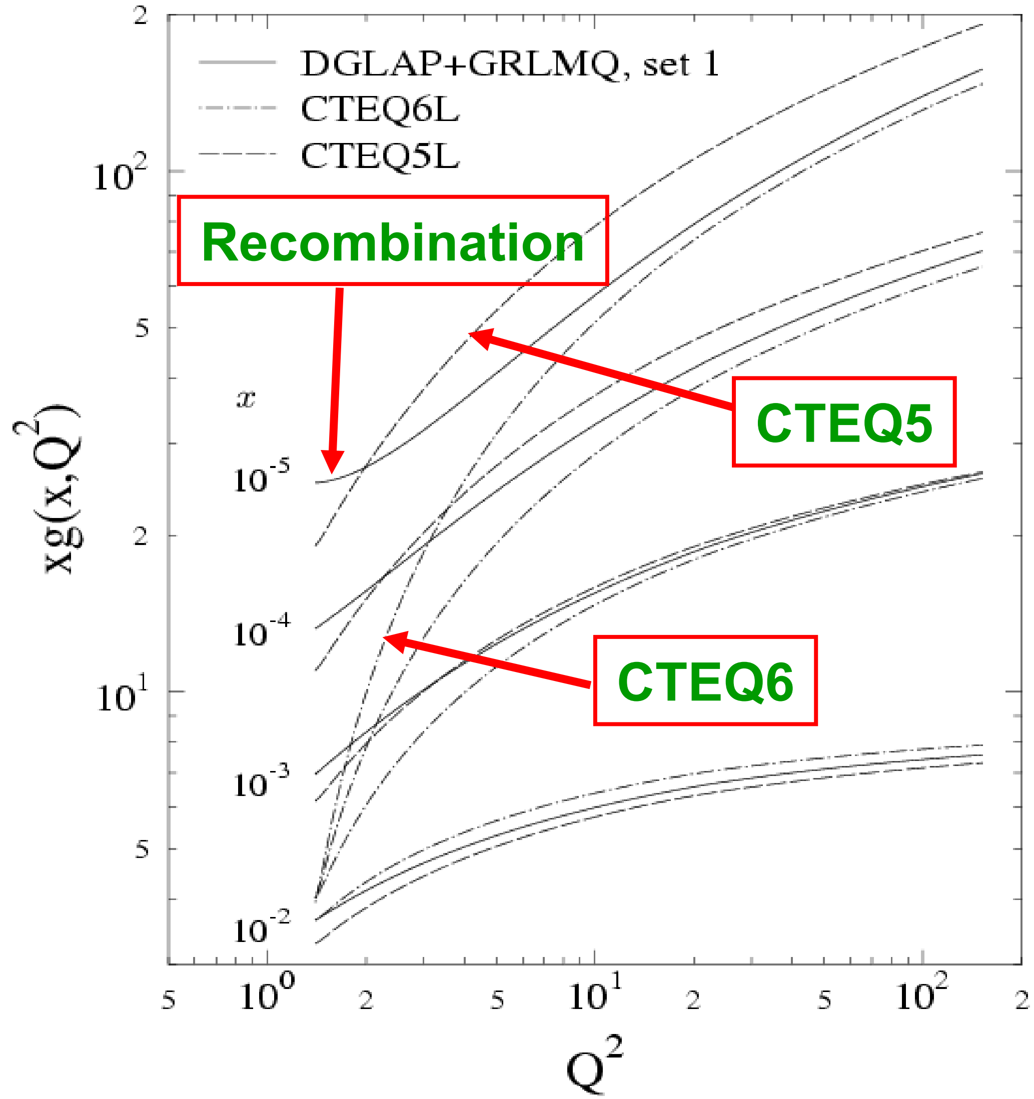
- Parton recombination slows down Q^2-dependence of DGLAP evolution



$$\frac{\partial g(x,\mu^2)}{\ell n \mu^2} \propto \frac{\alpha_s}{\pi}\gamma(x)\otimes g(x,\mu^2) - \frac{\alpha_s^2}{\mu^2}\bar{\gamma}(x)\otimes g^2(x,\mu^2)$$

