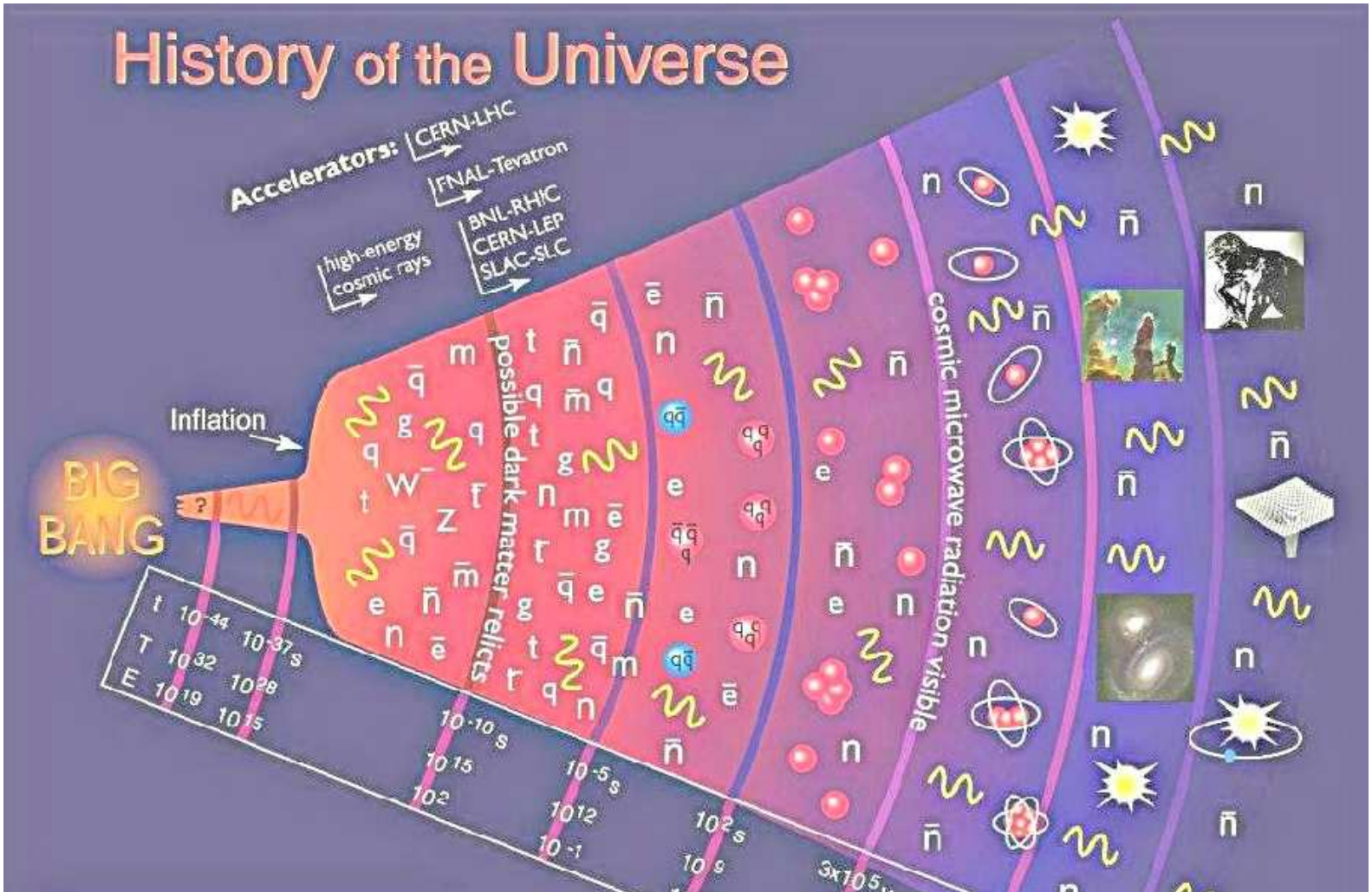
QCD EoS from LGT

- Introduction
  - QCD EoS and cosmology
  - QCD EoS and heavy ion collisions
- LGT results on the EoS
  - T_c, ϵ_c, EoS, v_s
  - comparison with resonance gas, quasi-particle gas, high-T pert. theory, HTL-resummation
- strongly coupled Coulomb phase or remnant of confinement?
  - heavy quark free energy and the running coupling

Dark matter and the QCD equation of state

- weakly interacting massive particles (WIMP) freeze out in the QGP at time $t_{freeze} \sim m_{WIMP}/25$ GeV $\sim (0.4 - 40)$GeV



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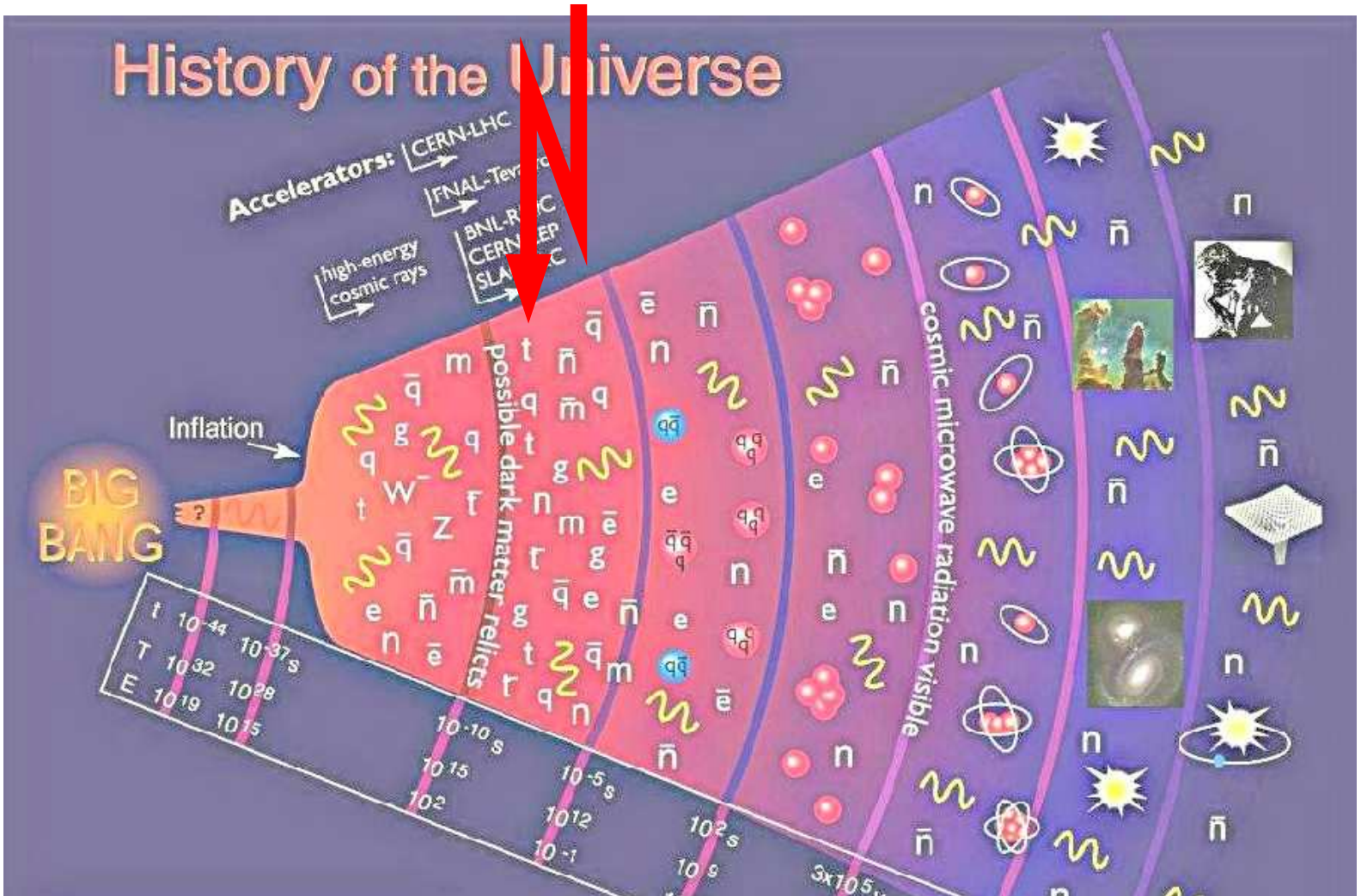


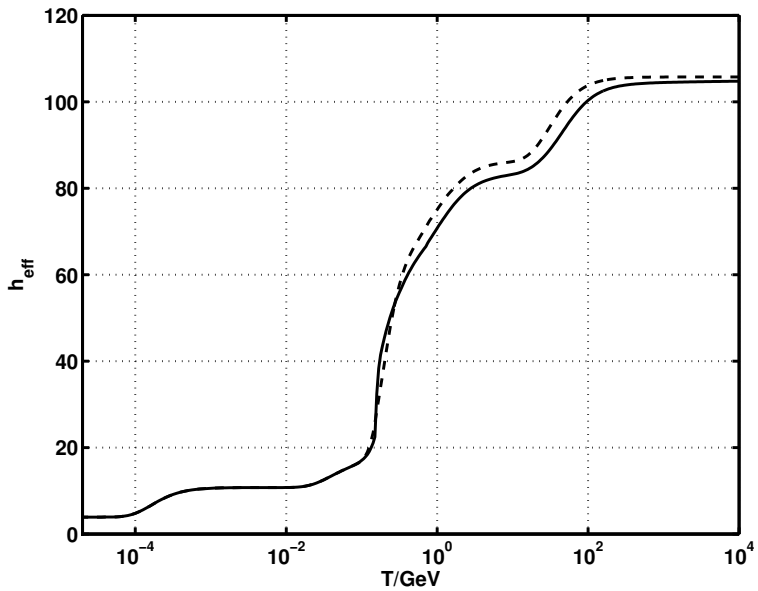
Dark matter and the QCD equation of state

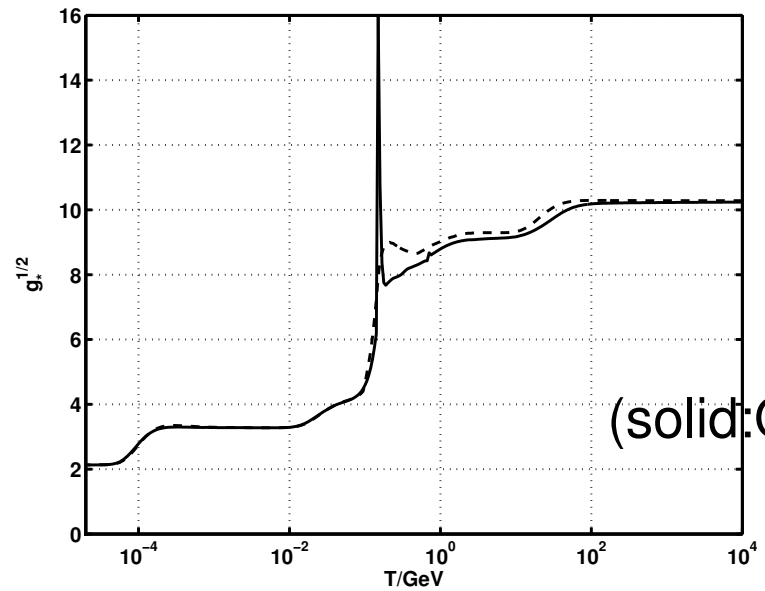
M.Hindmarsh and O. Philipsen, PR D71 (2005) 087302.