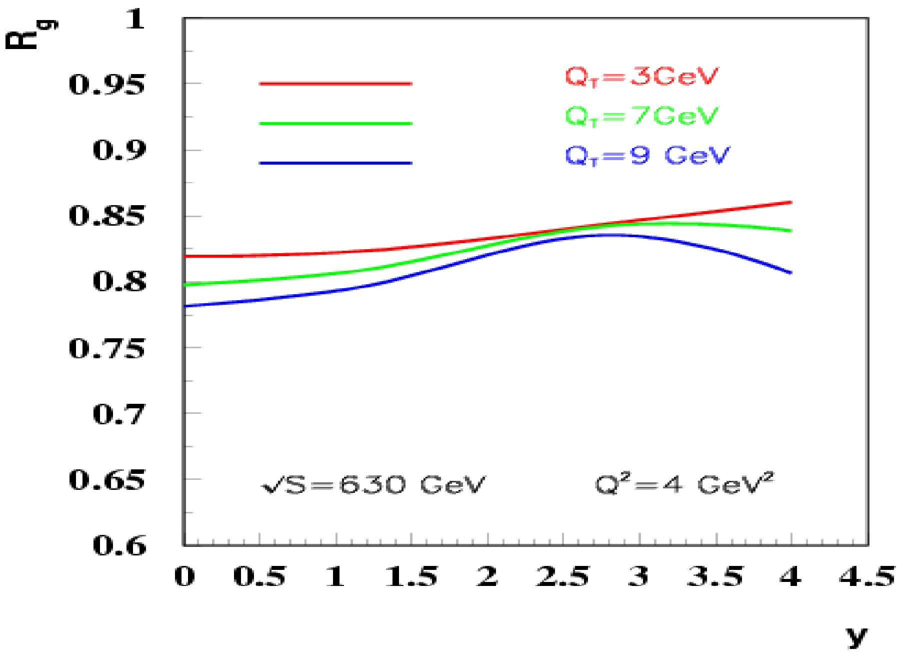
Recombination prevents negative gluon

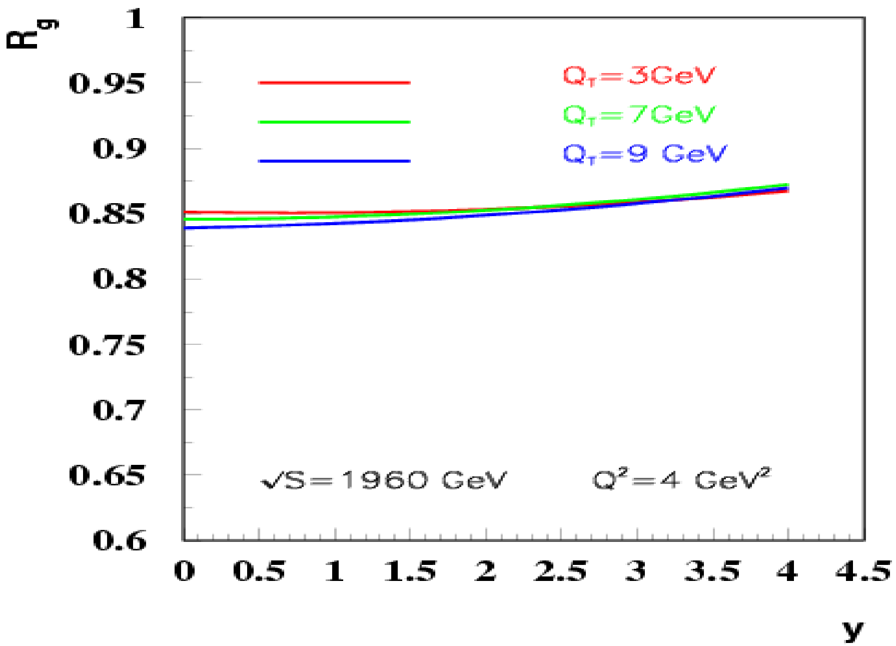
- In order to fit new HERA data, like MRST PDF’s, CTEQ6 gluon has to be much smaller than CTEQ5, even negative at Q = 1 GeV
- The power correction slows down the Q^2-dependence, prevents PDF’s to be negative
- Low mass DY to give direct information on gluon



Low-mass Drell-Yan in forward region

- ❑ **Gluon-initiated subprocesses also dominate the production rate in the forward region**





- ❑ Gluon initiated sub-processes contribute even more in the forward region
- ❑ Forward region in rapidity y to probe the small-x gluon distribution

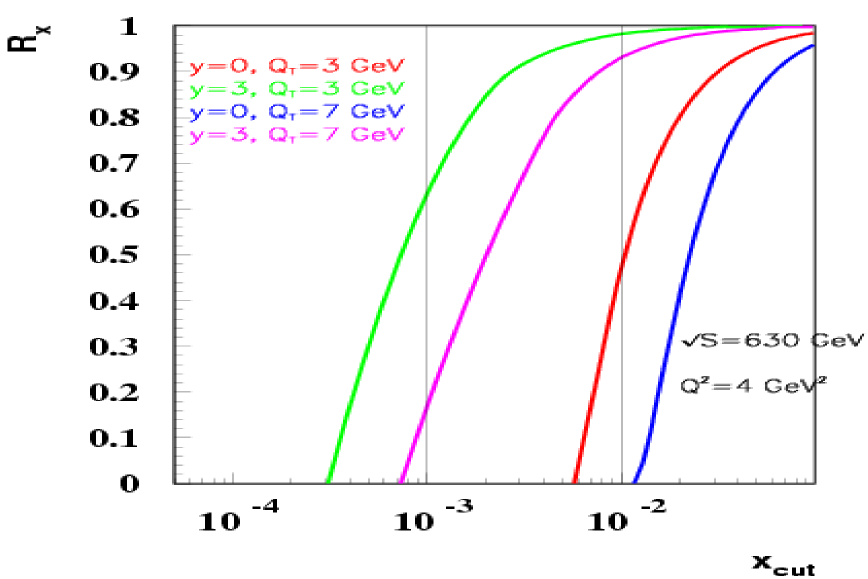
Fai, Qiu and Zhang

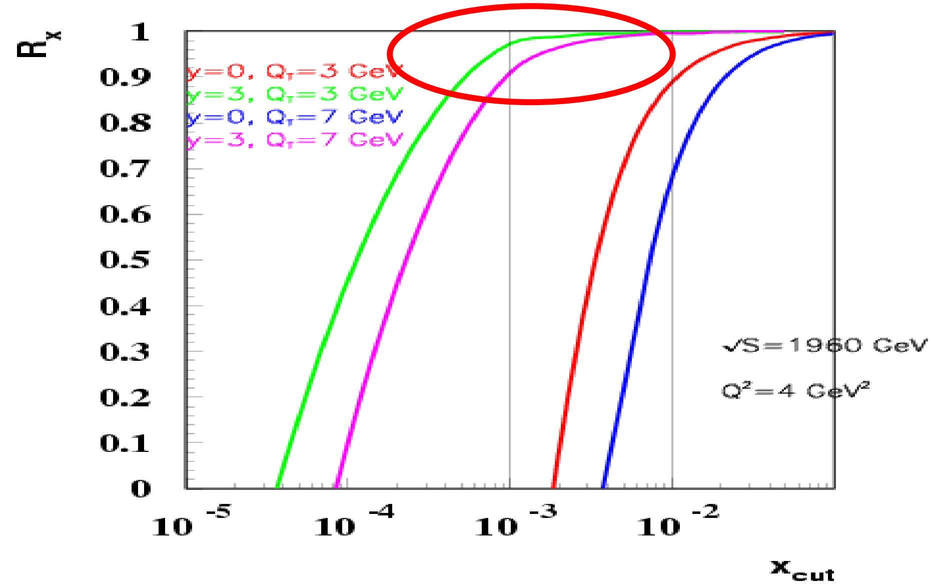
Drell-Yan dileptons in forward region

- Range of x probed:

Fai, Qiu and Zhang

$$R_x = \int_{x_{\min}}^{x_{\text{cut}}} dx_2 \left(\frac{d\sigma^{DY-LO}}{dx_2} \right) \Big/ \int_{x_{\min}}^{1} dx_2 \left(\frac{d\sigma^{DY-LO}}{dx_2} \right)$$





At √S=1.96 TeV and y=3, 90% of cross-section are given by gluon distribution with $x_2 < 0.001$!

At √S=630 GeV and y=3, 90% of cross-section are given by gluon distribution with $x_2 < 0.01$!

Nucleon is almost transparent

- **Number of gluons between x and x+Δx:**

$$n_g \equiv \int_x^{x+\Delta x} dx\, g(x,\mu^2) \approx xg(x,\mu^2) \approx 3 \text{ for } \mu^2 \approx 1\,\text{GeV}^2$$

$$\Rightarrow \quad \frac{n_g\left(1/\mu^2\right)}{r_0^2} \approx \frac{3}{25} \approx 0.12 \text{ for } \mu^2 \approx 1\,\text{GeV}^2$$

Higher μ^2 will not help much, n_g grows logarithmic in μ^2 unless x is very small

Large nuclei may increase n_g by as much as a factor of 6

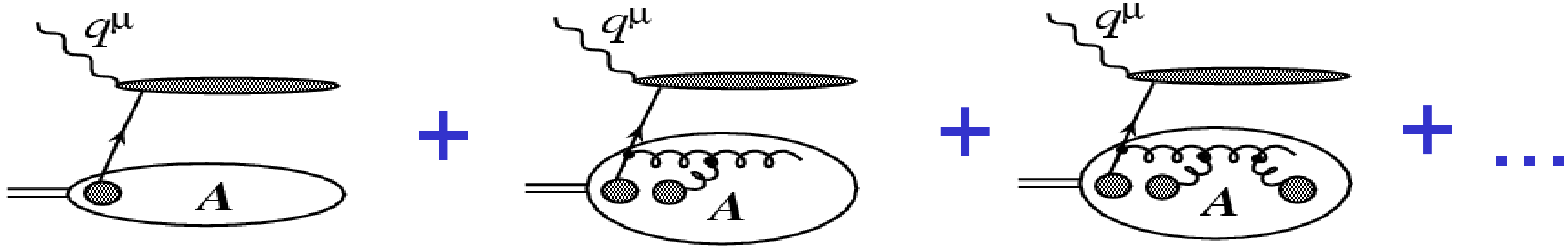
- Saturation: — Evolution of gluon distribution stops

$$\frac{\partial g(x,\mu^2)}{\ell n \mu^2} \propto \frac{\alpha_s}{\pi}\gamma(x) \otimes g(x,\mu^2) - \frac{\alpha_s^2}{\mu^2}\overline{\gamma}(x) \otimes g^2(x,\mu^2) \approx 0 \text{ at } \mu^2 = Q_s^2(x)$$

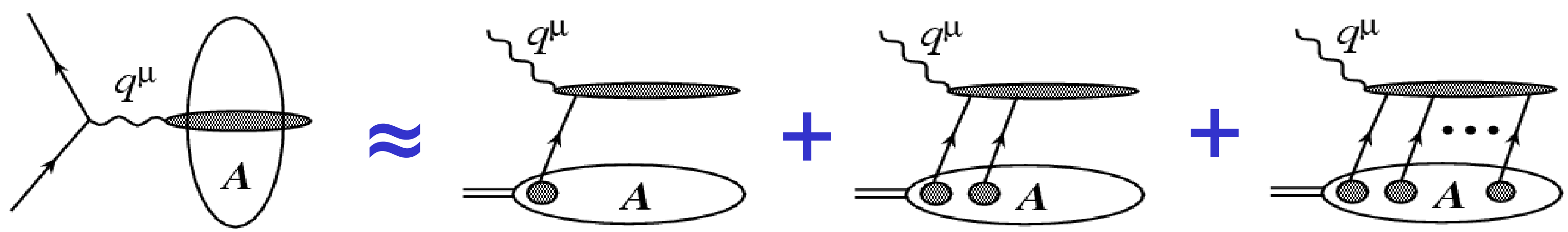
$\Rightarrow$ Saturation scale: $Q_s^2(x)$

Saturation and Color Glass Condensate

- If the coherent power corrections are so important to stop the evolution of parton distributions,



the coherent power corrections to the perturbative cross sections will be as large as the leading twist



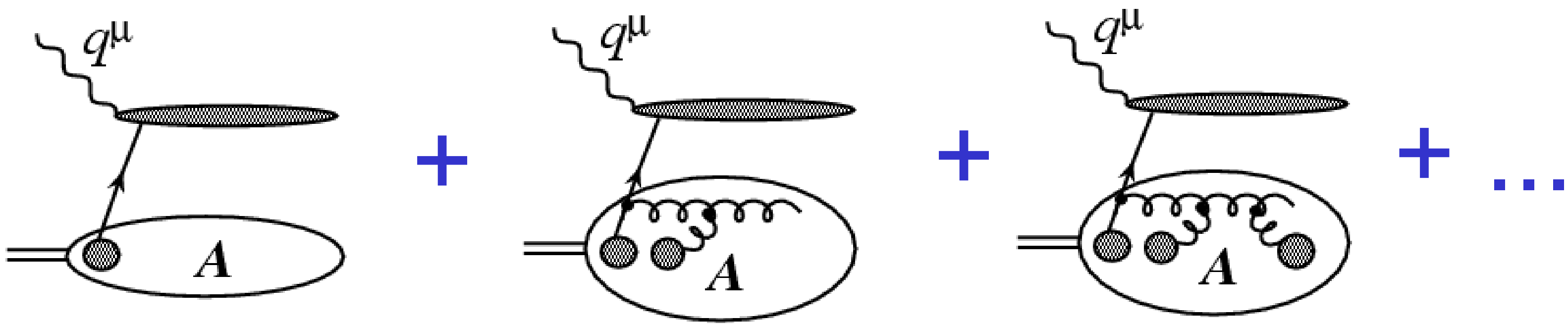
- Color glass condensate: —— Perturbative power expansion fails

Cannot extract the gluon distribution (defined here)
⇔ the probe will see more than one gluon at once

Coherent dynamical power corrections

When the probe size is larger than a Lorentz contracted large nucleus, the probe could interact coherently with any number of partons of the nucleus

- **If the probe interacts with only one parton,**

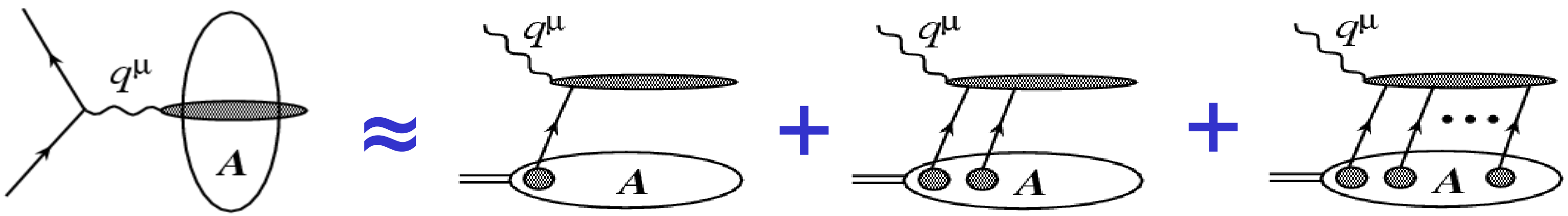


Nuclear dependence can only from the modified evolution equation and the input distributions needed to solve for the equation

→ **Shadowing in nPDFs ⇔ Leading twist shadowing**

Coherent dynamical power corrections (II)

- **If the probe interacts with many partons,**



Each additional coherent scattering is suppressed by a factor:

$$\frac{\xi^2}{Q^2}\left(A^{1/3}-1\right) \quad \text{with} \quad \xi^2 \propto \alpha_s \left\langle \tilde{F}^2 \right\rangle$$

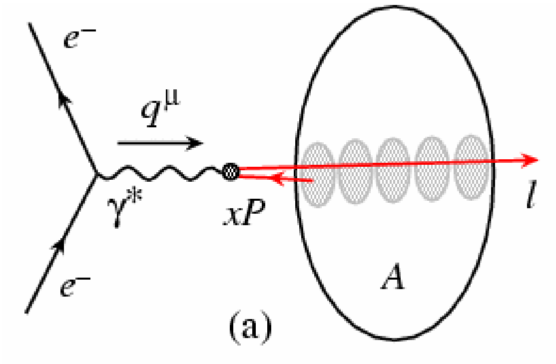
- **Resummation to connect partonic "hard" to high density "soft" physics**

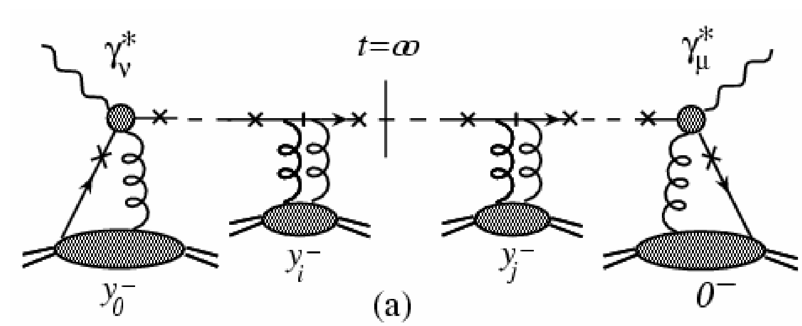
$$\sigma^h_{phys} = \hat{\sigma}^i_2 \otimes [1+\alpha_s+\alpha_s^2+\ldots] \otimes T_2^{i/h}(x) + \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1+\alpha_s+\alpha_s^2+\ldots] \otimes T_4^{i/h}(x) + \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1+\alpha_s+\alpha_s^2+\ldots] \otimes T_6^{i/h}(x) + \ldots$$

All power resummation

Dynamical power corrections in DIS

- Dynamical power corrections generated by the multiple **final state scattering** of the struck quark





- Coherence:

$$p^\mu = \left(p^+, p^-, p_T\right) = \left(p^+, 0, 0_T\right)$$

$$q^\mu = \left(q^+, q^-, q_T\right) = \left(-x_B p^+, Q^2/2x_B p^+, 0_T\right)$$

$$\text{If } \; q^+ = q^- \; \text{ and } x < x_c, \; \Delta y^+ \sim \frac{1}{xp} > 2r_0 \frac{m}{p}$$

The probe, virtual photon, interacts with all nucleons at a given impact parameter coherently

High twist shadowing

Resummed $A^{1/3}$-Enhanced Power Corrections

- Results:

$$F_T^A(x,Q^2) = \sum_{n=0}^{N} \frac{A}{n!}\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n \frac{d^n F_T^{(LT)}(x,Q^2)}{d^n x} \approx A\, F_T^{(LT)}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right)$$