- abundancy of weakly interacting massive particles (WIMP) sensitive to the evolution of the early universe ⇔ sensitive to the QCD EoS

energy density: $\frac{\epsilon}{T^4} = \frac{\pi^2}{30} g_{eff}(T)$; entropy density: $\frac{s}{T^4} = \frac{2\pi^2}{45} h_{eff}(T)$

$$\frac{n_{WIMP}}{s} \sim g_*^{-1/2}(T) = \frac{g_{eff}^{1/2}}{h_{eff}} \left(1 + \frac{T}{3} \frac{d \ln h_{eff}}{dT}\right)^{-1}$$





(solid:QGP)

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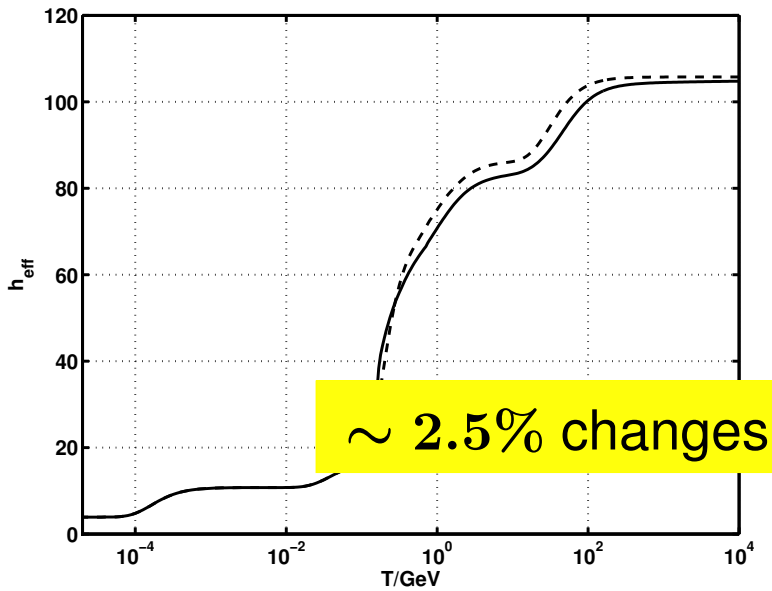
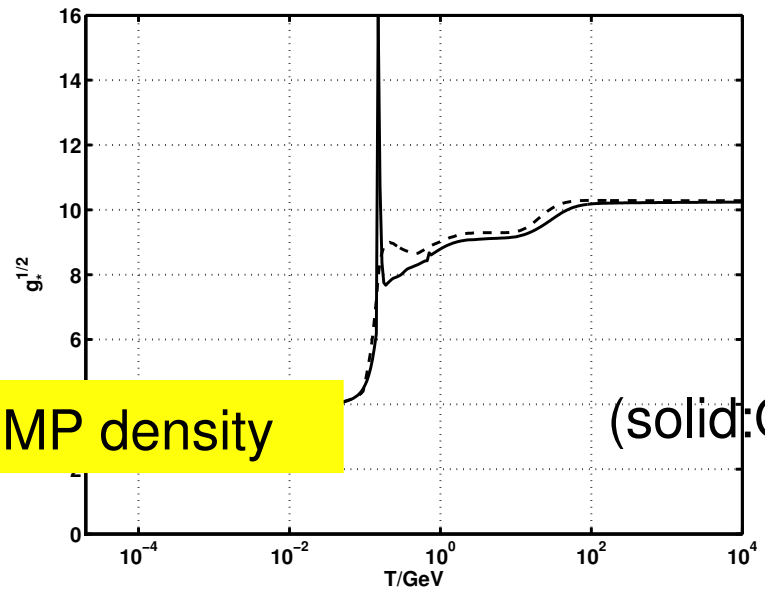




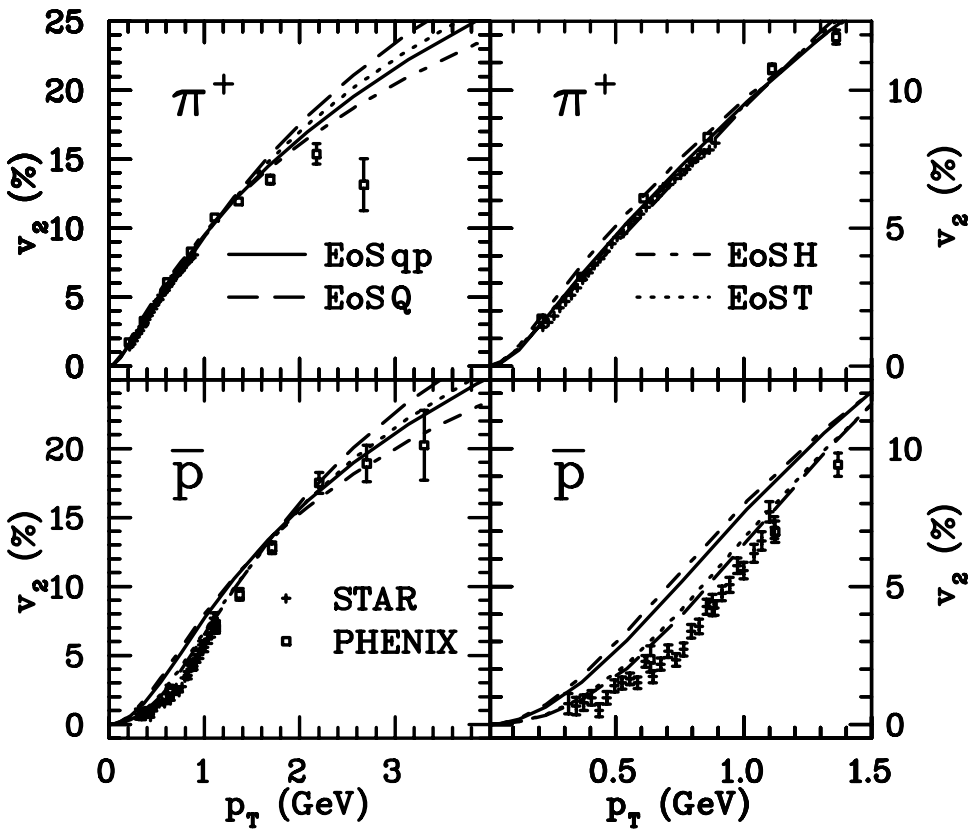
$\sim 2.5\%$ changes in WIMP density

(solid:QGP)

HIC-Hydro and the QCD equation of state

P. Huovinen, nucl-th/0505036

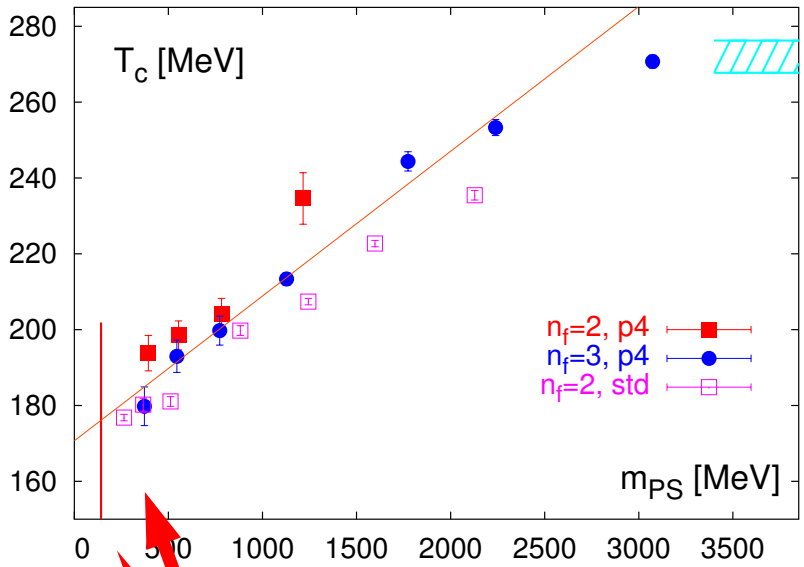
- hydrodynamic modeling of the elliptic flow is sensitive to details of the QCD EoS:
  steeper EoS (smaller velocity of sound) leads to smaller v_2 at low p_T



EoSqp: lattice EoS

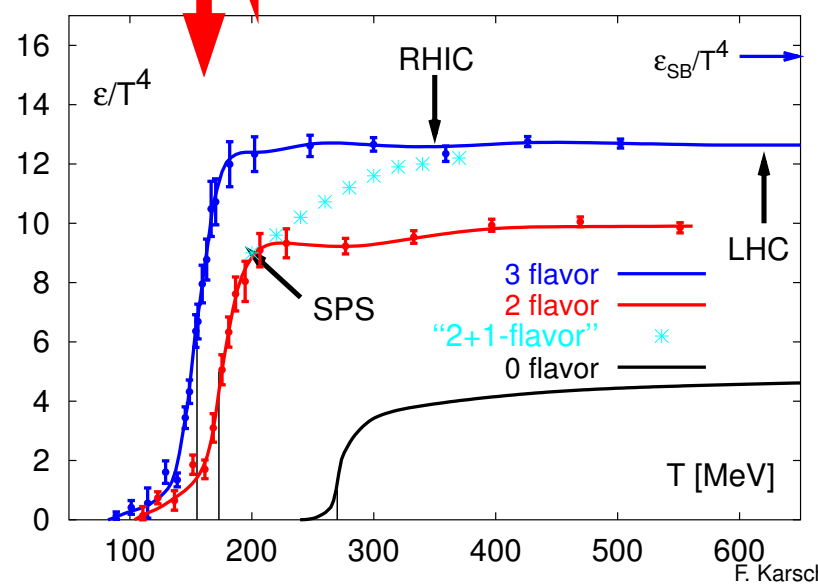
EoSQ: EoS with 1st order phase transition