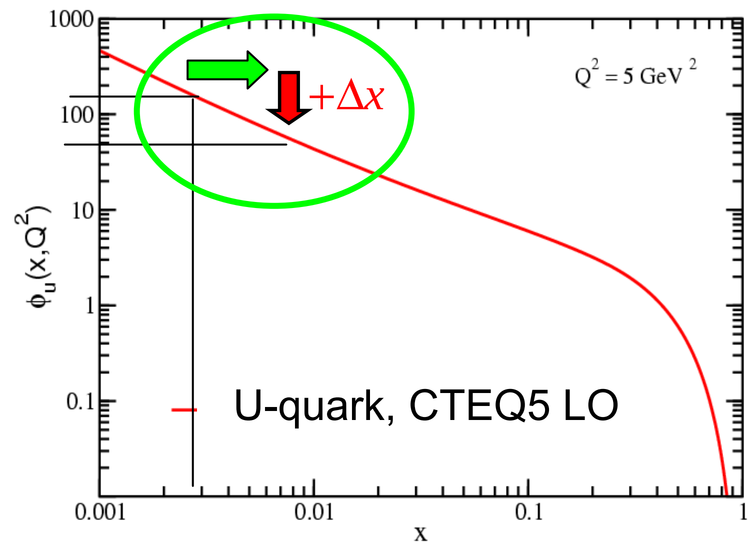
$$F_L^A(x,Q^2) = A\, F_L^{(LT)}(x,Q^2) + \sum_{n=0}^{N} \frac{A}{n!}\left(\frac{4\xi^2}{Q^2}\right)\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n \frac{d^n F_T^{(LT)}(x,Q^2)}{d^n x}$$

$$\approx A\, F_L^{(LT)}(x,Q^2) + \frac{4\xi^2}{Q^2} F_T^A(x,Q^2)$$

- One parameter – scale of power correction

$$\xi^2 = \left(\frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2}\right)\left\langle p\left|\hat{F}^2(\lambda_i)\right|p\right\rangle$$

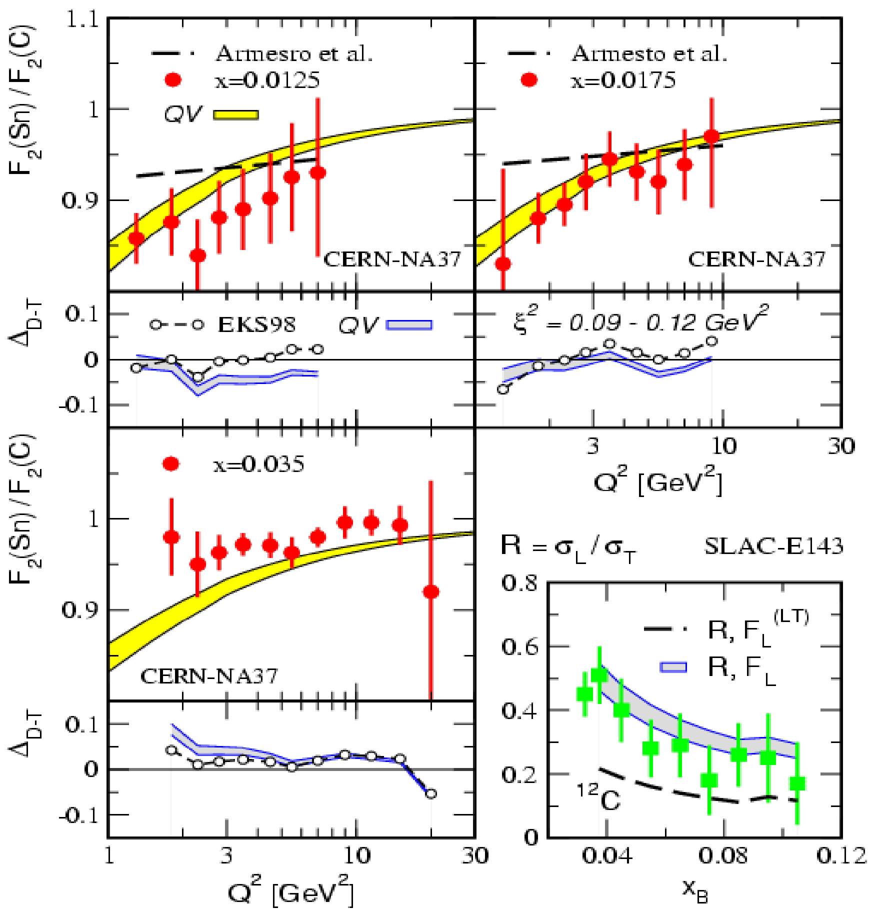
$$\frac{1}{2}\lim_{x\to 0} xG\left(x,Q^2\right)$$