LGT: Critical temperature, equation of state



$$\epsilon_c \simeq (6 \pm 2)\, T_c^4$$
$$\simeq (0.3 - 1.3)\, GeV/fm^3$$

energy density for 0, 2 and 3-flavor QCD



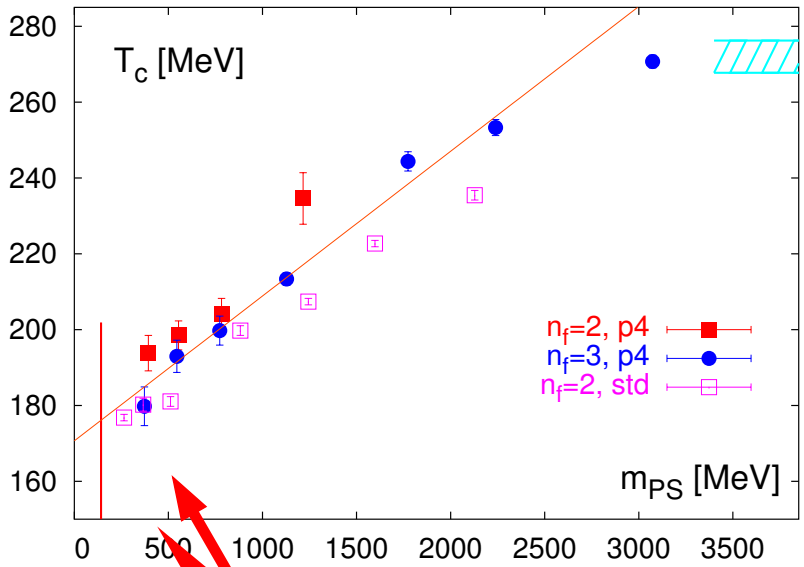
$$T_c = (173 \pm 8 \pm sys)\, MeV$$

FK, E. Laermann, A. Peikert,
Nucl. Phys. B605 (2001) 579

$$T_c = 167(13)\, [177(11)]\, MeV$$

MILC, hep-lat/0405029

LGT: Critical temperature, equation of state



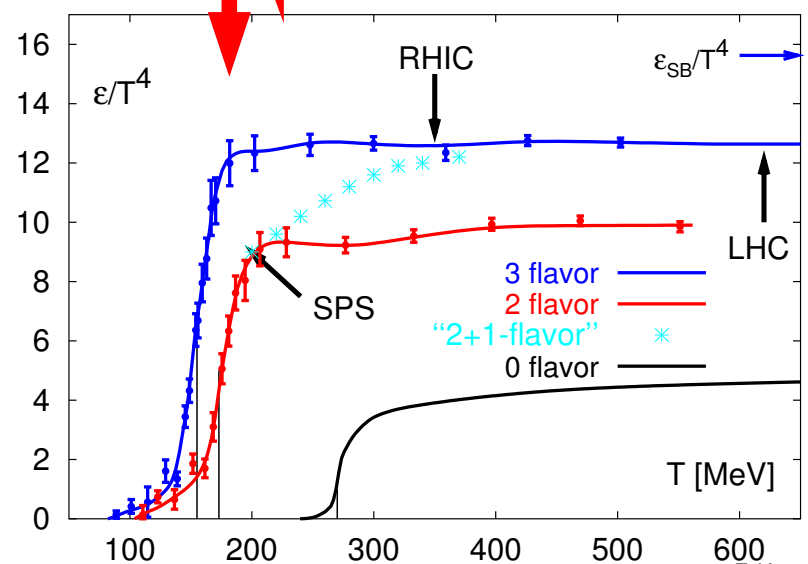
T_c

- $m_{PS} \gtrsim 300\ MeV$ (chiral limit??)
- $a \simeq 0.2\ fm$ (continuum limit??)
- improved staggered fermions,
  $\Rightarrow$ flavor symmetry breaking
  (need even better fermion actions)

ϵ_c

- $m_{PS} \simeq 770\ MeV$ (!!!)
- $V \simeq (4\ \text{fm})^3$ (thermodynamic limit)

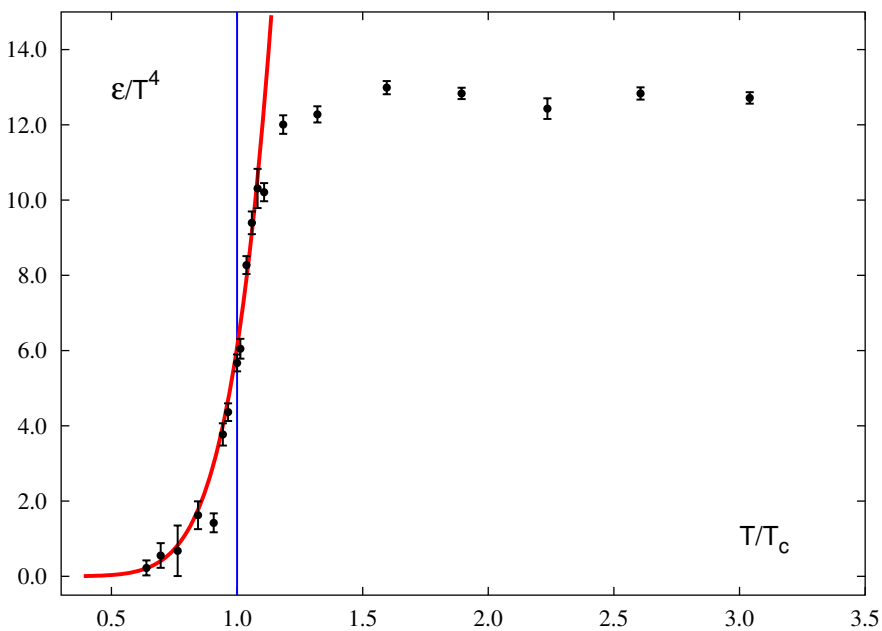
energy density for 0, 2 and 3-flavor QCD



Bulk thermodynamics

- lattice gauge theory provides basic results on the transition at $T \neq 0$ and properties of the high temperature phase
  - $T_c = (175 \pm 8 \pm sys)\ MeV$ (for $n_f = 2$)
  - $\epsilon_c = (6 \pm 2) T_c^4 \simeq (0.3 - 1.3)\ GeV/fm^3$

EoS: $T < T_c$ $\sim$ resonance gas



resonance gas (adjusted mass spectrum):

$\sim$ 1000 resonance d.o.f.

lattice calculation:

(2+1)-flavor QCD, $m_q/T = 0.4$, $\mu_B = 0$

continuous transition at T_c

FK, K. Redlich, A. Tawfik, hep-ph/0303108

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  - $T_c = (175 \pm 8 \pm sys)\ MeV$ (for $n_f = 2$)
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EoS: $T < T_c$ $\sim$ resonance gas

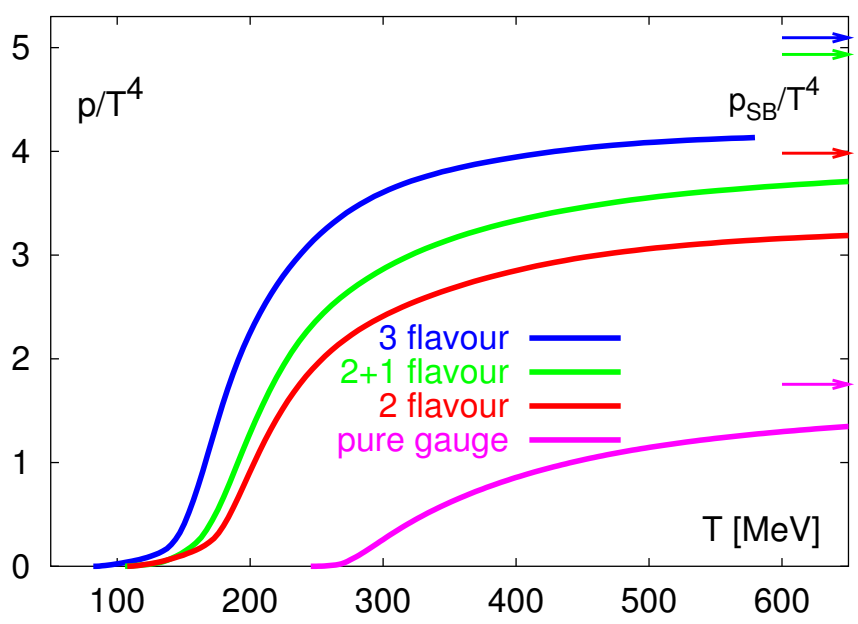
$T \gtrsim (3-4) T_c \sim$ "weakly" interacting quasi-particles

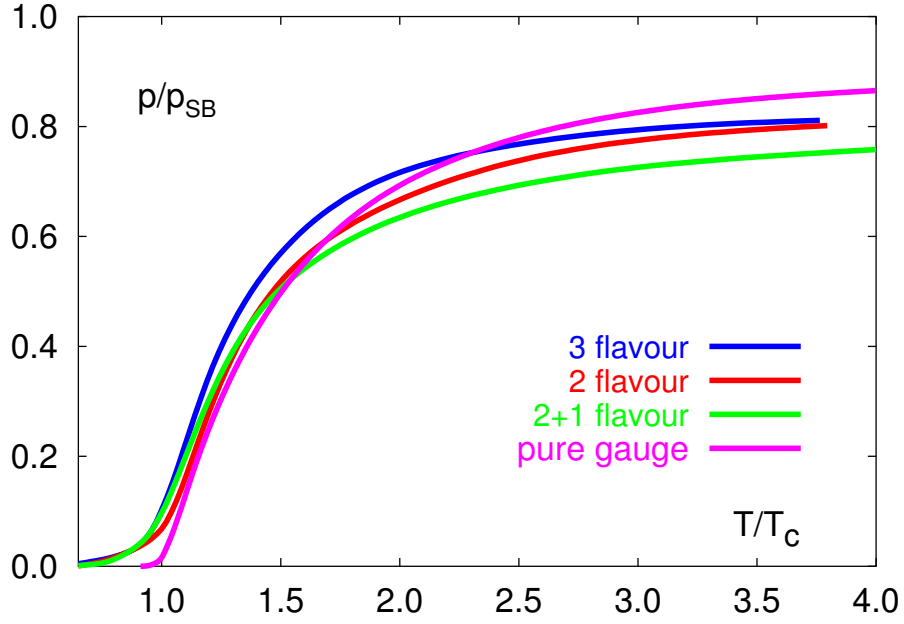
- for large T the EoS probes the large momentum sector of the plasma: $|\vec{q}| \sim 3T$