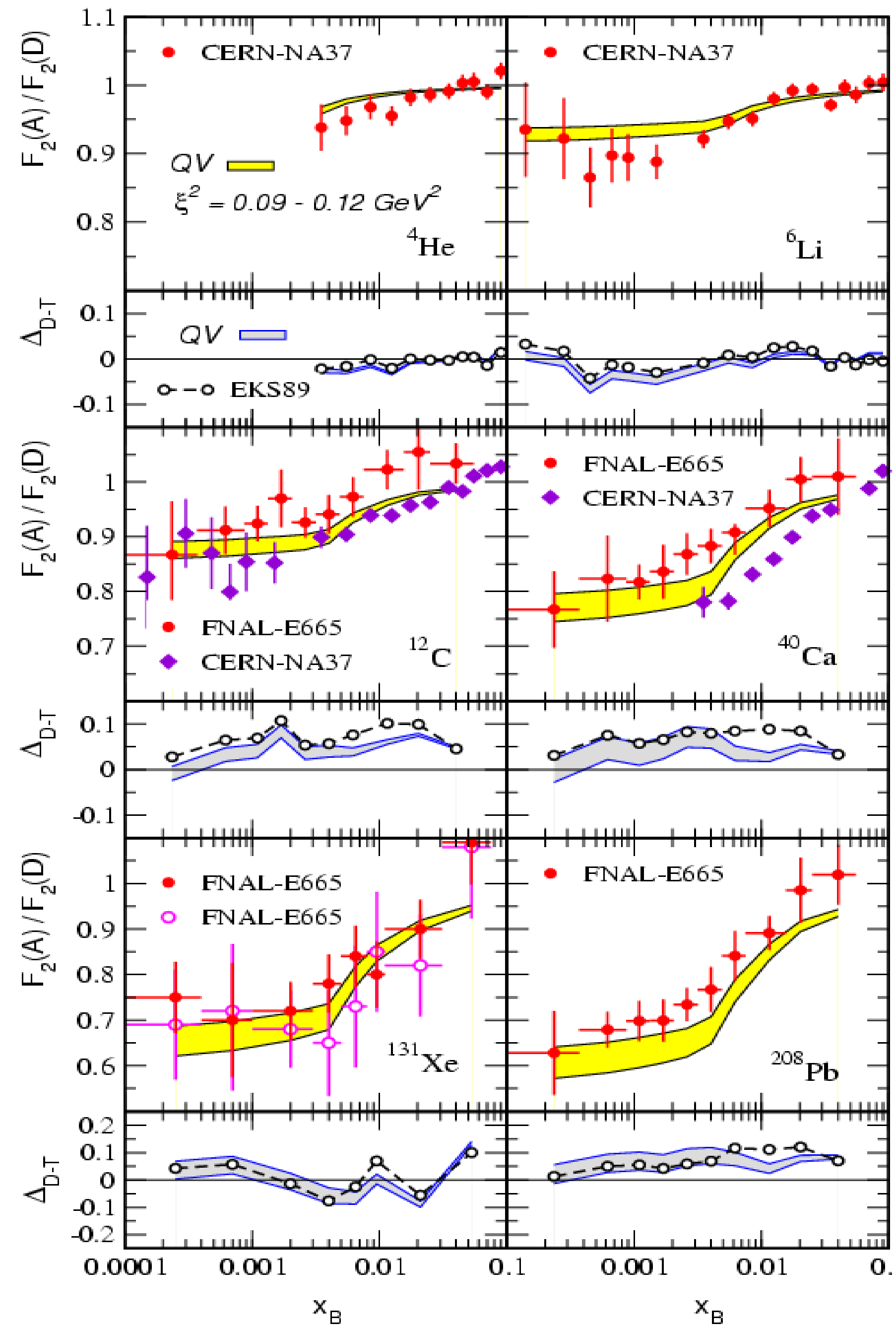


Qiu and Vitev, **hep-ph/0309094**

Scale for cold matter power corrections

$\xi^2 \sim$ 0.09 - 0.12 GeV^2





Power Corrections in Neutrino-Nucleus DIS

- **Coherent power corrections are process dependent:**

$$F_{1,3}^{(\nu W^+)}(x_B,Q^2) = \{2\}\, A \left(\sum_{D,U} |V_{DU}|^2\, \phi_D(x_B + x_{\xi^2} + x_{M_U}) \pm \sum_{\bar{U},\bar{D}} |V_{\bar{U}\bar{D}}|^2\, \phi_{\bar{U}}(x_B + x_{\xi^2} + x_{M_{\bar{D}}}) \right)$$

$$F_{1,3}^{(\bar{\nu} W^-)}(x_B,Q^2) = \{2\}\, A \left(\sum_{U,D} |V_{UD}|^2\, \phi_U(x_B + x_{\xi^2} + x_{M_D}) \pm \sum_{\bar{D},\bar{U}} |V_{\bar{D}\bar{U}}|^2\, \phi_{\bar{D}}(x_B + x_{\xi^2} + x_{M_{\bar{U}}}) \right)$$