  $\Rightarrow$ quasi-particle features of quark and gluon propagators (thermal poles) are probed:

  $$\sigma(\omega, \vec{q}) \sim \delta(\omega^2 - q^2 - m_\infty^2)\ ,\ m_\infty \sim g(T) T$$

The pressure revisited



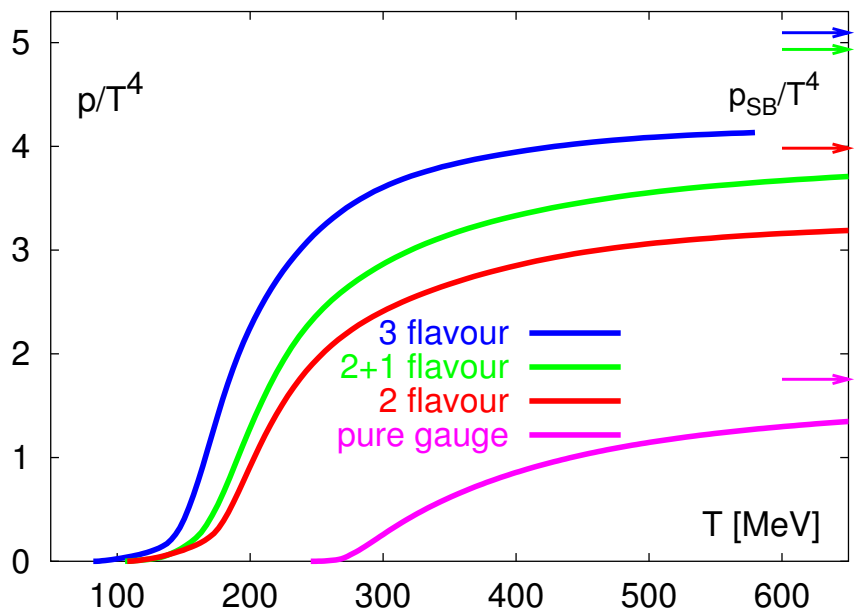


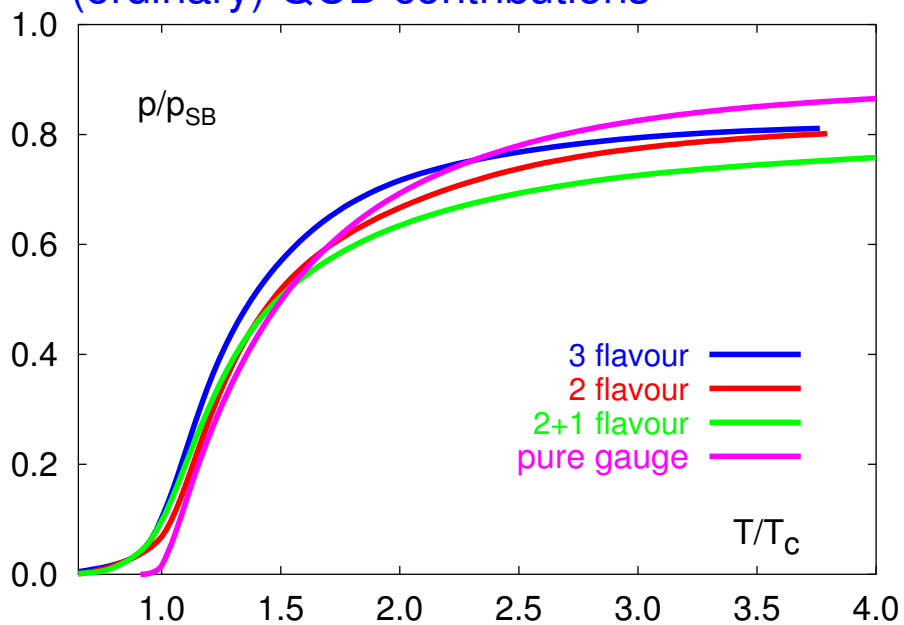
FK, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447.

status: 2000

The pressure revisited

$T \gtrsim (2\text{-}3) T_c$: deviations from ideal gas understood in terms of non-perturbative (ordinary) QCD contributions





FK, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447.

status: 2000

$T \lesssim 2T_c$: strong deviations from ideal gas

large screening masses,
remnants of confinement

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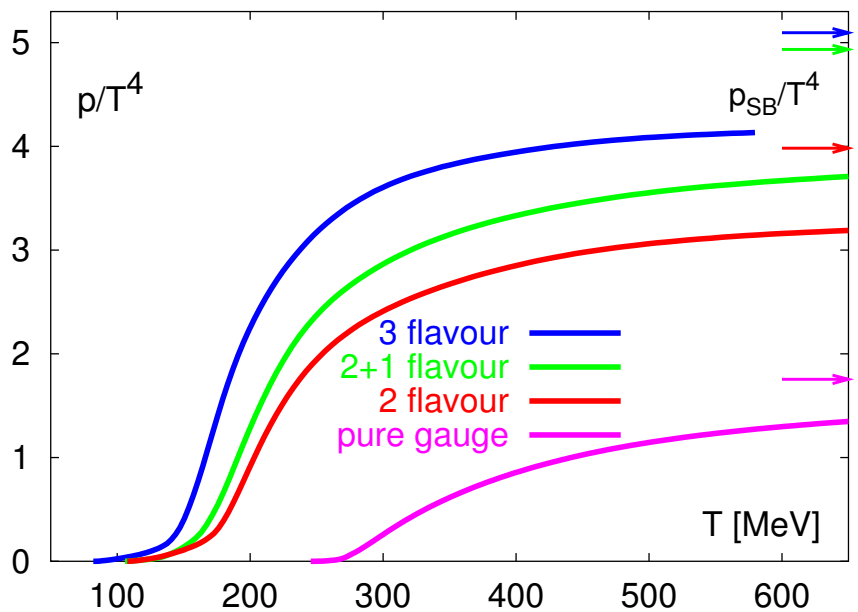

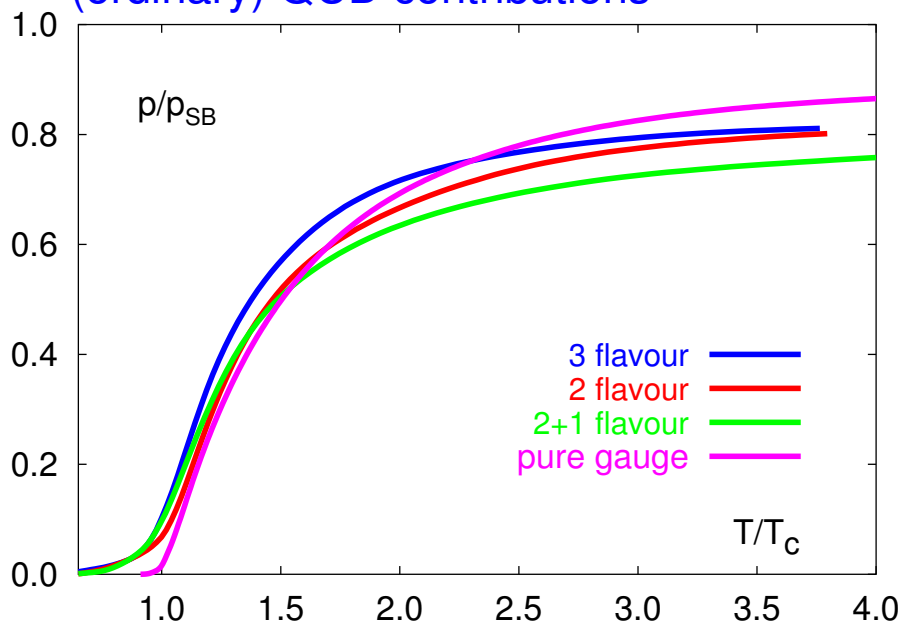

FK, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447.

status: 2000

deviations from p_{SB} almost flavor independent $\Rightarrow$ no need for colored $\bar{q}q$ bound states??

$T \lesssim 2T_c$: strong deviations from ideal gas

large screening masses, remnants of confinement

High temperature perturbation theory: complete result up to $\mathcal{O}(g^5)$

free energy density in the high temperature limit ($\alpha_s \equiv g^2/4\pi$):

$$\frac{f}{T^4} = -\frac{1}{V}\ln Z = -\frac{d_q \pi^2}{90}\left[1 + f_2\alpha_s + f_3\alpha_s^{3/2} + +f_4\alpha_s^2 + f_5\alpha_s^{5/2} + O(\alpha_s^3)\right]$$

P. Arnold and C. Zhai, Phys. Rev. D50 (1994) 7603; [SU(3)]

C. Zhai and B. Kastening, Phys. Rev. D52 (1995) 7232; [QCD, $\mu = 0$]

A. Vuorinen, hep-ph/0305183; [QCD, $\mu > 0$]

High temperature perturbation theory: complete result up to $\mathcal{O}(g^5)$

free energy density in the high temperature limit:

resummed self-energy;

electric screening; $m_e(T) = \sqrt{\frac{N_c}{3} + \frac{n_f}{6}}\, g(T)\, T$

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loops at all orders contribute

non-perturbative magnetic mass

$m_m(T) \sim \mathcal{O}(g^2\, T)$,

(A.D. Linde, 1980)

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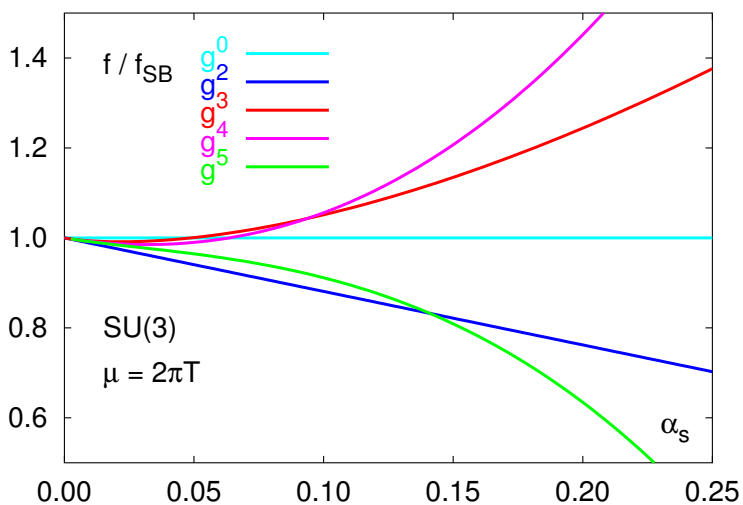


loops at all orders contribute

non-perturbative magnetic mass

$m_m(T) \sim \mathcal{O}(g^2\, T)$,

(A.D. Linde, 1980)

pure convergence properties; $g(T) < 1$ needed;

thermodynamics non-perturbative even at

$T >> \Lambda_{QCD} \simeq 200$ MeV

HTL quark propagator

E. Braaten, R.D. Pisarski, T.C. Yuan, PRL 64 (1990) 2242

- ϕ^4: screened perturbation theory shows better convergence

  FK, A. Patkos, P. Petreczky, PL B401 (1997) 69

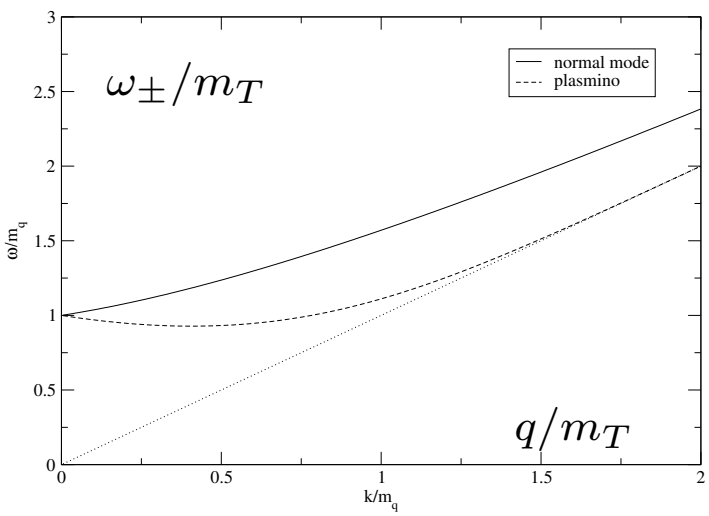
- HTL-resummed perturbation theory for pressure takes into account thermal masses of quark and gluon propagators

$$\rho_{\rm HTL}(\omega,\vec{q}) = \frac{1}{2}\rho_+(\omega,q)(\gamma_0 - i\,\hat{q}\cdot\vec{\gamma}) + \frac{1}{2}\rho_-(\omega,q)(\gamma_0 + i\,\hat{q}\cdot\vec{\gamma})$$

with

$$\rho_\pm(\omega,q) \;=\; \frac{\omega^2-q^2}{2m_T^2}[\delta(\omega-\omega_\pm)+\delta(\omega+\omega_\mp)] + \beta_\pm(\omega,q)\Theta(q^2-\omega^2)$$

pole cut



$$m_T \sim g(T)T$$

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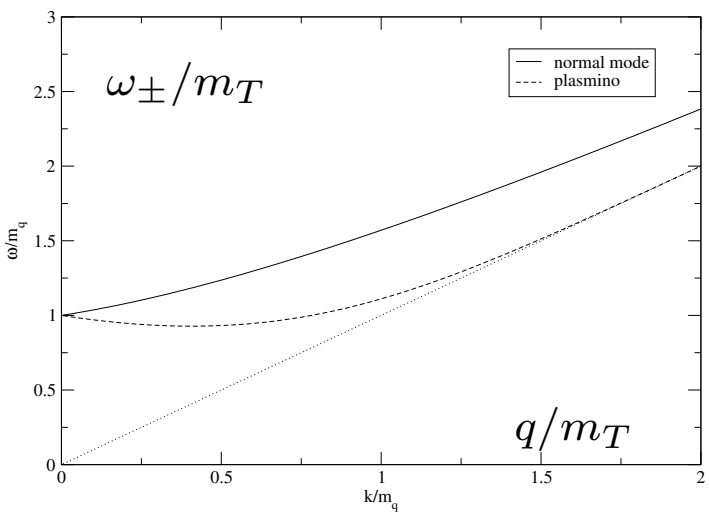
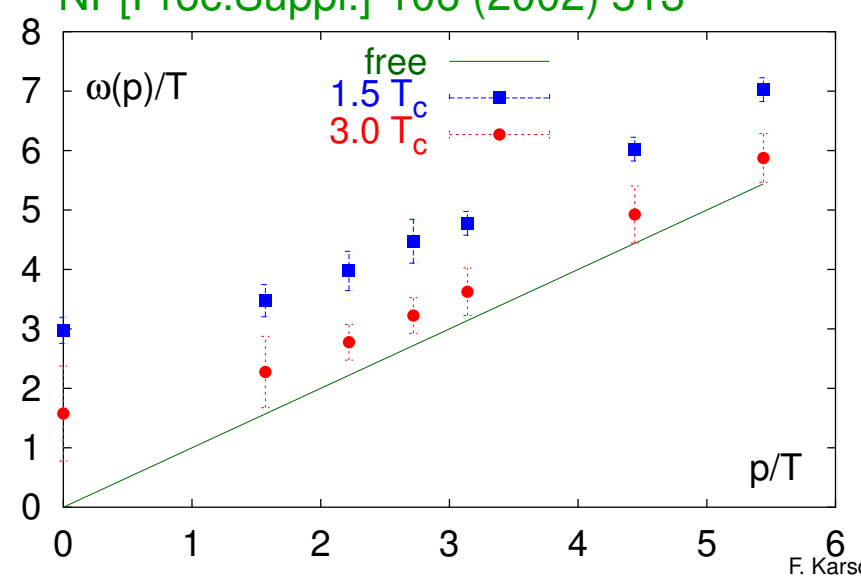


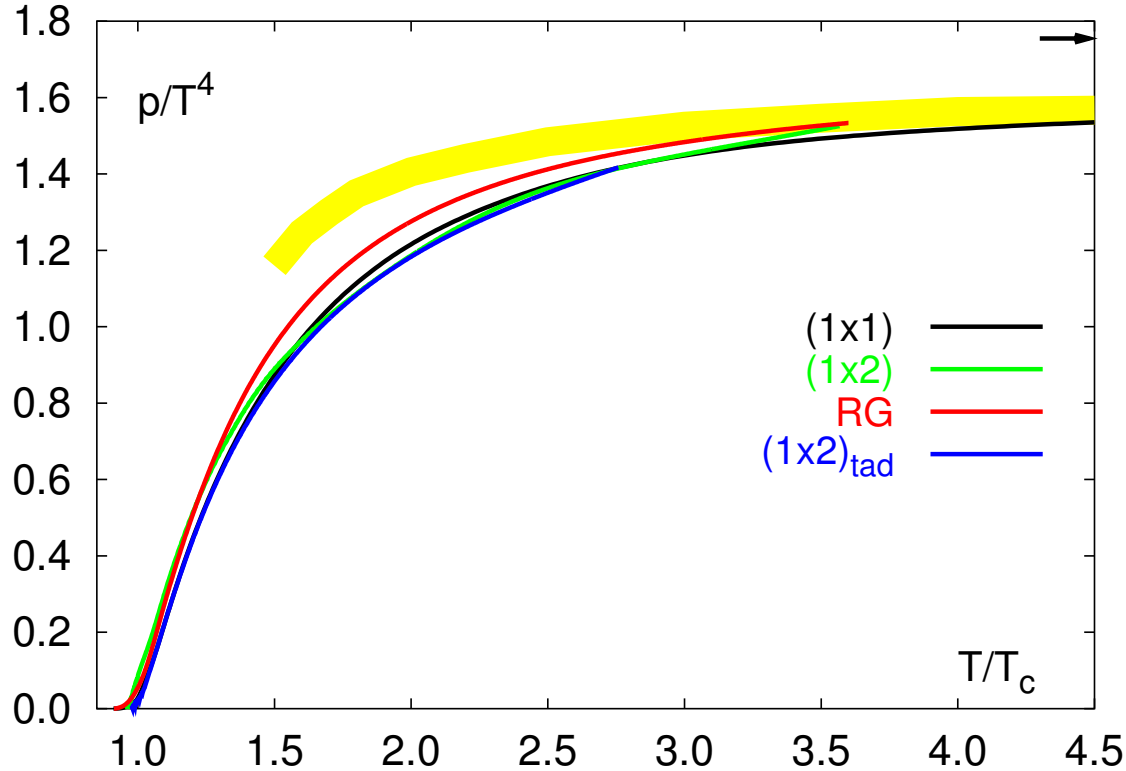
lattice quark dispersion relation:

P. Petreczky et al., NP[Proc.Suppl.] 106 (2002) 513



SU(3) Equation of State pressure: LGT vs. HTL

high T part of the pressure calculated on the lattice is in good agreement with HTL-resummed perturbation theory for $T \gtrsim 3T_c$



HTL: J.P. Blaizot, E. Iancu, A. Rebhan, PL B470 (99) 181

supports quasi-particle picture/models: A. Peshier et al, PRD54 (96) 2399
P. Lévai and U. Heinz, PR C57 (98); R.A. Schneider and W. Weise, PR C64 (01)

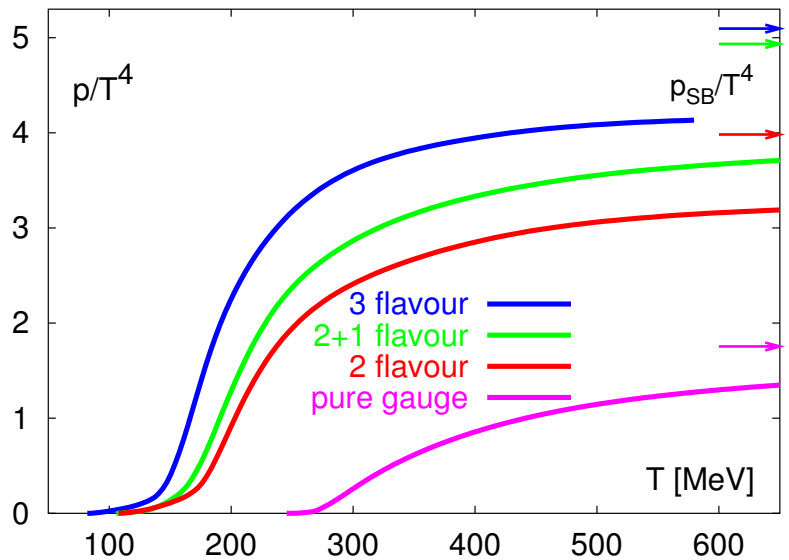
The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