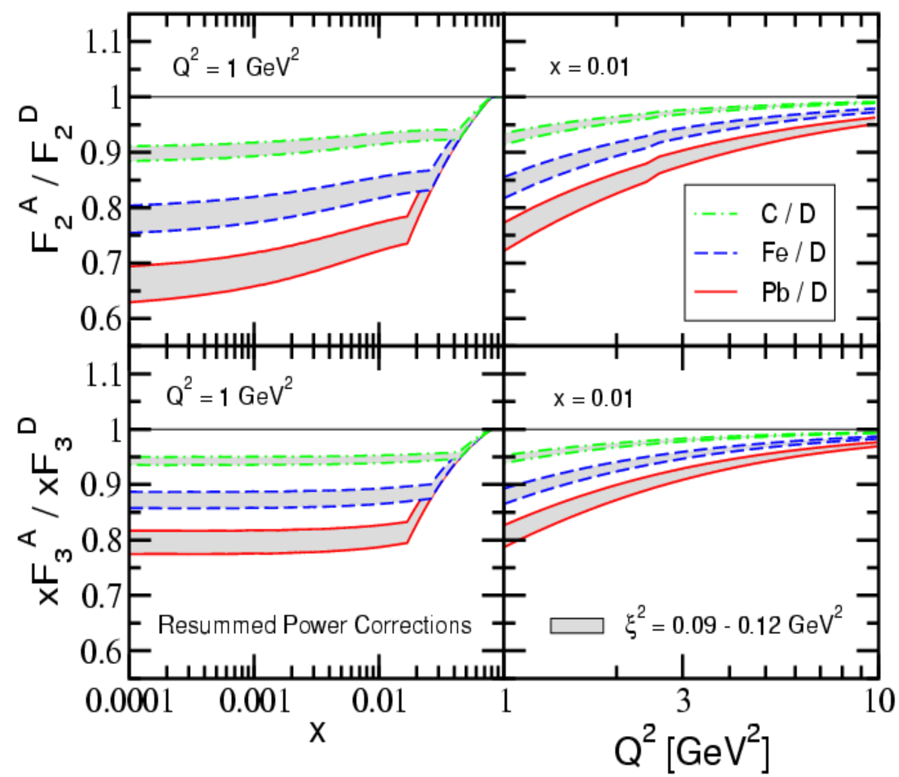
- **Power of factorization:**

Same nonperturbative ξ^2

➡ **Predict high twist shadowing In neutrino-nucleus structure Functions:**

$$\phi_{sea}(x) \propto x^{-\alpha_{sea}}, \quad \alpha_{sea} \simeq 1.0$$

$$\phi_{val.}(x) \propto x^{-\alpha_{val.}}, \quad \alpha_{val.} \simeq 0.5$$



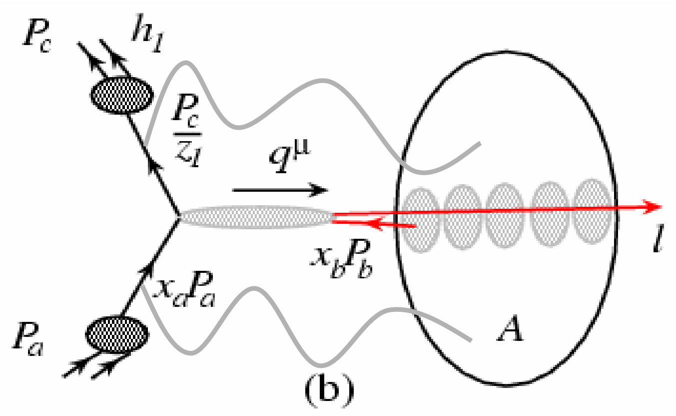
Qiu and Vitev, Phys.Lett.B 587 (2004)

Power Corrections in p+A Collisions

- Hadronic factorization fails for power corrections of the order of $1/Q^4$ and beyond

- Medium size enhanced dynamical power corrections in p+A could be factorized

⇨ **to make predictions for p+A collisions**



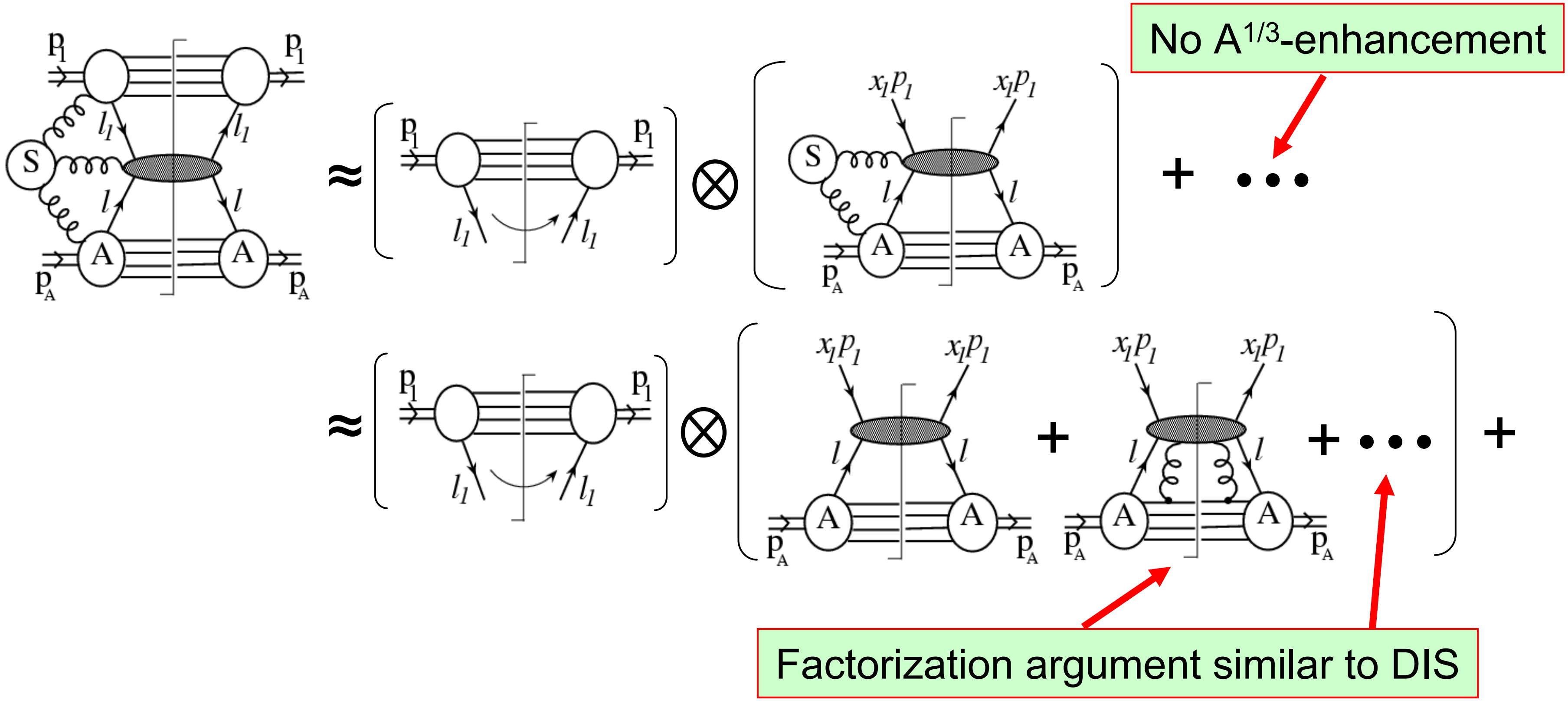
- Single hadron inclusive production:

Once we fix the incoming parton momentum from the beam and outgoing fragmentation parton, we uniquely fix the momentum exchange, q^μ, and the probe size