$\mu_q = 0, \quad 16^3 \times 4$ lattice

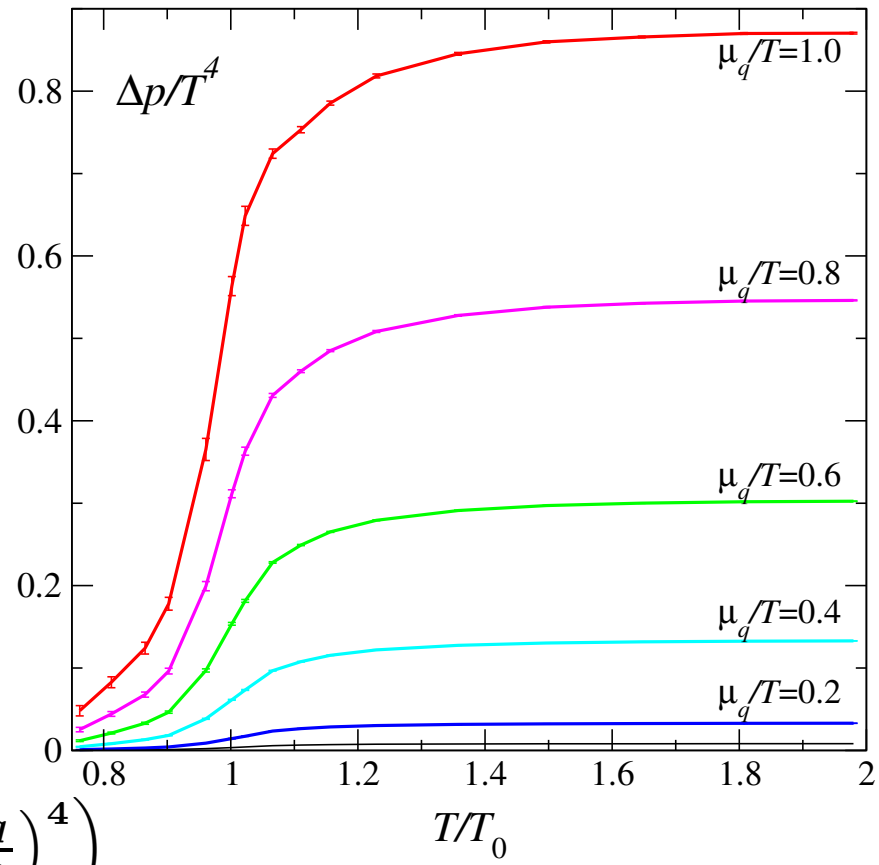
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$



contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$



high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$

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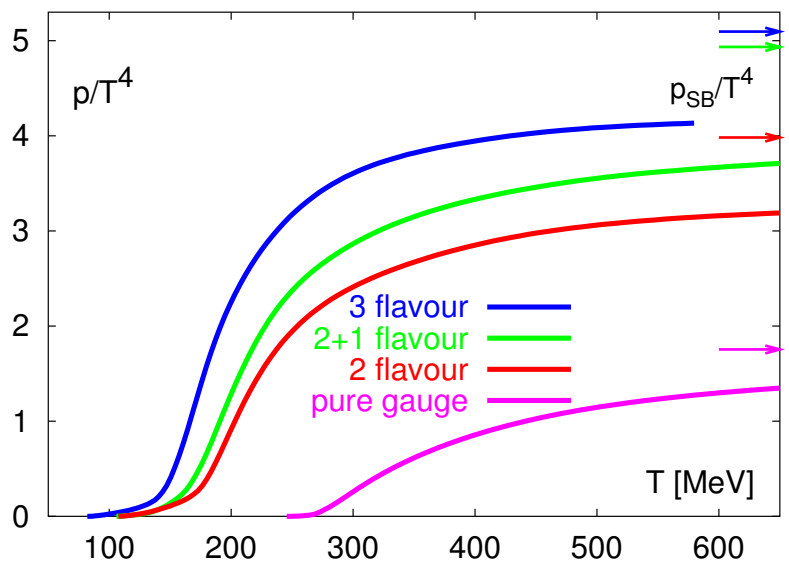
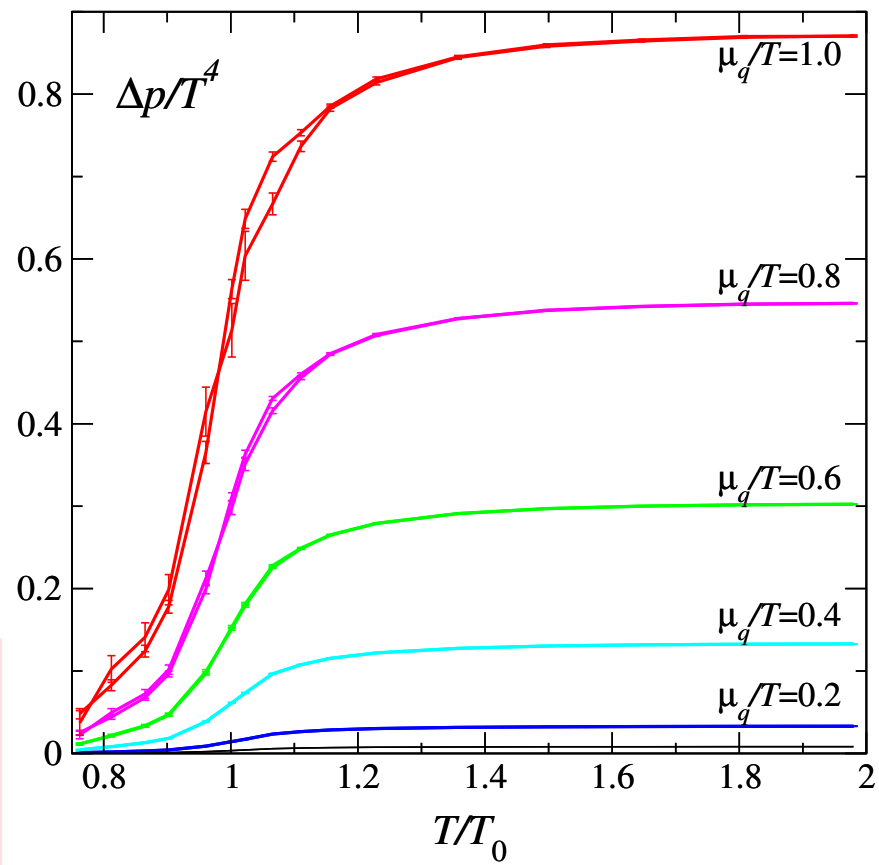


PRD71 (2005) 054508

contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$

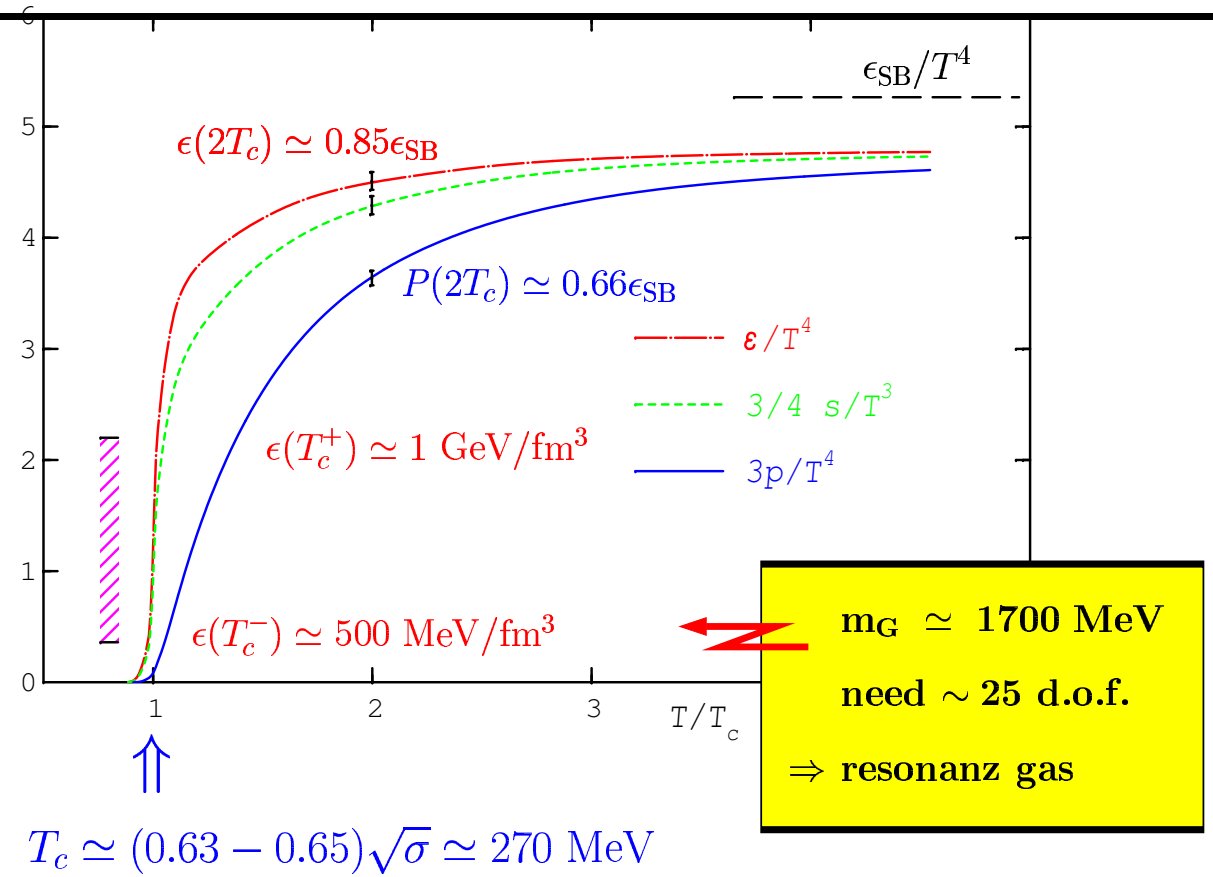


pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

RHIC: $\mu_q/T \lesssim 0.1$

SU(3) Equation of State

status: 2000

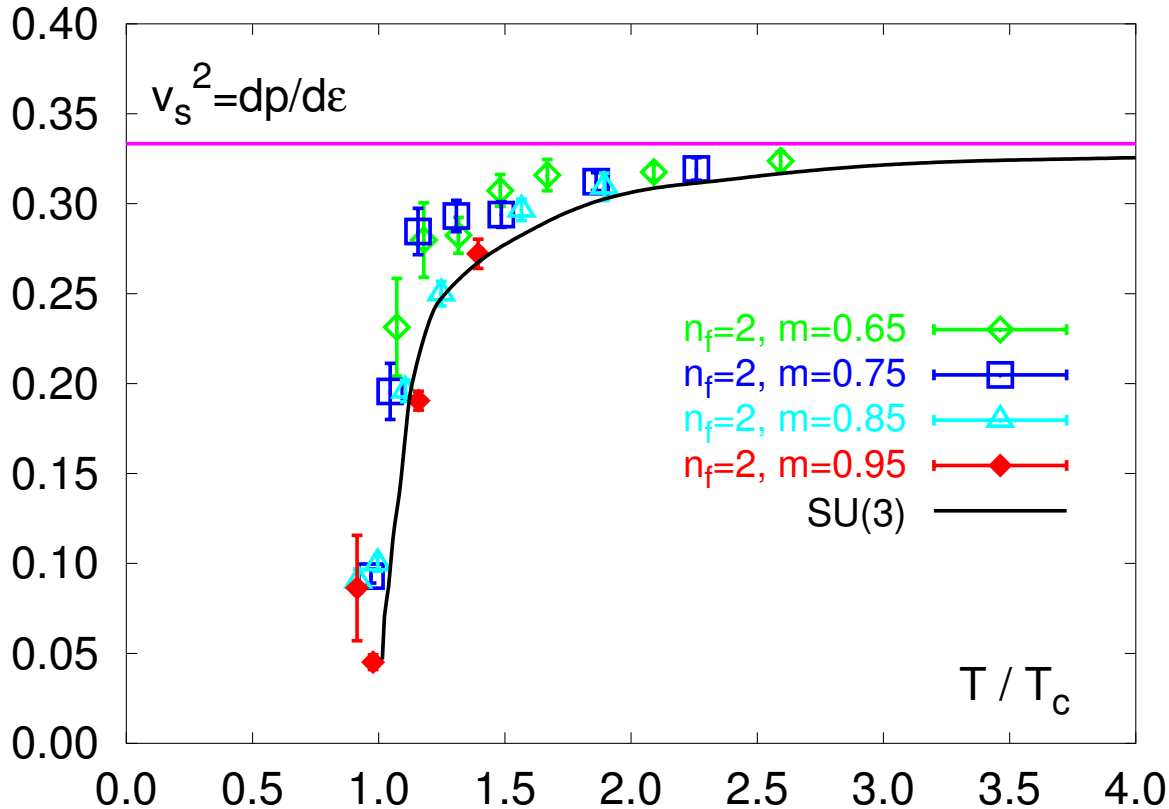


$T_c \simeq (0.63 - 0.65)\sqrt{\sigma} \simeq 270$ MeV

- substantial deviations from ideal gas even for $T \gtrsim 2T_c$

- continuum approaches (HTL-resummed perturbation theory) have difficulties in reproducing the EoS for $T \lesssim 2T_c$

Velocity of sound (pure gauge, $n_f = 2$)



$n_f = 2$:

A. Ali Khan et al.,

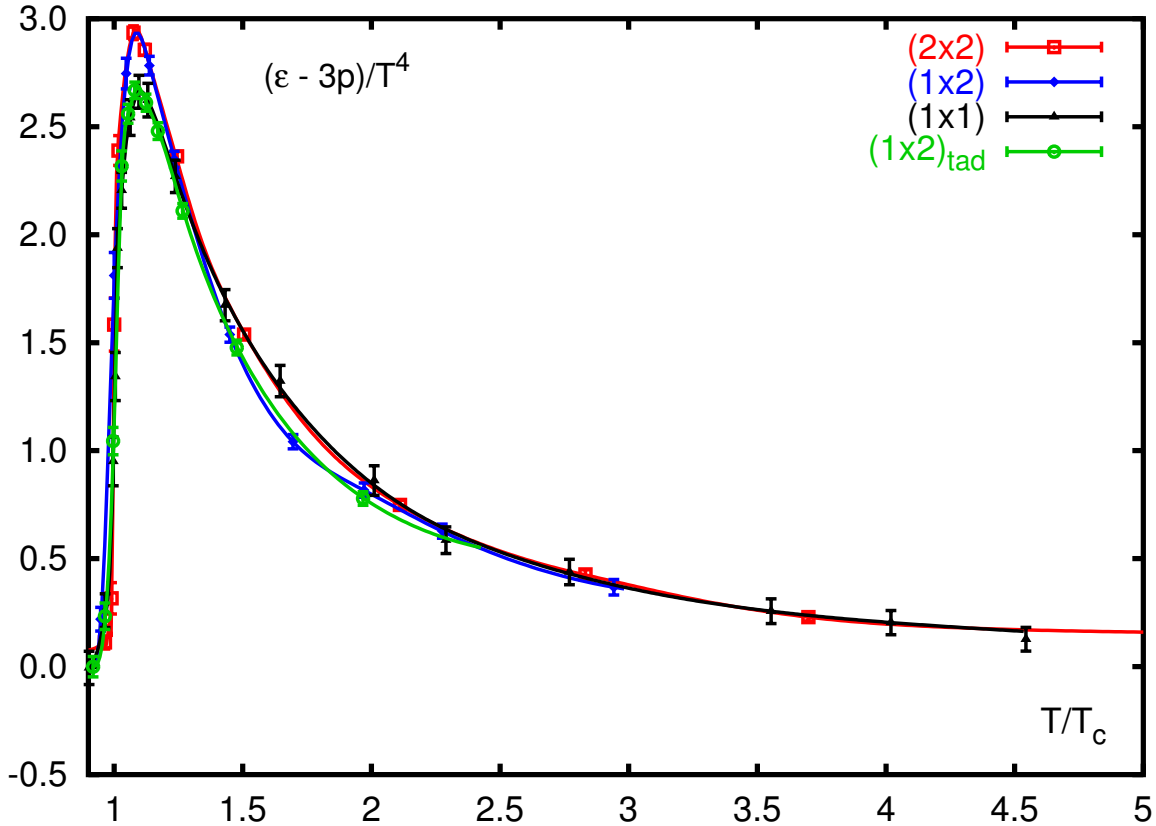
PR D64 2001

pure gauge theory:

G. Boyd et al., NPB 1996

Deviation from ideal gas (pure gauge)
– suggests strong interactions –

evidence for significant deviations from ideal gas behavior;
at least up to $T \simeq (2-3)\ T_c$; i.e. in the regime accessible to RHIC

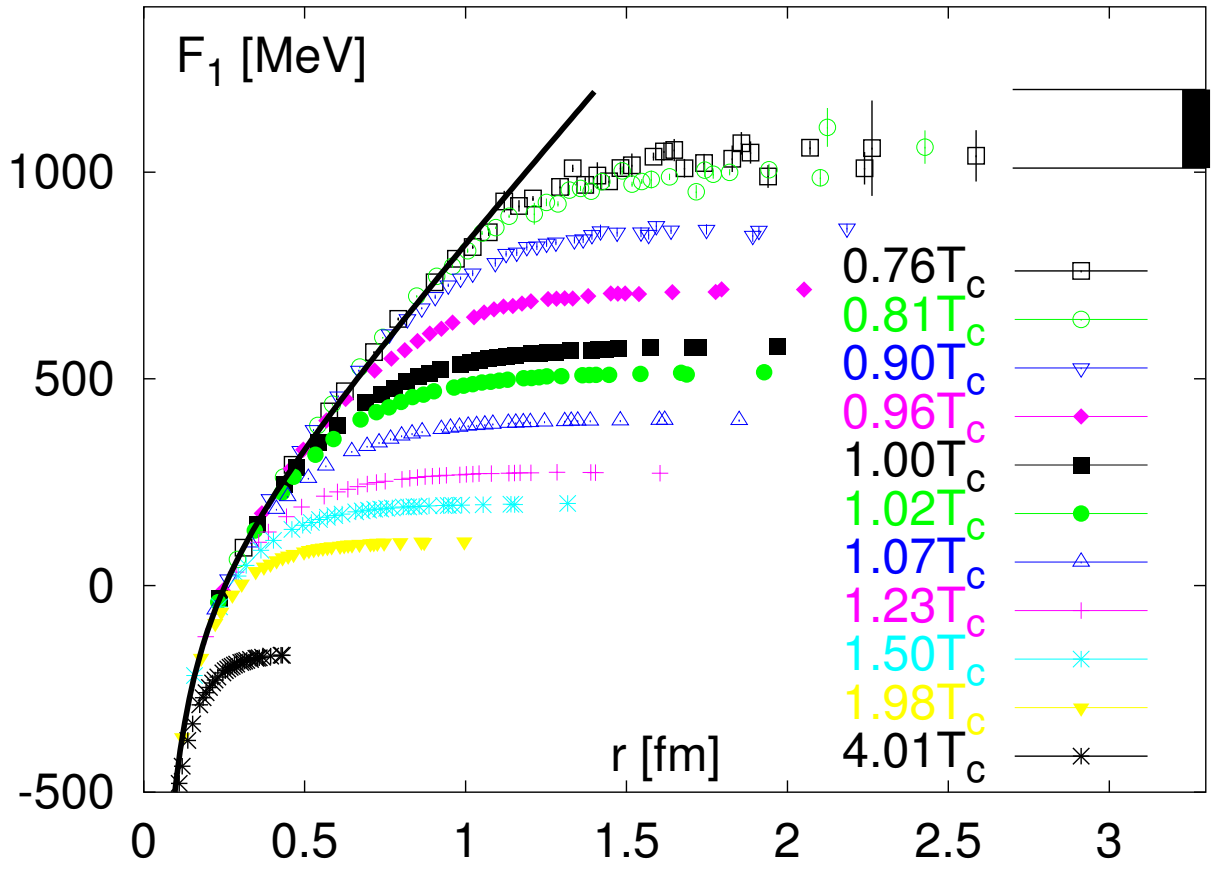


colored bound states above T_c ??