⬄ coherence along the direction of $q^\mu - p^\mu$

A-enhanced power corrections in pA

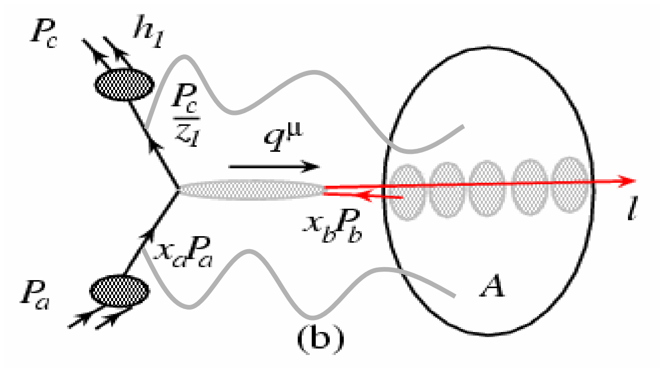
- A-enhanced power corrections, $A^{1/3}/Q^2$, are factorizable:



- But, power corrections are process-dependent, and they are different from DIS

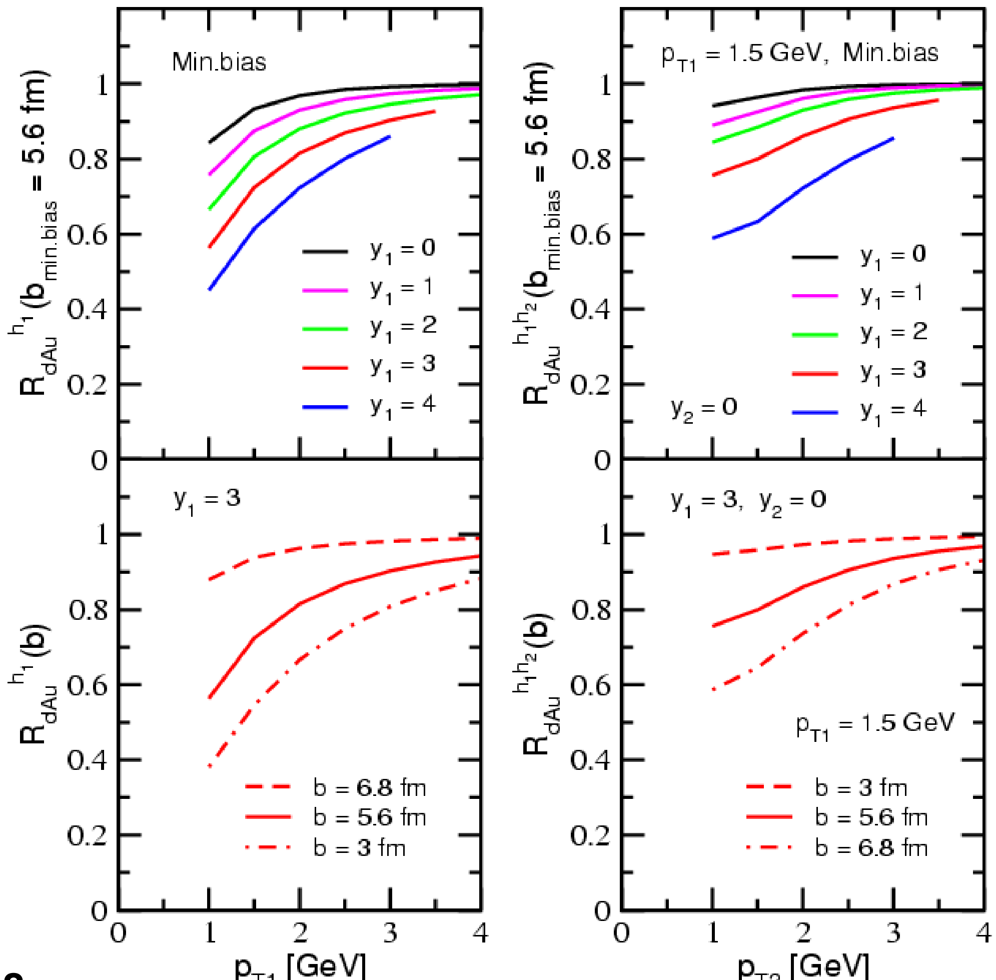
Single hadron inclusive production

- Resum the coherent final state multiple scattering of the parton of momentum ℓ with the remnants of the nucleus



- Other interactions are less coherent (elastic) and suppressed at forward rapidity by a large scale 1/u, or 1/s

more details see I. Vitev's talk in this users' meeting



Qiu and Vitev, hep-ph/0405068

Conclusions

- Although hard partonic collisions are localized in space-time, comparing to the rest size of a nucleon, the interaction area could be larger than a size of a Lorentz contracted nucleon
- **Low x partons can not only interact among themselves, but also interact coherently with the probe**
- **Coherent interactions lead to power corrections to DGLAP evolution equation and to physical cross sections:**
  - Power corrections to DGLAP ⇔ process independent corrections
  - Power corrections to physical cross sections ⇔ process dependent corrections
- **Interaction among low x partons produces a collective feature of whole nucleus**

Conclusions (II)

- Leading medium size enhanced power corrections are Infrared safe and can be systematically resummed into a translation operator acting on parton's momentum fraction, which leads to a shift in parton's momentum fraction without changing the leading twist factorized formula
  - The shift leads to a suppression in cross section – dynamical high twist shadowing
- Maximum characteristic scale of the dynamical power corrections: $\xi^2\,(A^{1/3}-1)\,C \sim 0.1\,(A^{1/3}-1)\,C$ GeV^2 with color factor C=1 (quark, antiquark), 9/4 (gluon)
- If there is saturation, it seems that corresponding hard probe is likely to interact coherently with more than one parton