Singlet free energy
– remnant of confinement vs. sQCD –

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

- singlet free energy



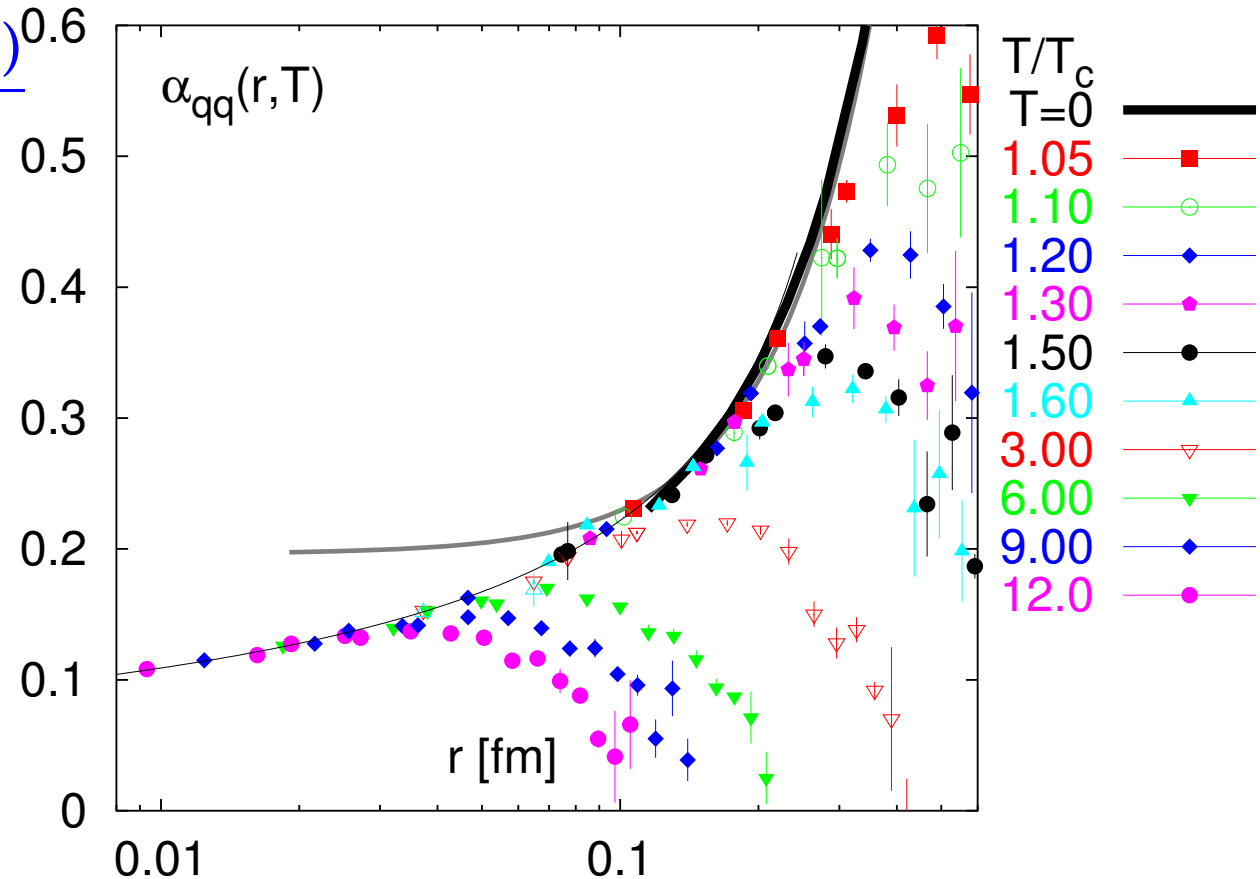
- $F_1(r,T)$ follows linear rise of $V_{\bar{q}q}(r,T=0) = -\frac{4\alpha(r,T=0)}{3r} + \sigma r$ for $T \lesssim 1.5T_c$, $r \lesssim 0.3$ fm

Singlet free energy and asymptotic freedom

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

- singlet free energy defines a running coupling:

$$\alpha_{\rm eff} = \frac{3r^2}{4}\,\frac{{\rm d}F_1(r,T)}{{\rm d}r}$$



Singlet free energy and asymptotic freedom

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

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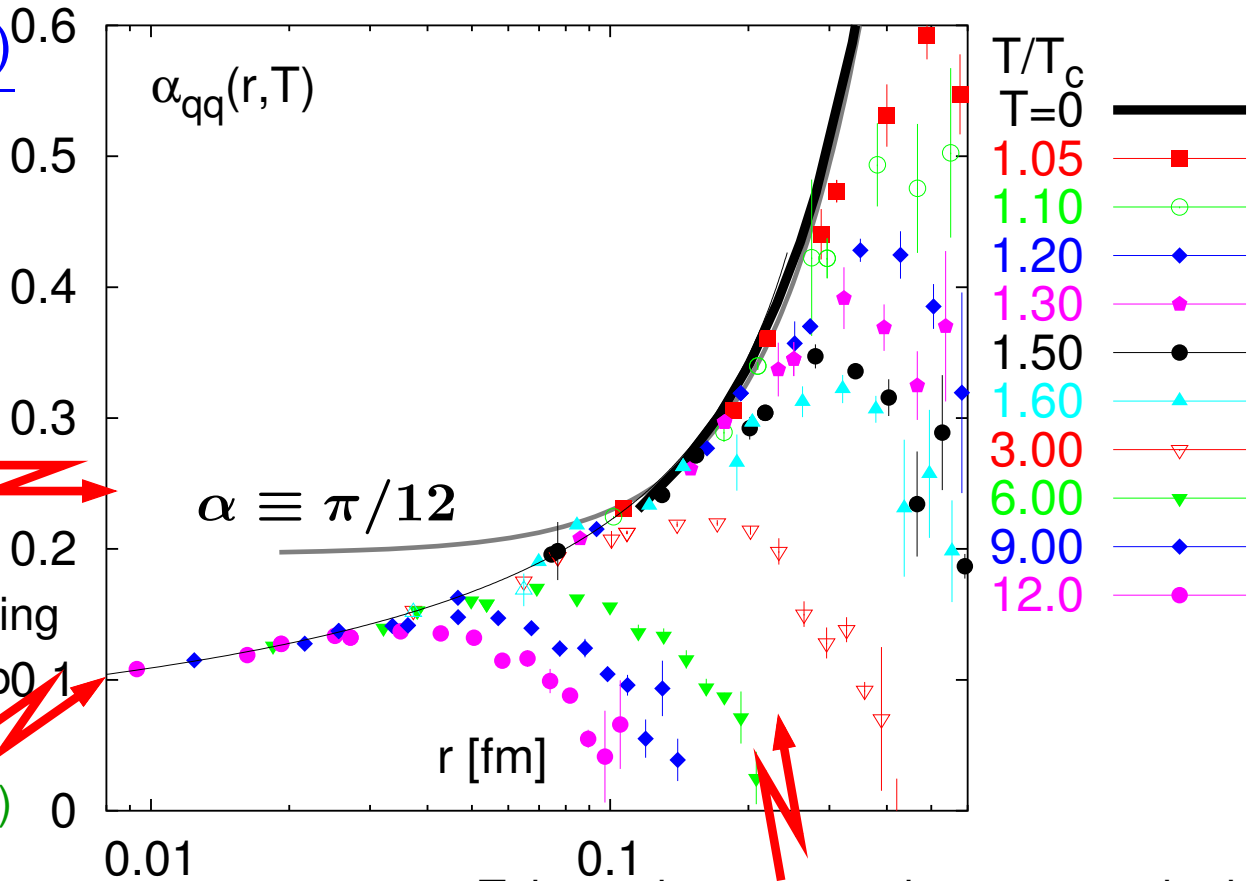


large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop (S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

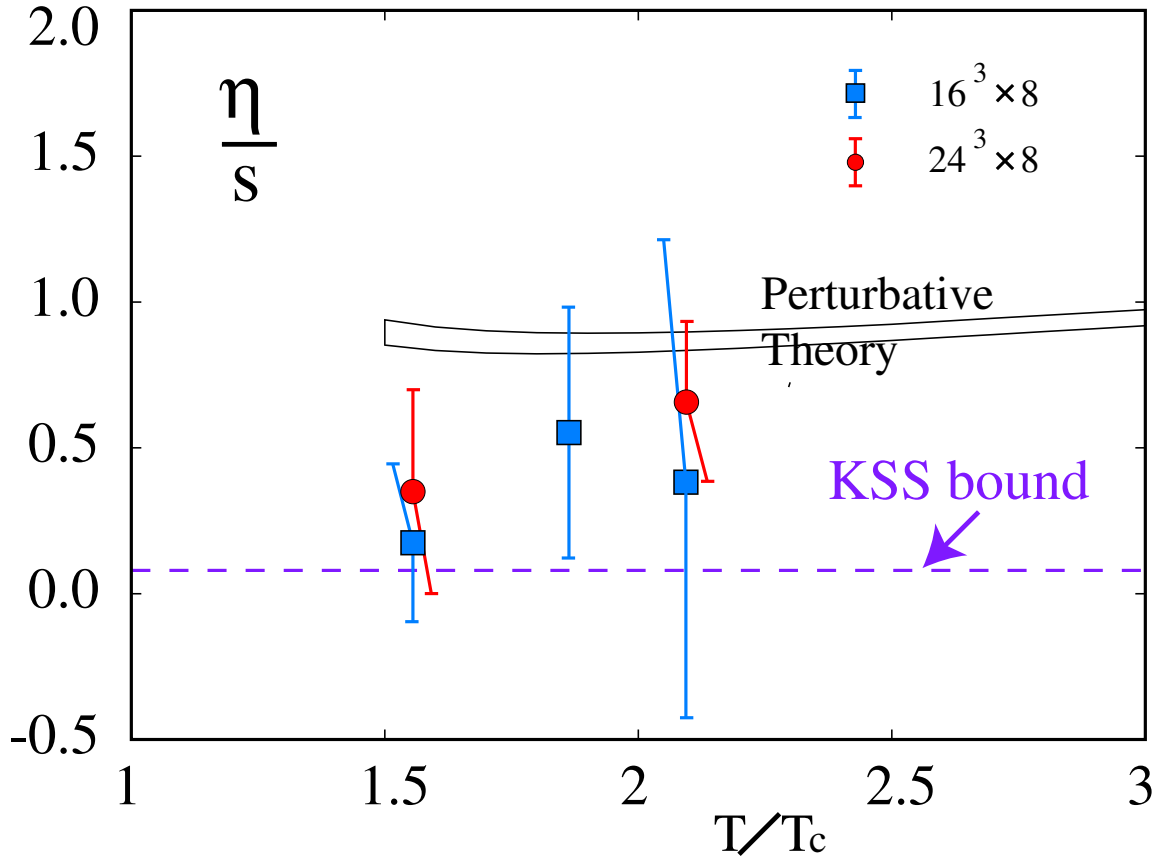
T-dependence starts in non-perturbative regime for $T \lesssim 3\ T_c$

- short distance physics $\Leftrightarrow$ vacuum physics

Conclusions

- lattice calculations (at $\mu = 0$) produced a lot of evidence for substantial deviations from ideal gas behavior for $T \lesssim (2-3)T_c$, *i.e.* in the temperature regime accessible to RHIC
- HTL-resummed perturbation theory and quasi-particle models descripe lattice results above $\sim 3\ T_c$ quite well
- heavy quark free energies and the running coupling change smoothly across T_c
- evidence for remnants of confinement
- no evidence for an unusually large Coulombic coupling at short distances

Viscosity (pure gauge, exploratory)



A. Nakamura and S. Sakai, PRL 94 (2005) 072305